



UNIVERSIDAD DE CÓRDOBA

comprometida con el desarrollo regional

TEORÍA DE GRAFOS



UNIVERSIDAD DE CÓRDOBA

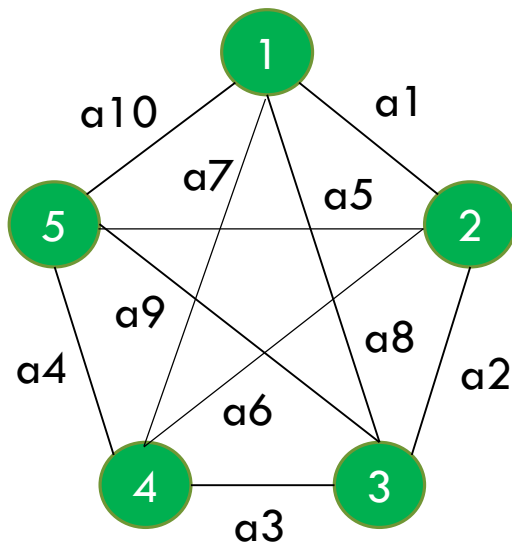
comprometida con el desarrollo regional

UNIDAD 2. SUBGRAFOS

SUBGRAFO

Definición: Se dice que un grafo H es un subgrafo de un grafo G si y sólo si, cada vértice en H es también un vértice en G , cada arista en H es también una arista en G y cada arista en H tiene los mismos puntos extremos de G .

Ejemplo



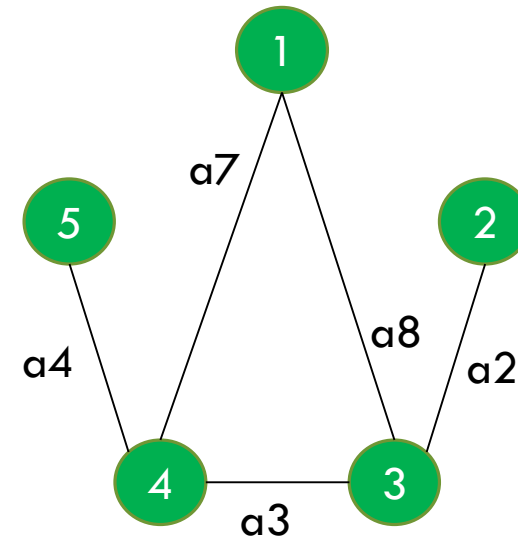
Grafo G

$$G = (V, A, f)$$

$$G(V) = \{1, 2, 3, 4, 5\}$$

$$f(A) = \{a1, a2, a3, a4, a5, a6, a7, a8, a9, a10\}$$

$$F_G = \begin{cases} f_G(a1) = (1, 2), f_G(a2) = (2, 3), \\ f_G(a3) = (3, 4), f_G(a4) = (4, 5), \\ f_G(a5) = (5, 1), f_G(a6) = (1, 3), \\ f_G(a7) = (1, 4), f_G(a8) = (1, 5), \\ f_G(a9) = (2, 4), f_G(a10) = (2, 5), \\ f_G(a11) = (3, 5) \end{cases}$$



Subgrafo H

$$G = (V, A, f)$$

$$G(V) = \{1, 2, 3, 4, 5\}$$

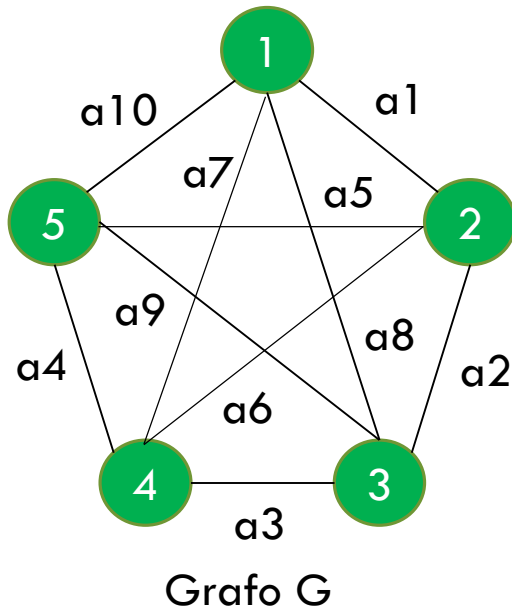
$$f(A) = \{a2, a3, a4, a7, a8\}$$

$$F_G = \begin{cases} f_G(a2) = (2, 3), f_G(a3) = (3, 4), \\ f_G(a4) = (4, 5), f_G(a7) = (4, 1), \\ f_G(a8) = (1, 3) \end{cases}$$

SUBGRAFO COBERTOR

Definición: un subgrafo H de G se llama cobertor si contiene a todos los vértices de G.

