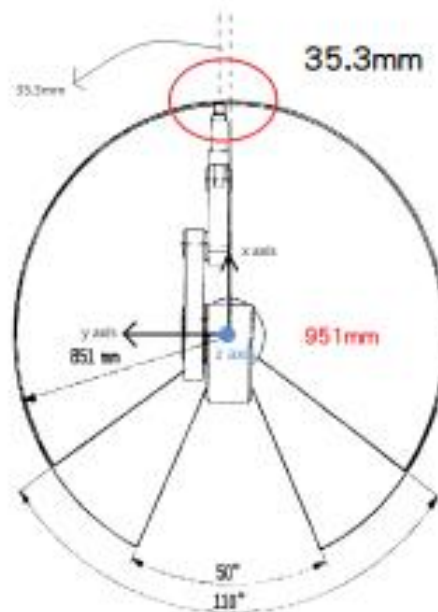
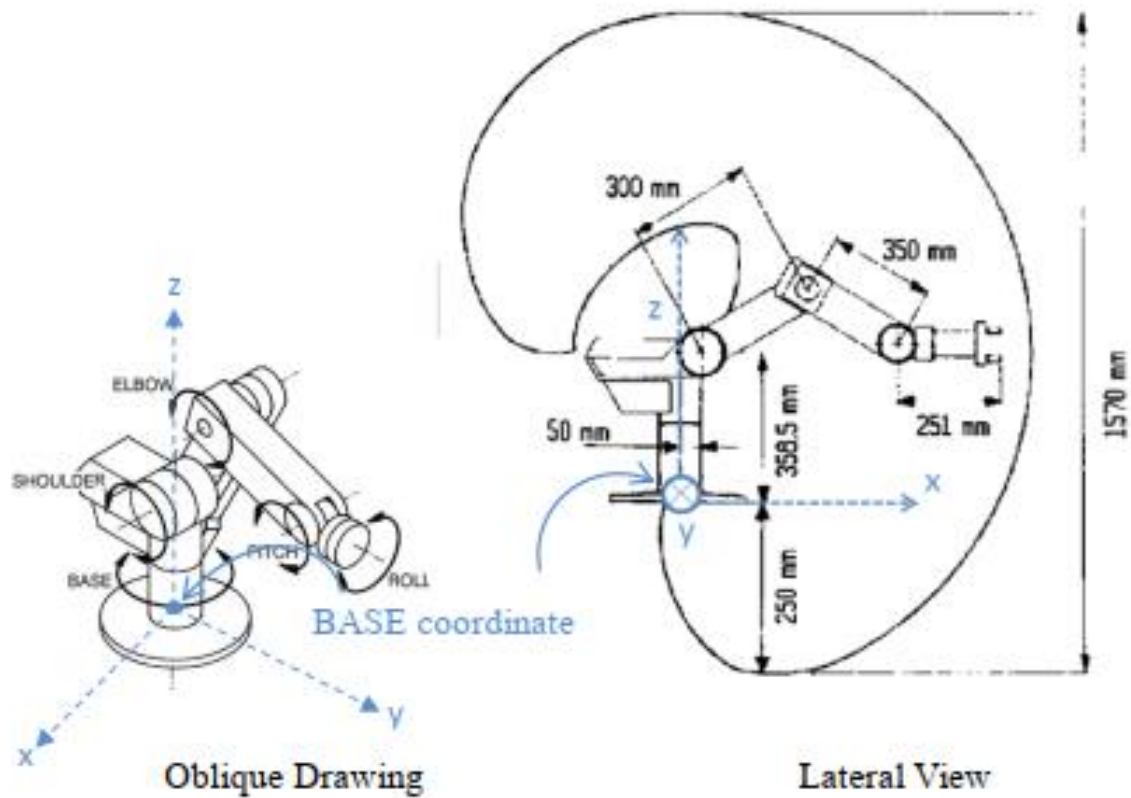


