CK0048 - MÉTODOS NUMÉRICOS II

Tarefa 19: Método das Diferenças Finitas

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Faça o que se pede.

1. Resolver o PVC1 com N = 8. Compare os seus resultados com aqueles apresentados em (13), montando uma tabela com os valores obtidos e os erros relativos à solução exata.

$$\text{PVC1:}\begin{cases}\dfrac{d^2y(x)}{dx^2} - y(x) = 0\\ y(0) = 0\\ y(1) = 1.\end{cases}$$

- **Divisão do Domínio**

Para N = 8,

$$\Delta x = \frac{1}{8} = 0.125$$

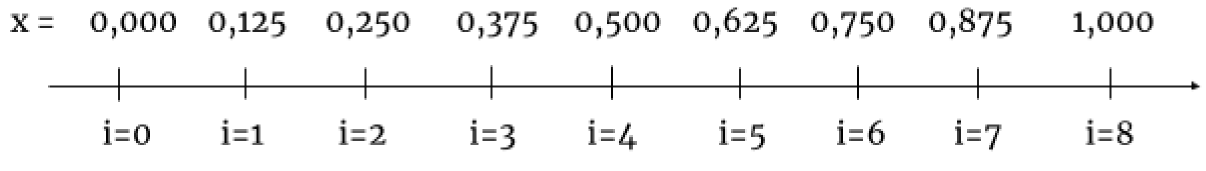


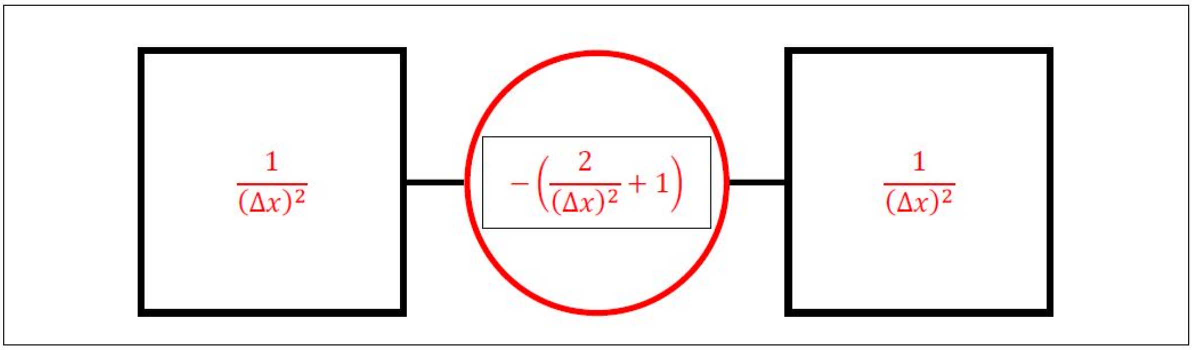
Figura 1. Divisão do domínio [0, 1] em oito partes iguais com nós nas posições i

- **Versão discreta da ED do PVC**

Usando a filosofia central, as derivadas que aparecem na ED do PVC aplicada no nó i ficam

$$
\begin{aligned}
\frac{d^2 y(x_i)}{dx^2} - y(x_i) &\approx \frac{1}{(\Delta x)^2}[y(x_{i-1}) - 2y(x_i) + y(x_{i+1})] - y(x_i) \\
&= \frac{1}{(\Delta x)^2} y(x_{i-1}) - \left(\frac{2}{(\Delta x)^2} + 1\right) y(x_i) + \frac{1}{(\Delta x)^2} y(x_{i+1}).
\end{aligned}
$$

- **Aplicação da versão discreta da ED do PVC nos nós do domínio.**



Célula esquerda = Célula direita = $\frac{1}{(\Delta x)^2} = \frac{1}{(\frac{1}{8})^2} = 64$

Célula central = $-\left(\frac{2}{(\Delta x)^2} + 1\right) = -\left(\frac{2}{(\frac{1}{8})^2} + 1\right) = -129$

