

A Fiscal Decomposition of Unexpected Inflation: Cross-Country Estimates and Theory

Livio C. Maya

November 2022

Introduction: The Fiscal Sources of Unexpected Inflation

- Connection between fiscal policy and inflation?
- Key Equilibrium Condition: **The Valuation Equation of Public Debt**

$$\frac{\text{Market Value of Debt (**Bond Prices**)}}{\text{Price Level}} = \text{Intrinsic Value (**Discounting, Surpluses**)}.$$

- Unexpected inflation must accompany news about:
 - Bond prices
 - Real surpluses
 - Real discounting
- If unexpected inflation exists, it must nowcast/forecast some of these variables
- What are these forecasts? How do they change across countries? How can theory explain them?

Introduction: Exercises, Motivation, Results

■ **Motivation.** How to read $\text{Debt/Price} = \text{Discounted Surpluses}$?

- Active monetary: *Inflation* \implies *Discounted Surpluses*
- Active fiscal: *Discounted Surpluses* \implies *Unexpected Inflation*
 - Fixed country: +1% inflation \implies +1% deficit/debt?
 - Cross country: +1% inflation in A relative to B \implies +1% deficit/debt in A compared to B?
 - Stylized facts to discipline monetary-fiscal theory
 - Fiscal role to monetary policy?

■ **This paper.**

1. Estimate a Bayesian-VAR to measure the decomposition for 25 countries
 - Variance decomposition: "What does 1% unexpected inflation forecast?"
 - Recession decomposition: "What do -0.5% inflation and -1% growth forecast?" (2008 Recession, deficits, etc...)
2. Estimate a New-Keynesian model to reproduce B-VAR decompositions

Introduction: Exercises, Motivation, Results

■ **Motivation.** How to read $\text{Debt/Price} = \text{Discounted Surpluses}$?

- Active monetary: *Inflation* \implies *Discounted Surpluses*
- Active fiscal: *Discounted Surpluses* \implies *Unexpected Inflation*
 - Fixed country: +1% inflation \implies +1% deficit/debt?
 - Cross country: +1% inflation in A relative to B \implies +1% deficit/debt in A compared to B?
 - Stylized facts to discipline monetary-fiscal theory
 - Fiscal role to monetary policy?

■ **This paper.**

1. Estimate a Bayesian-VAR to measure the decomposition for 25 countries
 - Variance decomposition: "What does 1% unexpected inflation forecast?"
 - Recession decomposition: "What do -0.5% inflation and -1% growth forecast?" (2008 Recession, deficits, etc...)
2. Estimate a New-Keynesian model to reproduce B-VAR decompositions

Introduction: Preview of Key Results

- The variance of unexpected inflation is **accounted for by discounted surpluses** (all countries)

$$\underset{> 0}{\text{var} [\Delta E \pi]} = \underset{< 0}{\text{cov} [\Delta E \pi, Q]} + \underset{> 0}{\text{cov} [\Delta E \pi, \{-s\} + \{R\}]}$$

- Most relevant terms (Surplus-to-GDP, GDP growth and Real discounting) vary
 - **Monetary policy reduces unexpected inflation variance** through bond prices
- Low inflation in recessions: low discounting + higher subsequent surpluses **relative to GDP**
- **Productivity shocks reproduce findings** in NK model
 - Policy shocks do not
- Different policy rules explain differences in variance decomposition across countries

■ **Fiscal Theory of the Price Level.** Cochrane (2022a) and Cochrane (2022b).

- Analysis of multiple countries + more general debt instruments
- NK model estimated to reproduce decompositions + Productivity shocks

Leeper (1991), Sims (1994), Cochrane (1998), Cochrane (2005), Sims (2011), Leeper and Leith (2016), Bassetto and Cui (2018), Cochrane (2022c), Brunnermeier et al. (2022).

■ **Monetary-Fiscal Interaction.**

Cagan (1956), Sargent and Wallace (1981), Hall and Sargent (1997), Hall and Sargent (2011), Jiang et al. (2019), Corsetti et al. (2019), Sunder-Plassmann (2020), Du et al. (2020), Akhmadieva (2022)

■ **Empirical Finance** (Decomposition of Returns)

Campbell and Shiller (1988), Cochrane (1992), Campbell and Ammer (1993), Chen and Zhao (2009).

Fiscal Decomposition: The Valuation Equation

- Environment with discrete time + single good (price P_t) + households and government
- One-period nominal public bonds (price Q_t)
- **In the morning**, the government:
 - redeems bonds B_{t-1} for currency
 - announces real taxes s_t (payable in currency)
 - announces sale of B_t new bonds (payable in currency)
- **In the afternoon**, households trade goods, purchase bonds, pay taxes
- Market clearing + No Money Holding $M = 0$:

$$B_{t-1} = P_t s_t + Q_t B_t$$

Fiscal Decomposition: The Valuation Equation

$$B_{t-1} = P_t s_t + Q_t B_t$$

- *Ex-post* real discounting $\beta_t = Q_t(P_{t+1}/P_t)$ $\beta_{t,t+k} = \prod_{\tau=t}^{t+k} \beta_\tau$
- Iterate law of motion forward:

$$\frac{B_{t-1}}{P_t} = \sum_{k=0}^p \beta_{t,t+k-1} s_{t+k} + \beta_{t,t+p} \frac{B_{t+p}}{P_{t+p+1}}$$

- **Key Assumption:** $\lim_{\tau \rightarrow \infty} \beta_{t,\tau} \frac{B_\tau}{P_{\tau+1}} = 0$
- Valuation equation of public debt:

$$\frac{B_{t-1}}{P_t} = \sum_{k=0}^{\infty} E_t [\beta_{t,t+k-1} s_{t+k}]$$

Fiscal Decomposition: In the Simplest Environment

- Nominal rate $1 + i_t = 1/Q_t$ and real interest $r_t = i_t - E_t\pi_{t+1}$
- End-of-period real debt v_t

$$\underbrace{\frac{1}{\beta} v_{t-1} + \frac{v}{\beta} (i_{t-1} - \pi_t)}_{B_{t-1}/P_t} = s_t + v_t = \sum_{k=0}^{\infty} \beta^k E_t s_{t+k} - \frac{v}{\beta} \sum_{k=1}^{\infty} \beta^k E_t r_{t+k}$$

Linearized valuation equation

- Take innovations $\Delta E_t = E_t - E_{t-1}$:

$$\Delta E_t \pi_t = -\frac{\beta}{v} \sum_{k=0}^{\infty} \beta^k \Delta E_t s_{t+k} + \sum_{k=0}^{\infty} \beta^k \Delta E_t r_{t+k}$$

Fiscal Decomposition: In the Simplest Environment

- Nominal rate $1 + i_t = 1/Q_t$ and real interest $r_t = i_t - E_t\pi_{t+1}$
- End-of-period real debt v_t

$$\underbrace{\frac{1}{\beta} v_{t-1} + \frac{v}{\beta} (i_{t-1} - \pi_t)}_{B_{t-1}/P_t} = s_t + v_t = \sum_{k=0}^{\infty} \beta^k E_t s_{t+k} - \frac{v}{\beta} \sum_{k=1}^{\infty} \beta^k E_t r_{t+k}$$

Linearized valuation equation

- Take innovations $\Delta E_t = E_t - E_{t-1}$:

$$\Delta E_t \pi_t = -\frac{\beta}{v} \sum_{k=0}^{\infty} \beta^k \Delta E_t s_{t+k} + \sum_{k=0}^{\infty} \beta^k \Delta E_t r_{t+k}$$

