Problem Chosen	2023	Team Control Number
A	MCM Summary Sheet	2320426

Drought-Stricken Plant Communities

Summary

Biodiversity is crucial for plant communities to cope with environmental stressors like drought, as different plant species have unique reactions to biotic and abiotic stress factors that shape the ecological characteristics of a community. This paper have shown that plant communities with diverse species are better equipped to adapt to drought conditions than those with only one species. In this paper, we analyze the impact of stress factors on the survivability of plant communities and determine the minimum number of species needed to ensure their long-term resilience.

In this paper, two models were utilized to quantify the impact of environmental factors on species populations. In the first model, we examine the competition, parasitic, and commensal relationships between plant species, constructing three systems of ordinary differential equations (ODE) by Lotka-Volterra equations to investigate how these relationships interact with their growth and the influence of drought. We also analyze the influence of various environmental factors on species growth using three different methods: Time Series analysis, logistic regression, and predator-prey modelling. The ultimate goal of model 1 is to explore the relationship between plant populations and their environment. In the second model, we classify the plant species into four types: arbors, shrubs, vines, and herbs, viewing this as an optimization problem. Hence, we attempt to find the number of species in a single plant community that both the environment and human beings can benefit from by constructing a linear programming model and solving it as an integer problem.

In Task 1, the East Sudan Savanna was chosen as a typical example, fitting the data with the exponential model and predicting with the Time Series method, investigating the impact of rainfall on plant growth in the Sudan drought area by simulating different precipitation scenarios. Then, we build the most common three relationships between plant species as systems of ordinary differential equations, estimating the solutions by the fourth Runge-Kutta method. On top of that, we combine the previous two models to observe how the population of each species performs under different drought situations. Finally, we concentrate on the competition relationship, which appears the most in natural environments, constructing the model with six species with competition relationships, setting them to interact with each other, and observing how their population respond to the climate.

Task 2 focused on the four types of species and 20 species samples, with the weather factor held constant, to formulate a model that identifies the minimum number of species required to optimize the utility associated with biomass. To consider other factors, such as pollution or any environmental change, a sensitivity analysis was conducted by either increasing or decreasing 10% of the total maximum carrying capacity of all the Species.

Keywords: Biodiversity, Plant communities, Ecological characteristics

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1 Introduction

1.1 Problem Background

Plants respond differently to environmental stresses, such as drought, highlighting the need to identify the factors affecting ecological systems and work towards improving them. Investigating the minimum number of plant species required to form a sustainable and resilient plant community is crucial for enhancing ecological communities. This raises important questions about how this phenomenon scales with increasing species and what it means for the long-term survival of plant communities. It is essential to determine whether individual plants can survive better in drought conditions or if a more diverse range of plant species can thrive under the same conditions.

Two major problems are discussed in this paper, which are:

- Different plant species react differently to stressors.
- The number of plant species in a community is crucial for its adaptation to droughts over time.

1.2 Restatement of the Problem

Investigating the drought adaptability of each species in a plant community and its potential impact on the number of species present. Given the background information and restricted conditions outlined in the problem statement, we aim to address the following problems:

1. Construct a mathematical model that can predict the impact of different weather cycles on a plant community, considering both drought and precipitation periods and the interaction between diverse species during drought cycles.
2. Considering the long-term interactions within a plant community and its relationship to the larger environment:
   - Determine the minimum number of species required to benefit the community and the impact of species growth on overall community health.
   - To what extent do the types of species in the community affect the results?
   - When adjusting the frequency and range, how does a greater occurrence of drought and a wider range of drought impact plant communities? Conversely, does the number of species similarly impact the overall population under different conditions?
   - Could other factors impact the conclusion, such as pollution or habitat reduction?
   - What actions does the model suggest to ensure the long-term viability of a plant community, and what are the impacts on the larger environment?"

1.3 Literature Review

In their 2014 study, Tilman, Isbell & Cowles and their 2017 follow-up investigation, Isbell et al. examined the role of biodiversity in driving ecosystem processes in non-manipulated, naturally assembled plant communities. Biodiversity was found to be non-randomly distributed across space and time and influenced by various environmental factors and ecosystem functions, such as plant biomass (Grime, 1973; Hautier, Niklaus & Hector, 2009) (Fig. 1C). In a 2017 study, John Harte and Johannes H. C. Cornelissen investigated the impact of species diversity on the competition for space in plant communities and how this can lead to increased diversity over time. At the same time, Wolfgang W. Weisser explored the relative importance of individual plant species or functional groups, or biodiversity itself, for ecosystem functioning in a study published in Basic and Applied Ecology. The results suggest that competition among species may also affect the community.

The research and discussions above demonstrate that many scholars have focused on the stressors faced by plant communities and have conducted extensive research in this field. Studies indicate that in drought-stricken conditions, diverse plant species perform better than monoculture communities. However, there still needs to be a knowledge gap in understanding how diverse environmental factors can affect the growth of plant species, highlighting the need for further research in this area.

1.4 Our work

As the question did not specify a particular country, we used Sudan's weather as a reference for our analysis. Based on the data available for Sudan, we built a mathematical model to predict the impact factors on plant communities. We analyzed the weather patterns of the past four years and examined five different plant species with three different relationships. Through this analysis, we aimed to gain insights into the ecological dynamics of plant communities and the factors that influence their growth and development in Sudan.

Through a thorough literature analysis, we have identified that biomass affects biodiversity, and competition between different species can also impact plant growth. Therefore, when estimating the number of plants, we take into consideration the biomass of each type of plant. Multiple variables must be considered and controlled to assess the impact on plant communities comprehensively. Thus, we analyze the multi-factor indicators and select relevant data from official websites. This paper selected 6 species based on three relationships: Striga asiatica, Ficus aurea, Usnea, Poplar, and Pine. Additionally, we examined the impact of different types of plants and selected representatives for each of the four types: Herb (Heteropogon contortus), Arbor (Acacia farnesiana), Shrub (Podranea ricasoliana), and Vines (Cissus rotundifolia).

Using the data from 2019 to 2022, we analyzed the weather and its impact on plant growth. However, due to the lack of data on the accurate number of each plant in Sudan, we estimated the number of plants in a given area or plant community based on the weight of the plants and the land they occupy.

