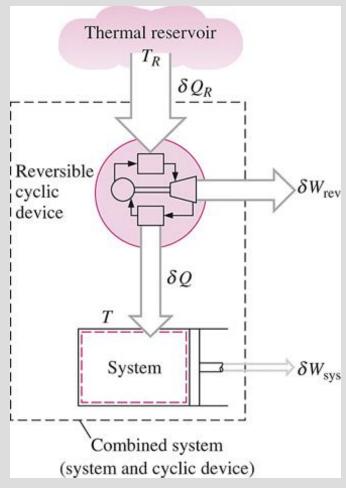
Thermodynamics: An Engineering Approach, 6th Edition Yunus A. Cengel, Michael A. Boles McGraw-Hill, 2008

Chapter 7 ENTROPY

Objectives

- Apply the second law of thermodynamics to processes.
- Define a new property called entropy to quantify the secondlaw effects.
- Establish the increase of entropy principle.
- Calculate the entropy changes that take place during processes for pure substances, incompressible substances, and ideal gases.
- Examine a special class of idealized processes, called isentropic processes, and develop the property relations for these processes.
- Derive the reversible steady-flow work relations.
- Develop the isentropic efficiencies for various steady-flow devices.
- Introduce and apply the entropy balance to various systems.

ENTROPY



The system considered in the development of the Clausius inequality.

$$\oint \frac{\delta Q}{T} \le 0 \qquad \delta W_C = \delta Q_R - dE_C$$

$$\frac{\delta Q_R}{T_R} = \frac{\delta Q}{T} \qquad \delta W_C = T_R \frac{\delta Q}{T} - dE_C$$

$$W_C = T_R \oint \frac{\delta Q}{T}$$

$$\oint \frac{\delta Q}{T} \le 0 \quad \text{Clasius}$$
inequality

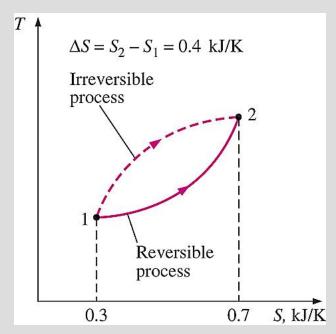
$$\oint \left(\frac{\delta Q}{T}\right)_{\text{int rev}} = 0$$

$$dS = \left(\frac{\delta Q}{T}\right)_{\text{int rev}} \qquad \text{(kJ/K)} \qquad \text{formal definition of entropy}$$

Formal of entropy

$$\Delta S = S_2 - S_1 = \int_1^2 \left(\frac{\delta Q}{T}\right)_{\text{int rev}}$$

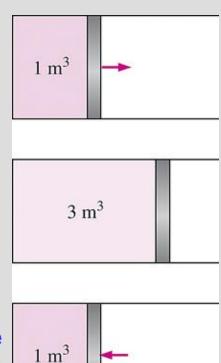
The equality in the Clausius inequality holds for totally or just internally reversible cycles and the inequality for the irreversible ones.



$$\oint \left(\frac{\delta Q}{T}\right)_{\text{int rev}} = 0$$

A quantity whose cyclic integral is zero (i.e., a property like volume)

Entropy is an extensive property of a system.



The entropy change between two specified states is the same whether the process is reversible or irreversible.

The net change in volume (a property) during a cycle is always zero.

A Special Case: Internally Reversible Isothermal Heat Transfer Processes

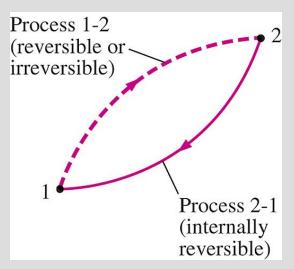
$$\Delta S = \int_{1}^{2} \left(\frac{\delta Q}{T}\right)_{\text{int rev}} = \int_{1}^{2} \left(\frac{\delta Q}{T_{0}}\right)_{\text{int rev}} = \frac{1}{T_{0}} \int_{1}^{2} (\delta Q)_{\text{int rev}}$$

$$\Delta S = \frac{Q}{T_{0}}$$

$$\Delta S = \frac{Q}{T_0}$$

This equation is particularly useful for determining the entropy changes of thermal energy reservoirs.

