Animal Movement Modeling

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Background

“Imputation Approaches for Animal Movement Modelling” by Henry Scharf, Mevin Hooten, and Devin Johnson

“An Introduction to Animal Movement Modeling with Hidden Markov Models using Stan for Bayesian Inference” by Vianey Leos-Barajas and Théo Michelot

Presentation Overview

- Brief overview of Bayesian Inference
- Continuous-time model
  - Overview of imputation for fitting model to data
  - Application to seal data
- Discrete-time model
  - Hidden Markov Model
  - Wild haggis example
- Takeaways

Bayesian Inference

- Use Bayes' Theorem to update hypothesis as more information becomes available
- Start with prior assumptions

Likelihood

Prior Probability

$$\underbrace{P(A|B)}_{\text{Posterior Probability}} = \frac{\overbrace{P(B|A)}^{\text{Likelihood}}\ \overbrace{P(A)}^{\text{Prior Probability}}}{\underbrace{P(B)}_{\text{Marginal Likelihood}}}$$

Posterior Probability

Marginal Likelihood

Continuous-Time Model

Telemetry for Animal Movement Modeling

- **Telemetry data**: Data collected from remote or inaccessible locations that is automatically transmitted for collection
- **Animal Telemetry**: tagging animals with radio or satellite sensors to track movements, biorhythms, and environmental conditions

Continuous-Time Model

- True latent position process at time t represented as $\mu(t_j)$, $j = 0, 1, 2, \ldots$
- μ is conditioned on model parameters Θ such that $\mu \sim [\mu \mid \Theta]$
- Telemetry position data at time t represented by $s(t_j)$

Continuous-Time Model

Stochastic Differential Equation (SDE) to describe Brownian particle in conservative force field:

velocity

$$d\boldsymbol{\mu}(t) = \mathbf{v}(t)dt$$

position

$$d\mathbf{v}(t) = -\nabla H\left(\boldsymbol{\mu}(t), t\right) dt - \sigma_v \mathbf{v}(t)dt + \sigma_v d\mathbf{b}(t)$$

$$H(\boldsymbol{\mu}(t), t) = \beta(t)||\boldsymbol{\mu}(t) - \mathbf{c}||_2$$

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Brownian motion

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Brownian motion

β(t) > 0 implies attraction towards c
β(t) < 0 implies repulsion

Euclidean distance between μ(t) and c, where c is point of interest in environment

Imputation Methods

- **Imputation**: statistical process of replacing missing data with substituted values
- Simplest approach is linear interpolation. This approach has a few drawbacks:
  - Linear path not representative
  - Need to consider uncertainty of telemetry accuracy
  - Need to consider uncertainty between observation points
- Process imputation
  - Developed by Hooten et al. (2010) and Hanks et al. (2011)
  - Involves approximating model parameters directly

Process Imputation

- Calculate posterior distribution with “complete data distribution” and imputation distribution

$$[\boldsymbol{\theta}|\mathbf{s}] = \int [\boldsymbol{\theta}|\boldsymbol{\mu},\mathbf{s}]\,[\boldsymbol{\mu}|\mathbf{s}]\,d\boldsymbol{\mu}.$$

Posterior distribution

Complete data distribution

Imputation distribution

Θ - SDE model parameters

μ - animal location data (all times)

s - telemetry data

Process Imputation

- Complete-data distribution is straightforward to compute
- Sampling from imputation distribution is difficult because it requires marginal distribution μ:

$$[\boldsymbol{\mu}] = \int [\boldsymbol{\mu}|\boldsymbol{\theta}]\,[\boldsymbol{\theta}]\,d\boldsymbol{\theta}.$$

- This integral can be intractable
- Instead of sampling from [μ|s], sample from the **approximate imputation distribution** [μ*|s]

Approximate Imputation Distribution

- Theoretically, in extreme case where measurements are made without error and measurements exist at all time, conditional distribution $[\mu|s]$ collapses to point mass and we can use any μ^*
- Realistically, choice of AID has significant impact on data inference

To Choose AID:

- Make an assumption about a family of distributions that $[\mu|s]$ might be in
- Fit $[\mu^*|s]$ to data for a best approximation

