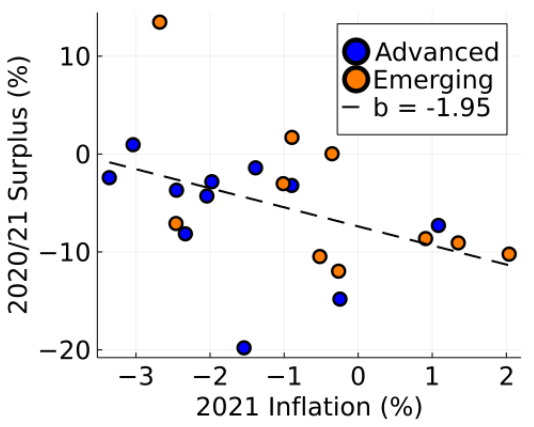
A Fiscal Decomposition of Unexpected Inflation: Cross-Country Estimates and Theory

Livio Maya

Introduction

- What drives innovations to the price level?
- Sources of inflation variation
- Focus on unexpected inflation $\Delta E_t \pi_t$
    - Campbell and Ammer (1993)
    - Internal consistency of expectations
- Breakdown of valuation equation of public debt

Fiscal Connection?



COVID Inflation - 21 countries in sample

Valuation Equation of Public Debt

- Stock market - Campbell and Ammer (1993)

$$\text{Stock price} = \text{Discounted Dividends}$$

$$\Delta E_t\,[\text{Stock price}] = \Delta E_t\,[\text{Dividends}] - \Delta E_t\,[\text{Disc Rates}]$$

- Micro-founded monetary models

$$\frac{\textbf{Bond Prices} \times \text{Bonds}}{\textbf{Price Level}} = \sum_t \frac{\textbf{Surpluses}_t}{\textbf{Discount}_t}$$

$$\Delta E_t\,[\text{Bond Price}] - \Delta E_t\,[\text{Price}] = \Delta E_t\,[\text{Surplus}] - \Delta E_t\,[\text{Disc}]$$

Exercises

1. Decomposition estimates
   - Bayesian VAR for 21 countries
   - Inflation shock $\Delta E_t \pi_t = 1$
   - Discounted surpluses shock: $\Delta E_t[\text{Disc Surp}] = -1$
2. FTPL, New-Keynesian Model
   - Volatile surpluses, no contribution to inflation?
   - GMM estimate to reproduce decompositions
   - Parametric model of partial debt repayment
   - Shocks to long-term growth

Motivation + Results

- Measures not structural
- Stylized facts to be matched by theory
- On average:
  - Discount rates → ~80% of total inflation
  - GDP growth → ~20% of total inflation
  - Surplus/GDP → ~0% of total inflation

Inflation has fiscal roots, even if fiscal policy is disconnected from the price level.

Motivation + Results

- Volatile surpluses, no inflation?
- ”Passive” vs ”active” fiscal policy
- No debt repayment inconsistent with decompositions

Discount-driven inflation and realistic surplus process preclude partial repayment.

Why unexpected inflation, not just inflation?

-

Literature

- **Monetary-Fiscal Interaction.** Cagan (1956), Sargent and Wallace (1981), Hall and Sargent (1997), Hall and Sargent (2011), Jiang et al. (2019), Corsetti et al. (2019), Sunder-Plassmann (2020), Du et al. (2020), Akhmadieva (2022)
- **Fiscal Theory of the Price Level.** Leeper (1991), Sims (1994), Woodford (1995), Cochrane (1998), Cochrane (2005), Sims (2011), Leeper and Leith (2016), Bassetto and Cui (2018), Cochrane (2022c), Brunnermeier et al. (2022), Cochrane (2022a), Cochrane (2022b)
- **Empirical Finance.** Campbell and Shiller (1988), Cochrane (1992), Campbell and Ammer (1993), Chen and Zhao (2009), Cochrane (2008), Jiang et al. (2019)

Environment

- 1 period = 1 year
- Consumption good price P_t
- Total output Y_t
- Nominal bonds $B^n_{N,t}$, price $Q^n_{N,t}$
  - Pay one unit of currency after n years
- Real bonds $B^n_{R,t}$, price $P_t Q^n_{R,t}$
  - Pay one unit of consumption good after n years
- Primary Surplus $P_t S_t$

Evolution of Public Debt

$$\overbrace{\left[B^1_{N,t-1} + P_t B^1_{R,t-1}\right]}^{\text{Issued Currency}} = \Delta M_t$$

$$\underbrace{+\left[P_t S_t + \sum_{n=1}^{\infty} Q^n_{N,t}\left(B^n_{N,t} - B^{n+1}_{N,t-1}\right) + P_t \sum_{n=1}^{\infty} Q^n_{R,t}\left(B^n_{R,t} - B^{n+1}_{R,t-1}\right)\right]}_{\text{Retired Currency}}$$

- This is a budget constraint
- Assumption 1: households do not value currency $M_t = 0$

Evolution of Public Debt

- Assumption 1: households do not value currency $M_t = 0$
- End-of-period debt $\mathscr{V}_{N,t}$ and $\mathscr{V}_{R,t}$

$$(1 + r_t^N)\mathscr{V}_{N,t-1} + (1 + r_t^R)(1 + \pi_t)\mathscr{V}_{R,t-1} = P_t S_t + \mathscr{V}_{N,t} + \mathscr{V}_{R,t}$$

- This is an equilibrium condition
- Price level adjusts so that

$$\text{currency issued} = \text{currency retired}$$

Evolution of Public Debt

- Constant structure of public debt: $\delta = \mathscr{V}_{N,t} / \mathscr{V}_t$

$$1 + r_t^n = \delta \left[(1 + r_{N,t}) \right] + (1 - \delta) \left[(1 + r_{R,t})(1 + \pi_t) \right]$$

