Deficit Financing Under Fiscal Dominance and Its Macroeconomic Implications

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Abstract

I estimate a small open economy model operating under fiscal dominance. Two types of fiscal shocks drive primary surpluses, "Ricardian" and "non-Ricardian". Ricardian fiscal shocks come accompanied by the implicit promisse of fiscal adjustments, so that the real value of public debt remains unchanged, hence the name. Non-Ricardian ones do not, and thus lead to unexpected inflation. I use a Metropolis-Hastings sampler to draw from the posterior distribution using Brazilian data. Ricardian and non-Ricardian shocks are negatively correlated (-95%). The combination of the two innovations with opposite signs can be interpreted as revaluations of public debt in the absence of surplus jumps. Using a Cholesky factorization, I show that such "surplus-neutral" fiscal innovation accounts for about 22% of inflation variation. Non-Ricardian innovations account for 48% of surplus variation, but only 2.5% of inflation variation.

1 Introduction

The fiscal theory of the price level posits that the distribution of future primary surpluses does not validate arbitrary or "off-equilibrium" fluctuations in the price level. On the contrary, the price level is fully determined by the extent to which the size of government liabilities, issued in nominal (or "currency") units, is backed by expected real (or "in goods") surpluses.

That is not to say that under the so-called fiscal dominance paradigm unexpected movements of the primary surplus process will be empirically associated to inflation. News about today's primary surplus may or may not come accompanied by news about the discounted value of future ones. Insofar as they are, the fiscal theory predicts we should observe unexpected movements in the price level and, therefore, in the inflation rate. In this paper, I make use of this prediction to estimate a fiscal policy rule that makes clear distinction between public borrowing that is backed by the implicit promisse of future surpluses and borrowing that is not.

Using Bayesian techniques and Brazilian data, I estimate a medium-scale New Keynesian model built to fit an emerging market economy[1]. I assume a small open economy with a representative agent and a government following a monetary-fiscal policy arrangement characterized by fiscal dominance. The model contains twenty-eight endogenous variables, eight being autoregressive disturbances, nine shocks, seven fixed parameters and thirty-three parameters estimated using a Metropolis-Hastings algorithm.

The key ingredient of the model is the existence of two types of primary surplus shocks, which differ in their financing source. One of them always comes associated with the promisse of future primary surpluses in the opposite direction. I call these shocks Ricardian. The other one does not - I call them non-Ricardian. Their effect on the price level and my naming choice can be understood

[1] For the importance of fiscal policy to emerging market business cycle, see fox instance Aguiar and Gopinath (2006) Arellano (2008) and Cuadra et al. (2010).

from a simple version of the valuation equation of public liabilities, a key equation in fiscal dominance models:

$$\frac{D_{t-1}}{P_t} = S_t + \sum_{i=1}^{\infty} \beta^i S_{t+i}.$$

D_{t-1} represents the nominal value of public debt outstanding at the beginnig of period t, P_t the price level and S_t real primary surplus in consumption good units; β is an exogenous discount parameter. The equation says that the price level is associated to the extent to which nominal debt is backed by its *real value*: the discounted sum of future real surpluses.

Ricardian shocks to today's surplus S_t do not affect the present discounted value of surpluses (the left-hand side). Hence their name. A deficit today, for instance, must be followed by future surpluses so that the real value of debt is unchanged. Since D_{t-1} is predetermined, the price level does not move in response to the shock.

Non-Ricardian shocks are not accompanied by changed surpluses in the future. Therefore, the real value of debt fluctuates (hence, non-Ricardian) in the same direction as the current surplus. In response, the price level fluctuates. Essentially, private owners of public liabilities finance the shock instead of future tax payers.

Because these shocks are a direct choice of the government, I allow them to be correlated in the model. The Metropolis-Hastings sampler converges to a region of the parameter space in which they have a very negative correlation (about -95%). The simultaneous occurrence of a Ricardian and a non-Ricardian shocks with opposite signs represents a shift in the perceived value of government debt - which leads to unexpected inflation movements - *as surpluses remain constant*. For this reason, I call such combination of shocks a *surplus-neutral* fiscal innovation.

Using the estimated mode of the posterior distribution, I perform a Cholesky decomposition of the structural shocks (they are correlated!). I find one the orthogonalized innovation components resulting from the decomposition to be a close approximation of the surplus-neutral fiscal shock. A variance decomposition exercise shows that such component accounts for about 22% of Brazilian inflation variance (monetary policy shocks account for 63.5%), but only 5.8% of primary surplus variance. A different orthogonalized component, which loads mostly on non-Ricardian fiscal shocks, accounts for most variation on primary surpluses (48%), but almost none of inflation's variation (2.5%).

2 Model Equations

I state the model equations in their linearized form to save space. The underlying framework is well known, and so I skip it and leave a fully specified model to appendix A.

There are two economies: home and foreign. Like in De Paoli (2009), I study the limit case in which the size of the home economy (the measure of households) approaches zero. It contains identical households, a government, firms and a risk-neutral financial intermediary. Households purchase differentiated consumption goods from domestic and the foreign firms, hold bonds issued in domestic currency by the financial intermediary and pay lump-sum taxes. While I use the word "currency", the economy is cashless. Agents trade using accounts held with the government. Taxes are payable in balances of such accounts.

I express all variables as deviations from their steady-state values. In addition, all variables are expressed in logs, except when otherwise indicated. Time subscripts refer to the period in which variables realize. State variables in period t thus have time subscript $t-1$.

2.1 Policy

Fiscal policy is the main object of study in this paper, so I start with the equations related it. The following block contains all equations related to policy.

$$\beta d_t = d_{t-1} + r_t^D - \pi_{H,t} - \beta(Y/D)sp_t \tag{1}$$
$$\beta d_t^o = d_{t-1}^o + r_t^D - \pi_{H,t}^o - \beta(Y/D)sp_t \tag{2}$$
$$sp_t = \theta_{sy}y_t + \theta_{sd}d_{t-1}^o + u_{s,t} \tag{3}$$
$$u_{s,t} = \rho_s u_{s,t-1} + \beta^{-1}(D/Y)\left(\varepsilon_{ss,t} + \varepsilon_{s\pi,t}\right) \tag{4}$$
$$\pi_{H,t}^o = E_{t-1}\pi_{H,t} - \varepsilon_{s\pi,t} \tag{5}$$
$$r_t^D = \omega\beta q_t^D - q_{t-1}^D \tag{6}$$
$$q_t^D = -i_t + \omega\beta E_t q_{t+1}^D \tag{7}$$
$$g_t = \theta_{gy}y_t + \theta_{gd}d_{t-1}^o + u_{g,t} \tag{8}$$
$$i_t = \theta_{i\pi}\pi_{H,t} + \theta_{iy}y_t + \theta_{ie}\Delta e_t + u_{i,t} \tag{9}$$

The government taxes home households (lump sum) and purchases the domestic goods. It sells nominal debt with a geometric maturity structure: the volume of bonds expiring in period t equals ω times the volume of bonds expiring in $t-1$.

Equations (1)-(7) follow Cochrane (2020). Parameter β represents intertemporal discounting. Equation (1) provides the law of motion for the real value of public debt. It depends on the revaluation of public bonds, captured by the *ex-post* return r_t^D on the public debt portfolio, the GDP inflation rate $\pi_{H,t}$ and the real surplus to output ratio[2] (*not* in log units) sp_t. The latent variable d_t^o accumulates past bond price revaluation and surpluses in the same way the actual public debt d_t does, but it responds only to the inflation target π_t^o. For this reason, I call d_t^o the debt target.

As you can see from (5), the inflation target π_t^o equals the sum of expected inflation - pinned down by households' Euler Equation and the monetary policy rule like most monetary models - and a shock $\varepsilon_{s\pi,t}$. This shock captures unexpected movements in the current primary surplus *and* revisions to the expected discounted value of future real surpluses, which constitute the fundamental value of public liabilities. For this reason, I call it a *non-Ricardian* innovation to the surplus. A negative realization, for example, corresponds to a decline in the real value of public bonds. Given the change in their nominal price (r^D) and their (predetermined) quantities, the inflation rate does the job of repricing public debt. Just like stock prices reprice equity shares given news about future dividends (Cochrane (2005)).

Equation (3) describes the primary surplus process. It responds to output (since tax proceeds tend to be procyclical) and, critically, to the debt *target* d_t^o. As discussed by Cochrane (2020), by having surpluses respond to the debt target instead of the actual debt, we prevent fiscal policy from validating arbitrary or "sunspot" changes to the inflation rate[3]. This is where fiscal dominance shows

[2] I normalize surplus by steady-state instead of actual output: $sp_t = SP_t/Y$, where SP is surplus in domestic good units and Y is steady state home output.

[3] To see this, replace (3) in (2) to get to

$$\beta d_t^o = (1 - \beta(Y/D)\theta_{sd})d_{t-1}^o + r_t^D - \pi_t^o - \beta(Y/D)(\theta_{sy}y_t + u_{s,t}).$$

If θ_{sd} is sufficiently large, the equation above contains a stable eigenvalue. Assuming r_t^D and π_t^o are not explosive, surplus process (3) therefore stabilizes the debt target process. Now, subtract (2) from (1):

$$\beta(d_t - d_t^o) = (d_{t-1} - d_{t-1}^o) - (\pi_{H,t} - \pi_{H,t}^o).$$

Since the equation above is backward looking and contains an unstable root, the only stable solution becomes $d_t = d_t^o$

up in the model.

Disturbances to the primary surplus $u_{s,t}$ follow an AR(1) structure with two innovations, $\varepsilon_{ss,t}$ and $\varepsilon_{s\pi,t}$. We already saw that non-Ricardian innovations $\varepsilon_{s\pi,t}$ account for changes in expected discounted surpluses. The other innovation $\varepsilon_{ss,t}$ therefore captures primary surplus variation that comes unaccompanied by changes to such discounted future surpluses (or the value of debt). I call them *Ricardian* innovations to the surplus process. For instance, a negative realization of $\varepsilon_{ss,t}$ indicates a lower surplus today but a higher expected surplus at some point in the future, so that the real value of debt remains unchanged. This is why $\varepsilon_{ss,t}$ does not lead to unexpected inflation. These two shocks - Ricardian and non-Ricardian - constitute the ways the government can finance its deficit in the model: it can either credibly borrow against future surpluses or tax private wealth *via* inflation[4]. Our goal is to measure the relative size of these two innovations and their importance for the dynamics of different macroeconomic variables.

Equation (6) defines the return on the public bond portfolio. Equation (7) calculates its price based on the expectations hypothesis $i_t = E_t r^D_{t+1}$, which holds since the financial intermediary is risk neutral[5]. Equation (8) provides a rule for public spending g_t. The government demands goods only from varieties produced in the home economy. Finally, equation (9) describes the monetary policy rule, which I allow to be a function of GDP inflation, output and nominal exchange rate depreciation Δe_t.

