A Fiscal Decomposition of Unexpected Inflation: Cross-Country Estimates and Theory*

Livio Maya

Link to the latest version.

1. Introduction

The use of the VAR to measure terms of the decomposition implicitly forces *consistency of expectations*: changes to surpluses or discount rates must change the real value of public debt; conversely, innovations to bond prices or the inflation rate must translate changes in expected surpluses or real discounting.

2. Fiscal Decompositions of Unexpected Inflation

2.1. General Setup

Consider an economy with a consumption good which households value. There is a government that manages a debt composed by nominal and real bonds, possibly with different maturities. Upon maturing, real bonds deliver one unit of consumption good, while nominal bonds deliver one unit of currency. Currency is a commodity that only the government can produce, at zero cost. Households do not value it and they cannot burn it. The price of the consumption good in terms of currency is P_t.

The government brings from period $t-1$ a schedule $\{B^n_{N,t-1}\}$ of nominal bonds and $\{B^n_{R,t-1}\}$ of real bonds, where n denotes maturity. In period t, the government pays for maturing debt $B^1_{N,t-1} + P_t B^1_{R,t-1}$ and public spending $P_t G_t$ using currency. It retires currency from circulation by charging taxes $P_t T_t$ and selling new issues of nominal $Q^n_{N,t}(B^n_{N,t} - B^{n+1}_{N,t-1})$ and real $P_t Q^n_{R,t}(B^n_{R,t} - B^{n+1}_{R,t-1})$ bonds (both can be negative). The difference between currency introduced and retired by government trading changes private sector's aggregate holdings of it, M_t. Therefore:

$$B^1_{N,t-1} + P_t B^1_{R,t-1} + P_t G_t = P_t T_t + \sum_{n=1}^{\infty} Q^n_{N,t}\left(B^n_{N,t} - B^{n+1}_{N,t-1}\right) + P_t \sum_{n=1}^{\infty} Q^n_{R,t}\left(B^n_{R,t} - B^{n+1}_{R,t-1}\right) + \Delta M_t$$

where $Q^n_{N,t}$ is the price of nominal bonds and $P_t Q^n_{R,t}$ is the price of real bonds (I state prices in currency units). I assume households do not hold currency, so $M_t = 0$.[1] The equation above can be written as

$$(1 + r^N_t)\mathcal{V}_{N,t-1} + (1 + r^R_t)(1 + \pi_t)\mathcal{V}_{R,t-1} = P_t S_t + \mathcal{V}_{N,t} + \mathcal{V}_{R,t}$$

where $S_t = T_t - G_t$ is the primary surplus, $1 + \pi_t = P_t/P_{t-1}$ is the inflation rate,

$$\mathcal{V}_{N,t} = \sum_{n=1}^{\infty} Q^n_{N,t} B^n_{N,t} \quad \text{and} \quad \mathcal{V}_{R,t} = P_t \sum_{n=1}^{\infty} Q^n_{R,t} B^n_{R,t}$$

*I thank John Cochrane for many suggestions on key aspects of the paper. I also thank Lucas Alves, Daniel Cuzzi, Vinícius Peçanha and Gustavo Pereira for useful comments.

[1] The main implication of $M = 0$ to us is the absence of seignorage revenues. These are small for the countries in our sample.

are the end-of-period nominal values of nominal and real debt, and

$$1+r_t^N = \frac{\sum_{n=1}^{\infty} Q_{N,t}^{n-1} B_{N,t-1}^n}{\mathcal{V}_{N,t-1}} \quad \text{and} \quad (1+\pi_t)(1+r_t^R) = \frac{P_t}{P_{t-1}} \frac{\sum_{n=1}^{\infty} Q_{R,t}^{n-1} B_{R,t-1}^n}{\mathcal{V}_{R,t-1}}$$

are nominal returns on holdings of the basket of nominal and real bonds.

Let $\mathcal{V}_t = \mathcal{V}_{N,t} + \mathcal{V}_{R,t}$ be end-of-period public debt and $\delta_t = \mathcal{V}_{N,t}/\mathcal{V}_t$ the relative share of nominal debt, both at market prices. We assume that governments keep this share constant at δ. Therefore, we can define the nominal return on the entire basket of public bonds as

$$1 + r_t^n = \delta(1 + r_t^N) + (1-\delta)(1 + r_t^R)(1 + \pi_t). \tag{1}$$

Since public debt and surpluses are not stationary in the data, I detrend both using output Y_t. Define $V_t = \mathcal{V}_t/(P_t Y_t)$ as the real debt-to-GDP ratio and $s_t = S_t/Y_t$ as the surplus-to-GDP ratio.[1] From the last flow equation for public debt, we get:

$$\frac{1 + r_t^n}{(1+\pi_t)(1+g_t)} V_{t-1} = s_t + \frac{\Delta M_t}{P_t} + V_t, \tag{2}$$

where g_t is the growth rate of GDP. Equation (2) provides a law of motion for the real market value of public debt. The left-hand side contains the beginning-of-period (but after bond prices change) real market value of debt. Define $\beta_t = (1+\pi_t)(1+g_t)/(1+r_t^n)$ as the *ex-post*, growth-adjusted real discount for public bonds, and $\beta_{t,t+j} = \prod_{\tau=t}^{t+j} \beta_\tau$. Since V_t satisfies (2), it also satisfies

$$\frac{V_{t-1}}{\beta_t} = \sum_{j=0}^{k} \beta_{t+1,t+j}\, s_{t+j} + \beta_{t+1,t+k} V_{t+k} \qquad \text{for any } k \geq 0$$

regardless of prices and choices. The key assumption I make in this paper is the following transversality condition:

$$\lim_{j\to\infty} E_t \beta_{t,t+j} V_{t+j} = 0 \text{ at every period } t. \tag{3}$$

The interpretation of (3) depends on whether the government uses nominal debt, that is, if $\delta > 0$.[2] If all debt is real, $\delta = 0$, (3) represents a no-default condition. If the limit is positive, there are paths of primary surpluses that lead public debt to explode. The government eventually defaults.

