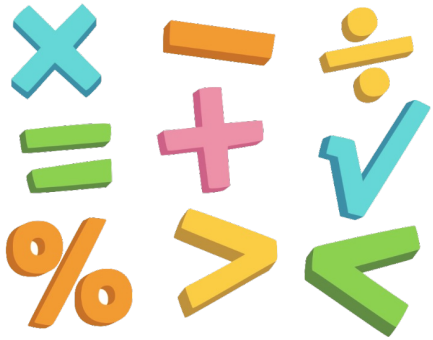
KIDS MATH BOOK

(Arithmetic)



KIDS MATH BOOK

(Arithmetic)

To Hope, Alice and Bruce.

- Love, mom

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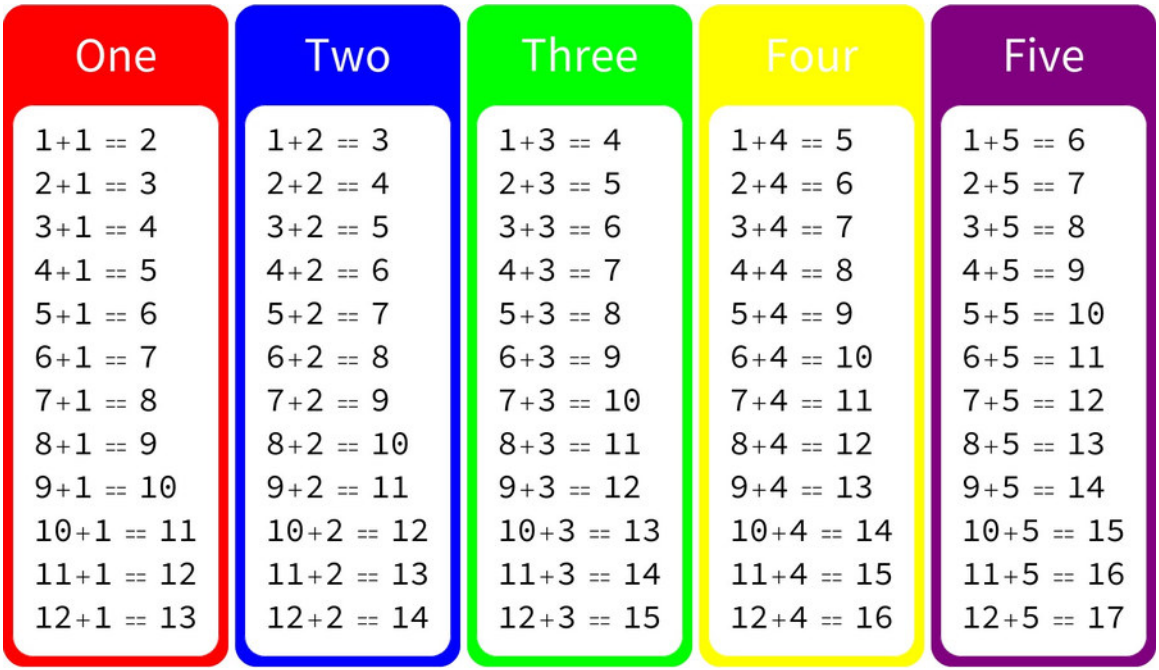
Addition tables



One	Two	Three	Four	Five
1+1 = 2	1+2 = 3	1+3 = 4	1+4 = 5	1+5 = 6
2+1 = 3	2+2 = 4	2+3 = 5	2+4 = 6	2+5 = 7
3+1 = 4	3+2 = 5	3+3 = 6	3+4 = 7	3+5 = 8
4+1 = 5	4+2 = 6	4+3 = 7	4+4 = 8	4+5 = 9
5+1 = 6	5+2 = 7	5+3 = 8	5+4 = 9	5+5 = 10
6+1 = 7	6+2 = 8	6+3 = 9	6+4 = 10	6+5 = 11
7+1 = 8	7+2 = 9	7+3 = 10	7+4 = 11	7+5 = 12
8+1 = 9	8+2 = 10	8+3 = 11	8+4 = 12	8+5 = 13
9+1 = 10	9+2 = 11	9+3 = 12	9+4 = 13	9+5 = 14
10+1 = 11	10+2 = 12	10+3 = 13	10+4 = 14	10+5 = 15
11+1 = 12	11+2 = 13	11+3 = 14	11+4 = 15	11+5 = 16
12+1 = 13	12+2 = 14	12+3 = 15	12+4 = 16	12+5 = 17

Zero

0 + 0 == 0	6 + 0 == 6
1 + 0 == 1	7 + 0 == 7
2 + 0 == 2	8 + 0 == 8
3 + 0 == 3	9 + 0 == 9
4 + 0 == 4	10 + 0 == 10
5 + 0 == 5	11 + 0 == 11

Six

1+6 == 7
2+6 == 8
3+6 == 9
4+6 == 10
5+6 == 11
6+6 == 12
7+6 == 13
8+6 == 14
9+6 == 15
10+6 == 16
11+6 == 17
12+6 == 18

Seven

1+7 == 8
2+7 == 9
3+7 == 10
4+7 == 11
5+7 == 12
6+7 == 13
7+7 == 14
8+7 == 15
9+7 == 16
10+7 == 17
11+7 == 18
12+7 == 19

Eight

1+8 == 9
2+8 == 10
3+8 == 11
4+8 == 12
5+8 == 13
6+8 == 14
7+8 == 15
8+8 == 16
9+8 == 17
10+8 == 18
11+8 == 19
12+8 == 20

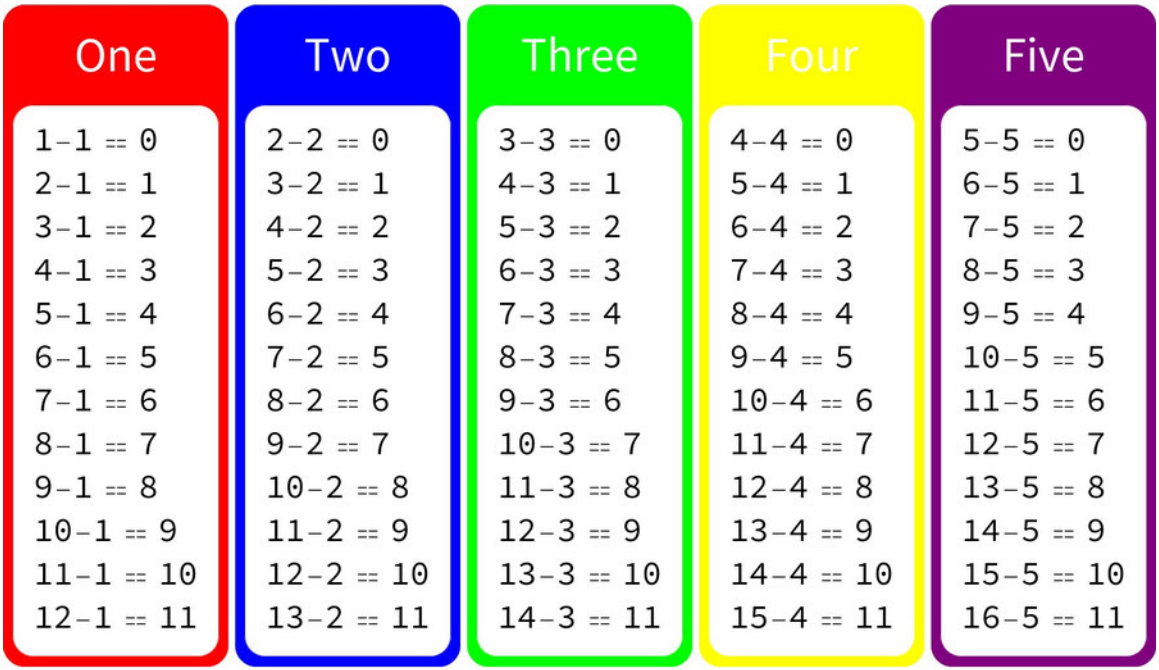
Nine

1+9 == 10
2+9 == 11
3+9 == 12
4+9 == 13
5+9 == 14
6+9 == 15
7+9 == 16
8+9 == 17
9+9 == 18
10+9 == 19
11+9 == 20
12+9 == 21

Ten

1+10 == 11
2+10 == 12
3+10 == 13
4+10 == 14
5+10 == 15
6+10 == 16
7+10 == 17
8+10 == 18
9+10 == 19
10+10 == 20
11+10 == 21
12+10 == 22

Subtraction tables



One	Two	Three	Four	Five
1-1 = 0	2-2 = 0	3-3 = 0	4-4 = 0	5-5 = 0
2-1 = 1	3-2 = 1	4-3 = 1	5-4 = 1	6-5 = 1
3-1 = 2	4-2 = 2	5-3 = 2	6-4 = 2	7-5 = 2
4-1 = 3	5-2 = 3	6-3 = 3	7-4 = 3	8-5 = 3
5-1 = 4	6-2 = 4	7-3 = 4	8-4 = 4	9-5 = 4
6-1 = 5	7-2 = 5	8-3 = 5	9-4 = 5	10-5 = 5
7-1 = 6	8-2 = 6	9-3 = 6	10-4 = 6	11-5 = 6
8-1 = 7	9-2 = 7	10-3 = 7	11-4 = 7	12-5 = 7
9-1 = 8	10-2 = 8	11-3 = 8	12-4 = 8	13-5 = 8
10-1 = 9	11-2 = 9	12-3 = 9	13-4 = 9	14-5 = 9
11-1 = 10	12-2 = 10	13-3 = 10	14-4 = 10	15-5 = 10
12-1 = 11	13-2 = 11	14-3 = 11	15-4 = 11	16-5 = 11

Zero

```
0-0 == 0  6-0 == 6
1-0 == 1  7-0 == 7
2-0 == 2  8-0 == 8
3-0 == 3  9-0 == 9
4-0 == 4  10-0 == 10
5-0 == 5  11-0 == 11
```

Six

```
6-6 == 0
7-6 == 1
8-6 == 2
9-6 == 3
10-6 == 4
11-6 == 5
12-6 == 6
13-6 == 7
14-6 == 8
15-6 == 9
16-6 == 10
17-6 == 11
```

Seven

```
7-7 == 0
8-7 == 1
9-7 == 2
10-7 == 3
11-7 == 4
12-7 == 5
13-7 == 6
14-7 == 7
15-7 == 8
16-7 == 9
17-7 == 10
18-7 == 11
```

Eight

```
8-8 == 0
9-8 == 1
10-8 == 2
11-8 == 3
12-8 == 4
13-8 == 5
14-8 == 6
15-8 == 7
16-8 == 8
17-8 == 9
18-8 == 10
19-8 == 11
```

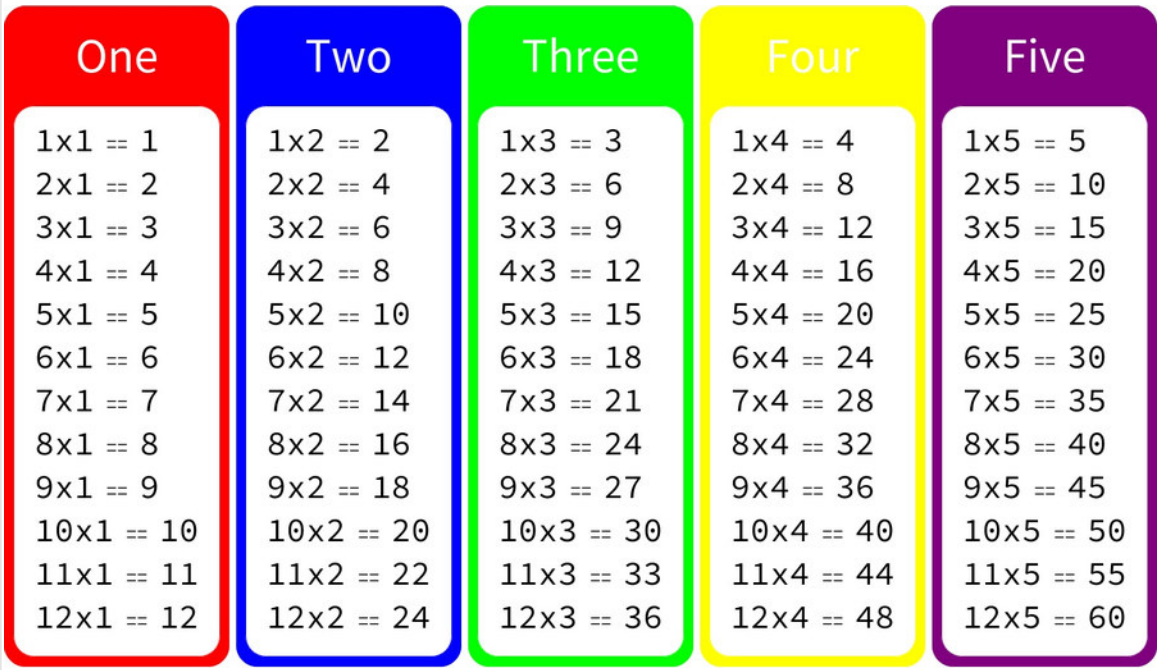
Nine

```
9-9 == 0
10-9 == 1
11-9 == 2
12-9 == 3
13-9 == 4
14-9 == 5
15-9 == 6
16-9 == 7
17-9 == 8
18-9 == 9
19-9 == 10
20-9 == 11
```

Ten

```
10-10 == 0
11-10 == 1
12-10 == 2
13-10 == 3
14-10 == 4
15-10 == 5
16-10 == 6
17-10 == 7
18-10 == 8
19-10 == 9
20-10 == 10
21-10 == 11
```

Multiplication tables



One	Two	Three	Four	Five
1x1 = 1	1x2 = 2	1x3 = 3	1x4 = 4	1x5 = 5
2x1 = 2	2x2 = 4	2x3 = 6	2x4 = 8	2x5 = 10
3x1 = 3	3x2 = 6	3x3 = 9	3x4 = 12	3x5 = 15
4x1 = 4	4x2 = 8	4x3 = 12	4x4 = 16	4x5 = 20
5x1 = 5	5x2 = 10	5x3 = 15	5x4 = 20	5x5 = 25
6x1 = 6	6x2 = 12	6x3 = 18	6x4 = 24	6x5 = 30
7x1 = 7	7x2 = 14	7x3 = 21	7x4 = 28	7x5 = 35
8x1 = 8	8x2 = 16	8x3 = 24	8x4 = 32	8x5 = 40
9x1 = 9	9x2 = 18	9x3 = 27	9x4 = 36	9x5 = 45
10x1 = 10	10x2 = 20	10x3 = 30	10x4 = 40	10x5 = 50
11x1 = 11	11x2 = 22	11x3 = 33	11x4 = 44	11x5 = 55
12x1 = 12	12x2 = 24	12x3 = 36	12x4 = 48	12x5 = 60

Zero

0 × 0 = 0	6 × 0 = 0
1 × 0 = 0	7 × 0 = 0
2 × 0 = 0	8 × 0 = 0
3 × 0 = 0	9 × 0 = 0
4 × 0 = 0	10 × 0 = 0
5 × 0 = 0	11 × 0 = 0

Six

1x6 = 6
2x6 = 12
3x6 = 18
4x6 = 24
5x6 = 30
6x6 = 36
7x6 = 42
8x6 = 48
9x6 = 54
10x6 = 60
11x6 = 66
12x6 = 72

Seven

1x7 = 7
2x7 = 14
3x7 = 21
4x7 = 28
5x7 = 35
6x7 = 42
7x7 = 49
8x7 = 56
9x7 = 63
10x7 = 70
11x7 = 77
12x7 = 84

Eight

1x8 = 8
2x8 = 16
3x8 = 24
4x8 = 32
5x8 = 40
6x8 = 48
7x8 = 56
8x8 = 64
9x8 = 72
10x8 = 80
11x8 = 88
12x8 = 96

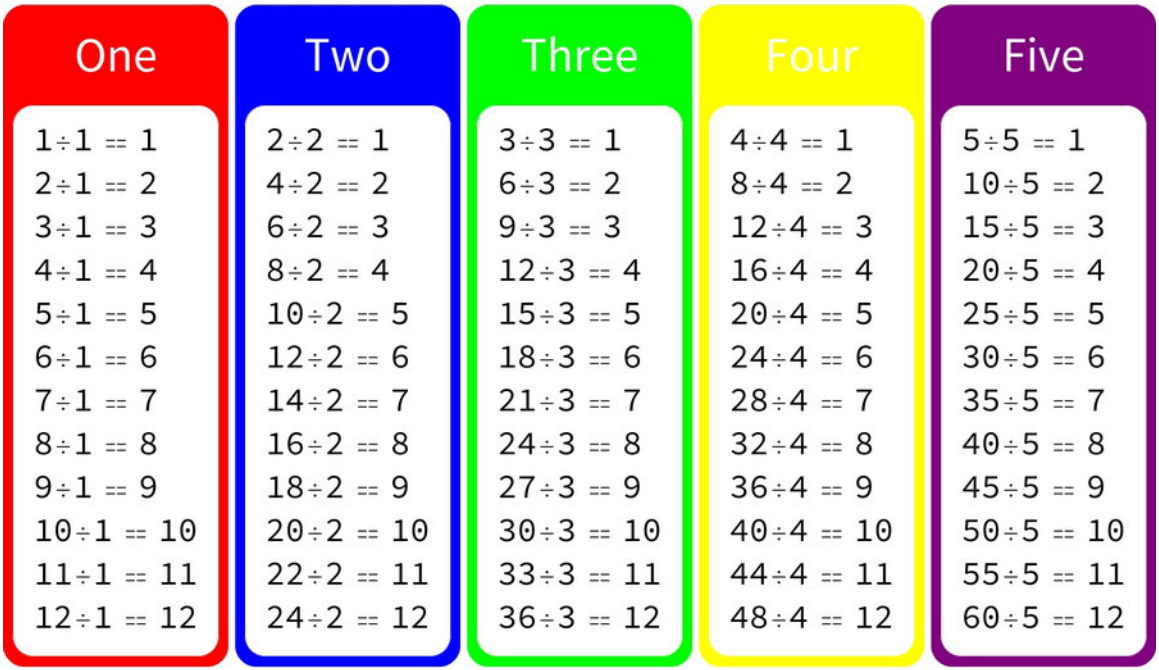
Nine

1x9 = 9
2x9 = 18
3x9 = 27
4x9 = 36
5x9 = 45
6x9 = 54
7x9 = 63
8x9 = 72
9x9 = 81
10x9 = 90
11x9 = 99
12x9 = 108

Ten

1x10 = 10
2x10 = 20
3x10 = 30
4x10 = 40
5x10 = 50
6x10 = 60
7x10 = 70
8x10 = 80
9x10 = 90
10x10 = 100
11x10 = 110
12x10 = 120

Division tables



One	Two	Three	Four	Five
1÷1 = 1	2÷2 = 1	3÷3 = 1	4÷4 = 1	5÷5 = 1
2÷1 = 2	4÷2 = 2	6÷3 = 2	8÷4 = 2	10÷5 = 2
3÷1 = 3	6÷2 = 3	9÷3 = 3	12÷4 = 3	15÷5 = 3
4÷1 = 4	8÷2 = 4	12÷3 = 4	16÷4 = 4	20÷5 = 4
5÷1 = 5	10÷2 = 5	15÷3 = 5	20÷4 = 5	25÷5 = 5
6÷1 = 6	12÷2 = 6	18÷3 = 6	24÷4 = 6	30÷5 = 6
7÷1 = 7	14÷2 = 7	21÷3 = 7	28÷4 = 7	35÷5 = 7
8÷1 = 8	16÷2 = 8	24÷3 = 8	32÷4 = 8	40÷5 = 8
9÷1 = 9	18÷2 = 9	27÷3 = 9	36÷4 = 9	45÷5 = 9
10÷1 = 10	20÷2 = 10	30÷3 = 10	40÷4 = 10	50÷5 = 10
11÷1 = 11	22÷2 = 11	33÷3 = 11	44÷4 = 11	55÷5 = 11
12÷1 = 12	24÷2 = 12	36÷3 = 12	48÷4 = 12	60÷5 = 12

Zero

0÷0 = Indeterminate	6÷0 = ∞
1÷0 = ∞	7÷0 = ∞
2÷0 = ∞	8÷0 = ∞
3÷0 = ∞	9÷0 = ∞
4÷0 = ∞	10÷0 = ∞
5÷0 = ∞	11÷0 = ∞

Six

6÷6 = 1
12÷6 = 2
18÷6 = 3
24÷6 = 4
30÷6 = 5
36÷6 = 6
42÷6 = 7
48÷6 = 8
54÷6 = 9
60÷6 = 10
66÷6 = 11
72÷6 = 12

Seven

7÷7 = 1
14÷7 = 2
21÷7 = 3
28÷7 = 4
35÷7 = 5
42÷7 = 6
49÷7 = 7
56÷7 = 8
63÷7 = 9
70÷7 = 10
77÷7 = 11
84÷7 = 12

Eight

8÷8 = 1
16÷8 = 2
24÷8 = 3
32÷8 = 4
40÷8 = 5
48÷8 = 6
56÷8 = 7
64÷8 = 8
72÷8 = 9
80÷8 = 10
88÷8 = 11
96÷8 = 12

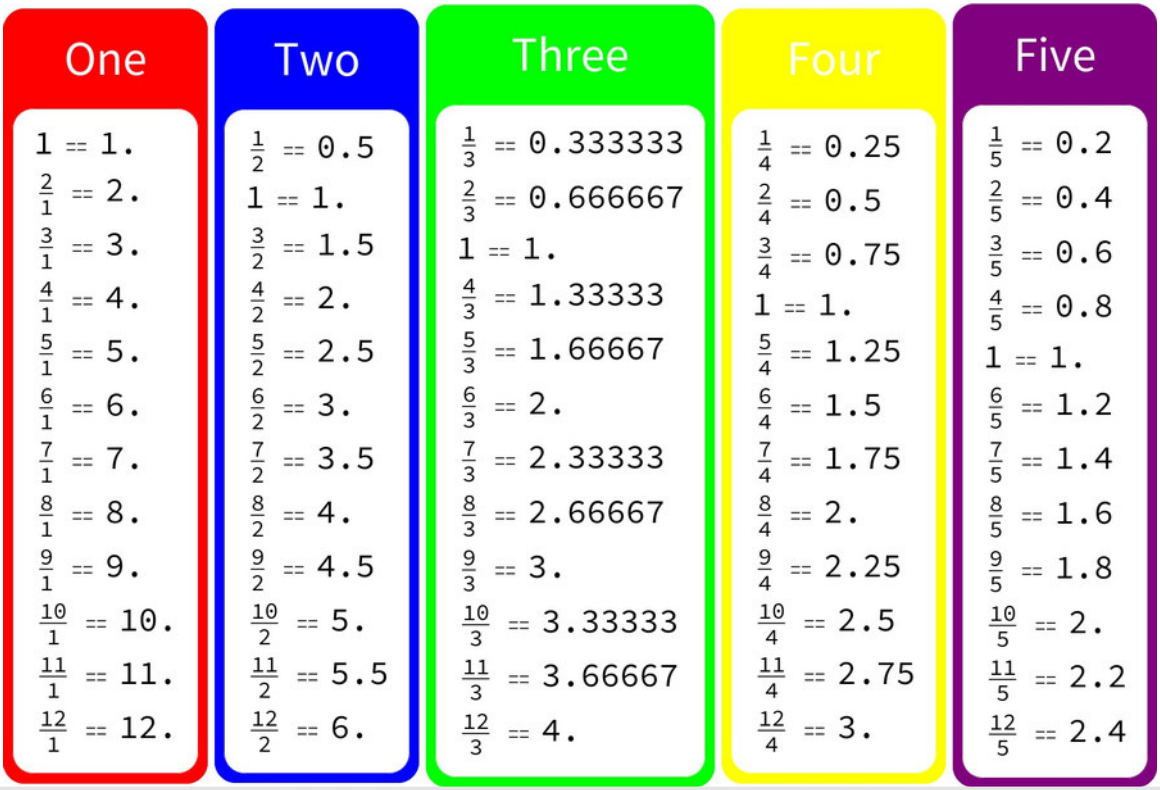
Nine

9÷9 = 1
18÷9 = 2
27÷9 = 3
36÷9 = 4
45÷9 = 5
54÷9 = 6
63÷9 = 7
72÷9 = 8
81÷9 = 9
90÷9 = 10
99÷9 = 11
108÷9 = 12

Ten

10÷10 = 1
20÷10 = 2
30÷10 = 3
40÷10 = 4
50÷10 = 5
60÷10 = 6
70÷10 = 7
80÷10 = 8
90÷10 = 9
100÷10 = 10
110÷10 = 11
120÷10 = 12

Fraction and decimal tables



One	Two	Three	Four	Five
$\frac{1}{1} = 1.$	$\frac{1}{2} = 0.5$	$\frac{1}{3} = 0.333333$	$\frac{1}{4} = 0.25$	$\frac{1}{5} = 0.2$
$\frac{2}{1} = 2.$	$\frac{2}{2} = 1.$	$\frac{2}{3} = 0.666667$	$\frac{2}{4} = 0.5$	$\frac{2}{5} = 0.4$
$\frac{3}{1} = 3.$	$\frac{3}{2} = 1.5$	$\frac{3}{3} = 1.$	$\frac{3}{4} = 0.75$	$\frac{3}{5} = 0.6$
$\frac{4}{1} = 4.$	$\frac{4}{2} = 2.$	$\frac{4}{3} = 1.33333$	$\frac{4}{4} = 1.$	$\frac{4}{5} = 0.8$
$\frac{5}{1} = 5.$	$\frac{5}{2} = 2.5$	$\frac{5}{3} = 1.66667$	$\frac{5}{4} = 1.25$	$\frac{5}{5} = 1.$
$\frac{6}{1} = 6.$	$\frac{6}{2} = 3.$	$\frac{6}{3} = 2.$	$\frac{6}{4} = 1.5$	$\frac{6}{5} = 1.2$
$\frac{7}{1} = 7.$	$\frac{7}{2} = 3.5$	$\frac{7}{3} = 2.33333$	$\frac{7}{4} = 1.75$	$\frac{7}{5} = 1.4$
$\frac{8}{1} = 8.$	$\frac{8}{2} = 4.$	$\frac{8}{3} = 2.66667$	$\frac{8}{4} = 2.$	$\frac{8}{5} = 1.6$
$\frac{9}{1} = 9.$	$\frac{9}{2} = 4.5$	$\frac{9}{3} = 3.$	$\frac{9}{4} = 2.25$	$\frac{9}{5} = 1.8$
$\frac{10}{1} = 10.$	$\frac{10}{2} = 5.$	$\frac{10}{3} = 3.33333$	$\frac{10}{4} = 2.5$	$\frac{10}{5} = 2.$
$\frac{11}{1} = 11.$	$\frac{11}{2} = 5.5$	$\frac{11}{3} = 3.66667$	$\frac{11}{4} = 2.75$	$\frac{11}{5} = 2.2$
$\frac{12}{1} = 12.$	$\frac{12}{2} = 6.$	$\frac{12}{3} = 4.$	$\frac{12}{4} = 3.$	$\frac{12}{5} = 2.4$

Zero

$1 == \text{Indeterminate}$

$\frac{1}{0} == \infty$

$\frac{2}{0} == \infty$

$\frac{3}{0} == \infty$

$\frac{4}{0} == \infty$

$\frac{5}{0} == \infty$

$\frac{6}{0} == \infty$

$\frac{7}{0} == \infty$

$\frac{8}{0} == \infty$

$\frac{9}{0} == \infty$

$\frac{10}{0} == \infty$

$\frac{11}{0} == \infty$

Six

$\frac{1}{6} == 0.166667$

$\frac{2}{6} == 0.333333$

$\frac{3}{6} == 0.5$

$\frac{4}{6} == 0.666667$

$\frac{5}{6} == 0.833333$

$1 == 1.$

$\frac{7}{6} == 1.16667$

$\frac{8}{6} == 1.33333$

$\frac{9}{6} == 1.5$

$\frac{10}{6} == 1.66667$

$\frac{11}{6} == 1.83333$

$\frac{12}{6} == 2.$

Seven

$\frac{1}{7} == 0.142857$

$\frac{2}{7} == 0.285714$

$\frac{3}{7} == 0.428571$

$\frac{4}{7} == 0.571429$

$\frac{5}{7} == 0.714286$

$\frac{6}{7} == 0.857143$

$1 == 1.$

$\frac{8}{7} == 1.14286$

$\frac{9}{7} == 1.28571$

$\frac{10}{7} == 1.42857$

$\frac{11}{7} == 1.57143$

$\frac{12}{7} == 1.71429$

Eight

$\frac{1}{8} == 0.125$

$\frac{2}{8} == 0.25$

$\frac{3}{8} == 0.375$

$\frac{4}{8} == 0.5$

$\frac{5}{8} == 0.625$

$\frac{6}{8} == 0.75$

$\frac{7}{8} == 0.875$

$1 == 1.$

$\frac{9}{8} == 1.125$

$\frac{10}{8} == 1.25$

$\frac{11}{8} == 1.375$

$\frac{12}{8} == 1.5$

Nine

$\frac{1}{9} == 0.111111$

$\frac{2}{9} == 0.222222$

$\frac{3}{9} == 0.333333$

$\frac{4}{9} == 0.444444$

$\frac{5}{9} == 0.555556$

$\frac{6}{9} == 0.666667$

$\frac{7}{9} == 0.777778$

$\frac{8}{9} == 0.888889$

$1 == 1.$

$\frac{10}{9} == 1.11111$

$\frac{11}{9} == 1.22222$

$\frac{12}{9} == 1.33333$

Ten

$\frac{1}{10} == 0.1$

$\frac{2}{10} == 0.2$

$\frac{3}{10} == 0.3$

$\frac{4}{10} == 0.4$

$\frac{5}{10} == 0.5$

$\frac{6}{10} == 0.6$

$\frac{7}{10} == 0.7$

$\frac{8}{10} == 0.8$

$\frac{9}{10} == 0.9$

$1 == 1.$

$\frac{11}{10} == 1.1$

$\frac{12}{10} == 1.2$

Power tables

Four

$1^4 = 1$
$2^4 = 16$
$3^4 = 81$
$4^4 = 256$
$5^4 = 625$
$6^4 = 1296$
$7^4 = 2401$
$8^4 = 4096$
$9^4 = 6561$
$10^4 = 10\,000$
$11^4 = 14\,641$
$12^4 = 20\,736$

Five

$1^5 = 1$
$2^5 = 32$
$3^5 = 243$
$4^5 = 1024$
$5^5 = 3125$
$6^5 = 7776$
$7^5 = 16\,807$
$8^5 = 32\,768$
$9^5 = 59\,049$
$10^5 = 100\,000$
$11^5 = 161\,051$
$12^5 = 248\,832$

Six

$1^6 = 1$
$2^6 = 64$
$3^6 = 729$
$4^6 = 4096$
$5^6 = 15\,625$
$6^6 = 46\,656$
$7^6 = 117\,649$
$8^6 = 262\,144$
$9^6 = 531\,441$
$10^6 = 1\,000\,000$
$11^6 = 1\,771\,561$
$12^6 = 2\,985\,984$

Seven

$1^7 = 1$
$2^7 = 128$
$3^7 = 2187$
$4^7 = 16\,384$
$5^7 = 78\,125$
$6^7 = 279\,936$
$7^7 = 823\,543$
$8^7 = 2\,097\,152$
$9^7 = 4\,782\,969$
$10^7 = 10\,000\,000$
$11^7 = 19\,487\,171$
$12^7 = 35\,831\,808$

Eight

$1^8 = 1$
$2^8 = 256$
$3^8 = 6561$
$4^8 = 65\,536$
$5^8 = 390\,625$
$6^8 = 1\,679\,616$
$7^8 = 5\,764\,801$
$8^8 = 16\,777\,216$
$9^8 = 43\,046\,721$
$10^8 = 100\,000\,000$
$11^8 = 214\,358\,881$
$12^8 = 429\,981\,696$

Nine

$1^9 = 1$
$2^9 = 512$
$3^9 = 19\,683$
$4^9 = 262\,144$
$5^9 = 1\,953\,125$
$6^9 = 10\,077\,696$
$7^9 = 40\,353\,607$
$8^9 = 134\,217\,728$
$9^9 = 387\,420\,489$
$10^9 = 1\,000\,000\,000$
$11^9 = 2\,357\,947\,691$
$12^9 = 5\,159\,780\,352$

Ten

$1^{10} = 1$
$2^{10} = 1024$
$3^{10} = 59\,049$
$4^{10} = 1\,048\,576$
$5^{10} = 9\,765\,625$
$6^{10} = 60\,466\,176$
$7^{10} = 282\,475\,249$
$8^{10} = 1\,073\,741\,824$
$9^{10} = 3\,486\,784\,401$
$10^{10} = 10\,000\,000\,000$
$11^{10} = 25\,937\,424\,601$
$12^{10} = 61\,917\,364\,224$

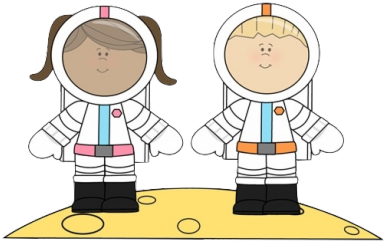
COUNTING NUMBERS


0 1 2 3 4 5 6 7 8 9
10 11 12 13 14 15 16 17 18 19
20 21 22 23 24 25 26 27 28 29
30 31 32 33 34 35 36 37 38 39
40 41 42 43 44 45 46 47 48 49
50 51 52 53 54 55 56 57 58 59
60 61 62 63 64 65 66 67 68 69
70 71 72 73 74 75 76 77 78 79
80 81 82 83 84 85 86 87 88 89
90 91 92 93 94 95 96 97 98 99
100