THE INCREASE OF ENTROPY PRINCIPLE



A cycle composed of a reversible and an irreversible process.

$$\oint \frac{\delta Q}{T} \le 0 \qquad \int_{1}^{2} \frac{\delta Q}{T} + \int_{2}^{1} \left(\frac{\delta Q}{T}\right)_{\text{int rev}} \le 0$$

$$\int_{1}^{2} \frac{\delta Q}{T} + S_{1} - S_{2} \le 0 \qquad S_{2} - S_{1} \ge \int_{1}^{2} \frac{\delta Q}{T}$$

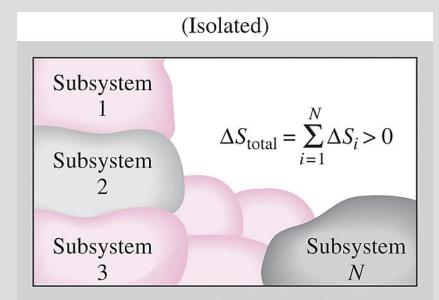
$$dS \ge \frac{\delta Q}{T}$$

 $\frac{\delta Q}{T}$ The equality holds for an internally reversible process and the inequality for an irreversible process.

$$\Delta S_{\text{sys}} = S_2 - S_1 = \int_1^2 \frac{\delta Q}{T} + S_{\text{gen}}$$
$$S_{\text{gen}} = \Delta S_{\text{total}} = \Delta S_{\text{sys}} + \Delta S_{\text{surr}} \ge 0$$

Some entropy is *generated* or *created* during an irreversible process, and this generation is due entirely to the presence of irreversibilities.

The entropy generation S_{qen} is always a *positive* quantity or zero. Can the entropy of a system during a process decrease?

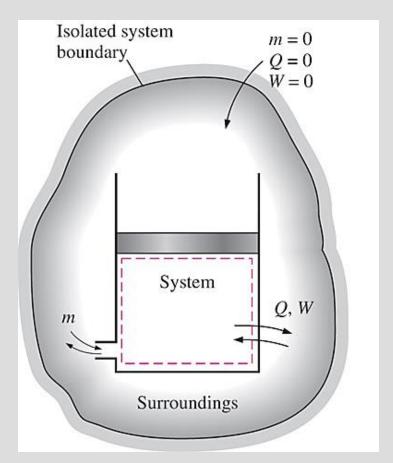


The entropy change of an isolated system is the sum of the entropy changes of its components, and is never less than zero.

$$\Delta S_{\text{isolated}} \ge 0$$

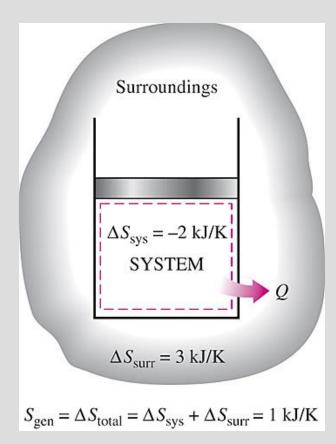
$$S_{\text{gen}} = \Delta S_{\text{total}} = \Delta S_{\text{sys}} + \Delta S_{\text{surr}} \ge 0$$

$$S_{\rm gen} \left\{ egin{array}{ll} > 0 & {\rm Irreversible\ process} \\ = 0 & {\rm Reversible\ process} \\ < 0 & {\rm Impossible\ process} \\ \end{array} \right.$$



A system and its surroundings form an isolated system.

Some Remarks about Entropy

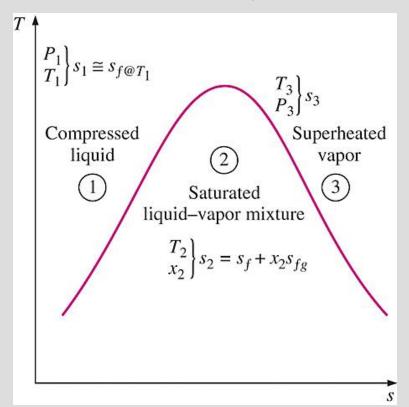


The entropy change of a system can be negative, but the entropy generation cannot.

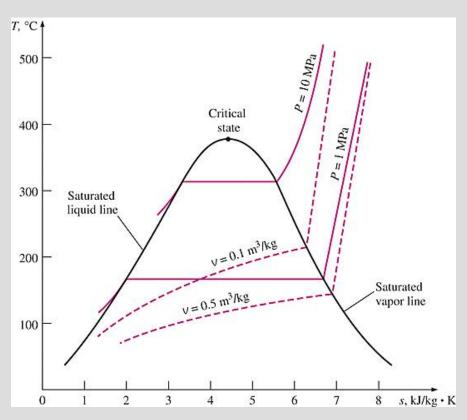
- 1. Processes can occur in a *certain* direction only, not in *any* direction. A process must proceed in the direction that complies with the increase of entropy principle, that is, $S_{gen} \ge 0$. A process that violates this principle is impossible.
- 2. Entropy is a *nonconserved property*, and there is *no* such thing as the *conservation of entropy principle*. Entropy is conserved during the idealized reversible processes only and increases during *all* actual processes.
- 3. The performance of engineering systems is degraded by the presence of irreversibilities, and *entropy generation* is a measure of the magnitudes of the irreversibilities during that process. It is also used to establish criteria for the performance of engineering devices.

ENTROPY CHANGE OF PURE SUBSTANCES

Entropy is a property, and thus the value of entropy of a system is fixed once the state of the system is fixed.



The entropy of a pure substance is determined from the tables (like other properties).



