Conduit Prevalence in the Woodville Karst Plain

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Abstract

The presence of subterranean conduits within the Woodville Karst Plain is inferred from statistical analysis of several sets of microgravity measurements. In addition to determining the locations of such conduits, their approximate relative depths and sizes are estimated as well. An iterated non-linear least squares method is used to fit observed gravity data to a proposed theoretical model for the conduit-gravity relation. The spatial density of the conduits is examined with respect to their relative cross-sectional areas.

Key words and phrases: non-linear least squares, wavelets, smoothing, microgravity

1 Introduction

The Woodville Karst Plain in northwest Florida is well known for possessing an extensive network of subterranean conduits (Loper et al. (2005), Veni (2002)). These conduits are important for a variety of reasons. They carry much of the area's fresh drinking water. They link the large Tallahassee metropolitan area with the protected natural springs to the south. They provide, at their outsources, unique environments for fragile species within the Florida panhandle coastal region and are a well-utilized venue for recreation.

Many of these conduits have been mapped by divers entering the system from several surface openings to the system. These include the entry conduits at the source of the Wakulla river in Wakulla Springs State Park and Leon Sinks State Geographical

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Area in southern Leon County, among others. Additionally, dye tracing experiments have shown that the transport of water through the region is much more rapid than would be expected through simple diffusion. This points to the presence of many unmapped conduits, with either unknown or non-existent large-scale surface entry points.

Determining the existence of such conduits is vitally important. The transport of pollutants from the urban area in the north (Tallahassee) to the nature preserves in the south is a major concern. If underground conduits directly link these areas, extreme care must be taken to maintain the health of the waterways in the southern portion of the Woodville Karst Plain, both above and below ground.

In order to determine the extent of such conduits, a method of detection, beyond physical exploration (such as diving or drilling), is needed. One such non-invasive method involves the collection and analysis of gravity measurements taken at the surface (Technos (2005)). The method described in this paper yields the locations, depths and sizes of conduits, making use of a theoretical model of the gravitational perturbation produced at the surface by a subterranean cylinder having a density different than the surrounding rock matrix. The theoretical model is the basis of a statistical model-fitting algorithm.

Using the results from the application of the proposed method, boreholes may be drilled in the most likely locations for the existence of conduits. These holes will allow for the placement of equipment in water-filled conduits that could monitor for several important factors: volume of water transported, pollutants in the water, speed of transport, etc.

This article is organized in four sections. Section 2 describes the collection of the gravity data in a portion of the Woodville Karst Plain in Wakulla and Leon counties. The statistical estimation method of the location, relative areas and depths of the conduits, and the results of applying this method to the gravity data are given in Section 3. Details on the theoretical model are given in a technical appendix. The appendix also includes a short description of a smoothing procedure applied to the gravity data.

2 Gravity and Subterranean Conduits

Spatial fluctuations in gravity at the surface are directly related to variations in density below ground. In particular, water-filled conduits produce negative deviations, with the magnitude of the deviation being proportional to the size of the conduit and the lateral scale of deviation increasing with conduit depth.

The primary data set used to locate conduits within the Woodville Karst Plain is a set of gravity measurements made at a total of 1444 points (spaced 30 meters apart) along nine survey lines throughout the region during 2006 and 2007; see Figures 1 and 2. Line 6 follows a north-south trend,while the remaining lines are east-west. Figure 1 shows the locations of these survey lines in a portion of the Woodville Karst Plain, Figure 2 shows the location of the survey area with respect the the surrounding area.

