Overlapping Generations and Pension Systems

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Course Content

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Precautionary Savings and Income Risk

Source: Aiyagari (1994)

$$\begin{aligned} \underset{c\geq 0,a}{\text{Max}} \quad & \sum_{t=0}^{\infty} \beta^t E u(c_t) \\ \text{s.t.} \quad & c_t + a_t = (1+r)a_{t-1} + wz_t \\ & a_t \geq \underline{a} \end{aligned}$$

$\log z$ follows AR(1)

$$\log z_t = \rho \log z_{t-1} + \sigma \epsilon_t \qquad \epsilon \sim N(0,1)$$

Precautionary Savings and Income Risk

TABLE II

A. Net return to capital in %/aggregate saving rate in % ($\sigma = 0.2$)

$\rho\backslash\mu$	1	3	5
0	4.1666/23.67	4.1456/23.71	4.0858/23.83
0.3	4.1365/23.73	4.0432/23.91	3.9054/24.19
0.6	4.0912/23.82	3.8767/24.25	3.5857/24.86
0.9	3.9305/24.14	3.2903/25.51	2.5260/27.36

B. Net return to capital in %/aggregate saving rate in % ($\sigma = 0.4$)

$\rho\backslash\mu$	1	3	5
0	4.0649/23.87	3.7816/24.44	3.4177/25.22
0.3	3.9554/24.09	3.4188/25.22	2.8032/26.66
0.6	3.7567/24.50	2.7835/26.71	1.8070/29.37
0.9	3.3054/25.47	1.2894/31.00	−0.3456/37.63

Source: Aiyagari (1994)

Overlapping Generations and Pension

- Generation A ($t = 0, 1$)

$$\underset{c^A \geq 0, a_0^A}{\text{Max}} \quad u(c_0^A) + v(1 - n_0^A) + \beta \left[\rho u(\tilde{c}_1^A) + (1 - \rho) u(c_1^A)\right]$$

$$\text{s.t.} \quad q_0 a_0^A + c_0^A \leq n_0^A$$

$$c_1^A \leq a_0^A + e \qquad \tilde{c}_1^A \leq a_0^A + e + \phi$$

- Generation B ($t = 1, 2$)

$$\underset{c^B \geq 0, a_1^B}{\text{Max}} \quad u(c_1^B) + v(1 - n_1^B) + \beta u(c_2^B)$$

$$\text{s.t.} \quad q_1 a_1^B + c_1^B \leq n_1^B - \tau_{L,1}$$

$$c_2^B \leq a_1^B + e$$

- Euler (capital choice), $v = 0$:

$$q_0 u'(1) = \beta \left[\rho u'(e + \phi) + (1 - \rho) u'(e)\right]$$

$$q_1 u'(1 - \rho\phi) = \beta u'(e)$$

Nishiyama and Smetters (2007)

- Does social security privatization lead to efficiency gains?
- Linearly reduce replacement rate by 50% over 40 years

$$(1)\quad v(\mathbf{s}, \mathbf{S}_t; \mathbf{\Psi}_t) = \max_{c,h} u_i(c,h) + \beta(1+\mu)^{\alpha(1-\gamma)}\phi_i \times E\left[v\left(\mathbf{s}', \mathbf{S}_{t+1}; \mathbf{\Psi}_{t+1}\right)|e\right]$$

subject to

$$(2)\quad a' = \frac{1}{1+\mu}\{w_t e h + (1+r_t)(a + tr_{\mathrm{LS},t}(\mathbf{s})) - \tau_{I,t}(w_t e h, r_t(a + tr_{\mathrm{LS},t}(\mathbf{s})), tr_{\mathrm{SS},t}(\mathbf{s})) - \tau_{P,t}(w_t e h) + tr_{\mathrm{SS},t}(\mathbf{s}) - c\} \geq a'_{\min,t}(\mathbf{s}),$$

$$a = 0 \quad \text{if} \quad i = 20, \quad a \geq 0 \quad \text{if} \quad i \geq 65,$$

where the utility function, $u_i(.)$, takes the Cobb-Douglas form nested within a time-separable isoelastic specification,

$$(3)\qquad u_i(c,h) = \frac{\{((1+n_i/2)^{-\zeta}c)^{\alpha}(h_{\max} - h)^{1-\alpha}\}^{1-\gamma}}{1-\gamma};$$