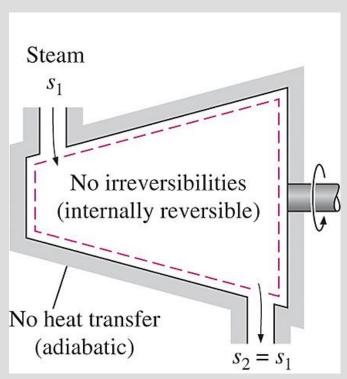
Schematic of the *T-s* diagram for water.

Entropy change

$$\Delta S = m\Delta s = m(s_2 - s_1) \qquad (kJ/K)$$

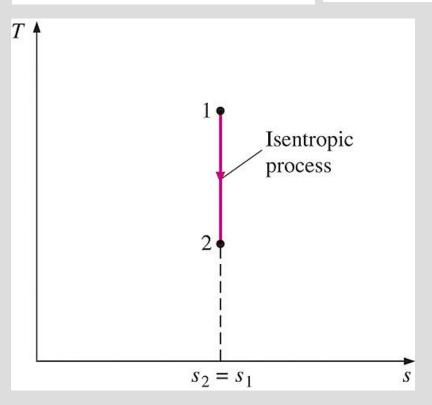
ISENTROPIC PROCESSES

A process during which the entropy remains constant is called an **isentropic process**.



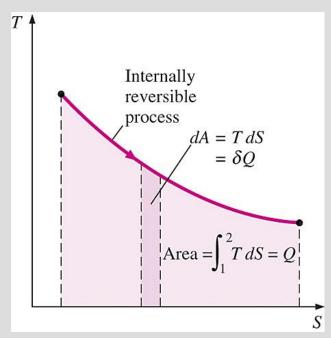
During an internally reversible, adiabatic (isentropic) process, the entropy remains constant.

$$\Delta s = 0$$
 or $s_2 = s_1 (kJ/kg \cdot K)$



The isentropic process appears as a *vertical* line segment on a *T-s* diagram.

PROPERTY DIAGRAMS INVOLVING ENTROPY



On a *T-S* diagram, the area under the process curve represents the heat transfer for internally reversible s processes.

$$\delta Q_{\rm int\,rev} = T \, dS$$

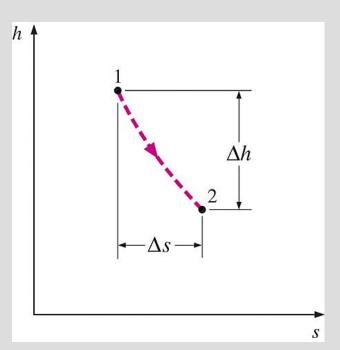
$$\delta Q_{\text{int rev}} = T \, dS \qquad Q_{\text{int rev}} = \int_{1}^{2} T \, dS$$

$$\delta q_{\rm int\,rev} = T \, ds$$

$$\delta q_{\rm int \, rev} = T \, ds \qquad q_{\rm int \, rev} = \int_{1}^{2} T \, ds$$

$$Q_{\rm int\,rev} = T_0 \, \Delta S \, | \, q_{\rm int\,rev} = T_0 \, \Delta s \, | \,$$

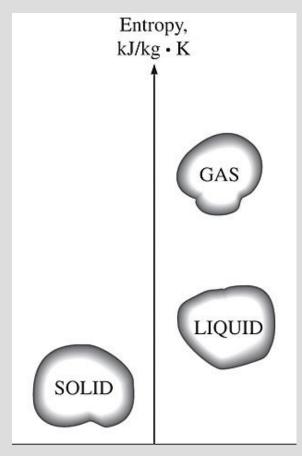
$$q_{\rm int\,rev} = T_0 \, \Delta s$$



For adiabatic steady-flow devices, the vertical distance Δh on an h-s diagram is a measure of work, and the horizontal distance Δs is a measure of irreversibilities.

Mollier diagram: The h-s diagram

WHAT IS ENTROPY?

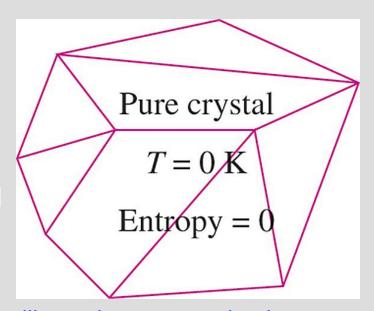


The level of molecular disorder (entropy) of a substance increases as it melts or evaporates.

