A Fiscal Decomposition of Unexpected Inflation: Cross-Country Estimates and Theory

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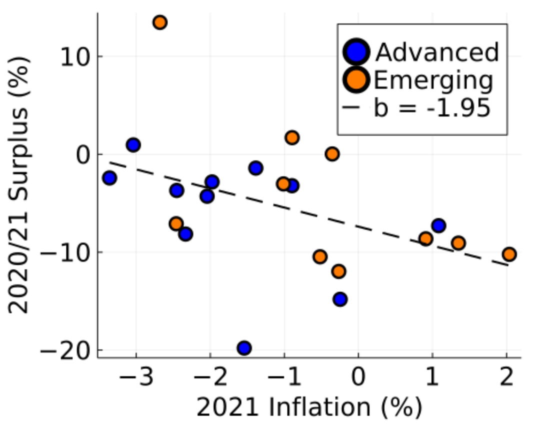
Fiscal Connection?



COVID Inflation - 21 countries in sample

Introduction

- Sources of inflation variation
- What drives innovations to the price level?
- Breakdown of valuation equation of public debt
- Focus on unexpected inflation $\Delta E_t \pi_t$
    - Campbell and Ammer (1993)
    - Internal consistency of expectations

Valuation Equation of Public Debt

- Stock market - Campbell and Ammer (1993)

$$\text{Stock price} = \text{Discounted Dividends}$$

$$\Delta E_t\,[\text{Stock price}] = \Delta E_t\,[\text{Dividends}] - \Delta E_t\,[\text{Disc Rates}]$$

- Micro-founded monetary models

$$\frac{\textbf{Bond Prices} \times \text{Bonds}}{\textbf{Price Level}} = \sum_t \frac{\textbf{Surpluses}_t}{\textbf{Discount}_t}$$

$$\Delta E_t\,[\text{Bond Price}] - \Delta E_t\,[\text{Price}] = \Delta E_t\,[\text{Surplus}] - \Delta E_t\,[\text{Disc}]$$

Exercises

1. Decomposition estimates
    - Bayesian VAR for 21 countries
    - Inflation shock $\Delta E_t \pi_t = 1$
    - Discounted surpluses shock: $\Delta E_t[\text{Disc Surp}] = -1$
2. FTPL, New-Keynesian Model
    - Volatile surpluses, no contribution to inflation?
    - Parametric model of partial debt repayment
    - GMM estimate to reproduce decompositions

Motivation + Results

- Measures not structural
- Stylized facts to be matched by theory
- On average:
  - Discount rates → ~80% of total inflation
  - GDP growth → ~20% of total inflation
  - Surplus/GDP → ~0% of total inflation

Inflation has fiscal roots, even if fiscal policy is disconnected from the price level.

Motivation + Results

- Structural interpretation?
- Volatile surpluses, no inflation?
- Partial debt repayment necessary
- On average, 0.78% of 1% GDP deficit is repaid
  - 0.96% in advanced economies
  - 0.59% in developing economies

Discount-driven inflation and realistic surplus process preclude partial repayment.

Why unexpected inflation, not just inflation?

- New Keynesian theory:
    - Fisher: monetary policy sets expected inflation
    - Fiscal policy sets unexpected inflation
- Measures do not depend on state of the economy
- Direct connection with impulse response functions

Literature

- **Monetary-Fiscal Interaction.** Cagan (1956), Sargent and Wallace (1981), Hall and Sargent (1997), Hall and Sargent (2011), Jiang et al. (2019), Corsetti et al. (2019), Sunder-Plassmann (2020), Du et al. (2020), Akhmadieva (2022)
- **Fiscal Theory of the Price Level.** Leeper (1991), Sims (1994), Woodford (1995), Cochrane (1998), Cochrane (2005), Sims (2011), Leeper and Leith (2016), Bassetto and Cui (2018), Cochrane (2022c), Brunnermeier et al. (2022), Cochrane (2022a), Cochrane (2022b)
- **Empirical Finance.** Campbell and Shiller (1988), Cochrane (1992), Campbell and Ammer (1993), Chen and Zhao (2009), Cochrane (2008), Jiang et al. (2019)

Environment

- 1 period = 1 year
- Consumption good price P_t
- Total output Y_t
- Nominal bonds $B^n_{N,t}$, price $Q^n_{N,t}$
  - Pay one unit of currency after *n* years
- Real bonds $B^n_{R,t}$, price $P_t Q^n_{R,t}$
  - Pay one unit of consumption good after *n* years
- Primary Surplus $P_t S_t$

Evolution of Public Debt

$$\overbrace{\left[B^1_{N,t-1} + P_t B^1_{R,t-1}\right]}^{\text{Issued Currency}} = \Delta M_t$$

$$\underbrace{+\left[P_t S_t + \sum_{n=1}^{\infty} Q^n_{N,t}\left(B^n_{N,t} - B^{n+1}_{N,t-1}\right) + P_t \sum_{n=1}^{\infty} Q^n_{R,t}\left(B^n_{R,t} - B^{n+1}_{R,t-1}\right)\right]}_{\text{Retired Currency}}$$

- This is a budget constraint
- Assumption 1: households do not value currency $M_t = 0$

Evolution of Public Debt

- Assumption 1: households do not value currency $M_t = 0$
- End-of-period debt $\mathscr{V}_{N,t}$ and $\mathscr{V}_{R,t}$

$$(1 + r_t^N)\mathscr{V}_{N,t-1} + (1 + r_t^R)(1 + \pi_t)\mathscr{V}_{R,t-1} = P_t S_t + \mathscr{V}_{N,t} + \mathscr{V}_{R,t}$$

- This is an equilibrium condition
- Price level adjusts so that

currency issued = currency retired

Evolution of Public Debt

- Constant structure of public debt: $\delta = \mathscr{V}_{N,t} / \mathscr{V}_t$

$$1 + r_t^n = \delta \left[(1 + r_{N,t}) \right] + (1 - \delta) \left[(1 + r_{R,t})(1 + \pi_t) \right]$$

