

# Statistical Analysis of Designs with Repeated Measures by Linear Mixed Models

Livio Finos

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## 1 Motivation

### 1.1 Import

```
load("./dataset/datiEEG_LMM_2x2.Rdata")
summary(dati)
```

```
##      Subj      Chan      Condition      Y
## s01      : 16    01:160    f:160      Min.      :-9.9964
## s02      : 16    02:160    n:160      1st Qu.   :-2.5505
## s03      : 16                                Median    :-0.8299
## s04      : 16                                Mean      :-0.6867
## s05      : 16                                3rd Qu.   : 1.3913
## s06      : 16                                Max.      : 5.5468
## (Other):224
```

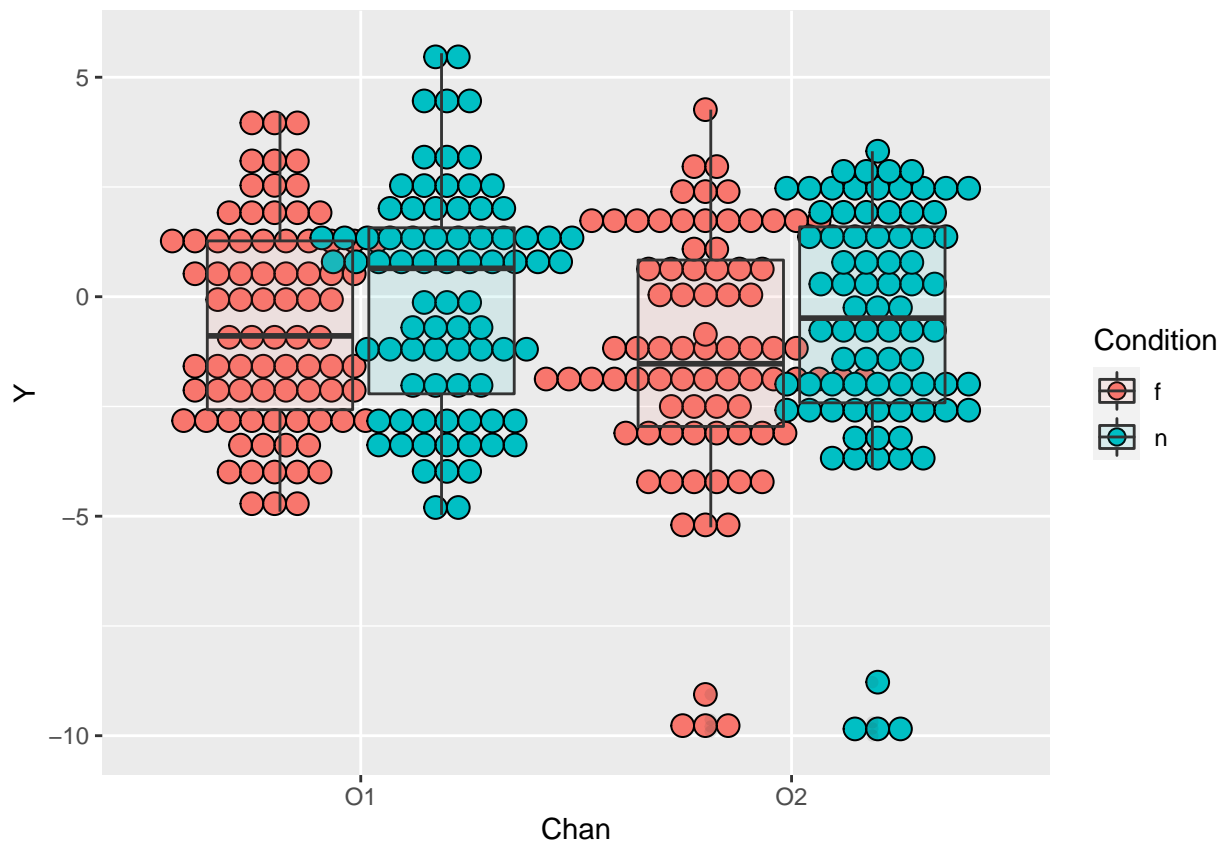
- 10 Subjects,
- 2-levels factor `Chan`
- 2-levels factor `Condition`

## 1.2 EDA

For Y: P300 Average Amplitude

```
library(ggplot2)
p <- ggplot(dati, aes(x=Chan, y=Y, fill=Condition))
p = p + geom_dotplot(binaxis = "y", position=position_dodge(0.8), stackdir = "center") + geom_boxplot(alpha=.1)
p
```

