The Fiscal Theory of the Price Level - A Short Introduction

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Precursors and Intellectual Landscape

Notes based on Cochrane (2022b): you should read yourself!

- Old-Keynesian Models (adaptive expectations, little economics)
 - Interest peg is unstable
 - Taylor rule $i_t = \phi \pi_t$ with $\phi > 1$ recovers stability by "adjusting aggregate demand"
- New-Keynesian Models (rational expectations, micro-founded)
 - Interest peg stable but indeterminate
 - Rule $i_t = i_t^* + \phi(\pi_t \pi_t^*)$ with $\phi > 1$ threats spiral, selects π_t^*
- Theoretical issues: How to rule out spirals? Where does $\Delta E_t \pi_t^*$ come from? Forward guidance puzzle?
- Empirical issues: Why rule out spirals? Do CBs threat spirals? Zero Lower Bound?

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- Household budget contraint: $B_0 + P_1y_1 = P_1c_1 + P_1s_1 + M_1$
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- "A prince, who should enact that a certain proportion of his taxes be paid in a paper money of a certain kind, might thereby give a certain value to this paper money" Wealth of Nation, Adam Smith

FTPL in a Two-Period Model

Now, let's consider decisions in period zero.

- Households inherit B_{-1} bonds, government charges s_0 in taxes and sells B_0 new bonds at discount Q_0
- Given equilibrium conditions $y_0 = c_0$ and $M_0 = 0$:

$$B_{-1} = P_0 s_0 + Q_0 B_0 \implies \frac{B_{-1}}{P_0} = s_0 + \frac{Q_0 B_0}{P_0}$$

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• Fisher equation $Q_0 = \frac{1}{1+i_0} = \frac{1}{R}E_0\left(\frac{P_0}{P_1}\right)$ and $\beta R = 1$:

Real Bond Sales Revenue =
$$\frac{Q_0B_0}{P_0} = \beta E_0 \left[\frac{B_0}{P_1}\right] = \beta E_0 \left[s_1\right]$$

■ The valuation equation becomes

$$\frac{B_{-1}}{P_0} = s_0 + \beta E_0 [s_1]$$

and the price level P_0 is again determined.

Monetary Policy sets Q_0 by changing B_0

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 (1)

$$\frac{B_{-1}}{P_0} = s_0 + \frac{Q_0 B_0}{P_0} \qquad (2)$$

$$Q_0 = \beta E_0 \left(\frac{P_0}{P_1}\right) \qquad (3)$$

$$\frac{Q_0 B_0}{P_0} = \beta E_0 [s_1] \tag{4}$$

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Monetary Policy sets Q_0 by changing B_0

- What if $B_0 \uparrow$?
 - Real bond sales revenue unchanged at $\beta E_0[s_1] \implies P_0$ constant
 - Since Q_0B_0 is constant, $Q_0 \downarrow$ (the government raises nominal interest)
 - By the Fisher equation, monetary policy controls **expected** inflation

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 - In t = 1: Lower surpluses soak up less $B_0 \implies P_1 \uparrow$ (unexpected inflation)
 - In t = 0: Real bond sales revenue $\beta E_0[s_1]$ declines $\implies P_0 \uparrow$ (unexpected inflation)
 - If monetary policy fixes Q_0 : expected inflation $E_0(P_0/P_1)$ constant

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FTPL: Infinite Periods

- Let $\beta_t = Q_t P_{t+1}/P_t$ be the *ex-post* real discount for public bonds, and $\beta_{t,t+k} \equiv \prod_{\tau=t}^{t+k} \beta_{\tau}$
- As long as $\lim_{k\to\infty} \beta_{t,t+k} \frac{B_{t+k}}{P_{t+k+1}} = 0$ at every t (No-Ponzi, optimality)

$$\frac{B_{t-1}}{P_t} = \sum_{k=0}^{\infty} E_t \left[\beta_{t,t+k-1} \, s_{t+k} \right]$$

- This is a valuation equation, not a budget constraint. It holds in all micro-founded models!
 - **Standard NK**: causality from left to right, $PDV(\{s, \beta\})$ adjusts to P_t
 - **FTPL**: causality from right to left, P_t adjusts to $PDV(\{s, \beta\})$
- Which theory asks more from the government? What about Japan? Reading of 2021/22 inflation?

