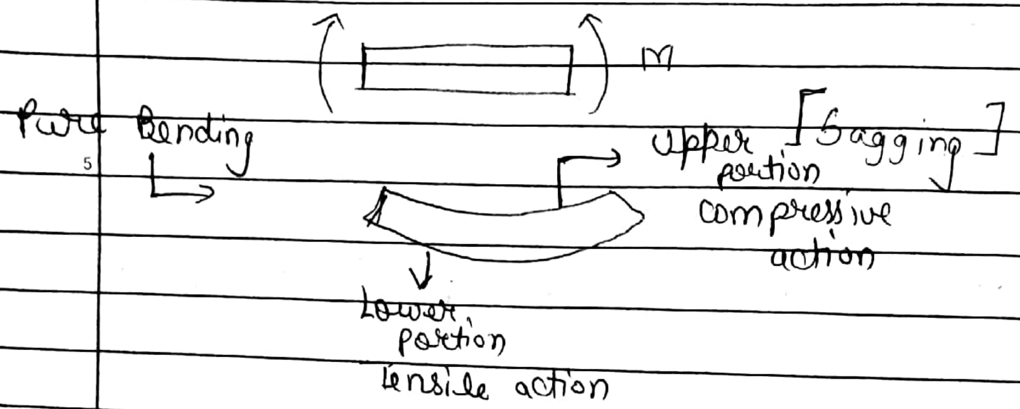
Isotropic → properties same all direction
Prismatic → Uniform cross section

★ Stresses in Beams :-



Pure Bending (Simple Bending) → If two couples (or moments) of equal magnitude & opposite direction act in the same longitudinal plane.

In pure Bending, shear force is zero & Bending moment is constant.

★ Theory of Pure Bending :-

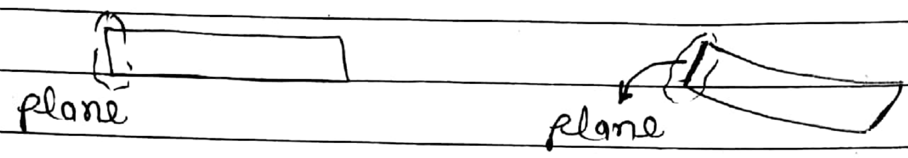
Assumptions →

(i) Material of beam is homogenous, isotropic & obeys hooke's law.

(ii) Initially, the beam is straight & all longitudinal filaments are bent into circular arcs with common centre of curvature which is very large as compared to cross-section.

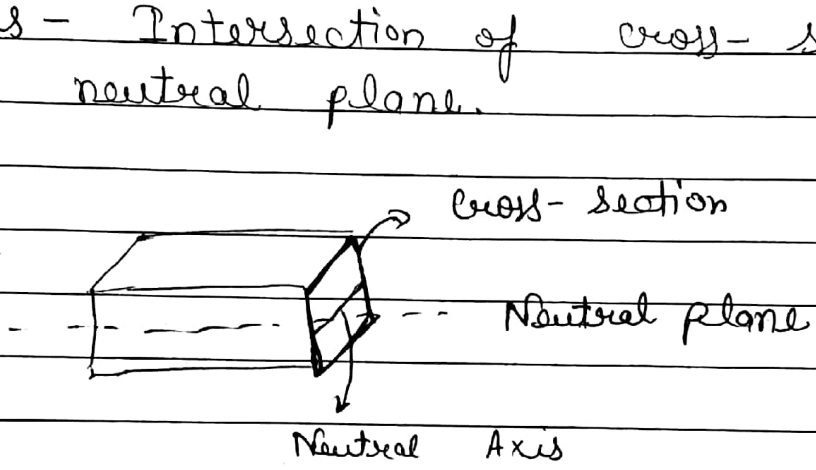
(iii) Beam is pristmatic & is symmatrical about a vertical plane through its centroid.

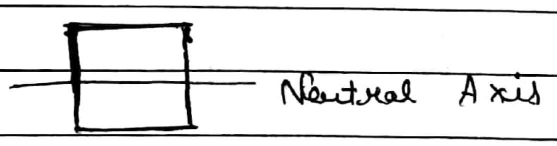
(iv) Transverse sections which are plane before bending remain plane after bending



Bending occurs about neutral ~~plane~~ Axis.

Neutral Axis – Intersection of cross-section & neutral plane.





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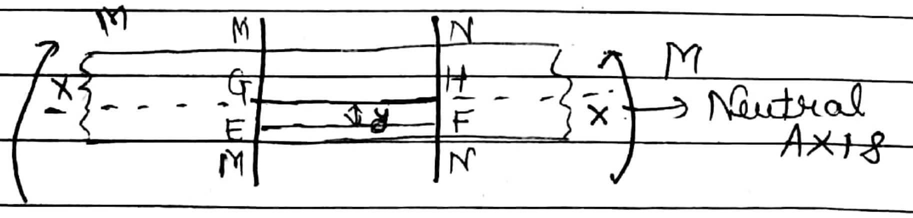
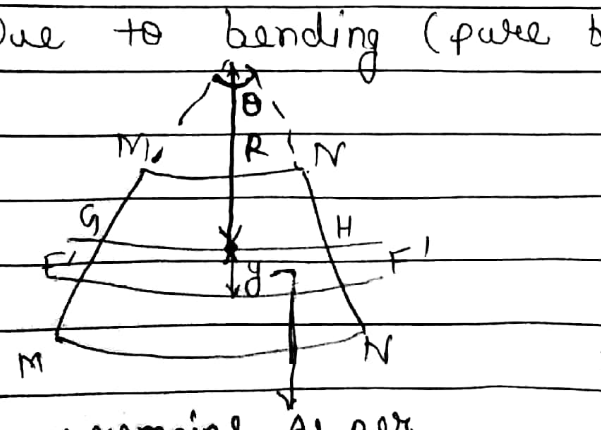


Symmetrical Prismatic

Due to bending (pure bending)



Strain in layer EF

$$= \frac{E'F' - EF}{EF}$$

$$E'F' = (R+y)\,\theta$$

$$GH = R\theta = EF$$

$$\therefore \quad \text{Strain} = \frac{(R+y)\theta - R\theta}{R\theta} = \frac{y}{R}$$

From Hooke's law →

$$\frac{\sigma}{E} = \frac{y}{R}$$

R → Radius of curvature

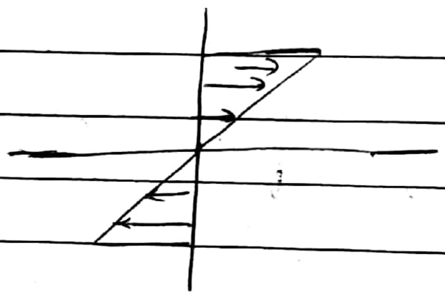
$$\Rightarrow \frac{\sigma}{y} = \frac{E}{R}$$

$$\Rightarrow \sigma = \frac{E}{R} y \quad \text{—} \ (1)$$

$$\sigma \propto y$$

At neutral axis $\sigma = 0$

At outermost layers $\sigma \rightarrow$ Max.



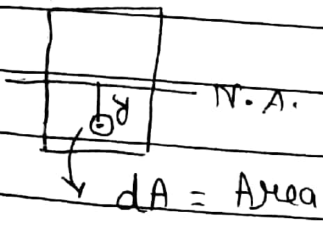
$\sigma \rightarrow +ve$

↳ Tensile

Q20 Why I - section is preferred than square one

Ans Since, outer surface bears more load.

Another view of the same beam →



↳ dA = Area of element

Force on the elemental area dA

$$= \int \sigma \, dA$$

$$= \int \frac{E}{R} y \, dA = \frac{E}{R} \int y \, dA$$

Area Moment → Area × ⊥r distance

$\int y\,dA$ → Area Moment

Force generated = Force external

$$\Rightarrow \frac{E}{R}\int y\,dA = 0$$

$$\Rightarrow \int y\,dA = 0 \qquad \rightarrow y = 0$$

↓ centroid coinciding with neutral axis

External moment = Moment due to force
↳ Force × distance

$$M = \frac{E}{R}\int y\,dA \cdot y$$

$$M = \frac{E}{R}\int y^2\,dA$$

Let $I = \int y^2\,dA$

= Area moment of inertia about neutral axis → Represents resistance to deformation

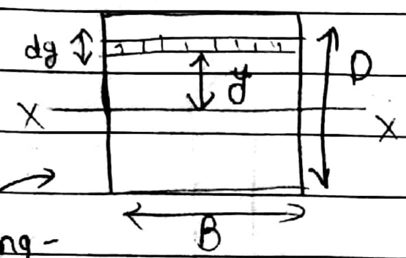
$$M = \frac{E}{R} I$$

$$\Rightarrow \frac{M}{I} = \frac{E}{R} \qquad — (2)$$

From (1) & (2)

Bending eqn →
$$\boxed{\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}}$$

#



Rectangular

$dA = B\,dy$

Area moment = $By\,dy$

$$I = \int_{-D/2}^{D/2} By^2\,dy$$

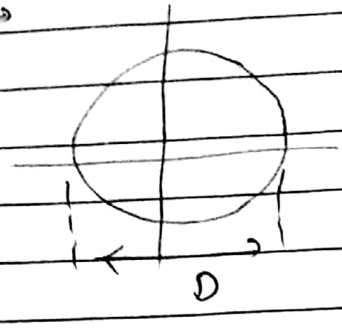
$$I_{xx} = \frac{BD^3}{12}$$

$$I_{yy} = \frac{DB^3}{12}$$

In real life cases → $M_{max} \rightarrow$

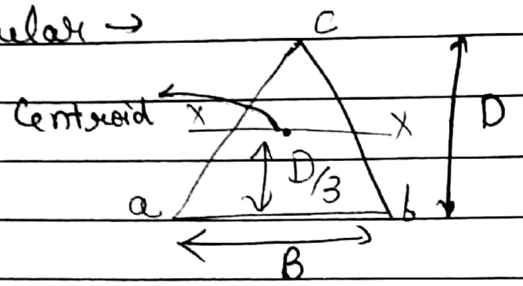
Bending eq^n → $\boxed{\dfrac{M_{max}}{I} = \dfrac{\sigma_{max}}{y_{max}} = \dfrac{E}{R}}$

Circular →



$$I_{xx} = I_{yy} = \frac{\pi}{64} D^4$$

Triangular →



$$I_{xx} = \frac{BD^3}{36}$$

Parallel Axis :-

$$I_{AB} = I_{xx} + Ah^2$$

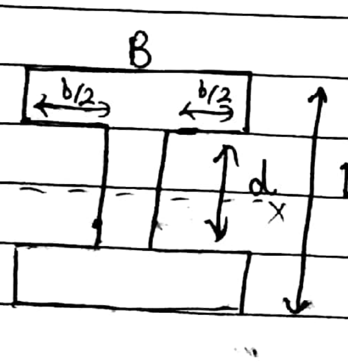
A → Area of Cross-section

h → height of shift of Axis from neutral axis

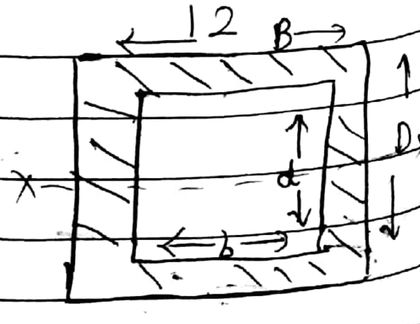
'I' about ab base in triangular section $\quad I_{ab} = \dfrac{BD^3}{36} + \dfrac{BD}{2}\left(\dfrac{D}{3}\right)^2$

$$= \frac{BD^3}{12}$$

Q (i)



(ii)



Ans I_{xx} in both cases,

$$I_{xx} = \frac{BD^3}{12} - \frac{bd^3}{12}$$

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★ Section Modulus:-

$$Z = \frac{I}{y_{max}}$$

$$\frac{M}{I} = \frac{\sigma}{y}$$

$$\Rightarrow M = \sigma Z$$

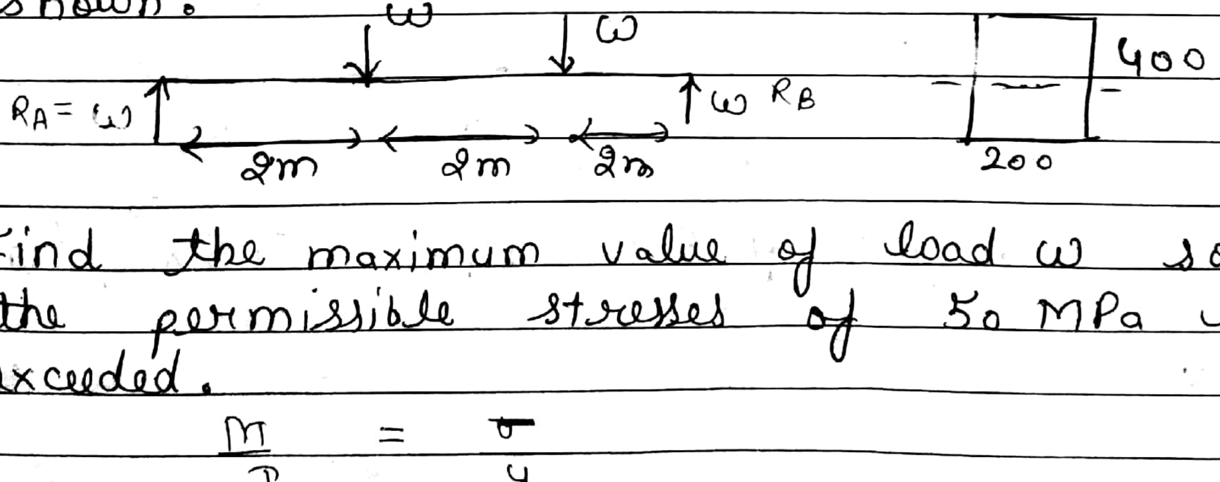
$$M \propto Z$$

↳ more section modulus → more strength

For circular section →

$$Z = \frac{\frac{\pi}{64} D^4}{D/2} = \frac{\pi}{32} D^3$$

Q. A rectangular beam of cross-section 200 mm × 400 mm is used on a span of 6 m as shown:



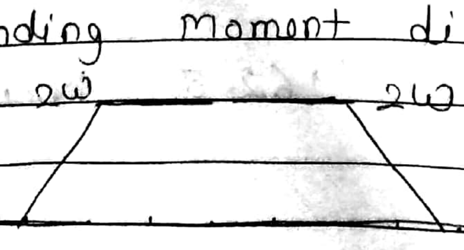
Find the maximum value of load w so that the permissible stresses of 50 MPa is not exceeded.

Ans

$$\frac{M}{I} = \frac{\sigma}{y}$$

$$\frac{M_{max}}{I} = \frac{\sigma_{max}}{y_{max}}$$

By Bending moment diagram →



$M_{max} = 2W$

Ξ & $\sigma = 50\ N/mm^2$

$I = \dfrac{200 \times 400^3}{12}$

$y_{max} = \dfrac{400}{2} = 200$

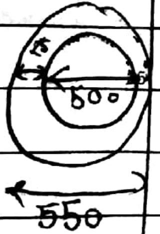
$\therefore \dfrac{2W}{\frac{200 \times 400^3}{12}} = \dfrac{50}{200}$

$\Rightarrow \dfrac{24W}{400^3} = 50$

$\Rightarrow W = 1.33\ KN$

Q ⇒ A water-pipe 500 mm internal diameter and 25 mm thickness, runs fully simply supported over a 20 meter. Determine maximum bending stress induced in the pipe if weight of water & pipe is taken into consideration
Specific weight of Steel $(w_s) = 75\ KN/m^3$
Specific weight of water $(w_w) = 10\ KN/m^3$

Ans



Weight of Steel Pipe $= 75 \times 10^3 \times \dfrac{\pi}{4}\left((0.55)^2 - (0.5)^2\right) \times 1$

$= 3090\ N/m$ → UDL

Weight of water $= 10 \times 10^3 \times \dfrac{\pi}{4} \times (0.5)^2$ ↳ inner diameter

$= 1962.5\ N/m$

$w = 3090 + 1962.5\ N/m$

Allowable stress → Allowable strength per unit Area
↳ property of a material
(~~applied~~ induced stress isn't property of material)

We know,

$$M = \frac{wl^2}{8} = 25262 \text{ N-m}$$

And, $$\frac{M}{I} = \frac{\sigma}{y}$$

$$I = \frac{\pi}{64}\left[(0.55)^4 - (0.5)^4\right] =$$

$$\& \; y = \frac{0.55}{2}$$

$$\text{So, } \sigma = 48.8 \text{ MPa}$$

Q. Three Beams have same length, same allowable stress & same bending moment. The cross-section of beams are square, one rectangular with depth twice the width and circle. Determine the ratio of weights of circular & rectangular beams w.r.t. square beam.

Ans
We know,

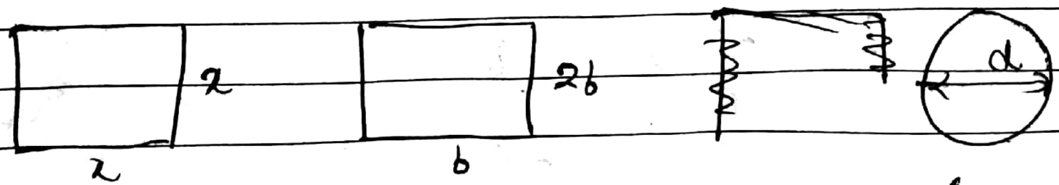
$$M = \sigma Z$$

$$\Rightarrow M \to \text{same}$$

$$\sigma \to \text{same}$$

$$\therefore Z \to \text{same}$$

$$\text{So, } Z_S = Z_R = Z_C$$



$$Z_S = \frac{\frac{x^4}{12}}{x/2} = \frac{x^3}{6}, \quad Z_R = \frac{b(2b)^3/12}{2b/2} = \frac{2b^3}{3}$$

$$Z_C = \frac{\pi/64\, d^4}{d/2} = \frac{\pi}{32} d^3$$

Allowable stress → same ~~m~~aterial → same density & same specific weight

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$$\Rightarrow \frac{x^3}{6} = \frac{2b^3}{3} \quad \text{①} \Rightarrow b = 0.63x \quad \text{②}$$

$$\& \quad \frac{x^3}{6} = \frac{\pi}{32} d^3 \quad \text{②} \Rightarrow d = 1.193x \quad \text{③}$$

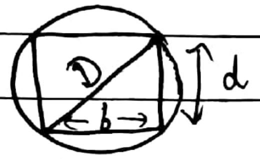
And, $\frac{W_C}{W_S} = \frac{A_C}{A_S} = \frac{\pi/4\, d^2}{x^2} = 1.118$

& $\frac{W_R}{W_S} = \frac{A_R}{A_S} = \frac{b(2b)}{x^2} = \frac{2b^2}{x^2} = 0.793$

Q Find the dimensions of the strongest rectangular beam that can be cut out of ~~stock of~~ log of 200 mm diameter.

Ans

$$D = \sqrt{b^2 + d^2}$$



$$M = \sigma Z$$

$$\Rightarrow M \propto Z$$

Z of rectangular part →

$$Z = \frac{I}{y} = \frac{bd^2}{6} = \frac{b(D^2 - b^2)}{6}$$

For maximum strength, [Z → maximum]

$$\frac{dZ}{db} = 0$$

$$\Rightarrow \frac{D^2}{6} - \frac{3b^2}{6} = 0$$

$$\Rightarrow D^2 = 3b^2$$

$$b = 0.577\, D$$

$$= 115.4 \text{ mm}$$

$$\therefore d = 163.2 \text{ mm}$$

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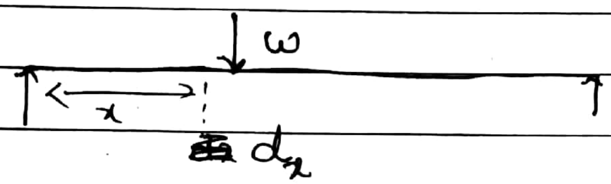
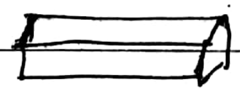
☆ Beam of uniform strength :-

A beam in which every section along the longitudinal axis has the same maximum bending stress is known as beam of uniform strength.

