

# A Fiscal Decomposition of Unexpected Inflation: Cross-Country Estimates and Theory

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# Introduction: The Fiscal Sources of Inflation Variation

## ■ The Valuation Equation of Public Debt

$$\frac{\text{Bond Prices} \times \text{Bonds}}{\text{Price Level}} = \sum_t \frac{\text{Surpluses}_t}{\text{Discount}_t}$$

## ■ Fiscal sources of inflation variation

- (Current) Bond prices
- (Path of) Primary surpluses
- (Path of) Real discount rates

## ■ Fiscal decomposition breaks down sources of **unexpected inflation** $(E_t - E_{t-1}) \pi_t$

- Vector autoregressions (VARs) measure future expectations
- How does unexpected inflation change our forecasts?

## ■ **This paper.** Empirical estimates for 25 countries + New-Keynesian model to reproduce them

## ■ Empirical Results

- Is inflation informative about fiscal policy? Does it accompany/anticipate deficits?
- Are forecasts similar across countries? Does  $\pi_A > \pi_B$  forecast  $s_A < s_B$ ?
- "Aggregate demand" recessions: huge deficits and... low inflation?
- **Fiscal policy.** 1% additional debt: how much is "paid for" by inflation?
- **Monetary policy.** Which fiscal determinants of inflation are affected by central banks?

## ■ Guidance for Theory Work

- Fiscal decomposition extremely general
- Stylized facts / **moment conditions** to discipline theory
- Model selection (structural shocks? policy instruments? frictions?)
- Parameter identification (policy: fiscal policy sustainability, monetary policy reaction, commitment vs discretion)
  - E.g. monetary disturbances can't be too volatile in NK models

# Introduction: Motivation

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# Introduction: Exercises

1. Fiscal decomposition of the unexpected inflation: derivation
2. Empirical estimates
  - Estimate Bayesian-VARs for 25 countries
  - **Unexpected Inflation:** "What does +1% unexpected inflation forecast?"
  - **Unexpected Demand:** "What does +1% unexpected inflation and +1% GDP growth forecast?"
  - **Unexpected Surpluses:** "What does -1% unexpected discounted surpluses forecast?"
3. The New-Keynesian Model
  - GMM estimation targeting empirical decompositions
  - AR(1) vs Trend productivity shocks
  - Parameter **identification**
    - Discretionary monetary policy
    - Central bank single vs dual mandates
  - Can policy explain cross-country heterogeneity in sources of inflation?

## Introduction: Related Literature

### ■ **Monetary-Fiscal Interaction.**

Cagan (1956), Sargent and Wallace (1981), Hall and Sargent (1997), Hall and Sargent (2011), Jiang et al. (2019), Corsetti et al. (2019), Sunder-Plassmann (2020), Du et al. (2020), Akhmadieva (2022)

### ■ **Fiscal Theory of the Price Level.** Leeper (1991), Sims (1994), Woodford (1995), Cochrane (1998), Cochrane (2005), Sims (2011), Leeper and Leith (2016), Bassetto and Cui (2018), Cochrane (2022c), Brunnermeier et al. (2022), Cochrane (2022a), Cochrane (2022b)

- Analysis of multiple countries
- Estimated NK model with productivity shocks

### ■ **Empirical Finance** (Drivers of Unexpected Returns)

Campbell and Shiller (1988), Cochrane (1992), Campbell and Ammer (1993), Chen and Zhao (2009), Cochrane (2008), Jiang et al. (2019).

- Unexpected return on basket of public debt

# **The Fiscal Decomposition of Unexpected Inflation**

## Fiscal Decomposition: The Valuation Equation

- Environment with discrete time + single good (price  $P_t$ ) + households and government
- One-period nominal public bonds (price  $Q_t$ )
- In each period, the government:
  - redeems bonds  $B_{t-1}$  for currency
  - soaks up currency through primary surpluses  $P_t S_t$  and bond sales  $Q_t B_t$
- Market clearing + No Currency Holdings  $M = 0$ :

$$B_{t-1} = P_t S_t + Q_t B_t$$



## Fiscal Decomposition: The Valuation Equation

- *Ex-post* real discounting  $\beta_t = Q_t(P_{t+1}/P_t)$   $\beta_{t,t+k} = \prod_{\tau=t}^{t+k} \beta_\tau$
- Iterate law of motion forward:

$$\frac{B_{t-1}}{P_t} = \sum_{k=0}^p \beta_{t,t+k-1} s_{t+k} + \beta_{t,t+p} \frac{B_{t+p}}{P_{t+p+1}}$$

- **Key Assumption:**  $\lim_{\tau \rightarrow \infty} \beta_{t,\tau} \frac{B_\tau}{P_{\tau+1}} = 0$
- Valuation equation of public debt:

$$\frac{B_{t-1}}{P_t} = \sum_{k=0}^{\infty} E_t [\beta_{t,t+k-1} s_{t+k}]$$

*"A prince who should enact that a certain proportion of his taxes should be paid in a paper money of a certain kind might thereby give a certain value to this paper money"* - Adam Smith

## Fiscal Decomposition: In the Simplest Environment

- End-of-period real debt  $v_t$
- Linearized flow condition + valuation equation

$$\underbrace{\frac{1}{\beta} v_{t-1} + \frac{v}{\beta} (i_{t-1} - \pi_t)}_{B_{t-1}/P_t} = s_t + v_t = \sum_{k=0}^{\infty} \beta^k E_t s_{t+k} - \frac{v}{\beta} \sum_{k=1}^{\infty} \beta^k E_t r_{t+k}$$

- Take innovations  $\Delta E_t = (E_t - E_{t-1})$

$$\Delta E_t \pi_t = -\frac{\beta}{v} \sum_{k=0}^{\infty} \beta^k \Delta E_t s_{t+k} + \sum_{k=0}^{\infty} \beta^k \Delta E_t r_{t+k}$$

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## Fiscal Decomposition: Generalizing

- GDP Growth
- Nominal, inflation-linked and dollar-denominated bonds
- Long-term bonds

$$\frac{\text{Bond Price in Home Currency} \times \text{Bonds}}{\text{Price Level}} = \sum_t \frac{\text{Surplus-to-GDP} \times \Delta \text{GDP}}{\text{Discounting}}$$

$$\frac{v_{t-1}}{\beta} + \frac{v}{\beta} \sum_j \delta_j (r x_{j,t} + i_{j,t-1} + \Delta h_{j,t} - \pi_{j,t}) = \sum_{k=0}^{\infty} \beta^k E_t s_{t+k} + \frac{v}{\beta} \sum_{k=0}^{\infty} \beta^k E_t g_{t+k} - \frac{v}{\beta} \sum_{k=1}^{\infty} \sum_j \delta_j \beta^k E_t r_{j,t+k}$$

[Details](#)[Currency Table](#)

## Fiscal Decomposition of Unexpected Inflation

- Ex-post real return  $r_{j,t} = rx_{j,t} + i_{j,t-1} + \Delta h_{j,t} - \pi_{j,t}$
- Take innovations  $\Delta E_t = E_t - E_{t-1}$

$$\Delta E_t \pi_t = \underbrace{\left[ \Delta E_t rx_t + \sum_{j \neq N} \frac{\delta_j}{\delta} \Delta E_t r_{j,t} \right]}_{\text{Innovation to Bond Prices}} - \frac{\beta}{\delta v} \underbrace{\left[ \sum_{k=0}^{\infty} \beta^k \Delta E_t s_{t+k} + \frac{v}{\beta} \sum_{k=0}^{\infty} \beta^k \Delta E_t g_{t+k} - \frac{v}{\beta} \sum_{k=1}^{\infty} \sum_j \delta_j \beta^k \Delta E_t r_{j,t+k} \right]}_{\text{Innovation to Discounted Surpluses}}$$

$$\equiv d(rx) + d(r_0) - d(s) - d(g) + d(r)$$

# Fiscal Decomposition: VAR-Based Measures

## ■ General VAR system

$$X_t = AX_{t-1} + Ke_t \quad e_t \sim N(0, \Omega)$$

$e_t$  can be reduced form or structural

## ■ How to measure terms of decomposition?

- Innovation to endogenous variables  $j$  periods ahead  $\Delta E_t X_{t+j} = A^j K e_t$  (Impulse Response Function)
- Therefore:

$$\sum_{j=0}^{\infty} \beta^j \Delta E_t X_{t+j} = \sum_{j=0}^{\infty} (\beta A)^j K e_t = (I - \beta A)^{-1} K e_t$$

## ■ How to build decomposition scenarios?

- Suppose we are interested in  $\Delta E_t X_t = x$  (e.g.  $\Delta E_t \pi_t = 1$ )
- Start by calculating the expected value of shocks  $e_t$  conditional on  $\Delta E_t X_t = x$

$$E[e \mid \Delta E_t X_t = x] = \Omega K' (K \Omega K')^{-1} x$$

- And then calculate the terms of the decomposition using  $e_t = E[e \mid \Delta E_t X_t = x]$

