European Journal of Operational Research 297 (2022) 977–985

Contents lists available at ScienceDirect

European Journal of Operational Research

journal homepage: www.elsevier.com/locate/ejor

Stochastics and Statistics

Eliminating transplant waiting time inequities – With an application to kidney allocation in the USA

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ARTICLE INFO

Article history:
Received 2 January 2021
Accepted 25 September 2021
Available online 13 October 2021

Keywords:
OR in health services
Transplant waitlist
Equity
Allocation

ABSTRACT

Inequities in waiting times for deceased donor organ transplantation have received considerable attention in the last three decades and have motivated allocation policy reforms. This holds particularly true for kidney transplantation in the United States, where more than 90,000 patients are wait listed and average waiting times vary considerably among patients from different blood types and ethnic groups. This research presents a novel approach to formally model, analyze, and optimize equity of transplant waiting times and probabilities using queuing models, network flows, and Rawls' Theory of Justice. The presented formal models address inequities resulting from blood type incompatibilities, which are interrelated to ethnic differences in patient and donor rates. Moreover, we present results of an application to the deceased donor kidney wait lists in the United States. The findings indicate that the allocation policies currently practiced red can virtually resolve blood type related inequities in average waiting time and transplant probability.

1. Introduction

Chronic kidney disease (CKD) is an increasingly prevalent disease, which forms the 11th most common cause of death globally, accounting for 2.53 percent of death worldwide, 2.19 percent in Europe and 3.63 percent of death in the United States (Institute for Health Metrics & Evaluation, 2019). The ultimate phase of CKD, End Stage Renal Disease (ESRD), is most commonly treated by dialysis. Compared to dialysis, the alternative of transplantation is viewed to be strictly preferable as it offers longer life expectancy, better quality of life, and lower average treatment cost (Axelrod et al., 2018; Haller, Gutjahr, Kramar, Harnoncourt, & Oberbauer, 2011; Sánchez-Escuredo et al., 2015; Wolfe, Roys, & Merion, 2010). Transplantation is considered to be life-saving (Steering Committee of the Istanbul Summit, 2008). A kidney for transplant can be retrieved from a living donor or from a deceased donor. In developed countries, deceased donor transplantation (DTx) is typically more prevalent than living donor transplantation (LTx). While slightly less cost-effective (Hart et al., 2017b; MacNeill, Casula, Shaw, & Castledine, 2016; Wolfe et al., 2010), DTx has the advantage of avoiding health risks to donors

Globally, the number of patients treated for ESRD amounted to 2.5 million with a comparable number of patients lacking access to treatment in 2017 (Bibkov et al., 2020). The treated ESRD prevalence in the United States amounted to 726,331 patients by December 31, 2016 where the wait list for transplantation peaked at a number of 99,120 subscribed patients in 2014 (Hart et al., 2017a; United States Renal Data System, 2018). Since then, a modest decline has set in and by the end of 2017 92,685 patients were wait listed. This decrease occurred despite the average yearly number of newly arriving patient of 30,816 being more than double the average yearly number of 12,862 transplants over the years 2014–2017 (Hart et al., 2017a; 2019). Over the same period, many patients left the wait list without DTx. Some patients found alternative treatments such as LTx (5.5 percent). Sadly however, 9.1 percent of patients had become too sick to transplant or died while on the wait list. Less than half of the patients wait listed before 2011 had left the wait list after receiving a DTx by 2014 (Hart et al., 2017a).

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https://doi.org/10.1016/j.ejor.2021.09.033

Organ transplant wait lists can be modeled as single server $M/M/1$ queues with patient arrival rate λ and organ arrival rate μ (Zenios, 1999). Unfortunately organ transplant wait lists often have a utilization rate of $\rho = \frac{\lambda}{\mu} >> 1$, as is illustrated by the case of the kidney transplant wait list in the United States. The queue length nevertheless stabilizes because of patients leaving the queue without receiving a DTx. Such behaviors have been modeled as abandonment or reneging in the queuing literature (Mandelbaum, Massey, Reiman, Stolyar, & Rider, 2002; Wang, Li, & Jiang, 2010) and have been studied in the context of organ allocation (Drekic, Stanford, Woolford, & McAlister, 2015; Stanford, Lee, Chandok, & McAlister, 2014; Zenios, 1999; Zenios, Chertow, & Wein, 2000) as addressed in more detail below. Fathi & Khakifirooz (2019) discuss a literature selection on queuing models for kidney transplantation and kidney-related operations research in general.

The persistent relative scarcity of organ supply has brought along challenging allocation problems and inequalities in waiting times and transplant probabilities (Melanson et al., 2017). Ethnic inequalities and inequities have particularly received attention and various policy improvements have been proposed to resolve them (Lee, Kanellis, & Mulley, 2019). Our research aim is to model existing policies and minimize the resulting ethnic inequities in DTx waiting times and probabilities. We model inequity following Rawls' Theory of Justice (Rawls, 2009) and first turn to addressing underlying blood type related inequities, which have received attention since the 80s (Port, Held, Wolfe, Garcia, & Rocher, 1991; Scandling & Norman, 2010; Williams et al., 2015).

With the objective to minimize inequity among sub populations, our study presents models and methods which consider the allocation of organs per blood type to patient sub populations which are also distinguished per blood type. As will become clear in Section 2, it will not be necessary to specify allocation decisions at the individual level. Hence, the models disregard various patient characteristics which are important in practice for organ allocation at the patient level. Specifically, HLA compatibility will not be considered. We now briefly review related literature on allocation at (blood type) sub population level.

Sönmez & Unver (2015) consider equity of blood type compatible DTx together with LTx and Kidney Exchange Programs (KEPs). The renege rates in our work form a simplified approach to modeling alternatives to DTx, such as LTx and KEPs. Focusing more exclusively on DTx, we extend the analysis from inequities among blood types to racial inequities. We explicitly and formally distinguish inequality and inequity based on the Theory of Justice (Rawls, 2009). Instead of using Continuum Fluid models for the wait lists (Sönmez & Unver, 2015), our analysis of patient and donor arrivals builds on previously developed queuing models for organ transplantation (Drekic et al., 2015; Stanford et al., 2014; Zenios, 1999; Zenios et al., 2000). Wait list mortality is therefore captured by reneging instead of more explicit survival functions. Lastly, our models appear the first to include blood type incompatible DTx as increasingly practiced.