- Debt-to-GDP = $V_t = \mathscr{V}_t / P_t Y_t$
- Surplus-to-GDP = $s_t = S_t / Y_t$

$$\frac{1 + r_t^n}{(1 + \pi_t)(1 + g_t)} V_{t-1} = s_t + V_t$$

Evolution of Public Debt

Linearized equations

$$v_t + \frac{s_t}{V} = \frac{1}{\beta}\left[v_{t-1} + r_t^n - \pi_t - g_t\right]$$

$$r_t^n = \delta\left[r_t^N\right] + (1-\delta)\left[r_t^R + \pi_t\right]$$

- v_t is log debt-to-GDP
- r_t^n is the nominal return on public debt

Valuation Equation of Public Debt

- Assumption 2: debt does not spiral $\lim_{j\to\infty} \beta^j v_{t+j} = 0$
- Solve flow equation forward:

$$\overbrace{v_{t-1} + r_t^n - \pi_t}^{\text{Real market value of debt}} = \underbrace{\frac{\beta}{V} \sum_{j=0}^{\infty} \beta^j \left[E_t s_{t+j} + E_t g_{t+j}\right] - \sum_{j=1}^{\infty} \beta^j \left[E_t r_{t+j}^n - E_t \pi_{t+j}\right]}_{\text{Discounted Surpluses}}$$

Marked-to-Market Decomposition

Take innovation on the valuation equation:

$$\boxed{\epsilon_{r^n,t} - \epsilon_{\pi,t} = \epsilon_{s,t} + \epsilon_{g,t} - \epsilon_{r,t}}$$

Terms:

$\epsilon_{r^n,t} = \Delta E_t r_t^n$

$\epsilon_{\pi,t} = \Delta E_t \pi_t$ (current inflation)

$\epsilon_{s,t} = (\beta / V) \sum_{j=0}^{\infty} \beta^j \Delta E_t s_{t+j}$

$\epsilon_{g,t} = \sum_{j=0}^{\infty} \beta^j \Delta E_t g_{t+j}$

$\epsilon_{r,t} = \sum_{j=1}^{\infty} \beta^j (\Delta E_t r_{t+j}^n - \Delta E_t \pi_{t+j})$

Public Finances Model

Why a public finances model?

1. We can do better: bond prices forecast future inflation
2. No historical data for bond price/return r_t^n
3. No data on market value of debt (only book value)

Public Finances Model

Key Assumptions

- Assumption: constant maturity structure
- Decays geometrically at rate ω:

$$B^n_{N,t} = \omega_N \, B^{n-1}_{N,t}$$
$$B^n_{R,t} = \omega_R \, B^{n-1}_{R,t}$$

- Assumption: constant (or no) risk premium

$$E_t r_{N,t} = E_t r_{R,t} + E_t \pi_t = i_t$$

Public Finances Model

- Bond prices:

$$q_{N,t} = (\omega_N \beta) E_t q_{N,t} - \left[i_t\right]$$

$$q_{R,t} = (\omega_R \beta) E_t q_{R,t} - \left[i_t - E_t \pi_{t+1}\right]$$

- Returns:

$$r_{j,t} = (\omega_j \beta) q_{j,t} - q_{j,t-1} \qquad j = N, R$$

Break down of bond price variation

Proposition: let $r_t = i_t - E_t\pi_{t+1}$ be the real interest. Then

$$\epsilon_{r^n,t} - \epsilon_{\pi,t} = -\delta \sum_{j=0}^{\infty} (\omega_N \beta)^j \Delta E_t \pi_{t+j} - \sum_{j=1}^{\infty} \beta^j [\delta \omega_N^j + (1-\delta)\omega_R^j] \Delta E_t r_{t+j}$$

- Higher real discount lowers real and nominal bond prices
- Higher inflation lowers nominal bond prices
- No long-term debt $\omega = 0$:

$$\epsilon_{r^n,t} - \epsilon_{\pi,t} = -\delta \Delta E_t \pi_t$$

Total Inflation Decomposition

Replace bond return decomp on marked-to-market decomp:

$$\boxed{-\varepsilon_{\pi,t} = \varepsilon_{s,t} + \varepsilon_{g,t} - \varepsilon_{r,t}}$$

Terms:

$\varepsilon_{\pi,t} = \delta \sum_{j=0}^{\infty} (\omega_N \beta)^j \Delta E_t \pi_{t+j}$ (current and future inflation)

$\varepsilon_{s,t} = \epsilon_{s,t} = (\beta / V) \sum_{j=0}^{\infty} \beta^j \Delta E_t s_{t+j}$

$\varepsilon_{g,t} = \epsilon_{g,t} = \sum_{j=0}^{\infty} \beta^j \Delta E_t g_{t+j}$

$\varepsilon_{r,t} = \sum_{j=1}^{\infty} \beta^j \left[1 - (\delta \omega_N^j + (1 - \delta) \omega_R^j) \right] \Delta E_t r_{t+j}$

Comparison of Decompositions

- **Marked-to-market**: $\boxed{\epsilon_{r^n,t} - \epsilon_{\pi,t} = \epsilon_{s,t} + \epsilon_{g,t} - \epsilon_{r,t}}$
  - Current inflation given current bond prices
  - Highlights effect of monetary policy
- **Total inflation**: $\boxed{-\varepsilon_{\pi,t} = \varepsilon_{s,t} + \varepsilon_{g,t} - \varepsilon_{r,t}}$
  - Path of inflation given path of discount rates
  - Sensitive to future inflation
  - Nets out effect of discount rates on bond prices

Build Market Value of Debt

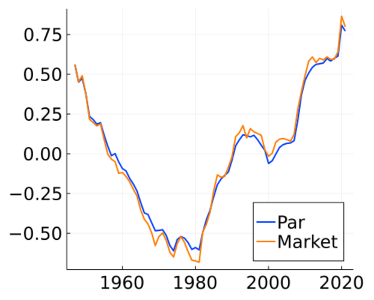
- Converting par to market value of debt
- Dallas Fed, Cox and Hirschhorn (1983) and Cox (1985)

$$\mathscr{V}_{j,t} = \mathscr{V}^b_{j,t} \times \frac{\text{market value of bonds}}{\text{book value of bonds}} = \mathscr{V}^b_{j,t} \frac{Q_{j,t}}{Q^b_{j,t}} \quad \text{for } j = N, R$$