Fiscal Decomposition: Generalizing

- Growth
- Nominal, Real and Dollar debt
- Long-term debt

$$\frac{\text{Bond Prices} \times \text{Bonds}}{\text{Price Level}} = \sum_t \frac{\text{Surplus-to-GDP} \times \text{GDP}}{\text{Discounting}}$$

$$\frac{v_{t-1}}{\beta} + \frac{v}{\beta} \sum_j \delta_j (r x_{j,t} + i_{j,t-1} + \Delta h_{j,t} - \pi_{j,t}) = \sum_{k=0}^{\infty} \beta^k E_t s_{t+k} + \frac{v}{\beta} \sum_{k=0}^{\infty} \beta^k E_t g_{t+k} - \frac{v}{\beta} \sum_{k=1}^{\infty} \sum_j \delta_j \beta^k E_t r_{j,t+k}$$

Details

Fiscal Decomposition of Unexpected Inflation

- Ex-post real return $r_{j,t} = rx_{j,t} + i_{j,t-1} + \Delta h_{j,t} - \pi_{j,t}$

$$\Delta E_t \pi_t = \underbrace{\left[\Delta E_t rx_t + \sum_{j \neq N} \frac{\delta_j}{\delta} \Delta E_t r_{j,t} \right]}_{\text{Innovation to Bond Prices}} - \frac{\beta}{\delta v} \underbrace{\left[\sum_{k=0}^{\infty} \beta^k \Delta E_t s_{t+k} + \frac{v}{\beta} \sum_{k=0}^{\infty} \beta^k \Delta E_t g_{t+k} - \frac{v}{\beta} \sum_{k=1}^{\infty} \sum_j \delta_j \beta^k \Delta E_t r_{j,t+k} \right]}_{\text{Innovation to the Intrinsic Value of Debt}}$$

$$\equiv d_1(rx) + d_1(r_0) - d_1(s) - d_1(g) + d_1(r)$$

- Variance decomposition.

$$\text{var} [\Delta E_t \pi_t] = \text{cov}_{\pi} [d_1(rx)] + \text{cov}_{\pi} [d_1(r_0)] - \text{cov}_{\pi} [d_1(s)] - \text{cov}_{\pi} [d_1(g)] + \text{cov}_{\pi} [d_1(r)]$$

Bayesian-VAR: Data and Model

- Annual data on **observables** \tilde{x}_t

$$x_t^{OBS} = \begin{bmatrix} i_t & \text{(Nominal Interest)} \\ \pi_t & \text{(CPI Inflation)} \\ v_t^b & \text{(Par-Value Debt-to-GDP)} \\ g_t & \text{(GDP growth)} \\ \Delta h_t & \text{(Chg. Real Exchange Rate)} \end{bmatrix}$$

- 25 countries (samples starting at 1945, 1960, 1975, 1998)
- General VAR

$$X_t = AX_{t-1} + Ke_t$$

- Decompose $X_t' = [x_t^{OBS'} \ x_t^{NOT'}]$

$$X_t = \begin{bmatrix} x_t^{OBS} \\ x_t^{NOT} \end{bmatrix} = \begin{bmatrix} a & 0 \\ b & c \end{bmatrix} \begin{bmatrix} x_{t-1}^{OBS} \\ x_{t-1}^{NOT} \end{bmatrix} + \begin{bmatrix} I \\ k \end{bmatrix} e_t$$

Bayesian-VAR: Empirical Challenges and Solutions

1. Explosive debt dynamics in 1960-2020 implies unstable OLS VARs

$$x_t^{OBS} = a x_{t-1}^{OBS} + e_t$$

- **United States:** Estimate model by OLS (stable!)
- **Others:** Estimate model with a Bayesian-Regression

$$a^{BAY} = (X'X + \lambda^{-1})^{-1}(X'X a^{OLS} + \lambda^{-1} a^{PRIOR})$$

λ maximizes the marginal distribution $p(\text{data})$ and ensures stability

2. Public finance data do not respect law of motion of public debt

- Define surplus from the law of motion: $s_t = \frac{v_{t-1}}{\beta} - v_t + \frac{v}{\beta} \left[-g_t + \sum_j \delta_j (rx_{j,t} + i_{j,t-1} + \Delta h_{j,t} - \pi_{j,t}) \right]$

3. No data on the market value of debt, only its par value (v_t^b) Public Finances Model

- Model for market vs par value (Cox (1985)): $v_t = v_t^b + \frac{v}{\beta} \sum_j \delta_j (rx_{j,t} + i_{j,t-1} - i_{j,t-1}^b)$

4. No data on bond returns Geometric Term Structure

- Geometric maturity structure: $rx_{j,t} + i_{j,t-1} = (\omega_j \beta) q_{j,t} - q_{j,t-1}$

Bayesian-VAR: Empirical Challenges and Solutions

1. Explosive debt dynamics in 1960-2020 implies unstable OLS VARs

$$x_t^{OBS} = a x_{t-1}^{OBS} + e_t$$

- **United States:** Estimate model by OLS (stable!)
- **Others:** Estimate model with a Bayesian-Regression

$$a^{BAY} = (X'X + \lambda^{-1})^{-1}(X'X a^{OLS} + \lambda^{-1} a^{PRIOR})$$

λ maximizes the marginal distribution $p(\text{data})$ and **ensures stability**

2. Public finance data do not respect law of motion of public debt

- Define surplus from the law of motion: $s_t = \frac{v_{t-1}}{\beta} - v_t + \frac{v}{\beta} \left[-g_t + \sum_j \delta_j (rx_{j,t} + i_{j,t-1} + \Delta h_{j,t} - \pi_{j,t}) \right]$

3. No data on the **market** value of debt, only its **par** value (v_t^b) Public Finances Model

- Model for market vs par value (Cox (1985)): $v_t = v_t^b + \frac{v}{\beta} \sum_j \delta_j (rx_{j,t} + i_{j,t-1} - i_{j,t-1}^b)$

4. No data on bond returns Geometric Term Structure

- Geometric maturity structure: $rx_{j,t} + i_{j,t-1} = (\omega_j \beta) q_{j,t} - q_{j,t-1}$

Bayesian-VAR: Empirical Challenges and Solutions

1. Explosive debt dynamics in 1960-2020 implies unstable OLS VARs

$$x_t^{OBS} = a x_{t-1}^{OBS} + e_t$$

- **United States:** Estimate model by OLS (stable!)
- **Others:** Estimate model with a Bayesian-Regression

$$a^{BAY} = (X'X + \lambda^{-1})^{-1}(X'X a^{OLS} + \lambda^{-1} a^{PRIOR})$$

λ maximizes the marginal distribution $p(\text{data})$ and ensures stability

2. Public finance data do not respect law of motion of public debt

- Define surplus from the law of motion: $s_t = \frac{v_{t-1}}{\beta} - v_t + \frac{v}{\beta} \left[-g_t + \sum_j \delta_j (rx_{j,t} + i_{j,t-1} + \Delta h_{j,t} - \pi_{j,t}) \right]$

3. No data on the market value of debt, only its par value (v_t^b) Public Finances Model

- Model for market vs par value (Cox (1985)): $v_t = v_t^b + \frac{v}{\beta} \sum_j \delta_j (rx_{j,t} + i_{j,t-1} - i_{j,t-1}^b)$

4. No data on bond returns Geometric Term Structure

- Geometric maturity structure: $rx_{j,t} + i_{j,t-1} = (\omega_j \beta) q_{j,t} - q_{j,t-1}$

