A Fiscal Decomposition of Unexpected Inflation: Cross-Country Estimates and Theory*

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Abstract

I estimate a variance decomposition of unexpected inflation for a set of twenty-five countries using Bayesian-VARs. The decomposition follows from the valuation equation of public debt. Unexpected inflation must be accounted for by news about: surplus-to-GDP ratios, real discounting, GDP growth, future inflation or, in the presence of dollar-denominated debt, real exchange rates and US inflation. Contributions from discounting and output growth are quantitatively large, which implies that unexpected inflation does not need to be accompanied by news of surplus-to-GDP ratios and fiscal policy. Future inflation, which translates the effects of monetary policy on bond prices, reduces unexpected inflation variability in all countries. Building on these results, I measure the sources of unexpected inflation following an "aggregate demand" recession scenario - in which low discounting is critical to account for low inflation - and a real exchange depreciation scenario. I compare the empirical findings with a two-country New-Keynesian model with active fiscal policy and trend shocks. With a strong Taylor rule, the model replicates the variance decomposition of unexpected inflation and its fiscal sources in the recession scenario only with productivity shocks.
Keywords: Inflation, Variance Decomposition, New-Keynesian, Fiscal Theory of the Price Level

1. Introduction

In the absence of financial bubbles, the real market value of the stock of public debt equals its intrinsic value, discounted primary surpluses:

Market Value of Debt (**Bond Prices**) / **Price Level** = Intrinsic Value (**Discounting**, **Surpluses**).

This valuation equation leads to a decomposition of unexpected inflation, defined as the difference between its realized and expected values. Surprise movements in the price level must be accounted for by news about bond prices, fiscal surpluses or real discounting. Unexpected inflation can only exist (*i.e.*, have non-zero variance) if it covaries with, and therefore forecasts, one or more of these variables. Which are they, and what are these forecasts? How do they compare with monetary theory? In this paper, I estimate a Bayesian-VAR for a set of twenty-five advanced and developing countries and use the estimated models to compute the terms of the decomposition. I then compare my results with those implied by an estimated two-country New-Keynesian (NK) model.

Empirically, I find that real discounting accounts for unexpected inflation as much as, and in many cases more than, surpluses. In addition, surplus contribution often stems from news about economic activity (GDP growth) rather than fiscal policy (surplus-to-GDP ratios). Given that the valuation equation holds in virtually any micro-founded macroeconomic model that precludes bubbles, these empirical results lead to general propositions. That the valuation equation links the price level to public

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budgets does not imply that unexpected inflation must follow from news about primary surpluses. Conversely, apparent disconnections between inflation and fiscal policy are not inconsistent with the valuation equation.

News about bond prices also enter the decomposition. Their role depends critically on the currency in which bonds are issued. In my setup, I allow governments to issue nominal, inflation-linked and dollar-linked bonds, a generalization not yet explored by previous literature, but necessary to describe the case of many economies, developing ones in particular.[1]

Nominal debt functions as a buffer for unexpected inflation. Central banks react to news of high inflation by raising interest. Lower bond prices reduce the market value of debt and thus absorb part of the inflationary impact of changing discounted surpluses. This pattern is common to all countries in my sample. Nevertheless, for all but one, lower bond prices forecast higher *future* inflation. Estimates therefore depict how monetary policy can effectively smooth unexpected inflation over time.

The role of inflation and dollar-linked bond prices is not as clear. Estimated covariances often imply a reduction of unexpected inflation variability. For example, surprise domestic inflation correlates with surprise inflation in the US. Higher US prices devalue the dollar portion of public debt, and thus reduce the required jump in the domestic price level. On the other hand, because domestic inflation can only devalue nominal bonds, the presence of non-nominal debt requires larger variations of the price level to re-establish the equality between market and intrinsic debt values.

By further assuming that governments keep a geometric term structure to public debt, we can express unexpected bond price variation as a function of unexpected future inflation (domestic and in the US), real interest and real exchange rates. A second decomposition follows. It involves only inflation and real variables, as promised by the abstract of this paper.

Besides accounting for unexpected inflation volatility, I apply the decomposition to study the fiscal sources of inflation following two reduced-form shocks, designed to simulate realistic scenarios: an "aggregate demand" recession, and an episode of currency depreciation.

In the demand recession scenario, domestic output and inflation decline. The combination is consistent with the experience of many countries in events like the Great Recession and COVID. So, where does lower inflation come from? For most economies (nine of the eleven advanced ones), I find that lower inflation is entirely accounted for by lower discounting. The effect of surpluses is typically neutral but, perhaps surprisingly, that of surplus-to-GDP ratios is *deflationary* and economically large. This suggests that, should governments react to a recession by credibly reducing current *and future* surplus-to-GDP ratios, the inflationary impact of such policies would be substantial. This is a plausible explanation for the worldwide inflation outbreak in the years following COVID.

The currency depreciation scenario is more inflationary in developing countries, as their governments rely more on dollar-linked debt. As a depreciated exchange rate raises the market value of debt in domestic currency, it counts as an inflationary force. Furthermore, the estimated responses resemble the effects of a "sudden stop" of foreign capital flows: growth declines and interest rates increase. I find that emerging markets often manage to revert the inflationary impact of a depreciated currency and lower growth with contractionary fiscal and monetary policies.

In terms of methodology, the analysis of multiple countries is challenging. Each country presents a unique economic experience that can cloud the common patterns we are interested in. In addition, available data is often considerably more limited than United States data. For instance, market-price measures of public debt and real and dollar bond price data are seldom available in a sufficiently large time span. I present an empirical model that circumvents these limitations and allows the estimation of the decomposition. The method requires five commonly available macroeconomic time series: output growth, the short-term interest rate, the inflation rate, the real exchange rate and the par value of public debt. For many countries, these time series go back decades in the past.

The use of Bayesian regressions ensures that estimated VARs are stationary, as necessary for the decomposition to hold. Because public debt is a highly persistent process, and because in many countries it has increased dramatically in the last fifty years, OLS estimation regularly returns unstable

[1] I use "dollar-linked" debt and the more common "dollar-denominated" debt interchangeably.

dynamics.[1] By properly tuning the prior distribution's hyperparameters, we can ensure stability at the same time we discipline parameter search by a goodness-of-fit criterion.

Alone, the estimated VARs have no structural content and tell no causal story. They simply measure the response of the economy to combinations of reduced-form shocks. Causal interpretation requires identification of structural shocks, which I do not pursue, or theory. How complex must a monetary model be to explain the empirical (VAR) findings? As it turns out, not too much. I present a simple two-country, complete markets New-Keynesian (NK) model, similar to Galí and Monacelli (2005), in which a Home economy is small and open and the Foreign economy is large and, in practice, "closed". The only non-standard ingredient I add is a common stochastic trend to both countries' productivity, which renders the economy non-stationary. The model follows the fiscal theory of the price level (FTPL, see Cochrane (2022c)) and selects equilibrium through active fiscal policy. I estimate its parameters through a method of moments that targets the VAR-measured decompositions of unexpected inflation.

With countercyclical fiscal policy (deficits in recessions) and a strong Taylor rule, standard AR(1) productivity shocks *alone* generate a decomposition of unexpected inflation variance with large contributions from surplus-to-output, growth and real discounting. The interpretation is straightforward. A negative productivity shock lowers growth and leads the government to reduce surpluses in response. Higher marginal costs faced by firms mean inflation today (π_t) is greater than inflation tomorrow ($E_t\pi_{t+1}$), by the Phillips curve. Given the strong Taylor rule, nominal interest $i_t \approx \pi_t$ and thus real interest is positive: $r_t = i_t - E_t\pi_{t+1} > 0$. Lower growth, lower surpluses and higher discounting combine to produce positive unexpected inflation. The mechanism is identical in the closed (Foreign) and the open (Home) economies.

We do not have to go too much further to also replicate the lower discounting that accounts for deflation in the recession scenario described above. Productivity shocks continue to be enough, but we have to go beyond AR(1) disturbances. The combination of a negative trend shock with a positive temporary AR(1) shock generates a slow and permanent decline in productivity. The recession comes but, contrary to the pure AR(1) case, firms experience a period of low marginal costs *relative to the economy's trend*. The argument of the last paragraph then holds in reverse and the economy experiences a period of reduced real interest which accounts for the low unexpected inflation. In the case of the open economy (Home), slowly-decaying productivity *abroad* (Foreign) can also generate low interest and the correct pattern of the fiscal decomposition through risk-sharing. The Backus and Smith (1993) condition entails a low (appreciated) and growing real exchange rate that translates into a growing inflation process in Home ($\pi_t < E_t\pi_{t+1}$), as its households import goods from Foreign. The strong Taylor rule then completes the argument for low discounting again.

I do not find policy shocks to be essential for the NK model to replicate the decompositions of unexpected inflation. For instance, an expansionary monetary policy shock leads to positive unexpected inflation, as it increases the market value of public debt (see the equation above). Therefore, if monetary policy shocks were the main drivers of the model's dynamics, its implied decomposition would not feature monetary policy functioning as a buffer of unexpected inflation, as the VARs suggest. Monetary policy would be *the source* of unexpected inflation.

If policy shocks are unimportant to match the decompositions, policy *rules* are critical. The Taylor rule creates quantitatively large contributions of real discounting and surplus rules ensure the correct cyclicality pattern of surplus-to-output ratios. Additionally, I show that by only changing policy rule parameters (thus keeping productivity parameters constant), the NK model can generate different profiles of the decomposition of unexpected inflation variance, with either lower (possibly negative) contributions from growth, surpluses or real discounting. In theory, cross-country differences in macroeconomic policy account for cross-country differences in the fiscal sources of unexpected inflation.

Related Literature

This paper follows Cochrane (2022a) and Cochrane (2022b). The former is the first to estimate the fiscal decomposition for the United States. Compared to it, I include estimates for other countries,

[1]See Bohn (1998), Uctum et al. (2006), Yoon (2012), Azzimonti et al. (2014) for analysis and estimates of public debt persistence.

and work with a slightly more general version of the decomposition that accommodates real and dollar-denominated public bonds. Cochrane (2022b) studies a FTPL, NK model of monetary policy and computes the theoretical decompositions following policy shocks. I also compute it, but with parameters estimated to replicate VAR estimates and in an environment with productivity shocks. The style of the decomposition and its use to discipline theory are borrowed from the finance literature (Campbell and Shiller (1988), Cochrane (1992), Campbell and Ammer (1993), Chen and Zhao (2009) and many others).

More broadly, this paper fits the literature of monetary-fiscal interactions, the beginning of which is hard to establish. Cagan (1956) attributes money growth to government's requirement of seignorage revenues, a proposition later formalized by Sargent and Wallace (1981)'s "unpleasant arithmetic". The theoretical model I present and my interpretation of the decompositions are inspired in the fiscal theory of the price level, kicked off by Leeper (1991), Sims (1994), Woodford (1995) and Cochrane (1998). Leeper (1991) and Cochrane (2022c) contain surveys of the literature and introductions to the FTPL. Studies of public finance aggregates, the components of the government's budget constraint and the evolution of debt includes Hall and Sargent (1997), Cochrane (1998), Hall and Sargent (2011), Cochrane (2019). Some of these papers infer components of the public debt's law of motion from accounting identities. I do that for primary surpluses. Akhmadieva (2022) studies fiscal policy adjustments in a panel of countries, but focuses on revenue *vs* spending dynamics. More recently, researchers have also considered the interaction between the fiscal policy and bond risk premia. These include Jiang et al. (2019) and Du et al. (2020). Still in the intersection between public economics and finance, Hilscher et al. (2022) measures the impact of inflation on real debt using options data and Jiang (2022) studies connections with currency risk. The estimates of the terms of the decomposition provided here can help to build macro-founded discount factors that correctly price unexpected inflation risk. Lastly, models that focus on violations of the no-bubble condition - which I assume to hold - and its implications include Reis (2021) and Brunnermeier et al. (2022). Kocherlakota (2022) considers also implications for policy.

2. Unexpected Inflation Decomposition

2.1. *Connection with the Value of Debt*

I start by establishing the connection between inflation and the value of debt. Consider an economy with a homogeneous final good which households value. The economy has a government that manages a debt composed by bonds that pay one unit of currency in the next period. Currency is a commodity that only the government can produce, at zero cost. There is nothing special about currency. Households do not value it, and it provides no convenience for trading. Households cannot burn currency.

At the beginning of period t, the face value of debt issued in the previous period is V_{t-1}. The government redeems it for currency, which moves to the hands of households. After redeeming bonds, the government announces that each household i must pay $T_{i,t}$ goods in taxes, payable in currency. It also announces it will sell V_t new bonds and purchase G_t units of the final good at market prices.[1] Nothing binds the government's choices of $T_{i,t}$ and V_t. Note the difference from the case with bonds payable in goods, in which the government *must* raise tax revenue or issue new debt to redeem old bonds. Additionally, if households accept currency as payment for final goods, nothing binds the government's choice of G_t either.

Let M_t be private holdings of currency at the end of t. As there is no free disposal of currency, the quantity used by the government to redeem $t-1$ bonds and purchase goods is used to either pay taxes, buy new bonds or increase currency holdings:

$$\begin{aligned} V_{t-1} + P_t G_t &= P_t T_t + Q_t V_t + \Delta M_t \\ \implies V_{t-1} &= P_t s_t + Q_t V_t + \Delta M_t \end{aligned} \tag{1}$$

[1] That the government forces households to pay for taxes and new bonds in currency is not necessary for the argument. All else follows if the government accepts payment in goods but stands ready to exchange these goods for currency at market prices.

where T_t are aggregate taxes, $s_t = T_t - G_t$ is the primary surplus, P_t is the final good's price and Q_t is the price of new bonds (I state prices in currency units). Equation (1) provides a low of motion for public debt. It holds for all prices and all choices of money holdings, new debt sales, and primary surplus.[1]

If $P_t = 0$, real public debt V_{t-1}/P_t equals infinity. For now, that is a possibility.

Suppose $P_t > 0$. Define $\beta_t \equiv Q_t P_{t+1}/P_t$ as the real discount for public bonds, and $\beta_{t,t+k} \equiv \prod_{\tau=t}^{t+k} \beta_\tau$. Since V satisfies (1), it also satisfies

$$\frac{V_{t-1}}{P_t} = \sum_{i=0}^{k} \beta_{t,t+i-1} \left(s_{t+i} + \Delta M_t\right) + \beta_{t,t+k} \frac{V_{t+k}}{P_{t+k+1}} \qquad \text{for any } k \geq 0 \tag{2}$$

regardless of prices and choices. Expressions (1) and (2) do not represent a constraint on the path of surpluses $\{s_t\}$ and bond sales $\{V_t\}$ the government chooses to follow. They merely express future debt given paths for quantities and prices.

So far, all we did was public finances accounting. I now move to economic behavior. If $P_t = 0$, non-satiable households demand infinite final goods and there is no equilibrium. Therefore $P_t > 0$. Given a utility function over consumption paths $U(\{c_t\})$, the optimal consumption-savings choice involves two conditions. First: $\beta_{t,t+k} = U$-implied marginal rate of substitution between time-t and time-$t+k$ consumption. Second, the transversality condition $\lim_{k\to\infty} \beta_{t,t+k} V_{t+k}/P_{t+k+1} \leq 0$. Coupled with a no-Ponzi condition that establishes the reverse inequality, it implies

$$\lim_{k\to\infty} \beta_{t,t+k} \frac{V_{t+k}}{P_{t+k+1}} = 0 \text{ at every period } t. \tag{3}$$

If bonds were redeemable in goods, (3) would represent a debt sustainability condition, as it forces the government to eventually raise the resources to pay for past borrowing.

With nominal debt, however, the government has no constraints on its choice of debt and surplus, as already pointed out. Replacing (3) on (2) and taking expectation yields

$$\frac{V_{t-1}}{P_t} = \sum_{i=0}^{\infty} E_t \left[\beta_{t,t+i-1} \left(s_{t+i} + \Delta M_{t+i}\right)\right]. \tag{4}$$

Given the face value of maturing public debt, the relative price of the final good is given by the expected β-discounted stream of real primary surpluses. This latter term - the right-hand side of (4) - I call the *intrinsic value of public debt*. My definition of debt value follows a "beginning-of-period" convention: it includes current period surplus s_t and starts discounting at $t+1$. The term on the left of (4) I call the *market value of public debt*, which, in the case of one-period bonds, equals their face value in units of goods. In the case of nominal debt, (4) is a *valuation equation*. Hence, expression (3) implies not a no-default condition, but a no-bubble condition: market prices should reflect intrinsic value.

Now, define the inflation rate $\Pi_t = P_t/P_{t-1}$, and take innovations $\Delta E_t = E_t - E_{t-1}$ on both sides of (4) to arrive at its innovation counterpart:

$$\frac{V_{t-1}}{P_{t-1}} \Delta E_t \Pi_t^{-1} = \sum_{i=0}^{\infty} \Delta E_t \left[\beta_{t,t+i-1} \left(s_{t+i} + \Delta M_{t+i}\right)\right]. \tag{5}$$

Equations (4) and (5) do not depend on equilibrium selection mechanisms. Both hold in all models in which the transversality condition (3) holds, including the canonical three-equation New-Keynesian model. In a FTPL interpretation, any shock can lead the government to change the conditional distribution of future surpluses (in units of goods) backing the stock of public nominal liabilities. The relative price of bonds in terms of goods $1/P_t$ then adjusts to reflect that change, much in the same way that news of future dividends (in currency units) per stock change the relative price of stocks in terms of

[1]Strictly speaking, (1) holds for all *feasible* choices of money holdings, that is, choices that respect households' budget constraint. Otherwise, one could point out that, for $V_{t-1} > 0$, $M_t = M_{t-1}$ and $s_t = V_t = 0$ violates (1). That would nevertheless involve households burning up currency.

currency (Cochrane (2005)). Unexpected inflation $\Delta E_t \Pi_t$ follows. Also like stocks, changes in stochastic discounting β affect fundamental value, and therefore affect prices. In models with "passive" fiscal policy, the government observes unexpected inflation and changes the path of surpluses so that the intrinsic value reflects the new real market value of debt.

2.2. Inflation Decomposition in the Simplest Environment

Start by linearizing the law of motion (2).

$$v_t + s_t = \frac{1}{\beta}\left(v_{t-1} + i_{t-1} - \pi_t\right) \tag{6}$$

where v_t is *end-of-period* stock of real debt, $i_t = -\log(Q_t)$ and $\pi_t = \log(\Pi_t)$. I assume $\Delta M_t = 0$ (households do not hold currency). Note that v and s are both in levels - I assume them to be stationary. Moreover, I linearize around an average debt of $v = 1$. The interpretation of (6) is the same as before. The expression on the right is the linearized beginning-of-period market value of debt, corresponding to V_{t-1}/P_t in the non-linear formulation. Previous period debt accrues by the mean real interest $(1/\beta)$ plus its local variation $i_t - \pi_t$. A 1% higher real interest raises debt by 1% on average because debt is on average one. The government redeems bonds for currency, and then sells new debt v_t and runs a surplus s_t to soak it up. Repeating the same steps as before, solve (6) forward:

$$\frac{1}{\beta}\left(v_{t-1} + i_{t-1} - \pi_t\right) = \sum_{k=0}^{\infty} \beta^k E_t s_{t+k} - \frac{1}{\beta}\sum_{k=1}^{\infty} \beta^k E_t r_{t+k}$$

where $r_t = i_{t-1} - \pi_t$ is the *ex-post* real interest rate. The expression on the right-hand side is the linearized intrinsic value of debt. It includes time-t surplus, and starts discounting at $t+1$. The inflation rate on the left represents the price level equalizing the beginning-of-period market value of debt to its intrinsic value.

Take innovation, and multiply both sides by β to find

$$\Delta E_t \pi_t = -\beta \sum_{k=0}^{\infty} \beta^k \Delta E_t s_{t+k} + \sum_{k=0}^{\infty} \beta^k \Delta E_t r_{t+k}. \tag{7}$$

In this simple environment, unexpected higher inflation is accounted for by news of larger deficits $-s$ or news of higher discounting r. That is, news about the intrinsic value of debt. This decomposition was introduced by Cochrane (2022a), and follows similar decompositions for stock returns and price-dividend ratios (Campbell and Shiller (1988), Campbell and Ammer (1993)). Continuing to follow the finance literature, take covariance with unexpected inflation on both sides. We arrive at an initial decomposition of unexpected inflation variance.

$$\text{var}(\Delta E_t \pi_t) = -\text{cov}\left[\Delta E_t \pi_t, \beta \sum_{k=0}^{\infty} \beta^k \Delta E_t s_{t+k}\right] + \text{cov}\left[\Delta E_t \pi_t, \sum_{k=0}^{\infty} \beta^k \Delta E_t r_{t+k}\right]. \tag{8}$$

In the simplest environment, if unexpected inflation "exists", it must forecast (or nowcast) deficits or higher discounting.

2.3. VAR-Based Measures

In this paper, I use models in VAR form $X_t = AX_{t-1} + Ke_t$ to measure (7) and (8). To measure the innovations decomposition (7), suppose we are interested in the response of the system to a shock e_t. Using $\Delta E_t s_{t+k} = \mathbf{1}_s' A^k K e_t$ (where $\mathbf{1}_s'$ selects the row of A corresponding to surpluses) and similar expressions to the other terms, (7) implies

$$\Delta E_t \pi_t = \mathbf{1}_\pi' K e_t = -\beta \mathbf{1}_s' (I - \beta A)^{-1} K e_t + \mathbf{1}_r' (I - \beta A)^{-1} K e_t.$$

More generally, suppose we want to measure the decomposition given only the innovations for a subset x_t of X_t. Start by projecting the entire vector of residuals using

$$\begin{aligned} \text{Proj}(e_t|\Delta E_t x_t) &= \text{cov}(e_t, \Delta E_t x_t)\text{var}(\Delta E_t x_t)^{-1}\Delta E_t x_t \\ &= \Omega K'\mathbf{1}_x \left(\mathbf{1}_x' K\Omega K'\mathbf{1}_x\right)^{-1} \Delta E_t x_t, \end{aligned}$$

where $\Omega = \text{cov}(e_t)$ and $\mathbf{1}_x$ are the columns of the identity matrix corresponding to variables in x (for instance, $\mathbf{1}_\pi' K$ selects the row of K corresponding to inflation).[1] We can then use the prior formula to measure the decomposition using $\text{Proj}(e_t|\Delta E_t x_t)$ in the place of e_t.

- *The variance decomposition* (8) *is equivalent to the innovations decomposition* (7) *applied to the shock* $\mathit{Proj}(e_t \mid \Delta E_t \pi_t = 1)$.

