Numerical Methods for Partial Differential Equations
A.Y. 2022/2023

Laboratory 05
Finite Element method for the diffusion-reaction equation in 3D

Exercise 1.

Let $\Omega = (0,1)^3$, be the unit cube and let us consider the following diffusion-reaction problem:

$$\begin{cases} -\boldsymbol{\nabla} \cdot (\mu \, \boldsymbol{\nabla} u) + \sigma u = f & \mathbf{x} \in \Omega, \quad \text{(1a)} \\ u = 0 & \text{on } \partial\Omega, \quad \text{(1b)} \end{cases}$$

where $\mathbf{x} = (x, y, z)^T$, $\sigma(\mathbf{x}) = 1$, $f(\mathbf{x}) = 5$ and

$$\mu(\mathbf{x}) = \begin{cases} 100 & \text{if } x < \dfrac{1}{2}, \\ 1 & \text{if } x \geq \dfrac{1}{2}. \end{cases}$$

1.1. Starting from the code of Laboratories 3 and 4, implement a finite element solver for problem (1). The solver should read the mesh from file (four differently refined meshes are provided as `mesh/mesh-cube-*.msh`).

1.2. Compute the solution on the mesh `mesh/mesh-cube-40.msh` (i.e. $N + 1 = 40$), with linear finite elements.