Optimal pricing in discrete choice models

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Logit-based discrete choice models are widely-used and are known to well-approximate more general discrete-choice problems ([1]). In this paper we derive in closed form optimal prices when demand is characterized by a Logit discrete choice model.

A seller has N products $i = 1, 2, \dots N$ available for sale. The buyer seeks to buy (at most) one product in a classical discrete-choice setting. The buyer values a purchase of product i at $v_i = u_i - \alpha_i f_i + \epsilon_i$, where the u_i and α_i are observable by the seller; f_i is the price set by the seller for product i (a function of the u_i and α_i), and ϵ_i are private buyer information distributed as i.i.d. $Gumbel(0, 1)$.

It is well known that in this setting, the buyer will choose product i with probability $p_i = \frac{exp(u_i - \alpha_i f_i)}{\sum_j exp(u_j - \alpha_j f_j)}$, where $i = 0$ corresponds to the option of not buying. The seller wants to set prices f_i to maximize its expected revenue $\sum_i f_i p_i$. We find a closed-form solution to this problem: $f_i^* = \frac{1}{\alpha_i} + V^*$, where V^* is the solution to $\frac{\sum_{i=1}^N (1/\alpha_i + V) exp(u_i - 1 - \alpha_i V)}{1 + \sum_{j=1}^N exp(u_j - 1 - \alpha_j V)} = V$.

We extend this result to a more general revenue optimization setting and show how the results apply to the dynamic case. We further apply the results to derive optimal personalized pricing for heterogeneous customers. This application is of particular importance due to the broad availability of customer data and the growing adoption of price targeting by sellers. ([2])

References

[1] Daniel McFadden and Kenneth Train. Mixed mnl models for discrete response. *Journal of applied Econometrics*, 15(5):447–470, 2000.

[2] Jean-Pierre Dubé and Sanjog Misra. Scalable price targeting. Technical report, National Bureau of Economic Research, 2017.