Pricing a Matching Marketplace

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This paper addresses the pricing problem of a matching marketplace under asymmetric information. It focuses on the matching function of a marketplace platform that engages in sequential search on behalf of a consumer using partially-observable consumer and supplier attributes. We find that when the participants' unobservable valuations are exponentially distributed, it is optimal to charge the *same* total fee for each match rather than engage in price discrimination, and that this entire fee should be levied on the less elastic side of the marketplace. We develop the optimal pricing policies for more general classes of matching platforms and extend the foregoing results to the case of a general distribution of unobservable valuations.

Key words: pricing, price discrimination, revenue management, multi-sided platforms, online marketplaces, matching, service systems, intermediation

1. Introduction

An increasing share of the economy is managed through platforms that leverage technology to match "consumers" looking for services with "suppliers" or service providers who provide these services (the "consumers" as well as the "suppliers" may be individuals or businesses). For example, Thumbtack matches home improvement professionals (as well as event planners and trainers) with consumers seeking a professional. Similarly, Upwork and Stackoverflow match organizations with freelance professionals looking for work, and TaskRabbit, Zaarly, Gigwalk, Fiverr and Eruditor, a multi-vertical online marketplace whose operating rules closely follow our model, match consumers with the providers of a wide array of services. In the business-to-business (B2B) domain, businesses may seek suppliers for goods and services using B2B matching marketplaces such as Alibaba.

These online marketplaces have been enabled by the Internet, which facilitates the aggregation of information and the efficient matching of suppliers for any given demand. Most marketplaces perform additional value added services such as route optimization and payments in the case of Uber or other consumer and supplier services ranging from shipping (in marketplaces for goods) to customer relationship management and escrow services. However, the core function common to virtually all online marketplace platforms is the matching of supply and demand, with the Internet playing an information aggregation and communication role. In this paper we focus on the pricing of this fundamental function, abstracting from the additional features of many specific marketplaces.

Marketplaces may provide a range of services, but finding an appropriate match comes first. The value of a prospective match is driven by the attributes of both consumers and suppliers. Some of these attributes are private attributes which the counterparties can discover only once they contemplate an actual match. Other attributes are observable by all participants (in particular, the marketplace) ex ante, and may be exploited by the marketplace to extract more of the surplus. How should the marketplace price each potential match given the observable attributes of both counterparties and the distributions of their unobservable attributes? Should it engage in price discrimination? How should it allocate its fees between consumers and suppliers? These are some key research questions addressed in this paper.

One simple approach to marketplace pricing is to charge a fixed percentage of the expected value of the service (or some other proxy) as a commission. However, this approach may be inefficient, as it fails to exploit the information provided by the counterparties. Further, it is important to determine how to split the total matching fee between the consumer and the supplier. We construct a micro-founded model that incorporates agents' heterogeneity and asymmetric information and derive the platform's optimal pricing policies.

Pricing problems of this type are complex and are often intractable. We first study the simpler case where all valuations are given by the sums of deterministic functions of the agents' known attributes and exponential random variables. We find that it is optimal to charge the *same* total fee for matches with *different* observable attributes, and that this fee should be levied in its entirety on the less elastic side of the platform, up to a threshold point. We then relax these distributional assumptions and derive the optimal prices for general (non-exponential and possibly correlated) random valuations. We further show that our results are applicable not only to the two-sided platform setting but also to other settings such as platforms that match consumers to multiple suppliers.

In what follows, we review the literature in Section 2, solve for exponential random valuations in Section 3, generalize our analytical framework in Section 4 and study the implications for matching marketplaces in Section 5. Our concluding remarks are in Section 6.

2. Literature Review

Research in the area of marketplace platforms is broad and growing, and multiple papers have addressed different aspects of these platforms. Our work may be placed within the broader area of peer-to-peer service platforms. A peer-to-peer service platform first matches incoming requests to available service providers (the pricing of which is the focus of this paper), and then performs additional, vertical-specific functions that are related to the actual service. These platforms often develop reputation systems to mitigate adverse selection problems that may result in market inefficiencies or failure (Akerlof (1970)). Multiple researchers (e.g., Pavlou and Dimoka (2006), Resnick et al. (2006), Yoganarasimhan (2013), Moreno and Terwiesch (2014)) empirically validated that user feedback aggregated by a reputation system provides a useful signal about supplier quality, leading to a higher willingness-to-pay for suppliers with better reputation. In a number of papers, Tadelis studies how reputation and feedback systems can support commerce in online marketplaces as well as the biases inherent in them (see Tadelis (2016) for a review). Cui et al. (2016) show empirically how racial biases affect rental behavior on the Airbnb marketplace, and how guest reviews significantly reduce these biases. Li et al. (2016) find differences in pricing behaviors between professional and non-professional hosts on Airbnb. Based on these differences, they develop a model for maximizing platform profits and social welfare, finding that the profit-maximizing platform operator should charge lower prices to nonprofessional hosts, whereas a social planner should charge the same prices to professionals and non-professionals. Zheng et al. (2016) find that higher uncertainty about the value of service perceived by either side of the market negatively impacts the probability of conversion to actual service. Mendelson and Tunca (2007) show theoretically how information asymmetries undermine the performance of B2B marketplace platforms in a supply chain context.

Other papers study how additional platform design characteristics affect performance. Bimpikis et al. (2017b) show how increasing market thickness and changing the listing policy and recommendation system of a B2B marketplace can be used to increase its revenues. Horton and Zeckhauser (2010) find that algorithmic wage negotiations reduce transaction costs and lead to more successful follow-up contracts. Fradkin (2015) proposes an improved search algorithm that leads to a 10% increase in AirBnb conversions. Kokkodis et al. (2015) find that data on previous interactions, countries of residence, skills and response speed largely improve the matching probability on oDesk (now Upwork). Snir and Hitt (2004) show that screening suppliers to avoid excessive bidding improves marketplace performance. Hong et al. (2016) report that making supplier bids visible leads to a 20% increase in the probability of conversion. Kanoria and Saban (2017) show how matching platforms can reduce wasteful search efforts by restricting agents' options, and how such restrictions can improve social welfare even when search costs are small. Horton (2017) finds that the ability of a supplier to signal her availability increases the market surplus by 6%.

A number of papers model marketplaces as queuing systems, showing that due to congestion effects, improving their operational efficiency may decrease the overall welfare they generate. Arnosti et al. (2014) show that lower search costs may decrease welfare as they increase the number of candidates considered through the platform. Focusing on the tradeoff between price and delay in a queuing model of a marketplace, Allon et al. (2012) show that greater operational efficiency of the marketplace may be detrimental to overall welfare. They further show how allowing communication among market participants may mitigate this effect. Shu et al. (2013) develop a stochastic network flow model for the related problem of bike sharing and apply it to data from the Singapore Mass Transit System. They show their model well-approximates the flow of bicycles in the system, and they use it to evaluate the effects of bicycle utilization, bicycle redistribution and the number of docks at each station. Using a stockout (rather than queuing) model to capture the effects of distance and bike availability, Kabra et al. (2016) estimate the demand for bikes at the Vélib' bike-share system in Paris and show the effects of station density and bike availability on ridership. They apply these results to show how to improve system performance.

The problem of revenue and capacity management for ride-sharing platforms such as Uber has attracted significant attention. These papers tie pricing decisions to the operational performance characteristics of the platform, providing results that are targeted to the specific platforms studied. Riquelme et al. (2015) focus on dynamic pricing to balance supply and demand in ride-sharing platforms. Bimpikis et al. (2017a) study price discrimination for a ride-sharing platform that serves a network of locations, focusing on the spatial dimension. They show how spatial price discrimination can be used to improve balance across the network, thereby increasing platform profits and consumer surplus. Taylor (2017) analyzes the impact of agents' independence (self-scheduling) and customers' delay sensitivities on the optimal prices. He finds that unlike one's intuitive expectations, the effects of delay sensitivity on both the optimal wage (paid to the driver) and price (paid by the passenger) are ambiguous and depend on the underlying uncertainties. Bai et al. (2017) consider the problem of optimally coordinating a queuing model with price- and delay-sensitive consumers on one side and profit-maximizing service providers on the other, showing the effects of demand increases and consumers' delay costs. Cachon et al. (2017) abstract from queuing effects, showing that both the service providers (drivers) and the consumers (passengers) are often better off with surge pricing, which makes the market more efficient: the providers benefit from higher utilization, and the consumers enjoy lower prices during off-peak demand and improved access during peak demand.

Our approach differs from the above for two related reasons. First, we focus on the matching function, which we believe is a core function of any marketplace, without modeling the services that follow, which differ across marketplaces. This allows us to derive more general results while abstracting from the actual details of the downstream vertical-specific services provided by many marketplaces. Second, we focus on the informational effects of agent heterogeneity in the matching process. As a result, our model does not address capacity and utilization considerations. Rather, we focus on how the platform exploits the information available to it to generate better matches and extract more surplus.

We find that the platform is better off using aggregate information without engaging in price discrimination in the cases where random valuations are exponentially distributed, or, for general distributions, when the matching demand functions for the different suppliers are proportional. We derive an explicit algorithm for solving the marketplace pricing problem and obtain closed-form solutions for the exponential case. Our results apply directly to marketplaces that focus on matching per se, such as Alibaba's B2B market, Eruditor or Thumbtack. For a marketplace like Uber, our results do not capture key aspects of marketplace operations, but they still apply as an approximation to the short-term matching problem faced by the platform. In the very short term, Uber observes which service providers are currently available for a given request and their locations, and it has to sequentially match them to a passenger based on observed data as well as price each match. Uber has employed a fixed-percentage pricing scheme with a potential surge multiplier, and it is reportedly experimenting with more complex price discrimination schemes (Newcomer (2017), Kominers (2017)).

A branch of the economics literature studies the pricing of two-sided platforms that are subject to network effects. This literature aims to find static equilibrium structures, typically assuming a linear relationship between value and number of agents (cf. Caillaud and Jullien (2003), Rochet and Tirole (2006), Armstrong (2006), Weyl (2010)). Within that literature, some researchers study price discrimination (e.g., Damiano and Li (2007)) but they do not base prices on agents' observable attributes; rather, they offer price schedules that induce agents to reveal their private information by self-selecting into designated tiers. Li and Netessine (2017) study the effects of market thickness on the efficiency of matching in a holiday rental platform, exploiting an exogenous thickness shock caused by a one-time listing migration from other platforms. They find that contrary to the dictum of most network effects models, increased thickness was associated with a significant decline in the matching probability, finding a "deadline effect." Our model does not assume the existence of network effects, and it does consider the effects of time constraints on the consumer's search. It starts from the micro level to find the optimal personalized, i.e., pairwise prices based on agents' observable attributes.

Our model may also be related to the task-assignment problem in the online mechanism design literature (Blum et al. (2004), Babaioff et al. (2012), Badanidiyuru et al. (2012), Singla and Krause (2013)). This literature proposes algorithms for dynamically posting prices for suppliers who bid for specific tasks. However, these papers do not derive closed-form pricing solutions because of their different objectives. In addition to their different model structures, the algorithms analyzed in these papers do not allow for price-discrimination based on observable attributes. Our approach is closer to the classic dynamic stochastic settings of Keilson (1970), Lippman and Ross (1971), and Mendelson and Whang (1990). Hu and Zhou (2015) study a more complex problem where demand types are dynamically matched to supply types to maximize total reward. Like the other papers in this stream, they do not consider the informational and pricing issues studied here.

Overall, we combine the timing dynamics, information asymmetry and participants' heterogeneity to find new quantitative and qualitative results. We also generalize our setting to multidimensional resource bundles.

