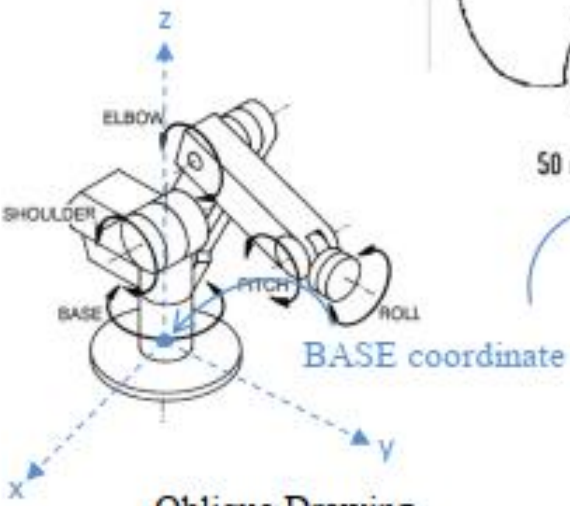
Robotics: Assignment II

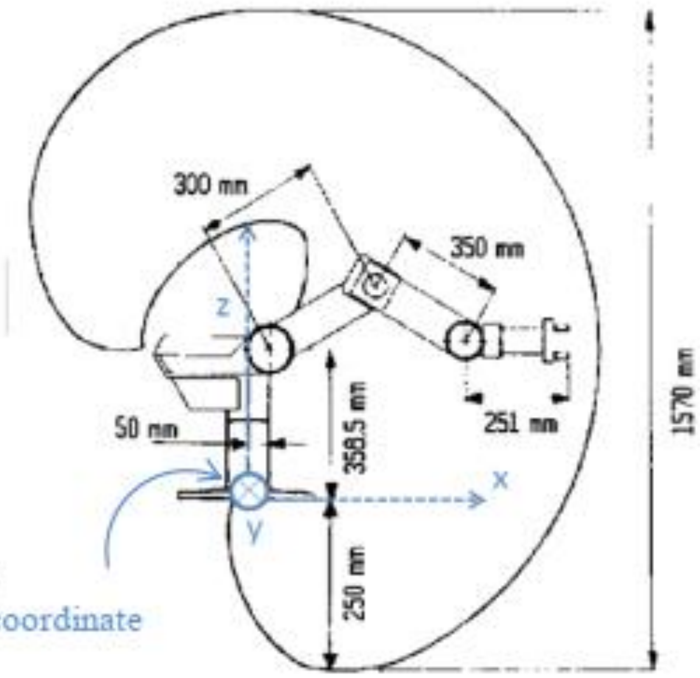
<u>Forward Kinematics and Inverse Kinematics</u>

Biomechatronics Engineering, Li-Wei Yang, B07611001

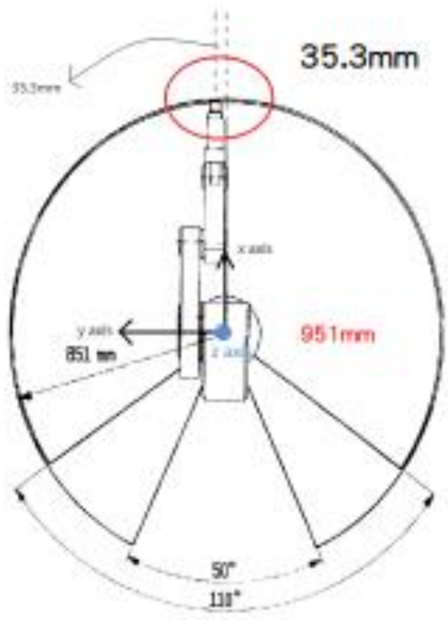
Consider the ER-7 robot arm shown in the following figures:



Oblique Drawing



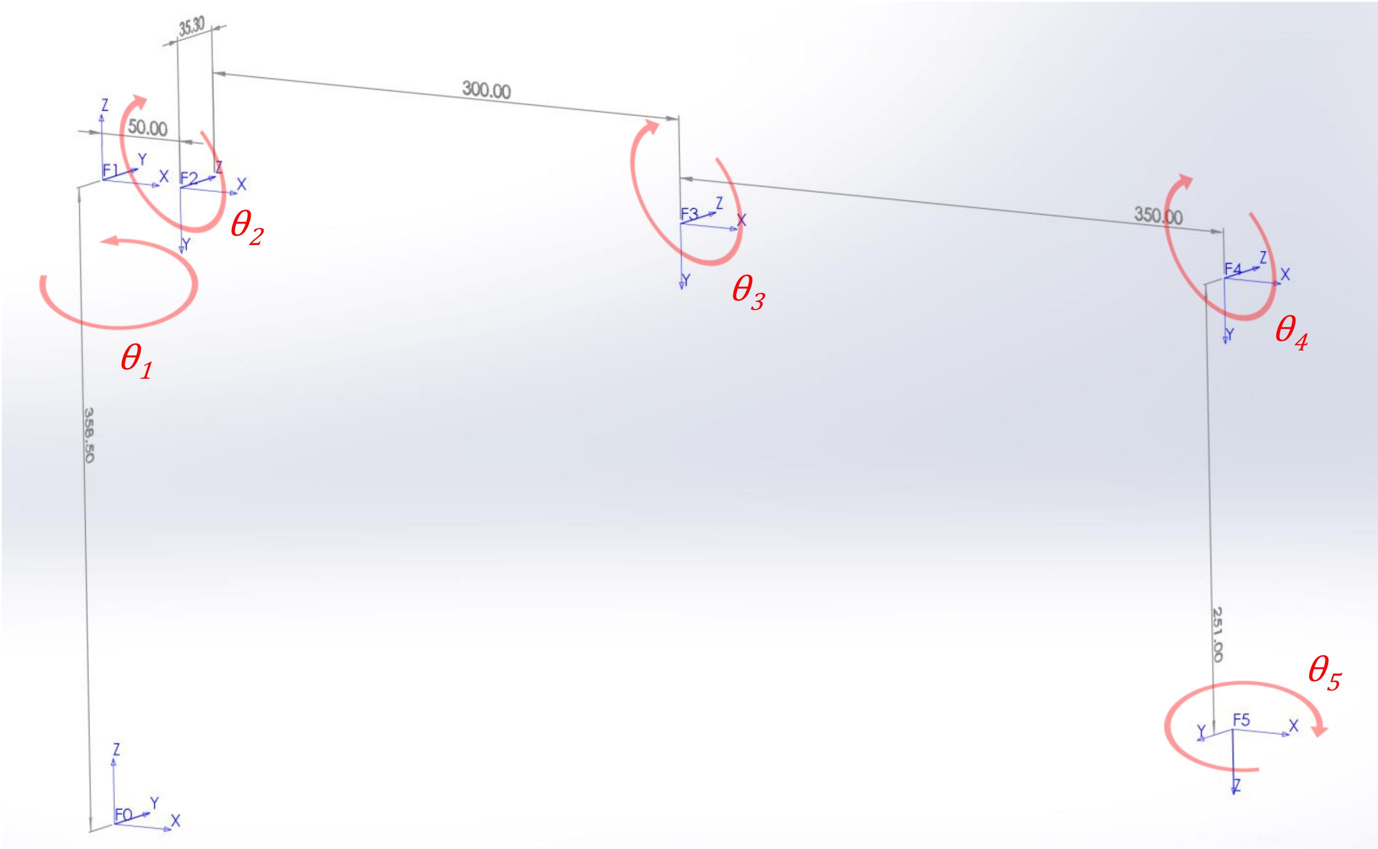
Lateral View



Top View

Part A (25%)

(1) According to ER-7 arm, draw the link coordinate diagram using D-H convention in Craig version from lecture slides page 34. (10%)



(2) Find the kinematics parameters of ER-7 and fill the table below: (15%)

ER-7				
Joint	$\alpha i-1$ (°)	$a i-1$ (mm)	$d i$ (mm)	θi
1	0	0	358.5	θ1
2	-90	50	0	θ2
3	0	300	35.3	θ3
4	0	350	0	θ4
5	-90	0	251	θ5

PART B (30%)

(1) Derive transformation matrices for each consecutive link, and also the transformation matrices T_5^0 (from frame 5 to frame 0).