To prove the proposition, divide the two sides of (8) by $\text{var}(\Delta E_t \pi_t)$ and replace the formulas implied by the VAR:

$$\begin{aligned} 1 &= -\beta \overbrace{\mathbf{1}_s'(I-\beta A)^{-1}K\Omega K'\mathbf{1}_\pi}^{\text{cov}\left[\Delta E_t\pi_t, \sum_k \beta^k \Delta E_t s_{t+k}\right]} \overbrace{\left(\mathbf{1}_\pi' K\Omega K'\mathbf{1}_\pi\right)^{-1}}^{\text{var}(\Delta E_t \pi_t)^{-1}} + \mathbf{1}_r'(I-\beta A)^{-1}K\Omega K'\mathbf{1}_\pi \left(\mathbf{1}_\pi' K\Omega K'\mathbf{1}_\pi\right)^{-1} \\ &= -\beta \mathbf{1}_s'(I-\beta A)^{-1}K\,\text{Proj}(e_t \mid \Delta E_t\pi_t = 1) + \mathbf{1}_r'(I-\beta A)^{-1}K\,\text{Proj}(e_t \mid \Delta E_t\pi_t = 1). \end{aligned}$$

The left-hand side above is $\Delta E_t \pi_t = 1$, and the right-hand side is the innovations decomposition applied to the projection of the residuals e_t onto $\Delta E_t \pi_t = 1$. We interpret each term as the share of unexpected inflation variance accounted for by the variable that enters the sum. The shares sum to one.

2.4. Generalizing Public Financing Instruments

2.4.1. Currency Denomination

The debt process considered so far is too unrealistic to be taken to the data. I generalize the financing instruments available to the government in two dimensions: currency denomination and maturity structure. I do not consider the case of debt with stage-contingent nominal payoffs.[2]

Starting here, I recycle part of the notation already established. Fix the case of a country's government. Let P_t be the price of the consumption basket in terms of domestic currency. The payoff of public bonds can be indexed to different currencies, enumerated by j. Let $P_{j,t}$ be the price of the consumption basket in units of currency j. Let $Q_{j,t}^n$ be the discount rate for a zero-coupon public bond paying one unit of currency j after n periods (so $Q_t^0 = 1$). Without loss of generality, all bonds are zero-coupon. Let $\mathcal{E}_{j,t}$ be the price of currency j in units of domestic currency.

The notation is general enough to accommodate currency-denominated bonds *per se*, but also inflation-linked securities such as American TIPS. In this case, one can interpret the "currency" as the consumption basket (so that $P_j = 1$ and $\mathcal{E}_j = P_t$). In the empirical exercises that follow, I consider only nominal bonds ($j = N$), inflation-linked (or real) bonds ($j = R$) and US-dollar-denominated bonds ($j = D$). Table 1 shows the value or interpretation of the variables defined above for these three cases.

I do not assume the price index for government's purchases (denoted P_t^S) and levied aggregates (such as income) to be the same as the price index for households' consumption (P_t). While governments do tend to tax private consumption, they also tax and transfer income, and purchase baskets of goods different than that of households. The fiscal effect of variations in the relative price of government to households' final goods basket must be accounted for.

With these changes, the law of motion of government debt (1) becomes

$$\sum_j \mathcal{E}_{j,t} B_{j,t-1}^1 = P_t^s S_t + \sum_j \mathcal{E}_{j,t} \sum_{n=2}^{\infty} Q_{j,t}^{n-1} \left(B_{j,t}^{n-1} - B_{j,t-1}^n\right),$$

[1] I assume invertibility of $\mathbf{1}_x' K\Omega K'\mathbf{1}_x$.

[2] Publicly-ran social security and pension systems can be modelled as cases of public debt with state-contingent payoffs or, in a fashion compatible with my framework, as an additional layer of uncertainty in the primary surplus process.

Symbol	Description	Nominal Debt	Real Debt	Dollar-Linked Debt
j	Index Symbol	N	R	D
	Notation	δ, ω	δ_R, ω_R	δ_D, ω_D
P_j	Price per Good	P	1	P_t^{US}
$\mathcal{E}_j$	Nominal Exchange Rate	1	P	Dollar NER
H_j	Real Exchange Rate	1	1	Dollar RER
π_j	Log Variation in Price	π	0	π_t^{US}
Δh_j	Log Real Depreciation	0	0	Δh_t

Notes: P = price of consumption basket in domestic currency. P^{US} = price of consumption basket in US dollars. NER = Nominal Exchange Rate. RER = Real Exchange Rate.

Table 1: Public Debt Denomination

where S_t is the real primary surplus and $B_{j,t}^n$ is the face value of bonds issued in currency j, period t, payable n periods in the future. The term on the left represents the cost of debt in period t; the second term on the right represents proceeds from the selling of new bonds.

Let $\mathcal{V}_{j,t} = \sum_{n=0}^{\infty} Q_{j,t}^n B_{j,t}^n$ be the end-of-period market value of nominal debt issued in currency j, $i_{j,t}$ the risk-free rate in bonds issued in currency j and $rx_{j,t} = \mathcal{V}_{j,t-1}^{-1} \sum_{n=1}^{\infty} Q_{j,t}^{n-1} B_{j,t-1}^n - i_{j,t-1}$ the realized excess return on portfolios that mimic the composition of j-currency debt. We can re-write the law of motion in terms of the $\mathcal{V}_j$ and its corresponding returns:

$$\sum_j (1 + rx_{j,t} + i_{j,t-1}) \mathcal{E}_{j,t} \mathcal{V}_{j,t-1} = P_t^s S_t + \sum_j \mathcal{E}_{j,t} \mathcal{V}_{j,t}$$

For empirical work, it is ideal to write the law of motion in terms of real and stationary variables. So, let Y_t be real GDP and let $g_t = Y_t / Y_{t-1} - 1$ be its growth rate. Define the real exchange rate $H_{j,t} = \mathcal{E}_{j,t} P_{j,t} / P_t$ and its growth rate $\Delta h_{j,t} = H_{j,t} / H_{j,t-1} - 1$. Define the detrended real value of j-indexed debt $V_{j,t} = \mathcal{V}_{j,t} / P_{j,t} Y_t$, the real value of total debt $V_t = \sum_j H_{j,t} V_{j,t}$ and the j-indexed share $\delta_{j,t} = H_{j,t} V_{j,t} / V_t$. I assume a constant currency structure $\delta_{j,t} = \delta_j$.

By properly dividing the whole above equation by $P_t Y_t$, and multiplying and dividing the j sum on the left by $P_{j,t-1}$, $P_{j,t}$, Y_{t-1} and $H_{j,t-1}$, we arrive at a final version for the law of motion of public debt:

$$V_{t-1} \sum_j \frac{(1 + rx_{j,t} + i_{j,t-1})(1 + \Delta h_{j,t})}{(1 + \pi_{j,t})(1 + g_t)} \delta_j = \frac{P_t^s}{P_t} s_t + V_t.$$

The law of motion above generalizes (2) for $k = 1$. The expression on the left is the beginning-of-period market price of public debt. During period t, the government must "pay" V_{t-1} plus realized returns and eventual changes to the relative value of currency j.[1]

I linearize the debt law of motion. Let $\beta_j = \exp\{-(rx_j + i_j - \pi_j - g)\}$ be the steady-state real and growth-adjusted discounting for public debt issued in currency j. I linearize around a steady state (assumed to exist) with $\beta_j = \beta$ for all j and $P^s = P$. This leads to a generalization of (6):

$$v_t + s_t + s(p_t^s - p_t) = \frac{v_{t-1}}{\beta} + \frac{v}{\beta} \left[\sum_j \delta_j \left(rx_{j,t} + i_{j,t-1} + \Delta h_{j,t} - \pi_{j,t} \right) - g_t \right], \quad (9)$$

where the right-hand side contains the linearized market value of public debt. I re-define rx_t, i_t, Δh_t and g_t to be log growth/return instead of percentual, and all variables are in deviations from steady state. Parameter v is the steady-state level (not log) of end-of-period public debt, and so $v_t = V_t - v$.

Let $r_{j,t} = rx_{j,t} + i_{j,t-1} + \Delta h_{j,t} - \pi_{j,t}$ be the *ex-post* real return on holdings of the j-currency portfolio

[1] "Pay" comes in parentheses here because, unlike in (1), the government does not actually redeem the entire term on the left at period t. It only pays for bonds maturing at t.

of public bonds, and define $s_t^p = s_t + s(p_t^s - p_t)$ as the price-adjusted surplus process.

Decomposition 1. *Solve the debt law of motion* (9) *forward and take innovations to arrive at*

$$\frac{v}{\beta}\left[\sum_{j\neq N}\delta_j \Delta E_t r_{j,t} + \delta\Big(\Delta E_t rx_t - \Delta E_t \pi_t\Big)\right] = \underbrace{\sum_{k=0}^{\infty}\beta^k \Delta E_t s_{t+k}^p + \frac{v}{\beta}\sum_{k=0}^{\infty}\beta^k \Delta E_t g_{t+k} - \frac{v}{\beta}\sum_{k=1}^{\infty}\sum_{j}\delta_j \beta^k \Delta E_t r_{j,t+k}}_{\text{Innovation to the Intrinsic Value of Debt}}.$$

Take covariance with unexpected inflation, and divide both sides by $\delta(v/\beta)$:

$$\begin{aligned} var(\Delta E_t \pi_t) = {} & cov\left[\Delta E_t \pi_t, \delta^{-1}\sum_{j\neq N}\delta_j \Delta E_t r_{j,t}\right] + cov\left(\Delta E_t \pi_t, \Delta E_t rx_t\right) \\ & - cov\left[\Delta E_t \pi_t, \left(\frac{\delta v}{\beta}\right)^{-1}\sum_{k=0}^{\infty}\beta^k \Delta E_t s_{t+k}\right] - cov\left[\Delta E_t \pi_t, \delta^{-1}\sum_{k=0}^{\infty}\beta^k \Delta E_t g_{t+k}\right] \\ & + cov\left[\Delta E_t \pi_t, \delta^{-1}\sum_{k=1}^{\infty}\sum_{j}\delta_j \beta^k \Delta E_t r_{j,t+k}\right]. \end{aligned} \tag{10}$$

Throughout the paper, I will refer to the innovations decomposition above as decomposition 1, and equation (10) as *variance* decomposition 1. Decomposition 1 generalizes (7). The right-hand side still contains the revision of the intrinsic value of debt. The left-hand side reveals the new terms Δr_t and Δrx_t containing time-t unexpected jumps in public bond prices, absent in the context of one-period bonds. Now, *given unexpected variation in the price of long-term debt*, unexpected inflation must be accounted for by news of surpluses or real discounting.

Compared to the $\delta = 1$ case with nominal debt only, the decomposition contains time-t price-adjustment terms (on the left) and future discounting terms (on the right) related to currency-linked real returns. For countries with dollar-linked debt $\delta_D > 0$, unexpected real exchange depreciation raises the home-currency value of debt and thus acts like an inflationary force (corresponding to the $\Delta E r_{D,t}$ term on the left). News of *future* real exchange depreciation also stimulate inflation by increasing real discounting ($\Delta r_{D,t+k}$ term on the right). The lower the share of nominal bonds on the stock of δ, the more the price levels must change to deflate total debt and account for innovations on the real value of debt. Expression (10) shows that, all else the same, lower δ leads to more volatile unexpected inflation.

2.4.2. Geometric Term Structure

I assume the government keeps a geometric maturity structure of its debt. The geometric term structure implies a tractable relationship between short-term interest rate $i_{j,t}$ and the excess returns that I explore to substitute the hard-to-interpret price adjustment terms of decomposition 1.

The term structure is constant over time, but can vary across the different currency portfolios $\{j\}$ of public debt. Specifically, for the slice of public debt linked to currency j, suppose the outstanding volume of bonds decays at a rate ω_j, so that $B_{j,t}^n = \omega_j B_{j,t}^{n-1}$. Define $Q_{j,t} = \sum_{n=1}^{\infty} Q_{j,t}^n \omega_j^{n-1}$ as the weighted-average price of currency j public bonds. Then, $\mathcal{V}_{j,t} = Q_{j,t}B_{j,t}^1$. The total return on currency-j bonds then is $1 + rx_{j,t} + i_{j,t-1} = (1+\omega_j Q_{j,t})/Q_{j,t-1}$, which I linearize as

$$rx_{j,t} + i_{j,t-1} = (\omega_j \beta) q_{j,t} - q_{j,t-1} \tag{11}$$

where $q_{j,t} = \log Q_{j,t}$ and I use the log approximation of percentage returns to re-define $rx_{j,t} + i_{j,t}$ again.

Equation (11) above defines the excess return on holdings of the j-currency portfolio of public debt. Given a model for the risk premium $E_t rx_{j,t+1}$, it also defines the price of the debt portfolio as a function of short-term interest:

$$
\begin{aligned}
q_{j,t} &= (\omega_j\beta)E_t q_{j,t+1} - E_t rx_{j,t+1} - i_{j,t} \\
&= -\sum_{k=0}^{\infty}(\omega_j\beta)^k E_t\left[rx_{j,t+1+k} + i_{j,t+k}\right].
\end{aligned}
\tag{12}
$$

The second equation in (12) shows the connection between short-term interest - hence monetary policy - and returns on debt holdings. News of higher interest lower public bond price q and leads to a low excess return. Now, lag equation (12) one period and take innovations to find

$$
\begin{aligned}
\Delta E_t rx_{j,t} &= -\sum_{k=1}^{\infty}(\omega_j\beta)^k\left[\Delta E_t rx_{j,t+k} + \Delta E_t i_{j,t+k-1}\right] \\
&= -\sum_{k=1}^{\infty}(\omega_j\beta)^k\left[\Delta E_t r_{j,t+k} + \Delta E_t \pi_{j,t+k} - \Delta E_t \Delta h_{j,t+k}\right].
\end{aligned}
\tag{13}
$$

Decomposition 2. *Replace (13) on decomposition 1 and gather terms to find*

$$
\begin{aligned}
\frac{\delta v}{\beta}\Delta E_t\pi_t = &-\frac{\delta v}{\beta}\sum_{k=1}(\omega\beta)^k\Delta E_t\pi_{t+k} - \sum_{k=0}^{\infty}\beta^k\Delta E_t s^p_{t+k} - \frac{v}{\beta}\sum_{k=1}^{\infty}\beta^k\Delta E_t g_{t+k} \\
&+ \frac{v}{\beta}\sum_{k=1}^{\infty}\sum_j \delta_j\beta^k(1-\omega_j^k)\Delta E_t r_{j,t+k} + \frac{\delta_D v}{\beta}\sum_{k=0}^{\infty}(\omega_D\beta)^k\Delta E_t\Delta h_{t+k} - \frac{\delta_D v}{\beta}\sum_{k=0}^{\infty}(\omega_D\beta)^k\Delta E_t\pi^{US}_{t+k}.
\end{aligned}
$$

Take covariance with unexpected inflation, and divide both sides by $\delta(v/\beta)$.

$$
\begin{aligned}
var(\Delta E_t\pi_t) = &-cov\left[\Delta E_t\pi_t, \sum_{k=1}^{\infty}(\omega\beta)^k\Delta E_t\pi_{t+k}\right] - cov\left[\Delta E_t\pi_t, \left(\frac{\delta v}{\beta}\right)^{-1}\sum_{k=0}^{\infty}\beta^k\Delta E_t s^p_{t+k}\right] \\
&- cov\left[\Delta E_t\pi_t, \delta^{-1}\sum_{k=1}^{\infty}\beta^k\Delta E_t g_{t+k}\right] + cov\left[\Delta E_t\pi_t, \delta^{-1}\sum_{k=1}^{\infty}\sum_j\delta_j\beta^k(1-\omega_j^k)\Delta E_t r_{j,t+k}\right] \\
&+ cov\left[\Delta E_t\pi_t, \frac{\delta_D}{\delta}\sum_{k=0}^{\infty}(\omega_D\beta)^k\Delta E_t\Delta h_{t+k}\right] - cov\left[\Delta E_t\pi_t, \frac{\delta_D}{\delta}\sum_{k=0}^{\infty}(\omega_D\beta)^k\Delta E_t\pi^{US}_{t+k}\right].
\end{aligned}
\tag{14}
$$

Decomposition 2 says that positive unexpected inflation must forecast either: *lower* future inflation (domestic or foreign), higher deficits, lower growth, higher real discounting or a more depreciated real exchange rate. In it, the ω terms give a clue of which terms derive from the time-t adjustment of bond prices. For example: an interest rate hike $\Delta E_t i_t$ can lead to a fall in nominal bond prices (negative $\Delta E_t rx_t$) and, by decomposition 1, a decline in current inflation. These lower bond prices in turn forecast higher inflation. This is why the decomposition prescribes future inflation as time-t *deflationary force*, like surpluses.[1] Another way to write decomposition 2 would be

$$
\begin{aligned}
&-\frac{v}{\beta}\left[\delta\sum_{k=0}(\omega\beta)^k\Delta E_t\pi_{t+k} + \delta_D\sum_{k=0}^{\infty}(\omega_D\beta)^k\Delta E_t\pi^{US}_{t+k}\right] = \\
&\sum_{k=0}^{\infty}\beta^k\Delta E_t s^p_{t+k} + \frac{v}{\beta}\left[\sum_{k=1}^{\infty}\beta^k\Delta E_t g_{t+k} - \sum_{k=1}^{\infty}\sum_j\delta_j\beta^k(1-\omega_j^k)\Delta E_t r_{j,t+k} - \delta_D\sum_{k=0}^{\infty}(\omega_D\beta)^k\Delta E_t\Delta h_{t+k}\right].
\end{aligned}
$$

Writing as above has the advantage of isolating nominal on the left and real on the right, like decomposition 1. The domestic inflation terms contains both unexpected current inflation $\Delta E_t\pi_t$ and unexpected "future inflation" $\Delta E_t\pi_{t+k}$, which means that the latter can absorb the impact of news about real variables on the former. Monetary policy can thus select the *timing* of inflation. Similar

[1] Of course, higher expected inflation means inflation is expected to grow after time t. Sims (2011) calls that mechanism "stepping on a rake" and argues it was part of the reason why 1970's US inflation kept coming back after temporary declines.

mechanisms apply to the exchange rate h_t and US inflation π_t^{US} terms that follow from dollar-linked debt. Lower dollar-bond prices can be disinflationary today but forecast higher (*i.e.*, more depreciated) real exchange in the future (along with higher US inflation).

Lastly, the $(1-\omega_j^k)$ term that multiplies $\Delta E_t r_{j,t+k}$ suggests that long-term bonds insulate inflation from real interest variation. We will that the evidence does not support that conclusion in all cases.

3. Empirical Model and Estimation

The main goal is to estimate the variance decompositions of unexpected inflation (10) and (14) for a set of twenty-five economies. To do this, I estimate a ten-equation VAR in which the debt law of motion (9) holds by construction. If the VAR is stationary, equation (3) will be satisfied, and the decompositions will hold.

Data is annual. Quarterly data is available, but it often does not go back as many years into the past. This is particularly true for emerging market variables and public debt measures from all countries. With a focus on long-term debt sustainability, going further into the past is more important than accounting for quarterly dynamics. Additionally, with annual data there is no danger of measurement errors due to seasonality adjustments.

I group countries in four sample categories. The first one contains only the United States. US data covers the period 1945-2019. The second group has six developed economies. You can check the list in table 2. Their samples start in 1960. The third group has four developed economies with the samples starting in 1973. The last group contains fourteen developing countries with samples starting in 1998. Grouping countries according to sample size helps to account for parameter volatility. Additionally, it provides control for international economic environment and historical events that affect inflation and its underlying causes.

I interpret parameters of the VAR as being random and estimate them using Bayesian methods. I establish a prior distribution, and then use data likelihood to compute the posterior.[1] One might ask why do Bayesian and not just OLS. First, parameter shrinkage reduces the volatility of estimated coefficients, an invaluable property when samples are relatively small. Second, provided that the prior distribution leads to a stable VAR, we can calibrate its tightness so as to ensure that the posterior centers around a stable VAR as well. OLS often returns unstable VARs in which the fiscal decomposition does not hold.

I base my prior on OLS-estimated US dynamics for two reasons. First, we already have results available in the literature (Cochrane (2022a), to the best of my knowledge the decompositions have not been estimated to other countries so far). Second, the US has the longest sample. Critically, it comprises the repayment of a major public borrowing event - World War II - that renders OLS estimates of the VAR stable and plausible.[2] I estimate the model for the US by OLS and use the resulting VAR to set the mean of the prior for other countries' estimation.

From the ten variables in the VAR, five are observed: the nominal interest (i_t), the inflation rate (π_t), par-value public debt (v_t^b), the first difference of the real exchange rate to the dollar (Δh_t) and GDP growth (g_t). I select these variables based on (9). Most time series data I collect from the St Louis Fed *FRED* website, the United Nations and the IMF. Details on appendix A.

I convert interest and inflation data to log. The change in dollar exchange rate is the nominal depreciation to the US dollar, plus US inflation minus domestic inflation. In the US case, I use exchange rate to the UK pound, which is available since the 1930s. GDP growth is the log difference of levels data. Public debt is provided as a ratio of GDP by the source (Ali Abbas et al. (2011)), and requires no transformation.

[1]See del Negro and Schorfheide (2011) or Karlsson (2013) for more on Bayesian estimation of VAR models.

[2]Including pre-1950 data in the sample proved necessary. Starting the sample after that leads to an unstable VAR estimate due to the large public debt equation root.

3.1. *Public Finances Model*

Governments typically report the par or book value of debt, and this is the data I gather. Because theory is based on the *market* value of debt, some adjustment is necessary. The Dallas Fed provides estimates of the market value of debt for the United States following Cox and Hirschhorn (1983) and Cox (1985).[1] I follow a similar methodology. Let $\mathcal{V}^b_{j,t}$ be the par value of the j-currency portfolio debt, and let $i^b_{j,t}$ be its average yield-to-maturity by which book values are updated from period to period. The adjustment equation is

$$\mathcal{V}_{j,t} = \mathcal{V}^b_{j,t} \times \frac{\text{market value of bonds}}{\text{book value of bonds}} = \frac{1+\omega_j Q_{j,t}}{(1+i^b_{j,t-1})Q_{j,t-1}} = \frac{1+rx_{j,t}+i_{j,t-1}}{1+i^b_{j,t-1}}.$$

I detrend the $\mathcal{V}$'s, convert to real, sum across portfolios and linearize to arrive at:

$$v_t = v^b_t + \frac{v}{\beta}\left[\sum_j \delta_j \left(rx_{j,t} + i_{j,t-1} - i^b_{j,t-1}\right)\right]. \tag{15}$$

The estimated VAR contains an equation for the par value of debt, which is the data I have. I use (15) to infer an equation for its market value. The average interest $i^b_{j,t}$ is not observed, so we cannot estimate an equation for it. Instead, I use a model for it too. With a geometric maturity structure of debt and in steady state, every period the government rolls over the volume of bonds coming to maturity (which had maturity $n = 1$ in the previous period). That accounts for a share $1-\omega_j$ of total outstanding bonds. In a steady state, the government then issues the same amount of new bonds ($1-\omega_j$ of total debt) at the prevailing interest rate i_t. The average interest therefore satisfies

$$i^b_{j,t} = (1-\omega_j)i_{j,t} + \omega_j i^b_{j,t-1} = (1-\omega_j)\sum_{k=0}^{\infty}\omega_j^k i_{j,t-k} \tag{16}$$

for $j \in \{N, R, D\}$.