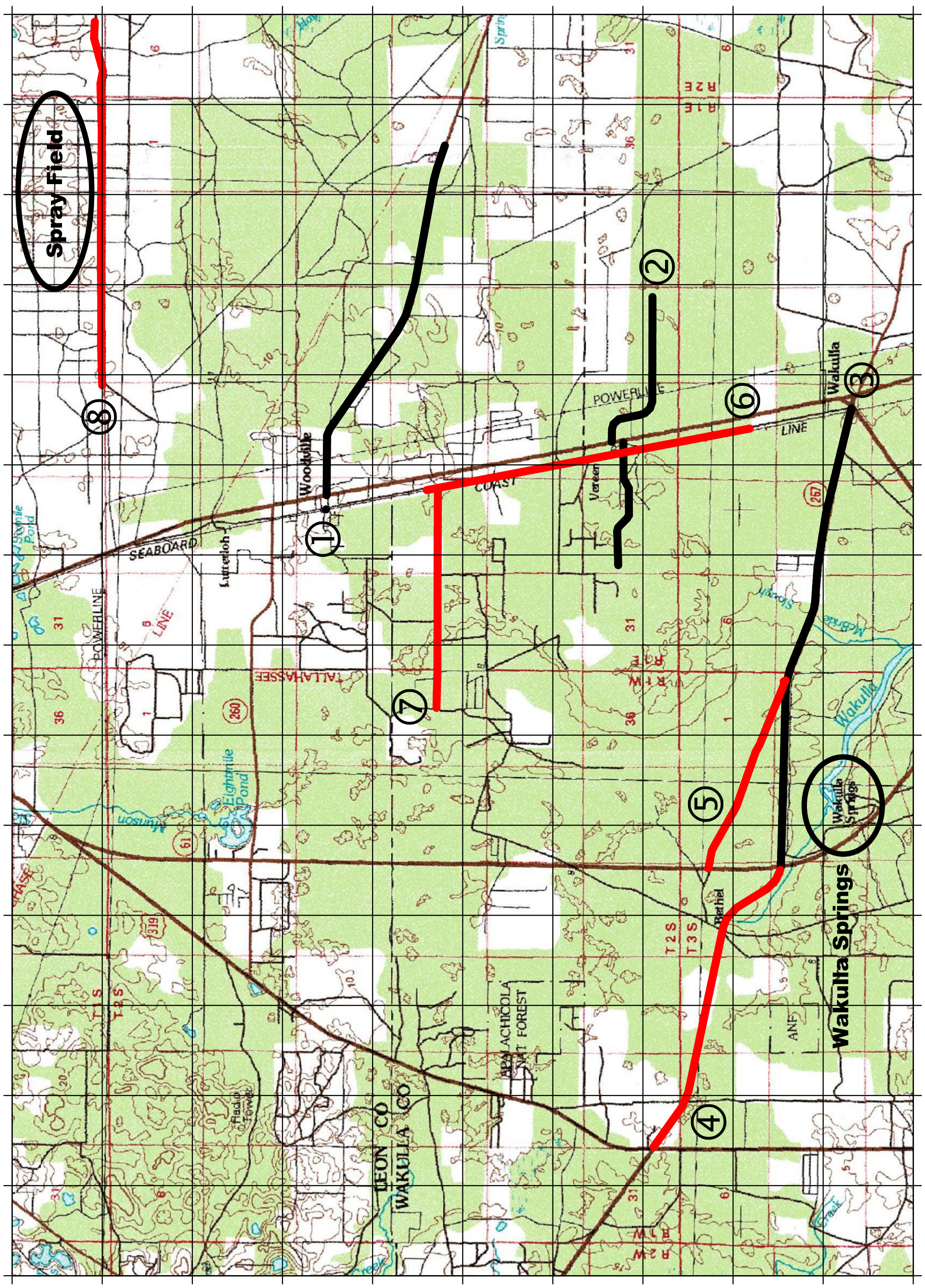

Figure 1: Location of survey lines. Note that east is to the top of the page, north is to the left.

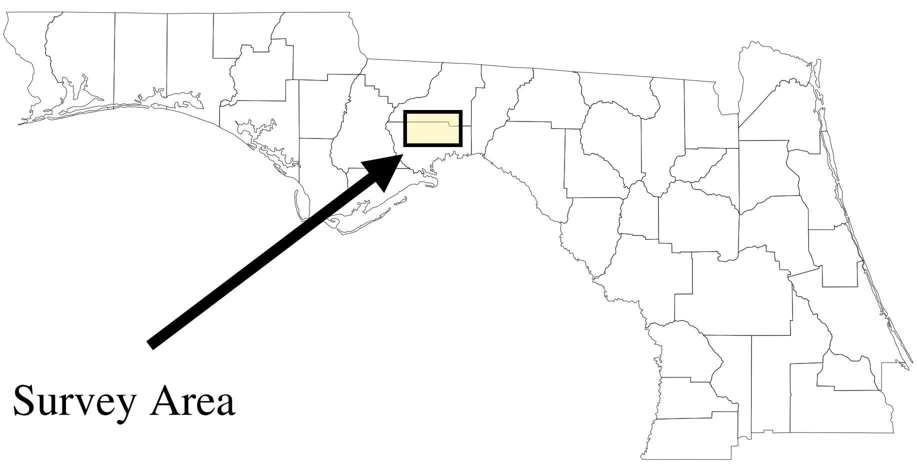

Figure 2: Survey area within Florida.

With uniform subsurface conditions, the gravity measurements should be uniform after accounting for variations due to known physical conditions: elevation, latitude, tidal effects, etc., see Technos (2006, 2007). The deviations in gravity due to the presence of conduits are very small relative to the earth's gravity. Therefore, the gravity is measured in this application as microGals (μGals). The average strength of the gravity of the earth is 9.8×10^8 μGals. Changes in gravity due to conduits are rarely larger than 10^2 μGals.

To better detect weak and small-scale deviations, the gravity measurements were transformed in the following way. First, the measurements were corrected to account for the physical parameters mentioned above: elevation, latitude, tidal effects, etc. Next, for each survey line these corrected gravity measurements, generally referred to as Bouguer gravity, had the regional linear (planar) trend removed to reduce the scale of the changes in gravity. Finally, a high-pass filter was applied to the detrended Bouguer gravity. This filter removed features from the survey line measurements that were on the order of 1500 meters or more (50 survey points or more). Such features in the survey lines' data were considered too wide to represent local changes in the gravity associated with conduits.

More specific information on the gravity data, including information on the equipment used and survey line details, may be found in Technos (2006, 2007).

3 Detection of Conduits by Statistical Methods

3.1 Method

The goal of the statistical method is to estimate the locations and relative sizes and depths of conduits using the theoretical relation between gravity and conduit existence given in the technical appendix. A complicating factor is that a given gravity deviation may be the result of two or more closely spaced conduits, requiring the estimation of multiple parameters using the data. Based on the model given in the appendix, the relation between the gravity perturbations g_i' measured at points x_i and the characteristics of a subterranean conduit is

$$g_i' = -Q \sum_{n=1}^{N} \frac{A_n H_n}{(x_i - l_n)^2 + H_n^2} - c \tag{1}$$

where $Q = 18.68\ \mu\text{Gal}\ m^{-1}$ is a scale factor and c is a zero-level constant that is needed to account for the smoothing processes described below. A negative value g_i' denotes the presence of a density deficit at depth. The parameters to be estimated are the number of conduits N, the conduit cross-sectional area A_n, the conduit depth H_n, and the location along the survey line (measured with respect to the line's starting point) l_n for $n = 1, 2, \ldots, N$. Hence, there are $3N + 1$ unknown parameters in this model.

The location parameters l_n are absolute: determined by the local minima of gravity. However, due to modifications applied to the gravity signal, the area and depth estimates will be relative to one another, rather than absolute. That is, if two conduits had estimated areas of 2 and 1, this would reflect the fact that one conduit's area was twice the others, rather than an indication of the actual size each conduit's area. A calibration between the relative measures provided here and true areas and depths can be determined later once actual physical conduits are measured.