Robotics: Assignment II

Forward Kinematics and Inverse Kinematics

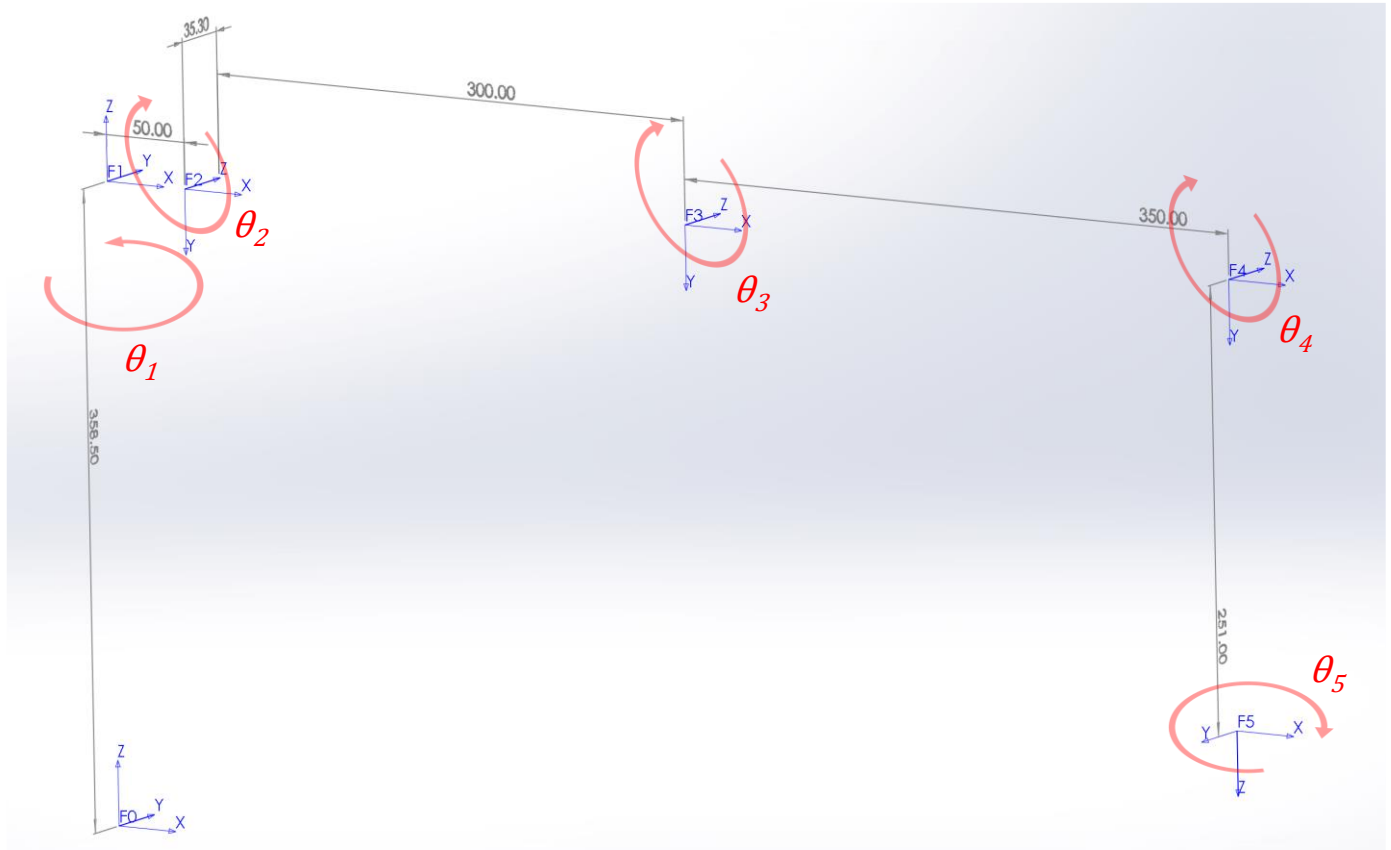
Biomechatronics Engineering, Li-Wei Yang, B07611001

Consider the ER-7 robot arm shown in the following figures:



Part A (25%)

(1) According to ER-7 arm, draw the link coordinate diagram using D-H convention in Craig version from lecture slides page 34. (10%)



(2) Find the kinematics parameters of ER-7 and fill the table below: (15%)

Joint	α_{i-1} (°)	a_{i-1} (mm)	d_i (mm)	θ_i
1	0	0	358.5	θ_1
2	-90	50	0	θ_2
3	0	300	35.3	θ_3
4	0	350	0	θ_4
5	-90	0	251	θ_5

PART B (30%)

(1) Derive transformation matrices for each consecutive link, and also the transformation matrices T_5^0 (from frame 5 to frame 0).