If $\delta > 0$ (the case we consider in this paper), the government has no constraint on its choice of surpluses, as long as households attribute value to currency in a given period.[3] Condition (3) becomes a no-bubble condition, which guarantees that the market value of debt equals discounted surpluses:

$$\frac{V_{t-1}}{\beta_t} = \sum_{j=0}^{\infty} E_t \left[\beta_{t+1,t+j} s_{t+j}\right]. \tag{4}$$

Equation (4) is the valuation equation of public debt. It is the condition upon which households accept to hold public bonds. Households redeem bonds for currency and can trade currency for taxes, which have real value. Therefore, the stream of surpluses provides value for currency and the public debt, and determines the price level.[4] A similar equation, stock price = discounted dividends, expresses the

[1] If $P_t = 0$, households demand infinite goods and there is no equilibrium. So $P_t > 0$ and ratios with the price level in the denominator are henceforth well defined.

[2] Typical models of intertemporal household choice do not imply (3), as the discounted sum here uses *ex-post* discounting $\beta_{t,t+j}$. They do imply instead that $E_t \Lambda_{t,t+j} V_{t+j}$ converges to zero, where Λ is the marginal rate of intertemporal substitution. *Ex-post* real returns and Λ coincide when markets are complete. Otherwise, equation (3) is not necessary for household optimality. See Bohn (1995).

[3] The government needs only to ensure that $\sum_{j=0}^{\infty} E_t \beta_{t+1,t+j} s_{t+j} > (1 + r_t^R)\mathcal{V}_{R,t-1}/Y_t$ for a positive price level.

[4] Again, the valuation equation determines the price level provided that $\delta > 0$. Note that time-t price level only shows up in the denominator of $\mathcal{V}_N$ on the left-hand side of (4):

$$\frac{V_{t-1}}{\beta_t} = (1 + r_t^N)\frac{\mathcal{V}_{N,t-1}}{P_t Y_t} + (1 + r_t^R)\frac{\mathcal{V}_{R,t-1}}{Y_t}.$$

condition for households to hold firms' equity shares (Cochrane (2005)).

The valuation equation is a rather general equilibrium condition. It does not depend on equilibrium selection mechanisms (fiscal theory or spiral threat) and it holds in any model in which the no-bubble condition (3) holds.

2.2. The Marked-to-Market Decomposition

The fiscal decomposition I study in this paper centers around the valuation equation (4). However, working with linearized equations is more tractable and allows estimates based on vector autoregressions. So I linearize (1) and (2) to find

$$r_t^n = \delta r_t^N + (1-\delta)\left(r_t^R + \pi_t\right) \tag{5}$$

$$\beta\left(v_t + \frac{s_t}{V}\right) = v_{t-1} + r_t^n - \pi_t - g_t, \tag{6}$$

where $\beta = (1+g)(1+\pi)/(1+r^n)$ and symbols without t subscripts (like V) correspond to steady-state values. To save notation, I re-define all variables above as deviations from the steady state. Additionally, I re-define growth rates r_t^n, r_t^N, r_t^R, π_t and g_t as log-growth rates. Finally, $v_t = \log(V_t) - \log(V)$.

Like before, I solve the flow equation (6) forward and impose (3).

$$v_{t-1} + r_t^n - \pi_t = \frac{\beta}{V}\sum_{j=0}^{\infty}\beta^j E_t s_{t+j} + \sum_{j=0}^{\infty}\beta^j E_t g_{t+j} - \sum_{j=1}^{\infty}\beta^j\left[E_t r_{t+j}^n - E_t\pi_{t+j}\right]$$

Above, I define $r_t = r_t^n - \pi_t$, the *ex-post* real return on holdings of public debt. The expression above is the linearized valuation equation of public debt.

Decomposition 1 (Marked-to-Market)**.** *Take innovations on the valuation equation of public debt to find the marked-to-market fiscal decomposition of unexpected inflation.*

$$\boxed{\epsilon_{r^n,t} - \epsilon_{\pi,t} = \epsilon_{s,t} + \epsilon_{g,t} - \epsilon_{r,t}} \tag{7}$$

The terms of the decomposition are $\epsilon_{r^n,t} = \Delta E_t r_t^n$, $\epsilon_{\pi,t} = \Delta E_t \pi_t$, $\epsilon_{s,t} = (\beta/V)\sum_{j=0}^{\infty}\beta^j\Delta E_t s_{t+j}$, $\epsilon_{g,t} = \sum_{j=0}^{\infty}\beta^j\Delta E_t g_{t+j}$ *and* $\epsilon_{r,t} = \sum_{j=1}^{\infty}\beta^j(\Delta E_t r_{t+j}^n - \Delta E_t\pi_{t+j})$.

