Assignment #06
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16UME017

Sol 1.

w N/mm

A ⌇⌇⌇⌇⌇⌇⌇⌇⌇⌇⌇ B

3m

$y_B = 40$ mm

A ⌇⌇⌇⌇⌇⌇⌇⌇⌇⌇⌇ B

x x

$$M_{x-x} = -\frac{w(l-x)^2}{2} = EI\,y''$$

$$\Rightarrow \frac{w(l-x)^3}{6} + c_1 = EI\,y'$$

$$\Rightarrow -\frac{w(l-x)^4}{24} + c_1 x + c_2 = EI\,y$$

at $x = 0$, $y = 0$ & $y' = 0$

$$\frac{wl^3}{6} + c_1 = 0 \Rightarrow c_1 = -\frac{wl^3}{6}$$

$$-\frac{wl^4}{24} + c_2 \Rightarrow c_2 = \frac{wl^4}{24}$$

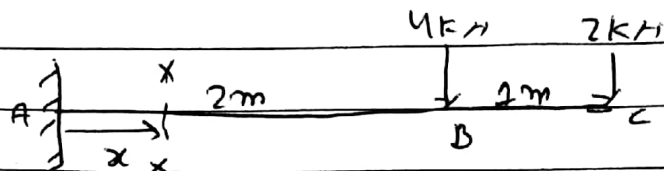$$y'_B = -\frac{wl^3}{6EI}, \quad y_B = -\frac{wl^4}{8EI}$$

here $y = 40$ mm, $l = 3 \times 10^3$ mm

$$40 = -\frac{w \times 81 \times 10^9}{8EI}, \quad \Rightarrow \frac{w}{EI} = \frac{-8 \times 40}{(81 \times 10^9) = l^4}$$

$$y'_B = \frac{8 \times 40}{l^4} \times \frac{l^3}{6} = \frac{8 \times 40}{3 \times 10^3 \times 6}$$

$$y'_B = 17.78 \times 10^{-3} \text{ Rad.}$$

Sol 2.

$$4KN \qquad 2KN$$

A ⊢ ——— x —— 2m ——— ↓ —— 2m —↓ c
      B

$$M_{xx} = -2\times10^3(l-x) - 4\times10^3\left(l-x-\frac{l}{3}\right)$$

$$M_{xx} = -2\times10^3\left[l-x+\frac{4l}{3}-2x\right]$$

$$M_{xx} = -2\times10^3\left(\frac{7l}{3}-3x\right) = (2\times10^{13})y''$$

$$\Rightarrow \quad -10^{10}\left(\frac{7l}{3}-3x\right) = y''$$

$$\Rightarrow \quad y' = -10^{-10}\left(\frac{7l}{3} - \frac{3x^2}{2}\right)+c_1$$

$$\Rightarrow \quad y = -10^{-10}\left(\frac{7lx^2}{6} - \frac{3x^3}{6}\right)+c_1x+c_2$$

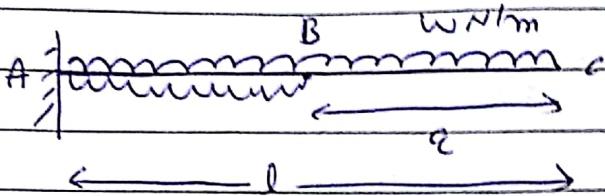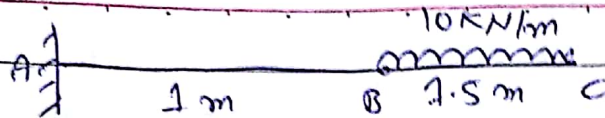at $x=0$ ⇒ $y=0$ & $y'=0$
$$\Rightarrow \quad c_1 = c_2 = 0$$
at $x=l$

$$y_2 = -10^{-10}\left(\frac{7l^3}{6} - \frac{3l^3}{6}\right)$$

$$\Rightarrow \quad y_2 = -10^{-10}\left(\frac{2l^3}{3}\right)$$

$$= \frac{-10^{-10}\times 2\times 27\times10^9}{3}$$

$$= 1.8\,mm = y_c$$

Sol. 3.

$$10\,kN/m$$

A ⊢——————1 m——————B 2.5 m C

B $w\,N/m$

A ⊢————————————————— C

←———— a ————→

←———————— l ————————→

$$W = 10\,kN/m = 10 \times 10^6\,N/mm$$

$$l = 2.5\,m$$

$$E = 210 \times 10^3\,N/mm^2$$

$$I = 9500 \times 10^4\,mm^4$$

$$\delta'_c = \frac{-Wl^3}{6EI} + \frac{W(l-a)^3}{6EI}$$

$$\delta'_c = \frac{W}{6EI}\left[(l-a)^3 - l^3\right]$$

$$\delta'_c = \frac{10}{6 \times 210 \times 10^3 \times 9500 \times 10^4}\left[(1\times10^3)^3 - (2.5\times10^3)^3\right]$$

$$\delta'_c = \frac{-14.625 \times 10^9}{1197 \times 10^4} = 0.12 \times 10^{-3}\,Rad$$

$$\delta_c = \frac{W}{EI}\left[\frac{-l^4}{8} + \frac{(l-a)^4}{8} + \frac{(l-a)^3.a}{6}\right]$$

$$\delta_c = \frac{10 \times 10^{12}}{6 \times 210 \times 10^3 \times 9500 \times 10^4}\left[-4.88 + 0.125 + 0.25\right]$$