Note: This should be revised to list all the transformation matrix, i.e. $T_1^{base}, T_2^1, T_3^2, T_4^3, T_5^4, T_5^{base}$.

T01

$$\begin{pmatrix} \cos(th_1) & -\sin(th_1) & 0 & 0 \\ \sin(th_1) & \cos(th_1) & 0 & 0 \\ 0 & 0 & 1 & \frac{717}{2} \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

T12

$$\begin{pmatrix} \cos(th_2) & -\sin(th_2) & 0 & 50 \\ 0 & 0 & 1 & 0 \\ -\sin(th_2) & -\cos(th_2) & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

T23

$$\begin{pmatrix} \cos(th_3) & -\sin(th_3) & 0 & 300 \\ \sin(th_3) & \cos(th_3) & 0 & 0 \\ 0 & 0 & 1 & \frac{353}{10} \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

T34

$$\begin{pmatrix} \cos(th_4) & -\sin(th_4) & 0 & 350 \\ \sin(th_4) & \cos(th_4) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

T45

$$\begin{pmatrix} \cos(th_5) & -\sin(th_5) & 0 & 0 \\ 0 & 0 & 1 & 251 \\ -\sin(th_5) & -\cos(th_5) & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

T05

$= {}^0r_{11} = C_5C_1C_{234} + S_1S_5$

${}^0r_{12} = C_5S_1 - S_5C_1C_{234}$

${}^0r_{13} = -C_1S_{234}$

${}^0r_{14} = 50C_1 - 35.3S_1 + 300C_1C_2 + 350C_1C_{23} - 251C_1S_{234}$

${}^0r_{21} = C_5S_1C_{234} - C_1S_5$

${}^0r_{22} = -S_5S_1C_{234} - C_1C_5$

${}^0r_{23} = -S_1S_{234}$

${}^0r_{24} = 50S_1 - 251S_1S_{234} + 300C_2S_1 + 350S_1C_{23} + 35.3C_1$

${}^0r_{31} = -C_5S_{234}$

${}^0r_{32} = S_5S_{234}$

${}^0r_{33} = -C_{234}$

${}^0r_{34} = -300S_2 - 251C_{234} - 350S_{23} + 358.5$

Part C (45%)

(1) Derive the inverse kinematics for ER-7. Given the target pose of the gripper tip (x, y, z, ϕ, θ, ψ) with respect to the base coordinate, calculate (θ1 , θ2 , θ3 , θ4 , θ5) . For the transformation from the base to the gripper tip, please refer to Inverse Kinematic slides. Let's assume the target is reachable in elbow-up configuration, and that the gripper tip pose is always vertically downward. (30%)

Because the tip pose is always vertically downward, the x y z is determined only by th1, th2, th3, and the z axis of frame 5 is always negative to the frame 0. So the x4 y4 z4 in T04 = x y z+251, with this correlation we can calculate th1 th2 th3. For elbow up configuration, we choose the [th1 th2 th3] set when th2 < 0 for afterward calculation.

$$50\cos(th_1) - \frac{353\sin(th_1)}{10} + 300\cos(th_1)\cos(th_2) - 350\cos(th_1)\sin(th_2)\sin(th_3) + 350\cos(th_1)\cos(th_2)\cos(th_3) = x$$

$$\frac{353\cos(th_1)}{10} + 50\sin(th_1) + 300\cos(th_2)\sin(th_1) - 350\sin(th_1)\sin(th_2)\sin(th_3) + 350\cos(th_2)\cos(th_3)\sin(th_1) = y$$

$$\frac{717}{2} - 350\cos(th_2)\sin(th_3) - 350\cos(th_3)\sin(th_2) - 300\sin(th_2) = z + 251$$

