
Single- and Three-Phase Power Measurements

Objective

The objectives of this experiment are to become familiar with single- and three-phase circuits. In particular, students will be exposed to instantaneous and average power as well as the three-wattmeter and two-wattmeter methods of measuring three-phase power.

Introduction

In a single-phase circuit, the instantaneous voltage and current are expressed as

$$v_a = \sqrt{2} V_{rms} \cos(\omega t) \quad (1)$$

$$i_a = \sqrt{2} I_{rms} \cos(\omega t - \theta) \quad (2)$$

where V_{rms} and I_{rms} are the rms values of voltage and current respectively and θ is the impedance angle of the load. The radian frequency is related to the frequency in Hertz by $\omega = 2\pi f$. The instantaneous power is then the product of (1) and (2) or

$$p = 2V_{rms} I_{rms} \cos(\omega t) \cos(\omega t - \theta) \quad (3)$$

This expression may be expanded by trigonometric identities to yield

$$p = V_{rms} I_{rms} \cos(\theta) + V_{rms} I_{rms} \cos(2\omega t - \theta) \quad (4)$$

Note that the instantaneous power has a constant component (average power) which depends on the rms voltage, rms current, and power factor $\cos(\theta)$ as well as a time-varying term which has a frequency that is twice that of the source. In a three-phase circuit the total instantaneous power is the sum of the power from all phases and the time-varying terms add to zero (since each has a 120° phase delay from the other). Therefore, the instantaneous power in a three-phase circuit is constant and is equal to three times the first term on the right-hand side of (4).

Figure 1 shows a single-phase circuit with a standard laboratory wattmeter. Therein, the phasor notation is used for the source voltage and current corresponding to (1-2). As can be seen, the wattmeter is a four-terminal device consisting of voltage and current measurement elements. The current coil has a low impedance and therefore a low voltage drop. The voltage coil has a high impedance and conducts a negligible current. The wattmeter will display the average power which is the product of the magnitudes of the voltmeter and ammeter elements times the cosine

of the phase angle between them. In this single-phase circuit, the wattmeter will display the power

$$P = |\hat{V}_a| |\hat{I}_a| \cos(\angle \hat{V}_a - \angle \hat{I}_a) \quad (5)$$

This is equivalent to the average real power delivered to the load given by the first term on the right-hand side of (4) or

$$P = V_{rms} I_{rms} \cos(\theta) \quad (6)$$

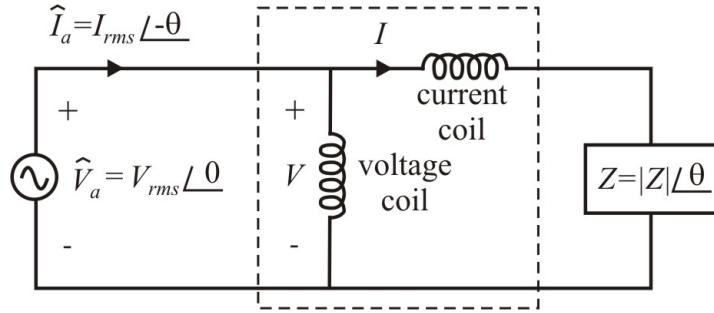


Figure 1. Single-phase power measurement.

For a three-phase system, a single-phase wattmeter can be connected in each phase as shown in Figure 2. Using this three-wattmeter configuration, the total real power can be obtained by adding the three wattmeter readings.