## **BVARs: Empirical Estimates**

## Bayesian-VAR: Data and Model

- Annual data on **observables**  $x_t^{OBS}$

$$x_t^{OBS} = \begin{bmatrix} i_t & \text{(Nominal Interest)} \\ \pi_t & \text{(CPI Inflation)} \\ v_t^b & \text{(Par-Value Debt-to-GDP)} \\ g_t & \text{(GDP growth)} \\ \Delta h_t & (\Delta \text{ Real Exchange to US Dollar}) \end{bmatrix}$$

- 25 countries (samples starting at 1945, 1960, 1973, 1998)
- General VAR

$$X_t = AX_{t-1} + Ke_t$$

- Decompose  $X_t' = [x_t^{OBS'} \ x_t^{NOT'}]$

$$x_t^{OBS} = a x_{t-1}^{OBS} + e_t$$

$$x_t^{NOT} = b x_{t-1}^{OBS} + c x_{t-1}^{NOT} + k e_t$$



# Bayesian-VAR: Empirical Challenges and Solutions

## 1. Explosive debt dynamics in 1960-2020 implies unstable OLS VARs

$$x_t^{OBS} = a x_{t-1}^{OBS} + e_t$$

- **United States:** Estimate model by OLS (stable!)
- **Others:** Estimate model with a Bayesian Linear Regression Bayesian Prior Hyperparameters

$$a^{BAY} = (X'X + \lambda^{-1})^{-1}(X'X a^{OLS} + \lambda^{-1} a^{PRIOR})$$

$\lambda$  maximizes the marginal distribution  $p(\text{data})$  (Giannone et al. (2015)) and ensures stability

## 2. Public finance data do not respect law of motion of public debt

- Define surplus from the law of motion:  $s_t = \frac{v_{t-1}}{\beta} - v_t + \frac{v}{\beta} \left[ -g_t + \sum_j \delta_j (rx_{j,t} + i_{j,t-1} + \Delta h_{j,t} - \pi_{j,t}) \right]$

## 3. No data on the market value of debt, only its par value ( $v_t^b$ ) Public Finances Model

- Model for market vs par value (Cox (1985)):  $v_t = v_t^b + \frac{v}{\beta} \sum_j \delta_j (q_{j,t} - q_{j,t-1}^b)$

## 4. No data on bond prices Geometric Term Structure

- Geometric maturity structure + constant risk premia:  $q_{j,t} = (\omega_j \beta) E_t q_{j,t+1} - i_{j,t}$

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# Bayesian-VAR: Unexpected Inflation Decomposition

- "Given 1% unexpected inflation, how do we change expectations over surplus, discounting, bond prices?"
- Reduced-form shock  $e_t = E[e \mid \Delta E_t \pi_t = 1]$

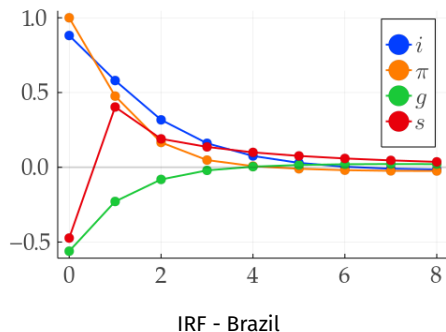
Country	$\Delta E_t \pi_t =$	$\Delta E_t(\text{Bond Prices})$		$-\Delta E_t(\text{Disc Surpluses})$		
		$d(r_0)$	$d(rx)$	$-d(s)$	$-d(g)$	$d(r)$
United States	1	0	* -0.8	0.6	0.2	1.0
<i>1960 Sample</i>						
Canada	1	* -0.1	* -1.6	0.6	* 1.2	0.9
Denmark	1	* -0.3	* -0.3	0.4	0	1.2
Japan	1	0	* -0.5	* 1.6	-0.4	0.3
Norway	1	0	* -0.4	0.6	0.5	0.3
Sweden	1	-0.2	* -0.9	-0.3	* 1.0	* 1.4
United Kingdom	1	* 0.5	* -0.7	* 2.9	* 1.0	* -2.7
<i>1973 Sample</i>						
Australia	1	* 0.1	* -0.8	* 2.1	0.7	-1.1
New Zealand	1	-0.1	* -0.9	0.4	* 0.9	0.7
South Korea	1	0	* -0.5	* 1.9	0.2	-0.6
Switzerland	1	0	* -0.7	0.9	* 0.9	-0.1

Advanced Markets

Country	$\Delta E_t \pi_t =$	$\Delta E_t(\text{Bond Prices})$		$-\Delta E_t(\text{Disc Surpluses})$		
		$d(r_0)$	$d(rx)$	$-d(s)$	$-d(g)$	$d(r)$
1998 Sample						
Brazil	1	-0.3	*-0.2	-1.5	1.1	1.9
Chile	1	-3.8	-1.3	9.0	-5.7	2.9
Colombia	1	1.5	*-1.0	1.4	-1.1	0.2
Czech Republic	1	*-0.2	*-0.4	-2.3	2.4	1.4
Hungary	1	*-0.6	*-0.9	-1.0	1.6	1.9
India	1	*0.2	*-0.5	1.5	0.1	-0.3
Indonesia	1	*-2.6	*-1.1	1.7	*2.6	0.4
Israel	1	-0.1	*-0.8	-0.6	*1.5	0.9
Mexico	1	0	*-0.7	1.4	0	0.3
Poland	1	*-0.5	*-1.2	0.9	-0.4	*2.1
Romania	1	-0.4	*-1.0	2.2	0.4	-0.3
South Africa	1	0.4	*-0.5	1.6	0.3	-0.7
Turkey	1	0.4	*-0.4	-1.2	-0.2	*2.3
Ukraine	1	0	*-0.8	0.7	0.4	*0.7

Emerging Markets

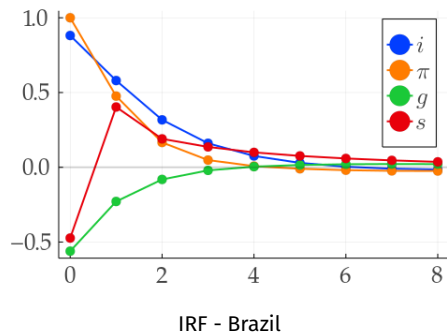
## Bayesian-VAR: Unexpected Inflation Decomposition - Takeaways



$$\begin{aligned} d(rx) < 0 & \quad -d(g) > 0 \\ d(r) > 0 & \quad -d(s) < 0 \end{aligned}$$

- $\Delta E\pi$  accounted for by discounted surpluses
- Surplus-to-GDP, GDP growth and real discounting...
  - ...account for unexpected inflation alone in 0/25
  - ...have a positive contribution in 18+/25
- Is inflation "fiscal"? Yes, but not only.
- Is inflation "fiscal" **cross-country**? Not at all.
- Bond price dynamics reduce  $\Delta E\pi$  in 25/25

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# Bayesian-VAR: Unexpected Demand Decomposition

- Environments of "strong aggregate demand": high inflation, high GDP and high surpluses.
- "Given +1% unexpected inflation and +1% GDP growth, how do we change forecast?"
- Reduced-form shock  $e_t = E[e \mid \Delta E_t \pi_t = 1, \Delta E_t g_t = 1]$

Country	$\Delta E_t \pi_t =$	$\Delta E_t(\text{Bond Prices})$		$-\Delta E_t(\text{Disc Surpluses})$		
		$d(r_0)$	$d(rx)$	$-d(s)$	$-d(g)$	$d(r)$
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1960 Sample						
Canada	1	*-0.2	*-2.9	0.8	0.3	*3.0
Denmark	1	*-0.4	*-1.1	3.0	*-2.9	2.3
Japan	1	0	*-1.2	*2.4	*-2.1	*1.8
Norway	1	0	*-0.9	1.8	*-1.7	1.8
Sweden	1	*-0.5	*-1.7	0.5	-0.4	*3.1
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Australia	1	0	*-2.1	2.6	-0.5	1.0
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South Korea	1	*-0.1	*-1.0	*4.4	*-1.9	-0.4
Switzerland	1	0	*-1.3	*1.3	0.6	0.4

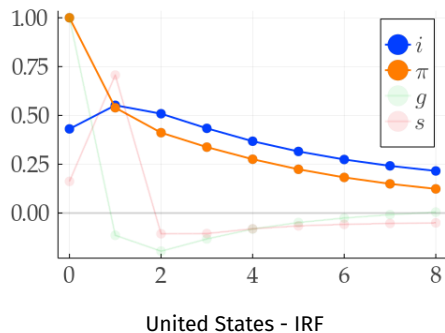
Advanced Markets

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1998 Sample						
Brazil	1	* -0.6	* -0.2	-2.9	0.2	4.3
Chile	1	* -18.4	* -3.7	36.4	-34.9	21.7
Colombia	1	-1.3	* -1.2	12.3	-8.6	-0.3
Czech Republic	1	* -0.5	* -0.8	-1.0	0.9	2.4
Hungary	1	* -1.3	* -1.1	-12.2	6.5	9.2
India	1	0.1	-0.4	2.0	-0.8	0
Indonesia	1	* -9.9	0.1	* 12.6	-0.2	-1.6
Israel	1	* -2.1	* -0.8	3.4	-0.7	1.1
Mexico	1	* -1.9	* -1.2	* 5.6	-2.1	0.6
Poland	1	* -1.0	* -1.5	0.6	-1.3	* 4.3
Romania	1	* -2.1	* -0.7	* 8.7	-1.7	-3.2
South Africa	1	0.3	-0.6	* 32.2	* -11.6	* -19.3
Turkey	1	-0.7	* -0.4	-1.2	-0.6	* 3.9
Ukraine	1	0	0.5	* 4.1	* -2.1	-1.4

