# An investigation on the Meta Portfolio Method

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***Abstract*-** This report examines the performance of my implementation of the Meta Portfolio Method (MPM). The effects of some important machine learning (ML) system choices are examined, and the best choices are used to generate performance results. Afterwards, we see if the correlation of the universe’ assets affect performance.

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6. *MPM Method*

The MPM suggests that a ML model can learn the optimal environments of the asset universe for each NRP and HRP strategies. Thereafter, the model chooses between NRP and HRP periodically, rebalancing the portfolio. A regression ML model is used. Features used to learn contain some measurements of the asset universe’ returns and the recent performance of each NRP and HRP. The label is the Sharpe ratio difference between HRP and NRP. This number gives information on whether HRP or NRP performed better.

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*Figure 1: Original MPM Timeline*

As we rebalance monthly, a model is trained monthly and a set of features is fed into it to predict the Sharpe difference. This will decide which strategy to choose. Tm represents the timeframe considered for feature-gathering. Th is the timeframe considered for calculating the Sharpe ratio. In this version, Tm=Th and is 21 trading days, one whole month. Data was obtained from 2003 November until 2022 November.

This work uses Equal Risk Contribution instead of Naïve Risk Parity.

1. *Daily Weights*

Some modifications were made to the original MPM to increase the size of the training data set. Originally, as in Figure 1, rebalancing occurs monthly and a data pair(feature-label pair) was only constructed once per month. In this version, rebalancing still occurs monthly but data pairs are constructed daily, hence this required that portfolio weights are constructed daily.

First, based on the previous year (Tr), a returns dataframe was generated containing the returns for all assets. A portfolio weight and correlation matrix was generated specific to this one-year timeframe. Then, this was done daily, so that daily weights and daily correlation matrixes were generated.

1. *Training Data*

Data pairs(feature-label) are generated daily. For every day, features includes statistics about the asset universe (Tm timeframe), results on each strategy’s recent performance, and some characteristics of the correlation matrix. The correlation matrix contains the correlations pairwise between all assets in the universe. The label is the Sharpe difference Th later.

1. *Model Training*

The training period represents the number of days we consider for each model, hence the timeframe containing the patterns for the model to learn. Since each day is one data point, this number also directly affects the size of the dataset. Hence, at each rebalancing date monthly the model has (Training period) number of data points and also looks back in time for (Training period) number of days. The most recent set of features is fed into the trained model and the predicted value of Sharpe ratio is used to decide between ERC and HRP.

1. *Differences with original MPM paper*

In this work, Equal Risk Contribution was used instead of Naïve Risk Parity. In ERC, equal marginal risk contribution was set for each asset and weights were decided, while in NRP weights were fixed by the formula. This was because the weights of NRP and HRP were almost identical, and much better performance was observed when ERC was used with HRP. [2] states ERC is closely related to NRP, except that it is more robust when assets have very different correlations.

Instead of monthly data points, daily data points were gathered. However, the portfolio was still rebalanced only monthly.

1. ML system design

In building the ML model, a few key choices significantly affected performance. A few tests were done on Universes 1 to 6 to see the effects of these choices and select the best combination for the final model. For a single metric to see the effects, we will use Performance Gain, measured as the percentage increase in Sharpe Ratio(daily data) of MPM over the average of ERC and HRP.

1. *Effect of Training Period*

Note that each year corresponds to 252 days and 252 data points worth. All training was done with no hyperparameter optimization and a 1 year covariance period.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Performance Gain | Universe 1 | Universe 2 | Universe 3 | Universe 4 | Universe 5 | Universe 6 |
| 1 year | +56% | +11% | +120% | +102% | +47% | 89% |
| 2 year | +52% | +11.3% | +144% | +118% | +41% | 125% |
| 4 year | +51% | +8% | +73% | +141% | +39% | 117% |
| 8 year | +42% | +9% | +151% | +122% | +40% | 183% |
| 12 year | +61% | +9.9% | +450% | +277% | +35% | 257% |

With one year, the feature importance plot suggests that the ML model might not be learning. Higher importance features had greater variance in their importances, where the lower bound will almost reach 0 for all features as in Figure 2. This suggests the ML model has low confidence that any of its features actually have the reported importances. In contrast, with 8 and 12 years the boxplots became shorter and the ML model was more confident about its feature importances, as in Figure 3.

