A Fiscal Decomposition of Unexpected Inflation: Cross-Country Estimates and Theory

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Introduction: The Fiscal Sources of Unexpected Inflation

- Connection between fiscal policy and inflation
- Key Equilibrium Condition: The Valuation Equation of Public Debt

$$\frac{\text{Market Value of Debt } (\textbf{Bond Prices})}{\textbf{Price Level}} = \text{Intrinsic Value } (\textbf{Discounting}, \textbf{Surpluses}).$$

- Unexpected inflation must accompany news about:
  - Bond prices
  - Real surpluses
  - Real discounting
- If unexpected inflation exists, it must nowcast/forecast some of these variables
- What are these forecasts? How do they change across countries? How can theory explain them?

Introduction: Exercises, Motivation, Results

- **This paper.**
  1. Estimate a Bayesian-VAR to measure the decomposition for 25 countries
     - Variance decomposition: *"What does +1% unexpected inflation forecast?"*
     - "Aggregate demand" shock: *"What does +1% unexpected inflation and +1% growth forecast?"*
     - Discounted surplus shock: *"What does +1% unexpected return on public debt forecast?"*
  2. GMM estimate of New-Keynesian model to reproduce BVAR decompositions
- **Motivation.** Valuation equation requires very weak assumptions (no bubbles!)
  - Does it mean inflation is "fiscal"?
    - Fixed country: +1% inflation $\Longrightarrow$ +1% deficit/debt?
    - Cross country: +1% inflation in A relative to B $\Longrightarrow$ +1% deficit/debt in A compared to B?
    - Fiscal role to monetary policy?
  - Guidance for monetary-fiscal theory (FTPL vs Spiral-Threat)

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Introduction: Preview of Key Results

- The variance of unexpected inflation is accounted for by discounted surpluses (all coutries)

$$\underbrace{\text{var}\left[\Delta E\pi\right]}_{>0} = \underbrace{\text{cov}\left[\Delta E\pi, \quad Q\right]}_{<0} + \underbrace{\text{cov}\left[\Delta E\pi, \quad \{-s\} + \{R\}\right]}_{>0}$$

  - Most relevant terms (Surplus-to-GDP, GDP growth and Real discounting) vary
  - Monetary policy reduces unexpected inflation variance through bond prices
- ”Aggregate demand” inflation: high discounting + lower future surplus-to-GDP
- Discount surplus shocks (return on public debt) driven by discounting
- Productivity shocks reproduce findings in NK model
  - Policy shocks do not
- Different policy rules explain differences in variance decomposition across countries

Introduction: Related Literature

- **Fiscal Theory of the Price Level.** Cochrane (2022a) and Cochrane (2022b).
  - Analysis of multiple countries + more general debt instruments
  - NK model estimated to reproduce decompositions + Productivity shocks

  Leeper (1991), Sims (1994), Woodford (1995), Cochrane (1998), Cochrane (2005), Sims (2011), Leeper and Leith (2016), Bassetto and Cui (2018), Cochrane (2022c), Brunnermeier et al. (2022).

- **Monetary-Fiscal Interaction.**
  Cagan (1956), Sargent and Wallace (1981), Hall and Sargent (1997), Hall and Sargent (2011), Jiang et al. (2019), Corsetti et al. (2019), Sunder-Plassmann (2020), Du et al. (2020), Akhmadieva (2022)

- **Empirical Finance** (Decomposition of Returns)
  Campbell and Shiller (1988), Cochrane (1992), Campbell and Ammer (1993), Chen and Zhao (2009).

The Fiscal Decomposition of Unexpected Inflation

Fiscal Decomposition: The Valuation Equation

- Environment with discrete time + single good (price P_t) + households and government
- One-period nominal public bonds (price Q_t)
- In each period, the government:
  - redeems bonds B_{t-1} for currency
  - soaks up currency through primary surpluses $P_t s_t$ and bond sales $Q_t B_t$
- Market clearing + No Currency Holdings $M = 0$:

$$\boxed{B_{t-1} = P_t s_t + Q_t B_t}$$

Fiscal Decomposition: The Valuation Equation

- *Ex-post* real discounting $\beta_t = Q_t(P_{t+1}/P_t)$ $\beta_{t,t+k} = \prod_{\tau=t}^{t+k} \beta_\tau$
- Iterate law of motion forward:

$$\frac{B_{t-1}}{P_t} = \sum_{k=0}^{p} \beta_{t,t+k-1} s_{t+k} + \beta_{t,t+p} \frac{B_{t+p}}{P_{t+p+1}}$$

- **Key Assumption**: $\lim_{\tau \to \infty} \beta_{t,\tau} \frac{B_\tau}{P_{\tau+1}} = 0$
- Valuation equation of public debt:

$$\boxed{\frac{B_{t-1}}{P_t} = \sum_{k=0}^{\infty} E_t \left[\beta_{t,t+k-1} s_{t+k} \right]}$$

"A prince who should enact that a certain proportion of his taxes should be paid in a paper money of a certain kind might thereby give a certain value to this paper money" - Adam Smith

Fiscal Decomposition: In the Simplest Environment

- End-of-period real debt v_t
- Linearized flow condition + valuation equation

$$\underbrace{\frac{1}{\beta}v_{t-1} + \frac{V}{\beta}\left(i_{t-1} - \pi_t\right)}_{B_{t-1}/P_t} = s_t + v_t \qquad = \sum_{k=0}^{\infty}\beta^k E_t s_{t+k} - \frac{V}{\beta}\sum_{k=1}\beta^k E_t r_{t+k}$$

- Take innovations $\Delta E_t = E_t - E_{t-1}$:

$$\boxed{\Delta E_t \pi_t = -\frac{\beta}{V}\sum_{k=0}^{\infty}\beta^k \Delta E_t s_{t+k} + \sum_{k=0}^{\infty}\beta^k \Delta E_t r_{t+k}}$$

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Fiscal Decomposition: Generalizing

- GDP Growth
- Nominal, inflation-linked and dollar-denominated bonds
- Long-term bonds

$$\frac{\textbf{Bond Price in Home Currency} \times \text{Bonds}}{\textbf{Price Level}} = \sum_t \frac{\textbf{Surplus-to-GDP} \times \Delta\textbf{GDP}}{\textbf{Discounting}}$$

$$\frac{v_{t-1}}{\beta} + \frac{v}{\beta} \sum_j \delta_j \left(rx_{j,t} + i_{j,t-1} + \Delta h_{j,t} - \pi_{j,t} \right) = \sum_{k=0}^{\infty} \beta^k E_t s_{t+k} + \frac{v}{\beta} \sum_{k=0}^{\infty} \beta^k E_t g_{t+k} - \frac{v}{\beta} \sum_{k=1}^{\infty} \sum_j \delta_j \beta^k E_t r_{j,t+k}$$

Details Currency Table

Fiscal Decomposition of Unexpected Inflation

- *Ex-post* real return $r_{j,t} = rx_{j,t} + i_{j,t-1} + \Delta h_{j,t} - \pi_{j,t}$

$$\Delta E_t \pi_t = \underbrace{\left[\Delta E_t rx_t + \sum_{j \neq N} \frac{\delta_j}{\delta} \Delta E_t r_{j,t}\right]}_{\text{Innovation to Bond Prices}} - \frac{\beta}{\delta V} \underbrace{\left[\sum_{k=0}^{\infty} \beta^k \Delta E_t s_{t+k} + \frac{V}{\beta} \sum_{k=0}^{\infty} \beta^k \Delta E_t g_{t+k} - \frac{V}{\beta} \sum_{k=1}^{\infty} \sum_j \delta_j \beta^k \Delta E_t r_{j,t+k}\right]}_{\text{Innovation to Discounted Surpluses}}$$

$$\equiv d(rx) + d(r_0) - d(s) - d(g) + d(r)$$

- Variance decomposition.