Emerging Markets



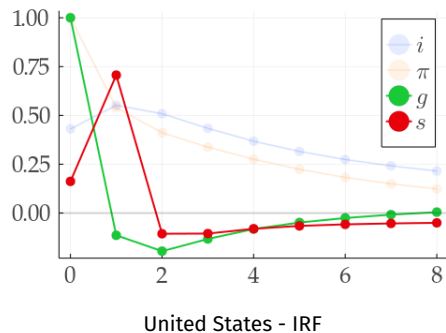
## Bayesian-VAR: Unexpected Demand Decomposition - Takeaways



$$\begin{aligned} d(rx) &< 0 & -d(g) &< 0 \\ d(r) &> 0 & -d(s) &> 0 \end{aligned}$$

- Higher inflation follows from...
  - higher discounting (monetary policy) in 19/25
  - lower surplus-GDP ratios, current or in the future in 21/25
- (Level) Surpluses increase in 23/25
- COVID inflation: decline in  $\{s\}$ ?

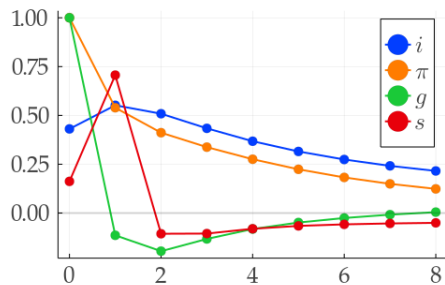
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## Bayesian-VAR: Unexpected Demand Decomposition - Takeaways



United States - IRF

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# Bayesian-VAR: Unexpected Surpluses Decomposition

- Unexpected inflation forecasts lower discounted surpluses. Is the converse true?
- "Given **-1% discounted surpluses**, how do we change forecast?"  
Reduced-form shock  $e_t = E[e | \Delta E_t \text{ Disc Surpluses} = -1]$
- $\Delta E_t \{\text{Disc Surpluses}\} = \Delta E_t \{\text{Bond Prices}\} - \Delta E_t \pi = \Delta E_t \{\text{Real Return on Public Debt}\}$

Country	$\Delta E_t \pi_t =$	$\Delta E_t(\text{Bond Prices})$		$-\Delta E_t(\text{Disc Surpluses})$		
		$d(r_0)$	$d(rx)$	$-d(s)$	$-d(g)$	$d(r)$
United States	<b>*0.4</b>	0	<b>*-0.6</b>	<b>0.2</b>	0	<b>*0.8</b>
<i>1960 Sample</i>						
Canada	<b>*0.2</b>	<b>*-0.1</b>	<b>*-0.8</b>	-0.1	0	<b>*1.2</b>
Denmark	<b>*0.2</b>	<b>*-0.2</b>	<b>*-0.6</b>	0.2	<b>*-0.6</b>	<b>*1.4</b>
Japan	<b>*0.5</b>	0	<b>*-0.5</b>	<b>0.7</b>	-0.2	<b>*0.5</b>
Norway	<b>*0.4</b>	0	<b>*-0.6</b>	-0.3	-0.1	<b>*1.4</b>
Sweden	<b>*0.2</b>	<b>*-0.3</b>	<b>*-0.5</b>	-0.1	0.1	<b>*1.0</b>
United Kingdom	<b>*0.1</b>	<b>-0.1</b>	<b>*-0.8</b>	0.2	-0.1	<b>0.9</b>
<i>1973 Sample</i>						
Australia	<b>*0.2</b>	0	<b>*-0.8</b>	-0.3	0	<b>*1.3</b>
New Zealand	<b>*0.3</b>	<b>*-0.1</b>	<b>*-0.5</b>	<b>-0.3</b>	<b>0.4</b>	<b>*0.9</b>
South Korea	<b>*0.5</b>	0	<b>*-0.5</b>	<b>1.5</b>	-0.2	<b>-0.3</b>
Switzerland	<b>*0.3</b>	0	<b>*-0.7</b>	0.3	0.2	<b>*0.5</b>

## Advanced Markets

Country	$\Delta E_t \pi_t =$	$\Delta E_t(\text{Bond Prices})$		$-\Delta E_t(\text{Disc Surpluses})$		
		$d(r_0)$	$d(rx)$	$-d(s)$	$-d(g)$	$d(r)$
1998 Sample						
Brazil	*0.4	*-0.4	*-0.1	*-2.4	0.3	*3.1
Chile	0	*-0.9	*-0.1	0.6	-0.4	0.9
Colombia	0	*-0.9	*-0.1	*1.7	-0.7	0
Czech Republic	*0.4	*-0.2	*-0.4	-1.0	0.8	1.1
Hungary	*0.2	*-0.4	*-0.3	-4.1	2.6	*2.6
India	*0.5	0	*-0.5	0.6	0.1	0.2
Indonesia	0	*-0.9	-0.1	0.5	0.2	0.3
Israel	*0.1	*-0.6	*-0.3	-0.9	0.1	*1.8
Mexico	*0.1	*-0.7	*-0.2	*1.4	-0.4	0.1
Poland	*0.2	*-0.4	*-0.3	-0.2	0.1	*1.0
Romania	*0.1	*-0.9	0	*1.6	-0.2	-0.4
South Africa	*0.2	*-0.5	*-0.3	-0.2	0.3	0.9
Turkey	*0.1	*-0.8	*-0.1	-0.1	0.1	*1.0
Ukraine	*0.4	0	*-0.6	0	*0.3	*0.6

## Emerging Markets

# Bayesian-VAR: Unexpected Surpluses Decomposition

- Unexpected inflation forecasts lower discounted surpluses. Is the converse true?
- "Given **-1% discounted surpluses**, how do we change forecast?"  
Reduced-form shock  $e_t = E[e | \Delta E_t \text{ Disc Surpluses} = -1]$
- $\Delta E_t\{\text{Disc Surpluses}\} = \Delta E_t\{\text{Bond Prices}\} - \Delta E\pi = \Delta E_t\{\text{Real Return on Public Debt}\}$

Country	$\Delta E_t \pi_t =$	$\Delta E_t(\text{Bond Prices})$		$-\Delta E_t(\text{Disc Surpluses})$		
		$d(r_0)$	$d(rx)$	$-d(s)$	$-d(g)$	$d(r)$
United States	<b>*0.4</b>	0	<b>*-0.6</b>	<b>0.2</b>	0	<b>*0.8</b>
<i>1960 Sample</i>						
Canada	<b>*0.2</b>	<b>*-0.1</b>	<b>*-0.8</b>	-0.1	0	<b>*1.2</b>
Denmark	<b>*0.2</b>	<b>*-0.2</b>	<b>*-0.6</b>	0.2	<b>*-0.6</b>	<b>*1.4</b>
Japan	<b>*0.5</b>	0	<b>*-0.5</b>	<b>0.7</b>	-0.2	<b>*0.5</b>
Norway	<b>*0.4</b>	0	<b>*-0.6</b>	-0.3	-0.1	<b>*1.4</b>
Sweden	<b>*0.2</b>	<b>*-0.3</b>	<b>*-0.5</b>	-0.1	0.1	<b>*1.0</b>
United Kingdom	<b>*0.1</b>	<b>-0.1</b>	<b>*-0.8</b>	0.2	-0.1	<b>0.9</b>
<i>1973 Sample</i>						
Australia	<b>*0.2</b>	0	<b>*-0.8</b>	-0.3	0	<b>*1.3</b>
New Zealand	<b>*0.3</b>	<b>*-0.1</b>	<b>*-0.5</b>	<b>-0.3</b>	<b>0.4</b>	<b>*0.9</b>
South Korea	<b>*0.5</b>	0	<b>*-0.5</b>	<b>1.5</b>	-0.2	<b>-0.3</b>
Switzerland	<b>*0.3</b>	0	<b>*-0.7</b>	0.3	0.2	<b>*0.5</b>

Advanced Markets

Country	$\Delta E_t \pi_t =$	$\Delta E_t(\text{Bond Prices})$		$-\Delta E_t(\text{Disc Surpluses})$		
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1998 Sample						
Brazil	*0.4	*-0.4	*-0.1	*-2.4	0.3	*3.1
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India	*0.5	0	*-0.5	0.6	0.1	0.2
Indonesia	0	*-0.9	-0.1	0.5	0.2	0.3
Israel	*0.1	*-0.6	*-0.3	-0.9	0.1	*1.8
Mexico	*0.1	*-0.7	*-0.2	*1.4	-0.4	0.1
Poland	*0.2	*-0.4	*-0.3	-0.2	0.1	*1.0
Romania	*0.1	*-0.9	0	*1.6	-0.2	-0.4
South Africa	*0.2	*-0.5	*-0.3	-0.2	0.3	0.9
Turkey	*0.1	*-0.8	*-0.1	-0.1	0.1	*1.0
Ukraine	*0.4	0	*-0.6	0	*0.3	*0.6