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- Which theory asks more from the government? What about Japan? Reading of 2021/22 inflation?
- Let v_t be *end-of-period* real debt. Linearize law of motion of public debt (around v = 1):

$$v_t + s_t = \underbrace{\frac{1}{\beta} (v_{t-1} + i_{t-1} - \pi_t)}_{\text{Beginning-of-period } V_{t-1}/P_t}$$

- Flexible prices, constant output, interest peg i^*
- From valuation equation:

$$\frac{B_{t-1}}{P_{t-1}} \Delta E_t \left[\Pi^{-1} \right] = \Delta E_t \left[\sum_{k=0}^{\infty} \beta_{t,t+k-1} s_{t+k} \right]$$

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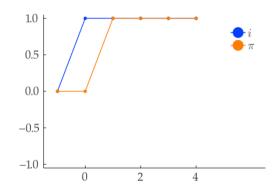
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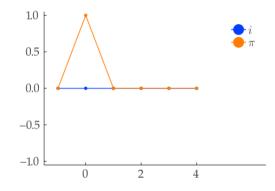
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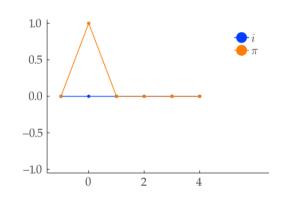
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- Spiral threat model:

$$i_t = i_t^* + \phi(\pi_t - \pi_t^*)$$
 $\phi > 1$
 $\pi_t^* = i_{t-1}^* + \Delta E_t \pi_t^*$



generates same equilibrium

Private sector and debt law of motion:

$$y_{t} = E_{t}y_{t+1} - \gamma (i_{t} - E_{t}\pi_{t+1})$$

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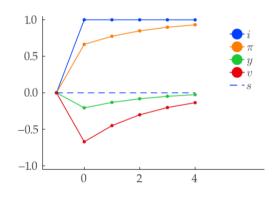
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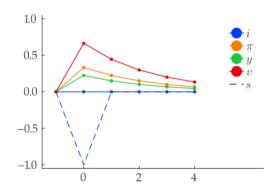
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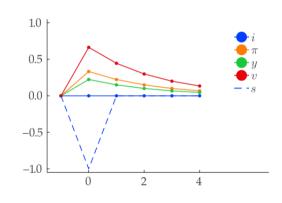
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- Inflation jumps at t = 0: SUPER-Fisherian model
- Spiral threat:

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 $\phi > 1$
 $s_t = \alpha v_t - \varepsilon_{s,t}$

Empirically, α > 0. Is that a problem for the FTPL?
 Cochrane (2022a)



- In practice, governments finance themselves through long-term debt
- This is important because, with long-term bonds, higher interest rate can reduce the market value of debt
- Multiple maturities n = 1, 2, 3, ... (until now, we only had n = 1)

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where $R_t^N = \sum Q_t^{n-1} B_{t-1}^n / \sum Q_{t-1}^n B_{t-1}^n$ is the *ex-post* nominal return on the public debt portfolio.

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• $v_t = V_t/P_t$ = real end-of-period market value of public debt

$$\frac{R_t^N}{\Pi_t} v_{t-1} = s_t + v_t \implies \frac{R_t^N}{\Pi_t} v_{t-1} = \sum_{k=0}^{\infty} E_t \left[\beta_{t,t+k-1} \, s_{t+k} \right]$$

Now: market value of debt = discounted surpluses

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$$R_t^N = \frac{\sum_{n=1}^{\infty} Q_t^{n-1} B_{t-1}^n}{V_{t-1}} = \frac{B_{t-1}^1 + \sum_{n=2}^{\infty} Q_t^{n-1} B_{t-1}^n}{Q_{t-1} B_{t-1}^1} = \frac{1 + \omega \sum_{n=1}^{\infty} \omega^{n-1} Q_t^n}{Q_{t-1}} = \frac{1 + \omega Q_t}{Q_{t-1}}$$

Linearizing:

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• Next, we need a pricing model. I assume a constant risk premium $E_t r_{t+1}^N = i_t$ (embeds expectations hypothesis)

$$q_t = \omega E_t q_{t+1} - i_t = -\sum_{k=0}^{\infty} E_t i_{t+k}$$
 (7)

- Monetary tightening: $i \uparrow \implies q \downarrow \implies r^{N} \downarrow \implies$ Market Value of Debt \downarrow
 - We can get deflation even if discounted surpluses decline!

Private sector, debt and bonds:

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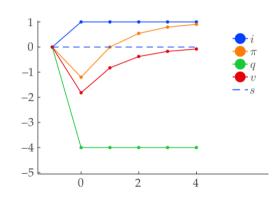
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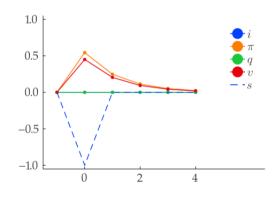
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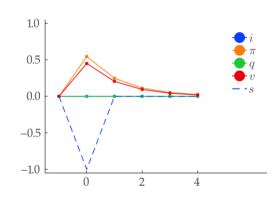
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Frame Title

References

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