Ejemplo

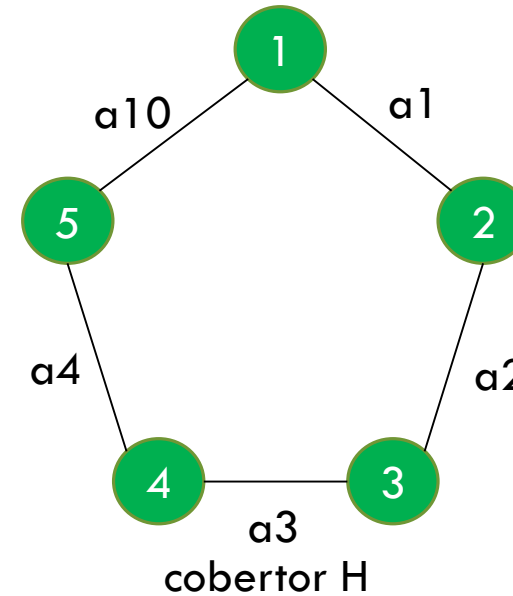


$$G = (V, A, f)$$

$$G(V) = \{1, 2, 3, 4, 5\}$$

$$f(A) = \{a1, a2, a3, a4, a5, a6, a7, a8, a9, a10\}$$

$$F_G = \begin{cases} f_G(a1) = (1, 2), f_G(a2) = (2, 3), \\ f_G(a3) = (3, 4), f_G(a4) = (4, 5), \\ f_G(a5) = (5, 2), f_G(a6) = (2, 4), \\ f_G(a7) = (4, 1), f_G(a8) = (1, 3), \\ f_G(a9) = (3, 5), f_G(a10) = (5, 1), \end{cases}$$



$$G = (V, A, f)$$

$$G(V) = \{1, 2, 3, 4, 5\}$$

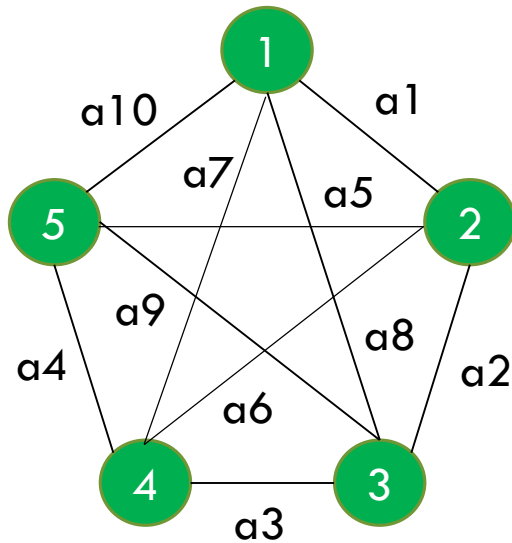
$$f(A) = \{a1, a2, a3, a4, a10\}$$

$$F_G = \begin{cases} f_G(a1) = (1, 2), f_G(a2) = (2, 3), \\ f_G(a3) = (3, 4), f_G(a4) = (4, 5), \\ f_G(a10) = (5, 1), \end{cases}$$

VÉRTICES DISYUNTOS

Definición: Dos o más subgrafo de un grafo G son vértices-disyuntos, si no poseen vértices comunes.

