CK0048 - Métodos Numéricos II

Tarefa 06: Desenvolver Quadraturas de Gauss-Hermite, Gauss-Laguerre e Gauss-Chebyshev para 4 pontos

Grupo 2

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Descobrindo x_k e w_k para completar a linha n = 4 da tabela

Gauss-Hermite

$$H_4(x) = (-1)^4 e^{x^2} \frac{\mathrm{d}^4}{\mathrm{d}x^4} e^{-x^2}$$

$$H_4(x) = 16x^4 - 48x^2 + 12$$

Fazendo $H_4(x) = 0$ temos as seguintes raízes:

$$x_1 = \frac{\sqrt{3-\sqrt{6}}}{\sqrt{2}} \qquad x_2 = -\frac{\sqrt{3-\sqrt{6}}}{\sqrt{2}}$$

$$x_3 = \frac{\sqrt{3+\sqrt{6}}}{\sqrt{2}} \qquad x_4 = -\frac{\sqrt{3+\sqrt{6}}}{\sqrt{2}}$$

Calculemos agora w_k

$$w_k = \frac{2^3 4! \sqrt{\Pi}}{4^2 [H_3(x_i)]^2}$$

$$= \frac{2^3 \cdot 2^2 \cdot 3 \cdot 2}{2^2 \cdot 2^2} \cdot \frac{\sqrt{\Pi}}{[H_3(x_i)]^2}$$

$$= \frac{12\sqrt{\Pi}}{[H_3(x_i)]^2}$$

Visitando a tabela de exemplos da seção 3.1.1 do documento fornecido, temos que $H_3(x) = 8x^3 - 12x$. Logo,

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Para $x_i = \frac{\sqrt{3-\sqrt{6}}}{\sqrt{2}} = x_1$

$$w_1 = \frac{12\sqrt{\Pi}}{[H_3\left(\frac{\sqrt{3-\sqrt{6}}}{\sqrt{2}}\right)]^2} \tag{1}$$

$$H_3\left(\frac{\sqrt{3-\sqrt{6}}}{\sqrt{2}}\right) = 8\left(\frac{\sqrt{3-\sqrt{6}}}{\sqrt{2}}\right)^3 - 12\left(\frac{\sqrt{3-\sqrt{6}}}{\sqrt{2}}\right) = (\sqrt{3-\sqrt{6}})(-4\sqrt{3}) \tag{2}$$

Aplicando (2) em (1):

$$w_1 = \frac{\sqrt{\Pi}}{4(3-\sqrt{6})}$$

Para $x_i = -\frac{\sqrt{3-\sqrt{6}}}{\sqrt{2}} = x_2$

$$w_2 = \frac{12\sqrt{\Pi}}{[H_3\left(-\frac{\sqrt{3-\sqrt{6}}}{\sqrt{2}}\right)]^2} \tag{3}$$

$$H_3\left(-\frac{\sqrt{3-\sqrt{6}}}{\sqrt{2}}\right) = 8\left(-\frac{\sqrt{3-\sqrt{6}}}{\sqrt{2}}\right)^3 - 12\left(-\frac{\sqrt{3-\sqrt{6}}}{\sqrt{2}}\right) = (-\sqrt{3-\sqrt{6}})(-4\sqrt{3}) \tag{4}$$

Aplicando (4) em (3):

$$w_2 = \frac{\sqrt{\Pi}}{4(3-\sqrt{6})}$$

Para $x_i = \frac{\sqrt{3+\sqrt{6}}}{\sqrt{2}} = x_3$

$$w_3 = \frac{12\sqrt{\Pi}}{[H_3\left(\frac{\sqrt{3+\sqrt{6}}}{\sqrt{2}}\right)]^2} \tag{5}$$

$$H_3\left(\frac{\sqrt{3+\sqrt{6}}}{\sqrt{2}}\right) = 8\left(\frac{\sqrt{3+\sqrt{6}}}{\sqrt{2}}\right)^3 - 12\left(\frac{\sqrt{3+\sqrt{6}}}{\sqrt{2}}\right) = (\sqrt{3+\sqrt{6}})(4\sqrt{3}) \tag{6}$$