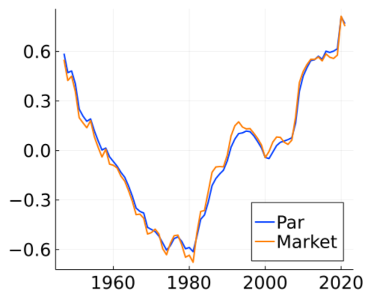
- Book price of bonds evolve according to average interest:

$$\begin{aligned} i^b_{N,t} &= (1 - \omega_N) i_t + \omega_N i^b_{N,t-1} \\ i^b_{R,t} &= (1 - \omega_R)(i_t - E_t \pi_{t+1}) + \omega_R i^b_{R,t-1} \end{aligned}$$

Comparison with Dallas Fed



(a) Dallas Fed



(b) Model

Vector Autoregression

- States X

$$X_t = AX_{t-1} + e_t \qquad e_t \sim N(0, \Sigma)$$

- Bayesian estimates of 21 countries
- Samples end in 2019 (no COVID!)
- Prior centered around US OLS estimates

$$\begin{bmatrix} i_t \text{ Nominal Interest} \\ \pi_t \text{ Inflation Rate} \\ g_t \text{ GDP Growth} \\ v_t \text{ Market Value Debt} \\ r_t^n \text{ Bond Return (model built)} \\ s_t \text{ Primary Surplus (model built)} \end{bmatrix}$$

VAR and Decomposition Measures

- VAR uses time series structure to identify revision of expectations

$$\Delta E_t X_{t+j} = A^j X_t$$

$$\sum_{j=0}^{\infty} \beta^j \Delta E_t X_{t+j} = (I - \beta A)^{-1} X_t$$

The Inflation Shock

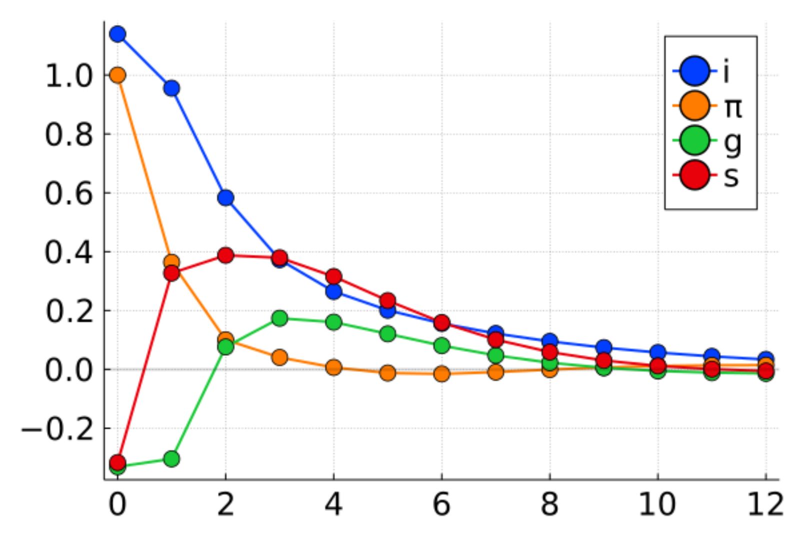
- Source of innovations to inflation $\Delta E_t \pi_t = 1$
- Reduced-form shock $e_t = E\left[e \mid \Delta E_t \pi_t = 1\right]$
- Proposition: MtM decomposition

$$\epsilon_{r^n,t} - \epsilon_{\pi,t} = \epsilon_{s,t} + \epsilon_{g,t} - \epsilon_{r,t}$$

same as variance decomposition

$$\frac{\text{cov}(\epsilon_{r^n,t}, \epsilon_{\pi,t})}{\text{var}(\epsilon_{\pi,t})} - 1 = \frac{\text{cov}(\epsilon_{s,t}, \epsilon_{\pi,t})}{\text{var}(\epsilon_{\pi,t})} + \frac{\text{cov}(\epsilon_{g,t}, \epsilon_{\pi,t})}{\text{var}(\epsilon_{\pi,t})} - \frac{\text{cov}(\epsilon_{r,t}, \epsilon_{\pi,t})}{\text{var}(\epsilon_{\pi,t})}$$

IRF - Brazil



IRF to $e_t = E\left[e \mid \Delta E_t \pi_t = 1\right]$

Inflation Shock - Marked-to-Market

Country	ϵ_{r^n}	$-\epsilon_\pi$	=	ϵ_s	$+\epsilon_g$	$-\epsilon_r$
1947 Sample (Advanced)						
United Kingdom	**-0.7	**-1	=	**-2.2	**-0.7	**1.2
United States	**-0.7	**-1	=	-0.3	**-0.5	**-0.9
1960 Sample (Advanced)						
Canada	**-2.8	**-1	=	0.3	*-1.4	**-2.8
Denmark	**-0.9	**-1	=	0.2	-0.2	**-1.9
Japan	**-0.6	**-1	=	**2.8	**-3.0	**-1.4
Norway	**-0.7	**-1	=	0.7	*3.0	**-5.4
Sweden	**-0.6	**-1	=	**0.9	**-0.9	**-1.6
1973 Sample (Advanced)						
Australia	**-2.2	**-1	=	0.2	0.1	**-3.5
New Zealand	**-1.0	**-1	=	*1.2	**-1.4	*-1.8
South Korea	**-0.6	**-1	=	**-2.4	0.2	*0.7
Switzerland	**-2.0	**-1	=	*-0.8	0.1	**-2.3