3. Base-case model

A consumer arrives at the marketplace, seeking a supplier to perform a service. The consumer a submits to the marketplace a service request identifying the service to be performed and provides other relevant information. In particular, the marketplace may manage a public profile with relevant information about the consumer. The consumer's request remains live in the system for an exponentially-distributed period of time with mean $1/\tau_A$ and then expires.

Suppliers respond randomly to the consumer's request. Suppliers whose observable characteristics can be summarized by index b, arrive from a Poisson stream with rate $1/\tau_B$[1]. Each arriving supplier examines the consumer's known attributes (from the service request and the consumer's profile) and decides whether to respond to the request. Once the supplier responds, both parties reveal their unobservable attributes to determine the actual value of a potential match. A match occurs if sufficient value is generated for both the supplier and the consumer, after subtracting the marketplace fees. If this is not the case, the supplier moves on and the process is repeated with subsequent suppliers until either a mutually acceptable match is found or the consumer's request expires. This stylized model is designed to capture the salient features of matching platforms such as Eruditor or Thumbtack[2], where a request is initiated by the consumer, multiple professionals or freelancers respond at random, and matches are made when they are mutually-beneficial.

We denote a match between consumer a and a supplier of type b (thereafter, "supplier b") by $i = (a, b)$. Upon a successful match i, the marketplace charges consumer a a matching fee f_i^a and supplier b a matching fee f_i^b. Each consumer valuation, u_i^a, is the sum of an observed valuation v_i^a and a random, unobservable valuation ϵ_i^a. Similarly, each supplier valuation u_i^b is the sum of an observed valuation v_i^b and the random unobserved valuation ϵ_i^b. Consumers (suppliers, respectively) have an outside option value (opportunity cost) of v_0^A (v_0^B). It follows that the match will be successful if for both the consumer and the supplier, the value of the match net of marketplace matching fees exceeds the value of the outside option: $u_i^a - f_i^a > v_0^A$ and $u_i^b - f_i^b > v_0^B$. The objective of the marketplace is to find the pricing policy $f_i = (f_i^a, f_i^b)$ that maximizes its expected profit. In some formulations of the problem, we will also consider the pricing policy that maximizes total surplus.

[1] The arrival rates of suppliers of different types are assumed identical here only for simplicity of exposition; this assumption is relaxed in the generalized model in Section 4.

[2] See *How Thumbtack Works* at www.thumbtack.com

In general, the valuations u_i^a and u_i^b have both observable (v) and unobservable (ϵ) components: $u_i^a = v_i^a + \epsilon_i^a$, $u_i^b = v_i^b + \epsilon_i^b$. We consider first the straightforward case of fully observable valuations.

3.1. Preliminary analysis: perfect information

As a preliminary analysis, we first consider the case where all the valuations are observable, i.e., $\epsilon_i^a = \epsilon_i^b = 0$. We define the matching compatibility indicators (between consumer a and supplier b) as follows: $I\left(a\ accepts\ b\right) = I\left(v_i^a \geq f_i^a + v_0^A\right)$ and $I\left(b\ accepts\ a\right) = I\left(v_i^b \geq f_i^b + v_0^B\right)$, where $I(\cdot)$ is the indicator function. A successful match $i = (a,b)$ is obtained only if $I\left(a\ accepts\ b\right) = I\left(b\ accepts\ a\right) = 1$ and supplier b arrives early enough (i.e., before other acceptable suppliers and before a's request expires). We show in Appendix A1 that the probability of a request being successfully matched is $\phi = \frac{1}{1+\frac{1}{k\sum_i (I(a\ accepts\ b) I(b\ accepts\ a))}}$, where $k = \frac{\tau_A}{\tau_B}$.

Maximizing total surplus. Assume first that the marketplace maximizes total surplus (rather than its own profit). For a given consumer request a, the problem is then to maximize

$$\phi \sum_{i\equiv(a,b)} \left(v_i^a + v_i^b\right) \frac{1}{\sum_i I\left(a\ accepts\ b\right) I\left(b\ accepts\ a\right)} \equiv \frac{\sum_i \left(v_i^a + v_i^b\right)}{\sum_i I\left(a\ accepts\ b\right) I\left(b\ accepts\ a\right) + \frac{1}{k}}$$

over all fee vectors f_i.

The following proposition establishes a threshold structure for the optimal policy. For a given valuation threshold V, let the admissible set $I(V)$ be the set of pairs $i = (a,b)$ generating pairwise values in excess of V, i.e., $V_i = v_i^a + v_i^b > V$. Then, we have

Proposition 1. *There exists a threshold level V^* such that the optimal policy is to match the first arriving supplier in $I(V^*)$. To find V^*, arrange the V_i in descending order and find the last V_i satisfying*[3]

$$V_i \geq \frac{\tau_A}{\tau_B} \sum_{j\in I} \left(V_j - V_i\right). \tag{1}$$

Because we are dealing with discrete values, V^* is not unique -- it can take on any value between the lowest valuation in the admissible set and the valuation that just follows. Any matching fees that admit only suppliers in $I(V^*)$ would achieve optimality. Formally, for $i \in I^*$, $f_i^{a*} \in (-\infty, v_i^a - v_0^A]$ and $f_i^{b*} \in (-\infty, v_i^b - v_0^B]$, and for $i \notin I^*$, $f_i^{a*} \in (v_i^a - v_0^A, \infty)$ or $f_i^{b*} \in (v_i^b - v_0^B, \infty)$.

Essentially, the optimal matching fees are used to exclude low-valuation suppliers at the expense of slowing down the matching process. Exclusion happens by raising prices on one or both sides so that the value from being matched is below the outside options for the low types and above the outside options for the high types. In other words, considering all possible matches for a particular consumer request, the marketplace defines the set of admissible pairs I^*. The first arrival from this set of suitable matches becomes the final match.

[3] Keilson (1970) and Lippman and Ross (1971) have solved a related problem showing that there exists a control limit V^* such that the customer will accept all suppliers whose expected valuations exceed V^*.

Maximizing revenue Since all information is perfectly observable, the marketplace may fully price-discriminate and extract the entire surplus. This gives rise to the following optimal fees:

COROLLARY 1. *The revenue-maximizing fees have the following structure*

$$\begin{cases} f_i^* = (v_i^a - v_0^A, v_i^b - v_0^B) & \text{if } i \in I^* \\ f_i^{a*} \in (v_i^a - v_0^A, \infty) \;\; \text{or} \;\; f_i^{b*} \in (v_i^b - v_0^B, \infty), & \text{if } i \notin I^*. \end{cases}$$

Note that some of the optimal prices may be negative, indicating that one side of the market may subsidize the other. This is a well-known phenomenon (cf., Caillaud and Jullien (2003), Kaiser and Wright (2006), Bolt and Tieman (2008)).

3.2. Imperfect information

Now, assume that the valuations are the sums of observed values (v_i^a and v_i^b) and unobserved (ϵ_i^a and ϵ_i^b) random components which are private information. We assume that the unobserved components, ϵ_i^a and ϵ_i^b, come from i.i.d. exponential distributions: $\epsilon_i^a \sim Exp\,(1/v^A)$, $\epsilon_i^b \sim Exp\,(1/v^B)$. As a result, from the marketplace perspective, the matching compatibility indicators $I\,(a \text{ accepts } b)$ and $I\,(b \text{ accepts } a)$ become random variables. In particular, the probability of b being acceptable to a is $Pr\,(a \text{ accepts } b) = Pr\,(u_i^a \geq f_i^a + v_0^A) = Pr\,(v_i^a + \epsilon_i^a > f_i^a + v_0^A) = exp\left(\frac{v_i^a - f_i^a - v_0^A}{v^A}\right)$. Similarly, the probability of b willing to serve a is $Pr\,(b \text{ accepts } a) = Pr\,(u_i^b \geq f_i^b + v_0^B) = Pr\,(v_i^b + \epsilon_i^b \geq f_i^b + v_0^B) = exp\left(\frac{v_i^b - f_i^b - v_0^B}{v^B}\right)$. Assume $v_i^a \leq v_0^A,\; v_i^b \leq v_0^B$ so that probabilities are well-defined for $f_i^a, f_i^b \geq 0$.

As we show in Appendix A2, the probability of a request being matched is

$$\phi = \frac{1}{1 + \frac{1}{k \sum_i \beta_i exp\left(-\alpha^A f_i^a - \alpha^B f_i^b\right)}},$$

where, as before, $i = (a, b)$ indexes the potential match between a and b, $k \equiv exp\left(-\frac{v_0^A}{v^A} - \frac{v_0^B}{v^B}\right)\frac{\tau_A}{\tau_B}$ is the decrease in the matching probability due to the outside options v_0^A and v_0^B, accounted for the exponential distribution and weighted by suppliers' arrival $1/\tau_B$ and consumers' departure $1/\tau_A$ rates, and $\beta_i \equiv exp\left(\frac{v_i^a}{v^A} + \frac{v_i^b}{v^B}\right)$ is the increase in the matching probability due to the observable valuations v_i^a and v_i^b. We interpret $\alpha^A \equiv \frac{1}{v^A}$ as the price sensitivity of demand and $\alpha^B \equiv \frac{1}{v^B}$ as the price sensitivity of supply.

Given the information available to all participants (and, in particular, the marketplace), the probability that the request results in a match $i = (a, b)$, is given by (see Appendix A3)

$$\phi_i = Pr\,(b \text{ serves } a) = \phi \frac{\beta_i e^{-\alpha f_i}}{\sum_j \beta_j e^{-\alpha f_j}} = \frac{\beta_i e^{-\alpha f_i}}{\sum_j \beta_j e^{-\alpha f_j} + \frac{1}{k}}, \tag{2}$$

where $\alpha \equiv (\alpha^A, \alpha^B)$, $f_i \equiv (f_i^a, f_i^b)$, $\alpha f_i \equiv \alpha^A f_i^a + \alpha^B f_i^b$.

Importantly, for any pair i, the fee vector f_i affects the matching probabilities of all pairs. In particular, the higher the fees for one particular match, the more likely are other matches to be successful. This interaction among matching probabilities and fees creates a $2 \times N$-dimenstional problem, where N is the number of different match types and 2 is the number of sides to be priced. However, we show below how to reduce the problem to N two-dimensional problems, which allows us to compute the solution in closed-form.

Total surplus optimization. As shown in Appendix B, the expected total surplus per request is given by

$$\begin{aligned} V &= E\sum_i \left(v_i^a + \epsilon_i^a + v_i^b + \epsilon_i^b\right) I\left(b \ serves \ a\right) \\ &= \sum_i \left(v^A + v_0^A + v^B + v_0^B + f_i^b + f_i^a\right)\phi_i. \end{aligned}$$

Let $W_0(\cdot)$ be the Lambert function which is the solution to the equation $z = W(ze^z)$, $z \geq -1$ (Corless et al. (1996)). Then, the pricing scheme that maximizes this total surplus is given in closed form by

PROPOSITION 2. *The pricing policy optimizing the total surplus is as follows:*

(a) If $\alpha^A > \alpha^B$, then

$$\begin{cases} f_i^{b^*} = & v^B W_0\left(exp\left(-\alpha^A v_0^A\right)\frac{\tau_A}{\tau_B}\sum_j exp\left(\alpha^A v_j^a + \alpha^B v_j^b + \alpha^B\left(v^A + v_0^A\right) - \left(\alpha^A - \alpha^B\right) f_j^{a^*}\right)\right) \\ & -v_0^B - v^A - v_0^A - f_i^{a^*} \\ f_i^{a*} = & v_i^a - v_0^A. \end{cases}$$

(b) If $\alpha^B > \alpha^A$,

$$\begin{cases} f_i^{a^*} = & v^A W_0\left(exp\left(-\alpha^B v_0^B\right)\frac{\tau_A}{\tau_B}\sum_j exp\left(\alpha^A v_j^a + \alpha^B v_j^b + \alpha^A\left(v^B + v_0^B\right) - \left(\alpha^B - \alpha^A\right) f_j^{b^*}\right)\right) \\ & -v_0^A - v^B - v_0^B - f_i^{b^*} \\ f_i^{b*} = & v_i^b - v_0^B \end{cases}$$

(c) If $\alpha^A = \alpha^B$, any $(f_i^a)^*$ and $(f_i^b)^*$ satisfying $f_i^{b^*} + f_i^{a^*} = v^B W_0\left(exp\left(-\alpha^A v_0^A\right)\frac{\tau_A}{\tau_B}\sum_j exp\left(\alpha^A v_j^a + \alpha^B v_j^b + \alpha^B\left(v^A + v_0^A\right)\right)\right) - v^B - v_0^B - v_0^A$, $f_i^{b^*} \geq v_i^b - v_0^B$ and $f_i^{a^*} \geq v_i^a - v_0^A$ are optimal.

