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BUST10134 Mathematical Programming in Advanced Analytics

Helping the Hebrides: A study into the potential efficiency gains of new regional distribution centres in the Scottish Highlands and Islands.

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Contents

1 Introduction

In the United Kingdom due to the increasing food price in 2022 (16.7%), 4.2 million people were suffered from food poverty and around 10% of people had to skip at least one meal per day [10]. Trussell Trust distribution centres recorded that there were 320 thousand more people having to turn to the Charity's distribution centres in the first half of this financial year and the charity provided more than 1.2 million emergency food parcels [18]. Additionally, about 89% of other food banks in the UK reported the demand for food parcels rose significantly. In Scotland 24.2% of households experience food insecurity issues, which makes it become the second worst region facing food poverty problems in UK [10](see Figure1). Considering the issues Scotland faces, this paper will aim to help food banks distribute more efficiently and equitably, which will in turn help more struggling people.

The problem of focus will consider whether the charity FareShare can increase the cost efficiency of distributing to Trussell Trust distribution centres located in the Highlands and Hebrides by building and operating new regional centres in Scotland. We consider The Trussell Trust as it currently operates 1,200 of the total 2,000 distribution centres in the UK and is the main supplier of food aid in the Highlands[20]. Unlike the Trussell Trust, FareShare is a food-aid charity including 18 organisations that operates as a middle man, distributing surplus food to front-line food-aid organisations - such as the Trussell Trust. We consider FareShare supply as the charity operates in every local authority in the UK and is the UK leader in charity collaboration [4] FareShare supported 10,542 charities in 2020 making it a major player in the UK fight against food poverty [4].

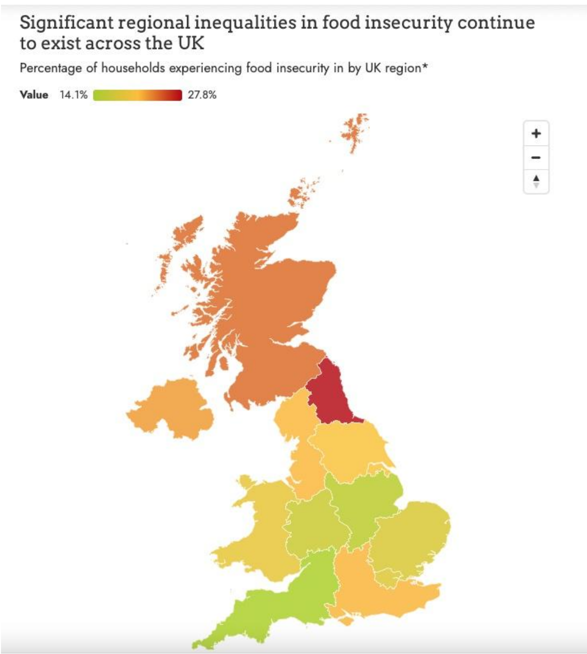

Figure 1: Regional Inequalities in Food Security. [10]

The Trussell Trust increased the number of parcels distributed by 17% in October-December 2021 compared to the same period in 2019, and increased parcels distributed by 22% from

January-February 2022 compared to the same period in 2020 [18]. In addition to this FareShare aims to double the amount of food surplus it delivered in the financial year 2021/2022 and has unrestricted reserves of £22.2 million that it intends to spend over the coming years in order to achieve this goal [4]. Given the trends and the goals of both the Trussell Trust and FareShare to ensure an end to food poverty, both companies must consider how to best allocate resources in order to maximise efficiency and equability. By considering specifically the largest organisations, any efficiency or equability gains our research may uncover will have the largest effect on aiding the fight against food poverty.

As both charities have limited resources, the cost efficiency of the distribution network is a key variable when trying to deliver food packages. Any inefficient expenditure for the charities has an opportunity cost of not helping vulnerable individuals. This problem presents a great opportunity for operational research to help decision makers assess how to efficiently allocate resources. In the problem presented mathematical programming will be used to determine the optimal number of additional FareShare centres, where they should be optimally located, and how food parcels should be distributed from them in order to minimise the costs of providing food-aid to the Hebrides and Highlands.

The motivation behind solving the cost efficiency problem for this specific region arises as, the Trussell Trust currently has 4 locations in the Highland region - which has a population of around 107 thousand - and no locations in the Hebrides [16] [11] - which has a population of around 28 thousand[1] [11]. However, despite there being only four current locations supported in this area, the historical data shows that there are 13 food banks that still operate and have previously been supported in the region, which demonstrates suggests there are unmet demands in these locations [17]. Therefore the model devised will find the most cost efficient way to ensure that food-aid is delivered to all of the 13 food-banks present in this region. Solving this problem will improve the current inadequate supply of food parcels and improve the cost efficiency of delivering to the pre-existing distribution centres.

Eskandarpour et al. argues that comparatively to economic issues, social issues are harder to quantify and utilise in mathematical models[3]. Despite this, there is a growing literature on how mathematical models can be used to solve the issue of food-bank supply chain networks [9]. The problem considered in this paper closely parallels that of Marins et al. which considers the problem of optimising a food bank distribution network in Portugal, and Reihaneh and Ghoniem, which considers the problem of routing-allocation problem with multi-start optimisation method in the UK [14]. The model that Reihaneh and Ghoniem have constructed was similar, to some extent, to the one that is presented in this paper since both aim to find the optimal location allocation while minimising costs [14]. The problems devised in this paper, however, will not follow the same model design as the above two. Instead the model proposed in this paper will focus on a one period decision problem concentrating on one product and specifically the interaction between regional centres and distribution centres in Scotland.

The structure of this paper will be as followed. First the problem description will aim to clarify the objective of this paper and will specify the relevant assumptions and limitations of the model. The Mathematical Formulation chapter will then formalise the model by introducing the decision variables, parameters and, objective function subject to given constraints. The model is then refined throughout this section, each rendition being solved with input data discussed in Appendix 1 using data from Trussell Trust and FareShare [17] [4]. The optimal solution to the Mathematical problems will be solved by using the GAMS programming language. The results and conclusions throughout are based on the output from computing the models. The conclusion will then aim to present any findings and recommendations on how the distribution network can be made more efficient.

2 Problem Description

2.1 Problem Description and Assumptions

The decision problem will consider the following 13 locations of Trussell Trust distribution centres that have existed in the Highland and Hebrides region: Broadford, Portree, Fort William, Oban, Tiree, Barra, Benbecula, Dornoch, Invergordon, Beauly, Inverness West, Inverness East, Nairn, and Elgin.