Inflation Shock: $e_t = E[e \mid \Delta E_t \pi_t = 1]$

Inflation Shock - Marked-to-Market

Country	ϵ_{r^n}	$-\epsilon_{\pi}$	=	ϵ_s	$+\epsilon_g$	$-\epsilon_r$
1997 Sample (Emerging)						
Brazil	**-0.7	**-1	=	**2.4	-0.1	**-4.0
Colombia	**-1.4	**-1	=	0.2	**-0.7	**-1.9
Czech Republic	*0.2	**-1	=	*0.7	**-1.3	-0.2
Hungary	**-0.8	**-1	=	0.0	-0.2	**-1.6
India	*-0.2	**-1	=	**-1.0	-0.1	-0.1
Israel	**-0.4	**-1	=	**0.8	*-0.4	**-1.8
Mexico	**-1.4	**-1	=	*-1.2	0.0	*-1.3
Poland	**-1.4	**-1	=	**1.0	*-0.3	**-3.0
South Africa	**-0.6	**-1	=	0.3	**-0.8	**-1.1
Ukraine	**-0.5	**-1	=	**-1.1	0.0	-0.3

Inflation Shock: $e_t = E[e \mid \Delta E_t \pi_t = 1]$

Inflation Shock - Total Inflation

Country	$-\varepsilon_\pi$	=	ε_s	$+\varepsilon_g$	$-\varepsilon_r$
1947 Sample (Advanced)					
United Kingdom	**-2.8	=	**-2.2	**-0.7	0.1
United States	**-1.5	=	-0.3	**-0.5	**-0.7
1960 Sample (Advanced)					
Canada	**-2.6	=	0.3	*-1.4	**-1.5
Denmark	**-1.6	=	0.2	-0.2	**-1.6
Japan	**-1.5	=	**2.8	**-3.0	**-1.3
Norway	**-2.0	=	0.7	*3.0	**-5.7
Sweden	**-1.6	=	**0.9	**-0.9	**-1.5
1973 Sample (Advanced)					
Australia	**-3.1	=	0.2	0.1	**-3.4
New Zealand	**-2.3	=	*1.2	**-1.4	**-2.1
South Korea	**-2.0	=	**-2.4	0.2	0.2
Switzerland	**-2.0	=	*-0.8	0.1	**-1.3

Inflation Shock: $e_t = E[e \mid \Delta E_t \pi_t = 1]$

Inflation Shock - Total Inflation

Country	$-\varepsilon_\pi$	=	ε_s	$+\varepsilon_g$	$-\varepsilon_r$
1997 Sample (Emerging)					
Brazil	**-0.8	=	**2.4	-0.1	**-3.1
Colombia	**-0.7	=	0.2	**-0.7	-0.2
Czech Republic	**-0.5	=	*0.7	**-1.3	0.1
Hungary	**-1.4	=	0.0	-0.2	**-1.3
India	**-1.4	=	**-1.0	-0.1	*-0.4
Israel	**-0.6	=	**0.8	*-0.4	**-1.0
Mexico	**-1.4	=	*-1.2	0.0	-0.3
Poland	**-1.4	=	**1.0	*-0.3	**-2.1
South Africa	**-0.8	=	0.3	**-0.8	*-0.3
Ukraine	**-1.2	=	**-1.1	0.0	-0.1

Inflation Shock: $e_t = E[e \mid \Delta E_t \pi_t = 1]$

Inflation Shock - Averages

Country	ϵ_{r^n}	$-\epsilon_\pi$	=	ϵ_s	$+\epsilon_g$	$-\epsilon_r$
Averages	**-1.0	**-1	=	0.1	**-0.4	**-1.7
1947 (Advanced)	**-0.7	**-1	=	**-1.2	**-0.6	0.1
1960 (Advanced)	**-1.1	**-1	=	*1.0	*-0.5	**-2.6
1973 (Advanced)	**-1.4	**-1	=	-0.4	-0.3	**-1.7
1997 (Emerging)	**-0.7	**-1	=	0.2	**-0.4	**-1.5

Marked-to-Market

Country	$-\varepsilon_\pi$	=	ε_s	$+\varepsilon_g$	$-\varepsilon_r$
Averages	**-1.6	=	0.1	**-0.4	**-1.3
1947 (Advanced)	**-2.2	=	**-1.2	**-0.6	-0.3
1960 (Advanced)	**-1.9	=	*1.0	*-0.5	**-2.3
1973 (Advanced)	**-2.3	=	-0.4	-0.3	**-1.6
1997 (Emerging)	**-1.0	=	0.2	**-0.4	**-0.9

Total Inflation

Inflation Shock - Takeaways

- Discount rates: 80% of inflation variation (average)
- The rest comes mostly from GDP growth
- Positive contribution of surplus-to-GDP 7/21
- Monetary policy reduces inflation in 20/21

Inflation has fiscal roots, even if fiscal policy is disconnected from the price level.

Discounted Surpluses Shock

- Discount rates drive innovations to inflation
- What drives discounted surpluses?
- Put differently: what drives unexpected returns on the basket of public bonds?

$$
\begin{aligned}
e_t &= E[e \mid \Delta E_t(\text{Disc Surpl}) = -1] \\
&= E[e \mid \Delta E_t(\epsilon_{s,t} + \epsilon_{g,t} - \epsilon_{r,t}) = -1]
\end{aligned}
$$