Section 2 reviews related literature and elaborates a network flow model for kidney allocation. Sections 3 develops a network flow algorithm minimizing inequity across blood type sub populations, which is shown to also minimize equity among ethnic sub populations in Section 4. All proofs are provided in the Appendix. Section 5 presents an application to kidney allocation in the United States. Further reflections are provided in Section 6.

2. Modelling and theoretical background

2.1. Allocation, blood types and ethnicity

As our research considers waiting time differences between patient sub populations, we model allocation policies at the level of sub populations. The allocation of specific organs to individual donors is therefore beyond the scope (see also Drekic et al., 2015; Stanford et al., 2014; Zenios, 1999; Zenios et al., 2000). We first investigate two traditional policies: *identical allocation* and *compatible allocation*. Identical allocation, as depicted in Fig. 1a, aims to avoid inequity by only allowing transplantations from donors to patients of the same type (Lee et al., 2019).

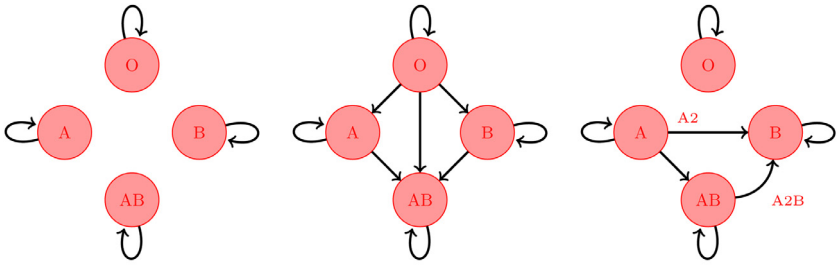
In comparison to identical allocation, the policy of compatible allocation additionally allows transplantations to patients with blood types that are compatible with the blood type of a donor. Fig. 1b presents the corresponding (and well known) ABO blood type compatibility graph. It reflects that type AB donors are only compatible with type identical AB patients. The arc emanating from the type A (B) node and incident to the type AB node reflects that type A (B) donors are compatible with and can therefore donate to type AB patients as well as to type identical A (B) patients. The type O node has three outgoing arcs which represent that type O donors are compatible with and can donate to patients of all types. Blood type (in)compatibility has been a main concern as cause of waiting time inequalities and inequities in larger scale transplant programs, (Port et al., 1991; Scandling & Norman, 2010; Williams et al., 2015).

A third policy considered captures the allocation policy as currently implemented in the kidney allocation system (KAS) of UNOS in the United States, in so far as related to blood types (Fig. 1c). This *KAS* policy allows for identical allocation and for allocation of *A* organs to *AB* patients. Moreover, since 2014, it allows for allocation of *A* sub type *A2* organs to *B* patients and *AB* sub type *A2B* organs to *B* patients since 2014 (Bryan, Cherikh, & Sesok-Pizzini, 2016).

Different policies may allocate organs differently and thus impact transplant waiting times and probabilities per blood type differently. As blood type distributions differ among ethnic groups, such policies may also cause waiting time and transplant probability differences among ethnic groups. Conversely - and closely related - the differences in disease prevalence and donation rates among ethnic groups importantly cause patient and organ arrival rate differences per blood type and hence per ethnic group. Fig. 2a and b illustrates such differences, based on patient level UNOS/OPTN data over the years 2014–2017.

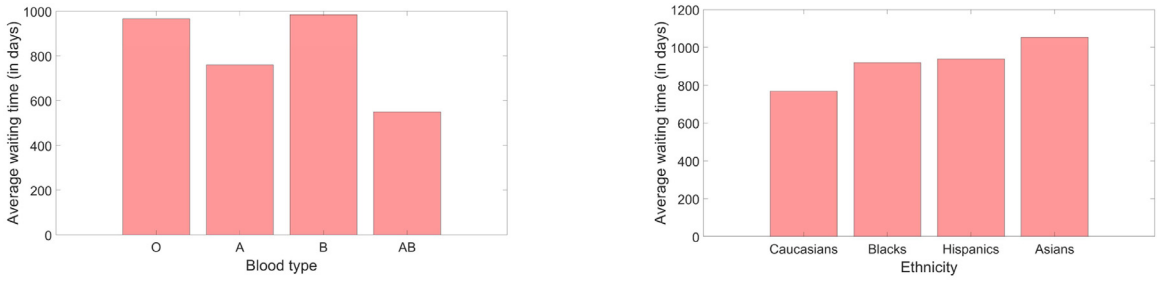
DTx programs in which patients subscribe to a deceased donor kidney transplant wait list have been modeled using queuing theory. Zenios (1999) develops queuing models with reneging for the case where $\rho = \frac{\lambda}{\mu} > 1$ which consider multiple patient and donor classes, with the objective to equalize waiting times and transplant probabilities. ABO blood types and ethnicities are not specifically considered in these generic queuing models. Zenios et al. (2000) model a problem which is closely related to ours, optimizing a multicriteria objective function that includes effectiveness (in quality-adjusted life years) as well as equality and equity in terms of transplant waiting time and transplant probability. They model the problem as an optimal control problem and present heuristics based on the approximate analysis of the control problem. Using simulation they show improvements over practiced policies are attainable. Stanford et al. (2014) present queuing models for the organ allocation problem and show that a restricted form of compatible allocation can resolve inequalities resulting from identical allocation in the case of stable queues, i.e. $\rho = \frac{\lambda}{\mu} < 1$. Drekic et al. (2015) present priority queuing models with reneging for ABO identical and ABO compatible allocation of livers from deceased donors, in which priority may change with patient health over time.