$$P = P_1 + P_2 + P_3 \quad (7)$$

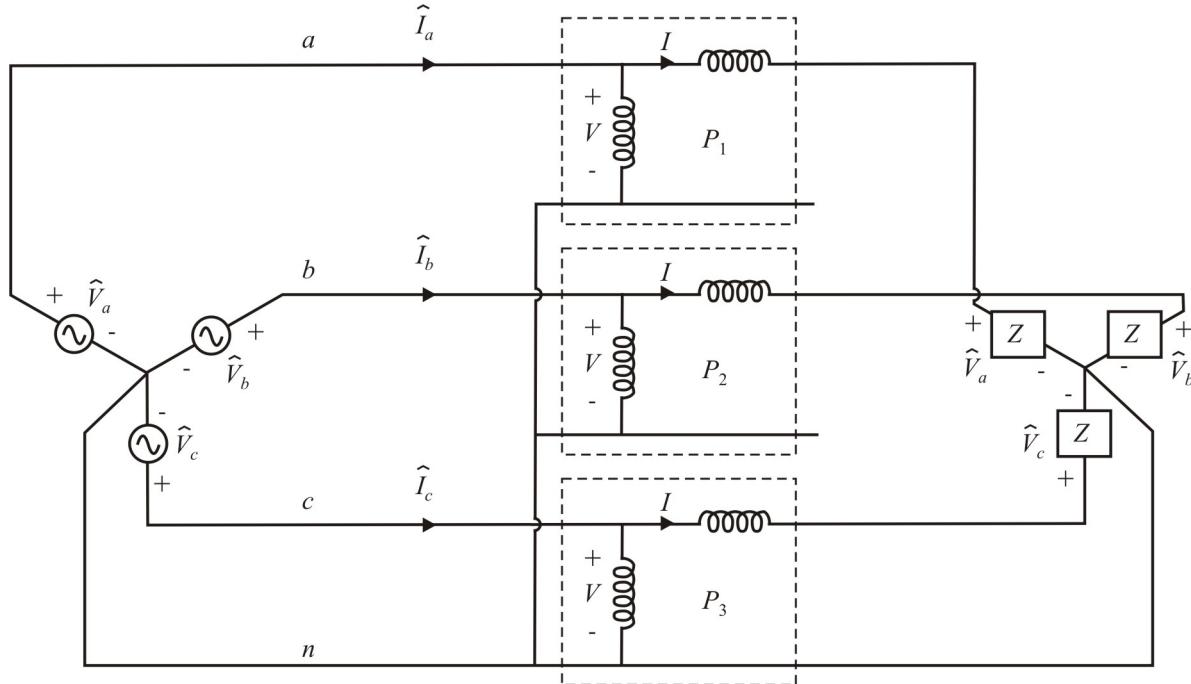


Figure 2. Three-phase power measurement using the three-watt-meter method.

As it turns out, it is more effective to use two wattmeters for measuring three-phase power. This two-wattmeter configuration is shown in Figure 3.

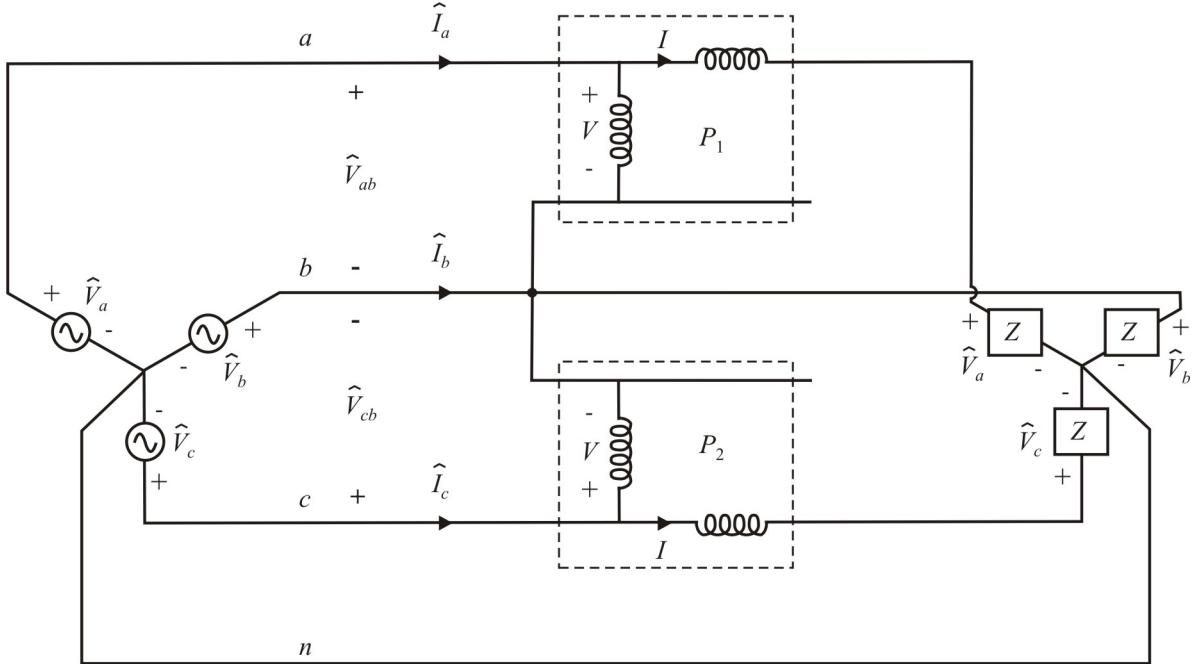


Figure 3. Three-phase power measurement using the two-watt-meter method.