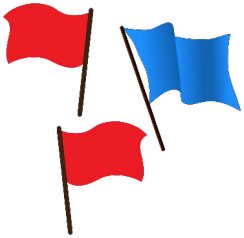
Numbers 1-10

1

2

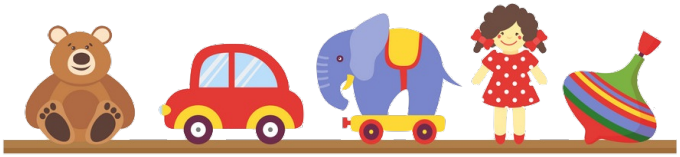


3



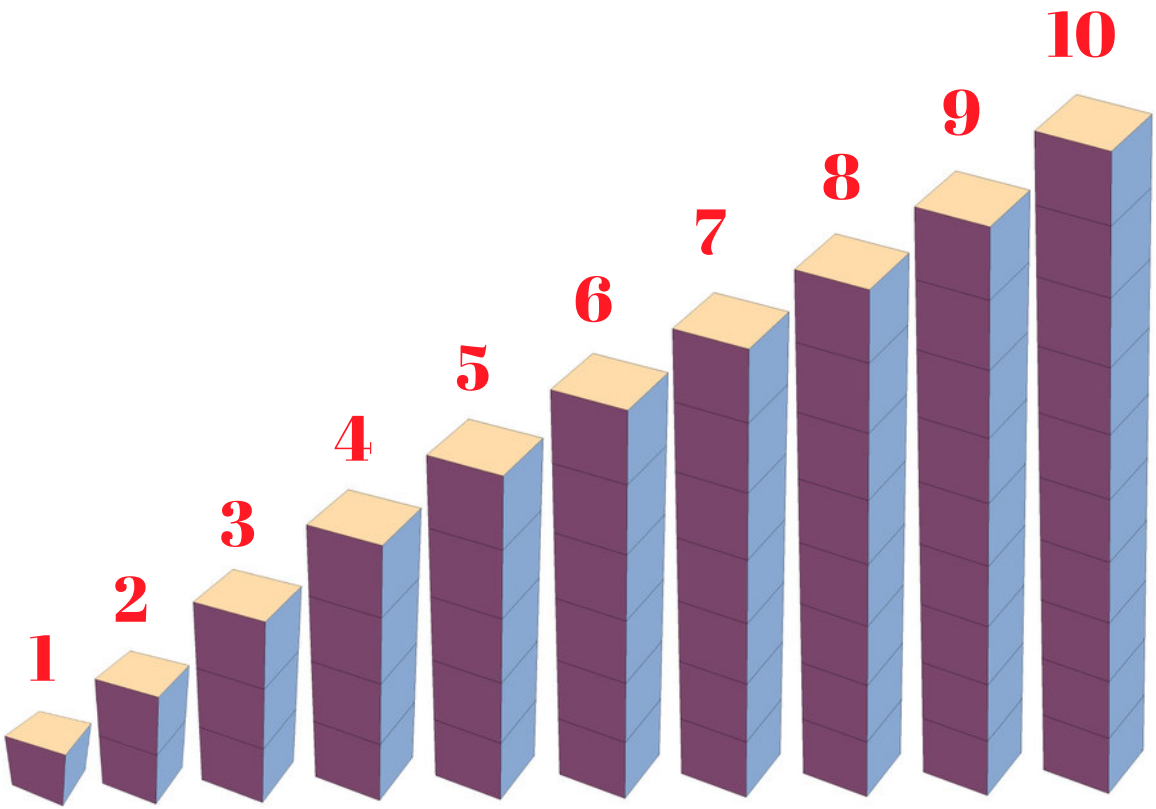
4

5

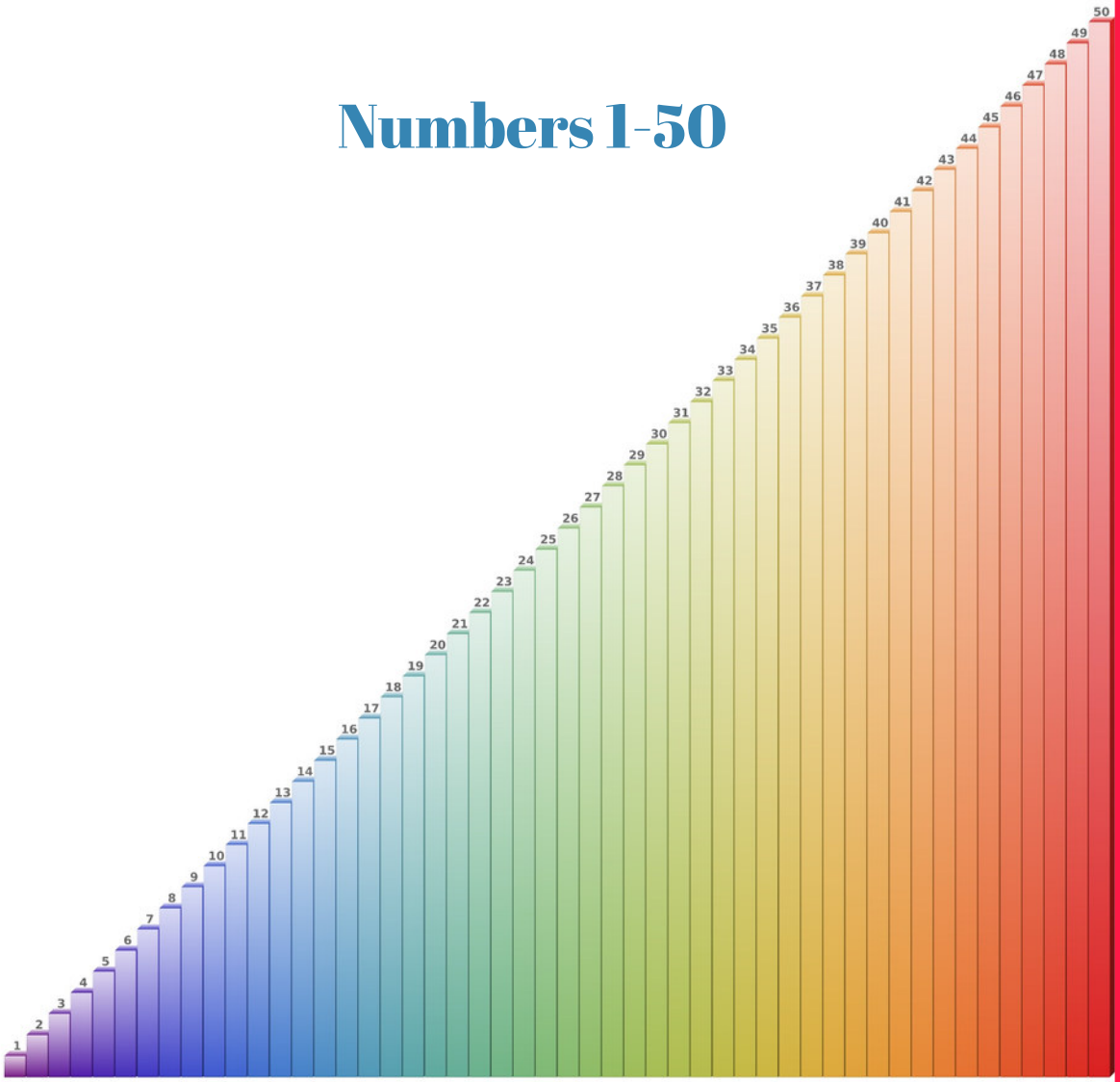



6
7
8
9
10

Numbers 1-10



Numbers 1-50


1
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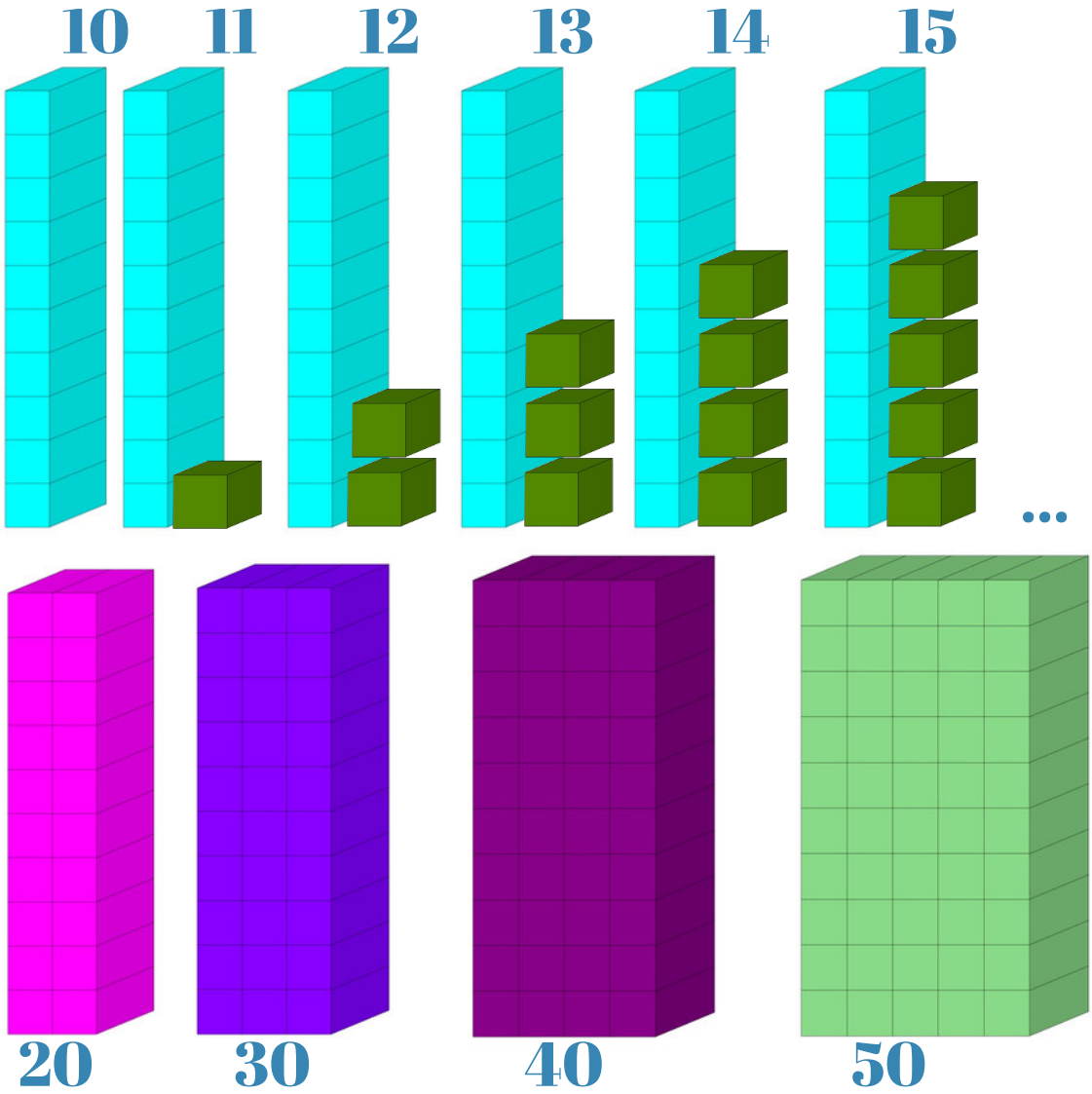
10
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12
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14
15
...
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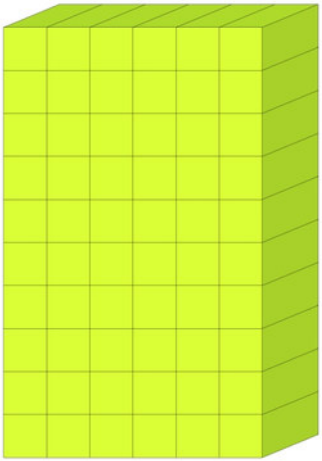
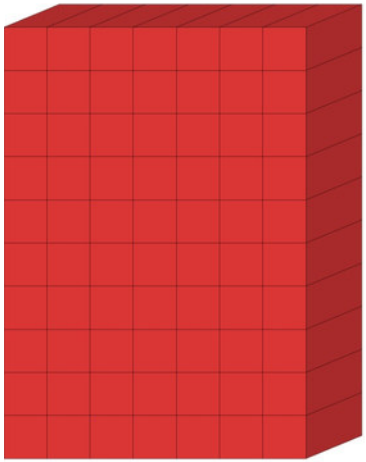
30
40
50

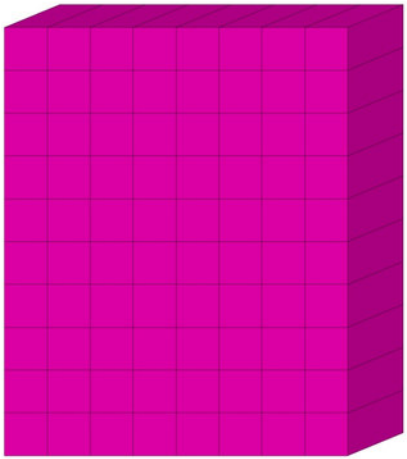
60

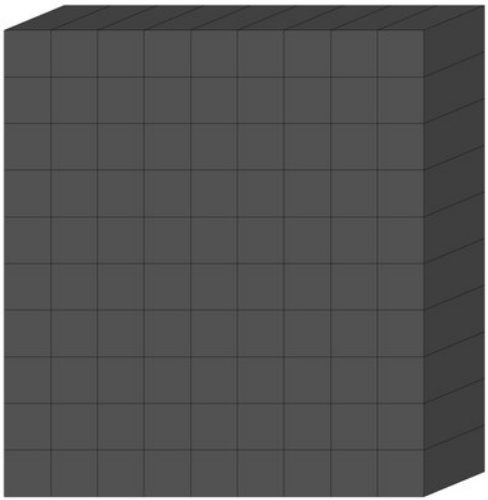


70



80





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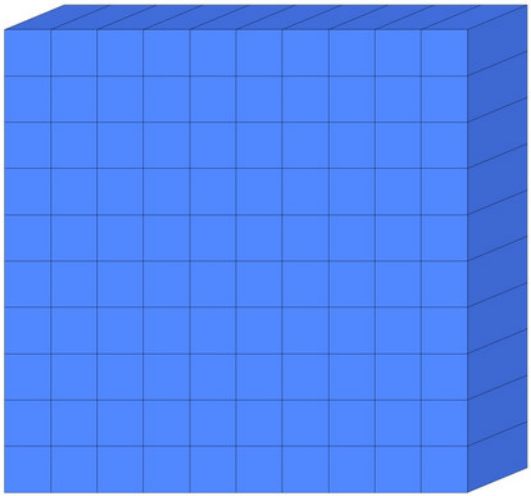
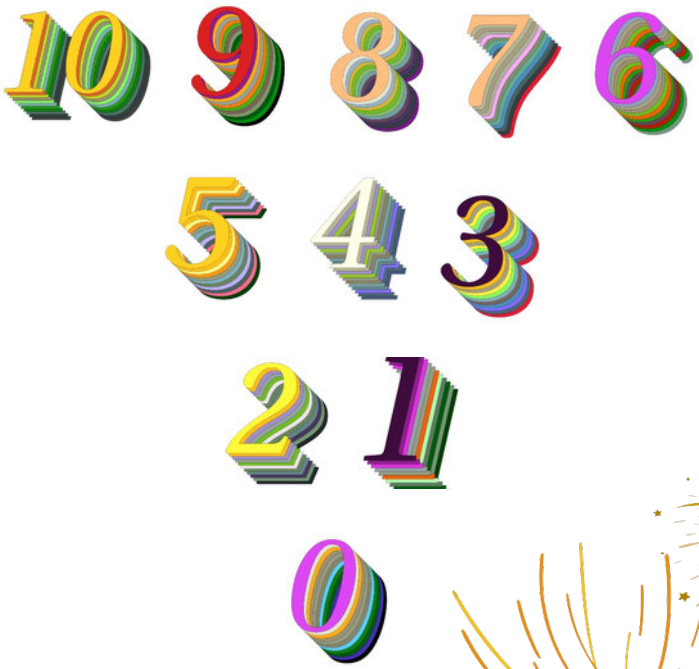
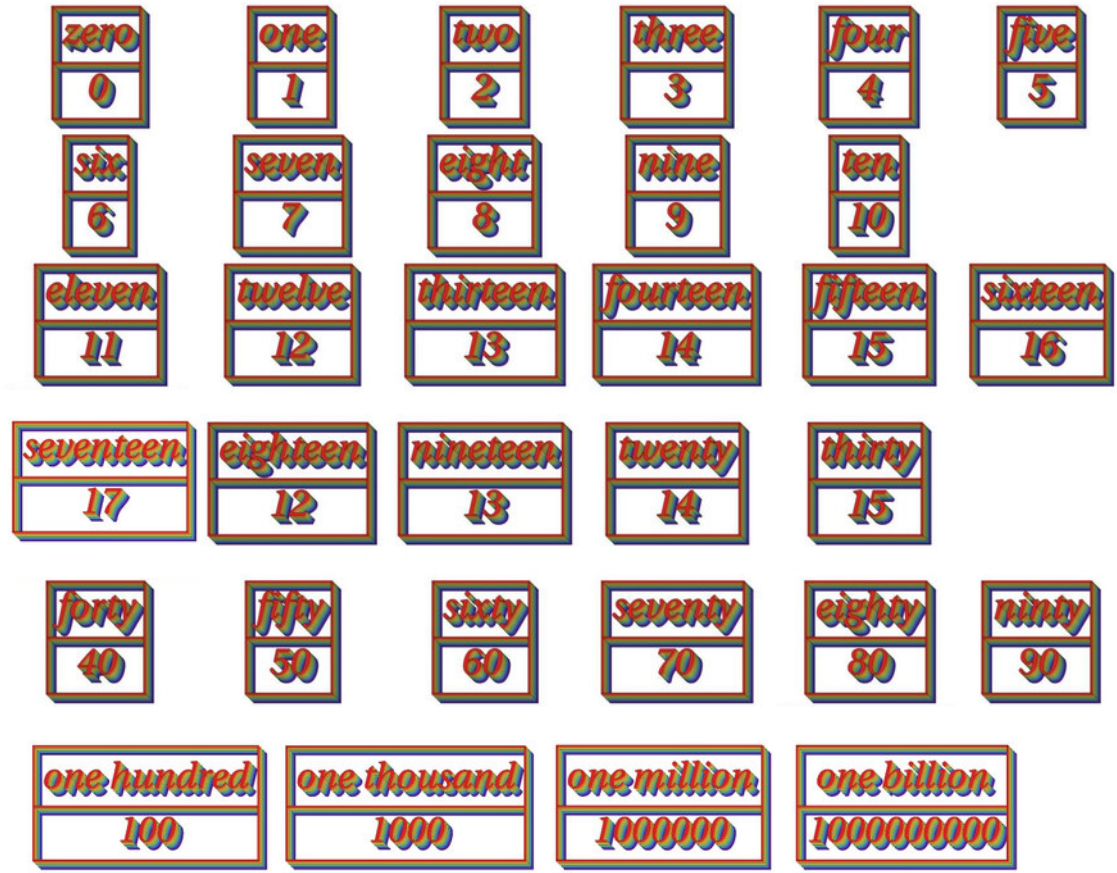


100

Numbers 10-0



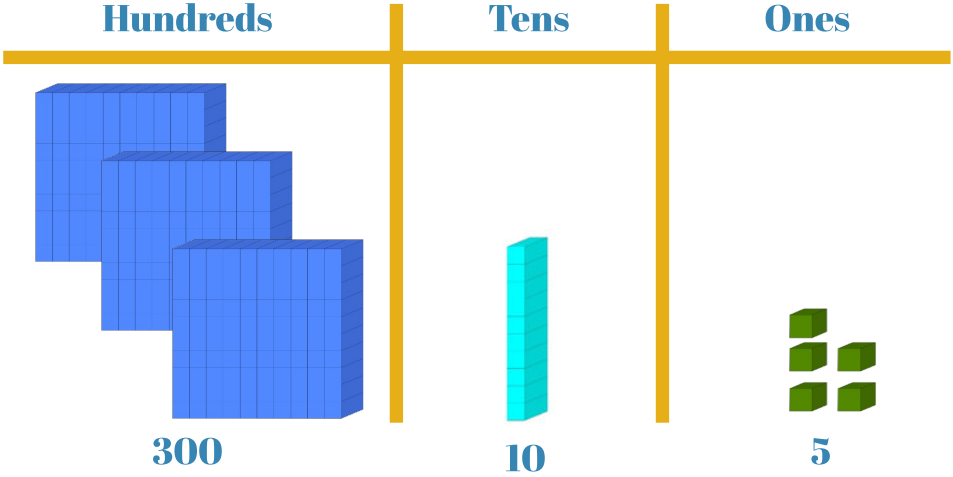
Written form

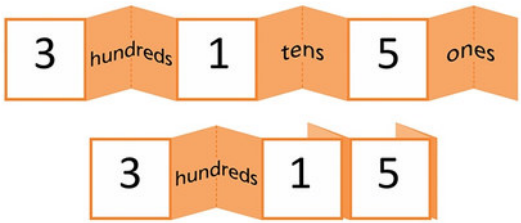


Numbers 0-200

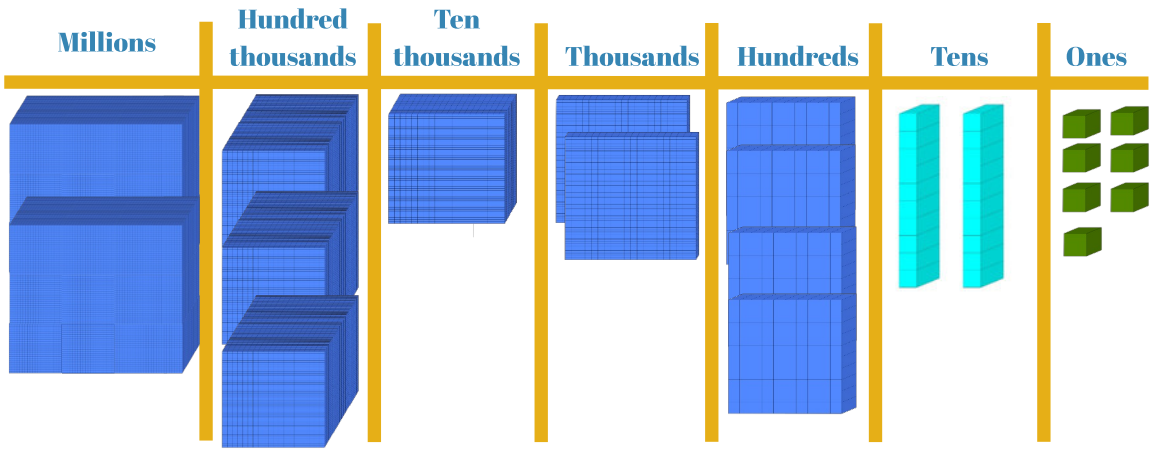
	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99


100 101 102 103 104 105 106 107 108 109
110 111 112 113 114 115 116 117 118 119
120 121 122 123 124 125 126 127 128 129
130 131 132 133 134 135 136 137 138 139
140 141 142 143 144 145 146 147 148 149
150 151 152 153 154 155 156 157 158 159
160 161 162 163 164 165 166 167 168 169
170 171 172 173 174 175 176 177 178 179
180 181 182 183 184 185 186 187 188 189
190 191 192 193 194 195 196 197 198 199
200





Three hundred and fifteen



2	3	1	2	4	2	7

Two million three hundred twelve thousand four hundred and twenty seven

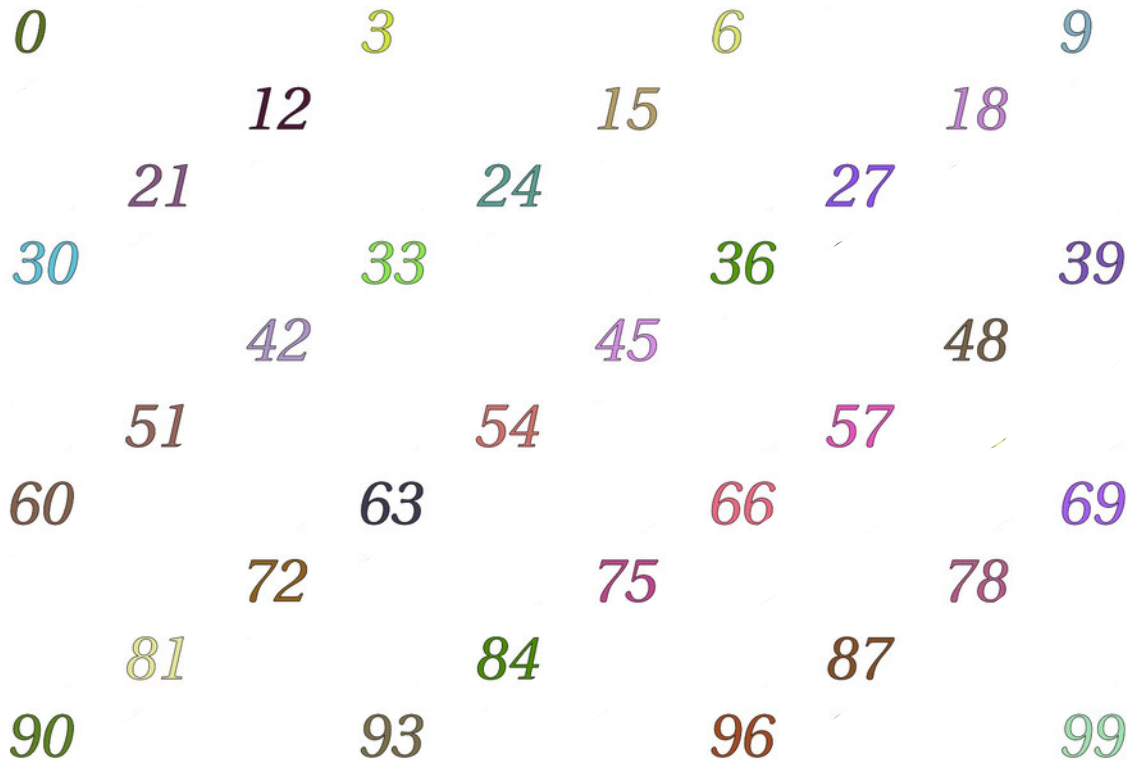
Count by 2s (even numbers)

0	2	4	6	8
10	12	14	16	18
20	22	24	26	28
30	32	34	36	38
40	42	44	46	48
50	52	54	56	58
60	62	64	66	68
70	72	74	76	78
80	82	84	86	88
90	92	94	96	98
100				

Count by 2s
(odd numbers)

1	3	5	7	9
11	13	15	17	19
21	23	25	27	29
31	33	35	37	39
41	43	45	47	49
51	53	55	57	59
61	63	65	67	69
71	73	75	77	79
81	83	85	87	89
91	93	95	97	99

Count by 3s


0 3 6 9
12 15 18
21 24 27
30 33 36 39
42 45 48
51 54 57
60 63 66 69
72 75 78
81 84 87
90 93 96 99

Count by 4s

0 4 8 12 16 20 24 28 32 36 40 44 48 52 56 60 64 68 72 76 80 84 88 92 96 100

Count by 5s

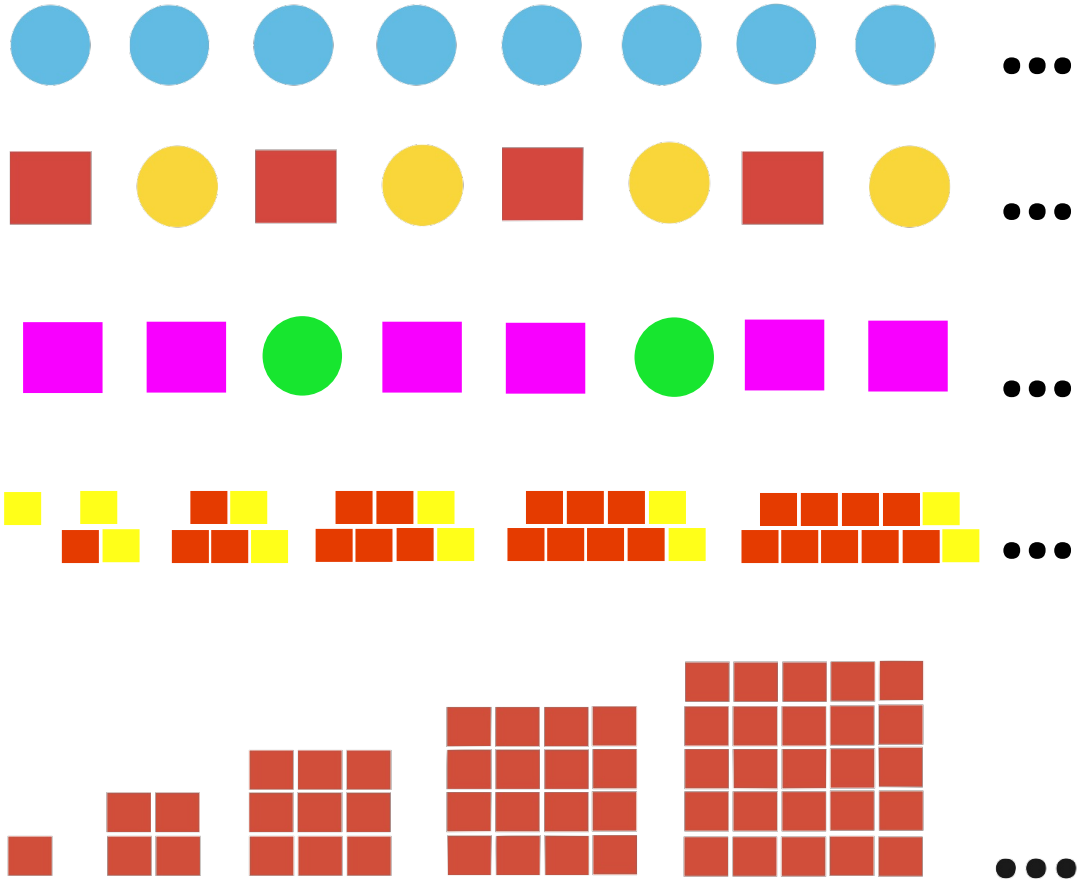
0	5
10	15
20	25
30	35
40	45
50	55
60	65
70	75
80	85
90	95
100	

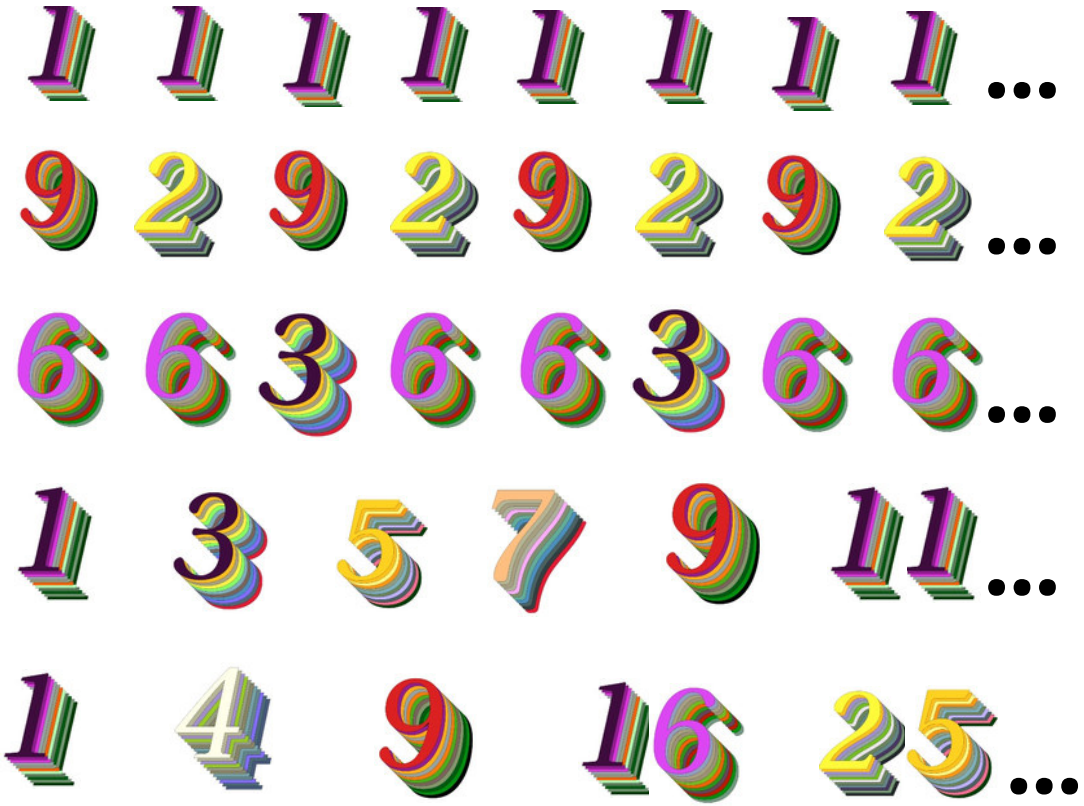
Count by 10s

0
10
20
30
40
50
60
70
80
90
100

Count by 100s

Sequences




1 1 1 1 1 1 1 1 ...
9 2 9 2 9 2 9 2 ...
6 6 3 6 6 3 6 6 ...
1 3 5 7 9 11 ...
1 4 9 16 25 ...

What is a number?

A number is a symbol assigned to each next number, starting at 0.

The Peano axioms

1) 0 is a counting number

0 is zero, nothing, none, nil

2) Every number has a next counting number:

Next(0) = 1, Next(1) = 2, Next(3) = 4, ... Next(n)=n+1

3) 0 does not have a previous counting number

There is no number n, such that 0 = Next(n)

4) If the next numbers of two numbers is the same, then the two original numbers are the same

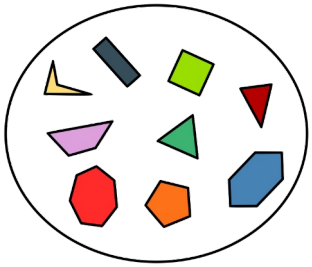
If Next(n) == Next(m), then n==m

5) If the set contains zero and the next number of every number is in the set, then the set contains all counting numbers

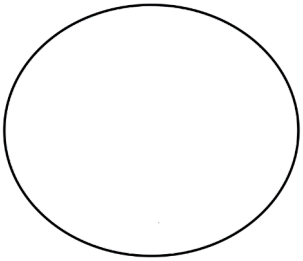
If s={0, Next(0)=1, Next(1)=2, Next(2)=3,...Next(n)}, s=$\mathbb{N}$

1)

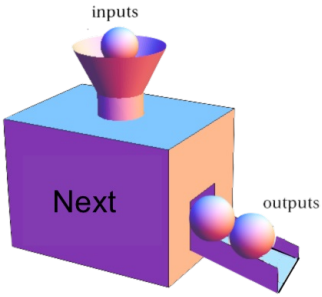
This set is not empty



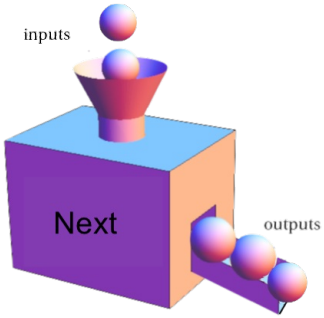
The empty set has 0 elements.



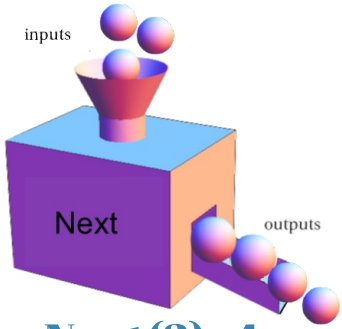
2)



Next(1)=2

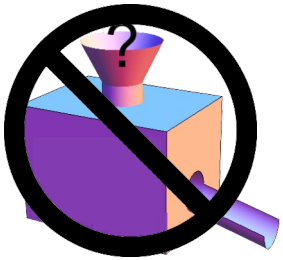


Next(2)=3



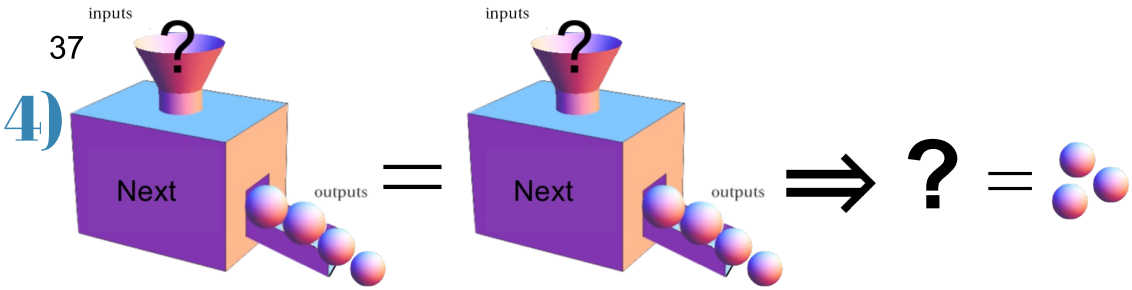
Next(3)=4

3)



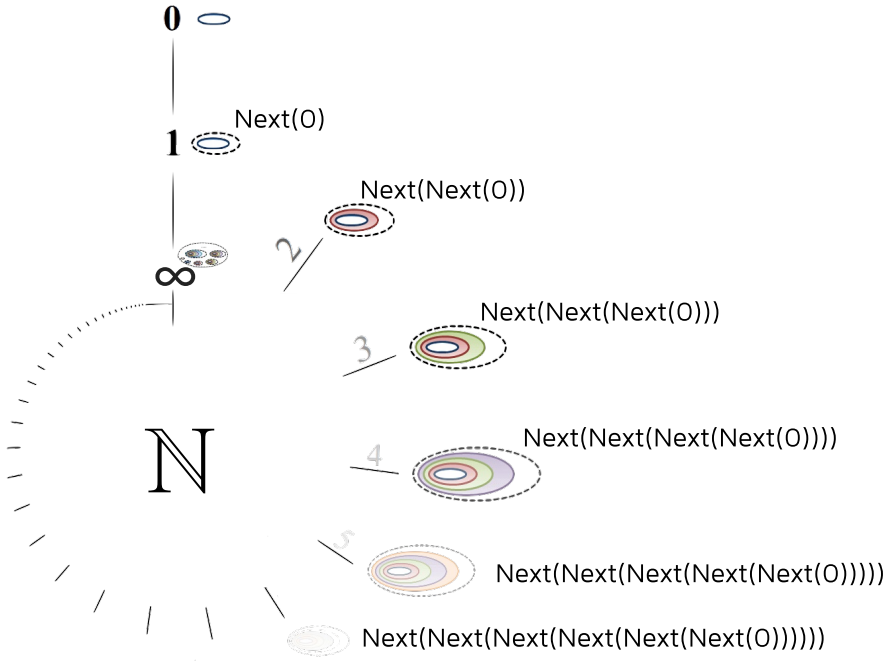
Next(?)≠0

4)

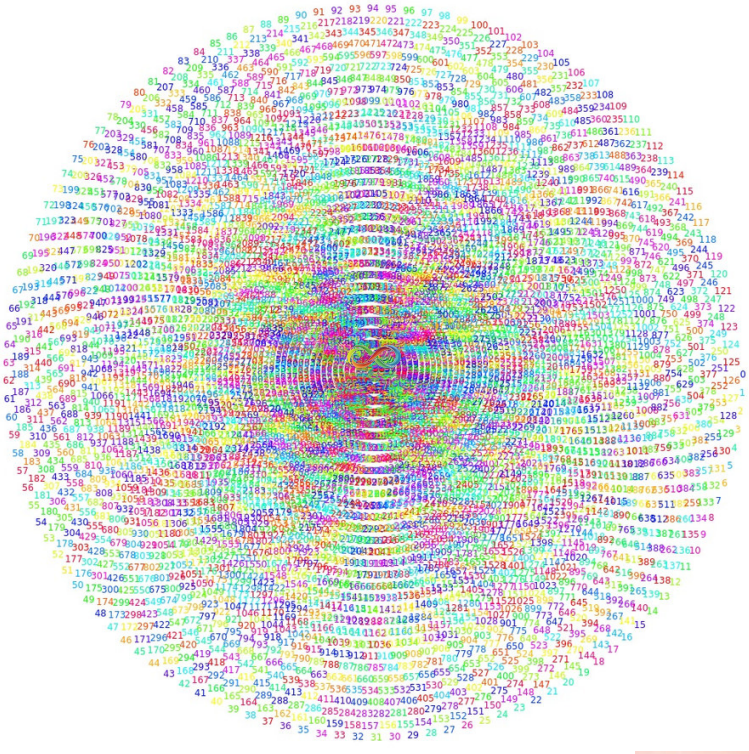


Next(n) = 4 **Next(m) = 4** **n=m=3**

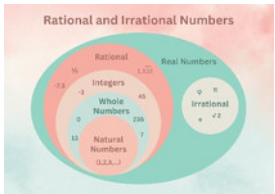
5)



$\mathbb{N}$



Counting numbers are called whole numbers, when you start at 0, and natural numbers when you start at 1.



