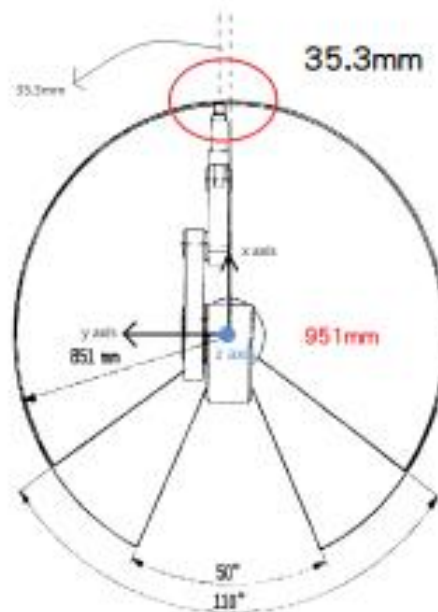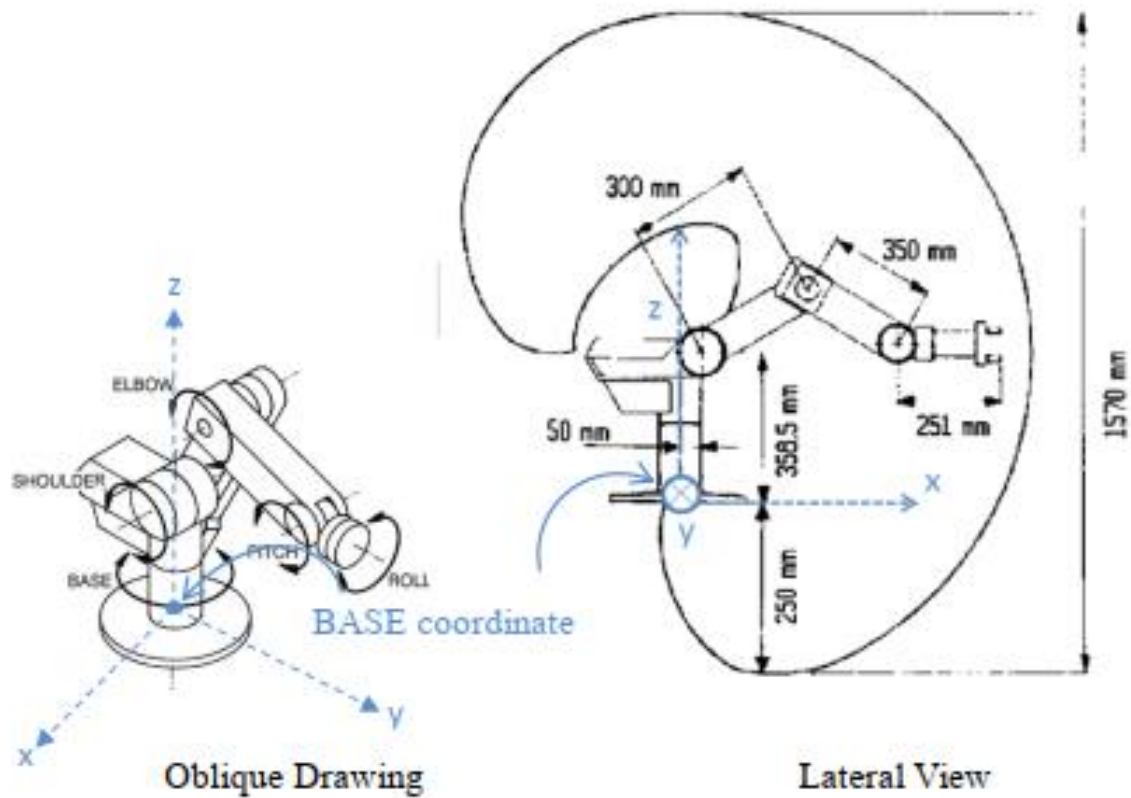# Robotics: Assignment II

