

# Discrete Structure

Assignment 1 on Sep 19<sup>th</sup>

## P.13

10. (c) The election is not decided and the votes have been counted.

(f) If the election is not decided then the votes have not been counted.

14. (b)  $p \wedge q \wedge r$

(d)  $p \wedge \neg q \wedge r$

16. (b) False.

20. (b) The statement uses exclusive OR. Since, it is used in the context that either soup or salad, but not both will be included in lunch.

(c) The statement uses inclusive OR. Since , it gives more information if one has both the documents.

32. (d)

p	q	$p \wedge q$	$p \vee q$	$(p \wedge q) \rightarrow (p \vee q)$
F	F	F	F	T
F	T	F	T	T
T	F	F	T	T
T	T	T	T	T

## P.23

12. Yes, these system specifications consistent.

To determine whether these specification are consistent, we first need to express those using logical expressions.

p: “the file system is locked”

q: “the new message will be queued”

r: “the system is functioning is normally ”

s : “ the new message will be sent to the message buffer”,

Those specifications can be written as

$$\neg p \rightarrow q,$$

$$\neg p \leftrightarrow r$$

$$, \neg q \rightarrow s ,$$

$$\neg p \rightarrow s ,$$

$$\neg s$$

An assignment of truth values that can makes all five specifications true must have

**s false, p true and q true and r is false.**

24. A says “C is the knave”

B says” A is the knight”

C says “I am the spy”

knight	knave	spy	A(F)	B(T)	C(T)
knight	spy	knave	A(T)	B(T)	C(F)
knave	knight	spy	A(F)	B(F)	C(T)
knave	spy	knight	A(F)	B(F)	C(F)
spy	knight	knave	A(T)	B(F)	C(F)

Spy	Knave	knight	A(F)	B(F)	C(F)
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So the only solution will be A is knight, B is Spy and C is Knave.

### Another Solution:

**P: A tells truth (C is knave)**

**Q: B tells truth (A is Knight)**

**R: C tells truth (I am Spy)**

**$\neg(P \wedge Q \wedge R)$  (it is impossible everybody tells truth)**

**$\neg(\neg P \wedge \neg Q \wedge \neg R)$  (it is impossible everybody tells lie)**

**$P \rightarrow \neg R$  (if A tells truth, C tells lie)**

P	Q	R	
F	F	F	F
F	F	T	F
F	T	F	F
F	T	T	F
T	F	F	F
T	F	T	F
T	T	F	T
T	T	T	F

**$Q \rightarrow \neg R$  (if Q tells truth, C tells lie)**

**$R \rightarrow (P \otimes Q)$  (If C tells truth, it is impossible that both A, B tell lies or both A, B tell truth, because one of them is Knight, one of them is Knave)**

**$Q \rightarrow P$  (If B tells truth, A is knight, A will tell the truth)**

**$\neg(P \wedge Q \wedge R) \wedge \neg(\neg P \wedge \neg Q \wedge \neg R) \wedge (P \rightarrow \neg R) \wedge (Q \rightarrow \neg R) \wedge (R \rightarrow (P \otimes Q)) \wedge (Q \rightarrow P)$  must be true**

**So A, B tell truth and C tells false.**

**A is knight, C is Knave and B is Spy**

36. (a) **P:** Alice said "Carlos did it."

**Q:** John said "I did not do it."

**R:** Carlos said "Diana did it."

**S:** Diana said "Carlos lied when he said that I did it."

Since the authorities know exactly one of the four is telling the truth,

Assume Alice is telling truth, Then John is also telling truth, it is not possible.

Assume John is telling truth, then Carlos should tell false, then Diana is telling truth, we can get two people are telling truth, it is not possible.

Assume Carlos is telling truth, then Diana and Alice and John are all lie, which can happens, then John did it.

Assume Diana is telling truth, Carlos and John and Alice are all lie, which can also happens, then John did it.

**Another way:** First of all we notice that Carlos and Diana cannot be both lying. If Carlos is lying, then Diana would be telling the truth, and if Carlos is telling the truth, then Diana would be lying.

So it is either Carlos or Diana who is telling the truth. John is lying, and John said “I did not do it”. Therefore John had the unauthorized access.

(b) Since the authorities know exactly one of the four is lying.

Assume Alice is lying, then John, Carols and Diana are all telling truth, while Carlos and Diana can't telling truth at the same time, which is not possible.

Assume John is lying, then Alice, Carols and Diana are all telling truth, Since Carlos and Diana can't telling truth at the same time, which is not possible.

Assume Carlos is lying, then Alice, John and Diana are all telling truth, it can be happened, then Carlos did it.

Assume Diana is lying, then Alice, John and Carlos are telling truth, while Alice and

Carlos can't telling truth at the same time. Which is not possible.

**Another Solution:** Again we see that Carlos and Diana cannot be both telling the truth, so the liar is either Carlos or Diana. This means that Alice is telling the truth. Alice said "Carlos did it". Therefore it was Carlos who had the unauthorized access.