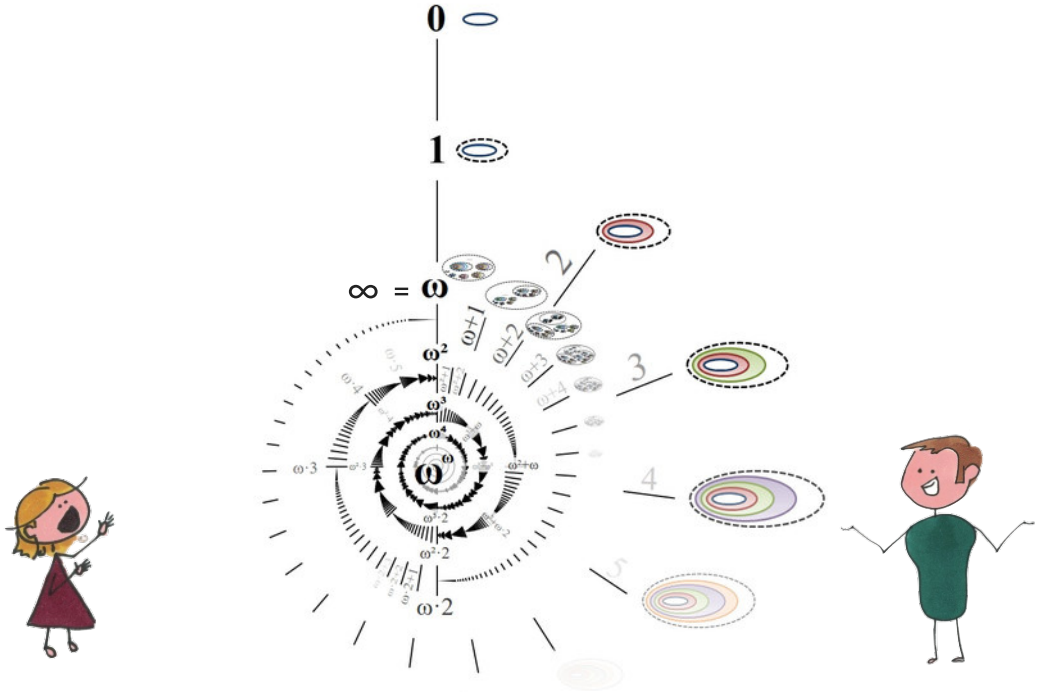
Infinity

Infinity (∞) is the smallest number you can NOT count to.

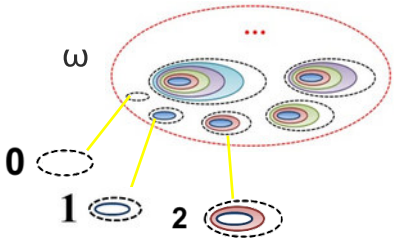
But, How can there be a number that you can't count to? Imagine all the largest numbers you can count to. Add one more to it. That's what infinity is.

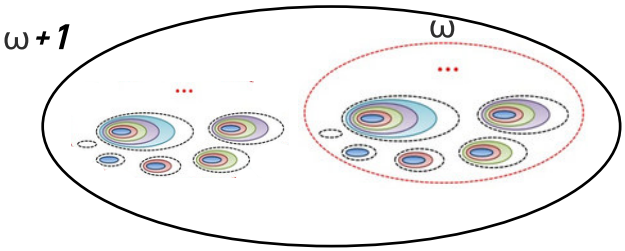
Can you count to 10, 100, 200? Yes. It takes a long time...
Can you count to 1000 by 100s starting at 0? Yes.
Can you count to 1000 starting at 900? Yes.
Can you count to 1 million by 100 000s? Yes.
Can you count to 1 000 000 000 by 100 000 000s? Yes.
Can you count to 10000000000000000000 by 1000000000000000s? Yes. By 1 000 000s? Yes. By 10s? Yes.

Can we go bigger...er? Yes. Let's go to infinity and beyond.

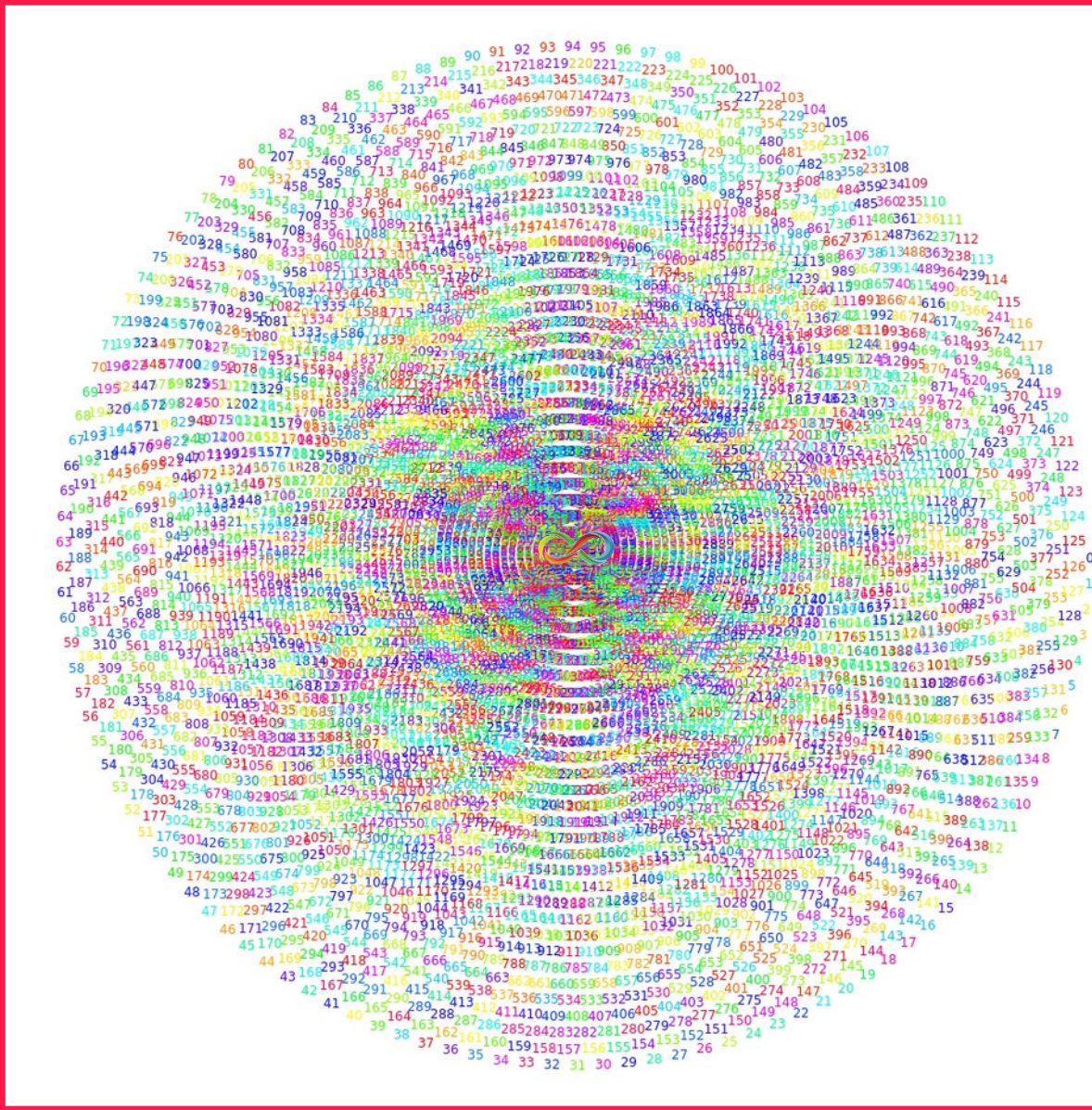


What is after ω ? ω + 1





We can go on indefinitely, and get ,⋯,ω+2, … ω·2, … ω·ω, ω·ω·ω, ⋯, etc.



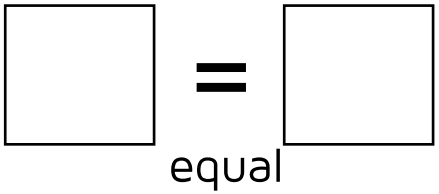
COMPARISON

$=, \neq, >, <$

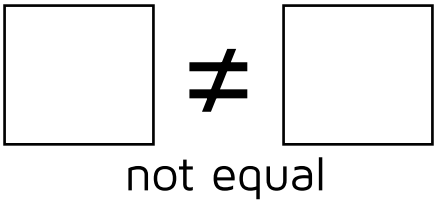
$1 = 1$	$1 < 2$	$1 < 3$	$1 < 4$	$1 < 5$	$1 < 6$	$1 < 7$
$2 \neq 1$	$2 = 2$	$2 < 3$	$2 < 4$	$2 < 5$	$2 < 6$	$2 < 7$
$3 > 1$	$3 \neq 2$	$3 = 3$	$3 < 4$	$3 < 5$	$3 < 6$	$3 < 7$
$4 > 1$	$4 > 2$	$4 \neq 3$	$4 = 4$	$4 < 5$	$4 < 6$	$4 < 7$
$5 > 1$	$5 > 2$	$5 > 3$	$5 \neq 4$	$5 = 5$	$5 < 6$	$5 < 7$
$6 > 1$	$6 > 2$	$6 > 3$	$6 > 4$	$6 \neq 5$	$6 = 6$	$6 < 7$
$7 > 1$	$7 > 2$	$7 > 3$	$7 > 4$	$7 > 5$	$7 \neq 6$	$7 = 7$
$8 > 1$	$8 > 2$	$8 > 3$	$8 > 4$	$8 > 5$	$8 > 6$	$8 \neq 7$
$9 > 1$	$9 > 2$	$9 > 3$	$9 > 4$	$9 > 5$	$9 > 6$	$9 > 7$
$10 \neq 1$	$10 \neq 2$	$10 \neq 3$	$10 \neq 4$	$10 \neq 5$	$10 \neq 6$	$10 \neq 7$

$1 < 8$	$1 < 9$	$1 < 10$	$1 < 11$	$1 < 12$	$1 < 13$	$1 < 14$
$2 < 8$	$2 < 9$	$2 < 10$	$2 < 11$	$2 < 12$	$2 < 13$	$2 < 14$
$3 < 8$	$3 < 9$	$3 < 10$	$3 < 11$	$3 < 12$	$3 < 13$	$3 < 14$
$4 < 8$	$4 < 9$	$4 < 10$	$4 < 11$	$4 < 12$	$4 < 13$	$4 < 14$
$5 < 8$	$5 < 9$	$5 < 10$	$5 < 11$	$5 < 12$	$5 < 13$	$5 < 14$
$6 < 8$	$6 < 9$	$6 < 10$	$6 < 11$	$6 < 12$	$6 < 13$	$6 < 14$
$7 < 8$	$7 < 9$	$7 < 10$	$7 < 11$	$7 < 12$	$7 < 13$	$7 < 14$
$8 = 8$	$8 < 9$	$8 < 10$	$8 < 11$	$8 < 12$	$8 < 13$	$8 < 14$
$9 \neq 8$	$9 = 9$	$9 < 10$	$9 < 11$	$9 < 12$	$9 < 13$	$9 < 14$
$10 > 8$	$10 \neq 9$	$10 = 10$	$10 < 11$	$10 < 12$	$10 < 13$	$10 < 14$

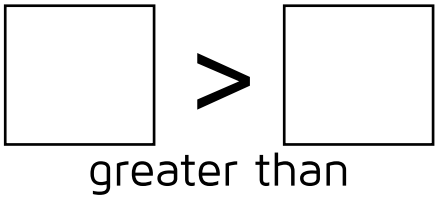
Comparison forms



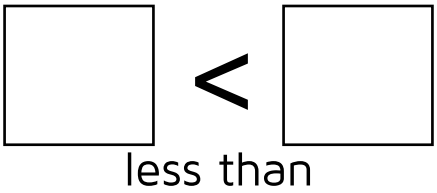
equal



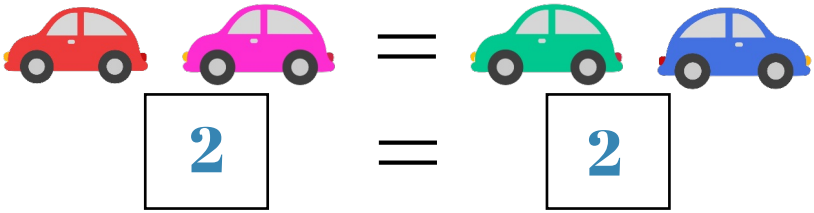
not equal

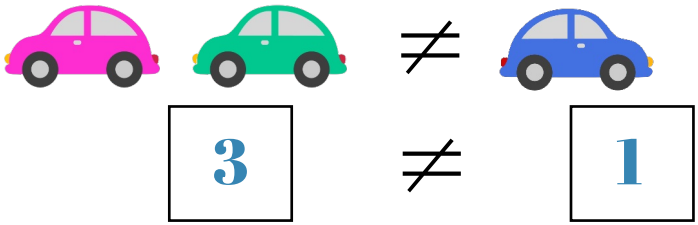


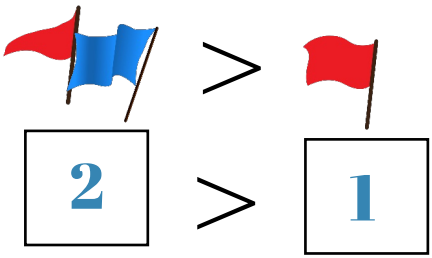
greater than

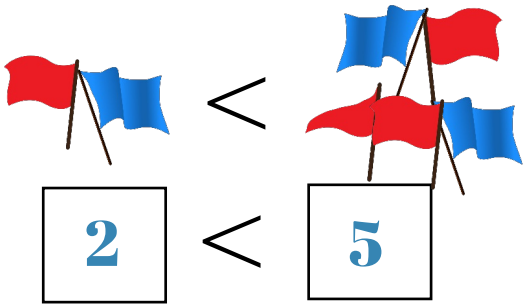


less than


=
2
=
2

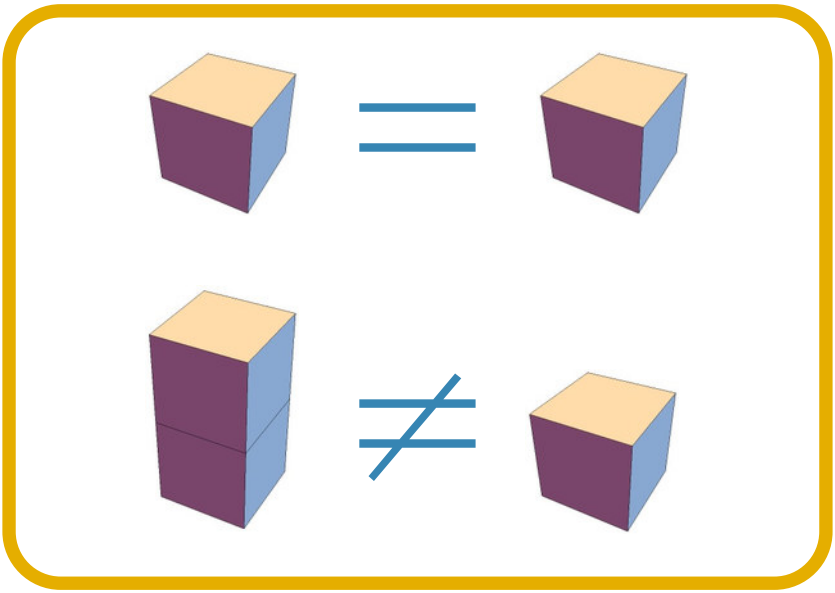

≠
3
≠
1

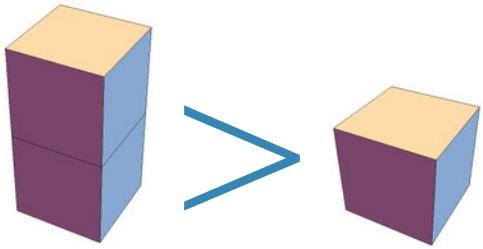

>
2
>
1

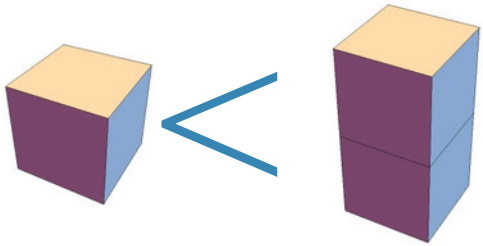

<
2
<
5

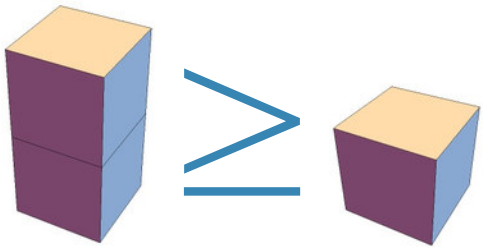
Examples

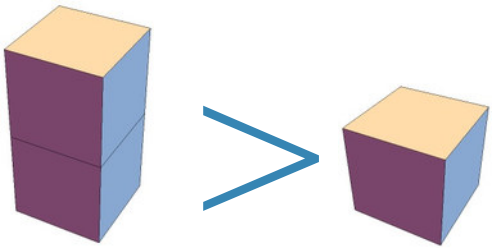
Operator	Name	Example
=	Equal	5 == 5
≠	Not equal	2 ≠ 3
>	Greater than	100 > 67
<	Less than	89 < 216
≥	Greater than or equal to	90 ≥ 54
≤	Less than or equal to	23 ≤ 77

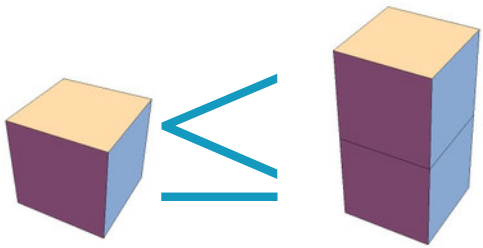


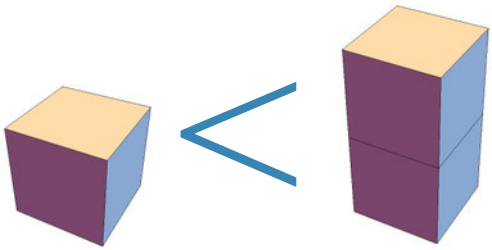


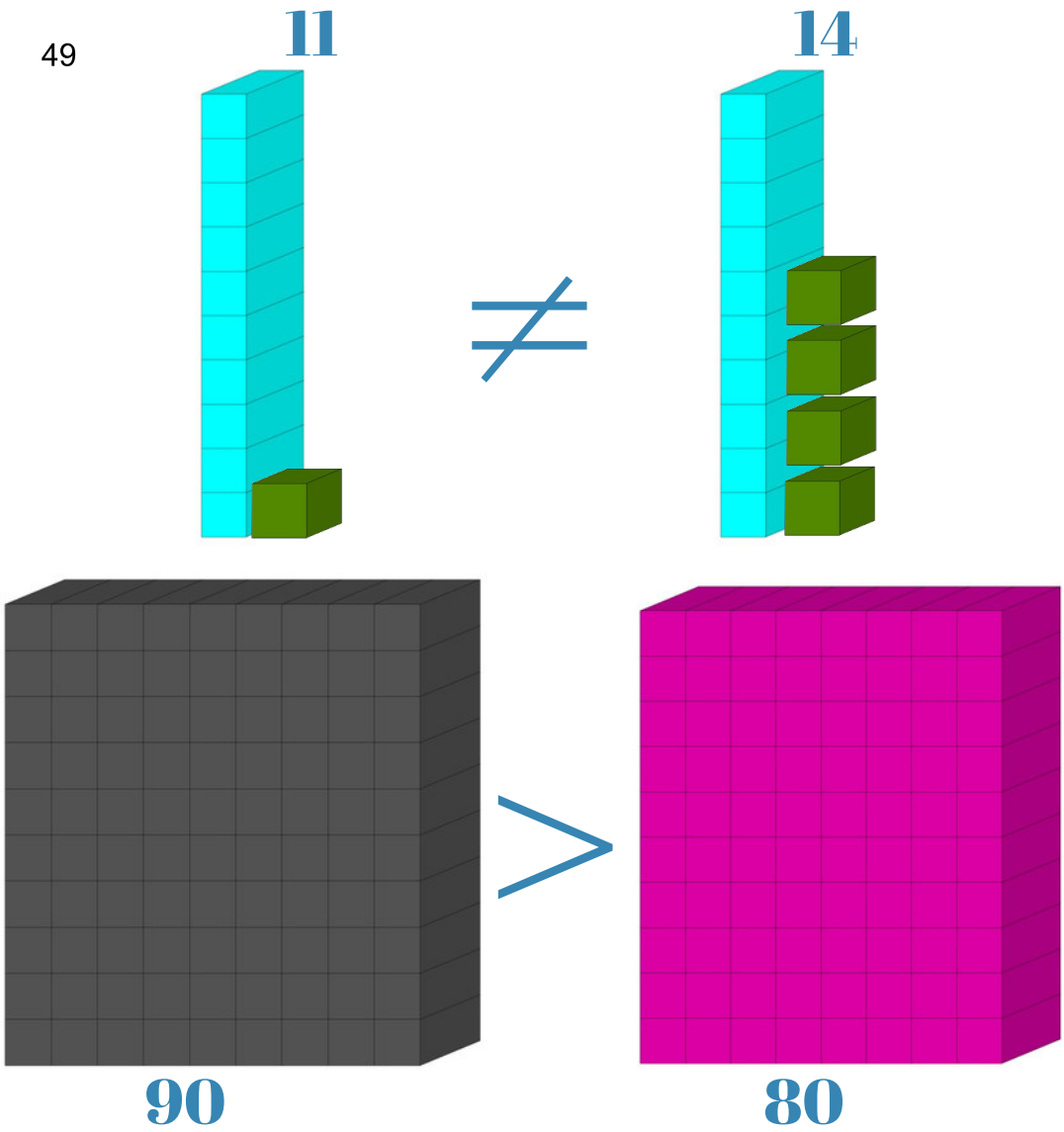











11
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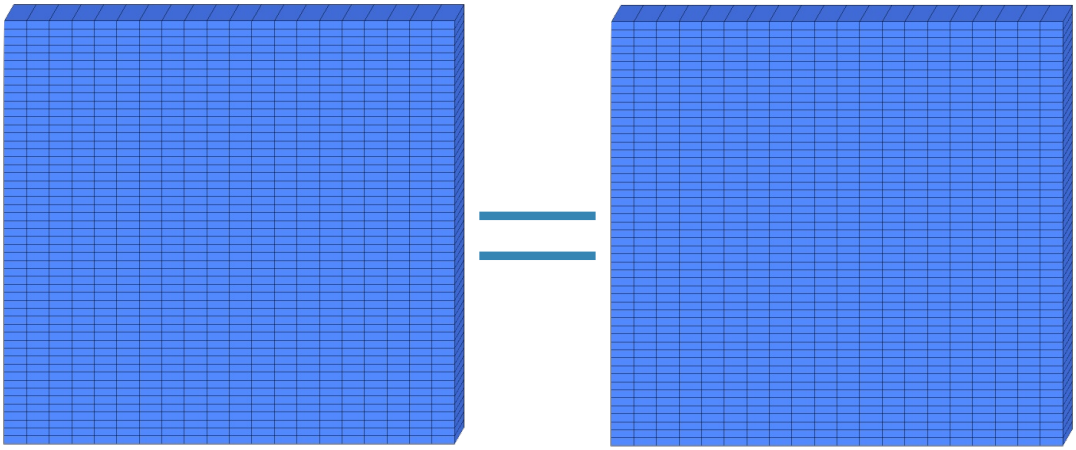
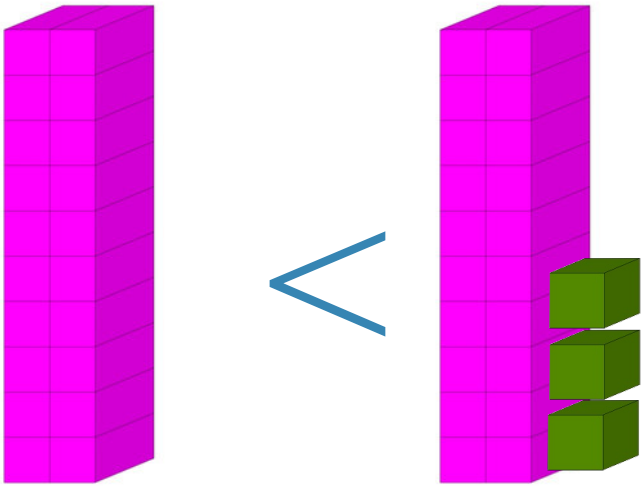
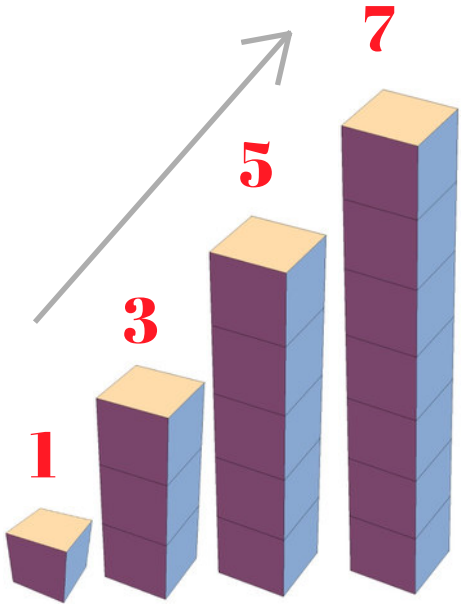
1000 1000



20 < 23

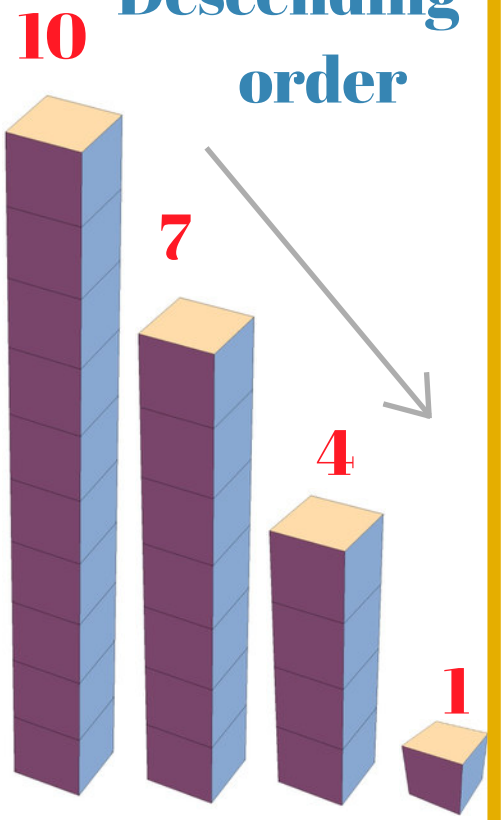


Ascending order



1 < 3 < 5 < 7

Descending order



10 > 7 > 4 > 1

Ascending order

1, 2, 3, 4, 5, 6, 7, 8, 9, 10

0, 5, 10, 15, 20, 25, 35, 40

1, 2, 4, 6, 7, 9, 13, 17, 18, 24

Descending order

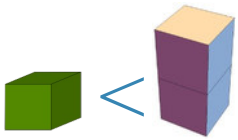
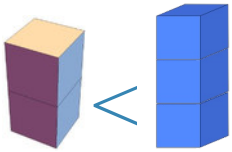
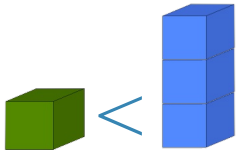
10, 9, 8, 7, 6, 5, 4, 3, 2, 1

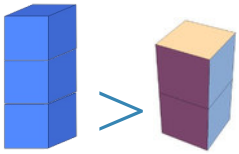
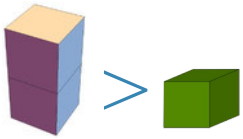
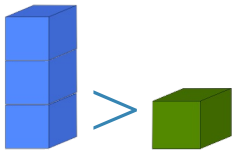
50, 45, 40, 35, 25, 20, 15, 10, 5, 0

24, 18, 17, 13, 12, 9, 7, 6, 4, 2, 1

Properties

1) If  and  then 

2) If  and  then 

3) If  and  then 

Transitive Properties

1) **Equal**
If a = b and b = c, then a = c

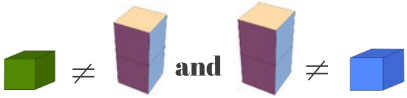
2) **Less than**
If a < b and b < c, then a < c

3) **Greater than**
If a > b and b > c, then a > c

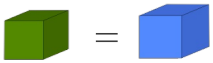
Examples

Inequality

It does not work for inequality:

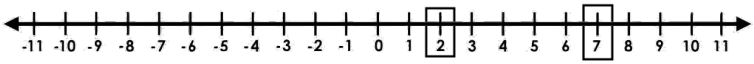


but:

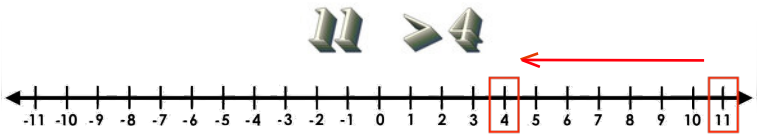


Number Line

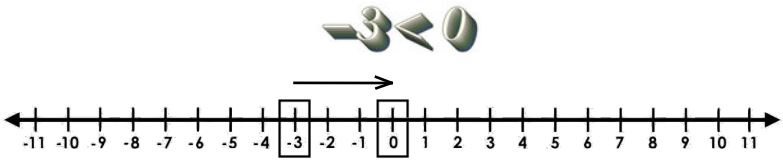
$2 < 7$



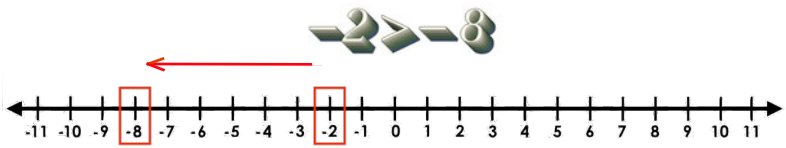
$11 > 4$



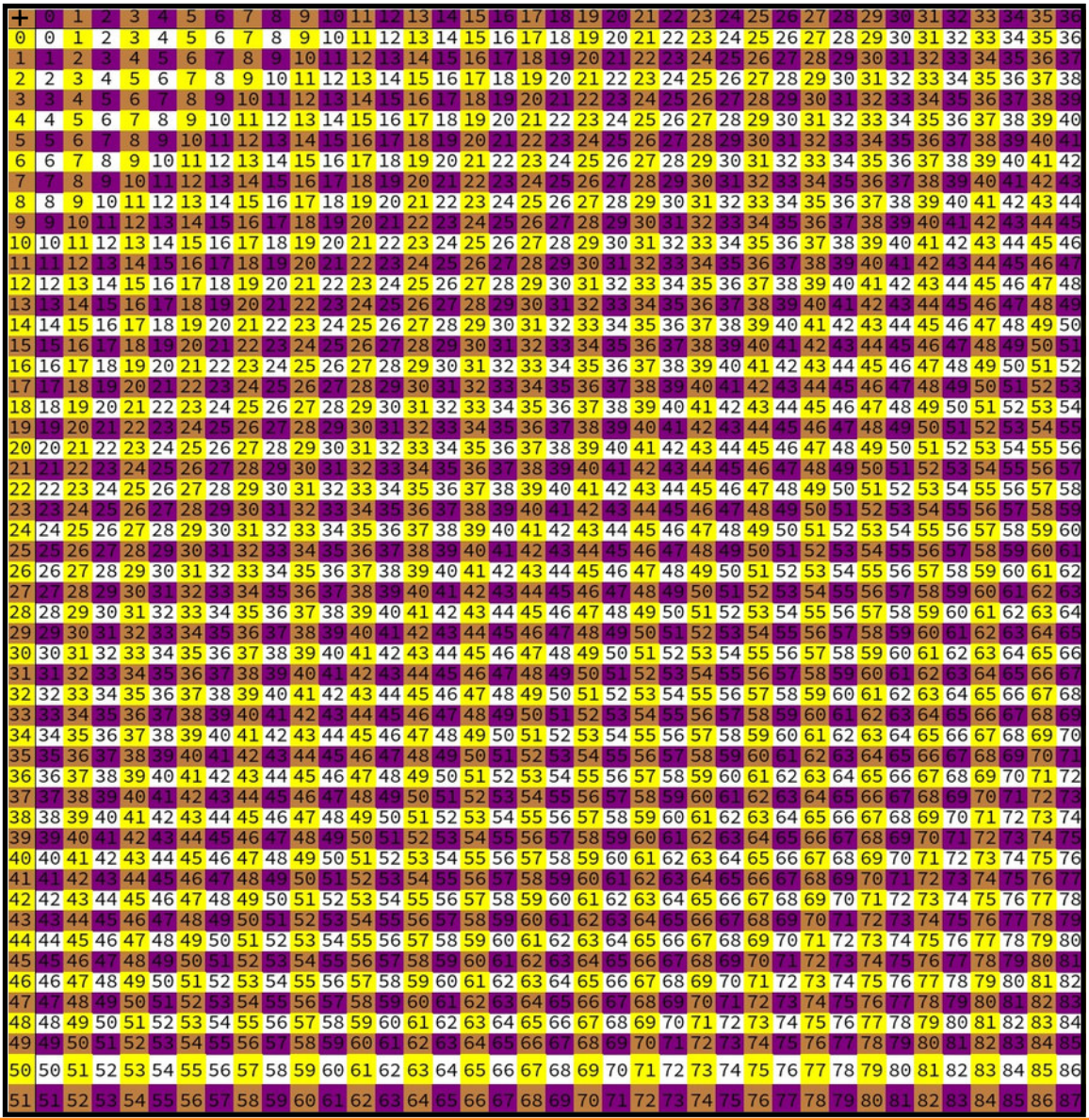
$-3 < 0$



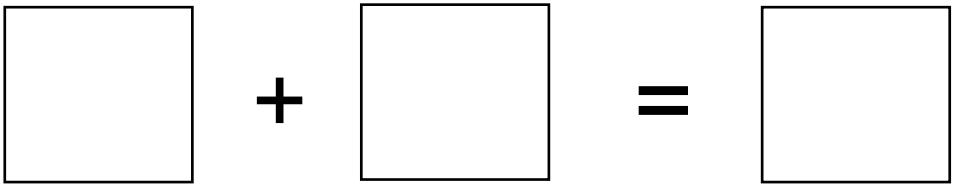
$-2 > -8$



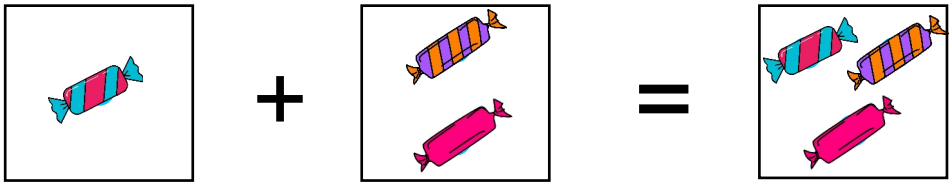
ADDITION



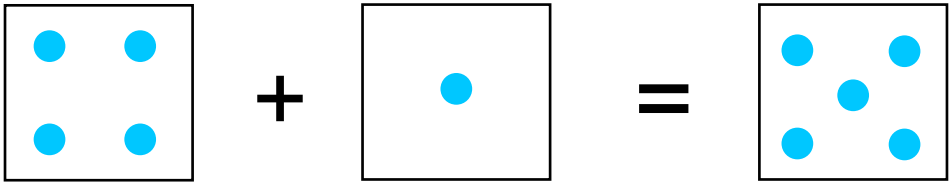
Addition form



addend addend sum

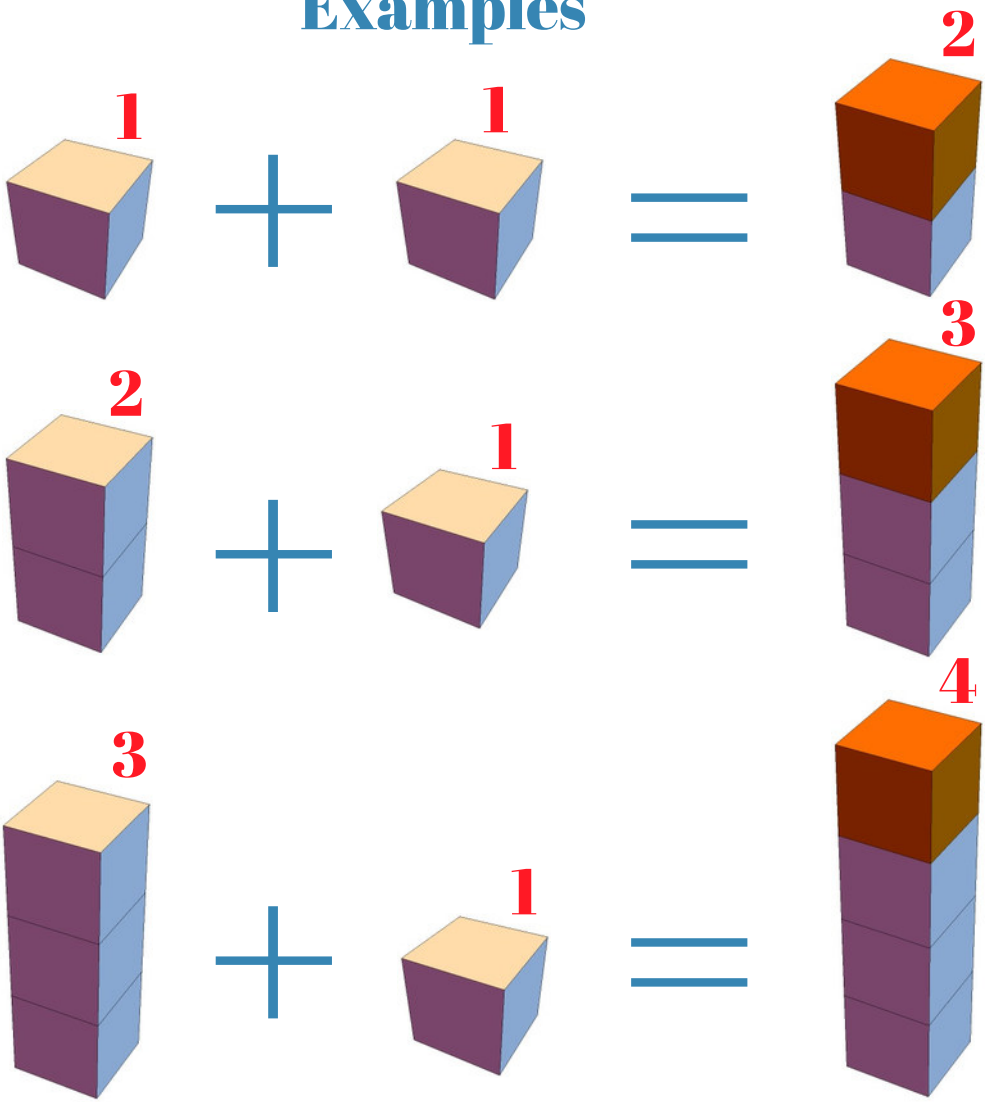


1 + 2 = 3

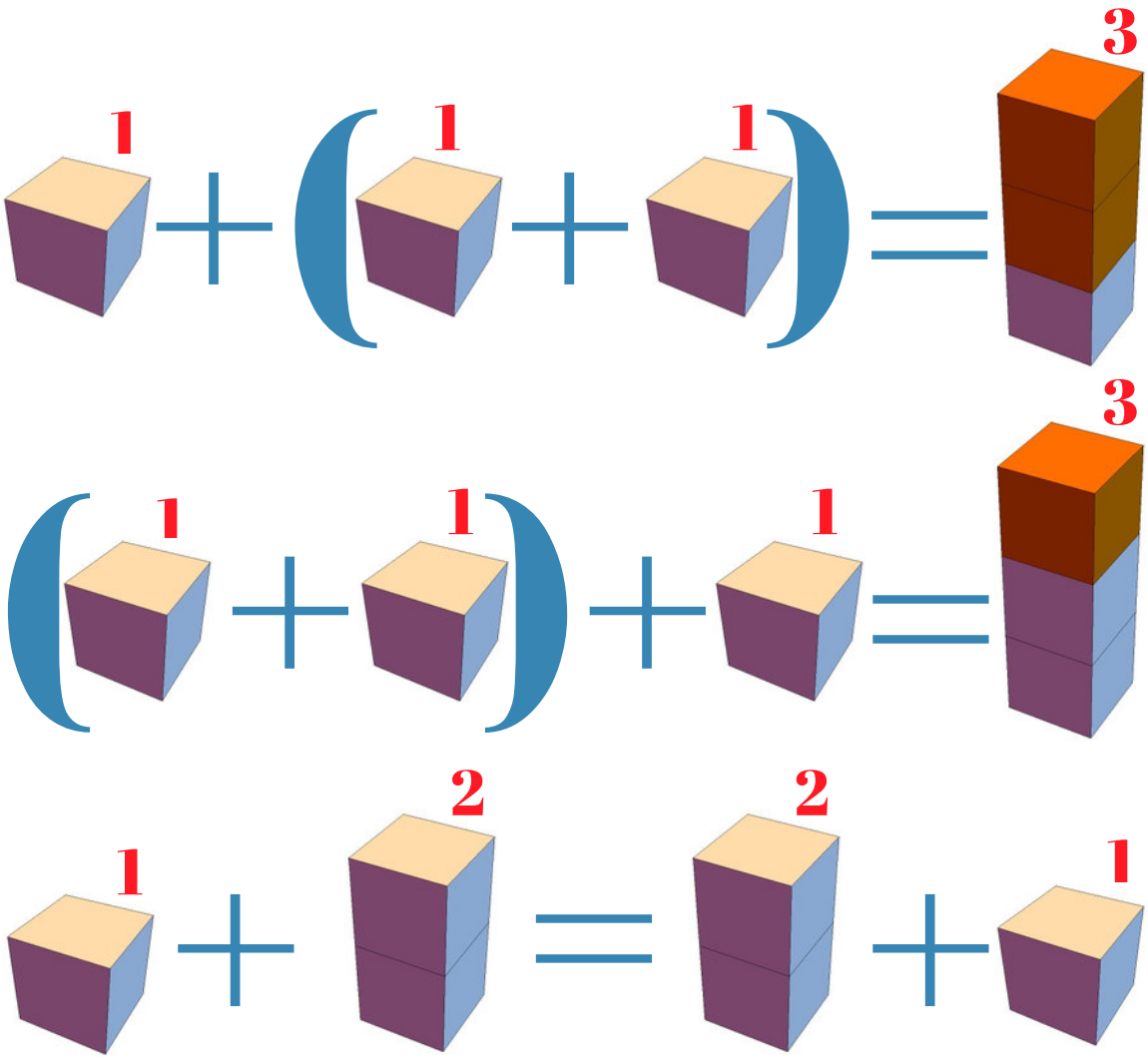


4 + 1 = 5

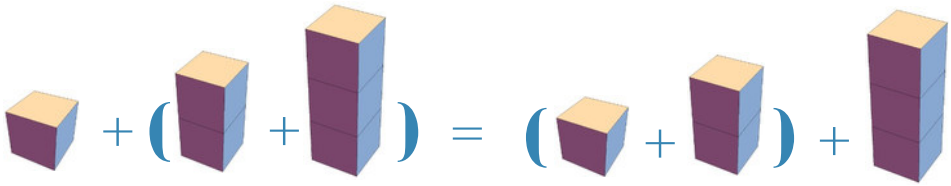
Examples

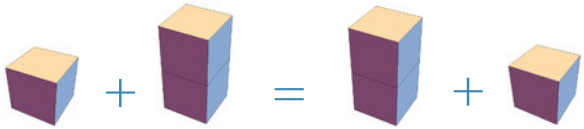


Addition Properties



Addition properties

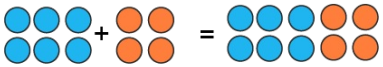
1) 

2) 

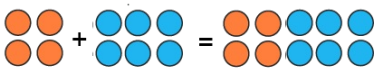
3) 

	Addition Properties	Examples
1)	**Commutative Property**	**Commutative Property**
	$a + b = b + a$	$6 + 2 = 2 + 6$
2)	**Associativity Property**	**Associativity Property**
	$(a + b) + c = a + (b + c)$	$(6 + 2) + 3 = 6 + (2 + 3)$
3)	**Identity Property**	**Identity Property**
	$a + 0 = a$	$6 + 0 = 6$

1)

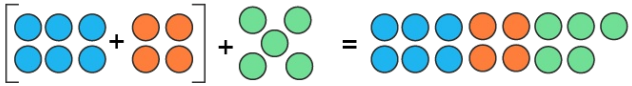


6 + 4 = 10

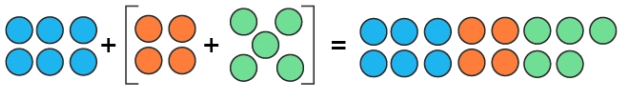


4 + 6 = 10

2)



(6 + 4) + 5 = 15



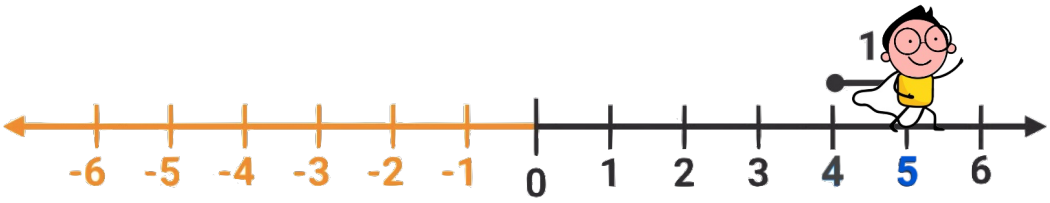
6 + (4 + 5) = 15

3)

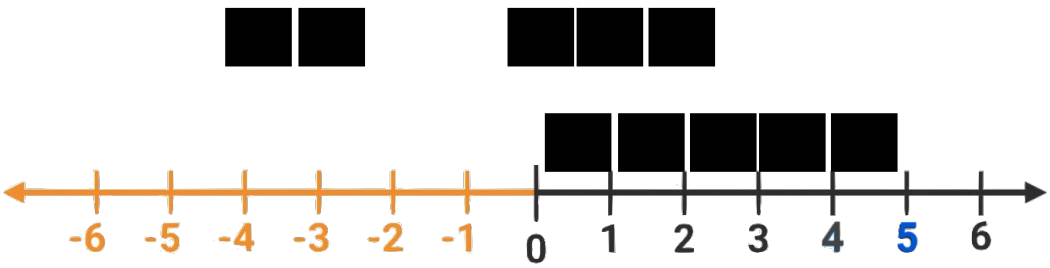
6 + 0 = 6

Number line

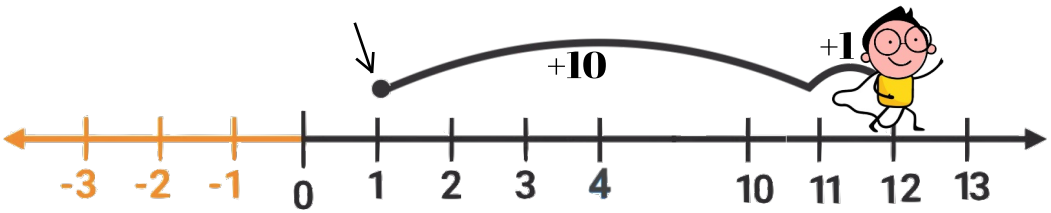
$$4 + 1 = 5$$



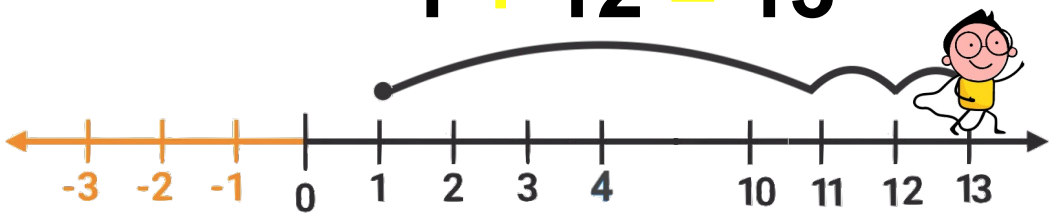
$$2 + 3 = 5$$



1 + 11 = 12



1 + 12 = 13



Step by step addition

1) $37 + 26$ — We want to add this.

2)
$$\begin{array}{r} 37 \\ +\ 26 \\ \hline \end{array}$$
Regroup the addition in column form.

3) $7 + 6 = 13$ — Sum up the 1's place.

4)
$$\begin{array}{r} {\scriptstyle 1}\ \downarrow \\ 37 \\ +\ 26 \\ \hline 3 \end{array}$$
Carry the one to the 10's place.

5)
$$\begin{array}{r} {\scriptstyle 1} \\ 37 \\ +\ 26 \\ \hline 63 \end{array}$$
Sum the 10s place.

6) $37 + 26 = 63$ — Write down the answer.

1) $237 + 116$ — We want to add this.

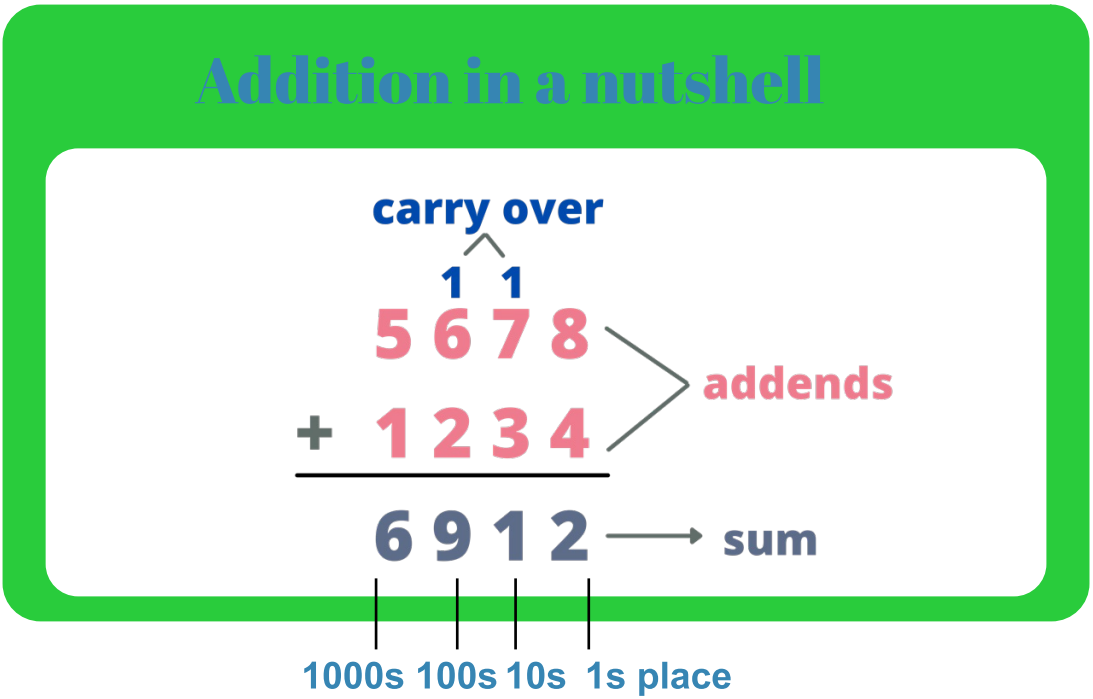
2)
$$\begin{array}{r} 237 \\ +\ 116 \\ \hline \end{array}$$
Regroup the addition in column form.

3)
$$\begin{array}{r} {\scriptstyle 1\ \downarrow} \\ 237 \\ +\ 116 \\ \hline 3 \end{array}$$
Sum up the 1's place and carry the one to 10s place..

4)
$$\begin{array}{r} {\scriptstyle 1} \\ 237 \\ +\ 116 \\ \hline 53 \end{array}$$
Sum up the 10s place.

5)
$$\begin{array}{r} {\scriptstyle \downarrow\ 1} \\ 237 \\ +\ 116 \\ \hline 353 \end{array}$$
Sum up the 100s place.

6) $237 + 116 = 353$ — Write down the answer.


Addition in a nutshell
carry over
1 1
5 6 7 8
+ 1 2 3 4
addends
6 9 1 2
sum
1000s 100s 10s 1s place

Addition in a nutshell

$$\begin{array}{r} {\scriptstyle 1\ \ 1\ \ \ \ } \\ 15365 \\ +\ 23499 \\ \hline 38864 \end{array}$$

SUBTRACTION

-	30	29	28	27	26	25	24	23	22	21	20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1
1	29	28	27	26	25	24	23	22	21	20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0
2	28	27	26	25	24	23	22	21	20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0	-1
3	27	26	25	24	23	22	21	20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0	-1	-2
4	26	25	24	23	22	21	20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0	-1	-2	-3
5	25	24	23	22	21	20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0	-1	-2	-3	-4
6	24	23	22	21	20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0	-1	-2	-3	-4	-5
7	23	22	21	20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0	-1	-2	-3	-4	-5	-6
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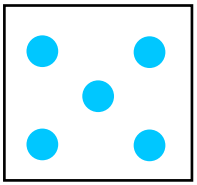
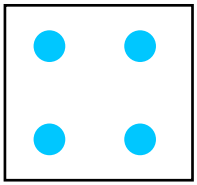
25	5	4	3	2	1	0	-1	-2	-3	-4	-5	-6	-7	-8	-9	-10	-11	-12	-13	-14	-15	-16	-17	-18	-19	-20	-21	-22	-23	-24
26	4	3	2	1	0	-1	-2	-3	-4	-5	-6	-7	-8	-9	-10	-11	-12	-13	-14	-15	-16	-17	-18	-19	-20	-21	-22	-23	-24	-25
27	3	2	1	0	-1	-2	-3	-4	-5	-6	-7	-8	-9	-10	-11	-12	-13	-14	-15	-16	-17	-18	-19	-20	-21	-22	-23	-24	-25	-26
28	2	1	0	-1	-2	-3	-4	-5	-6	-7	-8	-9	-10	-11	-12	-13	-14	-15	-16	-17	-18	-19	-20	-21	-22	-23	-24	-25	-26	-27
29	1	0	-1	-2	-3	-4	-5	-6	-7	-8	-9	-10	-11	-12	-13	-14	-15	-16	-17	-18	-19	-20	-21	-22	-23	-24	-25	-26	-27	-28
30	0	-1	-2	-3	-4	-5	-6	-7	-8	-9	-10	-11	-12	-13	-14	-15	-16	-17	-18	-19	-20	-21	-22	-23	-24	-25	-26	-27	-28	-29

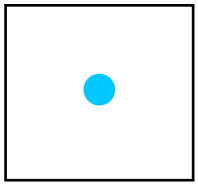
Subtraction form

minuend subtrahend difference

3 – 2 = 1

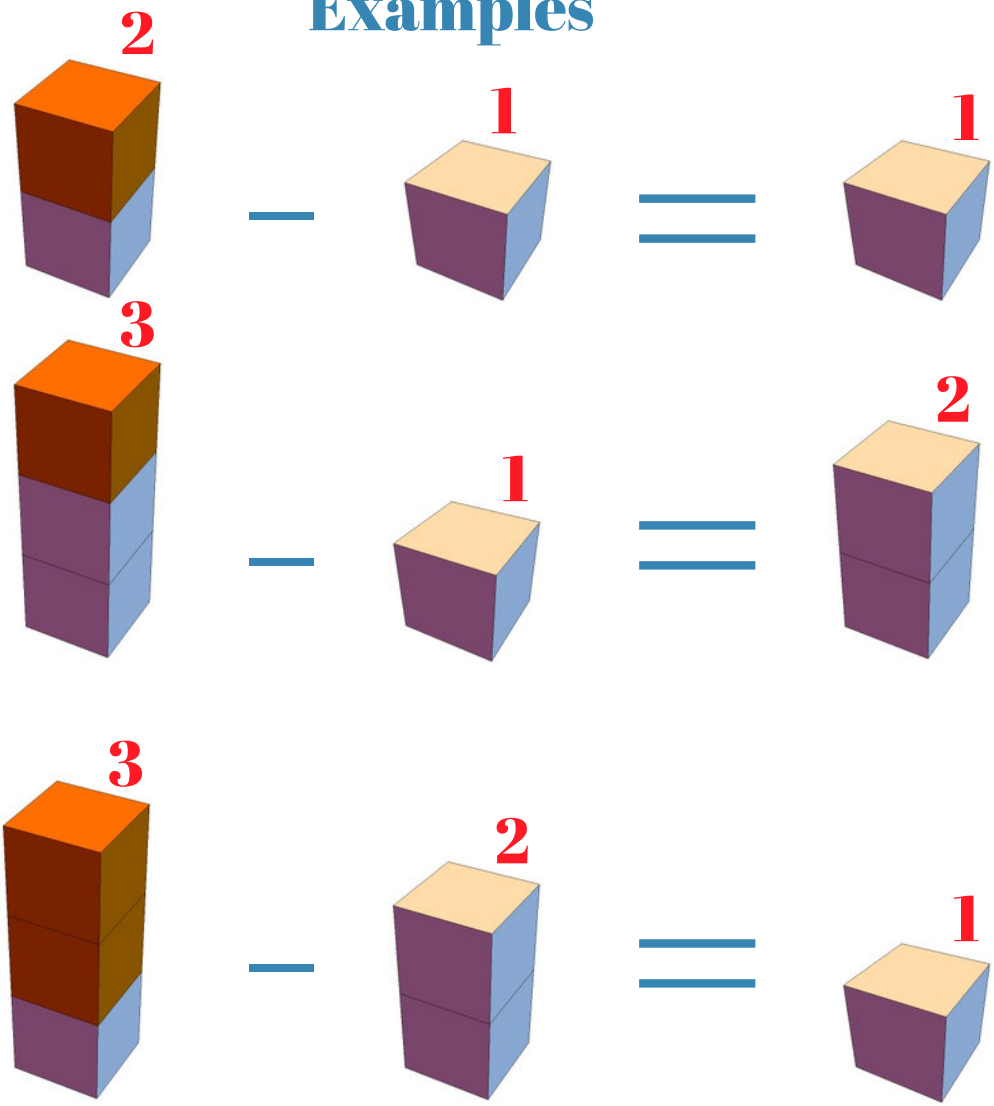




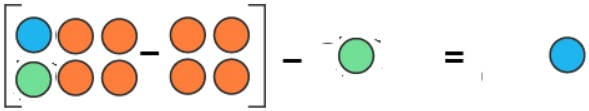


5 – 4 = 1

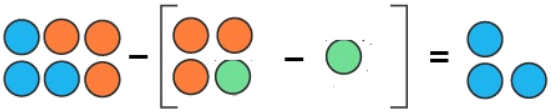
Examples



10 - 4 = 6



(6 – 4) – 1 = 1



6 – (4 – 1) = 3

6 – 0 = 6

Subtraction Properties

Identity Property

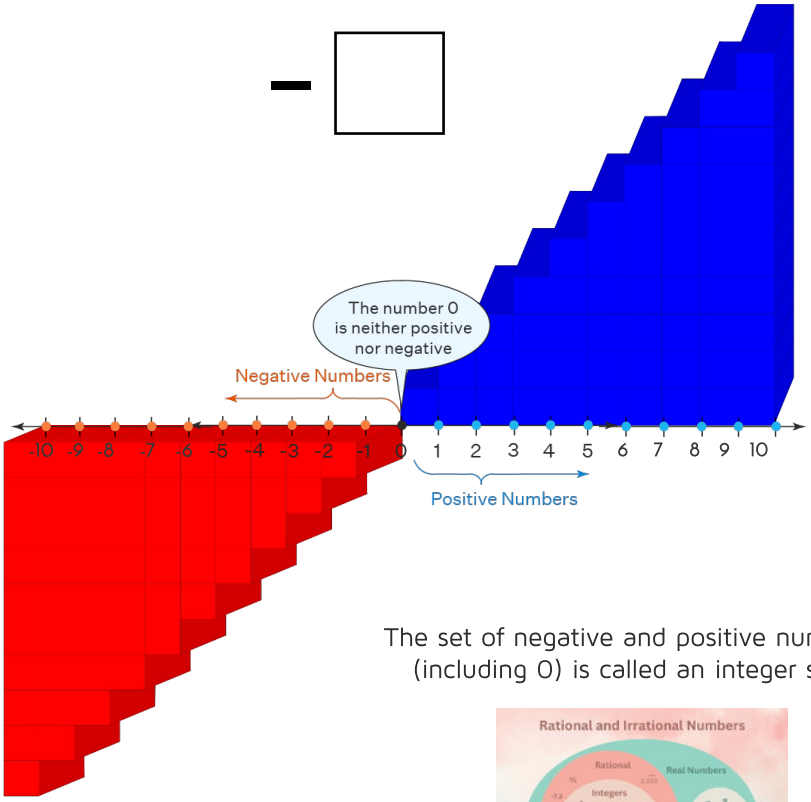
a - 0 = a

Commutative property and associative property do NOT apply here:

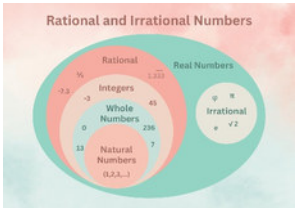
a - b ≠ b - a

(a - b) - c ≠ a - (b - c)

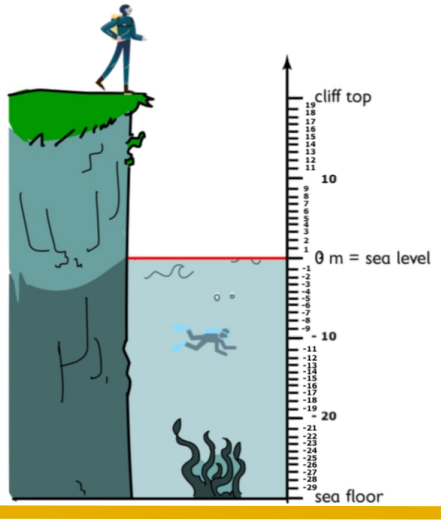
Negative numbers

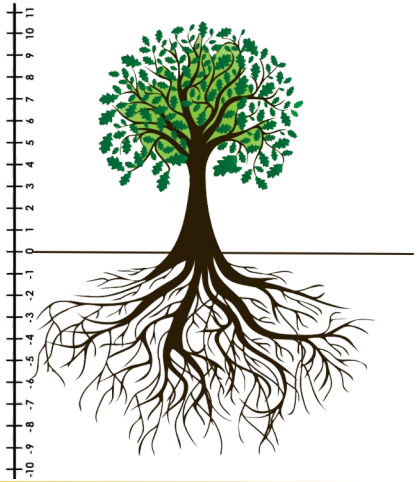


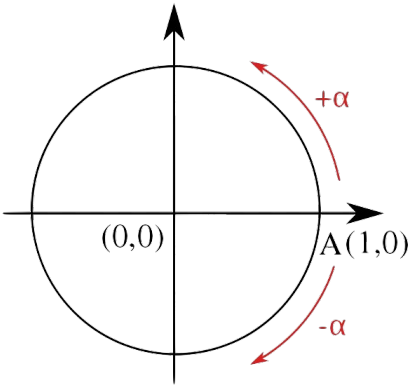
The set of negative and positive numbers (including 0) is called an integer set.

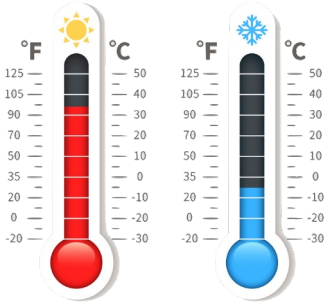


Examples









Properties

$+ - + = + + -$ example: $3 - 2 = 3 + (-2)$

$+ < +$ example: $2 < 3$

$- < -$ example: $-3 < -2$

Addition with negative numbers

$+ + + = +$

$- + - = -$

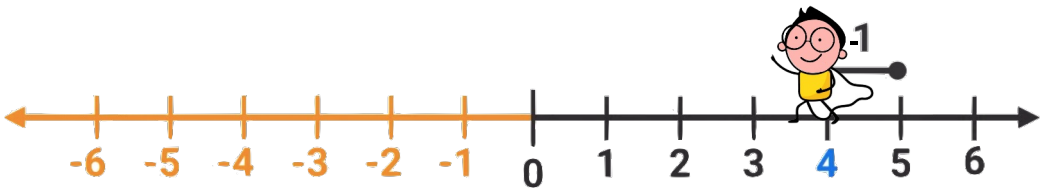
$+ + - = +$

$- + + = -$

Number line

Start at 5
Walk left
5 steps

$5 - 1 = 4$



Start at 13
Walk left
12 steps

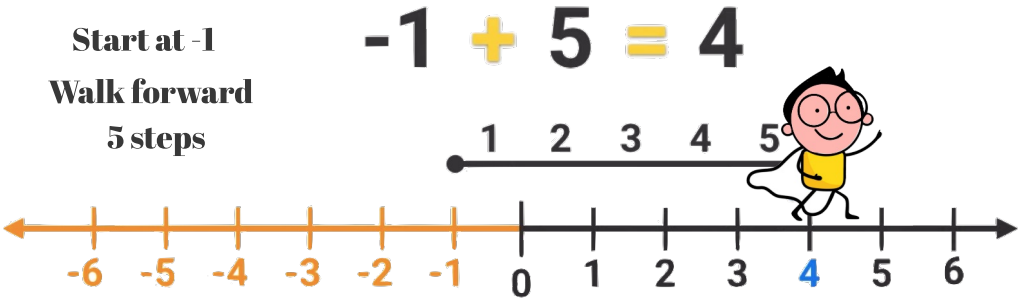
$13 - 12 = 1$

Negative numbers addition

Start at -1

Walk forward

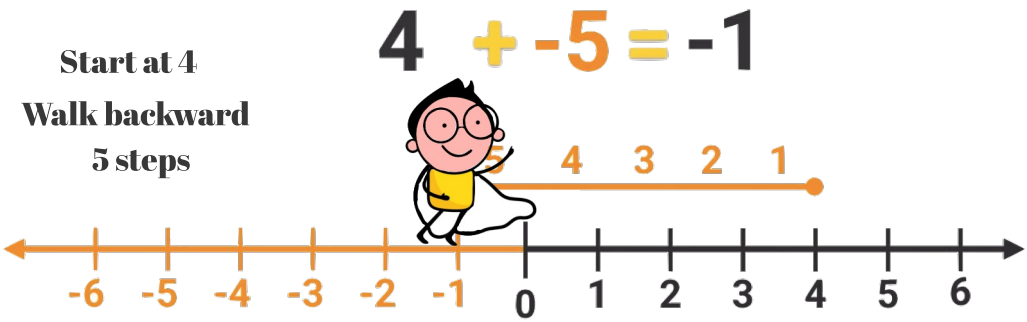
5 steps



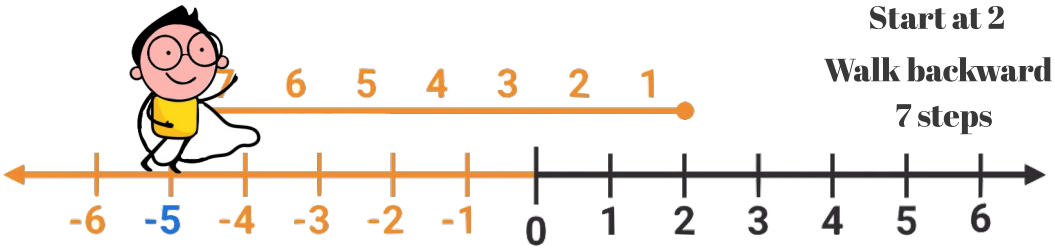
Start at 4

Walk backward

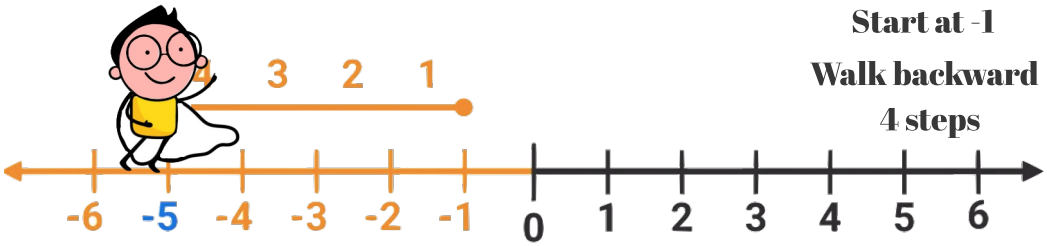
5 steps



$$2 + -7 = -5$$


7 6 5 4 3 2 1
-6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6
Start at 2
Walk backward
7 steps

$$-1 + -4 = -5$$


4 3 2 1
-6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6
Start at -1
Walk backward
4 steps

Step by step subtraction

1

1) 44 - 12

We want to subtract this.

2)
$$\begin{array}{r} 44 \\ -\ 12 \\ \hline \end{array}$$

Regroup the subtraction in column form.

3)
$$\begin{array}{r} \ 4\ 4 \\ -\ 1\ 2 \\ \hline \ 2 \end{array}$$

Subtract the 1's place.

4)
$$\begin{array}{r} \ 4\ 4 \\ -\ 1\ 2 \\ \hline 3\ 2 \end{array}$$

Subtract the 10s place.

5) 44 - 12 = 32

Write down the answer.

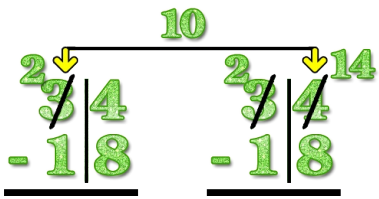
2

1) 34 - 18

We want to subtract this.

2)
$$\begin{array}{r} 34 \\ -\ 18 \\ \hline \end{array}$$

Regroup the subtraction in column form.

3)


Borrow a 1 from the 10's place and carry the one to 1s place.

4)
$$\begin{array}{r|l} 2 & 14 \\ -\ 1 & 8 \\ \hline 6 & \end{array}$$

Subtract the 1s place.

5)
$$\begin{array}{r|l} 2 & 14 \\ -\ 1 & 8 \\ \hline 1\ 6 & \end{array}$$

Subtract the 10s place.

6) 34 - 18 = 16

Write down the answer.

3

1) 334 - 118

We want to subtract this.

2)
$$\begin{array}{r} 334 \\ -\ 118 \\ \hline \end{array}$$

Regroup the subtraction in column form.

3)
$$\begin{array}{r} \overset{\ }{3}\overset{2}{\not 3}\overset{10}{4} \\ -\ 1\ 1\ 8 \\ \hline \end{array} \qquad \begin{array}{r} \overset{\ }{3}\overset{2}{\not 3}\overset{14}{\not 4} \\ -\ 1\ 1\ 8 \\ \hline \end{array}$$

Borrow a 1 from the 10's place and carry the one to 1s place.

4)
$$\begin{array}{r} 3\ 2\ 14 \\ -\ 1\ 1\ 8 \\ \hline 6 \end{array}$$

Subtract the 1s place.

5)
$$\begin{array}{r} 3\ 2\ 14 \\ -\ 1\ 1\ 8 \\ \hline 1\ 6 \end{array}$$

Subtract the 10s place.

6)

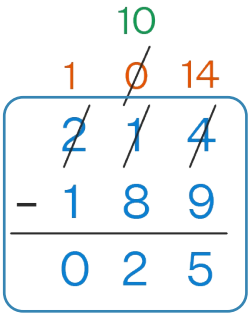
$$\begin{array}{r} 3\,|\,2\,|\,14 \\ -\,1\,|\,1\,|\,8 \\ \hline 2\;1\;6 \end{array}$$

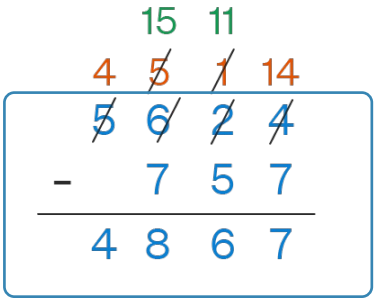
Subtract the 100s place.

7) 334 - 118 = 216

Write down the answer.