The relation given in (1) is in a very smooth functional form. Unfortunately, the gravity data is not particularly smooth, as can be seen in Figure 3, which shows the microgravity data for survey line 1. The vertical axis represents the microgravity measurements in microgals. Fluctuations above the zero line represent higher gravity than the surrounding area. This equates to denser material below the surface, such as isolated dense boulders below the surface. Fluctuations below the zero line represent lower gravity areas, possibly conduits. The data also exhibits noisy behavior.

The theoretical functional form derived in the appendix imply the deviation of gravity should be negative. Therefore, only the nonpositive part of the measured gravity signal is examined. To achieve optimal fits of the model to the data, smoothing techniques, particularly wavelet thresholding, are applied to the data. More details on the smoothing are given in the appendix. The result of these steps is shown in Figure 4. This figure shows the same data from survey line 1 used in 3. Note this is a smoothed version of the gravity signal, and is always zero or less.

The next step is to use a statistical method to fit this data to the theoretical model.

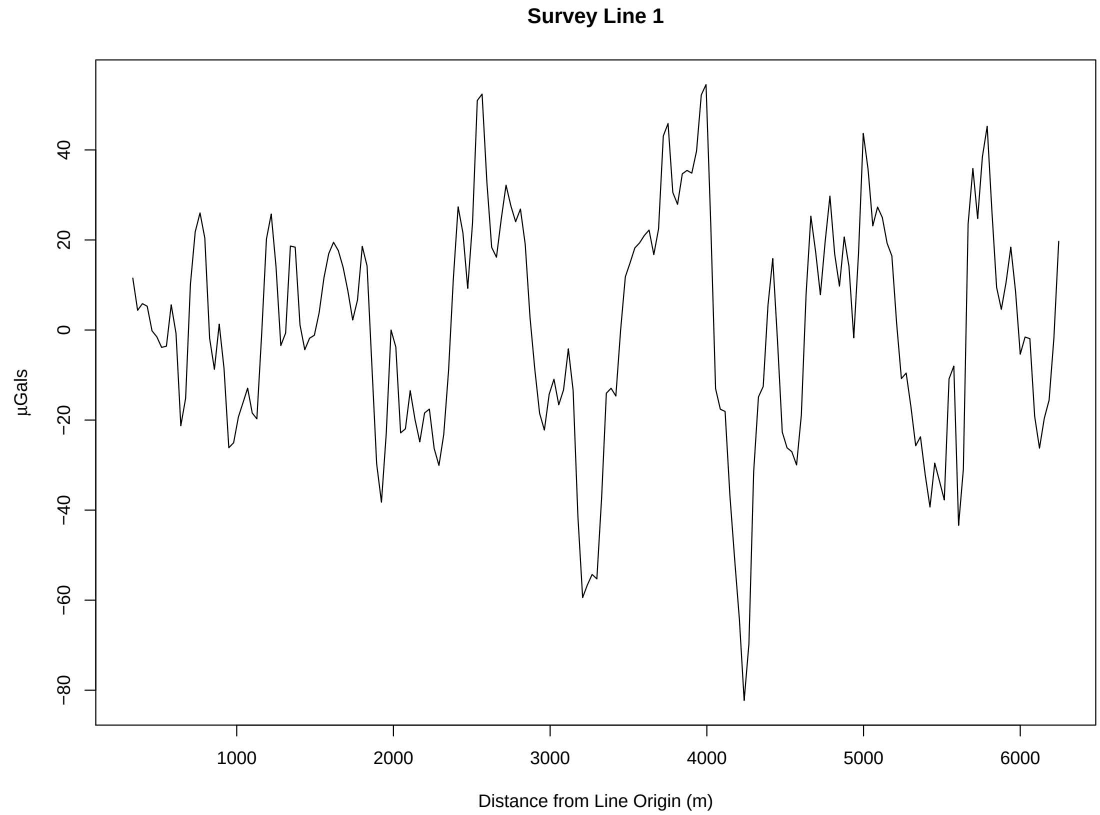

Figure 3: Microgravity data (in microgals) from survey line 1. Data points are spaced every 30 m.

Due to the nature of the theoretical model, there is no way to transform the data to allow the use of simple linear methods. Therefore, a non-linear least squares model is used to fit the data (Bates and Watts (1988)). The fit was implemented within the software package R (R Development Core Team (2006)). Since the number of conduits N is unknown, this method must be applied in multiple steps.

The first phase in our method is to identify the locations of conduits. This is done visually. The initial value of N is intentionally overspecified. Excess, unimportant parameters will be removed in a later step, reducing the value of N for a particular survey line.

The next step is to fit the data to equation (1) using non-linear least squares. This algorithm will fit the best combination of A_i, H_i, and l_i to the data in terms of minimizing the sum of squared errors.

Finally, after removing any conduits which have insignificant area estimates (the level of statistical significance used is $\alpha = 0.05$), the data are fit to the simplified model by the same non-linear least squares procedure. After this final step is complete, a