In addition to this, there are five existing FareShare regional centres running in the following locations: Glasgow, Edinburgh, Dundee, Aberdeen, and Alness (see Figure 3). These regional centres in our model will be responsible for supplying the food parcels to the Trussell Trust distribution centres.

We then plan to investigate how opening additional FareShare centres may impact the total operation cost. New FareShare centres in could be opened in any of the the following 19 proposed locations: Inverness, Portree, Mallaig, Fort William, Ullapool, Fort Augustus Aviemore, Nairn, Dornoch, Wick, Thurso, Culloden, Strone, Lairg, Tain, Cromarty, Dingwall, Black Isle, North Uist (see figure 3). Each proposed location represents the largest town in each of the Scottish Highland Wards and North Uist in order to represent Na h-Eileanan Siar. [23]. The number of new regional centres that can be open is only limited by the number of locations (19). However each center would have an associated running cost [4].

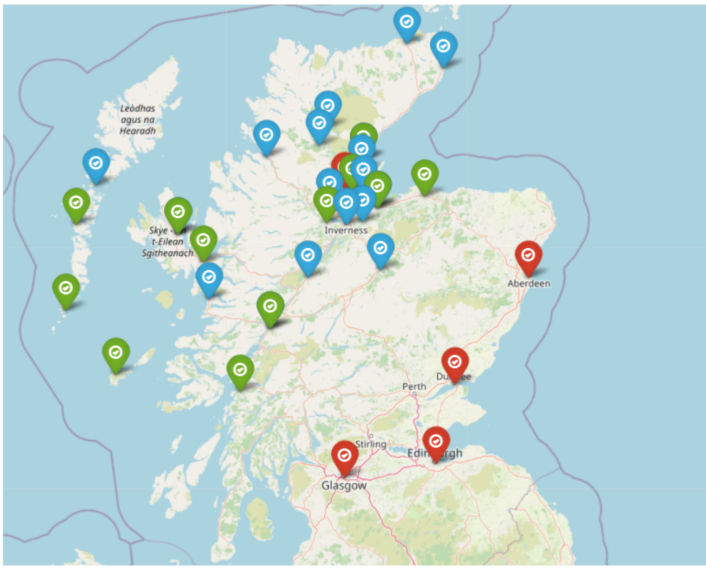

Figure 2: Food-banks and Distribution Centre Locations. [5] [17]

Figure 3: Possible New Locations of FareShare centres, Current Locations of FareShare centres, Existing Trussell Trust centres.[5] [17]

These Trussell Trust distribution centre and FareShare regional centre locations can be seen in Figure 1. Due to the principle aim of both charities - to end food poverty - we assume in both models that no Trussell Trust distribution centre can be closed down, as this would mean that people are unable to receive the food-packages they require.In addition both models will assume that no current FareShare regional centre can be closed down. This assumption is made as these regional centres do not exclusively serve the distribution centres that are considered in this model, and so closing any existing regional centre may have negative repercussions to distribution centres run by other charities, and in other regions in the United Kingdom that are external to the model. The demand for food parcels in both will be calculated from an maximum since 2017 of the total number of parcels distributed per year of the demands for each of the Trussell Trust centres, to help prepare for a worst case scenario. The demand at individual centres is then calculated by splitting the total demand by population[17] [11].

The remoteness of the centres means that the journeys made to supply these centres are not ones usually made in FareShare's operations. Therefore the cost of each journey must be considered carefully, including not only the distance between centres in calculations, but also cost of ferry crossing and vehicle hire to complete the journeys. We consider this in our models by first considering a simple cost, based only on distance. We then include a start up costs that are added for journeys proportional to their length and also incorporate ferry costs and vehicle hire costs.

Should demand not be met a penalty, equating to the cost of sourcing the food parcel, will be imposed. However, as one of the primary goals of this paper is improving both efficiency and equity to more remote centres in Scotland, the distribution of this unmet demand should be controlled and mitigated.

2.2 Problem Limitations

The first limitation of the proposed model is that each vehicle is only assumed to travel from one distribution centre directly to another distribution center. Therefore, it is not able to stop off at any distribution centres. Thus, a more complex and efficient transportation plan could be considered, with the delivery vehicle traveling via multiple centres, reducing the individual start up costs. Furthermore, the model is only based on a a singular time period for the food parcel transportation (1 year). A more advanced model could consider subdividing this into delivery periods e.g. monthly. This would allow for donations made throughout the year to be included in deliveries, and would account for perishable goods. A final limitation is that we assume that there is a single start up cost for sending a vehicle along a route, so the depreciation charge is not considered tentatively. In reality once a vehicle has its capacity reached a new additional vehicle would be needed. This could be implemented with a discontinuous piece-wise linear function for cost, instead of the current split into a start up cost and and additional per unit cost.

3 Mathematical Formulation and Results

In this section we define the mixed integer problems that our paper investigates.

3.1 Simple costs model (M_1):

We start by investigating the problem using a simple network configuration problem, combining a transportation problem with a facility location problem.

3.1.1 Input Parameters(M_1):

We introduce the following input parameters into our model.

$i \in \{1 \dots 24\}$: Index of FareShare centres
$k, k' \in \{1 \dots 13\}$: Index of Trussell Trust centres
a_i: Supply of parcels at FareShare centre i
d_k: Demand of parcels at Trussell Trust centre k
ϵ: Allowed variation in the proportion of unmet demand
α: Minimum service level of Trussell Trust centres
δ_k: Minimum requirement of parcels at centre k
c_{ik}: Cost per unit of shipping from centre i to centre k
p_k: Penalty per unit for not delivering to Trussell Trust centre k
R_i: Running cost of FareShare centre i

3.1.2 Decision Variables (M_1):

We explore the following decision variables in our model.

x_{ik}: Number of units shipped from centre i to centre k
y_i: Auxiliary binary variable taking value 1 if centre i is built and 0 otherwise
u_k: Auxiliary variable modelling unmet demand at centre k

3.1.3 Model and Constraints (M_1):

Our goal is to minimize the objective function of total cost. This is given by,

$$\text{Total cost } = \sum_i y_i R_i + \sum_k \sum_i x_{ik} c_{ik} + \sum_k u_k p_k.$$

That is the sum of the total running costs of open FareShare centres, total cost of transportation, and the sum of penalties at Trussell Trust centres.