The right-hand side of (7) contains revisions of discounted surpluses. These are divided between news about surplus-to-GDP ratios $\epsilon_{s,t}$, GDP growth $\epsilon_{g,t}$ and real discount rates $\epsilon_{r,t}$. The left-hand side contains the innovation to the price of public bonds $\epsilon_{r^n,t}$ in real terms. Given bond prices (this is why I call "marked-to-market"), surprise inflation $\epsilon_{\pi,t}$ devalues public debt so that its value coincides once again with discounted surpluses. We can replace equation (5) to highlight that inflation can only devalue the *nominal* portion of public debt:

$$\epsilon_{r^n,t} - \epsilon_{\pi,t} = \delta\left(\Delta E_t r_t^N - \Delta E_t\pi_t\right) + (1-\delta)\Delta E_t r_t^R.$$

A one percentage increase in the price level devalues total debt by δ%. The $1-\delta$ share of real bonds is not devalued because, in currency units, their prices grow along with the price level.

2.3. A Public Finances Model

I present a slightly more detailed public finances model, which I later use in the estimation. It also leads to a more general decomposition of unexpected inflation. The key assumption is that the government keeps a geometric maturity structure of its debt. The geometric term structure implies a tractable relationship between the short-term interest rate, inflation and bond returns. Specifically, for the slice of nominal public debt, suppose the outstanding volume of bonds decays at a rate ω_N, so that $B_{N,t}^n = \omega_N B_{N,t}^{n-1}$. Define $Q_{N,t} = \sum_{n=1}^{\infty} Q_{N,t}^n \omega_N^{n-1}$ as the weighted-average market price of nominal bonds.

Assume the same for real bonds, with decay rate of ω_R. Then, $\mathcal{V}_{N,t} = Q_{N,t}B^1_{N,t}$ and $\mathcal{V}_{R,t} = P_t Q_{R,t} B^1_{R,t}$. The linearized returns on public bonds are

$$\begin{aligned} r^N_t &= (\omega_N \beta) q_{N,t} - q_{N,t-1} \\ r^R_t &= (\omega_R \beta) q_{R,t} - q_{R,t-1} \end{aligned} \tag{8}$$

where $q_{N,t} = \log Q_{N,t}$ and variables are expressed as deviations from average.[1] Expression (8) defines the return on holdings of public bonds. It also defines the price of the public debt portfolios given models for expected returns $E_t r^N_t$ and $E_t r^R_t$. In this paper, we assume risk premia to be constant (which englobes the expectations hypothesis). Therefore:

$$\begin{aligned} q_{N,t} &= (\omega_N \beta) E_t q_{N,t+1} - i_t &&= -\sum_{j=0}^{\infty} (\omega_N \beta)^j E_t i_{t+j} \\ q_{R,t} &= (\omega_R \beta) E_t q_{R,t+1} - (i_t - E_t \pi_{t+1}) &&= -\sum_{j=0}^{\infty} (\omega_R \beta)^j \left[E_t i_{t+j} - E_t \pi_{t+j+1}\right]. \end{aligned} \tag{9}$$

Expression (9) guarantees that all bonds have an expected nominal return equal to the short-term risk-free rate: $E_t r^N_{t+1} = E_t r^R_{t+1} + E_t \pi_{t+1} = i_t$. Let $r_t = i_t - E_t \pi_{t+1}$ be the real interest rate. We can re-write the $\epsilon_{r,t}$ term of decomposition (7) as $\sum_{j=1}^{\infty} \beta^j \Delta E_t r_{t+j}$.

The second equalities in each line above show the connection between short-term interest (nominal or real) and returns on debt holdings. News of higher interest lower public bond prices and lead to low returns. In fact, equation (9) implies that we can decompose unexpected real returns on public debt holdings (the left-hand side of decomposition (7)) as follows:

$$\begin{aligned} \epsilon_{r^n,t} - \epsilon_{\pi,t} &= -\delta \Delta E_t \pi_t - \delta \sum_{j=1}^{\infty} (\omega_N \beta)^j \Delta E_t i_t - (1-\delta) \sum_{j=1}^{\infty} (\omega_R \beta)^j \Delta E_t r_t \\ &= -\delta \sum_{j=0}^{\infty} (\omega_N \beta)^j \Delta E_t \pi_t - \sum_{j=1}^{\infty} \beta^j [\delta \omega_N^j + (1-\delta) \omega_R^j] \Delta E_t r_t \end{aligned}$$

News of real bond prices must correspond to news about future real interest (which affect the price of all bonds) or current/future inflation (which affect the price of nominal bonds only; hence the δ). The ω's in the sum corresponding to real interest differs it from $\epsilon_{r,t}$ from decomposition (7). They govern duration, or the sensitiveness of bond prices to changes in future interest. When $\omega_N = \omega_R = 0$, all bonds have a one-period maturity. Their beginning-of-period nominal value is one (nominal) or P_t (real). It does not depend on future interest. When $\omega_N = \omega_R = 1$, public debt works as if it was constituted only of consols, whose price are most sensitivive to interest rate changes.

Decomposition 2 (Total Inflation)**.** *Replace the decomposition of bond prices on the marked-to-market decomposition.*

$$\boxed{-\varepsilon_{\pi,t} = \varepsilon_{s,t} + \varepsilon_{g,t} - \varepsilon_{r,t}} \tag{10}$$

The terms of the decomposition are $\varepsilon_{\pi,t} = \delta \sum_{j=0}^{\infty} (\omega_N \beta)^j \Delta E_t \pi_{t+j}$, $\varepsilon_{s,t} = \epsilon_{s,t}$, $\varepsilon_{g,t} = \epsilon_{g,t}$ *and* $\varepsilon_{r,t} = \sum_{j=1}^{\infty} \beta^j [1 - (\delta \omega_N^j + (1-\delta) \omega_R^j)] \Delta E_t r_{t+j}$.

