PSG COLLEGE OF TECHNOLOGY, COIMBATORE - 641 004

Department of Applied Mathematics and Computational sciences

Problem Sheet - 2

20XD48 - Scientific Computing Lab - Eigen values and Eigen vectors

Instructions:

- Don't use *linalg.inv*(), *linalg.dot*() and *linalg.solve*() methods in python *numpy* library.
- Don't use *linsolve*() method in python *sympy* library.
- Use *numpy*, *scipy*, *sympy* library.

1. Write a python program to find the eigen values and eigen vectors of the following using direct method? Also print the eigen values and corresponding eigen vectors.

   a. $\begin{bmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$

   b. $\begin{bmatrix} 1 & 0 & -2 \\ 0 & 0 & 0 \\ -2 & 0 & 4 \end{bmatrix}$

   c. $\begin{bmatrix} 6 & 3 & -8 \\ 0 & -2 & 0 \\ 1 & 0 & -3 \end{bmatrix}$

   d. $\begin{bmatrix} 3 & 0 & 0 \\ -2 & 7 & 0 \\ 4 & 8 & 1 \end{bmatrix}$

2. Write a python program to diagonalize the following matrix. Also print the eigen values, corresponding eigen vectors and diagonal matrix.

   a. $\begin{bmatrix} 1 & 0 \\ 6 & -1 \end{bmatrix}$

   b. $\begin{bmatrix} 2 & 0 & -2 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

   c. $\begin{bmatrix} -1 & 7 & -1 \\ 0 & 1 & 0 \\ 0 & 15 & -2 \end{bmatrix}$

3. Write a python program to change the following quadratic form into canonical form. Also print the matrix corresponding to QF, eigen values, corresponding eigen vectors and canonical form.

   a. $2x^2 + 6xy - 5y^2$

   b. $x_1^2 + 7x_2^2 - 3x_3^2 + 4x_1x_2 - 2x_1x_3 + 8x_2x_3$

   c. $x_1^2 - x_3^2 - 4x_1x_2 + 4x_2x_3$

4. Check whether the following matrices are diagonalizable or not using python program.

   a. $\begin{bmatrix} -10 & 11 & -6 \\ -15 & 16 & -10 \\ -3 & 3 & -2 \end{bmatrix}$

   b. $\begin{bmatrix} 3 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 3 \end{bmatrix}$

   c. $\begin{bmatrix} 1 & -7 & -1 \\ 0 & 1 & 0 \\ 0 & 15 & -2 \end{bmatrix}$

5. Write a python program for the following matrices to find the dominant eigen vector using power method and dominant eigen value using Rayleigh quotient? Also print the dominant eigen values and corresponding dominant eigen vectors.

   a. $\begin{bmatrix} 2 & 1 \\ 0 & -4 \end{bmatrix}$

   b. $\begin{bmatrix} -5 & 0 & 0 \\ 3 & 7 & 0 \\ 4 & -2 & 3 \end{bmatrix}$

   c. $\begin{bmatrix} 1 & 2 & -2 \\ -2 & 5 & -2 \\ -6 & 6 & -3 \end{bmatrix}$

6. Write a python program for the following matrices to find the smallest eigen vector using inverse power method and smallest eigen value using Rayleigh quotient? Also print the smallest eigen values and corresponding smallest eigen vectors.

   a. $\begin{bmatrix} 2 & 3 & 1 \\ 0 & -1 & 2 \\ 0 & 0 & 3 \end{bmatrix}$

   b. $\begin{bmatrix} 3 & 0 & 0 \\ 1 & -1 & 0 \\ 0 & 2 & 8 \end{bmatrix}$

c. $\begin{bmatrix} 1 & 2 & 0 \\ 0 & -7 & 1 \\ 0 & 0 & 0 \end{bmatrix}$