We must ensure that if a FareShare centre i is built, the total number of units sent out of this centre does not exceed the supply at this centre. We must also ensure that the total number of units sent out of a centre that does not exist should be 0. This gives us constraint;

$$\sum_k x_{ik} \leq a_i y_i \quad \forall i.$$

Similarly, for demand at Trussell Trust centre k, the total number of units arriving should not exceed the demand, as these parcels would then go to waste. Including the variable for unmet demand at centre k we get;

$$\sum_i x_{ik} + u_k = d_k \quad \forall k.$$

This means that $x_{ik} \geq 0$ forces y_i to be 1, and adds the associated running cost of center i to our total cost.

As a large part of the goal of this paper is to bring equity to remote centres, we introduce two constraints to ensure equity between the Trussell Trust centres, the first ensures a minimum service level to the Trussell Trust centres. We select constant $\alpha \in [0, 1]$ representing the minimum proportion of the demand at any Trussell Trust centres that must be fulfilled. Setting $\delta_k = \alpha d_k$ We achieve this with constraint;

$$\sum_i x_{ik} \geq \delta_k \quad \forall k.$$

Whilst this constraints allows us to make sure that each centre is delivered to, it does not currently let us control by how much this minimum is exceeded. For example if α was set to 0.5, it could be that remote centres receive the bare minimum of 50 of their demand, whilst more accessible centres have their full demand fulfilled. To mitigate this we introduce a maximum variation in the proportion of unmet demand, ϵ. We achieve this by adding a new equity constraint given by;

$$\left| \frac{u_k}{d_k} - \frac{u_{k'}}{d_{k'}} \right| \leq \epsilon, \quad \forall k, k'.$$

Here $0 \leq \epsilon \leq 1$ will be an additional input parameter determining the maximum allowed deviation in unfulfilled demand as a proportion of total demand. In our case as we set

$\delta_k = \alpha \cdot d_k$, ϵ should take value between 0 and $1 - \alpha$. An investigation into the choice of values of α and ϵ, and a discussion of the trade off between these two constraints is found in section 3.4.

We note that the previous constraint is not linear due to the presence of the absolute value. However as the constraint is true for all values of k and k', the absolute value can be dropped. This is as for any X and Y we have $X - Y \leq \epsilon$ and $Y - X \leq \epsilon$ if and only if $|X - Y| \leq \epsilon$. Therefore it is sufficient to have the linear constraint;

$$\frac{u_k}{d_k} - \frac{u_{k'}}{d_{k'}} \leq \epsilon, \quad \forall k, k'.$$

It is impossible to have a fractional or negative number of a food parcel delivered to a centre, therefore it is necessary to set x_{ik} to take non negative integer value. Also to ensure that the demand at a centre is not exceeded, we note that u_k must also take non negative value. This gives the constraint;

$$x_{ik}, u_k \in \mathbb{Z}^+ \quad \forall i, k.$$

For our Supply constraint to work we must also include the binary value of y_i with constraint;

$$y_i \in \{0, 1\} \quad \forall i.$$

Finally as the FareShare Centres at the first 5 locations already exist, and cannot be closed as per the previous discussion, we pre-process the value of y_i for $i \in \{1, ..., 5\}$ as follows;

$$y_i = 1, \quad \forall i \in \{1, ..., 5\}.$$

Combining this gives us model M_1 to be;

$$\text{Minimize} \sum_i y_i R_i + \sum_k \sum_i x_{ik} c_{ik} + \sum_k u_k p_k$$

$$\begin{aligned}
\text{Subject to} \sum_k x_{ik} &\leq a_i y_i \quad \forall i && \text{(Supply Constraints) (1)} \\
\sum_i x_{ik} + u_k &= d_k \quad \forall k && \text{(Total Demand Constraints) (2)} \\
\sum_i (x_{ik}) &\geq \delta_k \quad \forall k && \text{(Minimum Service Equity Constraints) (3)} \\
\frac{u_k}{d_k} - \frac{u_{k'}}{d_{k'}} &\leq \epsilon, \quad \forall k, k' && \text{(Maximum Variation Equity Constraints) (4)} \\
x_{ik}, u_k &\in \mathbb{Z}^+ \quad \forall i, k && \text{(Non-negative Integer Variable Constraint) (5)} \\
y_i &\in \{0, 1\} \quad \forall i && \text{(Binary Variable Constraint) (6)} \\
y_i &= 1, \quad \forall i \in \{1, ..., 5\} && \text{(Pre-Processing) (7)}
\end{aligned}$$

3.1.4 Results of Model M_1.

Figure 4 shows the transportation flow in the optimal solution using input data discussed in Appendix 1.

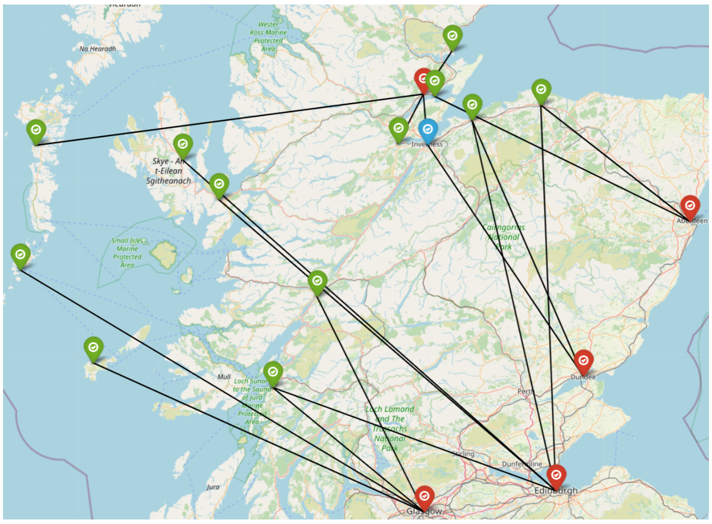

Figure 4: Transportation flow for simple cost model M_1

The above solution involves opening a new FareShare Regional centre in Inverness, as the demand at the Trussell Trust centre found here is highest.

We observe that, as suspected, that due to the remoteness of the Trussell Trust centres, simple costs do not effectively model the cost of transportation. The total cost of sending even a small shipment of parcels from for example from Glasgow to the island of Barra would still have to include; the hire of a vehicle to complete the transportation, the fuel cost of sending this vehicle (even if it is near empty) to the destination and, in the case of transportation to these remote islands by ferry, the cost of a return ferry ticket in order for the vehicle to cross onto the island then return to the mainland. This cannot be achieved using simple costs without introducing further constraints, as a per unit transportation price could lead to a very small number of parcels being sent which does not cover the cost of this vehicle hire and ferry transit.