Aplicando (6) em (5):

$$w_3 = \frac{\sqrt{\Pi}}{4(3+\sqrt{6})}$$

Para $x_i = -\frac{\sqrt{3+\sqrt{6}}}{\sqrt{2}} = x_4$

$$w_4 = \frac{12\sqrt{\Pi}}{[H_3\left(-\frac{\sqrt{3+\sqrt{6}}}{\sqrt{2}}\right)]^2} \tag{7}$$

$$H_3\left(-\frac{\sqrt{3+\sqrt{6}}}{\sqrt{2}}\right) = 8\left(-\frac{\sqrt{3+\sqrt{6}}}{\sqrt{2}}\right)^3 - 12\left(-\frac{\sqrt{3+\sqrt{6}}}{\sqrt{2}}\right) = (-\sqrt{3+\sqrt{6}})(4\sqrt{3}) \tag{8}$$

Aplicando (8) em (7):

$$w_4 = \frac{\sqrt{\Pi}}{4(3+\sqrt{6})}$$

Gauss-Laguerre

$$L_4(x) = \frac{e^x}{4!}\frac{\mathrm{d}^4}{\mathrm{d}x^4}(e^{-x}x^4)$$

$$L_4(x) = \frac{1}{24}(x^4 - 16x^3 + 72x^2 - 96x + 24)$$

Fazendo $L_4(x) = 0$ temos as seguintes raízes:

$$x_1 = 0.32254768961939$$
$$x_2 = 1.74576110115835$$
$$x_3 = 4.53662029692113$$
$$x_4 = 9.39507091230113$$

Calculemos agora w_k

$$w_k = \frac{x_i}{(5)^2[L_5(x_i)]^2}$$

Como precisamos de $L_5(x_i)$, vamos calculá-lo:

$$L_5(x) = \frac{1}{120}(-x^5 + 25x^4 - 200x^3 + 600x^2 - 600x + 120) \tag{9}$$

Para $x_i = x_1 = 0.32254768961939$

$$w_1 = \frac{0.32254768961939}{(5)^2[L_5(0.32254768961939)]^2}$$

Aplicando x_1 em (9), temos:

$$L_5(0.32254768961939) = -0.14625570635723736$$

Logo,

$$w_1 = 0.603154104341638$$

Para $x_i = x_2 = 1.74576110115835$

$$w_2 = \frac{1.74576110115835}{(5)^2[L_5(1.74576110115835)]^2}$$

Aplicando x_2 em (9), temos:

$$L_5(1.74576110115835) = 0.44201170588017363$$

Logo,

$$w_2 = 0.35741869243779295$$

Para $x_i = x_3 = 4.53662029692113$

$$w_3 = \frac{4.53662029692113}{(5)^2[L_5(4.53662029692113)]^2}$$

Aplicando x_3 em (9), temos:

$$L_5(4.53662029692113) = -2.1601749134539734$$

Logo,

$$w_3 = 0.03888790851500541$$

Para $x_i = x_4 = 9.39507091230113$

$$w_4 = \frac{9.39507091230113}{(5)^2[L_5(9.39507091230113)]^2}$$

Aplicando x_4 em (9), temos:

$$L_5(9.39507091230113) = 26.397752247264286$$

Logo,

$$w_4 = 0.0005392947055613309$$

Gauss-Chebyshev

$$T_4(x) = \frac{(-2)^4 4!}{(2\cdot 4)!}\sqrt{1-x^2}\frac{\mathrm{d}^4}{\mathrm{d}x^4}(1-x^2)^{\frac{7}{2}}$$

$$= \frac{1}{105}\cdot\sqrt{1-x^2}\cdot\frac{840x^4 - 840x^2 + 105}{\sqrt{1-x^2}}$$

$$= 8x^4 - 8x^2 + 1$$

$$T_4(x) = 8x^4 - 8x^2 + 1$$

Fazendo $T_4(x) = 0$ temos as seguintes raízes:

$$x_1 = \frac{\sqrt{2-\sqrt{2}}}{2} \qquad x_2 = -\frac{\sqrt{2-\sqrt{2}}}{2}$$

$$x_3 = \frac{\sqrt{2+\sqrt{2}}}{2} \qquad x_4 = -\frac{\sqrt{2+\sqrt{2}}}{2}$$

Calculemos agora w_k

$$w_k = \frac{\Pi}{n}$$

Logo,

$$w_1 = ... = w_4 = \frac{\Pi}{4}$$