Bayesian-VAR: Empirical Challenges and Solutions

1. Explosive debt dynamics in 1960-2020 implies unstable OLS VARs

$$x_t^{OBS} = a x_{t-1}^{OBS} + e_t$$

- **United States:** Estimate model by OLS (stable!)
- **Others:** Estimate model with a Bayesian-Regression

$$a^{BAY} = (X'X + \lambda^{-1})^{-1}(X'X a^{OLS} + \lambda^{-1} a^{PRIOR})$$

λ maximizes the marginal distribution $p(\text{data})$ and **ensures stability**

2. Public finance data do not respect law of motion of public debt

- Define surplus from the law of motion: $s_t = \frac{v_{t-1}}{\beta} - v_t + \frac{v}{\beta} \left[-g_t + \sum_j \delta_j (rx_{j,t} + i_{j,t-1} + \Delta h_{j,t} - \pi_{j,t}) \right]$

3. No data on the **market** value of debt, only its **par** value (v_t^b) Public Finances Model

- Model for market vs par value (Cox (1985)): $v_t = v_t^b + \frac{v}{\beta} \sum_j \delta_j (rx_{j,t} + i_{j,t-1} - i_{j,t-1}^b)$

4. No data on bond returns Geometric Term Structure

- Geometric maturity structure: $rx_{j,t} + i_{j,t-1} = (\omega_j \beta) q_{j,t} - q_{j,t-1}$

Bayesian-VAR: Variance Decomposition

- **Proposition.** The variance decomposition

$$1 = \frac{\text{cov}_\pi \left[d_1(rx) \right]}{\text{var} [\Delta E_t \pi_t]} + \frac{\text{cov}_\pi \left[d_1(r_0) \right]}{\text{var} [\Delta E_t \pi_t]} - \frac{\text{cov}_\pi \left[d_1(s) \right]}{\text{var} [\Delta E_t \pi_t]} - \frac{\text{cov}_\pi \left[d_1(g) \right]}{\text{var} [\Delta E_t \pi_t]} + \frac{\text{cov}_\pi \left[d_1(r) \right]}{\text{var} [\Delta E_t \pi_t]}$$

is equivalent to the innovations decomposition applied to VAR shock $\text{Proj}(e \mid \Delta E_t \pi_t = 1)$

$$1 = \Delta E_t \pi_t \equiv d_1(rx) + d_1(r_0) - d_1(s) - d_1(g) + d_1(r)$$

- *"Given 1% unexpected inflation, how do we change our nowcast/forecast of the surplus, discounting and bond prices?"*

Bayesian-VAR: Variance Decomposition

Country	$\Delta E_t \pi_t =$	$\Delta E_t(\text{Bond Prices})$		$-\Delta E_t(\text{Discounted Surpluses})$		
		$d(r_0)$	$d(rx)$	$-d(s)$	$-d(g)$	$d(r)$
United States	1	* 0.03	* -0.78	0.57	0.23	0.96
<i>1960 Sample</i>						
Canada	1	* -0.11	* -1.59	0.62	* 1.22	0.86
Denmark	1	* -0.29	* -0.30	0.42	-0.04	1.21
Japan	1	0	* -0.52	* 1.60	-0.38	0.30
Norway	1	* -0.01	* -0.36	0.60	0.47	0.30
Sweden	1	-0.15	* -0.93	-0.34	* 0.98	* 1.42
United Kingdom	1	* 0.52	* -0.73	* 2.89	* 0.97	* -2.65
<i>1973 Sample</i>						
Australia	1	* 0.07	* -0.76	* 2.09	0.66	-1.06
New Zealand	1	-0.10	* -0.86	0.40	* 0.87	0.68
South Korea	1	-0.01	* -0.45	* 1.91	0.17	-0.62
Switzerland	1	0	* -0.69	0.90	* 0.91	-0.12

Advanced Markets

Country	$\Delta E_t \pi_t =$	$\Delta E_t(\text{Bond Prices})$		$-\Delta E_t(\text{Discounted Surpluses})$		
		$d(r_0)$	$d(rx)$	$-d(s)$	$-d(g)$	$d(r)$
1998 Sample						
Brazil	1	-0.26	* -0.22	-1.46	1.05	1.89
Chile	1	-3.80	-1.33	8.95	-5.71	2.88
Colombia	1	1.51	* -0.96	1.39	-1.09	0.15
Czech Republic	1	* -0.16	* -0.37	-2.31	2.42	1.42
Hungary	1	* -0.57	* -0.93	-0.98	1.60	1.88
India	1	* 0.17	* -0.46	1.54	0.05	-0.30
Indonesia	1	* -2.59	* -1.07	1.69	* 2.61	0.35
Israel	1	-0.06	* -0.78	-0.55	* 1.51	0.88
Mexico	1	-0.02	* -0.74	1.41	0.03	0.32
Poland	1	* -0.45	* -1.15	0.87	-0.39	* 2.11
Romania	1	-0.40	* -0.96	2.24	0.42	-0.31
South Africa	1	0.36	* -0.51	1.58	0.25	-0.68
Turkey	1	0.37	* -0.37	-1.18	-0.15	* 2.33
Ukraine	1	0	* -0.77	0.65	0.41	* 0.70

Emerging Markets

Decomposition 2

Bayesian-VAR: Variance Decomposition - Takeaways

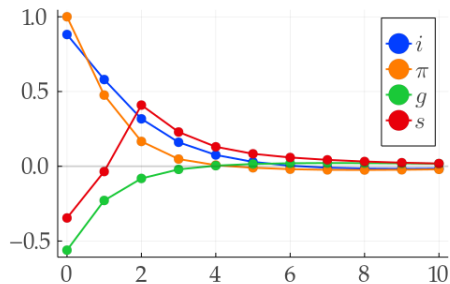


Figure: IRF - Brazil

- Unexpected inflation accounted for by variation in the intrinsic value of debt
- Surplus-to-GDP, GDP growth and real discounting...
 - ...account for unexpected inflation alone in 0/25
 - ...have a positive contribution in 18+/25
- Fiscal roots of inflation do not imply connection between fiscal policy and unexpected inflation
- Nominal bond price dynamics reduce unexpected inflation variance 25/25
 - Effects of monetary policy!

Bayesian-VAR: Variance Decomposition - Takeaways

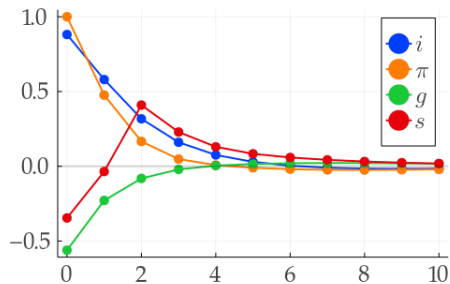


Figure: IRF - Brazil

- Unexpected inflation accounted for by variation in the intrinsic value of debt
- Surplus-to-GDP, GDP growth and real discounting...
 - ...account for unexpected inflation alone in 0/25
 - ...have a positive contribution in 18+/25
- Fiscal roots of inflation do not imply connection between fiscal policy and unexpected inflation
- Nominal bond price dynamics reduce unexpected inflation variance 25/25
 - Effects of monetary policy!