$$\text{var}\left[\Delta E_t \pi_t\right] = \text{cov}_\pi \left[d(rx)\right] + \text{cov}_\pi \left[d(r_0)\right] - \text{cov}_\pi \left[d(s)\right] - \text{cov}_\pi \left[d(g)\right] + \text{cov}_\pi \left[d(r)\right]$$

BVARs: Empirical Estimates

Bayesian-VAR: Data and Model

- Annual data on observables x_t^{OBS}

$$x_t^{OBS} = \begin{bmatrix} i_t & \text{(Nominal Interest)} \\ \pi_t & \text{(CPI Inflation)} \\ v_t^b & \text{(Par-Value Debt-to-GDP)} \\ g_t & \text{(GDP growth)} \\ \Delta h_t & \text{(}\Delta\text{ Real Exchange to US Dollar)} \end{bmatrix}$$

- 25 countries (samples starting at 1945, 1960, 1975, 1998)
- General VAR

$$X_t = AX_{t-1} + Ke_t$$

- Decompose $X_t' = [x_t^{OBS'} \; x_t^{NOT'}]$

$$\begin{aligned} x_t^{OBS} &= a\, x_{t-1}^{OBS} + e_t \\ x_t^{NOT} &= b\, x_{t-1}^{OBS} + c\, x_{t-1}^{NOT} + k\, e_t \end{aligned}$$

Bayesian-VAR: Empirical Challenges and Solutions

1. Explosive debt dynamics in 1960-2020 implies unstable OLS VARs

$$x_t^{OBS} = a\, x_{t-1}^{OBS} + e_t$$

   - **United States**: Estimate model by OLS (stable!)
   - **Others**: Estimate model with a Bayesian Linear Regression Bayesian Prior Hyperparameters

$$a^{BAY} = (X'X + \lambda^{-1})^{-1}(X'X\, a^{OLS} + \lambda^{-1}\, a^{PRIOR})$$

   λ maximizes the marginal distribution $p(\text{data})$ (Giannone et al. (2015)) and ensures stability

2. Public finance data do not respect law of motion of public debt
   - Define surplus from the law of motion: $s_t = \frac{v_{t-1}}{\beta} - v_t + \frac{v}{\beta}\left[-g_t + \sum_j \delta_j \left(rx_{j,t} + i_{j,t-1} + \Delta h_{j,t} - \pi_{j,t}\right)\right]$
3. No data on the market value of debt, only its par value (v_t^b) Public Finances Model
   - Model for market *vs* par value (Cox (1985)): $v_t = v_t^b + \frac{v}{\beta}\sum_j \delta_j \left(q_{j,t} - q_{j,t-1}^b\right)$
4. No data on bond prices Geometric Term Structure
   - Geometric maturity structure + constant risk premia: $q_{j,t} = (\omega_j \beta) E_t q_{j,t+1} - i_{j,t}$

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Bayesian-VAR: Variance Decomposition

- Variance decomposition $\iff$ Innovations decomposition applied to shock $E\,[\,e \mid \Delta E_t \pi_t = 1\,]$
- *"Given 1% unexpected inflation, how do we change expectations over surplus, discounting, bond prices?"*

Country	$\Delta E_t \pi_t =$	ΔE_t(Bond Prices)		$-\Delta E_t$(Disc Surpluses)		
		$d(r_0)$	$d(rx)$	$-d(s)$	$-d(g)$	$d(r)$
United States	1	0	*-0.8	0.6	0.2	1.0
1960 Sample						
Canada	1	*-0.1	*-1.6	0.6	*1.2	0.9
Denmark	1	*-0.3	-0.3	0.4	0	1.2
Japan	1	0	*-0.5	*1.6	-0.4	0.3
Norway	1	0	*-0.4	0.6	0.5	0.3
Sweden	1	-0.2	*-0.9	-0.3	*1.0	*1.4
United Kingdom	1	*0.5	*-0.7	*2.9	*1.0	*-2.7
1973 Sample						
Australia	1	*0.1	*-0.8	*2.1	0.7	-1.1
New Zealand	1	-0.1	*-0.9	0.4	*0.9	0.7
South Korea	1	0	*-0.5	*1.9	0.2	-0.6
Switzerland	1	0	*-0.7	0.9	*0.9	-0.1

Advanced Markets

Country	$\Delta E_t \pi_t =$	ΔE_t(Bond Prices)		$-\Delta E_t$(Disc Surpluses)		
		$d(r_0)$	$d(rx)$	$-d(s)$	$-d(g)$	$d(r)$
1998 Sample						
Brazil	1	-0.3	*-0.2	-1.5	1.1	1.9
Chile	1	-3.8	-1.3	9.0	-5.7	2.9
Colombia	1	1.5	*-1.0	1.4	-1.1	0.2
Czech Republic	1	*-0.2	*-0.4	-2.3	2.4	1.4
Hungary	1	*-0.6	*-0.9	-1.0	1.6	1.9
India	1	*0.2	*-0.5	1.5	0.1	-0.3
Indonesia	1	*-2.6	*-1.1	1.7	*2.6	0.4
Israel	1	-0.1	*-0.8	-0.6	*1.5	0.9
Mexico	1	0	*-0.7	1.4	0	0.3
Poland	1	*-0.5	*-1.2	0.9	-0.4	*2.1
Romania	1	-0.4	*-1.0	2.2	0.4	-0.3
South Africa	1	0.4	*-0.5	1.6	0.3	-0.7
Turkey	1	0.4	*-0.4	-1.2	-0.2	*2.3
Ukraine	1	0	*-0.8	0.7	0.4	*0.7

Emerging Markets

Decomposition 2 Proposition

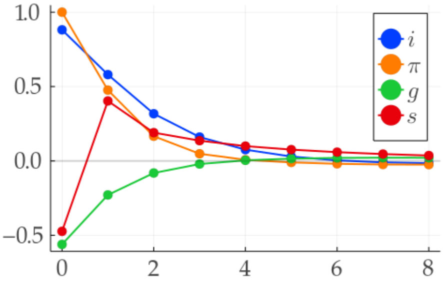


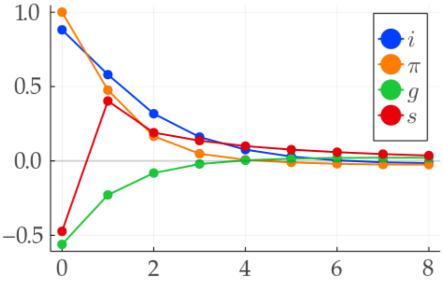
Figure: IRF - Brazil

$d(rx) < 0 \qquad -d(g) > 0$

$d(r) > 0 \qquad -d(s) < 0$

- $\Delta E\pi$ accounted for by discounted surpluses
- Surplus-to-GDP, GDP growth and real discounting...
  - ...account for unexpected inflation alone in 0/25
  - ...have a positive contribution in 18+/25
- Is inflation "fiscal"? Yes, but not only.
- Is inflation "fiscal" **cross-country**? Not at all.
- Bond price dynamics reduce $\Delta E\pi$ in 25/25

Bayesian-VAR: Variance Decomposition - Takeaways



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Bayesian-VAR: ”Aggregate Demand” Inflation

- ”Aggregate demand” recessions: low inflation, low growth, fiscal deficits. How come?
- Shock: $E\,[\,e \mid \Delta E_t \pi_t = 1, \Delta E_t g_t = 1\,]$
- *”Given +1% unexpected inflation and +1% growth, how do we change expectations?”*

