Code for Project 2(Here just give the code for using triangle element, the code of using rectangle element is similar)

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function [ ErrorMax,Error2 ] = Project_Parabolic2d_Triangle( n,dt,T )
N=n+1;h=1/n;
xe=linspace(0,1,N);ye=xe;

b=[1,-2];F=zeros(N*N,1);
nn=zeros(N*N,1);%to record for each node, how many elements of the coefficient matrix are not zero


for k=1:N*N
   i=mod(k-1,N)+1;j=floor((k-1)/N)+1;
   if(i~=1 && i~=N && j~=1 && j~=N)
      nn(k)=7;
%       A(k,k)=4;A(k,k+1)=-1;A(k,k-1)=-1;A(k,k+N)=-1;A(k,k-N)=-1;
%       B(k,k)=0.5*h*h;B(k,k+1)=h*h/12;B(k,k-1)=h*h/12;B(k,k+N)=h*h/12;B(k,k-N)=h*h/12;B(k,k-N-1)=h*h/12;B(k,k+N+1)=h*h/12;
%       C(k,k)=0;C(k,k+1)=h/3;C(k,k-1)=-h/3;C(k,k+N)=-h/6;C(k,k-N)=h/6;C(k,k-N-1)=-h/6;C(k,k+N+1)=h/6;
%       D(k,k)=0;D(k,k+1)=-h/6;D(k,k-1)=h/6;D(k,k+N)=h/3;D(k,k-N)=-h/3;D(k,k-N-1)=-h/6;D(k,k+N+1)=h/6;
      F(k)=f_Parabolic(xe(i)+0.5*h,ye(j),dt)+f_Parabolic(xe(i)+0.5*h,ye(j)+0.5*h,dt)+f_Parabolic(xe(i),ye(j)+0.5*h,dt)+f_Parabolic(xe(i)-0.5*h,ye(j),dt)+f_Parabolic(xe(i)-0.5*h,ye(j)-0.5*h,dt)+f_Parabolic(xe(i),ye(j)-0.5*h,dt);
      F(k)=h*h*F(k)/6; %To calculate the RHS integration
   else
      nn(k)=1;
%       B(k,k)=1;%to use the Dirichlet boundary condition at any time t
   end
end
pnn=zeros(N*N,1);pnn(1)=1;%to record the first position in coefficient matrix for each grid point
ii=zeros(sum(nn),1);%to construct the sparse matrix in matlab
jj=zeros(sum(nn),1);%to construct the sparse matrix in matlab
value_A=zeros(sum(nn),1);%to construct the sparse matrix A in matlab
value_B=zeros(sum(nn),1);%to construct the sparse matrix A in matlab
value_C=zeros(sum(nn),1);%to construct the sparse matrix A in matlab
value_D=zeros(sum(nn),1);%to construct the sparse matrix A in matlab

for k=2:N*N
   pnn(k)=nn(k-1)+pnn(k-1);
end

for k=1:N*N
   if(nn(k)==1)
      ii(pnn(k))=k;jj(pnn(k))=k;value_B(pnn(k))=1;
   else
      for i=pnn(k):(pnn(k+1)-1)
         ii(i)=k;
      end
      jj(pnn(k))=k-N-1;jj(pnn(k)+1)=k-N;jj(pnn(k)+2)=k-1;jj(pnn(k)+3)=k;jj(pnn(k)+4)=k+1;
      jj(pnn(k)+5)=k+N;jj(pnn(k)+6)=k+N+1;
      value_A(pnn(k))=0;value_A(pnn(k)+1)=-1;value_A(pnn(k)+2)=-1;value_A(pnn(k)+3)=4;value_A(pnn(k)+4)=-1;
      value_A(pnn(k)+5)=-1;value_A(pnn(k)+6)=0;
      value_B(pnn(k))=h*h/12;value_B(pnn(k)+1)=h*h/12;value_B(pnn(k)+2)=h*h/12;value_B(pnn(k)+3)=h*h/2;value_B(pnn(k)+4)=h*h/12;
      value_B(pnn(k)+5)=h*h/12;value_B(pnn(k)+6)=h*h/12;
      value_C(pnn(k))=-h/6;value_C(pnn(k)+1)=h/6;value_C(pnn(k)+2)=-h/3;value_C(pnn(k)+3)=0;value_C(pnn(k)+4)=h/3;
      value_C(pnn(k)+5)=-h/6;value_C(pnn(k)+6)=h/6;
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        value_D(pnn(k))=-h/6;value_D(pnn(k)+1)=-h/3;value_D(pnn(k)+2)=h/6;value_D(pnn(k)
+3)=0;value_D(pnn(k)+4)=-h/6;
        value_D(pnn(k)+5)=h/3;value_D(pnn(k)+6)=h/6;
    end
end
A=sparse(ii,jj,value_A,N*N,N*N);
B=sparse(ii,jj,value_B,N*N,N*N);
C=sparse(ii,jj,value_C,N*N,N*N);
D=sparse(ii,jj,value_D,N*N,N*N);

