Contrasting Contrasts

Livio Finos

Contents

Abstract I recall the importance of the use of zero-sum contrasts for categorical variables compared to the usual coding in dummy variables. The problem remains the same for quantitative variables. I tackle the problem with a synthetic dataset and a linear model with a factor (= categorical variable), a quantitative variable and their interaction.

1 The data + EDA

The Challenge:
Let's build a dataset where the effects are known, can you analyze it adequately?

The proposed model is a simple ANCOVA model: - normal response (lm) with errors with 0 mean and variance equal to 1 - linear model with predictors: * two groups (`A` and`B`), * a continuous variable e * their interaction - Effects: Intercept and group. The continuous variable has no relation to the answer for the group `A`, it has it instead in the group`B` (interaction).

These are the data created and their representation.

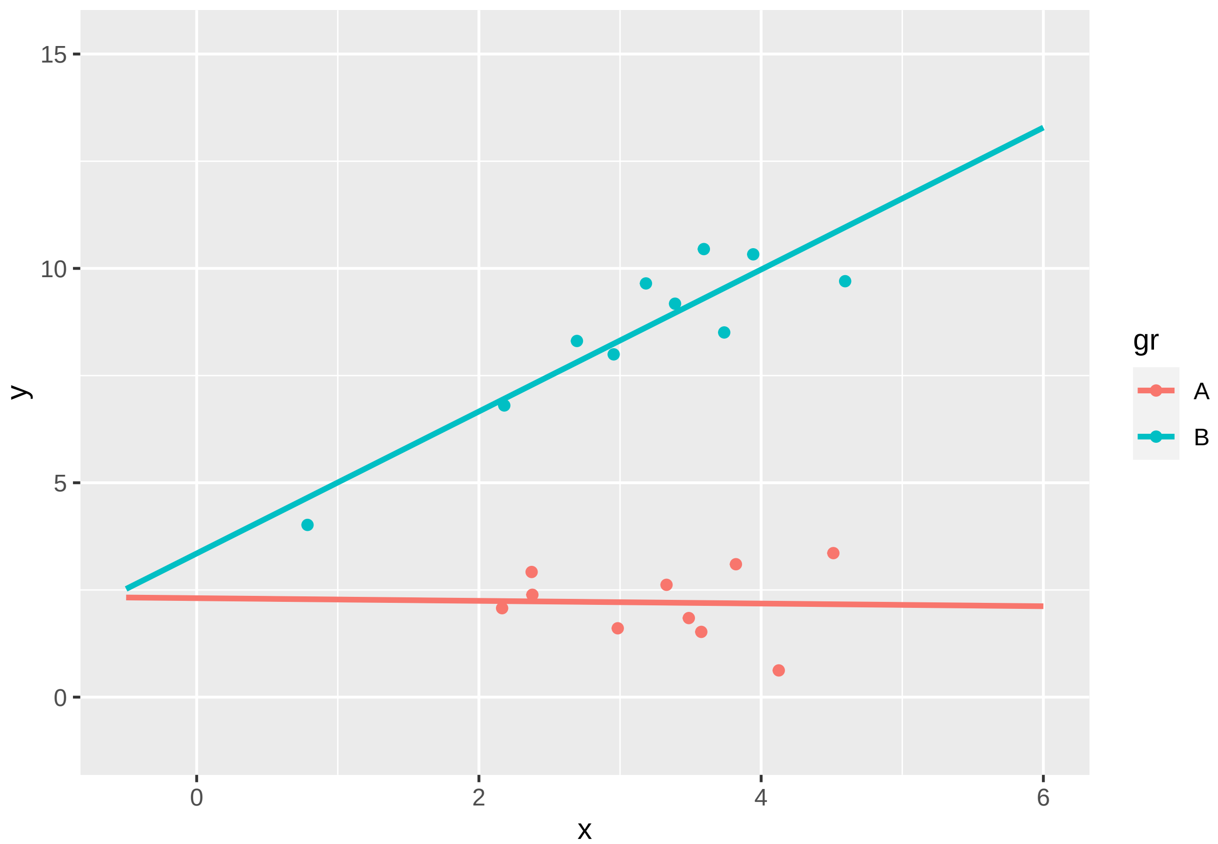
```
set.seed(1)
n0=10
D=data.frame(gr=as.factor(rep(LETTERS[1:2],n0)),
             x=rnorm(n0*2)+3)
mu=2+(D$gr=="B")*.5+D$x*(D$gr=="B")*2
D$y= mu+rnorm(n0*2)

D
```

```
##    gr         x          y
## 1   A 2.3735462  2.9189774
## 2   B 3.1836433  9.6494229
## 3   A 2.1643714  2.0745650
## 4   B 4.5952808  9.7012099
## 5   A 3.3295078  2.6198257
## 6   B 2.1795316  6.8029345
## 7   A 3.4874291  1.8442045
## 8   B 3.7383247  8.5058970
## 9   A 3.5757814  1.5218499
## 10  B 2.6946116  8.3071648
## 11  A 4.5117812  3.3586796
## 12  B 3.3898432  9.1768987
## 13  A 2.3787594  2.3876716
## 14  B 0.7853001  4.0167952
## 15  A 4.1249309  0.6229404
## 16  B 2.9550664  7.9951382
## 17  A 2.9838097  1.6057100
## 18  B 3.9438362 10.3283590
## 19  A 3.8212212  3.1000254
## 20  B 3.5939013 10.4509784
```

```
library(ggplot2)
ggplot(D,aes(x=x,y=y,color=gr))+geom_point()+
  geom_smooth(method = "lm", fill = NA,fullrange = TRUE)+xlim(-.5, 6)
```

```
## `geom_smooth()` using formula 'y ~ x'
```