A aplicação da máscara sobre os nós das incógnitas gera as seguintes equações:

$$
\begin{aligned}
64\,y(0.) - 129\,y_1 + 64\,y_2 &= 0 \\
64\,y_1 - 129\,y_2 + 64\,y_3 &= 0 \\
64\,y_2 - 129\,y_3 + 64\,y_4 &= 0 \\
64\,y_3 - 129\,y_4 + 64\,y_5 &= 0 \\
64\,y_4 - 129\,y_5 + 64\,y_6 &= 0 \\
64\,y_5 - 129\,y_6 + 64\,y_7 &= 0 \\
64\,y_6 - 129\,y_7 + 64\,y(1.) &= 0
\end{aligned}
$$

Os valores de y(0.) e y(1.) são especificados nas condições contorno, portanto:

$$
\begin{aligned}
-129\,y_1 + 64\,y_2 &= 0 \\
64\,y_1 - 129\,y_2 + 64\,y_3 &= 0 \\
64\,y_2 - 129\,y_3 + 64\,y_4 &= 0
\end{aligned}
$$

$$
\begin{aligned}
64\,y_3 - 129\,y_4 + 64\,y_5 &= 0\\
64\,y_4 - 129\,y_5 + 64\,y_6 &= 0\\
64\,y_5 - 129\,y_6 + 64\,y_7 &= 0\\
64\,y_6 - 129\,y_7 &= -64
\end{aligned}
$$

A fim de encontrar o vetor solução do sistema, podemos reescrevê-lo assim :

$$
\left(\begin{array}{cccccccc|c}
-129.0000 & 64.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000\\
64.0000 & -129.0000 & 64.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000\\
0.0000 & 64.0000 & -129.0000 & 64.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000\\
0.0000 & 0.0000 & 64.0000 & -129.0000 & 64.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000\\
0.0000 & 0.0000 & 0.0000 & 64.0000 & -129.0000 & 64.0000 & 0.0000 & 0.0000 & 0.0000\\
0.0000 & 0.0000 & 0.0000 & 0.0000 & 64.0000 & -129.0000 & 64.0000 & 0.0000 & 0.0000\\
0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 64.0000 & -129.0000 & 64.0000 & 0.0000\\
0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 64.0000 & -129.0000 & -64.0000
\end{array}\right)
$$

- **Resolução do sistema de equações algébricas.**

Resolvendo o sistema, temos os seguintes valores discretos e aproximados da função y(x)

$$
\begin{bmatrix} y_1\\ y_2\\ y_3\\ y_4\\ y_5\\ y_6\\ y_7\\ y_8 \end{bmatrix} = \begin{bmatrix} 0.0910\\ 0.1834\\ 0.2787\\ 0.3783\\ 0.4838\\ 0.5969\\ 0.7193\\ 0.8530 \end{bmatrix}
$$

*Erro relativo à solução exata obtida pela equação fornecida $\frac{1}{e^{-1}-e}(e^{-x} - e^{x})$

y_i	Valor Obtido	Erro Relativo
y_1	0.1067	0.0005438229773959
y_2	0.215	0.0002213963320694
y_3	0.3267	0.0002270210232090
y_4	0.4435	0.0002041894362187
y_5	0.5672	0.0001227959142972

y_6	0.6998	0.0001082961435946
y_7	0.8433	0.00004.0895586899

2. Resolver o PVC2 com N = 8 nas duas direções e usando $f(x, y)$ = 4 . Compare seus resultados com aqueles apresentados em (19), montando uma tabela com os valores obtidos e os erros relativos aos valores obtidos com N = 8.

$$\text{PVC2:} \begin{cases} \frac{\partial^2 u(x,y)}{\partial x^2} + \frac{\partial^2 u(x,y)}{\partial y^2} = f(x,y) \\ u(x,0) = 0: borda\ inferior \\ u(x,1) = 0: borda\ superior \\ u(0,y) = 0: borda\ esquerda \\ u(1,y) = 0: borda\ direita \end{cases}$$