Besides $u_{s,t}$, we have disturbances $u_{g,t}$ to public spending and $u_{i,t}$ to the target nominal rate. The law of motion for these two disturbances are:

$$u_{g,t} = \rho_g u_{g,t-1} + \varepsilon_{g,t} \tag{10}$$

$$u_{i,t} = \rho_i u_{i,t-1} + \varepsilon_{i,t}. \tag{11}$$

Shocks $\varepsilon_{ss,t}$, $\varepsilon_{s\pi,t}$, $\varepsilon_{g,t}$ and $\varepsilon_{i,t}$ are jointly Normal. Their standard deviations are, respectivelly, σ_{ss}, $\sigma_{s\pi}$, σ_g, σ_i. I allow for two non-zero correlations: $\text{cor}(\varepsilon_{s\pi,t}, \varepsilon_{ss,t})$ and $\text{corr}(\varepsilon_{s\pi,t}, \varepsilon_{i,t})$.

2.1.1 Shock Interpretation

Note that I state both $\varepsilon_{s\pi,t}$ and $\varepsilon_{ss,t}$ in log real public debt (or log expected discounted surplus) units. Hence, the $\beta^{-1}(D/Y)$ term in equation (4) (the surplus variable sp is *not* in logs).

It is useful to simplify for a moment the block of equations above to focus on the interpretation of the shocks and its propagation. Let us now assume a constant expected inflation rate, $E_t\pi_{t+1} = 0$, a constant nominal rate $i_t = 0$, and the following choice of parameters: $\beta = 1$, $Y/D = 1$, $\theta_{sy} = 0$, $\rho_s = 0$, $\omega = 0$. We then have the following three equation system

$$d_t = d_{t-1} - \pi_{H,t} - sp_t \tag{1'}$$

$$sp_t = \theta_{sd} d_{t-1} + \varepsilon_{ss,t} + \varepsilon_{s\pi,t} \tag{3'}$$

$$\pi_{H,t} = -\varepsilon_{s\pi,t}. \tag{5'}$$

determining real debt, inflation and the real primary surplus.

I now present the economy's response to three shocks. Figures 1, 2 and 3 present inflation, primary surplus and real debt dynamics following each of the three shocks. I set $\theta_{sd} = 0.5$.

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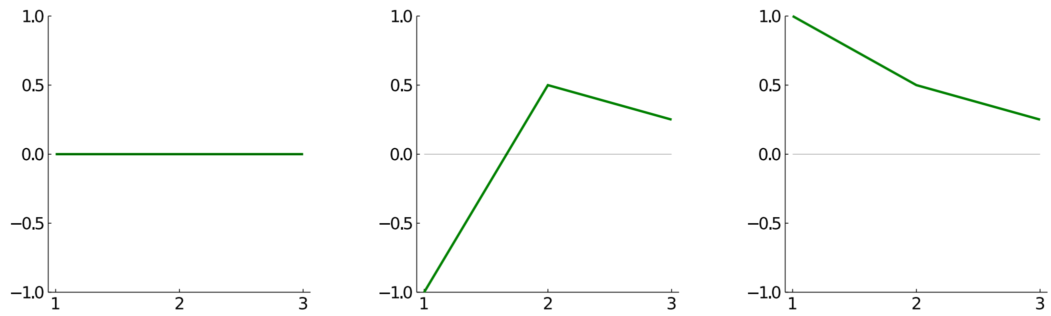
and $\pi_{H,t} = \pi^o_{H,t}$. In words, real public debt diverges for arbitrary values of inflation that do not coincide with the target. These solutions are ruled out in equilibria without (real) bubbles.

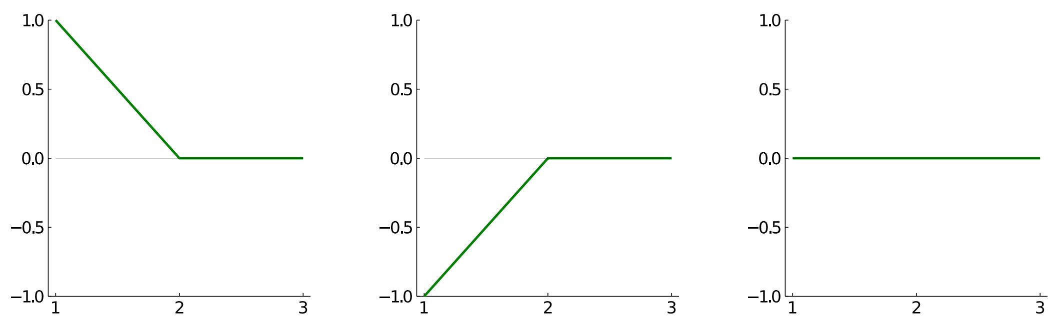
[4] I do not consider all policy tools potentially available to governments. We could consider the case of debt restructuring, haircuts or even systematic "fooling" of the private sector.

[5] Up to a first-order approximation, it would still hold if households bought public debt directly.



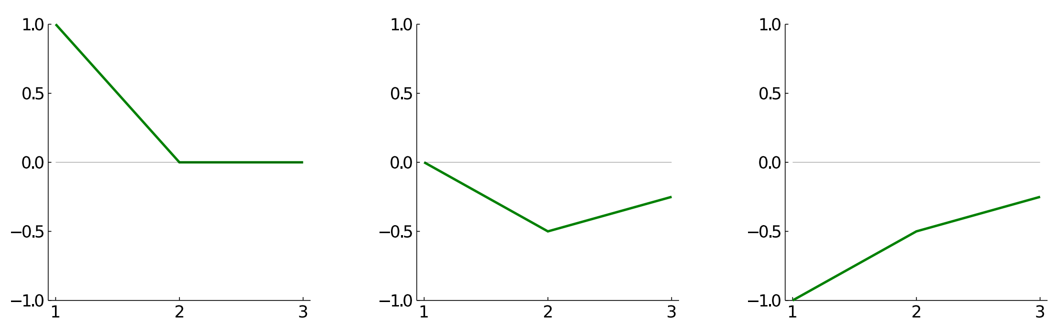
(a) Inflation (b) Surplus (c) Real Debt

Figure 1: Simplified Model - IRF to Ricardian Innovation



(a) Inflation (b) Surplus (c) Real Debt

Figure 2: Simplified Model - IRF to Non-Ricardian Innovation



(a) Inflation (b) Surplus (c) Real Debt

Figure 3: Simplified Model - IRF to "Surplus-Neutral" Innovations

Suppose the economy is in steady state in period zero: $d_0 = sp_0 = \pi_{H,0} = 0$. In $t = 1$, a Ricardian shock $\epsilon_{ss,1} = -1$ realizes. No other innovations take place. Figure 1 corresponds to this case. Because the shock is Ricardian, we know that discounted surpluses do not move - we will see how come. The fundamental value of existing debt stays constant, and hence we observe no inflation movements, $\pi_{H,1} = 0$. From equation (3') and $d_0 = 0$, we have $sp_1 = -1$: the government promotes a primary deficit in period one. By equation (1'), such deficit leads to an increase in real debt outstanding at the beginning of period 2 (d_1). Moving to period 2, we still see no inflation, but the primary surplus turns positive due to the feedback effect from the increase in real debt (the term $\theta_{sd} d_{t-1}$). The same happens in the following periods. The series of positive surpluses reduce real debt back to zero *and validate the Ricardian property of the initial shock*: the first deficit was financed by future surpluses.

Now suppose the initial shock is non-Ricardian: $\varepsilon_{s\pi,1} = -1$ and there are no other innovations. Figure 2 depicts this case. The shock represents a decrease in primary surpluses ($sp_1 = -1$) *which also decreases total discounted surpluses.* The government makes no change to promissed surpluses in the future. As beginning-of-period nominal debt is predetermined but the real value of debt goes down, inflation must increase: $\pi_{H,1} = 1$. Equation (1') indicates that beginning-of-period real debt in period two does not change, $d_1 = 0$. This is consistent with the fact that surpluses from period two onward stay the same, despite the initial period deficit. Curiously, while I call the innovation in this case "non-Ricardian", we observe no movement in the beginning-of-period real value of debt throughout the simulation. The revision of discounted surpluses in period one leads to an inflationary response which immediately re-prices public bonds.Effectively, bond holders pay for the deficit through inflation, and the government need not promote future surpluses to pay for the initial deficit.

In the absence of the other one, each of the two surplus innovations $\varepsilon_{ss,t}$ and $\varepsilon_{s\pi,t}$ have an intuitive interpretation. I now consider the economy's response to concomitant innovations in opposite directions. This case is of particular relevance as I estimate both shocks to be highly negatively correlated.

Starting from the steady state again, set $\varepsilon_{s\pi,1} = -1$ and $\varepsilon_{s\pi,1} = 1$. By (3'), the two shocks cancel each other in terms of primary surplus: $sp_1 = 0$. Hence, I call this combination of innovations "surplus-neutral". Figure 3 shows how variables respond. While in period one agents see no change to government surpluses, they perceive a decline in the discounted value of *future* surpluses: $\varepsilon_{s\pi,1} < 0$. The lower value of public debt leads to inflation $\pi_{H,1} = 1$ and, with no surplus or deficit to alter the volume of outstanding bonds, real debt at the beginning of period two declines $d_1 = -1$. Essentially, the same stock of debt is now worth less. Moving to period two, the government promotes a deficit in response to the decline in real debt: $sp_2 = \theta_{sd} d_1 = -0.5$. Such deficit increases the volume of outstanding bonds and, thus, real debt at the beginning of the period three. The same happens in the following periods so that real debt converges back to zero.

In all, surplus-neutral innovations indicate revaluations of the real value of public debt - up or down depending on the signs of the innovations - *without* observed changes to the surplus process in the same period. Through the feedback rule characterizing surpluses, such debt revaluation is later validated by the government. We can thus interpret such combination of innovations as a shock to future surpluses expected by private agents and validated by the government. The example above is also useful to make the correspondence between shock signs and the direction of perceived future surpluses. If the non-Ricardian component is negative (as in the example), agents forecast lower surpluses. A positive non-Ricardian component indicates higher expected surpluses.