Furthermore, we observe that in the case of Oban, despite having a relatively small demand of parcels, we deliver from both Edinburgh and Glasgow. This would involve hiring two vehicles and covering both their fuel cost. It would be preferable to reduce the total number of vehicles needed to complete the transportation, by reducing the total number of distinct journeys in our network. This can be modelled by implementing a start up cost for journeys, equal to the cost of sending an empty vehicle along this route including ferry transport where necessary. The per unit cost of transportation would then be reduced to cover only the additional fuel used by the vehicle during the transportation due to the weight of a food parcel. We implement this in Model M_2.

3.2 Journey Start Up Cost Model (M_2):

3.2.1 Input Parameters(M_2):

We introduce the following new input parameters into our model, in addition to those in M_1.

C_{ik}: Cost of sending an empty vehicle from FareShare centre i to Trussell Trust centre k
c_{ik}: Updated cost per unit of shipping from FareShare centre i to Trussell Trust centre k representing only the additional cost of fuel

3.2.2 Decision Variables (M_2):

We add the following decision variables to our existing model.

b_{ik}: Auxiliary binary variable for start up costs of using delivery route from i to k

3.2.3 Model and Constraints (M_2):

In our objective function we must now additionally include the sum of all the start up cost of the journeys in the transportation. Our updated objective function is given by;

$$\text{Total cost } = \sum_i y_i R_i + \sum_k \sum_i (x_{ik}c_{ik} + b_{ik}C_{ik}) + \sum_k u_k p_k.$$

To implement the journey start up costs we must introduce the following constraints;

$$x_{ik} \leq a_i b_{ik} \quad \forall i, k.$$

This implies that a journey being used ($x_{ik} > 0$) forces $b_{ik} = 1$ and adds the journey start up cost to the objective function. If a journey is used then the maximum value a_i is sufficient, as by the supply constraint we know that $x_{ik} \leq a_i$. Thus if $b_{ik} = 1$, the constraint does not restrict the value of x_{ik}.

For this to work we must add the constraint that b_{ik} as a binary variable using the following constraint;

$$b_{ik} \in \{0, 1\} \quad \forall i, k.$$

Combining this with model M_1 gives us model M_2 to be;

$$\text{Minimize} \sum_i y_i R_i + \sum_k \sum_i (x_{ik} c_{ik} + b_{ik} C_{ik}) + \sum_k u_k p_k$$

$$\text{Subject to} \sum_k x_{ik} \leq a_i y_i \quad \forall i \qquad \text{(Supply Constraints) (1)}$$

$$\sum_i x_{ik} + u_k = d_k \quad \forall k \qquad \text{(Total Demand Constraints) (2)}$$

$$x_{ik} \leq a_i b_{ik} \quad \forall i, k \qquad \text{(Journey Start Up Constraints) (3)}$$

$$\sum_i (x_{ik}) \geq \delta_k \quad \forall k \qquad \text{(Minimum Service Equity Constraints) (4)}$$

$$\frac{u_k}{d_k} - \frac{u_{k'}}{d_{k'}} \leq \epsilon, \quad \forall k, k' \qquad \text{(Maximum Variation Equity Constraints) (5)}$$

$$x_{ik}, u_k \in \mathbb{Z}^+ \quad \forall i, k \qquad \text{(Non-negative Integer Variable Constraint) (6)}$$

$$y_i, b_{ik} \in \{0, 1\} \quad \forall i, k \qquad \text{(Binary Variable Constraint) (7)}$$

$$y_i = 1, \quad \forall i \in \{1, ..., 5\} \qquad \text{(Pre-Processing) (8)}$$

3.2.4 Results of Model M_2.

Figure 5 below shows the transportation flow in the optimal solution using input data discussed in Appendix 1.

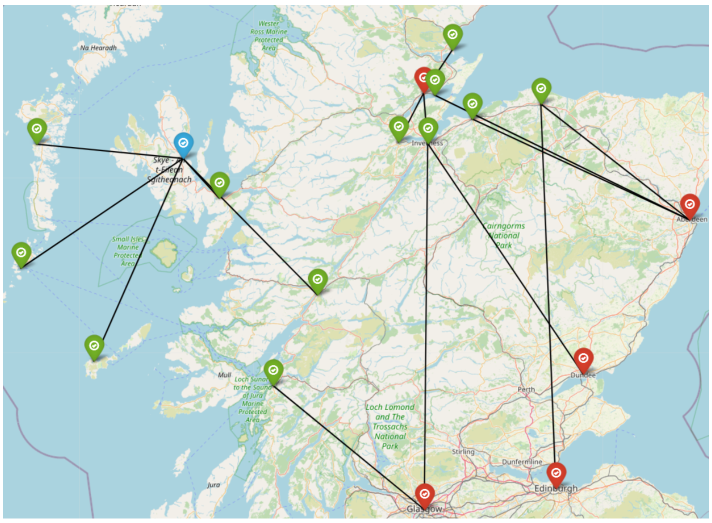

Figure 5: Transportation flow for model M_2

The above solution involves opening a new FareShare Regional centre in Portree, which supplies the Trussell Trust centres on the west coast and in the Hebrides. The total running cost when this solution is implemented is £285,970. This is a reduction from £315,895 which is the total running cost in the case we do not open the new centre at Portree. Not only this but the solution remains feasible for values of $\alpha = 0.92$ as further discussed in section 3.4, where as the value of £315,895 was found when the equity constraints are relaxed and some of the more remote centres had none of their demand fulfilled.

3.3 Predistribution Model M_3

We now investigate the potential improvements in efficiency when first allowing FareShare Centres to transport amongst one and other, redistributing the initial supply, before transporting to Trussell Trust Centres. The motivation to allow for such predistribution is that if two nearby FareShare centres both supply the same distant Trussell Trust centre, it makes sense to pay a start up cost for one short journey and one long journey. This is instead of paying start up cost for two long journeys . As the start up costs are journey length dependent, by including pre distribution we haven a lower transportation cost. Despite the Food parcels traveling a further distance, the total distance travelled by vehicles is shorter.

In the solution of M_2 we see that a potential location for such predistribution would be between FareShare centres in Glasgow and Dundee, as both of these supply Inverness. In fact we see that this predistriburtion decreases the total costs of transportation slightly, as seen in the results of model M_3.

3.3.1 Input Parameters(M_3):

We introduce the following new input parameters into our model, in addition to those in M_2, to model the costs predistribution.

