A Fiscal Decomposition of Unexpected Inflation: Cross-Country Estimates and Theory

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November 2022

Introduction: The Fiscal Sources of Unexpected Inflation

- Connection between fiscal policy and inflation?
- Key Equilibrium Condition: The Valuation Equation of Public Debt

 $\frac{\text{Market Value of Debt (Bond Prices)}}{\text{Price Level}} = \text{Intrinsic Value (Discounting, Surpluses)}.$

- Unexpected inflation must accompany news about:
 - Bond prices
 - Real surpluses
 - Real discounting
- If unexpected inflation exists, it must nowcast/forecast some of these variables
- What are these forecasts? How do they change across countries? How can theory explain them?

Introduction: Exercises, Motivation, Results

- **Motivation.** How to read Debt/Price = Discounted Surpluses?

 - - Fixed country: +1% inflation ⇒ +1% deficit/debt?
 - Cross country: +1% inflation in A relative to B ⇒ +1% deficit/debt in A compared to B?
 - Stylized facts to discipline monetary-fiscal theory
 - Fiscal role to monetary policy?
- This paper
 - Estimate a Bayesian-VAR to measure the decomposition for 25 countries
 - Variance decomposition: "What does 1% unexpected inflation forecast?"
 - Recession decomposition: "What do -0.5% inflation and -1% growth forecast?" (2008 Recession, deficits, etc...)
 - 2. Estimate a New-Keynesian model to reproduce B-VAR decompositions

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 - \circ Active monetary: Inflation \Longrightarrow Discounted Surpluses
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Introduction: Preview of Key Results

The variance of unexpected inflation is accounted for by discounted surpluses (all coutries)

$$\operatorname{var} \left[\Delta E \pi \right] = \operatorname{cov} \left[\Delta E \pi, \quad \mathbf{Q} \right] + \operatorname{cov} \left[\Delta E \pi, \quad \left\{ -\mathbf{S} \right\} + \left\{ \mathbf{R} \right\} \right]$$

- Most relevant terms (Surplus-to-GDP, GDP growth and Real discounting) vary
- Monetary policy reduces unexpected inflation variance through bond prices
- Low inflation in recessions: low discounting + higher subsequent surpluses relative to GDP
- Productivity shocks reproduce findings in NK model
 - Policy shocks do not
- Different policy rules explain differences in variance decomposition across countries

Introduction: Related Literature

- Fiscal Theory of the Price Level. Cochrane (2022a) and Cochrane (2022b).
 - Analysis of multiple countries + more general debt instruments
 - NK model estimated to reproduce decompositions + Productivity shocks

Leeper (1991), Sims (1994), Cochrane (1998), Cochrane (2005), Sims (2011), Leeper and Leith (2016), Bassetto and Cui (2018), Cochrane (2022c), Brunnermeier et al. (2022).

- Monetary-Fiscal Interaction.
 - Cagan (1956), Sargent and Wallace (1981), Hall and Sargent (1997), Hall and Sargent (2011), Jiang et al. (2019), Corsetti et al. (2019), Sunder-Plassmann (2020), Du et al. (2020), Akhmadieva (2022)
- Empirical Finance (Decomposition of Returns)
 Campbell and Shiller (1988), Cochrane (1992), Campbell and Ammer (1993), Chen and Zhao (2009).

Fiscal Decomposition: The Valuation Equation

- **Environment with discrete time + single good (price** P_t) + households and government
- One-period nominal public bonds (price Q_t)
- **In the morning**, the government:
 - redeems bonds B_{t-1} for currency
 - announces real taxes s_t (payable in currency)
 - \circ announces sale of B_t new bonds (payable in currency)
- In the afternoon, households trade goods, purchase bonds, pay taxes
- Market clearing + No Money Holding M = 0:

$$B_{t-1} = P_t s_t + Q_t B_t$$

Fiscal Decomposition: The Valuation Equation

$$B_{t-1} = P_t s_t + Q_t B_t$$

- **Ex-post real discounting** $\beta_t = Q_t(P_{t+1}/P_t)$ $\beta_{t,t+k} = \prod_{\tau=t}^{t+k} \beta_{\tau}$
- Iterate law of motion forward:

$$\frac{B_{t-1}}{P_t} = \sum_{k=0}^{p} \beta_{t,t+k-1} s_{t+k} + \beta_{t,t+p} \frac{B_{t+p}}{P_{t+p+1}}$$

- **Key Assumption**: $\lim_{\tau \to \infty} \beta_{t,\tau} \frac{B_{\tau}}{P_{\tau+1}} = 0$ almost surely (No bubbles)
 - In Macro models: transversality conditions + no Ponzi
- The valuation equation of public debt

$$\frac{B_{t-1}}{P_t} = \sum_{k=0}^{\infty} E_t \left[\beta_{t,t+k-1} s_{t+k} \right]$$