With the exception of Zenios et al. (2000) the aforementioned work exclusively focuses on equality in waiting times and transplant probabilities. Sönmez & Unver (2015) study minimization of DTx waiting time inequalities in combination with living donor



(a) Identical. (b) Compatible. (c) KAS.

Fig. 1. Overview of different allocation policies.



(a) Per blood type. (b) Per ethnicity.

Fig. 2. Realized average DTx waiting times for kidney transplantation in the USA, 2014–2017, original data source: UNOS/OPTN.

transplant treatments. With Zenios (1999) they point out that equality may not be attainable in practice. For such cases, we formally define and pursue how instead to minimize *inequity*, based on the Theory of Justice Rawls (2009). Using a network flow formulation that is closely related to the parametric network flow formulation of Sönmez & Unver (2015) we present necessary and sufficient conditions for equal waiting times and transplant probabilities for patients from different blood types in case of Poisson patient and organ arrivals. We subsequently develop an algorithm to minimize inequity when these conditions are violated. These models and methods are extended to address ethnic inequality and inequity. The algorithmic approach is closely related to earlier recursive algorithmic approaches to equitable allocation (Kominers, Pathak, Sönmez, & Unver, 2020; Luss, 1999; Megiddo, 1974; Sönmez & Unver, 2015).

2.2. *Equity in transplant waiting times*

A health inequality is an observable health difference between subgroups within a population (World Health Organization, 2017). Health equity refers to the absence of health inequalities in so far as avoidable, unnecessary, unfair and unjust (Whitehead, 1992). Instead of focusing on health status directly, our study considers waiting time and transplant probability, which are important determinants of health for ESRD patients.

Waiting time inequalities between sub populations may arise as a result of differences in the relative volumes of organs allocated. Such differences can be equalized by lowering the volumes for all sub populations that have shorter than maximum waiting time. An extreme allocation policy that achieves equality is the policy in which none of the organs is allocated. This extreme example illustrates the inaptness of simply minimizing inequalities. We adopt the view that it is unjust to leave any of the available organs unallocated and henceforth require that all available organs are allocated. Such allocations will be called *maximal*. If maximality and equality of waiting times cannot be simultaneously achieved, then inequalities are considered unavoidable and necessary. Within the set of maximal allocations, we pursue equity by avoiding other unnecessary, unfair and unjust inequalities.

We further formalize (in)equity based on Rawls' Theory of Justice (Rawls, 2009). This theory forms an anchor in the ongoing debate about definitions of equity, fairness and justice of allocation in health(care) (Peter, 2001). Applied to health of populations, the difference principle of the Theory of Justice implies to maximize the minimum health achieved over all sub populations considered. Translated to DTx waiting time, this principle entails to minimize the maximum waiting time and to maximize the minimum transplant probability over all patient populations considered. These measures will be defined more exactly below. The Theory of Justice can be interpreted to subsequently allow inequalities as follows. Provided that the difference principle is adhered to, inequalities that result from improving waiting time for some sub populations are fair and just if they recursively minimize the maximum waiting times for the remaining patient sub populations. Likewise, differences in transplant probabilities are fair and just if they recursively maximize minimum transplant probabilities. Hence, in order to minimize inequity, we set out to find maximal allocations that recursively minimize the maximum waiting time and maximize minimum transplant probability.

These equity definitions resemble the objectives defined in previous research on fair allocation of scarce resources. Megiddo (1974) considers the problem of seeking a fair maximum network flow in a network with multiple sources and sinks. He develops an optimal algorithm which relies on ordering the flows of the demand nodes (followed by the flows of the supply nodes) by increasing magnitude. Luss (1999, 2012) also develops such lexicographic algorithms to address more general resource allocation problems with the objective to maximize equity. Rather than analyzing the problem at hand on the basis of these general approaches, we present tailored models and methods which are more straightforward and insightful.

2.3. Queuing theory foundations of waiting times

$M/M/1$ queuing models have formed the most common formal approach to optimizing allocation of organs to patients for deceased donation (see Drekic et al., 2015; Stanford et al., 2014; Zenios, 1999; Zenios et al., 2000 and the references therein). The time between the arrival of consecutive organs forms the service time in these models. Hence, for a patient receiving a transplant, the actual time until transplant is the waiting time plus the service time, i.e. the sojourn time. In practice this (expected) service time is often much smaller (hours or days) than the preceding time in the queue (years), as illustrated in Section 5. Hence the waiting time until service is commonly considered for minimization.

For a generic queue, the expected number of newly arriving patients per time period is denoted by λ while μ denotes the expected number of donor organs per time period available to service the patients in the queue. We consider systems where the utilization rate $\rho = \frac{\lambda}{\mu} > 1$ yet in which the queue length does not grow indefinitely because patients renege, i.e patients leave the queue without DTx (Zenios, 1999). The renege probability per time unit is assumed to be constant over time and independent of queue length. Hence the time until reneging follows an exponential distribution. The invariant renege probability per time period is denoted by θ.

Now, let $L(t)$ be the length of the wait list in time period t (for $t \in \mathbb{Z}_{>0}$). Under the realistic assumptions that the queue is never empty and that patient arrivals continue to exceed donor organ arrivals ($\rho \geq 1$), $L(t)$ then transitions in expectation from time period t to time period $t+1$ by

$$L(t+1) = (1-\theta) \times L(t) + \lambda - \mu.$$

Thus, in case $\rho \geq 1$, a stable equilibrium can still be obtained for the queue length $L^* = \frac{\lambda-\mu}{\theta}$ due to the reneging. In this equilibrium, λ patients enter the waiting list, μ patients receive a DTx, and $\lambda - \mu$ patients renege per time period. For the equilibrium queue length of $\frac{\lambda-\mu}{\theta}$, the equilibrium time in the queue is

$$\frac{\lambda-\mu}{\theta} \times \frac{1}{\lambda} = \frac{1}{\theta}\left(1 - \frac{1}{\rho}\right).$$