To determine the optimal and healthy plant community, this paper employed the following methods:

- The 4th order Runge-Kutta model was utilized to solve the system of original differential

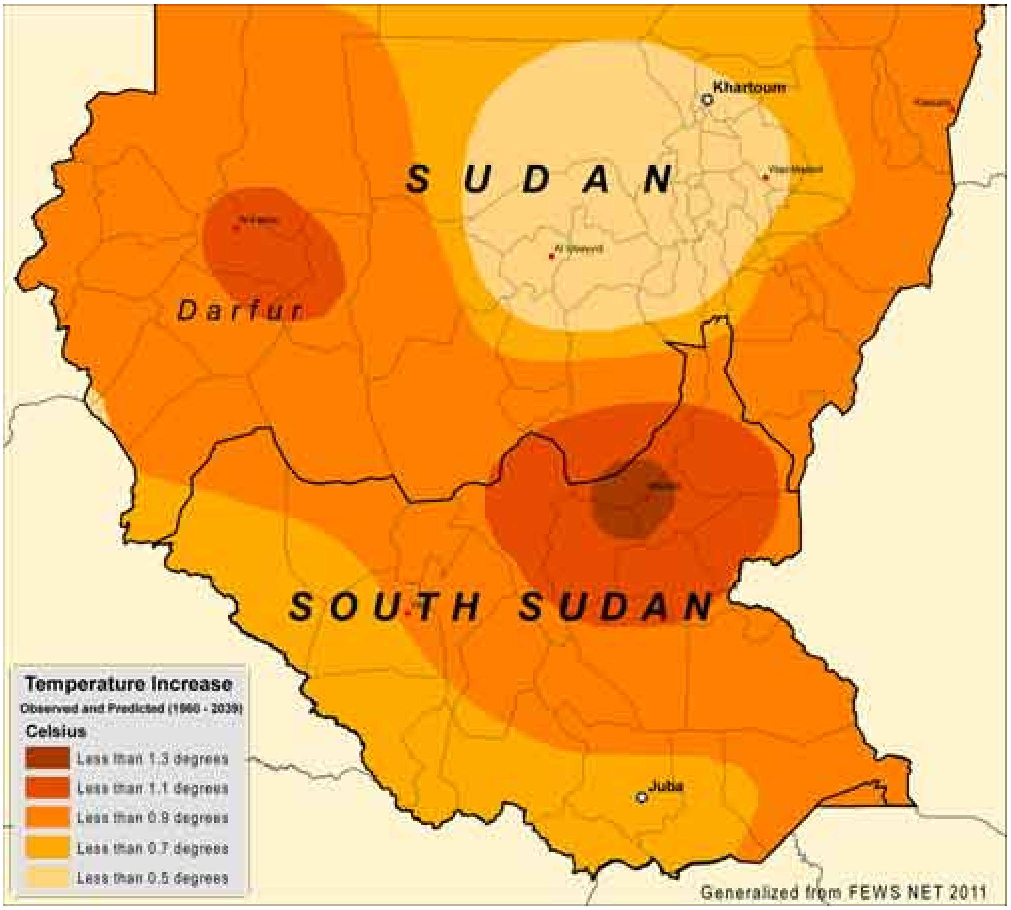

Figure 1: Sudan Region

equations.

- Time series analysis was used to predict the precipitation levels for a given year.
- Exponential functions and logistic regression were employed to model precipitation and establish its relationship with the growth rates of plants.
- The model is used to formulate the relationship between three different types of species by solving the Original Differential Equation system.
- The Model 2 formulated an optimization problem and it was solved by linear integer programming.

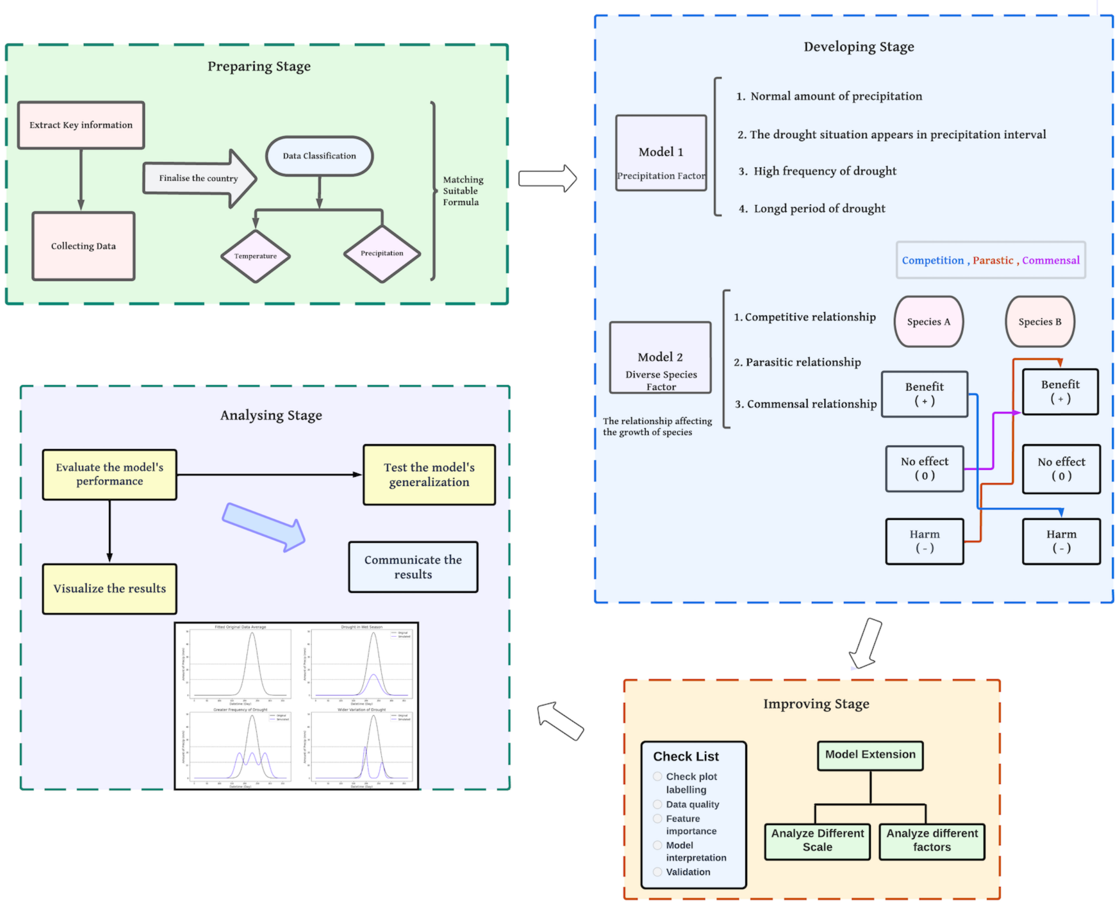

Figure 2: Flow Chart of Our Work

2 Assumptions and Justification

2.1 Assumptions

1. The precipitation amount will affect the species' population in a selected time interval. When we have the upper bond N for the extreme situation of precipitation condition, as the level of rain decrease, the plants will have a larger growth.

2. Warmer temperatures can lead to a higher growth rate of plants, as they are adapted to the climatic characteristics of tropical regions, including strong stalks and well-developed root systems. However, increased temperature also enhances evaporation, leading to a reduction in surface water and drying out of soils and vegetation, making it more suitable for plants that can thrive in drought conditions.

3. Competition relationships can promote plant growth because diverse plant communities are better at capturing essential plant nutrients from the soil, leading to more outstanding biomass production. Symbiotic relationships must be considered from both sides of the plant partnership, as the provider benefits the epiphyte and leads to a better community system. On the

other hand, parasitic relationships will decrease the population of plants, as one plant must grab nutrients from another to survive.

2.2 Notations

The primary notations used in this paper are listed in Table 2.