Boltzmann relation

$$S = k \ln p$$

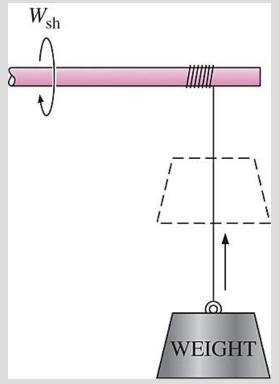
$$k = 1.3806 \times 10^{-23} \text{ J/K}$$

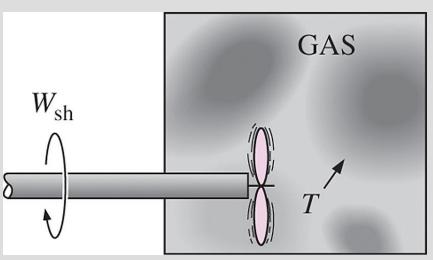


A pure crystalline substance at absolute zero temperature is in perfect order, and its entropy is zero (the third law of thermodynamics).



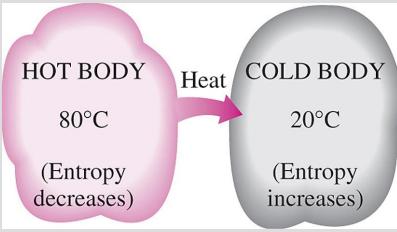
Disorganized energy does not create much useful effect, no matter how large it is.





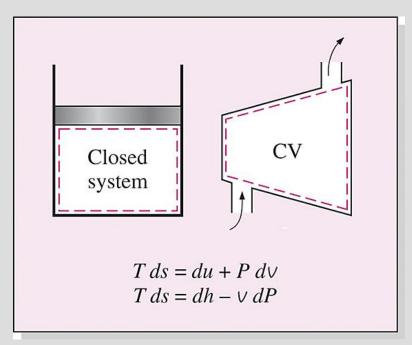
The paddle-wheel work done on a gas increases the level of disorder (entropy) of the gas, and thus energy is degraded during this process.

In the absence of friction, raising a weight by a rotating shaft does not create any disorder (entropy), and thus energy is not degraded during this process.



During a heat transfer process, the net entropy increases. (The increase in the entropy of the cold body more than offsets the decrease in the entropy of the hot body.)

THE T ds RELATIONS



The *T ds* relations are valid for both reversible and irreversible processes and for both closed and open systems.

$$\delta Q_{\rm int \, rev} - \delta W_{\rm int \, rev, out} = dU$$

$$\delta Q_{\rm int \, rev} = T \, dS$$

$$\delta W_{\rm int \, rev, out} = P \, dV$$

$$T dS = dU + P dV$$
 (kJ)

$$T ds = du + P dv$$
 (kJ/kg)

the first T ds, or Gibbs equation

the second T ds equation

$$ds = \frac{du}{T} + \frac{P \ dv}{T}$$
$$ds = \frac{dh}{T} - \frac{v \ dP}{T}$$

Differential changes in entropy in terms of other properties

ENTROPY CHANGE OF LIQUIDS AND SOLIDS

$$ds = \frac{du}{T} + \frac{P \ dV}{T}$$

Since $dv \approx 0$ for liquids and solids

$$ds = \frac{du}{T} = \frac{c \ dT}{T}$$

since $c_p = c_v = c$ and du = c dT

Liquids and solids can be approximated as incompressible substances since their specific volumes remain nearly constant during a process.

Liquids, solids:
$$s_2 - s_1 = \int_1^2 c(T) \frac{dT}{T} \approx c_{\text{avg}} \ln \frac{T_2}{T_1}$$
 (kJ/kg·K)

For and isentropic process of an incompressible substance

THE ENTROPY CHANGE OF IDEAL GASES

From the first T ds relation

$$ds = \frac{du}{T} + \frac{P}{T} \frac{dv}{T} \quad du = c_v dT$$
$$P = RT/V$$

$$ds = c_{v} \frac{dT}{T} + R \frac{dv}{v}$$

$$s_2 - s_1 = \int_1^2 c_v(T) \frac{dT}{T} + R \ln \frac{v_2}{v_1}$$

From the second *T ds* relation

$$ds = \frac{dh}{T} - \frac{\vee dP}{T}$$

$$dh = c_p dT$$
 $V = RT/P$

$$s_2 - s_1 = \int_1^2 c_p(T) \frac{dT}{T} - R \ln \frac{P_2}{P_1}$$

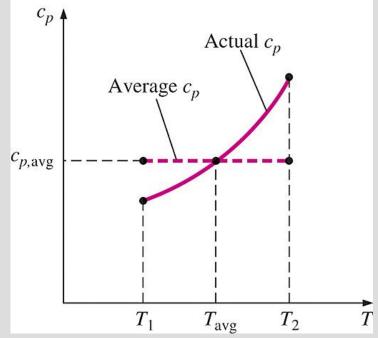
P v = RT $du = C_v dT$ $dh = C_p dT$ A broadcast from channel IG.