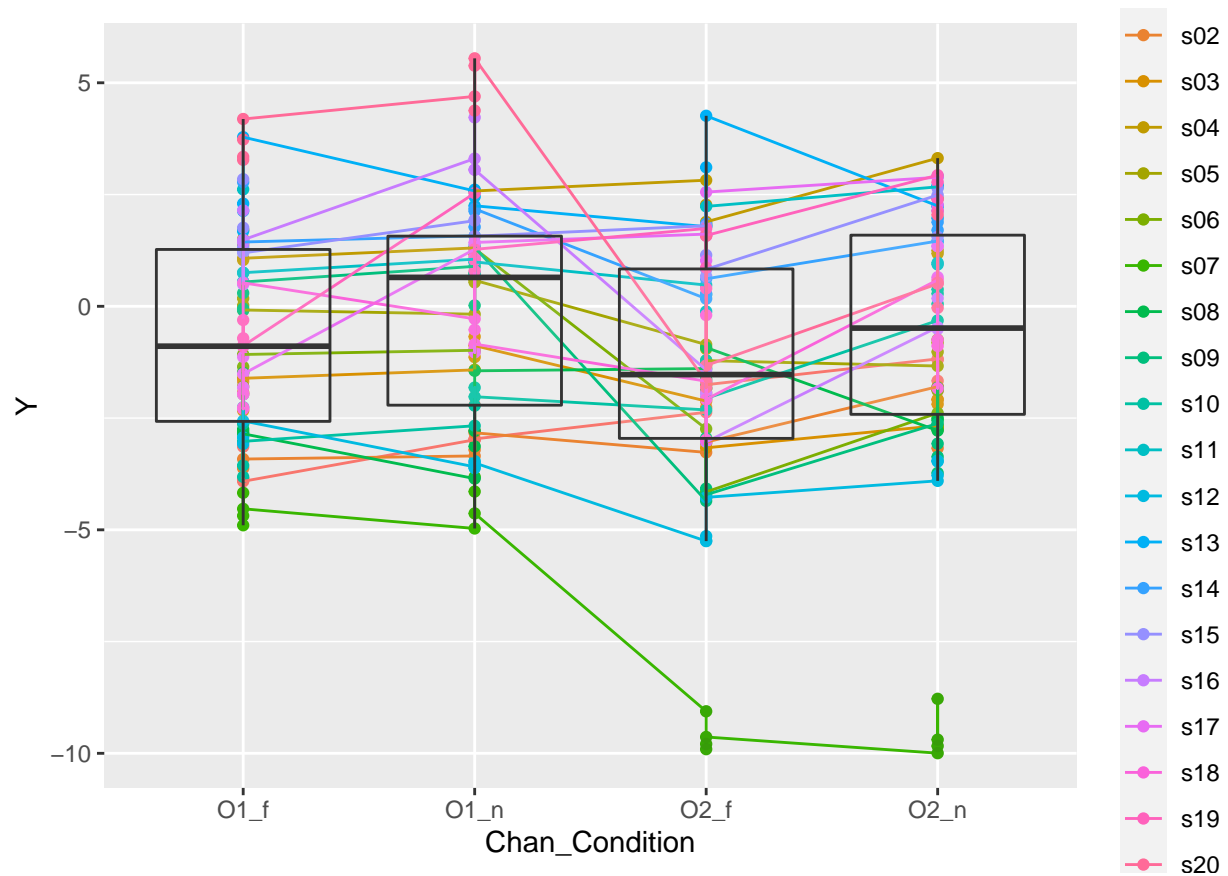
## Bin width defaults to 1/30 of the range of the data. Pick better value with 'binwidth'.