It should be pointed out that, in some systems, the neutral connections are not available. In this case, the neutral connection to the source and load in Figures 2 and 3 can be removed. With the two-wattmeter configuration, each wattmeter uses the *b*-phase line as its reference. In phasor notation, the line-to-neutral voltages, *a*- to *b*-phase voltage, and *a*-phase current are given by

$$\hat{V}_a = V_{rms} \angle 0 \quad (8)$$

$$\hat{V}_b = V_{rms} \angle -120^\circ \quad (9)$$

$$\hat{V}_c = V_{rms} \angle 120^\circ \quad (10)$$

$$\hat{V}_{ab} = V_{l-l} \angle 30^\circ \quad (11)$$

$$\hat{I}_a = I_{rms} \angle -\theta \quad (12)$$

where V_{l-l} is the RMS value of the line-to-line voltage which is related to the line-to-neutral voltage by

$$V_{l-l} = \sqrt{3} V_{rms} \quad (13)$$

These phasors are plotted in Figure 4.

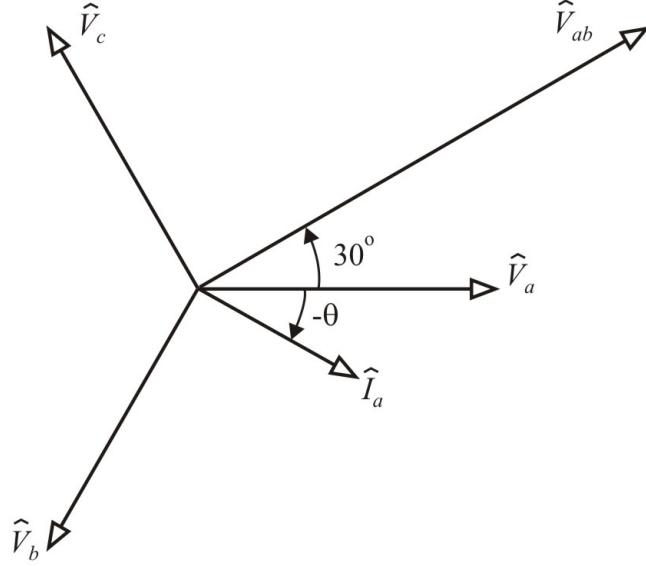


Figure 4. Voltage and current phasors in a three-phase power system.

Considering the connection diagram shown in Figure 3, the wattmeters readings are

$$P_1 = |\hat{V}_{ab}| |\hat{I}_a| \cos(\angle \hat{V}_{ab} - \angle \hat{I}_a) \quad (14)$$

$$P_2 = |\hat{V}_{cb}| |\hat{I}_c| \cos(\angle \hat{V}_{cb} - \angle \hat{I}_c) \quad (15)$$

As can be seen from Figure 4, and determined by combining (11-14)

$$P_1 = V_{l-l} I_{rms} \cos(\theta + 30^\circ) \quad (16)$$

Similarly,

$$P_2 = V_{l-l} I_{rms} \cos(\theta - 30^\circ) \quad (17)$$

Adding the two wattmeter readings yields

$$P_1 + P_2 = V_{l-l} I_{rms} [\cos(\theta + 30^\circ) + \cos(\theta - 30^\circ)] = \sqrt{3} V_{l-l} I_{rms} \cos(\theta) \quad (18)$$

Taking the difference of the wattmeter readings yields

$$P_2 - P_1 = V_{l-l} I_{rms} [\cos(\theta - 30^\circ) - \cos(\theta + 30^\circ)] = V_{l-l} I_{rms} \sin(\theta) \quad (19)$$

Recalling that three-phase real and reactive power are given by

$$P = \sqrt{3} V_{l-l} I_{rms} \cos(\theta) \quad (20)$$

$$Q = \sqrt{3} V_{l-l} I_{rms} \sin(\theta) \quad (21)$$

It can be seen that

$$P = P_1 + P_2 \quad (22)$$

$$Q = \sqrt{3}(P_2 - P_1) \quad (23)$$

From this information, the apparent power and load impedance angle can be found using

$$VA = \sqrt{P^2 + Q^2} \quad (24)$$

$$\theta = \tan^{-1}\left(\frac{Q}{P}\right) \quad (25)$$

It should be noted from (22-25) that *a-b-c* phase sequence is assumed for this derivation. If the phase sequence is *a-c-b*, then the wattmeter readings in (23) should be reversed.