(d) the optimal total price $f_i^{a^*} + f_i^{b^*}$ is the same for each matched pair.

Clearly, parts (a) and (b) of proposition 2 are symmetric, with A and B changing roles depending on which side has the larger price sensitivity. In part (c), since $\alpha^A = \alpha^B$, or, equivalently, $v^A = v^B$, only the total fee matters and the split between the demand and supply sides can be arbitrary, subject to feasibility constraints.

We next turn to the problem of optimizing marketplace revenue.

Revenue optimization. The expected revenue of the marketplace is given by

$$V = \sum_i \left(f_i^a + f_i^b\right)\phi_i.$$

The derivations follow the analysis for surplus optimization giving rise to the following result.

PROPOSITION 3. *The pricing policy maximizing marketplace revenue is as follows:*

(a) If $\alpha^A > \alpha^B$*, then*

$$\begin{cases} f_i^{b^*} &= v^B\left(1 + W_0\left(exp\left(-\alpha^A v_0^A - \alpha^B v_0^B - 1\right)\frac{\tau_A}{\tau_B}\sum_j exp\left(\alpha^A v_j^a + \alpha^B v_j^b - \left(\alpha^A - \alpha^B\right) f_j^{a^*}\right)\right)\right) - f_i^{a^*} \\ f_i^{a*} &= v_i^a - v_0^A \end{cases}$$

(b) If $\alpha^B > \alpha^A$*, the result is symmetric to (a), i.e.,*

$$\begin{cases} f_i^{a^*} &= v^A\left(1 + W_0\left(exp\left(-\alpha^A v_0^A - \alpha^B v_0^B - 1\right)\frac{\tau_A}{\tau_B}\sum_j exp\left(\alpha^A v_j^a + \alpha^B v_j^b - \left(\alpha^B - \alpha^A\right) f_j^{b^*}\right)\right)\right) - f_i^{b^*} \\ f_i^{b*} &= v_i^b - v_0^B \end{cases}$$

(c) If $\alpha^A = \alpha^B$*,*

$$\begin{cases} f_i^{b^*} + f_i^{a^*} = & v^A\left(1 + W_0\left(exp\left(-\alpha^A v_0^A - \alpha^B v_0^B - 1\right)\frac{\tau_A}{\tau_B}\sum_j exp\left(\alpha^A v_j^a + \alpha^B v_j^b\right)\right)\right) \\ f_i^{b^*} \geq v_i^b - v_0^B,\ f_i^{a^*} \geq v_i^a - v_0^A & \end{cases}$$

(d) the optimal total price $f_i^{a^*} + f_i^{b^*}$ *is the same for each matched pair.*

Part (d) of Propositions 2 and 3 is surprising: even though the marketplace has the ability to price-discriminate among the different pairs based on the particular information it observes about them, it does not take advantage of it. This does not mean, of course, that it does not take advantage of that information at all. In fact, an increase in valuations within a particular pair will increase the fees charged to all pairs. What the marketplace does not do is differentiate among the pairs based on the differences among their observed valuations. This result follows from the memoryless property of the exponential distribution: the expected ex-ante value of a match is given by $E\left[V + \epsilon \middle| V + \epsilon > v_0 + f\right] = E\epsilon + v_0 + f$, which is *independent* of the observable component V. Intuitively, the optimal price structure is driven by the probability distribution of the random valuation components. The observable component of the valuations affects the matching probabilities: the higher the valuation, the higher the probability of a match. Our finding means that the marketplace is better off engaging in quantity (or probability) differentiation (higher expected probability of a match for higher valuations) than in price discrimination.

Another important result is that the fee split between the supplier and consumer depends only on their respective price sensitivities and is always obtained at an extreme point: the side with the greater price sensitivity is charged the lowest price needed to attract any participants to the marketplace. In particular, when prices are non-negative, the less price-sensitive side of the market pays the entire marketplace fee while the other side pays nothing. We'll generalize this result in Section 5.

We next generalize our results to arbitrary distributions under a more general setting.

4. Optimal pricing in the general case

Consider the following setting. There are N mutually exclusive states of the world $i = 1, 2, \ldots, N$ that occur with probabilities $\phi_i \geq 0 : \sum_{i=1}^{N} \phi_i = 1$. The system reward (value) in state i is v_i. Then, the expected system reward is given by

$$\sum_{i=1}^{N} v_i \phi_i.$$

Both the probabilities and the rewards depend on the actions taken in each state i, which we denote by f_i[4]. We assume that for all $i = 1, 2, \ldots, N$

$$v_i = v_i(f_i)$$

and

$$\phi_i(f_1, \ldots, f_N) = \frac{h_i(f_i)}{\sum_{j=1}^{N} h_j(f_j)},$$

where $h_i(\cdot),\ i = 1 \ldots N$ are arbitrary non-negative matching demand functions. $h_i(f_i)$ reflects the relative supply rate for state (match in the case of a marketplace) i as a function of action f_i (price vector for a marketplace).

Under this structure,

$$\frac{\phi_j}{\phi_k} = \frac{h_j(f_j)}{h_k(f_k)} \text{ for all } j, k = 1, 2, \ldots N,$$

in other words, the relative likelihood of two states depends only on the actions taken in those two states and are independent of the actions taken in other states.

Consider now the problem of maximizing the expected system reward:

$$\max_{f_1, \ldots, f_N} \sum_{i=1}^{N} v_i(f_i)\, \phi_i(f_1, \ldots, f_N) \tag{3}$$
$$s.t.\ \ (f_1, \ldots, f_N) \in \mathcal{A},$$

where $\mathcal{A}$ is the set of feasible actions. As in our marketplace optimization problem in Section 3, the dimensionality of the problem is $N \times D$, where N is the number of states (supplier types for

[4] In the marketplace context, the system is in state i if the pair i is successfully matched; the probability ϕ_i is the matching probability of pair i; and the action f_i is the fee vector charged to the supplier and the consumer under match i.

a marketplace) and D is the dimensionality of the vector f (in our marketplace context, $D=2$). In general, this type of the problem cannot be decomposed due to the interdependencies among the state probabilities and actions. However, our problem structure allows us to decompose it into N D-dimensional optimization problems. And, if the D-dimensional problems can be solved as functions of a single parameter, the global solution is then reduced to solving one scalar equation. For example, when $D=1$, the solution involves inverting separately N scalar functions and solving a scalar equation.

Theorem 1. *Consider the optimization problem (3) where* $\phi_i(f_1,\ldots,f_N)=\frac{h_i(f_i)}{\sum_{j=1}^N h_j(a_j)}$ *and the functions* $v_i(\cdot)$, $h_i(\cdot)$ *are twice continuously-differentiable for* $i=1,2,\ldots,N$. *Also let* $T_i(\cdot)=\frac{(v_i(\cdot)h_i(\cdot))'}{h_i'(\cdot)}$. *Then, if problem (3) has an optimal solution, its value is given by the largest root of the scalar equation*

$$V=\frac{\sum_{i=1}^N v_i(f_i(V))\,h_i(f_i(V))}{\sum_{i=1}^N h_i(f_i(V))} \tag{4}$$

among all potential roots V *obtained by plugging in all possible combinations* $(f_1(V),\ldots,f_N(V))$, *where* $f_i(V)$ *is on the boundary of* $\mathcal{A}$ *or it solves* $T_i(f_i)=V$. *The vector* $(f_1(V^*),\ldots,f_N(V^*))$ *that yields the highest* V^* *is the optimal action vector.*

Remark 1. When actions are vector-valued, the equation $T_i(f_i)=V$ is understood as $\nabla_{f_i}(v_ih_i)=V\nabla_{f_i}h_i$.

We can simplify the interpretation of the Theorem and derive additional practical insights using the following proposition.

Proposition 4. *Assume that the state space* $\mathcal{A}$ *is decomposable, i.e.,* $\mathcal{A}=\Pi_{i=1}^N\mathcal{A}_i$ *and that the problem (3) has an optimal solution with value* V^*. *Then for* $i=1,2,\ldots,N$, *each component* f_i^* *of the solution* $(f_1^*,\ldots,f_N^*)$ *to problem (3) can be represented as a solution to the i-th subproblem*

$$\begin{gathered} max_{f_i} h_i(f_i)(v_i(f_i)-V^*) \\ s.t.\ f_i\in\mathcal{A}_i. \end{gathered} \tag{5}$$

That is, the optimal action in each state maximizes the weighted deviation from the global optimal value V^*.

As we illustrate below, Theorem 1 and Proposition 4 allow us to substantially generalize the results of Section 3. In addition, they allow us to consider the marketplace from a different perspective: whereas the building blocks of our underlying model start from the preferences of individual decision-makers, Theorem 1 and Proposition 4 examine them as statistical aggregates. In essence, we are replacing the primitives associated with the individual agents by matching demand functions $h_i(\cdot)$ which are proportional to the probabilities that a match will be generated in each state as a function of the pairwise price. We illustrate below the construction of these matching demand functions from the underlying parameters of our model, first for the exponential case studied in Section 3 and then for the case of general random valuations.

From a practical point of view, the matching demand functions of Theorem 1 can be actually estimated based on observable marketplace data whereas their individual building blocks are theoretical constructs that allow us to derive them. As a result, unlike the underlying parameters of our model, the matching demand functions $h_i(\cdot)$ have a direct operational meaning and they may be directly estimated. In practice, the operating characteristics of a marketplace are likely to be specified in terms of the matching demand functions $h_i(\cdot)$.

Theorem 1 applied to the base-case model

We illustrate the implementation of our general approach and the mapping from individual characteristics to matching demand functions by applying it in detail to the base-case model solved directly in Section 3. We first show how Theorem 1 applies to the marketplace context, where each state $i \in \{1,\ldots,N\}$ corresponds to a matched supplier. State $N+1$ is the state where the consumer gives up waiting for the match, which results in $v_{N+1}=0$. Each action in state $i=1,\ldots,N$ is a two-dimensional fee vector (f_i^a, f_i^b), which affects both the reward that the marketplace gets and the state probabilities $\phi_i = h_i(f_i^a, f_i^b) / \left[\frac{1}{k} + \sum_j h_i(f_i^a, f_i^b)\right]$. For a revenue-maximizing marketplace, the system reward in state i (the value of match i to the marketplace) is given by $v_i(f_i) = f_i^a + f_i^b$, where, as before, f_i^a and f_i^b are the fees charged to the consumer and supplier, respectively. Clearly, the marketplace may want to exclude certain suppliers, who do not produce enough value, from consideration. For simplicity of exposition, we specify the prices corresponding to these suppliers as infinite prices. This is shorthand for setting prices that are large enough to exclude them. In this case, it doesn't matter if the "infinite" price is charged to the supplier or to the consumer.