Forward Kinematics and Inverse Kinematics

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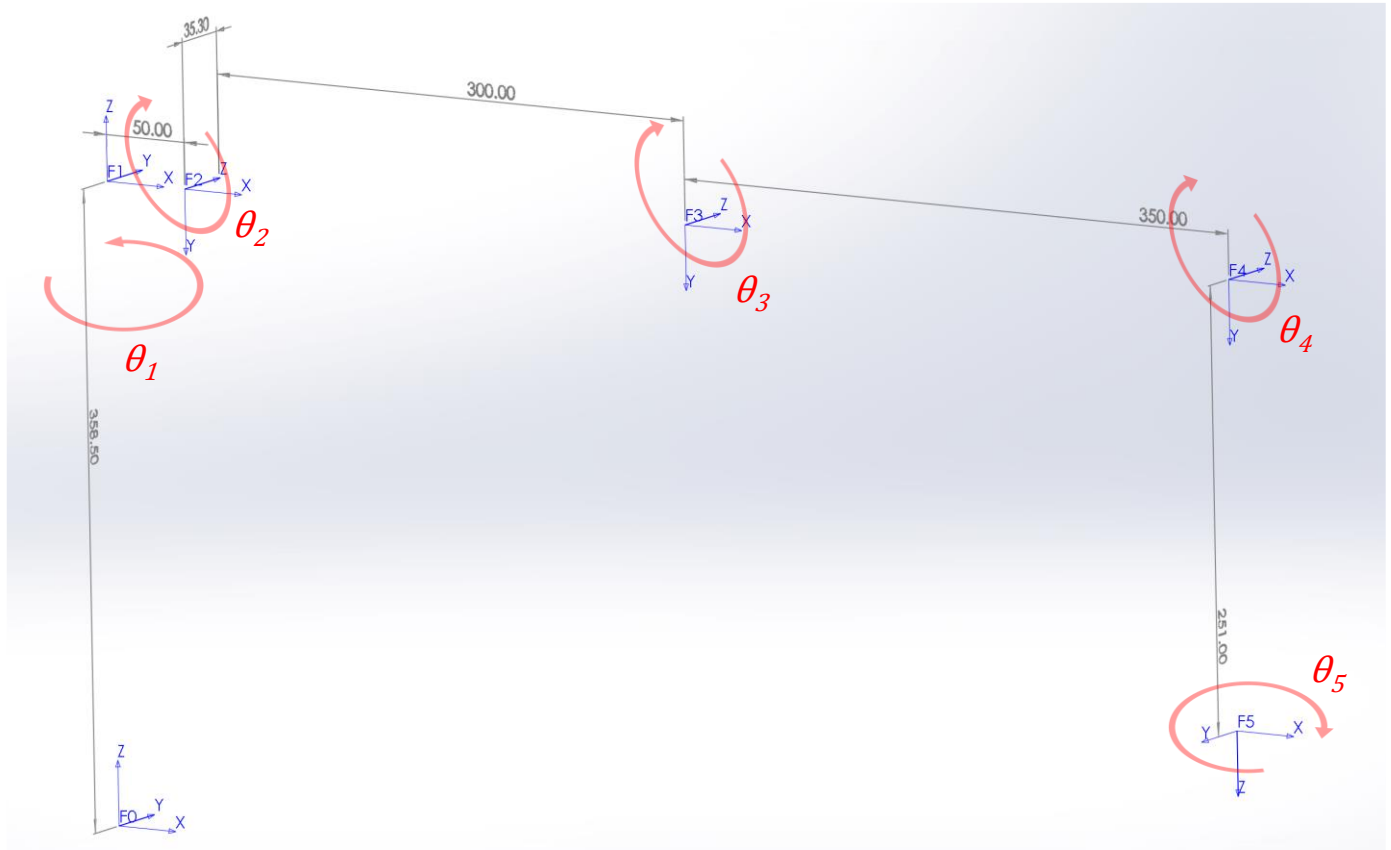
Consider the ER-7 robot arm shown in the following figures:

Oblique Drawing

Lateral View

Top View

Part A (25%)

(1) According to ER-7 arm, draw the link coordinate diagram using D-H convention in Craig version from lecture slides page 34. (10%)

(2) Find the kinematics parameters of ER-7 and fill the table below: (15%)

ER-7				
Joint	$\alpha i-1$ (°)	$ai-1$ (mm)	di (mm)	θi
1	0	0	358.5	$\theta 1$
2	-90	50	0	$\theta 2$
3	0	300	35.3	$\theta 3$
4	0	350	0	$\theta 4$
5	-90	0	251	$\theta 5$

PART B (30%)

(1) Derive transformation matrices for each consecutive link, and also the transformation matrices T_5^0 (from frame 5 to frame 0).

  Note: This should be revised to list all the transformation matrix, i.e. $T_1^{base}, T_2^1, T_3^2, T_4^3, T_5^4, T_5^{base}$.

T01
$$\begin{pmatrix} \cos(th_1) & -\sin(th_1) & 0 & 0 \\ \sin(th_1) & \cos(th_1) & 0 & 0 \\ 0 & 0 & 1 & \frac{717}{2} \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

T12
$$\begin{pmatrix} \cos(th_2) & -\sin(th_2) & 0 & 50 \\ 0 & 0 & 1 & 0 \\ -\sin(th_2) & -\cos(th_2) & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

T23
$$\begin{pmatrix} \cos(th_3) & -\sin(th_3) & 0 & 300 \\ \sin(th_3) & \cos(th_3) & 0 & 0 \\ 0 & 0 & 1 & \frac{353}{10} \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

T34
$$\begin{pmatrix} \cos(th_4) & -\sin(th_4) & 0 & 350 \\ \sin(th_4) & \cos(th_4) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

T45
$$\begin{pmatrix} \cos(th_5) & -\sin(th_5) & 0 & 0 \\ 0 & 0 & 1 & 251 \\ -\sin(th_5) & -\cos(th_5) & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

T05

$= O_{r_{11}} = C_5 C_1 C_{234} + S_1 S_5$

$O_{r_{12}} = C_5 S_1 - S_5 C_1 C_{234}$

$O_{r_{13}} = -C_1 S_{234}$

$O_{r_{14}} = 50 C_1 - 35.3 S_1 + 300 C_1 C_2 + 350 C_1 C_{23} - 251 C_1 S_{234}$

$O_{r_{21}} = C_5 S_1 C_{234} - C_1 S_5$

$O_{r_{22}} = -S_5 S_1 C_{234} - C_1 C_5$

$O_{r_{23}} = -S_1 S_{234}$

$O_{r_{24}} = 50 S_1 - 251 S_1 S_{234} + 300 C_2 S_1 + 350 S_1 C_{23} + 35.3 C_1$

$O_{r_{31}} = -C_5 S_{234}$

$O_{V_{32}} = S_5 S_{234}$

$O_{r_{33}} = -C_{234}$

$O_{r_{34}} = -300 S_2 - 251 C_{234} - 350 S_{23} + 358.5$

Part C (45%)

(1) Derive the inverse kinematics for ER-7. Given the target pose of the gripper tip (x, y, z, φ, θ, ψ) with respect to the base coordinate, calculate (θ1 , θ2 , θ3 , θ4 , θ5) . For the transformation from the base to the gripper tip, please refer to Inverse Kinematic slides. Let's assume the target is reachable in elbow-up configuration, and that the gripper tip pose is always vertically downward. (30%)

Because the tip pose is always vertically downward, the x y z is determined only by th1, th2, th3, and the z axis of frame 5 is always negative to the frame 0. So the x4 y4 z4 in T04 = x y z+251, with this correlation we can calculate th1 th2 th3. For elbow up configuration, we choose the [th1 th2 th3] set when th2 < 0 for afterward calculation.