Is there a specificity of the Subject?

```
dati$Chan_Condition=paste(sep = "_",dati$Chan,dati$Condition)

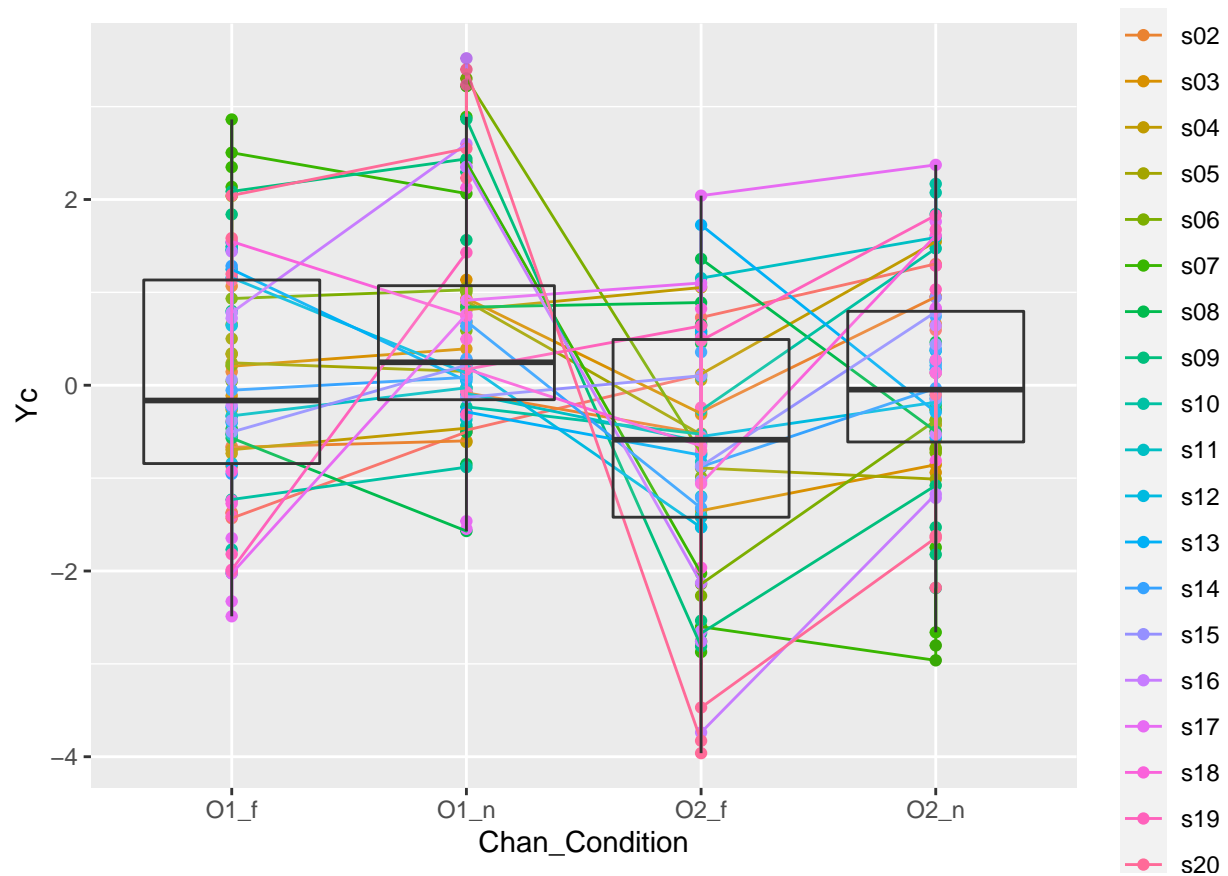
p <- ggplot(dati, aes(x=Chan_Condition, y=Y))
p+geom_point(aes(group = Subj, colour = Subj))+
  geom_line(aes(group = Subj, colour = Subj))+
  geom_boxplot(alpha=.1)
```



We subtract the Subject-specific effect (i.e. Subject's mean) to each observation.

```
mod=lm(Y~Subj,data=dati)
# summary(mod)
Y=residuals(mod)
dati$Yc=as.vector(Y)

library(ggplot2)
p <- ggplot(dati,aes(Chan_Condition,Yc))
p+geom_point(aes(group = Subj, colour = Subj))+
  geom_line(aes(group = Subj, colour = Subj))+
  geom_boxplot(alpha=.1)
```



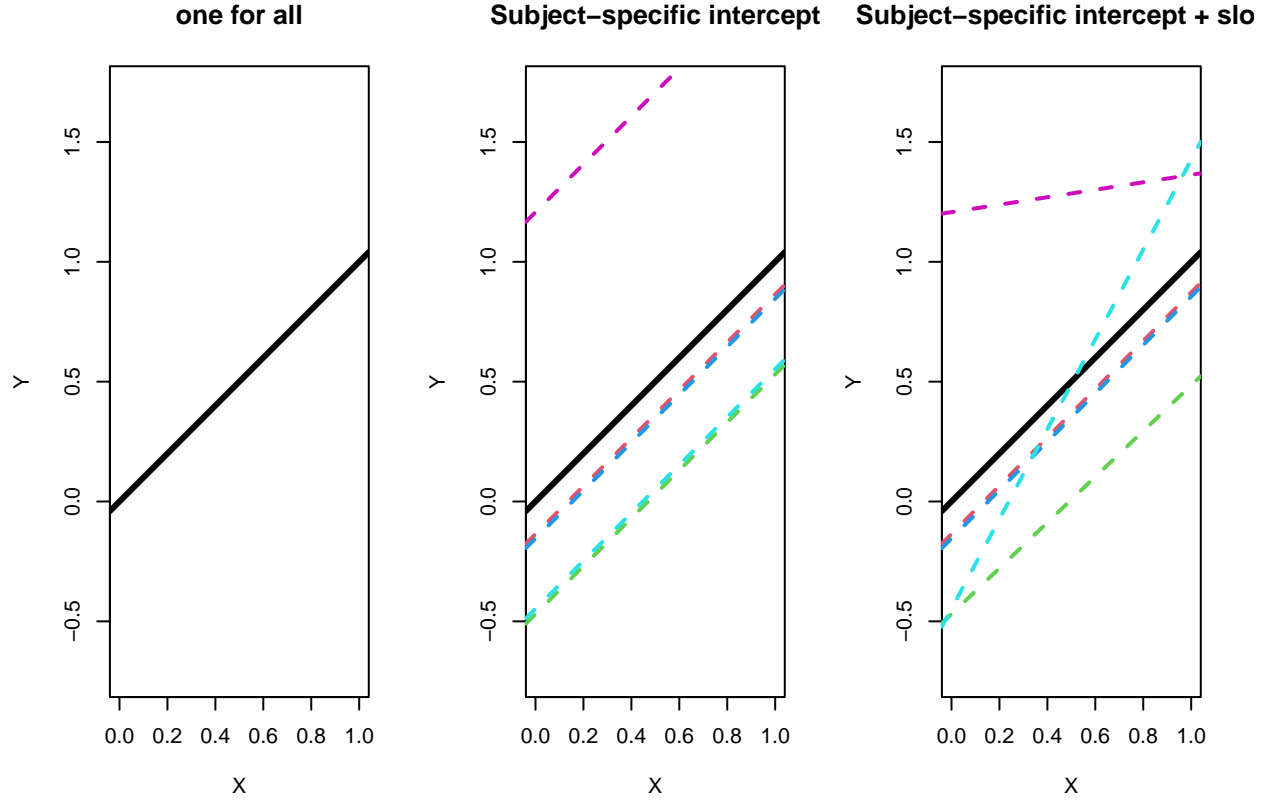
The dispersion of the data has been largely reduced. This effect is the one taken in account by the models for repeated measures.