$i, i' \in \{1 \dots 24\}$: Index of FareShare centres

$\tilde{C}_{ii'}$: Cost of sending an empty vehicle from FareShare centre i to FareShare centre i'

$\tilde{c}_{ii'}$: Additional fuel cost per unit of shipping from FareShare centre i to FareShare centre i'

3.3.2 Decision Variables (M_3):

We add the following decision variables to our existing model, to model the predistribution.

$\tilde{x}_{ii'}$: Number of units shipped from FareShare centre i to FareShare centre i'

$\tilde{b}_{ii'}$: Auxiliary binary variable for sending a vehicle using the delivery route from i to i'

3.3.3 Model and Constraints (M_3):

We must firstly add the total cost of predistribution to our objective function,

$$\text{Total cost } = \sum_{i'} \sum_{i} \left(\tilde{x}_{ii'} \tilde{c}_{ii'} + \tilde{b}_{ii'} \tilde{C}_{ii'} \right) + \sum_{k} \sum_{i} \left(x_{ik} c_{ik} + b_{ik} C_{ik} \right) + \sum_{k} u_k p_k + \sum_{i} y_i R_i.$$

We then add a supply constraint such that the total predistribution from a centre i to other centres i' is limited by the supply at centre i. This is achieved by constraint;

$$\sum_{i'} \tilde{x}_{ii'} \leq a_i y_i \quad \forall i.$$

Next we enforce that we only predistribute to centres that have been built. That is the total predistribution to a centre i is 0 if the centre is not built, but no additional restrictions are enforced if the centre is built. This is achieved by constraint;

$$\sum_{i'} \tilde{x}_{i'i} \leq M y_i \quad \forall i.$$

Here M must be sufficiently large as to not add any restrictions when y_i takes value 1. It is sufficient to use;

$$M = \sum_{i} a_i,$$

as this is the largest possible number of parcels that could be in the network (i.e if every potential centre was built).

Next we add an analogous start up cost constraint for $\tilde{b}_{ii'}$ given by;

$$\tilde{x}_{ii'} \leq M \tilde{b}_{ii'} \quad \forall i, i'.$$

Then we must amend our supply constraint from our previous models, enforcing that the total number of parcels that leaves a centre (including predistribution) is less than the total that arrives at a centre combined with the existing supply. This is achieved by the following constraint;

$$\sum_{k} x_{ik} + \sum_{i'} \tilde{x}_{ii'} \leq a_i y_i + \sum_{i'} \tilde{x}_{i'i} \quad \forall i.$$

Analogously to previous models we enforce $\tilde{b}_{ii'}$ to take binary value, and $\tilde{x}_{ii'}$ to take non negative integer value.

Finally to stop centres predistributing to themselves we add a preprocessing constraint;

$$\tilde{x}_{ii} = 0, \quad \forall i.$$

Combining this with what remains of M_2 gives us model M_3 to be;

$$
\begin{aligned}
\text{Minimize} \quad & \sum_{i'} \sum_{i} \left(\tilde{x}_{ii'} \tilde{c}_{ii'} + \tilde{b}_{ii'} \tilde{C}_{ii'} \right) + \sum_{k} \sum_{i} \left(x_{ik} c_{ik} + b_{ik} C_{ik} \right) + \sum_{k} u_k p_k + \sum_{i} y_i R_i \\
\text{Subject to} \quad & \sum_{i'} \tilde{x}_{ii'} \leq a_i y_i \quad \forall i && \text{(Predistribution Origin Constraints) (1)} \\
& \sum_{i'} \tilde{x}_{i'i} \leq M y_i \quad \forall i && \text{(Predistribution Destination Constraints) (2)} \\
& \tilde{x}_{ii'} \leq M \tilde{b}_{ii'} \quad \forall i, i' && \text{(Predistribution Journey Start Up Cost Constraints) (3)} \\
& \sum_{k} x_{ik} + \sum_{i'} \tilde{x}_{ii'} \leq a_i y_i + \sum_{i'} \tilde{x}_{i'i} \quad \forall i && \text{(Supply Constraints) (4)} \\
& \sum_{i} (x_{ik}) + u_k = d_k \quad \forall k && \text{(Total Demand Constraints) (5)} \\
& x_{ik} \leq M b_{ik} \quad \forall i, k && \text{(Journey Start Up Cost Constraints) (6)} \\
& \sum_{i} (x_{ik}) \geq \delta_k \quad \forall k && \text{(Minimum Service Equity Constraints) (7)} \\
& \frac{u_k}{d_k} - \frac{u_{k'}}{d_{k'}} \leq \epsilon, \quad \forall k, k' && \text{(Equity Constraints) (8)} \\
& \tilde{x}_{ii'}, x_{ik}, u_k \in \mathbb{Z}^+ \quad \forall i, i', k && \text{(Non-negative Integer Constraint) (9)} \\
& \tilde{b}_{ii'}, b_{jk}, y_i \in \{0, 1\} \quad \forall i, i', k && \text{(Binary Variable Constraint) (10)} \\
& y_i = 1, \quad \forall i \in \{1, ..., 5\} && \text{(Pre-Processing) (11)} \\
& \tilde{x}_{ii} = 0, \quad \forall i && \text{(Pre-Processing) (12)}
\end{aligned}
$$

3.3.4 Results of Model M_3.

The figure below shows the transportation flow in the optimal solution using input data discussed in Appendix 1.

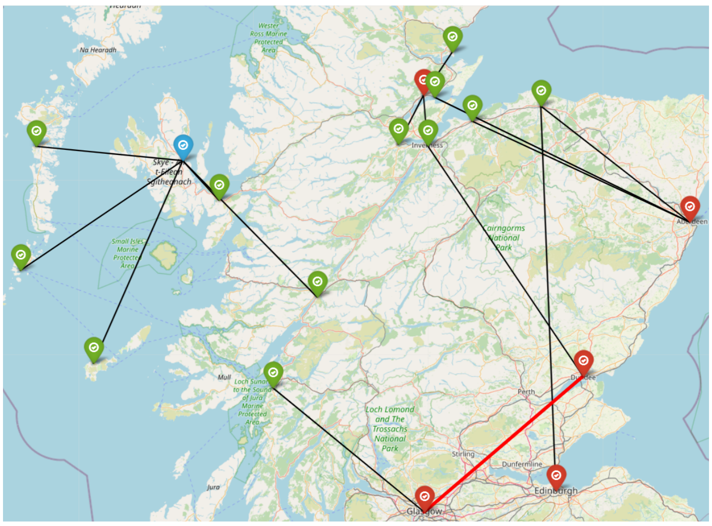

Figure 6: Transportation flow for model M_3

We note that allowing for predistribution results in the anticipated predistribution from Glasgow to Dundee. This reduces the total running cost to £285,947. Although in this case the reduction in cost is almost negligible, we suspect that in a larger problem with a greater number of high demand centres (e.g. FareShares UK wide operation), the savings found from introducing predistribution would be add up, and a larger number of such predistributions would be utilised. In fact when exploring our scenario based Stochastic Model (M_4), in one of our scenarios we see four separate predistributions even in our relatively small network.