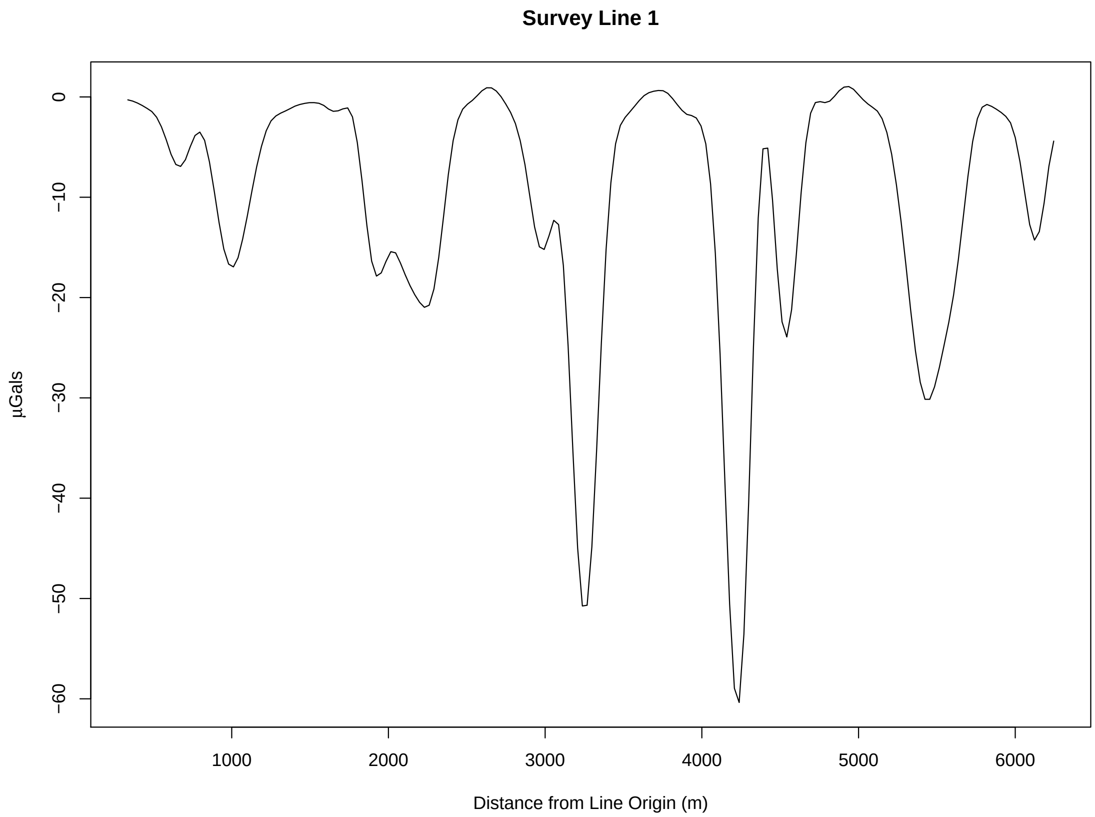

Figure 4: Smoothed, nonpositive microgravity data from survey line 1.

particular number of conduits, N, have been identified, and estimates of A_i, H_i, and l_i for $i = 1, 2, \ldots, N$ have been obtained. An example of a final fitted model is given in Figure 5. Again, this is the same data from survey line 1 shown in Figures 3 and 4. All the desired properties are now present: smooth, nonpositive function. Note that the gravity signal from Figure 4 is displayed as as the dotted line, the fitted model is solid.

3.2 Conduits in Woodville Karst Plain

Applying the methods from the previous section to the nine survey lines, 68 possible conduit locations were identified. The estimated locations are absolute, but the estimated areas and depths are relative to each other. The estimated parameters are given in Table 1.

The 68 possible conduits identified in the area North and North-East of Wakulla Springs are plotted geographically in Figure 6, with the size of the circle representing the estimated area. The dotted lines represent the survey lines; see Figure 1 for line numbers and geographical relations. Since the data was collected at approximately every

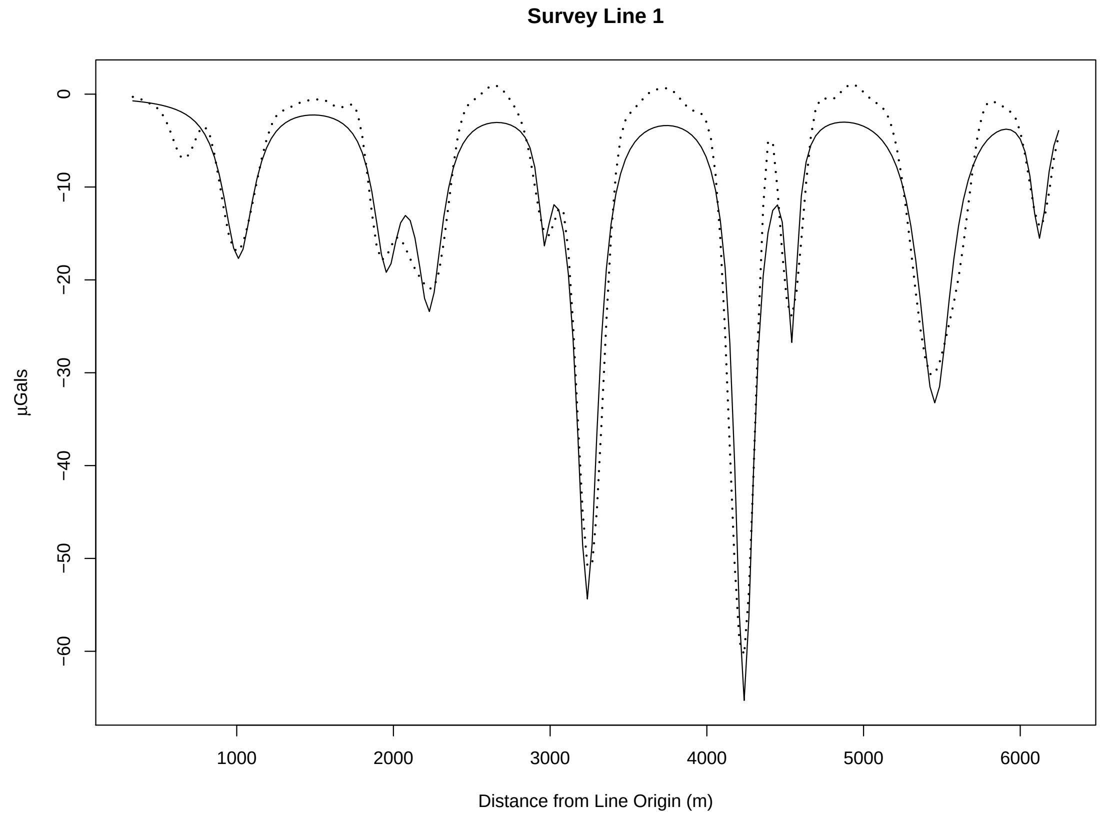

Figure 5: Microgravity data from survey line 1 fitted to (1) via non-linear least squares.