2 Linear Models

2.1 A linear model?

```
modDU=lm(y~gr*x,data=D)
summary(modDU)
```

```
##
## Call:
## lm(formula = y ~ gr * x, data = D)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1.55585 -0.61528 -0.00131  0.54234  1.19203
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  2.30818    1.24368   1.856  0.08198 .
## grB          1.04292    1.53659   0.679  0.50701
## x           -0.03137    0.37016  -0.085  0.93352
## grB:x        1.68703    0.46200   3.652  0.00215 **
## ---
```

```
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.8778 on 16 degrees of freedom
## Multiple R-squared:  0.9481, Adjusted R-squared:  0.9384
## F-statistic: 97.49 on 3 and 16 DF,  p-value: 1.708e-10
```

matrix of predictors (i.e. independent variables)

```
(mm <- model.matrix(~gr*x,data=D))
```

```
##    (Intercept) grB         x     grB:x
## 1            1   0 2.3735462 0.0000000
## 2            1   1 3.1836433 3.1836433
## 3            1   0 2.1643714 0.0000000
## 4            1   1 4.5952808 4.5952808
## 5            1   0 3.3295078 0.0000000
## 6            1   1 2.1795316 2.1795316
## 7            1   0 3.4874291 0.0000000
## 8            1   1 3.7383247 3.7383247
## 9            1   0 3.5757814 0.0000000
## 10           1   1 2.6946116 2.6946116
## 11           1   0 4.5117812 0.0000000
## 12           1   1 3.3898432 3.3898432
## 13           1   0 2.3787594 0.0000000
## 14           1   1 0.7853001 0.7853001
## 15           1   0 4.1249309 0.0000000
## 16           1   1 2.9550664 2.9550664
## 17           1   0 2.9838097 0.0000000
## 18           1   1 3.9438362 3.9438362
## 19           1   0 3.8212212 0.0000000
## 20           1   1 3.5939013 3.5939013
## attr(,"assign")
## [1] 0 1 2 3
## attr(,"contrasts")
## attr(,"contrasts")$gr
## [1] "contr.treatment"
```