Case 1:- Beam of uniform strength with constant width





$b \rightarrow$ width

Let 'd_x' be depth at length 'x'

Moment of resistance $(M) = \sigma Z_x$

$$= \frac{\sigma b d_x^2}{6}$$

For point load,

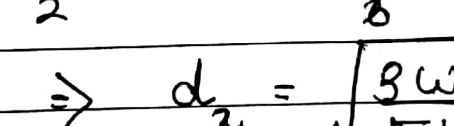
$$M_x = \frac{W}{2} \cdot x$$

$$\Rightarrow \frac{W}{2} \cdot x = \frac{\sigma b d_x^2}{6}$$

$$\Rightarrow d_x = \sqrt{\frac{3W}{\sigma b} x} = K\sqrt{x}$$

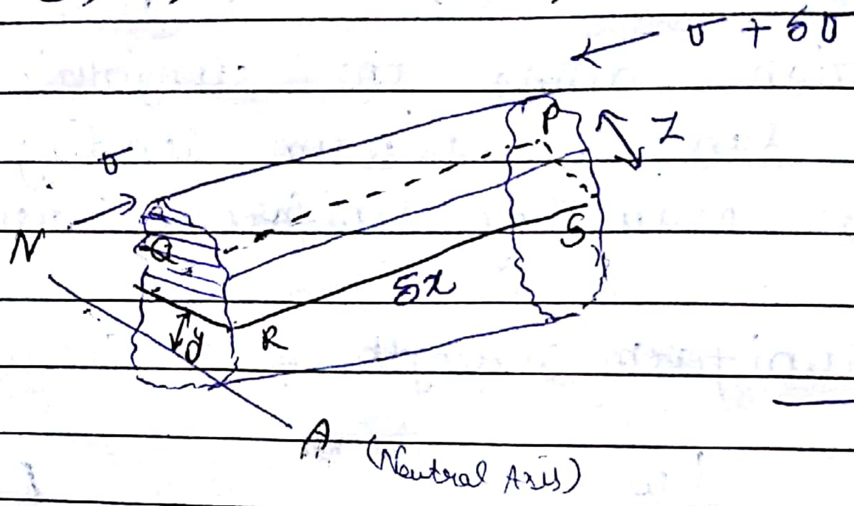
At $x = 0$, $d = 0$

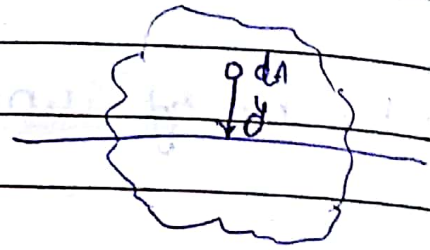
$x = \frac{l}{2}$, $d = \sqrt{\frac{3W}{2\sigma b}}$



← Variation of depth with length (or such a cross-section)

☆ Shear stresses in beams:-





Let 'σ' & 'σ + δσ' be stresses on left & right side respectively.

Force on left side $= \sigma \cdot dA$

Force on Right side $= (\sigma + \delta\sigma)\, dA$

Net force on dA

$$= \cancel{\sigma dA} - (\sigma + \delta\sigma)\, dA - \sigma\, dA$$
$$= \delta\sigma\, dA$$

$$\tau \cdot dx z = \int \frac{\delta M}{I}\, y\, dA$$

$\tau \rightarrow$ ~~Torque~~ Average shear stress

$$\Rightarrow \tau = \frac{\delta M}{\delta x} \cdot \frac{1}{z I} \int y\, dA$$

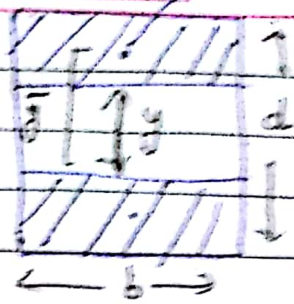
$$\Rightarrow \tau = F \cdot \frac{1}{z I} A \bar{y}$$

Here, $A \rightarrow$ Area of cross-section cut off by line parallel to the neutral axis

$\bar{y} \rightarrow$ Distance of centroid of Area 'A' from neutral Axis.

(a) For rectangular section $\qquad A \qquad \bar{y}$

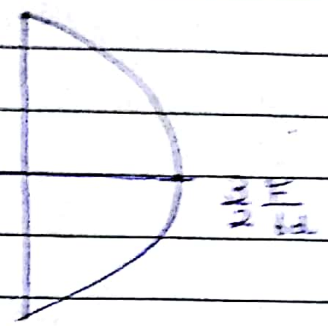
$$\tau = F \cdot \frac{1}{b\left(\frac{bd^3}{12}\right)} \cdot A\bar{y}\left(\frac{d}{2}-y\right)b \cdot \left(\frac{\frac{d}{2}+y}{2}\right)$$



$$\Rightarrow \tau = \frac{6F}{bd^3}\left(\frac{d^2}{4}-y^2\right)$$

At $y = 0$,

$$\tau_{max} = \frac{3}{2}\frac{F}{bd} \qquad \text{(Max.)}$$

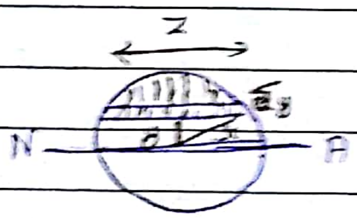


Average Shear Stress

$$\tau_{avg} = \frac{F}{bd} = \frac{\text{Force}}{\text{Area}}$$

$$\therefore \boxed{\tau_{max} = 1.5\,\tau_{avg}}$$

(b) For circular section :-



Area of elemental strip

$$= z\,\delta y$$

Area - Moment of elemental strip

$$= (z \cdot \delta y)\, y \qquad \text{— (1)}$$

And, $z = 2\sqrt{r^2 - y^2}$

$$z^2 = 4(r^2 - y^2)$$

Differentiate →

$$\Rightarrow 2z\,dz = -8y\,\delta y$$

$$\Rightarrow y\,\delta y = -\frac{z}{4}\,\delta z$$

Moment of Shaded Area about neutral axis (N.A)

limit of width varying ↓ z to 0.

$$A\bar{y} = \int_y^r z\left(-\frac{z}{4}\,\delta z\right)$$

$$= \int_z^0 -\frac{z^2}{4}\,dz$$

$$\therefore \quad A\bar{y} = \frac{z^3}{12}$$

So,

$$\tau = F \cdot \frac{1}{z \cdot I} \cdot \frac{z^3}{12}$$

$$= \frac{F z^2}{12 I}$$

$$= F \cdot \frac{1}{I} \cdot \frac{4(r^2 - y^2)}{12}$$

$$\tau = \frac{F}{3I}(r^2 - y^2)$$

At $y = 0$ (i.e. about NA)

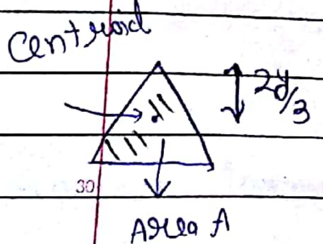
$$\tau_{max} = \frac{F r^2}{3I}$$

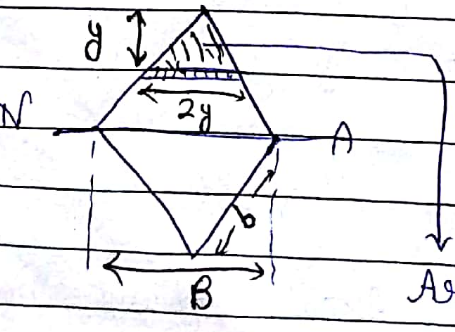
$$= \frac{F \cdot d^2/4}{3 \cdot \frac{\pi}{64} d^4}$$ $d \rightarrow$ diameter

$$\Rightarrow \tau_{max} = \frac{4}{3} \frac{F}{\left(\frac{\pi}{4} d^2\right)}$$

$$\boxed{\tau_{max} = \frac{4}{3} \tau_{avg}}$$ $\left[\tau_{avg} = \frac{\text{Force}}{\text{Area}} = \frac{F}{\left(\frac{\pi}{4} d^2\right)}\right]$

(c) Square with diagonal horizontal (Diamond Shape):-





$y \rightarrow$ distance of elemental strip from vertex

$2y \rightarrow$ width of strip

$$\text{Area } (A) = \frac{y(2y)}{2} = y^2$$

$$\bar{y} = \left(\frac{B}{2} - \frac{2y}{3}\right)$$

$$I = 2\left[\frac{B\cdot(B/2)^3}{12}\right] = \frac{B^4}{48}$$

Hence,

$$\tau = F \cdot \frac{1}{2y \cdot \frac{B^4}{48}} \cdot y^2\left(\frac{B}{2} - \frac{2y}{3}\right)$$

$$\tau = \frac{24Fy}{B^4}\left(\frac{B}{2} - \frac{2y}{3}\right)$$

$$\Rightarrow \tau = \frac{4Fy}{B^4}(3B - 4y)$$

About Neutral Axis i.e. $y = \frac{B}{2}$

$$\tau = \frac{2F}{B^2}$$

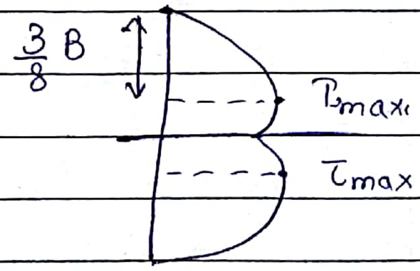
For maximum τ,

$$\frac{d\tau}{dy} = 0$$

Not at neutral axis

$$\Rightarrow y = \frac{3}{8}B$$

$$\therefore \tau_{max} = \frac{9F}{4B^2}$$



For Average,

$$\tau_{avg} = \frac{2F}{B^2}$$

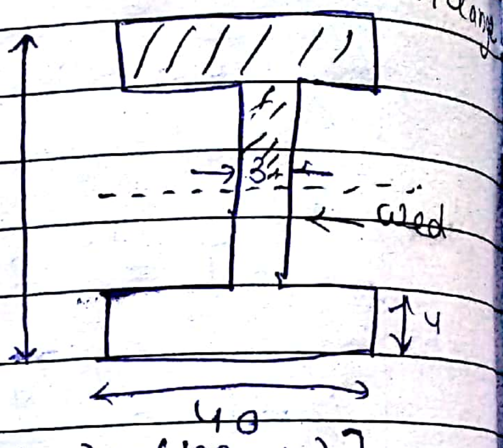
$$\boxed{\tau_{max} = \frac{9}{8}\tau_{avg.}}$$

Q. A 100mm x 40mm I-beam is subjected to shear force of 15 KN. Determine the transverse shear stress at:

(a) Neutral axis

(b) Top of wed

Compare it with beam stress
where web thickness = 3 mm
& flange thickness = 4 mm



Ans

$$\tau = \frac{F \cdot A\bar{y}}{Z\,I}$$

$F = 15\ KN$ (given)

At neutral axis ↳

$A\bar{y} \rightarrow$ Area moment of area under consideration

$$A\bar{y} = \left[(40\times4)\times\left(\frac{100-4}{2}\right) + (46\times3)\times\left(\frac{\frac{100}{2}-4}{2}\right)\right]$$

$$= \left[(40\times4)\times48 + (46\times3)\times23\right]$$

$Z = 3$

$$I = \left[\frac{40\times100^3}{12} - \frac{(40-3)\times(100-2\times4)^3}{12}\right]$$

At the top of web →

$$A\bar{y} = (40\times4)\times48$$

$I \rightarrow$ same

$\tau = 41.2\ MPa$

Q. A beam of rectangular cross-section 160 mm wide & 300 mm deep is of 4 m length & is loaded with a central point load of 50 KN. Determine the bending & shear stresses at top, 100 m & 40 mm from the neutral axis of section and at the neutral axis. Consider the bending moment at the mid-section of the beam. Also find the principal planes & principal stresses at these points.

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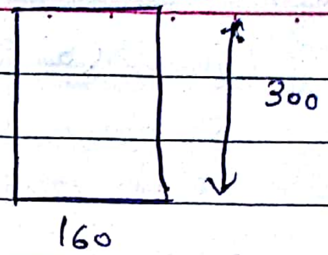
Ans ⇒ For Bending Stress



↳ $\frac{M}{I} = \frac{\sigma}{y}$

$\sigma = \frac{My}{I}$

$M = \frac{WL}{4} = \frac{50 \times 4}{4} = 50\ \text{KN-m}$

~~At top of beam~~

$I = \frac{bd^3}{12} = \frac{160 \times 300^3}{12} = 360 \times 10^6\ mm^4$

(a) At top of beam

$y = \frac{300}{2} = 150\ mm$

$\therefore \sigma_1 = \frac{50 \times 10^6 \times 150}{360 \times 10^6} = 20.83\ MPa$

(b) At ~~top~~ 100 from Neutral Axis (NA)

$y_2 = 100\ m$

$\sigma \propto y$

$\Rightarrow \frac{\sigma_2}{\sigma_1} = \frac{y_2}{y_1} \Rightarrow \sigma_2 = \sigma_1 \times \frac{y_2}{y_1}$

$= 20.83 \times \frac{100}{150} = 13.9\ MPa$

(c) At 40 mm from NA

$y_3 = 40$

$\therefore \sigma_3 = \sigma_1 \times \frac{y_3}{y_1} = 20.83 \times \frac{40}{150} = 5.55\ MPa$

(d) At NA, $y_4 = 0$

$\therefore \sigma_4 = 0$

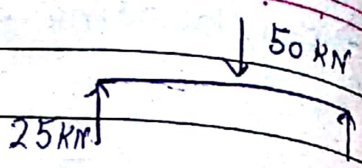
⇒ For Shear Stresses

$\tau = \frac{F \cdot A\bar{y}}{bI}$

(a) At top ~~$\bar{y} = 0$~~ $A = 0$

$\therefore \tau = 0$

(b) 100 mm from NA



$\cancel{F_2}$ F = 25 kN

$A\bar{y} = (50 \times 160) \times 125$

$z = 160$

$I = 360 \times 10^{6}$

$\therefore \tau_2 = 0.434$ MPa

(c) 40 mm from NA

$A\bar{y} = (160 \times 110) \times 95$

$z = 160$

$I = 360 \times 10^{6}$

$\tau_3 = 0.726$ MPa

(d) ~~At~~ NA

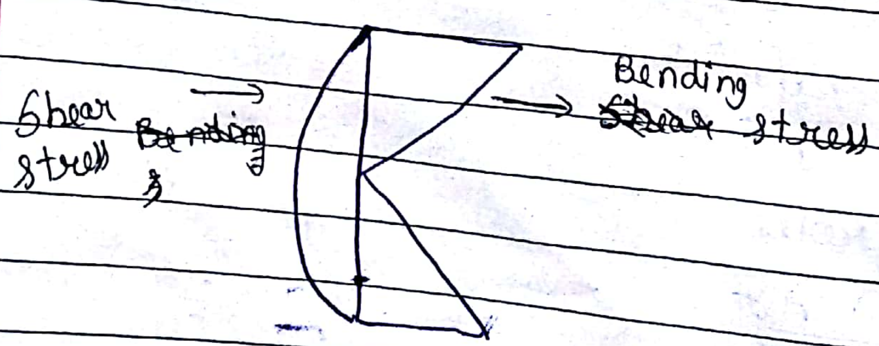
$\tau_4 = 0.78$ MPa

⇒ For Principal stresses

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2}$$

$\sigma_x = \sigma$

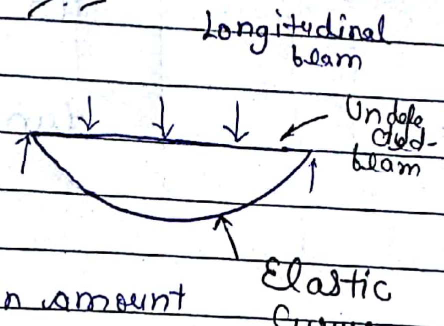
$\sigma_y = 0$



Note:- Bending moment → constant → Radius of Curvature same
→ Non-constant → 'R' varying

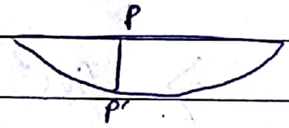
Slope & deflection of Beams

(i) Elastic Curve :-



Due to bending ~~sect~~ phenomenon of a longitudinal beam, when certain amount of load is applied onto it, beam ~~deflects~~ acts just to a form a curve named as ~~Bending~~ Elastic Curve.

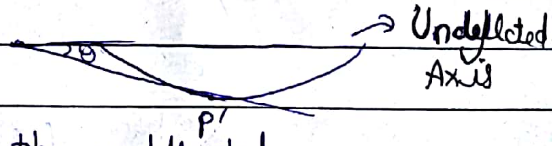
(ii) Deflection :-



Normal distance ~~b/w~~ from a point on a undeflected beam (P) to a point on ~~de~~ Elastic curve (P') is 'deflection'.

PP' → Deflection

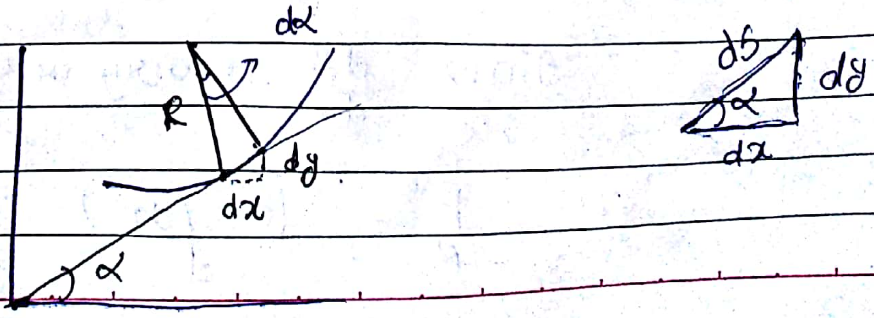
(iii) Slope :-



Slope is the angle formed by the tangent at P', with undeflected axis.

* Govering differential eq^n for deflection of beam :-

Assume a bending curve shown in x-y plane as follows:-



$$ds^2 = dx^2 + dy^2 \quad \text{--- (1)}$$

$$\tan\alpha = \frac{dy}{dx} \quad \text{--- (2)}$$

$$R\,d\alpha = ds$$

$$d\alpha = \frac{ds}{R} \quad \text{--- (3)}$$

Differentiating eq^n (2) w.r.t. x

$$\sec^2\alpha \frac{d\alpha}{dx} = \frac{d^2y}{dx^2}$$

$$\Rightarrow \sec^2\alpha \frac{ds}{R} \cdot \frac{1}{dx} = \frac{d^2y}{dx^2} \qquad \text{[From eq}^n \text{ (3)]}$$

$$\Rightarrow \sec^2\alpha \frac{ds}{dx} \cdot \frac{1}{R} = \frac{d^2y}{dx^2}$$

$$\Rightarrow \sec^3\alpha \times \frac{1}{R} = \frac{d^2y}{dx^2} \qquad \left[\sec\alpha = \frac{ds}{dx}\right]$$

$$\Rightarrow (1+\tan^2\alpha)^{3/2} \times \frac{1}{R} = \frac{d^2y}{dx^2}$$

$$\Rightarrow \left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2} \times \frac{1}{R} = \frac{d^2y}{dx^2}$$

$$\Rightarrow \frac{1}{R} = \frac{(d^2y/dx^2)}{\left[1+(dy/dx)^2\right]^{3/2}}$$

Since $\frac{dy}{dx}$ is very very small

$$\therefore \frac{1}{R} = \frac{(d^2y/dx^2)}{1} \quad \text{--- (4)}$$

Also,

$$\frac{E}{R} = \frac{M}{I}$$

$$\Rightarrow \frac{1}{R} = \frac{M}{EI}$$

$\therefore$

$$\frac{M}{EI} = \frac{d^2y}{dx^2}$$

$$\Rightarrow \boxed{M = EI\frac{d^2y}{dx^2}}$$ governing eqn for deflection of beam

(i) Double integration method
(ii) Macaulay's Method
(iii) Area-moment method

(i) Double integration method :- governing eqn is integrated twice

$$M = EI\frac{d^2y}{dx^2}$$

$$\int M\,dx + C_1 = EI\frac{dy}{dx}$$

$$\iint M\,dx + C_1x + C_2 = EIy$$

To calculate C_1 & C_2 :

↳ Boundary conditions are applied (or end conditions)

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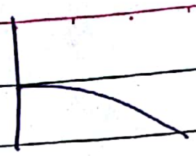
e.g.