Zenios (1999) proves that, as the queue length approaches infinity, the asymptotic stationary expected waiting time converges to $\frac{\lambda-\mu}{\theta} \times \frac{1}{\lambda}$ and the asymptotic stationary expected transplant probabilities converge to $\frac{\mu}{\lambda}$. As our problem is motivated by the very long queue lengths encountered in practice (up to 100.000 for kidney allocation in the USA), we adopt these two measures as our objectives. For compactness, we use the shorter *wait time (resp. Tx probability)* to refer to asymptotic stationary expected waiting time (resp. asymptotic stationary expected transplant probabilities) in the remainder. Thus our objectives are to (recursively) minimize the maximum wait time and to (recursively) maximize the minimum Tx probability. Zenios (1999) assumes a First Come First Served (FCFS) policy for the allocation of organs to individual patients within a blood type sub population. We may note that for the assumed constant renege rate θ and the wait time and Tx probability measures, patients of a same blood type are interchangeable and the results apply regardless of individual level allocation decisions among patients of a same blood type. Hence, the results remain valid if the order in which type identical patients receive allocated organs deviates from FCFS for reasons of HLA compatibility or otherwise.

2.4. Donor allocation as network flow

DTx allocation policies determine how many organs of each type to allocate to patients of each compatible type. The total number of organs allocated to each type then forms the organ arrival rates for the corresponding patients (as further explained below). Together with the patient arrival rates, these allocated organ arrival rates determine the wait times and Tx probabilities. We now show how to model the arising allocation problem as a network flow problem (see Fig. 3).

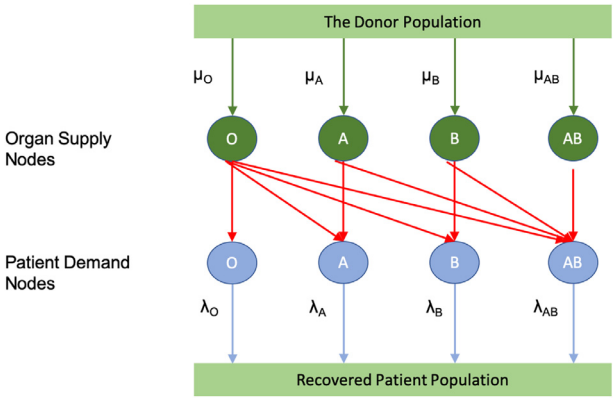


Fig. 3. Allocation graph G.

The red arcs in the *Allocation Graph* G in Fig. 3 correspond to the arcs in graph (Fig. 1b) for compatible allocation. (Section 5 covers the Allocation graph for KAS). They connect the organ supply nodes of each type to the patient demand nodes of the compatible types. For each blood type $x \in X$, available donor organs flow over the green arcs g_x emanating from a fictitious source to the type x supply nodes. For all $x \in X$, the capacity of arc g_x is set at μ_x, the number of donor organs of type x available per time unit. For any feasible flow f in G, let f'_x represent the corresponding flow on arc $g_x, x \in X$. Maximal allocations satisfy $f'_x = \mu_x$.

The red arcs have infinite capacity, modeling that for red arc r_{xy} any organ of type $x \in X$ that becomes available can be utilized for transplantation to patients of a compatible type $y \in X$. Because of flow conservation it must hold for any $x \in X$ that $\sum_{y \in X} f_{xy} = f'_x$, with f_{xy} the flow on r_{xy}.

For each type $y \in X$, there is a blue arc b_y for the flow of all organs allocated to patients of type y per time period, denoted by f_y. Flow conservation now implies that for each $y \in X$, $\sum_{x \in X} f_{xy} = f_y$. For any flow f and for all $y \in X$, the f_y represent Poisson organ arrivals to patients of type y in case for all $x \in X$ the organs of type x are randomly allocated over the patients type $z \in X$ compatible with x with probability $\frac{f_{xz}}{f'_x}$.

The values of λ_y, for $y \in X$ function as capacity limits for the arc flows f_y, as (in expectation) we cannot transplant more organs to type $y \in X$ than the number of patients arriving. The allocation problem then becomes to find a maximal feasible flow f in G that results in most equitable wait times and Tx probabilities.

3. Equity and blood types

The Max-Flow Min-Cut theorem specifies that the value of a maximum flow in G is equal to the value of a minimum cut in G (Ford & Fulkerson, 2009). A cut $C = (S, T)$ is defined as a partition of the vertices of G into two sets S and T, where S contains the source and T the sink. The cut-set is defined as the set of outgoing arcs from S to T. The value of a cut equals the sum of the capacities of the arcs included in the cut-set.

As the red arcs have infinite capacity, they will not be included in any minimum cut set. Any minimum cut C^* therefore necessarily consists of green and blue arcs. Among all cut sets of blue and green arcs disconnecting the source from the sink, a minimum cut is characterized by the blood types in the sets S^*_g and S^*_b that mini-

mizing $\sum_{x\in S_g}\mu_x+\sum_{x\in S_b}\lambda_x$. Here, $x\in S_g^*$ implicates that g_x is in the cut-set and $x\in S_b^*$ means that b_x is in the cut-set.

In pursuit of equity, the remainder of the analysis will not focus on the flows and cuts in G but in adjusted version of G as follows. For any feasible flow f in G, let f_x represent the corresponding flow on arc $b_x, x\in X$. Then, for each blood type x, f_x implies a wait time for the patients receiving a DTx of

$$\frac{1}{\theta}\left(1-\frac{f_x}{\lambda_x}\right).$$

Notice that by definition of the arc capacities, $f_x\le\lambda_x$. It then follows that f yields equal wait times if there exists a rational number $\rho^=\ge 1$ such that $\frac{\lambda_x}{f_x}=\rho^=$ for each $x\in X$.