Have a look to the correlations between predictors

```
cor(mm)
```

```
## Warning in cor(mm): la deviazione standard è zero
```

```
##             (Intercept)         grB           x     grB:x
## (Intercept)           1          NA          NA        NA
## grB                  NA  1.00000000 -0.09503104 0.9094708
## x                    NA -0.09503104  1.00000000 0.2451776
## grB:x                NA  0.90947082  0.24517762 1.0000000
```

and the Multiple R-squared of the first three columns to explain the interaction column:

```r
summary(lm(mm[,2]~mm[,-2]+0))
```

```
##
## Call:
## lm(formula = mm[, 2] ~ mm[, -2] + 0)
##
## Residuals:
##       Min        1Q    Median        3Q       Max
## -0.243736 -0.060709  0.005904  0.071867  0.265952
##
## Coefficients:
##                     Estimate Std. Error t value Pr(>|t|)
## mm[, -2](Intercept)  0.65509    0.11529   5.682 2.70e-05 ***
## mm[, -2]x           -0.19006    0.03590  -5.294 5.95e-05 ***
## mm[, -2]grB:x        0.29060    0.01871  15.528 1.79e-11 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.1385 on 17 degrees of freedom
## Multiple R-squared:  0.9674, Adjusted R-squared:  0.9616
## F-statistic:   168 on 3 and 17 DF,  p-value: 7.853e-13
```

2.2 Another linear model? (x 0-centered)

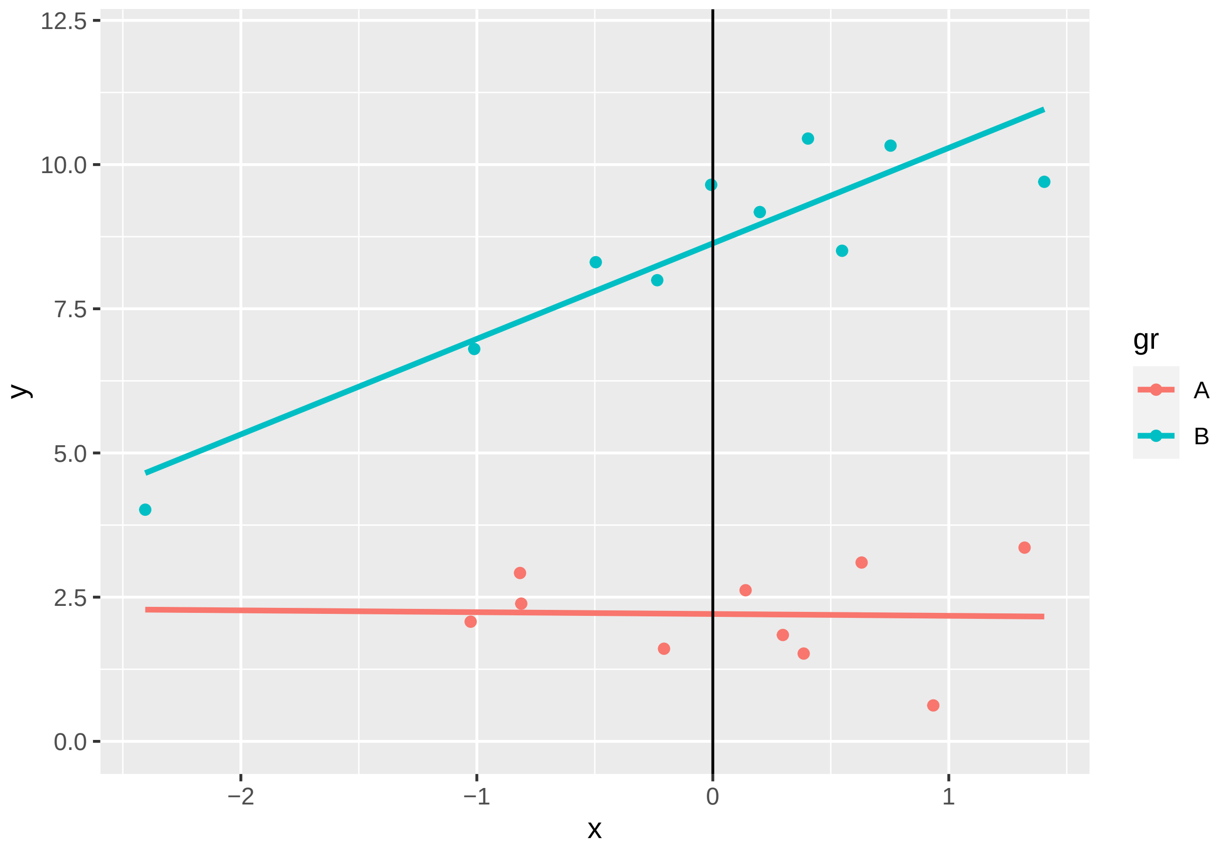
```r
D2=D
D2$x=D$x-mean(D$x)
modDUC=lm(y~gr*x,data=D2)
summary(modDUC)
```

```
##
## Call:
## lm(formula = y ~ gr * x, data = D2)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1.55585 -0.61528 -0.00131  0.54234  1.19203
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  2.20810    0.27933   7.905 6.47e-07 ***
## grB          6.42543    0.39449  16.288 2.21e-11 ***
## x           -0.03137    0.37016  -0.085  0.93352
## grB:x        1.68703    0.46200   3.652  0.00215 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.8778 on 16 degrees of freedom
## Multiple R-squared:  0.9481, Adjusted R-squared:  0.9384
## F-statistic: 97.49 on 3 and 16 DF,  p-value: 1.708e-10
```

