Mathematical Modeling of Behavior
(2024-2025)
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Binary logit — September 24, 2024

Question 1: Alternative specific constants

You have estimated the parameters of the following mode choice model, involving two transportation modes (index n has been dropped for notational convenience):

$$U_{\text{bicycle}} = ASC_{\text{bicycle}} + \beta_{\text{distance}} \cdot \text{distance} + \varepsilon_{\text{bicycle}}, \tag{1}$$

$$U_{\text{metro}} = ASC_{\text{metro}} + \beta_{\text{time}} \cdot \text{time}_{\text{metro}} + \beta_{\text{cost}} \cdot \text{cost}_{\text{metro}} + \varepsilon_{\text{metro}}, \tag{2}$$

where *distance* is the distance of the trip in kilometers, $\text{cost}_{\text{metro}}$ is the cost in Swiss francs (CHF) of the trip by metro and $\text{time}_{\text{metro}}$ is the time in minutes of the trip by metro. $\varepsilon_{\text{bicycle}}$ and $\varepsilon_{\text{metro}}$ are random terms.
In order to estimate the model, one of the two alternative specific constants must be normalized to zero. Table 1 reports the estimated parameters for each normalization. However, it is incomplete. First, complete the second column of Table 1 corresponding to the normalization $ASC_{\text{metro}} = 0$.

Parameters	Normalization 1	Normalization 2
ASC_{bicycle}	0	
ASC_{metro}	3	0
β_{distance}	-0.8	
β_{time}	-0.5	
β_{cost}	-1	

Table 1: Estimated parameters

Perform the following tasks for a respondent with a trip of 10 kilometers that takes 20 minutes and costs 2.2 CHF by metro:

1. calculate the choice probabilities in the case of a logit model with the parameter estimates with normalization 1, and the scale parameter set to one,

2. calculate the choice probabilities in the case of a probit model with the parameter estimates with normalization 1, and the scale parameter set to one,

3. calculate the choice probabilities in the case of a logit model with the parameter estimates with normalization 2, and the scale parameter set to one,
4. calculate the choice probabilities in the case of a probit model with the parameter estimates with normalization 2, and the scale parameter set to one.

Question 2: Scale

You have estimated the parameters of the following mode choice model, involving two transportation modes (index n has been dropped for notational convenience):

$$U_{\text{bicycle}} = ASC_{\text{bicycle}} + \beta_{\text{distance}} \cdot \text{distance} + \varepsilon_{\text{bicycle}}, \tag{3}$$

$$U_{\text{metro}} = ASC_{\text{metro}} + \beta_{\text{time}} \cdot \text{time}_{\text{metro}} + \beta_{\text{cost}} \cdot \text{cost}_{\text{metro}} + \varepsilon_{\text{metro}}, \tag{4}$$

where *distance* is the distance of the trip in kilometers, $\text{cost}_{\text{metro}}$ is the cost in Swiss francs (CHF) of the trip by metro and $\text{time}_{\text{metro}}$ is the time in minutes of the trip by metro. $\varepsilon_{\text{bicycle}}$ and $\varepsilon_{\text{metro}}$ are random terms. The parameter estimates are $ASC_{\text{bicycle}} = 0$, $ASC_{\text{metro}} = 3$, $\beta_{\text{distance}} = -0.8$, $\beta_{\text{time}} = -0.5$ and $\beta_{\text{cost}} = -1$.
Calculate the choice probabilities for a respondent with a trip of 10 kilometers that takes 20 minutes and costs 2.2 CHF by metro in the following cases:

1. using a logit model with scale parameter $\mu = 0.1$,
2. using a logit model with scale parameter $\mu = 10$,
3. using a probit model with scale parameter $\sigma = 0.1$,
4. using a probit model with scale parameter $\sigma = 10$.

Comment on these results including the implications of the scale parameter on the choice probabilities.

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Question 3: Normalization of the constants

We consider a route choice model with two alternatives for individual n. The utility functions are defined as follows:

$$\begin{aligned} U_{1n} &= ASC_1 + \beta_{length} \cdot \text{length}_{1n} + \varepsilon_{1n}, \\ U_{2n} &= ASC_2 + \beta_{length} \cdot \text{length}_{2n} + \varepsilon_{2n}, \end{aligned} \tag{5}$$

where alternatives 1 and 2 represent the two routes, ASC_1, ASC_2 and β_{length} are parameters to be estimated and $\text{length}_{in}, i \in \{1, 2\}$, is the length of each route in kilometers for individual n.
The estimation results of a binary logit model, where ASC_1 has been normalized to zero, are shown in the first column of Table 2. The second column corresponds to the same specification with ASC_2 normalized to zero instead. And the third column corresponds to the same specification where the sum of the two constants is constrained to be zero.

	Logit 1	Logit 2	Logit 3
ASC_1	0	x_2	x_3
ASC_2	-2	0	y_3
β_{length}	10	z_2	z_3

Table 2: Estimation results

1. Replace x_2, x_3, y_3, z_2 and z_3 in the table with the value of the corresponding parameter.

2. What are the distributions of ε_{1n}, ε_{2n} and $\varepsilon_{1n} - \varepsilon_{2n}$?

Hint: Please note that only the difference in utilities matters