Ejemplo



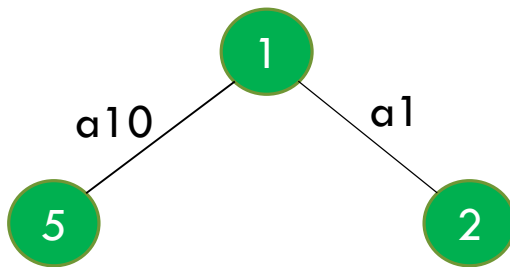
Grafo G

$$\begin{aligned} G &= (V, A, f) \\ G(V) &= \{1, 2, 3, 4, 5\} \\ f(A) &= \{a1, a2, a3, a4, a5, a6, a7, a8, a9, a10\} \\ F_G &= \left\{ \begin{array}{l} f_G(a1) = (1, 2), f_G(a2) = (2, 3), \\ f_G(a3) = (3, 4), f_G(a4) = (4, 5), \\ f_G(a5) = (5, 2), f_G(a6) = (2, 4), \\ f_G(a7) = (4, 1), f_G(a8) = (1, 3), \\ f_G(a9) = (3, 5), f_G(a10) = (5, 1), \end{array} \right. \end{aligned}$$

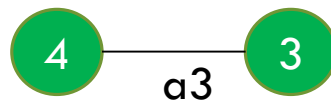




VÉRTICES DISYUNTOS



Subgrafo H3



Subgrafo H4

$$\begin{aligned} G &= (V, A, f) \\ G(V) &= \{1, 2, 5\} \\ f(A) &= \{a1, a10\} \end{aligned}$$

$$F_G = \left[f_G(a1) = (1, 2), f_G(a10) = (5, 1) \right]$$

$$\begin{aligned} G &= (V, A, f) \\ G(V) &= \{3, 4\} \\ f(A) &= \{a3\} \end{aligned}$$

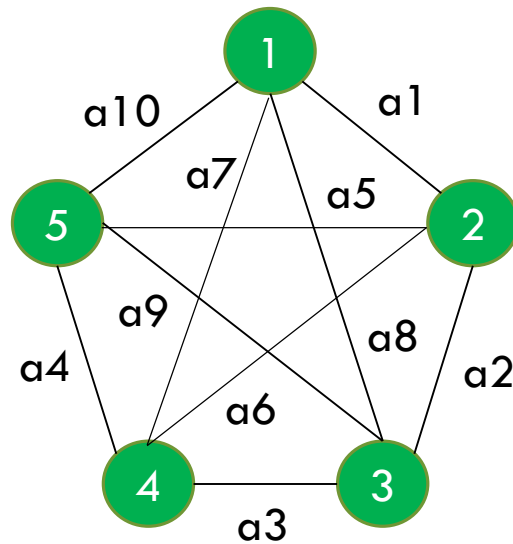
$$F_G = \left[f_G(a3) = (3, 4) \right]$$

El subgrafo H3 es vértice-disyunto del subgrafo H4

ARISTAS DISYUNTAS

Definición: Dos o más subgrafo de un grafo G son aristas-disyuntos, si ellos no poseen aristas comunes.

Ejemplo



Grafo G

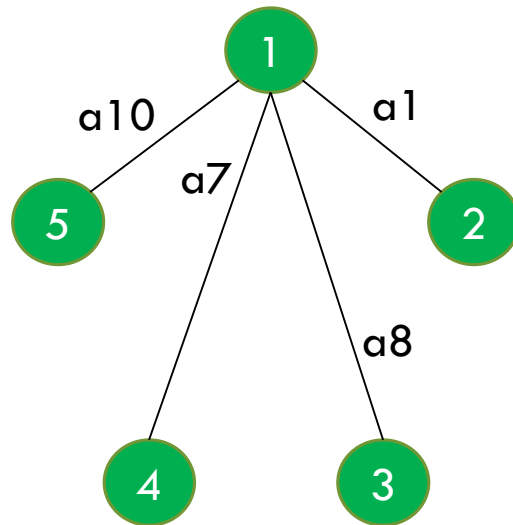
$$\begin{aligned} G &= (V, A, f) \\ G(V) &= \{1, 2, 3, 4, 5\} \\ f(A) &= \{a1, a2, a3, a4, a5, a6, a7, a8, a9, a10\} \\ F_G &= \left\{ \begin{array}{l} f_G(a1) = (1, 2), f_G(a2) = (2, 3), \\ f_G(a3) = (3, 4), f_G(a4) = (4, 5), \\ f_G(a5) = (5, 2), f_G(a6) = (2, 4), \\ f_G(a7) = (4, 1), f_G(a8) = (1, 3), \\ f_G(a9) = (3, 5), f_G(a10) = (5, 1), \end{array} \right. \end{aligned}$$