30 meters, the estimates of l are no more accurate than that.

An interesting phenomena is evident from inspection of Figure 6: large conduits are more common close to Wakulla Spring. By filtering the set of 68 conduits based on size, this becomes more apparent. Figure 7 displays the largest 20% (14 of 68) of the conduits. Note at this level of size filtering, there is only one large conduit in the north near the Tallahassee spray field. As the filtering percentage increases, the smaller conduits begin to appear in the northern part of the survey area. For example, Figure 8 shows the distribution of the largest 40% (27 of 68) of the tunnels. In fact, not until the size filter is set to 38% does a 2nd tunnel appear on the northernmost survey line (line 8). This line has 10 modeled conduit locations. Only 2 of them (20%) are in the top 40% of conduits with respect to size. For comparison, line 5 has 29% of its conduits in the top 20% by area, and 43% in the top 40% by area. Thus, it seems likely that large conduits are more prevalent in the south of the survey region.

With regard to the depths of the conduits, this same relation does not stand out. Figure 9 plots the relative depths in a similar fashion to Figures 6, 7 and 8. There does not appear to be a pattern to the distribution of tunnel depths over the survey area.

Survey Line 1					Survey Line 2				Survey Line 2a			
i	l_i	A_i	H_i	Sta	l_i	A_i	H_i	Sta	l_i	A_i	H_i	Sta
1	1011	2027	-117	2200	428	4820	-77	1400	244	2697	-71	600
2	1954	1542	-96	5300	1311	5836	-105	4200	883	577	-59	2700
3	2229	2098	-100	6200					1364	4586	-125	4300
4	2963	466	-44	8600					2246	957	-62	7200
5	3238	4570	-86	9500					2613	1246	-85	8400
6	4238	4778	-75	12800								
7	4542	949	-43	13800								
8	5454	4202	-128	16800								
9	6123	882	-62	19000								
Survey Line 3					Survey Line 4				Survey Line 5			
1	183	3707	-64	600	153	538	-60	500	579	1064	-63	1900
2	578	996	-55	1900	764	6835	-85	2500	793	2944	-89	2600
3	1308	4705	-97	4300	1008	2679	-61	3300	1038	2610	-56	3400
4	2434	2394	-73	8000	1679	1797	-67	5500	1833	7592	-114	6000
5	2739	5447	-94	9000	1984	2107	-104	6500	2048	2052	-64	6700
6	3713	3605	-101	12200	2228	1539	-88	7300	2809	2065	-65	9200
7	4260	2319	-110	14000	2777	420	-51	9100	3023	7057	-106	9900
8	5324	3568	-98	17500	3570	4391	-123	11700				
9	5659	1313	-62	18600	4058	1054	-49	13300				
10	6815	6895	-96	22400	4602	1161	-44	15100				
11	7728	879	-81	25400	4967	6468	-121	16300				
12					5211	1195	-82	17100				
Survey Line 6					Survey Line 7				Survey Line 8			
1	942	2252	-90	3000	641	2524	-67	2100	758	2347	-142	1800
2	1306	1463	-67	4200	823	5487	-78	2700	1008	2803	-111	2600
3	2188	2531	-143	7100	1703	2126	-76	5600	1227	617	-60	3300
4	2734	2878	-94	8900	1885	5097	-100	6200	1963	5567	-165	5700
5	3859	10039	-130	12600	2583	1295	-46	8500	2177	741	-49	6400
6	4952	5320	-112	16200	3280	7594	-67	10800	2972	790	-57	9000
7									3368	2301	-65	10300
8									3887	679	-102	12000
9									4558	1448	-75	14200
10									5139	1509	-79	16100

Table 1: Parameter estimates for the 9 survey lines. The “Sta” column refers to the station numbers given in Technos (2007).

Relative Conduit Areas

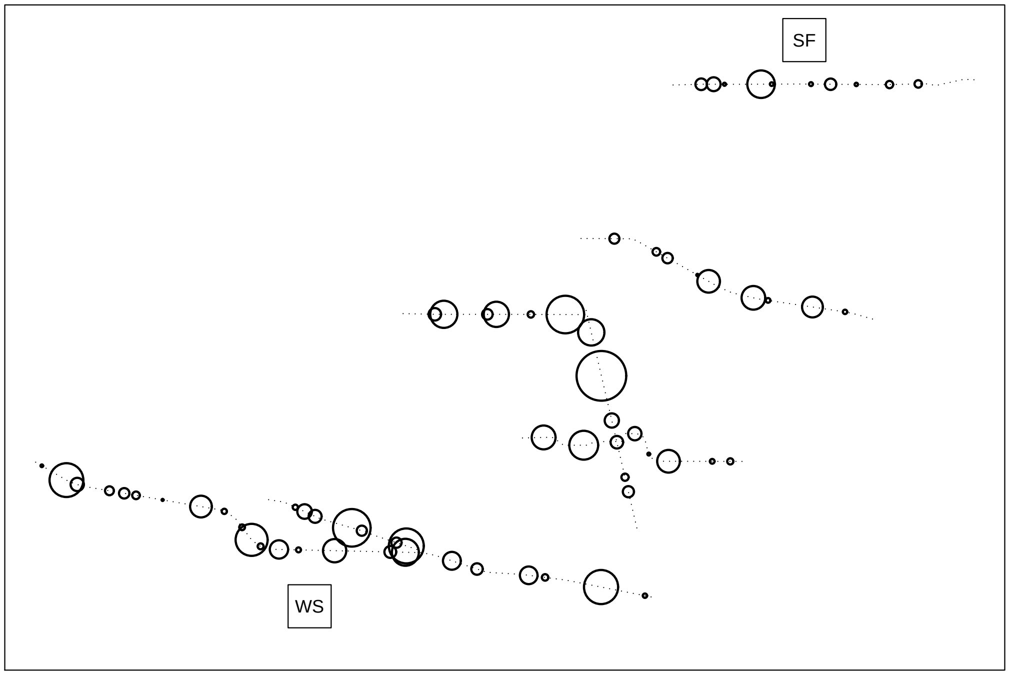

Figure 6: Estimated relative areas of possible conduits displayed geographically. The boxes labeled WS and SP represent the locations of Wakulla Spring in the southern portion of the survey area and the Tallahassee spray field at the northern edge of the area, respectively.