The marked-to-market decomposition (7) focuses on unexpected changes to current inflation *given* bond prices. Decomposition (10) recognizes that changes to bond prices coalesce from changes to perceived future inflation and real interest. The $\varepsilon_{\pi,t}$ term answers the question: given the path of real discount, how do news about the entire path of inflation affect the market value of debt? This is why I call it the *total inflation* decomposition. Like before, the terms $\varepsilon_{s,t}$ and $\varepsilon_{g,t}$ account for changes in primary surpluses. The $\varepsilon_{r,t}$ term captures the effect of discount rate on discounted surpluses *net of*

[1] In levels, the nominal return is $(B^1_{N,t-1} + \omega_N Q_{N,t} B^1_{N,t-1})/(Q_{N,t-1} B^1_{N,t-1})$. The analogous is true for the real return.

their effect on bond prices. If discount rates increase, they lower discounted surpluses, which calls for higher inflation. But they also lower bond prices, which reduces the required inflation adjustment. As discussed above, the tuple $(\delta, \omega_N, \omega_R)$ determines by how much prices decline, and therefore the net impact of discount rates on total inflation.

Lastly, governments typically report the par or book value of debt. Because theory is based on the market value of debt, some adjustment is necessary. The Dallas Fed provides estimates of the market value of debt for the United States following Cox and Hirschhorn (1983) and Cox (1985).[1] I follow a similar methodology. The adjustment equation is

$$\mathcal{V}_{j,t} = \mathcal{V}^b_{j,t} \times \frac{\text{market value of bonds}}{\text{book value of bonds}} = \mathcal{V}^b_{j,t} \frac{Q_{j,t}}{Q^b_{j,t}} \quad \text{for } j = N, R$$

where $Q^b_{j,t}$ is the weighted-average book price of bonds. We can linearize this adjustment equation to

$$v_t = v^b_t + \delta(q_{N,t} - q^b_{N,t}) + (1 - \delta)(q_{R,t} - q^b_{R,t}). \tag{11}$$

Let $i^b_{j,t}$ $(j = N, R)$ be the average interest rates by which book values $q^b_{j,t}$ are updated from period to period. They are not observed for most countries in a sufficiently large time span, so I use a model for it instead. With a geometric maturity structure of debt and in steady state, every period the government rolls over the volume of bonds coming to maturity. That accounts for a share $1 - \omega_j$ of total outstanding bonds. In a steady state, the government then issues the same amount of new bonds at the prevailing interest rate (i_t or $i_t - E_t\pi_{t+1}$). Therefore, the log book price of bonds and the average interest satisfy

$$\begin{aligned} q^b_{j,t} &= (\omega_j\beta)E_t q^b_{j,t+1} - i^b_{j,t} = -\sum_{k=0}^{\infty}(\omega_j\beta)^k E_t i^b_{j,t+k} \\ i^b_{N,t} &= (1 - \omega_N)i_t + \omega_N i^b_{N,t-1} \\ i^b_{R,t} &= (1 - \omega_R)(i_t - E_t\pi_{t+1}) + \omega_R i^b_{R,t-1}. \end{aligned} \tag{12}$$

I use equations in (11) and (12) to convert par value of debt data to market value.

3. Estimates

I measure the terms of the marked-to-market and total inflation decompositions for different combinations of shocks. To do this, I estimate a six-equation VAR in which the debt law of motion (6) holds by construction. If the estimated VAR systems are stationary, real debt will converge and the decompositions will hold. Keeping the same notation, the vector of variables is

$$x_t = [i_t \ \pi_t \ g_t \ v_t \ r^n_t \ s_t]'.$$

Data is annual. Quarterly data is available, but it often does not go back as many years into the past. This is particularly true in the case of emerging market variables and public debt measures from all countries. With a focus on long-term debt sustainability, using a large time span of data provides invaluable information regarding variables' covariances and autocovariances that is not worth forgoing to account for quarterly dynamics. Additionally, with annual data there is no danger of measurement errors due to seasonality adjustments.

I group countries in four categories according to when the sample begins: 1947, 1960, 1973 and 1997. The first group contains the United States and the United Kingdom. The next two groups contain developed economies. The last group (1997 sample) contains ten developing countries.

I interpret VAR parameters as being random and estimate them using Bayesian regressions. I establish a prior distribution, and then use data likelihood to compute the posterior.[2] I opt to use Bayesian shrinkage as it reduces the volatility of estimated coefficients, an invaluable property when

[1] Web link: https://www.dallasfed.org/research/econdata/govdebt.

[2] See del Negro and Schorfheide (2011) or Karlsson (2013) for more on Bayesian estimation of VAR models.

samples are relatively small. In addition, with a prior distribution that leads to a stable VAR, we can calibrate its tightness to ensure that the posterior centers around a stable VAR as well.