3.2. *The Bayesian-VAR*

Except for exchange rate depreciation, I demean all series. This is to assume that the stationary steady-state around which I linearize is given by the sample mean of the series of each country (with the exception of real exchange rate growth, which I assume to be zero). A note on language: I will sometimes refer to $\pi_t < 0$ as deflation or $s_t < 0$ as deficit for brevity, although the true meaning of these expressions being inflation or surplus below average.

The functional format of the VAR is

$$x_t = a x_{t-1} + b u_{t-1} + k\, e_t. \tag{17}$$

Both x and u are vectors with ten entries each. Five of them are the observed variables enumerated above. Vector u_t groups the same set of variables as x, but for the United States. I often use the "u" notation to refer to the US case or US variables. There are five shocks hitting the domestic economy: $\varepsilon \sim N(0,\Sigma)$, independent over time. The shocks hitting the US economy are $\varepsilon_u \sim N(0,\Sigma_u)$. I group them in $e_t = [\varepsilon'_t\ \varepsilon'_{u,t}]'$. Because the public debt process has a dollar component, and hence depends on dollar interest and inflation, u enters the regression of all countries. Matrix $k_{10\times 10}$ serves to correctly reproduce the law of motion governing unobserved variables.

The model for US variables is:

$$u_t = a_u u_{t-1} + k_u\, \varepsilon_{u,t} \tag{18}$$

(I use the same symbol x to denote the VAR of all countries and differentiate only in the US case). In (18), k_u is a 10×5 matrix.

[1] Web link: https://www.dallasfed.org/research/econdata/govdebt.

Cochrane (2022a) imposes the law of motion of public debt in the VAR by using primary surplus "data" calculated from it. Primary deficit equals the change in public debt net of realized return. This procedure bypasses the problem of dealing with accounting conventions in primary surplus reporting that, in most cases, make its data inconsistent with that of public debt. Since the OLS estimator is linear, his VAR satisfies the debt law of motion by construction.

In my VAR, the procedure needs to be adjusted. First, I do not measure excess returns or real interest rates. Second, the debt equation depends on US interest and inflation. Hence, the unrestrictive estimation of (17) spuriously projects these two US variables on domestic ones, which is inconsistent with (18). For these reasons, instead of measuring implied surpluses and then estimating the model, I estimate the model and then fill the equation for primary surpluses that ensures the debt law of motion (9) holds. Before doing that, I also need to include the adjustment equation for market-value debt (15) as well as the three definitions of average interest rates (16) required to do it. These five unobserved variables (surplus s_t^p, the market value of debt v_t, and the average interest $\{i_{j,t}^b\}$) complete the ten variables that compose x.

Let $X_t = [x_t' \; u_t']'$ be the vector that stacks domestic and US variables. In the United States case, $X_t = u_t$. The estimation has four steps.

Step 1. Estimate the Bayesian-VAR

$$\begin{aligned} \tilde{x}_t &= \tilde{a}\tilde{x}_{t-1} + \tilde{b}\tilde{u}_{t-1} + \varepsilon_t \\ \tilde{u}_t &= \tilde{a}_u\tilde{u}_{t-1} + \varepsilon_{u,t} \end{aligned} \tag{19}$$

along with Σ and Σ_u, where $\tilde{x}$ is a vector with the five observed variables, and $\tilde{u}$ is defined similarly. Matrices $\tilde{a}$, $\tilde{b}$ and $\tilde{a}_u$ are the submatrices of a, b and a_u corresponding to the rows and columns of these observed variables.

Step 2. Compute equations for excess returns and real interest. I assume a constant risk-premium: $E_t rx_{j,t+1} = 0$ for all j. I use the estimated VAR (19) to compute $E_t i_{j,t+i}$ and apply (12) to infer $q_{j,t}$. Equation (11) then yields expressions for excess returns of the form

$$rx_{j,t} = \varphi_j' e_t.$$

For real interest, I use the Fisher equation

$$i_{R,t} = i_t - E_t\pi_{t+1} = \zeta' X_t.$$

under the implied assumption of equal expected return on nominal and real debt, after accounting for risk premia. In appendix B, I present formulas for the φ's and ζ.

Step 3. Fill the equations of unobserved variables. Using the estimated model of step 1, I compute the equations for average interest using (16), and fill the corresponding rows of a, b and k (a_u and k_u in the US case). With the equations for average interest filled, I can do the same for the market-price debt using the par-value adjustment equation (15) which requires the return equations from step 2. With the equation for the market-price debt, I use the law of motion (9) and the expressions for excess return and real interest above to fill the equation for the primary surplus. The law of motion of public debt (9) therefore holds in the VAR.

This completes the estimation of a, b and k in the general case, and a_u and k_u in the US case. For each country, we can stack the equations into a single system for X:

$$X_t = AX_{t-1} + Ke_t. \tag{20}$$

If we order unobserved variables x^o at the top of the x, we can write (20) more explicitly:

$$\begin{pmatrix} x_t \\ u_t \end{pmatrix} = \begin{bmatrix} a & b \\ 0 & a_u \end{bmatrix} \begin{pmatrix} x_{t-1} \\ u_{t-1} \end{pmatrix} + \begin{bmatrix} & k \\ 0 & k_u \end{bmatrix} \begin{pmatrix} \varepsilon_t \\ \varepsilon_{u,t} \end{pmatrix}$$

$$\text{or yet} \quad \begin{pmatrix} x_t^o \\ \tilde{x}_t \\ u_t^o \\ \tilde{u}_t \end{pmatrix} = \begin{bmatrix} & & * & \\ 0 & \tilde{a} & 0 & \tilde{b} \\ 0 & 0 & & * \\ 0 & 0 & 0 & \tilde{a}_u \end{bmatrix} \begin{pmatrix} x_{t-1}^o \\ \tilde{x}_{t-1} \\ u_{t-1}^o \\ \tilde{u}_{t-1} \end{pmatrix} + \begin{bmatrix} & * \\ I & 0 \\ 0 & * \\ 0 & I \end{bmatrix} \begin{pmatrix} \varepsilon_t \\ \varepsilon_{u,t} \end{pmatrix}.$$

Symbol * indicates the coefficients are filled to ensure that (9), (15) and (16) hold. In appendix B I provide their formulas.

Step 4. Compute sample residuals $\hat{e}$ ($\hat{e}_u$ for the US) from (19), and estimate $\text{cov}(\varepsilon, \varepsilon_u) = \Sigma_{xu} = \sum_i \hat{e}_i \hat{e}'_{u,i}/(N-1)$, where N is the sample size. Then:

$$\Omega = \text{cov}(e) = \begin{bmatrix} \Sigma & \Sigma_{xu} \\ \Sigma_{ux} & \Sigma_u \end{bmatrix}.$$

3.3. The Prior Distribution

Step 1 requires a prior distribution to compute the posterior for parameters $\tilde{a}$, $\tilde{b}$, $\tilde{a}_u$, Σ and Σ_u. As the commonly used Litterman (1979) prior, I use a distribution of the Normal-Inverse-Wishart class, with general format

$$\Sigma \sim IW(\Phi; d)$$
$$\theta|\Sigma \sim N(\bar{\theta}, \Sigma \otimes \bar{\Omega}).$$

where $\theta \equiv [\text{vec}(\tilde{a}')' \ \text{vec}(\tilde{b}')']'$ and vec means stacking the columns. Given the Gaussian likelihood of the model, the posterior distribution is also of the Normal-Inverse-Wishart class. Giannone et al. (2015) provide formulas for the posterior distribution and marginal likelihood.

In the US case, the prior centers around zero, $\bar{\theta} = 0$, but since it has a very large variance, the posterior centers around the OLS estimate of $\tilde{a}_u$. The estimated VAR for the US is stationary. In the case of other countries, I center the prior around $\tilde{a} = \tilde{a}_u$, $\tilde{b} = 0$. The economic content of the prior is that the dynamics of the observed variables is the same as that we estimate for the United States. The surpluses process differs from that of the US only to account for the differences in public debt size and term and currency structures. With $\tilde{b} = 0$, the prior also translates the view that US variables do not affect the dynamics of domestic ones.

The mean of the IW distribution is $\Phi/(d - n - 1)$, where $n = 5$ is the dimension of ε and larger values of d represent tighter priors. I choose Φ to be the identity matrix (one percent standard deviation for all shocks, which are uncorrelated) and select $d = n + 2$, the lowest integer possible that leads to a well-defined distribution mean - which therefore equals Φ.

The conditional covariance between the coefficients in $\tilde{a}$ is

$$\text{cov}\left(\tilde{a}_{ij}, \tilde{a}_{kl} \mid \Sigma\right) = \begin{cases} \alpha^2 \frac{\Sigma_{ij}}{\Phi_{jj}} & \text{if } j = l \\ 0 & \text{otherwise.} \end{cases}$$

The format allows the loadings on the variable in different equations to be correlated. Loadings on the different variables on the same equation are independent. Hyperparameter α governs the overall tightness of the prior over linear coefficients, with lower values yielding tighter priors.

The conditional covariance of $\tilde{b}$ is

$$\operatorname{cov}\left(\tilde{b}_{ij}, \tilde{b}_{kl} \mid \Sigma\right) = \begin{cases} (\xi\alpha)^2 \frac{\Sigma_{ij}}{\Phi_{u,jj}} & \text{if } j = l \\ 0 & \text{otherwise} \end{cases}$$

where $\Phi_u = \Phi = I$ is the mean of the IW distribution in the US case. Hyperparameter ξ governs the tightness of the prior that US variables do not affect domestic ones more than the mechanical effect of US interest and inflation on domestic public debt. If $\xi = 1$, the prior is just as tight as that of $\tilde{a}$.

Finally, the covariance between $\tilde{a}$ and $\tilde{b}$ is zero. It is straightforward to build $\bar{\Omega}$ so that the conditional covariance structures above hold.

4. Empirical Results

4.1. Variance Decomposition

In the baseline specification, I calibrate $\beta = 0.98$ for all countries, and set b tightness parameter $\xi = 1/3$. I calibrate parameters δ and ω based on debt structure data gathered from various sources (see appendix A). They are reported in Table 2 along with average debt-to-output. The mean debt-to-GDP ratio in the sample is 48%, with developed countries slightly more indebted on average. Nominal debt tends to account for the bulk of sovereign debt, Chile being a notable exception. On average, emerging markets' governments tend to rely relatively more on real and especially dollar-denominated debt, and issue securities with higher maturity.

To ensure stability of the VAR, I start by finding the hyperparameter λ that maximizes the marginal likelihood.[1] Then, if the mode of the posterior leads to an unstable VAR, I progressively reduce λ in 0.001 steps until it leads to a stable VAR. Given the continuity of the posterior distribution on λ and the fact that $\lambda = 0$ leads to the stable US system, there must exist a non-zero value of λ that leads to a stationary model. You can check the resulting λ's in table 2.

Table 2 also reports the standard deviation of unexpected inflation, calculated at the mode of the posterior distribution. Unsurprisingly, unexpected inflation exists, with standard deviations ranging from 1% and 2% in the country group averages. The low figure for emerging markets reminds us of the importance of the sample time period. Emerging markets do not have less volatile unexpected inflation; we just sampled from a period in which inflation is known to be less volatile in most countries (see Stock and Watson (2002) and Coibion and Gorodnichenko (2011)).

Table 3 contains the estimated terms of variance decomposition 1, equation (10). I isolate unexpected inflation and establish the following notation:

$$\begin{aligned} \Delta E_t \pi_t &= \sum_{j \neq N} \frac{\delta_j}{\delta} \Delta E_t r_{j,t} + \Delta E_t r x_t - \frac{\beta}{\delta v} \sum_{k=0}^{\infty} \beta^k \Delta E_t s_{t+k}^p - \frac{1}{\delta} \sum_{k=0}^{\infty} \beta^k \Delta E_t g_{t+k} + \sum_{k=1}^{\infty} \sum_j \frac{\delta_j}{\delta} \beta^k \Delta E_t r_{j,t+k} \\ &\equiv d_1(r_0) + d_1(rx) - d_1(s) - d_1(g) + d_1(r). \end{aligned}$$

As discussed previously, the variance decomposition is algebraically equivalent to the innovations decomposition above evaluated at $\Delta E_t \pi_t = 1$. The d_1 terms then indicate the share of inflation variance explained by the corresponding term. They do not have to be in the $[0, 1]$ interval, but they must sum to one. Table 4 contains the estimated term of variance decomposition 2, equation (14). I again isolate unexpected inflation and define the d_2 terms accordingly:

$$\Delta E_t \pi_t = -d_2(\pi) - d_2(\pi^{US}) - d_2(s) - d_2(g) + d_2(r) + d_2(\Delta h).$$

The tables report cross-country averages and medians, conditional on country group. Highlighted

[1] Giannone et al. (2015) show that, in the case of Normal-Inverse-Wishart priors, the marginal likelihood can be decomposed in a goodness-of-fit term and a model-complexity term that penalizes conditional forecast variance. By maximizing the marginal likelihood, we ensure we cannot improve one of these terms without reducing the other. Similar methods for selecting the degree of informativeness of the prior distribution have been used. See, for example, Koop (2013) and Carriero et al. (2015).

Country	v (%)	δ_N (%)	δ_R (%)	δ_D (%)	Avg. Term (Years)	λ	$\sigma(\Delta E_t \pi)$ (%)
Averages	48	74	11	15	6.5		1.6
Advanced - 1960	58	87	8	6	6.4		1.6
Advanced - 1973	32	92	4	4	5.6		1.8
Emerging - 1998	47	63	14	23	6.9		1.6
Median	43	79	5	10	5.6		1.3
Advanced - 1960	53	88	3	2	5.6		1.5
Advanced - 1973	32	94	3	2	5.6		1.7
Emerging - 1998	43	67	6	23	7.6		1.2
United States	60	93	7	0	5	10	1.9
Advanced - 1960 Sample							
Canada	71	92	5	3	6.5	0.21	1.0
Denmark	37	84	0	16	5.6	0.18	1.2
Japan	98	100	0	0	5.5	0.01	2.2
Norway	35	99	0	1	3.7	0.19	1.5
Sweden	46	69	16	14	4.8	0.16	1.5
United Kingdom	61	76	24	0	12.3	0.17	2.0
Advanced - 1973 Sample							
Australia	24	90	10	0	7.2	0.18	1.5
New Zealand	41	82	6	13	4.3	0.15	2.0
South Korea	21	97	0	3	4	0.15	2.8
Switzerland	43	100	0	0	6.9	0.23	1.0
Emerging - 1998 Sample							
Brazil	70	70	25	5	2.6	0.12	1.4
Chile	14	10	57	33	12.8	0.27	1.0
Colombia	41	45	23	32	5.6	0.13	0.8
Czech Republic	31	91	0	9	5.6	0.15	1.1
Hungary	68	76	0	23	4.1	0.14	1.3
India	73	90	3	7	10.1	0.25	1.1
Indonesia	43	44	0	56	9.2	0.21	1.2
Israel	77	43	34	23	6.6	0.13	1.3
Mexico	45	65	10	26	5.5	0.15	1.0
Poland	47	79	1	20	4.2	0.10	1.3
Romania	28	50	0	50	4.8	0.10	1.9
South Africa	41	70	20	10	12.9	0.25	1.0
Turkey	43	47	23	30	3.6	0.13	2.1
Ukraine	43	100	0	0	9.1	0.07	5.7

Notes: v is the average public debt-to-GDP, δ_N is the share of nominal debt, δ_R is the share of real or inflation-linked debt, δ_D is the share of dollar-denominated debt (I also include debt denominated in other currencies, but the amounts are negligible in almost all cases). "Avg. Term" is the average maturity across currency portfolios, calculated as $\sum_j \delta_j (1-\omega_j)^{-1}$. λ is the prior tightness parameter, and $\sigma(\Delta E_t \pi)$ is the standard deviation of inflation innovations, evaluated at the mode of the posterior distribution.

Table 2: Debt Structure Parameters and Prior Tightness

figures indicate the statistical significance of the estimate's sign, based on ten-thousand simulations of the posterior distribution.[1] Bold font alone indicates that 75% of the simulated draws have the same sign as the posterior mode. I refer to that as a statistically significant estimate. Bold font with an asterisk indicates 90%. It is visible that, as sample size shrinks, parameters are estimated less precisely. The evidence is not conclusive to all countries. But a few patterns emerge.

- *Unexpected inflation variance is accounted for by variation in the intrinsic value of public debt. No single one of its components - surplus-to-GDP, GDP growth and real discounting - accounts for unexpected inflation variance alone. Nevertheless, each has a positive contribution in at least eighteen of the twenty-five countries.*

The right-hand side of decomposition 1 is the innovation to the intrinsic value of debt. That corresponds to the negative of the sum $-d_1(s) - d_1(g) + d_1(r)$ (times a positive constant $\delta v/\beta$), reported in table 3. In all countries in the sample, the sum is positive. In all countries but one (Colombia), the sum is greater than one. Therefore, with the pardon of a single exception, we can say that variation in the intrinsic value of debt fully accounts for unexpected inflation variance.

No single one of the three terms - surplus-to-GDP $d_1(s)$, growth $d_1(g)$ and real discounting $d_1(r)$ - is clearly more important quantitatively. In seven cases, all three terms are positive, or the surplus and growth terms are positive. In five cases, real discounting and surplus terms, or real discounting and growth terms are positive. Only in the case of Turkey we see two negative terms (surplus and growth). Counting positive contributions, real discounting accounts for a positive share of unexpected inflation variance in eighteen cases. Surplus-to-GDP and GDP growth terms are each positive in twenty cases. Focusing on statistically significant coefficients (at 75% confidence) does not change the overall message. The surplus term is positive and significant in ten cases, the GDP growth term in twelve cases and real discounting in eight cases.

- *In all country groups, concomitant nominal bond price shocks reduce the variance of unexpected inflation: $d_1(rx) < 0$. Monetary policy trades current for future inflation, $-d_2(\pi) < 0$.*

Unexpected inflation tends to come accompanied by a decline in nominal bond prices resulting from monetary policy. Central banks react to inflation spikes by raising interest (Goncalves and Guimaraes (2021)). As bond prices decline, the price level does not have to increase so much to equal the market value of debt to its intrinsic value. The covariance between $\Delta E_t rx_t$ and $\Delta E_t \pi_t$ is negative and statistically significant in all countries. They are also economically significant. I estimate $p_1(rx) = -0.78$ for the US. Cochrane (2022a) estimates -0.56. The effect is larger in several other cases. They suggest monetary policy is effective in preventing unexpected inflation.

However, lower bond prices forecast higher inflation. Decomposition 2 isolates the effect of future inflation. For the US, $d_2(\pi) = 1.12$ (Cochrane estimates 0.59). Inflation forecasts inflation. One can attribute that to price stickiness in the short run and, in the long run, Fisherian effects of monetary policy (Garín et al. (2018), Uribe (2022)). Estimates are similar for other advanced economies. Magnitudes are close -1 and somewhat larger for the United Kingdom. In the case of emerging markets, I also find negative figures, but with smaller magnitudes, as the median shows.

- *In advanced economies, unexpected inflation forecasts deficits, which contributes to its volatility ($-p_1(s) = -p_2(s) > 0$).*

With the exception of Sweden, inflation forecasts deficits (or nowcasts, since the surplus sum in the decompositions includes time t), in advanced economies. In a FTPL reading, unexpected inflation is caused by news of lower primary surpluses. Magnitudes are economically and, in most cases, statistically significant. I estimate $-d_2(s) = 0.57$ for the US (Cochrane finds -0.06). For other developed countries, it ranges from 0.50 to 3. The large estimates for Australia (2.1) and South Korea (1.9) can be partially attributed to their reduced indebtedness (δv enters the denominator of $p_2(s)$). The United

[1] I discard draws that lead to unstable VARs.

Country	$\Delta E_t \pi_t =$	Decomposition 1 - Variance decomposition (10)				
		ΔE_t(Bond Prices)		$-\Delta E_t$(Intrinsic Value of Debt)		
		$d_1(r_0)$	$d_1(rx)$	-$d_1(s)$	-$d_1(g)$	$d_1(r)$
Averages	1	-0.24	***-0.74**	1.02	**0.36**	0.60
Advanced - 1960	1	-0.01	***-0.74**	**0.97**	***0.54**	0.24
Advanced - 1973	1	-0.01	***-0.69**	***1.33**	***0.65**	-0.28
Emerging - 1998	1	-0.42	***-0.76**	0.99	0.22	0.97
Median	1	-0.02	***-0.76**	***0.87**	***0.42**	***0.68**
Advanced - 1960	1	***-0.06**	***-0.62**	***0.61**	***0.72**	**0.58**
Advanced - 1973	1	**-0.00**	***-0.72**	***1.40**	***0.76**	-0.37
Emerging - 1998	1	-0.11	***-0.77**	1.13	***0.33**	***0.79**
United States	1	***0.03**	***-0.78**	**0.57**	0.23	**0.96**
Advanced - 1960 Sample						
Canada	1	***-0.11**	***-1.59**	0.62	***1.22**	0.86
Denmark	1	***-0.29**	**-0.30**	0.42	-0.04	**1.21**
Japan	1	0	***-0.52**	***1.60**	-0.38	0.30
Norway	1	***-0.01**	***-0.36**	**0.60**	**0.47**	0.30
Sweden	1	**-0.15**	***-0.93**	-0.34	***0.98**	***1.42**
United Kingdom	1	***0.52**	***-0.73**	***2.89**	***0.97**	***-2.65**
Advanced - 1973 Sample						
Australia	1	***0.07**	***-0.76**	***2.09**	**0.66**	**-1.06**
New Zealand	1	**-0.10**	***-0.86**	0.40	***0.87**	0.68
South Korea	1	-0.01	***-0.45**	***1.91**	0.17	**-0.62**
Switzerland	1	0	***-0.69**	**0.90**	***0.91**	-0.12
Emerging - 1998 Sample						
Brazil	1	**-0.26**	***-0.22**	**-1.46**	**1.05**	**1.89**
Chile	1	-3.80	**-1.33**	8.95	-5.71	2.88
Colombia	1	**1.51**	***-0.96**	1.39	-1.09	0.15
Czech Republic	1	***-0.16**	***-0.37**	-2.31	**2.42**	**1.42**
Hungary	1	***-0.57**	***-0.93**	-0.98	1.60	1.88
India	1	***0.17**	***-0.46**	**1.54**	0.05	-0.30
Indonesia	1	***-2.59**	***-1.07**	1.69	***2.61**	0.35
Israel	1	-0.06	***-0.78**	-0.55	***1.51**	0.88
Mexico	1	-0.02	***-0.74**	**1.41**	0.03	0.32
Poland	1	***-0.45**	***-1.15**	0.87	-0.39	***2.11**
Romania	1	-0.40	***-0.96**	2.24	0.42	-0.31
South Africa	1	**0.36**	***-0.51**	1.58	0.25	-0.68
Turkey	1	**0.37**	***-0.37**	-1.18	-0.15	***2.33**
Ukraine	1	0	***-0.77**	**0.65**	**0.41**	***0.70**

Notes: The table reports the terms of variance decomposition (10) at the posterior distribution's mode. I divide each term by var($\Delta E_t \pi$). I simulate from the posterior to check the statistical significance of the sign of the estimate. Bold indicates 75% of results had the same sign. Bold with an asterisk indicates 90%.