4 Conclusion

The method presented in this paper determines locations and relative areas and depths of subterranean conduits. The method makes use of a theoretical relation between gravity measurements and conduit location, area and depth, and fits the gravity data to this model using a multistep statistical process.

When applied, this method determined 68 possible locations for conduits along the nine surveyed lines. Additionally, by plotting the data filtered by relative areas, an interesting pattern emerged: larger conduits are more prevalent near Wakulla Springs.

These locations are prime targets for drilling bores for the installation of water monitoring equipment. Such monitoring is essential to the continued safe use of the important fresh water resource in the region.

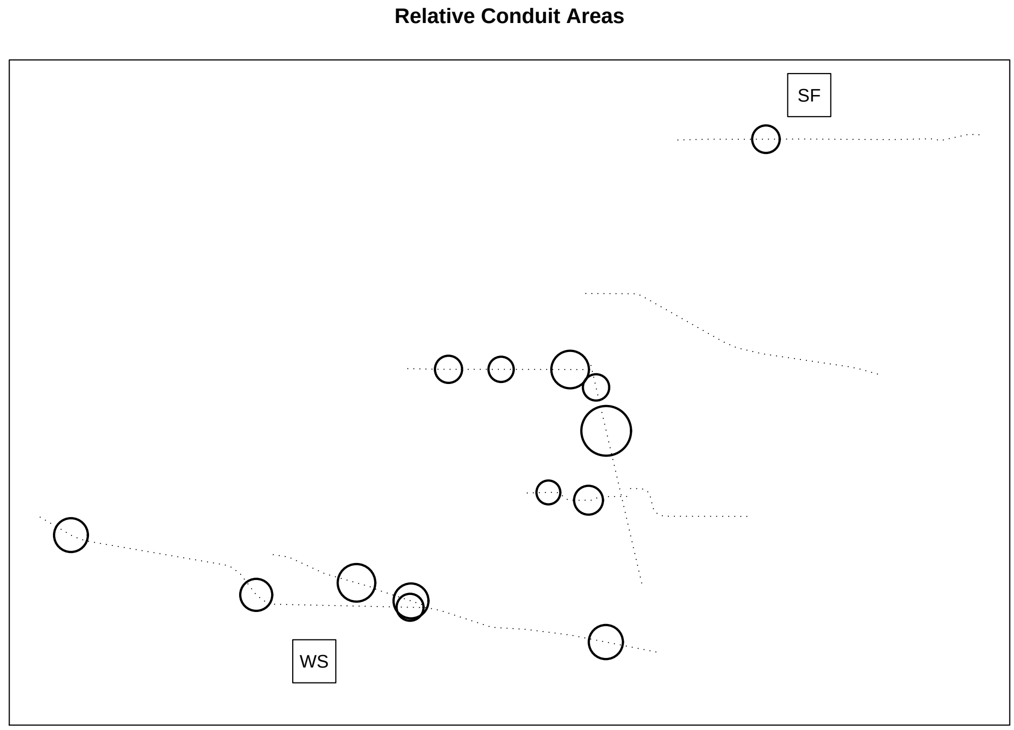

Figure 7: Largest 20% of estimated relative areas of possible conduits.

5 Appendix

5.1 Theoretical Model of Conduits using Gravity

The local acceleration of gravity, $\mathbf{g}$, at a specified point is given by the formula

$$\mathbf{g} = G \int \mathbf{r} \frac{dm}{r^3},$$

where $G = 6.67 \times 10^{-11} m^3 s^{-2} kg^{-1}$ is the gravitational constant, $\mathbf{r}$ is the position vector of a mass element dm (relative to the specified point) and $r = \|\mathbf{r}\|$. The perturbation of the vertical gravitational signal, g', produced by a mass anomaly (such as a conduit) is given by

$$g' = G \int \hat{\mathbf{z}} \cdot \mathbf{r} \frac{dm'}{r^3}$$

where dm' and $H = -\hat{\mathbf{z}} \cdot \mathbf{r}$ are the magnitude and depth of a mass anomaly.