Fiscal Decomposition: In the Simplest Environment

- Nominal rate $1 + i_t = 1/Q_t$ and real interest $r_t = i_t E_t \pi_{t+1}$
- End-of-period real debt vt

$$\underbrace{\frac{1}{\beta} v_{t-1} + \frac{v}{\beta} (i_{t-1} - \pi_t)}_{B_{t-1}/P_t} = s_t + v_t = \sum_{k=0}^{\infty} \beta^k E_t s_{t+k} - \frac{v}{\beta} \sum_{k=1} \beta^k E_t r_{t+k}$$

Linearized valuation equation (all variable in deviations from their mean)

■ Innovations $\Delta E_t = E_t - E_{t-1}$ decomposition:

$$\Delta E_t \pi_t = -\frac{\beta}{V} \sum_{k=0}^{\infty} \beta^k \Delta E_t s_{t+k} + \sum_{k=0}^{\infty} \beta^k \Delta E_t r_{t+k}$$

Variance decomposition

$$\mathsf{var}\left[\Delta E_t \pi_t
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$$\mathsf{var}\left[\Delta E_t \pi_t\right] \ = \ -\mathsf{cov}_{\pi}\left[\frac{\beta}{\mathsf{v}} \sum_{k=0}^{\infty} \beta^k \Delta E_t s_{t+k}\right] \ + \mathsf{cov}_{\pi}\left[\sum_{k=0}^{\infty} \beta^k \Delta E_t r_{t+k}\right]$$

Fiscal Decomposition: Currency and Term Structures + Growth

- **Real market value** debt-to-GDP v_t , surplus-to-GDP s_t and GDP growth g_t (stationarity!)
- Bonds (j, n) promisses one unit of currency j after n periods Currencies
 - Nominal bonds
 - Real bonds (currency denomination = final goods)
 - US Dollar bonds

Constant structure $\{\delta_j\}$, $\{\omega_j^n\}$

- Bond price $Q_{j,t}^n$, excess return $rx_{j,t}$ 1+ return_{j,t} = 1 + $rx_{j,t}$ + $i_{j,t-1} = \frac{\sum_n Q_{j,t-1}^n B_{j,t-1}^n}{\sum_n Q_{j,t-1}^n B_{j,t-1}^n}$ (one-period bonds $\implies rx = 0$)
- Debt law of motion:

$$\frac{\mathbf{v}_{t-1}}{\beta} + \frac{\mathbf{v}}{\beta} \left[-\mathbf{g}_t + \sum_j \delta_j \left(\mathbf{r} \mathbf{x}_{j,t} + \mathbf{i}_{j,t-1} + \Delta \mathbf{h}_{j,t} - \pi_{j,t} \right) \right] = \mathbf{v}_t + \mathbf{s}_t$$

Fiscal Decomposition of Unexpected Inflation

Ex-post real return $r_{j,t} = rx_{j,t} + i_{j,t-1} + \Delta h_{j,t} - \pi_{j,t}$

$$\Delta E_t \pi_t = \underbrace{\left[\Delta E_t r x_t + \sum_{j \neq N} \frac{\delta_j}{\delta} \Delta E_t r_{j,t} \right] - \frac{\beta}{\delta v} \underbrace{\left[\sum_{k=0}^{\infty} \beta^k \Delta E_t s_{t+k} + \frac{v}{\beta} \sum_{k=0}^{\infty} \beta^k \Delta E_t g_{t+k} - \frac{v}{\beta} \sum_{k=1}^{\infty} \sum_j \delta_j \beta^k \Delta E_t r_{j,t+k} \right]}_{}$$

Innovation to Bond Prices

Innovation to the Intrinsic Value of Debt

$$\equiv d_1(rx) + d_1(r_0) - d_1(s) - d_1(g) + d_1(r)$$

Variance decomposition.

$$\mathsf{var}\left[\Delta E_t \pi_t\right] = \mathsf{cov}_{\pi}\left[d_1(\mathit{rx})\right] + \mathsf{cov}_{\pi}\left[d_1(\mathit{r_0})\right] - \mathsf{cov}_{\pi}\left[d_1(s)\right] - \mathsf{cov}_{\pi}\left[d_1(g)\right] + \mathsf{cov}_{\pi}\left[d_1(r)\right]$$

Bayesian-VAR: Data and Model

■ Annual data on observables \tilde{x}_t

$$egin{aligned} \textit{x}_t^{ ext{OBS}} = \left[egin{array}{ll} i_t & (ext{Nominal Interest}) \\ \pi_t & (ext{CPI Inflation}) \\ \emph{v}_t^b & (ext{Par-Value Debt-to-GDP}) \\ \emph{g}_t & (ext{GDP growth}) \\ \Delta h_t & (ext{Chg. Real Exchange Rate}) \end{array}
ight] \end{aligned}$$

- 25 countries (samples starting at 1945, 1960, 1975, 1998)
- General VAR

$$X_t = AX_{t-1} + Ke_t$$

■ Decompose $X'_t = [x_t^{OBS'} x_t^{NOT'}]$

$$X_t = \left[\begin{array}{c} x_t^{OBS} \\ x_t^{NOT} \end{array} \right] = \left[\begin{array}{cc} a & 0 \\ b & c \end{array} \right] \left[\begin{array}{c} x_{t-1}^{OBS} \\ x_{t-1}^{NOT} \end{array} \right] + \left[\begin{array}{c} I \\ k \end{array} \right] e_t$$