I base my prior on OLS-estimated US dynamics for two reasons. First, we already have results available in the literature (Cochrane (2022), to the best of my knowledge the decompositions have not been estimated to other countries so far). Second, the US has the longest sample. Critically, it comprises the repayment of a major public borrowing event - World War II - that renders OLS estimates of the VAR stable and plausible.[1] I estimate the model for the US by OLS and use the resulting VAR to set the mean of the prior for other countries' estimation.

From the six variables in the VAR, three are directly observed: the nominal interest i_t, the inflation rate π_t and GDP growth g_t. These three rates are in logs. I also use log par debt-to-GDP v_t^b data to generate a series for market-value debt v_t (I describe the procedure next).[2] I demean each of these four time series.

3.1. Proxy Time Series

Country by country, the estimation contains two steps. In the first step, I build proxy time series for the market-debt to GDP (v_t), nominal returns on public bonds (r_t^n) and surplus-to-GDP (s_t). These proxy time series complete the vector of varibles x_t.

I start by running OLS on a three-equation VAR with nominal interest, inflation and GDP growth. I use the VAR to compute the expected values that enter the sums in (9) and then find proxy time series for the market price of bond portfolios $q_{N,t}$ and $q_{R,t}$. Equations (8) and (5) then give a series for nominal return r_t^n.

Next, I convert data on par-value public debt to market value. I begin by building time series for average interest $i_{N,t}^b$ and $i_{R,t}^b$ using the last two equations in (12).[3] Then, to find a proxy series for the book price of nominal bonds $q_{N,t}^b$, I estimate a four-equation VAR (with i_t, π_t, g_t and $i_{N,t}^b$) by OLS and use it to find the expected values in the top equation of (12). I follow the same procedure to estimate $q_{R,t}^b$, replacing $i_{N,t}^b$ with $i_{R,t}^b$ in the auxiliary VAR. With estimated proxy series for $q_{N,t}$, $q_{N,t}^b$, $q_{R,t}$ and $q_{R,t}^b$, I use (11) to convert par-value debt data to market-value debt v_t proxy data.

Finally, equation (6) gives a time-series for primary surplus s_t.

Is this procedure reasonable? In figures 1a and 1b I compare my data series for par value and proxy series for market value of public debt for the United States with the corresponding series reported by the Dallas Fed. The par value series are close to identical. Deviations implied by market price movements are also similar: public debt at market prices is lower than at par in periods of growing interest (such as the early 80s) and greater in periods of declining interest.

In panel 1c, I also compare my public debt series with the series used by Cochrane (2022) (provided by Hall et al. (2021), labeled "HPS" in the graph). They are broadly similar, except for a brief period in the early 2000s. Comparison with data from the Dallas Fed reveals that the difference comes from the fact that Cochrane uses private debt (debt in the hands of the public), while I use gross debt. Sadly, private debt data is not available in a satisfactory time span for most countries in the sample.

3.2. The Bayesian VAR

The functional format of the VAR is

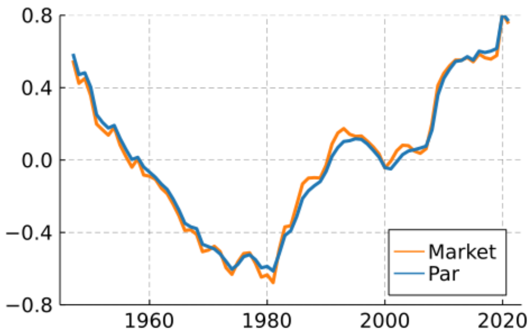
$$X_t = AX_{t-1} + e_t \qquad e_t \sim N(0, \Sigma).$$

I assume that the sample averages used to demean observed variables coincide with their model counterparts, and so we can ignore the constant term. In the second step of the procedure, I estimate

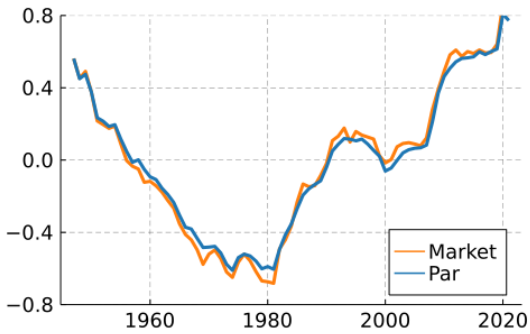
[1] Including pre-1950 data in the sample proved necessary. Starting the sample after that leads to an unstable VAR estimate due to the large public debt equation root.

[2] Most time series data I collect from the St Louis Fed *FRED* website, the United Nations and the IMF. Details on appendix **??**.

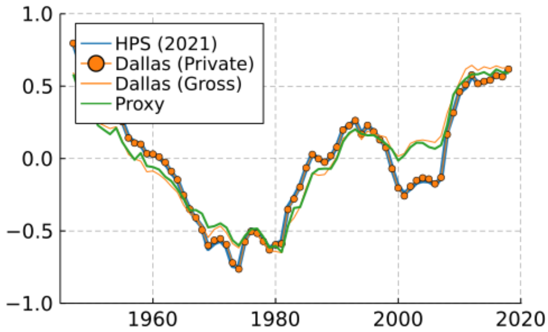
[3] The series for real interest $i_{R,t}^b = i_t - E_t\pi_{t+1}$ uses the expected inflation implied by the three-equation VAR. I set the starting condition for interest $i_{j,t-1}^b$ to be the first observation of each series (nominal i_t and real $i_t - E_t\pi_{t+1}$.)



