CK0048 - Métodos Numéricos II

Tarefa 02: Desenvolvimento de Fórmulas de Newton-Cotes

Grupo 2

Primeiramente devemos achar o polinômio $g(S)$ de substituição com grau 4

$$g(S) = \sum_{K=0}^{n=4} \binom{S}{K} \Delta^K y_0 = \sum_{K=0}^{n=4} \frac{S!}{K!(S-K)!} \Delta^K y_0$$

$$g(S) = \frac{S!}{0!(S-0)!} \Delta^0 y_0 + \frac{S!}{1!(S-1)!} \Delta^1 y_1 + \frac{S!}{2!(S-2)!} \Delta^2 y_2 + \frac{S!}{3!(S-3)!} \Delta^3 y_3 + \frac{S!}{4!(S-4)!} \Delta^4 y_4$$

$$g(S) = \Delta^0 y_0 + S \Delta^1 y_1 + \frac{S(S-1)}{2} \Delta^2 y_2 + \frac{S(S-1)(S-2)}{6} \Delta^3 y_3 + \frac{S(S-1)(S-2)(S-3)}{24} \Delta^4 y_4$$

Porém, temos que:

$$\Delta^0 y_0 = y(0)$$

$$\Delta^1 y_1 = \Delta^0 y_1 - \Delta^0 y_0 = y(1) - y(0)$$

$$\Delta^2 y_2 = \Delta^1 y_2 - \Delta^1 y_1 = (\Delta^0 y_2 - \Delta^0 y_1) - (\Delta^0 y_1 - \Delta^0 y_0)$$

$$= y(2) - y(1) - y(1) + y(0) = y(2) - 2y(1) + y(0)$$

$$\vdots$$

Portanto, podemos definir $\Delta^4 y_4$ da seguinte forma:

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$$\Delta^4 r_4 = \Delta^3 r_4 - \Delta^3 r_3 = (\Delta^2 r_4 - \Delta^2 r_3) - (\Delta^2 r_3 - \Delta^2 r_2)$$

$$= [\Delta^1 r_4 - \Delta^1 r_3] - [\Delta^1 r_3 - \Delta^1 r_2] - [\Delta^1 r_3 - \Delta^1 r_2] + [\Delta^1 r_2 - \Delta^1 r_1]$$

$$= \{\Delta^0 r_4 - \Delta^0 r_3\} - \{\Delta^0 r_3 - \Delta^0 r_2\} - \{\Delta^0 r_3 - \Delta^0 r_2\} + \{\Delta^0 r_2 - \Delta^0 r_1\}$$
$$- \{\Delta^0 r_3 - \Delta^0 r_2\} + \{\Delta^0 r_2 - \Delta^0 r_1\} + \{\Delta^0 r_2 - \Delta^0 r_1\} - \{\Delta^0 r_1 - \Delta^0 r_0\}$$

$$= r(4) - r(3) - r(3) + r(2) - r(3) + r(2) + r(2) - r(1) - r(3) + r(2)$$
$$+ r(2) - r(1) + r(2) - r(1) - r(1) + r(0)$$

$$= r(4) - 4r(3) + 6r(2) - 4r(1) + r(0)$$

Assim, podemos substituir todos os $\Delta^k r$ de $g(s)$:

$$g(s) = r(0) + s(r(1) - r(0)) + \frac{s(s-1)}{2}(r(2) - 2r(1) + r(0))$$
$$+ \frac{s(s-1)(s-2)}{6}(r(3) - 3r(2) + 3r(1) - r(0))$$
$$+ \frac{s(s-1)(s-2)(s-3)}{24}(r(4) - 4r(3) + 6r(2) - 4r(1) + r(0))$$

$$g(s) = r(0)\left(1 - s + \tfrac{1}{2}s(s-1) - \tfrac{1}{6}s(s-1)(s-2) + \tfrac{1}{24}s(s-1)(s-2)(s-3)\right)$$
$$+ r(1)\left(s - 2\left(\tfrac{1}{2}s(s-1)\right) + 3\left(\tfrac{1}{6}s(s-1)(s-2)\right) + (-4)\left(\tfrac{1}{24}s(s-1)(s-2)(s-3)\right)\right)$$
$$+ r(2)\left(\tfrac{1}{2}s(s-1) - 3\left(\tfrac{1}{6}s(s-1)(s-2)\right) + 6\left(\tfrac{1}{24}s(s-1)(s-2)(s-3)\right)\right)$$
$$+ r(3)\left(\tfrac{1}{6}s(s-1)(s-2) - 4\left(\tfrac{1}{24}s(s-1)(s-2)(s-3)\right)\right)$$
$$+ r(4)\left(\tfrac{1}{24}s(s-1)(s-2)(s-3)\right)$$