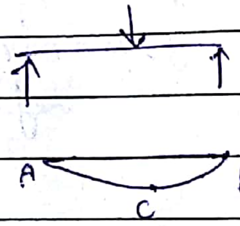


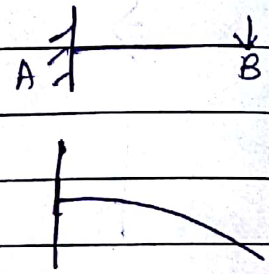
At free end
deflection = 0
slope = 0

Sign Convention for deflection & slope :-

Simply supported Beam

Cantilever Beam





~~In case~~ (a) In simply supported beam,

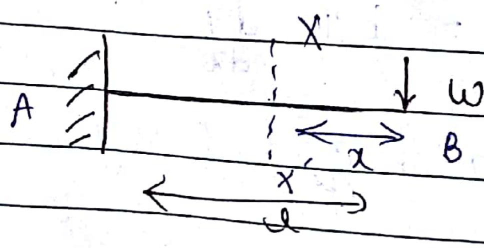
Origin at A: Deflection → −ve
Slope (b/w A & C) → −ve
(b/w C & B) → +ve

Origin at B: Deflection → −ve
Slope (b/w A & C) → +ve
(b/w B & C) → −ve

(b) In Cantilever beam

Origin at A: Deflection → −ve
Slope → −ve

Origin at B: Deflection → −ve ; Slope → +ve



Cantilever Beam
(load at free end)

$$M = EI \frac{d^2y}{dx^2}$$

(i) Take B as origin:

$$M = -Wx$$

$$\therefore \quad -wx = EI\frac{d^2y}{dx^2}$$

$$\Rightarrow \quad EI\,y' = -\frac{wx^2}{2} + C_1$$

At $x = l$, $y' = 0$ $\Rightarrow C_1 = \frac{wl^2}{2}$

$$y' = \frac{1}{EI}\left[-\frac{wx^2}{2} + \frac{wl^2}{2}\right]$$

$$\boxed{y' = \frac{w}{2EI}\left[l^2 - x^2\right]} \rightarrow \text{Eq}^n \text{ for slope}$$

And,

$$y = \frac{w}{2EI}\left[l^2x - \frac{x^3}{3}\right) + C_2$$

At $x = l$, $y = 0$

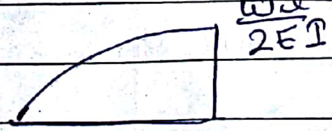
$$C_2 = -\frac{w}{2EI}\left(l^3 - \frac{l^3}{3}\right) = -\frac{wl^3}{3EI}$$

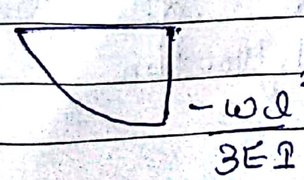
$$\boxed{y = -\frac{w}{6EI}\left[2l^3 - 3l^2x + x^3\right]} \rightarrow \text{Eq}^n \text{ for deflection}$$

At $x = 0$,

$y' = \frac{wl^2}{2EI}$ & $y = -\frac{wl^3}{3EI}$

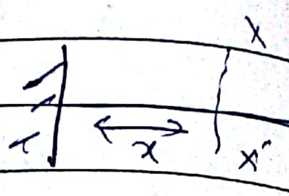
↓ Maximum ↓ Maximum





(ii)

Take A as origin :-



$$M = EI\frac{d^2y}{dx^2}$$

$$M = -w(l-x)$$

$$\therefore \quad -w(l-x) = EI\frac{d^2y}{dx^2}$$

$$\Rightarrow \quad -w\left(lx - \frac{x^2}{2}\right) = EI\frac{dy}{dx}$$

At $x = 0$, $y' = 0$, $C_1 = 0$

$$\boxed{y' = \frac{-w}{2EI}(2lx - x^2)} \rightarrow \text{Eq}^n \text{ of Slope}$$

$$y = \frac{-w}{2EI}\left(lx^2 - \frac{x^3}{3}\right) + C_2$$

At $x = 0$, $y = 0$, $C_2 = 0$

$$\boxed{y = \frac{-w}{6EI}(3lx^2 - x^2)} \rightarrow \text{Eq}^n \text{ of deflection}$$

At $x = l$ (free end)

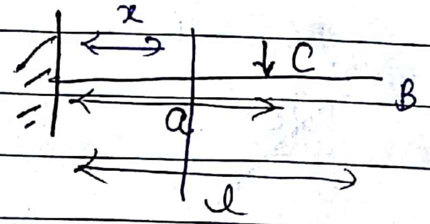
$$y' = \frac{-wl^2}{2E\cancel{2}I} \quad \& \quad y = -\frac{wl^3}{3EI}$$

$\downarrow$ Maximum $\qquad\qquad$ $\downarrow$ Maximum

<u>Note</u>:- In case of Cantilever Beam, if load is acted at free end, then maximum slope & deflection come out to be at free end.

Case 2: Load at a distance from fixed end (Cantilever Beam)

Assuming origin at A:



(i) Segment AC → $M = -\omega(a-x)$

$$\cancel{EIy''} \quad EIy'' = M = -\omega(a-x)$$

$$\Rightarrow IEy' = -\omega\left(ax - \frac{x^2}{2}\right) + C_1$$

At $x = 0$, $y' = 0$ $\therefore C_1 = 0$

$\therefore$ $\cancel{EI}\ \cancel{y'} =$

$$\boxed{y' = -\frac{\omega}{EI}\left(ax - \frac{x^2}{2}\right)}$$ → Slope eqⁿ

And, $EIy = -\omega\left(\frac{ax^2}{2} - \frac{x^3}{6}\right) + C_2$

At $x = 0$, $y = 0$ $\therefore C_2 = 0$

$$\boxed{y = -\frac{\omega}{EI}\left(\frac{ax^2}{2} - \frac{x^3}{6}\right)}$$ → Deflection eqⁿ

At point C, $x = a$

$$y' = -\frac{\omega a^2}{2EI} \qquad y = -\frac{\omega a^3}{3EI}$$

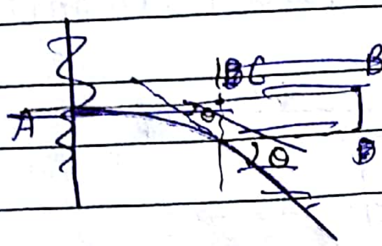
(ii) Segment BC →

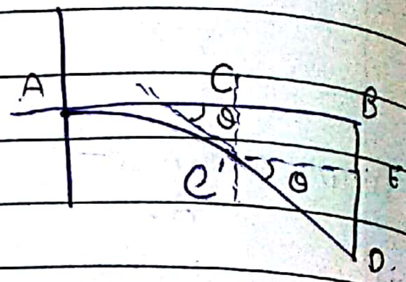
$M = 0$

$EIy'' = 0$

$\Rightarrow y' = C$

Slope is constant b/w B & C





Deflection at B,

$$BD = BE + DE$$

$$y' = \tan\theta = \frac{DE}{CE}$$

$$DE = y'\, C'E$$

$$DE = -\frac{wa^2}{2EI}(l-a)$$

$$\& \quad BE = -\frac{wa^3}{3EI}$$

Deflection at C

$$So, \quad BD = -\frac{wa^3}{3EI} - \frac{wa^2}{2EI}(l-a)$$

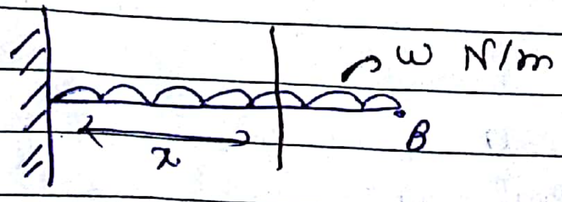
deflection due to slope at C

$y'_C (l-a)$

When $a = \frac{l}{2}$

Deflection at free end $\rightarrow$ $y = -\frac{5}{48}\frac{Wl^3}{EI}$

Case 3:- UDL & Cantilever beam (UDL at whole length of beam)



$$EIy'' = M$$

$$M_x = -\frac{w(l-x)^2}{2}$$

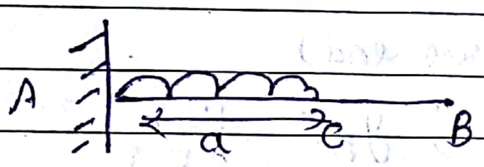
$$\therefore EIy'' = -\frac{wl^2}{2} + wlx - \frac{wx^2}{2}$$

Boundary Conditions $\rightarrow$ At $x = 0$, $y' = 0$
At $x = 0$, $y = 0$

$$y'_B = -\frac{wl^3}{6EI}$$

$$y_B = -\frac{wl^4}{8EI}$$

Case 4:- (a) UDL & Cantilever Beam (when UDL at only some length of beam)



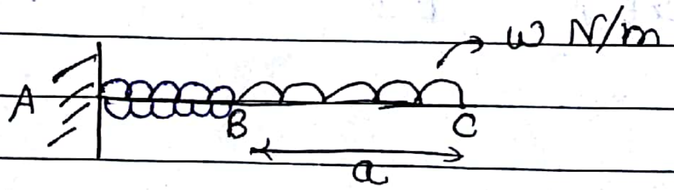
$$y'_C = -\frac{\cancel{?}wa^3}{6EI} = y'_B$$

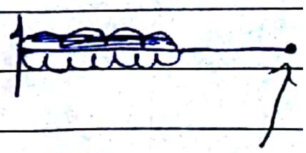
$$\cancel{y_B} = y_C = -\frac{wa^4}{8EI}$$

y_B = Deflection at C + Slope at C × length

$$= -\frac{wa^4}{8EI} - \frac{wa^3}{6\cancel{a}EI}(l-a)$$

(b)

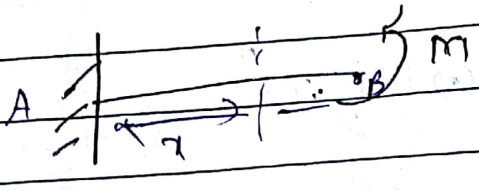




$$y'_C = -\frac{wl^3}{6EI} + \frac{w(l-a)^3}{6EI}$$

$$y_C = -\frac{wl^4}{8EI} + \cancel{w(l-a)^4} + \frac{w(l-a)^4}{8EI} + \frac{w(l-a)^3 \cdot a}{6EI}$$

Case 5:- External moment is applied at free end



$$y'' = \frac{M_x}{EI}$$

$$\Rightarrow M_x = M$$

$$\Rightarrow EI\,y'' = M$$

$$\Rightarrow EI\,y' = Mx + C_1$$

$$EI\,y = \frac{Mx^2}{2} + C_1 x + C$$

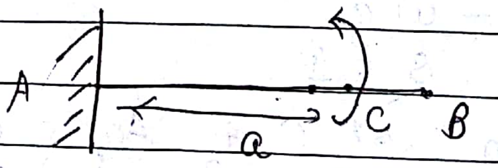
At $x = 0$, $y' = 0$, $C_1 = 0$

At $x = 0$, $y = 0$, $C_2 = 0$

At $x = l$ (free end)

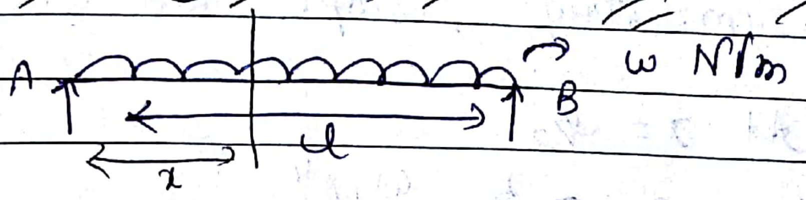
$$y'_B = \frac{Ml}{EI} \quad ; \quad y_B = \frac{Ml^2}{2EI}$$

Case 6:- External moment is applied at any length



$$y_B = \frac{Ma^2}{2EI} + \frac{Ma}{EI}(l-a)$$

★ Slope & deflection in simply supported system:-



Clearly, at ~~free~~ ends A & B slope is non-zero.

Boundary Conditions:- At $x = 0$, $y = 0$ [At A]
At $x = l$, $y = 0$ [At B]

$R_A = R_B = \frac{\omega l}{2}$

Now,

$$EI y'' = M_x = \frac{\omega l}{2} \cdot x - \frac{\omega x^2}{2}$$

$$\cancel{EI y' = \frac{\omega l}{4} x^3 - \frac{\omega}{24} x^4 + C_1}$$

$$\Rightarrow IEy' = \frac{\omega l}{4} x^2 - \frac{\omega}{6} x^3 + C_1$$

$$\Rightarrow EIy = \frac{\omega l}{12} x^3 - \frac{\omega}{24} x^4 + C_1 x + C_2$$

At $x = 0$, $y = 0 \Rightarrow C_2 = 0$

At $x = l$, $y = 0 \Rightarrow C_1 = -\frac{\omega l^3}{24}$

So, $$y' = \frac{1}{EI}\left[\frac{\omega l}{4} x^2 - \frac{\omega}{6} x^3 - \frac{\omega l^3}{24}\right]$$

$$y = \frac{1}{EI}\left[\frac{\omega l}{12} x^3 - \frac{\omega}{24} x^4 - \frac{\omega l^3}{24} x\right]$$

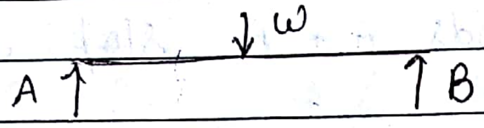
$$y'_A = \theta_A = -\frac{\omega l^3}{24 EI}$$

$$y'_B = \theta_B = \frac{\omega l^3}{24 EI}$$

Note:- If reactions at both ends are same, then by symmetry, slope too will be same.

At $x = l/2$

$$y_{max} = -\frac{5}{384}\frac{wl^4}{EI}$$



$$M_x = \frac{w}{2}x$$

$$IEy'' = \frac{w}{2}x$$

$$\Rightarrow y' = \frac{1}{EI}\left[\frac{w}{4}x^2 + C_1\right]$$

$$y'_A = \frac{-wl^2}{16EI}$$

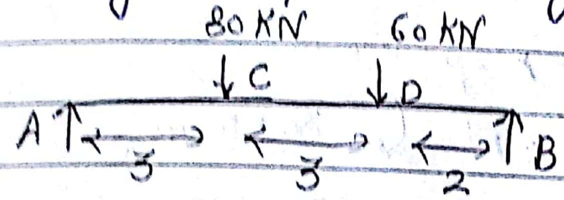
$$y'_B = \frac{wl^2}{16EI}$$

$$y_{max} = -\frac{wl^3}{48EI}$$

✱ Macaluay's Method:-

1. Take the origin at left most end of beam.
2. Write / Express the bending moment eq^n for the righmost segment of beam.
3. The terms in bracket should be integerated keeping them in brackets throughout. ~~It must be~~ ~~remem~~ The terms in bracket becomes zero ~~whoeve~~ whenever terms are negative.
4. Use the terms in bending moment eq^n selectively

depending about their validity. If the bracket term is +ve, it is included, if negative it becomes zero, while calculating the values of slope & deflection.

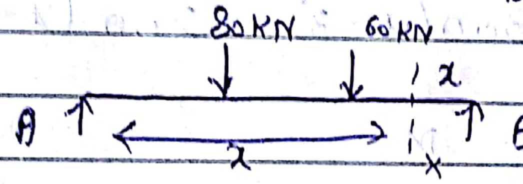
Q 

$E = 200\ GPa$

$I = 30,000\ cm^4$

Calculate the slopes at ends and deflection under the loads.

Ans $R_A = 65\ KN$, $R_B = 75\ KN$



$$M_x = 65x - 80(x-3) - 60(x-6)$$

$$EIy'' = M_x$$

$$\Rightarrow EIy' = 65\frac{x^2}{2} - \frac{80(x-3)^2}{2} - \frac{60(x-6)^2}{2} + C_1$$

$$EIy = \frac{65x^3}{6} - \frac{80(x-3)^2}{6} - \frac{60(x-6)^3}{6} + C_1 x + C_2$$

At $x = 0$, $y = 0$, $C_2 = 0$

At $x = 8$, $y = 0$, $C_1 = 475$

So,

$$y' = \frac{1}{EI}\left[65\frac{x^2}{2} - 40(x-3)^2 - 30(x-6)^2 + 475\right]$$

$$y = \frac{1}{EI}\left[\frac{65x^3}{6} - \frac{80(x-3)^3}{6} - 60\frac{(x-6)^3}{6} + 475x\right]$$

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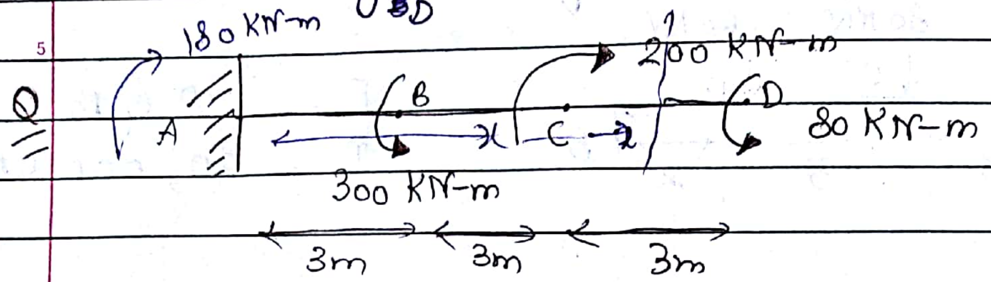
$\theta_A = \cdot 0079$ radian
$\theta_B = -0.0081$ radian
$y_C = 18.8$ mm
$y_D = 14.9$ mm



$EI = 90 \times 10^3 \ KN/m^2$

Calculate slope & deflection at couple-points

Ans

$$M_x = 180 - 300(x-3)^0 + 200(x-6)^0$$

$$EI y'' = M$$

$$\Rightarrow EIy' = 180x - 300(x-3) + 200(x-6) + C_1$$

$$\Rightarrow EIy = 180\frac{x^2}{2} - 300\frac{(x-3)^2}{2} + 200\frac{(x-6)^2}{2} + C_1 + C_2$$

At $x = 0,\ y' = 0 \Rightarrow C_1 = 0$

At $x = 0,\ y = 0 \Rightarrow C_2 = 0$

$$\therefore EIy' = 180x - 300(x-3) + 200(x-6)$$

$$EIy = 180\frac{x^2}{2} - 300\frac{(x-3)^2}{2} + 200\frac{(x-6)^2}{2}$$

At $x = 3,\ y'_B = \frac{180 \times 3}{EI}$

At $x = 6,\ y'_C = \frac{180 \times 6 - 300(6-3)}{EI}$

$\frac{M}{EI}$

P

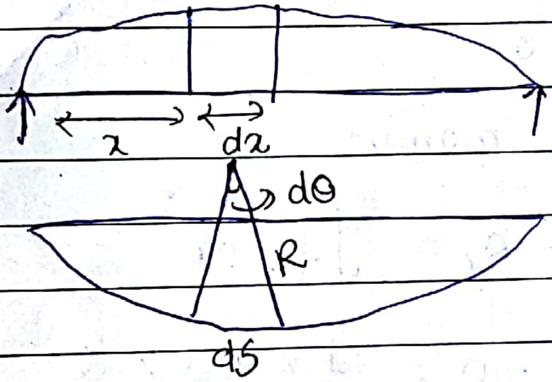
★ Area - moment method:-

(i) It does not directly give values of slope & deflection like previous methods.