Observe the 0 on the abscise: there is a clear difference between groups A and B.

```
ggplot(D2,aes(x=x,y=y,color=gr))+geom_point()+
  geom_smooth(method = "lm", fill = NA,fullrange = TRUE)+ geom_vline(xintercept = 0)
```

```
## `geom_smooth()` using formula 'y ~ x'
```



Correlations between predictors

It is also useful to evaluate the correlations between predictors.

```
(mm <- model.matrix(~gr*x,data=D2))
```

```
##    (Intercept) grB            x       grB:x
## 1            1   0 -0.816977687  0.000000000
## 2            1   1 -0.006880552 -0.006880552
## 3            1   0 -1.026152489  0.000000000
## 4            1   1  1.404756926  1.404756926
## 5            1   0  0.138983896  0.000000000
## 6            1   1 -1.010992260 -1.010992260
## 7            1   0  0.296905176  0.000000000
## 8            1   1  0.547800829  0.547800829
## 9            1   0  0.385257475  0.000000000
## 10           1   1 -0.495912263 -0.495912263
## 11           1   0  1.321257292  0.000000000
## 12           1   1  0.199319360  0.199319360
## 13           1   0 -0.811764457  0.000000000
```

```
## 14           1   1 -2.405223763 -2.405223763
## 15           1   0  0.934407042  0.000000000
## 16           1   1 -0.235457485 -0.235457485
## 17           1   0 -0.206714139  0.000000000
## 18           1   1  0.753312335  0.753312335
## 19           1   0  0.630697319  0.000000000
## 20           1   1  0.403377445  0.403377445
## attr(,"assign")
## [1] 0 1 2 3
## attr(,"contrasts")
## attr(,"contrasts")$gr
## [1] "contr.treatment"
```