- Debt-to-GDP = $V_t = \mathscr{V}_t / P_t Y_t$
- Surplus-to-GDP = $s_t = S_t / Y_t$

$$\frac{1 + r_t^n}{(1 + \pi_t)(1 + g_t)} V_{t-1} = s_t + V_t$$

Evolution of Public Debt

Linearized equations

$$v_t + \frac{s_t}{V} = \frac{1}{\beta}\left[v_{t-1} + r_t^n - \pi_t - g_t\right]$$

$$r_t^n = \delta\left[r_t^N\right] + (1-\delta)\left[r_t^R + \pi_t\right]$$

- v_t is log debt-to-GDP
- r_t^n is the nominal return on public debt

Valuation Equation of Public Debt

- Assumption 2: debt does not spiral $\lim_{j\to\infty} \beta^j v_{t+j} = 0$
- Solve flow equation forward:

$$\overbrace{v_{t-1} + r_t^n - \pi_t}^{\text{Real market value of debt}} = \underbrace{\frac{\beta}{V}\sum_{j=0}^{\infty}\beta^j\left[E_t s_{t+j} + E_t g_{t+j}\right] - \sum_{j=1}^{\infty}\beta^j\left[E_t r_{t+j}^n - E_t \pi_{t+j}\right]}_{\text{Discounted Surpluses}}$$

Marked-to-Market Decomposition

Take innovation on the valuation equation:

$$\boxed{\epsilon_{r^n,t} - \epsilon_{\pi,t} = \epsilon_{s,t} + \epsilon_{g,t} - \epsilon_{r,t}}$$

Terms:

$\epsilon_{r^n,t} = \Delta E_t r_t^n$

$\epsilon_{\pi,t} = \Delta E_t \pi_t$ (current inflation)

$\epsilon_{s,t} = (\beta / V) \sum_{j=0}^{\infty} \beta^j \Delta E_t s_{t+j}$

$\epsilon_{g,t} = \sum_{j=0}^{\infty} \beta^j \Delta E_t g_{t+j}$

$\epsilon_{r,t} = \sum_{j=1}^{\infty} \beta^j (\Delta E_t r_{t+j}^n - \Delta E_t \pi_{t+j})$

Public Finances Model

Why a public finances model?

1. We can do better: bond prices forecast future inflation
2. No historical data for bond price/return r_t^n
3. No data on market value of debt (only book value)

Public Finances Model

Key Assumptions

- Assumption: constant maturity structure
- Decays geometrically at rate ω:

$$B^{n}_{N,t} = \omega_N \, B^{n-1}_{N,t}$$
$$B^{n}_{R,t} = \omega_R \, B^{n-1}_{R,t}$$

- Assumption: constant (or no) risk premium

$$E_t r_{N,t} = E_t r_{R,t} + E_t \pi_t = i_t$$

Public Finances Model

- Bond prices:

$$q_{N,t} = (\omega_N \beta) E_t q_{N,t} - \left[i_t \right]$$

$$q_{R,t} = (\omega_R \beta) E_t q_{R,t} - \left[i_t - E_t \pi_{t+1} \right]$$

- Returns:

$$r_{j,t} = (\omega_j \beta) q_{j,t} - q_{j,t-1} \qquad j = N, R$$

Break down of bond price variation

Proposition: let $r_t = i_t - E_t\pi_{t+1}$ be the real interest. Then

$$\epsilon_{r^n,t} - \epsilon_{\pi,t} = -\delta \sum_{j=0}^{\infty} (\omega_N \beta)^j \Delta E_t \pi_{t+j} - \sum_{j=1}^{\infty} \beta^j [\delta \omega_N^j + (1-\delta)\omega_R^j] \Delta E_t r_{t+j}$$

- Higher real discount lowers real and nominal bond prices
- Higher inflation lowers nominal bond prices
- No long-term debt $\omega = 0$:

$$\epsilon_{r^n,t} - \epsilon_{\pi,t} = -\delta \Delta E_t \pi_t$$

Total Inflation Decomposition

Replace bond return decomp on marked-to-market decomp:

$$\boxed{-\varepsilon_{\pi,t} = \varepsilon_{s,t} + \varepsilon_{g,t} - \varepsilon_{r,t}}$$

Terms:

$\varepsilon_{\pi,t} = \delta \sum_{j=0}^{\infty} (\omega_N \beta)^j \Delta E_t \pi_{t+j}$ (current and future inflation)

$\varepsilon_{s,t} = \epsilon_{s,t} = (\beta / V) \sum_{j=0}^{\infty} \beta^j \Delta E_t s_{t+j}$

$\varepsilon_{g,t} = \epsilon_{g,t} = \sum_{j=0}^{\infty} \beta^j \Delta E_t g_{t+j}$

$\varepsilon_{r,t} = \sum_{j=1}^{\infty} \beta^j \left[1 - (\delta \omega_N^j + (1-\delta) \omega_R^j) \right] \Delta E_t r_{t+j}$

Comparison of Decompositions

- **Marked-to-market**: $\boxed{\epsilon_{r^n,t} - \epsilon_{\pi,t} = \epsilon_{s,t} + \epsilon_{g,t} - \epsilon_{r,t}}$
    - Current inflation given current bond prices
    - Highlights effect of monetary policy
- **Total inflation**: $\boxed{-\varepsilon_{\pi,t} = \varepsilon_{s,t} + \varepsilon_{g,t} - \varepsilon_{r,t}}$
    - Path of inflation given path of discount rates
    - Sensitive to future inflation
    - Nets out effect of discount rates on bond prices

Build Market Value of Debt

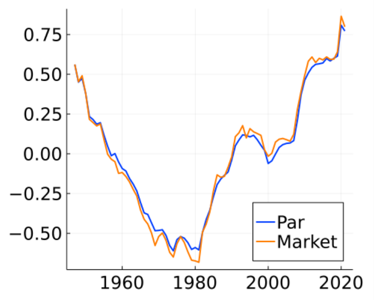
- Converting par to market value of debt
- Dallas Fed, Cox and Hirschhorn (1983) and Cox (1985)

$$\mathscr{V}_{j,t} = \mathscr{V}^{b}_{j,t} \times \frac{\text{market value of bonds}}{\text{book value of bonds}} = \mathscr{V}^{b}_{j,t} \frac{Q_{j,t}}{Q^{b}_{j,t}} \quad \text{for } j = N, R$$

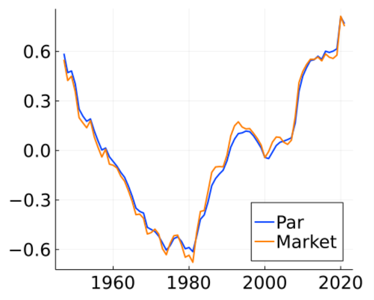
- Book price of bonds evolve according to average interest:

$$\begin{aligned} i^{b}_{N,t} &= (1 - \omega_N) i_t + \omega_N i^{b}_{N,t-1} \\ i^{b}_{R,t} &= (1 - \omega_R)(i_t - E_t \pi_{t+1}) + \omega_R i^{b}_{R,t-1} \end{aligned}$$