Laboratory Software

Figure 5 shows a screen-shot from the "LabPowerMeasurements" program used in this experiment. This program monitors the first three channels on the meter box and displays the rms voltage, rms current, and average power for each channel. The instantaneous voltages and currents as well as the instantaneous power from all three channels are also plotted in the oscilloscope displays. The total instantaneous power is also displayed along with the power from each channel. For different parts of the experiment, the data range will change. To accommodate these changes in the plots, the y-axis scale of any plot can be changed by clicking on the maximum or minimum y-axis number.

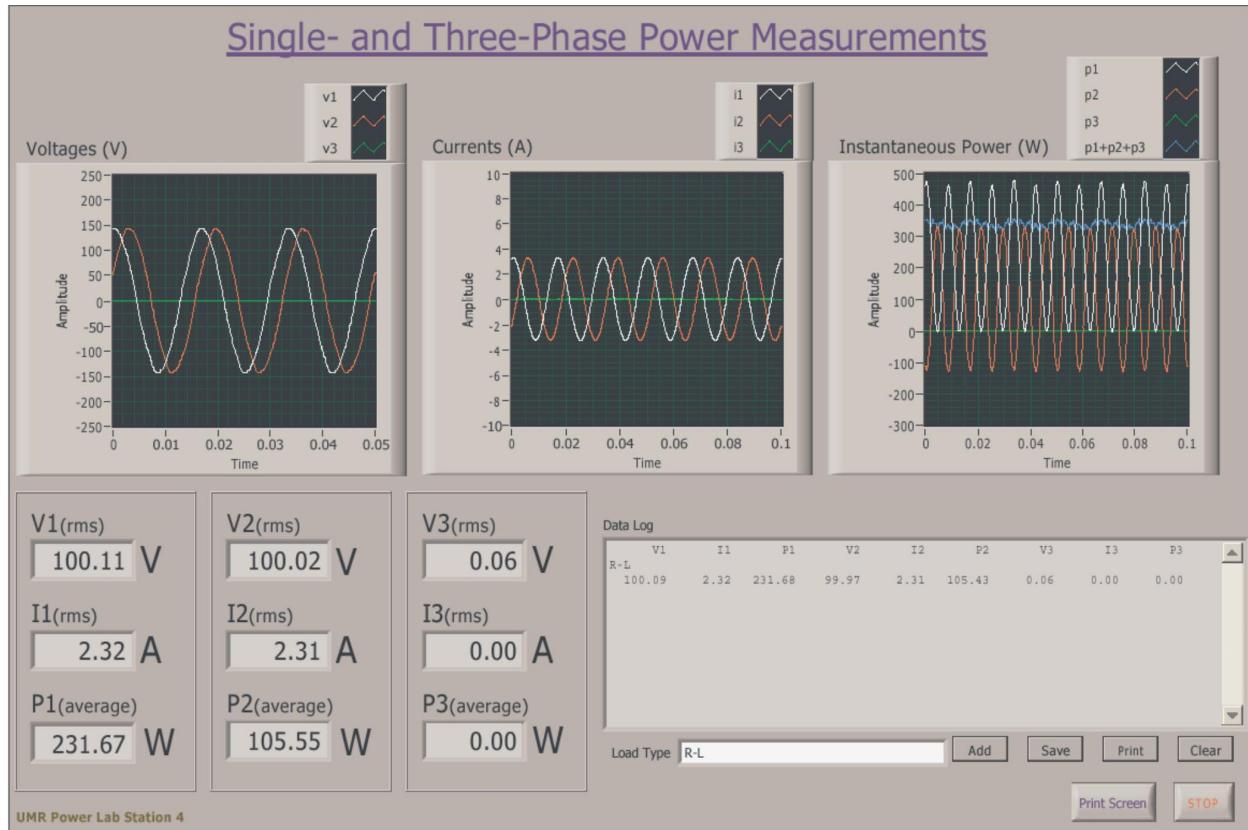


Figure 5. LabPowerMeasurements screen-shot.

By clicking the **Add** button the data from the rms voltage, rms current, and average power is saved in the data log. Students can type a title in the **Load Type** box which will also be logged along with the data. At the end of the experiment, the logged data can be saved to a file by clicking **Save** or the data can be printed by clicking **Print Screen**.