## 2 Mixed models

### 2.1 IMPORTANT REMARK about contrasts in (mixed) linear models

```
# VERY IMPORTANT:
contrasts(dati$Condition) <- contr.sum(2) #2 is the number of levels
contrasts(dati$Chan) <- contr.sum(2) #2 is the number of levels
```

## 2.2 Intuition



Mixed models allow to model Subject-specific (average) effect by assuming that it is randomly drawn from the distribution of the population (which is normal).

I assume you are expert on mixed models, if not [https://en.wikipedia.org/wiki/Mixed\\_model](https://en.wikipedia.org/wiki/Mixed_model)  
and much more on: [http://webcom.upmf-grenoble.fr/LIP/Perso/DMuller/M2R/R\\_et\\_Mixed/documents/Bates-book.pdf](http://webcom.upmf-grenoble.fr/LIP/Perso/DMuller/M2R/R_et_Mixed/documents/Bates-book.pdf)  
and  
<https://cran.r-project.org/web/packages/lme4/vignettes/lmer.pdf>

Due to the small size of the dataset, in our example we only explore the scenario with random intercept and fixed slope (i.e. a simpler model, less parameters).

## 2.3 The model

Models with random effects can be defined as:

$$Y_{n \times 1} = X_{n \times p} B_{p \times 1} + Z_{n \times q} b_{q \times 1} + \varepsilon_{n \times 1}$$

where

$$\varepsilon \sim \mathcal{N}(0, \sigma^2 I_n)$$

In the models we will consider, the random effects are modeled as a multivariate normal random variable:

$$b \sim \mathcal{N}(0, \Sigma_{q \times q}),$$

In a *linear mixed model* the Conditional distribution ( $Y|\mathcal{B} = b$ ) is a *spherical* multivariate Gaussian.

In our case  $n = \#Subj \times \#Chan \times \#Condition = 10 \times 2 \times 2 = 40$ .  $X$  is the matrix of (dummified) predictors.  $Z$  can take many dimensions and values. Examples follow.