Comparison with Dallas Fed



(a) Dallas Fed



(b) Model

Vector Autoregression

- States X

$$X_t = AX_{t-1} + e_t \qquad e_t \sim N(0, \Sigma)$$

- Bayesian estimates of 21 countries
- Samples end in 2019 (no COVID!)
- Prior centered around US OLS estimates

$$\begin{bmatrix} i_t \text{ Nominal Interest} \\ \pi_t \text{ Inflation Rate} \\ g_t \text{ GDP Growth} \\ v_t \text{ Market Value Debt} \\ r_t^n \text{ Bond Return (model built)} \\ s_t \text{ Primary Surplus (model built)} \end{bmatrix}$$

VAR and Decomposition Measures

- VAR uses time series structure to identify revision of expectations

$$\Delta E_t X_{t+j} = A^j X_t$$

$$\sum_{j=0}^{\infty} \beta^j \Delta E_t X_{t+j} = (I - \beta A)^{-1} X_t$$

The Inflation Shock

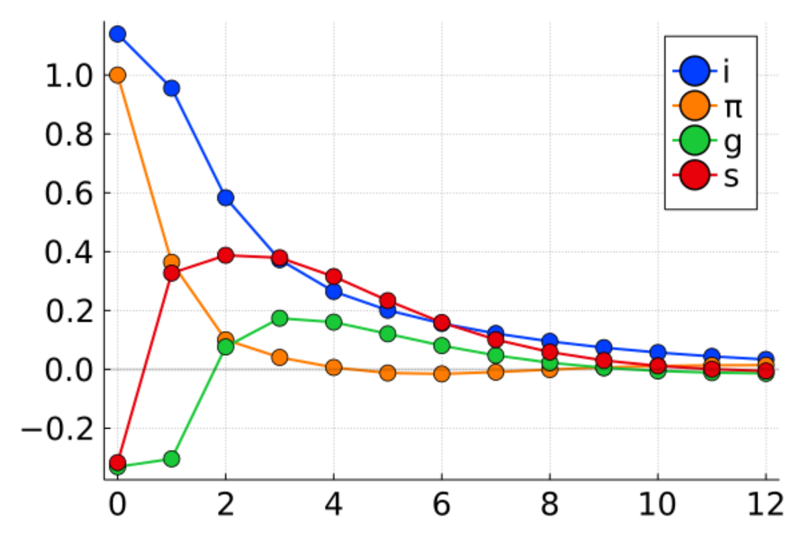
- Source of innovations to inflation $\Delta E_t \pi_t = 1$
- Reduced-form shock $e_t = E\left[e \mid \Delta E_t \pi_t = 1\right]$
- Proposition: MtM decomposition

$$\epsilon_{r^n,t} - \epsilon_{\pi,t} = \epsilon_{s,t} + \epsilon_{g,t} - \epsilon_{r,t}$$

same as variance decomposition

$$\frac{\mathrm{cov}(\epsilon_{r^n,t}, \epsilon_{\pi,t})}{\mathrm{var}(\epsilon_{\pi,t})} - 1 = \frac{\mathrm{cov}(\epsilon_{s,t}, \epsilon_{\pi,t})}{\mathrm{var}(\epsilon_{\pi,t})} + \frac{\mathrm{cov}(\epsilon_{g,t}, \epsilon_{\pi,t})}{\mathrm{var}(\epsilon_{\pi,t})} - \frac{\mathrm{cov}(\epsilon_{r,t}, \epsilon_{\pi,t})}{\mathrm{var}(\epsilon_{\pi,t})}$$

IRF - Brazil



IRF to $e_t = E\left[e \mid \Delta E_t \pi_t = 1\right]$

Inflation Shock - Marked-to-Market

Country	ϵ_{r^n}	$-\epsilon_\pi$	=	ϵ_s	$+\epsilon_g$	$-\epsilon_r$
1947 Sample (Advanced)						
United Kingdom	**-0.7	**-1	=	**-2.2	**-0.7	**1.2
United States	**-0.7	**-1	=	-0.3	**-0.5	**-0.9
1960 Sample (Advanced)						
Canada	**-2.8	**-1	=	0.3	*-1.4	**-2.8
Denmark	**-0.9	**-1	=	0.2	-0.2	**-1.9
Japan	**-0.6	**-1	=	**2.8	**-3.0	**-1.4
Norway	**-0.7	**-1	=	0.7	*3.0	**-5.4
Sweden	**-0.6	**-1	=	**0.9	**-0.9	**-1.6
1973 Sample (Advanced)						
Australia	**-2.2	**-1	=	0.2	0.1	**-3.5
New Zealand	**-1.0	**-1	=	*1.2	**-1.4	*-1.8
South Korea	**-0.6	**-1	=	**-2.4	0.2	*0.7
Switzerland	**-2.0	**-1	=	*-0.8	0.1	**-2.3

Inflation Shock: $e_t = E[e \mid \Delta E_t \pi_t = 1]$

Inflation Shock - Marked-to-Market

Country	ϵ_{r^n}	$-\epsilon_\pi$	=	ϵ_s	$+\epsilon_g$	$-\epsilon_r$
1997 Sample (Emerging)						
Brazil	**-0.7	**-1	=	**2.4	-0.1	**-4.0
Colombia	**-1.4	**-1	=	0.2	**-0.7	**-1.9
Czech Republic	*0.2	**-1	=	*0.7	**-1.3	-0.2
Hungary	**-0.8	**-1	=	0.0	-0.2	**-1.6
India	*-0.2	**-1	=	**-1.0	-0.1	-0.1
Israel	**-0.4	**-1	=	**0.8	*-0.4	**-1.8
Mexico	**-1.4	**-1	=	*-1.2	0.0	*-1.3
Poland	**-1.4	**-1	=	**1.0	*-0.3	**-3.0
South Africa	**-0.6	**-1	=	0.3	**-0.8	**-1.1
Ukraine	**-0.5	**-1	=	**-1.1	0.0	-0.3

Inflation Shock: $e_t = E[e \mid \Delta E_t \pi_t = 1]$

Inflation Shock - Total Inflation

Country	$-\varepsilon_\pi$	=	ε_s	$+\varepsilon_g$	$-\varepsilon_r$
1947 Sample (Advanced)					
United Kingdom	**-2.8	=	**-2.2	**-0.7	0.1
United States	**-1.5	=	-0.3	**-0.5	**-0.7
1960 Sample (Advanced)					
Canada	**-2.6	=	0.3	*-1.4	**-1.5
Denmark	**-1.6	=	0.2	-0.2	**-1.6
Japan	**-1.5	=	**2.8	**-3.0	**-1.3
Norway	**-2.0	=	0.7	*3.0	**-5.7
Sweden	**-1.6	=	**0.9	**-0.9	**-1.5
1973 Sample (Advanced)					
Australia	**-3.1	=	0.2	0.1	**-3.4
New Zealand	**-2.3	=	*1.2	**-1.4	**-2.1
South Korea	**-2.0	=	**-2.4	0.2	0.2
Switzerland	**-2.0	=	*-0.8	0.1	**-1.3