- **Divisão do Domínio**

Para N = 8,

$$\Delta x = \tfrac{1}{8} = 0.125$$
$$\Delta y = \tfrac{1}{8} = 0.125$$

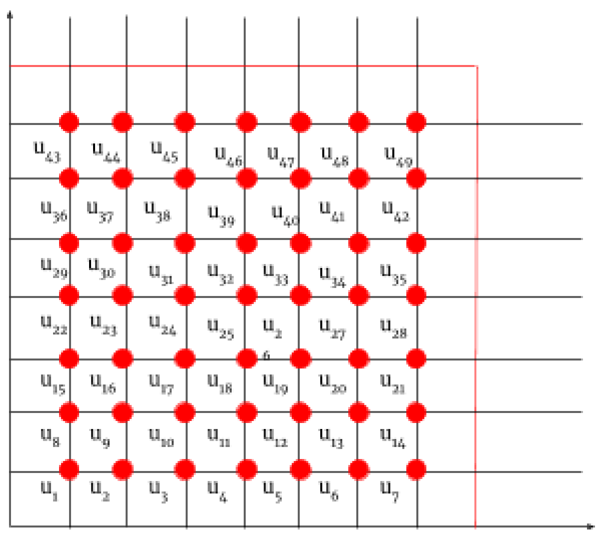


Figura 1. Partição do domínio quadrado em uma grade regular de espaçamento igual a 0.125.

- **Versão discreta da ED do PVC**

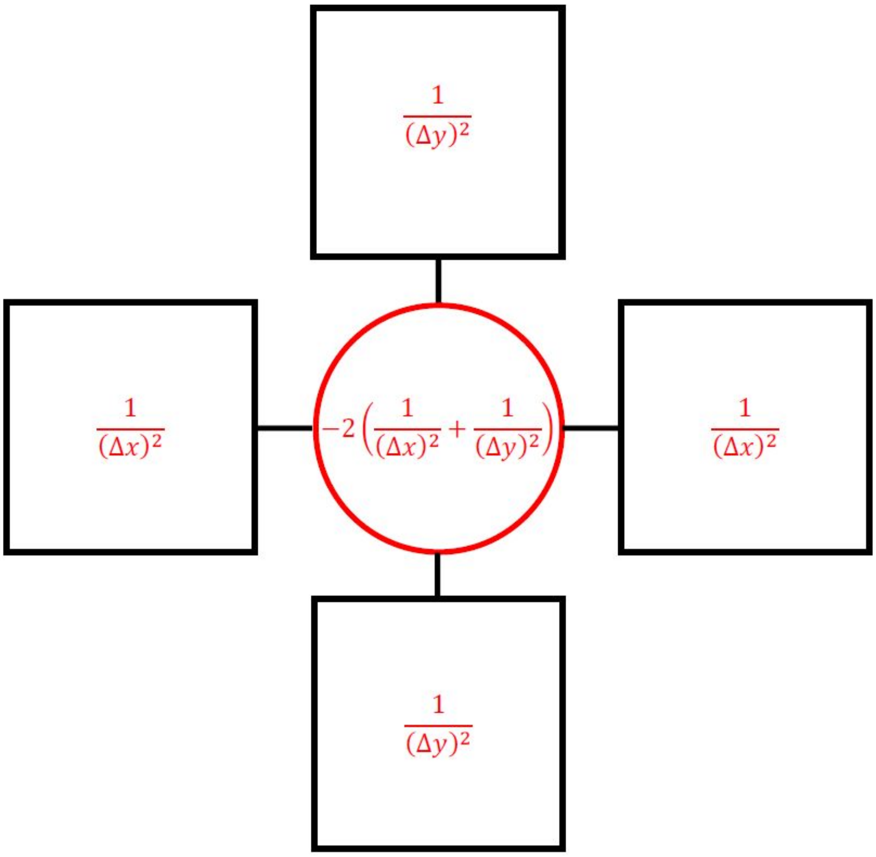
Usando a filosofia central, as derivadas que aparecem na ED do PVC aplicada no nó (x_i, y_i) ficam