Table 3: Unexpected Inflation - Variance Decomposition 1

Country	$\Delta E_t \pi_t =$	Decomposition 2 - Variance decomposition (14)					
		$-\Delta E_t$(Future Inflation)		$\pm\, \Delta E_t$(Real Variables)			
		$-d_2(\pi)$	-$d_2(\pi^{US})$	$-d_2(s)$	$-d_2(g)$	$d_2(r)$	$d_2(\Delta h)$
Averages	1	***-0.81**	***-0.22**	1.02	**0.36**	**0.63**	0.02
Advanced - 1960	1	***-1.20**	***-0.06**	**0.97**	***0.54**	**0.76**	0
Advanced - 1973	1	***-1.01**	***-0.03**	***1.33**	***0.65**	0.11	**-0.05**
Emerging - 1998	1	***-0.56**	***-0.37**	0.99	0.22	0.67	0.06
Median	1	***-0.76**	***-0.03**	***0.87**	***0.42**	***0.61**	0
Advanced - 1960	1	***-1.08**	***-0.04**	***0.61**	***0.72**	***0.85**	0
Advanced - 1973	1	***-0.91**	***-0.01**	***1.40**	***0.76**	-0.14	0
Emerging - 1998	1	***-0.61**	**-0.04**	1.13	***0.33**	**0.47**	-0.03
United States	1	***-1.12**	-	**0.57**	0.23	***1.32**	0
Advanced - 1960 Sample							
Canada	1	***-1.53**	***-0.07**	0.62	***1.22**	**0.78**	**-0.03**
Denmark	1	***-0.49**	***-0.20**	0.42	-0.04	**1.23**	0.08
Japan	1	***-1.14**	0	***1.60**	-0.38	***0.91**	0
Norway	1	***-0.70**	0	**0.60**	**0.47**	**0.64**	0
Sweden	1	***-1.02**	**-0.10**	-0.34	***0.98**	***1.54**	-0.07
United Kingdom	1	***-2.34**	0	***2.89**	***0.97**	**-0.52**	0
Advanced - 1973 Sample							
Australia	1	***-1.47**	0	***2.09**	***0.66**	-0.27	0
New Zealand	1	***-1.02**	***-0.08**	0.40	***0.87**	**1.04**	**-0.21**
South Korea	1	***-0.74**	***-0.03**	***1.91**	0.17	-0.33	**0.01**
Switzerland	1	***-0.79**	0	**0.90**	***0.91**	-0.02	0
Emerging - 1998 Sample							
Brazil	1	***-0.11**	0	**-1.46**	**1.05**	**1.46**	0.07
Chile	1	-0.76	**-2.75**	8.95	-5.71	-0.35	1.62
Colombia	1	***-0.61**	-0.04	1.39	-1.09	0.02	**1.34**
Czech Republic	1	-0.02	**-0.05**	-2.31	**2.42**	0.98	-0.03
Hungary	1	***-0.69**	***-0.15**	-0.98	1.60	1.83	***-0.61**
India	1	***-1.05**	***0.09**	**1.54**	0.05	0.41	-0.04
Indonesia	1	***-0.79**	***-1.33**	1.69	***2.61**	0.26	**-1.45**
Israel	1	***-0.54**	0.10	-0.55	***1.51**	0.61	-0.12
Mexico	1	***-0.60**	**0.17**	**1.41**	0.03	0.52	**-0.52**
Poland	1	***-0.59**	***-0.21**	0.87	-0.39	***1.43**	-0.11
Romania	1	***-1.14**	***-0.53**	2.24	0.42	-0.54	0.55
South Africa	1	0.05	-0.01	1.58	0.25	-0.79	-0.07
Turkey	1	***-0.76**	***-0.40**	-1.18	-0.15	***3.35**	0.14
Ukraine	1	**-0.29**	0	**0.65**	***0.41**	**0.23**	0

Notes: The table reports the estimated terms of variance decomposition (14) at the posterior distribution's mode. I divide each term by var($\Delta E_t \pi$). I simulate from the posterior to check the statistical significance of the sign of the estimate. Bold indicates 75% of results had the same sign. Bold with an asterisk indicates 90%.

Table 4: Unexpected Inflation - Variance Decomposition 2

Kingdom (1.89) has large debt but a low share of nominal debt. Sweden is a case of some inflation insulation from fiscal deficits ($-d_2(s)$=-0.34) despite the small size of nominal debt.

In the case of developing countries, estimated values are mostly positive too, but often not statistically different from zero. I estimate positive d_2 for Brazil certainly due to the large surpluses in the 2000s, when inflation was highest in the time series. Still, I do not draw definitive conclusions in the case of 1998 emerging market sample.

- *In advanced economies, unexpected inflation forecasts a growth decline, which contributes to its volatility ($-d_1(g) = -d_2(g) > 0$). In emerging markets, statistically significant figures indicate the same, but tend to be the exception.*

All countries with statistically non-zero estimates have a positive $-d_1(g) = -d_2(g)$. In these cases, inflation forecasts a GDP growth slowdown, which reduces the intrinsic value of debt and causes, in a FTPL interpretation, the unexpected inflation in the first place. The conclusion has more empirical support for developed countries. From them, only for Denmark and Japan I estimate negative and statistically insignificant terms $-d_2(g)$. Estimates are economically relevant. When positive, covariance with unexpected inflation accounts from 47% to 122% of its total variance. That is one of the differences I find between developed countries in general and the United States, to which I estimate $-d_1(g) = 0.23$, not statistically different from zero. Cochrane finds 0.49.

The pattern is similar in emerging markets: statistically significant figures are positive and economically significant too. But they are the exception. I estimate large growth contributions to unexpected inflation for Brazil, Czech Republic, Indonesia and Israel. Part of the explanation for the large magnitudes is the low share of nominal debt on the portfolio of developing countries' governments.

- *For countries with dollar debt, US inflation variation contributes to reduce the volatility of unexpected inflation ($-d_2(\pi^{US}) < 0$). The effect is stronger in emerging markets, since their governments tend to issue more dollar-linked bonds.*

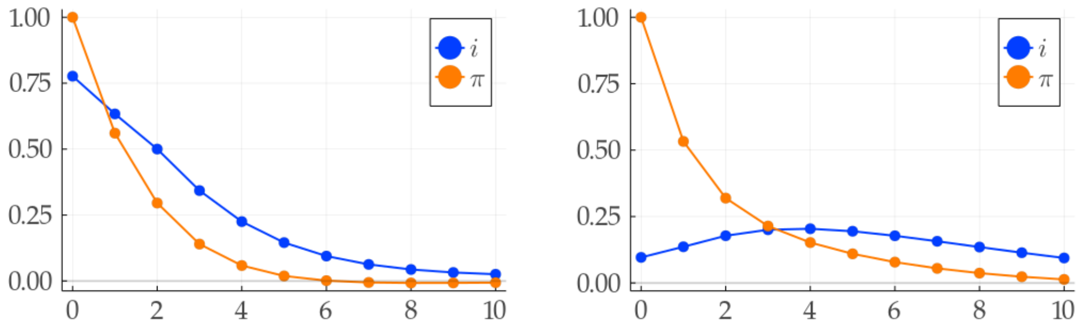
Decomposition 2 isolates the contributions of the real exchange rate and US inflation. They are only non-zero for countries with dollar debt. In all but three countries with dollar debt (India, Israel and Mexico), US inflation works as a deflationary force. Unexpected domestic inflation forecasts/nowcasts US inflation, which devalues dollar-linked bonds. Magnitudes are larger in the 1998 emerging markets sample, typically in the interval -0.10 to -0.60 (-2.75 for Chile derives from its large reliance on dollar debt, $\delta_D = 0.33$). This likely due to negative inflation shocks in 2008/2009, when inflation declined in the US due to the Great Recession. Indeed, in Mexico, for instance, inflation does not fall significantly in 2008/2009. Among advanced economies, conclusions are similar in both samples, but covariances are considerably smaller.

As for the real exchange rate, for only four of the thirteen developing countries with dollar debt I estimate statistically significant contributions to unexpected inflation variance. Signs are mixed, so there are no obvious conclusions to be made.

- *Long-term bonds often increase the contribution of discounting on unexpected inflation. After accounting for bond price variation, all statistically significant figures indicate such positive contribution: $d_2(r) > 0$. This result is particularly stronger for the United States, developed economies in the 1960 sample and emerging markets.*

Cochrane concludes that discount rate variation accounts for a large share of inflation variance in the United States ($d_2(r) = 1$). I find a similar conclusion for the United States ($d_2(r) = 1.32$) and advanced economies in the 1960 sample, with the exception of the United Kingdom ($d_2(r) = 0.76$ on average).

Both decompositions 1 and 2 have future real interest terms. Higher discounting lowers the intrinsic value of public debt, the right-hand side of decomposition 1. If $d_1 > 0$, it increases unexpected inflation variability; if $d_1 < 0$ it reduces. In either case, if the government finances itself with one-period bonds $\omega_j = 0$, this is the only effect to be considered, and $d_2(r) = d_1(r)$. With long-term bonds, (13) shows



(a) Poland: Lower Contribution $d_1 < d_2$

(b) Norway: Larger Contribution $d_2 > d_1$

Notes: IRFs implied by the estimated VAR model (17). The innovation vector e_t is projected on the condition $\Delta E_t \pi_t = 1$, which generates the fiscal decomposition of unexpected inflation variance.

Figure 1: Long-Term Bonds and the Contribution of Discounting to Unexpected Inflation

that higher discounting also leads to lower bond prices, which soak up part of such inflationary effect.[1] If the government borrows using perpetuities ($w_j = 1$), these two forces cancel out and real interest dynamics contributes neither for nor against unexpected inflation: $d_2 = 0$.

The intermediary case $0 < \omega_j < 1$ does *not* lead to the intermediary conclusion. Long-term bonds can *enhance* the impact, positive or negative, of real interest to inflation variance. With $0 < w_j < 1$, long-term bonds reflect (and cancel out) only short-term changes in real interest. If these short-term changes have an opposite signal to the overall covariance, then long-term bonds will insulate inflation from the "wrong" period of varying interest. Algebraically, if $\sum_k (1 - \omega^k) \Delta E_t r_k$ and $\sum_k \Delta E_t r_k$ have the same sign, then

$$\left| \sum_k \Delta E_t r_{t+k} \right| > \left| \sum_k (1 - \omega^k) \Delta E_t r_{t+k} \right|$$

if and only if $\sum_k \Delta E_t r_k$ and $\sum_k \omega^k \Delta E_t r_k$ also have the same sign.

In figures 1a and 1b, I plot the IRFs of nominal interest and inflation in the cases of Poland and Norway. The underlying shock is projected on $\Delta E_t \pi_t = 1$, as this condition leads to the variance decomposition. In Poland, unexpected inflation leads to a prolonged period of high real interest, a result of high rates in response to inflation in the 2000s. That is an inflationary force. As the inflation shock hits, the prices of nominal and real long-term bonds decline - a deflationary force that partially counteracts the first effect. Hence, $d_1(r) < d_2(r)$. In Norway, unexpected inflation leads to a smaller but more protracted increase of nominal interest. This is likely a consequence of the delayed response of monetary policy to 1970s inflation. Real interest is at first negative, then turns positive. Overall, the intrinsic value of debt declines, $d_1(r) > 0$. But the price of bonds *rises* - which is another inflationary effect - as it responds more to short-term real interest.[2] Hence, $d_2(r) > d_1(r)$.

Among advanced economies, in the six cases that long-term bonds enhance the contribution of real interest $\|d_2\| > \|d_1\|$, they contribute to make real interest dynamics *more inflationary*. Cochrane finds a similar result for the US, but in a much weaker magnitude: $d_1(r) = 1.0$, $d_2(r) = 1.04$. I find $d_1(r) = 0.96$, $d_2(r) = 1.32$.

In all countries of the 1960 sample, with the exception of the UK, unexpected inflation forecasts real interest, which accounts for 0.5 to 1.5 times unexpected inflation variance. Like for the US, *discounting matters*. In the 1973 sample, I estimate real interest to reduce inflation variance, probably due to delayed responses of monetary policy in the 1970s (which affect results more than in the 1960 sample). New Zealand, the single country in the group with a positive $d_2(r)$ raised interest in the 70s as inflation was

[1] In the case of nominal and dollar debt, this is *given* the long-term inflation and real exchange rate effects, which I am holding constant here.

[2] The more precise statement is: the effect of real interest is a rise in bond prices. Here, I am holding constant the effect of inflation, which is another term in the decomposition, $d_2(\pi)$. Nominal and dollar bond prices do not necessarily rise. They do not in figure 1b.

high but stable, and increased interest swiftly in 1985 during a new inflation spike. As for emerging markets, I again estimate economically significant, but statistically insignificant coefficients. In all but three cases (two being the volatile Chile and South Africa cases) they are positive. The four statistically significant coefficients are large and positive. In all, discounting and growth dynamics look more important drivers of unexpected inflation in emerging markets then surplus-to-GDP ratios.

4.2. Response to Reduced-Form Shocks

The variance decomposition is analogous to the innovations decomposition applied to a vector of shocks projected onto the condition $\Delta E_t \pi_t = 1$. I now consider the case of two different conditions, designed to simulate interesting scenarios: an "aggregate demand" recession scenario (similar to that studied by Cochrane (2022b)) and a real depreciation scenario.[1] Because I do not orthogonalize or attempt to build structural shocks, the decompositions do not tell a causal story and do not provide a structural interpretation. But, like the variance decompositions, they provide clues about inflation dynamics to support model building.

4.2.1. The "Aggregate Demand" Recession Scenario

I consider an "aggregate demand" shock, captured by a negative surprise in GDP growth and in the inflation rate: $\Delta E_t g_t = -1$ and $\Delta E_t \pi_t = -0.5$. The scenario I take from Cochrane (2022a). I choose a lower inflation shock based on the change in US inflation in the 2008 recession. The model is linear, so you can interpret values in percentage points too. I will henceforth refer to the terms of the decompositions in this recession scenario as recession decomposition. To save space, I report only decomposition 2 in table 5.[2] Notation is the same as before, with the d_2 terms calculated using the new innovation vector. Given the aggregate demand shock, unexpected inflation equals -0.5 in period zero by construction. But where does it come from?

- *Lower inflation in the recession scenario follows from lower real interest and larger surplus-to-output ratios. In many countries, like the US, the government reacts by raising deficits, but higher subsequent surpluses revert its inflationary effect. Lower growth, stimulative monetary policy and, for emerging markets, real exchange are inflationary.*

In all developed economies, the discounting effect contributes to the low inflation, although the result is stronger in the 1960 sample. Reported figures already incorporate the effect of higher long-term bond prices, but inspection of decomposition 1 (in the appendix) reveals that they are not driving the sign of decomposition 2 terms. Lower real interest raises the intrinsic value of debt as it raises the value of other assets. The effect on the price level is negative. Magnitudes are quantitatively significant, often larger than -1.5 (for a -0.5 inflation shock). Cochrane (2022a) finds this conclusion for the United States. It holds for other countries too.

In the case of emerging markets, like before, the conclusion is similar, but estimates are more often statistically insignificant or too volatile (Chile and South Africa are notorious). But most coefficients, significant or not, are negative. Monetary policy in South Africa and Ukraine - the two positive and significant coefficients - did not react strongly to recent recession shocks (2009 for both and, in the Ukrainian case, 2014/15 following the conflict involving Crimea, see figure 2b).

To all countries except Ukraine, the table suggests a strong response of monetary policy through lower interest. Higher bond prices forecast lower future inflation, which corresponds to a period-0 *inflationary* force in the decomposition, $-d_2(\pi) > 0$. Lower growth is also inflationary, as it raises the size of public debt relative to the GDP trend. Results indicate that fiscal policy, on the other hand, is *deflationary*, in that

$$\Delta E_t s_t + \sum_{k=1}^{\infty} \beta^k \Delta E_t s_{t+k} < 0.$$

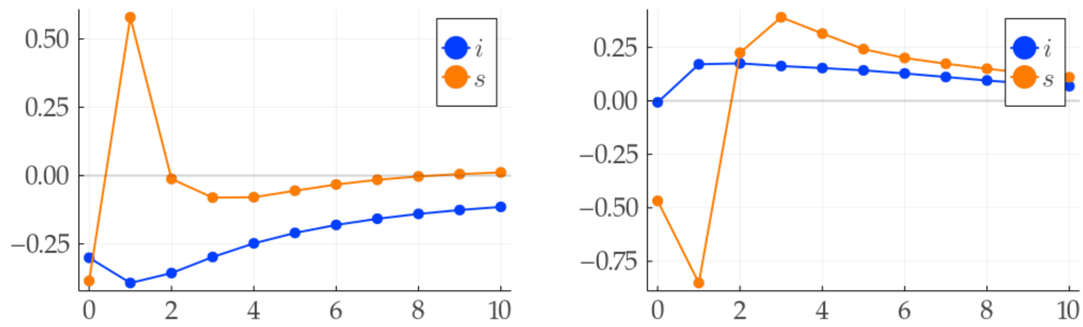
[1] In appendix C, I present the decomposition in two additional scenarios: a simple fiscal deficit ($\Delta E_t s_t = -1$), and a scenario of international monetary contraction in the US in response to inflation (like in the 1980s): $\Delta E_t i_t^{US} = \Delta E_t \pi_t^{US} = 1$.

[2] I report decomposition 1 in appendix F.

Country	$\Delta E_t \pi_t =$	Decomposition 2: $\Delta E_t \pi_t = -0.5, \Delta E_t g_t = -1$					
		$-\Delta E_t$(Future Inflation)		$\pm\ \Delta E_t$(Real Variables)			
		$-d_2(\pi)$	-$d_2(\pi^{US})$	$-d_2(s)$	$-d_2(g)$	$d_2(r)$	$d_2(\Delta h)$
Averages	-0.50	***0.78**	***0.54**	***-4.23**	***2.71**	-0.88	***0.59**
Advanced - 1960	-0.50	***1.24**	***0.05**	***-1.47**	***1.33**	***-1.70**	**0.04**
Advanced - 1973	-0.50	***0.83**	***0.02**	***-1.60**	***0.74**	**-0.56**	**0.06**
Emerging - 1998	-0.50	***0.58**	***0.93**	**-6.41**	***3.96**	-0.56	***1.01**
Median	-0.50	***0.64**	***0.11**	***-1.46**	***1.30**	***-1.23**	***0.18**
Advanced - 1960	-0.50	***1.13**	***0.03**	***-1.43**	***1.25**	***-1.78**	0
Advanced - 1973	-0.50	***0.82**	***0.01**	***-1.19**	**0.65**	**-0.43**	***0.03**
Emerging - 1998	-0.50	***0.48**	***0.61**	***-3.20**	***1.36**	-0.58	***0.51**
United States	-0.50	***0.57**	-	**-0.65**	***1.32**	***-1.75**	0
Advanced - 1960 Sample							
Canada	-0.50	***1.38**	***0.05**	-0.45	0.30	***-1.78**	-0.00
Denmark	-0.50	***1.06**	***0.15**	**-2.64**	***2.75**	**-1.78**	-0.04
Japan	-0.50	***0.60**	0	***-1.51**	***1.64**	***-1.23**	0
Norway	-0.50	***0.99**	0	**-1.36**	***1.72**	***-1.86**	0
Sweden	-0.50	***1.19**	***0.11**	-0.65	**0.87**	***-2.31**	***0.30**
United Kingdom	-0.50	***2.19**	0	**-2.20**	**0.73**	**-1.22**	0
Advanced - 1973 Sample							
Australia	-0.50	***0.96**	0	**-1.46**	**0.66**	-0.67	0
New Zealand	-0.50	***0.68**	***0.07**	**-0.84**	**0.63**	**-1.24**	**0.19**
South Korea	-0.50	***1.06**	***0.02**	***-3.17**	***1.74**	-0.20	***0.05**
Switzerland	-0.50	***0.64**	0	***-0.93**	-0.07	-0.13	0
Emerging - 1998 Sample							
Brazil	-0.50	**0.08**	***0.04**	1.87	0.13	**-2.85**	***0.23**
Chile	-0.50	***2.10**	***5.74**	-30.50	**30.54**	-7.76	-0.63
Colombia	-0.50	**0.49**	***1.26**	**-10.90**	***7.57**	-0.07	1.16
Czech Republic	-0.50	***0.51**	***0.14**	-0.07	**0.25**	**-1.61**	***0.27**
Hungary	-0.50	***0.64**	***0.54**	10.82	-5.29	**-7.91**	***0.70**
India	-0.50	0.45	-0.05	**-1.16**	**0.71**	-0.44	-0.02
Indonesia	-0.50	0.02	**1.73**	***-11.24**	**1.42**	0.76	***6.82**
Israel	-0.50	***0.47**	***0.79**	**-3.18**	**1.17**	-0.73	***0.98**
Mexico	-0.50	***0.48**	***0.68**	***-4.56**	***1.94**	0.04	***0.92**
Poland	-0.50	***0.50**	***0.42**	-0.14	**1.30**	**-2.90**	0.32
Romania	-0.50	**0.36**	***0.77**	***-8.16**	**2.05**	2.15	***2.33**
South Africa	-0.50	***1.59**	***0.30**	***-30.02**	***11.15**	***15.60**	**0.87**
Turkey	-0.50	***0.73**	***0.73**	0.64	**0.52**	***-3.31**	0.18
Ukraine	-0.50	-0.33	0	**-3.22**	***1.92**	**1.13**	0

Notes: I project VAR shocks on $\Delta E_t g_t = -1$ and $\Delta E_t \pi_t = -0.5$. The table shows the estimated terms of decomposition 2 at the posterior distribution's mode. I divide each term by $\delta v/\beta$, so the terms sum up to $\Delta E_t \pi$. I simulate from the posterior to check the statistical significance of the sign of the estimate. Bold indicates 75% of results had the same sign. Bold with an asterisk indicates 90%.

Table 5: Unexpected Inflation Decomposition 2 - Recession Scenario



(a) United States, $d_s = 0.65$ (b) Ukraine, $d_s = 3.22$

Notes: IRFs implied by the estimated VAR model (17). The innovation vector e_t is projected on the condition $\Delta E_t \pi_t = -0.5$, $\Delta E_t g_t = -1$ which defines the recession scenario.