Emerging Markets

## Bayesian-VAR: Summary of Results

		$\Delta E$ ( Bond Prices )		$-\Delta E$ ( Discounted Surpluses )		
	$\Delta E_t \pi_t =$	$\sum_{j \neq N} \delta_j \Delta E_t r_{j,t}$	$\Delta E_t r x_t$	$-\sum_k \beta^k \Delta E_t s_{t+k}$	$-\sum_k \beta^k \Delta E_t g_{t+k}$	$\sum_k \beta^k \Delta E_t r_{t+k}$
<b>Unexpected Inflation</b>	1	$< 0$		mostly $> 0$	mostly $> 0$	mostly $> 0$
<b>Unexpected Demand</b>	1	$< 0$		$> 0$	$< 0$	$> 0$
<b>Unexpected Surplus</b>	0.2-0.4	-0.8 to -0.6				$> 0$

- Is inflation informative about fiscal policy? Does it accompany/anticipate deficits?  
A: Yes. In most countries, it forecasts lower surplus-to-GDP, but also lower growth and higher discount rates.
- Are forecasts similar across countries? Does  $\pi_A > \pi_B$  forecast  $s_A < s_B$ ?  
A: No. Forecasts vary wildly across countries.
- "Aggregate demand" recessions: huge deficits and... low inflation?  
A: Low inflation follows from low discounting and the fact that surpluses recover faster than GDP.
- **Fiscal policy.** 1% additional debt: how much is "paid for" by inflation?  
A: 0.2%-0.4% in most countries (after accounting for monetary policy reaction)
- **Monetary policy.** Which fiscal determinants of inflation are affected by central banks?  
A: The price of long-term bonds, which changes the size public debt at market prices.

## **Theory: The New Keynesian Model**

# The New-Keynesian Model

Parameters

Why Trend? Growth

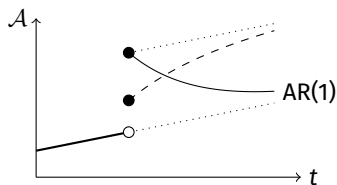
Equilibrium Selection

Comparative Statics

- Closed-economy New-Keynesian model
- **Trend Shocks.** Production function  $\mathcal{T}_t A_t N$

Trend component:  $\log \mathcal{T}_t = \log \mathcal{T}_{t-1} + u_{g,t}$

AR(1) component:  $a_t = \rho_a a_{t-1} + \varepsilon_{a,t}$



$$y_t = E_t y_{t+1} - \gamma (i_t - E_t \pi_{t+1}) + E_t u_{g,t}$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa y_t - \kappa_a a_t$$

$$g_t = \Delta y_t + u_{g,t}$$

$$i_t = \phi_\pi \pi_t + \phi_g g_t + u_{i,t}$$

$$s_t = \tau_\pi \pi_t + \tau_g g_t + u_{s,t}$$

$$\beta(v_t + s_t) = v_{t-1} + v \sum_j \delta_j [rx_{j,t} + i_{j,t-1} + \Delta h_{j,t} - \pi_{j,t}]$$

$$q_{j,t} = (\omega_j \beta) E_t q_{j,t+1} - i_{j,t}$$

$$rx_{j,t} = (\omega_j \beta) q_{j,t} - q_{j,t-1} - i_{j,t-1}$$

- Structural shocks:  $\varepsilon = [\varepsilon_a \ \varepsilon_g \ \varepsilon_i \ \varepsilon_s]$
- Method of moments:

$$\text{Min}_\Psi \quad \alpha_1 \|\mathcal{D}_{VAR} - \mathcal{D}_{NK}(\Psi)\| + \alpha_2 \|\mathcal{M} - \mathcal{M}_{NK}(\Psi)\|$$



## The New-Keynesian Model: Measuring the Fiscal Decomposition

- In the NK model, flow equation of public debt holds, so does fiscal decomposition
- Solution to NK model

$$X_t = AX_{t-1} + K\varepsilon$$

but  $\varepsilon$  is now **structural**

- So given innovation  $\Delta E_t X_t = x$  (e.g.  $\Delta E_t \pi_t = 1$ ), we compute

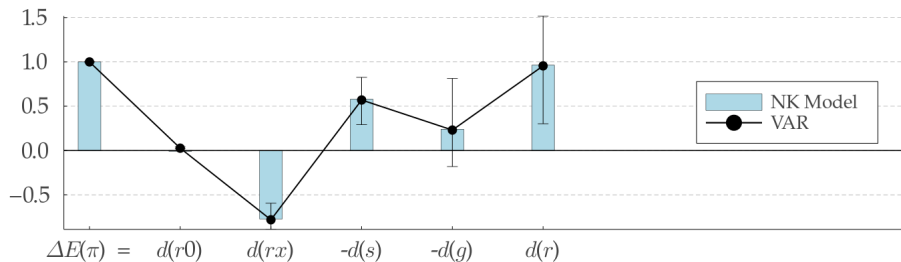
$$\varepsilon_t = E[\varepsilon \mid \Delta E_t X_t = x],$$

calculate the IFRs and the terms of the decomposition

# The New-Keynesian Model: Reproducing the Unexpected Inflation Decomposition

Simple version of the model. Target: unexpected inflation decomposition ( $\Delta E_t \pi_t = 1$ )

- **Result.** AR(1) productivity shocks  $\varepsilon_{a,t}$  **alone** reproduce the US **unexpected inflation decomposition**
- **Result.** Monetary, fiscal and trend shocks do not, even if combined.

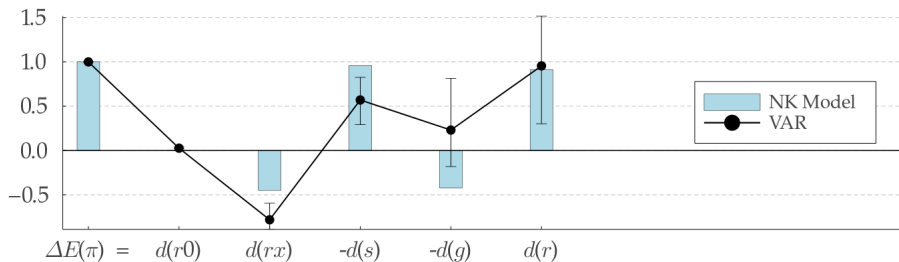


Target: United States. AR(1) productivity shocks. All others.

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Simple version of the model. Target: unexpected inflation decomposition ( $\Delta E_t \pi_t = 1$ )

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- **Result.** Monetary, fiscal and trend shocks do not, even if combined.



Target: United States. AR(1) productivity shocks. All others.

# The New-Keynesian Model: Reproducing the Unexpected Inflation Decomposition

$$1 = \Delta E_t \pi_t = d(rx) - d(s) - d(g) + d(r)$$

$$i_t = \phi_\pi \pi_t \quad s_t = \tau_g g_t$$

- Story: negative productivity shock

$$E[\varepsilon_a \mid \Delta E_t \pi_t = 1] = -0.85$$

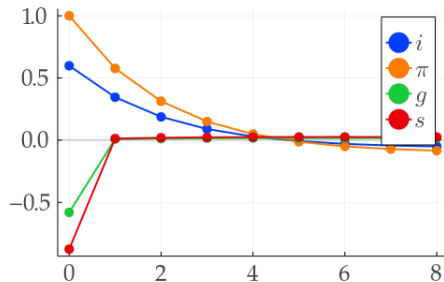
- Less productivity leads to less growth:  $-d(g) > 0$

- Procyclical surpluses:  $\tau_g = 1.5$   $-d(s) > 0$

- Strong Taylor rule:  $\phi_\pi = 0.6$

$$d(rx) < 0$$

$$d(r) > 0$$



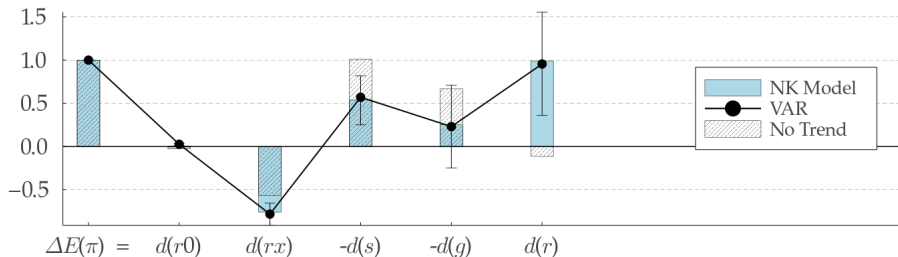
IRF to  $\varepsilon_{a,t} = -0.85$

# The New-Keynesian Model: Complete Model

## ■ Targets: three decompositions + second moments

- Unexpected Inflation:  $\Delta E_t \pi_t = 1$
- Unexpected Demand:  $\Delta E_t \pi_t = 1, \Delta E_t g_t = 1$
- Unexpected Surplus:  $\Delta E_t$  Discounted Surpluses =  $-1$