For th4, if we take some of the elements of T15 and do atan2, we can know θ2 + θ3 + θ4, as θ2 θ3 is already known, θ4 can be found by subtracting.

$${}^1r_{11} = c_5 c_{234}$$
$${}^1r_{12} = -s_5 c_{234}$$
$${}^1r_{13} = s_{234}$$
$${}^1r_{14} = 350 c_{23} - 251 s_{234} + 300 c_2 + 50$$
$${}^1r_{21} = -s_5$$
$${}^1r_{22} = -c_5$$
$${}^1r_{23} = 0$$
$${}^1r_{24} = 35.3$$
$${}^1r_{31} = -s_{234} c_5$$
$${}^1r_{32} = s_{234} s_5$$
$${}^1r_{33} = -1 \cdot [c_4 c_{23} - s_4 s_{23}] = -c_{234}$$
$${}^1r_{34} = -300 s_2 - 251 c_4 c_{23} + 251 s_4 s_{23} - 350 c_2 s_3 - 350 c_3 s_2$$
$$= -300 s_2 - 251 c_{234} - 350 s_{23}$$
$$\tan^{-1}\left(\frac{{}^1r_{13}}{-{}^1r_{33}}\right) = \theta_2 + \theta_3 + \theta_4$$

$$T_{15} = \begin{pmatrix} 0 & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 100 \\ -1 & 0 & 0 & 600 \\ 0 & -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & -\frac{519}{2} \\ 0 & 0 & 0 & ? \end{pmatrix}$$

$$\frac{\frac{\sqrt{2}}{2}}{-\frac{\sqrt{2}}{2}} = -1 \quad \tan^{-1}(-1)$$

$$\theta_2 + \theta_3 + \theta_4 = -\frac{\pi}{4}$$

$${}^1_5T$$

T15

```
th4_n = atan2(T15_subs(1,3),-T15_subs(3,3)) - S.th2(2,1) - S.th3(2,1);
```

For th5, because the tip is always vertically downward, the desired roll is done only by θ1 and θ5; so we can subtract θ1 from the roll and get θ5. As in my code written.

```
th5 = phi + S.th1(2,1);
```

Because the constraint of the question, the θ, ψ remain the same (0, pi).

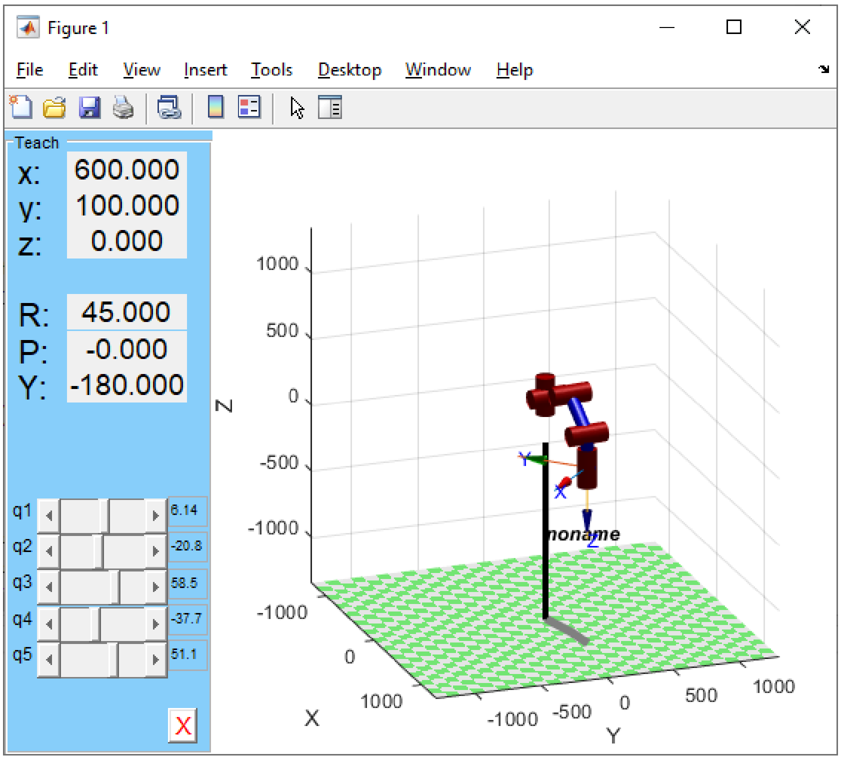
(2) Based on the previous question, please calculate (θ1 , θ2 , θ3 , θ4 , θ5) with the following target poses. The translation parameters (x, y, z) are in millimeter, and the rotation parameters (ϕ, θ, ψ) are in radian. (15%)