2.2 Preferences and Production

The following block of equations relate to households and firm's behavior. I assume the standard monopolistic competition setting, price rigidities as in Calvo (1983) and producer currency pricing.

$$
\begin{aligned}
mu_t &= -\gamma^{-1} c_t + u_{p,t} && (12)\\
mu_t &= E_t mu_{t+1} + (i_t - E_t \pi_{t+1}) && (13)\\
\pi_{H,t} &= \beta E_t \pi_{H,t+1} + \kappa \left[(\lambda/1-\lambda) q_t + mc_t \right] && (14)\\
n_t &= (1-\alpha)^{-1}(y_t - a_t) && (15)\\
mc_t &= \gamma^{-1} c_t + \psi n_t + (1-\alpha)^{-1}(\alpha y_t - a_t) && (16)
\end{aligned}
$$

Preferences are time separable, with period utility being

$$
e^{u_{p,t}} \left[\frac{C^{1-\gamma^{-1}}}{1-\gamma^{-1}} - \frac{N^{1+\psi}}{1+\psi} \right],
$$

where N is total hours of labor supplied and C is an isoelasitc Dixit and Stiglitz (1977) aggregator with bias towards home economy goods (see appendix A for the details). The term $u_{p,t}$ captures a preference or discounting disturbance. Equation (12) relates marginal utility mu_t to consumption c_t and equation (13) is the Euler Equation.

Turning to firm behavior, equation (14) is the New-Keynesian Phillips Curve. In it, q_t is the real exchange rate, and parameter $\lambda \in (0,1)$ captures the degree market openness (see below). In addition, we have

$$
\kappa = \frac{(1-\theta)}{\theta}(1-\beta\theta)\Theta \qquad \Theta = \frac{1-\alpha}{1-\alpha+\epsilon\alpha}
$$

where θ is the probability that a firm can't reset its variety price in a given period and ϵ represents households' elasticity of substitution between different goods produced in the same economy. Firms have access to a production function of the form $A_t N^{1-\alpha}$, where $A_t = e^{a_t}$ is a technology disturbance characterized later and N is total hours of labor employed. Equation (15) provides total hours of labor hired n_t, and equation (16) states the average real marginal cost mc_t after replacing the equilibrium wage rate.

The laws of motion for the two disturbances in this block are

$$
\begin{aligned}
a_t &= \rho_a a_{t-1} + \varepsilon_{a,t} && (17)\\
u_{p,t} &= \rho_p u_{p,t-1} + \varepsilon_{p,t}. && (18)
\end{aligned}
$$

Innovations $\varepsilon_{a,t}$ and $\varepsilon_{p,t}$ are i.i.d. over time and independent from each other and from other disturbanes in the model. Their standard deviations are σ_a and σ_p.

2.3 International Goods and Capital Markets

The final block of equations relates to trade of goods and assets between home and foreign economy agents.

$$\pi_t = \pi_{H,t} + (\lambda/(1-\lambda))\,\Delta q_t \tag{19}$$

$$i_t^* = \phi b_{F,t} + u_{i,t}^* \tag{20}$$

$$q_t = E_t q_{t+1} + i_t^* - E_t \pi_{t+1}^* - i_t + E_t \pi_{t+1} \tag{21}$$

$$nx_t = y_t - (G/Y)g_t - (C/Y)c_t - (C/Y)(\lambda/1-\lambda)q_t \tag{22}$$

$$\beta b_{F,t} = b_{F,t-1} - nx_t + \Gamma\left[\beta i_t^* + \Delta e_t - \pi_{H,t}\right] \tag{23}$$

$$\Delta e_t = \Delta q_t + \pi_t - \pi_t^* \tag{24}$$

$$y_t = (C/Y)\left[\nu q_t + (1-\lambda)c_t + \lambda c_t^*\right] + (G/Y)g_t \tag{25}$$

The construction of price indices is similar to Galí and Monacelli (2005), although I assume a single foreign economy. The law of one price holds for each good individually. However, households display home bias in their preferences, and thus the real exchange rate fluctuates. Parameter λ is the weight of foreign economy goods in the utility of home economy households and vice versa, as the size of the home economy approaches zero. We can interpret it as the degree of economic openness in the international trade environment of the model.

Equation (19) captures the pass-through of real exchange rate q_t movements to consumer price inflation π_t. The financial intermediary issues foreign debt in foreign currency at an interest rate i_t^*, which responds to foreign debt $b_{F,t}$ and to a disturbance term $u_{i,t}^*$ (equation (20)). We can interpret i_t^* as the sum of the international risk-free rate to a home economy specific risk premium. Equation (21) then states a "real" version of the uncovered interest rate parity. Variable π_t^* is the foreign economy's inflation rate.

Equation (22) states the expression for net exports nx_t and (23) states the law of motion for foreign debt b_F. Both net exports and foreign debt are stated as ratios of steady state output (not in logs) and deflated by the GDP price level. Parameter Γ equals the foreign debt to output ratio in steady state. Lastly, (24) links variation in the real exchange rate to variation in the nominal exchange rate using the definition of the former, and (25) states the market clearing condition for home economy goods basket. Variable c_t^* represents foreign economy households consumption. We have $\nu = \frac{\eta\lambda}{1-\lambda} + \lambda\eta$, where η is the household elasticity of substitution between the basket of goods of the home and foreign economy (and vice versa).

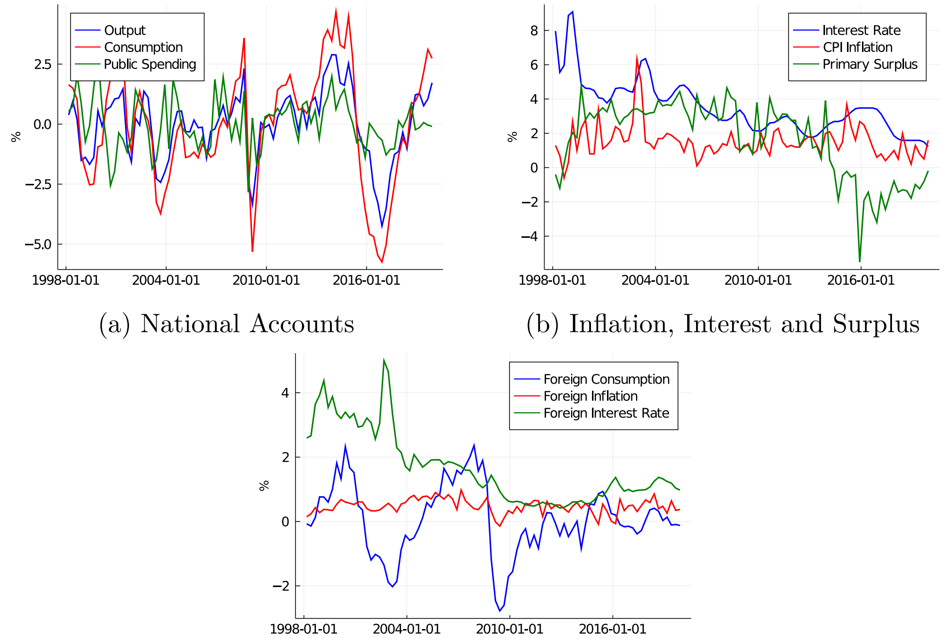
I assume an AR(1) structure to foreign consumption, foreign inflation and disturbances to foreign interest rate.

$$c_t^* = \rho_c^* c_{t-1}^* + \varepsilon_{c,t}^* \tag{26}$$

$$\pi_t^* = \rho_\pi^* \pi_{t-1}^* + \varepsilon_{\pi,t}^* \tag{27}$$

$$u_{i,t}^* = \rho_i^* u_{i,t-1}^* + \varepsilon_{i,t}^*. \tag{28}$$

The three innovations $\varepsilon_{c,t}^*$, $\varepsilon_{\pi,t}^*$ and $\varepsilon_{i,t}^*$ are i.i.d. over time and independent from each other and other shocks in the model. Their standard deviation parameters are, respectivelly, σ_c^*, σ_π^* and σ_i^*.



(a) National Accounts (b) Inflation, Interest and Surplus

(c) Foreign Sector

See appendix B for details on sources and data handling. In the plots, I multiply the two interest rate series in panels 4b and 4c by four. The two inflation series in the same panels and the primary surplus series are the rolling cumulative sum of the previous four periods.

Figure 4: Data

3 Data and Estimation Details

I estimate the model using Brazilian quaterly data for the period 1998Q1-2019Q4 (88 data points). The observed variables are: gross domestic product (y_t), aggregate consumption (c_t), public spending (g_t), CPI inflation rate (π_t), primary surplus (sp_t), nominal interest rate (i_t), foreign consumption (c_t^*), foreign inflation rate (π_t^*) and foreign interest rate (i_t^*). Section B of the appendix describes my sources, definitions and data handling in detail.

The three panels in figure 4 plot the nine observed variables, with the following modifications. I annualize the two interest rate series (panels 4b and 4c) by four. In the case of the two inflation series and the primary surplus series (panels 4b and 4c) I plot the rolling cumulative sum of the previous four periods.

Panel 4a contains national accounts data. In the computation of the consumption series I include gross capital formation, since the model does not contemplate physical capital. It is apparent that aggregate consumption is more volatile than output, a pattern found in other emerging market economies[6].

Panel 4b contains the interest rate, CPI inflation and primary surplus series. In the early 2010s, the Brazilian government starts to run primary deficits as public spending *falls*[7] This change coincides with an increase in the inflation rate and in nominal interest.

Panel 4c contains the proxies used for foreign economy variables. For consumption and inflation, I use the US GDP and GDP deflator growth. For the interest rate, I use the sum of the one-year

[6]See Aguiar and Gopinath (2007).

[7]The procyclicality of fiscal policy in developing countries is a phenomenon already identified by the macroeconomics literture. See, for instance, Kaminsky et al. (2004) or Alesina et al. (2008).

Treasury yield and the EMBI+ Brazil index.

3.1 Solution and Estimation

Given a choice for the parameters in the model, I use the generalized Schur decomposition to solve the linearized model as in Klein (2000). The procedure yields a state space representation of the system of equations. Since innovations are Normally distributed, I then use a standard Kalman filter to assess the likelihood of the observed data. There is no measurement error component in the empirical model. The (Gaussian) distribution of the initial hidden state of the Kalman filter has a mean of zero and a covariance matrix that coincides with the covariance matrix of the stationary distribution of the model.

To approximate draws from the posterior distribution, I use a random-block, random-walk Metropolis Hastings algorithm[8]. I detail in the next subsection which parameters I fix and which ones I estimate along with their prior distributions.

In each draw step of the sampler, I randomly divide the parameter space into five blocks. The composition of the five blocks changes from draw to draw. For each parameter block, I simulate a candidate point using the formula

$$\bar{\theta}^b = \theta^b_{i-1} + \eta_t \quad \text{where } \eta_t \sim N(0, k\Sigma^b)$$

where $\bar{\theta}$ is the candidate parameter vector, θ_{i-1} is the $i-1$ draw of the simulation, Σ is a positive definite matrix, k is positive scalar, and superscript b indicates that only block b parameters are being updated.

I adopt a two-step procedure. In the first step, I set Σ to be a diagonal matrix where each diagonal entry is given by the variance of the corresponding parameter's prior distribution; and $k = 0.001$ generates an acceptance rate[9] of 35%. I simulate 50,000 draws, with the initial parameter vector given by the mean of the prior distributions.