Country	$\Delta E_t \pi_t =$	ΔE_t(Bond Prices)		$-\Delta E_t$(Disc Surpluses)		
		$d(r_0)$	$d(rx)$	-$d(s)$	-$d(g)$	$d(r)$
United States	1	0	*-1.4	1.0	*-1.3	*2.8
1960 Sample						
Canada	1	*-0.2	*-2.9	0.8	0.3	*3.0
Denmark	1	*-0.4	*-1.1	3.0	*-2.9	2.3
Japan	1	0	*-1.2	*2.4	*-2.1	*1.8
Norway	1	0	*-0.9	1.8	*-1.7	1.8
Sweden	1	*-0.5	*-1.7	0.5	-0.4	*3.1
United Kingdom	1	0.1	*-3.2	*3.7	-0.4	0.8
1973 Sample						
Australia	1	0	*-2.1	2.6	-0.5	1.0
New Zealand	1	*-0.3	*-1.3	1.1	-0.3	1.9
South Korea	1	*-0.1	*-1.0	*4.4	*-1.9	-0.4
Switzerland	1	0	*-1.3	*1.3	0.6	0.4

Advanced Markets

Country	$\Delta E_t \pi_t =$	ΔE_t(Bond Prices)		$-\Delta E_t$(Disc Surpluses)		
		$d(r_0)$	$d(rx)$	-$d(s)$	-$d(g)$	$d(r)$
1998 Sample						
Brazil	1	*-0.6	*-0.2	-2.9	0.2	4.3
Chile	1	*-18.4	*-3.7	36.4	-34.9	21.7
Colombia	1	-1.3	*-1.2	12.3	-8.6	-0.3
Czech Republic	1	*-0.5	*-0.8	-1.0	0.9	2.4
Hungary	1	*-1.3	*-1.1	-12.2	6.5	9.2
India	1	0.1	-0.4	2.0	-0.8	0
Indonesia	1	*-9.9	0.1	*12.6	-0.2	-1.6
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Romania	1	*-2.1	*-0.7	*8.7	-1.7	-3.2
South Africa	1	0.3	-0.6	*32.2	*-11.6	*-19.3
Turkey	1	-0.7	*-0.4	-1.2	-0.6	*3.9
Ukraine	1	0	0.5	*4.1	*-2.1	-1.4

Emerging Markets

Bayesian-VAR: "Aggregate Demand" Inflation - Takeaways

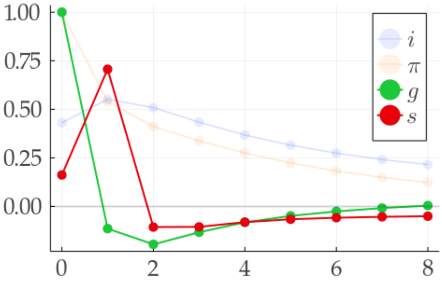


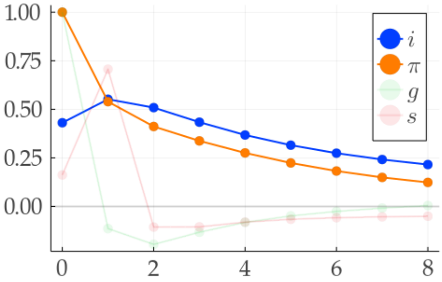
Figure: IRF - United States

$d(rx) < 0 \qquad -d(g) < 0$

$d(r) > 0 \qquad -d(s) > 0$

- Higher inflation follows from...
  - higher discounting (monetary policy) in 19/25
  - lower surplus-GDP **ratios**, current or in the future in 21/25
- (Level) Surpluses increase in 23/25
- COVID inflation: decline in $\{s\}$?

Bayesian-VAR: "Aggregate Demand" Inflation - Takeaways



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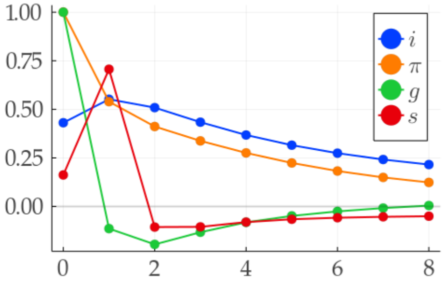

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Bayesian-VAR: Discounted Surplus Shock

- Unexpected inflation $\implies$ lower discounted surpluses (either s, g or r)
- Is the converse true? $E\,[\,e \mid \Delta E_t \text{Disc Surpluses} = -1\,]$
- $\Delta E_t\{\text{Disc Surpluses}\} = \Delta E_t\{\text{Bond Prices}\} - \Delta E\pi = \Delta E_t\{\text{Return on Public Debt}\}$

Country	$\Delta E_t \pi_t =$	ΔE_t(Bond Prices)		$-\Delta E_t$(Disc Surpluses)		
		$d(r_0)$	$d(rx)$	-$d(s)$	-$d(g)$	$d(r)$
United States	*0.4	0	*-0.6	0.2	0	*0.8
1960 Sample						
Canada	*0.2	*-0.1	*-0.8	-0.1	0	*1.2
Denmark	*0.2	*-0.2	*-0.6	0.2	*-0.6	*1.4
Japan	*0.5	0	*-0.5	0.7	-0.2	*0.5
Norway	*0.4	0	*-0.6	-0.3	-0.1	*1.4
Sweden	*0.2	*-0.3	*-0.5	-0.1	0.1	*1.0
United Kingdom	*0.1	-0.1	*-0.8	0.2	-0.1	0.9
1973 Sample						
Australia	*0.2	0	*-0.8	-0.3	0	*1.3
New Zealand	*0.3	*-0.1	*-0.5	-0.3	0.4	*0.9
South Korea	*0.5	0	*-0.5	1.5	-0.2	-0.3
Switzerland	*0.3	0	*-0.7	0.3	0.2	*0.5

Advanced Markets

Country	$\Delta E_t \pi_t =$	ΔE_t(Bond Prices)		$-\Delta E_t$(Disc Surpluses)		
		$d(r_0)$	$d(rx)$	-$d(s)$	-$d(g)$	$d(r)$
1998 Sample						
Brazil	*0.4	*-0.4	*-0.1	*-2.4	0.3	*3.1
Chile	0	*-0.9	*-0.1	0.6	-0.4	0.9
Colombia	0	*-0.9	*-0.1	*1.7	-0.7	0
Czech Republic	*0.4	*-0.2	*-0.4	-1.0	0.8	1.1
Hungary	*0.2	*-0.4	*-0.3	-4.1	2.6	*2.6
India	*0.5	0	*-0.5	0.6	0.1	0.2
Indonesia	0	*-0.9	-0.1	0.5	0.2	0.3
Israel	*0.1	*-0.6	*-0.3	-0.9	0.1	*1.8
Mexico	*0.1	*-0.7	*-0.2	*1.4	-0.4	0.1
Poland	*0.2	*-0.4	*-0.3	-0.2	0.1	*1.0
Romania	*0.1	*-0.9	0	*1.6	-0.2	-0.4
South Africa	*0.2	*-0.5	*-0.3	-0.2	0.3	0.9
Turkey	*0.1	*-0.8	*-0.1	-0.1	0.1	*1.0
Ukraine	*0.4	0	*-0.6	0	*0.3	*0.6

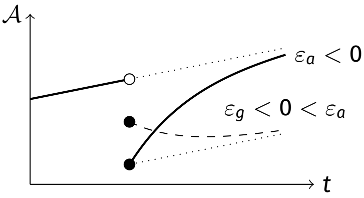
Emerging Markets

Theory: The New Keynesian Model

The New-Keynesian Model

- BVAR decompositions not structural
- Closed-economy New-Keynesian model
- **FTPL.** Decomposition determines $\Delta E_t \pi_t$
- **Trend Shocks.** Production function $\mathcal{A}_t N = \mathcal{T}_t \, A_t \, N$

$$\begin{aligned} &\text{(Trend component)} \quad \log \mathcal{T}_t = \log \mathcal{T}_{t-1} + u_{g,t} \\ &\text{(AR(1) component)} \quad a_t = \rho_a a_{t-1} + \varepsilon_{a,t} \end{aligned}$$