Process Imputation in Practice

Goal: Approximate SDE model parameter distributions

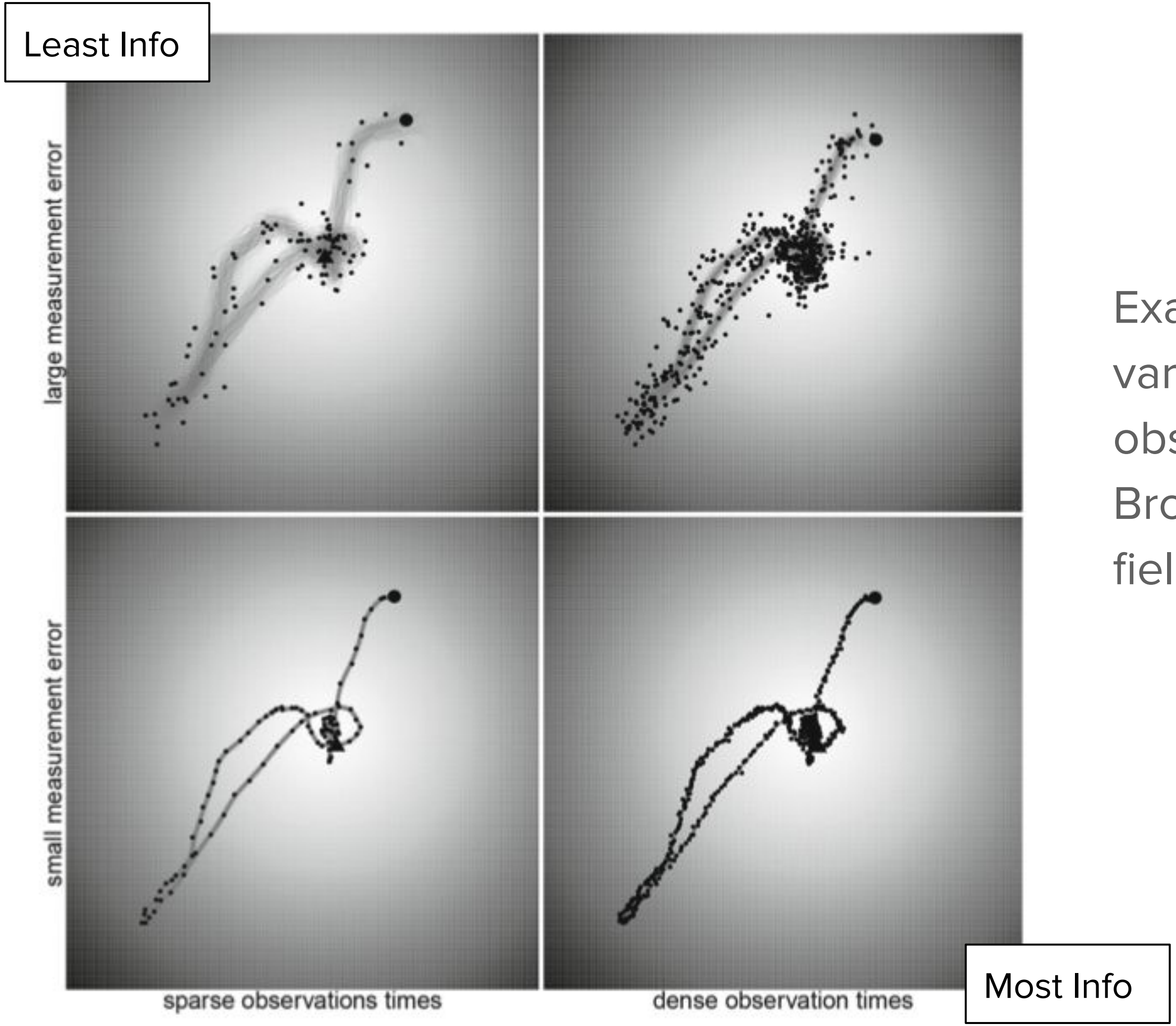
1. Draw realizations from the AID, $\boldsymbol{\mu}^{*(k)} \sim [\boldsymbol{\mu}^*|\mathbf{s}]$, for $k = 1, \ldots, K$.
2. MCMC procedure:
   (a) Randomly select one of the K samples, $\boldsymbol{\mu}^{*(k)}$, with probability $1/K$.
   (b) Update model parameters $\boldsymbol{\theta}|\boldsymbol{\mu} = \boldsymbol{\mu}^{*(k)}$ conditioned on the imputed path, $\boldsymbol{\mu}^{*(k)}$.
   (c) Repeat steps 2(a)–2(b) at each iteration of the MCMC algorithm.

$$[\boldsymbol{\theta}^*|\mathbf{s}] = \int [\boldsymbol{\theta}^*|\boldsymbol{\mu}^*, \mathbf{s}]\,[\boldsymbol{\mu}^*|\mathbf{s}]\,d\boldsymbol{\mu}^*.$$

Process Imputation in Practice

Validity affected by:

- Measurement error
- Density of observations in time
- Choice of approximate imputation distribution (AID) model
- Number of draws from AID



Example of imputed paths with varying measurement error and observation densities for a Brownian particle in an external field.