$$50\cos(\text{th}_1) - \frac{353\sin(\text{th}_1)}{10} + 300\cos(\text{th}_1)\cos(\text{th}_2) - 350\cos(\text{th}_1)\sin(\text{th}_2)\sin(\text{th}_3) + 350\cos(\text{th}_1)\cos(\text{th}_2)\cos(\text{th}_3) = X$$

$$\frac{353\cos(\text{th}_1)}{10} + 50\sin(\text{th}_1) + 300\cos(\text{th}_2)\sin(\text{th}_1) - 350\sin(\text{th}_1)\sin(\text{th}_2)\sin(\text{th}_3) + 350\cos(\text{th}_2)\cos(\text{th}_3)\sin(\text{th}_1) = Y$$

$$\frac{717}{2} - 350\cos(\text{th}_2)\sin(\text{th}_3) - 350\cos(\text{th}_3)\sin(\text{th}_2) - 300\sin(\text{th}_2) = Z + 251$$

For th4, if we take some of the elements of T15 and do atan2, we can know θ2 + θ3 + θ4, as θ2 θ3 is already known, θ4 can be found by subtracting.


```
th4_n = atan2(T15_subs(1,3),-T15_subs(3,3)) - S.th2(2,1) - S.th3(2,1);
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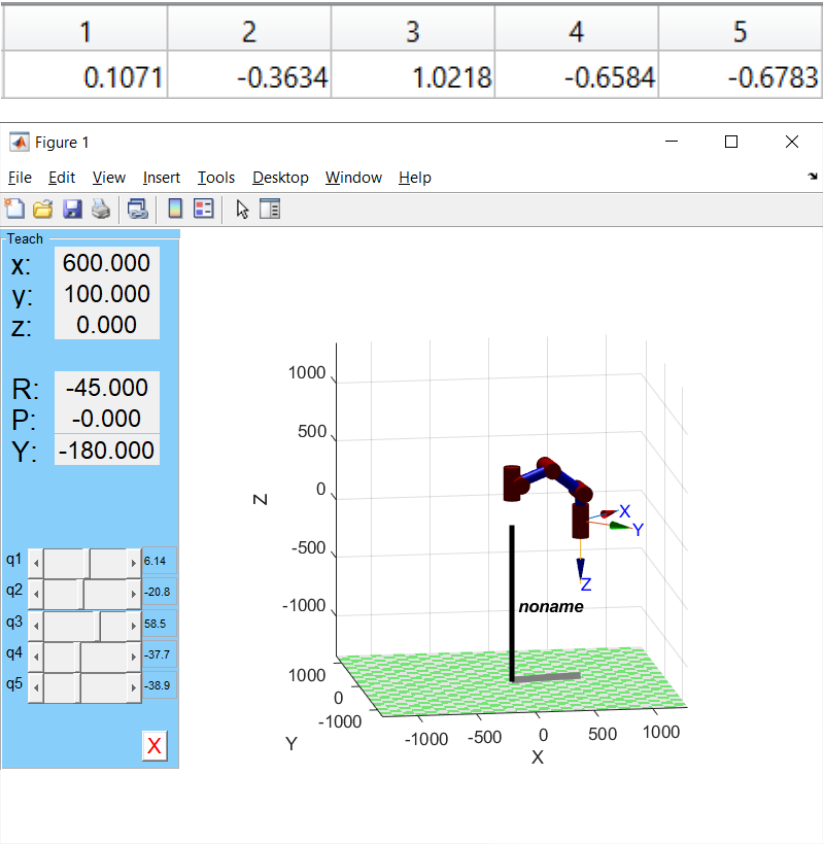
For th5, because the tip is always vertically downward, the desired roll is done only by θ1 and θ5; so we can subtract θ1 from the roll and get θ5. As in my code written.
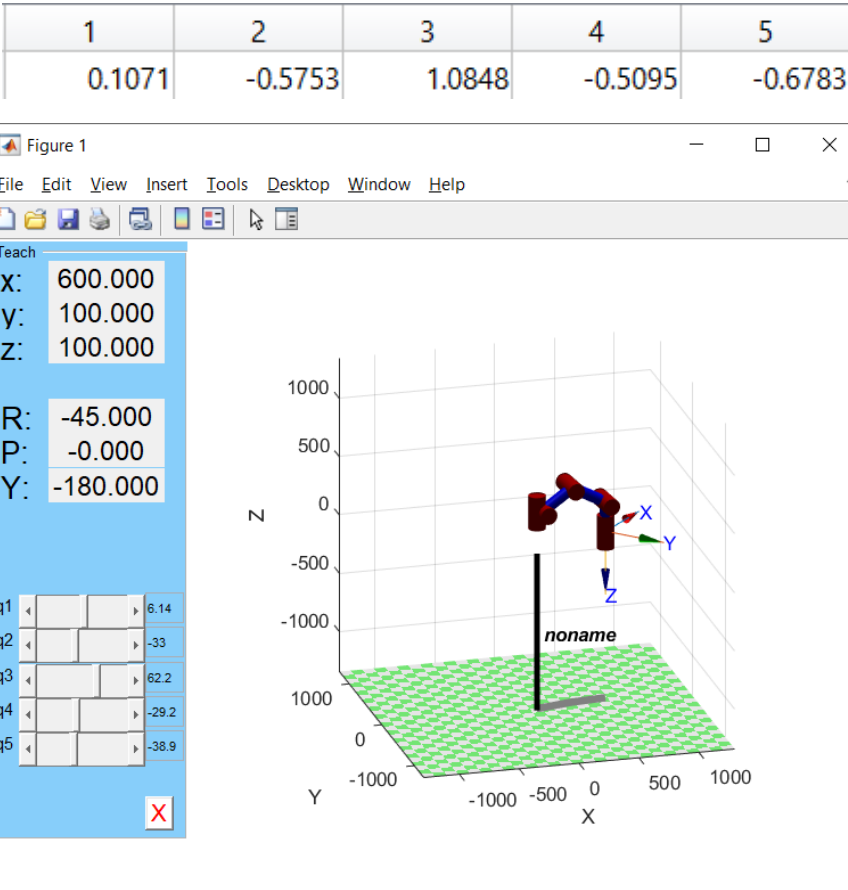
```
th5 = -phi + S.th1(2,1);
```

Because the constraint of the question, the θ, ψ remain the same (0, pi).