Relative Conduit Areas

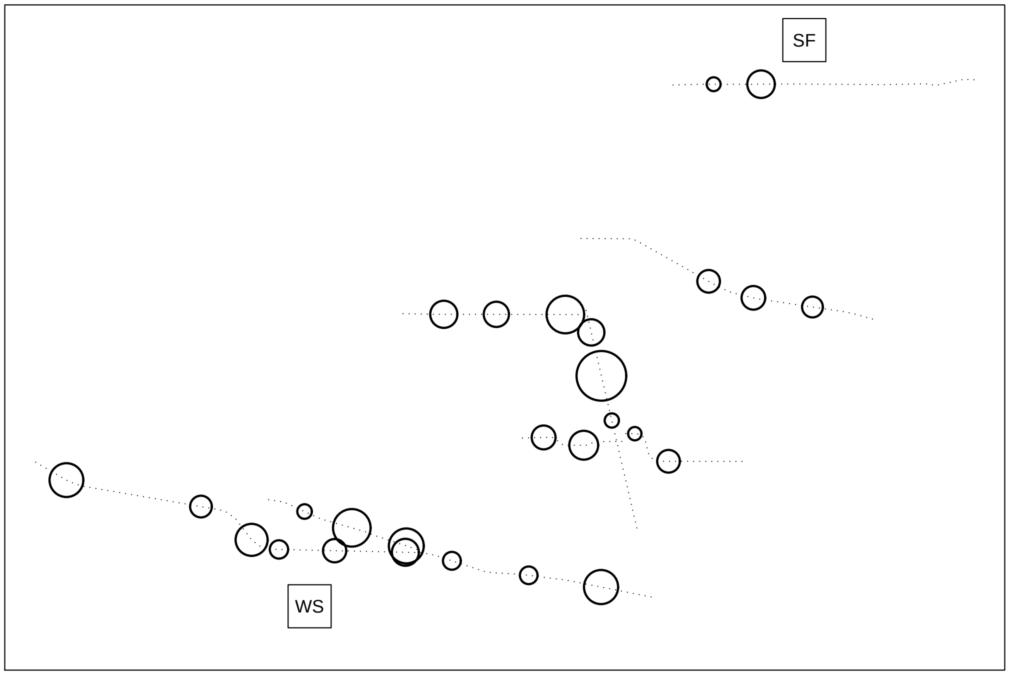

Figure 8: Largest 40% of estimated relative areas of possible conduits.

The minus signs arise since the z direction (upward) is, by convention, opposite to the direction of $\mathbf{g}$ and in the present context $\mathbf{r}$ points downward (so that H is positive).

Suppose that the mass anomaly is horizontal conduit full of water, having uniform area A and lying at constant depth H below the surface. We shall denote by x the horizontal distance along a survey line (where gravity data is available), hopefully at a large angle with respect to the conduit axis, and shall denote by y the horizontal distance perpendicular to x, see Figure 10. Let the angle between the conduit axis and the y axis be denoted by θ. If $\theta = 0$, the conduit is perpendicular to the survey line. Also, denote distance along the conduit by λ and suppose that the conduit passes under the roadway at position $x = x_0$. Now

$$r = x\hat{\mathbf{x}} + y\hat{\mathbf{y}} - H\hat{\mathbf{z}} = [x_0 + \lambda \sin(\theta)]\hat{\mathbf{x}} + \lambda \cos(\theta)\hat{\mathbf{y}} - H\hat{\mathbf{z}},$$

$$r^2 = x_0^2 + 2\lambda x_0 \sin(\theta) + \lambda^2 + H^2,$$

and

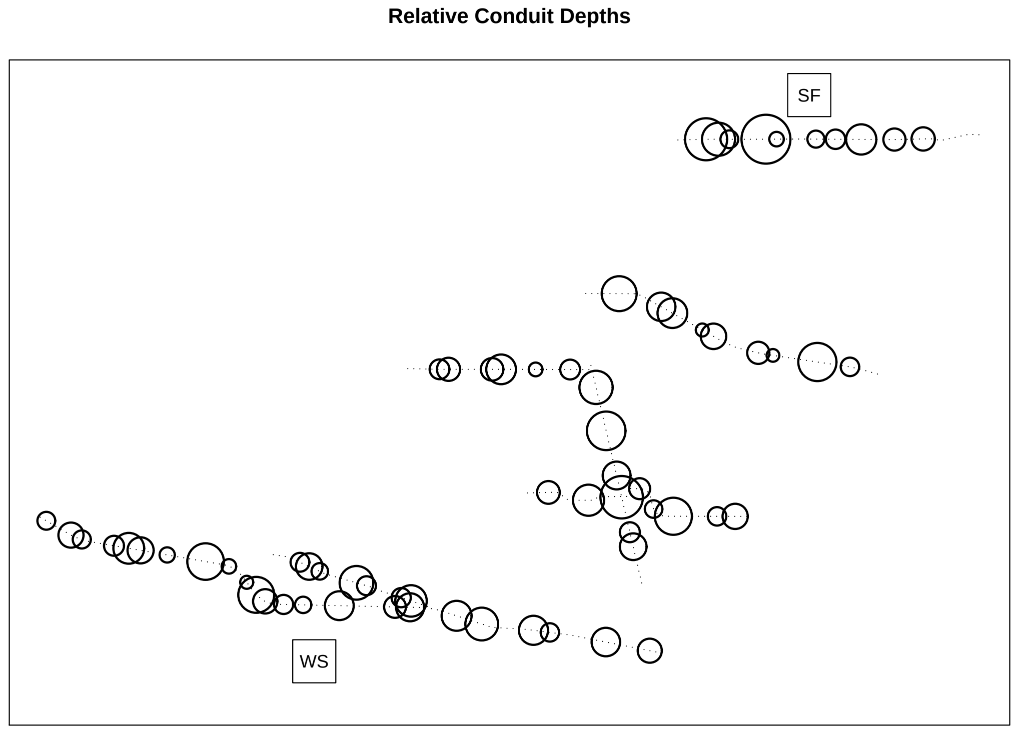

Figure 9: Estimated relative depths of possible conduits.

$$dm' = A\rho' d\lambda,$$

where ρ' is the density perturbation. If the perturbation is due to the presence of a water-filled conduit, then both ρ' and dm' are negative; a reasonable estimate for ρ' is $-1.4 \times 10^3 kg\ m^{-3}$. Substituting the above formulas into the integral yields

$$g' = A\rho' GH \int_{-\infty}^{\infty} \frac{d\lambda}{[x_0^2 + 2x_0\lambda\sin(\theta) + \lambda^2 + H^2]^{3/2}}$$