Table 1: Model 1 Notations

Symbol	Definition
α	Coefficient of competition
β	Coefficient of competition
γ	Coefficient of self-Inhibition
δ	Coefficient of symbiosis
$\mathbf{t}$	Time instant
$\mathbf{r}$	The rate of natural growth
$\mathbf{X}$	The number of species
$\mathbf{N}$	The maximum carrying capacity of species
RK4	The fourth-order Runge-Kutta method

Table 2: Model 2 Notations

Symbol	Definition
a_1, a_2, a_3, a_4	Biomass for Arbor, Shrub, Vines, Herb respectively
$\theta_1, \theta_2, \theta_3, \theta_4$	Group of Arbor, Shrub, Vines, Herb respectively
Θ	The whole space contains all the species
$\mathbf{t}$	Time instant (Days)
$\mathbf{r_i}$	The rate of natural growth for $Species_i$
$\mathbf{X_i}$	The number of $Species_i$
α_k^i	Coefficient of competition for $Species_i$ to $Species_k$
$x_i(0)$	Initial amount for the $Species_i$
$\mathbf{N}^*$	The total maximum carrying capacity of all the Species
$\mathbf{N_i}$	The maximum carrying capacity of $Species_i$
RK4	The fourth-order Runge-Kutta method

3 Model 1: Impact of Weather Cycles on Plant Communities

In the real world of nature, almost every specie is playing its own unique role, affecting other species and being affected. Natural scientists are devoted to figuring out how different species from the producers, consumers, and decomposers relate to each other and classify these relationships into mutualism, Commensalism, Competition, Parasitism, and semiparasitic, etc. Here, we only focus on the three plant relationships that are most common in nature, which are Competition, Parasitism, and Commensalism (Table 3).

Competition is a relationship in which organisms, including producers, consumers and decomposers, compete for specific limited resources, such as habitats, food, water, light, etc. For the plants, there are lots of examples of competitive relationships. As people commonly observe, many different kinds of flowers release their scents to be more competitive in attracting insects that help them spread their seeds.

Parasitism relationship exists between a parasite plant and a host plant. The parasite plants grow by feeding on the nutrition from their hosts' bodies, and the hosts die once the nutrition is insufficient for them to survive. Thus, this relationship is only harmful to the host plants. In most cases, parasite plants do not derive their nutrition from photosynthesis but only from the host, for example, the European dodder. However, in some particular species, the parasite plants, such as the mulberry parasite, have chlorophyll that feeds on their own.

Commensalism relationship also exists between a parasite plant and a host plant. However, in this case, the parasite plants do not hurt the hosts but only benefit from the latter. Some parasite plants can get from the hosts, like ferns climbing on the trees to gain more sunlight. Besides, some hosts can protect the parasites from being hurt. For example, garlic protects cotton by giving off a pungent smell that repels pests.

Table 3: An overview of the defining features and sample representatives for the three types of relationships in the plant community

Name	Description	Examples
Competitive	Competition is a type of relationship in which organisms compete for the same resources, such as food, water, or space, in the exact geographical location.	Usnea, Vanda Miss Joaquim, Tillandsia, Orchids, Cedars, Oaks, Phlox subulata, Agaricus
Parasitic	Parasitism affects host growth, allometry, and reproduction, leading to changes in competitive relationships, community structure, and population dynamics. This impact can extend to herbivores, pollinators, and seed vectors, closely linked to the abundance of parasitic plants.	Striga asiatic , Ficus aurea , Cuscuta , Striga , Sorghum bicolor , Orobanche , Helianthus annuus , Phoradendron

Commensal	Commensalism is a relationship between two species in which one species benefits without harming or benefiting the other. The commensal species receives nutrients, shelter, support, or locomotion from the host species unaffected by the interaction.	Poplar , Pine , Opuntia , Acacia , Artemisia tridentata , Bouteloua , Prosopis , Juniper

In this section, we will analyze how the populations of plant species with different relationships behave in stress factor drought in four stages:

Stage 1 Based on the climate database of East Sudan Savanna from 2019 to 2022, the time series used the forecasting method - algorithm average to predict the precipitation amount in the next year using the exponential model to fit the data values.

Stage 2 This stage focused on examining the impact of different relationships on the species' population without considering precipitation. To achieve this, we employ the RK4 method to solve the ODE system of equations. Our goal is to investigate the effect of plant relationships on the species' population in various scenarios.

Stage 3 Building on the previous stage, we aim to examine how both precipitation and the relationships between different plant species impact the population of the plant community.

Stage 4 We model the competitive relationships between the six species and investigate the impact of precipitation and inter-species Competition on their population growth.

3.1 Exploring the Role of Precipitation in Plant Community Dynamics

3.1.1 The Exponential Model of Amount of Precipitation

Many factors can cause drought, commonly the consequence of a comprehensive effect. We analyze the drought problem for the plants from two aspects: evaporation and soil moisture. Specifically, for evaporation, the potential factors may include the temperature, the ultraviolet radiation and the area of the leaves of plants. Additionally, for soil moisture, the potential factors may include precipitation, underground water sources, and soil materials. Moreover, combined with our data, precipitation was chosen as the main factor since the number of plants with roots is more than those without. The underground water sources and the soil moisture are difficult to be quantified consistently.

We extract the precipitation data of East Sudan Savanna in Africa from 2019 to 2022 (Figure 3 Figure 4). A flood happened in this place in 2021, as we can see that the total amount of precipitation is much more than in other years. Therefore, considering this, we use the algorithm average to calculate the average value of each day in four years,

$$Y_{n+1} = \frac{\sum_{i=1}^{n} Y_i}{n} = \frac{Y_1 + Y_2 + \cdots + Y_n}{n}$$

where Y is the amount of precipitation in millimetres daily, and n is the number of years. Here we only have four years of data, so our $n = 4$. However, since the amount of precipitation in 2021 is much higher than in other years, we add a weight coefficient ε, which takes value in $[0,1]$. Therefore, the predicted data for 2023 is

$$Y_5 = \frac{Y_1 + Y2 + \varepsilon Y_3 + Y_4}{4}.$$

Then, the exponential model was used

$$h(t) = Ae^{-B(t-C)} + D$$

to fit the data, where the coefficients A is the daily highest amount of precipitation, B is the wet season interval coefficient, C is the daytime precipitation peak. During the fitting process, we noticed that on most days, East Sudan Savanna does not rain, but the model needs to consider a more general case; therefore, we add the coefficient D as the fitted lowest amount of precipitation.