1. Explosive debt dynamics in 1960-2020 implies unstable OLS VARs

$$x_t^{OBS} = a x_{t-1}^{OBS} + e_t$$

- United States: Estimate model by OLS (stable!)
- Others: Estimate model with a Bayesian-Regression

$$a^{BAY} = (X'X + \lambda^{-1})^{-1}(X'X a^{OLS} + \lambda^{-1} a^{PRIOR})$$

- 2. Public finance data do not respect law of motion of public debt
- 3. No data on the market value of debt, only its par value (v_t^b) Public Finances Model
 - Model for market vs par value (Cox (1985)): $v_t = v_t^b + \frac{v}{\beta} \sum_j \delta_j \left(r x_{j,t} + i_{j,t-1} i_{j,t-1}^b \right)$
- 4 No data on bond returns Geometric Term Structure
 - Geometric maturity structure: $rx_{j,t} + i_{j,t-1} = (\omega_j \beta)q_{j,t} q_{j,t-1}$

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 - Define surplus from the law of motion: $\mathbf{s_t} = \frac{\mathbf{v}_{t-1}}{\beta} \mathbf{v}_t + \frac{\mathbf{v}}{\beta} \left[-g_t + \sum_j \delta_j \left(r x_{j,t} + i_{j,t-1} + \Delta h_{j,t} \pi_{j,t} \right) \right]$
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Bayesian-VAR: Variance Decomposition

Proposition. The variance decomposition

$$1 = \frac{\mathsf{cov}_{\pi} \bigg[d_1(rx) \bigg]}{\mathsf{var} \left[\Delta E_t \pi_t \right]} + \frac{\mathsf{cov}_{\pi} \bigg[d_1(r_0) \bigg]}{\mathsf{var} \left[\Delta E_t \pi_t \right]} - \frac{\mathsf{cov}_{\pi} \bigg[d_1(s) \bigg]}{\mathsf{var} \left[\Delta E_t \pi_t \right]} - \frac{\mathsf{cov}_{\pi} \bigg[d_1(g) \bigg]}{\mathsf{var} \left[\Delta E_t \pi_t \right]} + \frac{\mathsf{cov}_{\pi} \bigg[d_1(r) \bigg]}{\mathsf{var} \left[\Delta E_t \pi_t \right]}$$

is equivalent to the innovations decomposition applied to VAR shock $Proj(e \mid \Delta E_t \pi_t = 1)$

$$1 = \Delta E_t \pi_t \equiv d_1(rx) + d_1(r_0) - d_1(s) - d_1(g) + d_1(r)$$

"Given 1% unexpected inflation, how do we change our nowcast/forecast of the surplus, discounting and bond prices?"

Bayesian-VAR: Variance Decomposition

Country	$\Delta E_t \pi_t =$	$\Delta E_t(Bor$	d Prices)	$-\Delta E_t(In$	$-\Delta E_t$ (Intrinsic Value		
		$d_1(r_0)$	$d_1(rx)$	-d ₁ (s)	$-d_1(g)$	$d_1(r)$	
United States	1	*0.03	*-0.78	0.57	0.23	0.96	
Advanced - 1960 Sample							
Canada	1	*-0.11	*-1.59	0.62	*1.22	0.86	
Denmark	1	*-0.29	-0.30	0.42	-0.04	1.21	
Japan	1	0	*-0.52	*1.60	-0.38	0.30	
Norway	1	*-0.01	*-0.36	0.60	0.47	0.30	
Sweden	1	-0.15	*-0.93	-0.34	*0.98	*1.42	
United Kingdom	1	*0.52	*-0.73	*2.89	*0.97	*-2.65	
Advanced - 1973 Sample							
Australia	1	*0.07	*-0.76	*2.09	0.66	-1.06	
New Zealand	1	-0.10	*-0.86	0.40	*0.87	0.68	
South Korea	1	-0.01	*-0.45	*1.91	0.17	-0.62	
Switzerland	1	0	*-0.69	0.90	*0.91	-0.12	

Country	$\Delta E_t \pi_t =$	$\Delta E_t(Bon$	d Prices)	$-\Delta E_t(In$	$-\Delta E_t$ (Intrinsic Value		
		$d_1(r_0)$	$d_1(rx)$	-d ₁ (s)	$-d_1(g)$	$d_1(r)$	
Emerging - 1998 Sample							
Brazil	1	-0.26	*-0.22	-1.46	1.05	1.89	
Chile	1	-3.80	-1.33	8.95	-5.71	2.88	
Colombia	1	1.51	*-0.96	1.39	-1.09	0.15	
Czech Republic	1	*-0.16	*-0.37	-2.31	2.42	1.42	
Hungary	1	*-0.57	*-0.93	-0.98	1.60	1.88	
India	1	*0.17	*-0.46	1.54	0.05	-0.30	
Indonesia	1	*-2.59	*-1.07	1.69	*2.61	0.35	
Israel	1	-0.06	*-0.78	-0.55	*1.51	0.88	
Mexico	1	-0.02	*-0.74	1.41	0.03	0.32	
Poland	1	*-0.45	*-1.15	0.87	-0.39	*2.11	
Romania	1	-0.40	*-0.96	2.24	0.42	-0.31	
South Africa	1	0.36	*-0.51	1.58	0.25	-0.68	
Turkey	1	0.37	*-0.37	-1.18	-0.15	*2.33	
Ukraine	1	0	*-0.77	0.65	0.41	*0.70	

