

1. Environment

The economy is populated by households and a government. They live for two periods, $t = 0$ and $t = 1$, and trade identical consumption goods and public bonds. Public bonds promise their holder one unit of the consumption good in the following period. There is no money in this economy. Agents trade public bonds using consumption goods.

The government demands (g_0, g_1) consumption goods (*i.e.* g_0 in period zero and g_1 in period one). To finance its purchases, it charges lump-sum taxes (τ_0, τ_1) on households. Households cannot avoid paying taxes. The government also raises revenue from selling new public bonds. In period 0, the price of one bond is q_0 units of the consumption good. In period 1, agents have no incentive to save since the world ends in the following period. Since bonds have no demand we can set its equilibrium bond to zero $q_1 = 0$.

We make two critical assumptions on government behavior. First, it can *credibly* commit to fully repay previously issued debt. Second, it always does so. "Credibly" means that households believe the government will not default, and demand public bonds accordingly.

The government brings to period zero a debt of b_{-1} bonds, and must therefore come up with b_{-1} consumption goods to pay bondholders. To that end, it can either sell new bonds b_0 and raise $q_0 b_0$ goods in revenue, or run a *primary surplus*. The primary surplus is defined as the difference between tax proceeds and non-interest spending. In this model, it corresponds to the quantity $\tau_0 - g_0$. The government avoids a default in period zero if

$$q_0 b_0 + \tau_0 - g_0 = b_{-1}. \quad (1)$$

The revenue from selling new bonds plus the revenue from taxes in excess of public spending must be enough to redeem old bonds. Since the government will not default, condition (1) represents a budget constraint. It restricts the government's choice of how much to tax, how much to spend, and how much to borrow.

Like in period zero, in period one the government again must pay bondholders, which are now due b_0 units of the consumption good. But, in period one, the government cannot sell new bonds, since there is no demand for them (the bond price is zero $q_1 = 0$, so the government would not raise any revenues anyway). Therefore, to pay bondholders, the government must run a primary surplus of b_0 in period one:

$$\tau_1 - g_1 = b_0. \quad (2)$$

Like (1), expression (2) is a budget constraint for the government.

The consumption good is non-durable (households can only enjoy them for a single period), and non-perishable (agents cannot store them). Households value the consumption good in the period they make use of them. The utility function

$$u(c_0) + \beta u(c_1)$$

captures households' preferences over the amount consumed in period zero c_0 and period one c_1 .

$$\begin{aligned} \text{Max}_{c_0, c_1, b_0} \quad & u(c_0) + \beta u(c_1) \\ \text{s.t.} \quad & q_0 b_0 + c_0 = b_{-1} + y_0 - \tau_0 \end{aligned} \quad (3)$$

$$c_1 = b_0 + y_1 - \tau_1 \quad (4)$$

$$b_0 \geq \underline{b} \quad (5)$$

We assume income (y_0, y_1) and initial wealth b_{-1} are large enough so that the household can choose non-negative amounts of goods.

We assume that $b_{-1} + y_0 - \tau_0 \geq 0$

$y_0 > \tau_0$, so income high enough to pay taxes.

$b_{-1} \geq 0$

\underline{b} is the borrowing limit.