Parameters | Why Trend? Growth

$$\begin{aligned} y_t &= E_t y_{t+1} - \gamma \left(i_t - E_t \pi_{t+1} \right) + E_t u_{g,t} \\ \pi_t &= \beta \, E_t \pi_{t+1} + \kappa \, y_t - \kappa_a \, a_t \\ g_t &= \Delta y_t + u_{g,t} \\ i_t &= \phi_\pi \, \pi_t + \phi_g \, g_t + u_{i,t} \\ s_t &= \tau_\pi \, \pi_t + \tau_g \, g_t + u_{s,t} \\ \beta (v_t + s_t) &= v_{t-1} + v \sum_j \delta_j \left[rx_{j,t} + i_{j,t-1} + \Delta h_{j,t} - \pi_{j,t} \right] \end{aligned}$$

$$\begin{aligned} q_{j,t} &= (\omega_j \beta) \, E_t q_{j,t+1} - i_{j,t} \\ rx_{j,t} &= (\omega_j \beta) \, q_{j,t} - q_{j,t-1} - i_{j,t-1} \end{aligned}$$

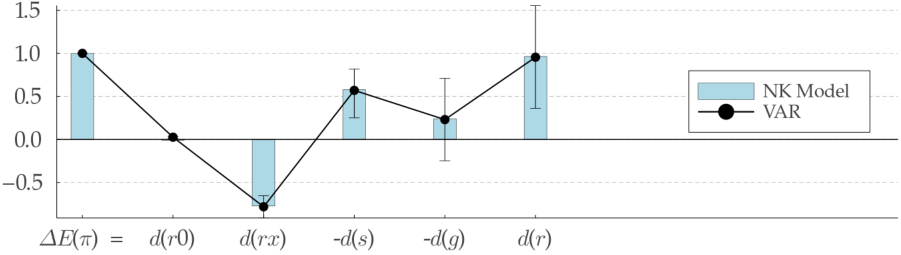
- Four shocks: ε_a, ε_g, ε_i, ε_s
- Method of moments:

$$\text{Min}_\Psi \quad \alpha_1 \left\| \mathcal{D}_{VAR} - \mathcal{D}_{NK}(\Psi) \right\| + \alpha_2 \left\| \mathcal{M} - \mathcal{M}_{NK}(\Psi) \right\|$$

The New-Keynesian Model: Reproducing the Variance Decomposition

Simple version of the model. Target: variance decomposition

- **Result.** AR(1) productivity shocks $\varepsilon_{a,t}$ alone reproduce the **variance decomposition** with positive contributions from surplus-to-output, growth and real interest terms
- **Result.** Monetary, fiscal and trend shocks do not, even if combined.



Target: United States. AR(1) productivity shocks. All others.

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Simple version of the model. Target: variance decomposition

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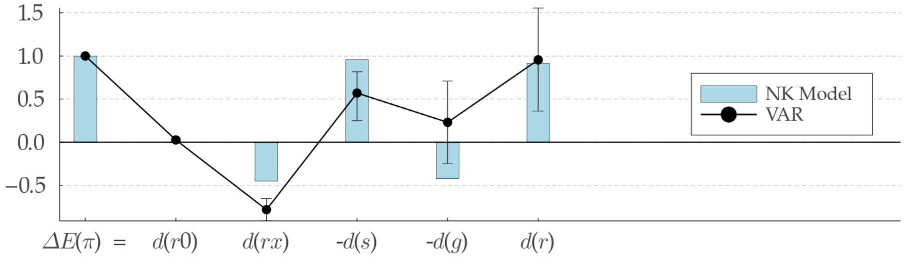


Target: United States. AR(1) productivity shocks. All others.

The New-Keynesian Model: Reproducing the Variance Decomposition

$$\boxed{1 = \Delta E_t \pi_t = d(rx) - d(s) - d(g) + d(r)}$$

- Story: negative productivity shock $\varepsilon_a < 0$
- Persistent shock: $\rho_a = 0.96$, low growth

$$-d(g) > 0$$

- Procyclical surpluses: $\tau_g = 1.5$

$$-d(s) > 0$$

- Strong Taylor rule: $\phi_\pi = 0.6$

$$d(rx) < 0$$
$$d(r) > 0$$

Marginal Costs | vs B-VAR IRF

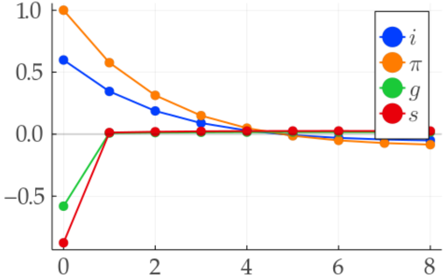
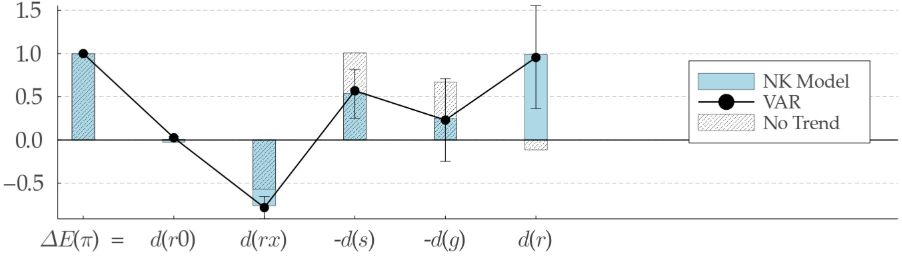


Figure: IRF to $\Delta E_t \pi_t = 1$ ($\varepsilon_{a,t} = -0.85$)

The New-Keynesian Model: Complete Model

Targets: three decompositions + second moments

- **Result.** In the absence of trend shocks, NK model fails to replicate the variance ($\Delta E\pi = 1$) and "aggregate demand" ($\Delta Eg = \Delta E\pi = 1$) decompositions. Policy shocks do not help.

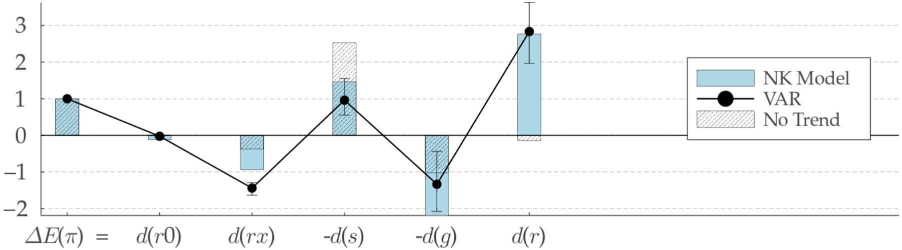


Target: United States - Variance "Agg Demand" Disc Surplus Structural Shocks

Moments Parameters

Targets: three decompositions + second moments

- **Result.** In the absence of trend shocks, NK model fails to replicate the variance ($\Delta E\pi = 1$) and "aggregate demand" ($\Delta Eg = \Delta E\pi = 1$) decompositions. Policy shocks do not help.