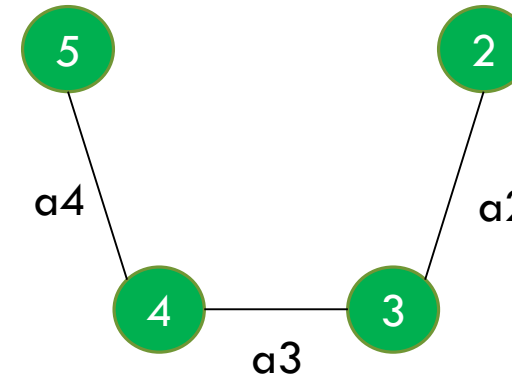
ARISTAS DISYUNTAS

Los subgrafos H5 y H6 son aristas-disyuntas ya que no tienen aristas en comunes



Subgrafo H5

$$\begin{aligned} G &= (V, A, f) \\ G(V) &= \{1, 2, 3, 4, 5\} \\ f(A) &= \{a1, a7, a8, a10\} \\ F_G &= \begin{cases} f_G(a1) = (1, 2), & f_G(a5) = (5, 2), \\ f_G(a7) = (4, 1), & f_G(a8) = (1, 3), \\ f_G(a10) = (5, 1), \end{cases} \end{aligned}$$



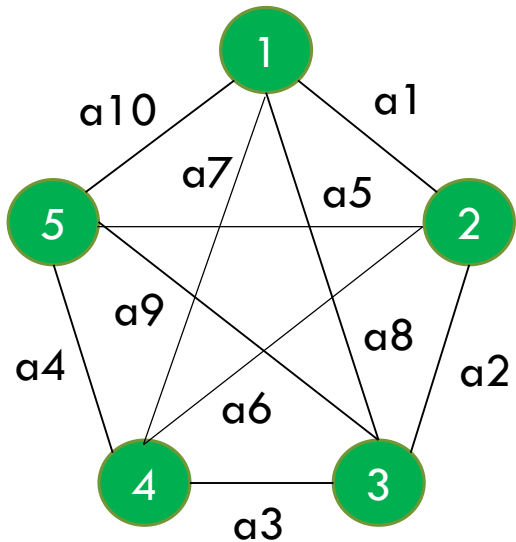
Subgrafo H6

$$\begin{aligned} G &= (V, A, f) \\ G(V) &= \{2, 3, 4, 5\} \\ f(A) &= \{a2, a3, a4\} \\ F_G &= \begin{cases} f_G(a2) = (2, 3), & f_G(a3) = (3, 4), \\ f_G(a4) = (4, 5), \end{cases} \end{aligned}$$

RESTANTE AL SUPRIMIR UN CONJUNTO DE VÉRTICES

Definición: Sea G un grafo con $|V| \geq 2$ y sea $V' \subseteq V$, la operación supresión de V' consiste en suprimir de G los vértices de V' y las aristas incidentes en ellos. Se denota $G - \{V'\}$.

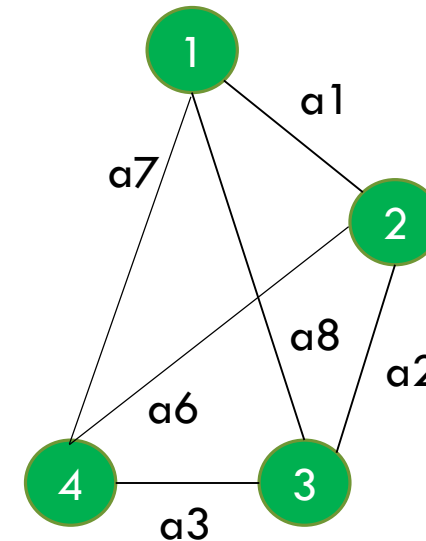
Ejemplo: $G - \{5\}$.