Note: This should be revised to list all the transformation matrix, i.e. $T_1^{base}, T_2^1, T_3^2, T_4^3, T_5^4, T_5^{base}$.

$$T_{01} = \begin{pmatrix} \cos(\theta_1) & -\sin(\theta_1) & 0 & 0 \\ \sin(\theta_1) & \cos(\theta_1) & 0 & 0 \\ 0 & 0 & 1 & \frac{717}{2} \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$T_{12} = \begin{pmatrix} \cos(\theta_2) & -\sin(\theta_2) & 0 & 50 \\ 0 & 0 & 1 & 0 \\ -\sin(\theta_2) & -\cos(\theta_2) & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$T_{23} = \begin{pmatrix} \cos(\theta_3) & -\sin(\theta_3) & 0 & 300 \\ \sin(\theta_3) & \cos(\theta_3) & 0 & 0 \\ 0 & 0 & 1 & \frac{353}{10} \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$T_{34} = \begin{pmatrix} \cos(\theta_4) & -\sin(\theta_4) & 0 & 350 \\ \sin(\theta_4) & \cos(\theta_4) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$T_{45} = \begin{pmatrix} \cos(\theta_5) & -\sin(\theta_5) & 0 & 0 \\ 0 & 0 & 1 & 251 \\ -\sin(\theta_5) & -\cos(\theta_5) & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

T05

$$\begin{aligned} O_{r11} &= C_1 C_2 C_3 + S_1 S_3 \\ O_{r12} &= C_1 S_2 C_3 + S_1 C_3 \\ O_{r13} &= -C_1 S_3 \\ O_{r14} &= S_0 C_1 - 35.3 S_1 + 300 C_1 C_2 + 350 C_1 C_3 - 251 C_1 C_2 C_3 \\ O_{r21} &= C_2 S_1 C_3 - C_1 S_3 \\ O_{r22} &= -S_2 S_1 C_3 - C_1 C_3 \\ O_{r23} &= -S_1 S_3 \\ O_{r24} &= S_0 S_1 - 251 S_1 C_2 + 300 C_2 S_1 + 350 S_1 C_3 + 35.3 C_1 \\ O_{r31} &= -C_2 S_3 \\ O_{r32} &= S_2 S_3 \\ O_{r33} &= -C_2 C_3 \\ O_{r34} &= -300 S_2 - 251 C_2 C_3 - 350 S_2 C_3 + 35.3 S_1 \end{aligned}$$

Part C (45%)

(1) Derive the inverse kinematics for ER-7. Given the target pose of the gripper tip ($x, y, z, \phi, \theta, \psi$) with respect to the base coordinate, calculate ($\theta_1, \theta_2, \theta_3, \theta_4, \theta_5$). For the transformation from the base to the gripper tip, please refer to Inverse Kinematic slides. Let's assume the target is reachable in elbow-up configuration, and that the gripper tip pose is always vertically downward. (30%)

Because the tip pose is always vertically downward, the x, y, z is determined only by $\theta_1, \theta_2, \theta_3$, and the z axis of frame 5 is always negative to the frame 0. So the x_4, y_4, z_4 in $T_{04} = x, y, z + 251$, with this correlation we can calculate $\theta_1, \theta_2, \theta_3$. For elbow up configuration, we choose the $[\theta_1, \theta_2, \theta_3]$ set when $\theta_2 < 0$ for afterward calculation.