Inflation Shock: $e_t = E[e \mid \Delta E_t \pi_t = 1]$

Inflation Shock - Total Inflation

Country	$-\varepsilon_\pi$	=	ε_s	$+\varepsilon_g$	$-\varepsilon_r$
1997 Sample (Emerging)					
Brazil	**-0.8	=	**2.4	-0.1	**-3.1
Colombia	**-0.7	=	0.2	**-0.7	-0.2
Czech Republic	**-0.5	=	*0.7	**-1.3	0.1
Hungary	**-1.4	=	0.0	-0.2	**-1.3
India	**-1.4	=	**-1.0	-0.1	*-0.4
Israel	**-0.6	=	**0.8	*-0.4	**-1.0
Mexico	**-1.4	=	*-1.2	0.0	-0.3
Poland	**-1.4	=	**1.0	*-0.3	**-2.1
South Africa	**-0.8	=	0.3	**-0.8	*-0.3
Ukraine	**-1.2	=	**-1.1	0.0	-0.1

Inflation Shock: $e_t = E[e \mid \Delta E_t \pi_t = 1]$

Inflation Shock - Averages

Country	ϵ_{r^n}	$-\epsilon_\pi$	=	ϵ_s	$+\epsilon_g$	$-\epsilon_r$
Averages	**-1.0	**-1	=	0.1	**-0.4	**-1.7
1947 (Advanced)	**-0.7	**-1	=	**-1.2	**-0.6	0.1
1960 (Advanced)	**-1.1	**-1	=	*1.0	*-0.5	**-2.6
1973 (Advanced)	**-1.4	**-1	=	-0.4	-0.3	**-1.7
1997 (Emerging)	**-0.7	**-1	=	0.2	**-0.4	**-1.5

Marked-to-Market

Country	$-\varepsilon_\pi$	=	ε_s	$+\varepsilon_g$	$-\varepsilon_r$
Averages	**-1.6	=	0.1	**-0.4	**-1.3
1947 (Advanced)	**-2.2	=	**-1.2	**-0.6	-0.3
1960 (Advanced)	**-1.9	=	*1.0	*-0.5	**-2.3
1973 (Advanced)	**-2.3	=	-0.4	-0.3	**-1.6
1997 (Emerging)	**-1.0	=	0.2	**-0.4	**-0.9

Total Inflation

Inflation Shock - Takeaways

- Discount rates: 80% of inflation variation (average)
- The rest comes mostly from GDP growth
- Positive contribution of surplus-to-GDP 7/21
- Monetary policy reduces inflation in 20/21

Inflation has fiscal roots, even if fiscal policy is disconnected from the price level.

Robustness - OLS Estimates

Country	ϵ_{r^n}	$-\epsilon_\pi$	=	ϵ_s	$+\epsilon_g$	$-\epsilon_r$
Averages	**-1.0	**-1	=	0.2	**-0.4	**-1.8
1947 (Advanced)	**-0.7	**-1	=	**-1.2	**-0.6	0.2
1960 (Advanced)	**-1.2	**-1	=	*1.0	*-0.5	**-2.6
1973 (Advanced)	**-1.4	**-1	=	-0.4	-0.3	**-1.7
1997 (Emerging)	**-0.7	**-1	=	0.4	*-0.3	**-1.8

Marked-to-Market

Country	$-\varepsilon_\pi$	=	ε_s	$+\varepsilon_g$	$-\varepsilon_r$
Averages	**-1.6	=	0.2	**-0.4	**-1.4
1947 (Advanced)	**-2.2	=	**-1.2	**-0.6	-0.3
1960 (Advanced)	**-1.9	=	*1.0	*-0.5	**-2.4
1973 (Advanced)	**-2.4	=	-0.4	-0.3	**-1.6
1997 (Emerging)	**-1.0	=	0.4	*-0.3	**-1.1

Total Inflation

Robustness - Minnesota Prior

Country	ϵ_{r^n}	$-\epsilon_\pi$	=	ϵ_s	$+\epsilon_g$	$-\epsilon_r$
Averages	**-1.0	**-1	=	0.2	**-0.4	**-1.8
1947 (Advanced)	**-0.7	**-1	=	**-1.2	**-0.6	0.2
1960 (Advanced)	**-1.1	**-1	=	*1.0	*-0.5	**-2.6
1973 (Advanced)	**-1.4	**-1	=	-0.5	-0.3	**-1.7
1997 (Emerging)	**-0.7	**-1	=	0.3	*-0.3	**-1.8

Marked-to-Market

Country	$-\varepsilon_\pi$	=	ε_s	$+\varepsilon_g$	$-\varepsilon_r$
Averages	**-1.6	=	0.2	**-0.4	**-1.4
1947 (Advanced)	**-2.2	=	**-1.2	**-0.6	-0.3
1960 (Advanced)	**-1.9	=	*1.0	*-0.5	**-2.4
1973 (Advanced)	**-2.3	=	-0.5	-0.3	**-1.6
1997 (Emerging)	**-1.1	=	0.3	*-0.3	**-1.1

Total Inflation

Discounted Surpluses Shock

- Discount rates drive innovations to inflation
- What drives discounted surpluses?
- Put differently: what drives unexpected returns on the basket of public bonds?

$$
\begin{aligned}
e_t &= E[e \mid \Delta E_t(\text{Disc Surpl}) = -1] \\
&= E[e \mid \Delta E_t(\epsilon_{s,t} + \epsilon_{g,t} - \epsilon_{r,t}) = -1]
\end{aligned}
$$