Have a look to the correlations between predictors

```
cor(mm)
```

```
## Warning in cor(mm): la deviazione standard è zero
```

```
##             (Intercept)         grB           x       grB:x
## (Intercept)           1          NA          NA          NA
## grB                  NA  1.00000000 -0.09503104 -0.05946962
## x                    NA -0.09503104  1.00000000  0.80181388
## grB:x                NA -0.05946962  0.80181388  1.00000000
```

and the Multiple R-squared of the first three columns to explain the interaction column:

```
summary(lm(mm[,2]~mm[,-2]+0))
```

```
##
## Call:
## lm(formula = mm[, 2] ~ mm[, -2] + 0)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.57782 -0.48222 -0.00215  0.50046  0.55697
##
## Coefficients:
##                     Estimate Std. Error t value Pr(>|t|)
## mm[, -2](Intercept)  0.50139    0.12127   4.135 0.000693 ***
## mm[, -2]x           -0.07448    0.22686  -0.328 0.746694
## mm[, -2]grB:x        0.03293    0.28393   0.116 0.909022
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.5397 on 17 degrees of freedom
## Multiple R-squared:  0.5049, Adjusted R-squared:  0.4175
## F-statistic: 5.779 on 3 and 17 DF,  p-value: 0.006501
```

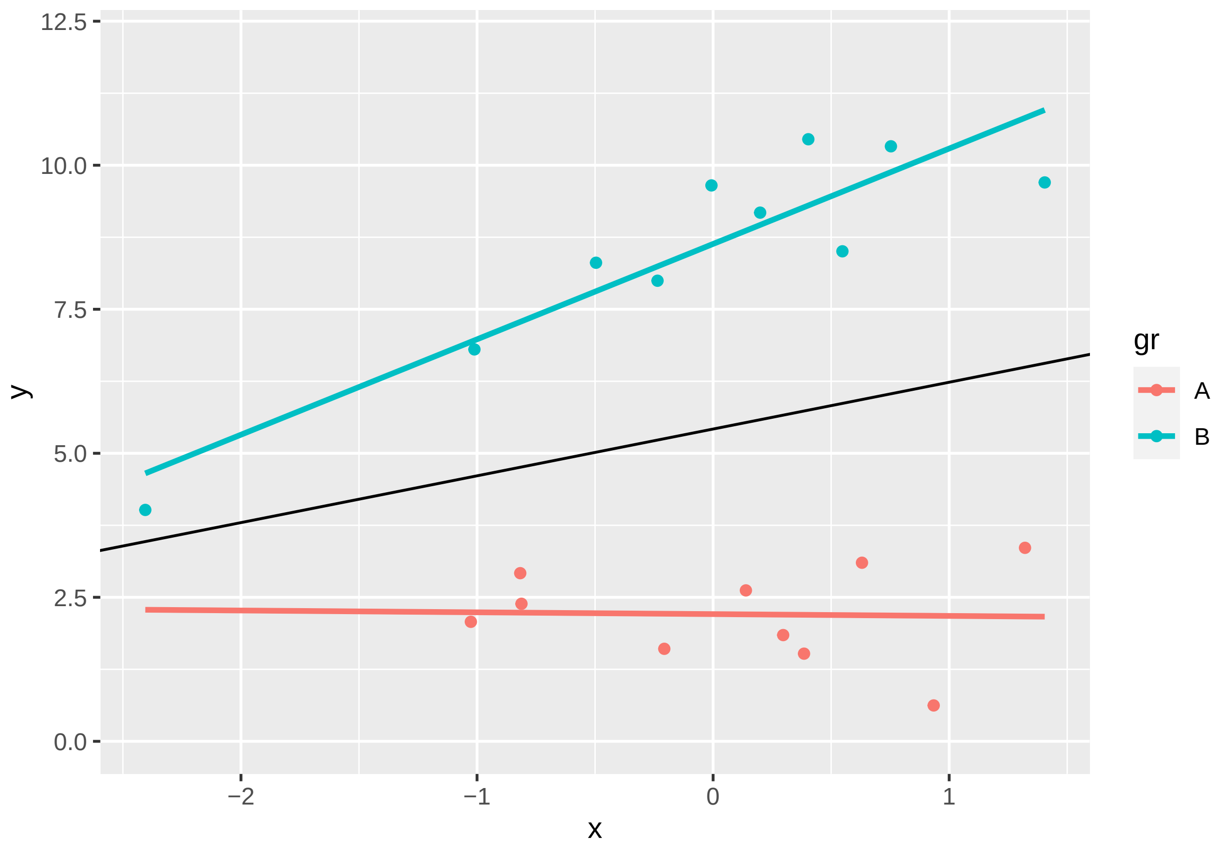
2.3 a third linear model?? (x 0-centered + gr 0-centered)

```
D3=D2
contrasts(D3$gr)=contr.sum(2)
modS0=lm(y~gr*x,data=D3)
summary(modS0)
```

```
##
## Call:
## lm(formula = y ~ gr * x, data = D3)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1.55585 -0.61528 -0.00131  0.54234  1.19203
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)   5.4208     0.1972  27.483 6.80e-15 ***
## gr1          -3.2127     0.1972 -16.288 2.21e-11 ***
## x             0.8121     0.2310   3.516  0.00287 **
## gr1:x        -0.8435     0.2310  -3.652  0.00215 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.8778 on 16 degrees of freedom
## Multiple R-squared:  0.9481, Adjusted R-squared:  0.9384
## F-statistic: 97.49 on 3 and 16 DF,  p-value: 1.708e-10
```

```
ggplot(D3,aes(x=x,y=y,color=gr))+geom_point()+
  geom_smooth(method = "lm", fill = NA,fullrange = TRUE)+ geom_abline(intercept = coef(modS0)[1], slope
```

```
## `geom_smooth()` using formula 'y ~ x'
```