Figure 2: Fiscal Shocks in Recessions: Deficits then Surpluses

This is not an indication that governments do not incur in deficits to "stimulate demand". First, in the case of some countries, estimates might be driven by periods of contractionary fiscal policy that *causes* recession and deflation. Second, to other countries a deficit in period zero ($\Delta E_t \pi_t < 0$ in the expression above) is followed by future surpluses. In figure 2, I plot the cases of the United States and Ukraine, which show this pattern. For the intrinsic value of debt, what matters is discounted surpluses, not current surpluses. Hence the deflationary effect $-d_2(s) < 0$. Again results are economically and statistically significant. Most estimates are larger than -1. In a FTPL causal reading, they suggest that, should governments manage to convince the public that large recession deficits would *not* be followed by future surpluses, the impact on unexpected inflation would be large. One can interpret the post-COVID, worldwide surge in inflation as an example of that (I do not include 2020-2022 data in the estimation).

The result of deflationary fiscal policy is common to all samples, but stronger in emerging markets, *despite* their lower share of nominal debt (recall that δ enters in the denominator of $d_2(s)$). More procyclical fiscal policy in developing countries has been previously identified by the macroeconomic literature. See, for instance Kaminsky et al. (2004), Alesina et al. (2008) and Ilzetzki (2011). Finally, I find for emerging markets an inflationary effect stemming from real exchange rate depreciation. I interpret these figures as capital exit and flight-to-quality movements in exchange rates (Jiang et al. (2020), Kekre and Lenel (2021)).

4.2.2. *Exchange Rate Depreciation Shock*

I next study the decomposition following episodes of real exchange rate depreciation. I consider a 10% shock to Δh. Such episodes can be due to shocks in the international economy, like Global Recessions, or shocks in the domestic economy, like sudden stops. I am going to focus on the latter, so I prevent US variables from jumping in the VAR:

$$\Delta E_t \Delta h_t = 10, \qquad \Delta E_t u_t = 0.$$

US variables are unaffected by domestic dynamics, so they do not respond to the shock at all. Table 6 reports the estimated terms of decomposition (2).

- *In emerging markets, real exchange depreciation shocks forecast low growth and contractionary monetary and fiscal policy. Higher nominal interest trades current for future inflation, $-d_2(\pi) < 0$; higher surpluses are deflationary. Lower growth and the depreciated currency are inflationary. Unexpected inflation is statistically zero in most cases.*

In emerging markets, real exchange rate shocks resemble "sudden stops", events in which foreign

Country	$\Delta E_t \pi_t =$	$-\Delta E_t$(Future Inflation)		$\pm \Delta E_t$(Real Variables)			
		Decomposition 2: $\Delta E_t \Delta h_t = 10$, $\Delta E_t u_t = 0$					
		$-d_2(\pi)$	-$d_2(\pi^{US})$	$-d_2(s)$	$-d_2(g)$	$d_2(r)$	$d_2(\Delta h)$
Averages	0.16	-0.27	0	***-4.48**	0.03	0.36	***4.53**
Advanced - 1960	**-0.39**	0.21	0	**-1.78**	-0.90	**1.29**	***0.80**
Advanced - 1973	**0.41**	0.15	0	**-1.41**	0.95	0.12	***0.61**
Emerging - 1998	0.29	**-0.57**	0	**-6.82**	0.03	-0.07	***7.57**
Median	0.02	**-0.16**	0	***-1.23**	-0.34	**0.37**	***1.53**
Advanced - 1960	***-0.69**	0.11	0	**-1.51**	**-1.29**	**1.90**	***0.27**
Advanced - 1973	**0.36**	0.11	0	-1.50	0.49	-0.09	***0.18**
Emerging - 1998	0.05	***-0.55**	0	***-2.90**	**1.29**	**0.49**	***3.25**
United States	***0.56**	**-0.75**	-	-0.19	**1.78**	-0.28	0
Advanced - 1960 Sample							
Canada	***-0.68**	0.81	0	**-2.64**	**-1.15**	1.82	***0.48**
Denmark	**-0.81**	0.32	0	**-3.87**	**-2.54**	**3.09**	***2.19**
Japan	**-0.36**	-0.10	0	***-1.80**	***2.15**	-0.61	0
Norway	***-1.29**	-0.20	0	-1.23	**-2.10**	**2.18**	***0.06**
Sweden	1.52	-0.83	0	-0.28	-1.43	1.98	***2.09**
United Kingdom	-0.69	1.24	0	-0.85	-0.34	-0.75	0
Advanced - 1973 Sample							
Australia	-0.10	**1.21**	0	-0.25	**-0.86**	-0.21	***0.03**
New Zealand	0.39	-0.84	0	**-3.91**	**1.76**	1.32	***2.07**
South Korea	**1.00**	0.40	0	**-2.74**	***3.67**	-0.66	***0.33**
Switzerland	**0.33**	-0.18	0	**1.25**	**-0.78**	0.02	0
Emerging - 1998 Sample							
Brazil	0.02	***-0.16**	0	-0.76	**1.43**	**-1.24**	***0.74**
Chile	0.41	**1.90**	0	**-87.99**	**29.66**	4.07	***52.78**
Colombia	-0.02	***-1.00**	0	***-11.74**	**3.06**	0.65	***9.00**
Czech Republic	0.08	-0.06	0	-6.64	3.88	1.36	***1.53**
Hungary	-0.85	0.03	0	**39.37**	**-29.48**	**-12.43**	**1.67**
India	-0.62	***3.87**	0	**-4.65**	-0.36	-0.02	***0.54**
Indonesia	-0.28	***-0.93**	0	***-12.74**	***2.34**	0.35	***10.71**
Israel	0.25	0.51	0	***-13.85**	**3.35**	***4.61**	***5.63**
Mexico	-0.69	**-0.94**	0	4.77	-7.65	-1.25	***4.39**
Poland	**-0.65**	0.26	0	**-0.08**	-3.56	0.62	***2.11**
Romania	2.58	**-3.18**	0	**-7.36**	**2.15**	0.37	***10.61**
South Africa	0.20	***-1.50**	0	3.52	-1.06	-2.10	***1.35**
Turkey	***2.26**	***-2.29**	0	-1.15	**-4.42**	***5.18**	***4.93**
Ukraine	**1.34**	***-4.45**	0	**3.79**	**1.16**	0.81	***0.02**

Notes: I project VAR shocks on $\Delta E_t \Delta h_t = 10$ and $\Delta E_t u_t = 0$. The table shows the estimated terms of decomposition 2 at the posterior distribution's mode. I divide each term by $\delta v / \beta$, so the terms sum up to $\Delta E_t \pi$. I simulate from the posterior to check the statistical significance of the sign of the estimate. Bold indicates 75% of results had the same sign. Bold with an asterisk indicates 90%.

Table 6: Unexpected Inflation Decomposition 2 - Real Exchange Depreciation

households abruptly sell holdings of domestic assets.[1] Nominal exchange depreciates and raises the in-domestic-currency value of public debt, which counts as an inflationary force by our decompositions. Its magnitude depends on the size of the dollar-linked portfolio of public debt but is positive and statistically significant to all countries.

Emerging markets respond to the depreciation shock by raising interest ($d_1(rx) < 0$, not reported). Lower bond prices forecast future inflation by (13). Fiscal policy response is contractionary, $-d_2(s) < 0$, with statistically and economically significant coefficients.[2] Median coefficients of growth and discounting effects suggest both are inflationary, but the growth effect is quantitatively and more often statistically significant. Indeed, sudden stops are characterized by output drops (Calvo et al. (2006)). Measured unexpected inflation figures following the depreciation shock suggest that policy efforts are often successful in mitigating it (although often at the cost of future inflation, $d_2(\pi) > 0$). From the fourteen cases, I estimate statistically significant $\Delta E_t \pi_t > 0$ in two.

The evidence for advanced economies is far less conclusive. From the set of developed countries, Canada, Denmark, Sweden, New Zealand and South Korea have a share of dollar debt greater than 1%. These five economies respond to the depreciation event by raising surpluses, like emerging markets. But depreciation shocks do not necessarily lead to a drop in output. They do the opposite in the 1960 sample (except for Japan), and the sign of coefficients split two and two in the 1973 sample. These numbers do not resemble the "sudden stops" in emerging markets. The case of South Korea is an exception. Its positive and large output coefficient $-d_2(g) > 0$ is driven by the 1998 recession following the Asian financial crisis. Finally the contribution of monetary policy through future inflation $d_2(\pi)$ is also ambiguous, like the overall effect on unexpected inflation.

5. The Decomposition in a New-Keynesian Framework

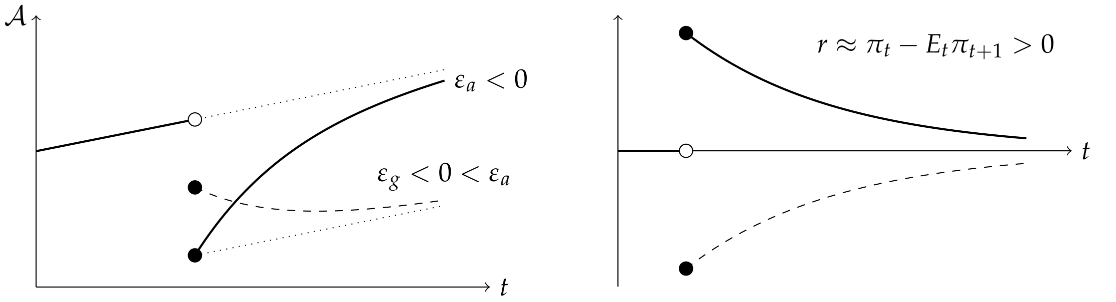
The estimates of the last section correspond to decompositions implied by reduced-form, non-structural shocks. Yet, in any theoretical model in which the transversality condition (3) holds, structural shocks will also lead to their own decompositions. In this section, I ask the question: how far do we need to go to find a model that can reproduce - with structural shocks - the measures implied by the estimated VARs?

I set a two-country New-Keynesian model and study how the terms of the decompositions behave in response to monetary, fiscal and technology (or productivity) shocks. The private sector framework is most similar to De Paoli (2009). There is a Foreign and a Home economy. The size of the Home economy converges to zero. In the limit, the Foreign economy behaves exactly as if it was closed, while the Home economy is open and "small". Firms produce differentiated goods using labor through a linear production function, and operate in a monopolistic competition environment. Prices are Calvo (1983)-sticky in the currency of the producing firm (producer-currency pricing), but not in the currency of the other country. The weight of Home goods in the basket of Foreign households converges to zero. But Home households have a bias towards consumption of Home goods, which we can interpret as consumption of services and other non-tradable commodities. As consumption baskets differ, the real exchange rate varies. Financial markets are complete, as in Galí and Monacelli (2005).

A theoretical model opens the door to causal interpretations of decompositions 1 and 2. Should we read them as "how the price level must respond to news of debt value" or "how the debt value must respond to news of the price level"? I assume a FTPL setup of public policy, which points to the former. Households observe the path of surpluses and attribute value to public debt in light of it. Surprise inflation or deflation ensues. Monetary policy is characterized by a Taylor (1993) rule.

[1] The literature on sudden stops is vast. See, for example, Calvo et al. (2006) and Verner and Gyöngyösi (2020) for empirical analysis, and Chari et al. (2005) and Mendoza (2010) for theoretical and quantitative accounts.

[2] Hungary and Ukraine are exceptions, although Hungarian estimates look suspiciously volatile for a country with low dollar debt, $\delta_D = 0.23$. Ukrainian result is driven by the 30% surge in inflation following the 2014 conflict in Crimea, which corresponds to an estimated fiscal *deficit* at the same time its currency depreciated.



(a) Productivity Path $\mathcal{A}_t$

(b) $-a_t$, or Mg. Cost $\approx -E_t\Delta\pi_{t+1}$ at fixed wages

Figure 3: Two Negative Technology Shocks with Different Implications for Real Interest

5.1. *Technology and Trend Shocks*

For reasons that will become clear, the economy is not stationary. The production function is $\mathcal{A}_t N = \mathcal{T}_t A_t N$ in the Home country and $\mathcal{A}_t^* N = \mathcal{T}_t A_t^* N$ in Foreign, where N is the number of hours of labor employed. Process $\mathcal{T}_t$ is the trend of the productivity processes and is common to both Home and Foreign. It introduces a unit root to productivity:

$$\log \mathcal{T}_t = \log \mathcal{T}_{t-1} + g + u_{g,t}$$
$$u_{g,t} = \rho_g u_{g,t-1} + \varepsilon_{g,t}.$$

A vast literature followed Nelson and Plosser (1982), who first tested and failed to reject the presence of a unit root in US GNP. Although a definitive conclusion has not been reached - testing for unit roots is hard! - I do not regard the assumption of trend shocks as unreasonable.

Processes A_t and A_t^* are stationary. They capture temporary fluctuations of productivity around the trend. The laws of motion for $a_t = \log A_t$ and $a_t^* = \log A_t^*$ are

$$a_t = \rho_a a_{t-1} + \varepsilon_{a,t}$$
$$a_t^* = \rho_a a_{t-1}^* + \varepsilon_{a,t}^*.$$

By themselves, $\mathcal{T}$ and A (or A^*) have a clear interpretation. Shock $e_{g,t}$ captures changes to long-term productivity; $e_{a,t}$ captures short-term deviations from the long term. Combinations of $e_{g,t}$ and $e_{a,t}$ are also "structural": like the individual shocks, all they do is change the path of productivity. In particular, by combining $e_{g,t}$ and $e_{a,t}$ we can generate permanent disturbances that are more general than $\log \mathcal{T}_t$ alone. Such generality will prove critical. Figure 3a sketches two relevant examples, which I will refer to later on. It plots the productivity process $\mathcal{A}_t$. The solid curve corresponds to a temporary negative shock $e_a < 0$. Productivity declines and then slowly grows as it reverts back to trend. The dashed curve depicts the combination $e_g < 0 < e_a$, with $|e_a| < |e_g|$. Productivity drops in period zero, and continues to decline until the total effect of the technological shock on productivity reaches e_g. Both disturbances will lead to recessions in the model, but the dynamic properties of marginal costs will imply different paths for inflation and, critically, real interest rates.

5.2. *The Foreign, "Closed" Economy*

The key ingredients of closed and open economy NK models are well known in the macroeconomic literature, so I state only the linearized version of the model. Appendix E presents a full derivation. I consider first the Foreign economy. As the measure of Home households and firms approaches zero, prices and quantities in the Foreign economy respond only to demand and supply of its own households and firms. Therefore, we can view the case of the Foreign country as the study of a closed economy - the equations are exactly the same. The private sector block is

$$\begin{aligned} y_t^* &= E_t y_{t+1}^* - \gamma \left[i_t^* - E_t \pi_{t+1}^* \right] + E_t u_{g,t+1} \\ \pi_t^* &= \beta E_t \pi_{t+1}^* + \kappa y_t^* - \kappa_a a_t^* \\ g_t^* &= y_t^* - y_{t-1}^* + u_{g,t}, \end{aligned} \tag{21}$$

where y_t^* is log detrended output, π_t^* is log inflation and g_t^* is the growth rate of output.[1] The asterisk indicates the variable corresponds to the Foreign economy. The top equation is the dynamic IS, which follows from households' intertemporal optimization and market clearing in the goods market. The second equation is the forward-looking Phillips curve, which follows from optimal price-setting behavior by firms. Its parameters are $\kappa = \lambda(\gamma^{-1} + \varphi) > 0$ and $\kappa_a = \lambda(1 + \varphi) > 0$. Parameter φ is the inverse of the Frisch elasticity of labor supply, and $\lambda = (1 - \beta\theta)(1 - \theta)/\theta$ is a function of intertemporal discounting β and the price-resetting rate $1 - \theta$.

By themselves, the IS and Phillips curve equations in (21) do not determine unexpected inflation. The Phillips curve pins *expected* inflation, or the expected change in the inflation rate. Any expectational shock is consistent with a stationary equilibrium. In Blanchard and Kahn (1980) language, the system contains two forward-looking variables (y^* and π^*) for a single explosive root: multiplicity of equilibrium ensues.[2] To solve it, the literature uses either a fiscal selection mechanism (the FTPL) or a monetary policy rule that implicitly threatens inflation spiraling out of control. I use the former, and defer to appendix D a discussion of the difference between the two.

The public policy block of the Foreign economy is[3]

$$\begin{aligned} i_t^* &= \phi_\pi \pi_t^* + \phi_g g_t^* + \varepsilon_{i,t}^* \\ s_t^* &= s_t^{p*} = \rho_s s_{t-1}^* + \tau_\pi \pi_t^* + \tau_g g_t^* + \varepsilon_{s,t}^*. \end{aligned} \tag{22}$$

The law of motion for public debt v^* is given by (9), except that all variables carry an asterisk. Leeper (1991) was the first to observe that such law of motion can provide the additional unstable root that the model requires for equilibrium uniqueness. That can only happen if surpluses do not respond strongly enough to debt. Note that is the case here: public debt v^* does not enter the equation for surpluses s^*. If it did, the government would adjust primary surpluses to stabilize real debt *for any possible value of the price level*, which would hence remain indeterminate. In a fiscal-active model like this one, that is not the case. Agents observe discounted surpluses and attribute value to public debt accordingly. Inflation jumps as a result of such re-evaluation. Therefore, in a FTPL model, we read decompositions 1 and 2 with causality running from right to left. News about the intrinsic value of public debt or bond prices *cause* unexpected inflation.

The equations in (21) and (22) form the canonical closed-economy New-Keynesian model with active fiscal policy, and augmented for non-stationary technology.[4] I compare the fiscal decompositions implied by it with VAR estimates for the United States. A subset of structural parameters I calibrate - see table 7. Parameters β, δ and ω are the same as the ones I use in the estimation of the VARs for the US. In particular, $\delta_D = 0$ implies that the real exchange rate and foreign inflation terms of decomposition 2 equal zero. Elasticities $\gamma = 0.4$ and $\varphi = 3$ follow literature standards. Price rigidity $\theta = 0.25$ follows the low-frequency estimate in Kehoe and Midrigan (2015) and the DSGE-estimate in Smets and Wouters (2007).

Let σ_a be the standard deviation of $\varepsilon_{a,t}$, σ_a^* the standard deviation of $\varepsilon_{a,t}^*$, and so on. I group all such parameters in a single vector σ. The ε and ε^* shocks are uncorrelated. I group model parameters that were not calibrated in $\Psi = (\rho_a, \rho_g, \phi_\pi, \phi_g, \rho, \tau_\pi, \tau_g, \sigma')$ and estimate them by a method of moments

[1] Output in the model corresponds to GDP in the data.

[2] The lack of determination is most easily seen in a model with flexible prices and constant output, in which the only equilibrium condition is the Fisher equation $i_t = E_t \pi_{t+1}$. An interest rate peg $i_{t-1} = 0$ leads to $E_{t-1}\pi_t = 0$. One cannot determine $\Delta E_t \pi_t$.

[3] By assumption public basket of goods is the same as that of households, so real surpluses s_t coincide with price-adjusted surpluses s^{p*}.

[4] I solve the model by solving forward-looking variables forward, using Klein (2000) method.

Parameter	Interpretation	Optim. Interval	Value No Trend	Value Full
A. Calibrated Parameters				
β	Intertemporal discount		0.98	
v	Average debt-to-GDP		country dependent	
δ	Currency structure of debt		country dependent	
ω	Term structure of debt		country dependent	
γ	Intertemporal elasticity		0.4	
φ^{-1}	Frisch elasticity of labor supply		$1/3$	
θ	Price rigidity		0.25	
α	Share Foreign goods in Home basket		0.45	
$\bar{\omega}$	Demand elasticity to terms of trade		γ^{-1}	
B. Estimated Parameters				
ρ_a	Temporary productivity persistence	$[0,1]$	0.98	0.84[a]
ρ_g	Trend disturbance persistence	$[0,1]$	-	0.27
ϕ_π	Interest sensitivity to inflation	$[0.3,1]$	0.80	0.93
ϕ_g	Interest sensitivity to GDP growth	$[0,1]$	0.34	0.61
ρ_s	Surplus persistence	$[0,1]$	0[a]	0.00
τ_π	Surplus sensitivity to inflation		0[a]	0.25
τ_g	Surplus sensitivity to GDP growth		0.70	0.15
σ_i^*, σ_i	Interest shock std	$(0,\infty)$	0[a]	0.05
σ_s^*, σ_s	Surplus shock std	$(0,\infty)$	0[a]	0.08
σ_a^*, σ_a	Temporary technology shock std	$(0,\infty)$	4.1	1.41
σ_g	Technology trend shock std	$(0,\infty)$	0[a]	2.62

Notes: Panel A reports parameters calibrated based on standard values used in the macroeconomic literature. Panel B contains parameters in Ψ that solve (23). The "Optim. Interval" column describes the constraints that define the optimization set Θ.
[a] The values indicated were fixed, not estimated.

Table 7: List of Model Parameters

procedure. The set of moments contains the terms of different decompositions we saw so far - I denote them $\mathcal{D}$. To avoid having the model yield completely irrealistic dynamics, I also include as targeted moments: the standard deviations of interest i_t and inflation π_t, both expressed as ratios of output growth standard deviation, growth volatility itself, as well as the three correlations between these variables. These six moments are calculated from the ergodic distribution of the system and grouped in vector $\mathcal{M}$. The optimization problem has the format

$$\text{Min}_\Psi \quad \frac{w}{n_\mathcal{D}} \|\mathcal{D}_{VAR} - \mathcal{D}_{NK}(\Psi)\| + \frac{1-w}{n_\mathcal{M}} \|\mathcal{M} - \mathcal{M}_{NK}(\Psi)\| \qquad \text{s.t. } \Psi \in \Theta \tag{23}$$

where $\|.\|$ is the Euclidian norm, w is a scalar governing the relative weight we give to decomposition moments (I use $w = 0.8$), and $n_\mathcal{D}$ and $n_\mathcal{M}$ are the sizes of vectors $\mathcal{D}$ and $\mathcal{M}$; D_{VAR} denotes VAR-implied decomposition terms, $\mathcal{M}$ denotes data moments and NK subscript denotes New-Keynesian model values. I restrict parameters to a subset Θ which limits them to reasonable values. Table 7 reports the list of parameters, estimated values and constraints. Table 8 reports estimated second moments $\mathcal{M}$.

- *Temporary technology shocks $e_{a,t}$ alone reproduce the decomposition of unexpected inflation variance with positive contributions from surplus-to-output, growth and real interest terms.*

For this first proposition, I turn off all shocks in the model, except for temporary technology shocks $\varepsilon_{a,t}$: $\sigma_g^* = \sigma_i^* = \sigma_s^* = 0$. Targeted decompositions include only decompositions 1 and 2 of unexpected inflation variance (tables 3 and 4); vector $\mathcal{D}$ thus contains thirteen elements. I choose ρ_a, ϕ_π, ϕ_g, τ_g and σ_a^* to solve (23).[1] Figure 4 displays results: bars correspond to the model, circles

[1]Surplus feedback parameters τ_π and τ_g are unidentified, since all the surplus equation does is select unexpected inflation in response to the single shock $e_{a,t}^*$. One feedback parameter suffices, so I fix $\tau_\pi = \rho_s = 0$ without affecting the estimation.