(ii) It is useful only where we know a point of zero slope.

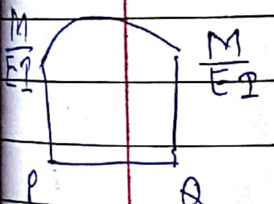
(iii) Only calculates slope & deflection at particular points.

This method is based on two theorems.



$R\, d\theta = ds$

$\Rightarrow d\theta = \frac{ds}{R} = \frac{dx}{R}$



$\frac{1}{R} = \frac{M}{EI}$

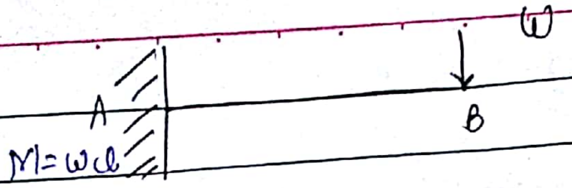
$\therefore \int_1^2 d\theta = \int_1^2 \frac{M}{EI}\, dx$

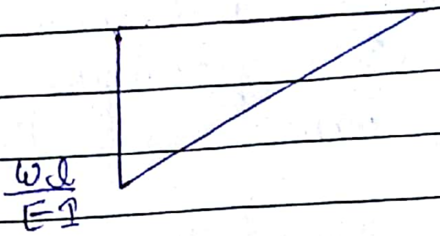
$\theta_2 - \theta_1$ = Area of $\frac{M}{EI}$ diagram b/w two points 1 & 2.

First Area moment Theorem:-

According to this theorem, the angle b/w the tangents to the elastic curve b/w any two points is equal to the area of $\frac{M}{EI}$ diagram b/w the two points.

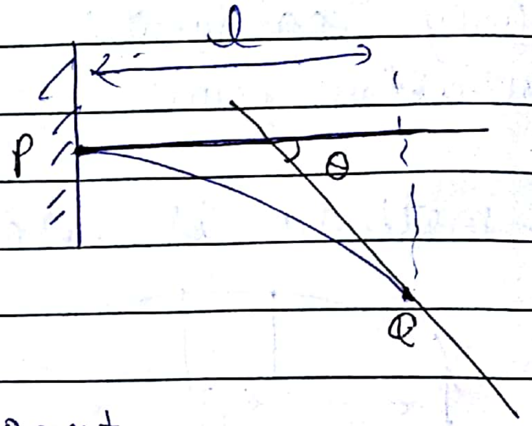
e.g.





← $\frac{M}{EI}$ diagram

Elastic curve (due to deformation of beam) →



Slope at P = 0

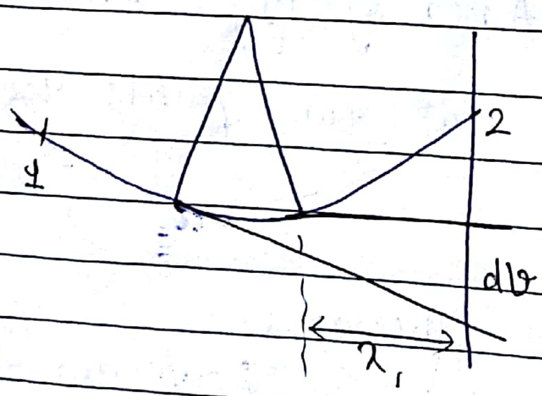
By first Area moment Theorem →

$$\theta_Q - \theta_P = \int \frac{M}{EI}\,dx$$

$$\Rightarrow \theta - 0 = \frac{1}{2} \times \frac{wl}{EI} \times l$$

$$= \frac{wl^2}{2EI}$$

Second Area-moment method:-



$$dv = x_1\,d\theta$$

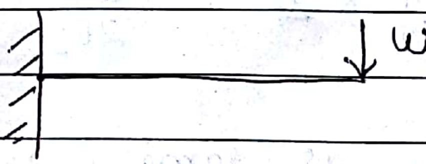
$$\Rightarrow \int_1^2 dv = \int_1^2 x_1 \frac{M}{EI}\,dx$$

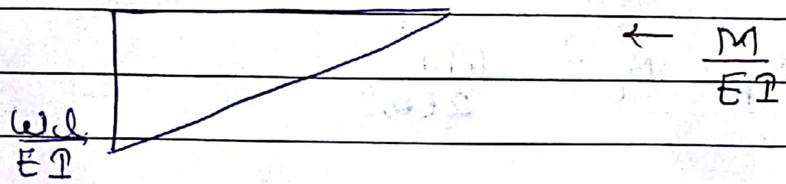
$V_{1,2}$ = Moment of Area of $\frac{M}{EI}$ diagram b/w point 1 & 2 about point 2.

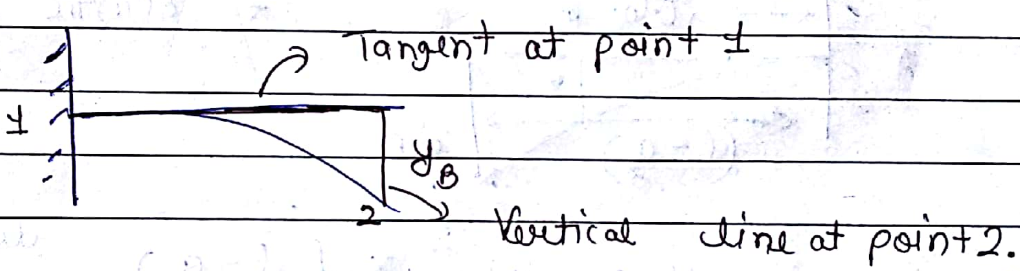
x_1 → distance of point 2 from elemental length ds of beam

Statement ↳ According to this, moment of area of $\frac{M}{EI}$ diagram b/w two points of a beam about one of these two points is equal to the vertical intersect made by the ~~tag~~ tangent drawn at one point ~~and~~ & a vertical line through a second point about which moment is taken.

e.g.

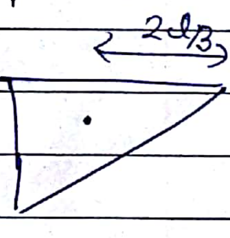


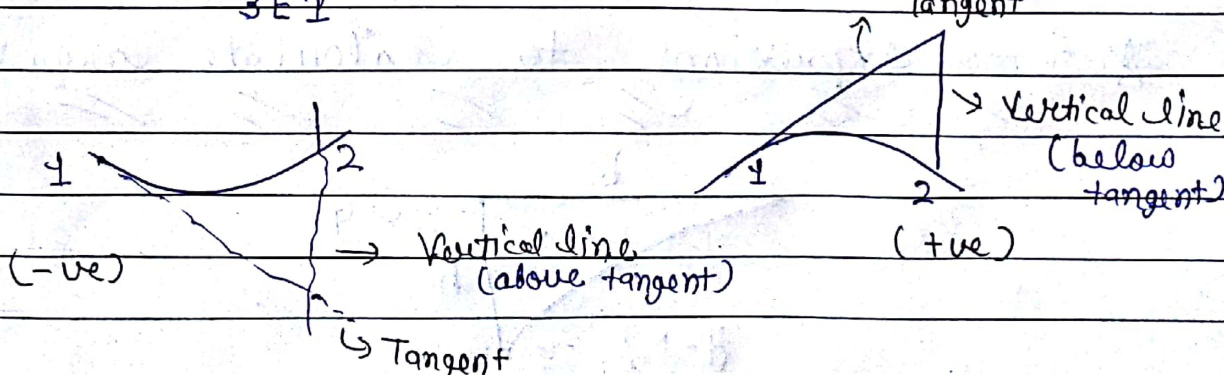


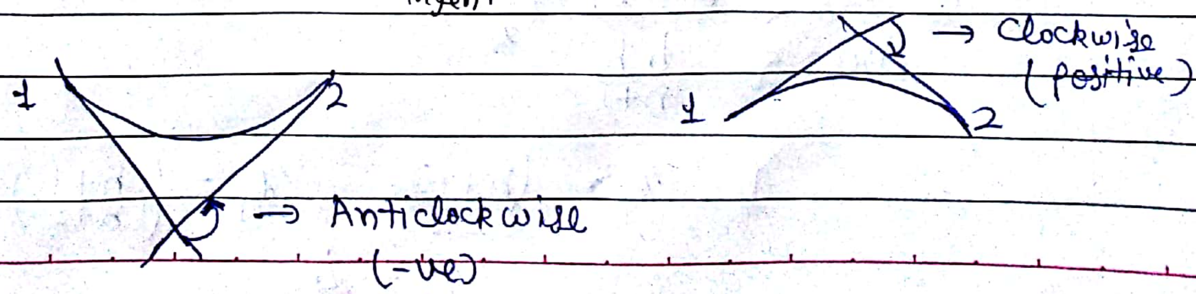


$$y_B = \frac{1}{2} \times \frac{Wl}{EI} \times l \times \frac{2l}{3}$$

$$= \frac{Wl^3}{3EI}$$







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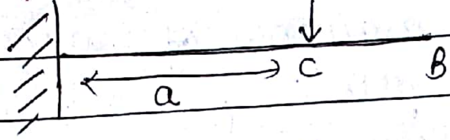
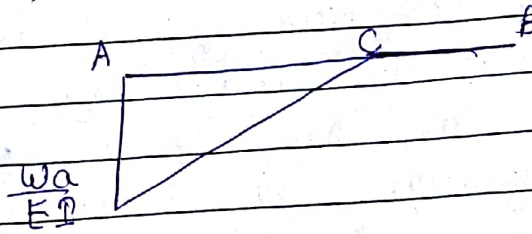
e.g. Deflection & Slope at free end?



Sol.

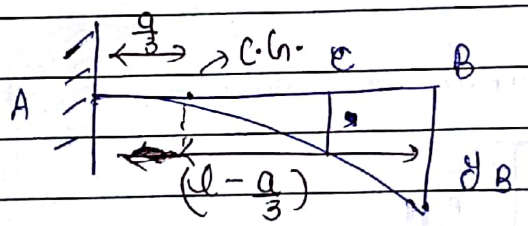


← $\frac{M}{EI}$ diagram

$$\theta_C = \frac{1}{2} \times \frac{Wa}{EI} \times a = \frac{Wa^2}{2EI}$$

B/w A & B → Area is same as b/w A & C.

$$\therefore \theta_B = \theta_C = \frac{Wa^2}{2EI}$$



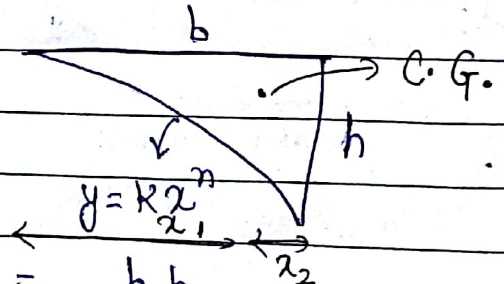
← Elastic Curve

$$y_B = \frac{Wa^2}{2EI} \times \left(l - \frac{a}{3}\right)$$

distance of C.G. from B.

$$= \frac{Wa^2(3l - a)}{6EI}$$

☆ Some expressions to calculate area :-

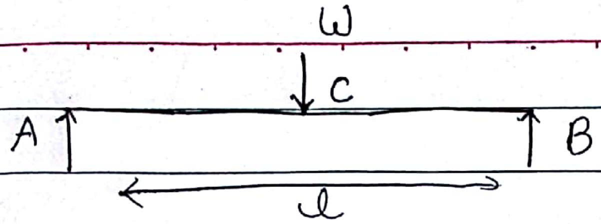


$$\text{Area} = \frac{bh}{n+1}$$

$$x_1 = \left(\frac{n+1}{n+2}\right) b$$

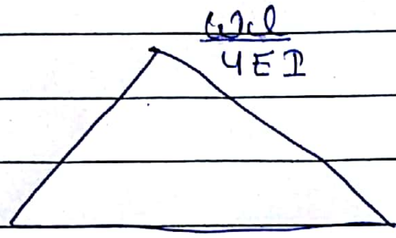
$$x_2 = \frac{b}{n+2}$$

Q

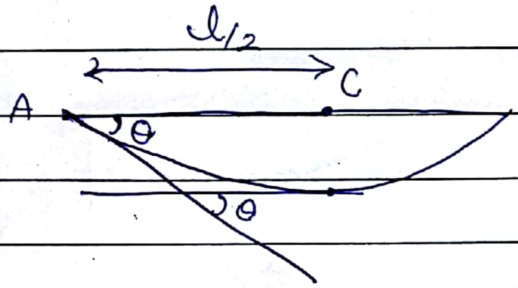


Calculate:
(i) Slope at end points (A & B)
(ii) Deflection at C.

Ans



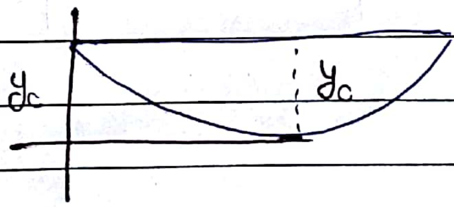
← $\frac{M}{EI}$ diagram

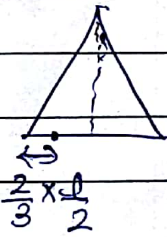


$$\theta_A = \theta = \frac{1}{2} \times \frac{Wl}{4EI} \times \frac{l}{2}$$

$$\theta_A = \frac{Wl^2}{16EI}$$

Draw a vertical line at A & at B a tangent

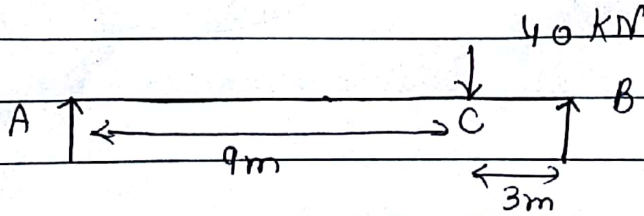




$$y_c = \frac{Wl^2}{16EI} \times \frac{2}{3} \times \frac{l}{2}$$

$$= \frac{Wl^3}{48EI}$$

Q



Calculate slope & deflection at A, B & C.

$I = 2 \times 10^9$ ~~m~~ mm^4

$E = 205$ GPa

Torsion

Shaft :- rotating system to transmit torque (or power) from one point to another.

	Torque	Moment
(i)	Tendecy is to rotate the body about longitudinal axis.	Tries to deflect the body
(ii)	Acts about longitudinal axis	acts in plane that contains longitudinal axis.
(iii)	produces torsional (or shear) stress	produces bending stress

★ Torsional Equation :-

Assumptions →

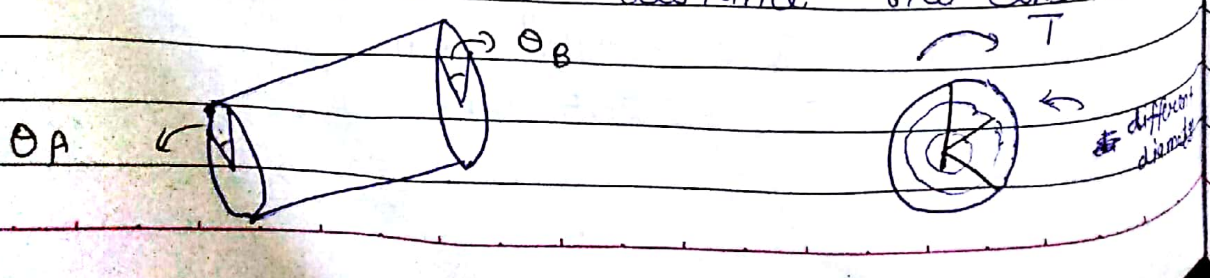
(i) Material is homogenous, isotropic and follows Hooke's law.

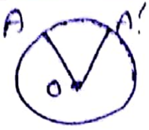
(ii) Twisting couple (moment) acts always in transverse plane (plane ⊥r to longitudinal ~~plane~~ axis)

(iii) The straight radial lines in transverse ~~line~~ plane remain same after twisting.

(iv) A cross-section at any axial length rotates as a rigid ~~length~~ plane. (all diameters rotate by same angle)

(v) The relative rotation b/w any two section is proportional to the distance b/w them

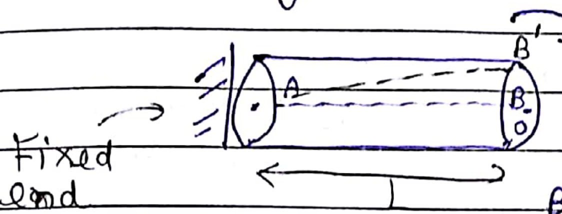


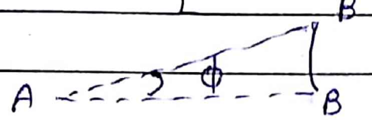


$(\theta_A - \theta_B) \propto L$

$\frac{\theta}{L} = C \rightarrow$ Angle of twist per unit length is constant

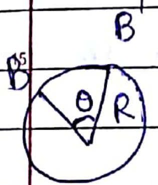
Let a shaft be drawn like this:





$\phi \rightarrow$ Angle b/w AB & AB'
↳ represents shear strain at the surface

$$\phi = \frac{BB'}{L} \quad — ①$$



$\theta \rightarrow$ Angle of twist (because A part is fixed here.
$\therefore \theta_A = 0$
$\theta_B = \theta$)

$R \rightarrow$ Radius of shaft

$$BB' = R\theta \quad — ②$$

$$\therefore \phi L = R\theta$$

$$\Rightarrow \phi = \frac{R\theta}{L}$$

If 'τ' is shear stress intensity at the surface. Then, from Hooke's law -

$\tau \propto \phi$

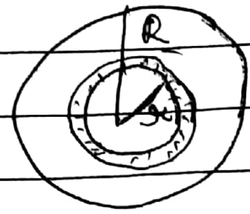
$\tau = C\phi$

$C \rightarrow$ Modulus of Rigidity

$$\Rightarrow \phi = \frac{\tau}{C} = \frac{R\theta}{L}$$

$$\Rightarrow \boxed{\frac{\tau}{R} = \frac{C\theta}{L}} \quad — (A)$$

Let there is small ring of thickness 'dr' at 'r' distance from centre in face B.