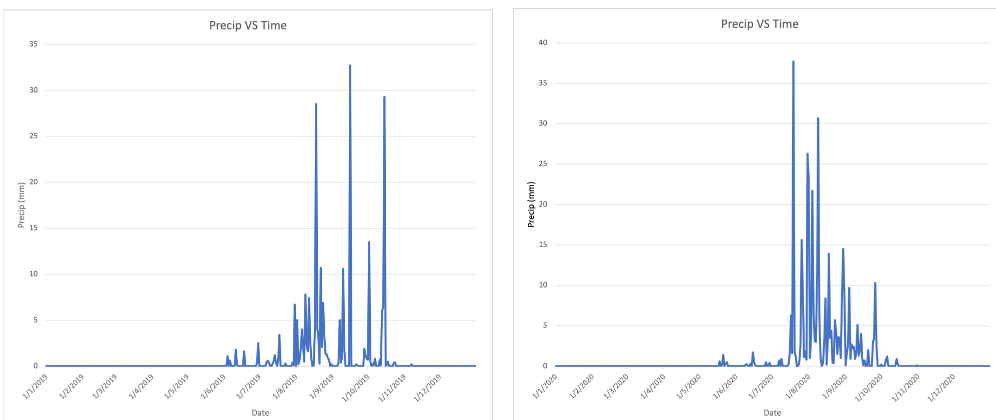

Figure 3: The precipitation from 2019 to 2020 in Sudan

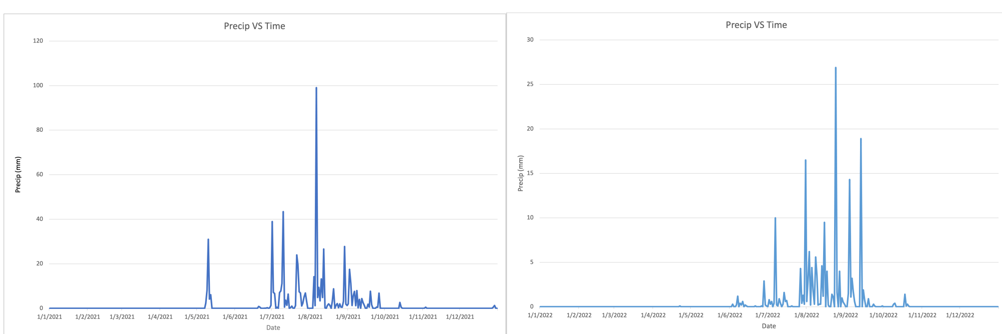

Figure 4: The precipitation from 2021 to 2022 in Sudan

Based on the temperature plot for the past four years, it is evident that the period between April and July experiences higher temperatures in Sudan.

Set four situations to generate the drought: 1. Original data fitted model; 2. Relative drought during the wet season; 3. Two drought periods in a wider wet season with the same variation; 4. One drought period with a wider variation.

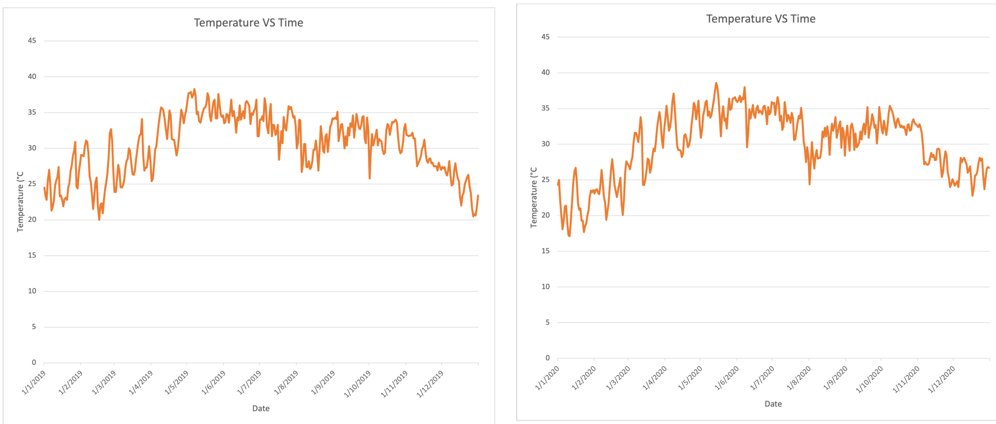

Figure 5: The temperature from 2019 to 2020 in Sudan

Figure 6: The temperature from 2021 to 2022 in Sudan

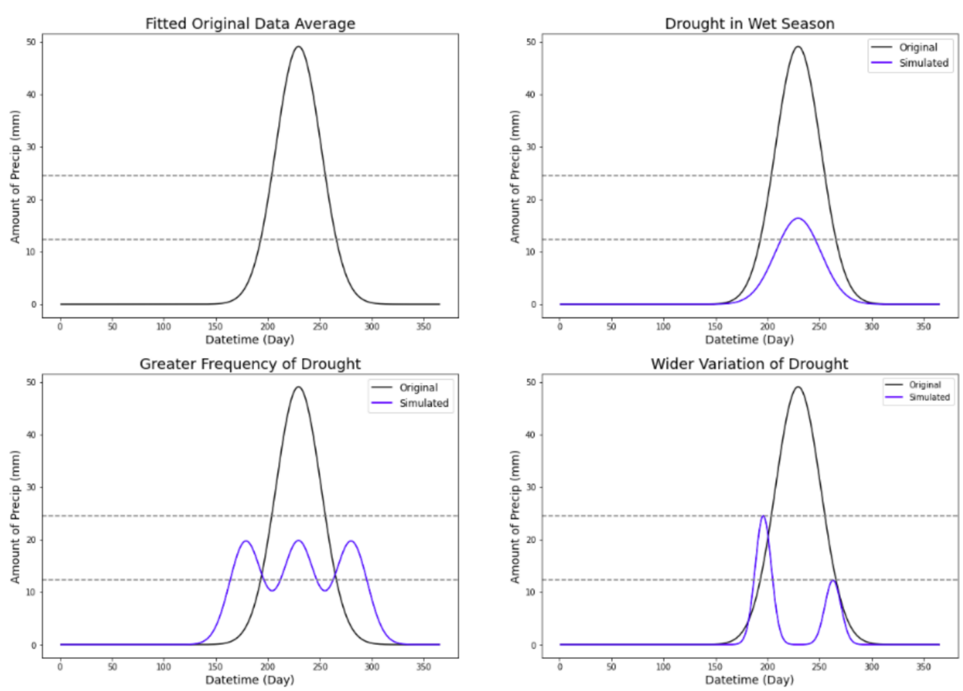

Figure 7: The various situation of precipitation

Based on the exponential precipitation model we fitted above, we assume the natural growth rates of plants are directly proportional to the amount of precipitation, with initial settings natural growth rate $r(t) = 0$ when the amount of precipitation $h(t) = 0$. We here construct a Logistic function which subtracts the value when the amount of precipitation is 0 :

$$r_i(t) = \frac{L_i}{1+e^{-k_i*h(t}} - \frac{L_i}{2}$$

where L_i is the most remarkable growth rate, k_i is the Logistic growth rate or the slope, and $h(t)$ is the amount of precipitation concerning time we defined above.

3.2 Interactions Among Three Plant Species and Their Impact on Growth

In this section, we construct three systems of ordinary differential equations of typical interactive relationships of plants, solving them by the fourth Runge-Kutta method and observing how the species' populations vary as time increases.