The requirement to allocate all available organs translates to finding maximal flows f satisfying $\sum_{x\in X}f_x=\sum_{x\in X}\mu_x$. We therefore define $G^=$ to be identical to G except for adjusting the capacities λ_x of the blue arcs $b_x, x\in X$ to $c_x^==\lambda_x\times\frac{\sum_{x\in X}\mu_x}{\sum_{x\in X}\lambda_x}$. This adjustment entails that the total capacity of the blue arcs $\sum_{x\in X}c_x^=$ equals $\sum_{x\in X}\mu_x$, the total capacity of the green arcs, while the $c_x^=$ are proportional to λ_x for $x\in X$. With graph $G^=$ at hand, we can determine whether a maximal allocation with equal wait times for all blood types exists:

Theorem 1. *A maximal allocation of donor organs to patients resulting in equal wait times exists if and only if a feasible flow f of value $\sum_{x\in X}\mu_x$ exists in $G^=$.*

Theorem 1 implies that blood type identical allocation yields equal and equitable wait times among blood types in case donor organ arrival rates and patient arrival rates are proportional to the blood type distributions in a population. Hence, the complexity of finding equitable allocations among sub populations of different blood types arises from disproportional arrival rates.

It is not hard to verify that in case a flow of value $\sum_{x\in X}\mu_x$ exists in $G^=$, both the set of green arcs $g_x, x\in X$ and the set of blue arcs $b_x, x\in X$ form a cut-set. We now turn to the case where the maximum flow in $G^=$ has value less than $\sum_{x\in X}\mu_x$.

Theorem 2. *If and only if for every minimum cut C^* in $G^=$ it holds that $S_g^*\neq X$ and $S_b^*\neq X$, then 1) $O\in S_g^*$, 2) $AB\in S_b^*$, and 3) the maximum flow has value less than $\sum_{x\in X}\mu_x$.*

Thus, if the maximum flow f has value strictly less than $\sum_{x\in X}\mu_x$, the cut partitions $G^=$ into two components, one of which contains the source and the other containing the sink. The component containing the source is connected to all supply nodes of types $x\in X\setminus S_g^*$ and subsequently via the red arcs to compatible demand nodes. It contains the supply and demand nodes of AB and will therefore be referred to as C_{AB}. Likewise, the other component symmetrically consists of the sink and the demand nodes of type $x\in X\setminus S_b^*$, together with the supply nodes of blood types compatible with these demand nodes. It necessarily contains the demand and supply nodes for type O and will therefore be referred to as C_O. For ease of notation, we define a cut $C=(C_O,C_{AB})$ by blood types, rather than by vertices. This means that $AB\in C_{AB}^*$ and $O\in C_O^*$ for minimum cut $C^*=(C_O^*,C_{AB}^*)$.

Fig. 4 shows an example of a minimum cut in case the maximum flow is strictly smaller than $\sum_{x\in X}\mu_x$. In this example the cut-set equals $\{g_O,g_A,b_B,b_{AB}\}$, meaning that $S_g^*=\{O,A\}$ and $S_b^*=\{B,AB\}$. The cut is then defined by $C^*=(C_O^*,C_{AB}^*)$ with $C_O^*=\{O,A\}$ and $C_{AB}^*=\{B,AB\}$.

Because C^* is a minimum cut, it must hold that

$$\sum_{x\in C_O^*}\mu_x\le\sum_{x\in C_O^*}c_x^==\sum_{x\in C_O^*}\left(\lambda_x\times\frac{\sum_{x\in C_O^*}\mu_x}{\sum_{x\in C_O^*}\lambda_x}\right)$$

and that conversely,

$$\sum_{x\in C_{AB}^*}\mu_x\ge\sum_{x\in C_{AB}^*}c_x^==\sum_{x\in C_{AB}^*}\left(\lambda_x\times\frac{\sum_{x\in C_{AB}^*}\mu_x}{\sum_{x\in C_{AB}^*}\lambda_x}\right).$$

As we are now considering the case for which the value of the minimum cut is strictly less than $\sum_{x\in X}\mu_x$, at least one of these inequalities is strict and hence at least one of the types in C_O^* must have longer wait times than at least one of the types in C_{AB}^* for any maximum allocation.

While we have thus established that maximality and equality may not be jointly attainable, one can still pursue equity for maximal allocations following Rawls' recursive minimax principles. In the remainder of this section we will use the results established so far to develop a recursive algorithm to obtain an equitable solution corresponding to these principles.

Let G_O be the subgraph of G induced by the types in C_O^*. As above, let $G_O^=$ be equal to G_O, except for adjusting the capacities λ_x of the blue arcs $b_x, x\in C_O^*$ to $\lambda_x\times\frac{\sum_{x\in C_O^*}\mu_x}{\sum_{x\in C_O^*}\lambda_x}$. Similarly to S_g^* and S_b^*, we let S_{Og}^* and S_{Ob}^* define the cut-set corresponding to a minimum cut in $G_O^=$.

Theorem 3. *If for every minimum cut in $G_O^=$ it holds that $S_{Og}^*\neq X$ and $S_{Ob}^*\neq X$, then (1) the type O demand and supply nodes are in the component which also contains the sink and (2) the maximum flow has value less than $\sum_{x\in C_O^*}\mu_x$.*

If the maximum flow in $G_O^=$ has value less than $\sum_{x\in C_O^*}\mu_x$, a minimum cut partitions it into two components, one of which contains the demand and supply nodes for blood type O. This component will be referred to as G_L and the other component as G_R. The same will recursively apply to G_L if this component contains nodes of blood type O and one other blood type A or B.

Likewise, let us symmetrically consider G_{AB}, the subgraph of G induced by the types $x\in C_{AB}^*$. As above, let $G_{AB}^=$ be equal to G_{AB}, except for adjusting the capacities λ_x of the blue arcs $b_x, x\in C_{AB}^*$ to $\lambda_x\times\frac{\sum_{x\in C_{AB}^*}\mu_x}{\sum_{x\in C_{AB}^*}\lambda_x}$ and let S_{ABg}^* and S_{ABb}^* define the cut-set corresponding to a minimum cut in $G_{AB}^=$.