Application - Northern Fur Seal

$$H(\boldsymbol{\mu}(t), t) = \beta(t)\|\boldsymbol{\mu}(t) - \mathbf{c}\|_2$$

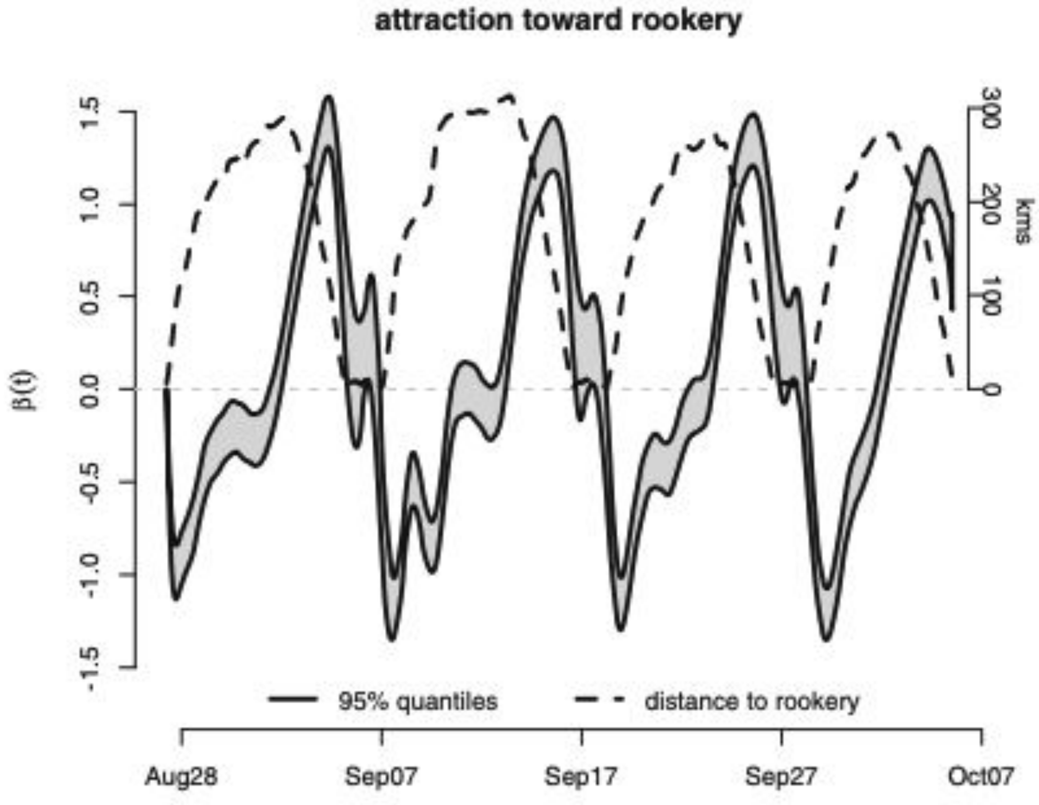
Photo: John Gibbens

- Telemetry data for single female, fall 2008
- GPS observations every 15 min for 6 weeks
- Female leaves rookery to forage, returns to nurse seal pups
- Rookery is the center of attraction

Figure 1. Movement of a female northern fur seal (*C.ursinus*). The *gray lines* are drawn from the approximate imputation distribution, $[\boldsymbol{\mu}^*|\mathbf{s}]$. The *black points* are the data.