Laboratory Work

Start the "LabPowerMeasurements" program on the station computer. There will be a 10 second delay while the software checks the data acquisition board. Connect the single-phase circuit as shown in Figure 6. Use the four-conductor cables with black (B), red (R), white (W), and green (G) connectors as shown in Figure 6. Note that the red and white connections will not be used for the single-phase measurements and must be left floating (not connected). The load should be set as an R-L using the switch positions shown in Figure 7. Also, turn on the load box fans (the fans should be on the entire time the load box is being used). With the source voltage set to zero, energize the circuit by turning on the breaker on the source panel. Increase the voltage so that the source panel meter reads 100-V. Increase the voltage slowly and watch the rms current on the computer screen to make sure it is not excessive (the current should be less than 3-A for this test). Note that the single-phase power waveform shows a positive average value as well as a pulsating component at twice the line frequency (120-Hz) as predicted by (4). Print the values and waveforms by clicking **Print Screen**. Reduce the voltage to zero and turn off the source panel breaker.

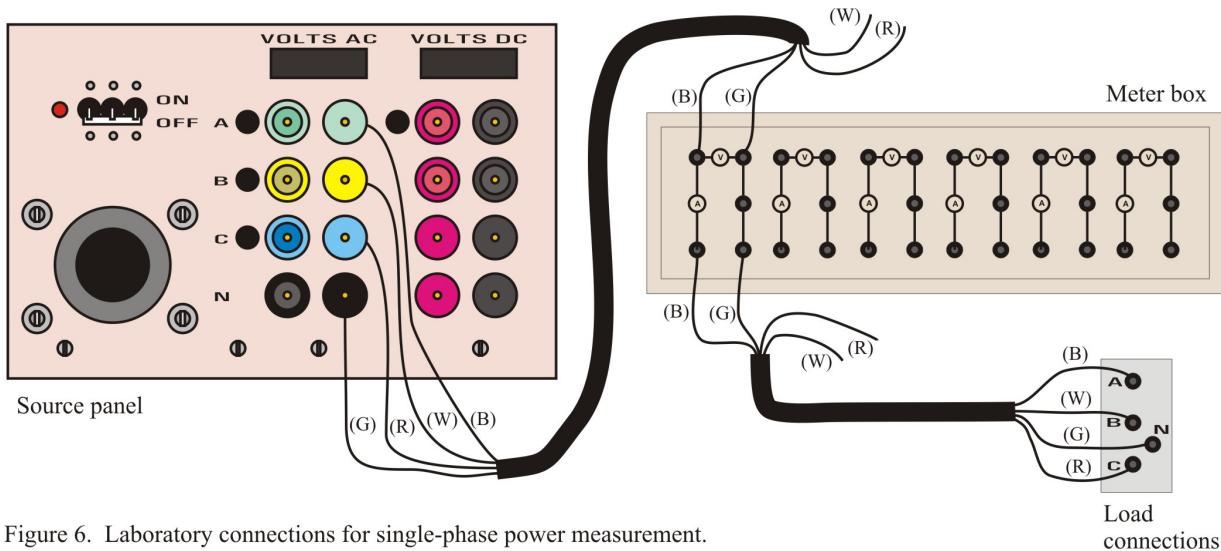


Figure 6. Laboratory connections for single-phase power measurement.

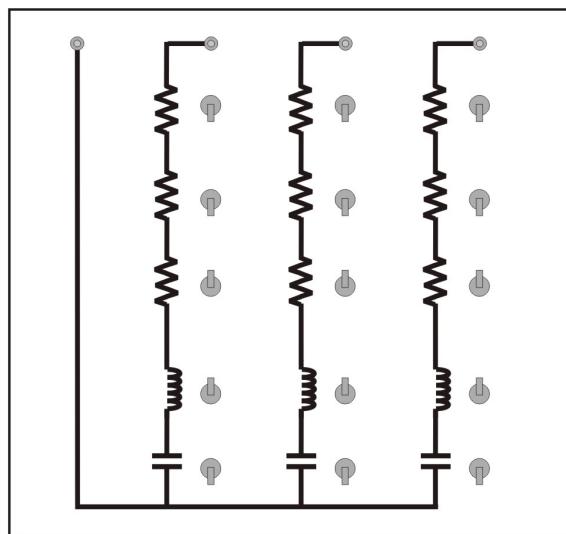


Figure 7. Load box switch settings for an R-L load.