Desenvolvendo $g(s)$, temos:

$$g(s) = r(0)\left(1 - \tfrac{25}{12}s + \tfrac{35}{24}s^2 - \tfrac{5}{12}s^3 + \tfrac{1}{24}s^4\right)$$
$$+ r(1)\left(4s - \tfrac{13}{3}s^2 + \tfrac{3}{2}s^3 - \tfrac{1}{6}s^4\right)$$
$$+ r(2)\left(-3s + \tfrac{19}{4}s^2 - 2s^3 + \tfrac{1}{4}s^4\right)$$
$$+ r(3)\left(\tfrac{4}{3}s - \tfrac{7}{3}s^2 + \tfrac{7}{6}s^3 - \tfrac{1}{6}s^4\right)$$
$$+ r(4)\left(-\tfrac{6}{24}s + \tfrac{11}{24}s^2 - \tfrac{6}{24}s^3 + \tfrac{1}{24}s^4\right)$$

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Abordagem Fechada

Nessa abordagem o x_i e x_f devem ser utilizados, logo

$$f(x_i) = f(x(s=0)) = g(0)$$
$$f(x_f) = f(x(s=4)) = g(4)$$

$$x(s) = x_i + sh$$

com $h = \Delta x / 4$

Mudança de variável

$$\int_{x_i}^{x_f} f(x)\,dx \approx \int_{x_i}^{x_f} P(x)\,dx = \int_{s_i}^{s_f} P(x(s)) \frac{dx(s)}{ds}\,ds$$

Como $x(s) = x_i + sh$, então $\frac{dx(s)}{ds} = h$. Logo

$$\int_{x_i}^{x_f} f(x)\,dx \approx h \int_{s_i}^{s_f} p(x(s))\,ds = h \int_0^4 g(s)\,ds$$

Substituindo $g(s)$ na equação acima temos

$$\int_{x_i}^{x_f} f(x)\,dx \approx h \Big(g(0) \int_0^4 \left(1 - \tfrac{25}{12}s + \tfrac{35}{24}s^2 - \tfrac{5}{12}s^3 + \tfrac{1}{24}s^4\right)$$
$$+ g(1) \int_0^4 \left(4s - \tfrac{13}{3}s^2 + \tfrac{3}{2}s^3 - \tfrac{1}{6}s^4\right)$$
$$+ g(2) \int_0^4 \left(-3s + \tfrac{19}{4}s^2 - 2s^3 + \tfrac{1}{4}s^4\right)$$
$$+ g(3) \int_0^4 \left(\tfrac{4}{3}s - \tfrac{7}{3}s^2 + \tfrac{7}{6}s^3 - \tfrac{1}{6}s^4\right)$$
$$+ g(4) \int_0^4 \left(-\tfrac{6}{24}s + \tfrac{11}{24}s^2 - \tfrac{6}{24}s^3 + \tfrac{1}{24}s^4\right) \Big)$$

$$\int_{x_i}^{x_f} f(x)\,dx \approx (2h/45)\left(7g(0) + 32g(1) + 12g(2) + 32g(3) + 7g(4)\right)$$

$$\int_{x_i}^{x_f} f(x)\,dx \approx (2h/45)\big(7f(x_i) + 32f(x_i+h) + 12f(x_i+2h)$$
$$+ 32f(x_i+3h) + 7f(x_i+4h)\big)$$

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Abordagem Aberta

Nessa abordagem o x_i e x_f não devem ser utilizados, logo

$$f(x_0) = f(x(s=0)) = g(0)$$
$$f(x_4) = f(x(s=4)) = g(4)$$

$$x(s) = x_i + h + sh$$
$$\text{com } h = \Delta x/6$$

Aplicando a mudança de variável,

$$\int_{x_i}^{x_f} f(x)\,dx \approx \int_{x_i}^{x_f} p(x)\,dx = \int_{s_i}^{s_f} p(x(s))\,ds\,\frac{dx(s)}{ds} = h\int_{-1}^{5} g(s)\,ds$$

Substituindo $g(s)$ na equação acima temos

$$\int_{x_i}^{x_f} f(x)\,dx \approx h\Big(g(0)\int_{-1}^{5}\left(1 - \tfrac{25}{12}s + \tfrac{35}{24}s^2 - \tfrac{5}{12}s^3 + \tfrac{1}{24}s^4\right)$$
$$+\, g(1)\int_{-1}^{5}\left(4s - \tfrac{13}{3}s^2 + \tfrac{3}{2}s^3 - \tfrac{1}{6}s^4\right)$$
$$+\, g(2)\int_{-1}^{5}\left(-3s + \tfrac{19}{4}s^2 - 2s^3 + \tfrac{1}{4}s^4\right)$$
$$+\, g(3)\int_{-1}^{5}\left(\tfrac{4}{3}s - \tfrac{7}{3}s^2 + \tfrac{7}{6}s^3 - \tfrac{1}{6}s^4\right)$$
$$+\, g(4)\int_{-1}^{5}\left(\tfrac{-6}{24}s + \tfrac{11}{24}s^2 - \tfrac{6}{24}s^3 + \tfrac{1}{24}s^4\right)\Big)$$

$$\int_{x_i}^{x_f} f(x)\,dx \approx (3h/10)\left(11\,g(0) - 14\,g(1) + 26\,g(2) - 14\,g(3) + 11\,g(4)\right)$$

$$\int_{x_i}^{x_f} f(x)\,dx \approx (3h/10)\big(11 f(x_i+h) - 14 f(x_i+2h) + 26 f(x_i+3h)$$
$$- 14 f(x_i+4h) + 11 f(x_i+5h)\big)$$