	σ_i/σ_g	σ_π/σ_g	cor(π, i)	cor(π, g)	cor(i, g)
United States					
Data	1.00	1.01	0.54	-0.24	0.16
Model (No Trend)	1.27	1.84	0.98	-0.70	-0.55
Model (Full)	0.98	1.07	0.81	-0.33	0.29
New Zealand (Not Targeted)					
Data	2.04	2.17	0.77	-0.39	-0.35
Model	0.89	0.87	0.75	-0.24	0.47

Notes: The table reports the second moments $\mathcal{M}$ that enter the optimization problem 23, with the exception of the standard deviation of output growth, which simply sets the scale of the σ parameters of model. Symbol σ_x stands for standard deviation of variable x; "cor" means contemporaneous correlation. All moments calculated from the stationary distribution implied by the NK model.

Table 8: Estimated Second Moments

correspond to the VAR. The style of the plot facilitates comparison between the decompositions - which correspond to combinations of structural shocks - and structural shocks themselves. Marker ribbons indicate percentiles 25 and 75 from the posterior distribution. The model successfully replicates the VAR decompositions.

Column "No Trend" of table 7 reports estimated parameters. Optimization requires a persistent technology shock ($\rho_a = 0.974$) to generate a large growth terms $d_1(g) = d_2(g) = \sum \beta^k \Delta E_t g_{t+k}$. This follows from the fact that this simplified model is stationary, and partially justifies the adoption of trend shocks in the complete specification. Since the model is stationary we can express output as $y_t = \mathbf{1}_y' a(L) e_t$ where $a(L)$ is a lag polynomial with finite $a(1)$.[1] In the absence of trend shocks,

$$g_t = (1-L)y_t = \mathbf{1}_y'(1-L)a(L)e_t \equiv \mathbf{1}_y' b(L) e_t,$$

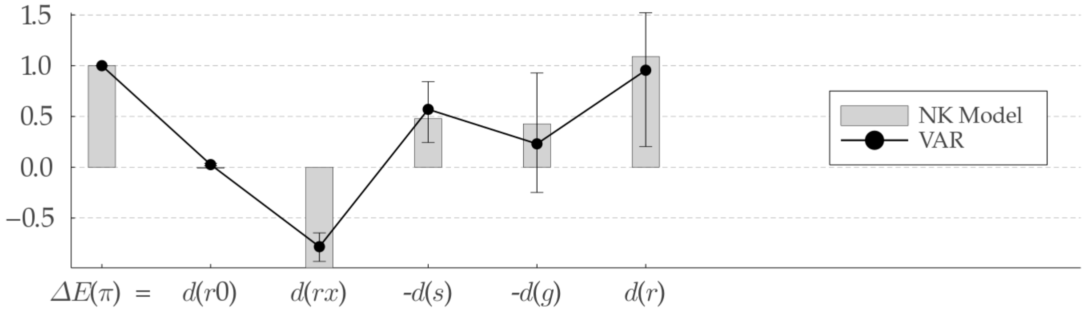
with $b(1) = 0$. Thus, $d_1(g) = \mathbf{1}_g' b(\beta) e_t \approx 0$. Intuitively, in a stationary model, positive growth today must be met by negative growth tomorrow so that output eventually returns to its long-term level. Therefore, the fiscal effect of unexpected growth will be small unless the initial impact takes a long time to fade and accrues interest β^{-1}.

The large interest rule parameters ϕ_π and ϕ_g matter to reproduce the negative contribution of excess returns $d_1(rx)$ and the positive contribution of real return $d_1(r)$ and $d_2(r)$. Figure 5a shows the response function to $\varepsilon^*_{a,t} = -1.15$, which yields the decomposition of unexpected variance. The solid curves correspond to the estimated $\phi_\pi = 0.81$. As inflation jumps, the central bank raises interest by the Taylor rule. Bond prices fall, so $d_1(rx) < 0$. As inflation starts to decline, real interest rates (plotted in figure 5b) becomes positive, and so $d_1(r), d_2(r) > 0$. The dashed lines in these two graphs represent the response to the same shock if we set $\phi_\pi = 0.3$. The initial jump in nominal interest is not as strong, so the model fails to generate large $d_1(rx)$. The real interest rate jumps down on spot and stays lower than the baseline for three periods, which precludes the model from generating large $d_1(r)$ and $d_2(rx)$. Note how the weaker response of nominal interest leads to higher unexpected inflation, but lower *future* inflation. The NK model captures the intertemporal trade-off faced by monetary policy.

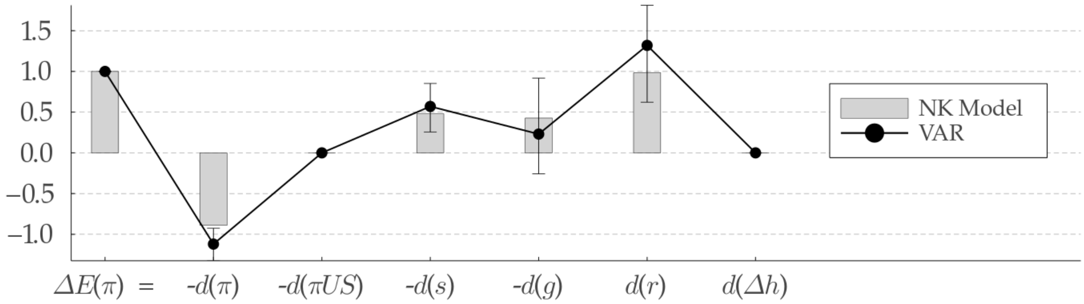
Finally, the optimization entails countercyclical fiscal policy (deficits in recessions), as the IRF shows. Negative growth in period one leads to a large deficit, which leads to the positive contribution of discounted surpluses to unexpected inflation variance, as in the VAR measures. Despite being able to replicate the empirical decompositions, this first simplified version of the NK model fails to generate realistic second moments, as table 8 shows. Growth is not volatile enough, and interest rates are countercyclical, contrary to the data.

- *In the absence of trend shocks, the NK model fails to replicate the decompositions of the "aggregate demand" recession scenario, even with fiscal and monetary shocks. The model with trend shocks accomplishes that by*

[1] The solution to the linear rational expectations model has the format $(I - AL)X = D(L)X = Ke_t$. A stationary solution implies that the roots of D are outside the unit circle, and so $\|D(1)\| \neq 0$. For $a = D^{-1}$, we then have $\|a(1)\| < \infty$ (a is absolutely summable) and thus $a(1) < \infty$ entry-by-entry.



(a) Variance Decomposition 1 - Equation (10)



(b) Variance Decomposition 2 - Equation (14)

Notes: The graphs depict the variance decomposition in the model's Foreign, "closed" economy. I shut down all shocks but temporary productivity shocks $\varepsilon^*_{a,t}$. The ribbons correspond to the 25 and 75 percentiles of the posterior distribution.

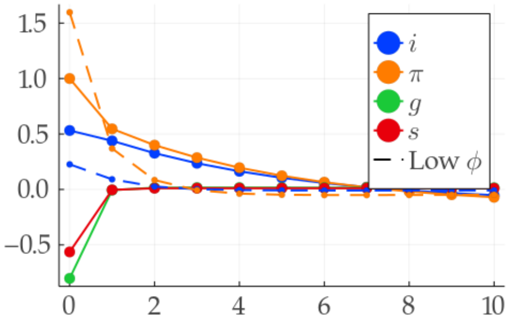
Figure 4: Decomposition of Unexpected Inflation - NK Model, Temporary Technology Shocks

combining strong Taylor rules (large ϕ_π) with recessions in which detrended marginal costs decline over time. Policy shocks are not necessary.

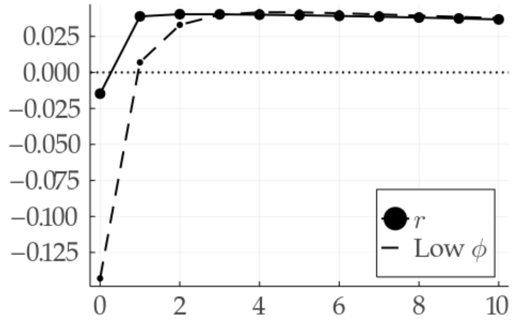
I bring back to the model monetary policy, fiscal policy and productivity trend shocks. I re-estimate parameters, but now including as a target in problem (23) the decompositions of unexpected inflation in the "aggregate demand" recession scenario - which I will refer to as recession decomposition -, considered in table 5. I calculate it exactly as in the empirical VARs, by projecting structural shocks on $\Delta E_t \pi_t = -0.5$ and $\Delta E_t g_t = -1$. Vector $\mathcal{D}$ now contains twenty-six entries. To properly characterize a_t as a measure of temporary shocks, I restrict $\rho_a = 0.84$ so that disturbances have a half-life of about four years. The gray bars in the four panels in figure 6 depict the fit of the model. It does a good job of reproducing the decompositions estimated for the United States. In the same panels, the hatched bars correspond to the same optimization, but under the constraint $\sigma_g = 0$ (no trend shocks). Even with disturbances to monetary and fiscal policy, it is clear that the NK model loses the ability of reproducing the two empirical decompositions of unexpected inflation at the same time.

To help understand these results, the two panels in figure 7 plot the terms of the decompositions in response to the four structural shocks. I normalize the size of the shocks so that the largest decomposition term equals one, and its sign so that the resulting unexpected inflation $\Delta E_t \pi_t$ is positive. The minus sign "-" in the legend of the plots indicate that the four shocks must be negative. Figures 5e-5f contain the IRFs to each of them. We saw in table 5 that the unexpected deflation $\Delta E_t \pi_t = -0.5$ characterizing the recession scenario followed from lower surplus-to-GDP ratios and lower real interest. In the absence of trend shocks, I argue that New-Keynesian model struggles in delivering the latter.

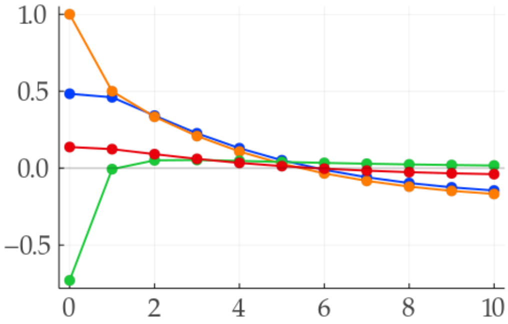
Consider first the effect of policy shocks. Inspection of the structural shocks' decompositions in figure 7 reveals that neither fiscal nor monetary policy shocks can lead to the correct pattern of real interest. An expansionary monetary policy shock (lower i_t, graph 5g) successfully yields lower real interest, but only through positive unexpected inflation - the recession scenario, on the contrary, asks for negative $\Delta E_t \pi_t$. The positive effect on inflation follows from the increase in bond prices, which



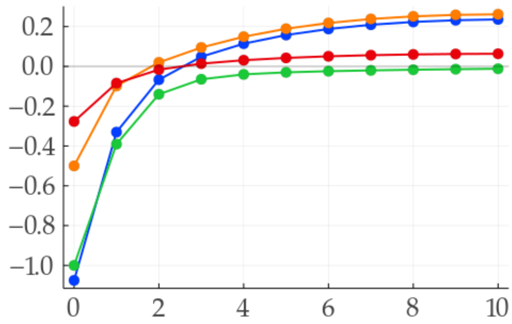
(a) No Trend Shocks, $\Delta E_t \pi_t = 1$



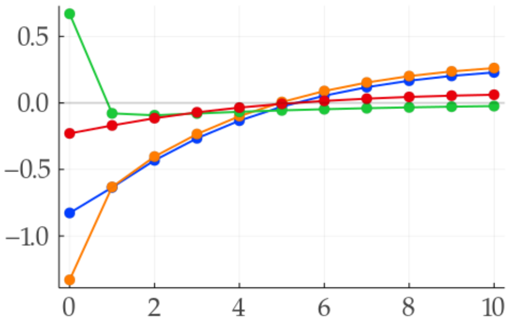
(b) No Trend Shocks, $\Delta E_t \pi_t = 1$, Real Interest



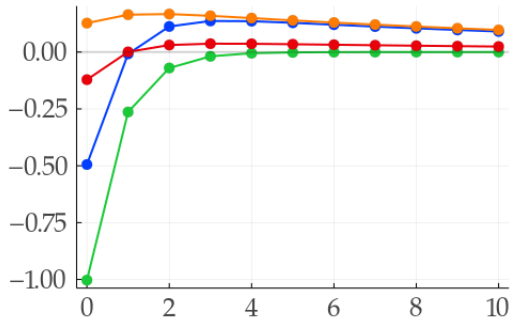
(c) Full Model, $\Delta E_t \pi_t = 1$



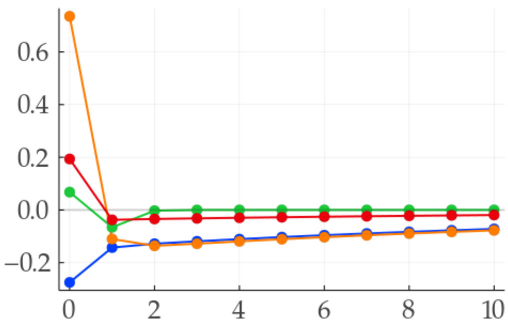
(d) Full Model, $\Delta E_t \pi_t = -0.5$, $\Delta E_t g_t = -1$



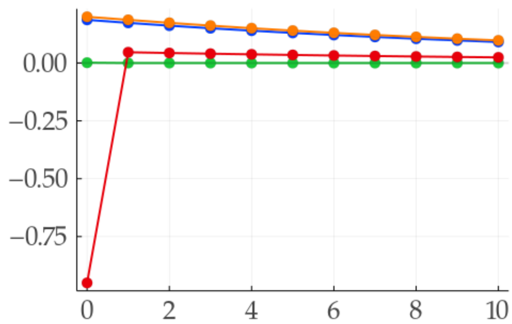
(e) Full Model, Productivity Shock $\varepsilon^*_{a,t} = 1$



(f) Full Model, Trend Shock $\varepsilon_{g,t} = -1$



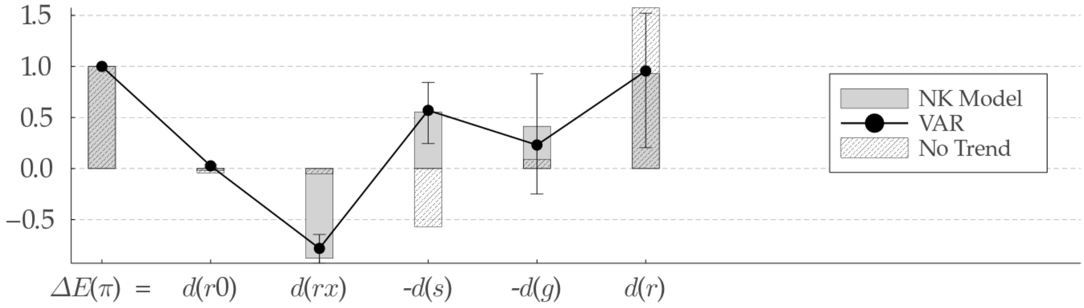
(g) Full Model, Interest Shock $\varepsilon^*_{i,t} = -1$



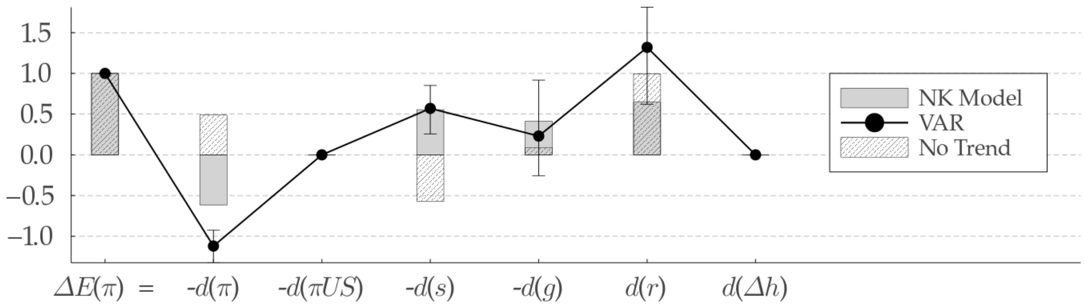
(h) Full Model, Surplus Shock $\varepsilon^*_{s,t} = -1$

Notes: IRFs of Foreign country of the NK model, which works as a closed economy. The "Full" model contains shocks to the trend of productivity. Plots 5c and 5d contain the responses to the structural shocks projected onto the innovations on variables indicated.

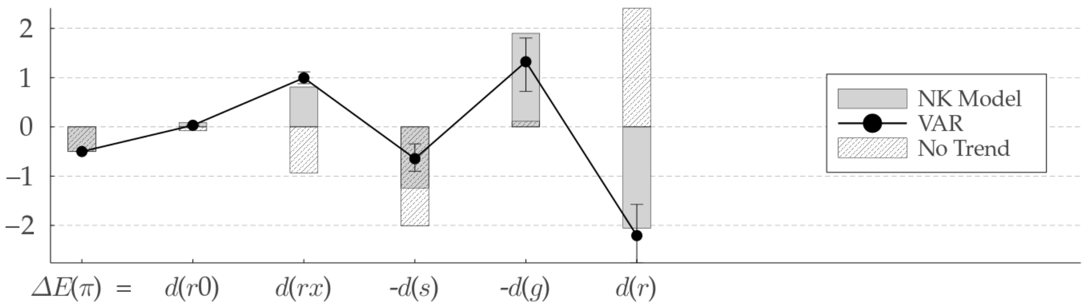
Figure 5: IRFs for the Foreign, "Closed" Economy



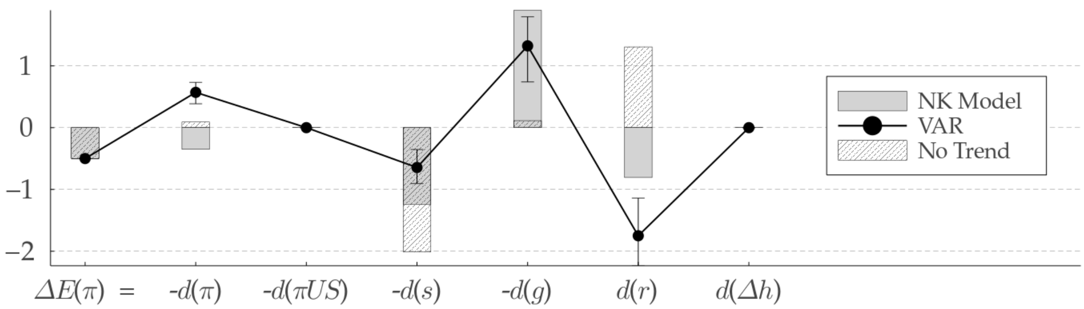
(a) Decomposition 1 - Variance Decomposition $\Delta E_t \pi_t = 1$



(b) Decomposition 2 - Variance Decomposition $\Delta E_t \pi_t = 1$



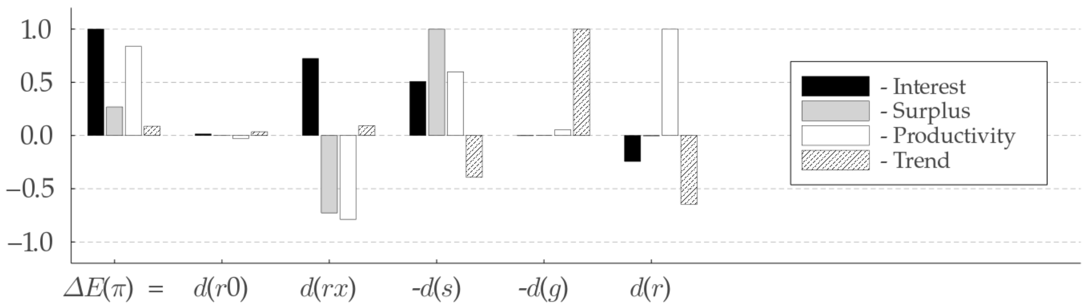
(c) Decomposition 1 - Recession Scenario $\Delta E_t \pi_t = -0.5$, $\Delta E_t g_t = -1$



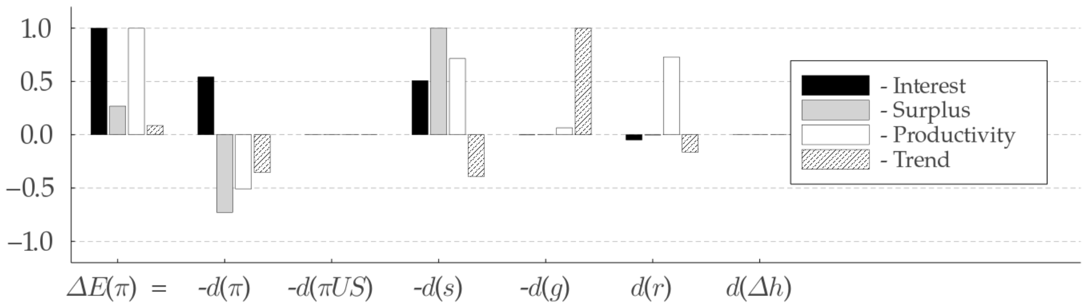
(d) Decomposition 2 - Recession Scenario $\Delta E_t \pi_t = -0.5$, $\Delta E_t g_t = -1$

Notes: The graphs depict the variance decomposition in the model's Foreign, "closed" economy. The hatched bars correspond to the model optimized subject to $\sigma_g = 0$ (no productivity trend shocks). The ribbons correspond to the 25 and 75 percentiles of the posterior distribution.

Figure 6: Fiscal Decomposition - NK Model with and without Trend Shocks



(a) Decomposition 1



(b) Decomposition 2

Notes: The graphs depict the fiscal decomposition of unexpected inflation resulting from each structural shocks of the Foreign, "closed" economy in the NK model. I normalize the size of each shock so that the largest term equals one in absolute value, and their sign (reported in the legend) so that unexpected inflation is positive.

Figure 7: Fiscal Decomposition of Structural Shocks

elevates the market price of public bonds. To restore the valuation equation (4), inflation jumps. An expansionary fiscal policy also yields positive real interest. However, as plot 5h shows, real interest is too small period by period, due to the strong Taylor rule coefficient $\varphi_\pi = 0.94$ combined with an absence of marginal cost movements that ensures $\pi_t \approx E_t \pi_{t+1}$.

Moving to technology shocks, the model cannot rely on temporary disturbances $e^*_{a,t}$ alone to replicate both the variance decompositions *and* the recession decompositions, since unexpected inflation responds to the underlying shocks in opposite directions. By adding trend shocks, the model reproduces both decompositions. Plots 5c and 5d show the economy's impulse response to shocks projected onto $\Delta E_t \pi_t = 1$ (the decomposition of unexpected inflation variance). It is similar to that of the stationary model, figure (5a), except that the primary surplus rises in the first period and only turns negative afterwards. The structural shocks that lead to it are

$$\varepsilon^*_{i,t} = \varepsilon^*_{s,t} \approx 0 \qquad \varepsilon^*_{a,t} = -0.73 \qquad \varepsilon_{g,t} = -0.24.$$

The similary between the IRFs in figures 5c and 5e reveal that productivity shocks can replicate the decomposition of unexpected inflation variance almost by themselves, as we verified to be the case.