Discounted Surpluses Shock - Marked-to-Market

Country	ϵ_{r^n}	$-\epsilon_\pi$	=	ϵ_s	$+\epsilon_g$	$-\epsilon_r$
1947 Sample (Advanced)						
United Kingdom	**-0.8	**-0.2	=	**-0.5	-0.1	*-0.4
United States	**-0.7	**-0.3	=	0.0	**0.2	**-1.2
1960 Sample (Advanced)						
Canada	**-0.8	**-0.2	=	*0.2	-0.1	**-1.1
Denmark	**-0.8	**-0.2	=	*0.6	*0.5	**-2.0
Japan	**-0.6	**-0.4	=	0.0	-0.2	**-0.8
Norway	**-0.6	**-0.4	=	*1.0	*1.9	**-3.9
Sweden	**-0.6	**-0.4	=	**0.7	-0.2	**-1.5
1973 Sample (Advanced)						
Australia	**-0.8	**-0.2	=	*0.5	*0.2	**-1.7
New Zealand	**-0.6	**-0.4	=	**0.8	**-0.5	**-1.3
South Korea	**-0.6	**-0.4	=	**-2.4	**1.3	0.2
Switzerland	**-0.8	**-0.2	=	-0.1	*0.2	**-1.1

Discounted Surpluses Shock: $E[e \mid \Delta E_t(\epsilon_{s,t} + \epsilon_{g,t} - \epsilon_{r,t}) = -1]$

Discounted Surpluses Shock - Marked-to-Market

Country	ϵ_{r^n}	$-\epsilon_\pi$	=	ϵ_s	$+\epsilon_g$	$-\epsilon_r$
1997 Sample (Emerging)						
Brazil	**-0.5	**-0.5	=	**1.4	0.1	**-2.6
Colombia	**-0.6	**-0.4	=	0.0	**-0.3	**-0.8
Czech Republic	**-0.4	**-0.6	=	-0.1	-0.3	**-0.6
Hungary	**-0.6	**-0.4	=	*0.4	-0.3	**-1.2
India	**-0.5	**-0.5	=	-0.1	*-0.2	**-0.7
Israel	**-0.7	**-0.3	=	**0.6	-0.1	**-1.5
Mexico	**-0.6	**-0.4	=	**-0.6	0.1	*-0.6
Poland	**-0.7	**-0.3	=	**0.5	-0.1	**-1.4
South Africa	**-0.7	**-0.3	=	*-0.2	0.0	**-0.8
Ukraine	**-0.5	**-0.5	=	**-0.4	*-0.1	**-0.6

Discounted Surpluses Shock: $E[e \mid \Delta E_t(\epsilon_{s,t} + \epsilon_{g,t} - \epsilon_{r,t}) = -1]$

Discounted Surpluses Shock - Total Inflation

Country	$-\varepsilon_\pi$	=	ε_s	$+\varepsilon_g$	$-\varepsilon_r$
1947 Sample (Advanced)					
United Kingdom	**-0.9	=	**-0.5	-0.1	*-0.3
United States	**-0.5	=	0.0	**0.2	**-0.7
1960 Sample (Advanced)					
Canada	**-0.5	=	*0.2	-0.1	**-0.6
Denmark	**-0.6	=	*0.6	*0.5	**-1.6
Japan	**-0.7	=	0.0	-0.2	**-0.5
Norway	**-0.9	=	*1.0	*1.9	**-3.8
Sweden	**-0.8	=	**0.7	-0.2	**-1.2
1973 Sample (Advanced)					
Australia	**-0.6	=	*0.5	*0.2	**-1.3
New Zealand	**-0.8	=	**0.8	**-0.5	**-1.2
South Korea	**-1.2	=	**-2.4	**1.3	0.0
Switzerland	**-0.5	=	-0.1	*0.2	**-0.6

Discounted Surpluses Shock: $E[e \mid \Delta E_t(\epsilon_{s,t} + \epsilon_{g,t} - \epsilon_{r,t}) = -1]$

Discounted Surpluses Shock - Total Inflation

Country	$-\varepsilon_\pi$	=	ε_s	$+\varepsilon_g$	$-\varepsilon_r$
1997 Sample (Emerging)					
Brazil	**-0.3	=	**1.4	0.1	**-1.9
Colombia	**-0.3	=	0.0	**-0.3	-0.1
Czech Republic	**-0.5	=	-0.1	-0.3	-0.2
Hungary	**-0.6	=	*0.4	-0.3	**-0.8
India	**-0.6	=	-0.1	*-0.2	**-0.3
Israel	**-0.2	=	**0.6	-0.1	**-0.7
Mexico	**-0.6	=	**-0.6	0.1	-0.1
Poland	**-0.5	=	**0.5	-0.1	**-0.9
South Africa	**-0.3	=	*-0.2	0.0	*-0.1
Ukraine	**-0.6	=	**-0.4	*-0.1	**-0.1

Discounted Surpluses Shock: $E[e \mid \Delta E_t(\epsilon_{s,t} + \epsilon_{g,t} - \epsilon_{r,t}) = -1]$

Discounted Surpluses Shock - Averages

Country	ϵ_{r^n}	$-\epsilon_\pi$	=	ϵ_s	$+\epsilon_g$	$-\epsilon_r$
Averages	**-0.6	**-0.4	=	0.1	0.1	**-1.2
1947 (Advanced)	**-0.8	**-0.2	=	*-0.2	0.1	**-0.8
1960 (Advanced)	**-0.7	**-0.3	=	*0.5	0.4	**-1.9
1973 (Advanced)	**-0.7	**-0.3	=	-0.3	0.3	**-1.0
1997 (Emerging)	**-0.6	**-0.4	=	*0.2	*-0.1	**-1.1

Marked-to-Market

Country	$-\varepsilon_\pi$	=	ε_s	$+\varepsilon_g$	$-\varepsilon_r$
Averages	**-0.6	=	0.1	0.1	**-0.8
1947 (Advanced)	**-0.7	=	*-0.2	0.1	**-0.5
1960 (Advanced)	**-0.7	=	*0.5	0.4	**-1.6
1973 (Advanced)	**-0.8	=	-0.3	0.3	**-0.8
1997 (Emerging)	**-0.4	=	*0.2	*-0.1	**-0.5

Total Inflation

Model Overview

- How hard to reproduce empirical findings? Interpretation?
- Fiscal theory of the price level, New-Keynesian model
- Partial debt repayment (but still FTPL!)
- Trend shocks

Model Equations

- Private sector

$$y_t = E_t y_{t+1} - \gamma(i_t - E_t \pi_{t+1}) + \rho_g u_{g,t}$$
$$\pi_t = \beta E_t \pi_{t+1} + \kappa y_t - \kappa_a a_t$$
$$g_t = y_t - y_{t-1} - u_{g,t}$$

- Central bank

$$i_t = \phi_\pi \pi_t + \phi_g g_t + u_{i,t}$$

Why trend shocks?