Application - Northern Fur Seal

- Fitting β(t) (attraction/repulsion parameter)
- We see periodic repulsion and attraction corresponding to foraging (repulsion) and returning to nurse pups (attraction)
- We also see exploratory behavior



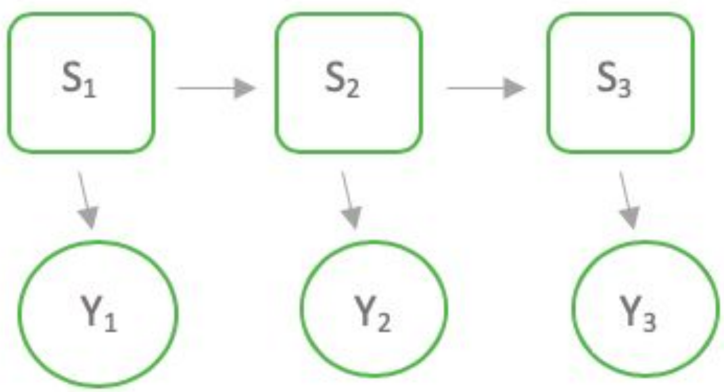
Hidden Markov Models

Hidden Markov Models

- Stochastic time series with observable process $\{Y\}^T_{t=1}$ dependent on a “hidden process” $\{S\}^T_{t=1}$
- Observations drawn from a distribution $\{f\}^N_{n=1}$
- Relies on transition matrix and initial distribution, $\Pr(S_1 = n)$:
- Use Y to deduce information about S

$$\boldsymbol{\Gamma}^{(t)} = \gamma_{i,j}^{(t)}, \text{ for } i, j = 1, \ldots, N$$

$$\boldsymbol{\delta} = \boldsymbol{\delta\Gamma}$$



Hidden Markov Models – Applications

General: Pattern Recognition

- Speech Recognition
- Part-of-speech Tagging

Animal Movement:

- Clusters of movements predict behaviors
- Predict areas of foraging, exploration, etc

Likelihood

- Used for predicting the probability of a sequence of observations
- Likelihood relies on hidden states

$$\mathcal{L}_m = \sum_{s_1=1}^{N} \cdots \sum_{s_T=1}^{N} \delta_{s_1}^{(1)} \prod_{t=2}^{T} \gamma_{s_{t-1},s_t} \prod_{t=1}^{T} f_{s_t}(y_t)$$

- Calculated recursively using forward algorithm

$$\boldsymbol{\alpha}_1 = \boldsymbol{\delta}^{(1)} \mathbf{P}(y_1), \qquad \boldsymbol{\alpha}_t = \boldsymbol{\alpha}_{t-1} \boldsymbol{\Gamma} \mathbf{P}(y_t),$$

$$\mathcal{L}_m = f(y_1, \ldots, y_T) = \sum_{i=1}^{N} \alpha_T(i) = \boldsymbol{\alpha}_T \mathbf{1}^\top.$$

Marginal Distribution and Temporal Dependence

- Two features of movement data that we aim to capture
- Marginal distribution y_t
  - The distribution of a given observation at time t unconditional on the states
  - Ex: give the ecologist an estimate of the proportion of time that the animal exhibits certain behaviors overall
- Temporal dependence
  - Complements information from marginal distribution
  - Autocorrelation structure is key feature for posterior predictive checking

Assessing Model Adequacy

- Forecast (Pseudo-) Residuals
  - Residuals -> mismatched prediction of states -> $r_t = \Phi^{-1}(u_t), \quad t \in \{1, \ldots, T\},$
  - If fitted HMM is true data-generating process, r_t follows normal distribution
  - QQ plot

- Posterior Predictive Checks
  - Generate M replicate data sets from posterior function
  - Assess the fitted model's ability to be interpreted as data generating mechanism
  - $f(\boldsymbol{y}^* \| \boldsymbol{y}) = f(\boldsymbol{y}^* | \boldsymbol{\theta}) f(\boldsymbol{\theta} \| \boldsymbol{y})$

State Estimation

- Not primary focus in animal movement modelling
- Can help visualize fitted models
- Local State Decoding
    - Consider t and S_t given observations and estimated parameters theta.
    - Obtained through the forward-backward algorithm
    - $Pr(S_t | y_1, \ldots, y_T, \boldsymbol{\theta})$
- Global State Decoding
    - Obtain the most likely state sequence given all observations
    - Viterbi algorithm
    - $Pr(S_1, \ldots, S_T | y_1, \ldots, y_T, \boldsymbol{\theta})$