Grafo G

$G = (V, A, f)$
 $G(V) = \{1, 2, 3, 4, 5\}$
 $f(A) = \{a1, a2, a3, a4, a5, a6, a7, a8, a9, a10\}$

$F_G = \left\{ \begin{array}{l} f_G(a1) = (1, 2), f_G(a2) = (2, 3), \\ f_G(a3) = (3, 4), f_G(a4) = (4, 5), \\ f_G(a5) = (5, 2), f_G(a6) = (2, 4), \\ f_G(a7) = (4, 1), f_G(a8) = (1, 3), \\ f_G(a9) = (3, 5), f_G(a10) = (5, 1), \end{array} \right.$



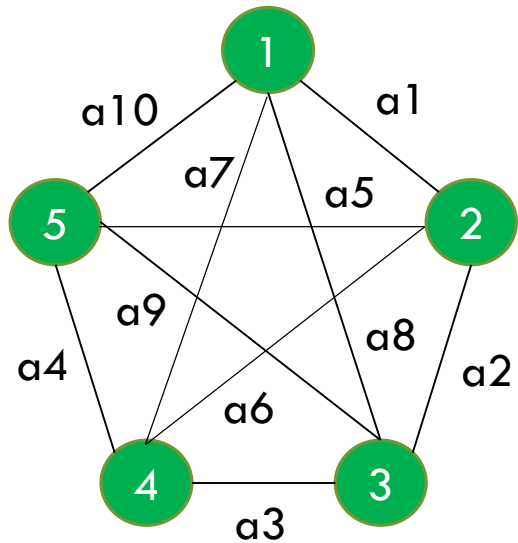
$G = (V, A, f)$
 $G(V) = \{1, 2, 3, 4\}$
 $f(A) = \{a1, a2, a3, a4, a6, a7, a8, a9, a10\}$

$F_G = \left\{ \begin{array}{l} f_G(a1) = (1, 2), f_G(a2) = (2, 3), \\ f_G(a3) = (3, 4), f_G(a6) = (2, 4), \\ f_G(a7) = (4, 1), f_G(a8) = (1, 3), \end{array} \right.$

SUPRIMIR UN CONJUNTO DE ARISTAS

Definición: Si $A' \in A$, la operación supresión de A' consiste en suprimir de G las aristas de A' . se denota $G - \{A'\}$

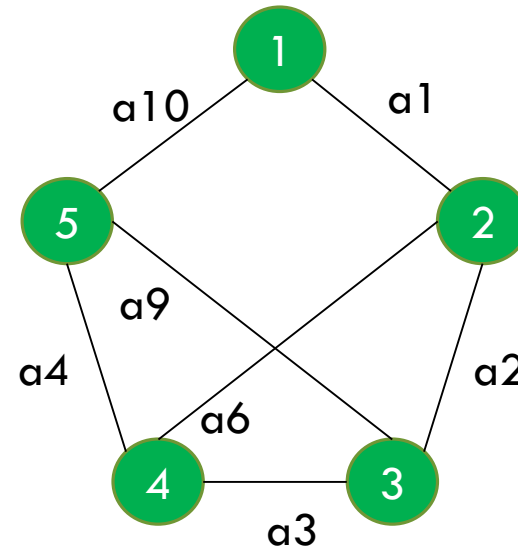
Ejemplo: $G - \{a5, a7, a8\}$.



Grafo G

$G = (V, A, f)$
 $G(V) = \{1, 2, 3, 4, 5\}$
 $f(A) = \{a1, a2, a3, a4, a5, a6, a7, a8, a9, a10\}$

$F_G = \left[\begin{array}{l} f_G(a1) = (1, 2), f_G(a2) = (2, 3), \\ f_G(a3) = (3, 4), f_G(a4) = (4, 5), \\ f_G(a5) = (5, 2), f_G(a6) = (2, 4), \\ f_G(a7) = (4, 1), f_G(a8) = (1, 3), \\ f_G(a9) = (3, 5), f_G(a10) = (5, 1), \end{array} \right.$



$G = (V, A, f)$
 $G(V) = \{1, 2, 3, 4, 5\}$
 $f(A) = \{a1, a2, a3, a4, a6, a9, a10\}$

$F_G = \left[\begin{array}{l} f_G(a1) = (1, 2), f_G(a2) = (2, 3), \\ f_G(a3) = (3, 4), f_G(a4) = (4, 5), \\ f_G(a6) = (2, 4), f_G(a9) = (3, 5), \\ f_G(a10) = (5, 1), \end{array} \right.$