In the second step, I set Σ to be the sample covariance matrix of the first-step simulation (after discarding the first 5,000 draws), and fix $k = 0.25$ to achieve an acceptance rate of 37.5%. I simulate a sample with 500,000 draws, and discard the first 50,000 ones.

3.2 Fixed Parameters and Prior Distributions

The model contains forty parameters. Seven of them (β, α, ω, Y/D, C/Y, G/Y, Γ) I fix to calibrate the model steady-state. The rest I estimate using the procedure above.

Intertemporal discounting $\beta = 0.98002$ generates a real interest rate of approximately two percentage points (per quarter), which I calculate as the difference between average nominal interest and average inflation rates. Labor share of output $\alpha = 0.33$ is a literature standard. The public debt maturity structure parameter $\omega = 0.87$ approximates a Macaulay Duration of 20 months, which is consistent with the Brazilian government's debt structure in the last two decades. The public debt to GDP ratio I set at 44% ($\times 4$, for quarterly conversion), and the foreign debt to GDP Γ I set at 33% ($\times 4$). These ratios, along with the duration of the Brazilian public debt are based on data available on the Brazilian Central Bank website. Finally, the aggregate consumption share $C/Y = 0.816$ and public spending share $G/Y = 0.186$ are the sample averages of the corresponding ratios in my dataset.

[8]See Herbst and Schorfheide (2015) for an introduction to Bayesian estimation of economic models.

[9]The reported acceptance rate is at the block level. The rate at the draw level is obviously higher.

Parameter	Prior Distribution			Posterior Distribution			
	Family			Mean	St. Dev.	Quantiles	
		Mean	St. Dev.			5%	95%
ρ_a	Beta	0.75	0.15	0.78	0.07	0.67	0.88
γ	Gamma	0.4	0.15	0.73	0.12	0.55	0.94
		Low Limit	Up Limit				
ρ_s	Uniform	0	0.99	0.89	0.07	0.75	0.98
ρ_p	Uniform	0	0.99	0.78	0.03	0.74	0.82
ρ_g	Uniform	0	0.99	0.34	0.11	0.17	0.52
ρ_i	Uniform	0	0.99	0.95	0.02	0.92	0.98
ρ_c^*	Uniform	0	0.99	0.87	0.05	0.8	0.95
ρ_π^*	Uniform	0	0.99	0.83	0.04	0.77	0.9
ρ_i^*	Uniform	0	0.99	0.98	0.01	0.97	0.99
$\text{cor}(\varepsilon_{ss}, \varepsilon_{s\pi})$	Uniform	-0.99	0.99	-0.95	0.01	-0.97	-0.92
$\text{cor}(\varepsilon_{i}, \varepsilon_{s\pi})$	Uniform	-0.99	0.99	0.04	0.04	-0.03	0.12
σ_{ss}	Uniform	0	5	2.25	0.22	1.91	2.63
$\sigma_{s\pi}$	Uniform	0	5	2.19	0.22	1.86	2.57
σ_a	Uniform	0	5	1.04	0.09	0.91	1.2
σ_p	Uniform	0	5	4.93	0.07	4.79	5.0
σ_g	Uniform	0	5	0.96	0.07	0.85	1.09
σ_i	Uniform	0	5	0.66	0.06	0.57	0.77
σ_c^*	Uniform	0	5	0.51	0.04	0.45	0.58
σ_π^*	Uniform	0	5	0.23	0.02	0.2	0.26
σ_i^*	Uniform	0	5	0.43	0.04	0.37	0.51
ψ	Uniform	0.2	10	7.0	2.01	3.34	9.75
λ	Uniform	0	0.5	0.15	0.02	0.13	0.18
ε	Uniform	1	6	3.47	1.45	1.24	5.74
η	Uniform	0	6	5.81	0.18	5.44	5.99
θ	Uniform	0	0.99	0.4	0.09	0.24	0.55
ϕ	Uniform	0.1	1	0.11	0.01	0.1	0.12
$\theta_{i\pi}$	Uniform	0	0.8	0.12	0.08	0.01	0.28
θ_{iy}	Uniform	0	0.8	0.05	0.05	0.0	0.14
θ_{ie}	Uniform	0.5	-0.5	-0.15	0.08	-0.29	-0.01
θ_{sy}	Uniform	-2	2	0.12	0.15	-0.12	0.37
θ_{sd}	Uniform	0.1	1	0.13	0.02	0.1	0.18
θ_{gy}	Uniform	-2	2	0.31	0.1	0.16	0.47
θ_{gd}	Uniform	-1	1	-0.01	0.01	-0.03	0.01

Notes: shocks' standard deviation parameters (the σs) are in percentage points.

Table 1: Prior and Posterior Distributions

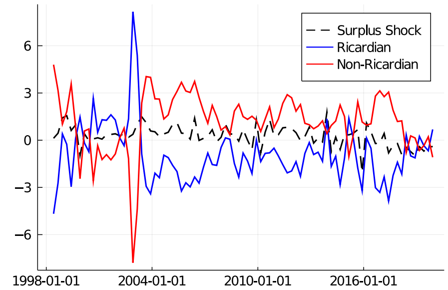


Figure 5: Fiscal Shocks Implied by Kalman Filter

Table 1 presents the prior distributions I use. Except for the intertemporal elasticity of substitution γ (Gamma centered on 0.4) and the persistence of technology shocks ρ_a (Beta around 0.75), which are well characterized by the existing macroeconomics literature, I opt for uniformly distributed priors. This choice keeps them not too informative and, thus, allows for a limited impact on my results. With a uniform distribution, these priors are completely characterized by a lower bound and an upper bound.

Starting with disturbance parameters, for the six ρ's I disregard the possibility of negative values, so that disturbances display positive autocovariance. The standard deviation of innovations (all reported in percentage points) also have a zero lower limit - they can't be negative - and an upper limit of 5%, which is sufficiently large for quarterly shocks. As for the two correlations $\varsigma_{ss,s\pi}$ and $\varsigma_{i,s\pi}$, I limit both to have an absolute value smaller than 0.99.

The Frisch elasticity of labor substitution ψ stays in the 0.2-10 range, which englobes most estimates from the literature. The degree of market openness λ has an upper limit of 50%, as the Brazilian economy is not yet too open to international trade. The two elasticity of substitution parameters have large upper limits, but in the case of the elasticity of substitution between goods of the same country ε, the lower limit I set to be one, which is a constraint of the model. The prior for the degree of price rigidity θ is completely loose: θ can take any value between 0 and 99% with equal density. As for the elasticity of the home economy's risk premium to foreign debt ϕ, I set a lower bound of 0.10 (a positive value is necessary for equilibrium determinacy) and an upper bound of 1.0.

4 Results

4.1 Posterior Sample

The right-hand side of table 1 describes the posterior distribution of estimated parameters. I report means, standard deviations and five percentage quantiles.

We are mainly interested in the properties of primary surplus shocks. Ricardian and non-Ricardin innovations have similar volatilities (2.25% *vs* 2.19% standard deviations) and are highly *negatively* correlated (-0.95). In all three cases, point estimates are highly significant compared to their estimated standard deviation. This combination of parameters suggests that innovations that approximate the surplus-neutral innovation of section 2 are of common occurrence. Thus, changes to the perceived intrinsic value of debt - expected current and future surpluses - tend to occur in the absence of forecast errors to the primary surplus series.

To better illustrate this, figure 5 plots the innovations implied by Kalman filter averages, computed using the posterior mode. Equation (5) provides the implied series for non-Ricardian innovations $\varepsilon_{s\pi,t}$. I then use (3) to infer the implied series for Ricardian ones $\varepsilon_{ss,t}$[10].

The red and blue curves represent non-Ricardian and Ricardian innovations. The dotted line represents innovations to the primary surplus series, or the sum of the blue and red curves. Clearly, implied innovations to the primary surplus series are less volatile than its two individual components. This is informative of the way the government performs fiscal policy in the model, according to our estimation. News about fiscal policy are not embedded in the forecast error of the surplus process, but in the revaluation of public liabilities *via* inflation. These revaluations are not "hollow" or pure sunspot. Recall from figure 3 that, while a surplus-neutral innovation does not lead to an unexpected surplus movement, primary surpluses *do move* in response to changes in real debt brought by higher or lower inflation. Those movements are not depicted in figure 5, but they are there!

Moreover, in a rational-expectations world such revaluations are a deliberate choice of policy by the government, not self-fulfilling prophecies. Private agents assign a lower value to public debt (leading to inflation) *when and because* the *actual* distribution of future surpluses changes to have a lower (discounted) expected value. What the results above says is that, when that happens we usually do not observe an immediate, same-period forecast error in the primary surplus series.

A few other parameters reported in table 1 deserve to be mentioned. First, all disturbances in the model are highly persistent, with the exception of those to public speding $u_{g,t}$. Second, we estimate that monetary policy shocks have (almost) no correlation with Ricardian shocks. Hence, I do not find that the government alters promissed surpluses to attain a desired unexpected inflation outcome following a monetary shock.

I estimate shocks to preferences to be the most volatile from the innovations included in the model. This is most likely a consequence of existing, but unmodelled, financial constraints to which Brazilian households are subject to. In addition, table 1 reports a moderate to low degree of price stickiness ($\theta = 0.40$ on average) and a low degree of market openness ($\lambda = 0.15$). Moving to policy parameters, I find feedback parameters low in absolute value, which suggests that policy is often dictated by discretionary choices of fiscal and monetary authorities. From these parameters, the largest one in absolute value turns out to be the feedback from output on public spending θ_{gy}. The fact that it is positive in the sample average highlights the pro-cyclical character of fiscal policy.

4.2 Policy Factors and Variance Decomposition

Which shocks drive the dynamics of the model's endogenous variables? To answer this question, I now perform a variance decomposition exercise. Because I allow policy innovations to be correlated, I first use the Cholesky factorization procedure to orthogonalize them[11].

The large negative correlation between Ricardian and non-Ricardian innovations suggests that one of the orthogonal shocks should approximate what I called in section 2 surplus-neutral innovations. It turns out that is exactly the case. Table 2 reports the weights we must use to recover the impulse response function to the orthogonalized shocks from the original, structural shocks. I do not report

[10] In the numerical computation of the model, I include expected GDP inflation $E_t\pi_{H,t+1}$ as a state variable, so that the Kalman filter returns a distribution series for it, which I use here.