$$
\begin{aligned}
\frac{\partial^2 u(x_i,y_j)}{\partial x^2} &\approx \frac{1}{(\Delta x)^2}\left[u(x_{i-1},y_j) - 2u(x_i,y_j) + u(x_{i+1},y_j)\right] \\
&= \frac{1}{(\Delta x)^2}u(x_{i-1},y_j) - \frac{2}{(\Delta x)^2}u(x_i,y_j) + \frac{1}{(\Delta x)^2}u(x_{i+1},y_j).
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial^2 u(x_i,y_j)}{\partial y^2} &\approx \frac{1}{(\Delta y)^2}\left[u(x_i,y_{j-1}) - 2u(x_i,y_j) + u(x_i,y_{j+1})\right] \\
&= \frac{1}{(\Delta y)^2}u(x_i,y_{j-1}) - \frac{2}{(\Delta y)^2}u(x_i,y_j) + \frac{1}{(\Delta y)^2}u(x_i,y_{j+1}).
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial^2 u(x_i,y_j)}{\partial x^2} + \frac{\partial^2 u(x_i,y_j)}{\partial y^2} &\approx \frac{1}{(\Delta x)^2}u(x_{i-1},y_j) - \frac{2}{(\Delta x)^2}u(x_i,y_j) + \frac{1}{(\Delta x)^2}u(x_{i+1},y_j) \\
&+ \frac{1}{(\Delta y)^2}u(x_i,y_{j-1}) - \frac{2}{(\Delta y)^2}u(x_i,y_j) + \frac{1}{(\Delta y)^2}u(x_i,y_{j+1})
\end{aligned}
$$

- **Aplicação da versão discreta da ED do PVC nos nós do domínio.**



Célula esquerda = Célula direita = $\frac{1}{(\Delta x)^2} = \frac{1}{(\frac{1}{8})^2} = 64$

Célula superior = Célula inferior= $\frac{1}{(\Delta y)^2} = \frac{1}{(\frac{1}{8})^2} = 64$

Célula central = $-2(\frac{1}{(\Delta x)^2} + \frac{1}{(\Delta y)^2}) = -2(\frac{1}{(\frac{1}{8})^2} + \frac{1}{(\frac{1}{8})^2}) = -256$

Podemos então formar o sistema que representa todos os pontos definidos. Organizando de outra forma, podemos separar os coeficientes do sistema em uma matriz e multiplicá-la por outra com os valores desconhecidos (u_1, ..., u_{49}) e igualar à matriz de resultados.