Discounted Surpluses Shock - Marked-to-Market

Country	ϵ_{r^n}	$-\epsilon_\pi$	=	ϵ_s	$+\epsilon_g$	$-\epsilon_r$
1947 Sample (Advanced)						
United Kingdom	**-0.8	**-0.2	=	**-0.5	-0.1	*-0.4
United States	**-0.7	**-0.3	=	0.0	**0.2	**-1.2
1960 Sample (Advanced)						
Canada	**-0.8	**-0.2	=	*0.2	-0.1	**-1.1
Denmark	**-0.8	**-0.2	=	*0.6	*0.5	**-2.0
Japan	**-0.6	**-0.4	=	0.0	-0.2	**-0.8
Norway	**-0.6	**-0.4	=	*1.0	*1.9	**-3.9
Sweden	**-0.6	**-0.4	=	**0.7	-0.2	**-1.5
1973 Sample (Advanced)						
Australia	**-0.8	**-0.2	=	*0.5	*0.2	**-1.7
New Zealand	**-0.6	**-0.4	=	**0.8	**-0.5	**-1.3
South Korea	**-0.6	**-0.4	=	**-2.4	**1.3	0.2
Switzerland	**-0.8	**-0.2	=	-0.1	*0.2	**-1.1

Discounted Surpluses Shock: $E[e \mid \Delta E_t(\epsilon_{s,t} + \epsilon_{g,t} - \epsilon_{r,t}) = -1]$

Discounted Surpluses Shock - Marked-to-Market

Country	ϵ_{r^n}	$-\epsilon_\pi$	=	ϵ_s	$+\epsilon_g$	$-\epsilon_r$
1997 Sample (Emerging)						
Brazil	**-0.5	**-0.5	=	**1.4	0.1	**-2.6
Colombia	**-0.6	**-0.4	=	0.0	**-0.3	**-0.8
Czech Republic	**-0.4	**-0.6	=	-0.1	-0.3	**-0.6
Hungary	**-0.6	**-0.4	=	*0.4	-0.3	**-1.2
India	**-0.5	**-0.5	=	-0.1	*-0.2	**-0.7
Israel	**-0.7	**-0.3	=	**0.6	-0.1	**-1.5
Mexico	**-0.6	**-0.4	=	**-0.6	0.1	*-0.6
Poland	**-0.7	**-0.3	=	**0.5	-0.1	**-1.4
South Africa	**-0.7	**-0.3	=	*-0.2	0.0	**-0.8
Ukraine	**-0.5	**-0.5	=	**-0.4	*-0.1	**-0.6

Discounted Surpluses Shock: $E[e \mid \Delta E_t(\epsilon_{s,t} + \epsilon_{g,t} - \epsilon_{r,t}) = -1]$

Discounted Surpluses Shock - Total Inflation

Country	$-\varepsilon_\pi$	=	ε_s	$+\varepsilon_g$	$-\varepsilon_r$
1947 Sample (Advanced)					
United Kingdom	**-0.9	=	**-0.5	-0.1	*-0.3
United States	**-0.5	=	0.0	**0.2	**-0.7
1960 Sample (Advanced)					
Canada	**-0.5	=	*0.2	-0.1	**-0.6
Denmark	**-0.6	=	*0.6	*0.5	**-1.6
Japan	**-0.7	=	0.0	-0.2	**-0.5
Norway	**-0.9	=	*1.0	*1.9	**-3.8
Sweden	**-0.8	=	**0.7	-0.2	**-1.2
1973 Sample (Advanced)					
Australia	**-0.6	=	*0.5	*0.2	**-1.3
New Zealand	**-0.8	=	**0.8	**-0.5	**-1.2
South Korea	**-1.2	=	**-2.4	**1.3	0.0
Switzerland	**-0.5	=	-0.1	*0.2	**-0.6

Discounted Surpluses Shock: $E[e \mid \Delta E_t(\epsilon_{s,t} + \epsilon_{g,t} - \epsilon_{r,t}) = -1]$

Discounted Surpluses Shock - Total Inflation

Country	$-\varepsilon_\pi$	=	ε_s	$+\varepsilon_g$	$-\varepsilon_r$
1997 Sample (Emerging)					
Brazil	**-0.3	=	**1.4	0.1	**-1.9
Colombia	**-0.3	=	0.0	**-0.3	-0.1
Czech Republic	**-0.5	=	-0.1	-0.3	-0.2
Hungary	**-0.6	=	*0.4	-0.3	**-0.8
India	**-0.6	=	-0.1	*-0.2	**-0.3
Israel	**-0.2	=	**0.6	-0.1	**-0.7
Mexico	**-0.6	=	**-0.6	0.1	-0.1
Poland	**-0.5	=	**0.5	-0.1	**-0.9
South Africa	**-0.3	=	*-0.2	0.0	*-0.1
Ukraine	**-0.6	=	**-0.4	*-0.1	**-0.1

Discounted Surpluses Shock: $E[e \mid \Delta E_t(\epsilon_{s,t} + \epsilon_{g,t} - \epsilon_{r,t}) = -1]$

Discounted Surpluses Shock - Averages

Country	ϵ_{r^n}	$-\epsilon_{\pi}$	=	ϵ_s	$+\epsilon_g$	$-\epsilon_r$
Averages	**-0.6	**-0.4	=	0.1	0.1	**-1.2
1947 (Advanced)	**-0.8	**-0.2	=	*-0.2	0.1	**-0.8
1960 (Advanced)	**-0.7	**-0.3	=	*0.5	0.4	**-1.9
1973 (Advanced)	**-0.7	**-0.3	=	-0.3	0.3	**-1.0
1997 (Emerging)	**-0.6	**-0.4	=	*0.2	*-0.1	**-1.1

Marked-to-Market

Country	$-\varepsilon_{\pi}$	=	ε_s	$+\varepsilon_g$	$-\varepsilon_r$
Averages	**-0.6	=	0.1	0.1	**-0.8
1947 (Advanced)	**-0.7	=	*-0.2	0.1	**-0.5
1960 (Advanced)	**-0.7	=	*0.5	0.4	**-1.6
1973 (Advanced)	**-0.8	=	-0.3	0.3	**-0.8
1997 (Emerging)	**-0.4	=	*0.2	*-0.1	**-0.5

Total Inflation

Model Overview

- How hard to reproduce empirical findings? Interpretation?
- Fiscal theory of the price level, New-Keynesian model
- Partial debt repayment (but still FTPL!)
- Trend shocks

Model Equations

- Private sector

$$y_t = E_t y_{t+1} - \gamma(i_t - E_t \pi_{t+1}) + \rho_g u_{g,t}$$
$$\pi_t = \beta E_t \pi_{t+1} + \kappa y_t - \kappa_a a_t$$
$$g_t = y_t - y_{t-1} - u_{g,t}$$

- Central bank

$$i_t = \phi_\pi \pi_t + \phi_g g_t + u_{i,t}$$

Why trend shocks?