Subtraction in a nutshell

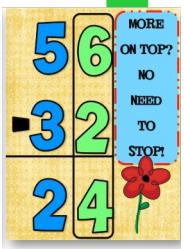


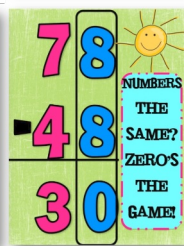


Subtraction in a nutshell

$$\begin{array}{r} {\scriptstyle 10} \\ 1\ \not{0}\ {}^{14} \\ \not{2}\ \not{1}\ \not{4} \\ -\ 1\ 8\ 9 \\ \hline 0\ 2\ 5 \end{array}$$

$$\begin{array}{r} {\scriptstyle 15\ \ 11} \\ 4\ \not{5}\ \not{1}\ {}^{14} \\ \not{5}\ \not{6}\ \not{2}\ \not{4} \\ -\ \ 7\ 5\ 7 \\ \hline 4\ 8\ 6\ 7 \end{array}$$



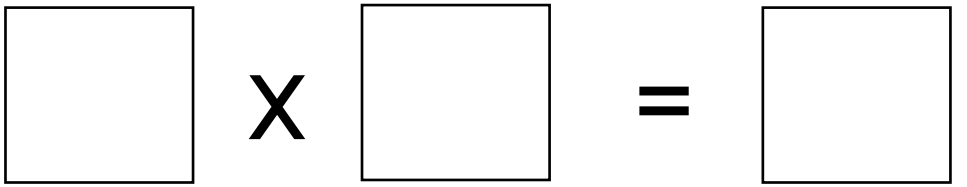


MULTIPLICATION

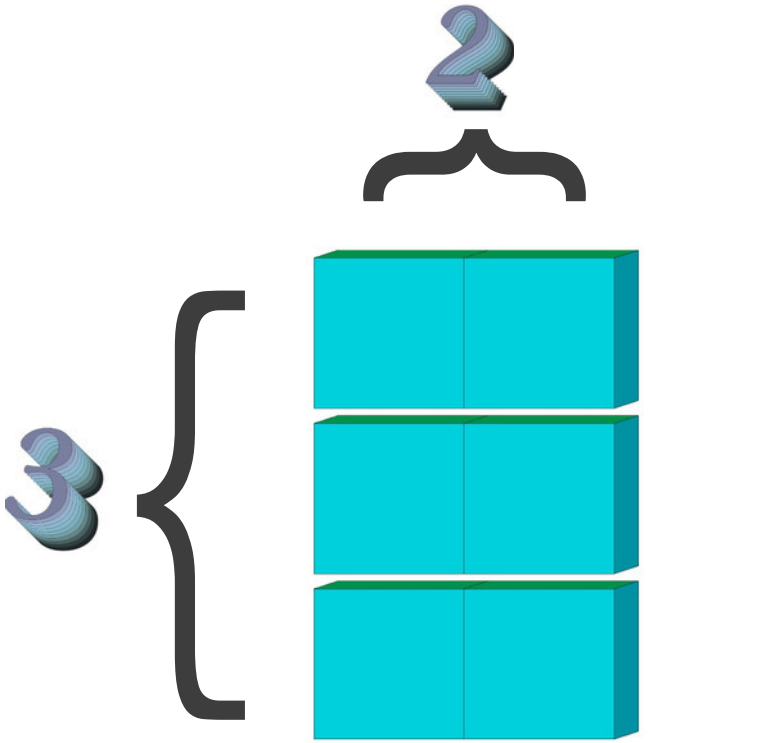
X

x	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
1	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
2	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38	40	42	44	46	48	50	52	54	56	58	60
3	3	6	9	12	15	18	21	24	27	30	33	36	39	42	45	48	51	54	57	60	63	66	69	72	75	78	81	84	87	90
4	4	8	12	16	20	24	28	32	36	40	44	48	52	56	60	64	68	72	76	80	84	88	92	96	100	104	108	112	116	120
5	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95	100	105	110	115	120	125	130	135	140	145	150
6	6	12	18	24	30	36	42	48	54	60	66	72	78	84	90	96	102	108	114	120	126	132	138	144	150	156	162	168	174	180
7	7	14	21	28	35	42	49	56	63	70	77	84	91	98	105	112	119	126	133	140	147	154	161	168	175	182	189	196	203	210
8	8	16	24	32	40	48	56	64	72	80	88	96	104	112	120	128	136	144	152	160	168	176	184	192	200	208	216	224	232	240
9	9	18	27	36	45	54	63	72	81	90	99	108	117	126	135	144	153	162	171	180	189	198	207	216	225	234	243	252	261	270
10	10	20	30	40	50	60	70	80	90	100	110	120	130	140	150	160	170	180	190	200	210	220	230	240	250	260	270	280	290	300
11	11	22	33	44	55	66	77	88	99	110	121	132	143	154	165	176	187	198	209	220	231	242	253	264	275	286	297	308	319	330
12	12	24	36	48	60	72	84	96	108	120	132	144	156	168	180	192	204	216	228	240	252	264	276	288	300	312	324	336	348	360
13	13	26	39	52	65	78	91	104	117	130	143	156	169	182	195	208	221	234	247	260	273	286	299	312	325	338	351	364	377	390
14	14	28	42	56	70	84	98	112	126	140	154	168	182	196	210	224	238	252	266	280	294	308	322	336	350	364	378	392	406	420
15	15	30	45	60	75	90	105	120	135	150	165	180	195	210	225	240	255	270	285	300	315	330	345	360	375	390	405	420	435	450
16	16	32	48	64	80	96	112	128	144	160	176	192	208	224	240	256	272	288	304	320	336	352	368	384	400	416	432	448	464	480
17	17	34	51	68	85	102	119	136	153	170	187	204	221	238	255	272	289	306	323	340	357	374	391	408	425	442	459	476	493	510
18	18	36	54	72	90	108	126	144	162	180	198	216	234	252	270	288	306	324	342	360	378	396	414	432	450	468	486	504	522	540
19	19	38	57	76	95	114	133	152	171	190	209	228	247	266	285	304	323	342	361	380	399	418	437	456	475	494	513	532	551	570
20	20	40	60	80	100	120	140	160	180	200	220	240	260	280	300	320	340	360	380	400	420	440	460	480	500	520	540	560	580	600
21	21	42	63	84	105	126	147	168	189	210	231	252	273	294	315	336	357	378	399	420	441	462	483	504	525	546	567	588	609	630
22	22	44	66	88	110	132	154	176	198	220	242	264	286	308	330	352	374	396	418	440	462	484	506	528	550	572	594	616	638	660
23	23	46	69	92	115	138	161	184	207	230	253	276	299	322	345	368	391	414	437	460	483	506	529	552	575	598	621	644	667	690
24	24	48	72	96	120	144	168	192	216	240	264	288	312	336	360	384	408	432	456	480	504	528	552	576	600	624	648	672	696	720
25	25	50	75	100	125	150	175	200	225	250	275	300	325	350	375	400	425	450	475	500	525	550	575	600	625	650	675	700	725	750
26	26	52	78	104	130	156	182	208	234	260	286	312	338	364	390	416	442	468	494	520	546	572	598	624	650	676	702	728	754	780
27	27	54	81	108	135	162	189	216	243	270	297	324	351	378	405	432	459	486	513	540	567	594	621	648	675	702	729	756	783	810
28	28	56	84	112	140	168	196	224	252	280	308	336	364	392	420	448	476	504	532	560	588	616	644	672	700	728	756	784	812	840
29	29	58	87	116	145	174	203	232	261	290	319	348	377	406	435	464	493	522	551	580	609	638	667	696	725	754	783	812	841	870
30	30	60	90	120	150	180	210	240	270	300	330	360	390	420	450	480	510	540	570	600	630	660	690	720	750	780	810	840	870	900

Multiplication form

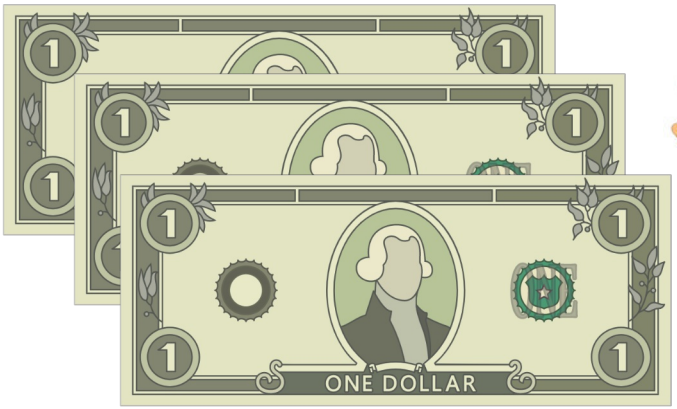


multiplier multiplicand product



$3 \times 2 = 6$

Examples



$3 \times 1 = 3$

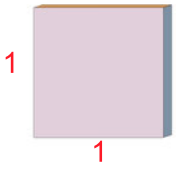
$5 \times 6 = 30$

$2 \times 6 = 12$

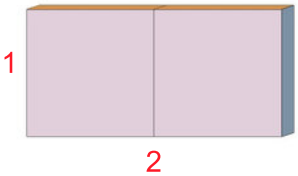
$3 \times 3 = 9$

93

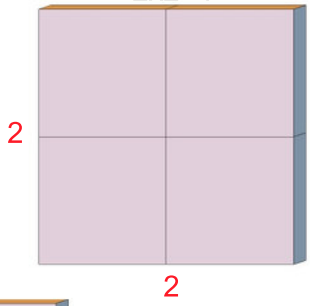
1x1=1



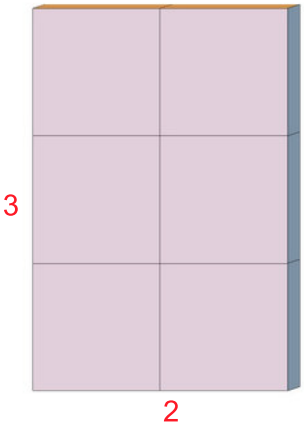
1x2=2



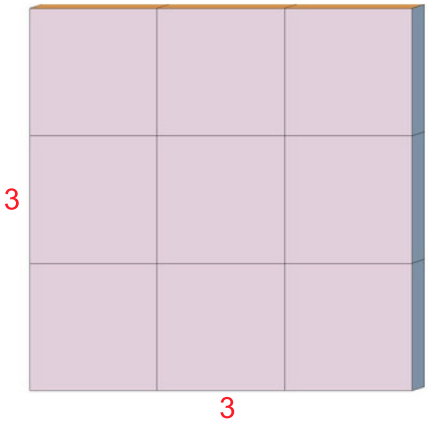
2x2=4



3x2=6

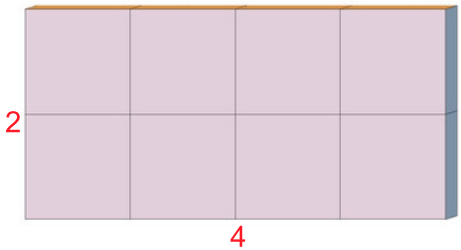


3x3=9

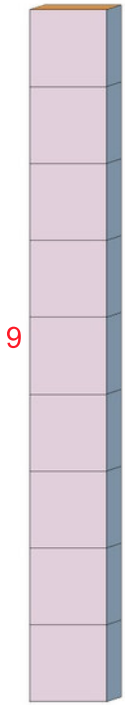


2x3=6

2x4=8



9x1=9



9

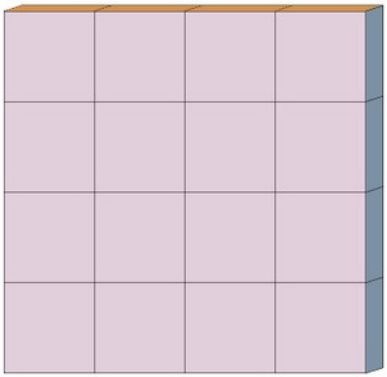
1

1x9=9

1

9

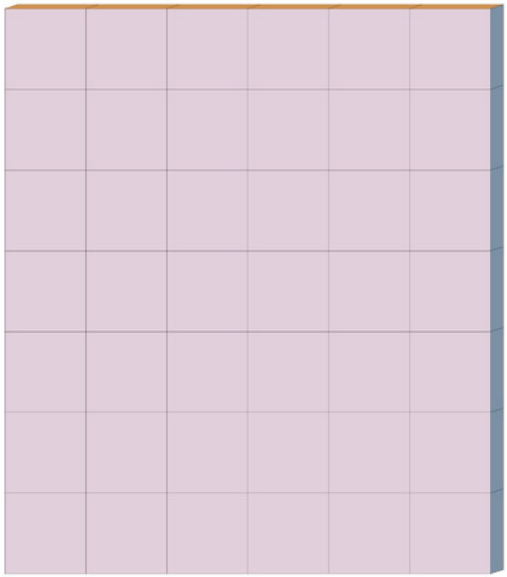
4x4=16



4

4

7x6=42

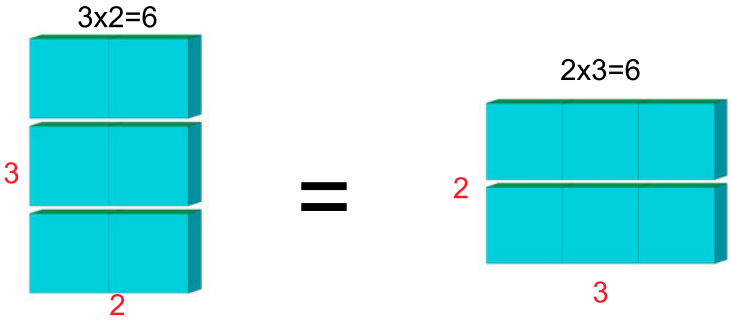


7

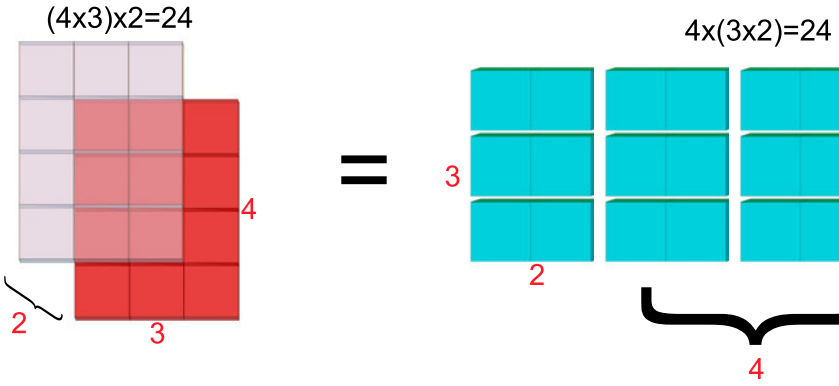
6

Multiplication properties

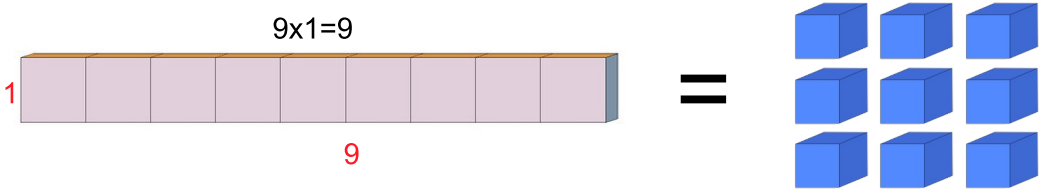
1)



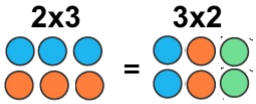
2)



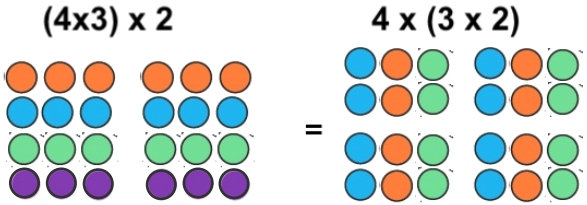
3)



1)



2)

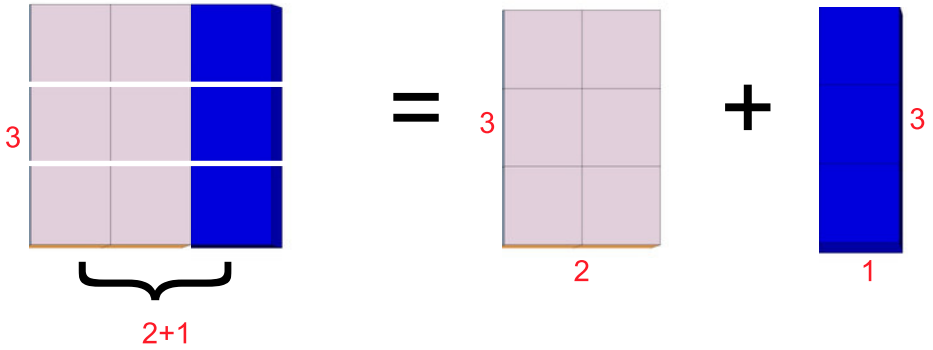


3)

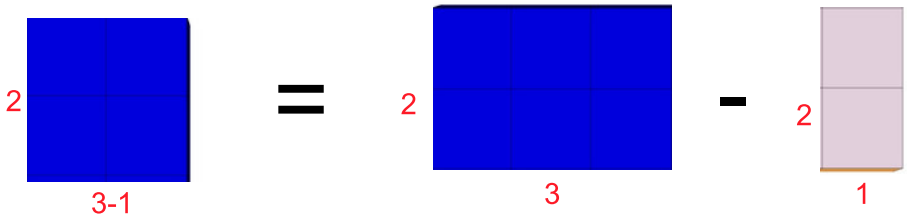
Multiplication Properties	Examples
1. Commutative Property	**1.Commutative Property**
$a \times b = b \times a$	$2 \times 3 = 3 \times 2$
2. Associativity Property	**2.Associativity Property**
$(a \times b) \times c = a \times (b \times c)$	$(4 \times 3) \times 2 = 4 \times (3 \times 2)$
3. Identity Property	**3. Identity Property**
$a \times 1 = a$	$9 \times 1 = 9$

Multiplication properties

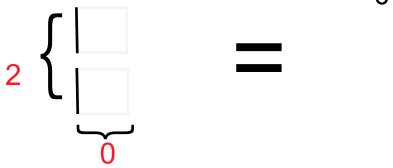
4) 3 x (2 + 1) = 9



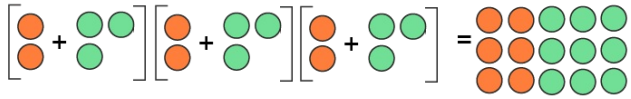
5) 2 x (3 - 1) = 4



6) 2 x 0 = 0

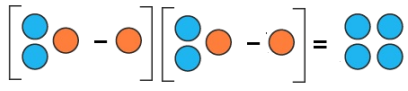


4)



3 x (2 + 3) 15

5)



2 x (3 - 1) = 4

2 x 3 - 2 x 1 4

6)

2 x 0 = 0

Multiplication Properties

4. Distributive Property

$a \times (b + c) = a \times b + a \times c$

5. Distributive Property

$a \times (b - c) = a \times b - a \times c$

6. Zero Property

$a \times 0 = 0$

Examples

4. Distributive Property

$3 \times (2 + 1) = 3 \times 2 + 3 \times 1$

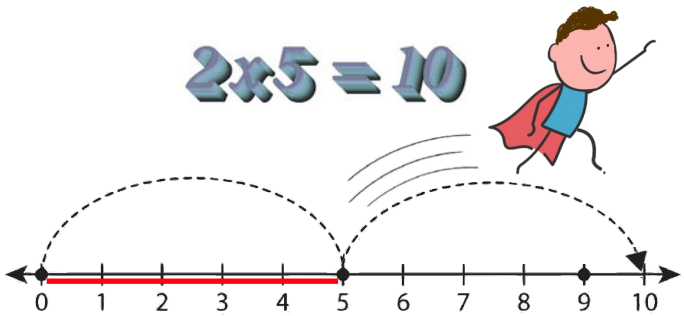
5. Distributive Property

$2 \times (3 - 1) = 2 \times 3 - 2 \times 1$

6. Zero Property

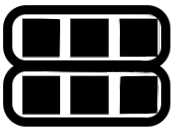
$2 \times 0 = 0$

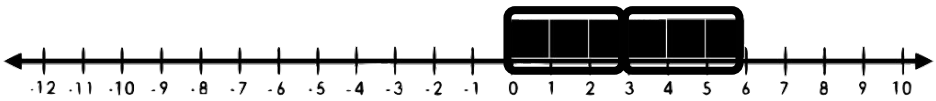
Number line



2x3 = 6

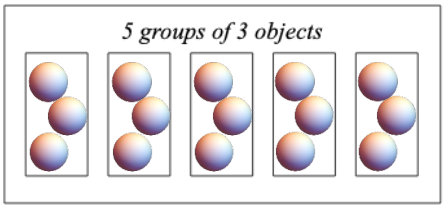
(2 groups of 3)= 6



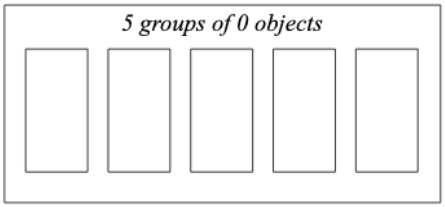


Multiplication by 0

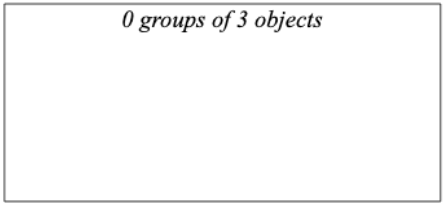
5x3



5x0



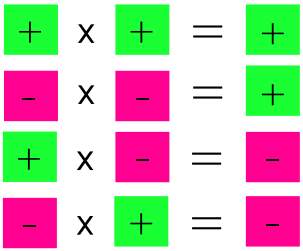
0x3



Zero, the hero.

Zero always wins.

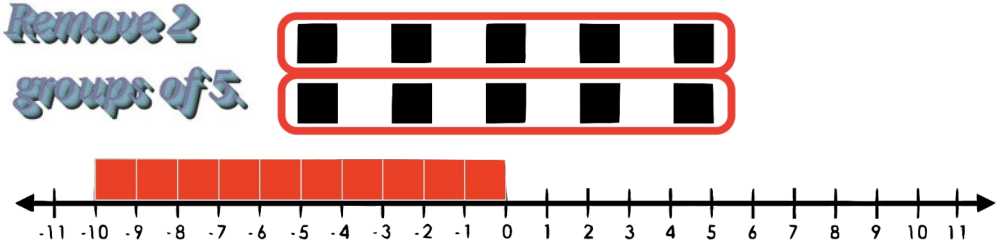
Multiplication with negative numbers



×	-3	-2	-1	0	1	2	3
-3	9	6	3	0	-3	-6	-9
-2	6	4	2	0	-2	-4	-6
-1	3	2	1	0	-1	-2	-3
0	0	0	0	0	0	0	0
1	-3	-2	-1	0	1	2	3
2	-6	-4	-2	0	2	4	6
3	-9	-6	-3	0	3	6	9

(−2 groups of 5) = −10

Remove 2 groups of 5.

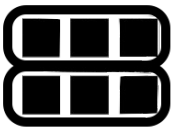


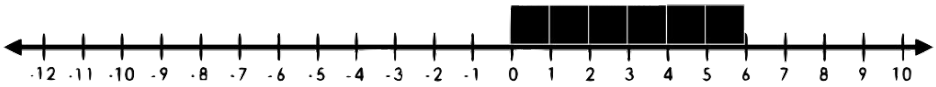
5 groups of (−2) = −10

Add 5 groups of −2.

−2 × (−3) = 6

[Remove 2 groups of (−3)] = 6





Step by step multiplication

1

1) 88 x 6

We want to multiply this.

2)
$$\begin{array}{r} 88 \\ \times \quad 6 \\ \hline \end{array}$$

Regroup the multiplication in column form.

3)
$$\begin{array}{r} {\scriptstyle 4} \\ 88 \\ \times \quad 6 \\ \hline 8 \end{array}$$

Multiply 6 with the 1's place. $6 \times 8 = 48$

4)
$$\begin{array}{r} {\scriptstyle 4} \\ 88 \\ \times \quad 6 \\ \hline 528 \end{array}$$

Multiply 6 with the 10s place. $(6 \times 8) + 4 = 48 + 4 = 52$

44 x 28

2

1) 44 x 28

Regroup the multiplication in column form.

2) 3
44
x 28
2

Multiply 8 with the 1s place. 8 x 4 = 32

3) 3
44
x 28
352

Multiply 8 with the 10s place. (8 x 4)+3= 32+3=35

4) 44
x 28
352
80

Add a zero below the 2. Multiply 2 with the 1s place and place below the 5. 2 x 4 =8.

5) 44
x 28
352
+ 880

Multiply 2 with the 10s place and place below the 3. 2 x 4 =8.

6) 44
x 28
1352
+ 880
1232

Now add 352 + 880.

3

463 x 53

1)

```
  463
x  53
-----
```

Regroup the multiplication in column form.

2)

```
  463
x  53
-----
    9
```

Multiply 3 with the 1s place. 3 x 3 = 9

3)

```
    1
  463
x  53
-----
 1389
```

Multiply 3 with the 10s place and 100s place. 3 x 6 = 18. (3 x 4) + 1 = 13.

4)

```
    1
  463
x  53
-----
 1389
   50
```

Add a zero below the 9. Multiply 5 with the 1s place and place below the 8. 5 x 3 =15.

5)

```
  3 1
  463
x  53
-----
 1389
  150
```

Multiply 5 with the 10s place(6) and place below the 3; carry over the 3 in 30 (5x6). (5 x 6) + 1 =31.

6)

```
  2 3 1
  463
x  53
-----
 1389
+23150
------
```

Multiply 5 with te 100s place (4) and place below the 1. We also place the carry over down now. (5x4)+ 3 = 23

7)

```
  2 3 1
  463
x  53
-----
 1389
+23150
------
 24539
```

Now add 1389 + 23150.