The **competition relationship** can be described by equation:

$$\begin{cases} \frac{\mathrm{dx}_1}{\mathrm{d}t} = r_1 \times x_1(1 - \frac{x_1}{N_1} - \alpha\frac{x_2}{N_2}) \\ \\ \frac{\mathrm{dx}_2}{\mathrm{d}t} = r_2 \times x_2(1 - \frac{x_2}{N_2} - \beta\frac{x_1}{N_1}) \end{cases} \tag{1}$$

where r_1, r_2 are the natural growth rate for each specie calculated by the above Logistic function, N_1, N_2 are the maximum capacity for each specie, and the competitive coefficients α, β represent the space of each of the current species takes than other species.

the parasitic relationship could be seen as a type of Predator-Prey relationship but for the plants. There are two reasons for choosing this model: Firstly, as we state above, most of the parasite plants cannot produce nutrition on their own because they do not have chlorophyll, so their roles in the natural world are quite similar to the consumers. Secondly, the host plants are hurt by the parasite plants until they die, similar to the prey eaten by the predator. Therefore, we select the Lotka-Volterra Predator-Prey Equations as our model of parasitism relationship. The **parasitic relationship** can be described by equation:

$$\begin{cases} \frac{\mathrm{d}x_3}{\mathrm{d}t} = r_3 x_3(1 - x_4) \\ \\ \frac{\mathrm{d}x_4}{\mathrm{d}t} = x_4(x_3 - 1) \end{cases} \tag{2}$$

where x_3 is the host specie, x_4 is the parasite specie, r_3 is the natural growth rate for the specie x_3, and here we set the natural growth rate for the specie x_4 is $r_4 = 1$.

The **commensal relationship** can be described by equation:

$$\begin{cases} \frac{\mathrm{dx}_5}{\mathrm{d}t} = r_5 \times N_5 - \gamma_5\, N_5^2 + \delta N_5 N_6 \\ \\ \frac{\mathrm{dx}_6}{\mathrm{d}t} = r_6 \times N_6 - \gamma_6\, N_6^2 \end{cases} \tag{3}$$

where x_5 is the host specie, x_6 is the parasite specie, r_5, r_6 are the natural growth rate of each specie, γ_5, γ_6 are the self-inhibition coefficients, and δ is the commensal coefficient of specie x_5 to specie x_6.

We solve these three systems of ODE by numerical RK4 method, which the algorithm is as follows:

$$\frac{\mathrm{d}y}{\mathrm{d}} = f(t,y), y(t_0) = y_0 \tag{4}$$

$$y_{n+1} = y_n + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)h \tag{5}$$

for $n = 0,1,2,3,...$

$$k_1 = f(t_n, y_n) \tag{6}$$

$$k_2 = f(t_n + frach2, y_n + h\frac{k_1}{2}) \tag{7}$$

$$k_3 = f(t_n + frach2, y_n + h\frac{k_2}{2}) \tag{8}$$

$$k_4 = f(t_n + h, y_n + hk_3) \tag{9}$$

The trend population of each specie with respective to time for each type of relationship without considering the amount of precipitation is shown as follows (Figure 8):

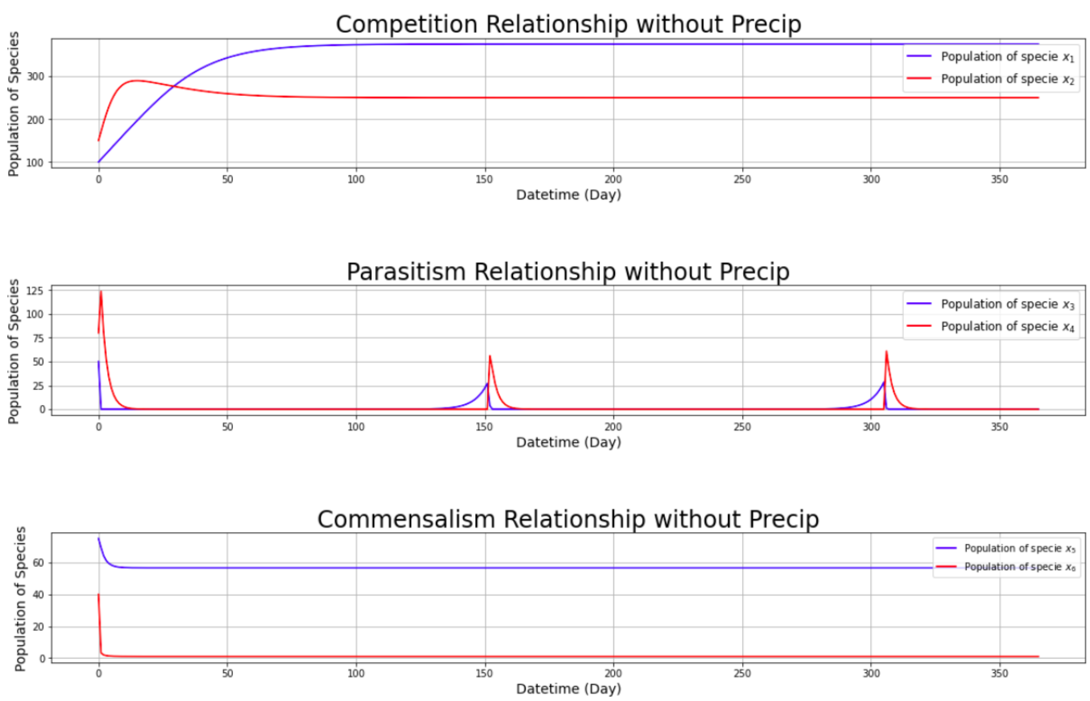

Figure 8: Two factors : Competition without Precipitation

Based on observation, the following could be found:

Firstly, in terms of the competitive relationship, we observed that both species eventually converged to a specific population level. Under this relationship, species A increased until reaching a maximum capacity, while the number of species B initially increased and then levelled off as the number of species A increased. Thus, the competitive relationship led to an increase in the population of species A but a decrease in the population of species B. Overall, this relationship helped to maintain a balanced ecological system.

Secondly, in the case of the parasitism relationship, we observed a periodic trend where the population of the parasite species increases immediately in response to an increase in the host species population. However, as the parasite population increases, the host species population declines rapidly, leading to the eventual decline of the parasite population due to the lack of nutrition sources. Overall, the population dynamics exhibit a cyclic pattern.

Thirdly, in the case of the commensalism relationship, we observed that both host and parasite species exhibited a decreasing trend and eventually converged to a particular population level. Specifically, as the population of the host species decreased, the parasite species exhibited a corresponding stronger decreasing trend. This suggests that the commensalism relationship between the two species may have a stabilizing effect on the population dynamics, as both species can coexist and maintain a relatively steady population size.

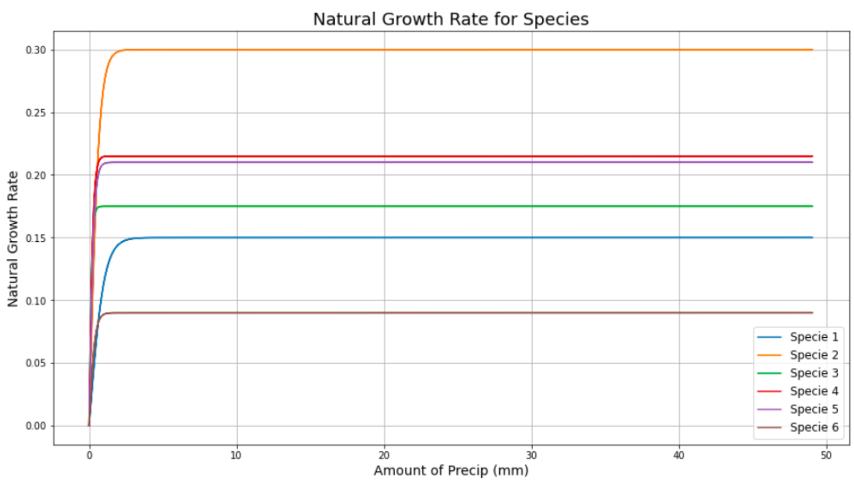

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Figure 9: The natural growth rate

Figure 9 shows that the natural growth rates r are pretty sensitive to the amount of precipitation when it increases a little from 0. However, as the precipitation increases, the growth rates tend to converge. This trend confirms our three systems of ODE performances above.