Nevertheless, trend shocks are necessary to replicate the recession decomposition ($\Delta E_t \pi_t = -0.5$, $\Delta E_t g_t = -1$). Figure 5d plots the economy's response to the projected shocks. They are:

$$\varepsilon^*_{i,t} = \varepsilon^*_{s,t} \approx 0 \qquad \varepsilon^*_{a,t} = 0.50 \qquad \varepsilon_{g,t} = -1.33.$$

The combination of these structural shocks leads to a productivity path similar to that depicted in plot 3a. Productivity falls in the initial period and, contrary to temporary shock $\varepsilon^*_{a,t}$, *continues to decline*. That leads to the critical feature of this particular productivity path, illustrated by plot 3b: the recession coincides with a period of relatively low *detrended* marginal costs.[1] Actual marginal costs do increase

[1]Figure 3b actually shows $-a$, not detrended marginal cost, which equals $\kappa y_t - \kappa_a a_t$. Detrended output y_t increases in

since productivity falls, but *relative to trend* they are low - that is the role of the positive $\varepsilon^*_{a,t}$ disturbance. In turn, by the Phillips curve, firms react to low marginal costs by setting current prices at relatively lower levels than future ones: inflation grows over time in expected value. The strong Taylor rule ensures $i_t \approx \pi_t$ and completes the argument, which I summarize below.

$$
\begin{aligned}
& E_t\pi_{t+1} \approx \beta E_t\pi_{t+1} > \pi_t \qquad \text{(by the Phillips curve, } \kappa y_t - \kappa_a a_t < 0\text{)} \\
\implies & E_t\pi_{t+1} > \phi_\pi^{-1}(i_t - \phi_g g_t) \approx i_t \qquad \text{(by the strong Taylor rule, } \phi_\pi \approx 1\text{)} \\
\implies & r_t < 0.
\end{aligned}
$$

In the absence of trend shocks or some other productivity process that can generate a recession with relatively low marginal utility, the basic NK model fails to replicate the negative contribution of real interest to the deflation that characterizes the "aggregate demand" recession scenario. Figure 7 summarizes the argument visually. Trend shocks (hatched bars) yield the required growth term of the decomposition, while temporary productivity shocks (unfilled bars) yield the discounting term. Lastly, trend shocks also help us through the IS equation. Since it has some persistence ($\rho_g = 0.26$), $\varepsilon_{g,t} < 0$ leads to multiple periods of negative growth. By the IS equation, that implies multiple periods of negative real interest, which is what we need.

The combination of structural shocks that lead to the two decompositions (variance and recession) require no policy shocks. As table 7 suggests with low values of σ^*_i and σ^*_s, policy shocks are also not critical for the model to reproduce empirical second moments. We see in table 8, row "Full", that it does so relatively well, although the correlation between nominal interest and inflation is somewhat larger than the evidence. Monetary and fiscal policy rules (22) are enough to ensure the required dynamics. Table 7 reports even larger estimated Taylor rule parameters ϕ_π and ϕ_g than in the simplified stationary model. It also requires a surplus process that responds more to inflation than to output growth.

5.3. The Home, Open Economy

I consider now the case of the Home country, which represents that of a small, open economy (I will call it the SOE-NK for brevity). The structural parameters (γ, φ, θ, ...) in Home are the same as those in Foreign; it turns out they need not be re-estimated. Parameters specific to the open economy case I calibrate based on the experience of New Zealand, which is the country I compare results with. My choice for New Zealand follows from the observation that its VAR decompositions are often close to the average of developed countries. Additionally, New Zealand public debt has a significant share of dollar debt, which will be important when we discuss the real depreciation shock. Given Foreign prices and quantities, the equilibrium conditions in the private sector of Home are:

$$
\begin{aligned}
y_t &= E_t y_{t+1} - \gamma\left[i_t - E_t\pi_{t+1} + \alpha\bar{\omega}E_t\Delta z_{t+1}\right] + E_t u_{g,t+1} && (24)\\
\pi_{H,t} &= \beta E_t \pi_{H,t+1} + \kappa y_t - \kappa_a a_t - \kappa_z z_t && (25)\\
g_t &= y_t - y_{t-1} + u_{g,t} && (26)\\
y_t &= y^*_t + \gamma_\alpha z_t && (27)\\
h_t &= (1-\alpha) z_t && (28)\\
\pi_t &= \pi_{H,t} + \alpha \Delta z_t && (29)
\end{aligned}
$$

The model contains two key international prices: terms of trade z_t and the real exchange rate h_t. Domestic output prices $p_{H,t}$ and the consumer price index p_t are related by $p_t = p_{H,t} + \alpha z_t$, where α is the weight of foreign goods on the basket of Home households. Equation (29) follows. I set $\alpha = 0.45$ following the Reserve Bank of New Zealand's KITT model (Beneš et al. (2009)). All prices are in logs.

Equation (28) follows from a decomposition of real exchange rate: Foreign-to-Home consumer price

response to the innovation, but not enough to switch the sign of the marginal cost process. Intuitively, if the marginal cost mc_t jumped up in period zero, inflation would go negative in period 1: $0 = \pi_0 = \beta E_0\pi_1 + mc_0 \implies E_0\pi_1 = -mc_0 < 0$. That would lead to a positive real interest $r_0 = i_0 - E_0\pi_1$ which, by the IS, leads to a negative output response $y_0 < 0$, not $y_0 > 0$.

ratio (h_t) equals Foreign-to-Home output price ratio (z_t) times (plus, in logs) Home output-to-consumer price ratio ($-\alpha z_t$). With complete markets, the Backus and Smith (1993) condition holds: domestic consumption rises relative to foreign consumption when h depreciates:

$$c_t = y_t^* + \gamma h_t. \tag{30}$$

Equation (24) is the intertemporal IS; it follows from households' Euler equation, added to the market-clearing condition

$$y_t = c_t + \alpha\bar{\omega}\gamma z_t, \tag{31}$$

which states that demand for home goods equals domestic consumption plus a term that adjusts for relative price variation of Home goods. Parameter $\bar{\omega}$ can be greater or lower than zero.[1] Depreciated terms of trade reduce the relative price for Home goods, which increases foreign demand. But they also correspond to lower foreign output due to the international risk-sharing rule (30). The net result on aggregate demand for domestic output can go either way. I set $\bar{\omega} = \gamma^{-1}$, which leads to zero net exports every period. Equation (27) follows from (28), (30) and (31). Parameter $\gamma_a = (1 - \alpha + \alpha\bar{\omega})\gamma$ adjusts intertemporal substitution γ for the presence of home bias and imperfect substitution between different varieties of goods. Lastly, the parameters of the Phillips curve (25) are $\kappa, \kappa_a > 0$ (same as Foreign) and $\kappa_z = \lambda\alpha(\bar{\omega} - 1)$ (positive in my calibration).

The problem of unexpected inflation determinacy that characterizes the closed-economy NK model is also present in this open-economy version. To see this, replace (27), (28) and (29) in the Phillips curve to reduce the system to

$$\begin{aligned} y_t &= E_t y_{t+1} - \gamma_\alpha \left[i_t - E_t \pi_{H,t+1}\right] + \alpha(\bar{\omega} - 1) E_t \Delta y_{t+1}^* + E_t u_{g,t+1} \\ \pi_{H,t} &= \beta E_t \pi_{H,t+1} + \kappa_\alpha y_t - \kappa_a a_t - \gamma_\alpha^{-1} \kappa_z y_t^*, \end{aligned}$$

where $\kappa_\alpha = \lambda(\gamma_\alpha^{-1} + \varphi)$. These two equations determine the distribution of y_t and $\pi_{H,t}$. Note the similarity with their closed-economy counterpart (21). Since trend growth $u_{g,t}$ and Foreign output-trend ratio y^* are determined elsewhere, they do not affect the determinacy/stability properties of the system, which again requires an additional unstable root.

The public policy block of the Home economy is

$$\begin{aligned} i_t &= \phi_\pi \pi_t + \phi_g g_t + \varepsilon_{i,t} \\ s_t &= \rho_s s_{t-1} + \tau_\pi \pi_t + \tau_g g_t + \varepsilon_{s,t} \\ s_t^p &= s_t - \alpha s z_t. \end{aligned} \tag{32}$$

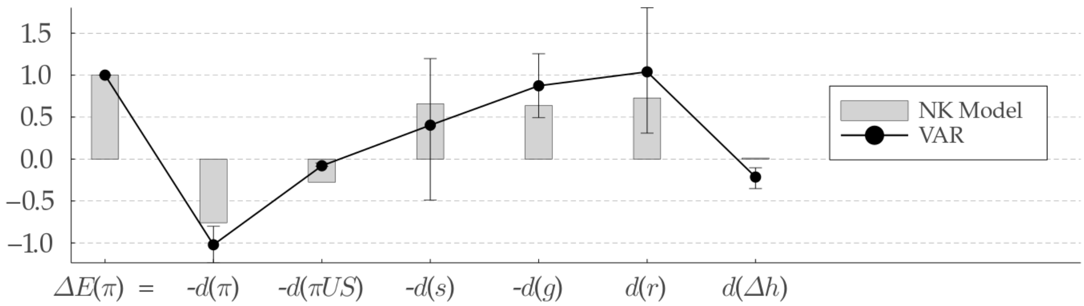
The equations are similar to those of the Foreign economy, except that surplus s_t and price-adjusted surplus s_t^p differ. I assume the government taxes domestic production, so the price index for its surplus is $p_{H,t}$, not p_t. I again prevent surpluses from stabilizing public debt for arbitrary values of unexpected inflation. Fiscal policy is therefore active, and provides the additional unstable root to determine unexpected inflation.

The fiscal decompositions of unexpected inflation variance and for the recession scenario are qualitatively similar for New Zealand and the United States. It turns out that we can have the open-economy model yield these decompositions for New Zealand using the same set of parameters estimated for the United States. Figure 8 shows the terms of decomposition 2.[2] In general, the model agrees with VAR estimates in the case of the first two (variance and recession), but not the third (real depreciation). Second moments are also partially at odds with the data, as New Zealand and US moments differ.

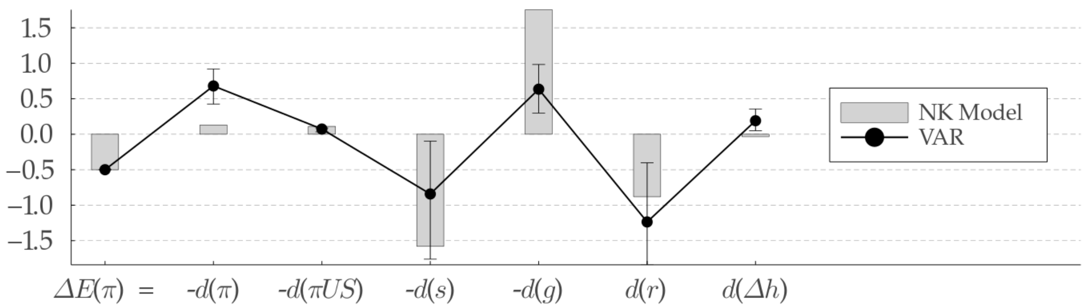
- *Like in a closed economy, domestic productivity shocks alone generate the variance and recession decompositions in the SOE-NK model. Due to the risk-sharing condition, temporary shocks on Foreign's productivity*

[1] I use the bar notation on $\bar{\omega}$ to differentiate it from the geometric term structure parameter of public debt.

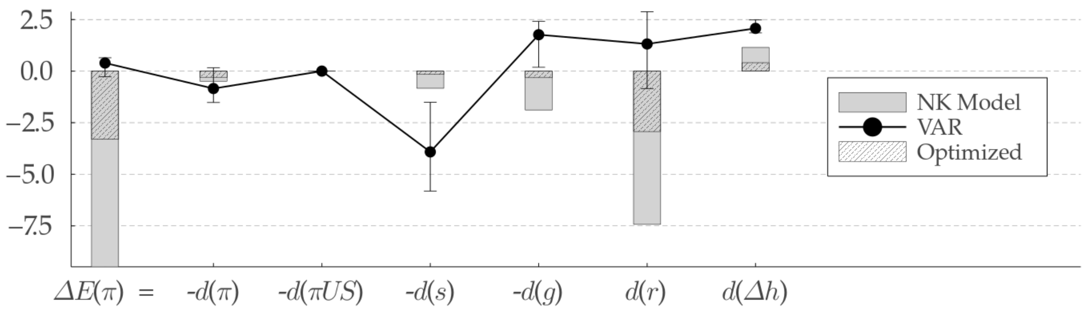
[2] You can check decomposition 1 in appendix F.



(a) Variance Decomposition $\Delta E_t \pi_t = 1$



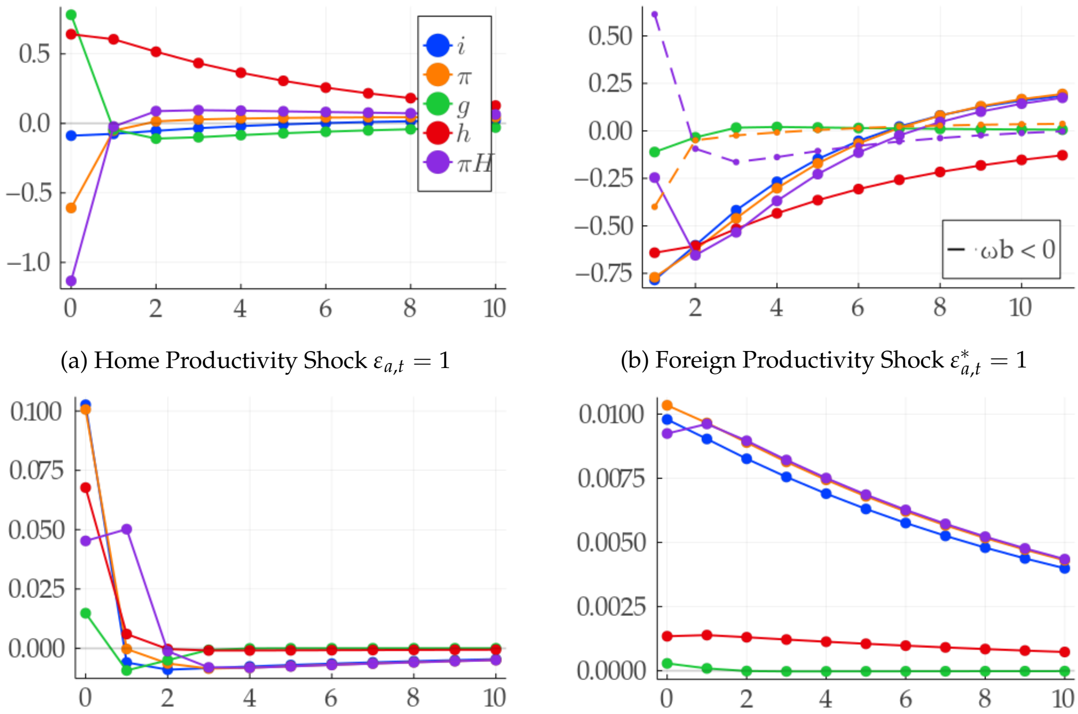
(b) Recession Scenario $\Delta E_t \pi_t = -0.5$, $\Delta E_t g_t = -1$



(c) Currency Depreciation Scenario $\Delta E_t h_t = 10$, $\Delta E_t i_t^* = \Delta E_t \pi_t^* = \Delta E_t g_t^* = 0$

Notes: The graphs depict the fiscal decompositions in the model's Home economy (SOE-NK). The hatched bars in plot 8c correspond to the model optimized to replicate the real depreciation scenario and nothing else ($w = 1$ in the objective function (23)). The ribbons correspond to the 25 and 75 percentiles of the posterior distribution.

Figure 8: Fiscal Decomposition 2 - Home Economy (SOE-NK Model)



(a) Home Productivity Shock $\varepsilon_{a,t} = 1$

(b) Foreign Productivity Shock $\varepsilon^*_{a,t} = 1$

(c) Foreign Interest Shock $\varepsilon^*_{i,t} = 1$

(d) Foreign Surplus Shock $\varepsilon^*_{s,t} = 1$

Notes: IRFs of the Home country in the NK model, which works as an open economy (SOE-NK). In plot 9b, the dashed lines correspond to $\bar{\omega} = -0.3$ (compared to baseline $\bar{\omega} = 2.5$).

Figure 9: IRFs for the Home Economy (SOE-NK)

generates:

a. unexpected inflation of equal sign in both countries, like in the VAR-based variance decomposition 2;

b. the same pattern of marginal costs in Home and Foreign, so that Foreign recessions with declining marginal costs replicate the recession decomposition in Home.

Foreign policy shocks move Foreign and Home inflation in opposite directions, contrary to the VAR-based variance decomposition.

Figure 9 plots Home's response to four shocks of the model. The responses to domestic policy shocks and to the trend shock are sufficiently similar to those of the closed economy to defer its graphs to appendix F. Figure 9a is the IRF to a Home productivity shock $\varepsilon_{a,t}$. The real exchange depreciates due to the risk-sharing condition (27). At least in the first few periods, the model keeps the property of negative marginal costs (figure 3b). Like in the closed-economy case, that allows for a period of negative real interest rates.[1]

The similar responses of the economy to domestic shocks does not imply that the terms of the decompositions will be the same in the Home economy. Foreign shocks can also lead to unexpected inflation and change its sources of variation. In estimating the parameters of the Home economy, we practically shut down policy shocks. So, it is really Foreign productivity shocks $\varepsilon^*_{a,t}$ we should worry about. Its IRF is shown in figure 9b. Like its effect in the Foreign country, the shock leads to a protracted deflation in Home. That the inflation rate responds in the same way at Home and Foreign for at least one structural shock is an important fact to replicate decomposition 2 of unexpected inflation variance (recall that $d_2(\pi^{US}) > 0$, unexpected inflation at Home coincides with some inflation abroad). As

[1]Compared to the closed-economy case, domestic output inflation $\pi_{H,t}$ reverts back to zero considerably faster in the open economy model. This is due to the lower impact of elevated output on firms' marginal costs ($\kappa < \kappa^*$), which in turn follows from $\bar{\omega} > 1$. Setting a lower value to $\bar{\omega}$ does not change the shape of the responses or the implications for the decompositions.

figures 9c and 9d show, unexpected inflation responds differently in the cases of policy shocks in Foreign. It is nice to verify that the one shock necessary to replicate the decomposition of unexpected inflation variance in the large, "closed" economy can also match the inflation term of decomposition 2 in the small, open one.[1]

The response of the Home economy to a Foreign productivity shock $\varepsilon^*_{a,t}$ also helps in replicating the recession decomposition. Returning to figure 9b, surprise deflation follows again from the effects of the appreciated currency on dollar-denominated bond prices and the price of imported goods. The appreciated currency in turn follows from the risk-sharing condition (27) and $y^*_t > 0$. In equilibrium, since $\bar{\omega}$ is positive and large, Home output falls, and so does marginal cost $\kappa y_t - \kappa_z z_t$ (the output term dominates, $\kappa = 12.45$, $\kappa_z = 1.53$).[2] This allows for the increasing inflation and negative real interest. Therefore, just like in the closed-economy case, we can combine a negative trend shock with a positive temporary shock abroad $\varepsilon^*_{a,t}$ to generate negative unexpected inflation and a negative real interest term $d_2(r) < 0$, as plot 8b shows. Indeed, the structural shocks required for the recession scenario $\Delta E_t \pi_t = -0.5$, $\Delta E_t g_t = -1$ are

$$\varepsilon^*_{a,t} = 0.64 \qquad \varepsilon_{a,t} = 0.18 \qquad \varepsilon_{g,t} = -1.06,$$

and (about) zero for the other shocks. Combined, they translate a permanent decline in the productivity of both Home and Foreign, with a faster convergence in the Home economy.

The argument above appears to hinge on the property that Home output falls in response to the *positive* productivity shock in Foreign. It does not. The dashed lines in figure (9b) correspond to the case $\bar{\omega} = -0.3$. The negative $\bar{\omega}$ inverts the effect of terms of trade on domestic output (equation (31)), which now increases (not plotted). The sign of marginal costs also switches, as evidenced by the declining Home output inflation $\pi_{H,t}$. CPI inflation π_t, nevertheless, continues to be increasing thanks to the positive slope of the real exchange rate (see equation (29)).

- *The SOE-NK model cannot replicate the VAR-based decomposition of the real exchange depreciation scenario. Home productivity and monetary policy shocks that depreciate the real exchange rate cause positive growth and a protracted period of negative real interest, both at odds with the VAR measures.*

The stylized SOE-NK model cannot replicate the pattern identified - especially for emerging markets - of table 6. The hatched area in figure 8c shows the model-implied decomposition for New Zealand, which conveniently follows the general patterns identified for emerging markets. The reduced-form shocks used to compute the real depreciation scenario are $\Delta E_t \Delta h_t = 10$ and $\Delta E_t a^*_t = \Delta E_t g^*_t = \Delta E_t i^*_t = \Delta E_t \pi^*_t = 0$. The zero equalities turn off Foreign structural shocks and the trend shock $\varepsilon_{g,t}$. To give the model the best possible chance, I set $w = 1$ in the objective function (23) and exclude second moments. I also allow arbitrary ρ_a. Foreign economy parameters are still the same as in table 7.

Even with all the "help", the model decomposition is embarrassingly different from the VAR measure. The first reason is the Backus-Smith anomaly (Backus et al. (1992), Backus and Smith (1993)): in the model, a real depreciation corresponds to an increase in output due to the risk-sharing condition (27). In the data, it does not. The VAR decompositions translate that through $d_2(g) < 0$. The NK model only gets us as far as $d_2(g) \approx 0$, through an estimated $\rho_a \approx 0$.

Home economy shocks can only generate real exchange rate innovation if they are productivity or (negative) monetary shocks. Fiscal shocks do not (see figure (5h)). In both of those cases, the shock leads to negative real interest, which is again at odds with the VAR measures. Finally, the NK model associates negative inflation with the depreciation scenario ($\Delta E_t \pi_t < 0$) because inflation declines following the productivity shock, which is the main driver of real exchange unexpected variation. As we saw, the VARs did not imply such a clear response in the real exchange depreciation scenario.

[1] This is not a coincidence, however. Since we have effectively shut down Foreign policy shocks, the only remaining one that can lead to Foreign unexpected inflation is productivity $\varepsilon^*_{a,t}$. Hence, the optimizing algorithm of problem (23) will try to generate, in response to $\varepsilon^*_{a,t}$, unexpected inflation in Home and Foreign of the same sign to match the VAR decomposition.

[2] There is risk-sharing: unplotted Home consumption increases as a result of the positive shock abroad.