The black line in the graph represents the equation: `Y = ` 5.4208153 `+` 0.8121474 `X` as estimated by the model. Actually, this is the effect for subjects with `gr == 0` which do not exist in reality, but represent an intermediate (somewhat null) value; are therefore an estimate *net of* the effects of `gr`.

Let's now observe how the predictor matrix has changed (in particular the interaction):

```
(mm <- model.matrix(~gr*x,data=D3))
```

```
##    (Intercept) gr1            x        gr1:x
## 1            1   1 -0.816977687 -0.816977687
## 2            1  -1 -0.006880552  0.006880552
## 3            1   1 -1.026152489 -1.026152489
## 4            1  -1  1.404756926 -1.404756926
## 5            1   1  0.138983896  0.138983896
## 6            1  -1 -1.010992260  1.010992260
## 7            1   1  0.296905176  0.296905176
## 8            1  -1  0.547800829 -0.547800829
## 9            1   1  0.385257475  0.385257475
## 10           1  -1 -0.495912263  0.495912263
## 11           1   1  1.321257292  1.321257292
## 12           1  -1  0.199319360 -0.199319360
## 13           1   1 -0.811764457 -0.811764457
## 14           1  -1 -2.405223763  2.405223763
## 15           1   1  0.934407042  0.934407042
## 16           1  -1 -0.235457485  0.235457485
## 17           1   1 -0.206714139 -0.206714139
```

```
## 18           1  -1  0.753312335 -0.753312335
## 19           1   1  0.630697319  0.630697319
## 20           1  -1  0.403377445 -0.403377445
## attr(,"assign")
## [1] 0 1 2 3
## attr(,"contrasts")
## attr(,"contrasts")$gr
##   [,1]
## A    1
## B   -1
```

and:

```
summary(lm(mm[,2]~mm[,-2]+0))
```

```
## 
## Call:
## lm(formula = mm[, 2] ~ mm[, -2] + 0)
## 
## Residuals:
##      Min       1Q   Median       3Q      Max 
## -1.11394 -1.00093  0.00431  0.96444  1.15564 
## 
## Coefficients:
##                      Estimate Std. Error t value Pr(>|t|)
## mm[, -2](Intercept) -0.002786   0.242535  -0.011    0.991
## mm[, -2]x            0.116024   0.282650   0.410    0.687
## mm[, -2]gr1:x        0.032933   0.283935   0.116    0.909
## 
## Residual standard error: 1.079 on 17 degrees of freedom
## Multiple R-squared:  0.009814,   Adjusted R-squared:  -0.1649 
## F-statistic: 0.05617 on 3 and 17 DF,  p-value: 0.9819
```