(2) Based on the previous question, please calculate (θ_1, θ_2, θ_3, θ_4, θ_5) with the following target poses. The translation parameters (x, y, z) are in millimeter, and the rotation parameters (ϕ, θ, ψ) are in radian. (15%)

For more information please refer my MATLAB code.
(Please install Peter Corke's robotics toolbox add-in to get a better experience).
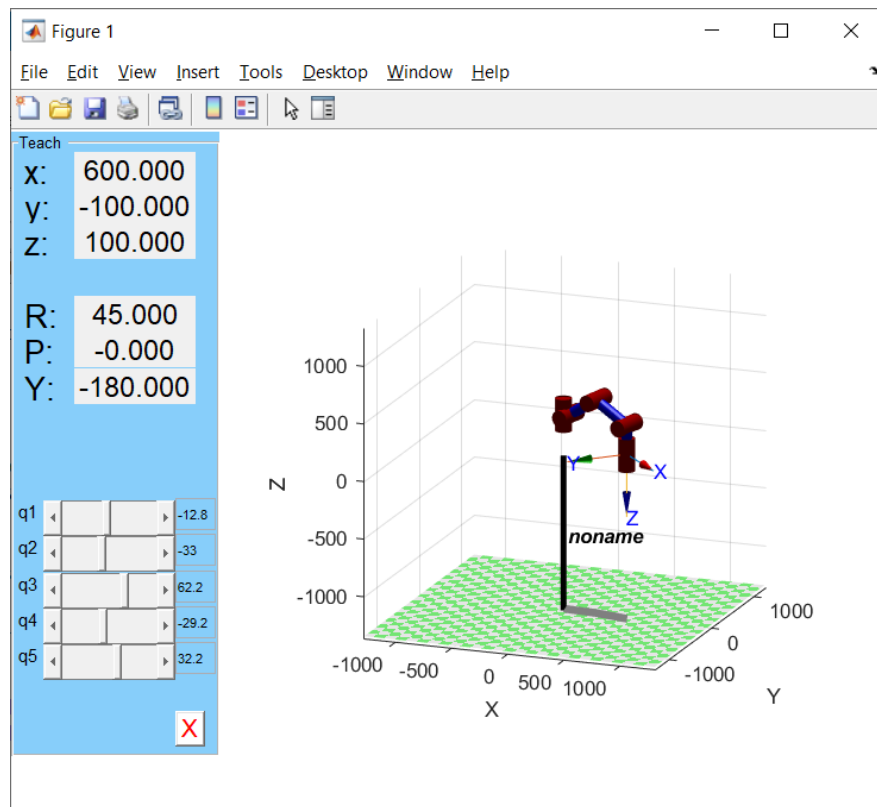(1) (x, y, z, ϕ, θ, ψ) = (600,100,0, $\pi/4$, 0, π)

1	2	3	4	5
0.1071	-0.3634	1.0218	-0.6584	-0.6783

(2) (x, y, z, ϕ, θ, ψ) = (600,100,100, $\pi/4$, 0, π)

1	2	3	4	5
0.1071	-0.5753	1.0848	-0.5095	-0.6783

(3) $(x, y, z, \phi, \theta, \psi) = (600, -100, 100, -\pi/4, 0, \pi)$