Nishiyama and Smetters (2007)

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TABLE VIII

PERCENT CHANGE IN SELECTED MACRO VARIABLES RELATIVE TO BASELINE

Run #	Year t	GNP	National wealth	Labor supply	Interest rate	Wage rate	Income tax rate[a]	Payroll tax rate
1	1	−0.2	0.0	−0.2	−0.3	0.1	0.4	0.2
Representative	10	0.8	2.1	0.2	−2.3	0.6	−2.2	−4.6
agent without	20	2.8	5.7	1.6	−4.7	1.2	−7.5	−15.9
wage shocks[b]	40	8.1	15.6	5.0	−11.4	2.9	−18.6	−42.2
	Long run	11.3	25.5	5.8	−19.8	5.3	−23.1	−51.9
2	1	0.1	0.0	0.2	0.2	−0.1	−0.4	0.1
Heterogenous	10	0.8	1.7	0.5	−1.5	0.4	−1.5	−4.3
agents with	20	2.2	4.6	1.2	−4.0	1.0	−3.8	−15.0
wage shocks[b]	40	5.7	12.8	2.9	−10.9	2.8	−8.9	−40.9
	Long run	7.9	20.0	3.1	−17.7	4.7	−11.2	−52.2

Nishiyama and Smetters (2007)

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$$(11) \qquad tr_{\mathrm{LS},t}(\mathbf{s}) = \begin{cases} tr_{\mathrm{CV},t}(\mathbf{s}) & \text{if } t = 1 \\ tr_{\mathrm{CV},t}(\mathbf{s}) + \Delta tr & \text{if } t > 1 \text{ and } i = 20 \\ 0 & \text{otherwise.} \end{cases}$$

If $\Delta tr > 0$ then privatization has produced net new resources and so we say that this reform "increases efficiency." Conversely, if $\Delta tr < 0$ then privatization "reduces efficiency."

Nishiyama and Smetters (2007)

- Does social security privatization lead to efficiency gains?
- Linearly reduce replacement rate by 50% over 40 years

CHANGE IN WELFARE PER HOUSEHOLD (1,000 DOLLARS IN 2001)

Run #	Age in year 1	Without LSRA[a] select productivities				With LSRA[b] for all productivities
		e^1	e^3	e^5	e^8	
1	79	—	−0.6	—	—	0.0
Representative	60	—	−30.8	—	—	0.0
agent without	40	—	−75.6	—	—	0.0
wage shocks	20	—	−36.3	—	—	0.0
	0	—	23.8	—	—	18.1
	−20	—	63.3	—	—	18.1
	−∞	—	73.2	—	—	18.1
2	79	−0.2	−0.2	−0.3	−0.5	0.0
Heterogenous	60	−22.6	−29.7	−37.5	−57.1	0.0
agents with	40	−27.6	−46.0	−75.8	−130.7	0.0
wage shocks	20	−5.4	−7.4	−11.2	−22.4	0.0
	0	34.3	37.0	38.4	30.7	−2.4
	−20	65.8	72.4	78.3	76.0	−2.4
	−∞	76.3	84.1	91.3	91.2	−2.4

Nishiyama and Smetters (2005)

- Reduce labor tax, increase consumption tax
- Less progressive taxation
- (Results from Nishiyama and Smetters (2014))

Table 2 The transition path of consumption tax reform (changes from the baseline economy).

Year	1	6	11	21	51	101	Long run
Capital stock (national wealth)	0.0	5.4	9.1	13.3	16.0	16.1	16.1
Labor supply (in efficiency units)	5.0	4.6	4.4	4.3	4.3	4.3	4.3
Gross domestic product	3.1	4.9	6.2	7.7	8.6	8.7	8.7
Private consumption	−1.5	1.1	2.9	4.9	6.5	6.6	6.6
Welfare of age-21 households[a]	−2.1	−1.2	−0.6	0.1	0.6	0.6	0.6
Interest rate	7.2	−1.1	−6.3	−11.7	−15.1	−15.2	−15.2
Average wage rate	−1.9	0.3	1.7	3.2	4.2	4.2	4.2
Income tax rates	−50.0	−50.0	−50.0	−50.0	−50.0	−50.0	−50.0
Consumption tax rate[b]	8.4	7.9	7.6	7.3	7.0	7.0	7.0

Frame title

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