[11] Let ε_t group the model's innovations in a single vector. Let $\Sigma = E(\varepsilon_t\varepsilon_t')$ be its covariance matrix. Since Σ is symmetric and positive definite, there is a unique Λ such that $\Sigma = \Lambda\Lambda'$. We can define $\eta_t = \Lambda^{-1}\varepsilon_t$, so that $E(\eta_t\eta_t') = I$ (the individual shocks in η are orthogonal to each other). Then, the following equations relate the moving average representations of the model using the original and orthogonalized innovations:

$$x_t = \sum_{i=0}^{\infty} C_i\varepsilon_t = \sum_{i=0}^{\infty} C_i\Lambda\eta_t.$$

Thus, the IRFs to the orthogonalized innovations are the linear combinations of the IRFS to the original shocks, where the weights are given by the columns of Λ.

	ε_{ss}	$\varepsilon_{s\pi}$	ε_i
Shock 1: "Surplus-Neutral Fiscal"	2.30	-2.12	
Shock 2: "Non-Ricardian Fiscal"		0.66	0.05
Shock 3: "Monetary"			0.60

Notes: The impulse response function to each of the three orthogonalized innovations above is given by the linear combination of the IRFs of the original innovation vector. The table reports the corresponding weights.

Table 2: Cholesky Orthogonalization - IRF Weights

the weights for the remaining orthogonal innovations, as each one only loads on one of the remaining non-policy structural shocks. Missing entries indicate a weight of zero.

The weights facilitate the interpretation of each of the orthogonalized innovations. For example, the economy's path in response to an isolated instance of "Shock 1" (first row of table 2) equals 2.3 times the response to the Ricardian shock minus 2.12 times the response to the non-Ricardian one. Hence, this shock approximates surplus-neutral innovations[12]. The difference rests on the slightly larger weight it puts on the Ricardian component. An analogous logic justifies my choice of names for orthogonal shock 2, described by table 2: "Non-Ricardian Fiscal Innovation". I highlight, nonetheless, that these two orthogonalized innovations are only *imperfect* depictions of their "structural" counterparts, as we see next. Shock 3 only loads on the monetary policy innovation, so it keeps its structural interpretation.

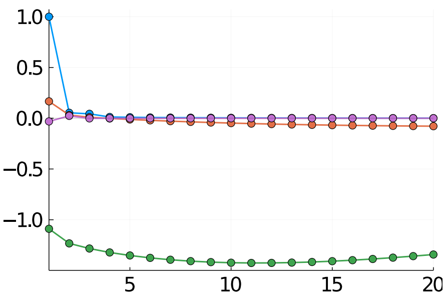
Figure 6 shows the impulse-response functions of these three orthogonalized innovations. I report the paths of four variables: inflation (blue), surplus (orange), real debt (green) and interest rate (pink). I normalize the values of the initial innovations in panel 6a and 6b to achieve an initial impact on inflation of 1%, and the value of panel 6c initial innovation to achieve a nominal interest impact of 1% as well.

Panel 6a plots the IRF to Shock 1. Except for the initial increase in surpluses (due to the larger weight we put on Ricardian innovations), the model simply adds persistence to the IRF of the simplified case depicted by figure 3. Inflation increases as the shock hits, and quickly (but not immediatly, there is price stickiness now) falls back to zero. Stated in real terms, public debt jumps down and, in response, the government runs primary deficits starting in period 4. The higher volume of bonds increases real debt over time so that it converges back to its steady-state level.

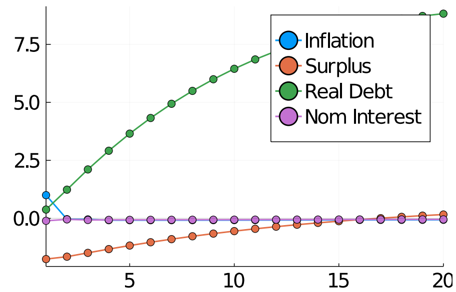
Such convergence is rather protracted (the shock half-life on debt is of 59 quarters). This is due to the low feedback effect from debt to surpluses ($\theta_{sd} = 0.1071$ in the posterior mode). Primary surpluses are never too affected by the shock in a single period. Quantitatively, for a surplus-neutral shock as depicted in figure 6a that leads to an inflation jump of one percentage point, the largest decrease in surpluses in a single period is of 0.08%. Therefore, in the economy described by the model, surplus-neutral fiscal shocks, which effectively change the perception of public debt's value, are hardly perceptible from observed government budgets. More than that, depending on the orthogonalization procedure adopted, inflation-inducing surplus-neutral shocks can come accompanied by an initial *increase* in observed surpluses, like in the case of figure 6a.

Panel 6b plots the IRFs to orthogonalized shock 2, which approximates non-Ricardian innovations. Again, the curves are similar to their counterpart in the simplified model, figure 3b. The most noticeable difference is in the persistence of the surplus process. Since the first primary deficit is followed by other deficits, real debt increases, unlike before. Panel 6c plots the IRF to the monetary policy shock. An increase in nominal interest rate leads to an increase in inflation. The large decline in real public debt follows from the decrease in the prices of public bonds, which highly affects the

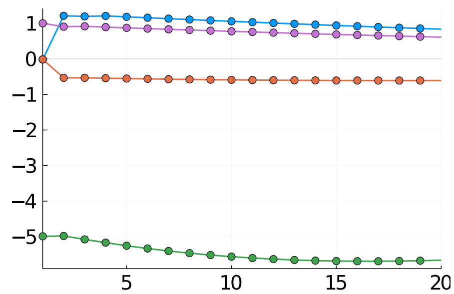
[12] Alternative ordering of the original innovations all lead to one combination which we could call "surplus-neutral".



(a) Surplus-Neutral



(b) Non-Ricardian



(c) Monetary Policy

Notes: I normalize the IRFs to the surplus-neutral and non-Ricardian fiscal shocks (panels 6a and 6b) so that the initial impact on inflation is of one percentage point. The IRF for the monetary policy shock (panel 6c) I normalize so that the nominal interest jumps one percentage point.

Figure 6: Impulse Response Function to Orthogonalized Innovations

value of the public debt portfolio given the large maturity structure.

With orthogonal shocks, we can perform a variance decomposition exercise. Table 3 reports the results for selected variables of the model and six of the nine orthogonalized shocks, which I denote η. The first two shocks η_{SN} and η_{NR} represent shocks 1 (surplus neutral) and 2 (non-Ricardian fiscal) described above. The other shocks load only on a single structural shock, in that I can recycle subscript without loss of precision. I consider two forecast horizons: a one-year horizon ($t = 5$ if shock hits in $t = 1$) to assess short-run variation, and a long-term horizon ($t = 500$ if shock hits in $t = 1$).

Starting by the inflation rate, over 75% of its variation comes from surplus-neutral fiscal shocks and monetary policy shocks, the importance of each varying depending on the forecast horizon. For short-run horizons, most variation comes from surplus-neutral fiscal innovations. Inflation response to these shocks are short-lived - it takes one period for bond holders to reassess public debt -, as figure 6a makes clear. On the other hand, forecast errors of longer horizons are mostly driven by monetary policy disturbances, which are highly persistent ($\rho_i = 0.976$ in the posterior mode).

Non-Ricardian fiscal shocks have a very low impact on inflation variation. They do, however, affect primary surpluses significantly, in both horizons reported. This is not too surprising, since surplus-neutral shocks almost perfectly combine the two fiscal shocks with opposite signs. The non-Ricardian orthogonal shock is the only remaining orthogonal shock that loads on any of the two structural fiscal innovations.

Moving on to real public debt, I find that 33.4% of one-year forecast errors are due to surplus-neutral innovations, while 45.3% follows from monetary policy ones. This is not too surprising also, as these innovations affect inflation and thus the price of public bonds. In the long run, 53.9% of their variation comes from non-Ricardian surplus innovations, which do *not* account for inflation

Variable	η_{SN}	η_{NR}	η_a	η_p	η_i	η_i^*
One Year Horizon						
Inflation Rate	52	5.1	0.8	6.1	24.6	8.9
Primary Surplus	2.3	88.6	0.1	0.4	7.9	0.6
Real Debt	33.4	11.5	0.7	3.5	45.3	4.2
Output	7.2	0.8	86.5	0.2	1	3.7
Consumption	0.8	0.1	22	16.9	0.1	54.3
Long Run						
Inflation Rate	22	2.5	0.7	5.4	63.5	4.6
Primary Surplus	5.8	47.9	1.2	9	31.1	4.3
Real Debt	12	53.9	0.9	6.6	23	3.2
Output	6.8	0.8	86.2	0.8	1	3.9
Consumption	0.6	0.7	20.7	27.8	0.1	46.5

Table 3: Forecast Error Variance Decomposition (%)

error variation too much. The second most relevant source of variation in real debt forecast error in the long run is monetary policy, which accounts for 23% of total variance. This explains why they also account for a significant 31.1% of surplus variation in the long term.

I also report in the table the decomposition of output and consumption forecast error variance. In the case of output, most variation comes from productivity shocks - a result in line with most estimates in the existing literature. This is true both in the short and long horizons. Surplus-neutral innovations account for about 7% of total variation. As for consumption, I find that shocks to the international interest rate η_i^* are responsible for about half of its total variance, in both horizons studied. Preference and productivity shocks each contribute to about 20%. The orthogonalized fiscal shocks are responsible for little variation of the consumption process. In no case studied their share is greater than one percentage point.

4.3 Model-Implied Moments

Still using the estimated posterior mode, I report in table 4 some key moments generated by the model and estimated from the data.

The model generates the empirically observed volatility of output and succeeds in generating a consumption process with higher volatility than that of output. However, it overestimates inflation and primary surplus volatilities.

In the data, GDP and primary surplus display a correlation of about 21%, while in the model such correlation is much closer to zero (an expression of the low estimated feedback from output to surpluses θ_{sd}). The model, on the other hand, generates a correlation between inflation and output similar to the one observed.

Perhaps the most flagrant flaw in the theoretical framework is the counterfactual predicted correlation between inflation and primary surpluses. While in the data, such correlation is low and positive (17.8%), the model generates a negative and large (in absolute value) correlation of -48%. As we saw, primary surpluses are mostly driven by non-Ricardian orthogonalized innovations, which drive inflation in an opposite direction. Hence, negative correlation.

The model also fails to correctly predict the correlation between inflation and nominal interest rates. The high correlation predicted by the equations follows in part from the estimated low

Moment	Data	Model	Moment	Data	Model
std(y)	1.5	1.6	cor(y, sp)	21.4	0.9
std(c)	2.4	3.6	cor(y, π)	14.1	17.3
std(π)	1.0	4.6	cor(π, sp)	17.8	-48.2
std(sp)	2.2	5.0	cor(π, i)	18.4	75.3
std(i)	1.6	2.8			

Table 4: Moments: Model *vs* Data

co-movement between monetary and non-Ricardian surplus shocks.

5 Conclusion

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Appendices

A Model Details

I present the detailed model leading to linear rational expectations model presented in the main text. Unless otherwise noted, lowecase letters indicate the log of their uppercase counterpart. For example, $p_t = \log P_t$.