## ■ **Result.** Trend shocks are necessary to reproduce unexpected demand decomposition.



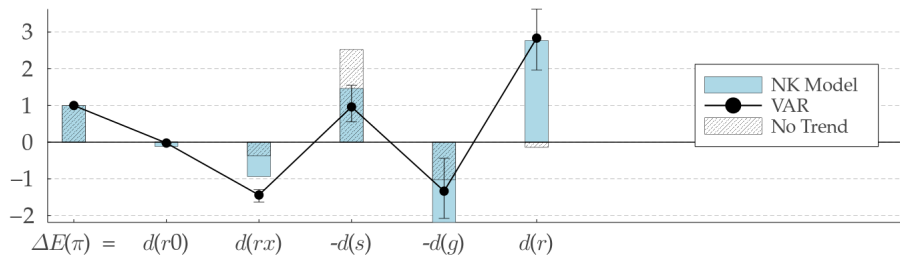
Structural Shocks Target: United States - Unexpected Inflation Demand Surpluses

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## ■ Targets: three decompositions + second moments

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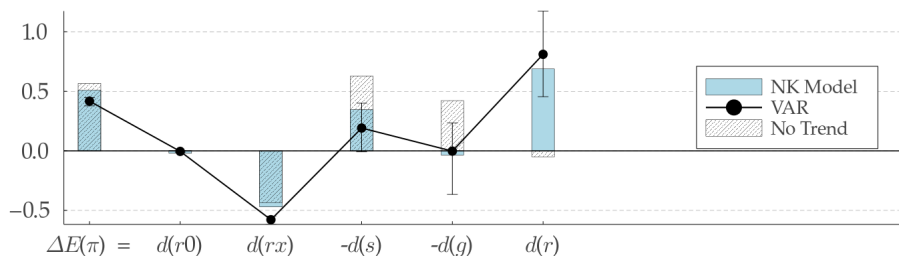
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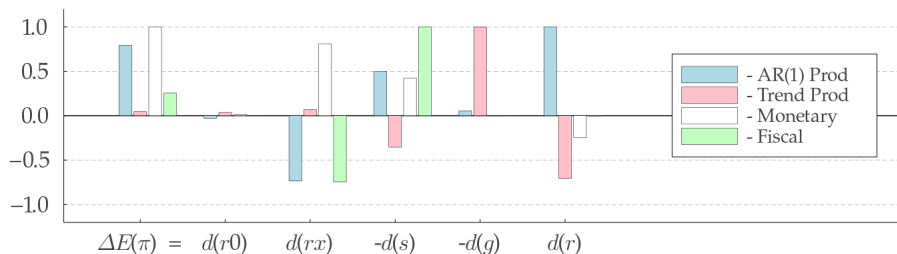
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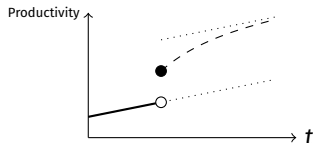


Structural Shocks Target: United States - Unexpected Inflation Demand Surpluses



# The New-Keynesian Model: Reproducing the Unexpected Demand Decomposition

$$1 = \Delta E_t \pi_t = d(rx) - d(s) - d(g) + d(r)$$



- High Marginal Costs + Positive Growth?

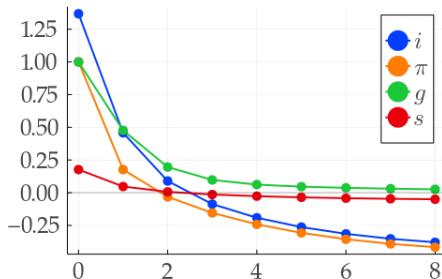
$$E[\varepsilon_g | \cdot] = 1.49 \quad E[\varepsilon_a | \cdot] = -0.76$$

- Protracted productivity growth

- Marginal costs high **relative to trend**

$$\pi_t = \beta E_t \pi_{t+1} + \kappa y_t - \kappa_a a_t \quad a_t < 0$$

$$g_t = \Delta y_t + u_{g,t} \quad u_{g,t} > 0$$



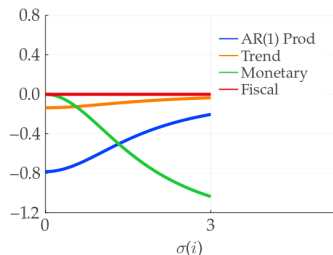
IRF to  $\Delta E_t g_t = 1$ ,  $\Delta E_t \pi_t = 1$

# The New-Keynesian Model: Comparative Statics and Identification

$$\Delta E\pi = 1$$

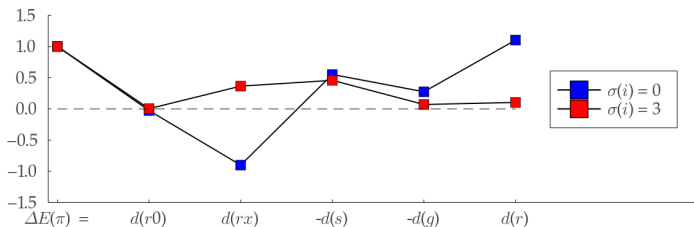
$$i_t = \phi_\pi \pi_t + \phi_g g_t + u_{i,t}$$

1. **(Discretionary Monetary Policy)**  $\sigma_i \uparrow \implies$  Unexpected inflation forecasts higher bond prices
2. **(Central Bank Dual Mandate)**  $\phi_g \uparrow \implies$  No "agg. demand" inflation;  $\Delta E\pi = 1$  forecasts lower growth



(a)  $E[\varepsilon \mid \Delta E\pi = 1]$

Figure: Comparative Statics:  $\sigma_i \phi_g$



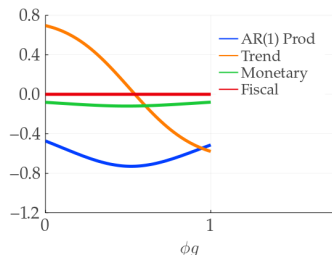
(b) Decomposition of Unexpected Inflation

# The New-Keynesian Model: Comparative Statics and Identification

$$\Delta E\pi = 1$$

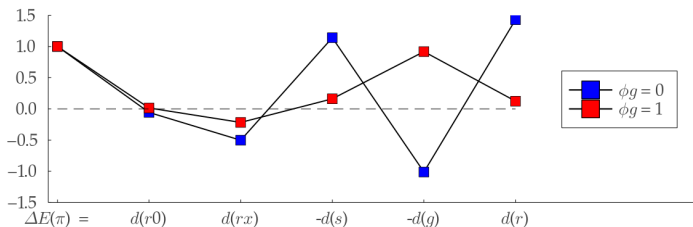
$$i_t = \phi_\pi \pi_t + \phi_g g_t + u_{i,t}$$

1. **(Discretionary Monetary Policy)**  $\sigma_i \uparrow \Rightarrow$  Unexpected inflation forecasts higher bond prices
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(a)  $E[\varepsilon \mid \Delta E\pi = 1]$

Figure: Comparative Statics:  $\sigma_i \phi_g$

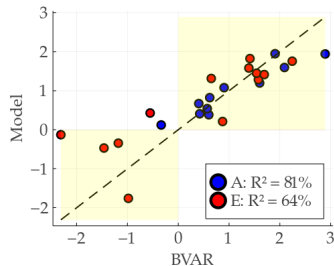


(b) Decomposition of Unexpected Inflation

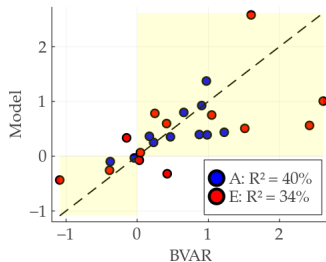
# The New-Keynesian Model: Unexpected Inflation Decomp. (Cross-Country)

$$\Delta E\pi = 1$$

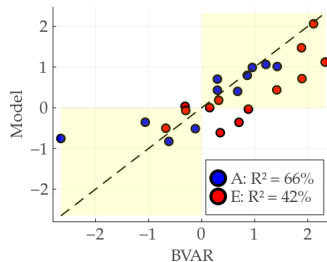
- Can cross-country differences in **policy** explain differences in unexpected inflation forecasts?
- Estimation.** Solve optimization problem to all countries; keep **productivity parameters constant**



(a)  $-d(s)$  ( $R^2 = 0.7$ )



(b)  $-d(g)$  ( $R^2 = 0.35$ )



(c)  $d(r)$  ( $R^2 = 0.6$ )

# The New-Keynesian Model: The Open Economy

$$y_t = E_t y_{t+1} - \gamma [i_t - E_t \pi_{H,t+1} + \alpha(\bar{\omega} - 1) E_t \Delta z_{t+1}] + E_t u_{g,t+1}$$