(a) Advanced Economies

(b) Emerging Economies

Decomposition 2

Bayesian-VAR: Variance Decomposition - Takeaways

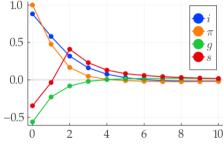


Figure: IRF - Brazil

- Unexpected inflation accounted for by variation in the intrinsic value of debt
- Surplus-to-GDP, GDP growth and real discounting...
 - ...account for unexpected inflation alone in 0/25
 - ...have a positive contribution in 18+/25
- Fiscal roots of inflation do not imply connection between fiscal policy and unexpected inflation
- Nominal bond price dynamics reduce unexpected inflation variance 25/25
 - Effects of monetary policy!

Bayesian-VAR: "Aggregate Demand" Recession

- "Aggregate demand" recessions (Great Recession in 2008) feature:
 - Low inflation
 - Low growth
 - Fiscal deficits (often)
- Does that deny the fiscal sources of inflation?
- Where does unexpected (dis)inflation come from?
- Scenario:

$$\Delta E_t g_t = -1$$
 $\Delta E_t \pi_t = -0.5$

VAR Shock: Proj($e \mid \Delta E_t g_t = -1, \ \Delta E_t \pi_t = -0.5$)

Bayesian-VAR: "Aggregate Demand" Recession

Country	$\Delta E_t \pi_t =$	Ī	$\Delta E_t(Bon$	d Prices)		$-\Delta E_t$ (Int	$-\Delta E_t$ (Intrinsic Value of			
		_	$d_1(r_0)$	$d_1(rx)$	Ι	$-d_1(s)$	$-d_1(g)$	$d_1(r$		
United States	-0.50	ı	*0.03	*1.00	1	-0.65	*1.32	*-2.21		
Advanced - 1960 Sample										
Canada	-0.50		*0.14	*2.21		-0.45	0.30	*-2.70		
Denmark	-0.50		*0.20	*0.86		-2.64	*2.75	-1.6		
Japan	-0.50		0	*0.83		*-1.51	*1.64	*-1.4		
Norway	-0.50		0	*0.63		-1.36	*1.72	-1.4		
Sweden	-0.50		*0.41	*1.22		-0.65	0.87	*-2.3		
United Kingdom	-0.50		0.11	*2.54		-2.20	0.73	-1.6		
Advanced - 1973 Sample										
Australia	-0.50		0.06	*1.54		-1.46	0.66	-1.3		
New Zealand	-0.50		*0.26	*0.87		-0.84	0.63	-1.4		
South Korea	-0.50		*0.10	*0.70		*-3.17	*1.74	0.1		
Switzerland	-0.50		0	*1.18		*-0.93	-0.07	-0.63		

(a) Advanced Economies

Country	$\Delta E_t \pi_t =$	$\Delta E_t(Bon d_1(r_0))$	d Prices)	$-\Delta E_t(Int -d_1(s))$	atrinsic Value of Debt) $-d_1(g) \qquad d_1(r)$			
Emerging - 1998 Sample								
Brazil	-0.50	*0.37	0.06	1.87	0.13	-2.39		
Chile	-0.50	*15.78	*2.94	-30.50	30.54	-19.26		
Colombia	-0.50			-30.50 -10.90	*7.57	0.31		
		1.86	*0.67					
Czech Republic	-0.50	*0.37	*0.61	-0.07	0.25	-1.65		
Hungary	-0.50	*0.99	*0.60	10.82	-5.29	-7.63		
India	-0.50	-0.03	0.13	-1.16	0.71	-0.15		
Indonesia	-0.50	*8.23	-0.55	*-11.24	1.42	1.64		
Israel	-0.50	*1.79	0.37	-3.18	1.17	-0.65		
Mexico	-0.50	*1.69	*0.81	*-4.56	*1.94	-0.38		
Poland	-0.50	*0.87	*1.00	-0.14	1.30	*-3.53		
Romania	-0.50	*2.08	0.21	*-8.16	2.05	3.31		
South Africa	-0.50	-0.10	0.35	*-30.02	*11.15	*18.13		
Turkey	-0.50	*0.99	*0.23	0.64	0.52	*-2.88		
Ukraine	-0.50	0	-0.68	-3.22	*1.92	1.48		

(b) Emerging Economies

Bayesian-VAR: "Aggregate Demand" Recession - Takeaways

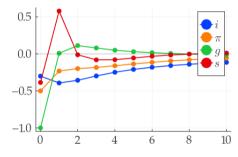


Figure: IRF - United States

- Lower inflation follows from...
 - lower discounting (monetary policy) in 19/25
 - larger surplus-GDP ratios, current or in the future in 22/25
- COVID: what if governments reacted to a recession by credibly reducing {s} permanently?
- Direction of causality?