Matriz de coeficientes:

```
[-256, 64, 0, 0, 0, 0, 0, 64, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]
[64, -256, 64, 0, 0, 0, 0, 0, 64, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]
[0, 64, -256, 64, 0, 0, 0, 0, 0, 64, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]
[0, 0, 64, -256, 64, 0, 0, 0, 0, 0, 64, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]
[0, 0, 0, 64, -256, 64, 0, 0, 0, 0, 0, 64, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]
[0, 0, 0, 0, 64, -256, 64, 0, 0, 0, 0, 0, 64, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]
[0, 0, 0, 0, 0, 64, -256, 0, 0, 0, 0, 0, 0, 64, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]
[64, 0, 0, 0, 0, 0, 0, -256, 64, 0, 0, 0, 0, 0, 64, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]
[0, 64, 0, 0, 0, 0, 0, 64, -256, 64, 0, 0, 0, 0, 0, 64, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]
[0, 0, 64, 0, 0, 0, 0, 0, 64, -256, 64, 0, 0, 0, 0, 0, 64, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]
[0, 0, 0, 64, 0, 0, 0, 0, 0, 64, -256, 64, 0, 0, 0, 0, 0, 64, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]
[0, 0, 0, 0, 64, 0, 0, 0, 0, 0, 64, -256, 64, 0, 0, 0, 0, 0, 64, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]
[0, 0, 0, 0, 0, 64, 0, 0, 0, 0, 0, 64, -256, 64, 0, 0, 0, 0, 0, 64, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]
[0, 0, 0, 0, 0, 0, 64, 0, 0, 0, 0, 0, 64, -256, 0, 0, 0, 0, 0, 0, 64, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]
[0, 0, 0, 0, 0, 0, 0, 64, 0, 0, 0, 0, 0, 0, -256, 64, 0, 0, 0, 0, 0, 64, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]
[0, 0, 0, 0, 0, 0, 0, 0, 64, 0, 0, 0, 0, 0, 64, -256, 64, 0, 0, 0, 0, 0, 64, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]
[0, 0, 0, 0, 0, 0, 0, 0, 0, 64, 0, 0, 0, 0, 0, 64, -256, 64, 0, 0, 0, 0, 0, 64, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 64, 0, 0, 0, 0, 0, 64, -256, 64, 0, 0, 0, 0, 0, 64, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 64, 0, 0, 0, 0, 0, 64, -256, 64, 0, 0, 0, 0, 0, 64, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 64, 0, 0, 0, 0, 0, 64, -256, 64, 0, 0, 0, 0, 0, 64, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 64, 0, 0, 0, 0, 0, 64, -256, 0, 0, 0, 0, 0, 0, 64, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]
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[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 64, 0, 0, 0, 0, 0, 64, -256, 64, 0, 0, 0, 0, 0, 64, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 64, 0, 0, 0, 0, 0, 64, -256, 64, 0, 0, 0, 0, 0, 64, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 64, 0, 0, 0, 0, 0, 64, -256, 64, 0, 0, 0, 0, 0, 64, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 64, 0, 0, 0, 0, 0, 64, -256, 0, 0, 0, 0, 0, 0, 64, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 64, 0, 0, 0, 0, 0, 0, -256, 64, 0, 0, 0, 0, 0, 64, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 64, 0, 0, 0, 0, 0, 64, -256, 64, 0, 0, 0, 0, 0, 64, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 64, 0, 0, 0, 0, 0, 64, -256, 64, 0, 0, 0, 0, 0, 64, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 64, 0, 0, 0, 0, 0, 64, -256, 64, 0, 0, 0, 0, 0, 64, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 64, 0, 0, 0, 0, 0, 64, -256, 64, 0, 0, 0, 0, 0, 64, 0, 0, 0, 0, 0, 0, 0, 0, 0]
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 64, 0, 0, 0, 0, 0, 64, -256, 64, 0, 0, 0, 0, 0, 64, 0, 0, 0, 0, 0, 0, 0, 0]
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 64, 0, 0, 0, 0, 0, 64, -256, 0, 0, 0, 0, 0, 0, 64, 0, 0, 0, 0, 0, 0, 0]
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 64, 0, 0, 0, 0, 0, 0, -256, 64, 0, 0, 0, 0, 0, 64, 0, 0, 0, 0, 0, 0]
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 64, 0, 0, 0, 0, 0, 64, -256, 64, 0, 0, 0, 0, 0, 64, 0, 0, 0, 0, 0]
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 64, 0, 0, 0, 0, 0, 64, -256, 64, 0, 0, 0, 0, 0, 64, 0, 0, 0, 0]
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 64, 0, 0, 0, 0, 0, 64, -256, 64, 0, 0, 0, 0, 0, 64, 0, 0, 0]
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[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 64, 0, 0, 0, 0, 0, 64, -256, 0, 0, 0, 0, 0, 0, 64]
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 64, 0, 0, 0, 0, 0, 0, -256, 64, 0, 0, 0, 0, 0]
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[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 64, 0, 0, 0, 0, 0, 64, -256, 64, 0, 0, 0]
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 64, 0, 0, 0, 0, 0, 64, -256, 64, 0, 0]
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 64, 0, 0, 0, 0, 0, 64, -256, 64, 0]
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 64, 0, 0, 0, 0, 0, 64, -256, 64]
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 64, 0, 0, 0, 0, 0, 64, -256]
```