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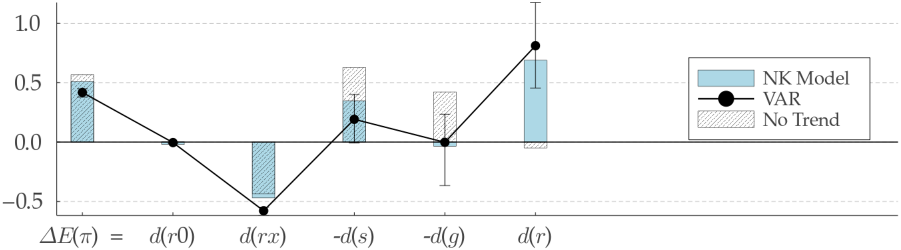

Target: United States - Variance "Agg Demand" Disc Surplus Structural Shocks

Moments Parameters

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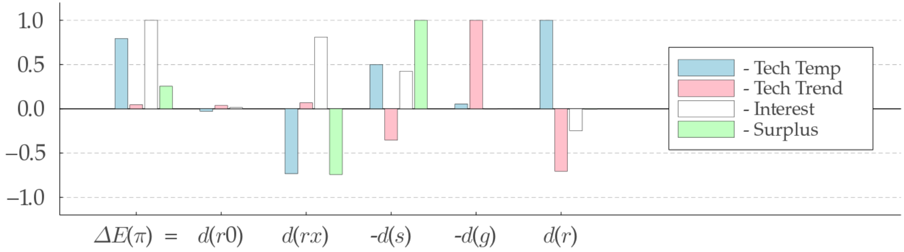
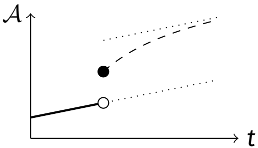


Target: United States - Variance ”Agg Demand” Disc Surplus Structural Shocks

Moments Parameters

The New-Keynesian Model: Reproducing the "Aggregate Demand" Shock

$$\boxed{1 = \Delta E_t \pi_t = d(rx) - d(s) - d(g) + d(r)}$$



- High Marginal Costs + Positive Growth?
- Protracted productivity growth

$$\varepsilon_g = 1.49 \qquad \varepsilon_a = -0.76$$

- Marginal costs high relative to trend

$$\pi_t = \beta E_t \pi_{t+1} + y_t - \kappa_a a_t \qquad a_t < 0$$
$$g_t = \Delta y_t + u_{g,t} \qquad u_{g,t} > 0$$

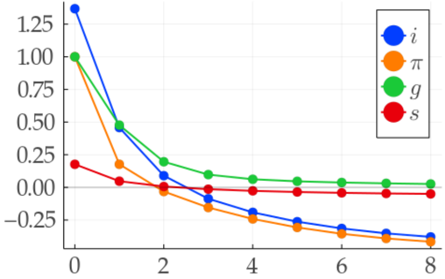


Figure: IRF to $\Delta E_t g_t = 1$, $\Delta E_t \pi_t = 1$

vs B-VAR IRF

$$\boxed{1 = \Delta E_t \pi_t = d(rx) - d(s) - d(g) + d(r)}$$

- Why less inflation? (Small) Monetary Shock
- Why more discounting? Milder recession.

Shock	Variance	Disc Surp
ε_a	-0.72	-0.40
ε_g	-0.13	0.05
ε_i	-0.12	-0.01

Structural Shocks

vs B-VAR IRF

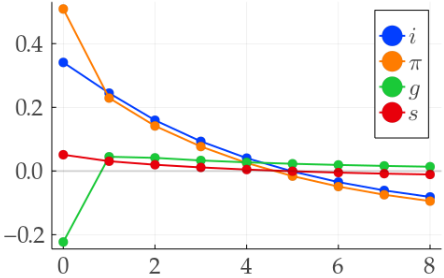


Figure: Disc Surp Variance

The New-Keynesian Model: Reproducing the Discounted Surplus Shock

$$\boxed{1 = \Delta E_t \pi_t = d(rx) - d(s) - d(g) + d(r)}$$

- Why less inflation? (Small) Monetary Shock
- Why more discounting? Milder recession.

Shock	Variance	Disc Surp
ε_a	-0.72	-0.40
ε_g	-0.13	0.05
ε_i	-0.12	-0.01

Structural Shocks

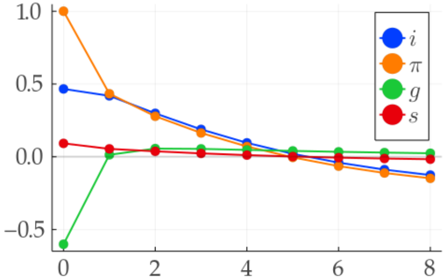
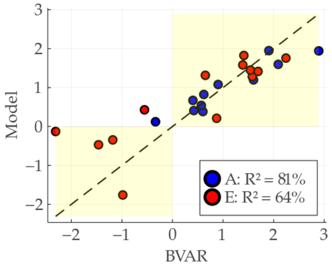


Figure: Disc Surp Variance

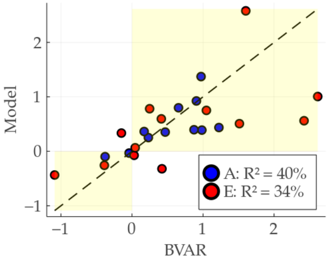
vs B-VAR IRF

The New-Keynesian Model: Variance Decompostion (Cross-Country)

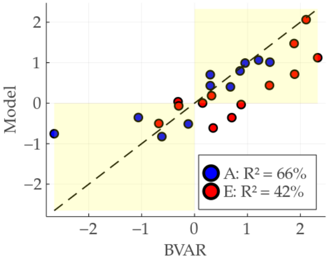
- Can cross-country differences in policy rules explain differences in variance decomposition?
- **Estimation.** Solve optimization problem to all countries; keep productivity parameters constant



(a) $-d(s)$ ($R^2 = 0.7$)



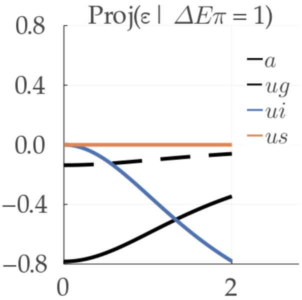
(b) $-d(g)$ ($R^2 = 0.35$)

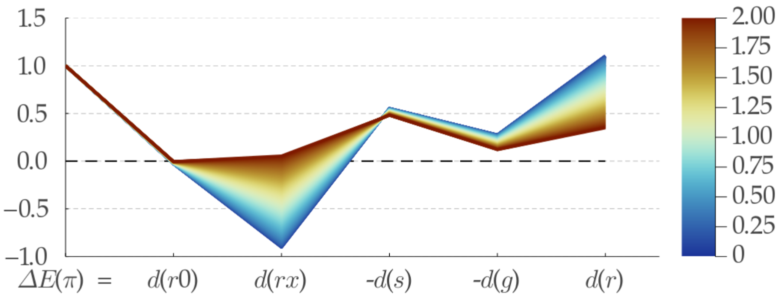


(c) $d(r)$ ($R^2 = 0.6$)

The New-Keynesian Model: Some Comparative Statics

- $\uparrow \sigma_i \implies$ Inflation driven by monetary policy. But bond price dynamics reduces $\Delta E_t \pi_t$ variance!
- $\uparrow \phi_g \implies$ Lower $\Delta E_t \pi_t$ during high growth. $\Delta E_t \pi_t$ not driven by "aggregate demand" shocks.





Comparative Statics: σ_i ϕ_g

Parameters

The New-Keynesian Model: Some Comparative Statics

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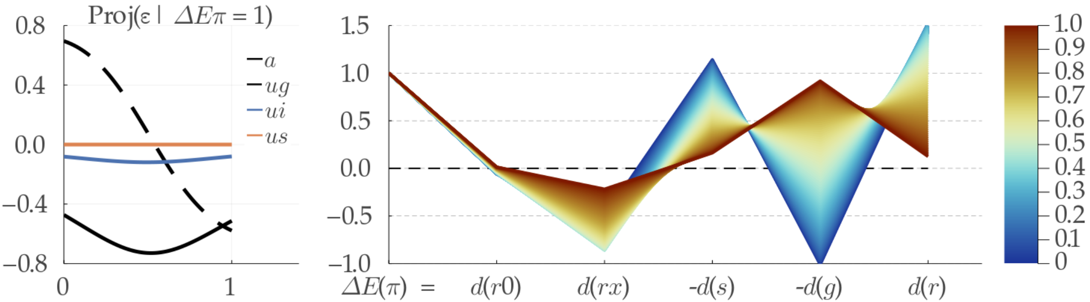