3.4 Equity discussion and Trade off between constraints.

In our models we have two equity constraints, these are the constraints for Minimum Service Level and Maximum Variation in Unmet Demand. We have chosen to include both in our model as they achieve different things, and including just one of the two does would not yield the same effect.

As we vary the values of α and ϵ in model M_3 we find the following results for the objective function.

Table 1: Table Caption

	1	0.8	0.6	0.4	0.2	0.1	0
0	£285,947	£285,947	£285,947	£285,947	£285,947	£285,947	£313,191
0.2		£285,947	£285,947	£285,947	£285,947	£285,947	£313,191
0.4			£285,947	£285,947	£285,947	£285,947	£313,191
0.6				£285,947	£285,947	£285,947	£313,191
0.8					£285,947	£285,947	£313,191
1							£313,206

We notice 3 distinct solutions. For values of α up to 0.935 and ϵ down to 0.065 we observe that the solution presented in figure 7, with objective value £285,947 is feasible and optimal. By having set up costs for journeys, once a journey does happen it usually fulfills as much of the demand as possible for that centre as the additional cost of taking more parcels is relatively small. Thus, in order to avoid the large penalties for not delivering to a centre at all, solutions immediately fulfill most of the demand, even without the equity constraints.

It is necessary to note that, when the equity constraints are pushed to their limits and we force no variation in proportional unmet demand, or force no unmet demand at all, we get two different solutions.

If we only have the Max Variation constraint and set $\epsilon = 0$, but do not have any constraint on the minimum service level, the optimal solution involves opening the new regional centres in Portree, but also open a new regional centre on the black isle.

However once we require no unmet demand by setting $\alpha = 1$, we instead open the second new regional centre in Cromarty. The distribution flow for this solution is illustrated in figure 7.

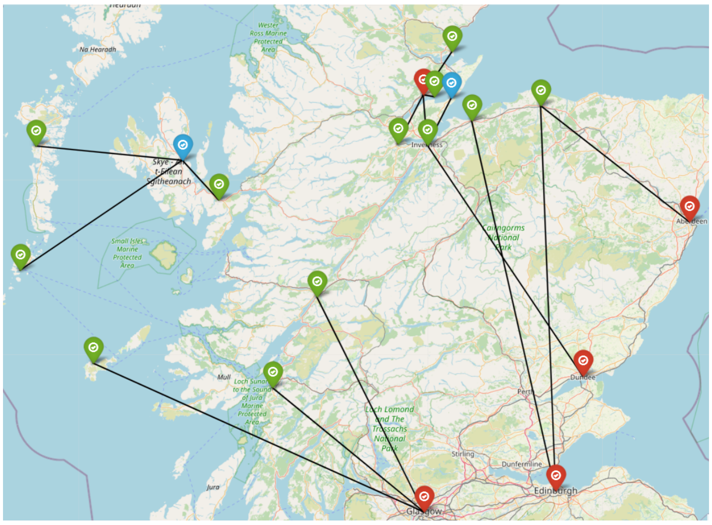

Figure 7: Transportation flow with new regional centre in Cromarty

That stated in both of the cases where we have enforced the extreme equity constraints, and open two centers we see a dramatic increase in the total running cost, so we therefore suggest that unless FareShare decides it crucial to enforce perfect equity or no unmet demand, then it is sufficient to just open the new center in Portree.

3.5 Stochastic model (M_4):

We now wish to investigate the stochastic nature of the problem, and investigate a scenario based stochastic model, rather than the deterministic model M_3.

We split our problem into two stages. Firstly our model selects the locations of the new FareShare centres, should any have to be built. Then the model evaluates the expected total transportation cost based on the probability of given scenarios

This first stochastic model we investigated is based on historical total demand from the last 4 years of Trussell Trust annual report data. [18]. This however gave a trivial solution. Varying the total demand simply scales all our demands in the individual centres, so either the centre at Portree is built, or no new centres are built. This is because the proportional distribution (that is the ratios of demands of Trussell Trust centres remains constant). We get a similar result when varying fuel prices for scenarios as we once again don't vary the proportional distribution.

Secondly we decided to investigate increasing the demands at individual centres one by one, thus changing the proportional distribution. This caused the first stage decisions to vary more drastically in the individual deterministic Wait and See problems. We call this a "stress test" where we individually stress the demand at each centre. We present these findings below using model M_4.

3.5.1 Input Parameters (M_4):

Based on the final deterministic model (M_3), we add the following new input parameters into our stochastic model:

$s \in \{1 \dots 4\}$: Index of four-year real data scenarios
π_s: Probability of scenario s happens
d_{ks}: Demand of parcels at Trussell Trust Centre k in scenario s
δ_{ks}: Minimum requirement of parcels at Trussell Trust centre k in scenario s

Here, all the probability parameters π_s are equal to

$$\frac{1}{\text{Total number of scenarios}}.$$

3.5.2 Decision Variables (M_4):

We must duplicate all decision variables associated to transportation once for each scenario. This is done as follows.