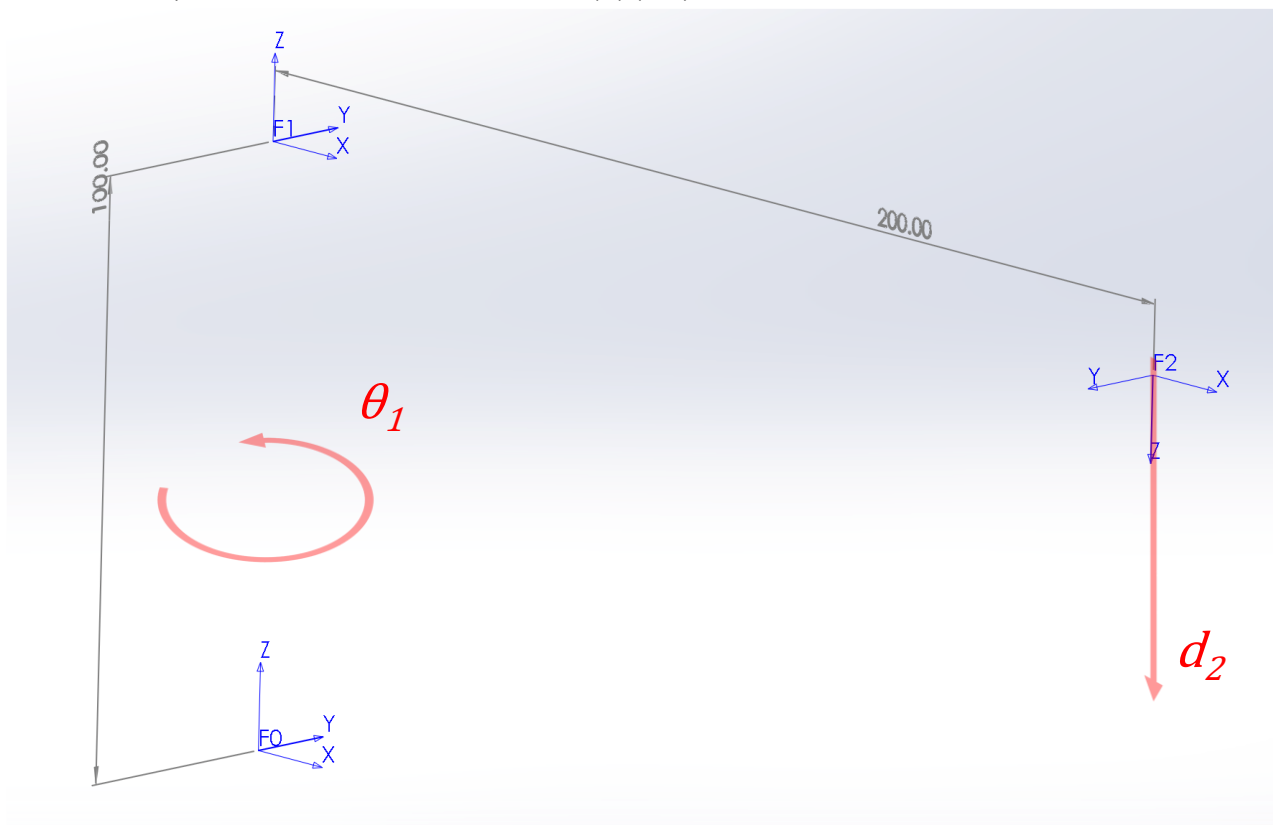| 1	2	3	4	5
-0.2232	-0.5753	1.0848	-0.5095	0.5622

PART D (10%) bonus

Consider the following robot arm which is consist of a revolute joint and prismatic joint:
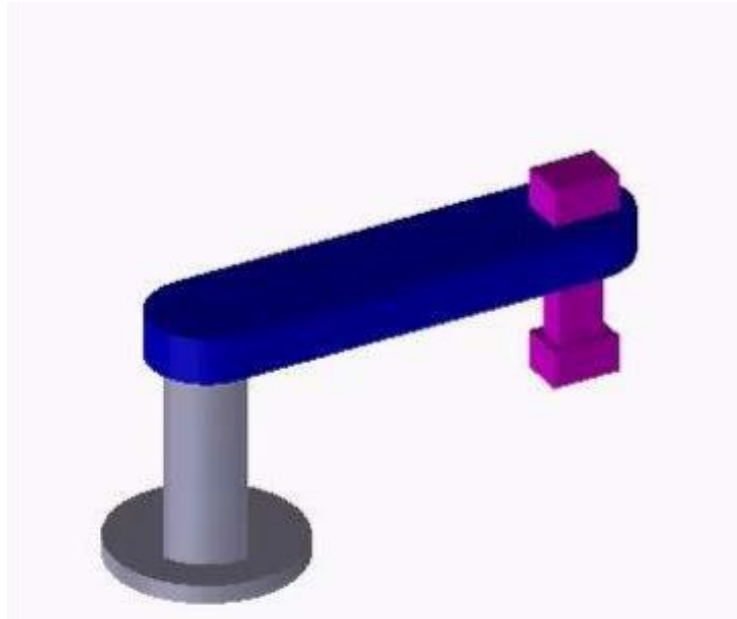
(1) Find the DH representation the same as Part A (1) (6%)

SCARA				
Joint	α_{i-1} (°)	a_{i-1} (mm)	d_i (mm)	θ_i
1	0	a1	0	θ_1
2	180	a2	d2	0

(2) For all DH parameters (α_{i-1}, a_{i-1}, d_i, θ_i), which two parameters are actuator joint (varying parameters)? (4%)

   θ_1 and d_2 are varying parameters, since joint 1 is a revolute joint, and joint 2 is a prismatic joint.