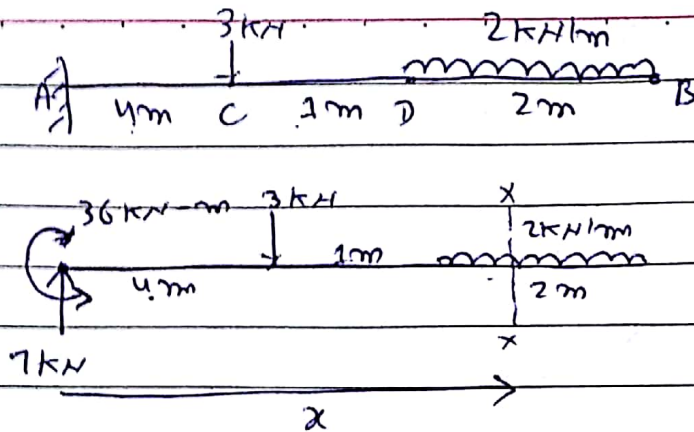$$\delta_c = \frac{-10^3 \times 4.505}{6 \times 21 \times 95} = -0.37\,mm$$

$$= 3.7\,mm$$

10^{13} mpi

10^{15}

10^{13} MPc

10^{16} N/m²

Sol 4.

$EJ = 10^{13}$ N/m

$EI = 1 \times 10^{10}$ kN/mm²

$= 10^{19}$ N/m²

$= 10^{16}$ kN/m²

$$M_{xx} = 7x - 36 - 3(x-4) - 2(x-5)^2 = EIy''$$

$$\textcircled{1}\; EIy' = \frac{7x^2}{2} - 36x - 3\left(\frac{x^2}{2} - 4x\right) - \frac{(x-5)^3}{3} + c_1$$

$$\textcircled{2}\; EIy = \frac{7x^3}{6} - 18x^2 - 3\left(\frac{x^3}{6} - 2x^2\right) - \frac{(x-5)^4}{12} + c_1 x + c_2$$

at $x = 0$, $y = 0$ & $y' = 0$

$$0 = -\frac{(-5)^3}{3} + c_1 \quad \Rightarrow \quad c_1 = \frac{-125}{3}$$

$$\& \quad 0 = -\frac{(-5)^4}{12} + c_2 \quad \Rightarrow \quad c_2 = \frac{625}{12}$$

$$EIy'' = \frac{7x^3}{6} - 18x^2 - 3\left(\frac{x^3}{6} - 2x^2\right) - \frac{(x-5)^4}{12} + \frac{625}{12} \cdot \frac{1}{12}$$

$\textcircled{•}$ $EJ = 10^{16}$ kN/m² & $x = 7m$

$$y = \frac{1}{10^{16}}\left[400.17 - 882 - 171.5 + 294 - 1.33 + 57.05\right]$$

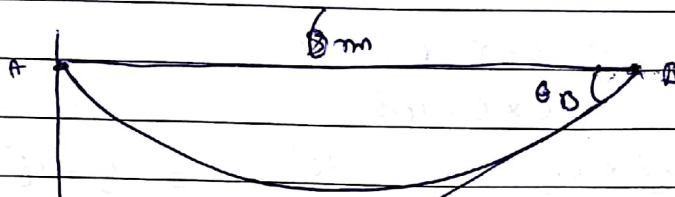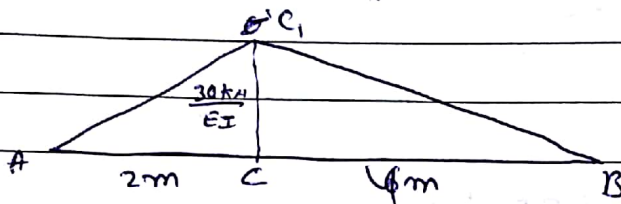$$= \frac{-308.61}{10^{16}} \times 10^{+6} \times (10^3)^3 = -3.08 \text{ mm}$$

Sols.

$P_A = 15 kN$

$R_B = 5 kN$
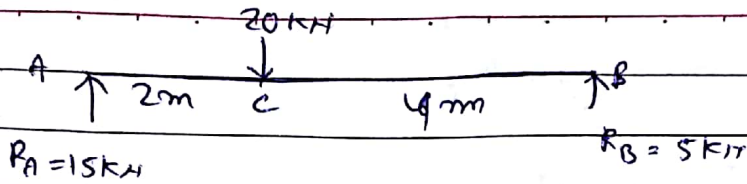
$I = 6 \times 10^8 \, mm^4$