$$50 \cos(\theta_1) - \frac{353 \sin(\theta_1)}{10} + 300 \cos(\theta_1) \cos(\theta_2) - 350 \cos(\theta_1) \sin(\theta_2) \sin(\theta_3) + 350 \cos(\theta_1) \cos(\theta_2) \cos(\theta_3) = x$$

$$\frac{353 \cos(\theta_1)}{10} + 50 \sin(\theta_1) + 300 \cos(\theta_2) \sin(\theta_1) - 350 \sin(\theta_1) \sin(\theta_2) \sin(\theta_3) + 350 \cos(\theta_2) \cos(\theta_3) \sin(\theta_1) = y$$

$$\frac{717}{2} - 350 \cos(\theta_2) \sin(\theta_3) - 350 \cos(\theta_3) \sin(\theta_2) - 300 \sin(\theta_2) = z + 251$$

For θ_4 , if we take some of the elements of T_{15} and do atan2 , we can know $\theta_2 + \theta_3 + \theta_4$, as θ_2, θ_3 is already known, θ_4 can be found by subtracting.

Handwritten derivation of the inverse kinematics for ER-7, showing the calculation of θ_4 from the transformation matrix T_{15} .

Transformation matrix T_{15} is given as:

$$T_{15} = \begin{pmatrix} 0 & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 100 \\ -1 & 0 & \frac{\sqrt{2}}{2} & 0 \\ 0 & -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & -517 \\ 0 & 0 & \frac{\sqrt{2}}{2} & 0 \end{pmatrix}$$

Key equations derived:

- $r_{11} = c_5 c_{234}$
- $r_{12} = -s_5 c_{234}$
- $r_{13} = s_{234}$
- $r_{14} = 350 c_{23} - 251 s_{234} + 300 c_2 + 50$
- $r_{21} = -c_5$
- $r_{22} = -s_5$
- $r_{23} = 0$
- $r_{24} = 35.3$ (crossed out)
- $r_{31} = -s_{234} c_5$
- $r_{32} = c_{234} c_5$
- $r_{33} = -1 \cdot [c_4 c_{23} - s_4 s_{23}] = -c_{234}$
- $r_{34} = -300 s_2 - 251 c_4 c_{23} + 251 s_4 s_{23} - 350 c_2 c_3 - 350 c_3 c_2$
- $r_{34} = -300 s_2 - 251 c_{234} - 350 s_{23}$

The final equation for θ_4 is:

$$\tan^{-1}\left(\frac{r_{13}}{-r_{33}}\right) = \theta_2 + \theta_3 + \theta_4$$

$$\theta_{4_n} = \text{atan2}(T_{15_subs}(1,3), -T_{15_subs}(3,3)) - S.\theta_2(2,1) - S.\theta_3(2,1);$$

For θ_5 , because the tip is always vertically downward, the desired roll is done only by θ_1 and θ_5 ; so we can add θ_1 and the roll to get θ_5 . As in my code written.

$$\theta_5 = \phi + S.\theta_1(2,1);$$

Because the constraint of the question, the θ, ψ remain the same (0, π).