The New-Keynesian Model: Setup

- B-VAR decompositions not structural: all shocks are reduced-form
- Theory: How far do we need to go? Not much!
- Two-country NK mode
 - \circ Home economy with $n \to 0$ households and firms (small and open)
 - \circ Foreign economy with $1-n \to 1$ households and firms (large and "closed")
- The Standard. Intertemporal substitution + Calvo rigidity
- **The New.** Production function $A_t N = \mathcal{T}_t A_t N$ (Home), $A_t^* N = \mathcal{T}_t A_t^* N$ (Foreign)

(Trend component)
$$\log \mathcal{T}_t = \log \mathcal{T}_{t-1} + u_{g,t}$$

(AR(1) component) $a_t = \rho_a a_{t-1} + \varepsilon_{a,t}$
 $a_t^* = \rho_a a_{t-1}^* + \varepsilon_{a,t}^*$



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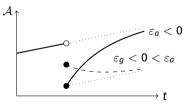
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$$\begin{array}{ll} \text{(Trend component)} & \log \mathcal{T}_t = \log \mathcal{T}_{t-1} + u_{g,t} \\ \text{(AR(1) component)} & a_t & = \rho_a a_{t-1} + \varepsilon_{a,t} \\ & a_t^* & = \rho_a a_{t-1}^* + \varepsilon_{a,t}^* \end{array}$$



The New-Keynesian Model: The Foreign, Closed Economy

Private Sector

$$y_{t}^{*} = E_{t}y_{t+1}^{*} - \gamma [i_{t}^{*} - E_{t}\pi_{t+1}^{*}] + E_{t}u_{g,t+1}$$

$$\pi_{t}^{*} = \beta E_{t}\pi_{t+1}^{*} + \kappa y_{t}^{*} - \kappa_{a} a_{t}^{*}$$

$$g_{t}^{*} = y_{t}^{*} - y_{t-1}^{*} + u_{g,t}$$

Why Trend? Growth

- Unexpected inflation indeterminacy? FTPL.
- Monetary and Fiscal Policy

$$\begin{aligned} & \textbf{\textit{i}}_{t}^{*} = \phi_{\pi} \; \pi_{t}^{*} + \phi_{g} \; \textbf{\textit{g}}_{t}^{*} + \varepsilon_{i,t}^{*} \\ & \textbf{\textit{s}}_{t}^{*} = \rho_{s} \; \textbf{\textit{s}}_{t-1}^{*} + \tau_{\pi} \; \pi_{t}^{*} + \tau_{g} \; \textbf{\textit{g}}_{t}^{*} + \varepsilon_{s,t}^{*} \end{aligned}$$

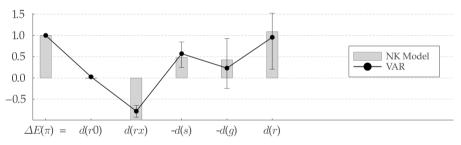
GMM for US moments

$$\mathsf{Min}_{\Psi} \quad {}_{\alpha_{1}} \| \mathcal{D}_{\mathsf{VAR}} - \mathcal{D}_{\mathsf{NK}}(\Psi) \| + {}_{\alpha_{2}} \| \mathcal{M} - \mathcal{M}_{\mathsf{NK}}(\Psi) \| \qquad \text{s.t. } \Psi \in \Theta$$

Parameters

The New-Keynesian Model: Reproducing the Variance Decomposition

Result. AR(1) productivity shocks $\varepsilon_{a,t}$ alone reproduce the **variance decomposition** with positive contributions from surplus-to-output, growth and real interest terms



Target: United States. Only AR(1) productivity shocks.



The New-Keynesian Model: Reproducing the Variance Decomposition

$$1 = \Delta E_t \pi_t = d(rx) - d(s) - d(g) + d(r)$$

- Key Ingredients
 - Persistent shock: $\rho_a = 0.98$
 - Countercyclical deficits: $\tau_q = 0.7$
 - Strong Taylor: $\phi_{\pi} = 0.8$
- What is the story?
 - Low productivity leads to a recession

Government raises deficit to fight recession

Monetary policy raises nominal interest

Marginal Costs

vs B-VAR IRF

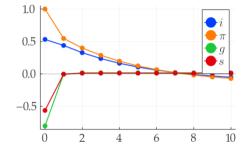


Figure: IRF to $\Delta E_t \pi_t = 1$ ($\varepsilon_{a,t} = -1.15$)

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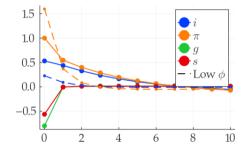
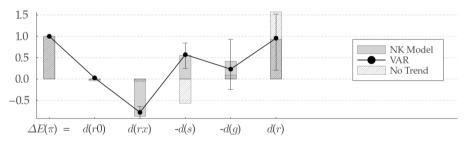


Figure: IRF to $\Delta E_t \pi_t = 1$ ($\varepsilon_{a,t} = -1.15$)

$$\Delta E_t q_t = -1$$
 $\Delta E_t \pi_t = -0.5$

- Result. In the absence of trend shocks, NK model fails to replicate the variance and recession decompositions. Policy shocks do not help.
- Result. The model with trend shocks reproduces the recession decomposition without policy shocks.