For more information please refer my MATLAB code.

(Please install Peter Corke's robotics toolbox add-in to get a better experience).

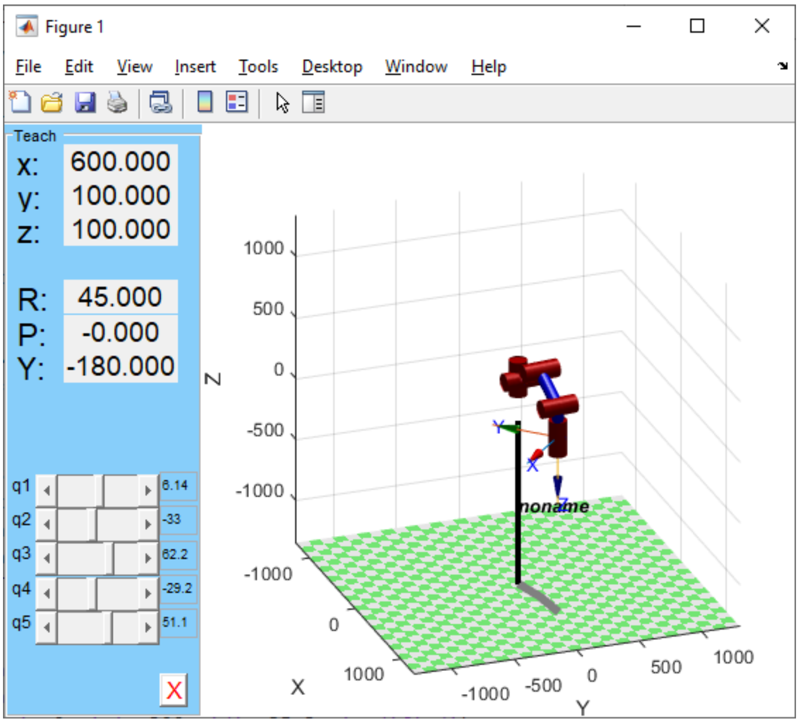
(1) $(x, y, z, \phi, \theta, \psi)$ = (600,100,0, $\pi/4$, 0, π)