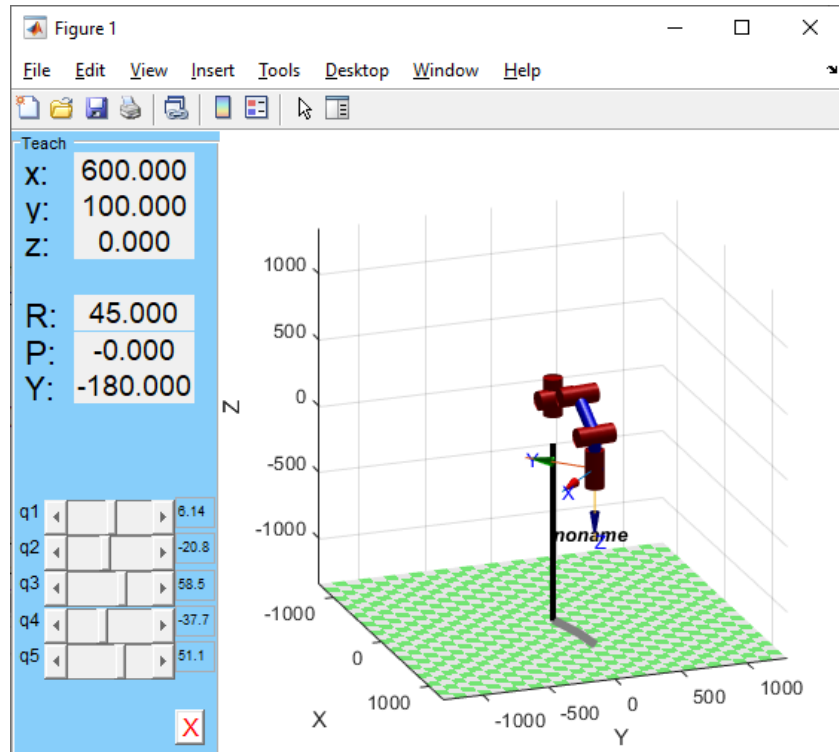
(2) Based on the previous question, please calculate $(\theta_1, \theta_2, \theta_3, \theta_4, \theta_5)$ with the following target poses. The translation parameters (x, y, z) are in millimeter, and the rotation parameters (ϕ, θ, ψ) are in radian. (15%)

For more information please refer my MATLAB code.

(Please install Peter Corke's robotics toolbox add-in to get a better experience).

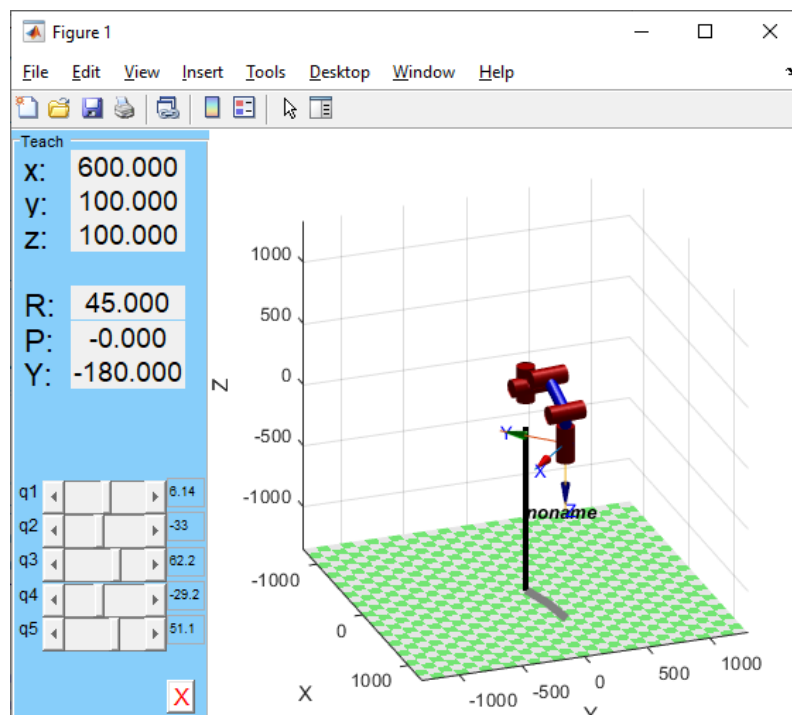
(1) $(x, y, z, \phi, \theta, \psi) = (600, 100, 0, \pi/4, 0, \pi)$