$\tau_r \rightarrow$ Shear stress at distance r

By (iv) assumption, & eqn A

$$\frac{\tau_r}{r} = \frac{C\theta}{L}$$

$C \rightarrow$ material property

$$\frac{\tau_r}{r} = \frac{\tau}{R}$$

$$\Rightarrow \tau_r = r \cdot \frac{\tau}{R}$$

Notably, Shear stress vary linearly with radial distance

$$\tau_{at\ r=0} = 0 = \tau_0$$

Max. $\rightarrow$ at $r = R$

If τ_r is the shear stress, then tangential force on the elemental (ring)

$$F = \tau_r \times 2\pi r dr \rightarrow \text{Area under action}$$

Moment due to this force

$$= (\tau_r \times 2\pi r dr) \times r$$

Torque induced to counteract against applied Torque

$$T = \int_0^R \tau_r \times 2\pi r dr \cdot r$$

$$T = \int_0^R \frac{rC\theta}{L} \times 2\pi r\, dr \cdot r$$

$$\Rightarrow \quad T = \frac{C\theta}{L} \cdot 2\pi \int_0^R r^3 \, dr$$

$$= \frac{C\theta}{L} \cdot 2\pi \frac{R^4}{4}$$

$$\Rightarrow \quad T = \frac{C\theta}{L} \cdot \frac{\pi}{32} D^4$$ [D → Diameter]

$$I_{xx} = I_{yy} = \frac{\pi}{64} D^4$$

$$I_{zz} = I_{xx} + I_{yy} = \frac{\pi}{32} D^4$$

[Polar moment of inertia]

$$J = \frac{\pi}{32} D^4$$

$$\Rightarrow \quad \frac{T}{J} = \frac{C\theta}{L}$$

$$\boxed{\frac{T}{J} = \frac{\tau}{R} = \frac{C\theta}{L}}$$ → Torsional eqn

Stiffness :- Load required per unit deflection.

$F = (K)x$

K → Stiffness of spring

$$\frac{T}{J} = \frac{\tau}{R} \quad \text{— (1)}$$ → Strength

$$\frac{T}{J} = \frac{C\theta}{L} \quad \text{— (2)}$$ → Stiffness or rigidity eqn

→ To design a shaft both eqn are required.

$\frac{T}{\theta}$ → Torque required per unit twist [Torsional stiffness]

→ θ is in radian.

$EI \rightarrow$ Flexure rigidity

Analogous to that,

$$CJ = \text{Torsional rigidity}$$

$$= \frac{T}{(\theta/L)} \rightarrow \text{Torque required angle of twist per unit length}$$

Just like section modulus $(Z) = \frac{I}{y_{max}}$

$$Z_p = \text{Polar modulus}$$

$$= \frac{J}{R} \rightarrow \text{Represents strength of shaft in torsion.}$$

☆ Power transmitting Capacity :- (In shaft)

$$\omega = \frac{2\pi N}{60} \ \text{~~radius~~ rad/sec.}$$

$T \rightarrow$ Torque in $\boxed{N\text{-}m}$ $\qquad N \rightarrow$ Rpm (rotations per minute)

$$\text{Power} = T\omega$$

$$P = \frac{2\pi NT}{60} \quad \frac{N\text{-}m}{s} \quad J/s \quad \text{Watt}$$

Solid shaft :-

$$\tau_{max} = \frac{T}{J} R$$

$$= \frac{16\,T}{\pi D^3}$$

$D \rightarrow$ Diameter

Hollow shaft :- (Inner diameter $\rightarrow d$; Outer diamer $\rightarrow D$)

$$\tau_{max} = \frac{16\,T\,D}{\pi(D^4 - d^4)}$$

$$R = \frac{D}{2}$$

$$J = \frac{\pi}{32}\left[D^4 - d^4\right]$$

Q. Calculate the maximum torque that can be transmitted by 35 cm diameter shaft, if the twist in the shaft does not exceed 1° in 15 diameter of length. Also shear stress is limited to 50 ~~150~~ MPa. [C = ~~7~~85 MPa]

Ans.

$$l = 15\,d \qquad d \rightarrow \text{diameter} = 85 \text{ cm}$$

$$l = 15 \times 35 \text{ cm}$$

$$\theta = 1^\circ = \frac{\pi}{180} \text{ radian}$$

$$\tau_{max.} = 50 \text{ MPa}$$

$$\because \quad \tau_{max.} = \frac{16T}{\pi D^3}$$

$$\Rightarrow \cancel{T = 420924 \text{ N-m}} \quad T = 420924 \text{ N-m}$$

With eq^n:

$$\frac{T}{J} = \frac{C\theta}{L}$$

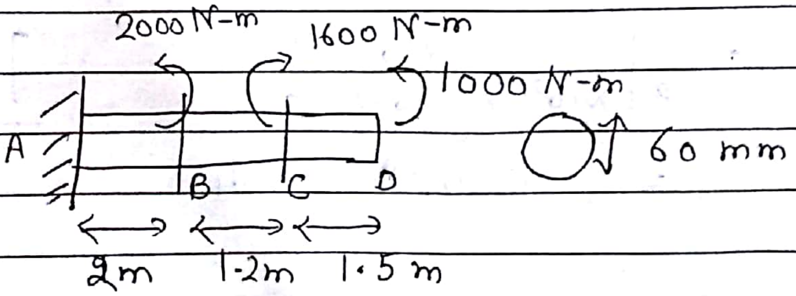
$$\Rightarrow T = 85 \times \frac{\pi}{180} \times \frac{\pi}{32} \times \frac{(350)^4}{15 \times 350} \rightarrow \text{in mm}$$

$$\cancel{T = 415837. \text{ N-m}}$$

$$T = 416302 \text{ N-m}$$

This is required Torque (Earlier one will produce $\theta > 1^\circ$)

Q.



G = 85 GPa

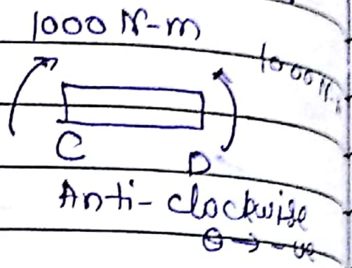
Ans

$$\frac{T}{J} = \frac{C\theta}{l}$$

$$\theta = \frac{Tl}{CJ}$$

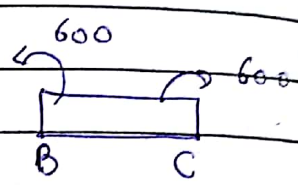
$\theta \rightarrow$ Angle w.r.t. another part that is [illegible] suppose

Angle of twist of point D with relative to C.



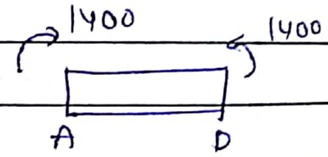
$$\theta_{D/C} = \frac{-1000 \times 1.5 \times 10^3 \times 10^3}{CJ}$$

Angle of twist C relative to B,



$$\theta_{C/B} = \frac{600 \times 1.2 \times 10^3 \times 10^3}{CJ}$$

Angle of twist of B relative to A



$$\theta_{B/A} = \frac{-1400 \times 2 \times 10^3 \times 10^3}{CJ}$$

Now,

Angle of twist of D relative to A

$$\theta_{D/A} = \theta_{D/C} + \theta_{C/B} + \theta_{B/A}$$

$$= \frac{1}{CJ}\left[10^9(-1.5 + 0.72 - 2.8)\right]$$

$$= \frac{1 \times 10^9}{85 \times 10^3 \times \frac{\pi}{32}(60)^4}\left[\quad\right]$$

$$= -0.0312 \text{ radian}$$

Q. If two shafts have same material & length & weight with configuration:

Hollow Shaft = R_o, R_i

Solid Shaft = R

Which shaft has more strength?

Ans.

$$\frac{T_h}{\frac{\pi}{2}(R_o^4 - R_i^4)} = \frac{\tau_h}{R_o}$$

$$\& \quad \frac{T_s}{\frac{\pi}{2} R^4} = \frac{\tau_s}{R}$$

Since, material is same.

$$\therefore \quad \tau_h = \tau_s$$

$$\Rightarrow \quad \frac{T_h}{T_s} = \frac{R_o^4 - R_i^4}{R_o R^3} \quad \text{--- (1)}$$

Both shaft have same weight & length

$$W_h = W_s$$

$$\Rightarrow \quad \pi(R_o^2 - R_i^2)\, l_h\, \rho_h = \pi R^2\, l_s\, \rho_s \quad \begin{bmatrix} l_h = l_s \\ \rho_h = \rho_s \end{bmatrix}$$

$$\Rightarrow \quad R_o^2 - R_i^2 = R^2 \quad \text{--- (2)}$$

From (1) & (2)

$$\frac{T_h}{T_s} = \frac{(R_o^2 + R_i^2)R^2}{R_o R^3} = \frac{(R_o^2 + R_i^2)}{R R_o}$$

$$\Rightarrow \quad \frac{T_h}{T_s} = \frac{R_o}{R}\left[1 + \frac{1}{(R_o/R_i)^2}\right]$$

Assume, $\frac{R_o}{R_i} = 2$

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$$\therefore \frac{T_h}{T_s} = \frac{R_o}{R}\left[1+\frac{1}{x^2}\right] \quad — ③$$

From eq ②

$$R_o^2\left(1-\frac{1}{x^2}\right) = R^2$$

$$\Rightarrow \frac{R_o}{R} = \frac{1}{\sqrt{1-1/x^2}}$$

In eq. ③

$$\frac{T_h}{T_s} = \frac{x}{\sqrt{x^2-1}}\left[1+\frac{1}{x^2}\right] = \frac{x^2+1}{x\sqrt{x^2-1}}$$

Since $x = \frac{R_o}{R_i} > 1$

$$\frac{T_h}{T_s} > 1$$

$$\Rightarrow \boxed{T_h > T_s}$$

If $R_o = 2R_i \Rightarrow x = 2$

$$\frac{T_h}{T_s} = 1.44$$

Q. Compare the above shafts on the basis of weight, if both have same Torque transmission capacity.

Ans Hollow shaft $\rightarrow R_o, R_i$

Solid Shaft $\rightarrow R$

Since, $T_h = T_s$, $\tau_h = \tau_s$

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$$\frac{T_h}{T_s} = \frac{R_o^4 - R_i^4}{R_o R^3} = 1$$

$$\Rightarrow R^3 = \frac{R_o^4 - R_i^4}{R_o}$$

$$\Rightarrow R^3 = R_o^3\left(1 - \frac{1}{x^4}\right) \quad \text{--- (1)} \qquad \left[x = \frac{R_o}{R_i}\right]$$

$$\frac{W_h}{W_s} = \frac{\pi(R_o^2 - R_i^2) l_h \cdot \rho_h}{\pi R^2 \rho_s \cdot l_s}$$

$$= \frac{R_o^2 - R_i^2}{R^2} \qquad \left[\because l_h = l_s, \; \rho_h = \rho_s\right]$$

$$= \frac{R_o^2}{R^2}\left(1 - \frac{1}{x^2}\right) \quad \text{--- (2)}$$

From eq. (1)

$$\frac{R_o}{R} = \frac{1}{\left(1 - \frac{1}{x^4}\right)^{1/3}}$$

$$\frac{W_h}{W_s} = \frac{\left(1 - 1/x^2\right)}{\left(1 - \frac{1}{x^4}\right)^{2/3}} = \frac{(x^2-1)\cdot x^{2/3}}{(x^4-1)^{2/3}}$$

If $x = 2$

$$\frac{W_h}{W_s} = 0.782$$

Clearly, for same Torque Transmitting Capacity if $R_o = 2R_i$ in hollow shaft, then weight of hollow shaft is 22 % lesser than solid shaft. Hence, hollow shaft is economical.

Q. A solid wrought iron shaft is to be replaced by steel hollow shaft with external diameter of hollow equal to diameter of solid one.

Calculate:

(i) Ratio to the external to internal diameter

(ii) % saving in weight by this replacement if steel is 30 % stronger & 2 % heavier than wrought iron.

Ans (i) $\tau_h = 1.3\, \tau_s$; $\omega_h = 1.02\, \omega_s$

$$T_h = T_s$$

Since solid one is to be replaced by hollow.
That means, advantage in terms of Torque transmission Capacity.

Torque transmitted by Solid Iron Shaft

$$T_1 = \tau_1 \times \frac{\pi}{16} d^3$$

Torque transmitted by hollow shaft

$$T_2 = \tau_2 \times \frac{\pi}{16}\left(\frac{d_o^4 - d_i^4}{d_o}\right)$$

$$T_1 = T_2 \qquad \tau_2 = 1.3\, \tau_1 \; ; \; d_o = d$$

$$\Rightarrow \tau_1 d_o^3 = \tau_2 \frac{d_o^4 - d_i^4}{d_o}$$

$$\Rightarrow \frac{\tau_2}{\tau_1} = \frac{d_o^4}{d_o^4 - d_i^4} = 1.3$$

$$\Rightarrow \frac{x^4}{x^4 - 1} = 1.3$$

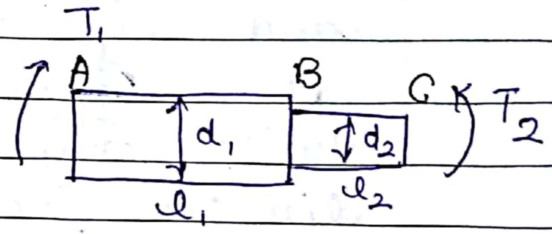
$$\Rightarrow x = 1.42$$

(ii) % saving $\rightarrow$ 47 %

$$W_h = 0.53\, W_s$$

* Composite shafts :-

(i) Connected in series -



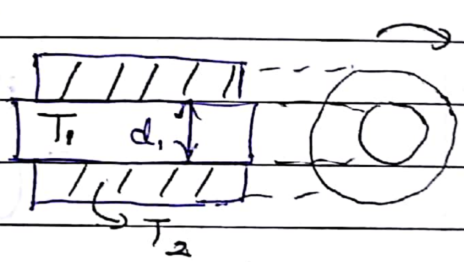
(a) Torque applied remains same. $T_1 = T_2$

(b) $\theta_{C/A} = \theta_{C/B} + \theta_{B/A}$

$\Rightarrow \theta = \theta_2 + \theta_1$

Angle of Twist

(ii) Connected in parallel



$T \rightarrow$ Total torque

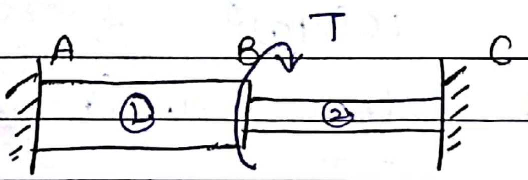
(a) Torque relation

↳ $T = T_1 + T_2$

(b) Angle of twist of the system remains same

$\theta_1 = \theta_2 = \theta$

e.g.



$T_1 + T_2 = T$

$\theta_{C/A} = \theta_{C/B} + \theta_{B/A} = 0$ → Since both ends are fixed.

$\Rightarrow \theta_{C/B} = -\theta_{B/A}$

↳ Same case as shafts connected in parallel

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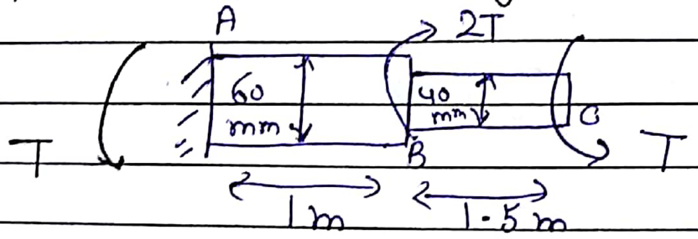
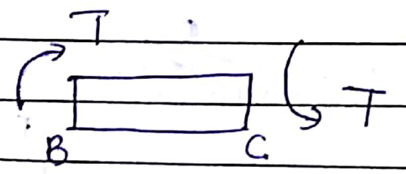
Q. A stepped steel shaft is subjected to a torque T at free end as follows and a torque 2T is opposite manner at junction. Determine total angle of twist if maximum shear stress is limited to 80 MPa.

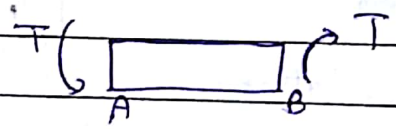
[C → 80 GPa]

[Given: Both shafts are made of same material]

Ans







Since, Torque applied on both section is same.

So, less - diameter → τ_{max}.

As, $\tau_{max} = \frac{16T}{\pi d^3}$

$\Rightarrow 80 = \frac{16T}{\pi (40)^3}$

$\Rightarrow T = 1005310$ N-mm

Total Angle of twist $\theta = \theta_1 + \theta_2$

$\theta_1 = \frac{-T_1 l_1}{C J_1}$

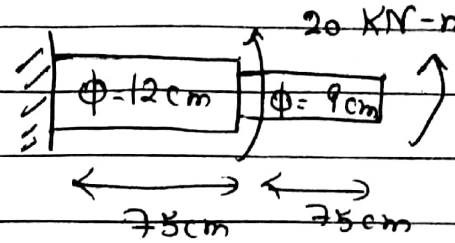
T → Anticlockwise
↳ θ → $-ve$
(But its upto us to decide sign convention)

$\theta_2 = \frac{T_2 l_2}{C J_2}$

$(T_1 = T_2)$

$\theta = 0.065$ radian

Q



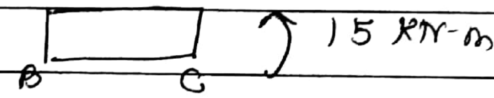
Calculate :

(i) Shear stress at each portion of the system.

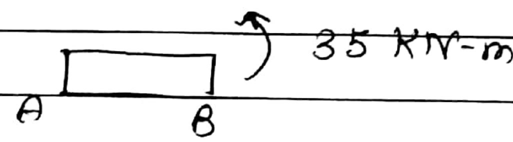
(ii) Angle of twist at the junction & at free end.

$$C = G = 82 \ GPa$$

Ans (i) $\tau_{BC} = \dfrac{16 \, T_{BC}}{\pi d^3}$



$$\tau_{AB} = \frac{16 \, T_{AB}}{\pi d^3}$$



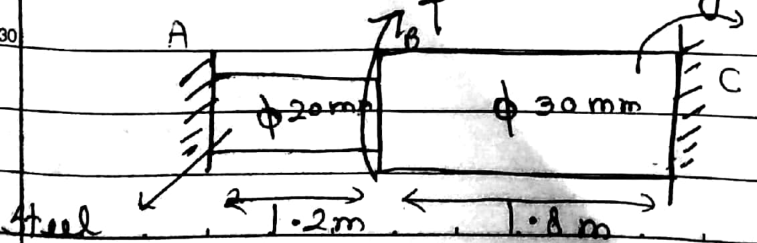
(ii) $\theta = \theta_1 + \theta_2$ → At free end

$$\theta_{C/B} = 0.0213 \text{ radian}$$

$$\theta_{B/A} = 0.0157 \text{ radian}$$

$$\theta_{C/A} = \theta_{C/B} + \theta_{B/A} = 0.037 \text{ radian}$$

Q For the shaft shown in figure, find the maximum value of Torque (T) such that stress in steel shaft is within 90 MPa & that in brass shaft is within 50 MPa



$C_{steel} = 85 \ GPa$

$C_{brass} = 40 \ GPa$

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Ans $\tau_{steel} = 90\ MPa$; $\tau_{brass} = 40\ MPa$

$$\theta_{A/C} = 0 \Rightarrow \theta_{A/B} + \theta_{B/C} = 0$$

$$\Rightarrow \theta_{A/B} = -\theta_{B/C}$$

Let T_S & T_B are the resisting torques at fixed end

$$\therefore T_S + T_B = T \quad — (1)$$

And, $\tau_S = \dfrac{16\, T_S}{\pi (20)^3} \leq 90$

$$\Rightarrow T_S \leq 141372\ N\text{-}mm$$

$$\tau_B = \dfrac{16\, T_B}{\pi (30)^3} \leq 50$$

$$\Rightarrow T_B \leq 265067\ N\text{-}mm$$

Also, $\theta_{B/A} = -\theta_{C/B}$

$$\theta_S = \theta_b$$

$$\Rightarrow \frac{T_S\, l_S}{J_S\, C_S} = \frac{T_b\, l_b}{J_b\, C_b}$$

$$\cancel{\Rightarrow T_S = 1.22\, T_b} \quad — (2)$$

$$T_S = 0.629\, T_b \quad — (2)$$

From (1) & (2)

So, $T_S = 0.387\, T \Rightarrow T = 365302$ Required Torque

$T_b = 0.613\, T \Rightarrow T = 432401$

If it is considered, T_S will be more & hence $\tau_S \leq 90$ eqn won't be satisfied.

