Certified Derivative-based Parsing of Regular Expressions

Raul Lopes[1] Rodrigo Ribeiro [2] Carlos Camarão[3]

DECOM, Universidade Federal de Ouro Preto (UFOP), Ouro Preto

DECSI, Universidade Federal de Ouro Preto (UFOP), João Monlevade

DCC, Universidade Federal de Minas Gerais (UFMG), Belo Horizonte

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Introduction

- Parsing is a fundamental activity in computing.
  - Used by compilers and string search tools
- In this work, we focus on parsing Regular Languages (RLs).
  - Languages accepted by (non)-deterministic finite automata and Regular Expressions (REs).

Introduction

- REs are an algebraic and compact way to specify RLs.
- RLs are used in lexical analyser generators and search tools.
- Our objective: Formalize an algorithm for RE based search using Dependently typed language Idris.

Introduction

- Our approach is to certify a search algorithm based on Brzozowski derivatives.
- Derivatives are an alternative method to compute a equivalent automaton from a given RE.
- Some recent works uses derivatives for parsing of both RE and even context free ones.

Contributions

- A complete formalization of derivative based parsing in Idris. The algorithm produces a parse tree that is an evidence that the input string is indeed on RE language or a proof that no such tree exists.
- A detailed formalization of the so-called "smart-constructors" used to quotient REs with respect to ACUI-axioms.

An Overview of Idris

- Idris is a dependently typed programming language which focus on programming rather than proving.
- Idris syntax is similar to Haskell syntax.
- Example: Natural numbers in Peano notation.

```
data Nat = Z | S Nat
```

An Overview of Idris

- Idris supports indexed type families using a Haskell GADT style syntax (similar to Agda's syntax).
- Example: Length-indexed lists (aka vectors) and safe head function.

```
data Vec : Nat -> Type -> Type where
    Nil : Vec Z a
    (::) : a -> Vec n a -> Vec (S n) a

head : Vec (S n) a -> a
head (x :: _) = x
```

Regular Expressions

- REs are defined with respect to a given alphabet, Σ, by the following context-free grammar:

$$e ::= \emptyset \mid \epsilon \mid a \mid e\,e \mid e + e \mid e^{\star}$$

Regular Expressions

- Idris representation

```
data RegExp : Type where
    Zero : RegExp
    Eps  : RegExp
    Chr  : Nat -> RegExp
    Cat  : RegExp -> RegExp -> RegExp
    Alt  : RegExp -> RegExp -> RegExp
    Star : RegExp -> RegExp
```

Regular Expression Semantics

- REs semantics as syntax directed judgement:

$$\frac{}{\epsilon \in [\![\epsilon]\!]} \qquad \frac{a \in \Sigma}{a \in [\![a]\!]}$$

$$\frac{w \in [\![e]\!] \quad w' \in [\![e']\!]}{ww' \in [\![ee']\!]} \qquad \frac{w \in [\![e]\!]}{w \in [\![e + e']\!]}$$

$$\frac{w \in [\![e']\!]}{w \in [\![e + e']\!]} \qquad \frac{w \in [\![\epsilon + ee^\star]\!]}{w \in [\![e^\star]\!]}$$

Regular Expression Semantics

```
data InRegExp : Word -> RegExp -> Type where
    InEps : InRegExp [] Eps
    InChr : InRegExp [ a ] (Chr a)
    InCat : InRegExp xs l ->
            InRegExp ys r ->
            zs = xs ++ ys ->
            InRegExp zs (Cat l r)
    InAltL : InRegExp xs l ->
             InRegExp xs (Alt l r)
    -- some rules omitted...
```

Nullability Test

- Brzozowski definition uses an auxiliar function, $\nu(e)$, which returns ϵ if $\epsilon \in [\![e]\!]$ or $\emptyset$, otherwise.

$$
\begin{aligned}
\nu(\emptyset) &= \emptyset \\
\nu(\epsilon) &= \epsilon \\
\nu(a) &= \emptyset \\
\nu(e\,e') &= \begin{cases} \epsilon & \text{if } \nu(e) = \nu(e') = \epsilon \\ \emptyset & \text{otherwise} \end{cases} \\
\nu(e + e') &= \begin{cases} \epsilon & \text{if } \nu(e) = \epsilon \text{ or } \nu(e') = \epsilon \\ \emptyset & \text{otherwise} \end{cases} \\
\nu(e^\star) &= \epsilon
\end{aligned}
$$

Nullability Test

- Idris implementation of nullability tests shows its decidability.
- Theorem `hasEmptyDec` either returns a proof that `InRegExp Eps e` or an evidence that such construction is impossible.
  - Theorem follows by induction over RE structure.