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*Figure 2: Feature Importance Plot for Universe 4, 1 year*

*Chart, box and whisker chart

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*Figure 3: Feature Importance plot for Universe 4, 12 years*

A training period of 8 years is selected so as to increase the size of dataset and confidence in features, but not capture patterns from too far behind in time.

1. *Effect of Bayesian Hyperparameter Optimization*

The effects of Bayesian Hyperparameter Optimization on the different universes are compared using MPM performance gain.

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Performance Gain | Univ 1 | Univ 2 | Univ 3 | Univ 4 | Univ 5 | Univ 6 | Univ7 | Uni8 | Uni9 | Uni10 |
| With Opt | +23% | +6% | +175% | +37% | +35% | +141% | +16.3% | +43.2% | +20.6% | +65% |
| Without Opt | +42% | +9% | +151% | +122% | +40% | +104% | +30.7% | +66.4% | +51% | +54% |

No significant performance improvements were observed with Bayesian Hyperparameter Optimization. However, similarly to increasing the training period, optimization made the model much more confident, albeit mainly reducing the right end(upper end) of the boxplots.

Chart

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*Figure 5: Feature Importance Plot for Universe 2, without Bayesian Opt*

Chart

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*Figure 6: Feature Importance Plot for Universe 2, with Bayesian Opt*

Without optimization, the boxes continued to increase in size with more important features. Optimization managed to alleviate this problem a little, mainly reducing the upper bound of importances while keeping the lower bounds about equal. However, even when the model was more confident in its features, performance did not improve. This is a worrying sign, perhaps a sign of overfitting when there is no optimization. Future work will seek to address this issue.

1. performance results

MPM. ERC and HRP are compared against each other in all 10 universes. Metrics used include the Compound Annual Growth Rate, Sharpe Ratio computed using daily, monthly and annual returns.

*Universe 1*

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | CAGR | Sharpe Ratio (Daily) | Sharpe Ratio (Monthly) | Sharpe Ratio (Yearly) |
| ERC | 0.036 | 0.54 | 0.49 | 0.87 |
| HRP | 0.028 | 0.49 | 0.44 | 0.87 |
| MPM | 0.038 | 0.63 | 0.59 | 0.97 |

*Universe 2*

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | CAGR | Sharpe Ratio (Daily) | Sharpe Ratio (Monthly) | Sharpe Ratio (Yearly) |
| ERC | 0.054 | 0.36 | 0.31 | 0.90 |
| HRP | 0.048 | 0.33 | 0.28 | 0.89 |
| MPM | 0.059 | 0.38 | 0.34 | 0.94 |

*Universe 3*

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | CAGR | Sharpe Ratio (Daily) | Sharpe Ratio (Monthly) | Sharpe Ratio (Yearly) |
| ERC | 0.0077 | 0.29 | 0.28 | 0.81 |
| HRP | 0.0046 | 0.34 | 0.32 | 0.83 |
| MPM | 0.017 | 0.86 | 0.74 | 1.1 |

*Universe 4*

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | CAGR | Sharpe Ratio (Daily) | Sharpe Ratio (Monthly) | Sharpe Ratio (Yearly) |
| ERC | 0.011 | 0.48 | 0.48 | 1.12 |
| HRP | 0.0048 | 0.44 | 0.40 | 0.87 |
| MPM | 0.014 | 0.64 | 0.66 | 1.10 |

*Universe 5*

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | CAGR | Sharpe Ratio (Daily) | Sharpe Ratio (Monthly) | Sharpe Ratio (Yearly) |
| ERC | 0.040 | 0.54 | 0.53 | 1.13 |
| HRP | 0.023 | 0.41 | 0.38 | 1.10 |
| MPM | 0.041 | 0.65 | 0.60 | 1.15 |

*Universe 6*

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | CAGR | Sharpe Ratio (Daily) | Sharpe Ratio (Monthly) | Sharpe Ratio (Yearly) |
| ERC | 0.014 | 0.46 | 0.46 | 1.03 |
| HRP | 0.0049 | 0.41 | 0.37 | 0.86 |
| MPM | 0.020 | 1.05 | 0.96 | 1.45 |