Bayesian-VAR: "Aggregate Demand" Recession

- "Aggregate demand" recessions (Great Recession in 2008) feature:
 - Low inflation
 - Low growth
 - Fiscal deficits (often)
- Does that deny the fiscal sources of inflation?
- Where does unexpected (dis)inflation come from?
- Scenario:

$$\Delta E_t g_t = -1 \quad \Delta E_t \pi_t = -0.5$$

VAR Shock: $\text{Proj}(e \mid \Delta E_t g_t = -1, \Delta E_t \pi_t = -0.5)$

Bayesian-VAR: "Aggregate Demand" Recession

Country	$\Delta E_t \pi_t =$	$\Delta E_t(\text{Bond Prices})$		$-\Delta E_t(\text{Discounted Surpluses})$		
		$d(r_0)$	$d(rx)$	$-d(s)$	$-d(g)$	$d(r)$
United States	-0.5	* 0.03	* 1.00	-0.65	* 1.32	* -2.21
<i>1960 Sample</i>						
Canada	-0.5	* 0.14	* 2.21	-0.45	0.30	* -2.70
Denmark	-0.5	* 0.20	* 0.86	-2.64	* 2.75	-1.67
Japan	-0.5	0	* 0.83	* -1.51	* 1.64	* -1.46
Norway	-0.5	0	* 0.63	-1.36	* 1.72	-1.49
Sweden	-0.5	* 0.41	* 1.22	-0.65	0.87	* -2.35
United Kingdom	-0.5	0.11	* 2.54	-2.20	0.73	-1.68
<i>1973 Sample</i>						
Australia	-0.5	0.06	* 1.54	-1.46	0.66	-1.31
New Zealand	-0.5	* 0.26	* 0.87	-0.84	0.63	-1.42
South Korea	-0.5	* 0.10	* 0.70	* -3.17	* 1.74	0.14
Switzerland	-0.5	0	* 1.18	* -0.93	-0.07	-0.67

Advanced Markets

Country	$\Delta E_t \pi_t =$	$\Delta E_t(\text{Bond Prices})$		$-\Delta E_t(\text{Discounted Surpluses})$		
		$d(r_0)$	$d(rx)$	$-d(s)$	$-d(g)$	$d(r)$
1998 Sample						
Brazil	-0.5	* 0.37	0.06	1.87	0.13	-2.39
Chile	-0.5	* 15.78	* 2.94	-30.50	30.54	-19.26
Colombia	-0.5	1.86	* 0.67	-10.90	* 7.57	0.31
Czech Republic	-0.5	* 0.37	* 0.61	-0.07	0.25	-1.65
Hungary	-0.5	* 0.99	* 0.60	10.82	-5.29	-7.63
India	-0.5	-0.03	0.13	-1.16	0.71	-0.15
Indonesia	-0.5	* 8.23	-0.55	* -11.24	1.42	1.64
Israel	-0.5	* 1.79	0.37	-3.18	1.17	-0.65
Mexico	-0.5	* 1.69	* 0.81	* -4.56	* 1.94	-0.38
Poland	-0.5	* 0.87	* 1.00	-0.14	1.30	* -3.53
Romania	-0.5	* 2.08	0.21	* -8.16	2.05	3.31
South Africa	-0.5	-0.10	0.35	* -30.02	* 11.15	* 18.13
Turkey	-0.5	* 0.99	* 0.23	0.64	0.52	* -2.88
Ukraine	-0.5	0	-0.68	-3.22	* 1.92	1.48

Emerging Markets

Bayesian-VAR: "Aggregate Demand" Recession - Takeaways

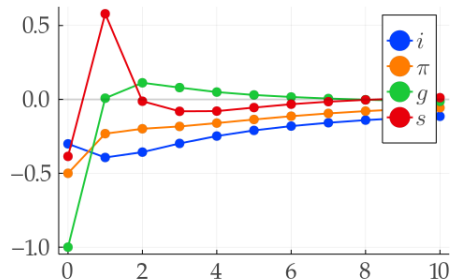


Figure: IRF - United States

- Lower inflation follows from...
 - lower discounting (monetary policy) in 19/25
 - larger surplus-GDP ratios, current or in the future in 22/25
- COVID: what if governments reacted to a recession by credibly reducing $\{s\}$ permanently?
- Direction of causality?

Bayesian-VAR: "Aggregate Demand" Recession - Takeaways

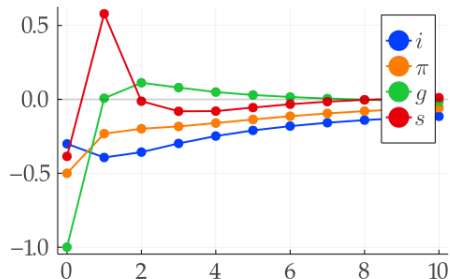


Figure: IRF - United States

- Lower inflation follows from...
 - lower discounting (monetary policy) in 19/25
 - larger surplus-GDP ratios, current or in the future in 22/25
- COVID: what if governments reacted to a recession by credibly reducing $\{s\}$ permanently?
- Direction of causality?

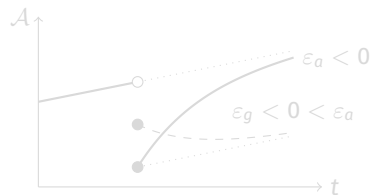
The New-Keynesian Model: Setup

- B-VAR decompositions not structural: all shocks are reduced-form
- Theory: How far do we need to go? **Not much!**
- Two-country NK model:
 - Home economy with $n \rightarrow 0$ households and firms (small and open)
 - Foreign economy with $1 - n \rightarrow 1$ households and firms (large and "closed")
- **The Standard.** Intertemporal substitution + Calvo rigidity
- **The New.** Production function $\mathcal{A}_t N = \tau_t A_t N$ (Home), $\mathcal{A}_t^* N = \tau_t^* A_t^* N$ (Foreign)

(Trend component) $\log \tau_t = \log \tau_{t-1} + u_{g,t}$

(AR(1) component) $a_t = \rho_a a_{t-1} + \varepsilon_{a,t}$

$a_t^* = \rho_a a_{t-1}^* + \varepsilon_{a,t}^*$



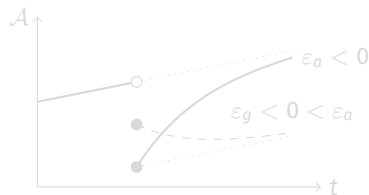
The New-Keynesian Model: Setup

- B-VAR decompositions not structural: all shocks are reduced-form
- Theory: How far do we need to go? **Not much!**
- Two-country NK model:
 - **Home** economy with $n \rightarrow 0$ households and firms (**small and open**)
 - **Foreign** economy with $1 - n \rightarrow 1$ households and firms (**large and "closed"**)
- **The Standard.** Intertemporal substitution + Calvo rigidity
- **The New.** Production function $\mathcal{A}_t N = \tau_t A_t N$ (Home), $\mathcal{A}_t^* N = \tau_t^* A_t^* N$ (Foreign)

(Trend component) $\log \tau_t = \log \tau_{t-1} + u_{g,t}$

(AR(1) component) $a_t = \rho_a a_{t-1} + \varepsilon_{a,t}$

$a_t^* = \rho_a a_{t-1}^* + \varepsilon_{a,t}^*$



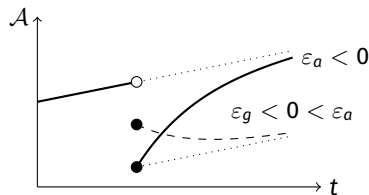
The New-Keynesian Model: Setup

- B-VAR decompositions not structural: all shocks are reduced-form
- Theory: How far do we need to go? **Not much!**
- Two-country NK model:
 - **Home** economy with $n \rightarrow 0$ households and firms (**small and open**)
 - **Foreign** economy with $1 - n \rightarrow 1$ households and firms (**large and "closed"**)
- **The Standard.** Intertemporal substitution + Calvo rigidity
- **The New.** Production function $\mathcal{A}_t N = \tau_t A_t N$ (Home), $\mathcal{A}_t^* N = \tau_t^* A_t^* N$ (Foreign)

(Trend component) $\log \tau_t = \log \tau_{t-1} + u_{g,t}$

(AR(1) component) $a_t = \rho_a a_{t-1} + \varepsilon_{a,t}$

$a_t^* = \rho_a a_{t-1}^* + \varepsilon_{a,t}^*$



The New-Keynesian Model: The Foreign, Closed Economy

■ Private Sector

$$y_t^* = E_t y_{t+1}^* - \gamma [i_t^* - E_t \pi_{t+1}^*] + E_t u_{g,t+1}$$

$$\pi_t^* = \beta E_t \pi_{t+1}^* + \kappa y_t^* - \kappa_a a_t^*$$

$$g_t^* = y_t^* - y_{t-1}^* + u_{g,t}$$

Why Trend? Growth

■ Unexpected inflation indeterminacy? FTPL.