Theorem 4. *If for every minimum cut in $G_{AB}^=$ it holds that $S_{ABg}^*\neq X$ and $S_{ABb}^*\neq X$, then 1) the type AB demand and supply nodes are in the component which also contains the source and 2) the maximum flow has value less than $\sum_{x\in C_{AB}^*}\mu_x$.*

If the maximum flow in $G_{AB}^=$ has value less than $\sum_{x\in C_{AB}^*}\mu_x$, a minimum cut partitions it into two components, one of which contains the demand and supply nodes for blood type AB. This component will be referred to as G_R and the other component as G_L. The same will recursively apply to G_R if this component contains nodes of blood type AB and one other blood type A or B.

The results of Theorems 2–4 are combined in Algorithm 1 in recursive function EquitableFLow() to obtain an equitable flow ac-

Algorithm 1 Equitable Flow.

```
input: graph G, blood types Y
output: equitable flow vector f̂
function EquitableFlow(G, Y)
    (G_L^=, G_R^=, C_L^*, C_R^*, f̂) ← MaxFlowMinCut(G, Y)
    if Σ_{y∈Y} f̂_y < Σ_{y∈Y} μ_y then
        f_L ← EquitableFlow(G_L^=, C_L^*)
        f_R ← EquitableFlow(G_R^=, C_R^*)
        f̂ ← flow values corresponding to f_L and f_R
    return f̂
```

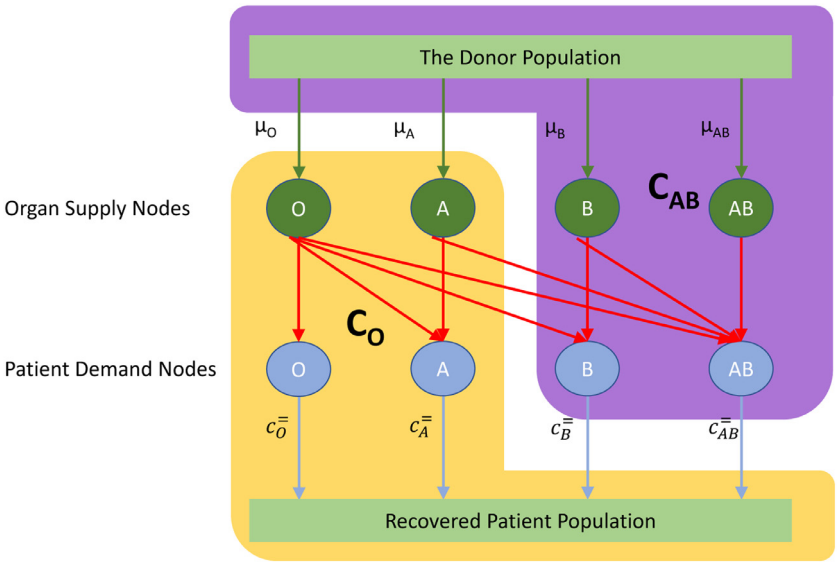


Fig. 4. Illustration of a Cut $C = (C_O, C_{AB})$.

Table 1
Percentages of arriving patients per blood type and ethnicity. Source: UNOS OPTN DTx data 2011–2017.

	Type O	Type A	Type B	Type AB	Total
Caucasians	20.3	18.1	5.0	1.8	45.1
Blacks	14.6	7.3	5.9	1.2	29.1
Hispanics	10.9	5.3	1.8	0.4	18.4
Asians	2.9	1.8	2.2	0.5	7.4
Total	48.7	32.6	14.9	3.8	

Table 2
Percentages of arriving donor organs per blood type and ethnicity: Source: UNOS OPTN DTx data 2011–2017.

	Type O	Type A	Type B	Type AB	Total
Caucasians	31.0	28.7	6.9	2.4	69.1
Blacks	6.9	3.5	2.8	0.6	13.8
Hispanics	8.8	4.3	1.2	0.2	14.6
Asians	1.0	0.7	0.6	0.2	2.6
Total	47.8	37.2	11.5	3.5	

cording to Rawls' Theory of Justice. The algorithm is initiated with a first call to EquitableFLow($G^=, X$), where $G^=$ is the graph constructed for Theorem 1 and X contains all blood types. The output is an equitable flow vector $\boldsymbol{f} = [f_O, f_A, f_B, f_{AB}]$. Recursively, EquitableFLow() is called on smaller instances when the flow is not maximal as described earlier. In this algorithm we do not specify the function MaxFlowMinCut(), this can be any maximum flow algorithm. Our algorithm requires a graph and a set of blood types as input and returns the maximum flow per blood type, the minimum cut $C^* = (C_L^*, C_R^*)$ in the graph and the two graphs $G_L^=$ and $G_R^=$, with adjusted capacities, corresponding to the cut. Note that the first call to MaxFlowMinCut() returns C_L^* containing blood type O and C_R^* containing blood type AB, as shown by Theorem 2. Two illustrative applications of the algorithm are provided in the Appendix.

4. Equity and ethnicity

As blood type prevalences vary over ethnic groups, equity among blood types is no guarantee for equity among ethnic groups. The next step is therefore to address ethnic inequities, which persist in practice (Melanson et al., 2017).

Let V be the set of all ethnic groups. For instance, for the United States we may define $V =$ {Caucasian, Black, Hispanic, Asian, Other}. In the remainder, we disregard Other, as the low number of patients in this group does not permit reliable quantitative analysis. Accordingly, let λ_{xv} be the arrival rate of patients of blood type $x \in X$ and ethnicity $v \in V$ and let μ_{xv} be the corresponding organ arrival rate. Then, by definition $\lambda_x = \sum_{v \in V} \lambda_{xv}$ and $\mu_x = \sum_{v \in V} \mu_{xv}$ for $x \in X$. Tables 1 and 2 present patient and donor organ arrival rates for the United States percentage wise.

Tables 1 and 2 teach us for instance that blood types O and A are approximately equally likely among Caucasians, while blood type A is considerably less prevalent than type O among the other ethnic groups. As Caucasians are relatively more prevalent among donors, identical allocation would result in a higher utilization rate for blood type A than for blood type O. This effect is even stronger for blood type B. These differences in rates are particularly pronounced for Blacks who experience longer average waiting times as already confirmed by Fig. 2b.