### Random effect for Subject (Random Intercept)

$Z$  is the matrix of dummy variables of the column `dati$Subj`.

```
library(lmerTest)

mod=lmer(Y~ Condition*Chan +(1|Subj),data=dati)

summary(mod)

## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula: Y ~ Condition * Chan + (1 | Subj)
## Data: dati
##
## REML criterion at convergence: 1165
##
## Scaled residuals:
##      Min       1Q   Median       3Q      Max
## -2.45740 -0.57314 -0.04347  0.69734  2.21259
##
## Random effects:
## Groups Name Variance Std.Dev.
## Subj (Intercept) 5.647 2.376
## Residual 1.709 1.307
## Number of obs: 320, groups: Subj, 20
##
## Fixed effects:
##              Estimate Std. Error      df t value Pr(>|t|)
## (Intercept) -0.68675 0.53639 19.00000 -1.280 0.216
## Condition1 -0.32723 0.07309 297.00000 -4.477 1.08e-05 ***
## Chan1 0.34713 0.07309 297.00000 4.749 3.18e-06 ***
## Condition1:Chan1 0.02160 0.07309 297.00000 0.295 0.768
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##              (Intr) Cndtn1 Chan1
## Condition1 0.000
## Chan1 0.000 0.000
## Cndtn1:Chn1 0.000 0.000 0.000

car::Anova(mod,type=3)

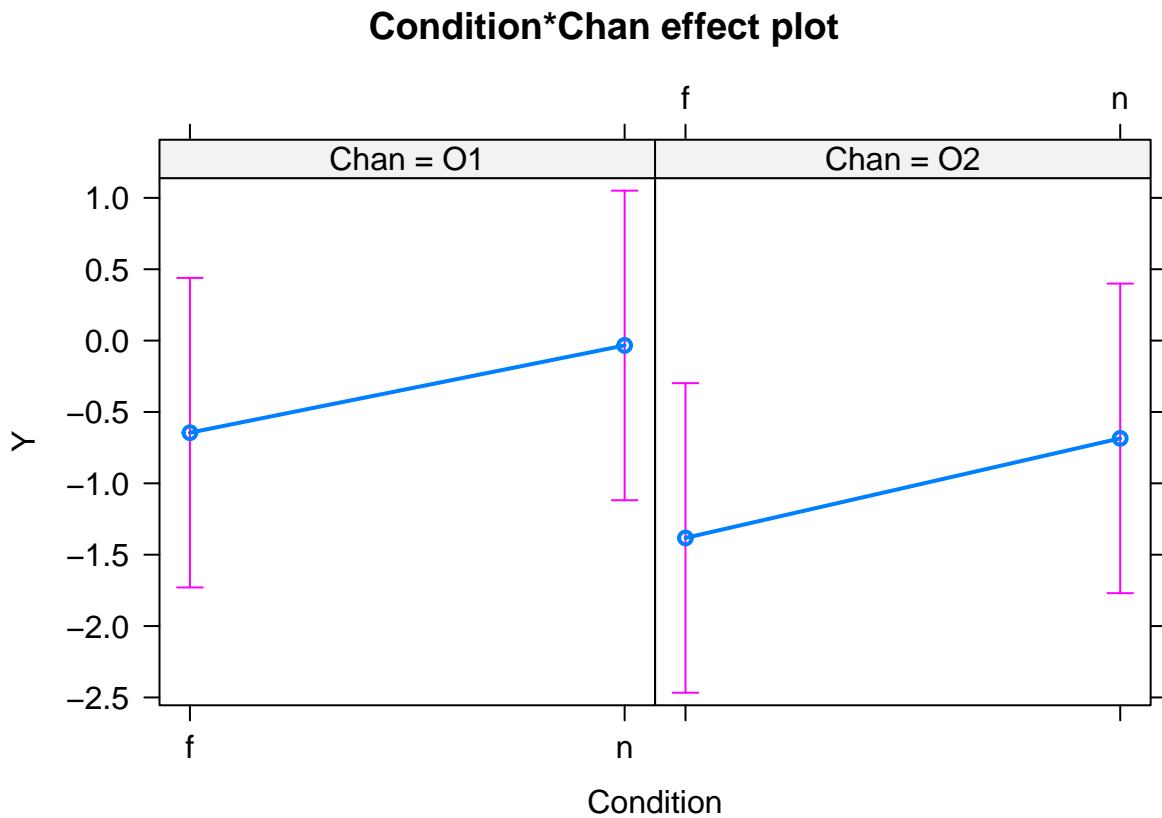
## Analysis of Deviance Table (Type III Wald chisquare tests)
##
## Response: Y
##              Chisq Df Pr(>Chisq)
## (Intercept) 1.6392 1 0.2004
```

```
## Condition      20.0447  1  7.565e-06 ***
## Chan           22.5567  1  2.040e-06 ***
## Condition:Chan  0.0873  1    0.7676
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

## 2.4 Plotting tools

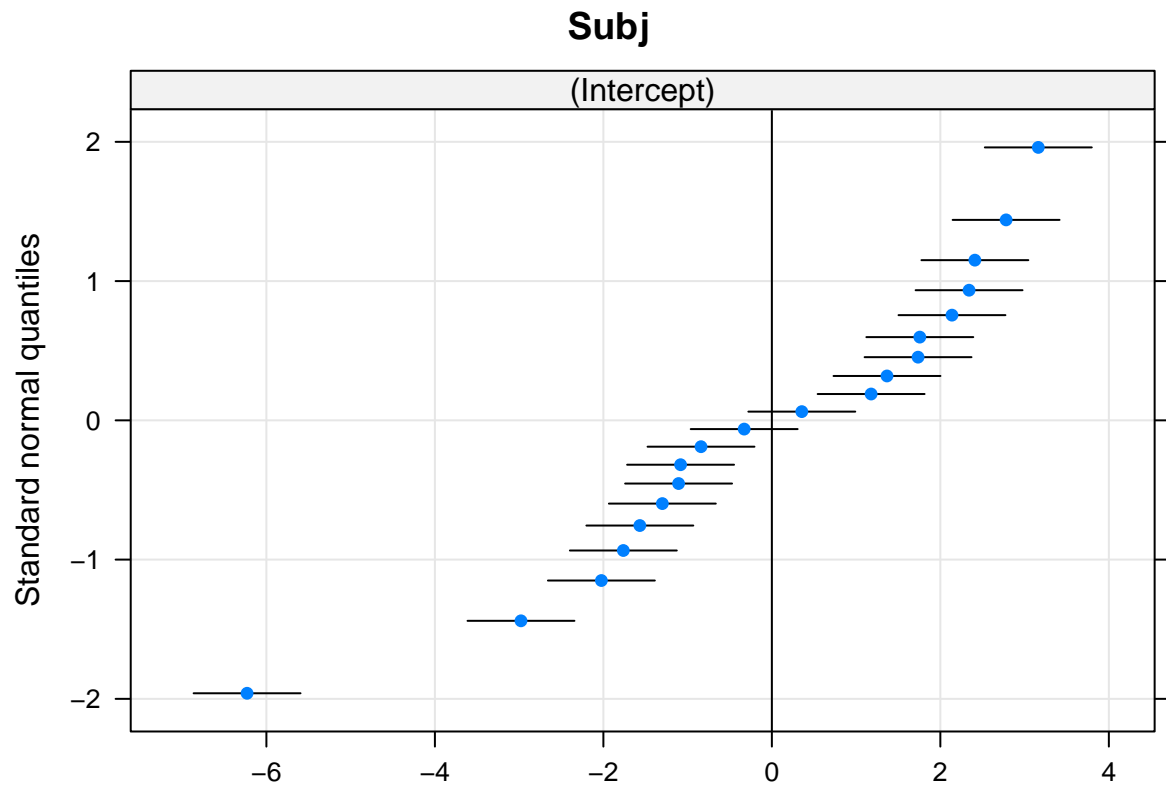
for the first model:

```
library(effects)
plot(allEffects(mod))
```



```
#plot random effects:
require(lattice)
qqmath(ranef(mod, condVar=TRUE))
```

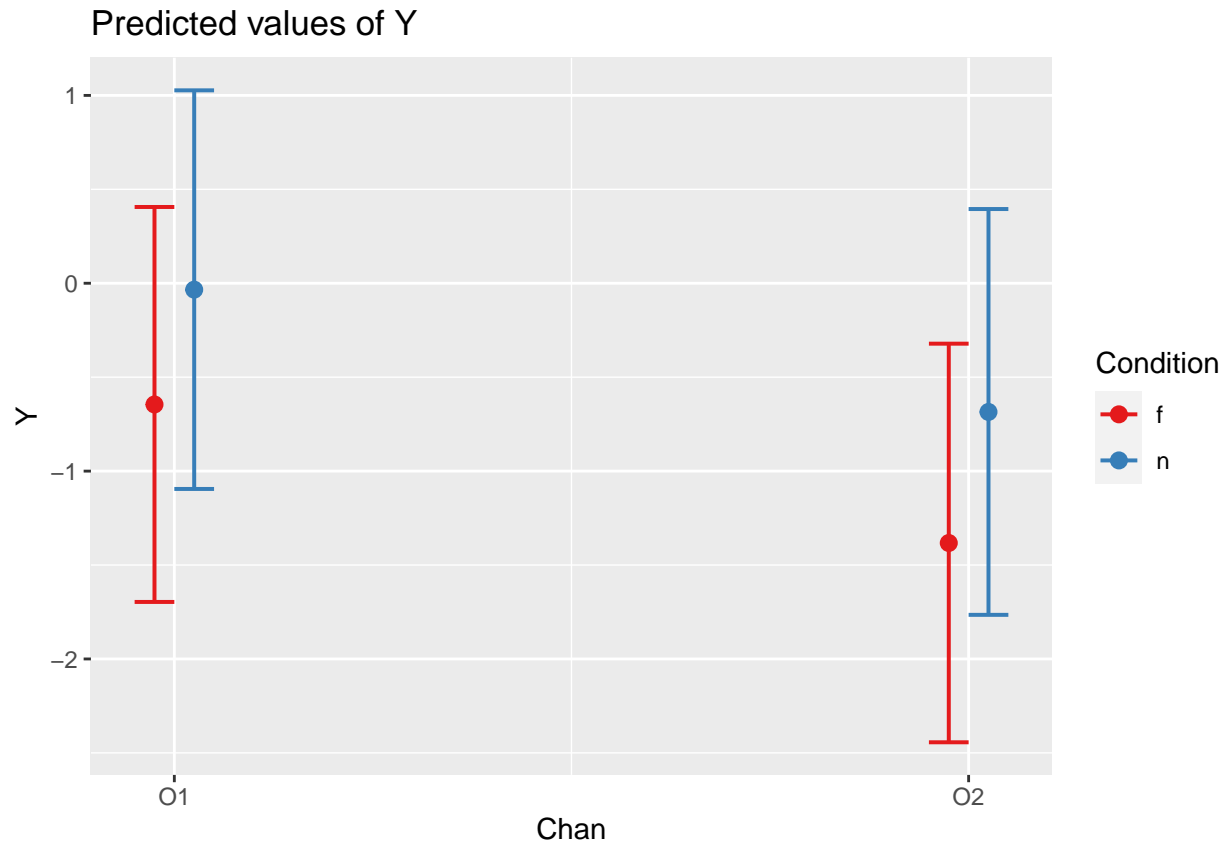
```
## $Subj
```



An alternative plotting tool:

```
library(sjPlot)
library(ggplot2)
plot_model(mod, type = "pred", terms = c("Chan", "Condition"))
```





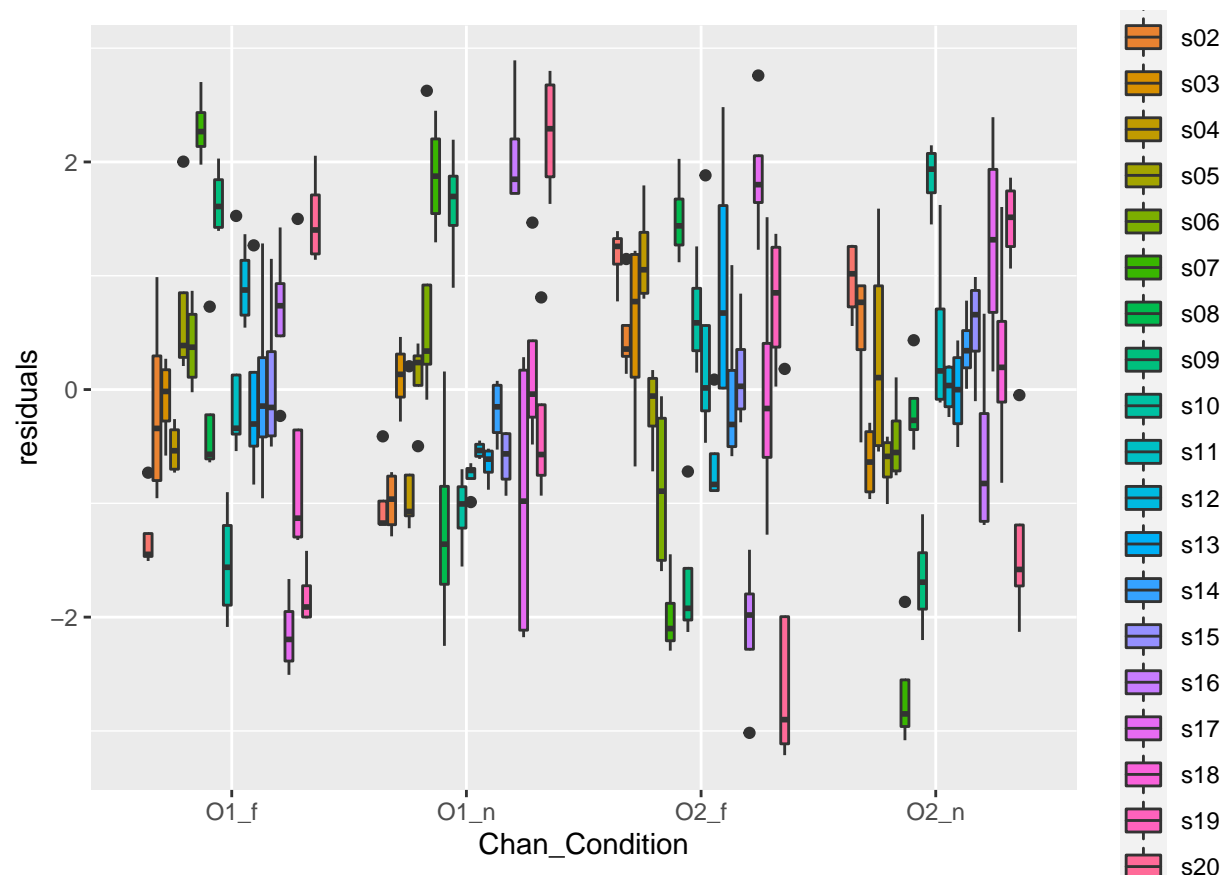
## 2.5 Validity of the assumptions

- Independence of the residuals?
- Normality of the residuals?
- Homoscedasticity of the residuals (i.e. same variance between Subject/Condition/Chan?)
- outliers?
- Leverage? (influential observations)

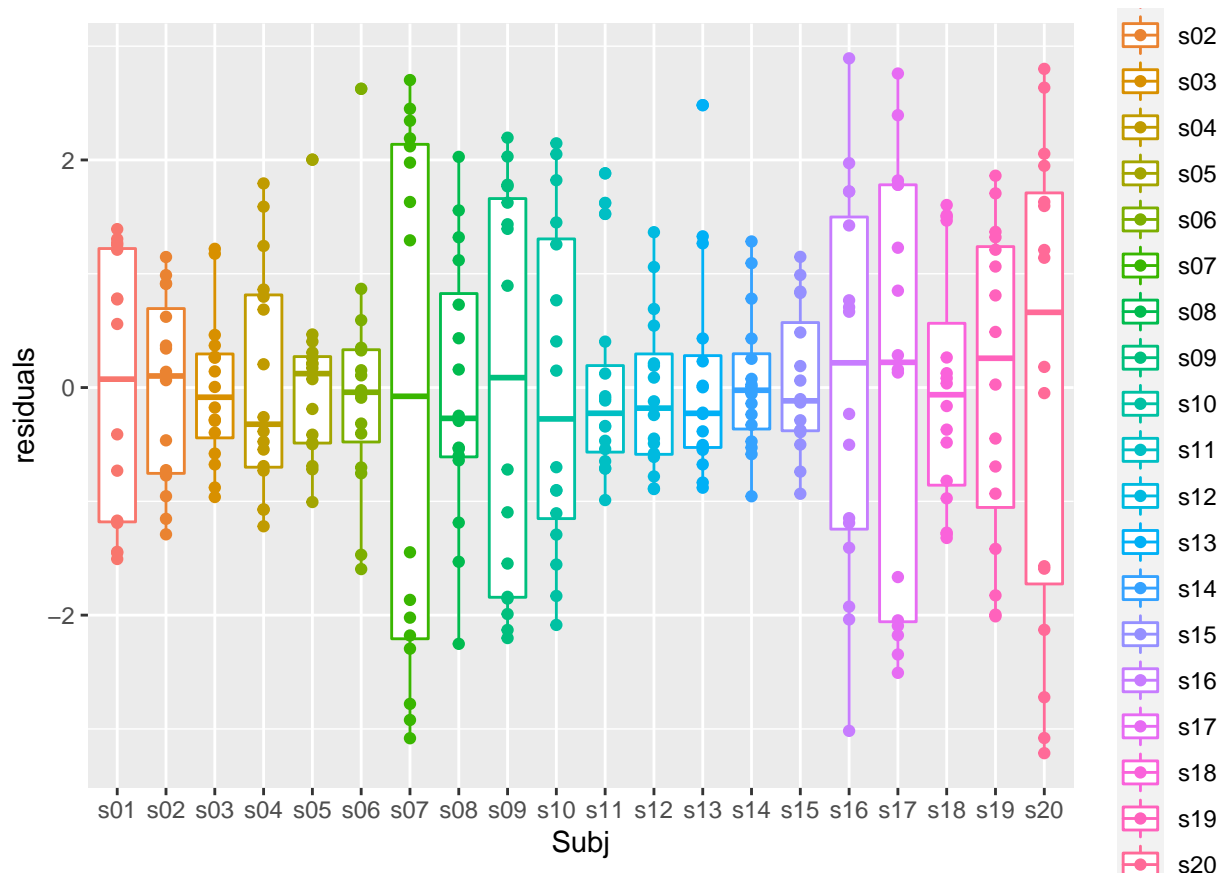
Please, do not test for normality, for homoscedasticity, sphericity etc.

Use Exploratory data Analysis, instead!