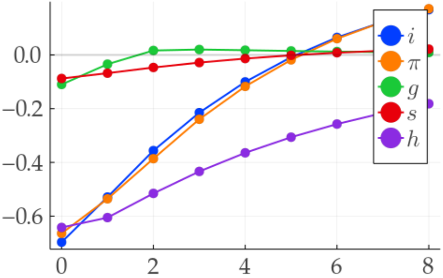
Comparative Statics: σ_i ϕ_g

Parameters

The New-Keynesian Model: The Open Economy

$$y_t = E_t y_{t+1} - \gamma\ [i_t - E_t\pi_{H,t+1} + \alpha(\bar{\omega} - 1)\ E_t\Delta z_{t+1}] + E_t u_{g,t+1}$$
$$\pi_{H,t} = \beta\ E_t\pi_{H,t+1} + \kappa\ y_t - \kappa_a\ a_t - \kappa_z\ z_t$$
$$\gamma_\alpha\ z_t = y_t - y_t^*$$
$$\pi_t = \pi_{H,t} + \alpha\ \Delta z_t$$
$$h_t = (1-\alpha)\ z_t$$

- Complete markets
- **Home**: small and open ($\alpha = 0.45$)
- **Foreign**: large and "closed"
- Same United States parameters:
  - Variance decomposition ✓ ($\varepsilon_a = -0.6$, $\varepsilon_a^* = -0.7$)
  - "Aggregate Demand" shock ✓
- Terms of trade dynamics and marginal costs:
  - ε_a and ε_a^*: same impact on Home's MC
  - Foreign Mon. Shocks: opposite $\Delta E_t\pi^*$ and $\Delta E_t\pi$



Shock to **Foreign**'s Productivity Interest

Foreign Policy Shocks New Zealand Decomp

The New-Keynesian Model: The Open Economy

$$y_t = E_t y_{t+1} - \gamma \left[i_t - E_t \pi_{H,t+1} + \alpha(\bar{\omega} - 1)\, E_t \Delta z_{t+1}\right] + E_t u_{g,t+1}$$

$$\pi_{H,t} = \beta\, E_t \pi_{H,t+1} + \kappa\, y_t - \kappa_a\, a_t - \kappa_z\, z_t$$

$$\gamma_\alpha\, z_t = y_t - y_t^*$$

$$\pi_t = \pi_{H,t} + \alpha\, \Delta z_t$$

$$h_t = (1 - \alpha)\, z_t$$

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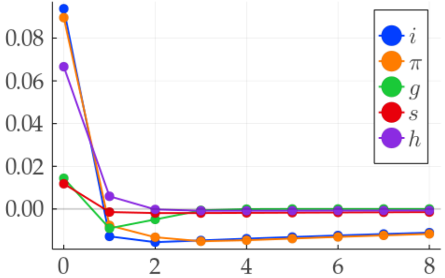

Shock to **Foreign**'s Productivity Interest

Foreign Policy Shocks | New Zealand Decomp

Conclusion

Conclusion

- **Variance Decomposition.**
  - Either discount rates or surpluses drive unexpected inflation
  - Inflation is partially ”fiscal”, but not cross-country
- **Discounted Surpluses Shock.**
  - Discount rates drive discounted surplus innovations (returns on public debt)
- **”Aggregate Demand” Shock.**
  - Discount rates and countercyclical future surpluses drive ”aggregate demand” inflation
- New-Keynesian models reproduce BVAR decompositions
  - Relevance of productivity shocks
  - Relevance of policy rules

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Appendix: Debt Instruments and Growth

Return

- Real **market value** debt-to-GDP v_t, surplus-to-GDP s_t and GDP growth g_t (stationarity!)
- Bonds (j, n) promisses one unit of currency j after n periods
  - Nominal bonds
  - Real bonds (currency denomination = final goods)
  - US Dollar bonds

  Constant structure $\{\delta_j\}$, $\{\omega_j^n\}$
- Bond price $Q_{j,t}^n$, excess return $rx_{j,t}$ $\quad 1 + \text{return}_{j,t} = 1 + rx_{j,t} + i_{j,t-1} = \frac{\sum_n Q_{j,t}^{n-1} B_{j,t-1}^n}{\sum_n Q_{j,t-1}^n B_{j,t-1}^n}$

  (one-period bonds $\implies rx = 0$)
- Debt law of motion:

$$\frac{v_{t-1}}{\beta} + \frac{v}{\beta}\left[-g_t + \sum_j \delta_j \left(rx_{j,t} + i_{j,t-1} + \Delta h_{j,t} - \pi_{j,t}\right)\right] = v_t + s_t$$

Appendix: Debt Instruments and Growth

Return

- Law of motion:

$$\sum_j \mathcal{E}_{j,t} B^1_{j,t-1} = P^s_t S_t + \sum_j \mathcal{E}_{j,t} \sum_{n=2}^{\infty} Q^{n-1}_{j,t} \left(B^{n-1}_{j,t} - B^n_{j,t-1} \right),$$

- $\mathcal{V}_{j,t} = \sum_{n=0}^{\infty} Q^n_{j,t} B^n_{j,t}$ (end-of-period market value of debt)

$$\sum_j (1 + rx_{j,t} + i_{j,t-1}) \mathcal{E}_{j,t} \mathcal{V}_{j,t-1} = P^s_t S_t + \sum_j \mathcal{E}_{j,t} \mathcal{V}_{j,t}$$

- $V_{j,t} = \mathcal{V}_{j,t} / P_{j,t} Y_t$ (real value of j-indexed debt)

$$V_{t-1} \sum_j \frac{(1 + rx_{j,t} + i_{j,t-1})(1 + \Delta h_{j,t})}{(1 + \pi_{j,t})(1 + g_t)} \delta_j = s_t + V_t.$$

Appendix: Public Debt Currency Denomination

Return

Symbol	Description	Nominal Debt	Real Debt	Dollar Debt
j	Index Symbol	N	R	D
	Notation	δ, ω	δ_R, ω_R	δ_D, ω_D
P_j	Price per Good	P	1	P_t^{US}
$\mathcal{E}_j$	Nominal Exchange Rate	1	P	Dollar NER
H_j	Real Exchange Rate	1	1	Dollar RER
π_j	Log Variation in Price	π	0	π_t^{US}
Δh_j	Log Real Depreciation	0	0	Δh_t

Table: Public Debt Denomination

Appendix: Bayesian Prior

Return

- Complete model (with US variables):

$$x_t^{OBS} = a\, x_{t-1}^{OBS} + b\, u_{t-1}^{OBS} + e_t$$
$$u_t^{OBS} = a_u\, u_{t-1}^{OBS} + e_{u,t}$$

- Group $\theta = [\mathrm{vec}(a)'\ \mathrm{vec}(b)']'$
- $\Sigma \sim IW(\Phi; d) \qquad \theta|\Sigma \sim N(\bar{\theta}, \Sigma \otimes \Omega)$
- $\Phi =$ Identity and $d = 7$ sets a loose prior
- $\bar{\theta}$ sets the mean of the prior for a to be OLS estimate of a_u

$$\mathrm{cov}\left(a_{ij}, a_{kl} \mid \Sigma\right) = \begin{cases} \lambda^2 \frac{\Sigma_{ij}}{\Phi_{jj}} & \text{if } j = l \\ 0 & \text{otherwise.} \end{cases} \qquad \mathrm{cov}\left(b_{ij}, b_{kl} \mid \Sigma\right) = \begin{cases} (\xi\lambda)^2 \frac{\Sigma_{ij}}{\Phi_{u,jj}} & \text{if } j = l \\ 0 & \text{otherwise} \end{cases}$$