Connect the three-phase system using the three-wattmeter method as shown in Figure 8. The R-L load from before, with settings shown in Figure 7, will be used. Notice that the three wattmeters are referenced to the system neutral. Also, the color sequence on the four-conductor cable black (B), white (W), red (R) is a common standard for low-voltage (208-V to 230-V) electrical connections. Switch on the power source breaker and increase the voltage to 100-V as read on the source panel meter. Notice from the power waveforms that the total instantaneous power is constant even though the power for the individual phases has a 120-Hz component. Print out the waveforms on the screen. Reduce the source voltage to zero and switch off the circuit breaker.

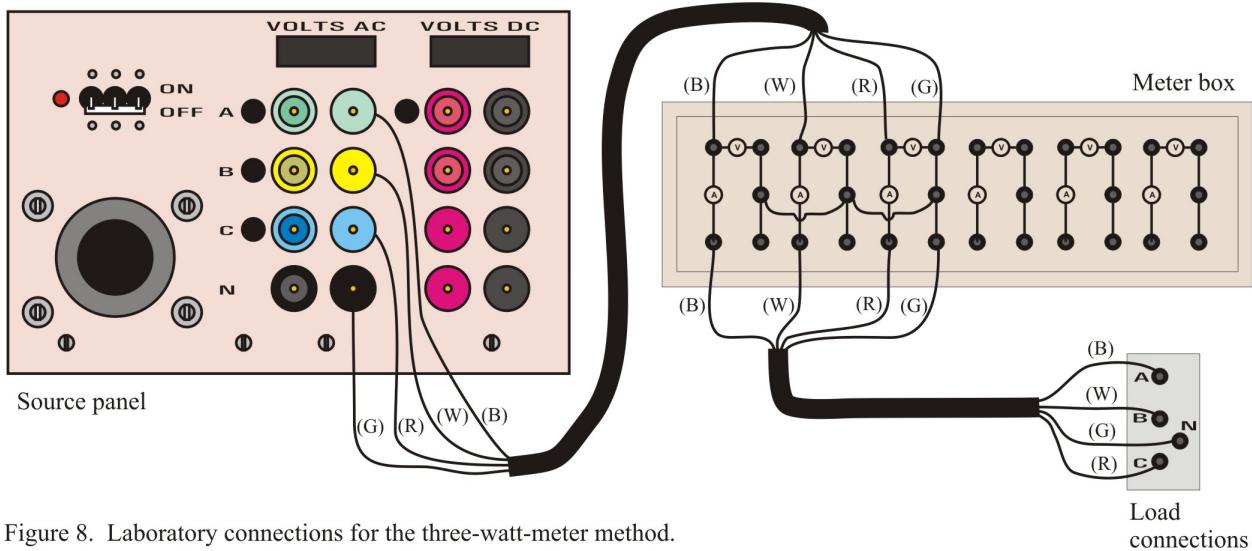


Figure 8. Laboratory connections for the three-watt-meter method.

Connect the three-phase system with the two-wattmeter connection as shown in Figure 9. Compare this configuration to that of Figure 3 to verify that they are the same. Leave the load settings in the R-L position as in Figure 7. Turn on the source circuit breaker and increase the voltage until the source panel meter reads 100-V. Enter "R-L Load" in the **Load Type** text box and log the data by clicking on **Add**. Reduce the voltage to zero and turn off the source circuit breaker.