3 A simulation

What would the three models tell me if I could repeat the experiment many times?

```
res_sim=replicate(1000,
          {
            D$y <- D2$y <- D3$y <- mu+rnorm(n0*2)
            modDU=lm(y~gr*x,data=D)
            p_DU=summary(modDU)$coeff[,4]
            modDUC=lm(y~gr*x,data=D2)
            p_DUC=summary(modDUC)$coeff[,4]
            modSO=lm(y~gr*x,data=D3)
            p_SO=summary(modSO)$coeff[,4]
            c(DU=p_DU,DUC=p_DUC,SO=p_SO)
          })

res_sim=t(res_sim)
```

(Empirical) Power:

```r
library(r41sqrt10)
```

```
##
## Caricamento pacchetto: 'r41sqrt10'

## Il seguente oggetto è mascherato da 'package:base':
##
##     mode
```

```r
## model Dummy
summaryResSim(res_sim[,1:4])
```

```
##       [,1]  [,2]  [,3]  [,4]  [,5]
## [1,] 0.004 0.036 0.081 0.468 0.723
## [2,] 0.016 0.064 0.119 0.532 0.777

##                 <=0.01 <=0.05 <=0.1 <=0.5 <=0.75
## DU.(Intercept)  0.095  0.244 0.364 0.773  0.904
## DU.grB          0.012  0.046 0.098 0.517  0.742
## DU.x            0.006  0.043 0.100 0.496  0.747
## DU.grB:x        0.783  0.940 0.974 0.999  1.000
```

```r
## model Dummy +  Centered X
summaryResSim(res_sim[,5:8])
```

```
##       [,1]  [,2]  [,3]  [,4]  [,5]
## [1,] 0.004 0.036 0.081 0.468 0.723
## [2,] 0.016 0.064 0.119 0.532 0.777

##                  <=0.01 <=0.05 <=0.1 <=0.5 <=0.75
## DUC.(Intercept)  0.997  1.000 1.000 1.000  1.000
## DUC.grB          1.000  1.000 1.000 1.000  1.000
## DUC.x            0.006  0.043 0.100 0.496  0.747
## DUC.grB:x        0.783  0.940 0.974 0.999  1.000
```

```r
## model SO +  Centered X
summaryResSim(res_sim[,9:12])
```

```
##       [,1]  [,2]  [,3]  [,4]  [,5]
## [1,] 0.004 0.036 0.081 0.468 0.723
## [2,] 0.016 0.064 0.119 0.532 0.777

##                 <=0.01 <=0.05 <=0.1 <=0.5 <=0.75
## SO.(Intercept)  1.000  1.000 1.000 1.000      1
## SO.gr1          1.000  1.000 1.000 1.000      1
## SO.x            0.781  0.934 0.971 0.997      1
## SO.gr1:x        0.783  0.940 0.974 0.999      1
```

```r
# prova anche con
# D3=D2
# contrasts(D3$gr)=contr.sum(2)
# modS0=lm(y~gr*x,data=D3)
# X non centrata e contr.sum(2)
```

4 Conclusion

The definition of an effect depends crucially on the way we encode variables (whether they are factors or continuous).

Their estimation depends crucially on the correlation between predictors. This point becomes crucial in interactions, where correlations are often high by nature (they are defined as a product of the columns of the experimental design).

When possible (and sensible), the recommendation is to center the contrasts around 0 (see using `contr.sum()`).