- Otherwise, output stationary $\implies \varepsilon_{g,t} \approx 0$
- Model solution: $X_t = a(L)e_t$ for finite $a(1)$
- Growth term of decomposition:

$$\epsilon_{g,t} = 1_g' a(\beta) e_t$$

- In the absence of trend shocks:

$$g_t = 1_g' a(L) e_t = 1_y'(1 - L) a(L) e_t$$
$$1_g' a(L) = 1_y'(1 - L) a(L)$$

- Therefore $\epsilon_{g,t} = 1_g' a(\beta) e_t \approx 1_g' a(1) e_t = 0$

Model Equations

- Flow of debt

$$v_t + \frac{s_t}{V} = \frac{1}{\beta}\left[v_{t-1} + r^n_t - \pi_t - g_t\right]$$
$$r^n_t = \delta\left[r^N_t\right] + (1-\delta)\left[r^R_t + \pi_t\right]$$

- Bond prices and return

$$q_{N,t} = (\omega_N\beta)E_t q_{N,t} - \left[i_t\right]$$
$$q_{R,t} = (\omega_R\beta)E_t q_{R,t} - \left[i_t - E_t\pi_{t+1}\right]$$
$$r_{j,t} = (\omega_j\beta)q_{j,t} - q_{j,t-1} \qquad j = N, R$$

Fiscal Policy

- Surplus-to-GDP could follow

$$h_t = \tau(\pi_t + g_t) + u_{s,t}$$

  where $u_{s,t}$ is a standard AR(1)
- No debt repayment
- News about surpluses always met by unexpected inflation

Fiscal Policy

- Surplus-to-GDP process

$$s_t = s_t^* + (1 - v)\, h_t$$
$$s_t^* = \alpha v_{t-1}^* + v\, h_t$$
$$v_{t-1}^* = \beta \left(v_t^* + s_t^* \right)$$

- s_t and s_t^* respond to ”debt value target” v^*

$$s_t = \alpha v_{t-1}^* + h_t$$

but not to actual debt v_t (or arbitrary $\Delta E_t \pi_t$)

Fiscal Policy

- What is the role of v_t^*?

$$s_t = s_t^* + (1 - v)\, h_t \tag{1}$$
$$s_t^* = \alpha v_{t-1}^* + v\, h_t \tag{2}$$
$$v_{t-1}^* = \beta \left(v_t^* + s_t^* \right) \tag{3}$$

- (2) and (3): v^* is stationary
- Solve (3) forward:

$$v_{t-1}^* = \beta \sum_{j=0}^{\infty} \beta^j E_t s_{t+j}^* = \beta \sum_{j=0}^{\infty} \beta^j \left[E_t s_{t+j} - (1 - v) E_t h_{t+j} \right]$$

Fiscal Policy

$$v^*_{t-1} = \beta \sum_{j=0}^{\infty} \beta^j \left[E_t s_{t+j} - (1-v) E_t h_{t+j} \right]$$

- Take innovations $\Delta E_t = E_t - E_{t-1}$

$$\beta \sum_{j=0}^{\infty} \beta^j \Delta E_t s_{t+j} = (1-v) \sum_{j=0}^{\infty} \beta^j \Delta E_t h_{t+j}$$

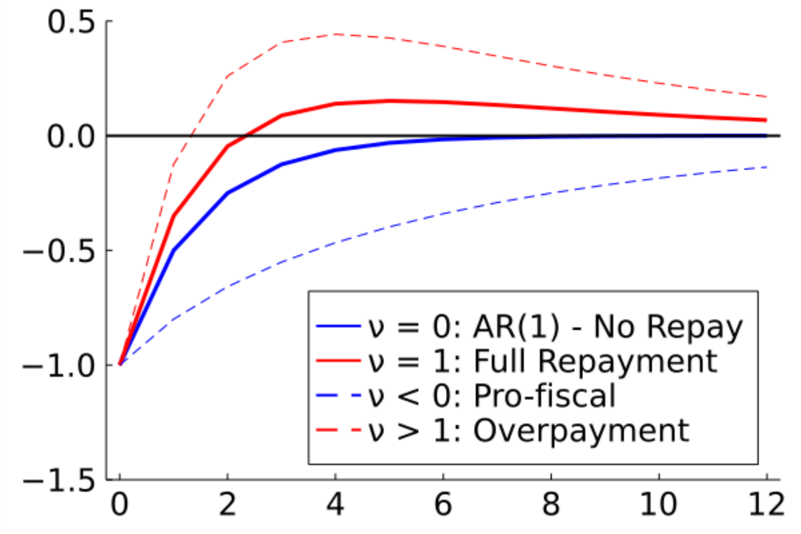
$$\boxed{\epsilon_{s,t} = (1-v) \sum_{j=0}^{\infty} \beta^j \Delta E_t h_{t+j}}$$

- v governs debt repayment

Partial debt repayment

- $\boxed{v = 0}$ No debt repayment: $\epsilon_{s,t} = \sum_{j=0}^{\infty} \beta^j \Delta E_t h_{t+j}$
  - $s_t = h_t$ (standard AR(1))
- $\boxed{v = 1}$ Full debt repayment: $\epsilon_{s,t} = 0$
  - $s_t = s_t^* = \alpha v_t^* + h_t$
- $\boxed{v < 0}$ ”Pro-fiscal” surplus: $\epsilon_{s,t} / \sum_{j=0}^{\infty} \beta^j \Delta E_t h_{t+j} > 1$
- $\boxed{v > 1}$ ”Overpayment”: $\epsilon_{s,t} / \sum_{j=0}^{\infty} \beta^j \Delta E_t h_{t+j} < 0$

Partial debt repayment - Cases



Response to deficit shock $u_{s,t} = -1$

GMM Estimation

- Method of moments:

$$\text{Min}_{\theta} \quad w \left\| \mathscr{D}_{VAR} - \mathscr{D}_{NK}(\theta) \right\| + (1-w) \left\| \mathscr{M} - \mathscr{M}_{NK}(\theta) \right\|$$

- $\mathscr{D}$ contains MtM decomposition for inflation shock
- $\mathscr{M}$ contains second moments
- Estimates for the United States