4

228 x 121

1)
```
 228
x 121
```
Regroup

2)
```
 228
x 121
   8
```
1x8=8

3)
```
 228
x 121
  28
```
1x2=2

4)
```
 228
x 121
 228
```
1x2=2

5)
```
   1
 228
x 121
 228
  60
```
Add 0.
2x8=16

6)
```
 228
x 121
 228
 560
```
(2x2) + 1 = 5

7)
```
  228
x 121
 228
4560
```
2x2 = 4

8)
```
  228
x 121
 228
4560
 800
```
Add two 0s.
1x8=8

9)
```
  228
x 121
 228
4560
2800
```
1x2=2

10)
```
  228
x 121
 228
4560
22800
```
1x2=2

11)
```
  228
x 121
 228
4560
22800
27588
```
Add:
228 + 4560 +22800

Lattice multiplication

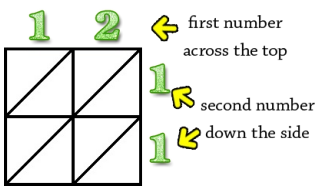
3 x 6 = 18

This is Tens Place

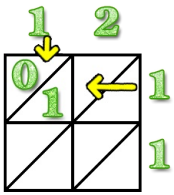
This is Ones Place

=>

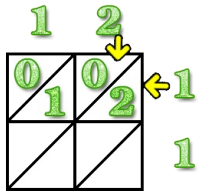
12 x 11



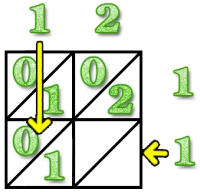
1 x 1



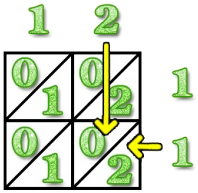
2 x 1



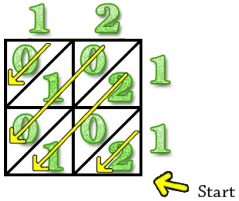
1 x 1

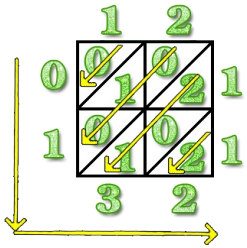


2 x 1



sum

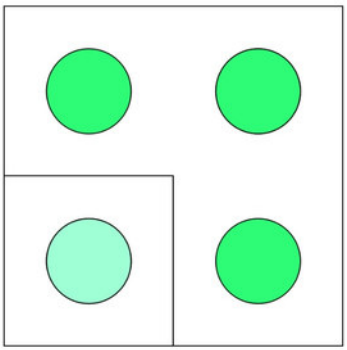




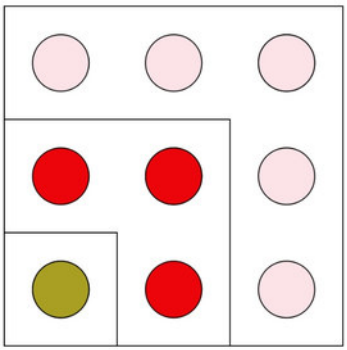
Series

$$S_n = \underbrace{\square + \square + \square + \square + \ldots}_{n}$$

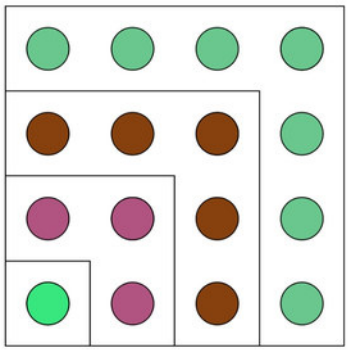
Example



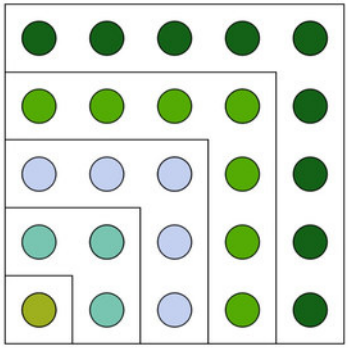
$S_2 = \underbrace{1+3}_{2} = 2x2 = 4$



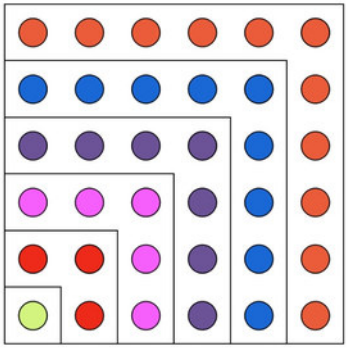
$S_3 = 1+3+5 = 3x3 = 9$



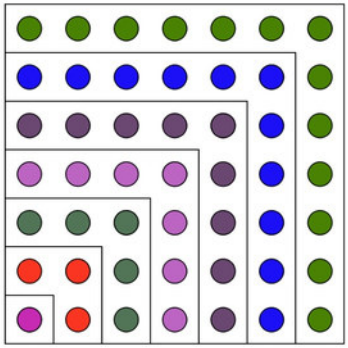
$S_4 = 1+3+5+7 = 4x4 = 16$



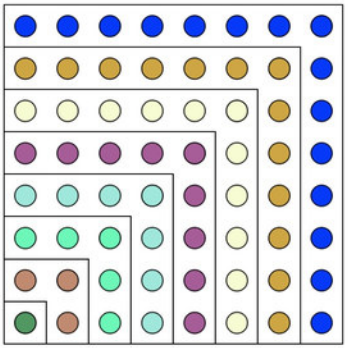
$S_5 = 1+3+5+7+9$
$= 5x5 = 25$



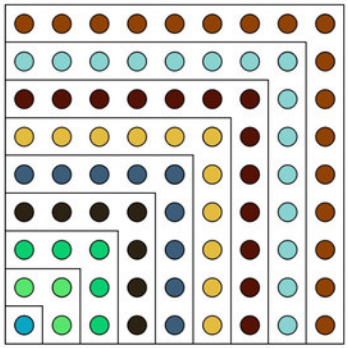
$S_6 = 1+3+5+7+9+11$
$= 6x6 = 36$



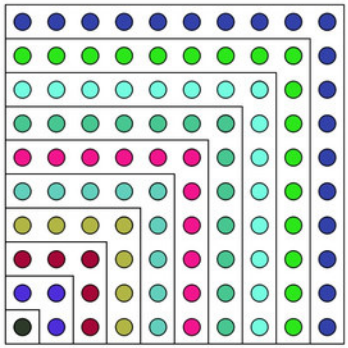
$S_7 = 1+3+5+7+9+11+13$
$= 7x7 = 49$



$S_8 = 1+3+5+7+9+11+13+15$
$= 8x8 = 64$



$S_9 = 1+3+5+7+9+11+13+15+17$
$= 9x9 = 81$



$S_{10} = 1+3+5+7+9+11+13+15+17+19$
$= 10x10 = 100$

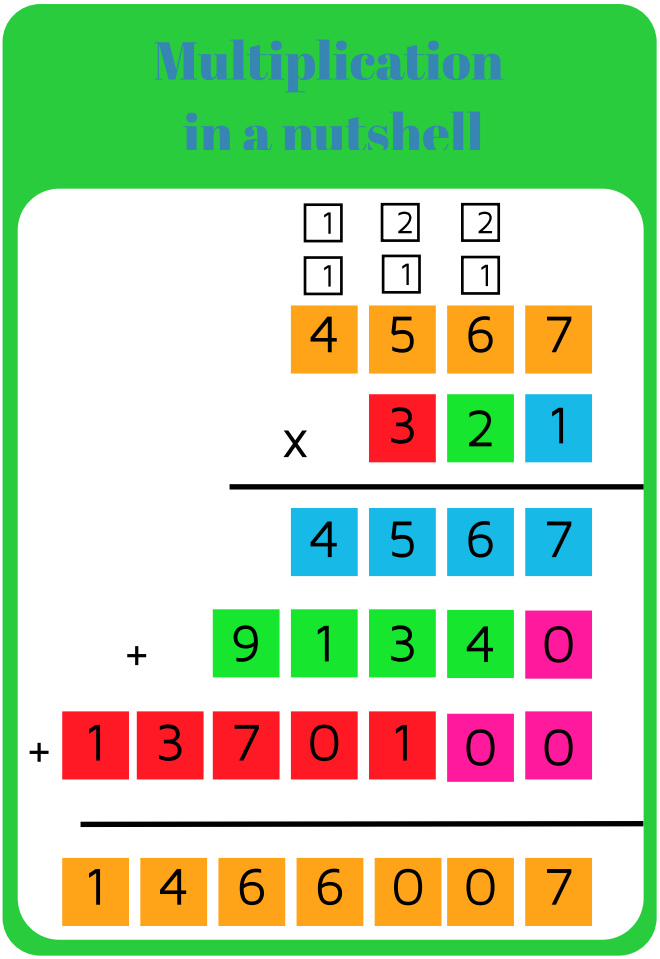
Sum of n Odd Numbers Formula

Sum of 'n' odd numbers = S_n

$S_n = 1 + 3 + 5 + \ldots\ldots\ldots + (2n - 1)$

$S_n = n^2$ where, n = number of terms

Multiplication in a nutshell



Division

÷	29	28	27	26	25	24	23	22	21	20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0
1	29	28	27	26	25	24	23	22	21	20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0
2	$\frac{29}{2}$	14	$\frac{27}{2}$	13	$\frac{25}{2}$	12	$\frac{23}{2}$	11	$\frac{21}{2}$	10	$\frac{19}{2}$	9	$\frac{17}{2}$	8	$\frac{15}{2}$	7	$\frac{13}{2}$	6	$\frac{11}{2}$	5	$\frac{9}{2}$	4	$\frac{7}{2}$	3	$\frac{5}{2}$	2	$\frac{3}{2}$	1	$\frac{1}{2}$	0
3	$\frac{29}{3}$	$\frac{28}{3}$	9	$\frac{26}{3}$	$\frac{25}{3}$	8	$\frac{23}{3}$	$\frac{22}{3}$	7	$\frac{20}{3}$	$\frac{19}{3}$	6	$\frac{17}{3}$	$\frac{16}{3}$	5	$\frac{14}{3}$	$\frac{13}{3}$	4	$\frac{11}{3}$	$\frac{10}{3}$	3	$\frac{8}{3}$	$\frac{7}{3}$	2	$\frac{5}{3}$	$\frac{4}{3}$	1	$\frac{2}{3}$	$\frac{1}{3}$	0
4	$\frac{29}{4}$	7	$\frac{27}{4}$	$\frac{13}{2}$	$\frac{25}{4}$	6	$\frac{23}{4}$	$\frac{11}{2}$	$\frac{21}{4}$	5	$\frac{19}{4}$	$\frac{9}{2}$	$\frac{17}{4}$	4	$\frac{15}{4}$	$\frac{7}{2}$	$\frac{13}{4}$	3	$\frac{11}{4}$	$\frac{5}{2}$	$\frac{9}{4}$	2	$\frac{7}{4}$	$\frac{3}{2}$	$\frac{5}{4}$	1	$\frac{3}{4}$	$\frac{1}{2}$	$\frac{1}{4}$	0
5	$\frac{29}{5}$	$\frac{28}{5}$	$\frac{27}{5}$	$\frac{26}{5}$	5	$\frac{24}{5}$	$\frac{23}{5}$	$\frac{22}{5}$	$\frac{21}{5}$	4	$\frac{19}{5}$	$\frac{18}{5}$	$\frac{17}{5}$	$\frac{16}{5}$	3	$\frac{14}{5}$	$\frac{13}{5}$	$\frac{12}{5}$	$\frac{11}{5}$	2	$\frac{9}{5}$	$\frac{8}{5}$	$\frac{7}{5}$	$\frac{6}{5}$	1	$\frac{4}{5}$	$\frac{3}{5}$	$\frac{2}{5}$	$\frac{1}{5}$	0
6	$\frac{29}{6}$	$\frac{14}{3}$	$\frac{9}{2}$	$\frac{13}{3}$	$\frac{25}{6}$	4	$\frac{23}{6}$	$\frac{11}{3}$	$\frac{7}{2}$	$\frac{10}{3}$	$\frac{19}{6}$	3	$\frac{17}{6}$	$\frac{8}{3}$	$\frac{5}{2}$	$\frac{7}{3}$	$\frac{13}{6}$	2	$\frac{11}{6}$	$\frac{5}{3}$	$\frac{3}{2}$	$\frac{4}{3}$	$\frac{7}{6}$	1	$\frac{5}{6}$	$\frac{2}{3}$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{6}$	0
7	$\frac{29}{7}$	4	$\frac{27}{7}$	$\frac{26}{7}$	$\frac{25}{7}$	$\frac{24}{7}$	$\frac{23}{7}$	$\frac{22}{7}$	3	$\frac{20}{7}$	$\frac{19}{7}$	$\frac{18}{7}$	$\frac{17}{7}$	$\frac{16}{7}$	$\frac{15}{7}$	2	$\frac{13}{7}$	$\frac{12}{7}$	$\frac{11}{7}$	$\frac{10}{7}$	$\frac{9}{7}$	$\frac{8}{7}$	1	$\frac{6}{7}$	$\frac{5}{7}$	$\frac{4}{7}$	$\frac{3}{7}$	$\frac{2}{7}$	$\frac{1}{7}$	0
8	$\frac{29}{8}$	$\frac{7}{2}$	$\frac{27}{8}$	$\frac{13}{4}$	$\frac{25}{8}$	3	$\frac{23}{8}$	$\frac{11}{4}$	$\frac{21}{8}$	$\frac{5}{2}$	$\frac{19}{8}$	$\frac{9}{4}$	$\frac{17}{8}$	2	$\frac{15}{8}$	$\frac{7}{4}$	$\frac{13}{8}$	$\frac{3}{2}$	$\frac{11}{8}$	$\frac{5}{4}$	$\frac{9}{8}$	1	$\frac{7}{8}$	$\frac{3}{4}$	$\frac{5}{8}$	$\frac{1}{2}$	$\frac{3}{8}$	$\frac{1}{4}$	$\frac{1}{8}$	0
9	$\frac{29}{9}$	$\frac{28}{9}$	3	$\frac{26}{9}$	$\frac{25}{9}$	$\frac{8}{3}$	$\frac{23}{9}$	$\frac{22}{9}$	$\frac{7}{3}$	$\frac{20}{9}$	$\frac{19}{9}$	2	$\frac{17}{9}$	$\frac{16}{9}$	$\frac{5}{3}$	$\frac{14}{9}$	$\frac{13}{9}$	$\frac{4}{3}$	$\frac{11}{9}$	$\frac{10}{9}$	1	$\frac{8}{9}$	$\frac{7}{9}$	$\frac{2}{3}$	$\frac{5}{9}$	$\frac{4}{9}$	$\frac{1}{3}$	$\frac{2}{9}$	$\frac{1}{9}$	0
10	$\frac{29}{10}$	$\frac{14}{5}$	$\frac{27}{10}$	$\frac{13}{5}$	$\frac{5}{2}$	$\frac{12}{5}$	$\frac{23}{10}$	$\frac{11}{5}$	$\frac{21}{10}$	2	$\frac{19}{10}$	$\frac{9}{5}$	$\frac{17}{10}$	$\frac{8}{5}$	$\frac{3}{2}$	$\frac{7}{5}$	$\frac{13}{10}$	$\frac{6}{5}$	$\frac{11}{10}$	1	$\frac{9}{10}$	$\frac{4}{5}$	$\frac{7}{10}$	$\frac{3}{5}$	$\frac{1}{2}$	$\frac{2}{5}$	$\frac{3}{10}$	$\frac{1}{5}$	$\frac{1}{10}$	0
11	$\frac{29}{11}$	$\frac{28}{11}$	$\frac{27}{11}$	$\frac{26}{11}$	$\frac{25}{11}$	$\frac{24}{11}$	$\frac{23}{11}$	2	$\frac{21}{11}$	$\frac{20}{11}$	$\frac{19}{11}$	$\frac{18}{11}$	$\frac{17}{11}$	$\frac{16}{11}$	$\frac{15}{11}$	$\frac{14}{11}$	$\frac{13}{11}$	$\frac{12}{11}$	1	$\frac{10}{11}$	$\frac{9}{11}$	$\frac{8}{11}$	$\frac{7}{11}$	$\frac{6}{11}$	$\frac{5}{11}$	$\frac{4}{11}$	$\frac{3}{11}$	$\frac{2}{11}$	$\frac{1}{11}$	0
12	$\frac{29}{12}$	$\frac{7}{3}$	$\frac{9}{4}$	$\frac{13}{6}$	$\frac{25}{12}$	2	$\frac{23}{12}$	$\frac{11}{6}$	$\frac{7}{4}$	$\frac{5}{3}$	$\frac{19}{12}$	$\frac{3}{2}$	$\frac{17}{12}$	$\frac{4}{3}$	$\frac{5}{4}$	$\frac{7}{6}$	$\frac{13}{12}$	1	$\frac{11}{12}$	$\frac{5}{6}$	$\frac{3}{4}$	$\frac{2}{3}$	$\frac{7}{12}$	$\frac{1}{2}$	$\frac{5}{12}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{6}$	$\frac{1}{12}$	0
13	$\frac{29}{13}$	$\frac{28}{13}$	$\frac{27}{13}$	2	$\frac{25}{13}$	$\frac{24}{13}$	$\frac{23}{13}$	$\frac{22}{13}$	$\frac{21}{13}$	$\frac{20}{13}$	$\frac{19}{13}$	$\frac{18}{13}$	$\frac{17}{13}$	$\frac{16}{13}$	$\frac{15}{13}$	$\frac{14}{13}$	1	$\frac{12}{13}$	$\frac{11}{13}$	$\frac{10}{13}$	$\frac{9}{13}$	$\frac{8}{13}$	$\frac{7}{13}$	$\frac{6}{13}$	$\frac{5}{13}$	$\frac{4}{13}$	$\frac{3}{13}$	$\frac{2}{13}$	$\frac{1}{13}$	0
14	$\frac{29}{14}$	2	$\frac{27}{14}$	$\frac{13}{7}$	$\frac{25}{14}$	$\frac{12}{7}$	$\frac{23}{14}$	$\frac{11}{7}$	$\frac{3}{2}$	$\frac{10}{7}$	$\frac{19}{14}$	$\frac{9}{7}$	$\frac{17}{14}$	$\frac{8}{7}$	$\frac{15}{14}$	1	$\frac{13}{14}$	$\frac{6}{7}$	$\frac{11}{14}$	$\frac{5}{7}$	$\frac{9}{14}$	$\frac{4}{7}$	$\frac{1}{2}$	$\frac{3}{7}$	$\frac{5}{14}$	$\frac{2}{7}$	$\frac{3}{14}$	$\frac{1}{7}$	$\frac{1}{14}$	0
15	$\frac{29}{15}$	$\frac{28}{15}$	$\frac{9}{5}$	$\frac{26}{15}$	$\frac{5}{3}$	$\frac{8}{5}$	$\frac{23}{15}$	$\frac{22}{15}$	$\frac{7}{5}$	$\frac{4}{3}$	$\frac{19}{15}$	$\frac{6}{5}$	$\frac{17}{15}$	$\frac{16}{15}$	1	$\frac{14}{15}$	$\frac{13}{15}$	$\frac{4}{5}$	$\frac{11}{15}$	$\frac{2}{3}$	$\frac{3}{5}$	$\frac{8}{15}$	$\frac{7}{15}$	$\frac{2}{5}$	$\frac{1}{3}$	$\frac{4}{15}$	$\frac{1}{5}$	$\frac{2}{15}$	$\frac{1}{15}$	0
16	$\frac{29}{16}$	$\frac{7}{4}$	$\frac{27}{16}$	$\frac{13}{8}$	$\frac{25}{16}$	$\frac{3}{2}$	$\frac{23}{16}$	$\frac{11}{8}$	$\frac{21}{16}$	$\frac{5}{4}$	$\frac{19}{16}$	$\frac{9}{8}$	$\frac{17}{16}$	1	$\frac{15}{16}$	$\frac{7}{8}$	$\frac{13}{16}$	$\frac{3}{4}$	$\frac{11}{16}$	$\frac{5}{8}$	$\frac{9}{16}$	$\frac{1}{2}$	$\frac{7}{16}$	$\frac{3}{8}$	$\frac{5}{16}$	$\frac{1}{4}$	$\frac{3}{16}$	$\frac{1}{8}$	$\frac{1}{16}$	0
17	$\frac{29}{17}$	$\frac{28}{17}$	$\frac{27}{17}$	$\frac{26}{17}$	$\frac{25}{17}$	$\frac{24}{17}$	$\frac{23}{17}$	$\frac{22}{17}$	$\frac{21}{17}$	$\frac{20}{17}$	$\frac{19}{17}$	$\frac{18}{17}$	1	$\frac{16}{17}$	$\frac{15}{17}$	$\frac{14}{17}$	$\frac{13}{17}$	$\frac{12}{17}$	$\frac{11}{17}$	$\frac{10}{17}$	$\frac{9}{17}$	$\frac{8}{17}$	$\frac{7}{17}$	$\frac{6}{17}$	$\frac{5}{17}$	$\frac{4}{17}$	$\frac{3}{17}$	$\frac{2}{17}$	$\frac{1}{17}$	0
18	$\frac{29}{18}$	$\frac{14}{9}$	$\frac{3}{2}$	$\frac{13}{9}$	$\frac{25}{18}$	$\frac{4}{3}$	$\frac{23}{18}$	$\frac{11}{9}$	$\frac{7}{6}$	$\frac{10}{9}$	$\frac{19}{18}$	1	$\frac{17}{18}$	$\frac{8}{9}$	$\frac{5}{6}$	$\frac{7}{9}$	$\frac{13}{18}$	$\frac{2}{3}$	$\frac{11}{18}$	$\frac{5}{9}$	$\frac{1}{2}$	$\frac{4}{9}$	$\frac{7}{18}$	$\frac{1}{3}$	$\frac{5}{18}$	$\frac{2}{9}$	$\frac{1}{6}$	$\frac{1}{9}$	$\frac{1}{18}$	0
19	$\frac{29}{19}$	$\frac{28}{19}$	$\frac{27}{19}$	$\frac{26}{19}$	$\frac{25}{19}$	$\frac{24}{19}$	$\frac{23}{19}$	$\frac{22}{19}$	$\frac{21}{19}$	$\frac{20}{19}$	1	$\frac{18}{19}$	$\frac{17}{19}$	$\frac{16}{19}$	$\frac{15}{19}$	$\frac{14}{19}$	$\frac{13}{19}$	$\frac{12}{19}$	$\frac{11}{19}$	$\frac{10}{19}$	$\frac{9}{19}$	$\frac{8}{19}$	$\frac{7}{19}$	$\frac{6}{19}$	$\frac{5}{19}$	$\frac{4}{19}$	$\frac{3}{19}$	$\frac{2}{19}$	$\frac{1}{19}$	0
20	$\frac{29}{20}$	$\frac{7}{5}$	$\frac{27}{20}$	$\frac{13}{10}$	$\frac{5}{4}$	$\frac{6}{5}$	$\frac{23}{20}$	$\frac{11}{10}$	$\frac{21}{20}$	1	$\frac{19}{20}$	$\frac{9}{10}$	$\frac{17}{20}$	$\frac{4}{5}$	$\frac{3}{4}$	$\frac{7}{10}$	$\frac{13}{20}$	$\frac{3}{5}$	$\frac{11}{20}$	$\frac{1}{2}$	$\frac{9}{20}$	$\frac{2}{5}$	$\frac{7}{20}$	$\frac{3}{10}$	$\frac{1}{4}$	$\frac{1}{5}$	$\frac{3}{20}$	$\frac{1}{10}$	$\frac{1}{20}$	0
21	$\frac{29}{21}$	$\frac{4}{3}$	$\frac{9}{7}$	$\frac{26}{21}$	$\frac{25}{21}$	$\frac{8}{7}$	$\frac{23}{21}$	$\frac{22}{21}$	1	$\frac{20}{21}$	$\frac{19}{21}$	$\frac{6}{7}$	$\frac{17}{21}$	$\frac{16}{21}$	$\frac{5}{7}$	$\frac{2}{3}$	$\frac{13}{21}$	$\frac{4}{7}$	$\frac{11}{21}$	$\frac{10}{21}$	$\frac{3}{7}$	$\frac{8}{21}$	$\frac{1}{3}$	$\frac{2}{7}$	$\frac{5}{21}$	$\frac{4}{21}$	$\frac{1}{7}$	$\frac{2}{21}$	$\frac{1}{21}$	0
22	$\frac{29}{22}$	$\frac{14}{11}$	$\frac{27}{22}$	$\frac{13}{11}$	$\frac{25}{22}$	$\frac{12}{11}$	$\frac{23}{22}$	1	$\frac{21}{22}$	$\frac{10}{11}$	$\frac{19}{22}$	$\frac{9}{11}$	$\frac{17}{22}$	$\frac{8}{11}$	$\frac{15}{22}$	$\frac{7}{11}$	$\frac{13}{22}$	$\frac{6}{11}$	$\frac{1}{2}$	$\frac{5}{11}$	$\frac{9}{22}$	$\frac{4}{11}$	$\frac{7}{22}$	$\frac{3}{11}$	$\frac{5}{22}$	$\frac{2}{11}$	$\frac{3}{22}$	$\frac{1}{11}$	$\frac{1}{22}$	0
23	$\frac{29}{23}$	$\frac{28}{23}$	$\frac{27}{23}$	$\frac{26}{23}$	$\frac{25}{23}$	$\frac{24}{23}$	1	$\frac{22}{23}$	$\frac{21}{23}$	$\frac{20}{23}$	$\frac{19}{23}$	$\frac{18}{23}$	$\frac{17}{23}$	$\frac{16}{23}$	$\frac{15}{23}$	$\frac{14}{23}$	$\frac{13}{23}$	$\frac{12}{23}$	$\frac{11}{23}$	$\frac{10}{23}$	$\frac{9}{23}$	$\frac{8}{23}$	$\frac{7}{23}$	$\frac{6}{23}$	$\frac{5}{23}$	$\frac{4}{23}$	$\frac{3}{23}$	$\frac{2}{23}$	$\frac{1}{23}$	0
24	$\frac{29}{24}$	$\frac{7}{6}$	$\frac{9}{8}$	$\frac{13}{12}$	$\frac{25}{24}$	1	$\frac{23}{24}$	$\frac{11}{12}$	$\frac{7}{8}$	$\frac{5}{6}$	$\frac{19}{24}$	$\frac{3}{4}$	$\frac{17}{24}$	$\frac{2}{3}$	$\frac{5}{8}$	$\frac{7}{12}$	$\frac{13}{24}$	$\frac{1}{2}$	$\frac{11}{24}$	$\frac{5}{12}$	$\frac{3}{8}$	$\frac{1}{3}$	$\frac{7}{24}$	$\frac{1}{4}$	$\frac{5}{24}$	$\frac{1}{6}$	$\frac{1}{8}$	$\frac{1}{12}$	$\frac{1}{24}$	0
25	$\frac{29}{25}$	$\frac{28}{25}$	$\frac{27}{25}$	$\frac{26}{25}$	1	$\frac{24}{25}$	$\frac{23}{25}$	$\frac{22}{25}$	$\frac{21}{25}$	$\frac{4}{5}$	$\frac{19}{25}$	$\frac{18}{25}$	$\frac{17}{25}$	$\frac{16}{25}$	$\frac{3}{5}$	$\frac{14}{25}$	$\frac{13}{25}$	$\frac{12}{25}$	$\frac{11}{25}$	$\frac{2}{5}$	$\frac{9}{25}$	$\frac{8}{25}$	$\frac{7}{25}$	$\frac{6}{25}$	$\frac{1}{5}$	$\frac{4}{25}$	$\frac{3}{25}$	$\frac{2}{25}$	$\frac{1}{25}$	0
26	$\frac{29}{26}$	$\frac{14}{13}$	$\frac{27}{26}$	1	$\frac{25}{26}$	$\frac{12}{13}$	$\frac{23}{26}$	$\frac{11}{13}$	$\frac{21}{26}$	$\frac{10}{13}$	$\frac{19}{26}$	$\frac{9}{13}$	$\frac{17}{26}$	$\frac{8}{13}$	$\frac{15}{26}$	$\frac{7}{13}$	$\frac{1}{2}$	$\frac{6}{13}$	$\frac{11}{26}$	$\frac{5}{13}$	$\frac{9}{26}$	$\frac{4}{13}$	$\frac{7}{26}$	$\frac{3}{13}$	$\frac{5}{26}$	$\frac{2}{13}$	$\frac{3}{26}$	$\frac{1}{13}$	$\frac{1}{26}$	0
27	$\frac{29}{27}$	$\frac{28}{27}$	1	$\frac{26}{27}$	$\frac{25}{27}$	$\frac{8}{9}$	$\frac{23}{27}$	$\frac{22}{27}$	$\frac{7}{9}$	$\frac{20}{27}$	$\frac{19}{27}$	$\frac{2}{3}$	$\frac{17}{27}$	$\frac{16}{27}$	$\frac{5}{9}$	$\frac{14}{27}$	$\frac{13}{27}$	$\frac{4}{9}$	$\frac{11}{27}$	$\frac{10}{27}$	$\frac{1}{3}$	$\frac{8}{27}$	$\frac{7}{27}$	$\frac{2}{9}$	$\frac{5}{27}$	$\frac{4}{27}$	$\frac{1}{9}$	$\frac{2}{27}$	$\frac{1}{27}$	0
28	$\frac{29}{28}$	1	$\frac{27}{28}$	$\frac{13}{14}$	$\frac{25}{28}$	$\frac{6}{7}$	$\frac{23}{28}$	$\frac{11}{14}$	$\frac{3}{4}$	$\frac{5}{7}$	$\frac{19}{28}$	$\frac{9}{14}$	$\frac{17}{28}$	$\frac{4}{7}$	$\frac{15}{28}$	$\frac{1}{2}$	$\frac{13}{28}$	$\frac{3}{7}$	$\frac{11}{28}$	$\frac{5}{14}$	$\frac{9}{28}$	$\frac{2}{7}$	$\frac{1}{4}$	$\frac{3}{14}$	$\frac{5}{28}$	$\frac{1}{7}$	$\frac{3}{28}$	$\frac{1}{14}$	$\frac{1}{28}$	0
29	1	$\frac{28}{29}$	$\frac{27}{29}$	$\frac{26}{29}$	$\frac{25}{29}$	$\frac{24}{29}$	$\frac{23}{29}$	$\frac{22}{29}$	$\frac{21}{29}$	$\frac{20}{29}$	$\frac{19}{29}$	$\frac{18}{29}$	$\frac{17}{29}$	$\frac{16}{29}$	$\frac{15}{29}$	$\frac{14}{29}$	$\frac{13}{29}$	$\frac{12}{29}$	$\frac{11}{29}$	$\frac{10}{29}$	$\frac{9}{29}$	$\frac{8}{29}$	$\frac{7}{29}$	$\frac{6}{29}$	$\frac{5}{29}$	$\frac{4}{29}$	$\frac{3}{29}$	$\frac{2}{29}$	$\frac{1}{29}$	0
30	$\frac{29}{30}$	$\frac{14}{15}$	$\frac{9}{10}$	$\frac{13}{15}$	$\frac{5}{6}$	$\frac{4}{5}$	$\frac{23}{30}$	$\frac{11}{15}$	$\frac{7}{10}$	$\frac{2}{3}$	$\frac{19}{30}$	$\frac{3}{5}$	$\frac{17}{30}$	$\frac{8}{15}$	$\frac{1}{2}$	$\frac{7}{15}$	$\frac{13}{30}$	$\frac{2}{5}$	$\frac{11}{30}$	$\frac{1}{3}$	$\frac{3}{10}$	$\frac{4}{15}$	$\frac{7}{30}$	$\frac{1}{5}$	$\frac{1}{6}$	$\frac{2}{15}$	$\frac{1}{10}$	$\frac{1}{15}$	$\frac{1}{30}$	0

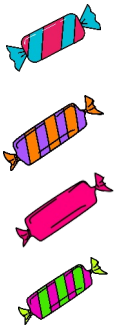
Division Form

dividend divisor quotient

÷

=

3	÷	3	=	1



÷

=

4	÷	2	=	2

Other division forms

dividend

divisor = quotient

divisor

quotient

divisor dividend

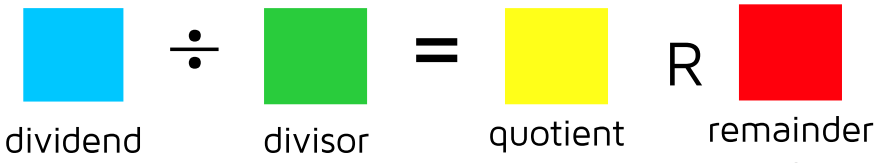
Examples

$24 \div 6 = 4$

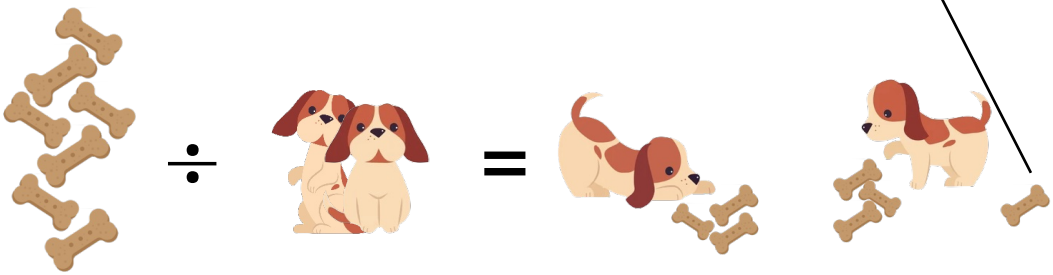
$\frac{24}{6} = 4$

$6 \overline{)\,24}$ $\quad 4$

Division with remainder



7 ÷ 2 = 3 R 1

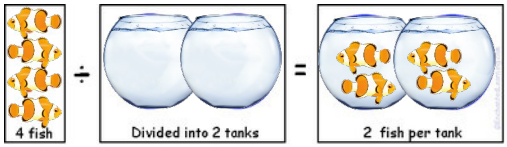


There are 7 bones to share with 2 puppies. But 7 cannot be divided exactly into 2 groups so we give each puppy 3 and 1 is left over.

It is also possible to cut the remaining bone in half: 7 ÷ 2 = 3 1/2

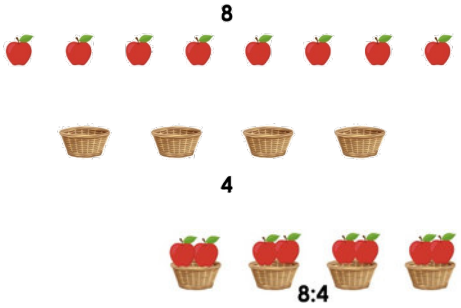
Examples

$4 \div 2 = 2$

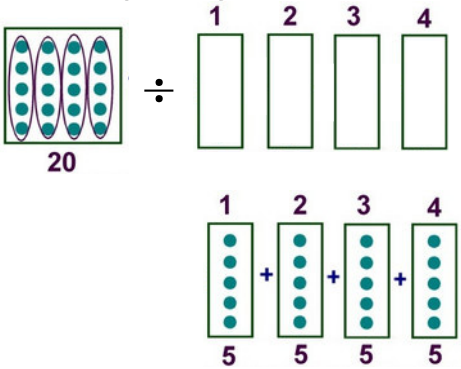


$8 \div 4 = 2$

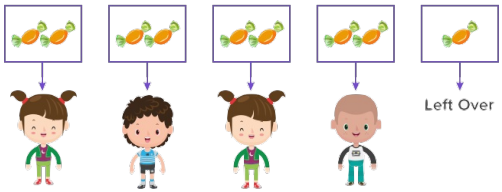
How many apples per basket?



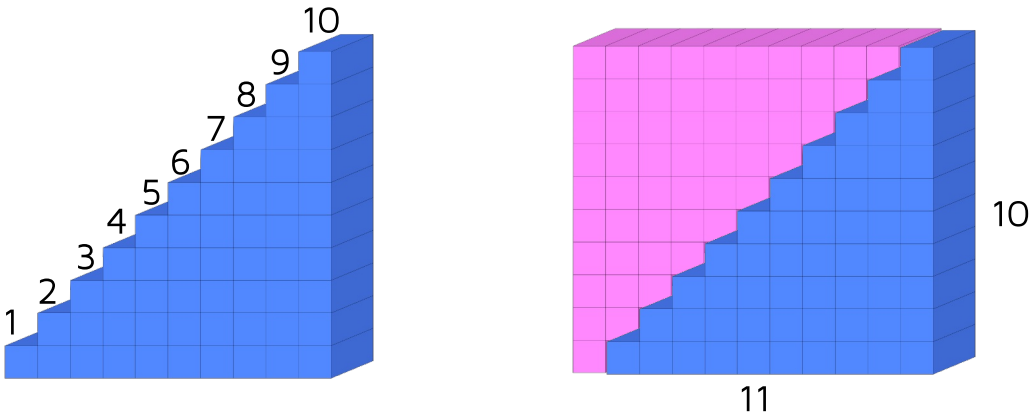
$20 \div 4 = 5$



$9 \div 4 = 2$ R 1



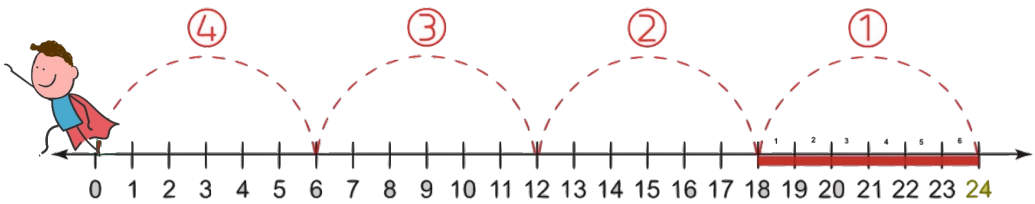
Gauss sum



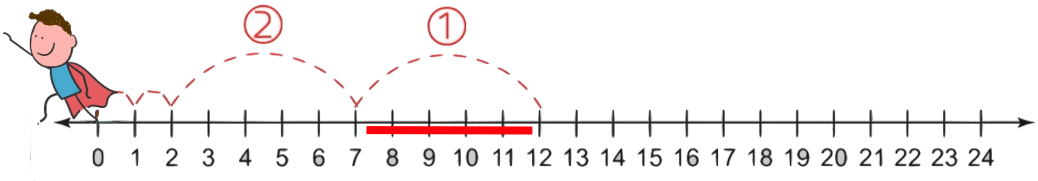
$$1+2+3+4+5+6+7+8+9+10 = \frac{(11 \times 10)}{2} = 55$$

Number line

$$24 \div 6 = 4$$

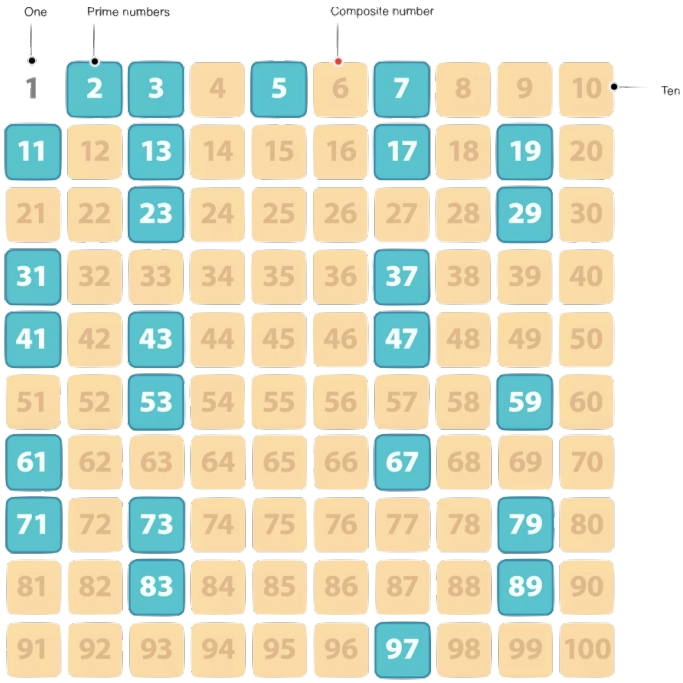


$$12 \div 5 = 2 \text{ R } 2$$



Prime numbers

Some whole numbers can only be divided, with a 0 remainder, by 1 or itself. A number that is not a prime is called a composite. You can divide a composite number by numbers other than 1 and itself. The table shows primes (in blue) and composites (in orange).

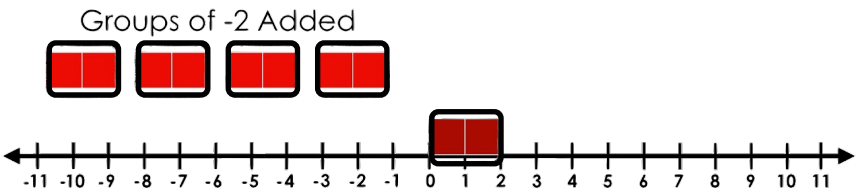


Division with negative numbers

+	÷	+	=	+
-	÷	-	=	+
+	÷	-	=	-
-	÷	+	=	-

8 ÷ (-2) = -4

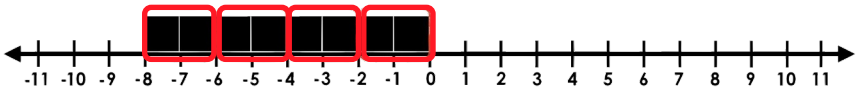
8 divided into groups of -2 results in -4 groups



-8 ÷ 2 = -4

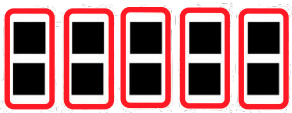
-8 divided into groups of 2 gives a result of -4 groups

-4 Groups of 2 Removed



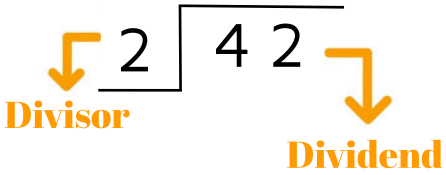
-10 ÷ (-5) = 2

-10 divided into -5 groups results in 2 per group

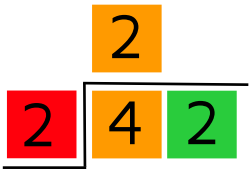


Step by step division

$42 \div 2 = ?$



1)

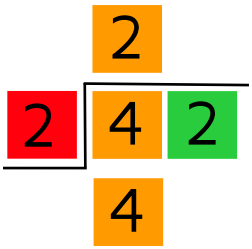


Find how many 2s (divisor) go into 4.

1 x 2 = 2.

2 x 2 = 4. ✓

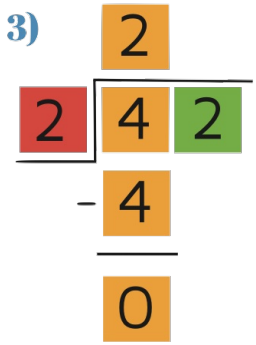
2)



Multiply.

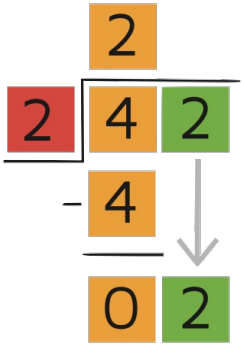
2 x 2 = 4.

3)



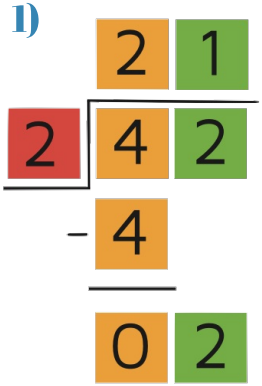
Subtract.

4 - 4 = 0.



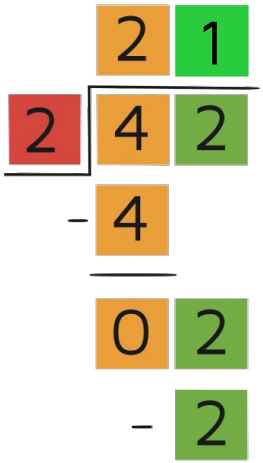
Move 2 down.

1)

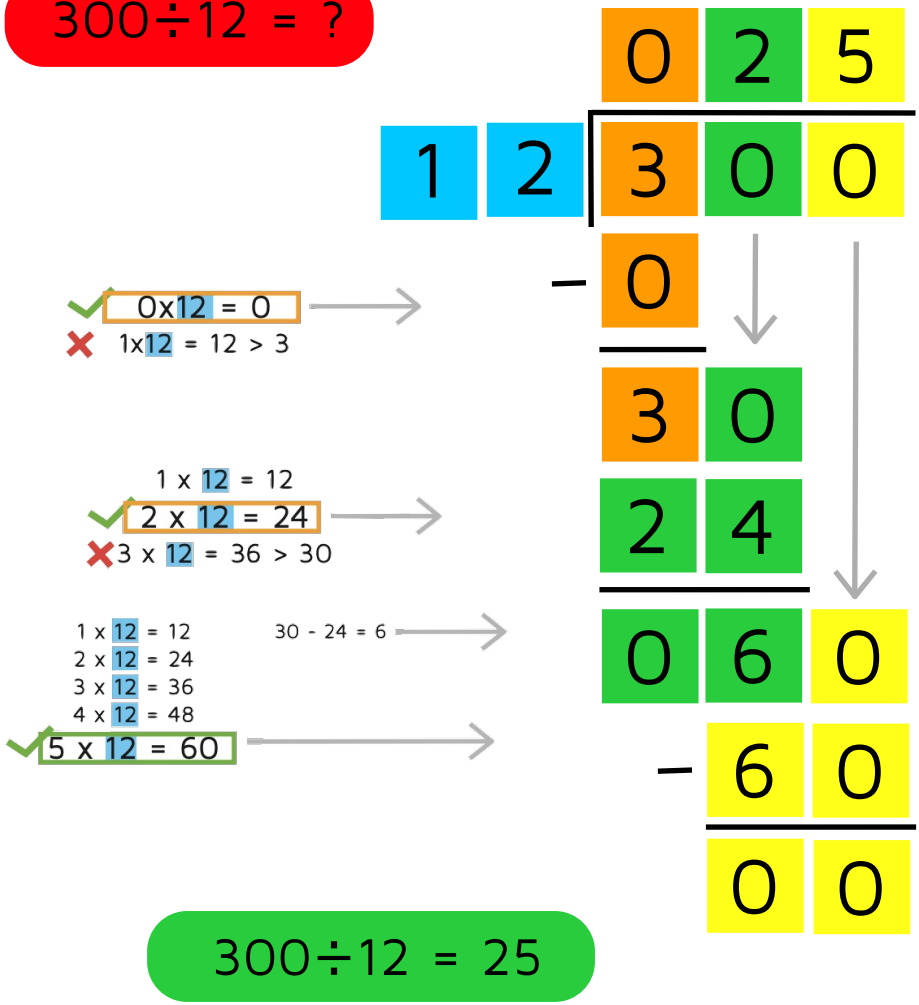


Find how many 2s go into 2.