(a) Data (Par) and Proxy (Market)



(b) Dallas Fed Estimates



(c) Comparison with Hall et al. (2021)

Figure 1: Proxy Time Series for the Market Value of US Public Debt

parameters A and Σ using Bayesian regressions.

The prior distribution belongs to the Normal-Inverse-Wishart (NIW) family. That is, letting $\theta = \text{vec}(A')$, where vec means stack columns,

$$\Sigma \sim IW(\Phi; d)$$
$$\theta|\Sigma \sim N(\bar{\theta}, \Sigma \otimes \bar{\Omega}).$$

With a Gaussian model, the NIW prior distribution is conjugate. Giannone et al. (2015) provide closed-form formulas for the posterior distribution and marginal likelihood.

The mean of the IW distribution is $\Phi/(d-n-1)$ where $n = 6$ is the dimension of the VAR and larger values of d represent tighter priors. I pick Φ to be the identity matrix (uncorrelated shocks, with a standard deviation of one percent) and select $d = n + 2 = 8$, the lowest integer that leads to a well-defined distribution mean (which equals Φ).

The prior for A centers around the coefficients estimated for the US via OLS, $\bar{\theta} = \text{vec}(A_{US}^{OLS})$.[1] The conditional covariance between coefficients is:

$$\text{cov}\left(\tilde{a}_{ij}, \tilde{a}_{kl} \mid \Sigma\right) = \begin{cases} \lambda^2 \frac{\Sigma_{ij}}{\Phi_{jj}} & \text{if } j = l \\ 0 & \text{otherwise.} \end{cases}$$

I build $\bar{\Omega}$ to reproduce the covariance structure above. It allows the loadings on a given variable in different equations to be correlated. The different loadings of any single equation are uncorrelated.

Hyperparameter λ governs the overall tightness of the prior. For each country, I choose the value of λ that maximizes the marginal likelihood of the sample.[2]

Finally, the Bayesian procedure breaks the flow equation of public debt (6), as it linearly combines the equation for the US with that of the estimated country. To restore when computing the posterior

[1]In the US case, this implies that the posterior distribution for A centers around the OLS estimate A_{US}^{OLS} itself.

[2]As Giannone et al. (2015) shows, the likelihood can be decomposed between a term that depends on in-sample model fit and a term that penalizes out-of-sample forecast imprecision, or model complexity.

mode and simulation draws, I manually change the loadings of the surplus equation in the VAR along with the covariance structure of its corresponding shock. The appendix provides details.

3.3. The Inflation Shock - Sources of Inflation Variation

In the baseline estimation, I set $\beta = 0.99$ for all countries and drop observations of the years 2020 and 2021. Parameter V is the average par debt-to-GDP ratio. I calibrate parameters δ and ω (reported in the appendix) based on debt structure data gathered from various sources.

I consider first the inflation shocks:

$$\text{Inflation Shock} = E\left[e \mid e_\pi = 1\right]$$

Inflation unexpectedly jumps by one and the other shocks move contemporaneously exactly as one would expected them to, conditional on the inflation change.[1]

The impulse-response function to the inflation shock tells us how the expected path of each variable changes given that inflation today is 1% greater than expected. Decomposition (7) and (10) measure which factors account, on average, for such increase from the point of view of the valuation equation of public debt.

Alternative interpretation: variance decomposition

Tables (1) and (2) present the terms of the marked-to-market and total inflation decompositions.

4. Concluding Remarks

References

Bohn, H. (1995). The Sustainability of Budget Deficits in a Stochastic Economy. *Journal of Money, Credit and Banking*, 27(1):257.

Cochrane, J. H. (2005). Money as stock. *Journal of Monetary Economics*, 52(3):501–528.

Cochrane, J. H. (2022). The fiscal roots of inflation. *Review of Economic Dynamics*, 45:22–40.

Cox, W. M. (1985). The behavior of treasury securities monthly, 1942–1984. *Journal of Monetary Economics*, 16(2):227–250.

Cox, W. M. and Hirschhorn, E. (1983). The market value of U.S. government debt; Monthly, 1942–1980. *Journal of Monetary Economics*, 11(2):261–272.

del Negro, M. and Schorfheide, F. (2011). Bayesian Macroeconometrics. In Geweke, J., Koop, G., and Van Dijk, H., editors, *The Oxford Handbook of Bayesian Econometrics*, pages 292–389. Oxford University Press.

Giannone, D., Lenza, M., and Primiceri, G. (2015). Prior Selection for Vector Autoregressions. *The Review of Economics and Statistics*, 97(2):436–451.

Hall, G., Payne, J., and Szoke, B. (2021). US Federal Debt 1776 - 1960: Quantities and Prices.

Karlsson, S. (2013). Forecasting with Bayesian Vector Autoregression. In *Handbook of Economic Forecasting*, volume 2, pages 791–897. Elsevier.

[1] To calculate projection like the expected value of the inflation shocks, I use $E\left[e \mid Ke = \epsilon\right] = \Sigma K'(K\Sigma K')^{-1}\epsilon$.