Integrating over λ from $-\infty$ to ∞ gives

$$g' = -Q\frac{A}{H}f$$

where

$$Q = -2\rho' G = 18.68\mu\text{Gal}\ m^{-1}$$

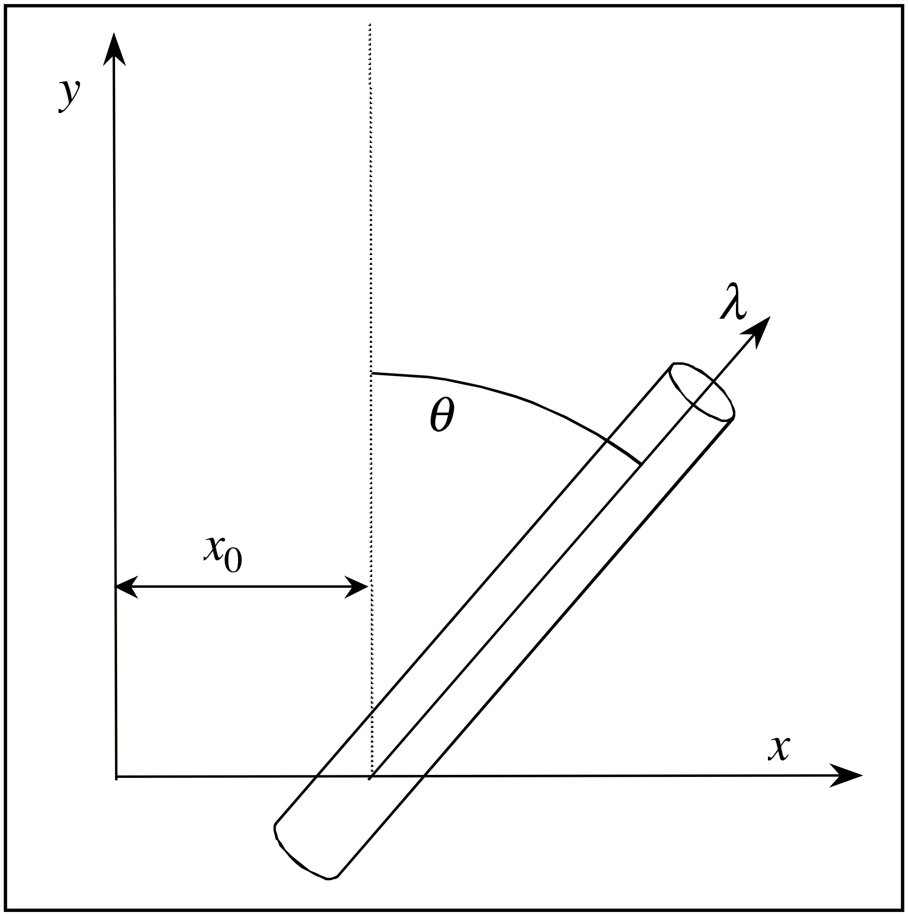

Figure 10: Diagram of parameters in theoretical model of conduits.

is a (constant) scaling factor and, dropping the subscript 0 on x

$$f(x) = \frac{H^2}{x^2\cos^2(\theta) + H^2}$$

gives the shape of the gravity perturbation. Note that a water-filled conduit (ρ' negative and Q positive) produces a weakening of gravity (g' negative). The maximum magnitude gravity deficit, g'_{max}, occurs when the observation point is directly over the conduit (i.e., when $x_0 = 0$):

$$g'_{\mathrm{max}} = -Q\frac{A}{H}.$$

The measured lateral shape of the signal (i.e., g' as a function of x), when fit to the curve $f(x)$, determines the effective depth:

$$H_{eff} = \frac{H}{\cos(\theta)}.$$

Since $|cos(\theta)| \leq 1$, the effective depth is at least as deep as the actual depth. Putting it the other way around, the actual depth of a conduit is shallower than the effective depth as determined by a gravity survey. This indeterminacy can be eliminated (or at least lessened) by completing two traverses offset by a small distance, thereby permitting the angle θ to be estimated. (Assuming, of course, that the conduit is straight.)

Knowing H and $g'_{\max}$, the equation for $g'_{\max}$ may be solved for the conduit area, A. If the conduit is circular with radius $R, (A = \pi R^2)$ this formula gives

$$R^2 = \frac{|g'_{\max}| H}{\pi Q}.$$

But H is not well known. The fit of f to the data gives H_{eff}, which is equal to $H/\cos(\theta)$. The problem is in determining or estimating θ. If the survey lines are well chosen so that θ is small, then $\cos(\theta) \approx 1$ and the uncertainty is resolved.

By way of illustration, supposing that $g'_{\max} = -58.67\mu\text{Gal}$ and the conduit is at depth of, say $81m$, then $R = 9m$. Alternatively, a conduit with $R = 5m$ at a depth of $80m$ gives a gravity signal $g'_{\max} = -18.3\mu\text{Gal}$.

The above formulas were used in the main text assuming that conduits cross the survey line perpendicularly. This is not unreasonable since it is known from exploration that the tunnels and caves in the region are generally north-south, and all lines save one run generally east-west.

Assuming that the conduits are perpendicular to the survey lines, $\theta = 0$, then $H_{eff}^2 = H^2$, and

$$g' = -Q\frac{A}{H}\frac{H^2}{x^2 + H^2}.$$

Since the microgravity data has been modified by application of a band-pass filter, the zero-level of the data has been compromised. Consequently, the above relation must be replaced by

$$g' = -Q\frac{A}{H}\frac{H^2}{x^2 + H^2} - c.$$

for some positive constant c.

Note that the estimates for the areas A and depths H are not absolute. Instead, these estimates are determined relative to each other.

5.2 Wavelet Smoothing of Microgravity Data

Smoothing was applied to the nonpositive part of the survey data using wavelet thresholding. The thresholding was implemented in the R software package (R Development Core Team (2006)). The wavelet basis used was the least asymmetric wavelet with support length 8 ("la8"). The thresholding was applied in two phases. First, soft VisuShrink (Donoho and Johnstone (1994)) was applied to the highest four levels of detail coefficients to remove noise. Then the top two levels of detail coefficients were set to zero to improve the visual appearance.

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