Set $\xi = (1/3)$

Appendix: Hyperparameters + Debt Structure

Return

Country	v (%)	δ_N (%)	δ_R (%)	δ_D (%)	Avg. Term (Years)	λ	$\sigma(\Delta E_t \pi)$ (%)
United States	60	93	7	0	5	10	1.9
Advanced - 1960 Sample							
Canada	71	92	5	3	6.5	0.21	1.0
Denmark	37	84	0	16	5.6	0.18	1.2
Japan	98	100	0	0	5.5	0.01	2.2
Norway	35	99	0	1	3.7	0.19	1.5
Sweden	46	69	16	14	4.8	0.16	1.5
United Kingdom	61	76	24	0	12.3	0.17	2.0
Advanced - 1973 Sample							
Australia	24	90	10	0	7.2	0.18	1.5
New Zealand	41	82	6	13	4.3	0.15	2.0
South Korea	21	97	0	3	4	0.15	2.8
Switzerland	43	100	0	0	6.9	0.23	1.0

(a) Advanced Economies

Country	v (%)	δ_N (%)	δ_R (%)	δ_D (%)	Avg. Term (Years)	λ	$\sigma(\Delta E_t \pi)$ (%)
Emerging - 1998 Sample							
Brazil	70	70	25	5	2.6	0.12	1.4
Chile	14	10	57	33	12.8	0.27	1.0
Colombia	41	45	23	32	5.6	0.13	0.8
Czech Republic	31	91	0	9	5.6	0.15	1.1
Hungary	68	76	0	23	4.1	0.14	1.3
India	73	90	3	7	10.1	0.25	1.1
Indonesia	43	44	0	56	9.2	0.21	1.2
Israel	77	43	34	23	6.6	0.13	1.3
Mexico	45	65	10	26	5.5	0.15	1.0
Poland	47	79	1	20	4.2	0.10	1.3
Romania	28	50	0	50	4.8	0.10	1.9
South Africa	41	70	20	10	12.9	0.25	1.0
Turkey	43	47	23	30	3.6	0.13	2.1
Ukraine	43	100	0	0	9.1	0.07	5.7

(b) Emerging Economies

Appendix: Public Finances Model

Return

- Convert par to market value of debt (Cox and Hirschhorn (1983))

$$\mathcal{V}_{j,t} = \mathcal{V}^b_{j,t} \times \frac{\text{market price of debt}}{\text{book price of debt}} = \mathcal{V}^b_{j,t} \times \frac{Q_{j,t}}{Q^b_{j,t}}.$$

- Linearized average interest follows

$$i^b_{j,t} = \omega_j i^b_{j,t-1} + (1-\omega_j) i_{j,t} = (1-\omega_j) \sum_{k=0}^{\infty} \omega_j^k i_{j,t-k}$$

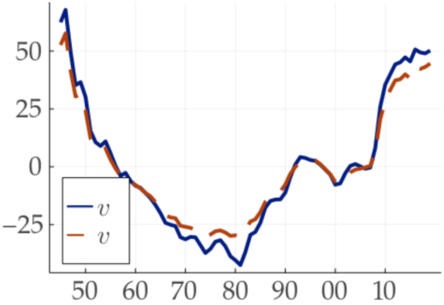
  since government rolls over share ω_j of public debt in steady state

- Linearized book price of debt:

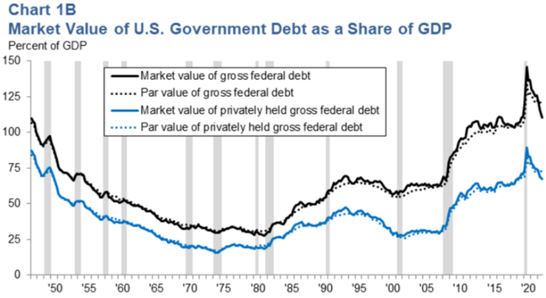
$$q^b_{j,t} = (\omega_j \beta) E_t q^b_{j,t+1} - i^b_{j,t}$$

Appendix: Public Finances Model

Return



(a) Model



(b) Emerging Economies

Appendix: Geometric Term Structure

Return | Decomposition 2

- To each currency portfolio j, fixed geometric maturity structure:

$$B^n_{j,t} = \omega_j B^{n-1}_{j,t}$$

- Total return on currency-j portfolio:

$$1 + rx_{j,t} + i_{j,t-1} = \frac{1 + \omega_j Q_{j,t}}{Q_{j,t-1}} \quad \Longrightarrow \quad \boxed{rx_{j,t} + i_{j,t-1} = (\omega_j \beta) q_{j,t} - q_{j,t-1}}$$

- Assume **constant risk premia** $E_t rx_{j,t+1} = 0$

$$\boxed{q_{j,t} = (\omega_j \beta) E_t q_{j,t+1} - i_{j,t}} = -\sum_{k=0}^{\infty} (\omega_j \beta)^k E_t i_{j,t+k}$$

Appendix: Second Decomposition

Return

- From geometric maturity structure Geometric Term Structure

$$\Delta E_t rx_{j,t} = -\sum_{k=1}^{\infty}(\omega_j\beta)^k\left[\Delta E_t r_{j,t+k} + \Delta E_t\pi_{j,t+k} - \Delta E_t\Delta h_{j,t+k}\right]$$

- Replace on the original fiscal decomposition

$$\Delta E_t\pi_t = \overbrace{\left[-\sum_{k=1}(\omega\beta)^k\Delta E_t\pi_{t+k} - \frac{\delta_D}{\delta}\sum_{k=0}^{\infty}(\omega_D\beta)^k\Delta E_t\pi^{US}_{t+k}\right]}^{\text{Innovation to Nominal Variables}}$$

$$\underbrace{-\frac{\beta}{\delta v}\left[\sum_{k=0}^{\infty}\beta^k\Delta E_t s_{t+k} + \frac{v}{\beta}\sum_{k=0}^{\infty}\beta^k\Delta E_t g_{t+k} - \frac{v}{\beta}\sum_{k=1}^{\infty}\sum_{j}\delta_j\beta^k(1-\omega^k)\Delta E_t r_{j,t+k} - \frac{\delta_D v}{\beta}\sum_{k=0}^{\infty}(\omega_D\beta)^k\Delta E_t\Delta h_{t+k}\right]}_{\text{Innovation to Real Variables}}$$

$$\equiv -d_2(\pi) - d_2(\pi^{US}) - d_2(s) - d_2(g) + d_2(r) + d_2(\Delta h)$$

Appendix: Second Decomposition

Return

Country	$\Delta E_t \pi_t =$	$-\Delta E_t$(Future Inflation)		$\pm\, \Delta E_t$(Real Variables)			
		$-d_2(\pi)$	$-d_2(\pi^{US})$	$-d_2(s)$	$-d_2(g)$	$d_2(r)$	$d_2(\Delta h)$
United States	1	***-1.12**	-	**0.57**	0.23	***1.32**	0
Advanced - 1960 Sample							
Canada	1	***-1.53**	***-0.07**	0.62	***1.22**	**0.78**	**-0.03**
Denmark	1	***-0.49**	***-0.20**	0.42	-0.04	**1.23**	0.08
Japan	1	***-1.14**	0	***1.60**	-0.38	***0.91**	0
Norway	1	***-0.70**	0	**0.60**	**0.47**	**0.64**	0
Sweden	1	***-1.02**	**-0.10**	-0.34	***0.98**	***1.54**	-0.07
United Kingdom	1	***-2.34**	0	***2.89**	***0.97**	**-0.52**	0
Advanced - 1973 Sample							
Australia	1	***-1.47**	0	***2.09**	***0.66**	-0.27	0
New Zealand	1	***-1.02**	***-0.08**	0.40	***0.87**	**1.04**	**-0.21**
South Korea	1	***-0.74**	***-0.03**	***1.91**	0.17	-0.33	**0.01**
Switzerland	1	***-0.79**	0	**0.90**	***0.91**	-0.02	0