1	2	3	4	5
0.1071	-0.3634	1.0218	-0.6584	0.8925



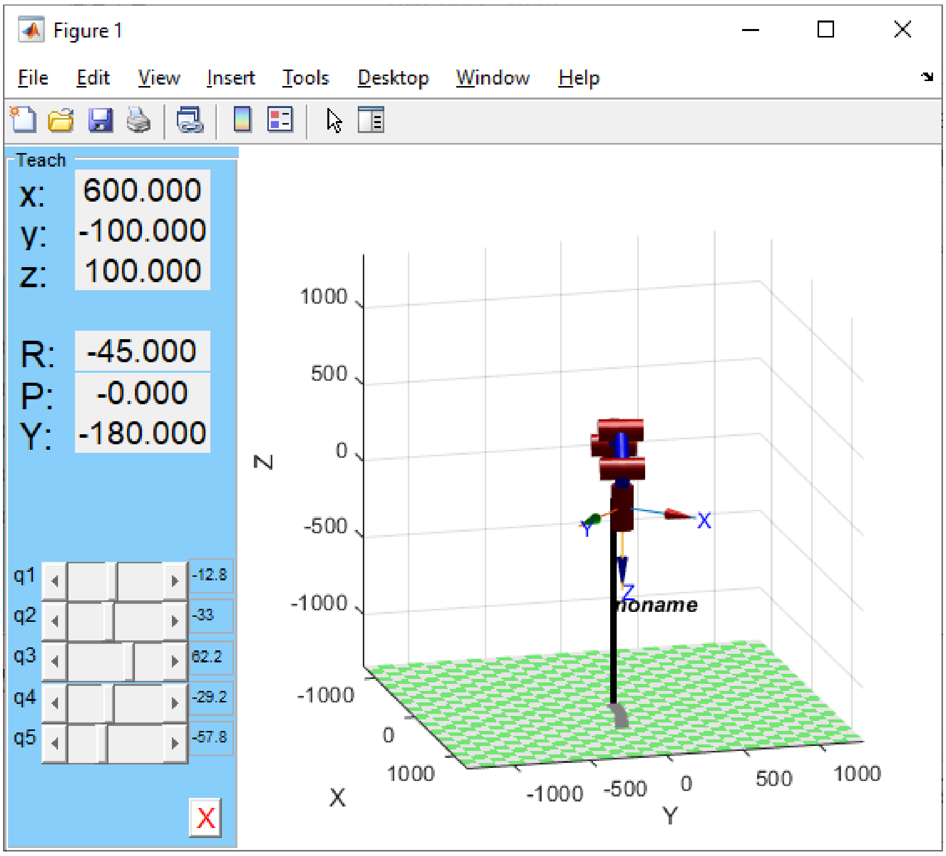
(2) $(x, y, z, \phi, \theta, \psi)$ = (600,100,100, $\pi/4$, 0, π)

1	2	3	4	5
0.1071	-0.5753	1.0848	-0.5095	0.8925



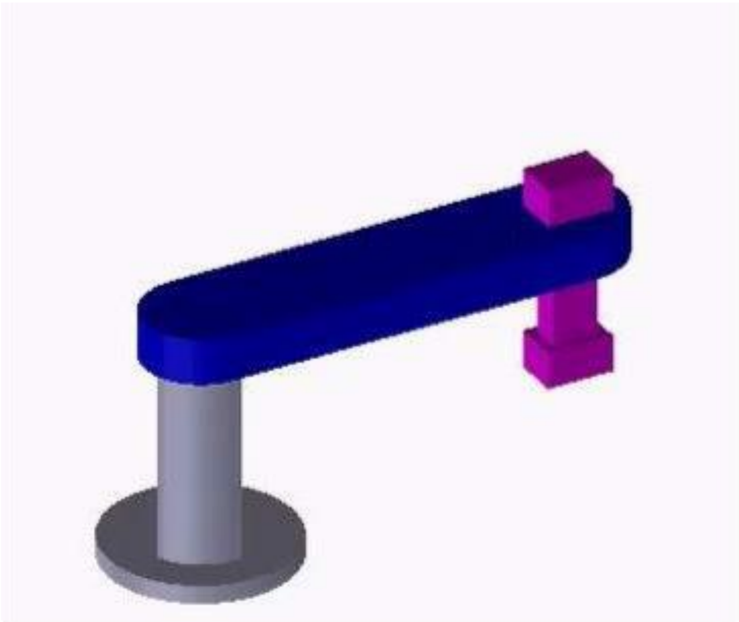
(3) $(x, y, z, \phi, \theta, \psi) = (600, -100, 100, -\pi/4, 0, \pi)$