For our base-case model with exponentially-distributed random valuations, the revenue that the marketplace generates from each request is given by

$$\sum_{i=1}^{N} \left(f_i^b + f_i^a\right)\phi_i = \sum_{i=1}^{N} \left(f_i^b + f_i^a\right) \frac{\beta_i e^{-\alpha^A f_i^a - \alpha^B f_i^b}}{\sum_j \beta_j e^{-\alpha^A f_j^a - \alpha^B f_j^b} + \frac{1}{k}}.$$

Thus, the reward in state i is given by

$$v_i\left(f_i^a, f_i^b\right) = \begin{cases} \left(f_i^b + f_i^a\right) & i=1,\ldots,N \\ 0 & i=N+1 \end{cases},$$

and the matching demand function is given by

$$h_i\left(f_i^a, f_i^b\right) = \begin{cases} \beta_i e^{-\alpha^A f_i^a - \alpha^B f_i^b} & i=1,\ldots,N \\ \frac{1}{k} & i=N+1 \end{cases}.$$

The revenue optimization problem of the marketplace is

$$\max_{\left(f_i^a, f_i^b\right)} \sum_{i=1}^{N} \left(v_i h_i / \sum_{j=1}^{N} h_j \right)$$
$$s.t.\ \left(f_i^a, f_i^b\right) \geq \left(v_i^a - v_0^A, v_i^b - v_0^B\right).$$

We solve the problem in four steps.

step 1. show that the optimal value V^* exists and is finite. The existence and finiteness of V^* follow from three facts: *(a)* the value function is linear in the marketplace fees, whereas *(b)* the matching probabilities decline exponentially with the marketplace fees, and *(c)* the marketplace can achieve zero revenue by rejecting all suppliers. It follows that there is an optimal solution with a finite, positive V^*.

The optimal prices are in a bounded set for the following reason. since V^* is finite, the gradient of the objective function with respect to at the optimal fee vector f^* is given by

$$\frac{\frac{\partial}{\partial f_i^a} h_i}{\sum_j h_j} \left[T\left(\begin{bmatrix} f_i^a \\ f_i^b \end{bmatrix} \right) - V^* \right] = \frac{\frac{\partial}{\partial f_i^a} h_i}{\sum_j h_j} \left[f_i^a + f_i^b - \begin{bmatrix} 1/\alpha^A \\ 1/\alpha^B \end{bmatrix} - V^* \right].$$

Since $\frac{\frac{\partial}{\partial f_i^a} h_i}{\sum_j h_j} < 0$ uniformly, $\frac{\partial V}{\partial f_i^a} < 0$ for large enough f_i^a. Similarly, $\frac{\partial V}{\partial f_i^b} < 0$ for large enough f_i^b. Thus, the optimal prices are contained in a bounded set.

step 2. decompose the global objective. By Proposition 4, we can decompose the problem into N optimization problems, one for each match i, $i = 1, 2, \ldots, N$:

$$max_{f_i} h_i\left(f_i\right)\left(v_i\left(f_i\right) - V^*\right)$$
$$s.t.\ \left(f_i^a, f_i^b\right) \geq \left(v_i^a - v_0^A, v_i^b - v_0^B\right),$$

which yields the following first-order conditions:

$$f_i^a + f_i^b - \begin{bmatrix} 1/\alpha^A \\ 1/\alpha^B \end{bmatrix} = \begin{bmatrix} V^* \\ V^* \end{bmatrix}.$$

step 3. filter the set of solution candidates.[5] There are 2 candidate points for the consumer-side fee f_i^a – one interior solution $V^* + 1/\alpha^A - f_i^b$ and one corner solution $(v_i^a - v_0^A)$. Similarly, there are 2 corresponding candidates for the supply-side fee f_i^b: $V^* + 1/\alpha^B - f_i^a$ and $(v_i^b - v_0^B)$. Thus, the optimal candidate fees (f_i^a, f_i^b) for state i constitute all four pair-wise combinations of these individual candidates. Then, we have the following candidate pair

[5] Using the results of Section 3 we can readily identify the structure of the optimal solution. However, to illustrate the general solution process, we perform the analysis *de novo*.

types: $(v_i^a - v_0^A, v_i^b - v_0^B)$, $(V^* + 1/\alpha^A - f_i^b,\ V^* + 1/\alpha^B - f_i^a)$, $(v_i^a - v_0^A,\ V^* + 1/\alpha^B - (v_i^a - v_0^A))$, and $(V^* + 1/\alpha^A - (v_i^b - v_0^B),\ v_i^b - v_0^B)$.

Rather than enumerate the results, we can eliminate some of the candidates upfront. For instance, both partial derivatives at $(v_i^a - v_0^A, v_i^b - v_0^B)$ are positive, hence this candidate be eliminated. The second candidate

$$\begin{bmatrix} f_i^a \\ f_i^b \end{bmatrix} = \begin{bmatrix} V^* + 1/\alpha^A - f_i^b \\ V^* + 1/\alpha^B - f_i^a \end{bmatrix}$$

can be eliminated if $\alpha^A \neq \alpha^B$. In fact, we can rewrite the expression as

$$\begin{bmatrix} f_i^a + f_i^b \\ f_i^a + f_i^b \end{bmatrix} = \begin{bmatrix} V^* + 1/\alpha^A \\ V^* + 1/\alpha^B \end{bmatrix},$$

which becomes inconsistent if $\alpha^A \neq \alpha^B$. Thus, if the price sensitivities are different, then we necessarily have a corner solution. Further, a variation in the fees $(\Delta f_i^a, \Delta f_i^b) = (\epsilon,\ -\epsilon)$, where $\epsilon > 0$, is profitable if $\alpha^B > \alpha^A$, that is, such variation increases the objective function value. Hence, if $\alpha^B > \alpha^A$, then the corner solution $(v_i^a - v_0^A,\ V^* + 1/\alpha^B - (v_i^a - v_0^A))$ cannot be optimal since it permits the above variation. As a result, when $\alpha^A \neq \alpha^B$ we are left with only one solution candidate, namely,

$$\left(f_i^a, f_i^b\right) = \begin{cases} (v_i^a - v_0^A,\ V^* + 1/\alpha^B - (v_i^a - v_0^A)) & \alpha^A > \alpha^B \\ (V^* + 1/\alpha^A - (v_i^b - v_0^B)\,,\ v_i^b - v_0^B) & \alpha^A < \alpha^B \end{cases}. \tag{6}$$

Finally, if $\alpha^A = \alpha^B$, the total price must be equal to $V^* + 1/\alpha^A$, and it may be allocated arbitrarily between the consumer and the supplier.

step 4. plug the solution candidates into the equation for V^*. Plugging (f_i^a, f_i^b) into the equation $V^* = \sum_{i=1}^{N+1} v_i\left(f_i^a, f_i^b\right) \frac{h_i\left(f_i^a, f_i^b\right)}{\frac{1}{k} + \sum_j h_i\left(f_i^a, f_i^b\right)}$, then solving for V^* and plugging it back into the expression (6) completes the solution. The exact formula was obtained in the previous section.

Frequently, the elimination of certain candidate solutions may come from additional constraints imposed by the nature or operating rules of the marketplace. For instance, for some platforms, charging the consumer (e.g., Yelp) or consumer and supplier (e.g., Stackoverflow) may be inappropriate, while the feasible price region for another side (e.g., advertisers) is unconstrained. In that case, constraints of the form of $f_i^a = 0$ or $f_i^b = 0$ may automatically eliminate a number of solution candidates.

5. Optimal marketplace pricing

In Section 3, we solved the marketplace pricing problem for the case where the random valuations were exponentially distributed. Specifically, we first solved the pricing problem and then studied the structural properties of the optimal solution finding that *(i)* the optimal total price charged from each pair is the same across all suppliers, and *(ii)* when prices are non-negative, the entire total price is charged to the less price-sensitive side of the market. Theorem 1 and Proposition 4 allow us to extend the analysis to general distributions. Further, they allow us to study market behavior from the perspective of both the underlying individual distributions and the aggregate matching demand functions.

Theorem 1 allows us to compute the optimal prices for a matching marketplace with a general distribution of the random valuations. Now, the marketplace objective function is given by

$$\sum_{i=1}^{N} E\left[v_i \middle| \text{pair } i \text{ matched}\right] Pr\left(\text{pair } i \text{ matched}\right) \equiv \sum_{i=1}^{N} v_i \phi_i,$$

where v_i is the reward from a successfully matched pair i. If arrivals follow a Poisson process, then the probability that the pair $i = (a, b)$ is successfully matched is given by

$$\phi_i\left(f_i^a, f_i^b\right) = \begin{cases} \frac{\lambda_i Pr\left(u_i^a \geq f_i^a, u_i^b \geq f_i^b\right)}{\sum \lambda_j Pr\left(u_i^a \geq f_i^a, u_i^b \geq f_i^b\right)} & i = 1, \ldots, N \\ \frac{\lambda_{N+1}}{\sum \lambda_j Pr\left(u_i^a \geq f_i^a, u_i^b \geq f_i^b\right)} & i = N+1 \end{cases},$$

where u_i^a is the value that the consumer a derives from being matched with supplier b, and i simultaneously indexes supplier b and the pair $i = (a, b)$. Similarly, u_i^b is the value that supplier b derives upon being matched with the consumer a, and λ_i is the arrival rate of type-b suppliers (corresponding to $i = (a, b)$). We allow the arrival rates λ_i, which were assumed the same in Section 3, to vary across suppliers. Finally, λ_{N+1} is the arrival rate of the outside option: if the outside option "arrives" before the consumer request is matched, the consumer gives up the search and exits the marketplace, leaving no revenue to the marketplace. For simplicity of exposition, we assume that prices are non-negative, although this assumption is not needed in the derivation of the optimal prices.

Our general marketplace formulation extends the base-case model to arbitrary random shock distributions $(\epsilon_i^a, \epsilon_i^b) \sim D_i$, where the individual shocks may be correlated, the distributions and arrival rates may differ across pair types i, and they may also depend on the marketplace fees (f_i^a, f_i^b). Moreover, this formulation allows us to compute the optimal prices not only for additive valuations of the form $u_i^a = v_i^a + \epsilon_i^a$, $u_i^b = v_i^b + \epsilon_i^b$ but also for general matching demand functions $h_i = \lambda_i P_i\left(f_i^a, f_i^b\right)$, where $P_i\left(f_i^a, f_i^b\right)$ are the probabilities of the suppliers and consumers accepting each other under the fee vectors (f_i^a, f_i^b). As discussed above, specifying the problem in terms of the matching demand functions $h_i(\cdot)$ is also attractive from a practical point of view, since these functions can be estimated and their properties (e.g., proportionality for different i's or specific functional forms) are amenable to direct empirical tests.

Under these general conditions, we can solve for the optimal prices using Theorem 1 and Proposition 4, as illustrated for the exponential case, by considering the candidate solutions obtained for each subproblem i and finding the one that leads to the largest V^* in equation (4). To what extent do our structural results generalize as well? The following Corollary shows that prices remain the same across matches as long as the matching demand functions $h_i\left(f_i^a, f_i^b\right)$ are proportional across the different suppliers.

COROLLARY 2. *If the matching demand functions $h_i\left(f_i^a, f_i^b\right)$ are proportional to each other for a subset of types I, i.e.,*

$$h_i\left(f_i^a, f_i^b\right) = C_i h\left(f_i^a, f_i^b\right),\ i \in I,$$

then the optimal fees charged by the marketplace to different pairs within the set I are the same.

The Corollary follows directly from Proposition 4, as the optimization subproblems solved for each match $i \in I$ are the same (if there are multiple optima, one of them will have the same fees).