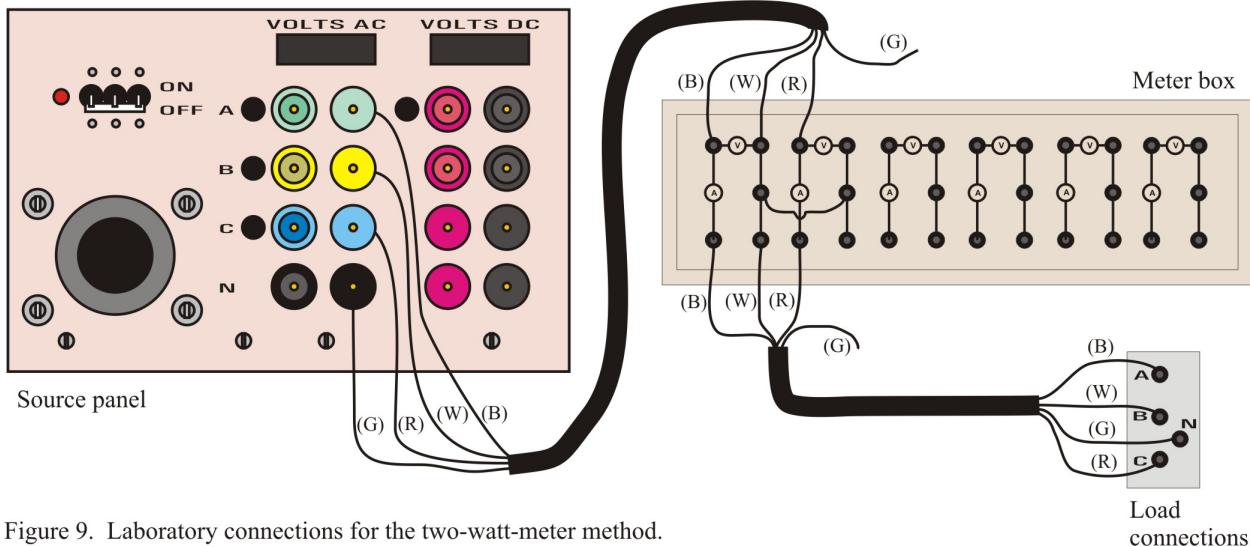


Figure 9. Laboratory connections for the two-watt-meter method.

Re-configure the load box for an R-C load by switching off the inductors and switching on the capacitors. The R-C switch settings are shown in Figure 10. Repeat the two-wattmeter test for the R-L load described above. Be sure to label the load type "R-C Load" before clicking **Add**.

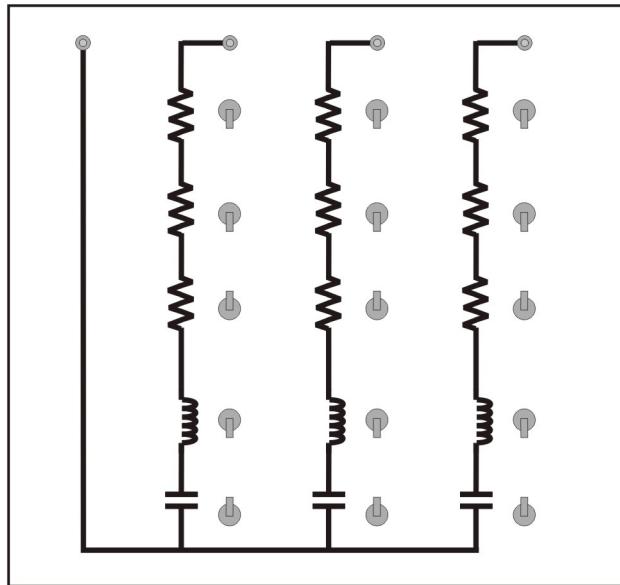


Figure 10. Load box switch settings for an R-C load.

Perform three more two-wattmeter tests on pure elements R, L, and C using the settings shown in Figures 11, 12, and 13, respectively. Save the data with file names "R Load", "L Load", and "C Load". **Be sure to reduce the voltage to zero and switch off the breaker before re-configuring the load box.** Use 100-V (line-to-line) for each test. Exceeding 100-V may exceed the rated current on the load box elements.

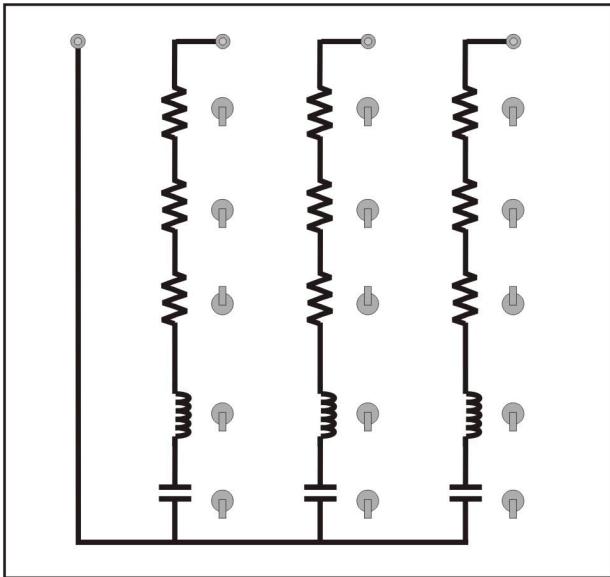


Figure 11. Load box switch settings for a resistive load.

