ENGINEERING TRIPOS PART II A

EIETL **MODULE EXPERIMENT 3F3**

RANDOM VARIABLES and RANDOM NUMBER GENERATION

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1 Uniform and normal random variables

1.1 Histogram of Gaussian random numbers

Histogram of Gaussian random numbers overlaid on exact Gaussian curve (scaled):

Include your graphic here

1.2 Histogram of Uniform random numbers

Histogram of Uniform random numbers overlaid on exact Uniform curve (scaled):

test

Include your graphic here

1.3 Kernel density estimate for Gaussian random numbers

Kernel density estimate for Gaussian random numbers overlaid on exact Gaussian curve:

Include your graphic here

1.4 Kernel density estimate for Uniform random numbers

Kernel density estimate for Uniform random numbers overlaid on exact Gaussian curve:

Include your graphic here

1.5 Comparison of kernel density and histogram methods

Comment on the advantages and disadvantages of the kernel density method compared with the histogram method for estimation of a probability density from random samples:

Text answer here

1.6 Theoretical analysis of uniform density

Theoretical mean and standard deviation calculation for uniform density as a function of N:

Text/maths answer here

Explain behaviour as N becomes large:

Text/maths answer here

1.7 Histograms for varying N

Plot of histograms for $N = 100$, $N = 1000$ and $N = 10000$ with theoretical mean and ± 3 standard deviation lines:

Include your graphic here

1.8 Consistency with multinomial distribution theory

Are your histogram results consistent with the multinomial distribution theory?

Text/maths answer here

2 Functions of random variables

2.1 Linear transformation of normal variables

For normally distributed $\mathcal{N}(x|0,1)$ random variables, take $y = f(x) = ax+b$. Calculate $p(y)$ using the Jacobian formula:

Text/maths answer here

Explain how this is linked to the general normal density with non-zero mean and non-unity variance:

Text/maths answer here

2.2 Verification of linear transformation

Verify this formula by transforming a large collection of random samples $x^{(i)}$ to give $y^{(i)} = f(x^{(i)})$, histogramming the resulting y samples, and overlaying a plot of your formula calculated using the Jacobian:

Include your graphic here

2.3 Quadratic transformation of normal variables

Now take $p(x) = \mathcal{N}(x|0, 1)$ and $f(x) = x^2$. Calculate $p(y)$ using the Jacobian formula:

Text/maths answer here

2.4 Verification of quadratic transformation

Verify your result by histogramming of transformed random samples:

Include your graphic here

3 Inverse CDF method

3.1 CDF and inverse CDF for exponential distribution

Calculate the CDF and the inverse CDF for the exponential distribution:

Text/maths answer here

3.2 Implementation of inverse CDF method

Matlab/Python code for inverse CDF method for generating samples from the exponential distribution:

Matlab/Python code here

3.3 Verification with histograms

Plot histograms/ kernel density estimates and overlay them on the desired exponential density:

Include your graphic here

4 Simulation from a non-standard density

4.1 Random number generation code

Matlab/Python code to generate N random numbers drawn from the distribution of X:

4.2 Histogram density estimates

Plot some histogram density estimates with $\alpha = 0.5$, 1.5 and several values of β.

4.3 Interpretation of parameters

Hence comment on the interpretation of the parameters α and β.