■ Monetary and Fiscal Policy

$$i_t^* = \phi_\pi \pi_t^* + \phi_g g_t^* + \varepsilon_{i,t}^*$$

$$s_t^* = \rho_s s_{t-1}^* + \tau_\pi \pi_t^* + \tau_g g_t^* + \varepsilon_{s,t}^*$$

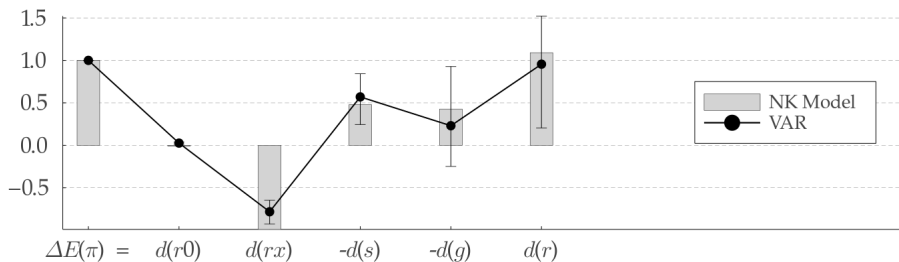
■ GMM for US moments

$$\text{Min}_{\Psi} \quad \alpha_1 \|\mathcal{D}_{VAR} - \mathcal{D}_{NK}(\Psi)\| + \alpha_2 \|\mathcal{M} - \mathcal{M}_{NK}(\Psi)\| \quad \text{s.t. } \Psi \in \Theta$$

Parameters

The New-Keynesian Model: Reproducing the Variance Decomposition

- **Result.** AR(1) productivity shocks $\varepsilon_{a,t}$ **alone** reproduce the **variance decomposition** with positive contributions from surplus-to-output, growth and real interest terms



Target: United States. **Only AR(1) productivity shocks.**

The New-Keynesian Model: Reproducing the Variance Decomposition

$$1 = \Delta E_t \pi_t = d(rx) - d(s) - d(g) + d(r)$$

■ Key Ingredients

- Persistent shock: $\rho_a = 0.98$
- Countercyclical deficits: $\tau_g = 0.7$
- Strong Taylor: $\phi_\pi = 0.8$

■ What is the story?

- Low productivity leads to a recession

$$d(g) < 0$$

- Government raises deficit to fight recession

$$d(s) < 0$$

- Monetary policy raises nominal interest

$$d(rx) < 0$$

$$d(r) > 0$$

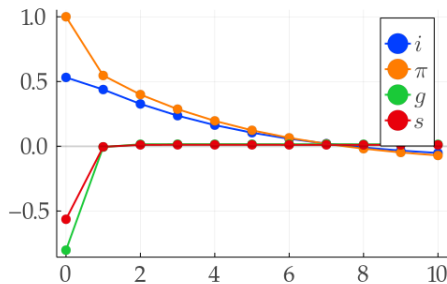


Figure: IRF to $\Delta E_t \pi_t = 1$ ($\varepsilon_{a,t} = -1.15$)

The New-Keynesian Model: Reproducing the Variance Decomposition

$$1 = \Delta E_t \pi_t = d(rx) - d(s) - d(g) + d(r)$$

■ Key Ingredients

- Persistent shock: $\rho_a = 0.98$
- Countercyclical deficits: $\tau_g = 0.7$
- Strong Taylor: $\phi_\pi = 0.8$

■ What is the story?

- Low productivity leads to a recession

$$d(g) < 0$$

- Government raises deficit to fight recession

$$d(s) < 0$$

- Monetary policy raises nominal interest

$$d(rx) < 0$$

$$d(r) > 0$$

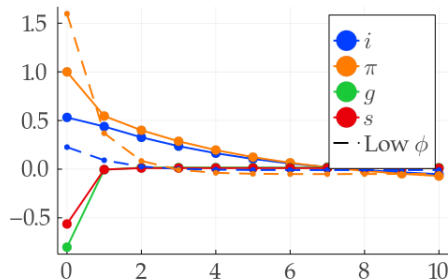
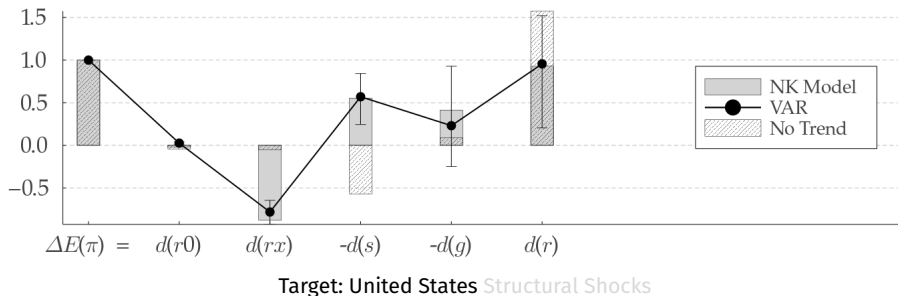


Figure: IRF to $\Delta E_t \pi_t = 1$ ($\varepsilon_{a,t} = -1.15$)

The New-Keynesian Model: Reproducing the Recession Decomposition

$$\Delta E_t g_t = -1 \quad \Delta E_t \pi_t = -0.5$$

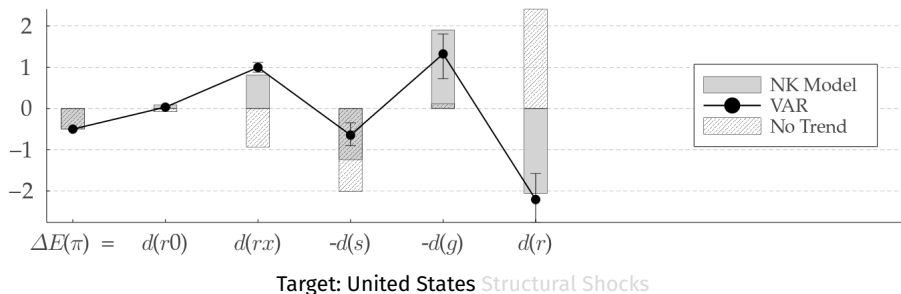
- **Result.** In the absence of trend shocks, NK model fails to replicate the variance and recession decompositions. Policy shocks do not help.
- **Result.** The model with trend shocks reproduces the **recession decomposition** without policy shocks.