Hence, the question rises whether these inequalities can be reduced to become as equitable as possible. In the remainder, we disregard the ethnic origin of the organs, and hence, the μ_{xv} and only consider $\mu_x = \sum_{v \in V} \mu_{xv}$ for $x \in X$, as before. The analysis therefore focuses on the λ_{xv}.

Theorem 5. *Let f be an equitable allocation as determined by Algorithm Equitable Flow, and let $f_{xv} = f_x \times \frac{\lambda_{xv}}{\lambda_x}$ for all $x \in X, v \in V$. Then, f_{xv} is an equitable allocation for the case with blood type and ethnicity.*

An important consequence of Theorem 5 is that for the case in which ethnic equity is explicitly considered together with blood type equity, equity can be maximized by firstly finding an allocation that maximizes equity among the blood types and subsequently allocating organs per blood type proportionally to demand of the ethnic groups for that blood type. This can be achieved by merging all patients into one queue per blood type and subsequent implementation of any policy for which allocation probabilities are subsequently independent of ethnicity. These findings stand in contrast to the observational data depicted in Fig. 5 which shows

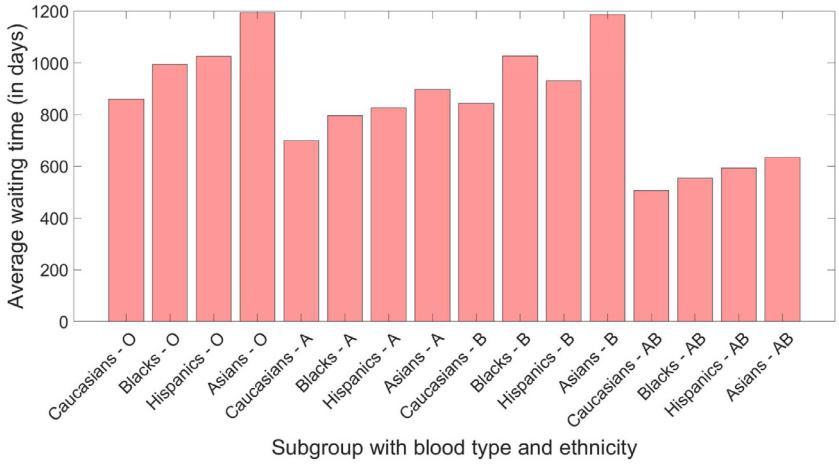


Fig. 5. Waiting time per blood type and ethnicity.

that considerable inequalities exist among ethnic groups for each blood type. In the discussion we consider whether these might be explained by reasons other than blood type compatibility.

While it may be possible to reduce ethnic inequalities beyond the levels attained by Theorem 5, the theorem implies that this necessarily implies increasing inequalities between some pair of pairs $(\{x, v\}, \{y, w\}), x, y \in X, v, w \in V$ and is therefore inequitable according to the Theory of Justice.

5. Equity of deceased donor kidney allocation in the United States

Tables 1 and 2 show that considerable differences exist between organ and patient arrival rates per ethnic group and subsequently per blood type, especially for blood types A and B. Combined with Fig. 2a, these numbers suggest that type O organs have been used for type B patients.

Let us now recall that over the years 2014–2017, 30,816 new patients entered the DTx wait list on average per year, and 12,862 patients received a DTx on average per year Hart et al. (2017a, 2019). The queue length started at 96,848 on January 1, 2014, peaked at 99,172 on December 31, of the same year, and then decreased to 92,685 by the end of 2017. Calculated over the average of queue lengths, the annual renege rate has been 0.196 (approximately one in five patients leaves the queue annually without transplant). Disregarding blood type incompatibilities for the time being, these total numbers would yield an equilibrium wait list length of $\frac{\lambda-\mu}{\theta} = \frac{30{,}816-12{,}862}{0.196} = 91,565$ with a corresponding wait time of $\frac{1}{\theta}(1-\frac{1}{\rho}) = 2.97$ years (1085 days). The Tx probability equals $\frac{\mu}{\lambda} = \frac{12{,}862}{30{,}816} = 0.42$ (close to the 0.44 reported in Hart et al. (2017a) for earlier years).

Table 3 presents the expected outcomes for each of the three policies identical allocation, compatible allocation and KAS. Figures 3,4, and 5 of the Appendix depict the corresponding flows.

For identical allocation, type B patients can expect to wait 287 days longer than type A patients. The differences among ethnic groups are much smaller, with a maximum of 51 days between Caucasians and Asian Americans. Given that compatible allocation is feasible and yields comparable patient outcomes, the inequalities resulting from identical allocation can be viewed to be avoidable and hence inequitable, in so far as they exceed the inequalities resulting from compatible allocation.

The wait times and Tx probabilities presented in Table 3 for blood type compatible allocation are obtained by Algorithm Equitable Flow, with one level of recursion (see Fig. 2 in the Appendix). The figure illustrates that Algorithm Equitable Flow separates component $C^*_O = \{O, B\}$ from component $C^*_{AB} = \{A, AB\}$. The corresponding equitable wait times are equal for types O and B (through allocating some type O organs to type B patients) and for types A and AB (through allocating some type A organs to type AB patients). The resulting wait time differences are much smaller than for the case of identical allocation, yet still considerable. Type O and B patients wait 144 days longer than type A and AB patients. Translated to ethnic groups however, the equitable wait time inequalities are at most 18 days and the Tx probabilities differ by 0.013 at most.

Table 3
Waiting times in days and deceased donor transplantation transplant probabilities for three allocation policies.