Next consider the allocation of the total marketplace fee between the consumer and the supplier. Using our aggregate matching demand formulation, the price sensitivity of the consumer (supplier, respectively) to its marketplace fee is $\frac{\partial h_i}{\partial f_i^a}$ ($\frac{\partial h_i}{\partial f_i^b}$, respectively). Transforming our variables to (f_i^a, f_i^Σ), where $f_i^\Sigma \equiv f_i^a + f_i^b$, in each subproblem i, the partial derivative of the objective function with respect to f_i^a is $(f_i^\Sigma - V^*)\left(\frac{\partial h_i}{\partial f_i^a} - \frac{\partial h_i}{\partial f_i^b}\right)$, and at the optimum, $V^* < \left(f_i^\Sigma\right)^*$. Thus, the fee split is determined by the sign of $\left(\frac{\partial h_i}{\partial f_i^a} - \frac{\partial h_i}{\partial f_i^b}\right)$. In particular, if $\left(-\frac{\partial h_i}{\partial f_i^a} > -\frac{\partial h_i}{\partial f_i^b}\right)$ for all $(f_i^a, f_i^b) \in \mathcal{A}_i$, then the entire fee should be paid by the supplier and the consumer pays nothing, and if $\left(-\frac{\partial h_i}{\partial f_i^a} < -\frac{\partial h_i}{\partial f_i^b}\right)$ for all $(f_i^a, f_i^b) \in \mathcal{A}_i$, then the entire fee is paid by the consumer and the supplier pays nothing. This directly generalizes the results we obtained in the exponential case. A similar analysis may be performed using the underlying preferences of marketplace participants. Here,

$$h_i\left(f_i^a, f_i^b\right) \equiv \lambda_i P_i\left(f_i^a, f_i^b\right) \equiv Pr\left(v_i^a + \epsilon_i^a \geq v_0^A + f_i^a, v_i^b + \epsilon_i^b \geq v_0^B + f_i^b\right)$$

For subproblem i, in transformed coordinates (f_i^a, f_i^Σ), the partial derivative of the objective function with respect to f_i^a is $(f_i^\Sigma - V^*)\left(\frac{\partial Pi}{\partial f_i^a} - \frac{\partial P_i}{\partial f_i^b}\right)$. Thus, similar to the above analysis, the fee split is determined by the sign of $\left(\frac{\partial h_i}{\partial f_i^a} - \frac{\partial h_i}{\partial f_i^b}\right)$: if $\left(-\frac{\partial h_i}{\partial f_i^a} > -\frac{\partial h_i}{\partial f_i^b}\right)$ for all $(f_i^a, f_i^b) \in \mathcal{A}_i$, then the entire marketplace fee is paid by the supplier, and if $\left(-\frac{\partial h_i}{\partial f_i^a} < -\frac{\partial h_i}{\partial f_i^b}\right)$ for all $(f_i^a, f_i^b) \in \mathcal{A}_i$, the entire marketplace fee is paid by the consumer[6].

Since we allowed for different matching demand functions for different matches, it is of course possible that unlike the more restrictive exponential case, one supplier will be uniformly *more* price sensitive than the consumer whereas another will be uniformly *less* price sensitive than the consumer. In this case, the former supplier will pay zero whereas the latter will pay the entire marketplace fee.

[6] If the fees can be negative, the more price-sensitive side of the market will pay the lowest possible fee, and the less price-sensitive side will pay the highest possible fee.

Our formulation also allows for more general fee splits. For example, if the matching demand functions are log-linear of the form $h_i\left(f_i^a,f_i^b\right)\equiv\lambda_i P_i\left(f_i^a,f_i^b\right)=C_i\left(A-f_i^a\right)^{\alpha}\left(B-f_i^b\right)^{\beta}$ for $f_i^a\in[0,A]\,,f_i^b\in[0,B]$, then $\frac{\partial h_i}{\partial f_i^a}=-\alpha C_i\left(A-f_i^a\right)^{\alpha-1}\left(B-f_i^b\right)^{\beta}$ and $\frac{\partial h_i}{\partial f_i^b}=-\beta C_i\left(A-f_i^a\right)^{\alpha}\left(B-f_i^b\right)^{\beta-1}$, implying that $\frac{A-\left(f_i^a\right)^*}{B-\left(f_i^b\right)^*}=\frac{\alpha}{\beta}$. Thus, the optimal fees are the same for all i (Corollary 2), and their ratio is inversely related to their elasticity coefficients, consistent with our intuition.

Composite matching marketplaces

In some applications, a demand requires not only a single supplier but a configuration of suppliers, each playing a different role. For example, there may be a request for a team comprising a programmer and a tester, with the marketplace matching an entire team rather than an individual supplier. Or, in a marketplace for goods or services, the demand may be for a complete bundle of resources rather than for one good or service. For example, the demand may be for computer components – a CPU, memory and a hard disk as a complete bundle. Similarly, in a cloud computing context, users require a bundle of resources (e.g., computing power, bandwidth and storage) which are to be priced as multidimensional goods (Angel et al. (2014), Kash and Key (2016)) whether they are provided by a single provider or by a cloud computing broker.

In this situation, rather than match a consumer's request with a single resource (supplier), the marketplace matches the request with a K-dimensional resource bundle. We model a match as a tuple $i=(c_1,\ldots,c_{K+1})$, where the first component c_1 represents the consumer and the K-dimensional resource bundle is represented by $(c_2,\ldots,c_{K+1})$. Within each tuple $(c_1,\ldots,c_{K+1})$, each resource component k $(k=2,\ldots,K+1)$ may take on different values which we call "types" as we did in our single-resource model. We denote the number of different type vectors (i.e., possible values of the vector i) by N. As before, tuple arrivals follow independent Poisson streams with rates $\lambda_i\left(f_i\right)$; the consumer's outside option arrives at rate λ_{N+1}. In other words, the consumer waits for a match for a random amount of time $t_{N+1}\sim Exp\left(\lambda_{N+1}\right)$ and then leaves the system.

We represent the valuation vector corresponding to resource bundle (vector) i by $\vec{u}_i=\vec{v}_i+\vec{\epsilon}_i,\;\; i=1,\ldots,N$. We assume that the tuple is successful if $\vec{u}_i\geq\vec{v}_0+\vec{f}_i$ component-wise, where $\vec{f}_i=\left(f_i^{c_1},\ldots,f_i^{c_{K+1}}\right)$ is the fee vector charged by the marketplace, and $f_i^{c_k}$ is the fee charged to the resource c_k in bundle i.

We define the matching demand functions by $h_i=\lambda_i P_i\left(f_i^{c_1},\ldots,f_i^{c_{K+1}}\right)\equiv\lambda_i Pr\left(\vec{u}_i\geq\vec{v}_0+\vec{f}_i\right)$. With this specification, we can apply Theorem 1 and Proposition 4 directly. In addition, we can directly generalize[7] the fee split result obtained above. In particular, if fees are non-negative and

[7] This directly follows upon computing the derivatives w.r.t. $f_i^{c_k}$ for each $k\in\{1,\ldots,k-1,k+1,\ldots K+1\}$ in the coordinates $\left(f_i^{c_1},\ldots,f_i^{c_{k-1}},f_i^{\Sigma},f_i^{c_{k+1}},\ldots,f_i^{c_{K+1}}\right)$, where $f_i^{\Sigma}=\sum_{j\in\left\{c_1,\ldots,c_{K+1}\right\}}f_i^{c_j}$.

there exists a k such that $-\frac{\partial P_i}{\partial f_i^{c_k}} = min_j \left(-\frac{\partial P_i}{\partial f_i^{c_j}}\right)$ for all $f_i \in \mathbb{R}^+_{K+1}$, where $k \in \{1, \ldots, K+1\}$, then the entire marketplace fee should be levied on that side of the marketplace.[8]

The methods of Sections 4 and 5 thus generalize directly to the multidimensional resource case. We illustrate their application by considering the case of independent exponential random valuations, which again gives us closed-form results. Assuming $\epsilon_i^{c_k} \sim Exp\,(1/v^{c_k})$ for all $i = 1, \ldots, N$, where $\epsilon_i^{c_k}$ are the components of the vector $\vec{\epsilon}_i$, we have the following Proposition.

PROPOSITION 5. *Let* $\frac{1}{v^m} = min\,(1/v^{c_k})$ *with* m *denoting the index of the agent (consumer or supplier) with minimal price sensitivity, then the* k*-th component of the optimal fee vector* $f_i^{c_k}$ *is given by*

$$v^m \left(1 + W_0 \left(exp\left(-1 - \sum_{k=1}^{K+1} \frac{v_0^{c_k}}{v^{c_k}}\right) \sum_{j=1}^{N} \frac{\lambda_j}{\lambda_{N+1}} exp\left(\sum_{k=1}^{K+1} \frac{v_j^{c_k}}{v^{c_k}} - \sum_{k=1}^{K+1} (\alpha^{c_k} - \alpha^m)\left(f_j^{c_k}\right)^*\right)\right)\right) - \sum_{k=1, c_k \neq m}^{K+1} \left(f_i^{c_k}\right)^*$$

if $c_k = m$*, and* $v_i^{c_k} - v_0^{c_k}$ *otherwise.*

Thus, the qualitative results from our single-resource setting carry over to the case of multi-dimensional resources. In particular, the total fee per match is the same across all tuples. Also, the resource with the lowest price sensitivity pays as much as possible up to the point, where lowering fees for other resources no longer increases their matching demand functions.

6. Concluding remarks

The Internet has spawned new forms of economic activity and gave rise to the development and growth of online marketplaces which in turn created new research challenges for both academics and practitioners (cf. Ghosh and Goel (2016)). This paper derives optimal pricing policies for a matching marketplace platform. We show that with exponentially-distributed random participants' valuations, it is optimal to charge the same total fee per match to participants with different observable attributes, and this fee should be levied on the less elastic side of the market up to a threshold. We generalize our approach to the case where the random valuations of marketplace participants follow an arbitrary distribution. We show that this is a special case of a specification where the outcome probabilities are proportional to arbitrary functions of the ex-ante valuations. We further generalize our results to the case of multi-dimensional resources.

[8] More generally, if the fee feasibility set $\mathcal{A}_i$ is a strict subset of $\mathbb{R}^+_{K+1}$ and is such that the fee for the side with the smallest price sensitivity is bounded from above, then this side is charged as much as possible and the argument applies recursively to the other sides.

Our results shed light on how marketplaces may be compensated for their matching function, and it will be interesting to consider them in conjunction with more elaborate specifications of particular vertical marketplaces. In a marketplace like Uber, for example, both capacity utilization and information aggregation are important. How do these two aspects interact? We believe our model provides a reasonable approximation under light relative demand conditions or for very short term decisions. However, a more intricate model is called for under more general conditions.

