

Problem Sheet - 1

Scientific Computing Lab - Solving system of Linear Equations

Instructions:

- Don't use *linalg.inv()*, *linalg.dot()* and *linalg.solve()* methods in python *numpy* library.
- Don't use *linsolve()* method in python *sympy* library.
- Use *numpy*, *scipy*, *sympy* library.

1. Write a python program to find augmented matrix corresponding to the following linear system. Also print the given linear system with its augmented matrix.

$$(a) \quad \begin{array}{rcl} 2x + 3y & = & 7; \\ 5x + 7y & = & 5. \end{array}$$

$$(b) \quad \begin{array}{rcl} 15x + 3y + 5z & = & 1; \\ 12x + 5y + 7z & = & 2; \\ 17x + 7y + 9z & = & 3. \end{array}$$

2. Write a python program to check which of the following matrices are in REF or RREF. Also print the given matrix and output result.

$$(a) \quad \begin{pmatrix} 1 & 2 & 0 & 3 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$(b) \quad \begin{pmatrix} 1 & -7 & 5 & 5 \\ 0 & 1 & 3 & 2 \end{pmatrix}$$

$$(c) \quad \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 0 & 7 & 1 & 3 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$(d) \quad \begin{pmatrix} 1 & 2 & 3 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

3. Write a python program for Gauss-Jordan Elimination method to find the RREF of the following matrices. Also print the given matrix and its RREF and rank?

$$(a) \begin{pmatrix} 0 & 3 & -6 & 6 & 4 & -5 \\ 3 & -7 & 8 & -5 & 8 & 9 \\ 3 & -9 & 12 & -9 & 6 & 15 \end{pmatrix}$$

$$(b) \begin{pmatrix} 1 & 1 & 1 & 6 \\ 2 & -1 & 1 & 3 \\ 1 & 0 & 1 & 4 \end{pmatrix}$$

$$(c) \begin{pmatrix} 1 & 3 & 4 & 7 \\ 2 & 4 & 6 & 8 \\ 3 & 6 & 9 & 12 \end{pmatrix}$$

$$(d) \begin{pmatrix} 0 & 1 & -2 & 3 \\ 1 & -3 & 4 & -6 \end{pmatrix}$$

$$(e) \begin{pmatrix} 1 & -3 & 7 & 1 \\ 0 & 1 & 4 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$(f) \begin{pmatrix} 1 & 1 & 1 & 8 \\ -1 & -2 & 3 & 1 \\ 3 & -7 & 4 & 10 \end{pmatrix}$$

4. Write a python program to find solutions for the following linear system using Gauss-Jordan elimination method. If it has unique solution, print the solution. If it has infinite solution, print the infinite solution in set form. If it has no solution, print no solution, rank of  $A$  and  $[A : B]$ .

$$(a) \begin{array}{rrcrcl} x_1 & + & x_2 & + & 2x_3 & = & 8 \\ -x_1 & - & 2x_2 & + & 3x_3 & = & 1 \\ 3x_1 & - & 7x_2 & + & 4x_3 & = & 10 \end{array}$$

$$(b) \begin{array}{rrcrcl} 2x_1 & - & 3x_2 & + & 4x_3 & - & 4x_4 & = & 0 \\ 7x_1 & + & x_2 & - & 8x_3 & + & 9x_4 & = & 0 \\ 2x_1 & - & 8x_2 & + & x_3 & - & x_4 & = & 0 \end{array}$$

$$(c) \begin{array}{rrcrcl} 2x_1 & - & x_2 & + & 3x_3 & - & 4x_4 & = & 9 \\ x_1 & + & x_2 & - & 2x_3 & + & 7x_4 & = & 11 \\ 3x_1 & - & 3x_2 & + & x_3 & + & 5x_4 & = & 8 \\ 2x_1 & + & x_2 & + & 4x_3 & + & 4x_4 & = & 10 \end{array}$$

5. Write a python program to solve the following linear system using Gauss-Seidel Method. Also print the initial values, the iteration table and final solution.

$$(a) \begin{array}{rrcrcl} 4x_1 & + & x_2 & + & x_3 & = & 2 \\ x_1 & + & 5x_2 & + & 2x_3 & = & -6 \\ x_1 & + & 2x_2 & + & 3x_3 & = & -4 \end{array}$$

$$(b) \begin{array}{rrcl} 2x & + & 5y & = & 21 \\ x & + & 2y & = & 8 \end{array}$$

$$\begin{array}{rclcl}
 2x & + & 3y & - & z & = & 5 \\
 \text{(c)} \quad 3x & + & 2y & + & z & = & 10 \\
 x & - & 5y & + & 3z & = & 0
 \end{array}$$

6. Write a python program to solve the following linear system using Gauss-Jacobi Method. Also print the initial values, the iteration table and final solution.

$$\begin{array}{rclcl}
 4x_1 & + & x_2 & + & x_3 & = & 2 \\
 \text{(a)} \quad x_1 & + & 5x_2 & + & 2x_3 & = & -6 \\
 x_1 & + & 2x_2 & + & 3x_3 & = & -4
 \end{array}$$

$$\begin{array}{rcl}
 \text{(b)} \quad 2x & + & 5y & = & 21 \\
 x & + & 2y & = & 8
 \end{array}$$

$$\begin{array}{rclcl}
 2x & + & 3y & - & z & = & 5 \\
 \text{(c)} \quad 3x & + & 2y & + & z & = & 10 \\
 x & - & 5y & + & 3z & = & 0
 \end{array}$$

7. Write python program to solve all above problems using any python library methods. like *linalg.inv()*, *linalg.dot()* and *linalg.solve()* methods in python *numpy* library. *linsolve()* method in python *sympy* library