	Type O	Type A	Type B	Type AB
Wait Time Identical	1099	975	1262	1146
Wait Time Compatible	1137	993	1137	993
Wait Time KAS	1099	1071	1071	1071
Tx Prob Identical	0.410	0.476	0.322	0.384
Tx Prob Compatible	0.389	0.467	0.389	0.467
Tx prob KAS	0.410	0.425	0.425	0.425
	Caucasian	Black	Hispanic	Asian American
Wait Time Identical	1069	1103	1080	1120
Wait Time Compatible	1074	1091	1092	1092
Wait Time KAS	1084	1081	1088	1082
Tx Prob Identical	0.426	0.408	0.420	0.398
Tx Prob Compatible	0.423	0.410	0.413	0.413
Tx Prob KAS	0.418	0.416	0.416	0.419

The inequalities of compatible allocation can still be regarded as avoidable (and hence inequitable) as they do not utilize type $A2$ to B and $A2B$ to B transplants, as practiced by UNOS since December 2014 (Bryan et al., 2016). The sub type $A2$ ($A2B$) makes up around 20 percent of the type A (AB) population (Bryan et al., 2016). The formal analysis of the KAS policy which includes such transplants requires to modify the compatibility graph. We modify it by adding donation by type $A2$ and $A2B$ donors to type B patients see also Fig. 1c. As type AB has a high utilization rate and low prevalence, we disregard $A2B$ to B donation in the remainder (without loss of optimality as we shall see). The reader may verify that Theorems 1–5 and Algorithm Equitable Flow still hold when allowing A to B allocation. In the presented results we applied Algorithm Equitable Flow allowing for allocations from type A to B, rather than the more restricted KAS which only allows for donations from type $A2$ to B. Afterwards we verified that the results complied with the KAS policy and the percentage of A to B allocations was below 20 percent.

The results obtained when allowing type $A2$ organs to be allocated to type B patients are also depicted in Table 3. Around 9.8

percent of type *A* donor organs are allocated to type *B* patients, well below the 20 percent threshold. The only other non-identical allocations are from *A* to *AB*. The types *A*, *B*, and *AB* have equal wait times without requiring *A*2*B* to *B* allocations. These findings confirm the currently practiced allocation policy. The equitable solution obtained when allowing for this additional allocation possibility has near perfect equality among blood types and ethnic groups. The total expected wait list length is 91,565.

6. Discussion and conclusions

We presented formal models to maximize equity of waiting times and transplant probabilities in deceased donor kidney transplantation based on Rawls' Theory of Justice, queuing theory and network flow theory. The models consider asymptotic stationary expected waiting times and transplant probabilities as equity measures. Allocations maximizing blood type equity (as derived in Section 3) can be straightforwardly translated to solutions jointly maximizing blood type and ethnic equity (as shown in Section 4). In so far as the resulting equitable allocations yield waiting time inequalities among ethnic groups, they cannot be further reduced without increasing inequalities among blood type. Our analysis also clarifies how the inequalities arise from differences in organ and patient arrival rates among ethnic groups. This confirms the benefit of increasing donation rates in ethnic groups which are under represented as donors.

Application to the 2014–2017 UNOS data on deceased donor kidney transplantation in the United States largely validate the presented models. The 2017 wait list length of 92,685 is close to the model outcome equilibrium of 91,565 and moving towards it. The renege rate of 0.42 is close to the rate of 0.44 for preceding years reported by Hart et al. (2017a). However, the models should not be expected to exactly replicate practice for a number of reasons. Let us mention already that the models assumes constant arrival and renege rates, while these vary in practice.

The results for blood type identical allocation yield considerable inequalities among blood types and rather modest ethnic inequalities. The results for compatible allocations yield quite limited inequalities for both. Inequalities, and hence inequities, virtually disappear when additionally allowing for *A*2 to *B* transplantation as currently practiced in KAS. The resulting solution reallocates 9.8 percent of A organs to B patients, higher than the 7.4 reported by Bryan et al. (2016). They also report a positive number of A2B allocations compared to zero in our optimal solution. Such modifications could be implemented through slight adjustments to current practice or by randomly assigning 9.8 percent of *A* organs to *B* patients (see Section 3).

The findings that blood type related inequalities can be eliminated do not contradict the persistent inequalities found in practice among ethnic groups (Melanson et al., 2017). There are other factors influencing the waiting times that are not included in the presented allocation model. Firstly, HLA profiles and HLA antibody profiles vary significantly among ethnic groups, creating subsequent differences in waiting time and transplant probabilities. Secondly, health state and time on dialysis at time of enrolment vary among ethnic groups, which may subsequently lead to differences in priority and renege rates, whereas we have assumed renege rates to be constant over time and independent of sub population (Hart, Salkowski, Snyder, Israni, & Kasiske, 2016). Thirdly, let us mention that there are significant differences in the likelihood of receiving a LTx among blood types and ethnic groups (Hart et al., 2019) that our model fails to capture as it assumes a constant θ. While this calls for further research to advance the models and analysis, it also raises new ethical questions. For example, should DTx allocation policies take HLA related and ethnic inequalities in determinants of waiting time and transplant probability more explicitly into account? Such questions are especially relevant now we have shown that blood type related inequities can be and are being addressed.

The modeling and methods presented have validity beyond DTx in the United States. They can assist to address inequality and equity allocation problems for other organs in the United States (Higgins & Fishman, 2006; Moylan et al., 2008) and for organ donation in other countries (Morgan, Hooper, Mayblin, & Jones, 2006; Stanford et al., 2014). Moreover, they can be applied when distinguishing sub populations based on other grounds than ethnicity, such as certain HLA types. It should be noted however that our analysis assumes $\mu < \lambda$, i.e. that the donor organ arrival rate is strictly smaller than the patient arrival rate. In the aspired future case for which this assumption is not valid, the objective function needs to be adjusted. If non-compatible allocations other than *A*2 to *B* are allowed, Algorithm Equitable Flow is not applicable, and more general approaches to fair resource allocation are required (see Luss, 1999; Luss, 2012; Megiddo, 1974).

Acknowledgement

We are grateful for discussions with Kristiaan Glorie, Jeroen Kremer, Chiel van Oosterom, and David Stanford.

Supplementary material

Supplementary material associated with this article can be found, in the online version, at 10.1016/j.ejor.2021.09.033.

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