(a) Advanced Economies

Country	$\Delta E_t \pi_t =$	$-\Delta E_t$(Future Inflation)		$\pm\, \Delta E_t$(Real Variables)			
		$-d_2(\pi)$	$-d_2(\pi^{US})$	$-d_2(s)$	$-d_2(g)$	$d_2(r)$	$d_2(\Delta h)$
Emerging - 1998 Sample							
Brazil	1	***-0.11**	0	**-1.46**	**1.05**	**1.46**	0.07
Chile	1	-0.76	**-2.75**	8.95	-5.71	-0.35	1.62
Colombia	1	***-0.61**	-0.04	1.39	-1.09	0.02	**1.34**
Czech Republic	1	-0.02	**-0.05**	-2.31	**2.42**	0.98	-0.03
Hungary	1	***-0.69**	***-0.15**	-0.98	1.60	1.83	***-0.61**
India	1	***-1.05**	***0.09**	**1.54**	0.05	0.41	-0.04
Indonesia	1	***-0.79**	***-1.33**	1.69	***2.61**	0.26	**-1.45**
Israel	1	***-0.54**	0.10	-0.55	***1.51**	0.61	-0.12
Mexico	1	***-0.60**	**0.17**	**1.41**	0.03	0.52	**-0.52**
Poland	1	***-0.59**	***-0.21**	0.87	-0.39	***1.43**	-0.11
Romania	1	***-1.14**	***-0.53**	2.24	0.42	-0.54	0.55
South Africa	1	0.05	-0.01	1.58	0.25	-0.79	-0.07
Turkey	1	***-0.76**	***-0.40**	-1.18	-0.15	***3.35**	0.14
Ukraine	1	**-0.29**	0	**0.65**	***0.41**	**0.23**	0

(b) Emerging Economies

Appendix: Variance Decomposition

Return

- **Proposition.** The variance decomposition

$$1 = \frac{\mathrm{cov}_\pi\left[d(rx)\right]}{\mathrm{var}\left[\Delta E_t \pi_t\right]} + \frac{\mathrm{cov}_\pi\left[d(r_0)\right]}{\mathrm{var}\left[\Delta E_t \pi_t\right]} - \frac{\mathrm{cov}_\pi\left[d(s)\right]}{\mathrm{var}\left[\Delta E_t \pi_t\right]} - \frac{\mathrm{cov}_\pi\left[d(g)\right]}{\mathrm{var}\left[\Delta E_t \pi_t\right]} + \frac{\mathrm{cov}_\pi\left[d(r)\right]}{\mathrm{var}\left[\Delta E_t \pi_t\right]}$$

is equivalent to the innovations decomposition applied to VAR shock $\mathrm{Proj}(e \mid \Delta E_t \pi_t = 1)$

$$1 = \Delta E_t \pi_t \equiv d(rx) + d(r_0) - d(s) - d(g) + d(r)$$

- Proof:

$$\begin{aligned} 1 &= -\beta\, \overbrace{\mathbf{1}_s'(I-\beta A)^{-1} K\Omega K' \mathbf{1}_\pi}^{\mathrm{cov}\left[\Delta E_t \pi_t, \sum_k \beta^k \Delta E_t s_{t+k}\right]} \overbrace{\left(\mathbf{1}_\pi' K\Omega K' \mathbf{1}_\pi\right)^{-1}}^{\mathrm{var}(\Delta E_t \pi_t)^{-1}} + \mathbf{1}_r'(I-\beta A)^{-1} K\Omega K' \mathbf{1}_\pi \left(\mathbf{1}_\pi' K\Omega K' \mathbf{1}_\pi\right)^{-1} \\ &= -\beta \mathbf{1}_s'(I-\beta A)^{-1} K\, \mathrm{Proj}(e_t \mid \Delta E_t \pi_t = 1) + \mathbf{1}_r'(I-\beta A)^{-1} K\, \mathrm{Proj}(e_t \mid \Delta E_t \pi_t = 1). \end{aligned}$$

Appendix: NK Model Parameters

Equations NK Complete Comparative Statics

Parameter	Value
β	0.98
γ	0.4
φ	3
θ	0.25
α	0.45
$\bar{\omega}$	γ^{-1}

Table: Fixed Parameters

Parameter	Simple	Complete
ρ_a	0.96	0.84
ρ_g		0.29
ρ_i		0
ρ_s		0.39
ϕ_π	0.60	0.95
ϕ_g		0.61
τ_π		0.12
τ_g	1.51	0.05
σ_a	1	1
σ_g		1.79
σ_i		0.53
σ_s		0

Table: Estimated Parameters

Appendix: Why Trend Shocks? The Growth Component

Return

- Empirical decompositions: often $d(g) \neq 0$
- But in the absence of trend shocks:

$$g_t = (1 - L)y_t = \mathbf{1}_y'(1 - L)a(L)e_t \equiv \mathbf{1}_y' b(L)e_t$$

- Stationary model $a(L)^{-1}X_t = e_t \implies$ the roots of $a(L)^{-1}$ are outside the unit circle
- Therefore $||a(1)|| < \infty$ and $b(1) = 0$
- Finally, note that

$$d(g) \propto \mathbf{1}_y' b(\beta)e_t \approx \mathbf{1}_y' b(1)e_t = 0$$

- With trend shocks:

$$g_t = (1 - L)y_t + u_{g,t}$$

Appendix: Estimated Moments

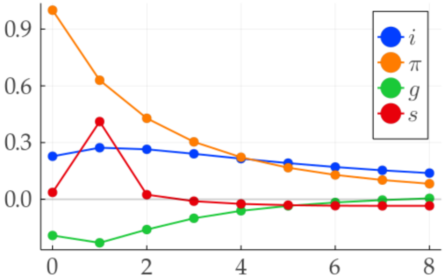
NK Simple NK Complete

Moments	Data	Model	Moments	Data	Model
σ_i/σ_g	1.00	1.16	cor(π, i)	0.54	0.84
σ_π/σ_g	1.01	1.24	cor(π, g)	-0.24	-0.25
$\sigma_{\Delta v}/\sigma_g$	1.43	0.90	cor(g, i)	0.16	0.27
a-cor(i)	0.92	0.75	cor(i, Δv)	0.02	-0.60
a-cor(π)	0.69	0.79	cor(π, Δv)	-0.29	-0.42
a-cor(g)	0.27	0.25	cor(g, Δv)	-0.39	-0.36
a-cor(Δv)	0.50	-0.13			

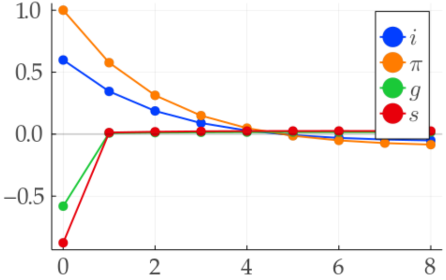
Table: Second Moment Fit - Complete Model ($\alpha_2 = 0.05$)

Appendix: Simple Model - US Data vs Model

NK Simple



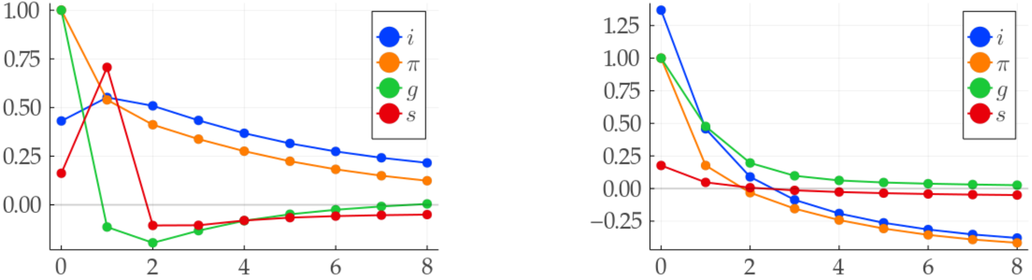
(a) B-VAR



(b) NK Model (Only Prod. Shocks)

Appendix: "Agg Demand" Shock - US Data vs Model

Return



(a) B-VAR (b) NK Model