3.3 Combined effects of precipitation and the three aforementioned relationships

Now, we combine the previous two sub-sections, observing the performance of each relationship during the drought situations we generate.

For the competitive relationship, based on the original data model we fitted, as the number of precipitation increases, the natural growth rates increase. Hence, the populations of the two species rise as well. In the situation of drought in the wet season, we see that the peak populations decreased, and at the same time, the wet season is shorter. In both of situations drought with greater frequency and wider variation, the two species are impacted a lot which shows their abilities to adapt to drought is relatively weak.

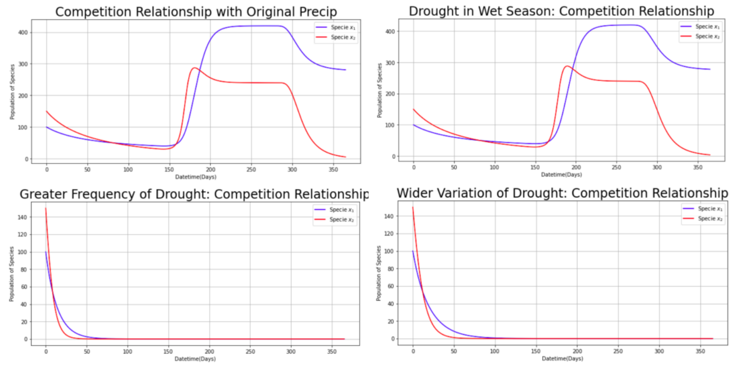


Figure 10: Two factors : Competition with Precipitation

For the parasitic relationship, comparing the performance of two species in a normal precipitation situation with the drought situation, we notice that the peak populations are lower when it is drought. When drought gets heavier, the populations of two species need to show good performance.

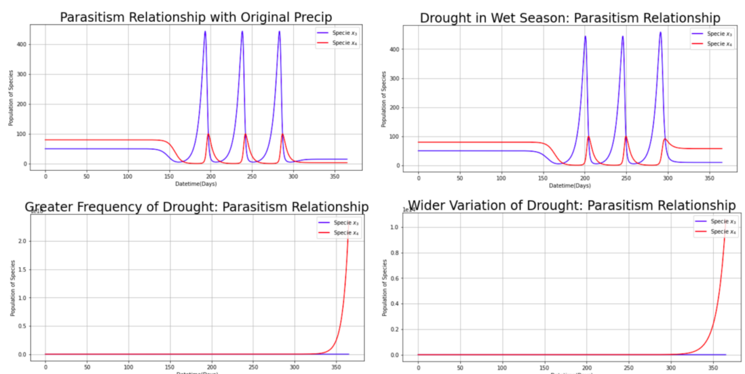


Figure 11: Two factors : Parasitism with Precipitation

For a commensal relationship, compare the first two situations; the populations of two species show a similar performance as the competitive relationship, which is that the peaks are lower and the wet seasons are shorter. For the latter two heavier situations, both species die immediately, showing a strong reaction to the decreasing precipitation. Additionally, when we zoom into the model's first month, we can see that the curves are sharper during the drought with greater frequency.

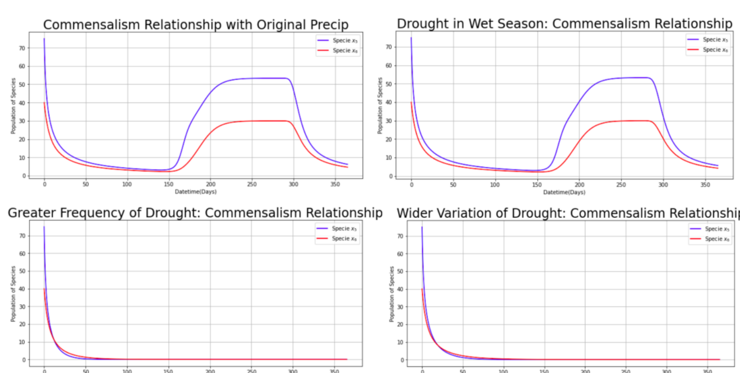


Figure 12: The affect in commensalism relationship

3.4 Competitive relationship with more species and performance based on precipitations

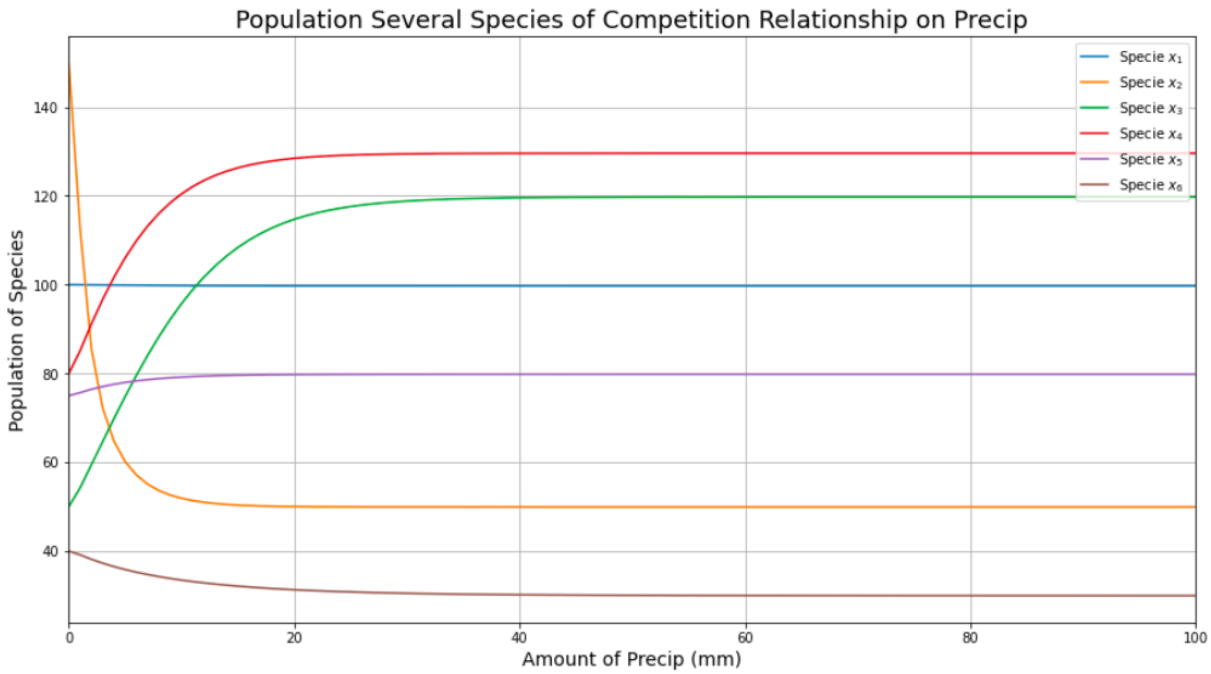


Figure 13: The competition relationship in a wider range of species

Here, it was only focused on the competitive relationship, the most common one in nature, to observe how the population of six species performed as the amount of precipitation. The model of the system of six ordinary differential equations is

$$\frac{dx_i}{dt} = r_i x_i (1 - \frac{x_i}{N_i} - \sum_{k=1}^{6} \alpha_k^i \frac{x_k}{N_k})$$

for $i = 1, 2, \ldots, 6$, where x_i are the species, r_i are the growth rates, N_i are the maximum growth capacity and α_k^i are competitive coefficients.