*Universe 7*

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | CAGR | Sharpe Ratio (Daily) | Sharpe Ratio (Monthly) | Sharpe Ratio (Yearly) |
| ERC | 0.043 | 0.64 | 0.58 | 1.04 |
| HRP | 0.033 | 0.59 | 0.54 | 1.01 |
| MPM | 0.044 | 0.71 | 0.67 | 1.10 |

*Universe 8*

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | CAGR | Sharpe Ratio (Daily) | Sharpe Ratio (Monthly) | Sharpe Ratio (Yearly) |
| ERC | 0.034 | 0.52 | 0.49 | 0.93 |
| HRP | 0.018 | 0.37 | 0.38 | 0.82 |
| MPM | 0.034 | 0.64 | 0.58 | 1.07 |

*Universe 9*

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | CAGR | Sharpe Ratio (Daily) | Sharpe Ratio (Monthly) | Sharpe Ratio (Yearly) |
| ERC | 0.047 | 0.68 | 0.73 | 1.21 |
| HRP | 0.025 | 0.54 | 0.55 | 1.14 |
| MPM | 0.038 | 0.73 | 0.74 | 1.54 |

*Universe 10*

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | CAGR | Sharpe Ratio (Daily) | Sharpe Ratio (Monthly) | Sharpe Ratio (Yearly) |
| ERC | 0.038 | 0.62 | 0.59 | 1.39 |
| HRP | 0.022 | 0.48 | 0.47 | 1.23 |
| MPM | 0.047 | 0.91 | 0.84 | 1.55 |

Without a doubt, MPM was able to significantly outperform both ERC and HRP all forms of measuring Sharpe Ratio. Whether or not this translates to live trades and profit is another issue however.

1. correlation data

Diversification works best when assets are as uncorrelated, or negatively correlated, with each other as possible. Below we investigate this relationship by looking at the average and standard deviation of correlations in each universe and the compound annual growth rate of ERC, HRP and MPM in each universe.

Mean\_Corr is the mean of all the upper triangular elements in the correlation matrix, while Std\_Corr is the standard deviation of all these elements. CAGR is the compound annual growth rate.

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | Univ1 | Univ2 | Univ3 | Univ4 | Univ5 | Univ6 | Univ7 | Univ8 | Univ9 | Univ10 |
| Mean\_Corr | 0.15 | 0.49 | 0.15 | 0.29 | 0.17 | 0.17 | 0.09 | 0.14 | 0.27 | 0.20 |
| Std\_Corr | 0.23 | 0.14 | 0.28 | 0.31 | 0.31 | 0.25 | 0.22 | 0.31 | 0.26 | 0.30 |
| CAGR | 0.038 | 0.059 | 0.017 | 0.014 | 0.041 | 0.19 | 0.044 | 0.034 | 0.038 | 0.047 |

Graphical user interface, calendar

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*Figure 7: Correlation matrix of Universe 6*

Interestingly, all assets in universe 6 are quite uncorrelated with SHY, and the model puts most of the weights in SHY, with the rest roughly equally split. This might explain why CAGR performance was much better.



*Figure 8: Correlation matrix of Universe 9*

Similarly, the model put about 50-70% of its weights in AGG, which was very uncorrelated with other assets.

Overall, I’m not really sure whether a lower correlation of all assets in the universe really led to any higher CAGR.

Appendix

*List of Tickers in each Universe*

1. ['IWM', 'LQD', 'EEM', 'GLD', 'TIP', 'SPY']
2. ['IWM', 'VNQ', 'VO', 'QQQ', 'VBR', 'EWJ']
3. ['EEM', 'AGG', 'EFA', 'IWD', 'LQD', 'SHY']
4. ['EFA', 'EWJ', 'IWM', 'SHY', 'SPY', 'XLY']
5. ['XLY', 'TLT', 'EFA', 'EEM', 'QQQ', 'TIP']
6. ['GLD', 'VNQ', 'IWM', 'IWD', 'SHY', 'XLV']
7. ['EWJ', 'TIP', 'GLD', 'XLV', 'VBR', 'TLT']
8. ['EFA', 'SPY', 'GLD', 'AGG', 'VO', 'TLT']
9. ['SPY', 'EEM', 'AGG', 'EWJ', 'XLV', 'QQQ']
10. ['AGG', 'VNQ', 'VBR', 'TIP', 'SPY', 'XLY']

References

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