Q. If two shafts are ~~of~~ connected in parallel, both of them are of same material & same length. Find out the ratio of their respective torque transmission capacity.

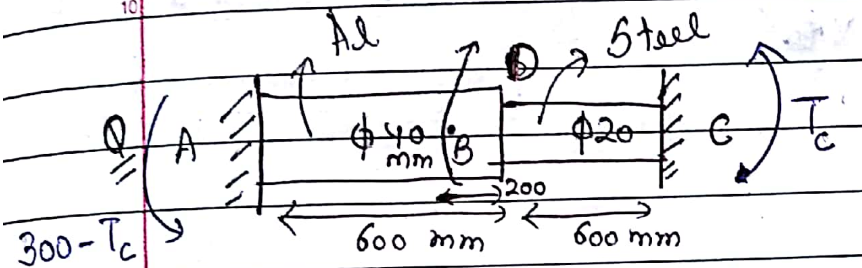
Ans

$$\theta_1 = \theta_2$$

$$\Rightarrow \frac{T_1 l_1}{J_1 C_1} = \frac{T_2 l_2}{J_2 C_2}$$

$$\Rightarrow \frac{T_1}{T_2} = \frac{J_1}{J_2} = \left(\frac{d_1}{d_2}\right)^4$$

Q.



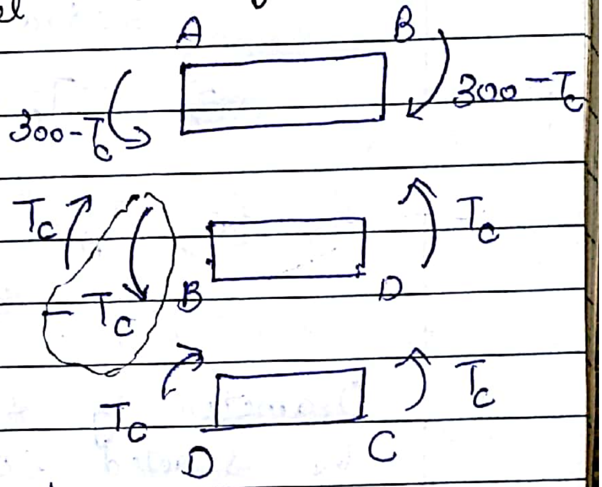
$C_S = 82$ GPa

$C_{Al} = 27$ GPa

Calculate total torque & angle of twist.
And maximum stresses in Al & Steel

Ans

Consider T_c torque at one end.



Net angle of twist $= 0$

$$\theta_{B/A} = \theta_{D/B} + \theta_{D/C}$$

$$\Rightarrow \frac{(300 - T_c) \times 400}{82 \times 10^3 \times \frac{\pi}{32}(40)^4} = \frac{T_c \times 200}{82 \times 10^3 \times \frac{\pi}{32}(40)^4} + \frac{T_c \times 600}{27 \times 10^3 \times \frac{\pi}{32}(20)^4}$$

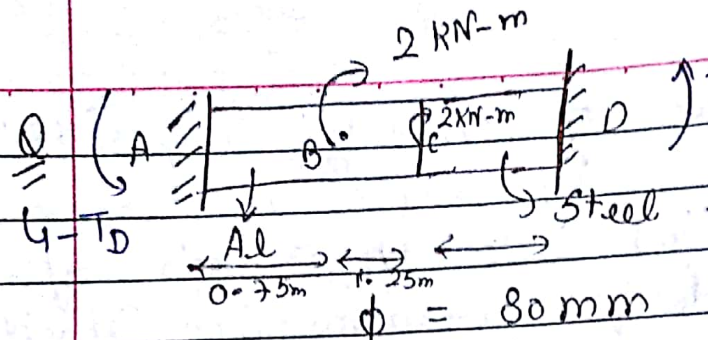
$$\Rightarrow T_c = 31.9 \text{ N-mm}$$

Consider signs of Torque in the eq^n, by free body diagram drawn.

Now, $\tau_{Al} = \frac{16 \times (300 - 31.9)}{\pi d^3}$

$\tau_{Steel} = \frac{16 \times 31.9}{\pi d^3}$

Q



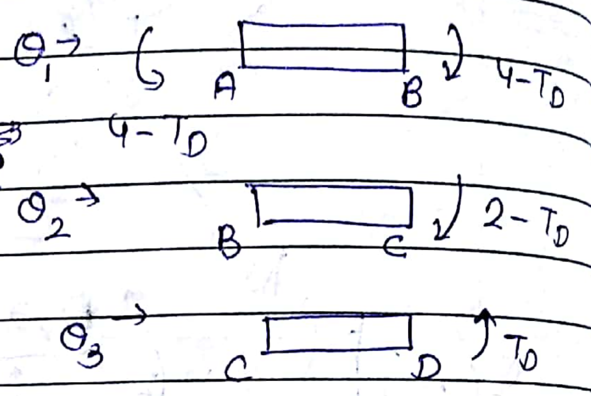
Find the maximum stress & angle of twist in the shaft.

$\phi = 80 mm$; $C_{Steel} = 85 GPa$

$C_{Al} = 30 GPa$

Ans

$\theta_1 + \theta_2 = \theta_3$



$\Rightarrow \cancel{\dfrac{(4-T_D)\times .75\times 10^3}{30\times 10^3 \times J} + \dfrac{(2-T_D)\times 1.25\times 10^3}{30\times 10^3 \times J}}$

$= \cancel{\dfrac{T_D \times 1.5\times 10^3}{80\times 10^3}}$

$\Rightarrow$ ~~T_D =~~

$\Rightarrow \dfrac{(4\times 10^6 - T_D)\times 0.75\times 10^3}{30\times 10^3\, J} + \dfrac{(2\times 10^6 - T_D)\times 1.25\times 10^3}{30\times 10^3 \times J} = \dfrac{T_D \times 1.5\times 10^3}{85\times 10^3 \times J}$

$\Rightarrow T_D = 2.175\times 10^6$ N-mm

So, $T_A = 4\times 10^6 - 2.175\times 10^6$
$= 1.825\times 10^6$ N-mm

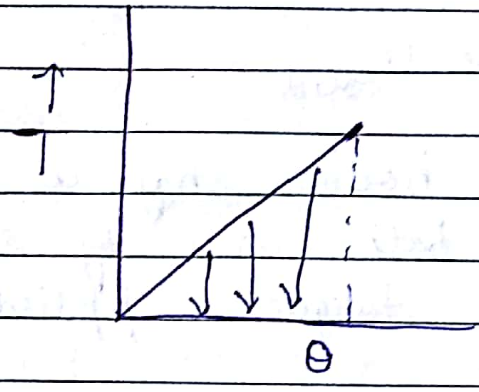
Diameter of each section is same, so τ_{max} will be selected so that Torque (T) is maximum for that section

$\tau_{max(Al)} = \dfrac{16\times 1.825\times 10^6}{\pi \times (80)^3}$

$\tau_{max(Steel)} = \dfrac{2.175\times 16\times 10^6}{\pi\, (80)^3}$

☆ Strain energy in Torsion :-

Total strain energy of a shaft under the action of torque is work done in twisting.



$$U = \frac{1}{2} T\theta$$

Replace T & θ → By stress

By Torsion eq^n — $\frac{T}{J} = \frac{\tau}{R} = \frac{G\theta}{l}$

$$\therefore \quad U = \frac{1}{2} \times \frac{\pi}{16} d^3 \times \tau \times \frac{2\tau l}{dC} \qquad \left[R = \frac{d}{2}\right]$$

$$\Rightarrow U = \frac{\tau^2}{4C} \underbrace{\left(\frac{\pi}{4} d^2 l\right)}_{\text{Volume of shaft}}$$

Total mean strain energy of a shaft (solid shaft) $\quad U = \frac{\tau^2}{4C} \times \text{Volume}$

For Hollow shaft -

$$U = \frac{1}{2} \times \frac{\pi}{32} \frac{(d_o^4 - d_i^4)}{d_o/2} \tau \times \frac{2\tau l}{d_o \times C}$$

$$= \frac{\pi}{16} \frac{(d_o^4 - d_i^4)}{d_o^2} \times \frac{\tau^2 l}{C}$$

$$= \frac{\tau^2}{4C} \times \frac{d_o^2 + d_i^2}{d_o^2} \times \left[\frac{\pi}{4}(d_o^2 - d_i^2) \times l\right]$$

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$$U = \frac{\tau^2}{4C} \times \text{Volume} \times \left[1 + \left(\frac{d_i}{d_o}\right)^2\right]$$ → Mean strain energy for hollow shaft

Clearly, $\left[1 + \left(\frac{d_i}{d_o}\right)^2\right] \geq 1$

$\therefore \quad U_{hollow} > U_{solid}$

Strain energy of hollow shaft is more than solid one, given both are of same material and same is torque applied.

Strain energy for pure shear → $U = \frac{\tau^2}{2C} \times \text{Volume}$

Strain energy for pure normal stress → $U = \frac{\sigma^2}{2E} \times \text{Volume}$

Columns → Stability of a Structure

Column :- Column is a vertical prismatic bar, designed for subjection to axial load liable to buckling.

* Behaviour & Classification :-

Behaviour can be determined by "Slenderness ratio"

Slenderness ratio → $\frac{l}{K}$

l → length of column
K → Radius of gyration

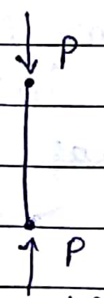
$$K = \sqrt{\frac{I}{A}}$$

Classification -

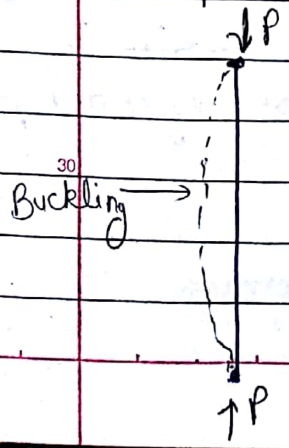
(i) $\frac{l}{K} < 32$ → Short column

(ii) $32 < \frac{l}{K} < 120$ → Medium Column

(iii) $\frac{l}{K} > 120$ → Large Column



In Short column, compressive stresses generate

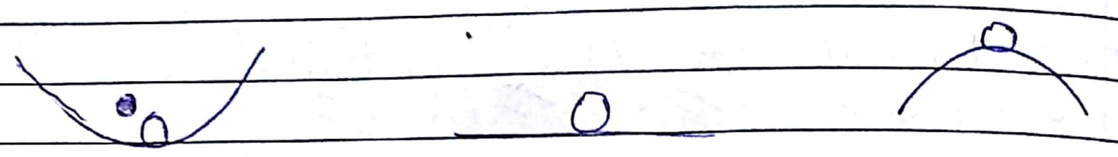


In medium or large column,

Bending stresses overpower (exceed) Normal stresses
↓
Buckling

Buckling :- It is the phenomenon by which a medium or long columns fails by combination of bending & compressive stresses.

Types of equilibrium :-



Stable ~~Unstable~~ Neutral Unstable

Stable equilibrium :- For small load P, column deflects but returns to its original position

Neutral equilibrium :- In this, column gains new position.

Unstable equilibrium :- In this, after a critical limit of load (P), the column fails or buckles up.

☆ Critical or buckling load :-

The maximum value of load upto the column remains in the neutral equilibrium is known as Critical or Buckling load.

At critical load, column can remain in equilibrium - either in straight or deflected position

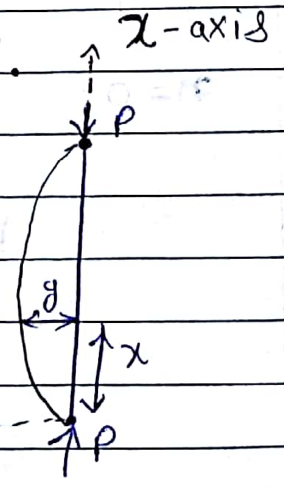
☆ Types of Columns :-

(i) Column with both ends hinged.
(ii) Column with one end free, another fixed.
(iii) Both ends are fixed.
(iv) One end fixed, other hinged.

☆ Euler's Theorem :-

Assumptions :-
(i) The column is initially ~~for~~ perfectly straight.
(ii) Load on the columns is truely axial.
(iii) The column is prismatic & material of column is perfectly elastic, homogenous & isotropic.
(iv) The column is long.
(v) The column fails by buckling only.

Case 1 :- Column with both ends hinged.



Here, $M = -Py$

$\Rightarrow EI\dfrac{d^2y}{dx^2} = M = -Py$ ← Deflection eqn

$\Rightarrow \dfrac{d^2y}{dx^2} + \dfrac{Py}{EI} = 0$

$\Rightarrow \dfrac{d^2y}{dx^2} + \alpha^2 y = 0$ [Consider $\alpha^2 = \dfrac{P}{EI}$]

Solution for this diff. eqn is -

$y = A\sin\alpha x + B\cos\alpha x$

At $x = 0$; $y = 0 \Rightarrow B = 0$

At $x = l$, $y = 0 \Rightarrow$ ~~A=0~~ $A\sin\alpha l = 0$

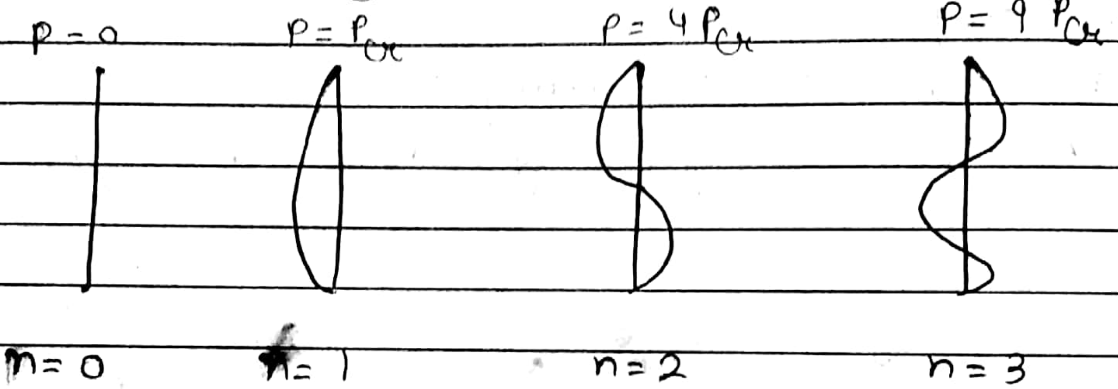
Note here, $A \neq 0$ because if $A = 0$ & $B = 0$ → $y = 0$ that means no bending → which is not the case here

$\therefore \sin \alpha l = 0$

$\Rightarrow \alpha l = n\pi$

$$\frac{P}{EI} = \alpha^2 = \frac{n^2\pi^2}{l^2}$$

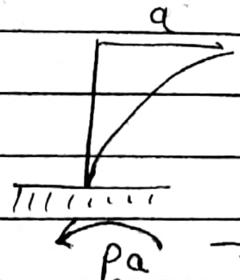
$$\Rightarrow \underbrace{P_{cr}}_{\text{Critical load}} = \frac{n^2\pi^2 EI}{l^2}$$



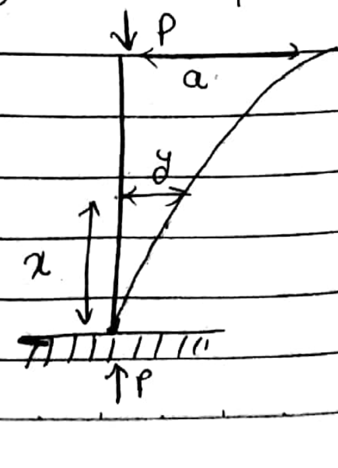
For $n=1$ → $P = P_{cr} = \dfrac{\pi^2 EI}{l^2}$

$$\boxed{P_{cr} = \frac{\pi^2 EI}{l^2}}$$ → depends on material, cross-sectional area

Case 2:- Column with one end fixed, other free.



Pa → due to fixed end's moment generated.



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At x distance,

$$M = Pa - Py$$

$$EI \frac{d^2y}{dx^2} = Pa - Py$$

$$\frac{d^2y}{dx^2} + \frac{Py}{EI} = \frac{Pa}{EI}$$

$$\frac{d^2y}{dx^2} + \alpha^2 y = \alpha^2 a \quad \text{— (1)}$$

↳ Solution for this eqn is –

$$y = C.I. + P.I.$$

↳ Particular sol.

To calculate C.I. → $\frac{d^2y}{dx^2} + \alpha^2 y = 0$

$$C.I. = A \sin \alpha x + B \cos \alpha x$$

P.I. is assumed to be –

$$y = Ax^2 + Bx + C$$

$$\frac{d^2y}{dx^2} = 2A$$

Put y & $\frac{d^2y}{dx^2}$ in (1) eqn

$$2A + \alpha^2 (Ax^2 + Bx + C) = \alpha^2 a$$

$$\Rightarrow \quad A = 0, \; B = 0 \; \& \; C = a$$

So, P.I. → $y = C = a$

$[y = a]$

So, solution of eqn (1)

$$y = A \sin \alpha x + B \cos \alpha x + a$$

At $x = 0, y = 0$

$$B = -a$$

$$y' = A\alpha \cos\alpha x + B\alpha \sin\alpha x$$

At $x = 0$, $y' = 0$

$\Rightarrow A = 0$

$$\therefore \quad y = -a\cos\alpha x + a$$

At $x = l$; $y = a$

$$a = -a\cos\alpha l + a$$

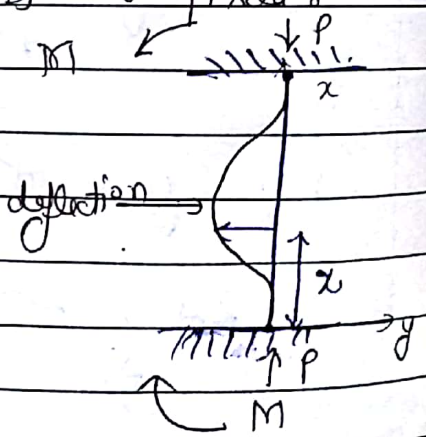
$$\Rightarrow \cos\alpha l = 0$$

$$\alpha l = \frac{\pi}{2} \rightarrow \text{For Critical case } (n=0)$$

$$\Rightarrow \alpha^2 = \frac{P}{EI} = \frac{\pi^2}{4l^2}$$

$$\boxed{P_{cr} = \frac{\pi^2 EI}{4l^2}}$$

Case 3: When both ends of columns are fixed :-



$$B.M. = M - Py$$

$$EI\frac{d^2y}{dx^2} = M - Py$$

$$\Rightarrow \frac{d^2y}{dx^2} + \frac{Py}{EI} = \frac{M}{EI}$$

$$\left[\alpha^2 = \frac{P}{EI}\right]$$

$$\frac{d^2y}{dx^2} + \alpha^2 y = \frac{M}{EI}$$

↳ Solution of this eqn : $y = A\sin\alpha x + B\cos\alpha x + \frac{M}{P}$

At $x = 0, \; y = 0 \Rightarrow B = \frac{M}{P}$

At $x = 0, \; y' = 0 \Rightarrow A = 0$

So, $y = -\frac{M}{P}\cos\alpha x + \frac{M}{P}$

$\Rightarrow y = \frac{M}{P}(1 - \cos\alpha x)$

At $x = l, \; y = 0$

$\Rightarrow 1 - \cos\alpha l = 0$

$\Rightarrow \cos\alpha l = 1 \Rightarrow 2n\pi = \alpha l$

Minimum value $\Rightarrow$

$\alpha l = 2\pi \quad (n = 1)$

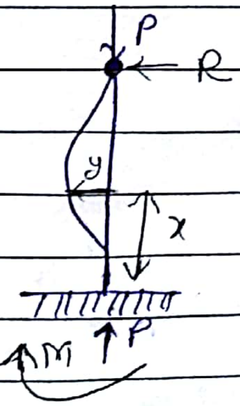
$\alpha = \frac{2\pi}{l}$

$\alpha^2 = \frac{P}{EI} = \frac{4\pi^2}{l^2}$

So, $\boxed{P_{cr} = \frac{4\pi^2 EI}{l^2}}$

Case 4: Column with one end fixed & other hinged.