The New-Keynesian Model: Reproducing the Recession Decomposition

$$\Delta E_t g_t = -1 \quad \Delta E_t \pi_t = -0.5$$

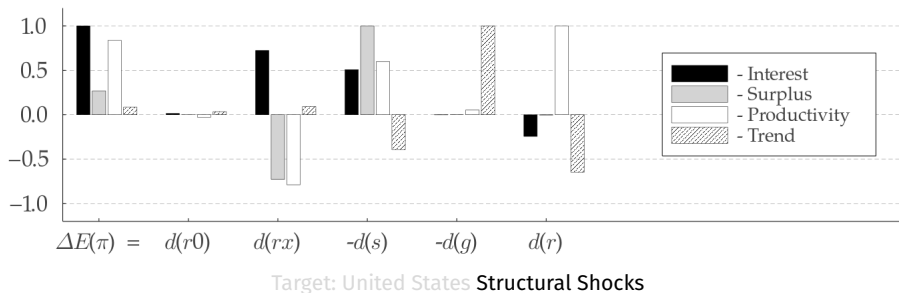
- **Result.** In the absence of trend shocks, NK model fails to replicate the variance and recession decompositions. Policy shocks do not help.
- **Result.** The model with trend shocks reproduces the **recession decomposition** without policy shocks.



The New-Keynesian Model: Reproducing the Recession Decomposition

$$\Delta E_t g_t = -1 \quad \Delta E_t \pi_t = -0.5$$

- **Result.** In the absence of trend shocks, NK model fails to replicate the variance and recession decompositions. Policy shocks do not help.
- **Result.** The model with trend shocks reproduces the **recession decomposition** without policy shocks.



The New-Keynesian Model: Reproducing the Recession Decomposition

$$-0.5 = \Delta E_t \pi_t = d(rx) - d(s) - d(g) + d(r)$$

■ Key Ingredients

- Quiet monetary shocks: $\sigma_i = 0.05$
- Trend + AR(1) shocks: $\sigma_g = 2.6$, $\sigma_a = 1.4$
- Strong Taylor: $\phi_\pi = 0.93$

■ Intuition

- AR(1) shocks reproduce variance decomposition
- Monetary policy shocks generate wrong $d(rx)$

■ Recession ($\varepsilon_a = 0.5$, $\varepsilon_g = -1.3$)

- Recession and lower interest

$$d(g) > 0 \quad d(rx) > 0$$

- Low **detrended** marginal costs: $\pi_t < E_t \pi_{t+1}$
- Low/increasing inflation + Taylor guarantee low real interest

$$r_t = i_t - E_t \pi_{t+1} \approx \pi_t - E_t \pi_{t+1} < 0$$

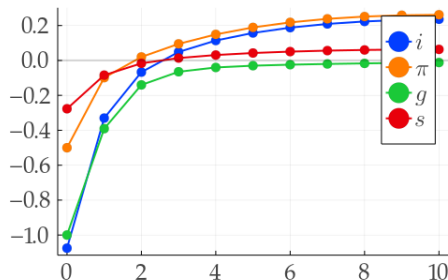


Figure: IRF to $\Delta E_t g_t = -1$, $\Delta E_t \pi_t = -0.5$

The New-Keynesian Model: The Open Economy

$$\begin{aligned}y_t &= E_t y_{t+1} - \gamma [i_t - E_t \pi_{t+1} + \alpha \bar{\omega} E_t \Delta z_{t+1}] + E_t u_{g,t+1} \\ \pi_{H,t} &= \beta E_t \pi_{H,t+1} + \kappa y_t - \kappa_a a_t - \kappa_z z_t \\ y_t &= y_t^* + \gamma_\alpha z_t \\ h_t &= (1 - \alpha) z_t \\ \pi_t &= \pi_{H,t} + \alpha \Delta z_t\end{aligned}$$

- **Home:** Open trade; Complete markets
- Same parameters of the US estimation
- Same combination of Home productivity shocks:
 - Variance decomposition ✓
 - Recession decomposition ✓

Foreign Policy Shocks

New Zealand Decomp

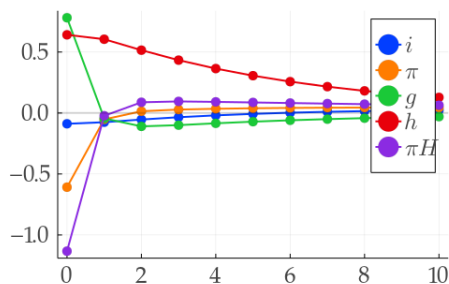


Figure: Productivity Shock in Home Foreign

The New-Keynesian Model: The Open Economy

$$\begin{aligned}y_t &= E_t y_{t+1} - \gamma [i_t - E_t \pi_{t+1} + \alpha \bar{\omega} E_t \Delta z_{t+1}] + E_t u_{g,t+1} \\ \pi_{H,t} &= \beta E_t \pi_{H,t+1} + \kappa y_t - \kappa_a a_t - \kappa_z z_t \\ y_t &= y_t^* + \gamma_\alpha z_t \\ h_t &= (1 - \alpha) z_t \\ \pi_t &= \pi_{H,t} + \alpha \Delta z_t\end{aligned}$$

- **Home:** Open trade; Complete markets
- Same parameters of the US estimation
- Same combination of Home productivity shocks:
 - Variance decomposition ✓
 - Recession decomposition ✓

Foreign Policy Shocks

New Zealand Decomp

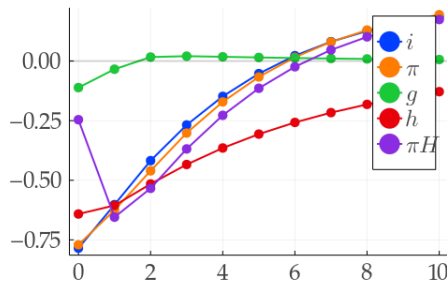


Figure: Productivity Shock in Home Foreign

The New-Keynesian Model: The Open Economy

$$y_t = E_t y_{t+1} - \gamma [i_t - E_t \pi_{t+1} + \alpha \bar{\omega} E_t \Delta z_{t+1}] + E_t u_{q,t+1}$$

$$\pi_{H,t} = \beta E_t \pi_{H,t+1} + \kappa y_t - \kappa_a a_t - \kappa_z z_t$$

$$y_t = y_t^* + \gamma_\alpha z_t$$

$$h_t = (1 - \alpha) z_t$$

$$\pi_t = \pi_{H,t} + \alpha \Delta z_t$$

- **Home:** Open trade; Complete markets
- Same parameters of the US estimation
- Same combination of Home productivity shocks:
 - Variance decomposition ✓
 - Recession decomposition ✓

Foreign Policy Shocks

New Zealand Decomp

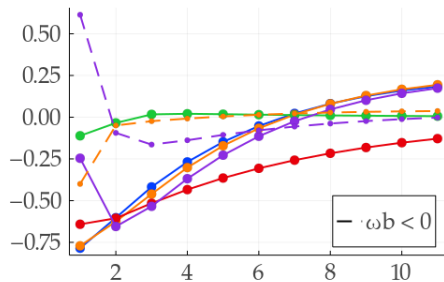


Figure: Productivity Shock in Home Foreign

The New-Keynesian Model: Variance Decomposition and Policy Rules

$$1 = \Delta E_t \pi_t = d(\text{BP}) - d(s) - d(g) + d(r)$$

- Can we match variance decomposition of different countries?
- Vary policy parameters
- Keep other structural parameters constant