$E = 200 \times 10^3 \, N/mm^2$

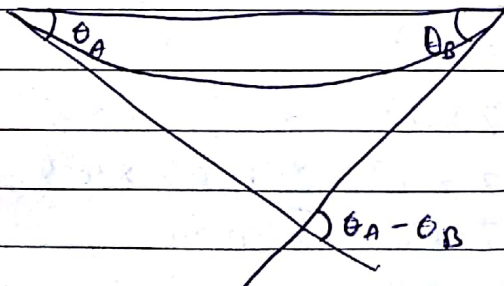$E = 200 \, kN/mm^2$

$AA' = $ area moment about A

$$AA' = \left[\frac{1}{2} \times (2 \times 10^3) \times \frac{30}{EI}\right] \times \frac{4 \times 10^3}{3}$$

$$+ \left[\frac{1}{2} \times (4 \times 10^3) \times \frac{30}{EI}\right] \times \left(\frac{2+4}{3}\right) \times 10^3$$

$$AA' = \frac{30 \times 10^6}{6 \times 10^8 2 \times 200} \,^3 \left(\frac{8}{3} + \frac{40}{3}\right) =$$

$$= 1.2 \times 10^6 \, mm \quad 0.6 \times 10^2 \, mm$$

$$0.2 \times 10^2 \, mm$$

$AB = 6 \times 10^3 \, mm$

$$\theta_B = tan^{-1}\left(\frac{0.2 \times 10^2}{6 \times 10^3}\right) = 1.56 \, Rad$$

$$3 \times 10^{-7} \, Rad$$
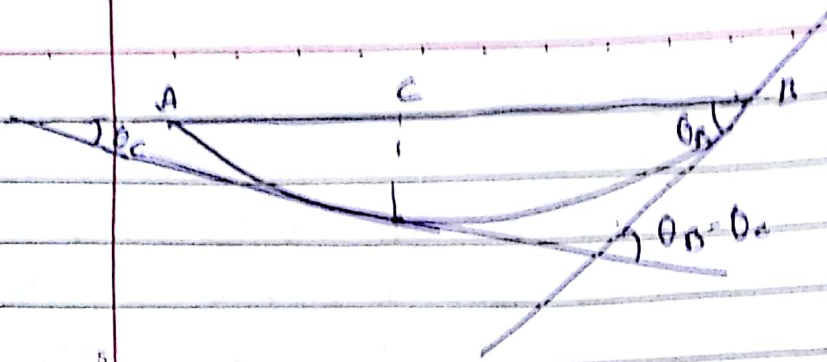

$Area = \theta_A - \theta_B$

$$= \frac{35}{EI}\left[(2 \times 10^3) + (4 \times 10^3)\right]$$

$$= \frac{15 \times 6 \times 10^3}{200 \times 6 \times 10^8}$$

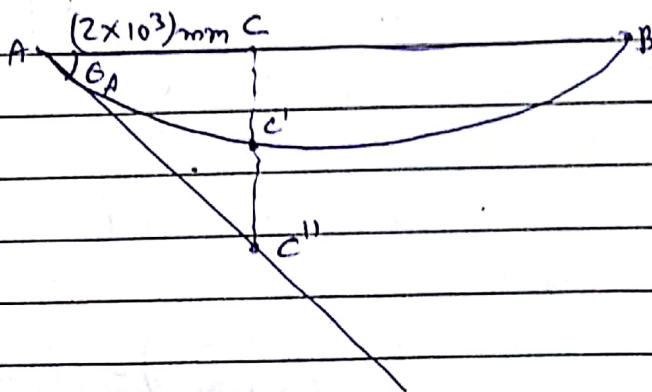$$= 7.5 \times 10^{-7} \, Rad$$

$$\theta_A = 10.5 \times 10^{-7} \, Rad$$

$$|\theta_B - \theta_C| = Area(BCC^1)$$

$$|\theta_B - \theta| = \frac{1}{I} \times \frac{24 \times 10^3 \times 30}{200 \times 6 \times 10^8}$$

$$= \frac{10^3 \times 3\phi}{60\phi \times 10^8} = \frac{10^3}{2 \times 10^9} = 5 \times 10^{-7} \, Rad$$

$$\theta_C = \theta + (5 \times 10^{-7}) \div \theta_B$$
$$= 2 \times 10^{-7} \, Rad$$

$$C \rlhd\!\!= \quad \tan \theta_A = \frac{CC^{11}}{2 \times 10^3}$$

$$\Rightarrow \quad CC^{11} = 2 \times 10^3 \times \tan(10 \approx 5 \times 10^{-7} \, Rad)$$
$$= 2 \times 10^3 \times 1.05 \times 10^{-6}$$
$$= 2.1 \times 10^{-3} \, mm$$

$$C^1 C^{11} = area \; moment \; about \; C$$
$$= \frac{1}{2} \times (2 \times 10^3) \times \frac{30}{EI} \times \frac{2 \times 10^3}{3}$$

$$c'c'' = \frac{6\cancel{0} \ast\ 2\cancel{0} \times 10^6}{\frac{20\cancel{0} \times 6 \times 10^8}{10}}$$

$$= 0.16 \times 10^{-3} \ mm$$

deflection at pt c (cc') = $cc'' - c'c''$

$$= (7.1 - 0.16) \times 10^{-3}$$

$$= 1.94 \times 10^{-3} \ mm$$