Based on figure 13, we observe that at the beginning of precipitation increments, two of six species grow, two stay flat, and two decrease. All of the six species converge to some certain population, which means that the plant community with six species is relatively stable.

4 Model 2: Factors Affecting Plant Communities in the Broader Environment

4.1 The Impact of Species Diversity on Plant Growth in Communities

4.1.1 Model Formulation

Since the majority relationship between species are the competition. The second model would only consider competitions between 20 typical species in this area; the sample space Θ would be separated into 4 categories: Arbor(θ_1), Shrub(θ_2), Vines(θ_3), Herb(θ_4), each of the group had a size 3, 5, 6, 6 respectively. By considering an upper limit of total number of all the species $N*$ in the whole space space Θ, one of the essential question could be considered is how many species needed to maximise the utility.

4.1.2 Maximizing Efficiency in Ecological Communities: Determining the Minimum Species Richness Required

The model could be formulated as an optimization question as follow:

Objective Function:

$$Max\, Z = a_1 \times \sum_{i=1}^{\theta_1} b_i x_i(T) + a_2 \times \sum_{i=\theta_1+1}^{\theta_1+\theta_2} b_i x_i(T) + a_3 \times \sum_{i=\theta_1+\theta_2+1}^{\theta_1+\theta_2+\theta_3} b_i x_i(T) + a_4 \times \sum_{i=\theta_1+\theta_2+\theta_3+1}^{\theta_1+\theta_2+\theta_3+\theta_4} b_i x_i(T) \tag{10}$$

$x_i(t)$ is satisfied by:

$$\begin{cases} for\; t = 0 \to T, \\ \frac{\mathrm{d}x_i(t)}{\mathrm{d}t} = (\frac{L_i}{1+e^{-k\times h(t)}} - \frac{L_i}{2}) x_i(t)(1 - \frac{x_i(t)}{N_i} - \sum \alpha_k^i \frac{x_k(t)}{N_k}), \\ x_i(0) = [x_1(0), x_2(0), \dots, x_{20}(0)] \end{cases} . \tag{11}$$

Subject to:

$$\begin{cases} \theta_1 + \theta_2 + \theta_3 + \theta_4 \leq \Theta, \\ b_i\ = [0,1]\; binary\; decision\; variable, \\ for\; i = 1,2,3...20, \\ 0 < \theta_i < \Theta, \\ r_i \leq\ L_i, \\ \sum_{i=1}^{\theta_1+\theta_2+\theta_3+\theta_4} b_i x_i(T) \leq\ N_* \\ x_i(t) \leq\ N_i\; for\; any\; t. \end{cases} . \tag{12}$$

4.1.3 Optimization Solution

Due to the lack of data, some fundamental constants are missing for some specific species x_i. Any missing constants would be generated from a relatively reasonable interval. The optimization model was tested 6 times with different constants, and the mean value of the above 6 experiments was accepted as an optimal solution in this case. The table below shows the optimal solution in each trial.

Optimized Solution

Experiment	Optimized θ1	Optimized θ2	Optimized θ3	Optimized θ4	Optimized Z
1	3	3	3	2	153.779
2	2	2	6	2	102.63
3	3	4	3	1	153.987
4	2	4	5	1	103.037
5	3	4	4	2	154.028
6	2	3	4	1	102.788

	Optimized Z
Max	153.779
Min	102.63
Mean	128.3748333

Figure 14: Optimization at original N^*

4.2 The Impact of Other Factors on Plant Communities

4.2.1 N^* decreased by 10%

By taking into consideration of other factors, one typical factor is pollution. Pollution would lead to damage to either air or soil resources. Thus, the total maximum carrying capacity N^* would decrease due to changes in essential resources such as air and soil. In this case, assuming there is a 10% decrease in N^*.The constraint for N^* would be reformulated as
$\sum_{i=1}^{\theta_1+\theta_2+\theta_3+\theta_4} b_i x_i(T) \leq 90\% N^*$.
Thus, the optimal solution would be expressed in the following table:

Optimized Solution

Experiment	Optimized θ1	Optimized θ2	Optimized θ3	Optimized θ4	Optimized Z
1	2	3	2	2	102.754
2	2	2	3	2	102.555
3	3	3	5	2	153.829
4	2	4	5	3	103.069
5	3	2	3	1	153.539
6	2	5	4	1	103.236

	Optimized Z
Max	153.539
Min	120.555
Mean	119.830

Figure 15: Optimization at 90% N*

4.2.2 N^* increased by 10%

From an alternative review, other factors may lead N^* to increase. In this case, wind and extreme temperature were two major factors to be considered. If wind and temperature are smooth enough rather than extreme: that could increase the survival rate for some species. Thus, a case of an increase in N^* by 10% would be considered. In this case, the constraint for N^* would be reformulated as
$\sum_{i=1}^{\theta_1+\theta_2+\theta_3+\theta_4} b_i x_i(T) \leq\ 110\%\ N_*$.
Thus, the optimal solution would be expressed in the following table:

Optimized Solution

Experiment	Optimized θ1	Optimized θ2	Optimized θ3	Optimized θ4	Optimized Z
1	3	5	5	4	153.302
2	3	5	4	2	153.245
3	2	5	4	2	102.245
4	3	4	5	1	153.2314
5	3	4	4	2	153.2224
6	3	4	6	2	153.2724

	Optimized Z
Max	153.302
Min	102.245
Mean	144.7530333

Figure 16: Optimization at 110% N*

4.2.3 Sensitivity Analysis: Model Comparison

The following comparison could be drawn by evaluating this model at different levels of N^*.