1	2	3	4	5
-0.2232	-0.5753	1.0848	-0.5095	-1.0086

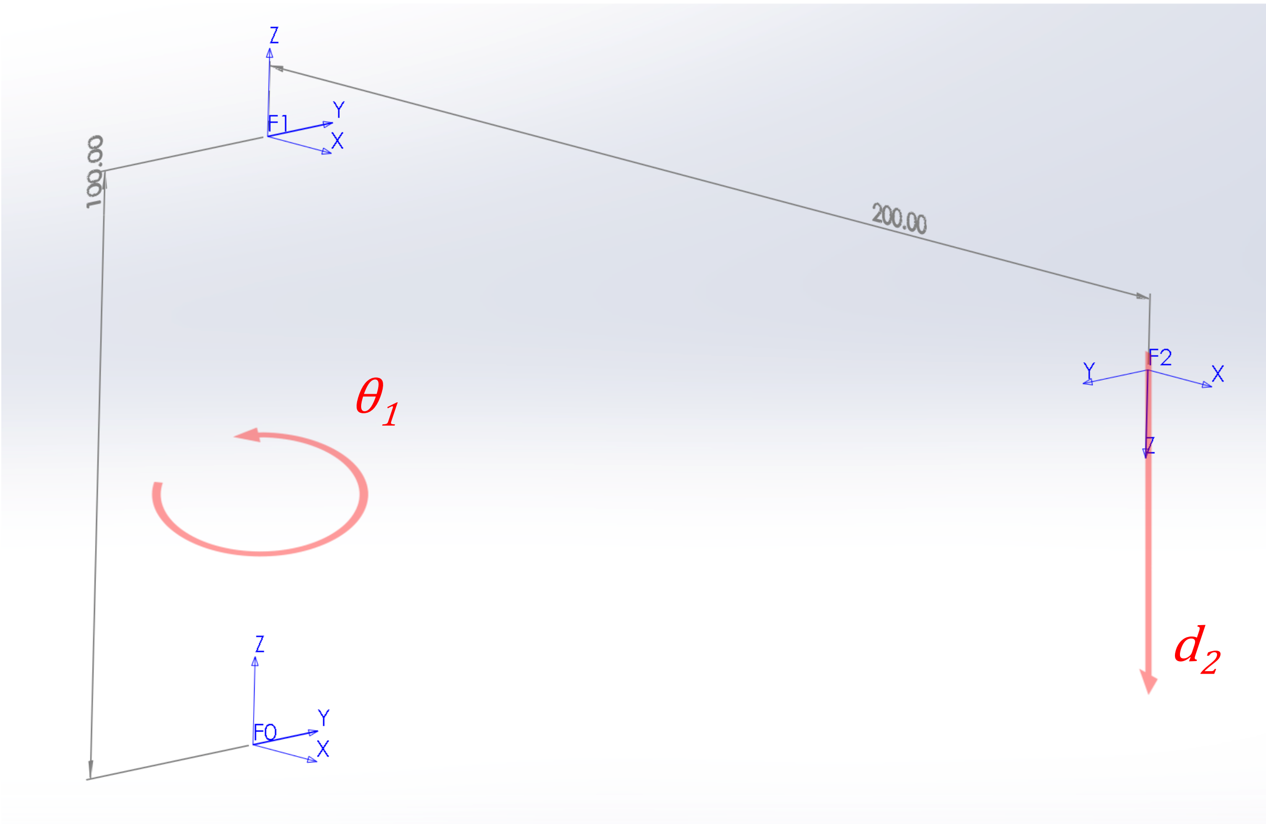


PART D (10%) bonus

Consider the following robot arm which is consist of a revolute joint and prismatic joint:



(1) Find the DH representation the same as Part A (1) (6%)



SCARA				
Joint	α_{i-1} (°)	a_{i-1} (mm)	d_i (mm)	θ_i
1	0	a1	0	θ_1
2	180	a2	d2	0

(2) For all DH parameters (α_{i-1}, a_{i-1}, d_i, θ_i), which two parameters are actuator joint (varying parameters)? (4%)

θ_1 and d_2 are varying parameters, since joint 1 is a revolute joint, and joint 2 is a prismatic joint.