Constant Specific Heats (Approximate Analysis)

$$s_2 - s_1 = \int_1^2 c_v(T) \frac{dT}{T} + R \ln \frac{v_2}{v_1} \longrightarrow s_2 - s_1 = c_{v,avg} \ln \frac{T_2}{T_1} + R \ln \frac{v_2}{v_1}$$

$$s_{2} - s_{1} = \int_{1}^{2} c_{p}(T) \frac{dT}{T} - R \ln \frac{P_{2}}{P_{1}} \longrightarrow s_{2} - s_{1} = c_{p,\text{avg}} \ln \frac{T_{2}}{T_{1}} - R \ln \frac{P_{2}}{P_{1}}$$

$$(kJ/kg \cdot K)$$



Entropy change of an ideal gas on a unit-mole basis

$$\overline{s}_2 - \overline{s}_1 = \overline{c}_{v,avg} \ln \frac{T_2}{T_1} + R_u \ln \frac{v_2}{v_1}$$
 (kJ/kmol·K)

$$\overline{s}_2 - \overline{s}_1 = \overline{c}_{p,\text{avg}} \ln \frac{T_2}{T_1} - R_u \ln \frac{P_2}{P_1}$$
 (kJ/kmol·K)

Under the constant-specificheat assumption, the specific heat is assumed to be constant at some average value.

Variable Specific Heats (Exact Analysis)

We choose absolute zero as the reference temperature and define a function s° as

$$s^{\circ} = \int_{0}^{T} c_{p}(T) \frac{dT}{T}$$

$$\int_{1}^{2} c_p(T) \frac{dT}{T} = s_2^{\circ} - s_1^{\circ}$$

On a unit-mass basis

$$s_2 - s_1 = s_2^{\circ} - s_1^{\circ} - R \ln \frac{P_2}{P_1}$$
 (kJ/kg·K)

On a unit-mole basis

$$\overline{s}_2 - \overline{s}_1 = \overline{s}_2^{\circ} - \overline{s}_1^{\circ} - R_u \ln \frac{P_2}{P_1}$$
 (kJ/kmol·K)

<i>T</i> , K	s° , kJ/kg • K
	<u> </u>
300	1.70203
310	1.73498
320	1.76690
•	•
	•
	(5.1.1.1.1.5)
(Table A-17)	

The entropy of an ideal gas depends on both *T* and *P*. The function *s* represents only the temperature-dependent part of entropy.

Isentropic Processes of Ideal Gases

Constant Specific Heats (Approximate Analysis)

$$s_2 - s_1 = c_{v,avg} \ln \frac{T_2}{T_1} + R \ln \frac{v_2}{v_1}$$

Setting this eq. equal to zero, we get

$$\ln \frac{T_2}{T_1} = -\frac{R}{c_v} \ln \frac{v_2}{v_1}$$

$$\ln \frac{T_2}{T_1} = \ln \left(\frac{V_1}{V_2}\right)^{R/c_v}$$

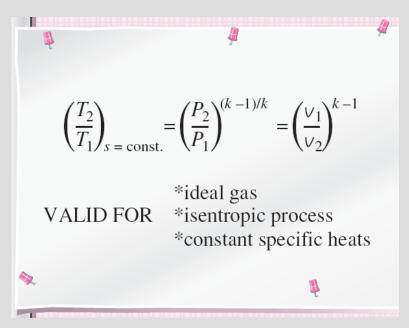
$$R = c_p - c_v$$
, $k = c_p/c_v$
and thus $R/c_v = k - 1$

$$\left(\frac{T_2}{T_1}\right)_{s=\text{const.}} = \left(\frac{V_1}{V_2}\right)^{k-1}$$

$$\left(\frac{T_2}{T_1}\right)_{s=\text{const.}} = \left(\frac{P_2}{P_1}\right)^{(k-1)/k} \left(\frac{P_2}{P_1}\right)_{s=\text{const.}} = \left(\frac{V_1}{V_2}\right)^k \quad TP^{(1-k)/k} = \text{constant}$$

$$PV^k = \text{constant}$$

$$\left(\frac{P_2}{P_1}\right)_{s=\text{const.}} = \left(\frac{V_1}{V_2}\right)^k$$



The isentropic relations of ideal gases are valid for the isentropic processes of ideal gases only.

$$T \vee^{k-1} = \text{constant}$$

 $T P^{(1-k)/k} = \text{constant}$
 $P \vee^k = \text{constant}$

Isentropic Processes of Ideal Gases

Variable Specific Heats (Exact Analysis)

$$0 = s_2^{\circ} - s_1^{\circ} - R \ln \frac{P_2}{P_1} \longrightarrow s_2^{\circ} = s_1^{\circ} + R \ln \frac{P_2}{P_1}$$

Relative Pressure and Relative Specific Volume

$$\frac{P_2}{P_1} = \exp \frac{s_2^{\circ} - s_1^{\circ}}{R} \text{ exp(s°/R) is the relative}$$

$$\frac{P_2}{P_1} = \frac{\exp(s_2^{\circ}/R)}{\exp(s_1^{\circ}/R)}$$

the relative pressure
$$P_r$$

The use of P_r data for calculating the final temperature process.