- Otherwise, output stationary $\implies \varepsilon_{g,t} \approx 0$
- Model solution: $X_t = a(L)e_t$ for finite $a(1)$
- Growth term of decomposition:

$$\epsilon_{g,t} = 1'_g a(\beta)e_t$$

- In the absence of trend shocks:

$$g_t = 1'_g a(L)e_t = 1'_y(1 - L)a(L)e_t$$
$$1'_g a(L) = 1'_y(1 - L)a(L)$$

- Therefore $\epsilon_{g,t} = 1'_g a(\beta)e_t \approx 1'_g a(1)e_t = 0$

Model Equations

- Flow of debt

$$v_t + \frac{s_t}{V} = \frac{1}{\beta}\left[v_{t-1} + r_t^n - \pi_t - g_t\right]$$
$$r_t^n = \delta\left[r_t^N\right] + (1-\delta)\left[r_t^R + \pi_t\right]$$

- Bond prices and return

$$q_{N,t} = (\omega_N \beta) E_t q_{N,t} - \left[i_t\right]$$
$$q_{R,t} = (\omega_R \beta) E_t q_{R,t} - \left[i_t - E_t \pi_{t+1}\right]$$
$$r_{j,t} = (\omega_j \beta) q_{j,t} - q_{j,t-1} \qquad j = N, R$$

Fiscal Policy

- Surpluses could follow

$$h_t = \tau(\pi_t + g_t) + u_{s,t}$$

  where $u_{s,t}$ is a standard AR(1)
- No debt repayment
- News about surpluses always met by unexpected inflation

Fiscal Policy

- Surplus process

$$
\begin{aligned}
s_t &= s_t^* + (1 - v)\, h_t \\
s^* &= \alpha v_{t-1}^* + v\, h_t \\
v_{t-1}^* &= \beta \left(v_t^* + s_t^* \right)
\end{aligned}
$$

- s_t and s_t^* respond to ”debt value target” v^*

$$s_t = \alpha v_{t-1}^* + h_t$$

but not to actual debt v_t (or arbitrary $\Delta E_t \pi_t$)

Fiscal Policy

- What is the role of v_t^*?

$$s_t = s_t^* + (1 - v)\, h_t \tag{1}$$
$$s^* = \alpha v_{t-1}^* + v\, h_t \tag{2}$$
$$v_{t-1}^* = \beta \left(v_t^* + s_t^*\right) \tag{3}$$

- (2) and (3): v^* is stationary
- Solve (3) forward:

$$v_{t-1}^* = \beta \sum_{j=0}^{\infty} \beta^j \Delta E_t s_{t+j}^* = \beta \sum_{j=0}^{\infty} \beta^j \left[E_t s_{t+j} - (1 - v) E_t h_{t+j}\right]$$

Fiscal Policy

$$v^*_{t-1} = \beta \sum_{j=0}^{\infty} \beta^j \left[E_t s_{t+j} - (1 - v) E_t h_{t+j} \right]$$

- Take innovations $\Delta E_t = E_t - E_{t-1}$

$$\beta \sum_{j=0}^{\infty} \beta^j \Delta E_t s_{t+j} = (1 - v) \sum_{j=0}^{\infty} \beta^j \Delta E_t h_{t+j}$$

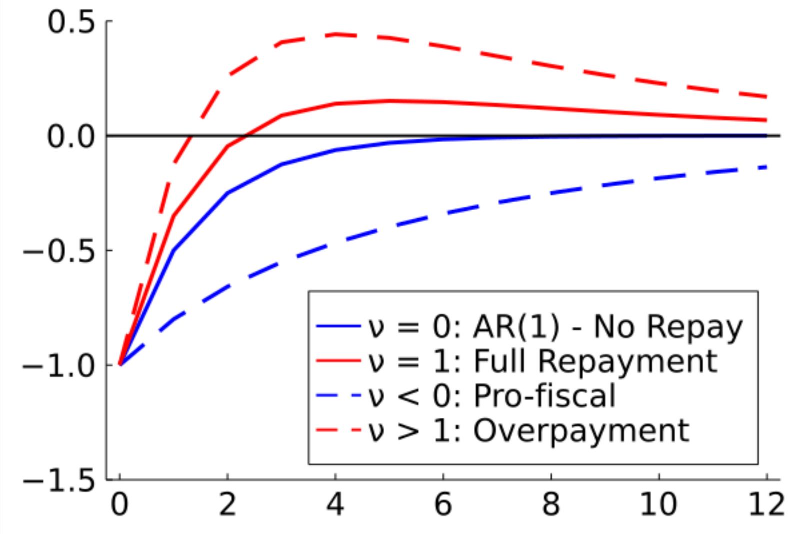
$$\boxed{\epsilon_{s,t} = (1 - v) \sum_{j=0}^{\infty} \beta^j \Delta E_t h_{t+j}}$$

- v governs debt repayment

Partial debt repayment

- $\boxed{v = 0}$ No debt repayment: $\epsilon_{s,t} = \sum_{j=0}^{\infty} \beta^j \Delta E_t h_{t+j}$
    - $s_t = h_t$ (standard AR(1))
- $\boxed{v = 1}$ Full debt repayment: $\epsilon_{s,t} = 0$
    - $s_t = s_t^* = \alpha v_t^* + h_t$
- $\boxed{v < 0}$ "Pro-fiscal" surplus: $\epsilon_{s,t} / \sum_{j=0}^{\infty} \beta^j \Delta E_t h_{t+j} > 1$
- $\boxed{v > 1}$ "Overpayment": $\epsilon_{s,t} / \sum_{j=0}^{\infty} \beta^j \Delta E_t h_{t+j} < 0$