bb=zeros(N*N,1);%the source terms
for k=1:N*N
    i=mod(k-1,N)+1;j=floor((k-1)/N)+1;
    if(i~=1 && i~=N && j~=1 && j~=N)
        bb(k)=u0_Parabolic(xe(i)+0.5*h,ye(j))+u0_Parabolic(xe(i)+0.5*h,ye(j)+0.5*h)
+u0_Parabolic(xe(i),ye(j)+0.5*h)+u0_Parabolic(xe(i)-0.5*h,ye(j))+u0_Parabolic(xe(i)-0.5*h,ye(j)-
0.5*h)+u0_Parabolic(xe(i),ye(j)-0.5*h);
        bb(k)=h*h*bb(k)/6;%To calculate the RHS integration
    else
        bb(k)=u_Parabolic( xe(i),ye(j),0 );
    end
end
u0=B\bb;%u_{h}^{0} is taken to be the interpolation of u_{0}

E1=B+A*dt;%use (3.69) to calculate u1
E=B+2*dt*A/3;%use (3.68);E(k,k) need to equals B(k,k)=1

bb=dt*F+B*u0-dt*b(1)*C*u0-dt*b(2)*D*u0;%use to calculate u1
for k=1:N*N
    i=mod(k-1,N)+1;j=floor((k-1)/N)+1;
    if(i==1 || i==N || j==1 || j==N)
        bb(k)=u_Parabolic( xe(i),ye(j),dt );
    end
end

u1=E1\bb;

iter=2;
while((iter-1)*dt<T) %Pay attention to iter! If use 'iter*dt<T', just calculate to T-dt, not T!
    F=zeros(N*N,1);
    for k=1:N*N
        i=mod(k-1,N)+1;j=floor((k-1)/N)+1;
        if(i~=1 && i~=N && j~=1 && j~=N)
        F(k)=f_Parabolic(xe(i)+0.5*h,ye(j),dt*iter)+f_Parabolic(xe(i)+0.5*h,ye(j)+0.5*h,dt*iter)
+f_Parabolic(xe(i),ye(j)+0.5*h,dt*iter)+f_Parabolic(xe(i)-0.5*h,ye(j),dt*iter)+f_Parabolic(xe(i)-
0.5*h,ye(j)-0.5*h,dt*iter)+f_Parabolic(xe(i),ye(j)-0.5*h,dt*iter);
        F(k)=h*h*F(k)/6; %To calculate the RHS integration
        end
    end
    bb=2*dt*F+4*B*u1-B*u0-2*dt*b(1)*C*(2*u1-u0)-2*dt*b(2)*D*(2*u1-u0);
    bb=bb/3;
    for k=1:N*N
        i=mod(k-1,N)+1;j=floor((k-1)/N)+1;
        if(i==1 || i==N || j==1 || j==N)
            bb(k)=u_Parabolic( xe(i),ye(j),dt*iter );
        end
    end
    u2=E\bb;
    u0=u1;u1=u2;
    iter=iter+1;
end
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u_exact=zeros(N*N,1);
for k=1:N*N
    i=mod(k-1,N)+1;j=floor((k-1)/N)+1;
    u_exact(k)=u_Parabolic( xe(i),ye(j),dt*(iter-1) );
end

ErrorMax=norm(u2-u_exact,inf);
Error2=norm(u2-u_exact,2)*h;


function u = u_Parabolic( x,y,t )
u=cos(t).*cos(6*x).*sin(6*y).*exp(x-y);

function u = u0_Parabolic( x,y )
u=cos(6*x).*sin(6*y).*exp(x-y);

function f = f_Parabolic( x,y,t )
%diff(cos(t)*cos(6*x)*sin(6*y)*exp(x-y),t,1)+diff(cos(t)*cos(6*x)*sin(6*y)*exp(x-y),x,1)-
2*diff(cos(t)*cos(6*x)*sin(6*y)*exp(x-y),y,1)-(diff(cos(t)*cos(6*x)*sin(6*y)*exp(x-
y),x,2)+diff(cos(t)*cos(6*x)*sin(6*y)*exp(x-y),y,2))
f=73*cos(6*x).*sin(6*y).*exp(x - y).*cos(t) - cos(6*x).*sin(6*y).*exp(x - y).*sin(t) +
6*sin(6*x).*sin(6*y).*exp(x - y).*cos(t);
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