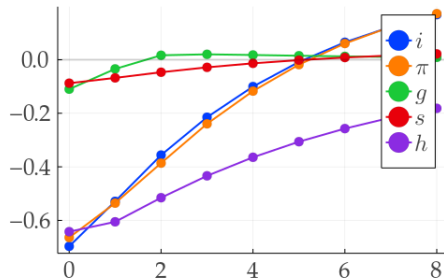
$$\pi_{H,t} = \beta E_t \pi_{H,t+1} + \kappa y_t - \kappa_a a_t - \kappa_z z_t$$

$$\gamma_\alpha z_t = y_t - y_t^*$$

$$\pi_t = \pi_{H,t} + \alpha \Delta z_t$$

$$h_t = (1 - \alpha) z_t$$

- **Complete markets**
- **Home:** small and open ( $\alpha = 0.45$ )
- **Foreign:** large and "closed"
- Same United States parameters:
  - Unexpected Inflation ✓ ( $\varepsilon_a = -0.6$ ,  $\varepsilon_a^* = -0.7$ )
  - Unexpected Demand ✓
- Terms of trade dynamics and marginal costs:
  - $\varepsilon_a$  and  $\varepsilon_a^*$ : same impact on Home's MC
  - Foreign Mon. Shocks: opposite  $\Delta E_t \pi^*$  and  $\Delta E_t \pi$



Shock to **Foreign's Productivity** Interest

# The New-Keynesian Model: The Open Economy

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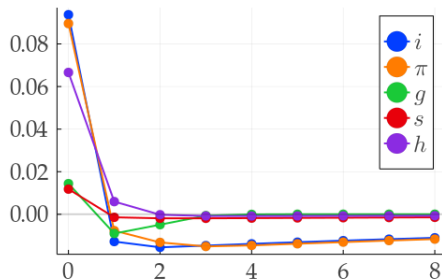
$$\pi_{H,t} = \beta E_t \pi_{H,t+1} + \kappa y_t - \kappa_a a_t - \kappa_z z_t$$

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- Terms of trade dynamics and marginal costs:
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  - Foreign Mon. Shocks: opposite  $\Delta E_t \pi^*$  and  $\Delta E_t \pi$



Shock to **Foreign's** Productivity Interest

## Conclusion

## Conclusion: What have we learned?

### ■ Empirical Results

- Is inflation informative about fiscal policy? Does it accompany/anticipate deficits?  
A: Yes. In most countries, it forecasts lower surplus-to-GDP, but also lower growth and higher discount rates.
- Are forecasts similar across countries? Does  $\pi_A > \pi_B$  forecast  $s_A < s_B$ ?  
A: No. Forecasts vary wildly across countries.
- "Aggregate demand" recessions: huge deficits and... low inflation?  
A: Low inflation follows from low discounting and the fact that surpluses recover faster than GDP.
- **Fiscal policy.** 1% additional debt: how much is "paid for" by inflation?  
A: 0.2%-0.4% in most countries (after accounting for monetary policy reaction)
- **Monetary policy.** Which fiscal determinants of inflation are affected by central banks?  
A: The price of long-term bonds, which changes the size public debt at market prices.

### ■ New-Keynesian Model

- New Keynesian model reproduces empirical decomposition with productivity shocks
- Fiscal decomposition can be used to discipline model selection and parameters
- Policy parameters account (to a large extent) differences in the fiscal decomposition of unexpected inflation



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## Appendix: Debt Instruments and Growth

### Return

- Real **market value** debt-to-GDP  $v_t$ , surplus-to-GDP  $s_t$  and GDP growth  $g_t$  (stationarity!)
- Bonds  $(j, n)$  promises one unit of currency  $j$  after  $n$  periods
  - Nominal bonds
  - Real bonds (currency denomination = final goods)
  - US Dollar bonds

Constant structure  $\{\delta_j\}, \{\omega_j^n\}$

- Bond price  $Q_{j,t}^n$ , excess return  $rx_{j,t}$   $1 + \text{return}_{j,t} = 1 + rx_{j,t} + i_{j,t-1} = \frac{\sum_n Q_{j,t}^{n-1} B_{j,t-1}^n}{\sum_n Q_{j,t-1}^n B_{j,t-1}^n}$   
(one-period bonds  $\implies rx = 0$ )
- Debt law of motion:

$$\frac{v_{t-1}}{\beta} + \frac{v}{\beta} \left[ -g_t + \sum_j \delta_j (rx_{j,t} + i_{j,t-1} + \Delta h_{j,t} - \pi_{j,t}) \right] = v_t + s_t$$

## Appendix: Debt Instruments and Growth

### Return

- Law of motion:

$$\sum_j \mathcal{E}_{j,t} B_{j,t-1}^1 = P_t^s S_t + \sum_j \mathcal{E}_{j,t} \sum_{n=2}^{\infty} Q_{j,t}^{n-1} (B_{j,t}^{n-1} - B_{j,t-1}^n),$$

- $\mathcal{V}_{j,t} = \sum_{n=0}^{\infty} Q_{j,t}^n B_{j,t}^n$  (end-of-period market value of debt)

$$\sum_j (1 + rx_{j,t} + i_{j,t-1}) \mathcal{E}_{j,t} \mathcal{V}_{j,t-1} = P_t^s S_t + \sum_j \mathcal{E}_{j,t} \mathcal{V}_{j,t}$$

- $V_{j,t} = \mathcal{V}_{j,t} / P_{j,t} Y_t$  (real value of  $j$ -indexed debt)

$$V_{t-1} \sum_j \frac{(1 + rx_{j,t} + i_{j,t-1})(1 + \Delta h_{j,t})}{(1 + \pi_{j,t})(1 + g_t)} \delta_j = s_t + V_t.$$

## Appendix: Public Debt Currency Denomination

Return

Symbol	Description	Nominal Debt	Real Debt	Dollar Debt
$j$	Index Symbol Notation	$N$ $\delta, \omega$	$R$ $\delta_R, \omega_R$	$D$ $\delta_D, \omega_D$
$P_j$	Price per Good	$P$	1	$P_t^{US}$
$\mathcal{E}_j$	Nominal Exchange Rate	1	$P$	Dollar NER
$H_j$	Real Exchange Rate	1	1	Dollar RER
$\pi_j$	Log Variation in Price	$\pi$	0	$\pi_t^{US}$
$\Delta h_j$	Log Real Depreciation	0	0	$\Delta h_t$

Table: Public Debt Denomination

## Appendix: Bayesian Prior

Return

- Complete model (with US variables):

$$x_t^{OBS} = a x_{t-1}^{OBS} + b u_{t-1}^{OBS} + e_t$$

$$u_t^{OBS} = a_u u_{t-1}^{OBS} + e_{u,t}$$

- Group  $\theta = [\text{vec}(a)' \text{vec}(b)']'$
- $\Sigma \sim IW(\Phi; d) \quad \theta | \Sigma \sim N(\bar{\theta}, \Sigma \otimes \Omega)$
- $\Phi = \text{Identity}$  and  $d = 7$  sets a loose prior
- $\bar{\theta}$  sets the mean of the prior for  $a$  to be **OLS estimate of  $a_u$**

$$\text{cov}(a_{ij}, a_{kl} | \Sigma) = \begin{cases} \lambda^2 \frac{\Sigma_{ij}}{\Phi_{ij}} & \text{if } j = l \\ 0 & \text{otherwise.} \end{cases}$$

$$\text{cov}(b_{ij}, b_{kl} | \Sigma) = \begin{cases} (\xi \lambda)^2 \frac{\Sigma_{ij}}{\Phi_{u,jj}} & \text{if } j = l \\ 0 & \text{otherwise} \end{cases}$$

Set  $\xi = (1/3)$

# Appendix: Hyperparameters + Debt Structure

## Return

Country	$v$ (%)	$\delta_N$ (%)	$\delta_R$ (%)	$\delta_D$ (%)	Avg. Term (Years)	$\lambda$	$\sigma(\Delta E_t \pi)$ (%)
United States	60	93	7	0	5	10	1.9
<i>Advanced - 1960 Sample</i>							
Canada	71	92	5	3	6.5	0.21	1.0
Denmark	37	84	0	16	5.6	0.18	1.2
Japan	98	100	0	0	5.5	0.01	2.2
Norway	35	99	0	1	3.7	0.19	1.5
Sweden	46	69	16	14	4.8	0.16	1.5
United Kingdom	61	76	24	0	12.3	0.17	2.0
<i>Advanced - 1973 Sample</i>							
Australia	24	90	10	0	7.2	0.18	1.5
New Zealand	41	82	6	13	4.3	0.15	2.0
South Korea	21	97	0	3	4	0.15	2.8
Switzerland	43	100	0	0	6.9	0.23	1.0

(a) Advanced Economies

Country	$v$ (%)	$\delta_N$ (%)	$\delta_R$ (%)	$\delta_D$ (%)	Avg. Term (Years)	$\lambda$	$\sigma(\Delta E_t \pi)$ (%)
<i>Emerging - 1998 Sample</i>							
Brazil	70	70	25	5	2.6	0.12	1.4
Chile	14	10	57	33	12.8	0.27	1.0
Colombia	41	45	23	32	5.6	0.13	0.8
Czech Republic	31	91	0	9	5.6	0.15	1.1
Hungary	68	76	0	23	4.1	0.14	1.3
India	73	90	3	7	10.1	0.25	1.1
Indonesia	43	44	0	56	9.2	0.21	1.2
Israel	77	43	34	23	6.6	0.13	1.3
Mexico	45	65	10	26	5.5	0.15	1.0
Poland	47	79	1	20	4.2	0.10	1.3
Romania	28	50	0	50	4.8	0.10	1.9
South Africa	41	70	20	10	12.9	0.25	1.0
Turkey	43	47	23	30	3.6	0.13	2.1
Ukraine	43	100	0	0	9.1	0.07	5.7

