Mathematics-III
(MTH 213)

Title of the course: Mathematics-III

Target Audience: Core course for B. Tech. 02^{nd} Year

Pre-requisites: Mathematics-I

Total contact hours: Lectures 40

Total number of credits: 04

Course Objectives: The main objective of this course is to introduce the undergraduate students to Complex Analysis and partial differential equations. Complex Analysis and partial differential equations are widely used in the fields of Science and Technology. In contrast with Calculus, students are novice in learning Complex Analysis and partial differential equations at the undergraduate level. Realizing this fact, these topics of Applied Mathematics are kept among the main ingredients of Engineering Mathematics.

Course Outcomes: At the end of the course, students will be able to

- Carry out computations with complex numbers and the geometry of the complex plane.
- Determine if a function is harmonic and find a harmonic conjugate via the Cauchy-Riemann equations.
- Find the images of lines, circles in the complex plane under the exponential, trigonometric and hyperbolic & other elementary mappings.
- Evaluate contour integrals using the Cauchy Integral Theorem and the Cauchy Integral Formula in basic and extended form. Find Taylor or Laurent Series for complex valued functions. Identify and classify zeros and singular points of functions. Compute residues. Use residues to evaluate various contour integrals.
- Identify & classify PDEs of different types, solve first order PDEs and give their geometric interpretations. Compute canonical form a second order PDE and solve Heat, Wave & Laplace equations on rectangular and spherical domains using Fourier Method.

Course Outline: This course has been designed as a compulsory course for undergraduate students. It mainly consists of *complex analysis*, and *Partial Differential Equations*.

Unit I

Field of Complex Numbers; Absolute values, Conjugates, Polar form, Argument, Demoivre's Theorem, nth roots, geometrical interpretations.Complex valued functions, limits, continuity, differentiability, analyticity; Cauchy-Riemann Equations; harmonic functions; conjugate harmonic functions.Elementary complex functions (exponential, logarithmic, trigonometric/hyperbolic, inverse trigonometric, complex powers)’ principle values of multi-valued functions.

Unit II

Conformality of analytic functions, mappings of plane regions by elementary analytic functions; Mobeus transformations; Mobeus transformation as a composition of translation, dilation, rotation, inversion; fixed points; cross ratios; mapping of disk/half plane using Mobeus transformations; determination of Mobeus transformation which map a given region onto another prescribed region.

Unit III

Line integrals; ML-inequality, dependence of integral upon the end points of a path; Cauchy Integral Theorem for simply connected domains and its extension to multiply connected domains; Cauchy’s Integral formula; existence of derivatives of all orders of an analytic function; Cauchy’s inequality; Liouville’s Theroem, Morera’s Theorem; Fundamental Theorem of Algebra, Gauss’s Theorem (zeros of the derivative of a polynomial lie within the convex hull of the zeros of the polynomial).

Unit IV

Convergence of sequences and series; Power series; radius of convergence of a power series; identity theorem for power series; analyticity of power series; Taylor Series of an analytic function; Laurent Series; methods of obtaining Laurent Series; Zeroes of an analytic function are isolated; Isolated/non-isolated singularities; classification of isolated singularities as removable, pole, essential; Residues, Cauchy Residue Theoroem; Rouche’s Theorem; Evaluation of some real integrals by using Cauchy’s Residue Theorem.

Unit V

Introduction to PDE, basic concepts, Linear and quasi-Linear first order PDE, 2nd order semi-linear PDE(Canonical form), D'Alemberts formula and Duhamel's principle for one dimensional wave equation., Laplace and Poisson's equation., maximum principle with application, Fourier Method for IBV problem for wave and heat equation rectangular region, Fourier Series method for Laplace equation in three dimensions.

Suggested Reading: Following books are referred for this course:

- E. Kreyszig, *Advanced Engineering Mathematics*, Wiley India (08th Edition).
- J.W. Brown & R.V. Churchill, *Complex Variables and Applications* , McGraw Hill
- An Elementary Course in Partial Differential Equations by Amarnath T.