Matriz de variáveis:

Matriz de Resultados:

u_1	4
u_2	4
u_3	4
u_4	4
u_5	4
u_6	4
u_7	4
u_8	4
u_9	4
u_10	4
u_11	4
u_12	4
u_13	4
u_14	4
u_15	4
u_16	4
u_17	4
u_18	4
u_19	4
u_20	4
u_21	4
u_22	4
u_23	4
u_24	4
u_25	4
u_26	4
u_27	4
u_28	4
u_29	4
u_30	4
u_31	4
u_32	4
u_33	4
u_34	4
u_35	4
u_36	4
u_37	4
u_38	4
u_39	4
u_40	4
u_41	4
u_42	4
u_43	4
u_44	4
u_45	4
u_46	4
u_47	4
u_48	4
u_49	4

Resolvendo o sistema, temos os seguintes valores discretos e aproximados da função u(x,y), isto é

```
u_1 =  -0.07111672794117646
u_2 =  -0.11098345588235295
u_3 =  -0.13166360294117646
u_4 =  -0.1380974264705882
u_5 =  -0.13166360294117646
u_6 =  -0.11098345588235295
u_7 =  -0.07111672794117647
u_8 =  -0.11098345588235294
u_9 =  -0.17865349264705882
u_10 =  -0.21507352941176472
u_11 =  -0.22656250000000006
u_12 =  -0.21507352941176472
u_13 =  -0.17865349264705882
u_14 =  -0.11098345588235294
u_15 =  -0.13166360294117646
u_16 =  -0.2150735294117647
u_17 =  -0.26091452205882354
u_18 =  -0.27550551470588236
u_19 =  -0.26091452205882354
u_20 =  -0.21507352941176477
u_21 =  -0.1316636029411765
u_22 =  -0.13809742647058823
u_23 =  -0.22656249999999997
u_24 =  -0.27550551470588236
u_25 =  -0.29113051470588236
u_26 =  -0.27550551470588236
u_27 =  -0.22656250000000006
u_28 =  -0.13809742647058823
u_29 =  -0.1316636029411765
u_30 =  -0.21507352941176472
u_31 =  -0.260914522058235
u_32 =  -0.27550551470588236
u_33 =  -0.260914522058236
u_34 =  -0.21507352941176475
u_35 =  -0.13166360294117646
u_36 =  -0.11098345588235294
u_37 =  -0.17865349264705882
u_38 =  -0.21507352941176466
u_39 =  -0.22656249999999997
u_40 =  -0.2150735294117647
u_41 =  -0.1786534926470588
u_42 =  -0.1109834558823529
u_43 =  -0.07111672794117645
u_44 =  -0.11098345588235291
u_45 =  -0.13166360294117643
u_46 =  -0.1380974264705882
u_47 =  -0.13166360294117643
u_48 =  -0.11098345588235292
u_49 =  -0.07111672794117646
```

Olhando para os pontos originais para N=4 como mostrado no arquivo da aula “2020_Aula#27” podemos fazer as comparações, mostradas na tabela:

y_i	Valor Exato	Valor Obtido	Erro Relativo
u_1	-0.171875	-0.1786534926470588	0.03794212218649516
u_2	-0.21875	-0.226562500000000	0.03448275862068989
u_3	-0.171875	-0.1786534926470588	0.03794212218649516
u_4	-0.21875	-0.226562499999999	0.03448275862068954
u_5	-0.28125	-0.29113051470588236	0.0339384372533544
u_6	-0.21875	-0.226562500000000	0.03448275862068989
u_7	-0.171875	-0.1786534926470588	0.03794212218649516
u_8	-0.21875	-0.226562499999999	0.03448275862068954
u_9	-0.171875	-0.1786534926470588	0.03794212218649501