```
dati$residuals=residuals(mod)
p <- ggplot(dati, aes(x=Chan_Condition, y=residuals, fill=Subj)) + geom_boxplot()
p
```



```
p <- ggplot(dati, aes(x=Subj, y=residuals,col=Subj)) + geom_boxplot()+ geom_point(aes(group = interaction(Subj, Chan_Condition)))
```



### 3 (minimal) Bibliography

Jonathan Baron (2011) Notes on the use of R for psychology experiments and questionnaires [https://www.sas.upenn.edu/~baron/from\\_cattell/rpsych/rpsych.html](https://www.sas.upenn.edu/~baron/from_cattell/rpsych/rpsych.html)

and Course material of

ST 732, Applied Longitudinal Data Analysis, NC State University by Marie Davidian <https://www.stat.ncsu.edu/people/davidian/courses/st732/notes/chap5.pdf> from <https://www.stat.ncsu.edu/people/davidian/courses/st732/>

About Type I, II, III SS: <https://mcfromnz.wordpress.com/2011/03/02/anova-type-iiiiii-ss-explained/>

About Mixed models:

[http://webcom.upmf-grenoble.fr/LIP/Perso/DMuller/M2R/R\\_et\\_Mixed/documents/Bates-book.pdf](http://webcom.upmf-grenoble.fr/LIP/Perso/DMuller/M2R/R_et_Mixed/documents/Bates-book.pdf)

and <https://cran.r-project.org/web/packages/lme4/vignettes/lmer.pdf>