• ← Hinged



In this case, to balance the the moment, there must be counter moment caused by a reaction force (R) generated.

$M = R(l - x)$

B.M. $= M - Py$
$= R(l - x) - Py$

$$\therefore \frac{d^2y}{dx^2} = \frac{R(l-x)}{EI} - \frac{Py}{EI}$$

$$\Rightarrow \frac{d^2y}{dx^2} + \alpha^2 y = \frac{R(l-x)}{EI}$$

↳ Solution of this eqn : $y = A \sin\alpha x + B \cos\alpha x + \frac{R(l-x)}{P}$

At $x = 0$, $y = 0$

$$B = -\frac{R \cdot l}{P}$$

At $x = 0$, $\frac{dy}{dx} = 0$

$$\Rightarrow A = \frac{R}{P\alpha}$$

$$\therefore y = \frac{R}{P\alpha} \cdot \sin\alpha x - \frac{Rl}{P} \cos\alpha x + \frac{R}{P}(l-x)$$

At $x = l$, $y = 0$

$$\therefore \frac{R}{P\alpha} \sin\alpha l = \frac{Rl}{P} \cos\alpha l$$

$$\Rightarrow \tan\alpha l = \alpha l$$

↳ Minimum value

$\alpha l = 4.49$ radian

$$\therefore \frac{P}{EI} = \alpha^2 = \frac{20.2}{l^2}$$

$$\boxed{P_{cr.} = \frac{20.2\, EI}{l^2} \approx \frac{2\pi^2 EI}{l^2}}$$

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Equivalent / Effective length for coloumn :-

In general for all cases → $\boxed{P_{cr} = \dfrac{\pi^2 EI}{l_e^2}}$

$l_e \rightarrow$ effective length

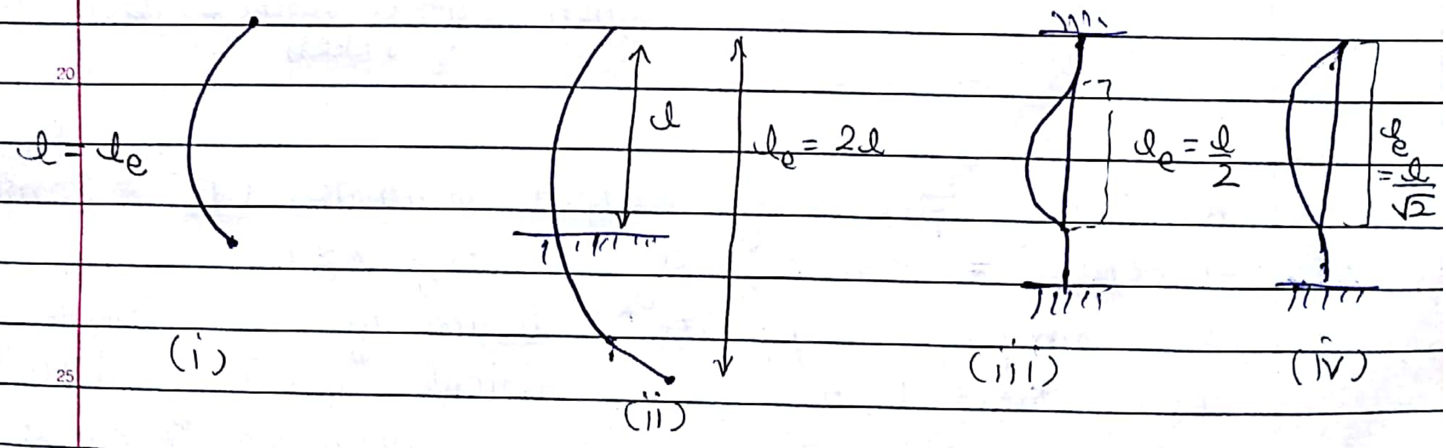
Effective length is the length b/w the points of inflection on the deflected shape.

(i) Both ends ~~fixed~~ hinged $\quad P_{cr} = \dfrac{\pi^2 EI}{l^2} \rightarrow l_e = l$

(ii) One end fixed, other free $\quad P_{cr} = \dfrac{\pi^2 EI}{4l^2} \rightarrow l_e = 2l$

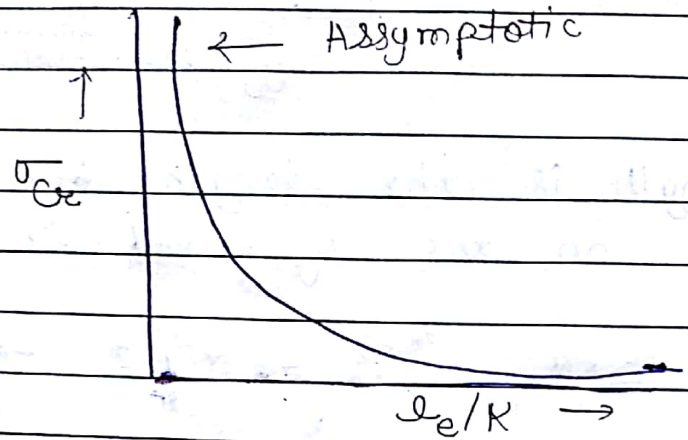
(iii) Both ends fixed $\quad P_{cr} = \dfrac{4\pi^2 EI}{l^2} \rightarrow l_e = \dfrac{l}{2}$

(iv) One end fixed, other hinged. $\quad P_{cr} = \dfrac{2\pi^2 EI}{l^2} \rightarrow l_e = \dfrac{l}{\sqrt{2}}$



Critical Stress → $\sigma_{cr} = \dfrac{P_{cr}}{Area} = \dfrac{\pi^2 E A K^2}{l^2 A}$ $[I = AK^2]$

$\sigma_{cr} = \dfrac{\pi^2 E}{(l_e/K)^2}$ ⇒ Euler



The above formula gives correct results for medium or large columns, but not for small columns. (small columns fail due to compressive stress, but graph tells it fails due to critical stress

Limiting value for Euler's formula →
When Critical value of stress = Yield stress of material

Q A steel pipe of output diameter (D_o) = 20mm, thickness = 3 mm, deflects by 3 mm when used as simply supported beam of 1 m length is subjected to a critical load of 170 N. Find the buckling load when the pipe is used as a column with both ends hinged.

Ans

↓W

$\delta_{max} = \dfrac{W l^3}{48EI}$

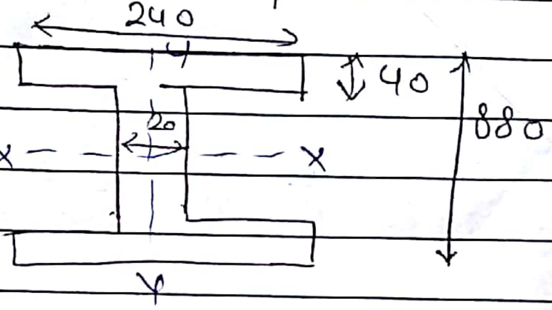
$\Rightarrow 3 = \dfrac{170 \times 10^3}{48EI}$ $\Rightarrow EI = ?$

$$P_{cr} = \frac{\pi^2 EI}{l^2}$$

$$\Rightarrow P_{cr} = 11.63 \text{ KN}$$

Q. If a simply supported beam of I section (shown in figure) is deflected by 12 mm, when subjected to UDL of 50 KN/m. Determine the safe load if the beam is used as column with both ends fixed that factor of safety is equal to 5.

$E = 205$ GPa



Ans:
$\delta = 12$ mm ; $w = 50$ KN/m ; F.O.S. = 5

$E = 205$ GPa

$$\delta = \frac{5}{384}\frac{wl^4}{EI} \quad \text{— (1)} \rightarrow \text{In case of UDL}$$

$$I_{xx} = \frac{1}{12}\left[240 \times 880^3 - 220 \times 800^3\right]$$

$$I_{yy} = \frac{1}{2}\left[2 \times (40 \times 240^3) + 800 \times 20^3\right]$$

Smaller I would be considered.

Put it into (1) & calculate l.

Mechani

Solid

$$P_{cr} = \frac{4\pi^2 EI}{l^2}$$

$$\boxed{\text{Safe load} = \frac{P_{cr}}{\text{F.O.S.}}}$$

★ Rankine's Empirical formula:-

$$\frac{1}{P_{cr}} = \frac{1}{P_c} + \frac{1}{P_e}$$

$P_{cr} \rightarrow$ Critical buckling load

$P_c \rightarrow$ Direct load due to compression/crushing $= \sigma_y \cdot A$

$P_e \rightarrow$ Euler's critical load $= \dfrac{\pi^2 E A}{(l/K)^2}$

(a) For short column-

$$P_{cr} = P_c$$

[Because $l/K \rightarrow$ small $\therefore \frac{1}{P_e} \rightarrow$ small]

(b) For long column-

$$P_{cr} = P_e$$

Again,

$$\frac{1}{P_{cr}} = \frac{1}{P_c} + \frac{1}{P_e}$$

$$\Rightarrow P_{cr} = \frac{P_c \cdot P_e}{P_c + P_e}$$

$$= \frac{\sigma_y \cdot A}{(1 + P_c/P_e)} = \frac{\sigma_y \cdot A}{1 + \frac{\sigma_y \cdot A}{\pi^2 E A}(l/K)^2}$$

$$\boxed{P_{cr} = \frac{\sigma_y \cdot A}{1 + a \cdot (l/K)^2}}$$

where $a = \dfrac{\sigma_y}{\pi^2 E} \longrightarrow$ Rankine constant [is determined experimentally]

$\hookrightarrow$ depends on material.

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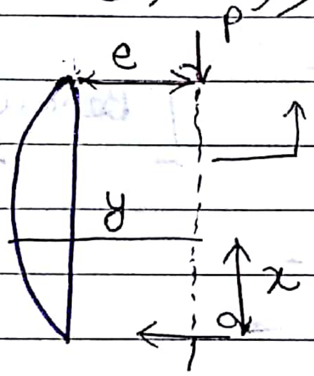
Limitations (Rankine's empirical formula) :-

(i) Load does not always passing through axial axis (eccentricity)

(ii) Column, in general, has curvature initially, but as per our assumption, we assume it straight.

(iii) 'a' has to be determined by values of σ_y & E, experimentally.

★ Column with eccentric load (Secant formula)



[Both ends hinged]

$$B.M. = -Py$$

$$\Rightarrow EI\frac{d^2y}{dx^2} + Py = 0$$

Sol. → $y = A\sin\alpha x + B\cos\alpha x$

At $x = 0$, $y = e$

$\therefore B = e$

At $x = \frac{l}{2}$, $\frac{dy}{dx} = 0$

$$\frac{dy}{dx} = \alpha A\cos\alpha x - B\alpha\sin\alpha x$$

$\therefore A = e \cdot \tan\frac{\alpha l}{2}$

$$\therefore y = \left(e\tan\frac{\alpha l}{2} \cdot \sin\alpha x + e\cos\alpha x\right)$$

Deflection will be maximum at $x = \frac{l}{2}$

$$\therefore \; y_{max} = e \tan\frac{\alpha l}{2} \cdot \sin\frac{\alpha l}{2} + e \cos\frac{\alpha l}{2}$$

$$y_{max} = e \sec\frac{\alpha l}{2}$$

Max. Bending moment (B.M.) $= P \, y_{max}$.

$$= P e \sec\frac{\alpha l}{2}$$

Maximum stress

$$\hookrightarrow \sigma_{max} = \underbrace{\frac{P}{A}}_{\text{Compressive stress}} + \underbrace{\frac{M y_c}{I}}_{\text{Bending stress}}$$

$$= \frac{P}{A} + \frac{P \cdot e \sec\left(\frac{\alpha l}{2}\right) \cdot y_c}{A \cdot K^2}$$

$$= \frac{P}{A}\left[1 + \frac{e y_c}{K^2} \cdot \sec\left(\frac{\alpha l}{2}\right)\right]$$

(i) If P & e are given σ_{max} can be calculated from this formula.

(ii) If σ_{max} & e are given, P can't be directly calculated using this formula. Because - $\sec\frac{\alpha l}{2}$ term makes it difficult.

$$\alpha^2 = \frac{P}{EI}$$

So, Perry gave simpler version of this formula.

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* Perry's formula:-

Now,

$$\sec\frac{\alpha l}{2} = \sec\frac{l}{2}\sqrt{\frac{P}{EI}} = \sec\frac{\sqrt{P}}{2}\cdot\frac{l}{\sqrt{EI}}$$

$$= \sec\frac{\sqrt{P}}{2}\cdot\frac{\pi}{\sqrt{\frac{\pi^2 EI}{l^2}}}$$

$$= \sec\frac{\pi}{2}\sqrt{\frac{P}{P_e}}$$

$P_e \rightarrow$ Euler's load

$$\sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_e}}\right) \approx \frac{1.2 P_e}{P_e - P}$$

So,

$$\sigma_{max.} = \frac{P}{A}\left[1 + \frac{\sigma \cdot y_c}{K^2} \times \frac{1.2 P_e}{P_e - P}\right]$$

let $\frac{P}{A} = \sigma_o$ & $\frac{P_e}{A} = \sigma_e$

$$\frac{\sigma_{max.}}{\sigma_o} = 1 + \frac{e \cdot y_c}{K^2} \times \frac{1.2}{1 - \left(\frac{\sigma_o}{\sigma_e}\right)}$$

$$\Rightarrow \left(\frac{\sigma_{max.}}{\sigma_o} - 1\right)\left(1 - \frac{\sigma_o}{\sigma_e}\right) = \frac{1.2\, e\, y_c}{K^2} \quad \text{— (A)}$$

Q. A hollow steel column 2.4 m long is hinge joined at both ends. It has outer diameter 40 mm, thickness 5 mm, if the yield stress is 320 MPa & $E = 2 \times 10^5$ MPa. Compare the buckling load (crippling load) given by Euler or Rankine, if the rankine constant

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(a) $= \frac{1}{7500}$. Also determine minimum $\frac{l}{K}$ ratio for which Rankine Euler formula may apply.

Ans

$l = 2.4 m$, $d_o = 40 mmm$; $d_i = 30mm$

$\sigma_y = 320 MPa$; $E = 2 \times 10^5 MPa$

$a = \frac{1}{7500}$

$$P_e = \frac{\pi^2 E I}{l^2} = 2937 N$$

$$I = \frac{\pi}{64}\left[40^4 - 30^4\right] = 8.59 \times 10^4 mm^4$$

$$A = \frac{\pi}{4}(40^2 - 30^2) = 550 mm^2$$

$$K = \sqrt{\frac{I}{A}}$$

$$P_r = \frac{\sigma_y \cdot A}{1 + a \cdot \left(\frac{l}{K}\right)^2} = 29754 N$$

$$\frac{P_r}{P_e} \simeq 10$$

$$P_c = \sigma_y \cdot A = 320 \times 550$$

$$= 176000 N$$

It is neither a short column, nor even long

$[P_r \neq P_c$ & $P_r \neq P_e]$

So, it is medium column.

↳ (Buckling action + Crushing action)

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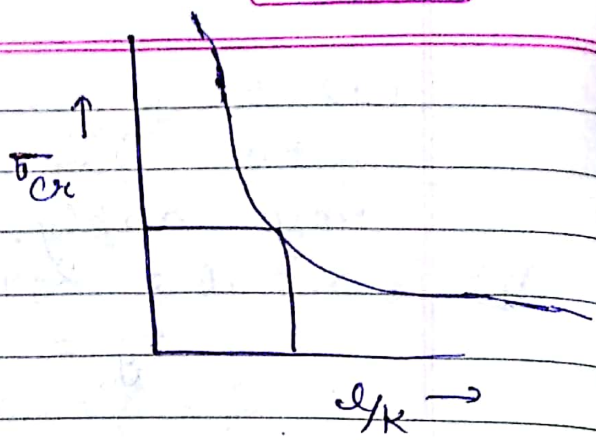
For minimum $\left(\frac{l}{K}\right)$

$$P_e = P_c \rightarrow \text{upto yield stress}$$



$$\Rightarrow \frac{\pi^2 E A}{(l/K)^2} = \sigma_y \cdot A$$

$$\Rightarrow \frac{l}{K} = \pi\sqrt{\frac{E}{\sigma_y}}$$

$$= 78.5 \quad \text{(minimum)}$$

↳ after this value, Euler's formula can be applied (buckling dominant)

Q. A metal column of 300 mm diameter & 20 mm thickness and There is load of 400 N, at an eccentricity of 50 mm. Determine the max. & min. stress in the column if the length is 5 m, both the ends of column are fixed. E = 95 GPa.

Ans

$$I = \frac{\pi}{64}\left(300^4 - 260^4\right) = 1.732\times10^4 \text{ mm}^4$$

$$A = \frac{\pi}{4}\left(300^2 - 260^2\right) = 17590 \text{ mm}^2$$

$$\sigma_{max., min.} = \frac{P}{A} \pm \frac{Pe\ \mathrm{Sec}\left(\frac{\alpha l}{2}\right)\cdot y_c}{I}$$

$$= 22.74 \pm 17.67$$

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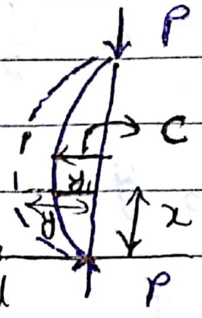
★ Column with initial curvature:-

— curve → initial curvature (Sinusoidal, parabola etc.)