1	2	3	4	5
0.1071	-0.3634	1.0218	-0.6584	0.8925



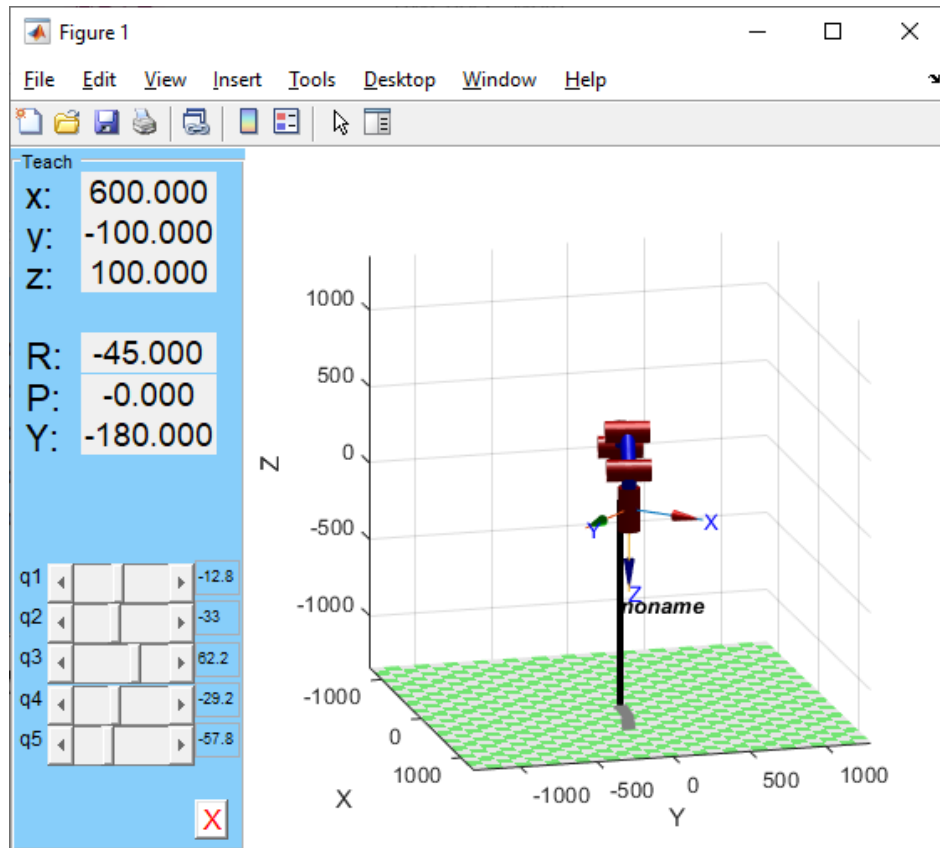
(2) $(x, y, z, \phi, \theta, \psi) = (600, 100, 100, \pi/4, 0, \pi)$

1	2	3	4	5
0.1071	-0.5753	1.0848	-0.5095	0.8925



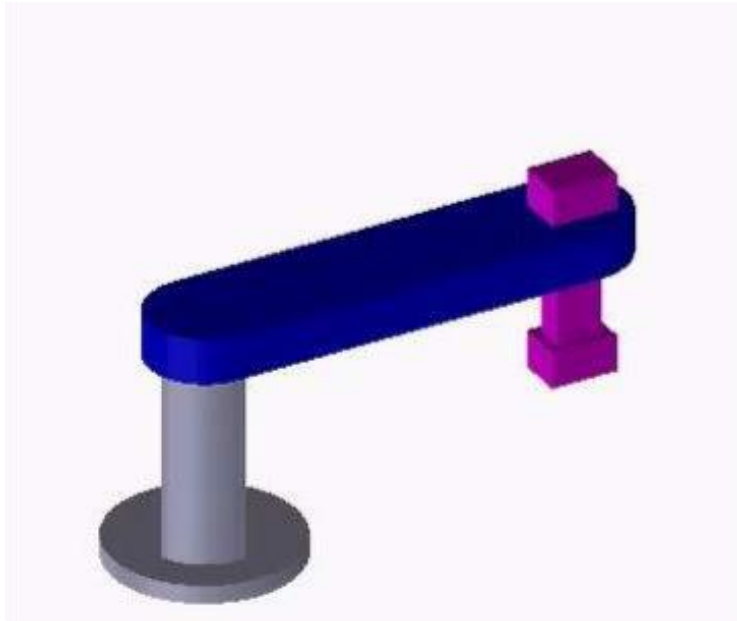
(3) $(x, y, z, \phi, \theta, \psi) = (600, -100, 100, -\pi/4, 0, \pi)$