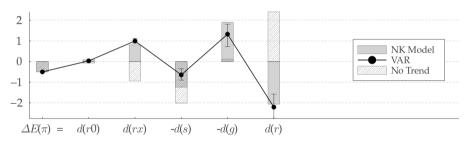


Target: United States Structural Shocks



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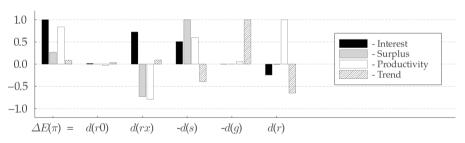


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Target: United States Structural Shocks



$$-0.5 = \Delta E_t \pi_t = d(rx) - d(s) - d(g) + d(r)$$

- Kev Ingredients
 - Quiet monetary shocks: $\sigma_i = 0.05$
 - Trend + AR(1) shocks: $\sigma_0 = 2.6$, $\sigma_0 = 1.4$
 - Strong Taylor: $\phi_{\pi} = 0.93$
- Intuition
 - AR(1) shocks reproduce variance decomposition
 - Monetary policy shocks generate wrong d(rx)
- Recession ($\varepsilon_a = 0.5$, $\varepsilon_a = -1.3$)
 - Recession and lower interest

$$d(g)>0 \qquad d(rx)>0$$

- Low detrended marginal costs: $\pi_t < E_t \pi_{t+1}$
- Low/increasing inflation + Taylor guarantee low real interest

$$r_t = i_t - E_t \pi_{t+1} \approx \pi_t - E_t \pi_{t+1} < 0$$

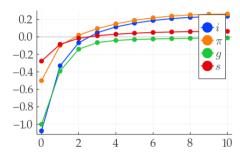


Figure: IRF to
$$\Delta E_t g_t = -$$
1, $\Delta E_t \pi_t = -$ 0.5

Marginal Costs vs B-VAR IRF

The New-Keynesian Model: The Open Economy

$$y_{t} = E_{t}y_{t+1} - \gamma \left[i_{t} - E_{t}\pi_{t+1} + \alpha \overline{\omega} E_{t}\Delta z_{t+1}\right] + E_{t}u_{g,t+1}$$

$$\pi_{H,t} = \beta E_{t}\pi_{H,t+1} + \kappa y_{t} - \kappa_{a} a_{t} - \kappa_{z} z_{t}$$

$$y_{t} = y_{t}^{*} + \gamma_{\alpha} z_{t}$$

$$h_{t} = (1 - \alpha) z_{t}$$

$$\pi_{t} = \pi_{H,t} + \alpha \Delta z_{t}$$

- **Home**: Open trade; Complete markets
- Same parameters of the US estimation
- Same combination of Home productivity shocks:
 - Variance decomposition ✓
 - Recession decomposition ✓

Foreign Policy Shocks

New Zealand Decomp

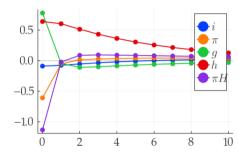


Figure: Productivity Shock in Home Foreign

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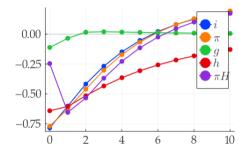


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Foreign Policy Shocks New Zealand Decomp

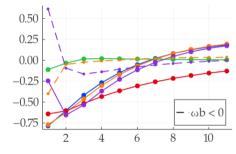


Figure: Productivity Shock in Home Foreign

The New-Keynesian Model: Variance Decomposition and Policy Rules

$$1 = \Delta E_t \pi_t = d(BP) - d(s) - d(g) + d(r)$$

- Can we match variance decomposition of different countries?
- Vary policy parameters
- Keep other structural parameters constant