$\tilde{x}_{ii's}$: Number of units shipped from FareShare centre i to FareShare centre i' in scenario s
x_{iks}: Number of units then shipped from FareShare centre i to Trussell Trust centre k in scenario s
$\tilde{b}_{ii's}$: Auxiliary binary variable for sending a vehicle using the delivery route from i to i' in scenario s
b_{iks}: Auxiliary binary variable for sending a vehicle using the delivery route from i to k in scenario s
u_{ks}: Auxiliary variable modelling unmet demand at Trussell Trust centre k in scenario s

3.5.3 Stochastic Model of Demand Scenarios (M_4)

Once these variables have been added our problem becomes;

$$
\begin{aligned}
\text{Minimize } & \sum_i y_i R_i + \sum_s \left(\pi_s \left(\sum_{i'} \sum_i \left(\tilde{x}_{ii's} \tilde{c}_{ii'} + \tilde{b}_{ii's} \tilde{C}_{ii'} \right) + \sum_k \sum_i (x_{iks} c_{ik} + b_{iks} C_{ik}) \right) \right) \\
& + \sum_s \left(\pi_s \left(\sum_k u_{ks} p_k \right) \right) \\
\text{Subject to } & \sum_{i'} \tilde{x}_{ii's} \leq a_i y_i \quad \forall i, s && \text{(Predistribution Origin Constraints) (1)} \\
& \sum_{i'} \tilde{x}_{i'is} \leq M y_i \quad \forall i, s && \text{(Predistribution Destination Constraints) (2)} \\
& \tilde{x}_{ii's} \leq M \tilde{b}_{ii's} \quad \forall i, i', s && \text{(Predistribution Journey Start Up Cost Constraints) (3)} \\
& \sum_k x_{iks} + \sum_{i'} \tilde{x}_{ii's} \leq a_i y_i + \sum_{i'} \tilde{x}_{i'is} \quad \forall i, s && \text{(Supply Constraints) (4)} \\
& \sum_i (x_{iks}) + u_{ks} = d_{ks} \quad \forall k, s && \text{(Total Demand Constraints) (5)} \\
& x_{iks} \leq M b_{iks} \quad \forall i, k, s && \text{(Journey Start Up Cost Constraints) (6)} \\
& \sum_i (x_{iks}) \geq \delta_{ks} \quad \forall k, s && \text{(Minimum Service Equity Constraints) (7)} \\
& \frac{u_{ks}}{d_{ks}} - \frac{u_{k's}}{d_{k's}} \leq \epsilon, \quad \forall k, k', s && \text{(Equity Constraints) (8)} \\
& \tilde{x}_{i'is}, x_{iks}, u_{ks} \in \mathbb{Z}^+ \quad \forall i, i', k, s && \text{(Non-negative Integer Variable Constraint) (9)} \\
& \tilde{b}_{ii's}, b_{iks}, y_i \in \{0, 1\} \quad \forall i, i', k, s && \text{(Binary Variable Constraint) (10)} \\
& y_i = 1, \quad \forall i \in \{1, ..., 5\} && \text{(Pre-Processing) (11)} \\
& \tilde{x}_{iis} = 0, \quad \forall i, s && \text{(Pre-Processing) (12)}
\end{aligned}
$$

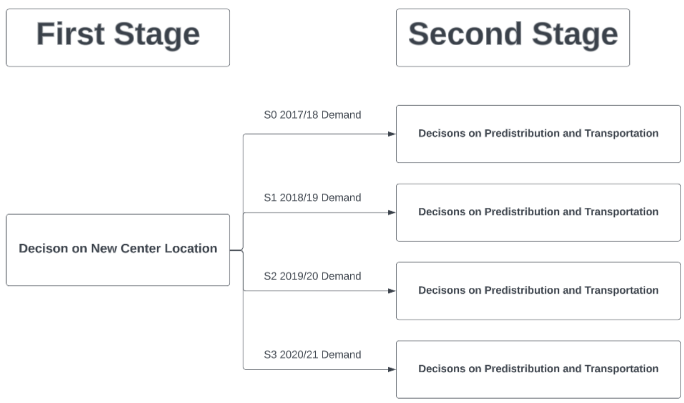

Figure 8: Two-Stage Stochastic Model Flow Map

3.5.4 Result for Historical Stochastic Model

To make a variable conclusion on the cost-effectiveness analysis.Ẇe introduce the concept of EVPI (Expected Value of Perfect Information). Defining it as follow:

$$EVPI = \frac{RP - WS}{RP},$$

where; RP stands for the "Recourse Problem" (the optimal objective value of the two-stage stochastic model) and WS stands for "Wait and See" (the optimal objective value with perfect information) . There are wait-and-see problems (and ws solutions) associated with each scenarios. The expression $RP - WS$ from the numerator would simply indicate the loss of profit due to the presence of uncertainty.

To compute WS, we take an average of all wait-and-see solutions. It will be computed as follow:

$$WS = \sum_{s=1}^{S} p_s z_s^{ws}.$$

in which z_s^{ws} is the (optimal) objective value associated with the wait-and-see problem s. Since this is a minimization problem, WS can be seen as lower bound on RP.

Thus, the wait-and-see solutions associated with each scenario would be computed through the previous deterministic model. The result will be presented in the following table;

ws Results		
Scenarios	Objective	New Center Location
S_0	£285,947	Portree
S_1	£269,863	Portree
S_2	£274,462	Portree
S_3	£269,893	Portree

Table 1: Wait and See Results for Historical Data Model.

Results	
RP	£275,136
WS	£275,091
EVPI	0.01624%

Table 2: Summary of Historical Data Stochastic Model.

In table 1 and 2 scenarios were based on considering the historical total demand data for different years. As the demand at each location k was computed using population proportions, a scaling up or a scaling down on the total demand would make the same changes in terms of proportions for each location. In this case, varying total demand would not significantly change the final decision. From the solution of the previous deterministic model, the optimal solution is required to open a new center at Portree. Since the previous deterministic model used total demand from 2019-2020, considered the maximum demands, the solution would already consider a worst demand scenario. As expected, to solve the previous deterministic model with each individual scenario, it always returned a solution that opened a centre at Portree. Thus, it would lead an $EVPI$ equal to zero. Our value EVPI is almost, but not exactly zero, due to our GAMS solutions being close upper bounds. Since the $EVPI$ was almost 0, WS is a good approximation for RP in this case.

3.6 Results for Stress Test Stochastic Model

In the previous section, the solution of the stochastic model does not vary much from the deterministic, and is somewhat a trivial case. At this stage, the stress test was introduced to investigate the impact of considerable variation in location specific demand. The stress test stochastic model formulation remains the same as the historical stochastic model but based on a larger number of scenarios corresponding to different demand data, which gives the new scenario index $s \in \{0 \dots 13\}$. In each scenario we individually double the demand for a given centre k. In addition, we have the input data used in model M_3 as a base scenario s_0.