1	2	3	4	5
-0.2232	-0.5753	1.0848	-0.5095	-1.0086

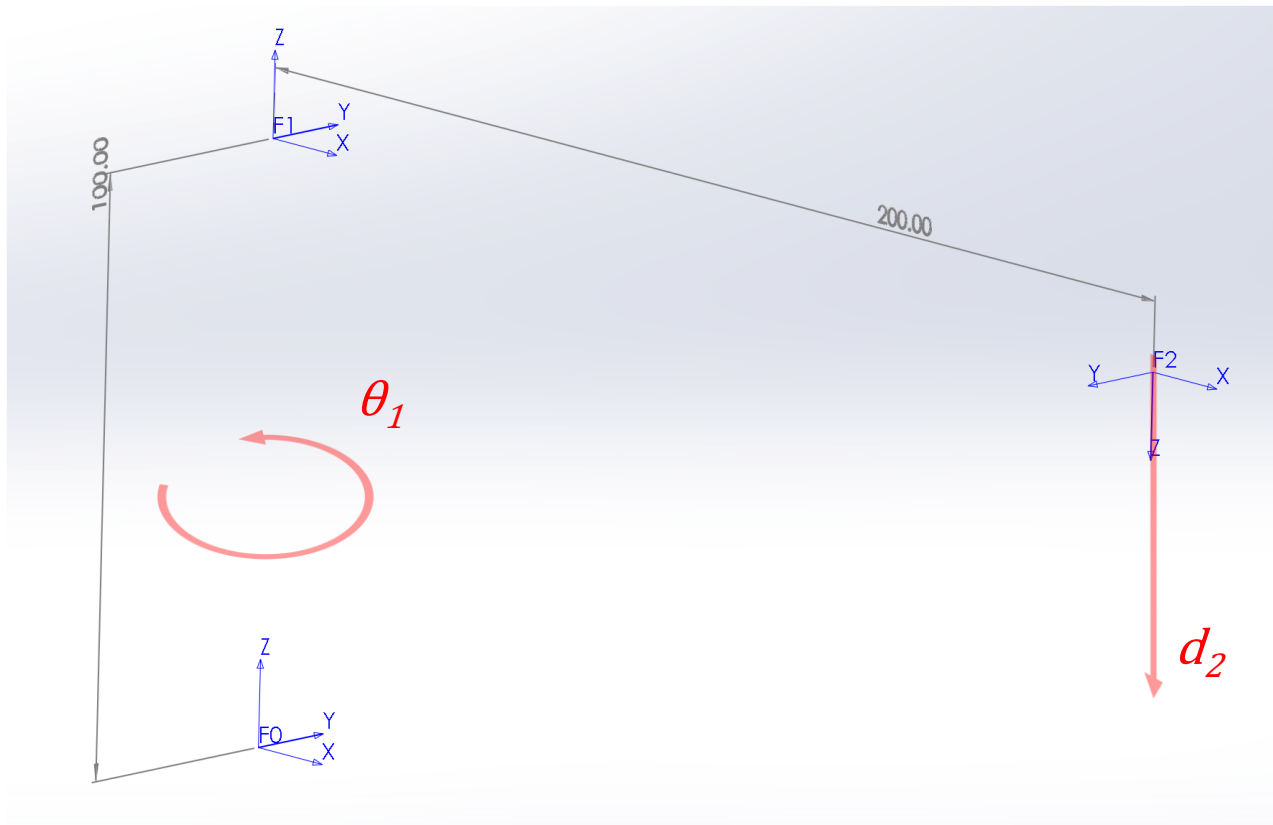


PART D (10%) bonus

Consider the following robot arm which is consist of a revolute joint and prismatic joint:



(1) Find the DH representation the same as Part A (1) (6%)



SCARA				
Joint	α_{i-1} (°)	a_{i-1} (mm)	d_i (mm)	θ_i
1	0	a1	0	θ_1
2	180	a2	d2	0

(2) For all DH parameters (α_{i-1} , a_{i-1} , d_i , θ_i), which two parameters are actuator joint (varying parameters)? (4%)

θ_1 and d_2 are varying parameters, since joint 1 is a revolute joint, and joint 2 is a prismatic joint.