- - - curve → curvature due to load applied



Assume → Initial curvature is sinusoidal given by eqⁿ:

$$y_1 = C \sin\frac{\pi x}{l}$$

Let 'y' be deflection from axis of load.

$$B.M. = -Py$$

$$EI\frac{d^2(y-y_1)}{dx^2} = -Py \quad \text{— (1)}$$

$$\Rightarrow \frac{d^2 y_1}{dx^2} = -C\frac{\pi^2}{l^2}\sin\frac{\pi x}{l}$$

From eq (1)

$$\hookrightarrow \frac{d^2y}{dx^2} + \frac{P}{EI}\cdot y = \frac{d^2y_1}{dx^2}$$

$$\Rightarrow \frac{d^2y}{dx^2} + \alpha^2 y = -C\frac{\pi^2}{l^2}\sin\left(\frac{\pi x}{l}\right)$$

↳ Solution of the ODE is -

$$y = A\sin\alpha x + B\cos\alpha x + \frac{C\pi^2/l^2 \cdot \sin\left(\frac{\pi x}{l}\right)}{\frac{\pi^2}{l^2} - \alpha^2}$$

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At $x = 0$; $y = 0 \Rightarrow B = 0$

At $x = \frac{l}{2}$; $\frac{dy}{dx} = 0 \Rightarrow A = 0$

So, $$y = \frac{C \frac{\pi^2}{l^2} \cdot \sin\left(\frac{\pi x}{l}\right)}{\frac{\pi^2}{l^2} - \alpha^2}$$

$$\alpha^2 = \frac{P}{EI}$$

$$\therefore \quad y = \frac{C \frac{\pi^2 EI}{l^2} \sin\left(\frac{\pi x}{l}\right)}{\frac{\pi^2 EI}{l^2} - P}$$

We know, $P_e = \frac{\pi^2 EI}{l^2}$ [For hinged case]

$$\Rightarrow y = \frac{C P_e \sin\left(\frac{\pi x}{l}\right)}{P_e - P}$$

At $x = \frac{l}{2}$; $y = y_{max}$.

$$\therefore y_{max} = \frac{C P_e}{P_e - P}$$

Maximum Bending Moment $\rightarrow$ $B.M. = P \cdot y_{max} = \frac{C P P_e}{P_e - P}$

Maximum Compressive Stress $\rightarrow$ $\sigma_{max} = \frac{P}{A} + \frac{M y_c}{I}$

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Note that y_c & $y_{max.}$ are different.
$y_c \rightarrow$ depends on cross-section
for square (side $\rightarrow b$) [$y_c = \frac{b}{2}$]

$$\Rightarrow \sigma_{max.} = \frac{P}{A} + \frac{(C \cdot P \cdot P_e)\, y_c}{(P_e - P) \cdot A k^2}$$

$$\Rightarrow \sigma_{max.} = \frac{P}{A}\left[1 + \frac{C \cdot P_e\, y_c}{(P_e - P)\, k^2}\right]$$

If $\frac{P}{A} = \sigma_0$ & $\frac{P_e}{A} = \sigma_e$

$$\sigma_{max.} = \sigma_0\left[1 + \frac{C \cdot y_c}{\left(1 - \frac{\sigma_0}{\sigma_e}\right) k^2}\right]$$

$$\left(\frac{\sigma_{max.}}{\sigma_0} - 1\right)\left(\frac{\sigma_e - \sigma_0}{\sigma_e}\right) = \frac{c\, y_c}{k^2} \quad — ②$$

Compare it with eq^n derived in eccentric load case

$$\left(\frac{\sigma_{max.}}{\sigma_0} - 1\right)\left(\frac{\sigma_e - \sigma_0}{\sigma_e}\right) = \frac{1 \cdot 2\, e\, y_c}{k^2} \quad — ③$$

$$C = 1 \cdot 2e$$

If a column is initially bent & is also eccentrically loaded, then the combined initial equivalent curvature can be taken as -

$$C_{eq.} = C + 1 \cdot 2\, e$$

And, equivalent initial eccentricity can be taken as

$$e_{eq.} = e + \frac{C}{1 \cdot 2}$$

Q. An initially curved column of section 36×48 mm^2 is 1·2 m long. It is fixed at both the ends & has the equation of curve as - $y = 0 \cdot 006 \sin \frac{\pi x}{2}$ m. A compressive load of 20 kN is applied parallel to the axis at eccentricity of

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20 mm. Find the maximum stress induced.
$E = 204\ GPa$

Ans

$$\left(\frac{\sigma_{max}}{\sigma_0} - 1\right)\left(1 - \frac{\sigma_0}{\sigma_e}\right) = \frac{1.2\, e\, y_c}{k^2}$$

$$e = 20 + \frac{c}{1.2} = 20 + \frac{6}{1.2} = 25\ mm$$

$$k^2 = \frac{I}{A} \quad ; \quad I = \frac{48 \times 36^3}{12}$$ → lesser has to be considered.

$$y_c = \frac{36}{2} = 18 \qquad \boxed{\frac{36 \times 48^3}{12}} \times$$

$$\sigma_0 = \frac{P}{A} =$$

$$\sigma_e = \frac{P_e}{A} \quad ; \quad P_e = \frac{4\pi^2 EI}{l^2}$$ [In case of hinged] at both ends.

$$\sigma_{max} = 70.55\ MPa$$

Pressure Vessels

↳ vessel that contains pressurized fluids
  ↳ Forces onto walls → Stresses & strains developed.

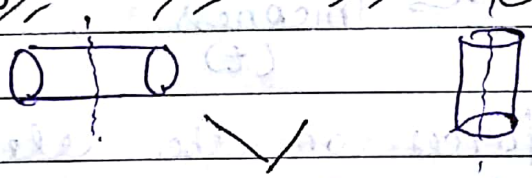
★ Classification of Pressure Vessels:-

Ratio → $\frac{t}{d}$ (t → Thickness, d → diameter)

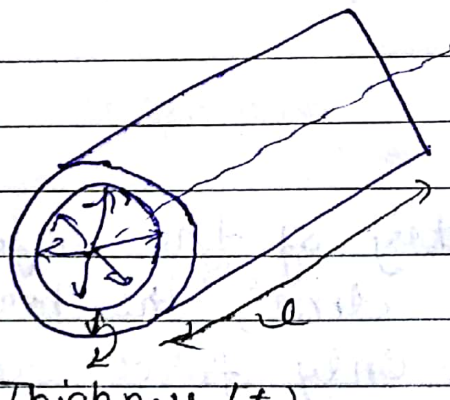
(a) $\frac{t}{d} < \frac{1}{20}$ → Thin pressure Vessel

(b) $\frac{t}{d} > \frac{1}{20}$ → Thick pressure Vessel

★ Thin Cylindrical Pressure Vessel:-



Modes of ~~of the~~ failure of pressure Vessels



Assumption :-

(i) Only positive internal pressure is considered (weight of content, self-weight of cylinders, external pressure not considered.)

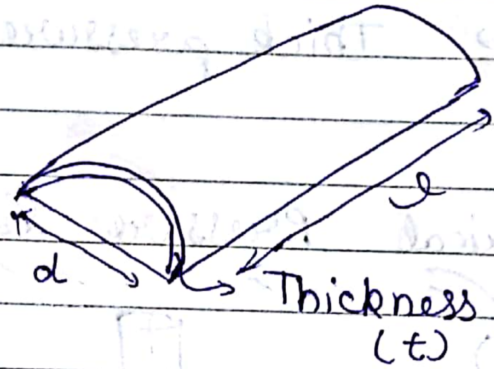
(ii) Radial planes remain radial & whole thickness does not change due to internal pressure.

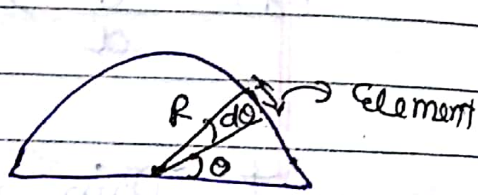
(iii) Fluid is flowing in a streamlined manner (There will be no shearing)

(iv) The ends of cylinders are flat.

(v) Material is elastic (linearly)

Case 1 :-
Around (along) circumference failure





Net forces on the element

$$dF = P_i \, r \, d\theta \times l$$

Due to the symmetry of the surface about a vertical radial line, the horizontal forces tend to cancel. Only force remaining is vertical.

Vertical force $\longrightarrow$ $dF_v = P_i \, r \, d\theta \, l \sin\theta$

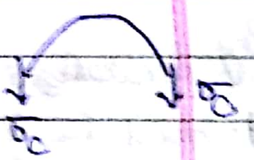
$$F_\theta = \int_0^\pi P_i\, r\, l \sin\theta\, d\theta$$

$$F_\theta = P_i \frac{d}{2}\, l\, [-\cos\theta]_0^\pi \qquad \left[d = \frac{r}{2}\right]$$

$$F_\theta = P_i\, d\, l$$

This force has to be resisted by the stress resultants along the the two edges.

$\sigma_c \rightarrow$ Stresses produced.



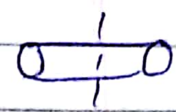
$$2(\sigma_c \times t \times l) = F_\theta$$

Resisting area $\rightarrow t \times l$

$$\Rightarrow 2\sigma_c\, t\, l = P_i\, d\, l$$

$$\Rightarrow \boxed{\sigma_c = \frac{P_i d}{2t}}$$ → Hooke's stresses or Circumferencial stresses.

Case 2:- Failure along plane ⊥r to longitudinal axis -



$$\text{Longitudinal force} = P_i \times \frac{\pi}{4} d^4$$

$P_i \rightarrow$ Internal pressure

$$\text{Resisting area} = 2\pi r t$$
$$= \pi d t$$

$\sigma_l \rightarrow$ longitudinal stress

$$\sigma_l \times \pi d t = P \times \frac{\pi}{4} d^2$$

$$\boxed{\sigma_l = \frac{Pd}{4t}}$$

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Maximum Shear stress:-

$$\sigma_c = \frac{Pd}{2t} \rightarrow \sigma_1$$

$$\sigma_l = \frac{Pd}{4t} \rightarrow \sigma_2$$

$$\text{Max. shear stress} = \frac{\sigma_c - \sigma_l}{2} = \frac{Pd}{8t}$$

☆ Hoop Strain:-

$$\varepsilon_h = \frac{\sigma_h}{E} - \underbrace{\mu}_{\text{Poisson's ratio}} \frac{\sigma_l}{E}$$

[By generalized Hooke's law]

$$= \frac{Pd}{2tE} - \mu \frac{Pd}{4tE}$$

$$= \frac{Pd}{2tE}\left(1 - \frac{\mu}{2}\right)$$

Longitudinal strain:-

$$\varepsilon_l = \frac{\sigma_l}{E} - \mu \frac{\sigma_h}{E}$$

$$= \frac{Pd}{4tE} - \frac{\mu Pd}{2tE}$$

$$= \frac{Pd}{2tE}\left[\frac{1}{2} - \mu\right]$$

Volumetric Strain:-

$$\varepsilon_v = \frac{\delta V}{V}$$

$$V = \frac{\pi}{4} d^2 l$$

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$$\delta V = \frac{\pi}{4}\left[l \cdot 2d \cdot \delta d + d^2 \cdot \delta l \right]$$

$$\therefore \quad \epsilon_V = \frac{l \cdot 2d\, \delta d + d^2 \delta l}{d^2 l}$$

$$\epsilon_V = \underbrace{2\frac{\delta d}{d}}_{\text{Hoop Strain}} + \underbrace{\frac{\delta l}{l}}_{\text{longitudinal strain}}$$

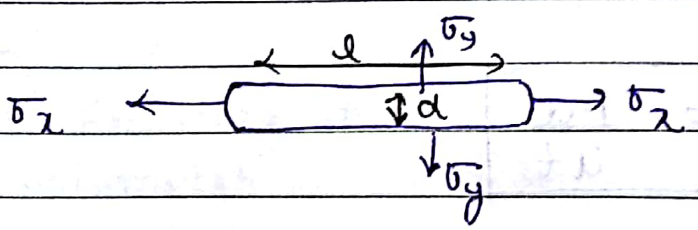
$$\therefore \quad \epsilon_V = 2\epsilon_h + \epsilon_l$$

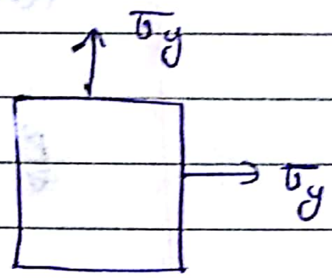
$$= 2 \cdot \frac{Pd}{2tE}\left(1 - \frac{\mu}{2}\right) + \frac{Pd}{2tE}\left(\frac{1}{2} - \mu\right)$$

$$\epsilon_V = \frac{Pd}{4tE}\left[5 - 4\mu\right]$$

★ **Generalized Hooke's law :-**

(when multiple stresses act)





Strain in x- direction due to σ_x

$$= \frac{\sigma_x}{E}$$

Strain in y- direction due to σ_x

$$= -\mu \frac{\sigma_x}{E}$$

$\mu \rightarrow$ Poisson's ratio

$$\mu = \frac{\text{Strain in } y \text{ dir.}}{\text{Strain in } x\text{-dir.}} = \frac{\text{Lateral strain}}{\text{Long. Strain}}$$

Strain in y- direction due to σ_y

$$= \frac{\sigma_y}{E}$$

Strain in x- direction due to σ_y

$$= -\mu \frac{\sigma_y}{E}$$

Therefore,

Total strain in x- direction $\rightarrow \epsilon_x = \frac{\sigma_x}{E} - \mu \frac{\sigma_y}{E}$

Total strain in y- direction $\epsilon_y = \frac{\sigma_y}{E} - \mu \frac{\sigma_x}{E}$

★ <u>Thin Spherical Pressure Vessel :-</u>

Force $= P \times \frac{\pi}{4} d^2$

→ Resisting area

$$P \times \frac{\pi}{4} d^2 = \pi d t \times \sigma$$

σ_θ & σ_h → $\boxed{\sigma = \frac{Pd}{4t}}$

Both hoop & longitudinal $\rightarrow \epsilon = \frac{Pd}{4tE}(1-\mu)$

$$\epsilon_v = 3\epsilon$$

$$= 3\frac{Pd}{4tE}(1-\mu)$$

$\left[V = \frac{\pi d^3}{6} \right.$, $\delta V = \frac{\pi \cdot 3d^2 \cdot \delta d}{6}$

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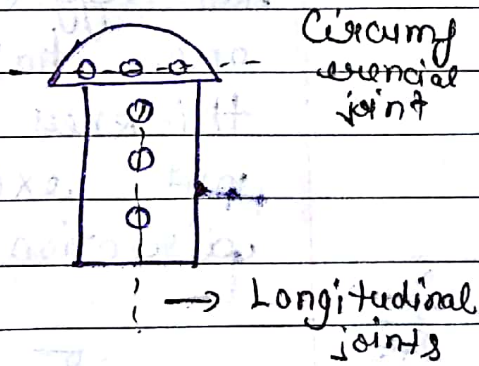
$$\therefore \ \epsilon_v = \frac{3\delta d}{d}$$ ↳ Circumferencial stress

$$= 3\epsilon \ ]$$

★ Joint in Thin Cylindrical Vessel :-



Efficiency of joints

$$\eta = \frac{\text{Strength of joint}}{\text{Strength of solid plate}}$$

$\cancel{\eta_h}$ η_c & η_l

$$\sigma_h = \frac{Pd}{2t\,\eta_l} \quad ; \quad \sigma_l = \frac{Pd}{4t\,\eta_c}$$

Q. A 6m long thin cylindrical shell is 800 mm in diameter & 10 mm thick. It is subjected to an internal pressure of 400 MPa. Determine the change in diameter, change in length & change in Volume. Take $\mu = 0.3$

$E = 205$ GPa

Ans

$$\frac{\delta d}{d} = \epsilon_h = \frac{Pd}{2tE}\left(1 - \frac{\mu}{2}\right)$$

$$\delta d = \epsilon_h \cdot d = 0.53 \text{ mm}$$

$$\delta l = \epsilon_l \cdot l = 0.93 \text{ mm}$$

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$\delta V = (2\epsilon_h + \epsilon_l) V = 4.47 \times 10^{-6}$

Q A ~~cyl~~ ~~cyl~~ cylinder has internal diameter 1.2 m & length 2.5 m. The internal pressure 1.5 MPa. The longitudinal joint has efficiency 80 %, the ~~cir~~ circumferencial one has 50 %. Find the minimum thickness required if the stresses are not exceed 48 MPa in circumferencial direction & 32 MPa in longitudinal direction.

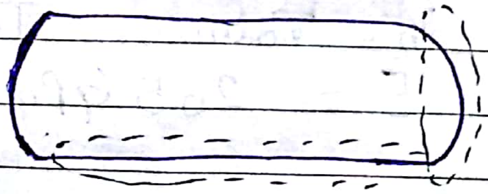
Ans

$\sigma_h = \frac{Pd}{2t\,\eta_l} \Rightarrow t = 23.43\ mm$

$\sigma_l = \frac{Pd}{4t\,\eta_c} \Rightarrow t = 28.12\ mm$ Minimum

(Because if we take $t = 23.43$ then $\sigma_l > 32$)

★ Thin cylindrical shell with hemispherical ends:-



If max. allowable stress is same (both are made up of same material)

$\frac{Pd}{2t_c} = \frac{Pd}{4t_s}$

$t_c \rightarrow$ thickness of cylindrical part

$t_s \rightarrow$ thickness of spherical part

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$$\Rightarrow \boxed{t_c = 2t_s}$$

To avoid distortion →
Hoop strain has to be same.

$$\frac{Pd}{2t_c E}\left(1 - \frac{\mu}{2}\right) = \frac{Pd}{4t_s E}(1-\mu)$$

$$\boxed{\frac{t_c}{t_s} = \frac{2-\mu}{1-\mu}}$$

For steel $\mu = 0.3$

$$\frac{t_c}{t_s} = \frac{17}{7}$$

↳ Max. stress in spherical part (of steel material)

$$\sigma_c = \frac{17\,Pd}{28\,t_c}$$

Q. A thin cylindrical shell of 1.8 m diameter with hemi-spherical ends is subjected to internal pressure of 2 MPa. (a) Determine the thickness required if the stress is not to exceed 120 MPa in both the parts.

$\boxed{\mu = 0.25}$ (b) Determine the thickness of two parts, if distortion at the junction is to be avoided.

(c) Determine the stresses if thickness to satisfy the above condition, is provided.

Ans (a) $\sigma = \frac{Pd}{2t_c} \Rightarrow 2t_c = \frac{2 \times 1.8 \times 10^3}{120}$

$t_c = 15$ mm

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$\therefore \; t_s = 7.5 \text{ mm}$

(b) $\dfrac{t_c}{t_s} = \dfrac{2-\mu}{1-\mu} = \dfrac{1.75}{0.75} = \dfrac{7}{3}$

Take $t_c = 15 \text{ mm} \Rightarrow t_s = \cancel{7.5 \text{ mm}}$ 6.43 mm

But with $t_s = 6.43$ mm

$\sigma_s > 120$ MPa

$\therefore \; t_c = 7.5 \times \frac{7}{3} = 17.5 \text{ mm}$

with $t_c = 17.5$ mm

$\sigma_c < 120$ MPa

Q. A cylindrical boiler drum has semi-spherical ends. The cylindrical portion is 1.26 m long, 8 mm in diameter & 20 mm thickness. After filling it with water, at atmospheric pressure, it's put on hydraulic test & pressure is raised to 12 MPa. Find the volume of water (additional) required to fill in drum. Assume there is no distortion.

Take $E = 205$ GPa ; $\mu = 0.3$

$K = 2080$ MPa (Bulk Modulus)

Ans. Change in volume of cylindrical portion

$$\delta V_c = \frac{Pd}{4tE}(5-4\mu)V$$

$$= 1.78 \times 10^6 \text{ mm}^3$$

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Change in Volume of spherical portion →

$$\delta V_s = 3\,\epsilon_v \cdot V$$

$$= 3 \times \frac{204}{E} \times \frac{\pi}{6} (800)^3$$

{ Since distortion is to be avoided. ∴ $\epsilon_v = \epsilon_{hc}$ }

[In cylindrical portion →

$$\sigma_h = \frac{Pd}{2t} = \frac{12 \times 800}{2t} = 240\ N/mm^2$$

$$\sigma_l = 120\ N/mm^2$$

$$\epsilon_{hc} = \frac{1}{E}\left[\sigma_c - \mu\sigma_l\right] = \frac{204}{E}$$]

$$\therefore\ \delta V_s = 0.8 \times 10^6\ mm^3$$

Now, To calculate decrease in Volume of water →

$$K = \frac{-P}{(dV/V)}$$

$$\delta V_w = \frac{-P(V)}{K}$$ → Volume of drum (cylindrical + spherical)

$$= 6.187 \times 10^6\ mm^3$$

So, $\underbrace{\delta V}_{\text{Water to be added}} = \delta V_c + \delta V_s + \delta V_w$

=

Q A thin cylindrical shell of diameter D, length l & thickness t. What is the ratio of longitudinal strain to Hoop strain in terms of poisson's ratio.

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Ans $\dfrac{\epsilon_v}{\epsilon} = \dfrac{2-\mu}{1-\mu}$

Q For a power transmission shaft, power transmitting P at N rpm. It's diameter is proportional to -

Ans $$d \propto \left(\frac{P}{N}\right)^{\frac{1}{3}}$$

$\tau_{max} \rightarrow$ Allowable stress (Material property)

Q Maximum shear stress developed in solid circular shaft under pure torsion is 240 MPa. If shaft diameter is doubled, what is the max. shear stress developed corresponding to same torque.

Ans 30