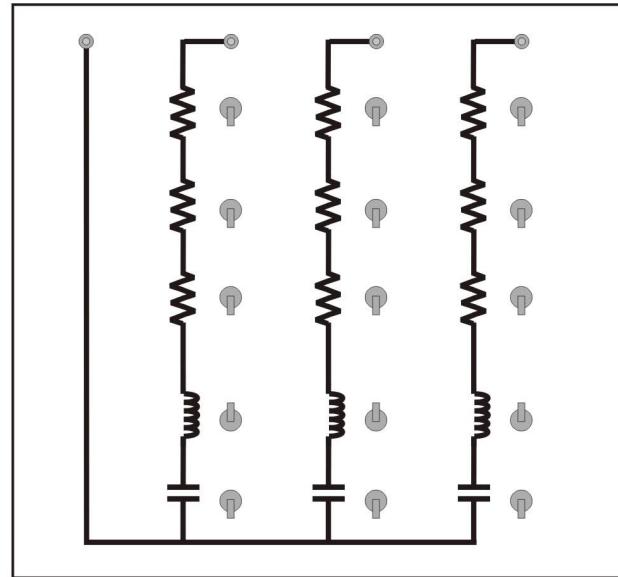


Figure 12. Load box switch settings for an inductive load.

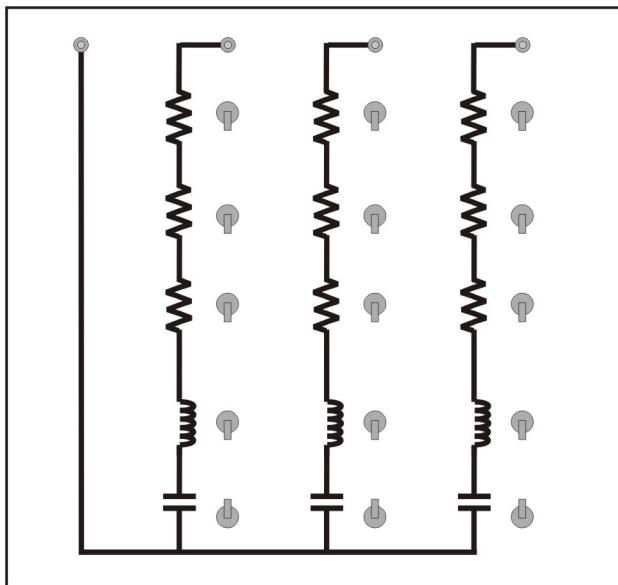


Figure 13. Load box switch settings for a capacitive load.

Calculations and Questions

1. For a three-phase system with voltages and currents of

$$v_a = \sqrt{2} V_{rms} \cos(\omega t)$$

$$v_b = \sqrt{2} V_{rms} \cos(\omega t - 120^\circ)$$

$$v_c = \sqrt{2} V_{rms} \cos(\omega t + 120^\circ)$$

$$i_a = \sqrt{2} I_{rms} \cos(\omega t - \theta)$$

$$i_b = \sqrt{2} I_{rms} \cos(\omega t - \theta - 120^\circ)$$

$$i_c = \sqrt{2} I_{rms} \cos(\omega t - \theta + 120^\circ)$$

Derive equations for the instantaneous power for each phase. Using these expressions, show that the total instantaneous power is $p = 3V_{rms}I_{rms} \cos(\theta)$. Discuss how the waveforms from the single-phase and three-wattmeter measurements validate the instantaneous power equations.

2. Show equations for obtaining the average real power P and the average reactive power Q based on the three-wattmeter measurements P_1 , P_2 , and P_3 as well as the voltage and current measurements V_{l-l} and I_{rms} . With this information, is it possible to determine the sign of the reactive power (and thus the power factor of the load).
3. Discuss in generic terms, the effects of load power factor and source phase sequence on wattmeter readings in the two-wattmeter method.
4. Using the two-wattmeter method measurements, calculate P , Q , VA , θ , and power factor (indicate leading or lagging) for each set of data using only the wattmeter readings. Compile these results in a table. Discuss how the real power, reactive power, power factor, and impedance angle for each case compare to theoretical expectations.
5. From the measurements on the pure elements R, L, and C, calculate the resistance, inductance, and capacitance. Also compute the series resistance for the L and C elements.