Mathematical Modeling of Behavior (2024-2025)

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Binary logit – supplement — September 24, 2024

Question 1: Probit

Consider the utility functions of individual n for two alternatives i and j as follows:

$$U_{in} = V_{in} + \varepsilon_{in}, \tag{1}$$
$$U_{jn} = V_{jn} + \varepsilon_{jn}. \tag{2}$$

The binary probit model is obtained based on the assumption that the error terms are i.i.d. normally distributed across n (not necessarily across i). Derive the binary probit model $P_n(i)$.
Hints:

- Remember that the utility difference matters.
- Remember the definition of a cumulative distribution function (CDF).

Question 2: Binary logit

In a case study of transportation mode choice, the parameters of the utility functions have been estimated as follows:

$$\begin{aligned} U_{1n} &= 1 - 0.03 \cdot tt_{1n} - 0.06 \cdot c_{1n} + 0.5 \cdot \text{income}_n + \varepsilon_{1n}, \\ U_{2n} &= -0.02 \cdot tt_{2n} - 0.0375 \cdot c_{2n} + 0.5 \cdot \text{university}_n + \varepsilon_{2n}, \end{aligned} \tag{3}$$

where tt_{in} is the travel time in minutes and c_{in} is the cost in CHF for respondent n, with $i \in \{\text{car}, \text{train}\}$. income_n takes value 1 if the respondent's monthly income is larger than 6000CHF and 0 otherwise, and university_n takes value 1 if the respondent went to the university and 0 otherwise. $\varepsilon_{1n}, \varepsilon_{2n} \overset{\text{iid}}{\sim} \text{EV}(0, 1)$.
Compute the probability (with two significant digits) to choose each mode for the following individuals:

Name	tt_1	tt_2	c_1	c_2	monthly income	university
Eva	22	18	2	2.1	7000	yes
Matthieu	120	100	10	15.0	3000	yes
Michel	10	50	3	5.0	10000	no
Meri	25	9	7	2.1	5000	no

Question 3: Specification of the utility function

In a case study of transportation mode choice, the parameters of the utility functions have been estimated as follows:

$$U_{1n} = 1 - 0.03 \cdot tt_{1n} - 0.06 \cdot c_{1n} + 0.5 \cdot \text{income}_n + \varepsilon_{1n},$$
$$U_{2n} = -0.02 \cdot tt_{2n} - 0.0375 \cdot c_{2n} + 0.5 \cdot \text{university}_n + \varepsilon_{2n},$$

where tt_{in} is the travel time in minutes and c_{in} is the cost in CHF for respondent n, with $i \in \{\text{car}, \text{train}\}$. income_n takes value 1 if the respondent's monthly income is larger than 6000CHF and 0 otherwise, and university_n takes value 1 if the respondent went to the university and 0 otherwise. $\varepsilon_{1n}, \varepsilon_{2n} \overset{\text{iid}}{\sim} EV(0, 1)$.

Which of the following specifications are equivalent to the proposed one?

1. $$U_{1n} = -0.03 \cdot tt_{1n} - 0.06 \cdot c_{1n} + 0.5 \cdot \text{income}_n + \varepsilon_{1n},$$
   $$U_{2n} = 1 - 0.02 \cdot tt_{2n} - 0.0375 \cdot c_{2n} + 0.5 \cdot \text{university}_n + \varepsilon_{2n}.$$

2. $$U_{1n} = -0.03 \cdot tt_{1n} - 0.06 \cdot c_{1n} + 0.5 \cdot \text{income}_n + \varepsilon_{1n},$$
   $$U_{2n} = -1 - 0.02 \cdot tt_{2n} - 0.0375 \cdot c_{2n} + 0.5 \cdot \text{university}_n + \varepsilon_{2n}.$$

3. $$U_{1n} = 1 - 0.03 \cdot tt_{1n} - 0.06 \cdot c_{1n} + 0.5 \cdot \text{income}_n - 0.5 \cdot \text{university}_n + \varepsilon_{1n},$$
   $$U_{2n} = -0.02 \cdot tt_{2n} - 0.0375 \cdot c_{2n} + \varepsilon_{2n}.$$

4. $$U_{1n} = 1 - 0.03 \cdot tt_{1n} - 0.06 \cdot c_{1n} - 0.5 \cdot \text{university}_n + \varepsilon_{1n},$$
   $$U_{2n} = -0.02 \cdot tt_{2n} - 0.0375 \cdot c_{2n} - 0.5 \cdot \text{income}_n + \varepsilon_{2n}.$$

5. $$U_{1n} = 1 - 0.03 \cdot tt_{1n} - 0.06 \cdot c_{1n} + 0.5 \cdot \text{university}_n + \varepsilon_{1n},$$
   $$U_{2n} = -0.02 \cdot tt_{2n} - 0.0375 \cdot c_{2n} + 0.5 \cdot \text{income}_n + \varepsilon_{2n}.$$

6. $$U_{1n} = 10 - 0.3 \cdot tt_{1n} - 0.6 \cdot c_{1n} + 5 \cdot \text{income}_n + \varepsilon_{1n},$$
   $$U_{2n} = -0.2 \cdot tt_{2n} - 0.375 \cdot c_{2n} + 5 \cdot \text{university}_n + \varepsilon_{2n},$$