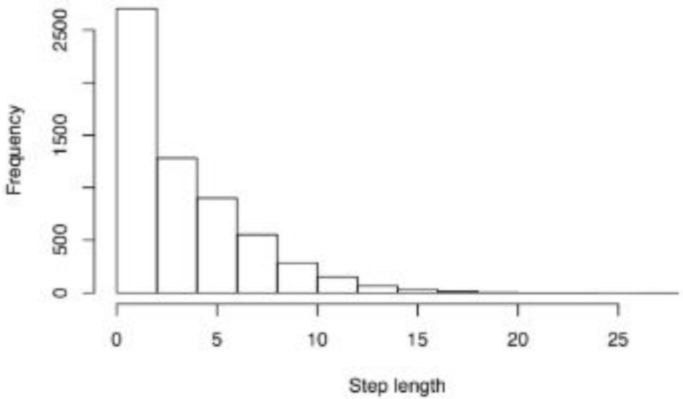
Wild Haggis

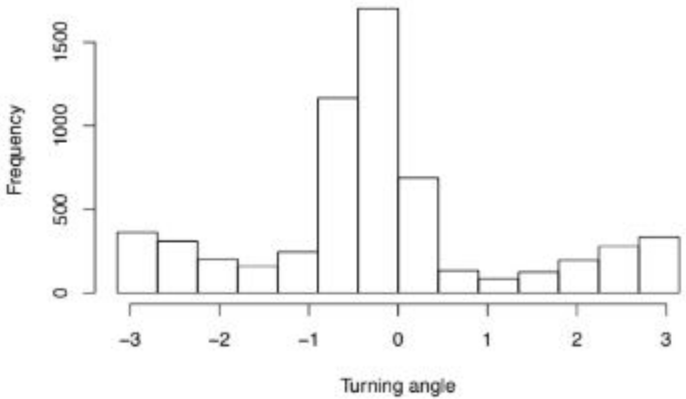
- Fictional creature of Scottish folklore
- One leg is shorter than the other
- Can run downhill very quickly
- Consider applications to animal movement tracks
    - Analyze movement patterns with respect to step length and turning angle
    - moveHMM package

Image from Leos-Barajas & Michelot

Example: Wild Haggis

- Observation states: location
- Hidden states: behavioral states

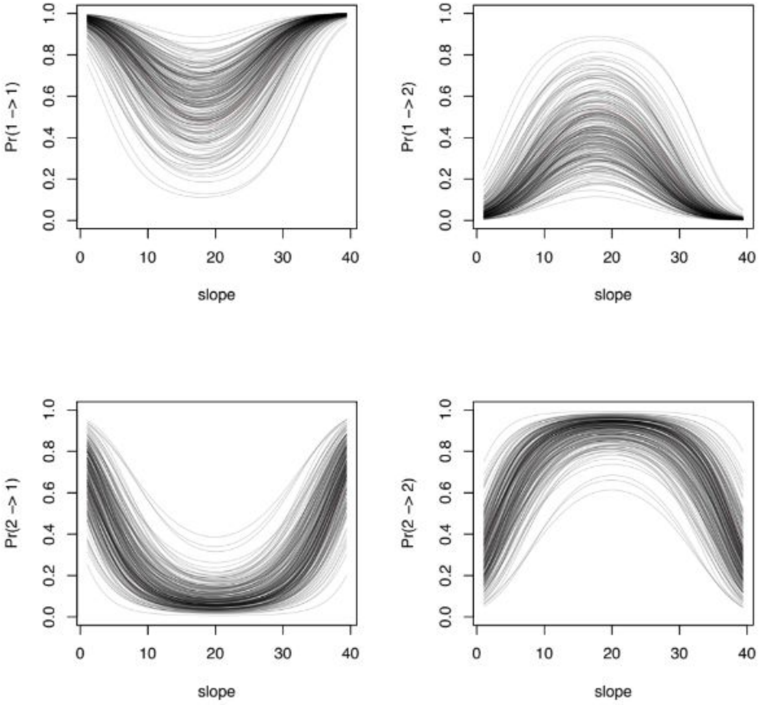




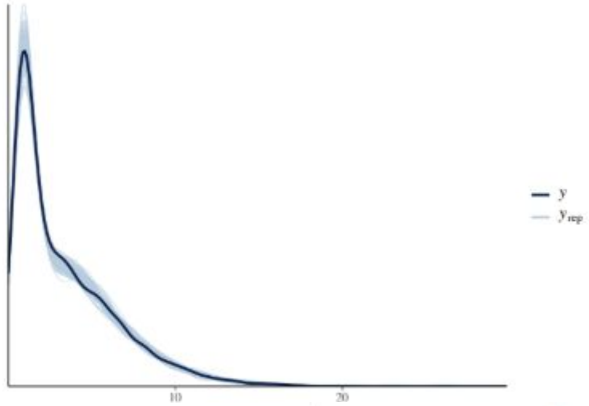
$$L_t|S_t = j \sim \text{gamma}(\alpha_j, \beta_j)$$
$$\varphi_t|S_t = j \sim \text{von Mises}(\mu_j, \kappa_j),$$