Partial debt repayment - Cases



Response to deficit shock $u_{s,t} = -1$

GMM Estimation

- Method of moments:

$$\text{Min}_\theta \quad {\scriptstyle w} \left\| \mathscr{D}_{VAR} - \mathscr{D}_{NK}(\theta) \right\| + {\scriptstyle (1-w)} \left\| \mathscr{M} - \mathscr{M}_{NK}(\theta) \right\|$$

- $\mathscr{D}$ contains MtM decomposition for inflation shock
- $\mathscr{M}$ contains second moments
- Estimates for the United States

GMM Estimation

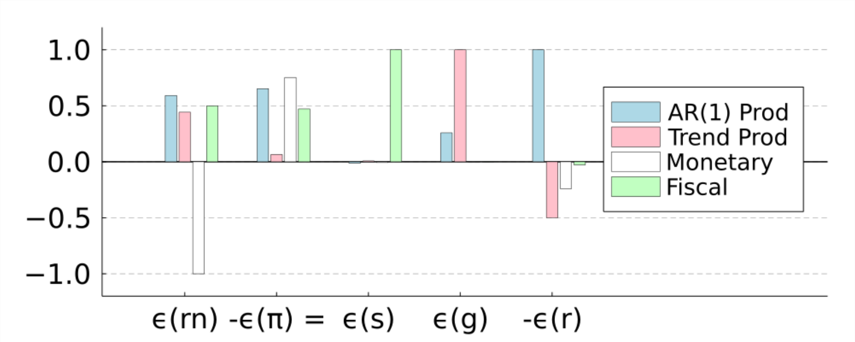
United States Estimates

Fixed		Estimated	
Parameter	Value	Paramater	Value
β	0.99	ρ_a	0.98
γ	0.4	ρ_g	0.23
φ	3	ρ_i	0.00
θ	0.25	ρ_s	0.72
$\bar{\omega}$	γ^{-1}	ϕ_π	0.68
σ_a	1	ϕ_g	0.00
		τ	-0.06
		ν	0.89
		α	0.01
		σ_g	1.21
		σ_g	0.53
		σ_g	1.07

US Model Parameters

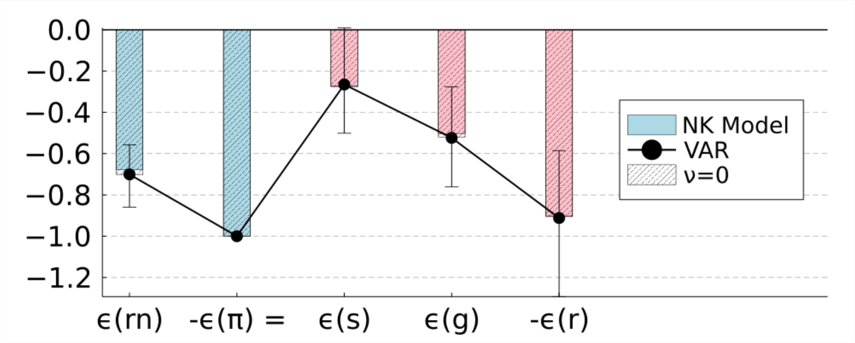
GMM Estimation

United States Estimates



Fiscal decomposition of structural shocks

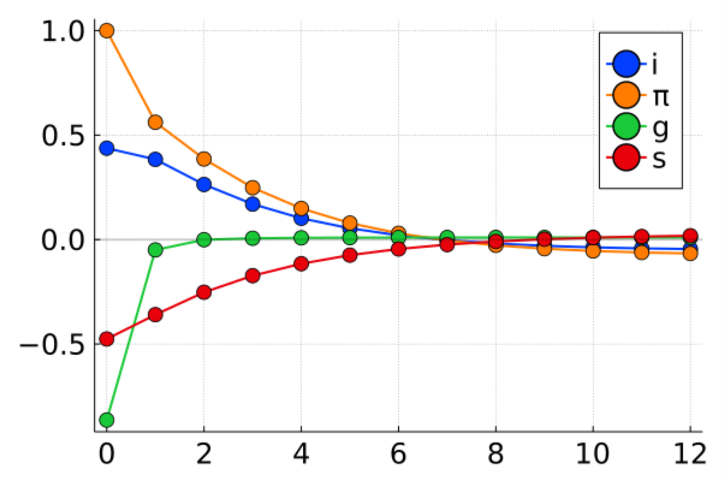
Is AR(1) surplus a good model?



MtM decomposition of Inflation Shock $e_t = E\left[e \mid \Delta E_t \pi_t = 1\right]$

Is AR(1) surplus a good model?

Structural shocks: $\varepsilon_a = -1$, $\varepsilon_g = -0.2$, $\varepsilon_i = -0.3$, $\varepsilon_s = -0.5$



Inflation Shock

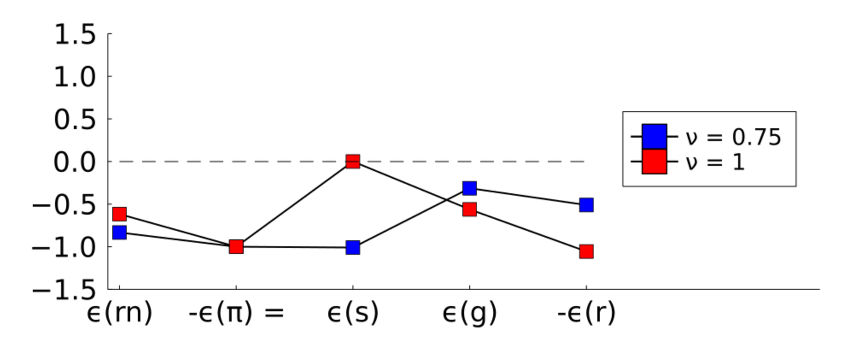
Is AR(1) surplus a good model?