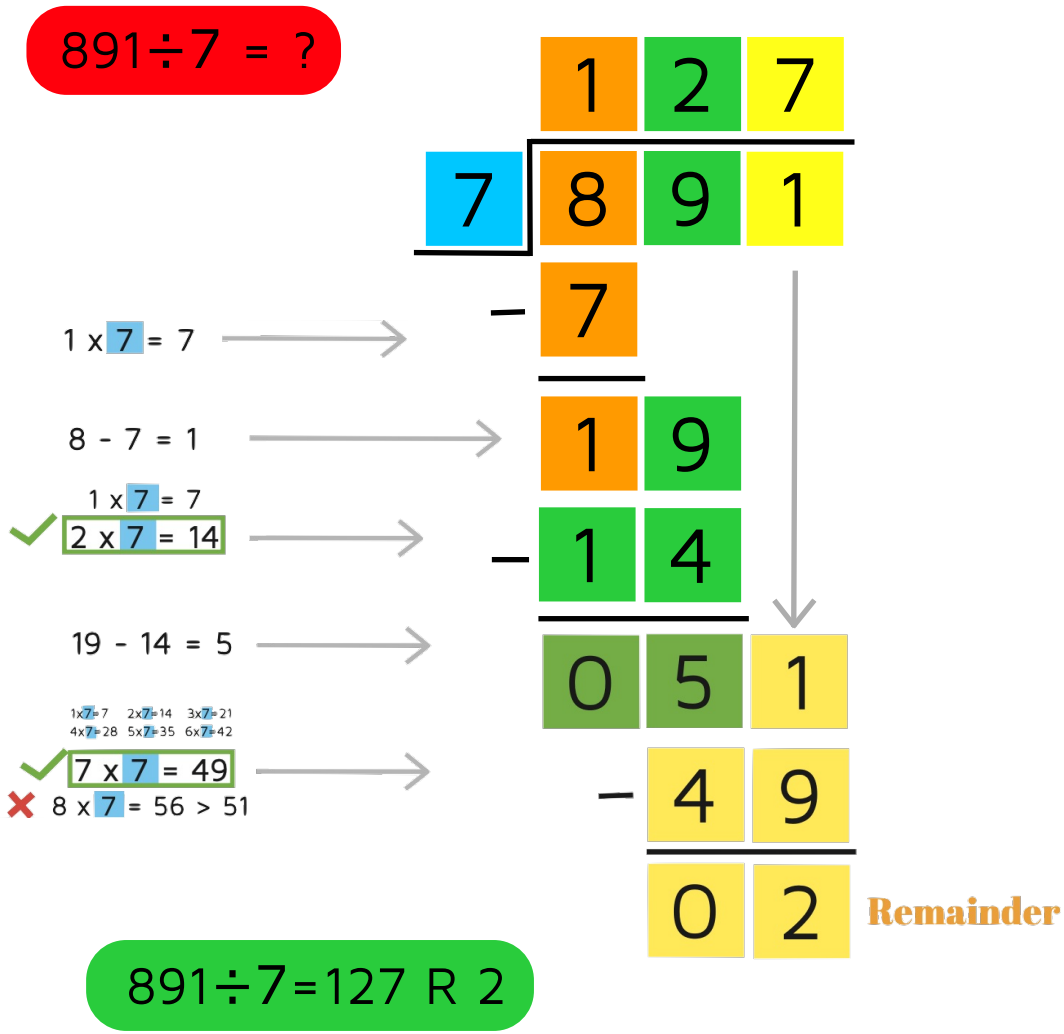
1 x 2 = 2. ✓



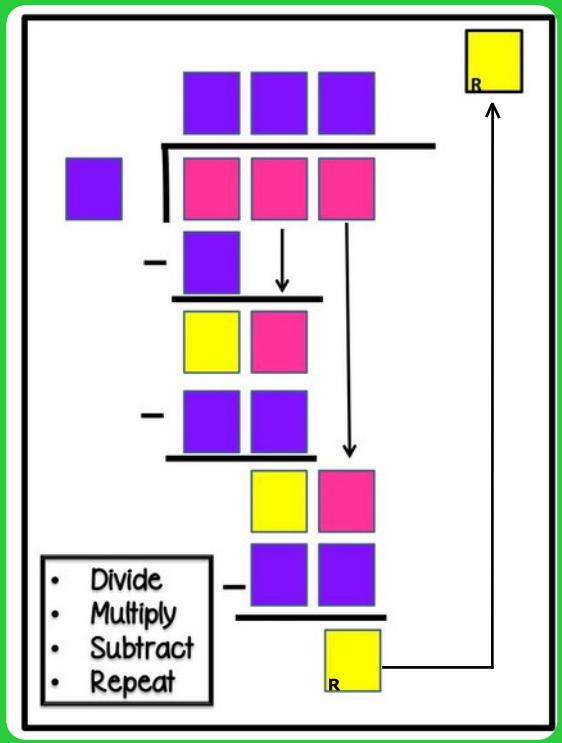
300÷12 = ?



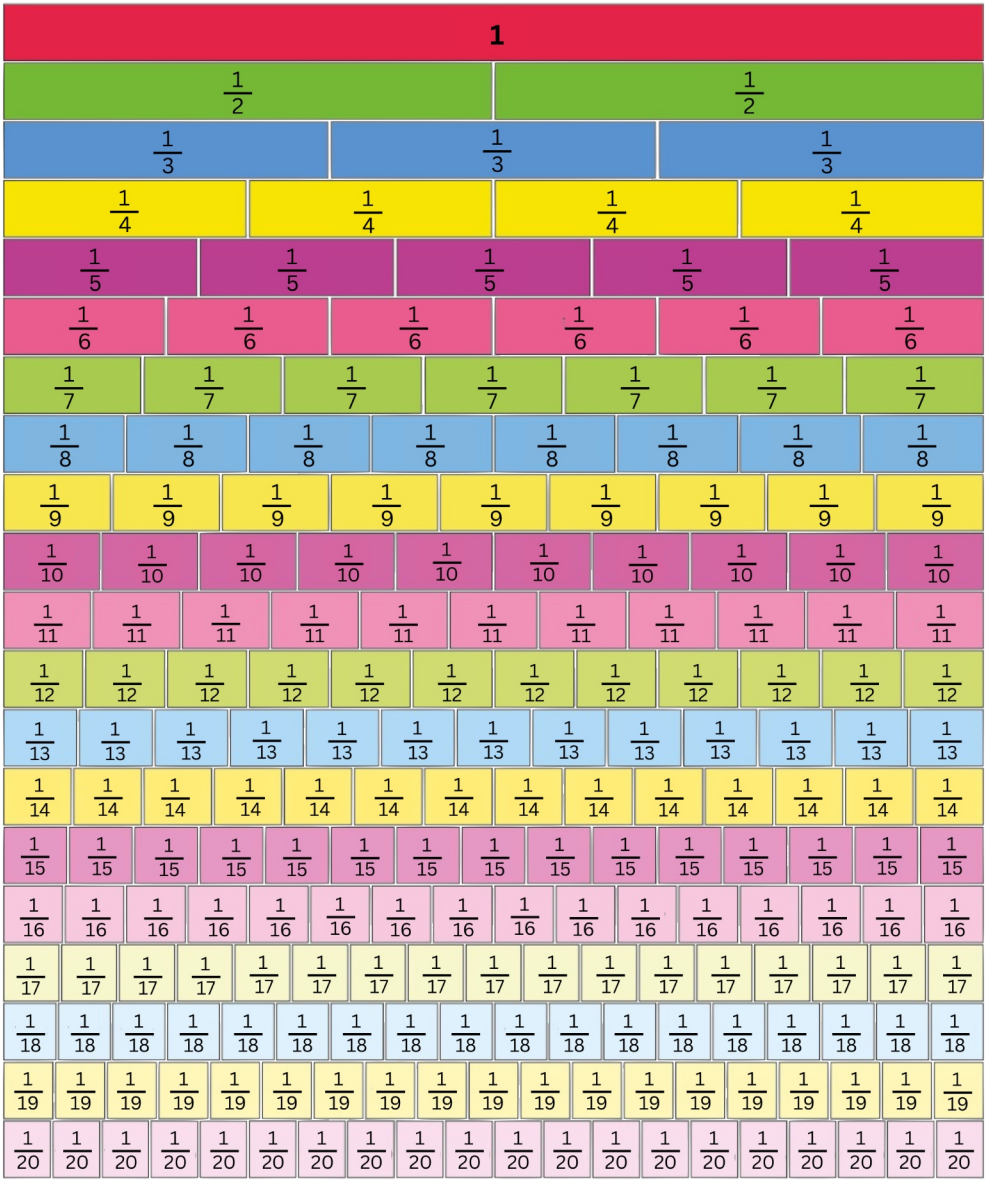
300÷12 = 25


891÷7 = ?
1 2 7
7 8 9 1
− 7
1 x 7 = 7
1 9
8 - 7 = 1
1 x 7 = 7
2 x 7 = 14
− 1 4
19 - 14 = 5
0 5 1
1x7=7 2x7=14 3x7=21
4x7=28 5x7=35 6x7=42
7 x 7 = 49
8 x 7 = 56 > 51
− 4 9
0 2
Remainder
891÷7=127 R 2

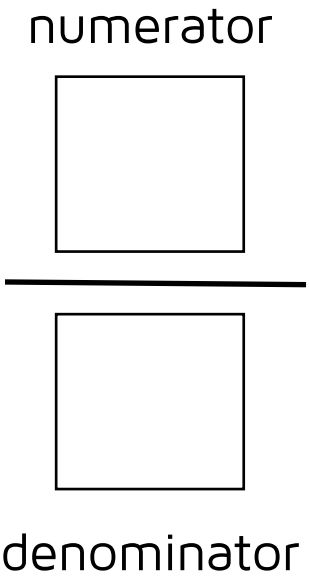
Division in a nutshell

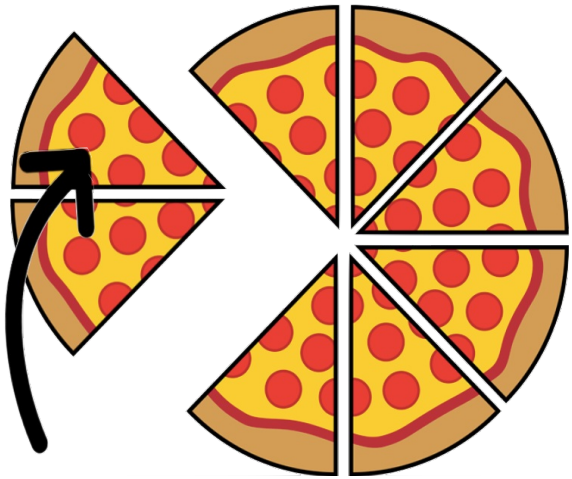


Fraction

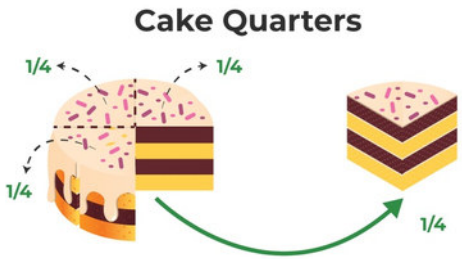

1
1/2 1/2
1/3 1/3 1/3
1/4 1/4 1/4 1/4
1/5 1/5 1/5 1/5 1/5
1/6 1/6 1/6 1/6 1/6 1/6
1/7 1/7 1/7 1/7 1/7 1/7 1/7
1/8 1/8 1/8 1/8 1/8 1/8 1/8 1/8
1/9 1/9 1/9 1/9 1/9 1/9 1/9 1/9 1/9
1/10 1/10 1/10 1/10 1/10 1/10 1/10 1/10 1/10 1/10
1/11 1/11 1/11 1/11 1/11 1/11 1/11 1/11 1/11 1/11 1/11
1/12 1/12 1/12 1/12 1/12 1/12 1/12 1/12 1/12 1/12 1/12 1/12
1/13 1/13 1/13 1/13 1/13 1/13 1/13 1/13 1/13 1/13 1/13 1/13 1/13
1/14 1/14 1/14 1/14 1/14 1/14 1/14 1/14 1/14 1/14 1/14 1/14 1/14 1/14
1/15 1/15 1/15 1/15 1/15 1/15 1/15 1/15 1/15 1/15 1/15 1/15 1/15 1/15 1/15
1/16 1/16 1/16 1/16 1/16 1/16 1/16 1/16 1/16 1/16 1/16 1/16 1/16 1/16 1/16 1/16
1/17 1/17 1/17 1/17 1/17 1/17 1/17 1/17 1/17 1/17 1/17 1/17 1/17 1/17 1/17 1/17 1/17
1/18 1/18 1/18 1/18 1/18 1/18 1/18 1/18 1/18 1/18 1/18 1/18 1/18 1/18 1/18 1/18 1/18 1/18
1/19 1/19 1/19 1/19 1/19 1/19 1/19 1/19 1/19 1/19 1/19 1/19 1/19 1/19 1/19 1/19 1/19 1/19 1/19
1/20 1/20 1/20 1/20 1/20 1/20 1/20 1/20 1/20 1/20 1/20 1/20 1/20 1/20 1/20 1/20 1/20 1/20 1/20 1/20

Fraction form



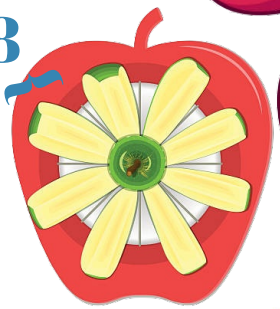


$$\frac{2}{8}$$

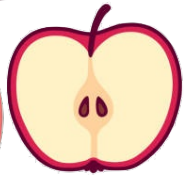


Examples

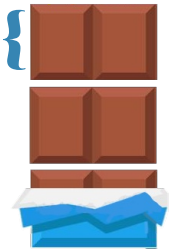
1



1/8



1/2



1/3

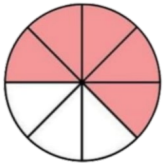
1/6

1/8

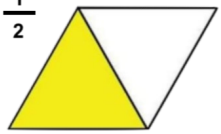
1/2

$\frac{3}{4}$

$\frac{5}{8}$

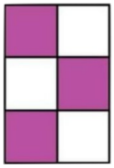


$\frac{1}{2}$



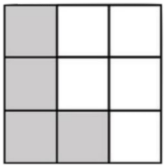
$\frac{2}{4}$

$\frac{3}{6}$

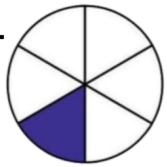


$\frac{5}{6}$

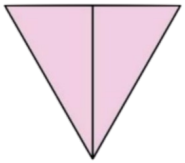
$\frac{4}{9}$



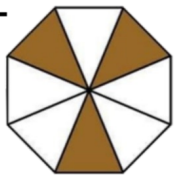
$\frac{1}{6}$



$\frac{2}{2}$

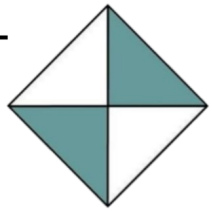


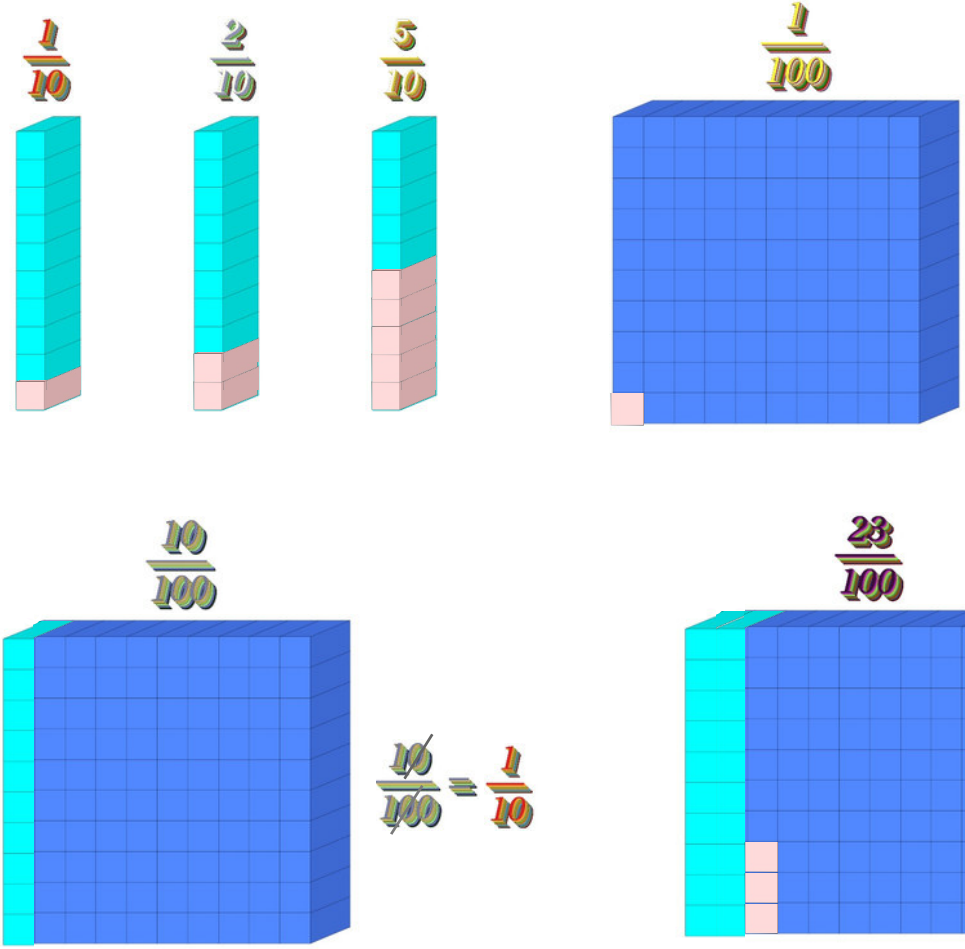
$\frac{3}{8}$



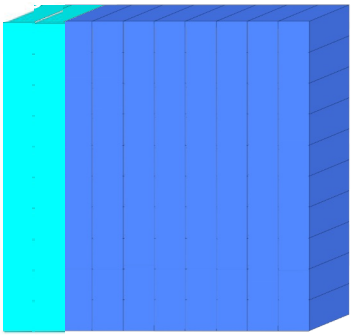
$\frac{2}{5}$

$\frac{2}{4}$

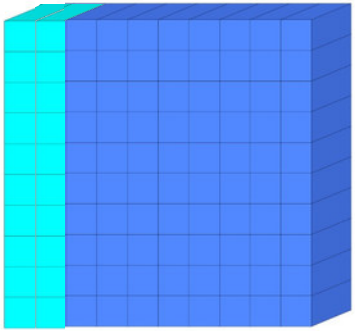



1/10
2/10
5/10
1/100
10/100
10/100 = 1/10
23/100

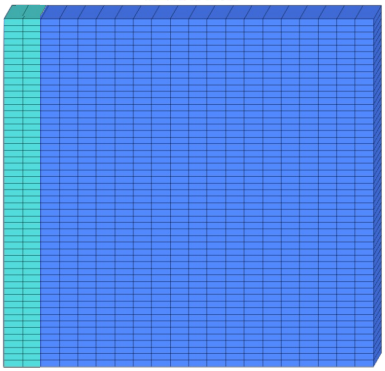
$\frac{2}{10}$



$\frac{20}{100}$



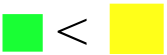
$\frac{200}{1000}$

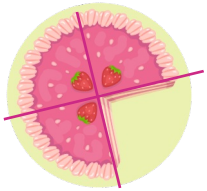


$\frac{200}{1000} = \frac{20}{100} = \frac{2}{10}$

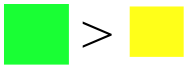
Types of fractions

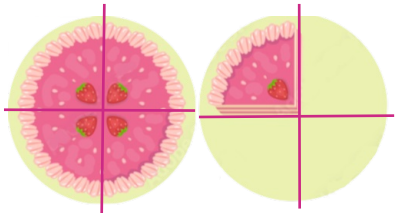
Proper fraction


where 
example: $\frac{3}{4}$

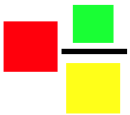
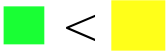


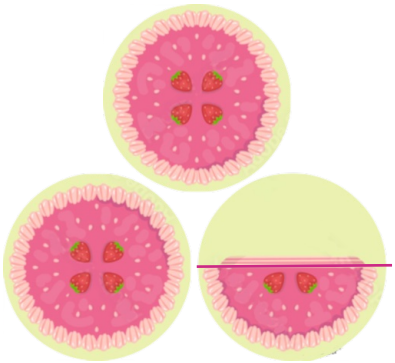
Improper fraction


where 
example: $\frac{5}{4}$

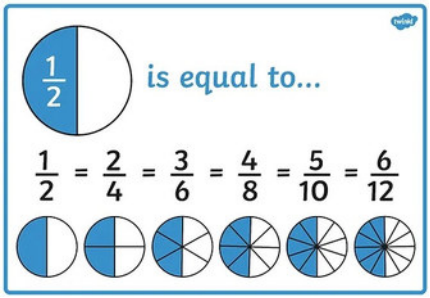


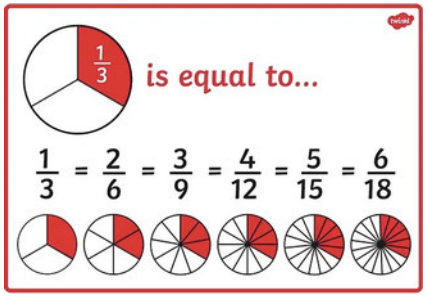
Mixed fraction

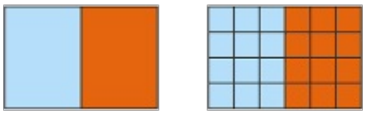

where 
example: $2\frac{1}{2}$



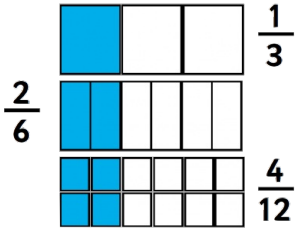
Equivalent fractions







$$\frac{1}{2} = \frac{12}{24}$$



Multiply

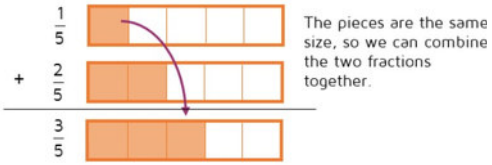
$$\frac{2}{3} \overset{\times 2}{=} \frac{4}{6} \overset{\times 2}{=} \frac{8}{12}$$

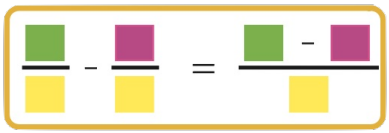
Divide

$$\frac{2}{3} \overset{\div 2}{=} \frac{4}{6} \overset{\div 2}{=} \frac{8}{12}$$

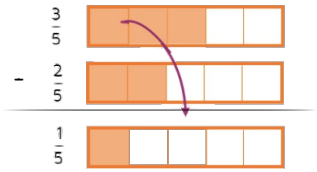
Fraction addition and subtraction

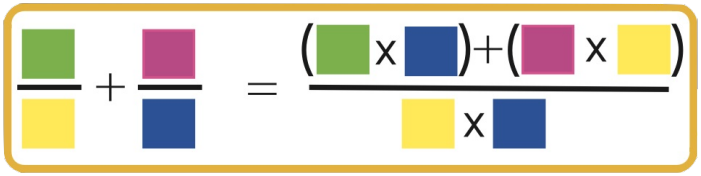
example: $\frac{1}{5} + \frac{2}{5} = \frac{1+2}{5} = \frac{3}{5}$





example: $\frac{3}{5} - \frac{2}{5} = \frac{3-2}{5} = \frac{1}{5}$

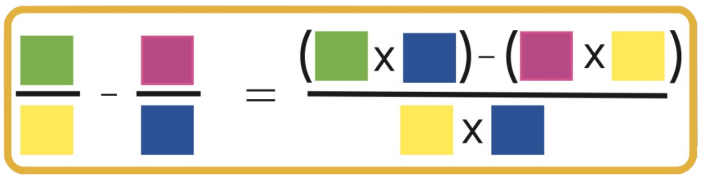




example:

$$\frac{2}{5} + \frac{1}{2} = \frac{(2 \times 2) + (1 \times 5)}{2 \times 5}$$

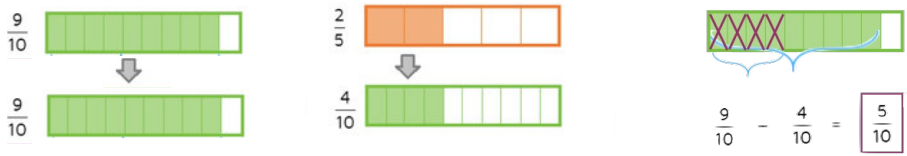
$$= \frac{9}{10}$$



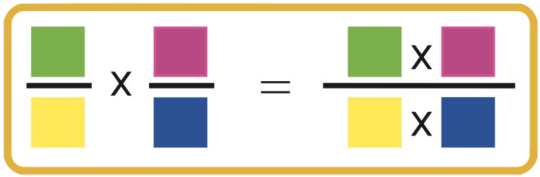
$$\frac{9}{10} - \frac{2}{5} = \frac{9 - (2 \times 2)}{2 \times 5}$$

$$= \frac{5}{10}$$

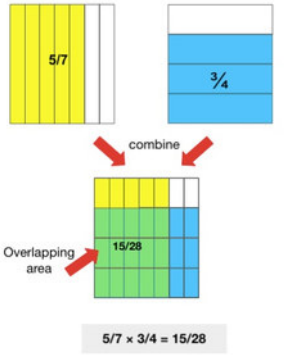
The least common denominator is 10 here.

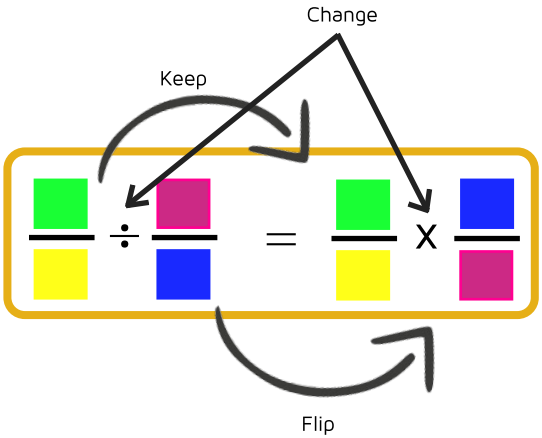


Fraction multiplication and division



$$\frac{5}{7} \times \frac{3}{4} = \frac{5 \times 3}{7 \times 4} = \frac{15}{28}$$



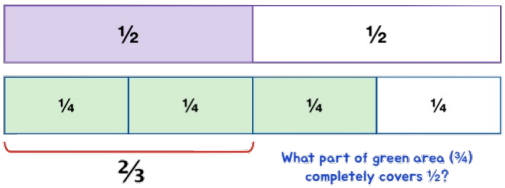


$$\frac{1}{2} \div \frac{3}{4} = \frac{1}{2} \times \frac{4}{3} = \frac{4}{6} = \frac{2}{3}$$

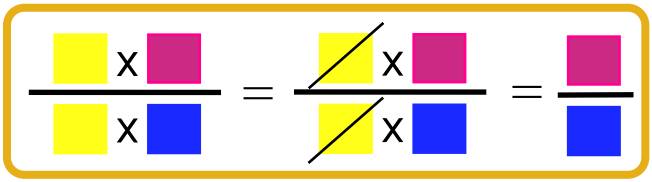
÷2 ÷2

How many groups of ¾ cover ½?

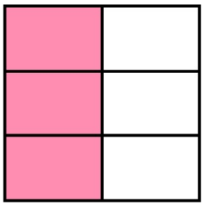
$$\frac{1}{2} \div \frac{3}{4} = \frac{2}{3}$$

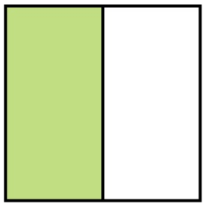


Cancellation property



$$\frac{10}{15} = \frac{\cancel{5} \times 2}{\cancel{5} \times 3} = \frac{2}{3}$$





$$\frac{3}{6} = \frac{\cancel{3} \times 1}{\cancel{3} \times 2} = \frac{1}{2}$$

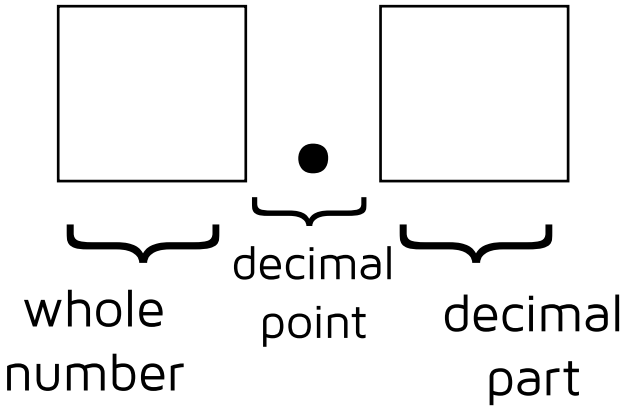
Fractions in a nutshell

Decimals

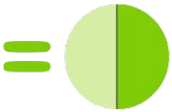
÷	1	2	3	4	5	6	7	8	9	10
1	1.	0.5	0.333333	0.25	0.2	0.166667	0.142857	0.125	0.111111	0.1
2	2.	1.	0.666667	0.5	0.4	0.333333	0.285714	0.25	0.222222	0.2
3	3.	1.5	1.	0.75	0.6	0.5	0.428571	0.375	0.333333	0.3
4	4.	2.	1.33333	1.	0.8	0.666667	0.571429	0.5	0.444444	0.4
5	5.	2.5	1.66667	1.25	1.	0.833333	0.714286	0.625	0.555556	0.5
6	6.	3.	2.	1.5	1.2	1.	0.857143	0.75	0.666667	0.6
7	7.	3.5	2.33333	1.75	1.4	1.16667	1.	0.875	0.777778	0.7
8	8.	4.	2.66667	2.	1.6	1.33333	1.14286	1.	0.888889	0.8
9	9.	4.5	3.	2.25	1.8	1.5	1.28571	1.125	1.	0.9
10	10.	5.	3.33333	2.5	2.	1.66667	1.42857	1.25	1.11111	1.

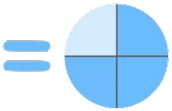
÷	1	2	3	4	5	6	7	8	9	10
11	11.	5.5	3.66667	2.75	2.2	1.83333	1.57143	1.375	1.22222	1.1
12	12.	6.	4.	3.	2.4	2.	1.71429	1.5	1.33333	1.2
13	13.	6.5	4.33333	3.25	2.6	2.16667	1.85714	1.625	1.44444	1.3
14	14.	7.	4.66667	3.5	2.8	2.33333	2.	1.75	1.55556	1.4
15	15.	7.5	5.	3.75	3.	2.5	2.14286	1.875	1.66667	1.5
16	16.	8.	5.33333	4.	3.2	2.66667	2.28571	2.	1.77778	1.6
17	17.	8.5	5.66667	4.25	3.4	2.83333	2.42857	2.125	1.88889	1.7
18	18.	9.	6.	4.5	3.6	3.	2.57143	2.25	2.	1.8
19	19.	9.5	6.33333	4.75	3.8	3.16667	2.71429	2.375	2.11111	1.9
20	20.	10.	6.66667	5.	4.	3.33333	2.85714	2.5	2.22222	2.

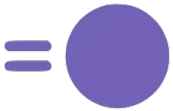
Decimals



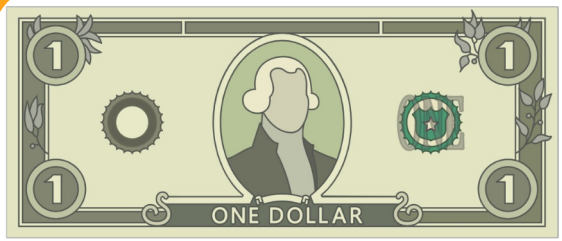
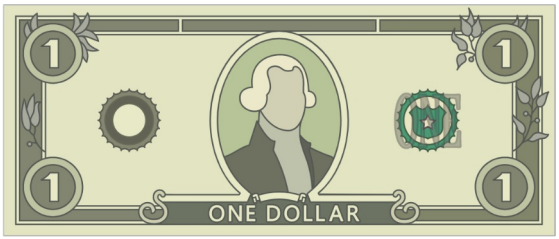
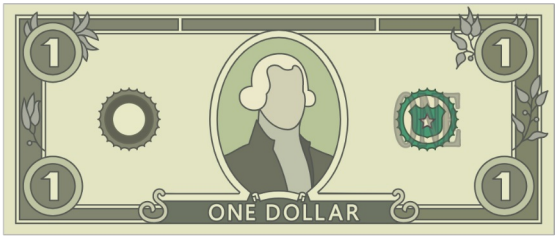
0,25 = 

0,5 = 

0,75 = 

1,0 = 

Examples

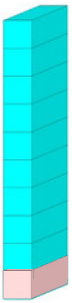
 . 	1.1
 . 	1.11
 . 	1.01


Menu
Cotton Candy $1.33
Moose Tracks $1.24
Strawberry Sorbet $1.00
Lemon Custard $1.00
Cookies N' Cream $1.99


Vegetable Market
4.50
1.25
0.2
0.25
2.99
1.0

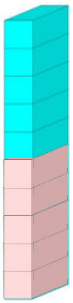

00:37:88

$\frac{1}{10} = 0.1$

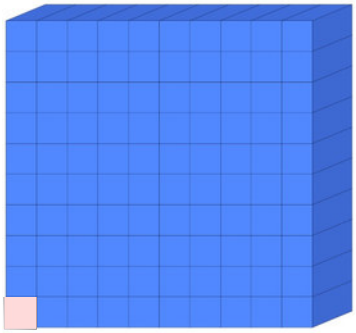


$\frac{2}{10} = 0.2$

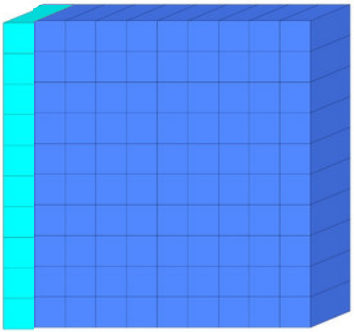
$\frac{5}{10} = 0.5$



$\frac{1}{100} = 0.01$



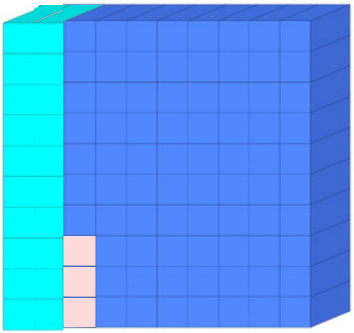
$\frac{10}{100} = 0.10$



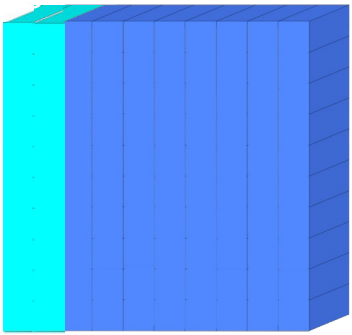
$\frac{\cancel{10}}{\cancel{100}} = \frac{1}{10} = 0.1$