during an isentropic

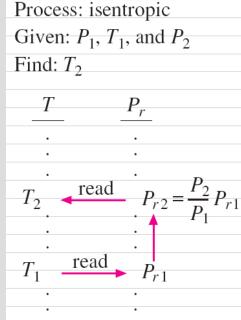
$$\left(\frac{P_2}{P_1}\right)_{s=\text{const.}} = \frac{P_{r2}}{P_{r1}}$$

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2} \rightarrow \frac{V_2}{V_1} = \frac{T_2}{T_1} \frac{P_1}{P_2} = \frac{T_2}{T_1} \frac{P_{r1}}{P_{r2}} = \frac{T_2/P_{r2}}{T_1/P_{r1}}$$

$$\left(\frac{V_2}{V_1}\right)_{s=\text{const.}} = \frac{V_{r2}}{V_{r1}}$$

 T/P_r is the relative specific volume v_r

The use of v_r data for calculating the final temperature during an isentropic process



Process: isentropic Given: v_1 , T_1 , and v_2 Find: T_2

ıЭ

REVERSIBLE STEADY-FLOW WORK

$$\delta q_{\text{rev}} - \delta w_{\text{rev}} = dh + d\text{ke} + d\text{pe}$$

$$\begin{cases} \delta q_{\text{rev}} = T \, ds & \text{(Eq. 7-16)} \\ T \, ds = dh - v \, dP & \text{(Eq. 7-24)} \end{cases} \quad \delta q_{\text{rev}} = dh - v \, dP$$

$$-\delta w_{\text{rev}} = v dP + d\text{ke} + d\text{pe}$$

$$w_{\text{rev}} = -\int_{1}^{2} v \, dP - \Delta \text{ke} - \Delta \text{pe}$$

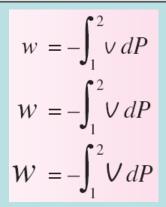
$$w_{\text{rev}} = -\int_{1}^{2} v \, dP$$
 When kinetic and potential energies are negligible

$$w_{\text{rev,in}} = \int_{1}^{2} v \, dP + \Delta \text{ke} + \Delta \text{pe}$$

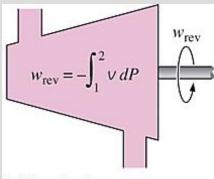
$$w_{\text{rev}} = -v(P_2 - P_1) - \Delta \text{ke} - \Delta \text{pe}$$

For the steady flow of a liquid through a device that involves no work interactions (such as a pipe section), the work term is zero (Bernoulli equation):

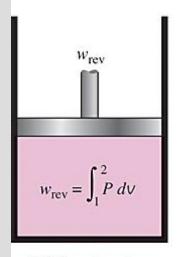
$$v(P_2 - P_1) + \frac{V_2^2 - V_1^1}{2} + g(z_2 - z_1) = 0$$



The larger the specific volume, the greater the work produced (or consumed) by a steady-flow device.



(a) Steady-flow system



(b) Closed system

Reversible work relations for steady-flow and closed systems.

Proof that Steady-Flow Devices Deliver the Most and Consume the Least Work when the Process Is Reversible

Taking heat input and work output positive:

$$\delta q_{\rm act} - \delta w_{\rm act} = dh + d{\rm ke} + d{\rm pe}$$
 Actual
$$\delta q_{\rm rev} - \delta w_{\rm rev} = dh + d{\rm ke} + d{\rm pe}$$
 Reversible

$$\delta q_{\rm act} - \delta w_{\rm act} = \delta q_{\rm rev} - \delta w_{\rm rev}$$

$$\delta w_{\rm rev} - \delta w_{\rm act} = \delta q_{\rm rev} - \delta q_{\rm act}$$

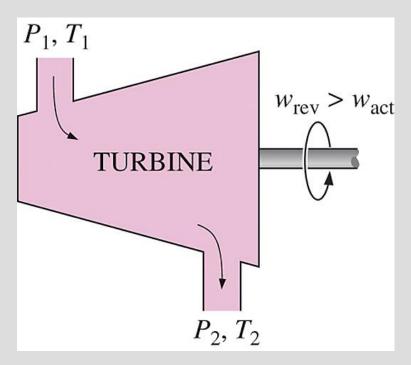
$$\delta q_{\rm rev} = T \ ds$$
 $ds \ge \frac{\delta q_{\rm act}}{T}$

$$\frac{\delta w_{\text{rev}} - \delta w_{\text{act}}}{T} = ds - \frac{\delta q_{\text{act}}}{T} \ge 0$$

$$\partial w_{\rm rev} \ge \partial w_{\rm act}$$

$$w_{\rm rev} \ge w_{\rm act}$$

Work-producing devices such as turbines deliver more work, and workconsuming devices such as pumps and compressors require less work when they operate reversibly.



A reversible turbine delivers more work than an irreversible one if both operate between the same end states.

