CK0048 - MÉTODOS NUMÉRICOS II

TAREFA 05

GRUPO 02

4 pontos de interpolação

1) Descobrir $\alpha_1, \dots, \alpha_4$

$$P_4(\alpha) = \frac{1}{2^4 \cdot 4!} \cdot \frac{d^4}{d\alpha^4} \left[(\alpha^2 - 1)^4 \right]$$

$$= \frac{1}{384} \cdot \frac{d^4}{d\alpha^4} \left(\alpha^8 - 4\alpha^6 + 6\alpha^4 - 4\alpha^2 + 1 \right)$$

$$= \frac{1}{384} \cdot \left(1680\alpha^4 - 1440\alpha^2 + 144 \right)$$

$$= \frac{1}{8} \left(35\alpha^4 - 30x^2 + 3 \right)$$

Resolvendo a equação, encontramos as raízes:

$$\alpha_1 = \sqrt{\frac{15 + 2\sqrt{30}}{35}}, \quad \alpha_2 = -\sqrt{\frac{15 + 2\sqrt{30}}{35}},$$

$$\alpha_3 = \sqrt{\frac{15 - 2\sqrt{30}}{35}}, \quad \alpha_4 = -\sqrt{\frac{15 - 2\sqrt{30}}{35}}$$

$\alpha_1 = 0,8611363115$; $\alpha_2 = -0,8611363115$; $\alpha_3 = 0,3399810435$; $\alpha_4 = -0,3399810435$

2) descobrir $x(\alpha_1), \dots, x(\alpha_4)$

$$x(\alpha_1) = \frac{x_i + x_f}{2} - \left(\frac{x_f - x_i}{2}\right) \cdot 0{,}8611363115$$

$$x(\alpha_2) = \frac{x_i + x_f}{2} + \left(\frac{x_f - x_i}{2}\right) 0{,}8611363115$$

$$x(\alpha_3) = \frac{x_i + x_f}{2} - \left(\frac{x_f - x_i}{2}\right) 0{,}3399810435$$

$$x(\alpha_4) = \frac{x_i + x_f}{2} + \left(\frac{x_f - x_i}{2}\right) 0{,}3399810435$$

3) Descobrir w_1, w_2, w_3 e w_4

Sabemos que a área $L_2(\alpha) = L_1(\alpha)$ e $L_4(\alpha)$ é igual a $L_3(\alpha)$, portanto não precisamos calcular w_2 e w_4.

$$L_1(\alpha) = \frac{(\alpha - \alpha_2).(\alpha - \alpha_3).(\alpha - \alpha_4)}{(\alpha_1 - \alpha_2)(\alpha_1 - \alpha_3)(\alpha_1 - \alpha_4)}$$

$$L_3(\alpha) = \frac{(\alpha - \alpha_1)(\alpha - \alpha_2)(\alpha - \alpha_4)}{(\alpha_3 - \alpha_1)(\alpha_3 - \alpha_2)(\alpha_3 - \alpha_4)}$$

Substituindo os α nas equações:

$$L_1(\alpha) = \frac{(\alpha + 0,8611363115)(\alpha - 0,3399810435)(\alpha + 0,3399810435)}{1,078088646}$$

$$L_3(\alpha) = \frac{(\alpha - 0,8611363115)(\alpha + 0,8611363115)(\alpha + 0,3399810435)}{-0,4256349408}$$

Aplicando na fórmula do w:

$$w_1 = w_2 = \int_{-1}^{1} L_1(\alpha)\, d\alpha = 0,3478548451$$

$$w_3 = w_4 = \int_{-1}^{1} L_3(\alpha)\, d\alpha = 0,6521451548$$

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4) Solução → $\int_0^1 (\text{sen}(2x) + 4x^2 + 3x)^2 \, dx$

α_k	$x(\alpha_k) = 1/2 + \alpha_k/2$	$f(x(\alpha_k))$
$\alpha_1 = 0,8611363115$	$x(\alpha_1) = 0,93056815$	$F(x(\alpha_1)) = 52,03717103$
$\alpha_2 = -\alpha_1$	$x(\alpha_2) = 0,06943184$	$F(x(\alpha_2)) = 0,13395342$
$\alpha_3 = 0,3399810435$	$x(\alpha_3) = 0,66999052$	$F(x(\alpha_3)) = 22,83885025$
$\alpha_4 = -\alpha_3$	$x(\alpha_4) = 0,33000947$	$F(x(\alpha_4)) = 4,15664543$

w_k	$f(x(\alpha_k))\, w_k$
$w_1 = 0,3478548451$	18,10138207
$w_2 = w_1$	0,04659634
$w_3 = 0,6521451548$	14,89424553
$w_4 = w_3$	2,71073618
	TOTAL = 35,75296013

$$I = \int_0^1 (\text{sen}(2x) + 4x^2 + 3x)^2 \, dx$$

$$\approx \frac{x_f - x_i}{2} \left[\sum_{k=1}^{4} f(x(\alpha_k))\, w_k \right]$$

$$= \frac{1}{2}(35,75296013) = 17,87648007$$

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