Smart Constructors

- Smart constructors ensures the following RE equivalences

$$1)\ e + \emptyset \approx e \qquad 2)\ \emptyset + e \approx e$$
$$3)\ e\,\emptyset \approx \emptyset \qquad 4)\ e\,\epsilon \approx e$$
$$5)\ \emptyset\,e \approx \emptyset \qquad 6)\ \epsilon\,e \approx e$$
$$7)\ \emptyset^\star \approx \epsilon \qquad 8)\ \epsilon^\star \approx \epsilon$$

Smart Constructors

- Smart constructor for union.
  - Such smart constructor is sound and complete w.r.t. RE semantics.
  - Same structure for smart constructors for concatenation and star.

```
(.|.) : RegExp -> RegExp -> RegExp
Zero .|. e = e
e .|. Zero = e
e .|. e'   = Alt e e'
```

Derivative of Regular Expression

- Idris version uses nullability test function and smart constructors for concatenation, union and star.

$$
\begin{aligned}
\partial_a(\emptyset) &= \emptyset \\
\partial_a(\epsilon) &= \emptyset \\
\partial_a(b) &= \begin{cases} \epsilon & \text{if } b = a \\ \emptyset & \text{otherwise} \end{cases} \\
\partial_a(e\, e') &= \partial_a(e)\, e' + \nu(e)\, \partial_a(e') \\
\partial_a(e + e') &= \partial_a(e) + \partial_a(e') \\
\partial_a(e^\star) &= \partial_a(e)\, e^\star
\end{aligned}
$$

Derivative Properties

Theorem (Soundness of derivative operation)

For all RE `e`*, string* `xs` *and symbol* `x`*, if* `InRegExp xs (deriv x e)` *then* `InRegExp (x :: xs) e`.

Proof.

By induction on the structure of `e`, using the soundness lemmas for smart constructors and decidability of the emptiness test. □

Derivative Properties

Theorem (Completeness of derivative operation)

For all RE `e`*, string* `xs` *and symbol* `x`*, if* `InRegExp (x :: xs) e` *then* `InRegExp xs (deriv e x)`.

Proof.

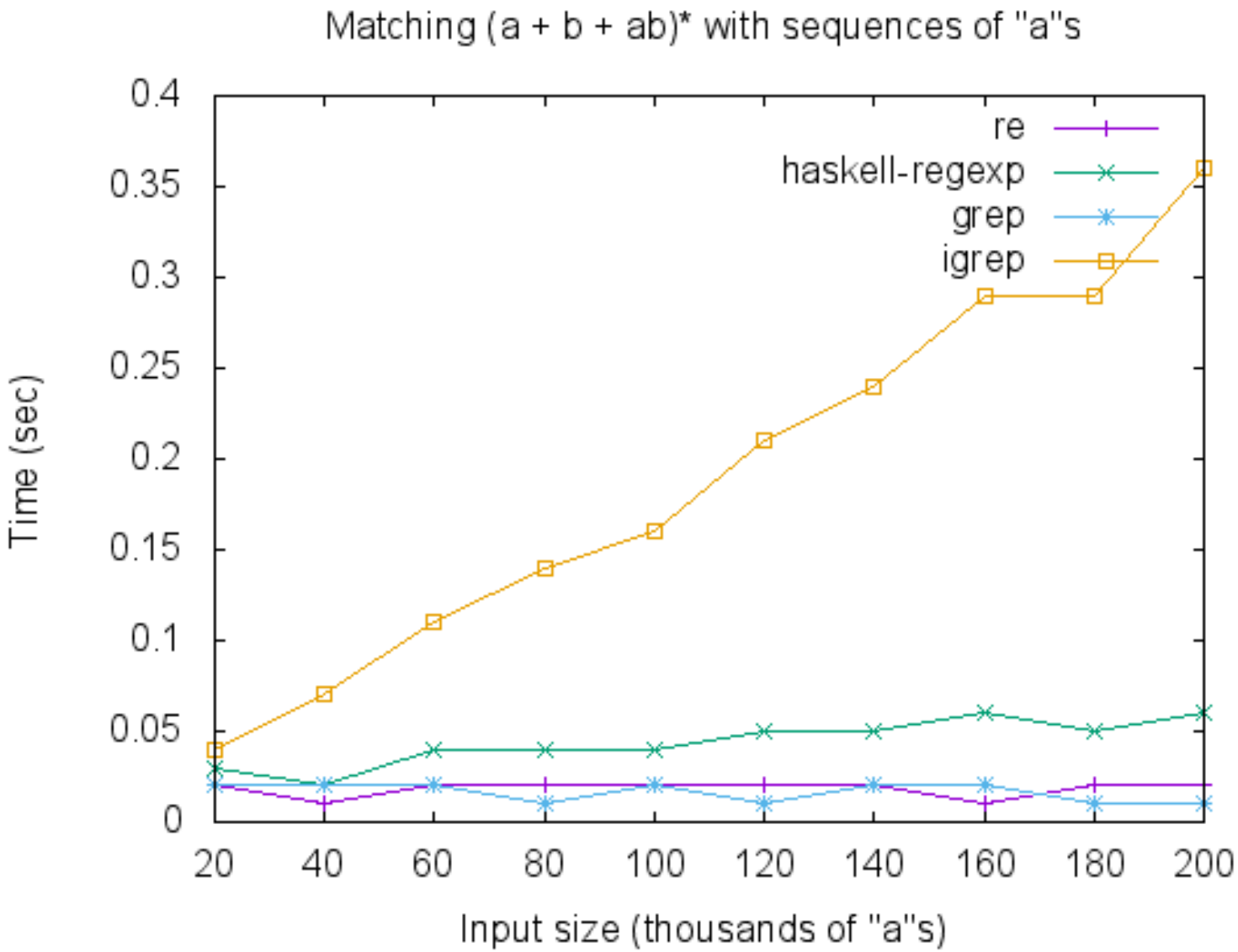
By induction on the structure of `e` using the completeness lemmas for smart constructors and decidability of the emptiness test. □