MINIMIZING THE COMPRESSOR WORK

$$w_{\rm rev,in} = \int_{1}^{2} v \ dP$$

 $w_{\rm rev,in} = \int_{1}^{2} v \, dP$ When kinetic and potential energies are negligible

Isentropic ($Pv^k = \text{constant}$):

$$w_{\text{comp,in}} = \frac{kR(T_2 - T_1)}{k - 1} = \frac{kRT_1}{k - 1} \left[\left(\frac{P_2}{P_1} \right)^{(k-1)/k} - 1 \right]$$

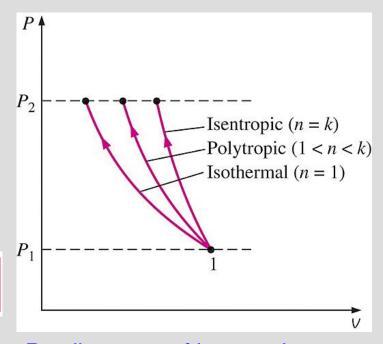
Polytropic ($Pv^n = constant$):

$$w_{\text{comp,in}} = \frac{nR(T_2 - T_1)}{n - 1} = \frac{nRT_1}{n - 1} \left[\left(\frac{P_2}{P_1} \right)^{(n-1)/n} - 1 \right]$$

Isothermal (Pv = constant):

$$w_{\text{comp,in}} = RT \ln \frac{P_2}{P_1}$$

The adiabatic compression ($Pv^k = \text{constant}$) requires the maximum work and the isothermal compression (T = constant) requires the minimum. Why?

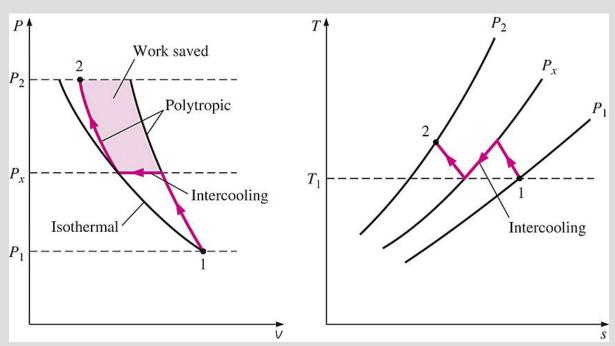


P-v diagrams of isentropic, polytropic, and isothermal compression processes between the same pressure limits.

Multistage Compression with Intercooling

The gas is compressed in stages and cooled between each stage by passing it through a heat exchanger called an intercooler.

> P-v and T-s diagrams for a two- $_{P_1}$ stage steady-flow compression process.



$$w_{\text{comp,in}} = w_{\text{comp I,in}} + w_{\text{comp II,in}}$$

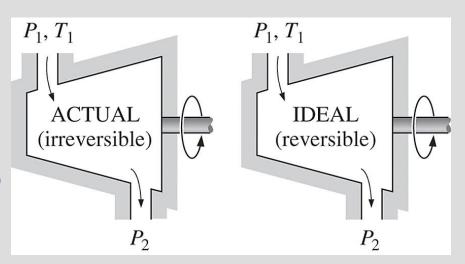
$$= \frac{nRT_1}{n-1} \left[\left(\frac{P_x}{P_1} \right)^{(n-1)/n} - 1 \right] + \frac{nRT_1}{n-1} \left[\left(\frac{P_2}{P_x} \right)^{(n-1)/n} - 1 \right]$$

$$P_x = (P_1 P_2)^{1/2}$$
 or $\frac{P_x}{P_1} = \frac{P_2}{P_x}$

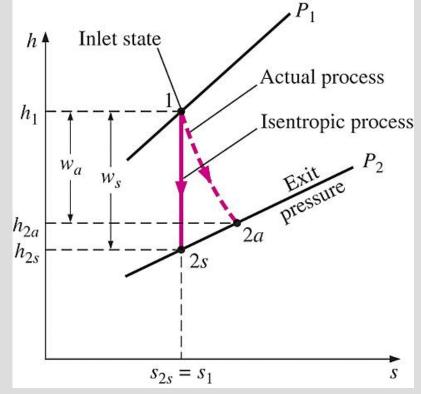
 $P_x = (P_1 P_2)^{1/2}$ or $\frac{P_x}{P_1} = \frac{P_2}{P_x}$ To minimize compression work during two-stage compression, the pressure ratio across each stage of the compressor must be the same.

ISENTROPIC EFFICIENCIES OF STEADY-FLOW DEVICES

The isentropic process involves no irreversibilities and serves as the ideal process for **adiabatic devices**.



Isentropic Efficiency of Turbines



$$\eta_T = \frac{\text{Actual turbine work}}{\text{Isentropic turbine work}} = \frac{w_a}{w_s}$$

$$\eta_T \cong \frac{h_1 - h_{2a}}{h_1 - h_{2s}}$$

The *h*-s diagram for the actual and isentropic processes of an adiabatic turbine.