- v = 0 precludes realistic fiscal policy and discount-driven inflation at the same time

	Data	$v = 0.9$	$v = 0$		Data	$v = 0.9$	$v = 0$
σ_i/σ_g	1.29	0.77	1.25	$\text{cor}(\pi, i)$	0.70	0.88	0.89
σ_π/σ_g	1.20	1.10	1.56	$\text{cor}(\pi, g)$	-0.11	-0.35	-0.40
σ_s/σ_g	1.08	1.09	0.45	$\text{cor}(g, i)$	0.04	-0.35	-0.04
$\text{acor}(i)$	0.91	0.75	0.87	$\text{cor}(i, s)$	-0.26	-0.28	-0.46
$\text{acor}(\pi)$	0.69	0.72	0.81	$\text{cor}(\pi, s)$	-0.28	-0.29	-0.41
$\text{acor}(g)$	0.14	0.14	0.16	$\text{cor}(g, s)$	0.01	-0.04	-0.05
$\text{acor}(s)$	0.64	0.72	0.27				

Second Moment Fit

Is AR(1) surplus a good model?



Comparative Statics for ν

Frametitle

-

References I

Akhmadieva, V. (2022). Fiscal adjustment in a panel of countries 1870–2016. *Journal of Comparative Economics*, 50(2):555–568.

Bassetto, M. and Cui, W. (2018). The fiscal theory of the price level in a world of low interest rates. *Journal of Economic Dynamics and Control*, 89:5–22.

Brunnermeier, M. K., Merkel, S., and Sannikov, Y. (2022). The Fiscal Theory of the Price Level With a Bubble.

Cagan, P. (1956). The Monetary Dynamics of Hyperinflation. In *Studies in the Quantity Theory of Money*, pages 25–117. University of Chicago Press, milton friedman edition.

Campbell, J. Y. and Ammer, J. (1993). What Moves the Stock and Bond Markets? A Variance Decomposition for Long-Term Asset Returns. *The Journal of Finance*, 48(1):3–37.

References II

Campbell, J. Y. and Shiller, R. J. (1988). The Dividend-Price Ratio and Expectations of Future Dividends and Discount Factors. *The Review of Financial Studies*, 1(3):195–228.

Chen, L. and Zhao, X. (2009). Return Decomposition. *Review of Financial Studies*, 22(12):5213–5249.

Cochrane, J. H. (1992). Explaining the Variance of Price–Dividend Ratios. *The Review of Financial Studies*, 5(2):243–280.

Cochrane, J. H. (1998). A Frictionless View of U.S. Inflation. *NBER Macroeconomics Annual*, 13:323–384.

Cochrane, J. H. (2005). Money as stock. *Journal of Monetary Economics*, 52(3):501–528.

Cochrane, J. H. (2008). The Dog That Did Not Bark: A Defense of Return Predictability. *The Review of Financial Studies*, 21(4):1533–1575.

References III

Cochrane, J. H. (2022a). The fiscal roots of inflation. *Review of Economic Dynamics*, 45:22–40.

Cochrane, J. H. (2022b). A fiscal theory of monetary policy with partially-repaid long-term debt. *Review of Economic Dynamics*, 45:1–21.

Cochrane, J. H. (2022c). The Fiscal Theory of the Price Level.

Corsetti, G., Dedola, L., Jarociński, M., Maćkowiak, B., and Schmidt, S. (2019). Macroeconomic stabilization, monetary-fiscal interactions, and Europe's monetary union. *European Journal of Political Economy*, 57:22–33.

Cox, W. M. (1985). The behavior of treasury securities monthly, 1942–1984. *Journal of Monetary Economics*, 16(2):227–250.

Cox, W. M. and Hirschhorn, E. (1983). The market value of U.S. government debt; Monthly, 1942–1980. *Journal of Monetary Economics*, 11(2):261–272.

References IV

Du, W., Plueger, C. E., and Schreger, J. (2020). Sovereign Debt Portfolios, Bond Risks, and the Credibility of Monetary Policy. *The Journal of Finance*, 75(6):3097–3138.

Hall, G. J. and Sargent, T. J. (1997). Accounting for the federal government's cost of funds. *Economic Perspectives*, 21(4).

Hall, G. J. and Sargent, T. J. (2011). Interest Rate Risk and Other Determinants of Post-WWII US Government Debt/GDP Dynamics. *American Economic Journal: Macroeconomics*, 3(3):192–214.

Jiang, Z., Lustig, H., Van Nieuwerburgh, S., and Xiaolan, M. (2019). The U.S. Public Debt Valuation Puzzle. Technical Report w26583, National Bureau of Economic Research, Cambridge, MA.

Leeper, E. and Leith, C. (2016). Chapter 30 - Understanding Inflation as a Joint Monetary–Fiscal Phenomenon. In *Handbook of Macroeconomics*, volume 2, pages 2305–2415. Elsevier.

References V

Leeper, E. M. (1991). Equilibria under ‘active’ and ‘passive’ monetary and fiscal policies. *Journal of Monetary Economics*, 27(1):129–147.

Sargent, T. J. and Wallace, N. (1981). Some Unpleasant Monetarist Arithmetic. *Quarterly Review*, 5.

Sims, C. A. (1994). A simple model for study of the determination of the price level and the interaction of monetary and fiscal policy. *Economic Theory*, 4(3):381–399.

Sims, C. A. (2011). Stepping on a rake: The role of fiscal policy in the inflation of the 1970s. *European Economic Review*, 55(1):48–56.

Sunder-Plassmann, L. (2020). Inflation, default and sovereign debt: The role of denomination and ownership. *Journal of International Economics*, 127:103393.

Woodford, M. (1995). Price-level determinacy without control of a monetary aggregate. *Carnegie-Rochester Conference Series on Public Policy*, 43:1–46.