$$\dot{i}_t = \phi_\pi \pi_t + \phi_g g_t + \varepsilon_{i,t}$$

$$s_t = \rho_s s_{t-1} + \tau_\pi \pi_t + \tau_g g_t + \varepsilon_{s,t}$$

Parameters	New Zealand	Sweden	Denmark	Australia
<i>A. Variance Decomposition - Value of Debt Contribution</i>				
$-d(s)$	0.40 (0.66)	-0.34 (0.04)	0.42 (0.48)	2.09 (2.04)
$-d(g)$	0.87 (0.63)	0.98 (1.03)	-0.04 (-0.02)	0.66 (0.97)
$d(r)$	0.68 (0.90)	1.42 (1.10)	1.21 (1.17)	-1.06 (-0.52)
<i>B. Estimated Parameters</i>				
ρ_a	0.84			
ρ_g	0.27			
ϕ_π	0.93	0.99	0.96	0.91
ϕ_g	0.61	0.63	0.71	1
ρ_s	0	0.26	0.61	0.05
τ_π	0.25	0.01	-0.02	-0.03
τ_g	0.15	-0.10	-0.13	0.25
σ_i	0.05	0	0	0.51
σ_s	0.08	0	0.12	0.50
σ_a	1.41			
σ_g	2.62			
<i>C. Productivity Shocks Projected by $\Delta E_t \pi_t = 1$</i>				
$\varepsilon_{g,t}$	-0.35	-0.48	0.12	-0.74
$\varepsilon_{a,t}$	-0.77	-0.82	-0.06	-0.08
$\varepsilon_{s,t}$	-0.61	-0.76	-1.49	-0.16

Variance Decomps and Policy Rules

- Valuation equation of public debt \implies decomposition of unexpected inflation
- B-VAR based estimation of two versions of the decomposition
- In most countries:
 - **Variance** of unexpected inflation stems from discounted surpluses (all of its components)
 - **Recessions**: low inflation follow from low discounting and "not so expansionary" fiscal policy
- Stylized New-Keynesian models reproduce VAR decompositions
 - Relevance of productivity shocks
 - Relevance of policy rules

References

- Akhmadieva, V. (2022). Fiscal adjustment in a panel of countries 1870–2016. *Journal of Comparative Economics*, 50(2):555–568.
- Bassetto, M. and Cui, W. (2018). The fiscal theory of the price level in a world of low interest rates. *Journal of Economic Dynamics and Control*, 89:5–22.
- Brunnermeier, M. K., Merkel, S., and Sannikov, Y. (2022). The Fiscal Theory of the Price Level With a Bubble.
- Cagan, P. (1956). The Monetary Dynamics of Hyperinflation. In *Studies in the Quantity Theory of Money*, pages 25–117. University of Chicago Press, milton friedman edition.
- Campbell, J. Y. and Ammer, J. (1993). What Moves the Stock and Bond Markets? A Variance Decomposition for Long-Term Asset Returns. *The Journal of Finance*, 48(1):3–37.
- Campbell, J. Y. and Shiller, R. J. (1988). The Dividend-Price Ratio and Expectations of Future Dividends and Discount Factors. *The Review of Financial Studies*, 1(3):195–228.
- Chen, L. and Zhao, X. (2009). Return Decomposition. *Review of Financial Studies*, 22(12):5213–5249.
- Cochrane, J. H. (1992). Explaining the Variance of Price–Dividend Ratios. *The Review of Financial Studies*, 5(2):243–280.
- Cochrane, J. H. (1998). A Frictionless View of U.S. Inflation. *NBER Macroeconomics Annual*, 13:323–384.
- Cochrane, J. H. (2005). Money as stock. *Journal of Monetary Economics*, 52(3):501–528.
- Cochrane, J. H. (2022a). The fiscal roots of inflation. *Review of Economic Dynamics*, 45:22–40.
- Cochrane, J. H. (2022b). A fiscal theory of monetary policy with partially-repaid long-term debt. *Review of Economic Dynamics*, 45:1–21.
- Cochrane, J. H. (2022c). The Fiscal Theory of the Price Level.

Appendix: Debt Instruments and Growth

Return

- Real **market value** debt-to-GDP v_t , surplus-to-GDP s_t and GDP growth g_t (stationarity!)
- Bonds (j, n) promises one unit of currency j after n periods
 - Nominal bonds
 - Real bonds (currency denomination = final goods)
 - US Dollar bonds

Constant structure $\{\delta_j\}, \{\omega_j^n\}$

- Bond price $Q_{j,t}^n$, excess return $rx_{j,t}$ $1 + \text{return}_{j,t} = 1 + rx_{j,t} + i_{j,t-1} = \frac{\sum_n Q_{j,t}^{n-1} B_{j,t-1}^n}{\sum_n Q_{j,t-1}^n B_{j,t-1}^n}$
(one-period bonds $\implies rx = 0$)
- Debt law of motion:

$$\frac{v_{t-1}}{\beta} + \frac{v}{\beta} \left[-g_t + \sum_j \delta_j (rx_{j,t} + i_{j,t-1} + \Delta h_{j,t} - \pi_{j,t}) \right] = v_t + s_t$$

Appendix: Public Debt Currency Denomination

Symbol	Description	Nominal Debt	Real Debt	Dollar Debt
j	Index Symbol Notation	N δ, ω	R δ_R, ω_R	D δ_D, ω_D
P_j	Price per Good	P	1	P_t^{US}
\mathcal{E}_j	Nominal Exchange Rate	1	P	Dollar NER
H_j	Real Exchange Rate	1	1	Dollar RER
π_j	Log Variation in Price	π	0	π_t^{US}
Δh_j	Log Real Depreciation	0	0	Δh_t

Table: Public Debt Denomination

Appendix: Public Finances Model

[Return](#)

Appendix: Geometric Term Structure

Return

Decomposition 2

- To each currency portfolio j , fixed geometric maturity structure:

$$B_{j,t}^n = \omega_j B_{j,t}^{n-1}$$

- Total return on currency- j portfolio:

$$1 + rx_{j,t} + i_{j,t-1} = \frac{1 + \omega_j Q_{j,t}}{Q_{j,t-1}} \implies \boxed{rx_{j,t} + i_{j,t-1} = (\omega_j \beta) q_{j,t} - q_{j,t-1}}$$

- Assume **constant risk premia** $E_t rx_{j,t+1} = 0$

$$\boxed{q_{j,t} = (\omega_j \beta) E_t q_{j,t+1} - i_{j,t}} = - \sum_{k=0}^{\infty} (\omega_j \beta)^k E_t i_{j,t+k}$$

Appendix: Second Decomposition

Return

- From geometric maturity structure Geometric Term Structure

$$\Delta E_t r x_{j,t} = - \sum_{k=1}^{\infty} (\omega_j \beta)^k [\Delta E_t r_{j,t+k} + \Delta E_t \pi_{j,t+k} - \Delta E_t \Delta h_{j,t+k}]$$

- Replace on the original fiscal decomposition

$$\begin{aligned} \Delta E_t \pi_t &= \overbrace{\left[- \sum_{k=1}^{\infty} (\omega \beta)^k \Delta E_t \pi_{t+k} - \frac{\delta_D}{\delta} \sum_{k=0}^{\infty} (\omega_D \beta)^k \Delta E_t \pi_{t+k}^{US} \right]}^{\text{Innovation to Nominal Variables}} \\ &\quad - \frac{\beta}{\delta v} \underbrace{\left[\sum_{k=0}^{\infty} \beta^k \Delta E_t s_{t+k} + \frac{v}{\beta} \sum_{k=0}^{\infty} \beta^k \Delta E_t g_{t+k} - \frac{v}{\beta} \sum_{k=1}^{\infty} \sum_j \delta_j \beta^k (1 - \omega^k) \Delta E_t r_{j,t+k} - \frac{\delta_D v}{\beta} \sum_{k=0}^{\infty} (\omega_D \beta)^k \Delta E_t \Delta h_{t+k} \right]}_{\text{Innovation to Real Variables}} \\ &\equiv -d_2(\pi) - d_2(\pi^{US}) - d_2(s) - d_2(g) + d_2(r) + d_2(\Delta h) \end{aligned}$$