Parsing with Derivatives

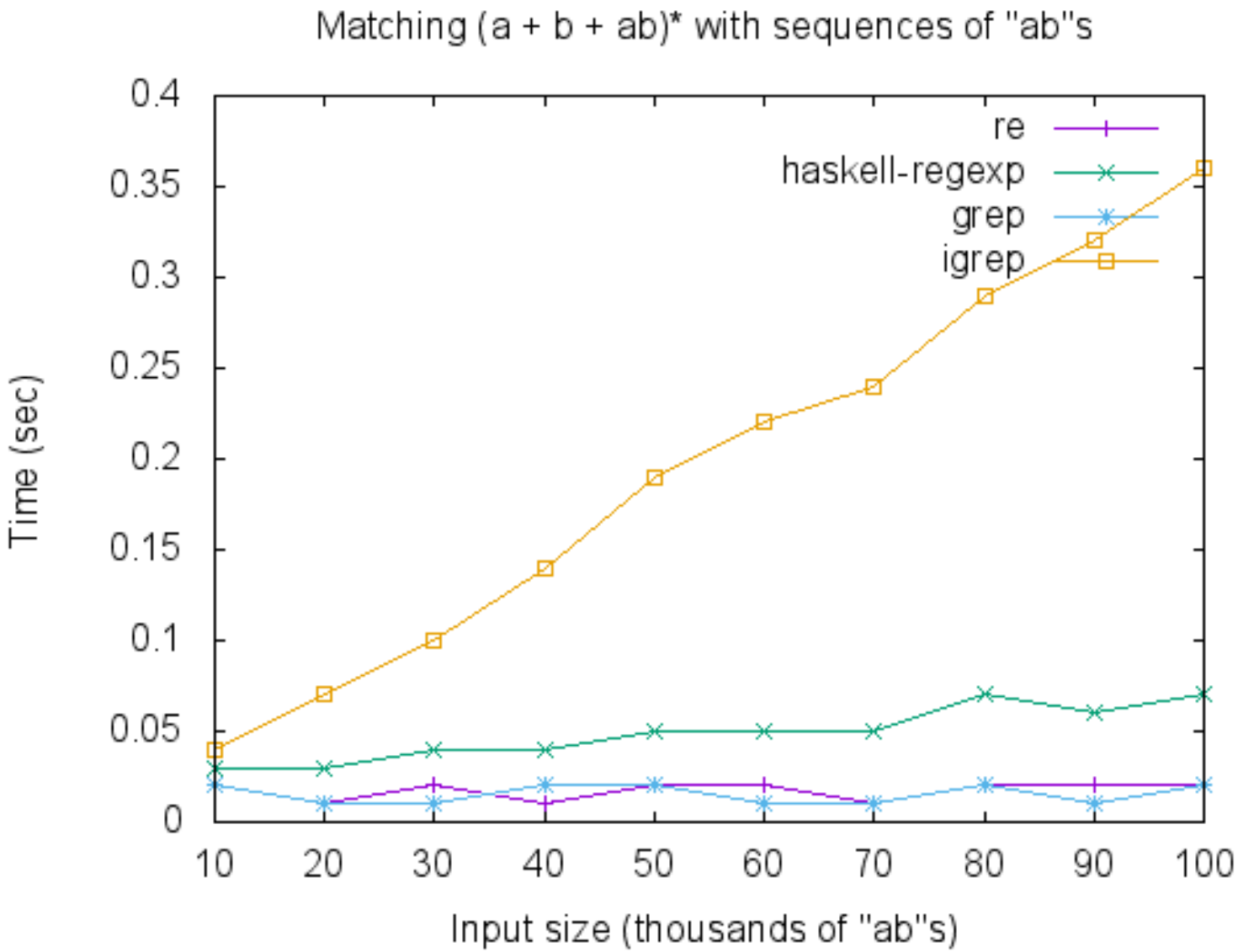
- Just apply the derivative operation recursively on each symbol of input.
- String accepted if resulting RE is nullable. Our implementation keeps track if a prefix or a substring of input is matched by input RE.

$$\begin{aligned} \partial^{\star}_{\epsilon}(e) &= e \\ \partial^{\star}_{a\,w}(e) &= \partial^{\star}_{w}(\partial_{a}(e)) \end{aligned}$$

Experiments



Experiments



Conclusion and Future Work

- Complete formalization of derivative based RE parsing algorithm in Idris.
- A tool, named iGrep, is produced from the formalization and experiments were conducted.
- Future work:
  - Improve efficient by using disambiguation strategies like greedy or POSIX.
  - Compile RE to an automaton and use it to parse input string and compare the efficency with derivative based algorithm.