(b) Emerging Economies

## Appendix: Public Finances Model

### Return

- Convert par to market value of debt (Cox and Hirschhorn (1983))

$$\mathcal{V}_{j,t} = \mathcal{V}_{j,t}^b \times \frac{\text{market price of debt}}{\text{book price of debt}} = \mathcal{V}_{j,t}^b \times \frac{Q_{j,t}}{Q_{j,t}^b}.$$

- Linearized average interest follows

$$i_{j,t}^b = \omega_j i_{j,t-1}^b + (1 - \omega_j) i_{j,t} = (1 - \omega_j) \sum_{k=0}^{\infty} \omega_j^k i_{j,t-k}$$

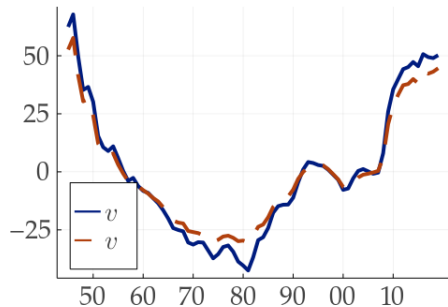
since government rolls over share  $\omega_j$  of public debt in steady state

- Linearized book price of debt:

$$q_{j,t}^b = (\omega_j \beta) E_t q_{j,t+1}^b - i_{j,t}^b$$

## Appendix: Public Finances Model

Return



(a) Model

**Chart 1B**  
**Market Value of U.S. Government Debt as a Share of GDP**



(b) Emerging Economies



## Appendix: Geometric Term Structure

Return

Decomposition 2

- To each currency portfolio  $j$ , fixed geometric maturity structure:

$$B_{j,t}^n = \omega_j B_{j,t}^{n-1}$$

- Total return on currency- $j$  portfolio:

$$1 + rx_{j,t} + i_{j,t-1} = \frac{1 + \omega_j Q_{j,t}}{Q_{j,t-1}} \implies \boxed{rx_{j,t} + i_{j,t-1} = (\omega_j \beta) q_{j,t} - q_{j,t-1}}$$

- Assume **constant risk premia**  $E_t rx_{j,t+1} = 0$

$$\boxed{q_{j,t} = (\omega_j \beta) E_t q_{j,t+1} - i_{j,t}} = - \sum_{k=0}^{\infty} (\omega_j \beta)^k E_t i_{j,t+k}$$

## Appendix: Second Decomposition

Return

- From geometric maturity structure Geometric Term Structure

$$\Delta E_t r x_{j,t} = - \sum_{k=1}^{\infty} (\omega_j \beta)^k [\Delta E_t r_{j,t+k} + \Delta E_t \pi_{j,t+k} - \Delta E_t \Delta h_{j,t+k}]$$

- Replace on the original fiscal decomposition

$$\begin{aligned} \Delta E_t \pi_t &= \overbrace{\left[ - \sum_{k=1}^{\infty} (\omega \beta)^k \Delta E_t \pi_{t+k} - \frac{\delta_D}{\delta} \sum_{k=0}^{\infty} (\omega_D \beta)^k \Delta E_t \pi_{t+k}^{US} \right]}^{\text{Innovation to Nominal Variables}} \\ &\quad - \frac{\beta}{\delta v} \underbrace{\left[ \sum_{k=0}^{\infty} \beta^k \Delta E_t s_{t+k} + \frac{v}{\beta} \sum_{k=0}^{\infty} \beta^k \Delta E_t g_{t+k} - \frac{v}{\beta} \sum_{k=1}^{\infty} \sum_j \delta_j \beta^k (1 - \omega^k) \Delta E_t r_{j,t+k} - \frac{\delta_D v}{\beta} \sum_{k=0}^{\infty} (\omega_D \beta)^k \Delta E_t \Delta h_{t+k} \right]}_{\text{Innovation to Real Variables}} \\ &\equiv -d_2(\pi) - d_2(\pi^{US}) - d_2(s) - d_2(g) + d_2(r) + d_2(\Delta h) \end{aligned}$$

# Appendix: Second Decomposition

## Return

Country	$\Delta E_t \pi_t =$	$-\Delta E_t(\text{Future Inflation})$		$\pm \Delta E_t(\text{Real Variables})$			
		$-d_2(\pi)$	$-d_2(\pi^{US})$	$-d_2(s)$	$-d_2(g)$	$d_2(r)$	$d_2(\Delta h)$
United States	1	<b>*-1.12</b>	-	<b>0.57</b>	0.23	<b>*1.32</b>	0
<i>Advanced - 1960 Sample</i>							
Canada	1	<b>*-1.53</b>	<b>*-0.07</b>	0.62	<b>*1.22</b>	<b>0.78</b>	<b>-0.03</b>
Denmark	1	<b>*-0.49</b>	<b>*-0.20</b>	0.42	-0.04	<b>1.23</b>	0.08
Japan	1	<b>*-1.14</b>	0	<b>*1.60</b>	-0.38	<b>*0.91</b>	0
Norway	1	<b>*-0.70</b>	0	<b>0.60</b>	<b>0.47</b>	<b>0.64</b>	0
Sweden	1	<b>*-1.02</b>	<b>-0.10</b>	-0.34	<b>*0.98</b>	<b>*1.54</b>	-0.07
United Kingdom	1	<b>*-2.34</b>	0	<b>*2.89</b>	<b>*0.97</b>	<b>-0.52</b>	0
<i>Advanced - 1973 Sample</i>							
Australia	1	<b>*-1.47</b>	0	<b>*2.09</b>	<b>*0.66</b>	-0.27	0
New Zealand	1	<b>*-1.02</b>	<b>*-0.08</b>	0.40	<b>*0.87</b>	<b>1.04</b>	<b>-0.21</b>
South Korea	1	<b>*-0.74</b>	<b>*-0.03</b>	<b>*1.91</b>	0.17	-0.33	<b>0.01</b>
Switzerland	1	<b>*-0.79</b>	0	<b>0.90</b>	<b>*0.91</b>	-0.02	0

(a) Advanced Economies

Country	$\Delta E_t \pi_t =$	$-\Delta E_t(\text{Future Inflation})$		$\pm \Delta E_t(\text{Real Variables})$			
		$-d_2(\pi)$	$-d_2(\pi^{US})$	$-d_2(s)$	$-d_2(g)$	$d_2(r)$	$d_2(\Delta h)$
<i>Emerging - 1998 Sample</i>							
Brazil	1	<b>*-0.11</b>	0	<b>-1.46</b>	<b>1.05</b>	<b>1.46</b>	0.07
Chile	1	-0.76	<b>-2.75</b>	8.95	-5.71	-0.35	1.62
Colombia	1	<b>*-0.61</b>	-0.04	1.39	-1.09	0.02	<b>1.34</b>
Czech Republic	1	-0.02	<b>-0.05</b>	-2.31	<b>2.42</b>	0.98	-0.03
Hungary	1	<b>*-0.69</b>	<b>*-0.15</b>	-0.98	1.60	1.83	<b>*-0.61</b>
India	1	<b>*-1.05</b>	<b>*0.09</b>	<b>1.54</b>	0.05	0.41	-0.04
Indonesia	1	<b>*-0.79</b>	<b>*-1.33</b>	1.69	<b>*2.61</b>	0.26	<b>-1.45</b>
Israel	1	<b>*-0.54</b>	0.10	-0.55	<b>*1.51</b>	0.61	-0.12
Mexico	1	<b>*-0.60</b>	<b>0.17</b>	<b>1.41</b>	0.03	0.52	<b>-0.52</b>
Poland	1	<b>*-0.59</b>	<b>*-0.21</b>	0.87	-0.39	<b>*1.43</b>	-0.11
Romania	1	<b>*-1.14</b>	<b>*-0.53</b>	2.24	0.42	-0.54	0.55
South Africa	1	0.05	-0.01	1.58	0.25	-0.79	-0.07
Turkey	1	<b>*-0.76</b>	<b>*-0.40</b>	-1.18	-0.15	<b>*3.35</b>	0.14
Ukraine	1	<b>-0.29</b>	0	<b>0.65</b>	<b>*0.41</b>	<b>0.23</b>	0