Country	ϵ_{r^n}	$-\epsilon_\pi$	$=$	ϵ_s	$+\epsilon_g$	$-\epsilon_r$
Averages	**-1.0	**-1	=	0.1	**-0.4	**-1.7
1947 (Advanced)	**-0.7	**-1	=	**-1.2	**-0.6	0.1
1960 (Advanced)	**-1.1	**-1	=	*1.0	*-0.5	**-2.6
1973 (Advanced)	**-1.4	**-1	=	-0.4	-0.3	**-1.7
1997 (Emerging)	**-0.7	**-1	=	0.2	**-0.4	**-1.5
1947 Sample (Advanced)						
United Kingdom	**-0.7	**-1	=	**-2.2	**-0.7	**1.2
United States	**-0.7	**-1	=	-0.3	**-0.5	**-0.9
1960 Sample (Advanced)						
Canada	**-2.8	**-1	=	0.3	*-1.4	**-2.8
Denmark	**-0.9	**-1	=	0.2	-0.2	**-1.9
Japan	**-0.6	**-1	=	**2.8	**-3.0	**-1.4
Norway	**-0.7	**-1	=	0.7	*3.0	**-5.4
Sweden	**-0.6	**-1	=	**0.9	**-0.9	**-1.6
1973 Sample (Advanced)						
Australia	**-2.2	**-1	=	0.2	0.1	**-3.5
New Zealand	**-1.0	**-1	=	*1.2	**-1.4	*-1.8
South Korea	**-0.6	**-1	=	**-2.4	0.2	*0.7
Switzerland	**-2.0	**-1	=	*-0.8	0.1	**-2.3
1997 Sample (Emerging)						
Brazil	**-0.7	**-1	=	**2.4	-0.1	**-4.0
Colombia	**-1.4	**-1	=	0.2	**-0.7	**-1.9
Czech Republic	*0.2	**-1	=	*0.7	**-1.3	-0.2
Hungary	**-0.8	**-1	=	0.0	-0.2	**-1.6
India	*-0.2	**-1	=	**-1.0	-0.1	-0.1
Israel	**-0.4	**-1	=	**0.8	*-0.4	**-1.8
Mexico	**-1.4	**-1	=	*-1.2	0.0	*-1.3
Poland	**-1.4	**-1	=	**1.0	*-0.3	**-3.0
South Africa	**-0.6	**-1	=	0.3	**-0.8	**-1.1
Ukraine	**-0.5	**-1	=	**-1.1	0.0	-0.3

Notes: The table reports the terms of the fiscal decomposition to the shock $\Delta E_t \pi_t = 1$. VAR coefficients fixed at the posterior distribution's mode. One asterisk indicates that 75% of the values out of 10,000 draws from the posterior had the same sign as the figure reported. Two asterisks indicate 90%.

Table 1: Marked-to-market decomposition of the shock $E\left[e_t \mid \Delta E_t \pi_t = 1\right]$

Country	$-\varepsilon_\pi$	=	ε_s	$+\varepsilon_g$	$-\varepsilon_r$
Averages	**-1.6	=	0.1	**-0.4	**-1.3
1947 (Advanced)	**-2.2	=	**-1.2	**-0.6	-0.3
1960 (Advanced)	**-1.9	=	*1.0	*-0.5	**-2.3
1973 (Advanced)	**-2.3	=	-0.4	-0.3	**-1.6
1997 (Emerging)	**-1.0	=	0.2	**-0.4	**-0.9
1947 Sample (Advanced)					
United Kingdom	**-2.8	=	**-2.2	**-0.7	0.1
United States	**-1.5	=	-0.3	**-0.5	**-0.7
1960 Sample (Advanced)					
Canada	**-2.6	=	0.3	*-1.4	**-1.5
Denmark	**-1.6	=	0.2	-0.2	**-1.6
Japan	**-1.5	=	**2.8	**-3.0	**-1.3
Norway	**-2.0	=	0.7	*3.0	**-5.7
Sweden	**-1.6	=	**0.9	**-0.9	**-1.5
1973 Sample (Advanced)					
Australia	**-3.1	=	0.2	0.1	**-3.4
New Zealand	**-2.3	=	*1.2	**-1.4	**-2.1
South Korea	**-2.0	=	**-2.4	0.2	0.2
Switzerland	**-2.0	=	*-0.8	0.1	**-1.3
1997 Sample (Emerging)					
Brazil	**-0.8	=	**2.4	-0.1	**-3.1
Colombia	**-0.7	=	0.2	**-0.7	-0.2
Czech Republic	**-0.5	=	*0.7	**-1.3	0.1
Hungary	**-1.4	=	0.0	-0.2	**-1.3
India	**-1.4	=	**-1.0	-0.1	*-0.4
Israel	**-0.6	=	**0.8	*-0.4	**-1.0
Mexico	**-1.4	=	*-1.2	0.0	-0.3
Poland	**-1.4	=	**1.0	*-0.3	**-2.1
South Africa	**-0.8	=	0.3	**-0.8	*-0.3
Ukraine	**-1.2	=	**-1.1	0.0	-0.1

Notes: The table reports the terms of the fiscal decomposition to the shock $\Delta E_t \pi_t = 1$. VAR coefficients fixed at the posterior distribution's mode. One asterisk indicates that 75% of the values out of 10,000 draws from the posterior had the same sign as the figure reported. Two asterisks indicate 90%.