There are two economies: home and foreign. The home economy contains a continuum of size $\tilde{n}$ of households and firms, a government and a representative financial intermediary. The foreign economy contains a continuum of size $1 - \tilde{n}$ of households. Agents in the model trade goods of different varieties, with each firm responsible for the production of a different one, labor hours, and different classes of assets. As currency in the home economy (henceforth "home currency"), agents use the balances in accounts held with the government. You can think that each agent has a private account with the central bank - which is part of the government - and they can use these accounts to make payments to each other *and to pay taxes*. The government accepts tax payments only in the form account balances[13], and that makes the currency have an intrinsic value. I do not model the currency in the foreign economy ("foreign currency"). Instead, I assume that the foreign inflation follows an exogenous process (equation (27)).

Turning to the asset structure, the financial intermediary issues one-period claims of two types. The first type is issued in home currency only to home economy households. The second type is issued in foreign currency to international investors. These claims finance the purchase of public debt sold by the government and equity shares of home economy firms.

The home economy is "small" in two senses. First, we consider the limit $\tilde{n} \to 0$, so that home inflation and production do not affect foreign inflation and aggregate consumption. Second, international investors demand the intermediary's claims at inelastic rate i_t^*.

[13] In fact, the fiscal theory of the price level does not preclude the government from charging taxes in goods or different currencies, as long as it is willing to trade these goods or currencies for public account balances later on.

A.1 Households

Period utility is a function of consumption indices aggregated as in Dixit and Stiglitz (1977) as well as labor hours supplied to firms. Households solve the following sequential problem:

$$
\begin{aligned}
\max_{\{C_H(j)\},\{C_F(j)\}} \quad & E_t \sum_{i=0}^{\infty} \beta^i e^{u_{p,t}} \left[\frac{C_{t+i}^{1-\gamma^{-1}}}{1-\gamma^{-1}} - \frac{N_{t+i}^{1+\psi}}{1+\psi} \right] \\
\text{s.t.} \quad & C_t = \left[(1-(1-\tilde{n})\lambda)^{\frac{1}{\eta}} C_{H,t}^{\frac{\eta-1}{\eta}} + ((1-\tilde{n})\lambda)^{\frac{1}{\eta}} C_{F,t}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \\
& C_{H,t} = \left(\frac{1}{\tilde{n}} \int_0^{\tilde{n}} C_{H,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}} \\
& C_{F,t} = \left(\frac{1}{1-\tilde{n}} \int_{\tilde{n}}^{1} C_{F,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}} \\
& \int_0^{\tilde{n}} P_{H,t}(j) C_{H,t}(j) dj + \int_{\tilde{n}}^{1} P_{F,t}(j) C_{F,t}(j) dj + \frac{B_{H,t}}{1+i_t} \leq B_{H,t-1} + W_t N_t - \hat{T}_t \\
& \frac{B_{H,t}}{1+i_t} \geq \text{Natural Debt Limit}
\end{aligned}
\tag{29}
$$

where $C_H(j)$ indicates consumption of variety j, produced by the home economy, $C_F(j)$ indicates consumption of variety j, produced by the foreign economy, B_H is the amount of bonds puchased from the intermediary at a price of $1/(1+i_t)$ and $\hat{T}$ are lump-sum taxes.

We can define the following price indices, which define the minimum cost P_t of purchasing one aggregate consumption unit in period t.

$$
\begin{aligned}
P_{H,t} &= \left(\frac{1}{\tilde{n}} \int_0^{\tilde{n}} P_{H,t}(j)^{1-\varepsilon} ds \right)^{\frac{1}{1-\varepsilon}} \qquad P_{F,t} = \left(\frac{1}{1-\tilde{n}} \int_{\tilde{n}}^{1} P_{F,t}(j)^{1-\varepsilon} ds \right)^{\frac{1}{1-\varepsilon}} \\
P_t &= \left[(1-(1-\tilde{n})\lambda) P_{H,t}^{1-\eta} + (1-\tilde{n})\lambda P_{F,t}^{1-\eta} \right]^{\frac{1}{1-\eta}}
\end{aligned}
$$

First-order conditions for $C_{H,t}$, $C_{H,t}(j)$, $C_{F,t}$ and $C_{F,t}(j)$ yield:

$$
\begin{aligned}
C_{H,t}(j) &= \left(\frac{P_{H,t}(j)}{P_{H,t}} \right)^{-\varepsilon} C_{H,t} \qquad C_{F,t}(j) = \left(\frac{P_{F,t}(j)}{P_{F,t}} \right)^{-\varepsilon} C_{F,t} \\
C_{H,t} &= (1-(1-\tilde{n})\lambda) \left(\frac{P_{H,t}}{P_t} \right)^{-\eta} C_t \qquad C_{F,t} = (1-\tilde{n})\lambda \left(\frac{P_{F,t}}{P_t} \right)^{-\eta} C_t.
\end{aligned}
$$

Along with the definition of the price indices, they imply that $\int_0^{\tilde{n}} P_{H,t}(j) C_{H,t}(j) dj = P_{H,t} C_{H,t}$, $\int_0^{\tilde{n}} P_{F,t}(j) C_{F,t}(j) dj = P_{F,t} C_{F,t}$ and $P_t C_t = P_{H,t} C_{H,t} + P_{F,t} C_{F,t}$.

The remaining first-order conditions are:

$$
\begin{aligned}
MU_t &\equiv e^{u_{p,t}} C_{t+i}^{-\frac{1}{\gamma}} \\
MU_t &= \beta(1+i_t) E_t \left\{ MU_{t+1} / \Pi_{t+1} \right\} \\
\frac{W_t}{P_t} &= C_t^{\frac{1}{\gamma}} N_t^{\psi},
\end{aligned}
$$

which linearize to

$$
\begin{aligned}
mu_t &= -\frac{1}{\gamma} c_t + u_{p,t} \\
mu_t &= E_t \; mu_{t+1} - \gamma \left[i_t - E_t \pi_{t+1} - \rho \right] \\
w_t - p_t &= \frac{1}{\gamma} c_t + \psi n_t
\end{aligned}
\tag{15}
$$

using $\rho = -\log \beta$ and $i_t \approx \log(1+i_t)$.

A.2 The Real Exchange Rate

Preferences for foreign economy households are analogous to home economy households'. Therefore, their demand schedule satisfy:

$$C^*_{H,t}(j) = \left(\frac{P^*_{H,t}(j)}{P^*_{H,t}}\right)^{-\varepsilon} C^*_{H,t} \qquad C^*_{F,t}(j) = \left(\frac{P^*_{F,t}(j)}{P^*_{F,t}}\right)^{-\varepsilon} C_{F,t}$$
$$C^*_{H,t} = \tilde{n}\lambda \left(\frac{P^*_{H,t}}{P^*_t}\right)^{-\eta} C^*_t \qquad C^*_{F,t} = (1-\tilde{n}\lambda)\left(\frac{P^*_{F,t}}{P^*_t}\right)^{-\eta} C^*_t.$$

We can also define price indices P^*_H, P^*_F and P^* for the foreign economy the same we do for the home economy.

Let $\mathcal{E}_t$ be the nominal exchange rate: the price of one unit foreign currency in units of home currency. The law of one price $P_{i,t}(j) = \mathcal{E}_t P^*_{i,t}(j)$, $i = H, F$ holds for each individual good. This leads to $P_{H,t} = \mathcal{E}_t P^*_{H,t}$ and $P_{F,t} = \mathcal{E}_t P^*_{F,t}$. However, due to home bias, $P_t \neq \mathcal{E}_T P^*_t$, and so the real exchange $Q_t = \mathcal{E}_t P^*_t / P_t$ fluctuates. The foreign economy's price level is $P^*_t = \lambda\tilde{n}P^*_{H,t} + (1-\lambda\tilde{n})P^*_{F,t}$. As the size of the home economy $\tilde{n}$ converges to zero, we have $P^*_t = P^*_{F,t} = P_{F,t}/\mathcal{E}_t$. Therefore, in this limit we have $Q_t = P_{F,t}/P_t$.

Define the home economy's terms of trade as $ToT_t = P_{F,t}/P_{H,t}$, and $tot_t = \log ToT_t = p_{F,t} - p_{H,t}$. Linearizing the definition of P_t and taking the limit $\tilde{n} \to 0$ yields

$$p_t = p_{H,t} + \lambda tot_t, \tag{30}$$

an equation that captures terms of trade passthrough to consumer prices. It also allows a connection between terms of trade and the real exchange rate. Linearization of Q_t gives $q_t = p_{F,t} - p_t$, which combines with (30) to yield

$$q_t = (1-\lambda)tot_t. \tag{31}$$

We state the model mostly in terms of the real exchange rate, and so equation (31) effectively defines terms of trade. Differentiate (30) and use (31) to arrive at

$$\pi_t = \pi_{H,t} + \frac{\lambda}{1-\lambda}\Delta q_t, \tag{19}$$

(remember that we assume the existence of home bias: $\lambda < 1$). If instead $\lambda = 1$, there is no home bias, $\pi_t = \pi_{F,t}$ and thus $q_t = 0$.

Finally, we again use the linearization of the real exchange rate definition $q_t = e_t + p^*_t - p_t$, where $e_t = \log \mathcal{E}_t$. Taking first differences, we have

$$\Delta q_t = \Delta e_t + \pi^*_t - \pi_t, \tag{24}$$

an equation we use to determine nominal exchange rate depreciation Δe_t (the nominal exchange rate itself is a non-stationary variable).

A.3 The Government

The government is responsible for fiscal and monetary policy. It uses proceeds from taxation $\hat{T}_t = P_{H,t}T_t$ and from borrowing at different maturities to finance a stream $P_{H,t}G_t$ of public spending. Let $SP_t = T_t - G_t$ be the real primary surplus.

Let $D_{i,t}$ be the volume of public bonds outstanding in period t to be paid i periods ahead and $Q^D_{i,t}$ its price. In their due period, each promisses the payment of one unit of home currency, and so $Q^D_{0,t} = 1$ for any t. Public debt evolves according to

$$\sum_{i=1}^{\infty} Q^D_{i-1,t} D_{i,t-1} = P_{H,t} SP_t + \sum_{i=1}^{\infty} Q^D_{i,t} D_{i,t}.$$

Let $D_t = \sum_{i=1}^{\infty} Q^D_{i,t} D_{i,t}$ be the end-of-period value of public debt. The return on the public debt portfolio is

$$1 + r^D_t = \frac{\sum_{i=1}^{\infty} Q^D_{i-1,t} D_{i,t-1}}{\sum_{i=1}^{\infty} Q^D_{i,t-1} D_{i,t-1}} = \frac{\sum_{i=1}^{\infty} Q^D_{i-1,t} D_{i,t-1}}{D_{t-1}}.$$

Therefore, we can re-write the law of motion of public debt:

$$D_{t-1}(1+r_t^D) = P_{H,t}SP_t + D_t$$
$$\implies \hat{D}_{t-1}\frac{1+r_t^D}{\Pi_{H,t}} = SP_t + \hat{D}_t$$

where $\hat{D}_t = D_t/P_{H,t}$ is the value of the public debt in terms of home goods and $\Pi_{H,t} = P_{H,t}/P_{H,t-1}$ is GDP inflation. Let $sp_t = SP_t/Y$ be the primary surplus expressed as a ratio of steady state output. Throughout our linearizations, we use the approximation $\log(1+i) = i$ for the nominal interest rate and similar approximations for other asset returns. Linearization of the equation above yields

$$\beta d_t = (1-\beta)(i+d) + d_{t-1} + r_t^D - \pi_{H,t} - \beta\frac{Y}{D}sp_t.$$

The expression above corresponds to (1).