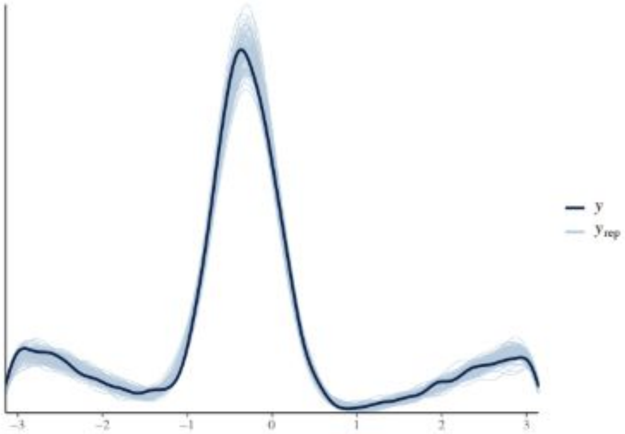
Example: Wild Haggis

Transition probabilities



Posterior Predictive Checks

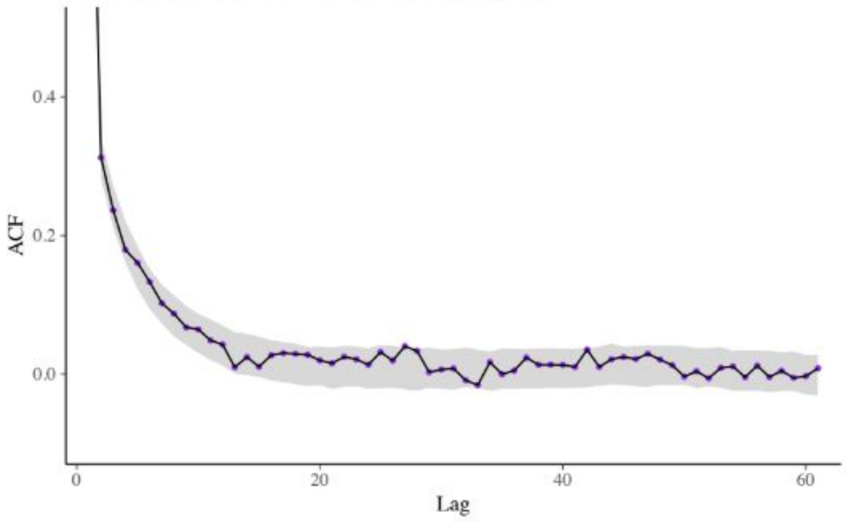


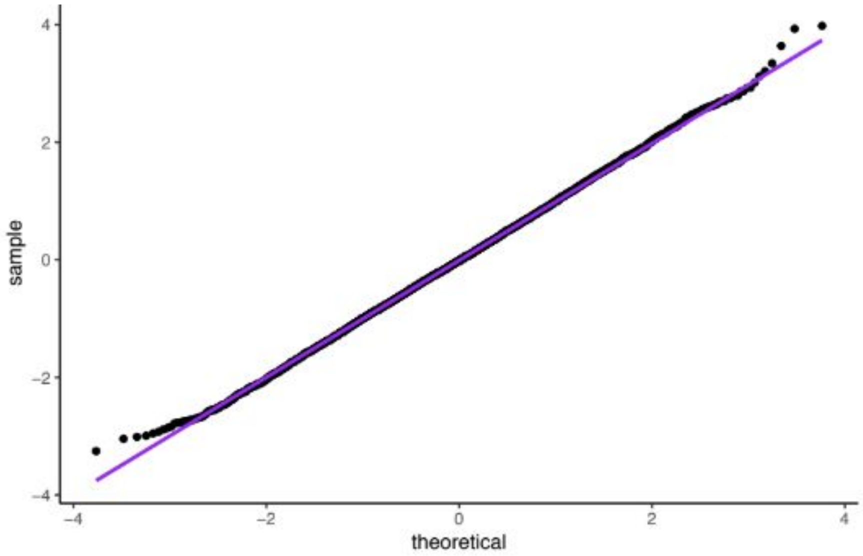


Example: Wild Haggis

Q-Q Residual Plot

Observed Autocorrelation





Takeaways

- Discrete time and continuous time models can model a variety of animal behavior using information about movement
  - Continuous time model: can track movement of animals around some point of interest
  - HMM: can deduce “hidden” behavioral states
- Model parameters can be deduced with Bayesian inference
  - Imputation can be used to fit a continuous model to discrete telemetry data