Isentropic Efficiencies of Compressors and Pumps

$$\eta_C = \frac{\text{Isentropic compressor work}}{\text{Actual compressor work}} = \frac{w_s}{w_a}$$

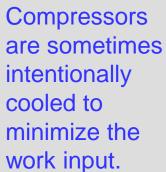
$$\eta_{C} \cong rac{h_{2s} - h_{1}}{h_{2a} - h_{1}}$$
 When kinetic and potential energies are negligible

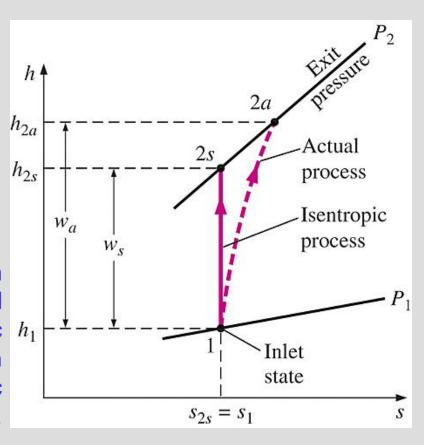
$$\eta_P = rac{w_s}{w_a} = rac{V(P_2 - P_1)}{h_{2a} - h_1}$$
 For a pump

$$\eta_{C} = \frac{w_{t}}{w_{a}}$$
 Isothermal efficiency

Cooling water

The h-s diagram of the actual and isentropic h_1 processes of an adiabatic compressor.





Can you use isentropic efficiency for a non-adiabatic compressor?

Can you use isothermal efficiency for an adiabatic compressor?

ENTROPY BALANCE

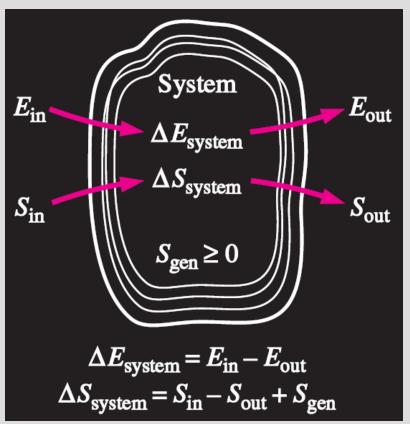
Entropy Change of a System, ΔS_{system}

$$\Delta S_{\text{system}} = S_{\text{final}} - S_{\text{initial}} = S_2 - S_1$$

When the properties of the system are not uniform

$$S_{\text{system}} = \int s \, \delta m = \int_{V} s \rho \, dV$$

Energy and entropy balances for a system.

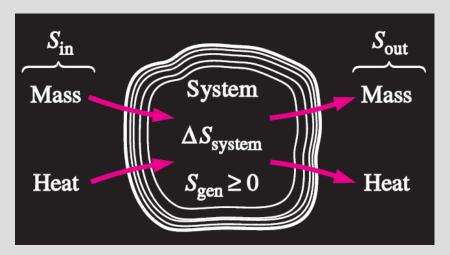


Entropy Generation, S_{gen}

$$\underbrace{S_{\text{in}} - S_{\text{out}}}_{\text{Net entropy transfer by heat and mass}} + \underbrace{S_{\text{gen}}}_{\text{Entropy generation}} = \underbrace{\Delta S_{\text{system}}}_{\text{Change in entropy}} \tag{kJ/K}$$

$$\underbrace{\dot{S}_{\text{in}} - \dot{S}_{\text{out}}}_{\text{Rate of net entropy transfer by heat and mass}} + \underbrace{\dot{S}_{\text{gen}}}_{\text{Rate of entropy generation}} = \underbrace{dS_{\text{system}}/dt}_{\text{Rate of change in entropy}}$$

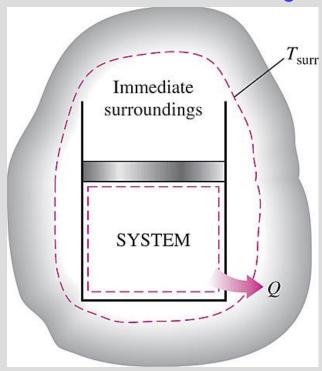
$$(s_{\rm in} - s_{\rm out}) + s_{\rm gen} = \Delta s_{\rm system}$$
 $(kJ/kg \cdot K)$



Mechanisms of entropy transfer for a general system.

Entropy generation outside system boundaries can be accounted for by writing an entropy balance on an extended system that includes the system and its immediate surroundings.

(kW/K)



Summary

- Entropy
- The Increase of entropy principle
- Some remarks about entropy
- Entropy change of pure substances
- Isentropic processes
- Property diagrams involving entropy
- What is entropy?
- The T ds relations
- Entropy change of liquids and solids
- The entropy change of ideal gases
- Reversible steady-flow work
- Minimizing the compressor work
- Isentropic efficiencies of steady-flow devices
- Entropy balance