Appendix A: Probability computations

A1 Probability of a successful match under perfect information

Proof. $I_{ab} \equiv \{i=(a,b):\ a\ accepts\ b\}$, $J_{ba} \equiv \{i=(a,b):\ b\ accepts\ a\}$. Consumer a's request expires within time $t_A \sim Exp\left(\frac{1}{\tau_A}\right)$ and the first acceptable supplier arrives within time $t_B = min_b\,(t_b)$ for $(a,b) \in I_{ab} \cap J_{ba}$. Since each supplier b arrives within time $t_b \sim Exp\left(\frac{1}{\tau_B}\right)$, then $t_B \sim Exp\left(\frac{1}{\tau_B}\left|I_{ab} \cap J_{ba}\right|\right)$. Thus the probability of the request resulting in the actual service is given by

$$\phi = Pr\,(t_A < t_B) = Pr\left(Exp\left(\frac{1}{\tau_A}\right) > Exp\left(\frac{1}{\tau_B}\left|I_{ab}\cap J_{ba}\right|\right)\right)$$
$$= \frac{\tau_A}{\tau_A + \frac{\tau_B}{|I_{ab}\cap J_{ba}|}} = \frac{1}{1+\frac{1}{\frac{\tau_A}{\tau_B}|I_{ab}\cap J_{ba}|}}$$

□

A2 Probability of a successful match under imperfect information

Proof. Let $I_{ab} \equiv \{i=(a,b):\ a\ accepts\ b\}$, $J_{ba} \equiv \{i=(a,b):\ b\ accepts\ a\}$. The customer request expires within time $t_A \sim Exp\left(\frac{1}{\tau_A}\right)$ and the first acceptable supplier arrives within time $t_B = min_b\,(t_b)$ for $(a,b) \in I_{ab} \cap J_{ba}$. Since $Pr\,(i \in I_{ab}) = exp\left(\frac{v_i^a - f_i^a - v_0^A}{v^A}\right)$ and $Pr\,(i \in J_{ba}) = exp\left(\frac{v_i^b - f_i^b - v_0^B}{v^B}\right)$, and each supplier b arrives within time $t_b \sim Exp\left(\frac{1}{\tau_B}\right)$, then $t_B \sim Exp\left(\frac{1}{\tau_B}\sum_i Pr\,(i\in I_{ab})\,Pr\,(i \in J_{ba})\right)$. Hence the probability of the request being matched is given by

$$\phi = Pr\,(t_A > t_B) = Pr\left(Exp\left(\frac{1}{\tau_A}\right) > Exp\left(\frac{1}{\tau_B}\sum_i Pr\,(i\in I_{ab})\,Pr\,(i\in J_{ba})\right)\right)$$
$$= \frac{\tau_A}{\tau_A + \frac{\tau_B}{\sum_i Pr(i\in I_{ab})Pr(i\in J_{ba})}} = \frac{1}{1+\frac{1}{\frac{\tau_A}{\tau_B}\sum_i Pr(i\in I_{ab})Pr(i\in J_{ba})}}$$
$$= \left(1+\left(exp\left(-\frac{v_0^A}{v^A}-\frac{v_0^B}{v^B}\right)\frac{\tau_A}{\tau_B}\sum_i exp\left(\frac{v_i^a}{v^A}+\frac{v_i^b}{v^B}-\frac{f_i^a}{v^A}-\frac{f_i^b}{v^B}\right)\right)^{-1}\right)^{-1}$$
$$= \frac{1}{1+\frac{1}{k\sum_i \beta_i exp\left(-\alpha^B f_i^b - \alpha^A f_i^a\right)}},$$

where $k \equiv exp\left(-\frac{v_0^A}{v^A}-\frac{v_0^B}{v^B}\right)\frac{\tau_A}{\tau_B}$, $\beta_i \equiv exp\left(\frac{v_i^a}{v^A}+\frac{v_i^b}{v^B}\right)$, $\alpha^B \equiv \frac{1}{v^B}$, $\alpha^A \equiv \frac{1}{v^A}$.

□

A3 Probability of a successful request being served by a supplier of a particular type

Proof. Let $I_{ab} \equiv \{i=(a,b):\ a\ accepts\ b\}$, $J_{ba} \equiv \{i=(a,b):\ b\ accepts\ a\}$. In addition, let the request expiration time $t_A \sim Exp\left(\frac{1}{\tau_A}\right)$ and $t_i \sim Exp\left(\frac{1}{\tau_i}\right)$ – the arrival time for supplier corresponding to the match indexed i and who accepts a and is accepted by a, i.e. $i \in I_{ab} \cap J_{ba}$, and $t_{-i} \sim$

$Exp\left(\frac{1}{\tau_i}\right)$ – arrival time of any supplier $j \in I_{ab} \cap J_{ba}/\{i\}$, i.e. the supplier other than i, who accepts a and is accepted by a. Then $\phi_i = Pr\left(t_i < t_{-i}, min\left(t_i, t_{-i}\right) < t_A\right) = Pr\left(t_i = min\left(t_i, t_{-i}, t_A\right)\right) = Pr\left(t_i = min\left(t_i, min\left(t_{-i}, t_A\right)\right)\right)$. Due to t_{-i} and t_A being exponentially distributed $min\left(t_{-i}, t_A\right)$ is also exponentially distributed, therefore $\phi_i = \frac{1/\tau_i}{1/\tau_i + 1/\tau_{-i} + 1/\tau_A}$. Further, substituting $\frac{1}{\tau_i} = \frac{1}{\tau_B} Pr\left(i \in I_{ab}\right) Pr\left(i \in J_{ba}\right) = \frac{1}{\tau_B} exp\left(\frac{v_i^a}{v^A} + \frac{v_i^b}{v^B} - \frac{f_i^a}{v^A} - \frac{f_i^b}{v^B} - \frac{v_0^B}{v^B} - \frac{v_0^A}{v^A}\right) = \frac{k}{\tau_A}\beta_i e^{-\alpha^B f_i^b - \alpha^A f_i^a}$ into the expression for ϕ_i we get

$$\phi_i = \frac{\frac{k}{\tau_A}\beta_i e^{-\alpha f_i}}{\frac{k}{\tau_A}\beta_i e^{-\alpha f_i} + \sum_{-i} \frac{k}{\tau_A}\beta_j e^{-\alpha f_j} + \frac{1}{\tau_A}} = \frac{\beta_i e^{-\alpha f_i}}{\sum_j \beta_j e^{-\alpha f_j} + \frac{1}{k}},$$

where $k \equiv exp\left(-\frac{v_0^A}{v^A} - \frac{v_0^B}{v^B}\right)\frac{\tau_A}{\tau_B}$, $\beta_i \equiv exp\left(\frac{v_i^a}{v^A} + \frac{v_i^b}{v^B}\right)$, $\alpha \equiv (\alpha^A, \alpha^B)$, $f_i \equiv (f_i^a, f_i^b)$, $\alpha f_i \equiv \alpha^B f_i^b + \alpha^A f_i^a$.
□

Appendix B: Proofs

Proof of Proposition 1:

Let $I_{ab} \equiv \{i = (a,b):\ a \text{ accepts } b\}$, $J_{ba} \equiv \{i = (a,b):\ b \text{ accepts } a\}$. The expected value maximization problem is given by

$$\max_{\{(f_i^a, f_i^b)\}} \phi \sum_i V_i \frac{1}{|I_{ab} \cap J_{ba}|} =$$
$$\max_{\{(f_i^a, f_i^b)\}} \frac{\sum_i V_i}{|I_{ab} \cap J_{ba}| + 1/k},$$

where $V_i \equiv v_i^a + v_i^b$, $k \equiv \tau_A/\tau_B$. The fees $f_i = \{(f_i^a, f_i^b)\}$ apply only to the set of admissible pairs i; all other pairs will be excluded. Let I^* denote the optimal admissible set with fees f_i^* for each $i \in I^*$. That is,

$$I^* = \left\{i:\ v_i^a \geq f_i^{a^*} + v_0^A, v_i^b \geq f_i^{b^*} + v_0^B\right\}.$$

Then, due to the optimality of I^*,

$$\frac{\sum_{I^*} V_i}{|I^*| + \frac{1}{k}} > \frac{\sum_{I^*} V_i + V_j}{|I^*| + 1 + \frac{1}{k}}\ \forall j \notin I^*.$$

Thus,

$$V_j < k \sum_{I^*} (V_i - V_j)\ \forall j \notin I^*.$$

Similarly, it can be shown that

$$V_j \geq k \sum_{I^*/j} (V_i - V_j)\ \forall j \in I^*.$$

That is, in order to be in the admissible set, pair j should add enough value to the group. Intuitively, if the absolute value added per unit of request expiration time $\frac{V_j}{\tau_A}$ is higher than the value decreased by adding a pair per unit of non-availability time $\frac{\sum_{I^*}(V_i - V_j)}{\tau_B}$, then the pair should be in the admissible set.

We now identify the structure of I^*. In particular, we show that there exists V^* such that for all pairs j, $j \in I^*$ if and only if $V_j > V^*$. Without loss of generality, assume that I^* does not contain suppliers that create the total value of 0.

First, we show that I^* is not empty. The proof is by contradiction. Assume that I^* is empty. Then, take any supplier b_1 and set the fees such that $I\left(a \text{ accepts } b_1\right) I\left(b_1 \text{ accepts } a\right) = 1$, which makes the success probability of the match (a, b_1) positive. This, in turn, makes the expected value positive. It follows that the empty set (which produces zero value) cannot be a solution. Since the number of pairs in the admissible set is finite, the maximum is achieved with the nonempty set I^*.

Second, we show that highest valuation type belongs to I^*. In particular, let

$$V_{j^*} = \max_i V_i,$$

and assume that $j^* \notin I^*$. This implies

$$V_{j^*} < k \sum_{I^*} (V_i - V_{j^*}) \leq 0.$$

If the set of valuations is non-trivial, that is, its maximum is greater than 0, this leads to a contradiction.

Third, we demonstrate that I^* corresponds to a contiguous interval in V space, i.e., if $m \in I^*$ and $p \in I^*$, then $V_n \in [V_m, V_p]$ implies $n \in I^*$. The proof is by contradiction.

Assume $n \notin I^*$. Then

$$\begin{aligned} V_n &< k \sum_{I^*} (V_i - V_n) \\ &= k \sum_{I^*/m} (V_i - V_n) + V_m - V_n \\ &= k \sum_{I^*/m} (V_i - V_m) + (k+1) \sum (V_m - V_n) \\ &\leq k \sum_{I^*/m} (V_i - V_m). \end{aligned}$$

Thus, since $V_m \leq V_n$, we have

$$V_m < k \sum_{I^*/m} (V_i - V_m).$$

This contradicts the optimality criterion (1), hence $n \in I^*$.

Proof of Proposition 2:

Using the memorylessness property of the exponential distribution, i.e., $E\left(y \middle| y > x\right) = Ey + x$, we derive the expected total surplus

$$\begin{aligned} V &= E \sum_{i \equiv (a,b)} \left(v_i^a + \epsilon_i^a + v_i^b + \epsilon_i^b\right) I\left(b \text{ } serves \text{ } a\right) \\ &= \sum_i \left(v_i^a + v_i^b\right) E\left(I\left(b \text{ } serves \text{ } a\right)\right) + \sum_i E\left(\left(\epsilon_i^a + \epsilon_i^b\right) I\left(b \text{ } serves \text{ } a\right)\right) \\ &= \sum_i \left(v_i^a + v_i^b\right) \phi_i + \sum_i E\left(\left(\epsilon_i^a + \epsilon_i^b\right) \middle| I\left(b \text{ } serves \text{ } a\right)\right) \phi_i \\ &= \sum_i \left(v_i^a + v_i^b\right) \phi_i + \sum_i E\left(\epsilon_i^a + \epsilon_i^b \middle| \epsilon_i^b > f_i^b + v_0^B - v_i^b, \epsilon_i^a > f_i^a + v_0^A - v_i^a\right) \phi_i \\ &= \sum_i \left(v_i^a + v_i^b + E\left(\epsilon_i^a\right) + f_i^a + v_0^A - v_i^a + E\left(\epsilon_i^b\right) + f_i^b + v_0^B - v_i^b\right) \phi_i \\ &= \sum_i \left(v^A + v_0^A + f_i^a + v^B + v_0^B + f_i^b\right) \phi_i. \end{aligned} \tag{7}$$