I assume a geometric decaying maturity structure, at a rate ω, meaning that $D_{i,t} = \omega D_{i-1,t}$ for any given $D_{1,t}$. Let

$$Q_t^D = \sum_{i=1}^{\infty}\omega^{i-1}Q_{i,t}^D$$

be the price of the public bond portfolio. Thus, we have $D_t = Q_t^D D_{1,t}$. In addition, its return respects the following equations.

$$\begin{aligned}
1+r_{t+1}^D &= \frac{\sum_{i=1}^{\infty}Q_{i-1,t+1}^D D_{i,t}}{\sum_{i=1}^{\infty}Q_{i,t}^D D_{i,t}} \\
&= \frac{\sum_{i=1}^{\infty}Q_{i-1,t+1}^D D_{i,t}}{D_t} \\
&= \frac{D_{1,t} + D_{1,t}\sum_{i=2}^{\infty}Q_{i-1,t+1}^D\omega^{i-1}}{Q_t^D D_{1,t}} \\
&= \frac{D_{1,t} + \omega D_{1,t}\sum_{j=1}^{\infty}Q_{j,t+1}^D\omega^{j-1}}{Q_t^D D_{1,t}} \\
&= \frac{1+\omega Q_{t+1}^D}{Q_t^D}.
\end{aligned}$$

Linearize it to find

$$r_{t+1}^D = i - \left(\frac{1+i-\omega}{1+i}\right)q^D + \frac{\omega}{1+i}q_{t+1}^D - q_t^D,$$

which corresponds to (6) in the demeaned model.

A.3.1 Monetary and Fiscal Policy

The system of equations needs two equations determining what fiscal and monetary policy look like. We write these equations directly in log-linearized form.

Log-linerization of $SP_t = T_t - G_t$ leads to

$$sp_t = \frac{T}{Y}\log T_t - \frac{G}{Y}g_t.$$

We assume that g_t follows an independent AR(1). Given the g process, lump-sum tranfers are such that primary surpluses follow

$$sp_t = sp + \theta_{sy}y_t + \theta_{sd}d_t^o + u_{s,t}$$

where $u_{s,t}$ is the disturbance process specified in the main text (with the two innovation types).

Monetary policy is described by a feedback rule of the form

$$i_t = i + \theta_{i\pi}\pi_{H,t} + \theta_{iy}y_t + \theta_{ie}\Delta e_t + u_{i,t}$$

where $u_{i,t}$ is a disturbance term.

A.4 Financial Intermediary

The representative financial intermediary is risk neutral. It sells one period, risk-free bonds to households at a price $(1+i_t)^{-1}$ and foreign debt to international investors at a price $(1+i_t^*)^{-1}$, which incorporates a risk premium component. It then uses the revenue to purchase government bonds of different maturities and firms' equity.

The intermediary's optimization problem is

$$\max_{\{D_{i,t}\},B_{H,t},B_{F,t},F_t} \quad E_t\left\{P_t C_t^I\right\} = E_t\left\{\sum_{i=1}^{\infty} Q_{i-1,t+1}^D D_{i,t} + (P_{t+1}^Q + \Pi_{t+1}^Q)F_t\right\} - B_{H,t} - \mathcal{E}_{t+1}B_{F,t}$$

$$\text{s.t.} \quad \sum_{i=1}^{\infty} Q_{i,t}^D D_{i,t} + P_t^Q F_t = \frac{B_{H,t}}{1+i_t} + \mathcal{E}_t\frac{B_{F,t}}{1+i_t^*}$$

where P^Q is the price of the basket of firms' shares and Π_t^Q is aggregate profit in period t.

Unless the four assets have the same expected return i_t, the solution to the intermediary's problem leads to infinite demand of the asset with largest expected return, which is inconsistent with equilibrium. Therefore, all assets must provide an expected return i_t:

$$1+i_t = \frac{1}{Q_{1,t}^D} = \frac{Q_{i-1,t}^D}{Q_{i,t}^D} = (1+i_t^*)\frac{E_t\mathcal{E}_{t+1}}{\mathcal{E}_t} = \frac{E_t(P_{t+1}^Q + \Pi_{t+1}^Q)}{P_t^Q}. \tag{32}$$

The relationship above implies that the expected and steady-state period profit for the financial intermediary equals zero. Unexpected profits are distributed to households. We call those the financial intermediary consumption.

Equation (32) implies that the expected yield of the public bonds portfolio is also equal to i_t:

$$E_t(1+r_{t+1}^D) = 1+i_t,$$

which leads to equation (7) when we combine it with (6).

Equation (32) also contains the uncovered interest parity. Along with (24), it implies

$$(i_t - E_t\pi_{t+1}) = i_t^* - E_t\pi_{t+1}^* + E_t\Delta q_{t+1},$$

which corresponds to equation (21) in the main text.

Finally, international investors inelastically demand a return on their investment of i_t^* given by the expression below.

$$1+i_t^* = (1+i^*)\left(\frac{B_{F,t}Q_t}{P_t^*}\frac{P}{B_F}\right)^{\phi}\exp(u_{i,t}^*),$$

The specification above allows us to represent a risk-premium demanded by foreign investors that varies with the size of the foreign debt relative to its steady state level. I do not provide a structural model of risk for the claims issued by the financial intermediary and there is no default in the model. However, different studies (such as Neumeyer and Perri (2005), Calvo et al. (2006), Mendoza (2010)) argue that changes in international financing conditions are critical to explain economic activity in emerging markets. The model captures these changes with the disturbance term $u_{i,t}^*$ and the sensitivity of demanded interest to outstanding foreign debt. In addition, such sensitivity is also important to leave the foreign debt process stationary. Linearization gives

$$i_t^* = i^* + \phi\left(b_{F,t} - bF\right) + u_{i,t}^*$$

where $b_{F,t} = \log(Q_t B_{F,t}/P_t^*)$ is the log real face value of foreign debt, expressed in units of home consumption goods.

A.5 Price-Setting and Firm Behavior

Each firm has access to a production function $A_t N^{1-\alpha}$ to produce its own variety, where N is total labor hours employed. Prices are sticky like in Calvo (1983). It is useful to state the demand for a home firm's variety sold at price p in period t:

$$\begin{aligned} Y_t^p &= \left(\frac{p}{P_{H,t}}\right)^{-\varepsilon}\left\{\left(\frac{P_{H,t}}{P_t}\right)^{-\eta}\left[(1-(1-n)\lambda)C_t + n\lambda\left(\frac{1-n}{n}\right)\left(\frac{1}{Q_t}\right)^{-\eta}C_t^*\right] + G_t\right\} \\ &= \left(\frac{p}{P_{H,t}}\right)^{-\varepsilon}\chi_t \end{aligned} \tag{33}$$

where χ_t is the demand for a variety whose price equals the average price level in the home economy.

Firms readjust their price with probability θ. The resetting firm chooses the new price level to solve the optimization problem[14]

$$\begin{aligned} \max_p \quad & \sum_{i=0}^{\infty} (\theta\beta)^i E_t \left[\frac{P_t}{P_{t+i}} \left(p Y_{t+i}^p - \Psi_{t+i}(Y_{t+i}) \right) \right] \\ \text{s.t.} \quad & Y_{t+i}^p = \left(\frac{p}{P_{H,t+i}} \right)^{-\varepsilon} \chi_{t+i} \end{aligned}$$

where $\Psi_t(y) = W_t(y/A_t)^{\frac{1}{1-\alpha}}$ is the nominal cost function. The first-order condition is

$$\sum_{i=0}^{\infty} (\theta\beta)^i E_t \left[\frac{P_t}{P_{t+i}} Y_{t+i}^{\hat{P}} \left(\hat{P} - \frac{\varepsilon}{\varepsilon - 1} P_{t+i} MC'_{t+i}(Y_{t+i}^{\hat{P}}) \right) \right] = 0$$

where $MC_t(y) = \Psi'_t(y)/P_t$ is the real marginal cost function (in terms of the household consumption goods). Its log-linearized version is

$$(1 - \beta\theta)^{-1} (\hat{p} - \mathcal{M}) = \sum_{i=0}^{\infty} (\beta\theta)^i E_t \left\{ p_{t+i} + mc_{t+i}^{\hat{p}} \right\} \tag{34}$$

where $\mathcal{M} = \log(\varepsilon/(\varepsilon - 1))$ is the desired markup over marginal costs.

A.6 Equilibrium

In equilibrium, the supply of each variety equals households' demand. Using (33):

$$Y_t(j) = \left(\frac{P_{H,t}(j)}{P_{H,t}} \right)^{-\varepsilon} \chi_t.$$

$Y(j)$ refers to home firms' production; I drop the subscript H since I do not work with foreign output. Integrate the equation above to arrive at

$$Y_t = \frac{1}{\tilde{n}} \int_0^{\tilde{n}} Y_t(j) dj = \chi_t = \left\{ \left(\frac{P_{H,t}}{P_t} \right)^{-\eta} \left[(1 - (1 - \tilde{n})\lambda) C_t + \tilde{n}\lambda \left(\frac{1-n}{n} \right) \left(\frac{1}{Q_t} \right)^{-\eta} C_t^* \right] + G_t \right\}, \tag{35}$$

which implies $Y_t(i) = (P_{H,t}(i)/P_{H,t})^{-\varepsilon} Y_t$. We log linearize (35) around a symmetric steady state, meaning a steady state in which $Q = 1$ and $C = C^*$:

$$y_t = \left[y - \frac{C}{Y} c - \frac{G}{Y} g \right] + \frac{C}{Y} \left[\nu q_t + (1 - \lambda) c_t + \lambda c_t^* \right] + \frac{G}{Y} g_t, \tag{36}$$

where parameter $\nu = \frac{\eta\lambda}{1-\lambda} + \lambda\eta$ captures the effect of real exchange rate depreciation on total demand for home goods. The first term $\eta\lambda/(1 - \lambda)$ represents the change in the relative price of output to consumption goods $P_t/P_{H,t}$. The second term $\eta\lambda$ represent the change in the relative price of consumption goods between the two economies (P_t and P_t^*) *given* the change in $P_t/P_{H,t}$, which exists due to home bias.