(b) Emerging Economies

## Appendix: Variance Decomposition

Return

■ **Proposition.** The variance decomposition

$$1 = \frac{\text{cov}_\pi [d(rx)]}{\text{var} [\Delta E_t \pi_t]} + \frac{\text{cov}_\pi [d(r_0)]}{\text{var} [\Delta E_t \pi_t]} - \frac{\text{cov}_\pi [d(s)]}{\text{var} [\Delta E_t \pi_t]} - \frac{\text{cov}_\pi [d(g)]}{\text{var} [\Delta E_t \pi_t]} + \frac{\text{cov}_\pi [d(r)]}{\text{var} [\Delta E_t \pi_t]}$$

is equivalent to the innovations decomposition applied to VAR shock  $E[ e \mid \Delta E_t \pi_t = 1 ]$

$$1 = \Delta E_t \pi_t \equiv d(rx) + d(r_0) - d(s) - d(g) + d(r)$$

■ **Proof:**

$$\begin{aligned} 1 &= -\beta \underbrace{\mathbf{1}'_s (I - \beta A)^{-1} K \Omega K' \mathbf{1}_\pi}_{\text{cov}[\Delta E_t \pi_t, \sum_k \beta^k \Delta E_t s_{t+k}]} \underbrace{(\mathbf{1}'_\pi K \Omega K' \mathbf{1}_\pi)^{-1}}_{\text{var}(\Delta E_t \pi_t)^{-1}} + \mathbf{1}'_r (I - \beta A)^{-1} K \Omega K' \mathbf{1}_\pi (\mathbf{1}'_\pi K \Omega K' \mathbf{1}_\pi)^{-1} \\ &= -\beta \mathbf{1}'_s (I - \beta A)^{-1} K \text{Proj}(e_t \mid \Delta E_t \pi_t = 1) + \mathbf{1}'_r (I - \beta A)^{-1} K \text{Proj}(e_t \mid \Delta E_t \pi_t = 1). \end{aligned}$$

## Appendix: NK Model Parameters

Equations

NK Complete

Comparative Statics

Parameter	Value
$\beta$	0.98
$\gamma$	0.4
$\varphi$	3
$\theta$	0.25
$\alpha$	0.45
$\bar{\omega}$	$\gamma^{-1}$

**Table:** Fixed Parameters

Parameter	Simple	Complete
$\rho_a$	0.96	0.84
$\rho_g$		0.29
$\rho_i$		0
$\rho_s$		0.39
$\phi_\pi$	0.60	0.95
$\phi_g$		0.61
$\tau_\pi$		0.12
$\tau_g$	1.51	0.05
$\sigma_a$	1	1
$\sigma_g$		1.79
$\sigma_i$		0.53
$\sigma_s$		0

**Table:** Estimated Parameters

## Appendix: Why Trend Shocks? The Growth Component

### Return

- Empirical decompositions: often  $d(g) \neq 0$
- But in the absence of trend shocks:

$$g_t = (1 - L)y_t = \mathbf{1}'_y(1 - L)a(L)e_t \equiv \mathbf{1}'_yb(L)e_t$$

- Stationary model  $a(L)^{-1}X_t = e_t \implies$  the roots of  $a(L)^{-1}$  are **outside** the unit circle
- Therefore  $\|a(1)\| < \infty$  and  $b(1) = 0$
- Finally, note that

$$d(g) \propto \mathbf{1}'_yb(\beta)e_t \approx \mathbf{1}'_yb(1)e_t = 0$$

- With trend shocks:

$$g_t = (1 - L)y_t + u_{g,t}$$

## Appendix: FTPL vs Spiral Threat

Return

- In NK models, private sector equations do not determine  $\Delta E_t \pi_t$ .

$$y_t = E_t y_{t+1} - \gamma (\bar{i} - E_t \pi_{t+1})$$
$$\pi_t = \beta E_t \pi_{t+1} + \kappa y_t$$

- In FTPL models, the valuation equation of public debt determines unexpected inflation

$$\Delta E_t \pi_t = \Delta E_t r x_t - \sum_{k=0}^{\infty} \beta^k \Delta E_t s_{t+k} + \sum_{k=1}^{\infty} \beta^k \Delta E_t r_{t+k}$$

- In Spiral Threat models, fiscal decomposition determines  $\Delta E_t s_t$ , not  $\Delta E_t \pi_t$

$$i_t = i_t^* + \phi(\pi_t - \pi_t^*) \quad \phi > 1 \quad \implies \quad \Delta E_t \pi_t = \Delta E_t \pi_t^*$$

- **Observational Equivalence Theorem:** FTPL and Spiral Threat generate the same set of equilibria

## Appendix: Estimated Moments

NK Simple

NK Complete

Moments	Data	Model	Moments	Data	Model
$\sigma_i/\sigma_g$	1.00	1.16	$\text{cor}(\pi, i)$	0.54	0.84
$\sigma_\pi/\sigma_g$	1.01	1.24	$\text{cor}(\pi, g)$	-0.24	-0.25
$\sigma_{\Delta v}/\sigma_g$	1.43	0.90	$\text{cor}(g, i)$	0.16	0.27
$\text{a-cor}(i)$	0.92	0.75	$\text{cor}(i, \Delta v)$	0.02	-0.60
$\text{a-cor}(\pi)$	0.69	0.79	$\text{cor}(\pi, \Delta v)$	-0.29	-0.42
$\text{a-cor}(g)$	0.27	0.25	$\text{cor}(g, \Delta v)$	-0.39	-0.36
$\text{a-cor}(\Delta v)$	0.50	-0.13			

**Table:** Second Moment Fit - Complete Model ( $\alpha_2 = 0.05$ )



## Appendix: Conditional Shocks

NK Complete

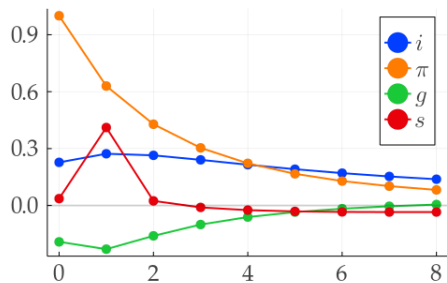
Unexpected Demand

Shock		Inflation	Demand	Surplus
AR(1) Productivity	$\varepsilon_a$	-0.72	-0.76	-0.40
Trend Productivity	$\varepsilon_g$	-0.13	1.49	0.05
Monetary	$\varepsilon_i$	-0.12	-0.19	-0.01
Fiscal	$\varepsilon_g$	0	0	0

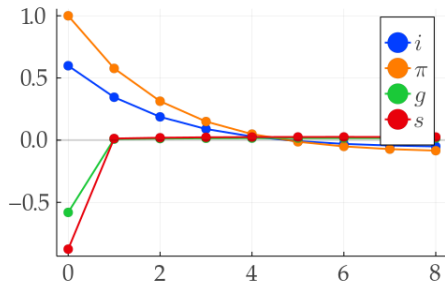
**Table:** Conditional Expected Shocks  $E[\varepsilon | \cdot]$

## Appendix: Simple Model - US Data vs Model

NK Simple



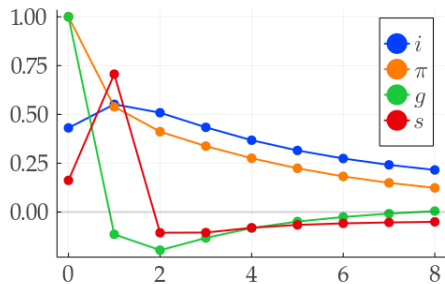
(a) B-VAR



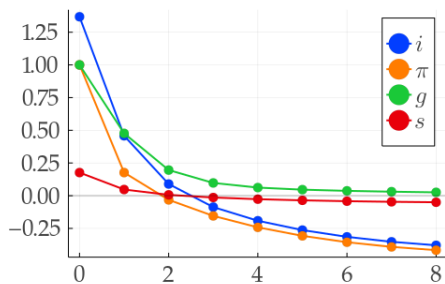
(b) NK Model (Only Prod. Shocks)

## Appendix: Unexpected Demand - US Data vs Model

Return



(a) B-VAR



(b) NK Model

## Appendix: Reproducing the Unexpected Surplus Decomposition

Return

$$1 = \Delta E_t \pi_t = d(rx) - d(s) - d(g) + d(r)$$

Shock	Decomposition	
	Surplus	Inflation
$\varepsilon_a$	-0.40	-0.72
$\varepsilon_g$	0.05	-0.13
$\varepsilon_i$	<b>-0.01</b>	<b>-0.12</b>

Expected Structural Shocks

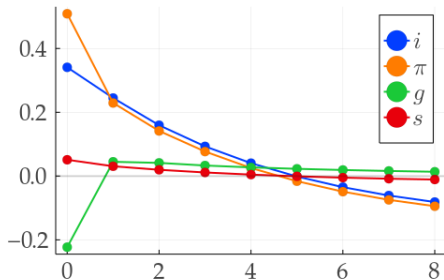


Figure: Disc Surp Variance

Why discount rates? Monetary policy disturbances.

## Appendix: Reproducing the Unexpected Surplus Decomposition

Return

$$1 = \Delta E_t \pi_t = d(rx) - d(s) - d(g) + d(r)$$

Shock	Decomposition	
	Surplus	Inflation
$\varepsilon_a$	-0.40	-0.72
$\varepsilon_g$	0.05	-0.13
$\varepsilon_i$	<b>-0.01</b>	<b>-0.12</b>

Expected Structural Shocks

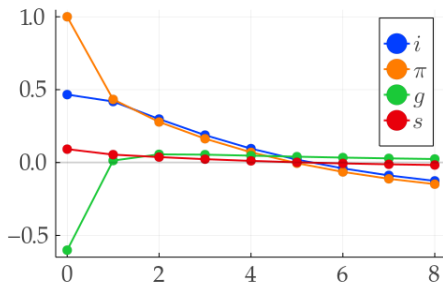


Figure: Disc Surp Variance

Why discount rates? Monetary policy disturbances.