$$i_t = \phi_\pi \ \pi_t + \phi_g \ g_t + \varepsilon_{i,t}$$

 $S_t = \rho_S \ S_{t-1} + \tau_\pi \ \pi_t + \tau_g \ g_t + \varepsilon_{s,t}$

	N	6	S	A	
Parameters	New Zealand	Sweden	Denmark	Australia	
A. Variance De	composition - Val	ue of Debt Contri	bution		
-d(s)	0.40 (0.66)	-0.34 (0.04)		2.09 (2.04)	
-d(g)	0.87 (0.63)	0.98 (1.03)		0.66 (0.97)	
d(r)	0.68 (0.90)	1.42 (1.10)	1.21 (1.17)	-1.06 (-0.52)	
B. Estimated F	arameters				
ρa	0.84				
ρg	0.27				
ϕ_{π}	0.93	0.99	0.96	0.91	
$\phi oldsymbol{g}$	0.61	0.63	0.71	1	
ρ_{S}	0	0.26	0.61	0.05	
τ_{π}	0.25	0.01	-0.02	-0.03	
τ_g	0.15	-0.10	-0.13	0.25	
σ_i	0.05	0	0	0.51	
σ_{S}	0.08	0	0.12	0.50	
σa	1.41				
σg	2.62				
C. Productivity	Shocks Projected	by $\Delta E_t \pi_t = 1$			
εa.t	-0.35	-0.48	0.12	-0.74	
$arepsilon^{arepsilon}_{arepsilon^*_{m{a},m{t}}}$	-0.77	-0.82	-0.06	-0.08	
εa,t	-0.61	-0.76	-1.49	-0.16	

Variance Decomps and Policy Rules

Conclusion

- lacktriangle Valuation equation of public debt \Longrightarrow decomposition of unexpected inflation
- B-VAR based estimation of two versions of the decomposition
- In most countries:
 - Variance of unexpected inflation stems from discounted surpluses (all of its components)
 - Recessions: low inflation follow from low discounting and "not so expansionary" fiscal policy
- Stylized New-Keynesian models reproduce VAR decompositions
 - Relevance of productivity shocks
 - Relevance of policy rules

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Appendix: Public Debt Currency Denomination

Return

Symbol	Description	Nominal Debt	Real Debt	Dollar Debt
j	Index Symbol	N	R	D
	Notation	δ , ω	$\delta_{ extsf{R}}$, $\omega_{ extsf{R}}$	$\delta_{ extsf{D}}$, $\omega_{ extsf{D}}$
P _i	Price per Good	Р	1	P _t ^{US}
$\dot{\mathcal{E_i}}$	Nominal Exchange Rate	1	Ρ	Dollar NER
H_j	Real Exchange Rate	1	1	Dollar RER
π_i	Log Variation in Price	π	0	$\pi_t^{ extsf{US}}$
Δh_j	Log Real Depreciation	0	0	Δh_t

Table: Public Debt Denomination

Appendix: Public Finances Model



Appendix: Geometric Term Structure

Return Decomposition 2

■ To each currency portfolio j, fixed geometric maturity structure:

$$B_{j,t}^n = \omega_j B_{j,t}^{n-1}$$

■ Total return on currency-*j* portfolio:

$$1+r\mathsf{x}_{j,t}+i_{j,t-1}=\frac{1+\omega_jQ_{j,t}}{Q_{i,t-1}}\qquad\Longrightarrow\qquad \boxed{\mathsf{rx}_{j,t}+i_{j,t-1}=(\omega_j\beta)q_{j,t}-q_{j,t-1}}$$

Assume constant risk premia $E_t r x_{i,t+1} = 0$

$$\boxed{\mathbf{q}_{j,t} = (\omega_j \beta) E_t q_{j,t+1} - i_{j,t}} = -\sum_{k=0}^{\infty} (\omega_j \beta)^k E_t i_{j,t+k}$$

Appendix: Second Decomposition

Return

■ From geometric maturity structure Geometric Term Structure

$$\Delta E_t r x_{j,t} = -\sum_{i=0}^{\infty} (\omega_j \beta)^k \left[\Delta E_t r_{j,t+k} + \Delta E_t \pi_{j,t+k} - \Delta E_t \Delta h_{j,t+k} \right]$$

Replace on the original fiscal decomposition

Innovation to Nominal Variables

$$\Delta E_{t}\pi_{t} = \boxed{-\sum_{k=1}^{\infty} (\omega\beta)^{k} \Delta E_{t}\pi_{t+k} - \frac{\delta_{D}}{\delta} \sum_{k=0}^{\infty} (\omega_{D}\beta)^{k} \Delta E_{t}\pi_{t+k}^{US}} \\ - \frac{\beta}{\delta v} \boxed{\sum_{k=0}^{\infty} \beta^{k} \Delta E_{t}S_{t+k} + \frac{v}{\beta} \sum_{k=0}^{\infty} \beta^{k} \Delta E_{t}g_{t+k} - \frac{v}{\beta} \sum_{k=1}^{\infty} \sum_{j} \delta_{j}\beta^{k} (1 - \omega^{k}) \Delta E_{t}r_{j,t+k} - \frac{\delta_{D}v}{\beta} \sum_{k=0}^{\infty} (\omega_{D}\beta)^{k} \Delta E_{t}\Delta h_{t+k}} }$$
Innovation to Real Variables
$$\equiv -d_{2}(\pi) - d_{2}(\pi^{US}) - d_{2}(s) - d_{2}(g) + d_{2}(r) + d_{2}(\Delta h)$$