$\Rightarrow 0.10 = 0.1$

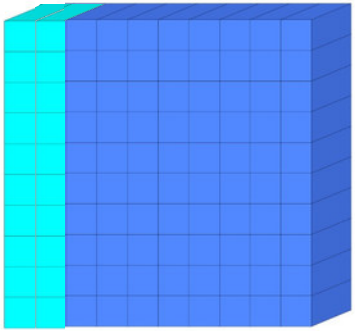
$\frac{23}{100} = 0.23$



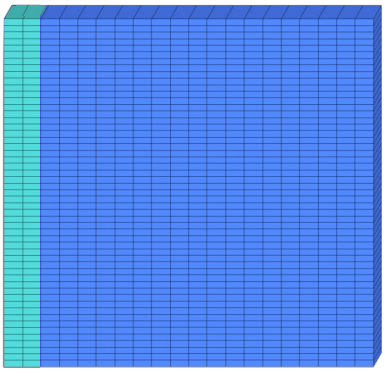
$\frac{2}{10} = 0.2$



$\frac{20}{100} = 0.20$

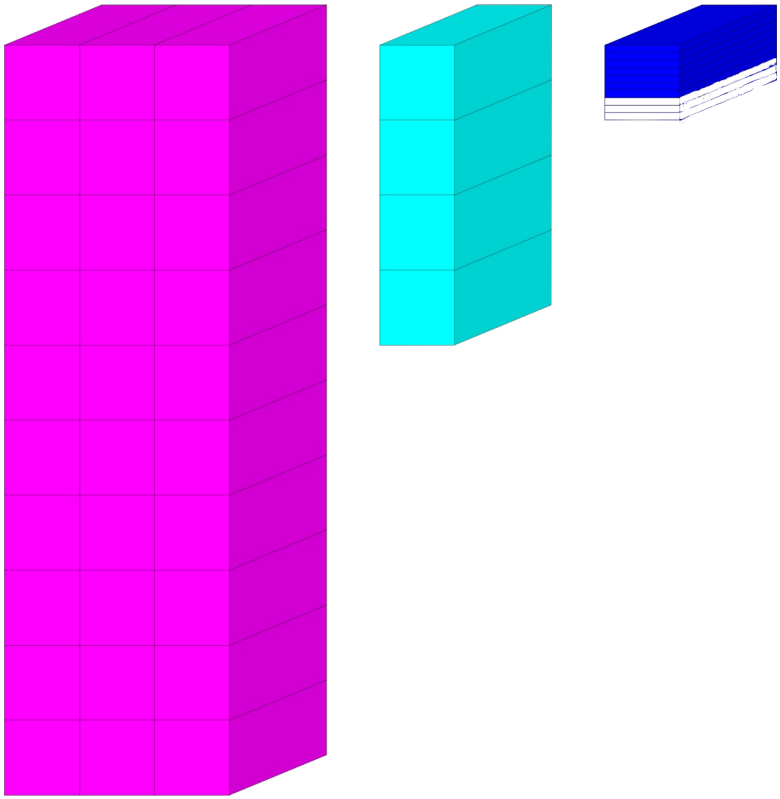


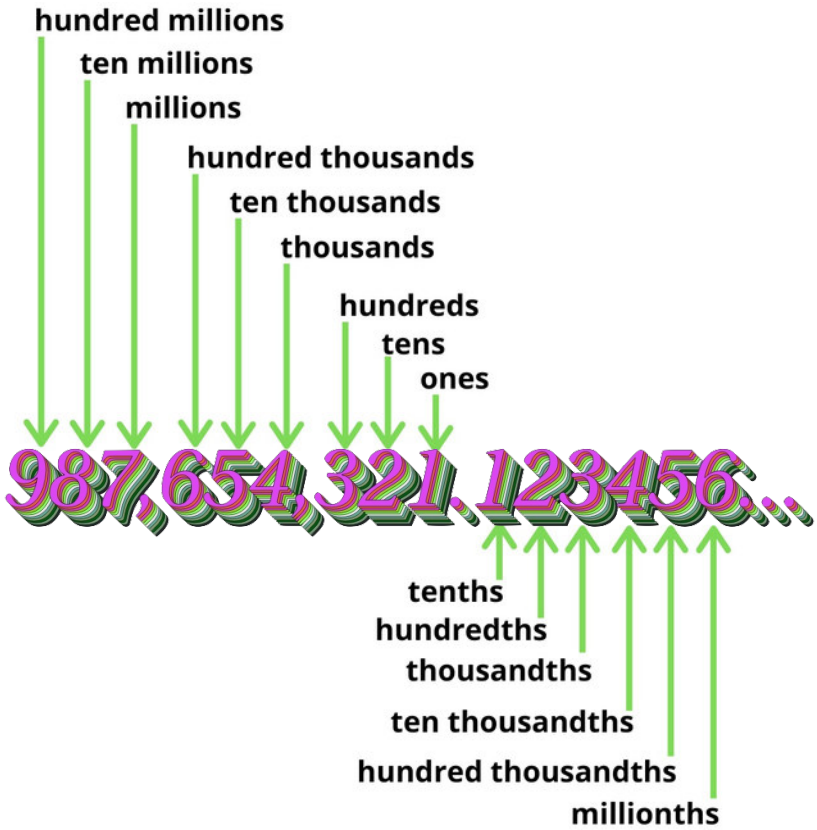
$\frac{200}{1000} = 0.200$



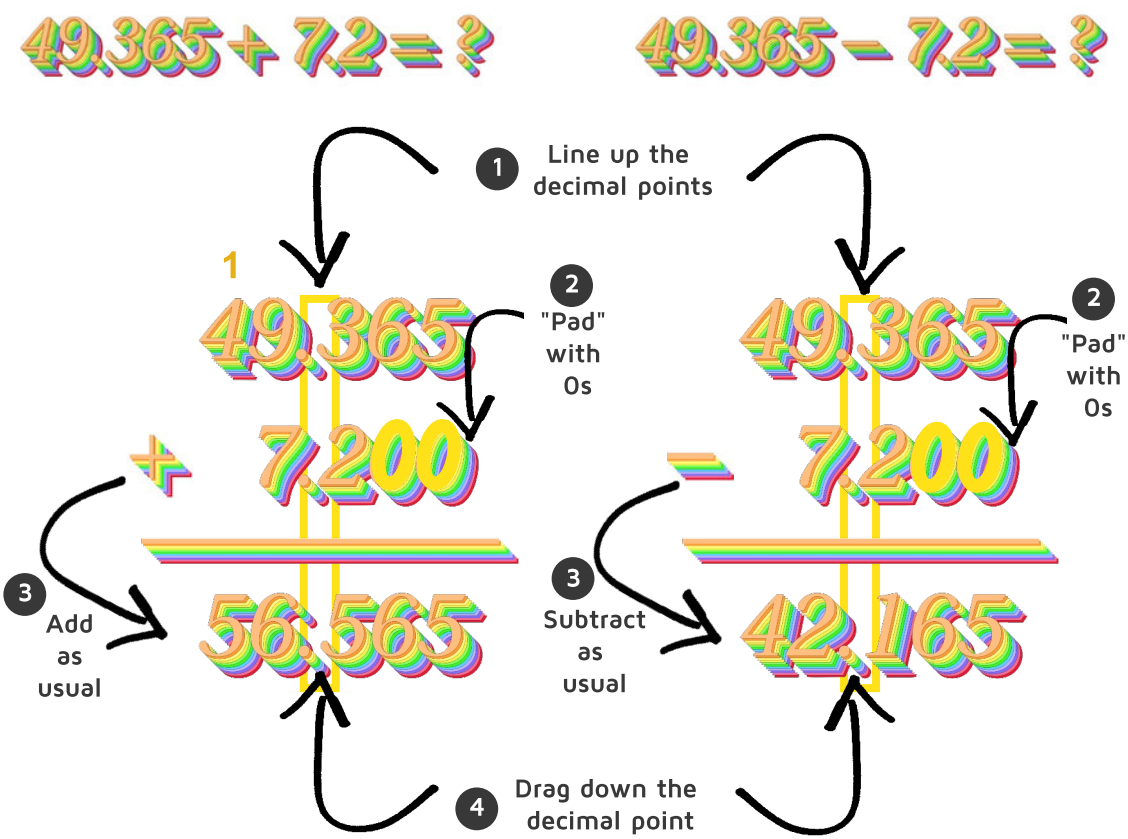
$0.2 = 0.20 = 0.200$

$$34.7 = 30 + 4 + \frac{7}{10}$$

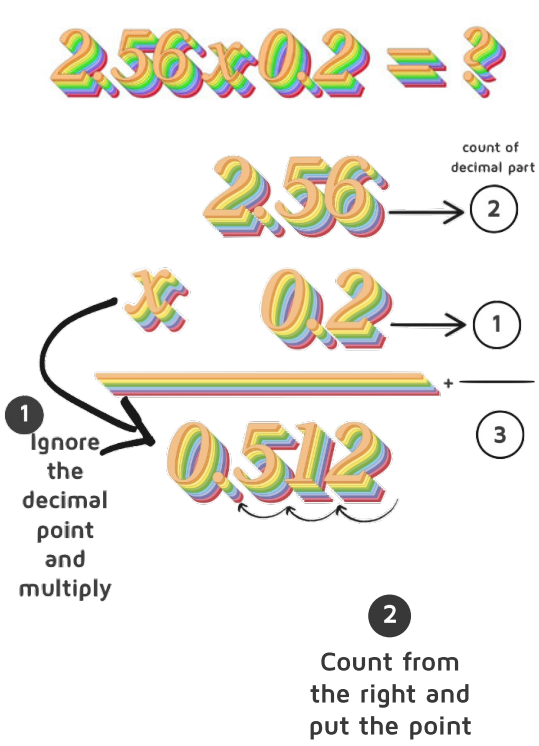


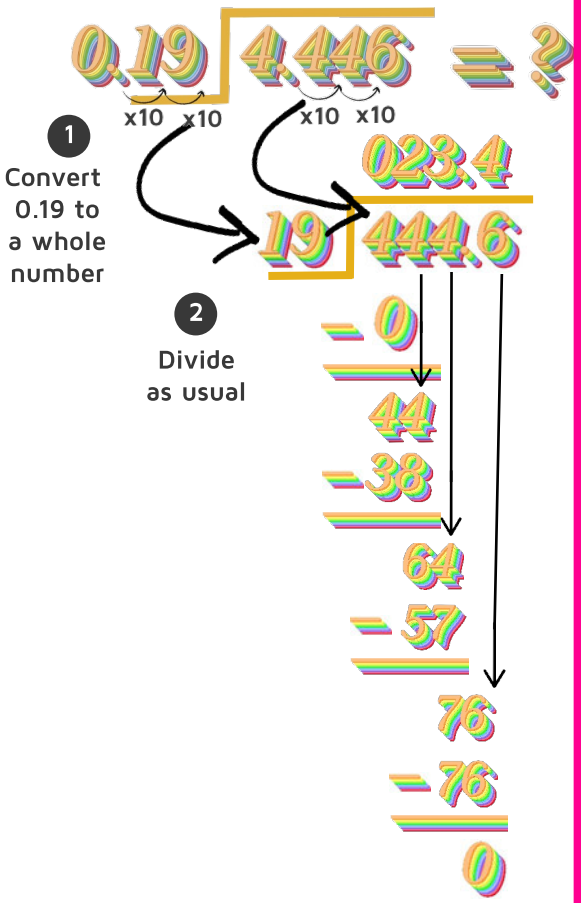

hundred millions
ten millions
millions
hundred thousands
ten thousands
thousands
hundreds
tens
ones
987,654,321.123456...
tenths
hundredths
thousandths
ten thousandths
hundred thousandths
millionths

Decimal addition and subtraction



Decimal multiplication and division



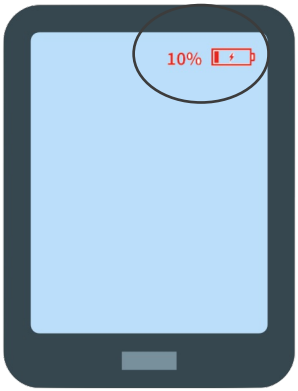


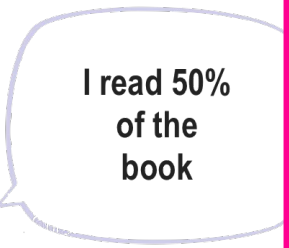
Percent

$$0 \leq \square\% \leq 100$$

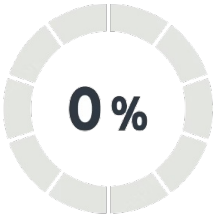
Examples

$$x\% = \frac{x}{100}$$

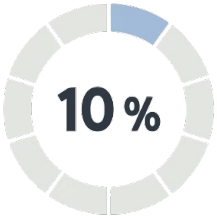




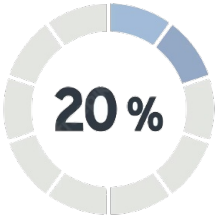
$\frac{0}{100}$



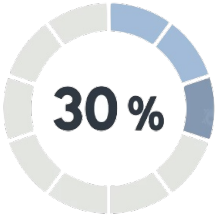
$\frac{10}{100}$



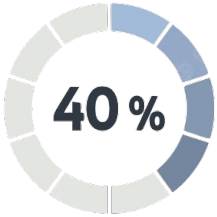
$\frac{20}{100}$



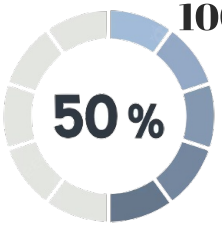
$\frac{30}{100}$



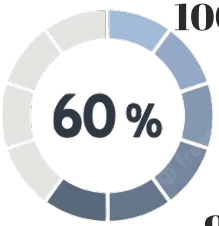
$\frac{40}{100}$ 160



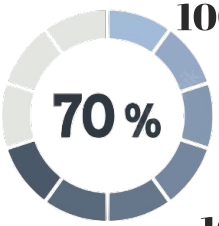
$\frac{50}{100}$



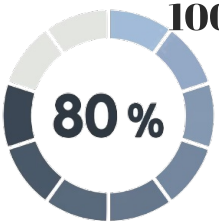
$\frac{60}{100}$



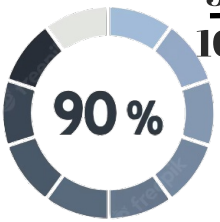
$\frac{70}{100}$



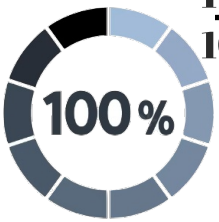
$\frac{80}{100}$



$\frac{90}{100}$



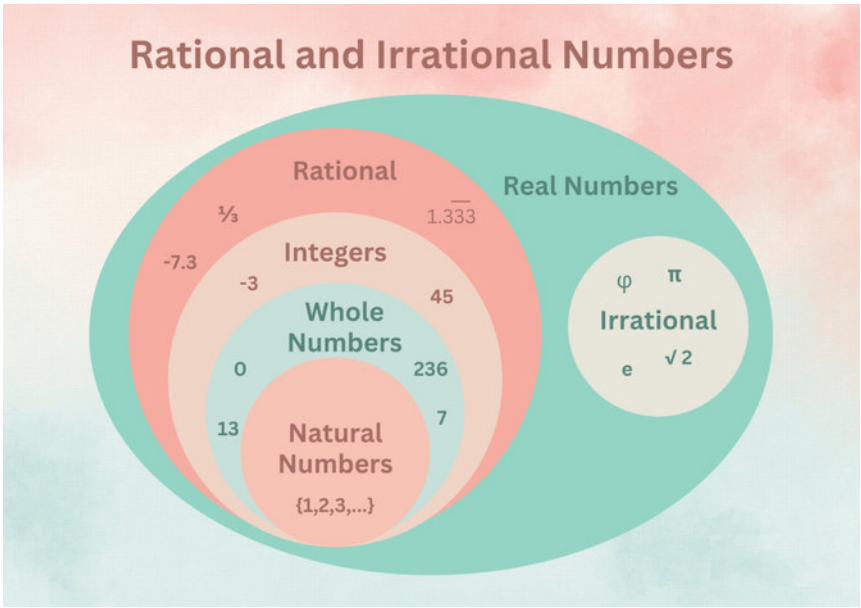
$\frac{100}{100}$

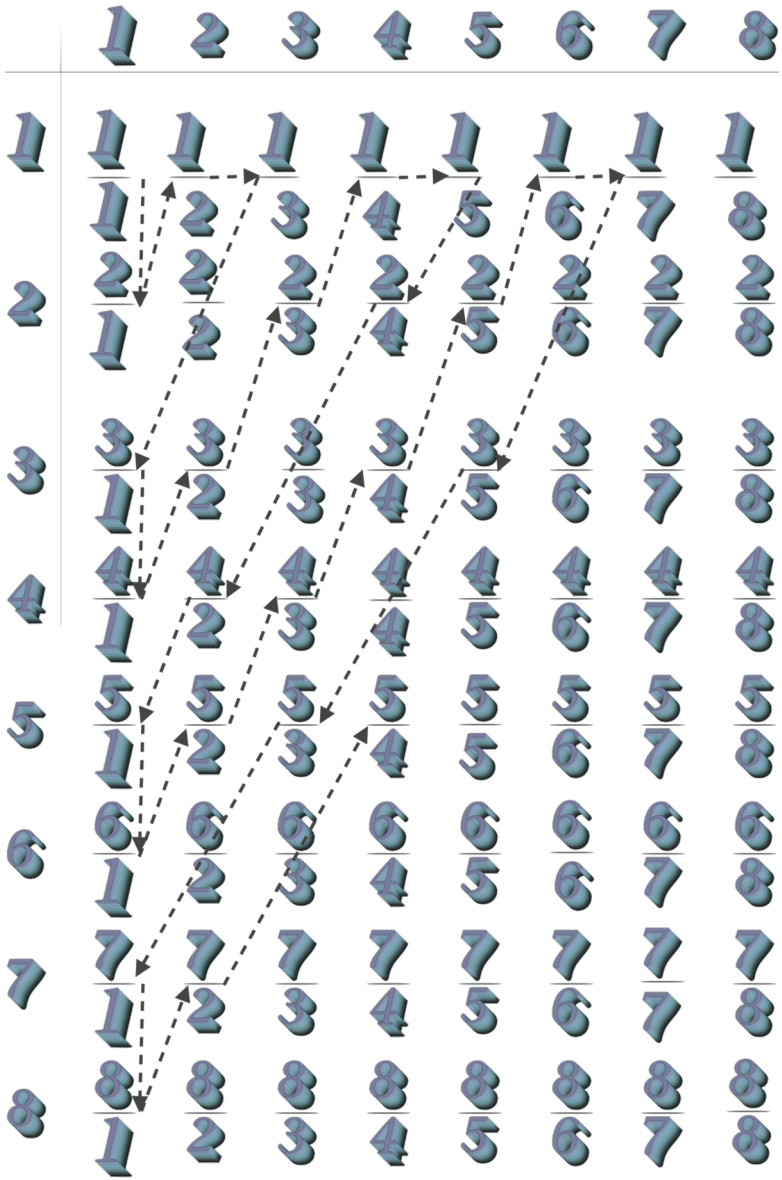


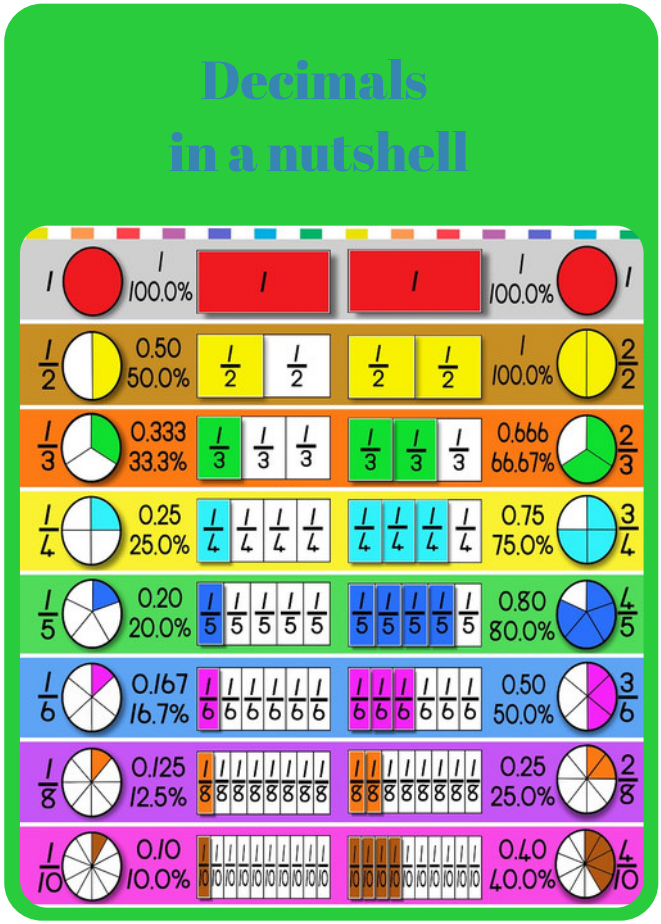
$\frac{8 \times 50}{100} = 4$ $ off

Rational Numbers

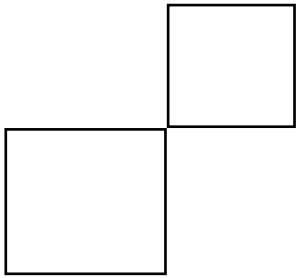
A rational number is any number that can be written as a fraction, where the denominator is not equal to zero. Rational numbers are countable, just like natural numbers are. This means we can arrange them in a way (like in the next page) and count them just like natural numbers..




1 2 3 4 5 6 7 8
1
1 1 1 1 1 1 1 1
1 2 3 4 5 6 7 8
2
2 2 2 2 2 2 2 2
1 2 3 4 5 6 7 8
3
3 3 3 3 3 3 3 3
1 2 3 4 5 6 7 8
4
4 4 4 4 4 4 4 4
1 2 3 4 5 6 7 8
5
5 5 5 5 5 5 5 5
1 2 3 4 5 6 7 8
6
6 6 6 6 6 6 6 6
1 2 3 4 5 6 7 8
7
7 7 7 7 7 7 7 7
1 2 3 4 5 6 7 8
8
8 8 8 8 8 8 8 8
1 2 3 4 5 6 7 8

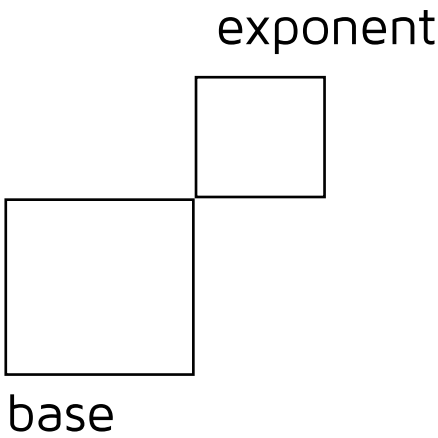

Decimals
in a nutshell
1
1
100.0%
1
1
1
100.0%
1
1/2
0.50
50.0%
1/2
1/2
1/2
1/2
1
100.0%
2/2
1/3
0.333
33.3%
1/3
1/3
1/3
1/3
1/3
1/3
0.666
66.67%
2/3
1/4
0.25
25.0%
1/4
1/4
1/4
1/4
1/4
1/4
1/4
1/4
0.75
75.0%
3/4
1/5
0.20
20.0%
1/5
1/5
1/5
1/5
1/5
1/5
1/5
1/5
1/5
1/5
0.80
80.0%
4/5
1/6
0.167
16.7%
1/6
1/6
1/6
1/6
1/6
1/6
1/6
1/6
1/6
1/6
1/6
1/6
0.50
50.0%
3/6
1/8
0.125
12.5%
1/8
1/8
1/8
1/8
1/8
1/8
1/8
1/8
1/8
1/8
1/8
1/8
1/8
1/8
1/8
1/8
0.25
25.0%
2/8
1/10
0.10
10.0%
1/10
1/10
1/10
1/10
1/10
1/10
1/10
1/10
1/10
1/10
1/10
1/10
1/10
1/10
1/10
1/10
1/10
1/10
1/10
1/10
0.40
40.0%
4/10

Power

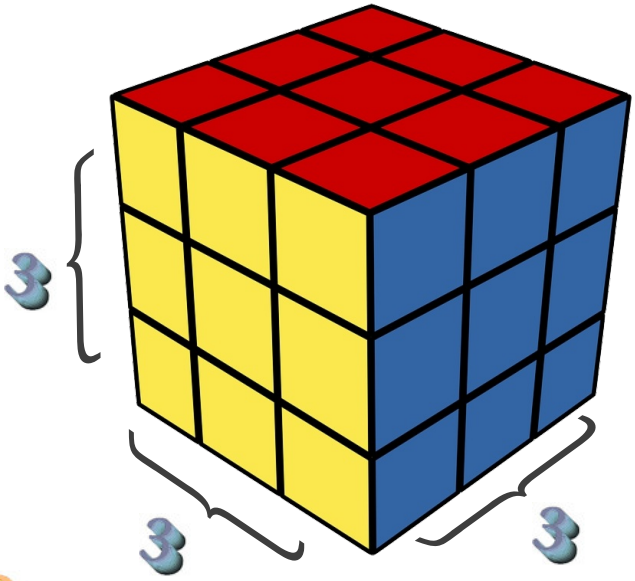


X	1	2	3	4	5	6	7	8	9	10	11	12
1	1	2	3	4	5	6	7	8	9	10	11	12
2	2	4	6	8	10	12	14	16	18	20	22	24
3	3	6	9	12	15	18	21	24	27	30	33	36
4	4	8	12	16	20	24	28	32	36	40	44	48
5	5	10	15	20	25	30	35	40	45	50	55	60
6	6	12	18	24	30	36	42	48	54	60	66	72
7	7	14	21	28	35	42	49	56	63	70	77	84
8	8	16	24	32	40	48	56	64	72	80	88	96
9	9	18	27	36	45	54	63	72	81	90	99	108
10	10	20	30	40	50	60	70	80	90	100	110	120
11	11	22	33	44	55	66	77	88	99	110	121	132
12	12	24	36	48	60	72	84	96	108	120	132	144

Power

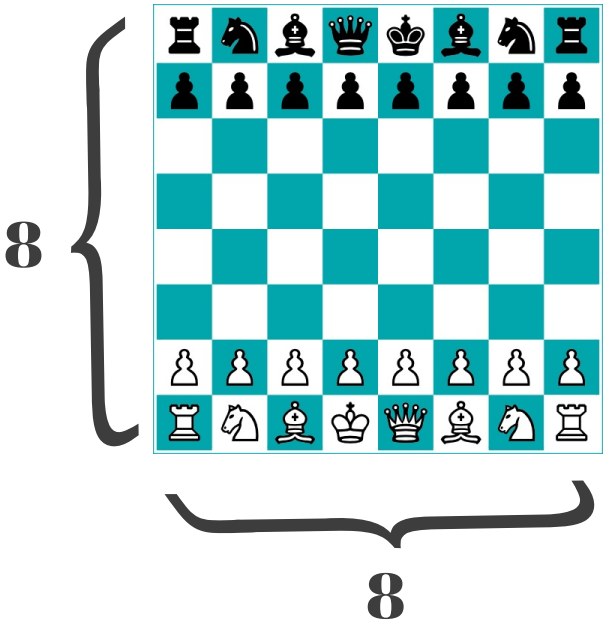


$$x^n = \underbrace{x \cdot x \cdot x \cdot \text{-----} \cdot x}_{n \text{ times}}$$



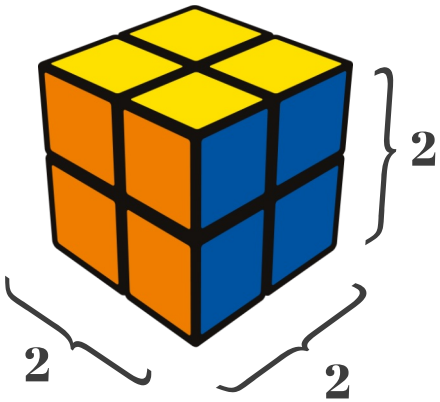
$$3^3 = \underbrace{3 \times 3 \times 3}_{3} = 27$$

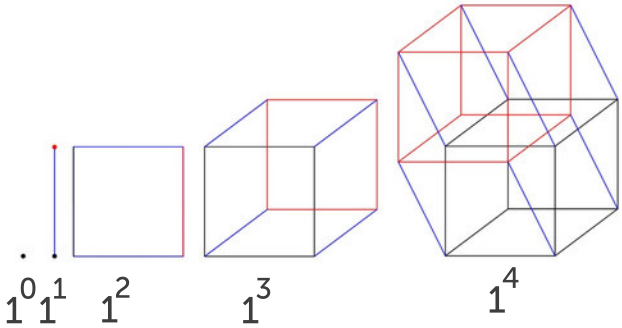
Examples

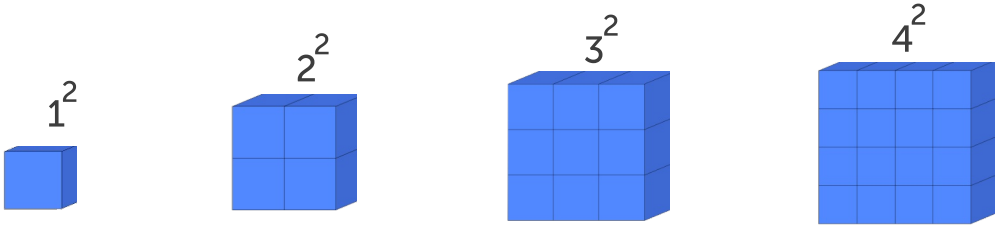


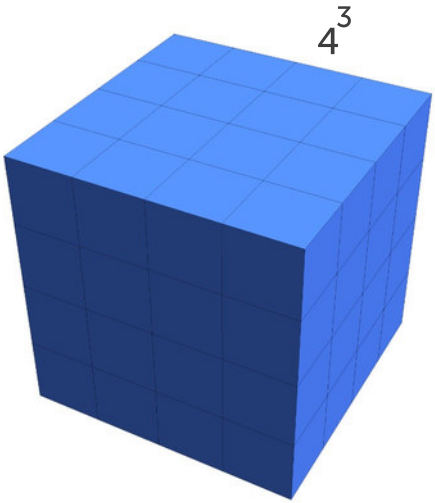
$8^2 = 8 \times 8 = 64$

$2^3 = 2 \times 2 \times 2 = 16$



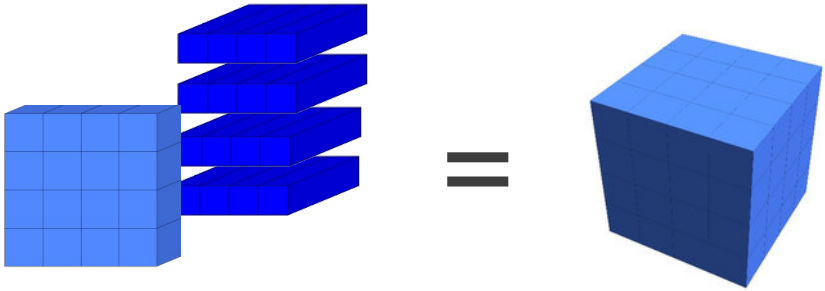

1^0 1^1 1^2 1^3 1^4


1^2 2^2 3^2 4^2


4^3

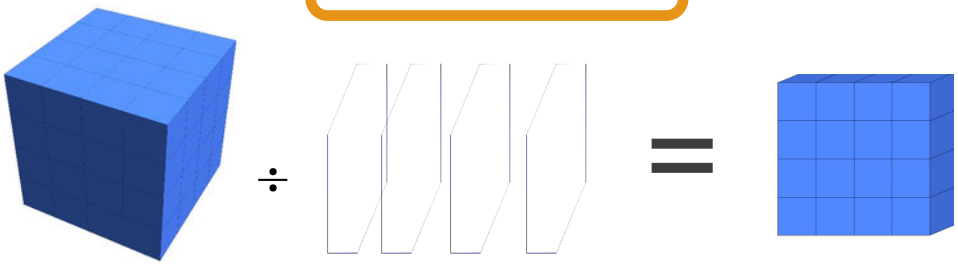
Properties of exponents

$$a^m \times a^n = a^{m+n}$$

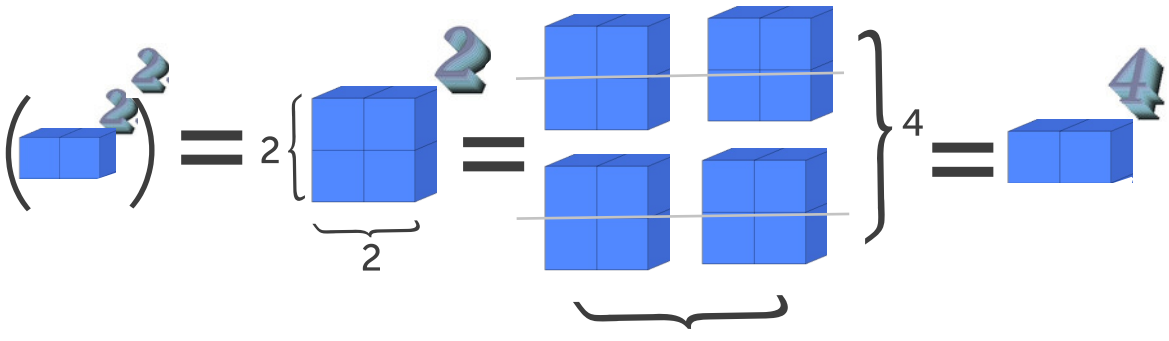


$$4^2 \times 4^1 = 16 \times 4 = 64 = 4^{2+1} = 4^3$$

$$a^m \div a^n = a^{m-n}$$

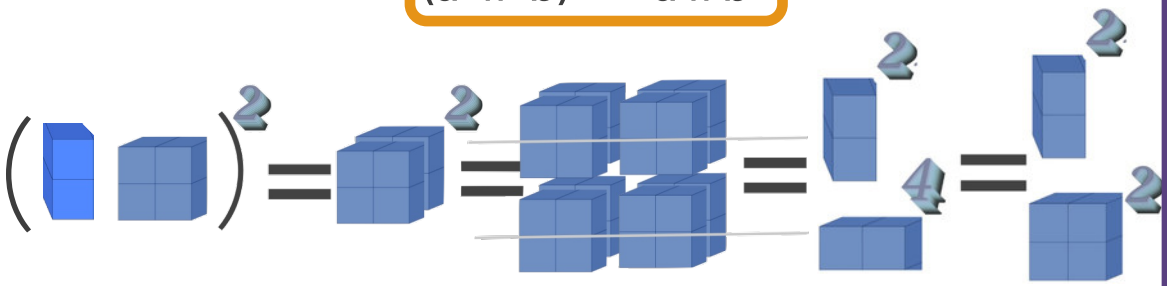


$$(a^m)^n = a^{m \times n}$$



$(2^2)^2 = 4^2 = 16 = 2^{2x2} = 2^4$

$$(a \times b)^n = a^n \times b^n$$



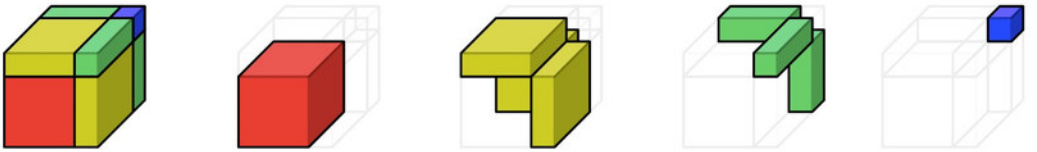
$(2 \times 4)^2 = 8^2 = 64 = 4 \times 16 = 2^2 \times 4^2$

Binomial formula

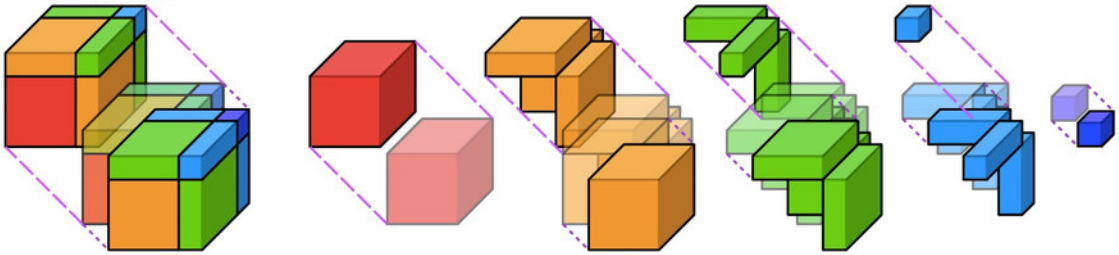
$(a+b)^1 = a + b$

$(a+b)^2 = a^2 + 2ab + b^2$

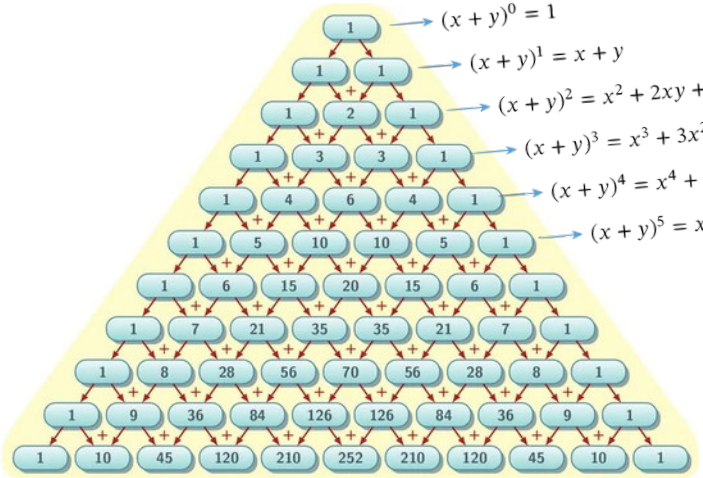
$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$



$(a+b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$



Pascal triangle



$(x+y)^0 = 1$

$(x+y)^1 = x+y$

$(x+y)^2 = x^2 + 2xy + y^2$

$(x+y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$

$(x+y)^4 = x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$

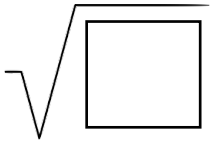
$(x+y)^5 = x^5 + 5x^4y + 10x^3y^2 + 10x^2y^3 + 5xy^4 + y^5$

...

Examples

$$(3+4)^2 = 3^2 + 2x3x4 + 4^2$$

Square root



Examples

$\sqrt{4} = 2$  2^2 or 2 x 2 = 4

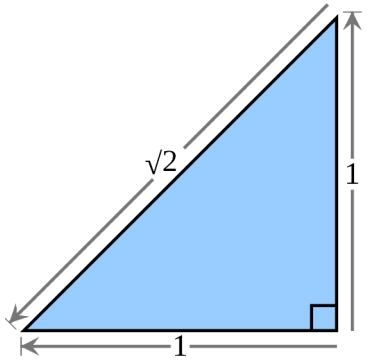
$\sqrt{9} = 3$  3^2 or 3 x 3 = 9

$\sqrt{16} = 4$  4^2 or 4 x 4 = 16

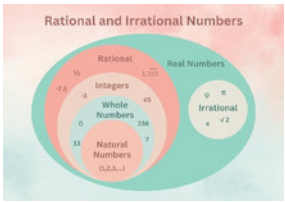
$\sqrt{25} = 5$  5^2 or 5 x 5 = 25

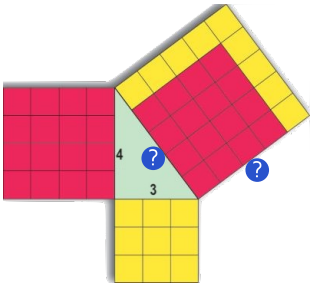
$\sqrt{36} = 6$  6^2 or 6 x 6 = 36

$\sqrt{1}$	= 1	$\sqrt{11}$	= 3.3166...
$\sqrt{2}$	= 1.4142 ...	$\sqrt{12}$	= 3.4641...
$\sqrt{3}$	= 1.732 ...	$\sqrt{13}$	= 3.6055...
$\sqrt{4}$	= 2	$\sqrt{14}$	= 3.7416...
$\sqrt{5}$	= 2.236 ...	$\sqrt{15}$	= 3.8729...
$\sqrt{6}$	= 2.4494 ...	$\sqrt{16}$	= 4
$\sqrt{7}$	= 2.6457...	$\sqrt{17}$	= 4.1231...
$\sqrt{8}$	= 2.8284...	$\sqrt{18}$	= 4.2426...
$\sqrt{9}$	= 3	$\sqrt{19}$	= 4.3588...
$\sqrt{10}$	= 3.1622...	$\sqrt{20}$	= 4.4721...



$\sqrt{2}$ is an irrational number.





? $= \sqrt{3^2 + 4^2}$

? $= \sqrt{9 + 16}$

? $= \sqrt{25}$

? $= 5$

Power in a nutshell