GMM Estimation

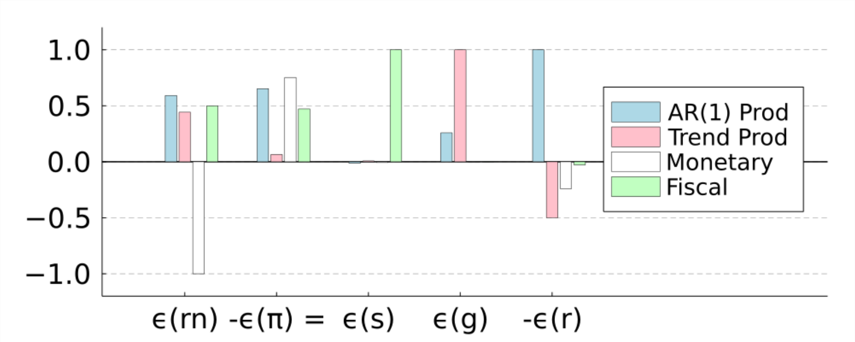
United States Estimates

Fixed		Estimated	
Parameter	Value	Paramater	Value
β	0.99	ρ_a	0.98
γ	0.4	ρ_g	0.23
φ	3	ρ_i	0.00
θ	0.25	ρ_s	0.72
$\bar{\omega}$	γ^{-1}	ϕ_π	0.68
σ_a	1	ϕ_g	0.00
		τ	-0.06
		ν	0.89
		α	0.01
		σ_g	1.21
		σ_g	0.53
		σ_g	1.07

US Model Parameters

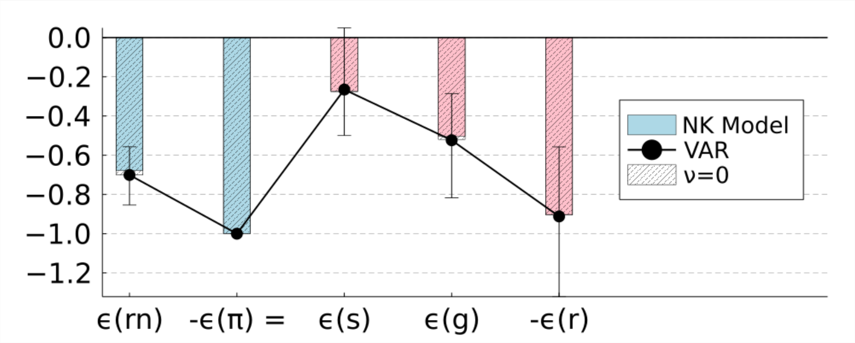
GMM Estimation

United States Estimates



Fiscal decomposition of structural shocks

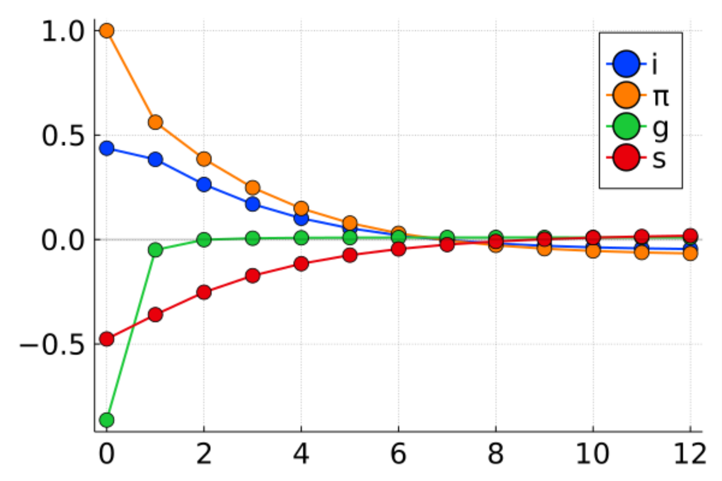
Is AR(1) surplus a good model?



MtM decomposition of Inflation Shock $e_t = E\left[e \mid \Delta E_t \pi_t = 1\right]$

Is AR(1) surplus a good model?

Structural shocks: $\varepsilon_a = -1$, $\varepsilon_g = -0.2$, $\varepsilon_i = -0.3$, $\varepsilon_s = -0.5$



Inflation Shock

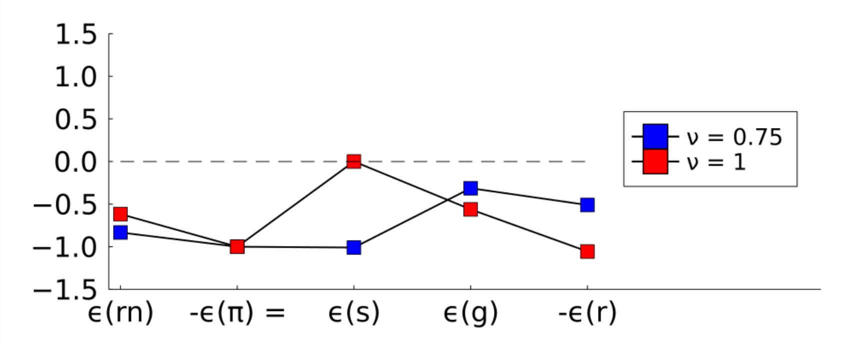
Is AR(1) surplus a good model?

- $v = 0$ precludes realistic fiscal policy and discount-driven inflation at the same time

	Data	$v = 0.9$	$v = 0$		Data	$v = 0.9$	$v = 0$
σ_i/σ_g	1.29	0.77	1.25	$\text{cor}(\pi, i)$	0.70	0.88	0.89
σ_π/σ_g	1.20	1.10	1.56	$\text{cor}(\pi, g)$	-0.11	-0.35	-0.40
σ_s/σ_g	1.08	1.09	0.45	$\text{cor}(g, i)$	0.04	-0.35	-0.04
acor(i)	0.91	0.75	0.87	$\text{cor}(i, s)$	-0.26	-0.28	-0.46
acor(π)	0.69	0.72	0.81	$\text{cor}(\pi, s)$	-0.28	-0.29	-0.41
acor(g)	0.14	0.14	0.16	$\text{cor}(g, s)$	0.01	-0.04	-0.05
acor(s)	0.64	0.72	0.27				

Second Moment Fit

Is AR(1) surplus a good model?