Substituting ϕ_i from (2) we get $V=\frac{\sum_i\left(v^A+v_0^A+v^B+v_0^B+f_i^b+f_i^a\right)\beta_i e^{-\alpha^B f_i^b-\alpha^A f_i^a}}{\sum_i\beta_i e^{-\alpha^B f_i^b-\alpha^A f_i^a}+\frac{1}{k}}$, where $\beta_i\equiv exp\left(\alpha^A v_i^a+\alpha^B v_i^b\right)$, $k\equiv exp\left(-\alpha^A v_0^A-\alpha^B v_0^B\right)\frac{\tau_A}{\tau_B}$, $\alpha^B\equiv\frac{1}{v^B}$, $\alpha^A\equiv\frac{1}{v^A}$. We notice that f_i^b and f_i^a enter this expression in a symmetric way. Then, assuming that $\alpha^A>\alpha^B$, we rearrange the expression as follows: $V=\frac{\sum_i\left(v^A+v_0^A+v^B+v_0^B+f_i^b+f_i^a\right)\beta_i e^{-\alpha^B\left(f_i^b+f_i^a\right)-\left(\alpha^A-\alpha^B\right)f_i^a}}{\sum_i\beta_i e^{-\alpha^B f_i^b-\alpha^A f_i^a}+\frac{1}{k}}$. Substituting $F_i=v^A+v_0^A+v^B+v_0^B+f_i^b+f_i^a$, we obtain $V=\frac{\sum_i F_i\beta_i e^{\alpha^B\left(v^A+v_0^A+v^B+v_0^B\right)}e^{-\alpha^B F_i-\left(\alpha^A-\alpha^B\right)f_i^a}}{\sum_i\beta_i e^{\alpha^B\left(v^A+v_0^A+v^B+v_0^B\right)}e^{-\alpha^B F_i-\left(\alpha^A-\alpha^B\right)f_i^a}+\frac{1}{k}}$. Further, we notice that if $f_i^a\geq v_i^a-v_0^A$, i.e. when $Pr\left(\epsilon_i^a>f_i^a+v_0^A-v_i^a\right)<1$, then $\frac{\partial V}{\partial f_i^a}<0$. However, if $f_i^{a^*}<v_i^a-v_0^A$, then $Pr\left(\epsilon_i^a>f_i^a+v_0^A-v_i^a\right)=1\neq exp\left(-\alpha^A\left(f_i^a+v_0^A-v_i^a\right)\right)$ and thus $V=\frac{\sum_i F_i\beta_i e^{\alpha^B\left(v^A+v_0^A+v^B+v_0^B\right)}e^{-\alpha^B F_i+\alpha^B f_i^a}}{\sum_i\beta_i e^{\alpha^B\left(v^A+v_0^A+v^B+v_0^B\right)}e^{-\alpha^B F_i+\alpha^B f_i^a}+\frac{1}{k}}$, hence $\frac{\partial V}{\partial f_i^a}>0$. Thus the optimal value of V^* is achieved at $f_i^{a^*}=v_i^a-v_0^A$. Plugging in the optimal values of f_i^a yields $V=\frac{\sum_i\tilde\beta_i F_i e^{-\alpha^B F_i}}{\sum_i\tilde\beta_i e^{-\alpha^B F_i}+\frac{1}{k}}$, where $\tilde\beta_i=\beta_i exp\left(\alpha^B\left(v^A+v_0^A+v^B+v_0^B\right)-\left(\alpha^A-\alpha^B\right)f_i^{a^*}\right)$. We now transform the expression for value as follows:

$$
\begin{aligned}
V&=k\frac{\sum_{-i}F_j\tilde\beta_j e^{-\alpha F_j}+F_i\tilde\beta_i e^{-\alpha F_i}}{1+k\sum_{-i}\tilde\beta_j e^{-\alpha F_j}+k\tilde\beta_i e^{-\alpha F_i}}\\
&=k\frac{\sum_{-i}\tilde\beta_j F_j e^{-\alpha F_j}}{1+k\sum_{-i}\tilde\beta_j e^{-\alpha F_j}}\cdot\frac{1+\left(\tilde\beta_i F_i e^{-\alpha F_i}\right)/\sum_{-i}\tilde\beta_j F_j e^{-\alpha F_j}}{1+k\tilde\beta_i e^{-\alpha F_i}/\left(1+k\sum_{-i}\tilde\beta_j e^{-\alpha F_j}\right)}\\
&=\frac{\alpha_2}{\alpha_1}\frac{1+\alpha_1 F_i e^{-\alpha F_i}}{1+\alpha_2 e^{-\alpha F_i}},
\end{aligned}
$$

where $\alpha_1=\tilde\beta_i/\sum_{-i}\tilde\beta_j F_j e^{-\alpha F_j}$ and $\alpha_2=k\tilde\beta_i/\left(1+k\sum_{-i}\tilde\beta_j e^{-\alpha F_j}\right)$ are factors independent of F_i. It is trivial to verify that $|V|<\frac{\alpha_2}{\alpha_1}\max_{F_i}\left(1+\alpha_1 F_i e^{-\alpha F_i}\right)=\frac{\alpha_2}{\alpha_1}\left(1+e^{-1}\right)$. Thus, if $V\left(F_i\right)$, as a bounded function, has an optimal value, it is either reached at $F_i=0$, $F_i=\infty$ or at F_i such that $\frac{dV}{dF_i}=0$.

Differentiating V w.r.t. F_i:

$$
\begin{aligned}
\frac{dV}{dF_i}&=\frac{\alpha_2}{\alpha_1}e^{-\alpha F_i}\left[\alpha_1\left(1-\alpha F_i\right)\left(1+\alpha_2 e^{-\alpha F_i}\right)+\alpha_2\alpha\left(1+\alpha_1 F_i e^{-\alpha F_i}\right)\right]\left(1+\alpha_2 e^{-\alpha F_i}\right)^{-2}\\
&=\frac{\alpha_2}{\alpha_1}e^{-\alpha F_i}\alpha_1\alpha\left(1+\alpha_2 e^{-\alpha F_i}\right)\left[\left(\frac{1}{\alpha}-F_i\right)+\frac{\alpha_2}{\alpha_1}\frac{\left(1+\alpha_1 F_i e^{-\alpha F_i}\right)}{\left(1+\alpha_2 e^{-\alpha F_i}\right)}\right]\left(1+\alpha_2 e^{-\alpha F_i}\right)^{-2}\\
&=\frac{\alpha_2}{\alpha_1}e^{-\alpha F_i}\alpha_1\alpha\left(1+\alpha_2 e^{-\alpha F_i}\right)^{-1}\left[\left(\frac{1}{\alpha}-F_i\right)+V\right].
\end{aligned}
$$

Since $\frac{dV}{dF_i}(0)=\frac{\alpha_2}{\alpha_1}\alpha_1\alpha\left(1+\alpha_2\right)^{-1}\left[V+\frac{1}{\alpha}\right]>0$, then $F_i^*>0$. Since also

$$
\begin{aligned}
\frac{dV}{dF_i}&=\frac{\alpha_2}{\alpha_1}e^{-\alpha F_i}\alpha_1\alpha\left(1+\alpha_2 e^{-\alpha F_i}\right)^{-1}\left[V-\left(F_i-\frac{1}{\alpha}\right)\right]\\
&\leq\frac{\alpha_2}{\alpha_1}e^{-\alpha F_i}\alpha_1\alpha\left(1+\alpha_2 e^{-\alpha F_i}\right)^{-1}\left[\frac{\alpha_2}{\alpha_1}\left(1+e^{-1}\right)-\left(F_i-\frac{1}{\alpha}\right)\right],
\end{aligned}
$$

then for $F_i>\frac{1}{\alpha}\beta_i\frac{\alpha_2}{\alpha_1}\left(1+e^{-1}\right)\frac{dV}{dF_i}<0$. Thus $F_i^*<\infty$. As a result, the optimal value $V^*=\max_{F_i}V\left(F_i\right)$ is reached when $\frac{dV}{dF_i}=0$, which implies $f_i^{D^*}+f_i^{S^*}=F_i^*-v^A-v_0^A-v^B-v_0^B=\frac{1}{\alpha^B}+V^*-v^A-v_0^A-v^B-v_0^B$. Since $\alpha^B\equiv\frac{1}{v^B}$, then $f_i^{a^*}+f_i^{b^*}\equiv f^*=V^*-v_0^B-v^A-v_0^A$. To find the exact value of V^* and, in turn f^* we plug F_i^* into the expression for revenue:

$$
V^*=k\frac{\sum_i F_i^*\tilde\beta_i e^{-\alpha^B F_i^*}}{1+k\sum_i\tilde\beta_i e^{-\alpha F_i^*}}=k\frac{\sum_i\left(\frac{1}{\alpha^B}+V^*\right)\tilde\beta_i e^{-\alpha^B F_i^*}}{1+k\sum_i\tilde\beta_i e^{-\alpha^B F_i^*}}.
$$

Multiplying both sides by $1+k\sum_i\tilde{\beta}_i e^{-\alpha^B F_i^*}$ yields $V^*+V^*k\sum\tilde{\beta}_i e^{-\alpha^B F_i^*}=\frac{k}{\alpha^B}\sum_i\tilde{\beta}_i e^{-\alpha^B F_i^*}+V^*k\sum\tilde{\beta}_i e^{-\alpha^B F_i^*}$. Thus, $V^*=\frac{k}{\alpha^B}\sum_i\tilde{\beta}_i e^{-\alpha^B F_i^*}=\frac{k}{\alpha^B}\sum_i\tilde{\beta}_i e^{-\alpha^B\left(\frac{1}{\alpha^B}+V^*\right)}=\frac{k}{\alpha^B}\sum_i\tilde{\beta}_i e^{-\left(1+\alpha^B V^*\right)}$. It follows that $\alpha^B V^* e^{\alpha^B V^*}=k\sum\frac{\tilde{\beta}_i}{e}$. Applying the principal branch of Lambert function $W_0(\cdot)$ which is the solution to the equation $z=W(ze^z),\ z\geq -1$ (Corless et al. (1996)) to both sides we get

$$\begin{aligned} W_0\left(\alpha^B V^* e^{\alpha^B V^*}\right) &= W_0\left(\frac{k}{e}\sum\tilde{\beta}_i\right)\\ \alpha^B V^* &= W_0\left(\frac{k}{e}\sum\tilde{\beta}_i\right)\\ V^* &= \frac{1}{\alpha^B}W_0\left(\frac{k}{e}\sum\tilde{\beta}_i\right).\end{aligned}$$

Hence,

$$\begin{aligned} f_i^{b^*} &= \frac{W_0\left(\frac{k}{e}\sum_i\tilde{\beta}_i\right)}{\alpha^B}-v_0^B-v^A-v_0^A-f_i^{a^*}\\ &= v^B W_0\left(exp\left(-\frac{v_0^A}{v^A}\right)\frac{\tau_A}{\tau_B}\sum_{i\equiv(a,b)}exp\left(\frac{v_i^a}{v^A}+\frac{v_i^b}{v^B}+\frac{v^A+v_0^A}{v^B}-\left(\alpha^A-\alpha^B\right)f_i^{a^*}\right)\right)-v_0^B-v^A-v_0^A-f_i^{a^*},\end{aligned}$$

where $f_i^{a^*}=v_i^a-v_0^A$. Due to symmetry, if $\alpha^A<\alpha^B$, we have

$$f_i^{a^*}=v^A W_0\left(exp\left(-\frac{v_0^B}{v^B}\right)\frac{\tau_A}{\tau_B}\sum_{i\equiv(a,b)}exp\left(\frac{v_i^a}{v^A}+\frac{v_i^b}{v^B}+\frac{v^B+v_0^B}{v^A}-\left(\alpha^B-\alpha^A\right)f_i^{b^*}\right)\right)-v_0^A-v^B-v_0^B-f_i^{b^*},$$

where $f_i^{b^*}=v_i^b-v_0^B$. The case $\alpha^A=\alpha^B$ follows directly from the above.