Appendix: Second Decomposition

Return

Country	$\Delta E_t \pi_t =$		$-\Delta E_t$ (Futi	ire Inflation)			$\pm \Delta E_t$ (Real	Variables)				
			$-d_2(\pi)$	$-d_2(\pi^{US})$	I	$-d_2(s)$	$-d_2(g)$	$d_2(r)$	$d_2(\Delta h)$			
United States	1		*-1.12		ı	0.57	0.23	*1.32	0			
Advanced - 1960 Sample												
Canada	1	1	*-1.53	*-0.07		0.62	*1.22	0.78	-0.03			
Denmark	1		*-0.49	*-0.20		0.42	-0.04	1.23	0.08			
Japan	1	1	*-1.14	0		*1.60	-0.38	*0.91	(
Norway	1	1	*-0.70	0		0.60	0.47	0.64	0			
Sweden	1	1	*-1.02	-0.10		-0.34	*0.98	*1.54	-0.07			
United Kingdom	1		*-2.34	0		*2.89	*0.97	-0.52	(
Advanced - 1973 Sample												
Australia	1	1	*-1.47	0		*2.09	*0.66	-0.27	(
New Zealand	1		*-1.02	*-0.08	1	0.40	*0.87	1.04	-0.21			
South Korea	1		*-0.74	*-0.03	1	*1.91	0.17	-0.33	0.01			
Switzerland	1	1	*-0.79	0		0.90	*0.91	-0.02	0			

Country	$\Delta E_t \pi_t = $	$-\Delta E_t$ (Future Inflation)				$\pm \Delta E_t$ (Real	(Real Variables)	
		$-d_2(\pi)$	$-d_2(\pi^{US})$	1	$-d_2(s)$	$-d_{2}(g)$	$-d_2(g)$ $d_2(r)$	$d_2(\Delta h)$
Emerging - 1998 Sample								
Brazil	1	*-0.11	0	- 1	-1.46	1.05	1.46	0.07
Chile	1	-0.76	-2.75		8.95	-5.71	-0.35	1.62
Colombia	1	*-0.61	-0.04		1.39	-1.09	0.02	1.34
Czech Republic	1	-0.02	-0.05		-2.31	2.42	0.98	-0.03
Hungary	1	*-0.69	*-0.15		-0.98	1.60	1.83	*-0.61
India	1	*-1.05	*0.09		1.54	0.05	0.41	-0.04
Indonesia	1	*-0.79	*-1.33		1.69	*2.61	0.26	-1.45
Israel	1	*-0.54	0.10		-0.55	*1.51	0.61	-0.12
Mexico	1	*-0.60	0.17		1.41	0.03	0.52	-0.52
Poland	1	*-0.59	*-0.21		0.87	-0.39	*1.43	-0.11
Romania	1	*-1.14	*-0.53		2.24	0.42	-0.54	0.55
South Africa	1	0.05	-0.01		1.58	0.25	-0.79	-0.07
Turkey	1	*-0.76	*-0.40		-1.18	-0.15	*3.35	0.14
Ukraine	1	-0.29	0		0.65	*0.41	0.23	0

(a) Advanced Economies

(b) Emerging Economies

Appendix: NK Model Parameters

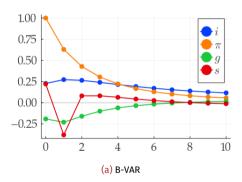
Appendix: Why Trend Shocks? The Growth Component

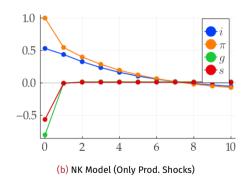
Appendix: Estimated Moments

NK Simple NK Full

Appendix: Simple Model - US Data vs Model





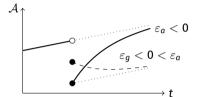


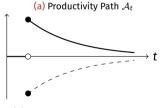
Appendix: Marginal Costs

NK Simple

- AR(1) Productivity Shock
 - High marginal costs + strong Taylor rule ($\phi_{\pi} \approx$ 1):

$$i_t \approx \underbrace{\pi_t > E_t \pi_{t+1}}_{m_{t+2}} \implies r_t = i_t - E_t \pi_{t+1} > 0$$

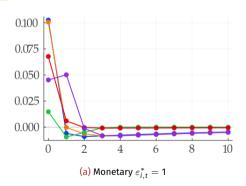


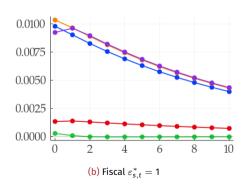


(b) $-a_t$ or Mg. Cost at fixed wages

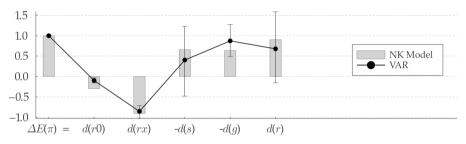
Appendix: NK Open - Foreign Policy Shocks





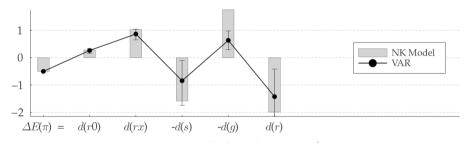


Appendix: NK Open - New Zealand Decomposition



Data: New Zealand - Variance Recession

Appendix: NK Open - New Zealand Decomposition



Data: New Zealand - Variance Recession