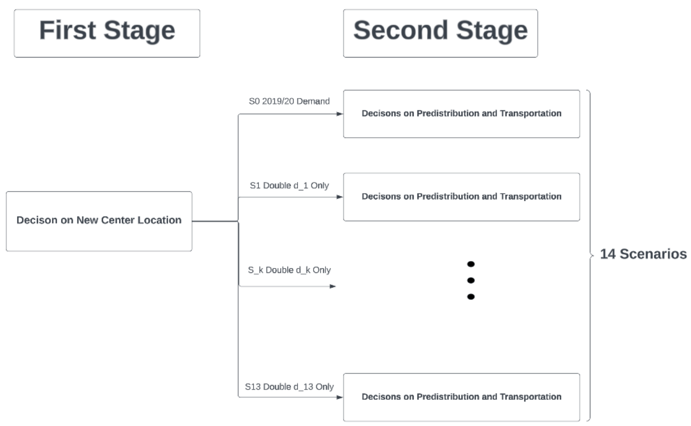

Figure 9: Stress Test Flow Map.

Similarly, in order to compute $EVPI$ discussed above, 14 wait-and-see problems needed to be computed separately through the previous deterministic model, we display these solutions in Table 3.

ws Results		
Scenarios	Objective	New Center Location
S_0	£285,947	Portree
S_1	£291,096	Portree
S_2	£296,095	Portree
S_3	£310,710	Nairn
S_4	£313,334	Portree, Inverness
S_5	£294,882	Portree
S_6	£300,593	Portree
S_7	£302,956	Portree
S_8	£292,247	Portree
S_9	£292,247	Portree
S_{10}	£292,104	Portree
S_{11}	£401,856	Portree, Narin, Inverness, Culloden
S_{12}	£313,829	Portree, Culloden
S_{13}	£357,389	Portree, Narin, Inverness, Culloden

Table 3: Wait and See Results for Stress Test Model.

Results	
RP	£321,393
WS	£310,377
EVPI	3.427%

Table 4: Summary of Stress Test Model.

For this model $EVPI$ was calculated as 3.427 % corresponding to savings of £11,015.

Conclusion

This paper has focused on improving the efficiency and equity of food parcel distribution in the Scottish Highland and Islands region. We look specifically at the distribution network between charities FareShare and Trussell Trust. In order to produce a more efficient distribution network we use both mixed integer programming and stochastic modelling techniques. The results of the mixed integer programming model M_3 show that the efficiency of the current system can be improved by utilising pre-distribution processes whereby FareShare centres can distribute parcels between each other before distributing to the Trussell Trust centres. By allowing for pre-distribution we see that the FareShare centre in Glasgow sends to the FareShare centre in Dundee, before sending to the Trussell Trust centre in Inverness. In addition to this result regarding pre-distribution, we find that the model suggest an improvement in efficiency that arises from opening a new FareShare distribution centre in Portree. This new centre allows for more efficient distribution specifically to the Scottish Islands and West Coast. Our formulation also results in a large minimum service level of 93.5 % to all Trussell Trust Centers, and corresponding to the largest variation of unmet demand of 0.065. All things considered implementing the new transportation network leads to significant costs savings as discussed the individual results sections. Following up on these findings, we run two stochastic models. The first stochastic model considered the impact of varying demand using historical total demand data (see Appendix 1 for demand data discussion). We found the results to be somewhat trivial with a centre being built in Portree. We then run a second model in order to stress test the results from the first stochastic model by investigating larger variations in demand at any given centre. The results from the second stochastic model show that in all scenarios considered bar one a centre is built in Portree and other centres are only built in the most extreme scenarios. This stress test suggests that in some extreme cases other centres may have to be built but even under extreme demand variation the likely optimal solution is to build only one centre in Portree. The building of such centre will therefore improve efficiency within the network, even under conditions of varying demand. As such we recommend that to improve efficiency within the FareShare Trussell Trust distribution network a new centre in Portree should be opened.

Appendix 1: Input Parameter Data

This appendix will justify the input parameters outlined in section 3.1.1 and reference data sources.

First we consider the Index of FareShare centres. We index these centres $i \in \{1, \ldots, 24\}$ as we have 5 current centres and 19 proposed locations [5]. We propose 19 locations as this allows us to set up a centre in any of the 19 largest town in each of the Scottish Highland Wards, and North Uist in order to represent Na h-Eileanan Siar [23].

We index the Trussell Trust centres $k \in \{1, \ldots, 13\}$ to represent both past and present running Trussell Trust centres [16].

The values for supply of parcels at FareShare centre i (a_i) were calculated from FareShare end of year statistics and population data for the regions supplied relative to the rest of the UK. [4] [11]. We assume equal demand $a_i = 1599$ for any opened centre that is dedicated to supplying our network.

The demands for parcels at Trussell Trust centre k,(d_k) were calculated using data from both Trussell Trust end of year statistics, government data on food bank demand and population data [11] [17] [7].

To calculate the start up cost of sending an empty vehicle along a journey we calculate C_{ik} as;

$$C_{ik} = \text{fuel efficiency} * \text{distance}_{ik} + \text{ferry cost} \cdot \text{ferry binary}_{ik} + \text{van hire cost}.$$

Here fuel efficiency is a constant 7.370929079 measured in pence for kilometers - this values is gathered using Volkswagen and Government data [24] [2].

The distances are gathered using Google MyMaps [6].

The ferry cost is a constant equal to 147 was gathered using information from the north link ferry website [13] and the ferry binary variable is a binary variable equal to 1 if travelling on a given route requires ferry transportation.

The van hire cost was £497.60 [12].

We then calculate an additional cost per unit of shipping (c_{ik}) from centre i to centre k. We calculate this as an extra parcel causes a 0.25 percent increase in total journey fuel cost. This value was based on US government data and data on food parcel weight. [22][8].

Analogous definitions are made for $\tilde{C}_{ii'}$ and $\tilde{c}_{ii'}$.

The value of the penalty per unit not delivered is £50 given by Trussell Trust data on the value of a parcel [15].

Finally the per year running cost of A FareShare centre i (R_i) was calculated using data from FareShare Annual Reports and population data [11] [4]. This takes value £43,526.

Appendix 2: Optimal value for transportation variables in solution to M_3.

Below is a table of the distribution quantities between the centres for the solution to M_3 presented previously. Predistribution is shown in *italic* font.

Origin	**Destination**	**Number of Parcels**
Glagow	*Dundee*	*884*
Dundee	Inverness	2483
Edinburgh	Elgin	1599
Alness	Inverness	1350
Aberdeen	Nairn	896
Glasgow	Oban	715
Aberdeen	Elgin	577
Portree	Fort William	492
Portree	Benbecula	340
Portree	Barra	293
Portree	Portree	203
Portree	Tiree	168
Aberdeen	Invergordon	126
Alness	Dornoch	126
Alness	Beauly	123
Portree	Broadford	103

Similar tables were generated for all other solutions presented in the paper, however due to space limitations these are not presented here.

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