- Animal movement models are helpful for tracking large-scale trends
- Stochastic models help capture inherent randomness in animal behavior data

Appendix

Study Results

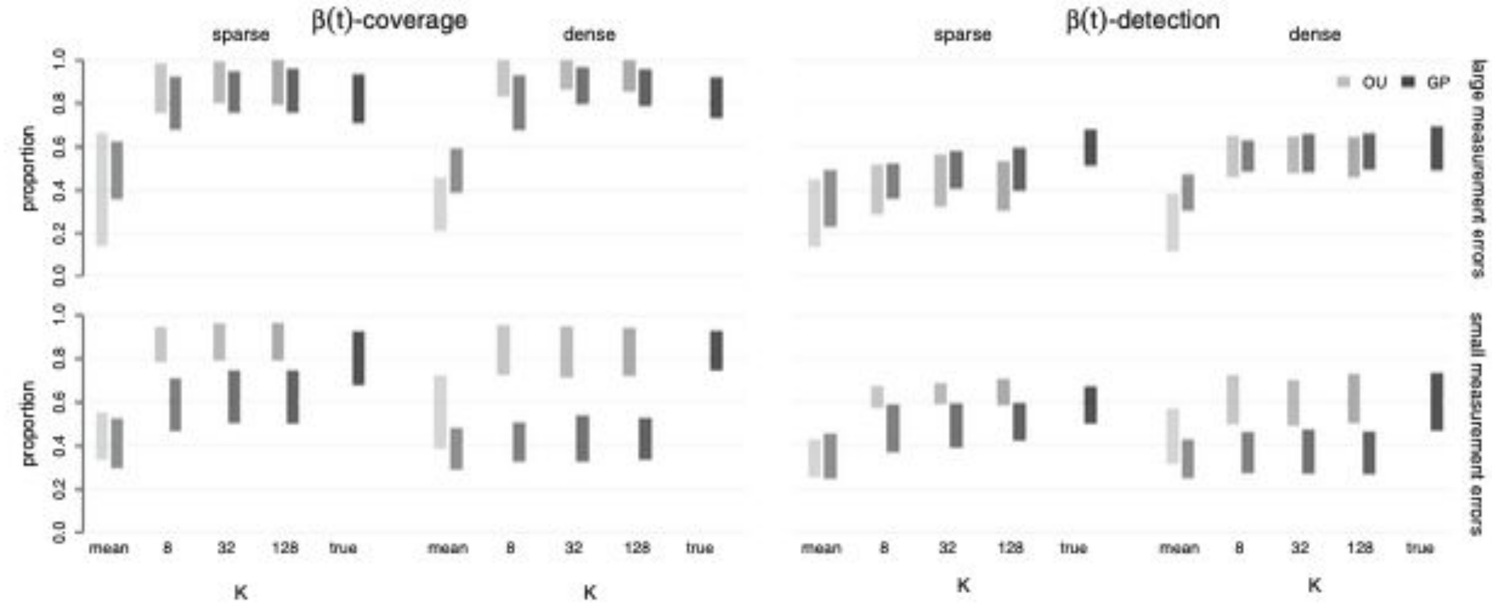


Figure 4. Proportion of $\boldsymbol{\beta}$ covered/detected by the equal-tailed 95% credible interval. The range of each bar represents the 12.5–87.5% quantiles across the 24 simulations. See Fig. 3 for details on the definition of the "coverage" region.

SDE Movement Model & Cucker-Smale - Comparison

$$(5)\quad \begin{cases} \dot{x}_i = v_i \\ \dot{v}_i = \frac{1}{N} \sum_{j=1}^{N} \underbrace{H(|x_i - x_j|)}_{>0} (v_j - v_i) \end{cases}$$

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