Mathematical Modeling of Behavior (2024-2025)

Michel Bierlaire.

TAs: Cloe Cortes, Pavel Ilinov, Evangelos Paschalidis, Anne-Valérie Preto, Negar Rezvany

Choice theory (correction) — September 17, 2024

Question 1: Specification of the utility function

If we want to capture the idea that an additional minute spent in the bus may not be perceived the same way as spending one more minute driving the car, then the utility functions associated with each alternative have to be written as

$$U_i = -\beta_{ti} t_i - \beta_c c_i, \tag{1}$$

$$U_j = -\beta_{tj} t_j - \beta_c c_j, \tag{2}$$

where $\beta_{ti} > 0$, $\beta_{tj} > 0$, and $\beta_c > 0$ are parameters.

Question 2: Utility functions

1. There is no alternative that is both cheaper and faster than all other alternatives. Therefore, there is no dominating alternative. However, the bus alternative is both slower and more expensive than the metro alternative. The bus alternative is therefore dominated by the metro alternative. The bus alternative will never be chosen and can be removed from the choice set of the commuter.

2. If $\beta_t = \beta_c = 0.5$, the utility for each alternative is

   - $U_{Car} = -0.5 \times 10 - 0.5 \times 18 = -14$,
   - $U_{Metro} = -0.5 \times 14 - 0.5 \times 12 = -13$,
   - $U_{Bike} = -0.5 \times 30 - 0.5 \times 2 = -16$.

   The highest utility is associated with the metro alternative.

3. If $\beta_t = 1.0$ and $\beta_c = 0.5$, the utility for each alternative is

   - $U_{Car} = -1.0 \times 10 - 0.5 \times 18 = -19$,
   - $U_{Metro} = -1.0 \times 14 - 0.5 \times 12 = -20$,
   - $U_{Bike} = -1.0 \times 30 - 0.5 \times 2 = -31$.

   The highest utility is associated with the car alternative.

4. If $\beta_t = 0.5$ and $\beta_c = 1.0$, the utility for each alternative is

   - $U_{Car} = -0.5 \times 10 - 1.0 \times 18 = -23$,
   - $U_{Metro} = -0.5 \times 14 - 1.0 \times 12 = -19$,
   - $U_{Bike} = -0.5 \times 30 - 1.0 \times 2 = -17$.

   The highest utility is associated with the bike alternative.

5. If the individual has no driving license, the car is not an alternative and her choice set contains only two alternatives: the metro and the bike. In all the above questions, the car alternative disappears. In consequence, the metro alternatives becomes the alternative with the highest utility in question 3.

Question 3: Data representation

1. The value of time is calculated as follows (notice the required conversion of units as the time is expressed in minutes in this exercise):

$$\text{VOT} = \frac{\beta_t}{\beta_c} \cdot 60 = \frac{0.363}{1.38} \cdot 60 = 15.78\ [\text{CHF/h}].$$

2. The differences in time are the following:

Individual	Choice	$\text{time}_{\text{pmm}} - \text{time}_{\text{pt}}$	$\text{cost}_{\text{pmm}} - \text{cost}_{\text{pt}}$
1	pmm	-10	1.3
2	pt	-5	1.8
3	pmm	5	-3
4	pmm	-2	-0.5
5	pt	-1.5	0.75
6	pt	-5	1.5
7	pt	3	1
8	pt	1	-1
9	pt	3	-1.5
10	pmm	-0.5	-6

The plot is reported in Figure 1 on the following page, where the observation corresponding to the choice "pmm" are represented in blue, and those corresponding to the choice "pt" are represented in red.

3. Add to the previous plot the line $-\beta_c \cdot \text{cost}_{\text{pmm}} - \beta_t \cdot \text{time}_{\text{pmm}} = -\beta_c \cdot \text{cost}_{\text{pt}} - \beta_t \cdot \text{time}_{\text{pt}}$. What does its slope represent?

Solution: The line can be written as follows:

$$\text{cost}_{\text{pmm}} - \text{cost}_{\text{pt}} = -\frac{\beta_t}{\beta_c}(\text{time}_{\text{pmm}} - \text{time}_{\text{pt}}),$$

and is represented in green in Figure 1 on the next page. The slope represents the change in the y-value ($\text{cost}_{\text{pmm}} - \text{cost}_{\text{pt}}$) per unit change in the x-value ($\text{time}_{\text{pmm}} - \text{time}_{\text{pt}}$). Thus, it represents the value of time, as each minute less in time corresponds to an additional cost of $\frac{\beta_t}{\beta_c}$.

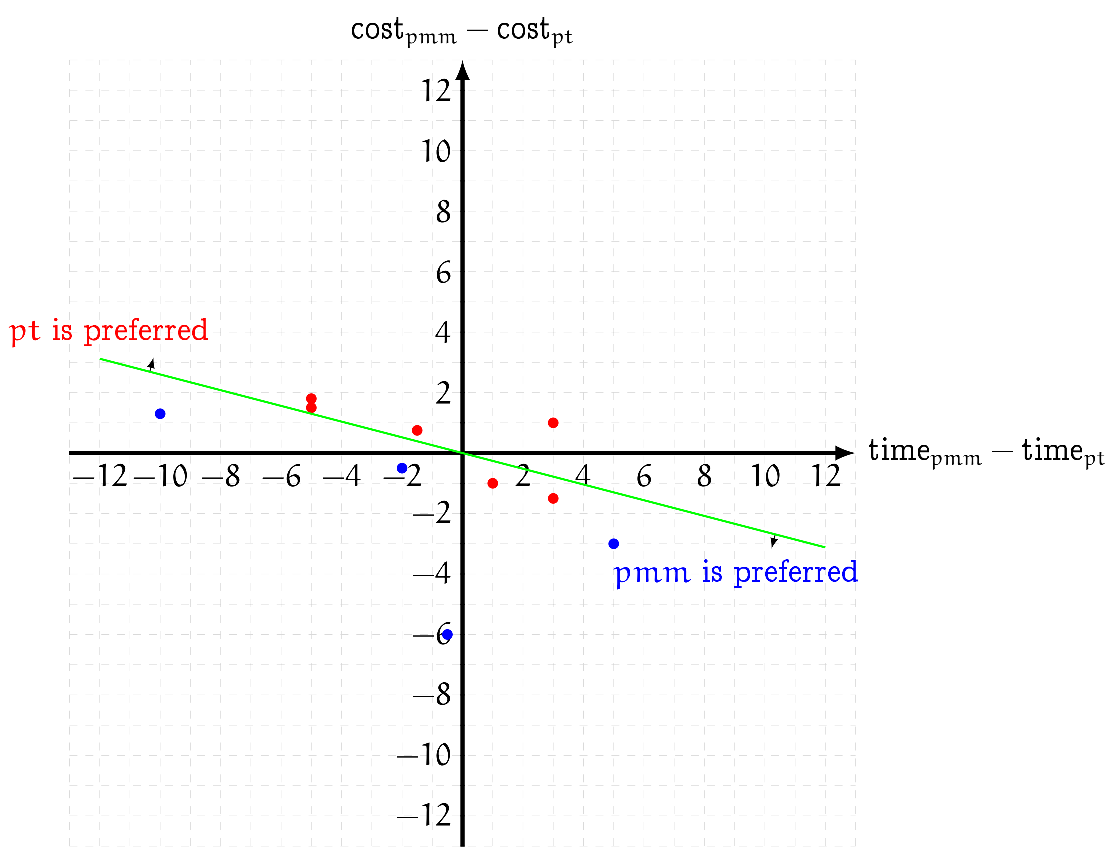

Table 1: Representation of choice data