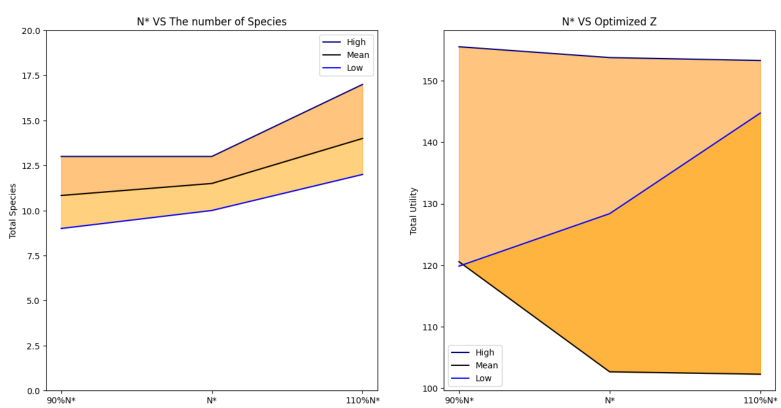


Figure 17: Model Comparison at differnet % N*

The graph on the left exposed the total number of species that would achieve the optimal solution as N^* increases. Furthermore, the graph on the right illustrates how objective value Z would change as N^* varies. As N^* increases, the mean value of Z would increase as expected. However, the highest values decreased slightly and asymptotically converged to an equilibrium. Also, the total number of species increases. In other words, bio-diversity increases as N^* increases.

5 Sensitivity Analysis and Model Promotion

5.1 Sensitivity Analysis

5.1.1 Sensitivity Analysis of the weather factor

Model 1 aims to identify the key parameters, namely Growth rate, Carrying Capacity, and Competition Coefficient, that influence the population of six plant species in East Sudan under varying drought scenarios. We aim to adjust the Carrying Capacity and Competition Coefficient within a specific range to test the accuracy of the results. Our analysis will focus on a combined model that integrates an exponential model and time series method to predict plant growth under different precipitation scenarios and a competition model that simulates interactions between the plant species.

To achieve this, we gather weather data from official websites and use the existing data to run simulations for 4 years, estimating the population of each species every year. Based on different scenarios, we compare the population of each species and the growth rate to identify the most impactful variables for population growth. Finally, we analyze the results to conclude the relationship between the identified parameters and the population dynamics of different plant species under different drought conditions.

5.1.2 Sensitivity Analysis of the type of species factor

Task 2 aimed to identify the minimum number of species required to optimize biomass production from 20 available species, with constant weather conditions. To assess the model's sensitivity to different species types that may impact plant growth, we conducted a sensitivity analysis by increasing or decreasing the total maximum carrying Capacity of all the species by 10%. By analyzing the corresponding changes in biomass production and minimum species number, we identified potential threshold values for carrying Capacity beyond which the model may no longer be effective in optimizing biomass production. These findings provide insights for making informed decisions about managing different species types to achieve optimal biomass production.

5.1.3 Sensitivity Analysis of the type of precipitation factor

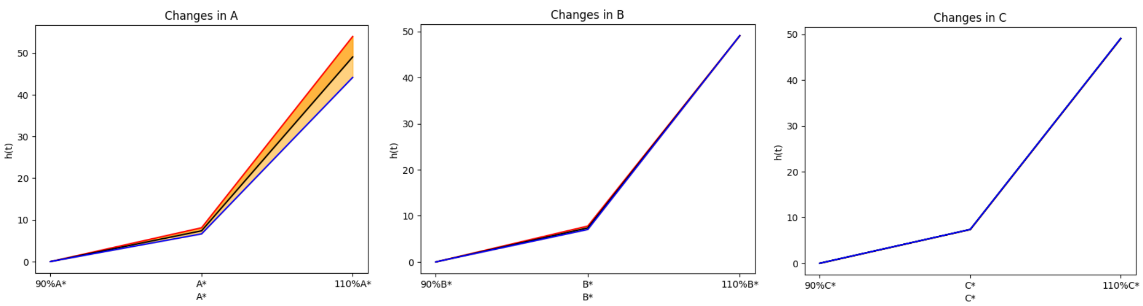
In the first part of Task 1, we fitted the model of precipitation exponentially with three main parameters A, B, C, which are the daily highest amount of precipitation, the wet season interval coefficient, and the day with the highest amount of precipitation. We analyzed the model's sensitivity to each parameter by varying their values by 10% and examining the resulting changes in the predicted precipitation.

Upon comparing the three plots, it becomes apparent that the parameter A, representing the maximum amount of precipitation, has the most significant impact on the model, followed by B, the wet season interval coefficient. On the other hand, C has a minor effect on the model.

The highest daily amount of precipitation, A, strongly depends on the location chosen. In a tropical rainforest climate, the peak amount is higher, and the values throughout the year are much greater than those in a subrigid tundra climate. Therefore, it is reasonable that A affect the model the most.

The wet season interval coefficient controls the general shape of our bell diagram of the amount of precipitation. It should affect the model sensitively, but since the original B value is relatively small, it is 10% floats that are much smaller.

Generally, for most areas, the day with the highest precipitation is during the wet or rainy season, which means the days near the peak day have very similar amounts of precipitation. Therefore, C affects the model the least.



(a) Changes in A (b) Changes in B (c) Changes in C

5.2 Model Imrpovement

- In Model 1, the constructed h(t) is an exponential function that is only suitable for predicting a one-year period. To extend the prediction period, it is possible to reconstruct the future as a periodic function with a period of 365 days. This approach may enable better simulations for longer periods, and improve the accuracy of the model's predictions.

- For model 2, it only considered 20 species with 4 groups. The more advanced formulation would have more complete $alpha_k^i$ and add more complicated and realistic constraints between the groups. Furthermore, by applying a more advanced optimization algorithm, time dimension T should consider as a variable rather than only simply taking the average value of $x_i(T)$ over 1 year period.

- Both models consider natural factors, such as weather and competition among species, but they do not account for the significant impact of human activities like deforestation and land use changes on plant communities in the real world. Incorporating the effect of human activities in future research can lead to a more accurate representation of the ecosystem and provide better predictions. Therefore, next time we can focus on including the impact of human activities in the model to make it more applicable to real-world scenarios.

6 Model Evaluation and Further Discussion

6.1 Strengths

- Constructed a competition model that simulates the interactions between different plant species, thereby enabling a more realistic simulation and examination of the impact of drought on competition dynamics.
- In model 1, the formulation not only considered the competition but also simulated the other essential relationship, such as parasitic and commensal.
- To indicate how the growth rate would be affected by rainfall, it formulated r_i by a logistic function rather than a constant, which may lead to a more realistic growth rate curve.
- The actual weather data and simulations were used for 4 years, which provides a more accurate representation of the plant population growth in the region.
- Conducted a sensitivity analysis to identify the most critical variables that affect population growth under different drought scenarios, which can help develop more effective optimization strategies.

6.2 Weaknesses

- This model is based on the situation in Sudan, and it may not apply to other drought-prone areas or countries with different environmental and climatic conditions. As a result, the findings of this model should not be generalized beyond the specific context of Sudan.
- This model only considers the impact of carrying capacity on biomass production; however, it does not consider other environmental factors such as pollution, climate change, or soil quality, which can also significantly affect plant growth.
- This model assumes that the weather conditions are held constant, which may not accurately reflect real-world conditions where weather cannot be variable and unpredictable.
- This model only considers a limited set of 20 available species and may not apply to other plant species or ecosystems with different characteristics.

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Appendix A: Program Codes

For the full code and the data set, please access via GitHub

`https://github.com/liuzs00/2023-MCM-A`