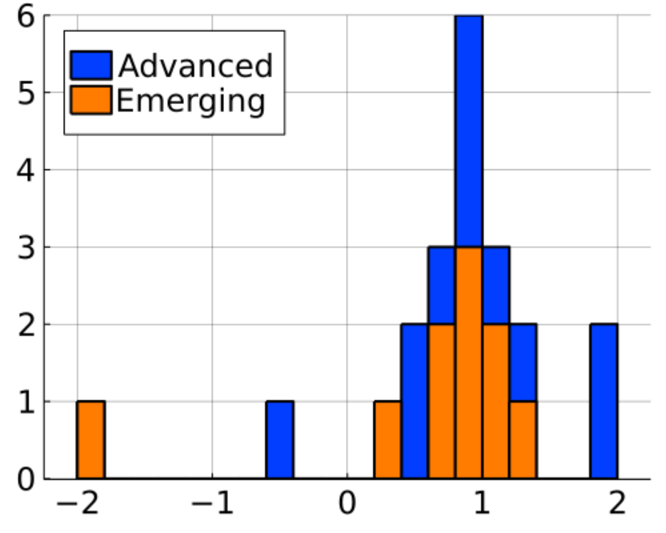
Comparative Statics for ν

Cross-Country Estimates

$$\text{Min}_{\theta} \quad w \left\| \mathscr{D}_{VAR} - \mathscr{D}_{NK}(\theta) \right\| + (1-w) \left\| \mathscr{M} - \mathscr{M}_{NK}(\theta) \right\|$$

- Repeat procedure for each country in the sample
- Use corresponding debt profile $(\delta, \omega_N, \omega_R)$

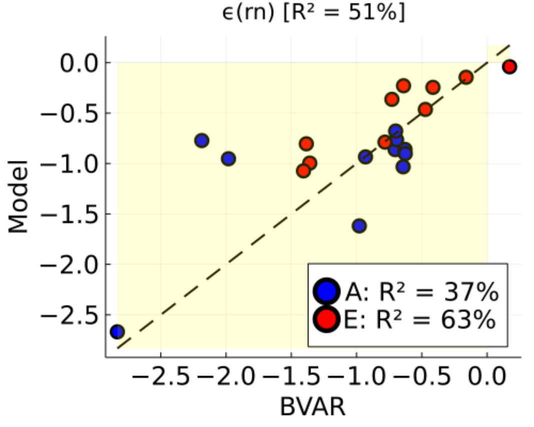
Cross-Country Estimates of Debt Repayment v



Histogram of v estimates

Cross-Country Fit of Fiscal Decomposition

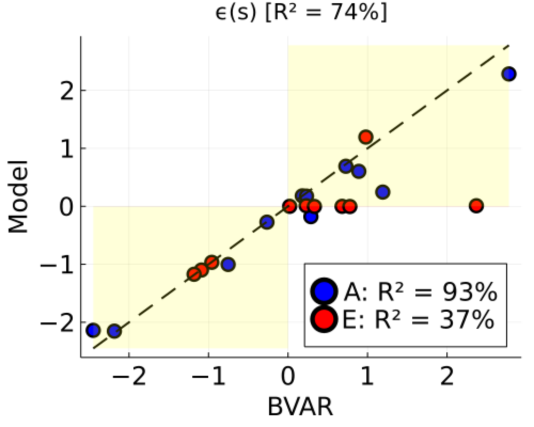
$$\boxed{\epsilon_{r^n,t} - \epsilon_{\pi,t} = \epsilon_{s,t} + \epsilon_{g,t} - \epsilon_{r,t}}$$



Fit of Bond Price Term $\epsilon_{r^n,t}$

Cross-Country Fit of Fiscal Decomposition

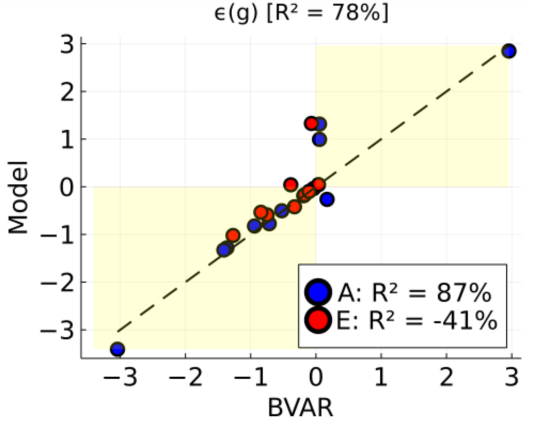
$$\boxed{\epsilon_{r^n,t} - \epsilon_{\pi,t} = \epsilon_{s,t} + \epsilon_{g,t} - \epsilon_{r,t}}$$



Fit of Surplus Term $\epsilon_{s,t}$

Cross-Country Fit of Fiscal Decomposition

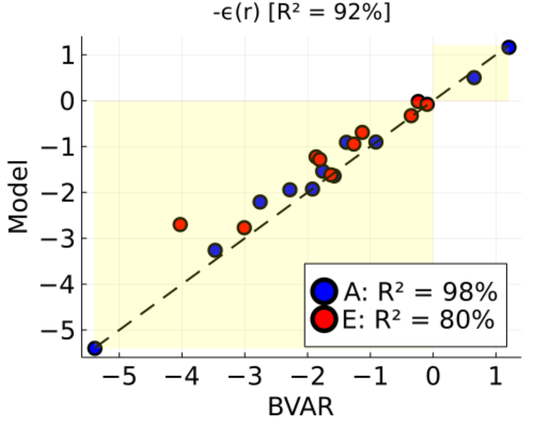
$$\boxed{\epsilon_{r^n,t} - \epsilon_{\pi,t} = \epsilon_{s,t} + \epsilon_{g,t} - \epsilon_{r,t}}$$



Fit of Growth Term $\epsilon_{g,t}$

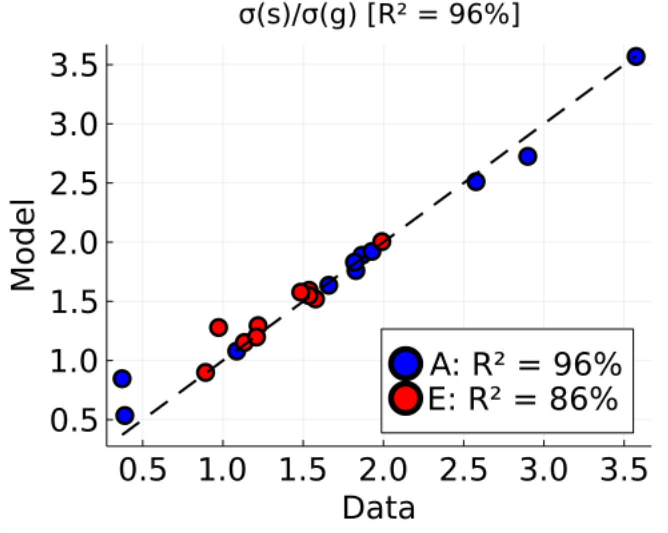
Cross-Country Fit of Fiscal Decomposition

$$\boxed{\epsilon_{r^n,t} - \epsilon_{\pi,t} = \epsilon_{s,t} + \epsilon_{g,t} - \epsilon_{r,t}}$$



Fit of Discount Term $\epsilon_{r,t}$

Cross-Country Fit of Fiscal Policy Volatility



Fit of Fiscal Policy Volatility

Frametitle

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