Table 2: Total inflation decomposition of the shock $E\left[e_t \mid \Delta E_t \pi_t = 1\right]$

Country	ϵ_{r^n}	$-\epsilon_\pi$	$=$	ϵ_s	$+\epsilon_g$	$-\epsilon_r$
Averages	**-0.6	**-0.4	=	0.1	0.1	**-1.2
1947 (Advanced)	**-0.8	**-0.2	=	*-0.2	0.1	**-0.8
1960 (Advanced)	**-0.7	**-0.3	=	*0.5	0.4	**-1.9
1973 (Advanced)	**-0.7	**-0.3	=	-0.3	0.3	**-1.0
1997 (Emerging)	**-0.6	**-0.4	=	*0.2	*-0.1	**-1.1
1947 Sample (Advanced)						
United Kingdom	**-0.8	**-0.2	=	**-0.5	-0.1	*-0.4
United States	**-0.7	**-0.3	=	0.0	**0.2	**-1.2
1960 Sample (Advanced)						
Canada	**-0.8	**-0.2	=	*0.2	-0.1	**-1.1
Denmark	**-0.8	**-0.2	=	*0.6	*0.5	**-2.0
Japan	**-0.6	**-0.4	=	0.0	-0.2	**-0.8
Norway	**-0.6	**-0.4	=	*1.0	*1.9	**-3.9
Sweden	**-0.6	**-0.4	=	**0.7	-0.2	**-1.5
1973 Sample (Advanced)						
Australia	**-0.8	**-0.2	=	*0.5	*0.2	**-1.7
New Zealand	**-0.6	**-0.4	=	**0.8	**-0.5	**-1.3
South Korea	**-0.6	**-0.4	=	**-2.4	**1.3	0.2
Switzerland	**-0.8	**-0.2	=	-0.1	*0.2	**-1.1
1997 Sample (Emerging)						
Brazil	**-0.5	**-0.5	=	**1.4	0.1	**-2.6
Colombia	**-0.6	**-0.4	=	0.0	**-0.3	**-0.8
Czech Republic	**-0.4	**-0.6	=	-0.1	-0.3	**-0.6
Hungary	**-0.6	**-0.4	=	*0.4	-0.3	**-1.2
India	**-0.5	**-0.5	=	-0.1	*-0.2	**-0.7
Israel	**-0.7	**-0.3	=	**0.6	-0.1	**-1.5
Mexico	**-0.6	**-0.4	=	**-0.6	0.1	*-0.6
Poland	**-0.7	**-0.3	=	**0.5	-0.1	**-1.4
South Africa	**-0.7	**-0.3	=	*-0.2	0.0	**-0.8
Ukraine	**-0.5	**-0.5	=	**-0.4	*-0.1	**-0.6

Notes: The table reports the terms of the fiscal decomposition to the shock ΔE_t(Disc Surpluses) $= -1$. VAR coefficients fixed at the posterior distribution's mode. One asterisk indicates that 75% of the values out of 10,000 draws from the posterior had the same sign as the figure reported. Two asterisks indicate 90%.

Table 3: Marked-to-market decomposition of the shock $E\left[e_t \mid \Delta E_t(\text{Disc Surpluses}) = -1\right]$

Country	$-\varepsilon_\pi$	=	ε_s	$+\varepsilon_g$	$-\varepsilon_r$
Averages	**-0.6	=	0.1	0.1	**-0.8
1947 (Advanced)	**-0.7	=	*-0.2	0.1	**-0.5
1960 (Advanced)	**-0.7	=	*0.5	0.4	**-1.6
1973 (Advanced)	**-0.8	=	-0.3	0.3	**-0.8
1997 (Emerging)	**-0.4	=	*0.2	*-0.1	**-0.5
1947 Sample (Advanced)					
United Kingdom	**-0.9	=	**-0.5	-0.1	*-0.3
United States	**-0.5	=	0.0	**0.2	**-0.7
1960 Sample (Advanced)					
Canada	**-0.5	=	*0.2	-0.1	**-0.6
Denmark	**-0.6	=	*0.6	*0.5	**-1.6
Japan	**-0.7	=	0.0	-0.2	**-0.5
Norway	**-0.9	=	*1.0	*1.9	**-3.8
Sweden	**-0.8	=	**0.7	-0.2	**-1.2
1973 Sample (Advanced)					
Australia	**-0.6	=	*0.5	*0.2	**-1.3
New Zealand	**-0.8	=	**0.8	**-0.5	**-1.2
South Korea	**-1.2	=	**-2.4	**1.3	0.0
Switzerland	**-0.5	=	-0.1	*0.2	**-0.6
1997 Sample (Emerging)					
Brazil	**-0.3	=	**1.4	0.1	**-1.9
Colombia	**-0.3	=	0.0	**-0.3	-0.1
Czech Republic	**-0.5	=	-0.1	-0.3	-0.2
Hungary	**-0.6	=	*0.4	-0.3	**-0.8
India	**-0.6	=	-0.1	*-0.2	**-0.3
Israel	**-0.2	=	**0.6	-0.1	**-0.7
Mexico	**-0.6	=	**-0.6	0.1	-0.1
Poland	**-0.5	=	**0.5	-0.1	**-0.9
South Africa	**-0.3	=	*-0.2	0.0	*-0.1
Ukraine	**-0.6	=	**-0.4	*-0.1	**-0.1

Notes: The table reports the terms of the fiscal decomposition to the shock $\Delta E_t(\text{Disc Surpluses}) = -1$. VAR coefficients fixed at the posterior distribution's mode. One asterisk indicates that 75% of the values out of 10,000 draws from the posterior had the same sign as the figure reported. Two asterisks indicate 90%.

Table 4: Total inflation decomposition of the shock $E\left[e_t \mid \Delta E_t(\text{Disc Surpluses}) = -1\right]$

A. Data Sources and Treatment

B. Restoring the Flow Equation in the VAR