Appendix: Second Decomposition

Return

Country	$\Delta E_t \pi_t =$	$\pm \Delta E_t(\text{Real Variables})$					
		$-\Delta E_t(\text{Future Inflation})$		$\pm \Delta E_t(\text{Real Variables})$			
		$-d_2(\pi)$	$-d_2(\pi^{US})$	$-d_2(s)$	$-d_2(g)$	$d_2(r)$	$d_2(\Delta h)$
United States	1	*-1.12	-	0.57	0.23	*1.32	0
<i>Advanced - 1960 Sample</i>							
Canada	1	*-1.53	*-0.07	0.62	*1.22	0.78	-0.03
Denmark	1	*-0.49	*-0.20	0.42	-0.04	1.23	0.08
Japan	1	*-1.14	0	*1.60	-0.38	*0.91	0
Norway	1	*-0.70	0	0.60	0.47	0.64	0
Sweden	1	*-1.02	-0.10	-0.34	*0.98	*1.54	-0.07
United Kingdom	1	*-2.34	0	*2.89	*0.97	-0.52	0
<i>Advanced - 1973 Sample</i>							
Australia	1	*-1.47	0	*2.09	*0.66	-0.27	0
New Zealand	1	*-1.02	*-0.08	0.40	*0.87	1.04	-0.21
South Korea	1	*-0.74	*-0.03	*1.91	0.17	-0.33	0.01
Switzerland	1	*-0.79	0	0.90	*0.91	-0.02	0

(a) Advanced Economies

Country	$\Delta E_t \pi_t =$	$\pm \Delta E_t(\text{Real Variables})$					
		$-\Delta E_t(\text{Future Inflation})$		$\pm \Delta E_t(\text{Real Variables})$			
		$-d_2(\pi)$	$-d_2(\pi^{US})$	$-d_2(s)$	$-d_2(g)$	$d_2(r)$	$d_2(\Delta h)$
<i>Emerging - 1998 Sample</i>							
Brazil	1	*-0.11	0	-1.46	1.05	1.46	0.07
Chile	1	-0.76	-2.75	8.95	-5.71	-0.35	1.62
Colombia	1	*-0.61	-0.04	1.39	-1.09	0.02	1.34
Czech Republic	1	-0.02	-0.05	-2.31	2.42	0.98	-0.03
Hungary	1	*-0.69	*-0.15	-0.98	1.60	1.83	*-0.61
India	1	*-1.05	*0.09	1.54	0.05	0.41	-0.04
Indonesia	1	*-0.79	*-1.33	1.69	*2.61	0.26	-1.45
Israel	1	*-0.54	0.10	-0.55	*1.51	0.61	-0.12
Mexico	1	*-0.60	0.17	1.41	0.03	0.52	-0.52
Poland	1	*-0.59	*-0.21	0.87	-0.39	*1.43	-0.11
Romania	1	*-1.14	*-0.53	2.24	0.42	-0.54	0.55
South Africa	1	0.05	-0.01	1.58	0.25	-0.79	-0.07
Turkey	1	*-0.76	*-0.40	-1.18	-0.15	*3.35	0.14
Ukraine	1	-0.29	0	0.65	*0.41	0.23	0

(b) Emerging Economies

Appendix: NK Model Parameters

[Return](#)

Appendix: Why Trend Shocks? The Growth Component

[Return](#)

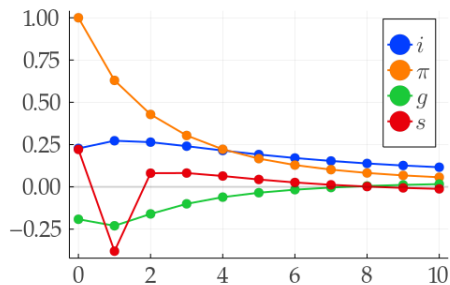
Appendix: Estimated Moments

NK Simple

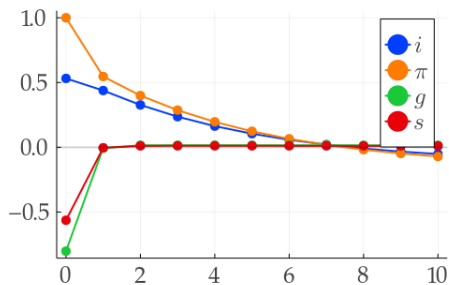
NK Full

Appendix: Simple Model - US Data vs Model

NK Simple



(a) B-VAR



(b) NK Model (Only Prod. Shocks)

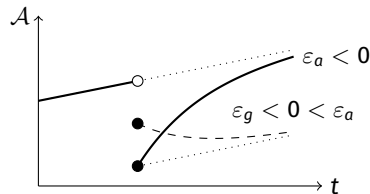
Appendix: Marginal Costs

NK Simple

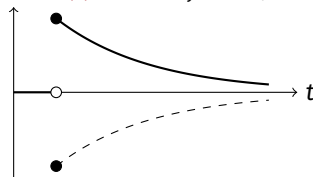
■ AR(1) Productivity Shock

- High marginal costs + strong Taylor rule ($\phi_\pi \approx 1$):

$$i_t \approx \underbrace{\pi_t > E_t \pi_{t+1}}_{mc_t > 0} \implies r_t = i_t - E_t \pi_{t+1} > 0$$



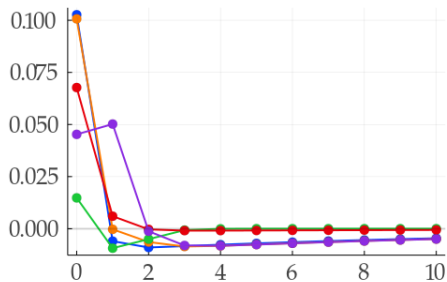
(a) Productivity Path \mathcal{A}_t



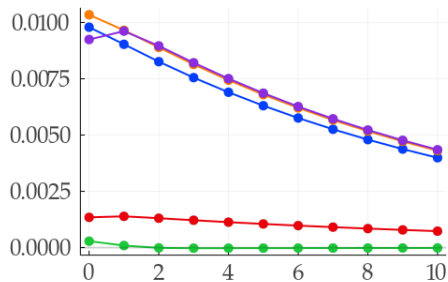
(b) $-a_t$ or Mg. Cost at fixed wages

Appendix: NK Open - Foreign Policy Shocks

Return



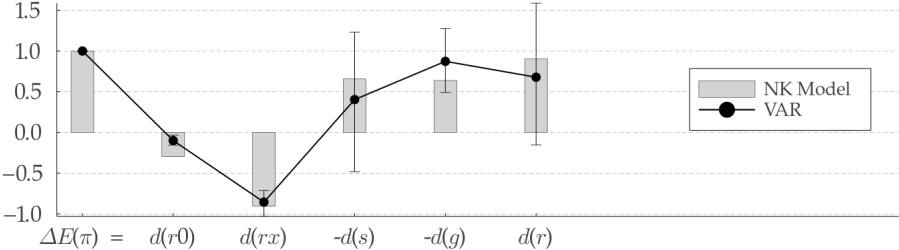
(a) Monetary $\varepsilon_{i,t}^* = 1$



(b) Fiscal $\varepsilon_{s,t}^* = 1$

Appendix: NK Open - New Zealand Decomposition

Return



Data: New Zealand - Variance Recession

Appendix: NK Open - New Zealand Decomposition

Return