In equilibrium, we also have that supply of labor hours must be equal to firms' demand:

$$N_t = \frac{1}{\tilde{n}} \int_0^{\tilde{n}} N_t(i) di = \left(\frac{Y_t}{A_t} \right)^{\frac{1}{1-\alpha}} \left[\frac{1}{\tilde{n}} \int_0^{\tilde{n}} \left(\frac{P_{H,t}(j)}{P_{H,t}} \right)^{-\frac{\varepsilon}{1-\alpha}} dj \right]. \tag{37}$$

Log-linearization of (37) yields

$$n_t = (1 - \alpha)^{-1} (y_t - a_t), \tag{15}$$

as the dispersion term in brackets in (37) equals zero up to a first-order approximation around the perfect foresight steady state. Equation (15) determines hired labor hours.

[14] I am ignoring the existence of any non-degenerate stochastic discount factor. To a first-order approximation, such factor would be irrelevant to the optiomal choice of price. Moreover, firms in the model are owned by the risk-neutral financial intermediary.

The log real marginal cost of the individual firm with price $\hat{p}$ in t is

$$\begin{aligned} mc_t^{\hat{p}} &= -\log(1-\alpha) + w_t - p_t + (1-\alpha)^{-1}\left[\alpha y_t^{\hat{p}} - a_t\right] \\ &= mc_t - \frac{\varepsilon\alpha}{1-\alpha}\left(\hat{p} - p_{H,t}\right) \end{aligned}$$

where mc_t is the average log real marginal cost of domestic firms:

$$\begin{aligned} mc_t &= -\log(1-\alpha) + w_t - p_t + (1-\alpha)^{-1}(\alpha y_t - a_t) \\ &= -\log(1-\alpha) + \frac{1}{\gamma}c_t + \psi n_t + (1-\alpha)^{-1}(\alpha y_t - a_t) \end{aligned}$$

(I use equation (15)).

Replacing the expression for the firm's marginal cost above in its first-order condition (34), we get

$$(1-\beta\theta)^{-1}\hat{p}_t = \sum_{i=0}^{\infty}(\beta\theta)^i E_t\left\{p_{H,t+i} + \Theta\left[\frac{\lambda}{1-\lambda}q_{t+i} + mc_{t+i} + \mathcal{M}\right]\right\}$$

where $\Theta = \frac{1-\alpha}{1-\alpha+\varepsilon\alpha} \leq 1$ captures the existence of decreasing returns to scale in firms' production function, and the $(\lambda/(1-\lambda))q = p_t - p_{H,t}$ term accounts for the fact that we state firms' real marginal cost *in units of household consumption good*, not domestic firms' output (we divide nominal cost by P_t, not $P_{H,t}$). For any given price of the aggregate domestic goods P_H, real depreciations increase firms' real marginal cost measured in units of consumption goods.

The pricing condition above, together with the first-order approximation $\pi_{H,t} = (1-\theta)(\hat{p}_t - p_{H,t-1})$ gives the New-Keynesian Phillips Curve.

$$\pi_{H,t} = \beta E_t \pi_{H,t+1} + \kappa\left[\frac{\lambda}{1-\lambda}q_t + mc_t + \mathcal{M}\right],$$

where parameter $\kappa = \frac{(1-\theta)(1-\beta\theta)\Theta}{\theta}$ increases for less sticky prices (low θ). This is equation (14) in the main system.

Integrating the household's budget contraint yields

$$P_{H,t}C_{H,t} + P_{F,t}C_{F,t} + \frac{B_{H,t}}{1+i_t} = B_{H,t-1} + W_tN_t - \hat{T}_t.$$

Replace the financial intermediary's consumption C_t^I:

$$P_{H,t}C_{H,t} + P_{F,t}C_{F,t} + P_tC_t^I - W_tN_t + \hat{T}_t = \left[(1+r_t^D)D_{t-1} - D_t\right] + \left[(P_t^Q + \Pi_t^Q)F_{t-1} - P_t^Q F_t\right] + \left[\frac{\mathcal{E}_t B_{F,t}}{1+i_t^*} - \mathcal{E}_t B_{F,t-1}\right]$$

Share of firms' equity is in $(1/\tilde{n})$ supply, so aggregate profits are $\Pi_t^Q/\tilde{n} = P_{H,t}Y_t - W_tN_t$. Replacing above, along with the law of motion of the government's debt, we have:

$$P_{H,t}C_{H,t} + P_{F,t}C_{F,t} + P_tC_t^I + P_{H,t}G_t = P_{H,t}Y_t + \left[\frac{\mathcal{E}_t B_{F,t}}{1+i_t^*} - \mathcal{E}_t B_{F,t-1}\right]$$

which we can re-write as

$$\frac{P_t}{P_{H,t}}(C_t + C_t^I) + G_t = Y_t + (1+i_t^*)^{-1}\frac{\mathcal{E}_t B_{F,t}}{P_{H,t}} - \frac{\mathcal{E}_{t-1}B_{F,t-1}}{P_{H,t-1}}\Delta\mathcal{E}_t\frac{P_{H,t-1}}{P_{H,t}}$$

or even

$$-nx_t = (1+i_t^*)^{-1}b_{F,t} - b_{F,t-1}\Delta\mathcal{E}_t\frac{P_{H,t-1}}{P_{H,t}} \tag{38}$$

where $nx_t = (Y_t - G_t - P_tC_t/P_{H,t})/Y$ and $b_{F,t} = \mathcal{E}_t B_{F,t}/(P_{H,t}Y)$ are, respectively, the trade balance and the stock of foreign debt, both expressed in terms of domestic output and as a ratio of steady-state output. As these quantities can take negative values, I do not take logs. We linearize (38) around a symmetric perfect foresight steady state. Perfect foresight implies that the financial intermediary's consumption C^I equals zero.

$$\beta b_{F,t} = -nx_t + b_{F,t-1} + \Gamma\left(\beta i_t^* + \Delta e_t - \pi_{H,t}\right), \tag{23}$$

where $\Gamma = \mathcal{E}B_F/PY$ is the steady-state level of foreign debt to output.

Finally, we linearize the trade balance formula, stated in the previous paragraph.

$$nx_t = [Y - C - G - Yy + Gg + Cc]/Y + y_t - \frac{G}{Y}g_t - \frac{C}{Y}c_t - \frac{C}{Y}\frac{\lambda}{1-\lambda}q_t.$$

A.7 The Perfect Foresight Steady State

I consider a zero inflation steady state, symmetric in the sense that $Q = 1$ and $C = C^*$. In it, $(1+i)^{-1} = (1+i^*)^{-1} = \beta$. Demanded return on foreign debt i^* can contain risk premium component compared to international risk-free rate.

B Data Sources and Handling

Data from the Brazilian Institute of Geography and Statistics (IBGE) and the Brazilian Central Bank (BCB).

National Accounts From IBGE. Output measured in 1995 Brazilian reals, with seasonality adjustment. I define consumption as the sum of household consumption and fixed capital formation. The statistic discrepancy between output and the sum of its demand components is 2.5% of its measured value, on average. To keep consistency, I define gross domestic product as the sum of consumption, public spending and net exports. To convert data national account series into model units, I take logs and apply a standard HP filter with smoothing parameter 1600 to the output, consumption and public spending series. The net exports series is given by the division of the observed net exports series and the output trend calculated by the HP filter.

GDP Deflator IBGE does not provide a deflator series. I calculate the deflator as the ratio between nominal GDP and GDP measured in 1995 Brazilian reals. Since the nominal GDP series is not adjusted for seasonality, I use the constant price series that is also unadjusted. Finally, I use the U.S. Census Bureau's *X-13 ARIMA-SEATS* to filter out the seasonality effect from the log deflator series.

Primary Surplus Series provided by the BCB in its time series database[15] under code 4649. I divide the raw data by the unadjusted deflator series (see above), and remove seasonal effects using the X-13 ARIMA-SEATS algorithm. Finally, I divide the resulting series by the output trend (calculation described in the main text).

CPI I use the percentage change in *IPCA*, calculated and adjusted for seasonality by IBGE. I transform the monthly series into a quarterly one by converting percentage to log growth and summing the rates of the three months contained in any given quarter.

Labor Hours Lacking better data, I define $N = \log(1-u)$, where u is unemployment. Measures of unemployment in Brazil have changed over time. Current estimates come from PNADc (National Survey of Household Samples), which starts in 2012. For prior months, I use the predicted values of the linear model $u_t^{PNAD} = a + bu_t^{SP} + \varepsilon$, where u_t^{SP} is unemployment measured in the metropolitan region of Sao Paulo, which is available starting December 1984. After that, I remove the seasonal component - which the sources don't - and define the quarter value as the average of the three corresponding months.

Nominal Interest I use the effective SELIC rate, the policy rate targetted by the BCB. The SELIC rate is the average rate of interest on interbank loans backed by Brazilian public debt. You can find the daily series in the BCB database under code 1178. I take the quarter average of the daily series and perform the transformation $i_t = \log(1 + i_t^{data}/4)$ (the raw interest rate is annualized, my transformation converts it to quarterly rate).

Exchange Rates I collect real and nominal exchange rate data from the BCB, codes 11752 and 20360 respectivelly. Both series are indices, the growth rates of which are calculated as the weighted averages of bilateral exchange rates, using as weights the share of commerce of each country with the Brazilian economy. I use data starting in June 1994. That is the first date to which nominal exchange rate data is available, probably due to the adoption of the Brazilian Real around the same. The data for a given quarter is the value prevailing in the last month of the corresponding quarter. In the case of the real exchange rate, I divide the entire series by its average and take log to arrive at the series for q_t. In the case of the nominal exchange, I calculate log growth to arrive at the series corresponding to Δe_t.

Foreign Interest I take the EMBI+ Brazil index - a measure of return demanded on Brazilian debt instruments in excess to US public bonds - and sum to the yields of one-year Treasuries with constant maturity, which I get from the St. Louis FRED website. The value for a quarter is the average over yields in each day. Also like I did in the case of nominal interest, I build the series after transforming annual into quarterly yields and taking logs.

[15] You can find the database in www3.bcb.gov.br/sgspub.

Foreign Consumption My proxy for foreign consumption is the US GDP, which I collect from the St. Louis FRED website (under code *GDPC1*). I take logs and apply the HP filter just like the I do with Brazilian national accounts data. In the case of the US, I collect data from 1990 and 2019.

Foreign Inflation I use the log growth in the US deflator, again provided by the St. Louis Fed (code *USAGDPDEFQISMEI*). The series I download is adjusted for seasonality and quaterly.