Proof of Proposition 3:

The expected revenue is given by

$$V=\sum_i\left(f_i^a+f_i^b\right)\phi_i.$$

Without loss of generality, assume that $\alpha^A>\alpha^B$. Upon substituting $F_i=f_i^a+f_i^b$, we get

$$V=\frac{\sum_i F_i\beta_i e^{-\alpha^B F_i-\left(\alpha^A-\alpha^B\right)f_i^a}}{\sum_i\beta_i e^{-\alpha^B F_i-\left(\alpha^A-\alpha^B\right)f_i^a}+\frac{1}{k}},$$

where $\beta_i\equiv exp\left(\alpha^A v_i^a+\alpha^B v_i^b\right)$, $k\equiv exp\left(-\alpha^A v_0^A-\alpha^B v_0^B\right)\frac{\tau_A}{\tau_B}$, $\alpha^B\equiv\frac{1}{v^B}$, $\alpha^A\equiv\frac{1}{v^A}$. As in the proof of Proposition 2, $f_i^{a^*}=v_i^a-v_0^A$ and, hence, $V=\frac{\sum_i\tilde{\beta}_i F_i e^{-\alpha^B F_i}}{\sum_i\tilde{\beta}_i e^{-\alpha^B F_i}+\frac{1}{k}}$, where $\tilde{\beta}_i=\beta_i exp\left(-\left(\alpha^A-\alpha^B\right)f_i^{a^*}\right)$. Repeating the steps of the proof of Proposition 2 exactly yields

$$V^*=\frac{1}{\alpha^B}W_0\left(\frac{k}{e}\sum\tilde{\beta}_i\right).$$

Hence,

$$\left(f_i^b\right)^*=v^B\left[1+W_0\left(exp\left(-\frac{v_0^A}{v^A}-\frac{v_0^B}{v^B}-1\right)\frac{\tau_A}{\tau_B}\sum_{i\equiv(a,b)}exp\left(\frac{v_i^a}{v^A}+\frac{v_i^b}{v^B}-\left(\alpha^A-\alpha^B\right)f_i^{a^*}\right)\right)\right]-\left(f_i^a\right)^*,$$

where $f_i^{a^*}=v_i^a-v_0^A$. Due to symmetry, if $\alpha^A<\alpha^B$, we have

$$\left(f_i^a\right)^*=v^A\left[1+W_0\left(exp\left(-\frac{v_0^A}{v^A}-\frac{v_0^B}{v^B}-1\right)\frac{\tau_A}{\tau_B}\sum_{i\equiv(a,b)}exp\left(\frac{v_i^a}{v^A}+\frac{v_i^b}{v^B}-\left(\alpha^B-\alpha^A\right)f_i^{b^*}\right)\right)\right]-\left(f_i^b\right)^*,$$

where $f_i^{b^*}=v_i^b-v_0^B$. The case $\alpha^A=\alpha^B$ follows directly from the above.

Proof of Theorem 1:

The first-order condition w.r.t. f_i is given by

$$\frac{\nabla(v_i h_i)\sum_{i=1}^N h_i - \nabla(h_i)\sum_{i=1}^N (h_i v_i)}{\left(\sum_{i=1}^N h_i\right)^2} = 0,$$

which can be rewritten as

$$\frac{1}{\sum_{i=1}^N h_i}\left[\nabla(v_i h_i) - \nabla(h_i)\frac{\sum_{i=1}^N h_i v_i}{\sum_{i=1}^N h_i}\right] = 0,$$

which implies

$$\nabla(v_i h_i)\Big|_{f_i} = \frac{\sum_{i=1}^N h_i v_i}{\sum_{i=1}^N h_i}\Big|_{f_i} \nabla h_i\Big|_{f_i} = V^*\nabla h_i\Big|_{f_i}, \tag{8}$$

where $V^* = V(f_1, \ldots, f_N)$ is the optimal value of the objective function. Let $f^* = (f_1^*, \ldots, f_N^*)$ denote the optimal action vector, then $f_i^*(V^*)$ either solves (8) or is on the boundary of $\mathcal{A}$. As a result, there is a set of candidates $\{f^j(V)\}$ for an optimal action vector f^*, where $f^j(V) = \left(f_1^j(V), \ldots, f_N^j(V)\right)$, and $f_i^j \in \mathcal{A}_i$ or f_i^j solves (8) for $i = 1, \ldots, N$. For each j we plug $f^j(V)$ in the expression for the objective $V = \frac{\sum_{i=1}^N v_i\left(f_i^j(V)\right)h_i\left(f_i^j(V)\right)}{\sum_{i=1}^N h_i\left(f_i^j(V)\right)}$ and solve for V. Then we arrive at the set of values $\{V^j\}$ corresponding to optimal action candidates $\{f^j(V)\}$. Obviously, the largest value $V^* \equiv V^{j^*} = \max_j V^j$ is the optimal value of the objective function, and, hence, the corresponding vector $f^{j^*}\left(V^{j^*}\right)$ is the optimal action vector.

Proof of Proposition 4:

If f_i^* is the solution to (5), then for all f_i

$$(v_i^* - V^*)\,h_i^* \geq (v_i - V^*)\,h_i, \tag{9}$$

where $v_i^* = v_i(f_i^*)$, $h_i^* = h_i(f_i^*)$, $v_i = v_i(f_i)$, $h_i = h_i(f_i)$.

Plugging in

$$V^* = \frac{v_i^* h_i^* + (VH)_{-i}^*}{h_i^* + H_{-i}^*},$$

where $(VH)_{-i} = \sum_{-i} v_j h_j$, $H_{-i} = \sum_{-i} h_j$, into (9) we get

$$\left(v_i^* - \frac{v_i^* h_i^* + (VH)_{-i}^*}{h_i^* + H_{-i}^*}\right) h_i^* \geq \left(v_i - \frac{v_i^* h_i^* + (VH)_{-i}^*}{h_i^* + H_{-i}^*}\right) h_i$$

Rearranging this expression yields

$$\left(\frac{v_i^* h_i^* + (VH)_{-i}^*\, h_i^*}{h_i^* + H_{-i}^*}\right) \geq \frac{v_i h_i^* h_i + v_i H_{-i}^* h_i - v_i^* h_i^* h_i + (VH)_{-i}^*\, h_i}{h_i^* + H_{-i}^*}$$

$$(v_i^* - v_i)\, h_i^* h_i + \left(v_i^* H_{-i}^* - (VH)_{-i}^*\right) h_i^* - \left(v_i H_{-i}^* - (VH)_{-i}^*\right) h_i \geq 0$$

$$\left(v_i^* h_i^* + (VH)_{-i}^*\right)\left(h_i + H_{-i}^*\right) \geq \left(v_i h_i + (VH)_{-i}^*\right)\left(h_i^* + H_{-i}^*\right)$$

$$\frac{\left(v_i^* h_i^* + (VH)_{-i}^*\right)}{h_i^* + H_{-i}^*} \geq \frac{\left(v_i h_i + (VH)_{-i}^*\right)}{h_i + H_{-i}^*}.$$

Thus, f_i^* is the optimal solution for (3) as well.

Proof of Proposition 5:

The objective function is given by

$$V=\sum_{i=1}^{N}\left[\sum_{k=1}^{K+1} f_i^{c_k}\frac{\lambda_i exp\left(\sum_{k=1}^{K+1}\alpha^{c_k}\left(v_i^{c_k}-v_0^{c_k}-f_j^{c_k}\right)\right)}{\lambda_{N+1}+\sum_{j=1}^{N}\lambda_j exp\left(\sum_{k=1}^{K+1}\alpha^{c_k}\left(v_j^{c_k}-v_0^{c_k}-f_j^{c_k}\right)\right)}\right]$$

Recall that m is the index of the agent (consumer or supplier) with minimal price sensitivity, and let α^m denote the corresponding price sensitivity (for simplicity we assume there is a unique minimum; the extension to multiple minima is immediate). Set $F_i=\left(\sum_{k=1}^{K+1} f_i^{c_k}\right)$ and rewrite the objective function as

$$\begin{aligned}V&=\sum_{i=1}^{N}\left[F_i\frac{\lambda_i exp\left(-\alpha^m F_i+\sum_{k=1}^{K+1}\left[\alpha^{c_k}\left(v_i^{c_k}-v_0^{c_k}\right)-\left(\alpha^{c_k}-\alpha^m\right)f_i^{c_k}\right]\right)}{\lambda_{N+1}+\sum_{j=1}^{N}\lambda_j exp\left(-\alpha^m F_j+\sum_{k=1}^{K+1}\left[\alpha^{c_k}\left(v_j^{c_k}-v_0^{c_k}\right)-\left(\alpha^{c_k}-\alpha^m\right)f_j^{c_k}\right]\right)}\right]\\&=\sum_{i=1}^{N}\left[F_i\frac{\lambda_i\beta_i exp\left(-\alpha^m F_i-\sum_{k=1}^{K+1}\left(\alpha^{c_k}-\alpha^m\right)f_i^{c_k}\right)}{\lambda_{N+1}+\sum_{j=1}^{N}\lambda_j\beta_j exp\left(-\alpha^m F_j-\sum_{k=1}^{K+1}\left(\alpha^{c_k}-\alpha^m\right)f_i^{c_k}\right)}\right],\end{aligned}$$

where $\beta_i=\sum_{k=1}^{K+1}\left[\alpha^{c_k}\left(v_i^{c_k}-v_0^{c_k}\right)\right]$ for $i=1,\dots,N$.

Proposition 4 implies the following subproblem for each i:

$$\max_{f_i^{c_k},F_i}\left(exp\left(-\alpha^m F_i-\sum_{k=1}^{K+1}\left(\alpha^{c_k}-\alpha^m\right)f_i^k\right)\left(F_i-V^*\right)\right)$$
$$s.t.\ f_i^{c_k}\geq v_i^{c_k}-v_0^{c_k}.$$

It immediately follows that the optimal fees for all but one resource are at the boundary points: $f_i^{c_k^*}=v_i^{c_k}-v_0^{c_k}$ for $c_k\neq m$. Plugging these fees into the objective function, we obtain

$$V^*=\sum_{i=1}^{N}\left[F_i^*\frac{\lambda_i\tilde{\beta}_i exp\left(-\alpha^m F_i^*\right)}{\lambda_{N+1}+\sum_{j=1}^{N}\lambda_j\tilde{\beta}_j exp\left(-\alpha^m F_j^*\right)}\right],\tag{10}$$

where $\tilde{\beta}_i\equiv\beta_i exp\left(-\sum_{k=1}^{K+1}\left(\alpha^{c_k}-\alpha^m\right)f_i^{c_k^*}\right)$. Then, as in Proposition 2,

$$F_i^*=V^*+\frac{1}{\alpha^m}.\tag{11}$$

Plugging F_i^* into (10), solving for V^* and plugging the solution into (11), we get

$$\begin{aligned}\left(f_i^m\right)^*=v^m&\left(1+W_0\left(exp\left(-1-\sum_{k=1}^{K+1}\frac{v_0^{c_k}}{v^{c_k}}\right)\sum_{j=1}^{N}\frac{\lambda_j}{\lambda_{N+1}}exp\left(\sum_{k=1}^{K+1}\frac{v_j^{c_k}}{v^{c_k}}-\sum_{k=1}^{K+1}\left(\alpha^{c_k}-\alpha^m\right)\left(f_j^{c_k}\right)^*\right)\right)\right)\\&-\sum_{k=1,c_k\neq m}^{K+1}\left(f_i^{c_k}\right)^*.\end{aligned}$$

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