Parameters	New Zealand[c]	Sweden	Denmark	Australia
A. Variance Decomposition - Value of Debt Contribution[a]				
$-d(s)$	0.40 (0.66)	**-0.34 (0.04)**	0.42 (0.48)	2.09 (2.04)
$-d(g)$	0.87 (0.63)	0.98 (1.03)	**-0.04 (-0.02)**	0.66 (0.97)
$d(r)$	0.68 (0.90)	1.42 (1.10)	1.21 (1.17)	**-1.06 (-0.52)**
B. Estimated Parameters				
ρ_a	0.84[b]			
ρ_g	0.27[b]			
ϕ_π	0.93	0.99	0.96	0.91
ϕ_g	0.61	0.63	0.71	1
ρ_s	0	0.26	0.61	0.05
τ_π	0.25	0.01	-0.02	-0.03
τ_g	0.15	-0.10	-0.13	0.25
σ_i	0.05	0	0	0.51
σ_s	0.08	0	0.12	0.50
σ_a	1.41[b]			
σ_g	2.62[b]			
C. Productivity Shocks Projected by $\Delta E_t \pi_t = 1$				
$\varepsilon_{g,t}$	-0.35	-0.48	0.12	-0.74
$\varepsilon^*_{a,t}$	-0.77	-0.82	-0.06	-0.08
$\varepsilon_{a,t}$	-0.61	-0.76	-1.49	-0.16

[a] The table reports the terms of decomposition 1 and, in parentheses, that generated by the model after optimization (23).
[b] Parameters governing the productivity process are kept constant.
[c] In the case of New Zealand, I use the parameters estimated to reproduce United States decomposition.

Table 9: Alternative Decomposition Profiles - Fit and Estimates

5.4. *Policy Shocks and the Fit of Alternative Decompositions*

Table 7 parameters were estimated to replicate the terms of the decomposition for the United States. They lead to positive contributions of surplus-to-output, output growth and real discounting to unexpected inflation volatility, in both decompositions 1 and 2. But VAR estimates imply that this is not the case for all countries. The overall contribution from intrinsic debt value variation can result from one negative and two positive terms. Can the SOE-NK model replicate these alternative decompositions? How should we change structural parameters? I answer this question by re-solving problem (23) for three different countries (Sweden, Denmark and Australia), each containing a negative term among the three that form the value of debt contribution to unexpected inflation. Parameters estimated to the Foreign economy remain the same. I also keep constant parameters related to technology, so that changes are only attributable to different monetary and fiscal policy. I also change the public debt profile (v, δ, ω) to match the case of each country. I continue to include as targeted moments both the variance and the recession decompositions in $\mathcal{D}$ (although I focus on the former) as well as second moments $\mathcal{M}$.

Table 9 reports results.[1] Panel A shows the terms of decomposition 1 in the VAR (they repeat table 3) and, in parentheses, those implied by the re-optimized models. Panel B reports estimated parameters. How to interpret the changes in estimated parameters? Different policy rules change the response of the model to structural shocks. Critically, they change the associated *unexpected inflation*. Hence, given the condition $\Delta E_t \pi_t = 1$ that defines the decomposition of unexpected inflation variance, the new parameters select different combinations of structural shocks that lead to it (the same is true for the recession shock). The optimization then tries to make more "inflationary" the structural shocks that best reproduce the terms of the VAR decompositions $\mathcal{D}_{VAR}$.

The case of Denmark is instructive. VAR estimates imply that unexpected inflation forecasts *positive* total growth, although the estimate is close to zero ($d_1(g) = 0.04$). The NK model can replicate the

[1] I report in figures 12-14 of appendix F the data/model decompositions, as in figure 6.

decomposition with a surplus-to-output process that is more persistent then in the US/New Zealand baseline, and that responds negatively to output growth. Contrary to the baseline, a positive trend shock now leads to *positive* unexpected inflation. With the trend shock, surplus declines ($\tau_g < 0$) for a long time (large ρ_s), which reduces the value of debt and generates inflation. Hence, unexpected inflation will sometimes follow from positive trend shocks and thus forecast positive total growth.

In Australia, unexpected inflation forecasts negative real interest ($d_1(r) < 0$). To replicate this decomposition, the key parametric change is the increase of output growth on the Taylor rule ϕ_g, which leaves the negative trend shocks considerably more inflationary. This is why projected $\varepsilon^*_{a,t} = -0.08$ and $\varepsilon^*_{a,t} = -0.16$ in the variance decomposition decrease. The decline in output following the trend shock leads the central bank to reduce interest more strongly. As bond prices rise, the market value of debt grows and causes higher unexpected inflation. Like in the baseline case (see figure 11c), the negative trend shock leads to a few periods of negative real interest. The result follows.

Lastly, in the case of Sweden, unexpected inflation forecasts larger surplus-to-GDP ratios ($d(s) > 0$). With a more persistent surplus process that responds less to inflation and negatively to growth, we reduce the model-implied coefficient from 0.4 to 0.04, about halfway to the required change to -0.34. In this case, the projected structural shocks are similar to those of the New Zealand case, but discounted surpluses are higher for each of them.

6. Conclusion

In a large class of macroeconomic models, the price level is such that the market value of debt equals its intrinsic value, discounted surpluses. Following Cochrane (2022a), I explore that equilibrium condition to estimate a decomposition of unexpected inflation. The key contribution of this paper is to present a method that allows for the estimation of such decomposition despite the many limitations imposed by currently available data. I apply the method for a group of twenty-five countries, focusing on the sources of unexpected inflation variability, and the sources of unexpected inflation responses in "aggregate demand" recessions scenarios (featuring low growth and low inflation) and real exchange rate depreciation scenarios. I find that, despite the fiscal nature of the decomposition, the surplus-to-GDP ratio - a typical gauge of fiscal policy - is far from the only driver of unexpected inflation. Growth, discounting and, for countries with dollar-denominated debt, foreign prices are also quantitatively important. In addition, our estimates point to a key role of monetary policy. By affecting bond prices, central banks can smooth inflation over time and reduce the size of unexpected price level jumps.

I have also shown how simple NK models, of closed and small open economies, can reproduce the main empirical regularities. My analysis highlights the importance of productivity shocks in reproducing the VAR measures of the decompositions, although the common AR(1) assumption has proven to be too restrictive. I do not find policy *shocks* to be of critical importance, although policy *rules* certainly are. The right cyclicality of surplus-to-output ratios is necessary to reproduce the contributions of surpluses in the decomposition. Strong Taylor rules do the same for real discounting. Additionally, different combinations of policy rule parameters can account for the decompositions measured for different countries by the VARs. Lastly, the model presented in this paper follows the fiscal theory of the price level. It shows how active fiscal policy does *not* imply a perfect connection between surpluses and inflation, and how it can reproduce the VAR measures of the decomposition (although observational equivalency implies that any statements like these holds equally for models with "active monetary policy").

There are many venues available for future literature to expand both the empirical and theoretical results described here. Empirically, models that account for time-varying structures of public debt can reveal new mechanisms. Alternative scenarios to the ones I have considered could also prove interesting, and help the development of new models. Time-varying risk premia can be shown to be quantitatively important, particularly for emerging markets. The experience of COVID deficits, recessions and posterior worldwide inflation - and how they eventually unfold in the broader economy - will also provide valuable information to future estimations of the decompositions. More and better data is always helpful. As for theory, new mechanisms can be considered to reproduce the VAR measures. For instance, I have argued that the assumption of complete markets is too strong to replicate the decompositions in the real depreciation scenario. How would an incomplete markets model succeed in doing so? Alternatively, what would be the fiscal decompositions for other structural shocks, not captured in the stylized environment I presented? In addition, the models described in this paper are admittedly simple and, more importantly, estimated to reproduce the decompositions. It is not clear if and how more complex models - designed for alternative purposes such as data fitting - will continue to reproduce the decompositions. They do, in any case, provide helpful benchmarks that can discipline the development of realistic monetary models.

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A. Data Sources and Treatment

A.1. Sources

I collect a significant share of the data from the St. Louis Fed's *FRED* website. In the case of countries with sample starting after 1970 I get data from the United Nations's National Accounts Main Aggregates Database. Their database also contains exchange rate data, which I use only in the case of emerging markets (with sample starting after 1998).

Whenever omitted in the list below, the source for interest rate data is the FRED; and the source of debt structure data is the OECD's Central Government Debt database. Finally, unless otherwise noted, public debt data I get from the database from Ali Abbas et al. (2011), which is kept up-to-date.

Australia 1973-2021. All except GDP and public debt from FRED.

Brazil 1998-2021. Debt structure data I collect from the Brazilian Central Bank.

Canada 1960-2021. All except public debt from FRED.

Chile 1998-2021.

Colombia 1998-2021. Debt structure data I collect from the Internal Debt Profile report, available at the Investor Relations Colombia webpage.

Czech Republic 1998-2021.

Denmark 1960-2021. All except public debt from FRED.

Hungary 1998-2021.

India 1998-2021. Debt structure data collect from the Status Paper on Government Debt report, 2019-2020, available at the Department of Economic Affairs.

Indonesia 1998-2021. Debt structure data I gather from the 2014 "Central Government Debt Profile" report and the 2018 "Government Securities Management" report, both from the Ministry of Finance.

Israel 1998-2021.

Japan 1960-2021. All except public debt from FRED.

Mexico 1998-2021.

Norway 1960-2021. All except public debt and interest rates from FRED. I interpolate the debt data for the year 1966. FRED interest data goes back to 1979, I splice it with historical data from Eitrheim et al. (2007), available at the website of the Norges Bank.

New Zealand 1973-2021. All except GDP and public debt from FRED.

Poland 1998-2021.

Romania 1998-2021. Interest rate is the deposit rate series from IMF's International Finance Statistics. Debt structure data I collect from the 2018 "Flash Report on the Romanian Public Debt" and the 2019-2021 and 2021-2023 "Government Debt Management Strategy" report, all from the Treasury and Public Debt Deparment (Ministry of Public Finance).

South Africa 1998-2021. Debt structure data from the 2020/2021 Debt Management Report, from the National Treasury Department.

South Korea 1973-2021. All except GDP and public debt from FRED.

Sweden 1960-2021. All except public debt from FRED. I interpolate the debt data for the year 1965 and 1966.

Switzerland 1973-2021. Interest, CPI and exchange rate from FRED.

Turkey 1998-2021.

Ukraine 1998-2021. Interest rate data from National Bank of Ukraine (NBU Key Policy Rate). Debt structure data I collect from "Ukraine's Public Debt Performance in 2021 and Local Market Update", from the Ministry of Finance of Ukraine.

United Kingdom 1960-2021. Interest data from the Bank of England (Base Rate); I splice it with discount rate data from the FRED. Inflation and GDP data from FRED.

United States 1950-2021. All data collected from the FRED. I use real exchange rate to the United Kingdom, since nominal exchange rate is available since before 1950.

B. Additional Details of the BVAR Estimation

This section describes in more detail the estimation steps of the VARs (17) and (18). The first step involves estimation of the posterior distribution of the parameters $\tilde{a}$, $\tilde{b}$, $\tilde{a}_u$, Σ and Σ_u. These parameters govern the law of motion (19) of observable variables $\tilde{x}_t$ (and $\tilde{u}_t$ in the US case). These are: the interest rate, inflation, the par-value of public debt, the real exchange rate and the growth rate of real GDP. Consider the stacked VAR $X_t = [x_t' \; u_t']'$ and its law of motion

$$X_t = AX_{t-1} + Ke_t.$$

The mode of the posterior distribution determines the coefficients of A in the rows corresponding to the observed variables, both for the country whose VAR we are estimating and of the US. The corresponding rows of K form the identity matrix. In the description of the following steps, I will refer to A and K as if we had them fully estimated - this will simplify exposition a lot. In each step, consider that the rows corresponding to the equations not yet described to be zero. For example, after only estimating the rows corresponding to observed variables, incomplete matrices A and K equal

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \tilde{a} & 0 & \tilde{b} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \tilde{a}_u \end{bmatrix} \qquad \begin{bmatrix} 0 & 0 \\ I & 0 \\ 0 & 0 \\ 0 & I \end{bmatrix}$$

(I order variables in x and u with unobserved ones on top, observed ones at the bottom).

The next step involves computation of the ϕ_j parameters used to calculate excess returns $rx_{j,t} = \phi_j' e_t$. Starting with nominal debt, applying the VAR to equation (12) yields

$$q_t = -\omega\beta \mathbf{1}_i'(I - \omega\beta A)^{-1} X_t,$$

where I is the identity matrix (recall that I omit the "N" subscript when referring to nominal bonds, and that $\mathbf{1}_y$ is a vector of zeros and one in the entry of variable y). Replacing the expression above in the definition of excess return (11) leads to

$$\begin{aligned} rx_t &= -\omega\beta \mathbf{1}_i'(I - \omega\beta\tilde{a})^{-1} K e_t \\ &\equiv \phi_{N,t}' e_t. \end{aligned}$$

By construction, the expectations hypothesis holds: $E_t rx_{t+1} = 0$ does not depend on X_t. An analogous procedure leads to the formula for the other two ϕ's:

$$\begin{aligned}\phi_{R,t} &= -\omega_R\beta(\mathbf{1}'_i - \mathbf{1}'_\pi A)(I-\omega_R\beta A)^{-1}K\\ \phi_{D,t} &= -\omega_D\beta\mathbf{1}'_{i^{US}}(I-\omega_D\beta A)^{-1}K.\end{aligned}$$

Symbol i^{US} refers to the US interest rate, which enters X_t. The text also defines $\zeta' = \mathbf{1}'_i - \mathbf{1}'_\pi A$, which computes the real interest rate, $i_{R,t} = i_t - E_t\pi_{t+1} = \zeta' X_t$.

Step 3 of the estimation procedure involves the filling of the rows of A and K relative to the average interest rates $\{i^b_{j,t}\}$, the market value of debt v_t and primary surplus s_t. Using (16) and VAR equation, we have:

$$i^b_t = (1-\omega)i_t + \omega i^b_{t-1} = (1-\omega)\mathbf{1}'_i X_t + \omega\mathbf{1}'_{i^b}X_{t-1} = (1-\omega)\mathbf{1}'_i\left[AX_{t-1}+Ke_t\right] + \omega\mathbf{1}'_{i^b}X_{t-1}.$$

So, we can fill the rows of A and K storing the i^b_t equation with

$$\begin{aligned}\mathbf{1}'_{i^b}A &= (1-\omega)\mathbf{1}'_iA + \omega\mathbf{1}'_{i^b}\\ \mathbf{1}'_{i^b}K &= (1-\omega)\mathbf{1}'_iK.\end{aligned}$$

Analogously, the rows corresponding to the average real interest rate are

$$\begin{aligned}\mathbf{1}'_{i^b_R}A &= (1-\omega_R)\zeta' A + \omega_R\mathbf{1}'_{i^b_R}\\ \mathbf{1}'_{i^b_R}K &= (1-\omega_R)\zeta' K,\end{aligned}$$

and those corresponding to the average interest on dollar-denominated debt are

$$\begin{aligned}\mathbf{1}'_{i^b_D}A &= (1-\omega_D)\mathbf{1}'_{i^{US}}A + \omega_D\mathbf{1}'_{i^b_D}\\ \mathbf{1}'_{i^b_D}K &= (1-\omega_D)\mathbf{1}'_{i^{US}}K.\end{aligned}$$

To fill the equation of the market-value of debt, I use (15). Re-write it using the VAR as

$$\begin{aligned}v_t = \mathbf{1}'_{v^b}AX_{t-1} + \mathbf{1}'_{v^b}Ke_t + \frac{v}{\beta}\Big[&\delta\left(\varphi'_N e_t + \mathbf{1}'_iX_{t-1} - \mathbf{1}'_{i^b}X_{t-1}\right)\\ &+ \delta_R\left(\varphi'_R e_t + \zeta' X_{t-1} - \mathbf{1}'_{i^b_R}X_{t-1}\right) + \delta_D\left(\varphi'_D e_t + \mathbf{1}'_{i^{US}}X_{t-1} - \mathbf{1}'_{i^b_D}X_{t-1}\right)\Big].\end{aligned}$$

Gathering terms:

$$\begin{aligned}\mathbf{1}'_vA &= 1'_{v^b}A + \frac{v}{\beta}\left[\delta\left(\mathbf{1}'_i - \mathbf{1}'_{i^b_R}\right) + \delta_R\left(\zeta' - \mathbf{1}'_{i^b_R}\right) + \delta_D\left(\mathbf{1}'_{i^{US}} - \mathbf{1}'_{i^b_D}\right)\right]\\ \mathbf{1}'_vK &= 1'_{v^b}K + \frac{v}{\beta}\left[\delta\varphi'_N + \delta_R\varphi'_R + \delta_D\varphi'_D\right].\end{aligned}$$

Lastly, I do a similar procedure using the law of motion for public debt (9) to fill the row corresponding to primary surpluses.

$$\begin{aligned}\mathbf{1}'_sA &= \frac{1}{\beta}\mathbf{1}'_v - \mathbf{1}'_vA + \frac{v}{\beta}\left[-\mathbf{1}'_gA + \delta\left(\mathbf{1}'_i - \mathbf{1}'_\pi A\right) + \delta_R\zeta' + \delta_D\left(\mathbf{1}'_{i^{US}} + \mathbf{1}'_{\Delta h}A - \mathbf{1}'_{\pi^{US}}A\right)\right]\\ \mathbf{1}'_sK &= -\mathbf{1}'_vK + \frac{v}{\beta}\left[-\mathbf{1}'_gK + \delta\left(\varphi'_N - \mathbf{1}'_\pi K\right) + \delta_R\varphi'_R + \delta_D\left(\varphi'_D + \mathbf{1}'_{\Delta h}K - \mathbf{1}'_{\pi^{US}}K\right)\right]\end{aligned}$$

C. Alternative Reduced-Form Shocks

C.1. Primary Deficit Shock

I project VAR shocks onto $\Delta E_t s_t = -1$, which captures a scenario of fiscal deficit - for any possible reason. Estimates of decomposition 2 reported in table 10.

C.2. International Monetary Policy Shock

I project VAR shocks onto $\Delta E_t i_t^{US} = \Delta E_t \pi_t^{US} = 1$. The shocks are designed to capture events in which the large economies tighten monetary policy to fight inflation. I model the scenario by setting unexpected United States interest and inflation to +1. Estimates of decomposition 2 reported in table 11.

D. Equilibrium Selection in the NK Model

The New-Keynesian model (21) does not determine unexpected inflation $\Delta E_t \pi_t$. Any value of unexpected inflation is consistent with a stationary equilibrium. There are two existing selection mechanisms: fiscal selection and spiral threat selection. Both pin down $\Delta E_t \pi_t$ while leaving other equations unchanged. This feature characterizes the *observational equivalence proposition* (Cochrane (2011), Cochrane (1998)), which states that, in the absence of further assumptions, one cannot use data to test selection mechanisms.

Fiscal Selection. Fiscal selection, or the fiscal theory of the price level, determines unexpected inflation by means of (5), with causality running from right to left. Any economic shock can change the conditional distribution of discounted future surpluses (in units of goods) backing the stock of public nominal liabilities. It can thus change its real value. The relative price of public debt in terms of goods $1/P_t$ then adjusts to reflect that change, much in the same way that news of future dividends (in currency units) per stock change the relative price of stocks in terms of currency (Cochrane (2005)).

Spiral-Threat Selection. Spiral threat selection is the approach that most of the monetary economics literature has adopted so far. It starts by attributing causality in equation (5) from left to right: to any price level, no matter how large or small, the government alters its surplus choice to reflect the new value of public debt. It then pins down unexpected inflation by means of an explosive root introduced by an interest policy equation of the format $i_t = \phi \pi_t, \phi > 1$. The equation was associated to the celebrated Taylor (1993) rule, but its role in the NK model is not to stabilize "demand" shocks via rapidly-adjusted, pro-cyclical real interest rates. On the contrary, the policy rule here introduces the instability required by the NK model to pin down unexpected inflation. Assuming muted monetary policy $i_t = 0$, the system of equations (21) is "too stable": it contains one explosive eigenvalue for two forward-looking variables. Any choice of unexpected inflation forms a stable equilibrium path that converges to the zero steady state.[1] Equation $i_t = \phi \pi_t$ solves that issue when $\phi > 1$.

Importantly, the *selection of equilibrium* is completely unrelated to the *observed* interest rate. For instance, $i_t =$ white noise would be a perfectly valid specification for *observed* interest. More rigorously, consider the simplest possible NK model

$$\begin{aligned} x_t &= E_t y_{t+1} - \gamma(i_t - E_t \pi_{t+1}) \\ \pi_t &= \beta E_t \pi_{t+1} + \kappa x_t, \end{aligned} \tag{NK}$$

[1] Economists have interpreted this feature as admissibility of "sunspot" shocks. Without a selection mechanism, (21) will only determine the unexpected component of one variable, *if* it is fed the unexpected component of the other.

Country	$\Delta E_t \pi_t =$	Decomposition 2: $\Delta E_t s_t = -1$					
		$-\Delta E_t$(Future Inflation)		$\pm\, \Delta E_t$(Real Variables)			
		$-d_2(\pi)$	-$d_2(\pi^{US})$	$-d_2(s)$	$-d_2(g)$	$d_2(r)$	$d_2(\Delta h)$
Averages	***0.13**	-0.02	***-0.08**	-0.89	***0.92**	0.18	0.02
Advanced - 1960	**0.02**	**0.12**	***-0.01**	-0.13	**0.31**	-0.28	**0.02**
Advanced - 1973	***0.33**	***-0.27**	***-0.01**	1.04	0.39	-0.79	**-0.02**
Emerging - 1998	***0.13**	-0.02	**-0.13**	**-1.84**	**1.30**	**0.78**	0.04
Median	***0.08**	-0.02	0	0.01	***0.41**	0.16	0
Advanced - 1960	0	***0.08**	0	**-0.19**	**0.27**	-0.16	***0.01**
Advanced - 1973	***0.29**	-0.31	***-0.01**	0.66	0.35	-0.57	0
Emerging - 1998	***0.13**	-0.03	-0.01	-0.13	**0.47**	**0.47**	-0.07
United States	***-0.11**	***0.10**	-	0.09	***1.24**	***-1.55**	-
Advanced - 1960 Sample							
Canada	0.01	***0.32**	0	-0.47	0.26	-0.16	***0.05**
Denmark	***0.12**	***0.29**	***-0.04**	**-0.72**	**0.41**	0.16	0.02
Japan	0.04	-0.01	0	**-0.30**	***1.27**	***-0.92**	0
Norway	0	**0.08**	0	-0.04	0.12	-0.16	0
Sweden	-0.03	**0.07**	***-0.03**	-0.09	***-0.45**	**0.44**	0.03
United Kingdom	0	-0.06	0	**0.83**	0.27	**-1.04**	0
Advanced - 1973 Sample							
Australia	**0.08**	**-0.51**	0	**2.80**	0.63	**-2.84**	0
New Zealand	***0.41**	***-0.40**	***-0.03**	0.05	0.06	**0.81**	**-0.10**
South Korea	***0.65**	-0.22	***-0.01**	1.10	0.79	-1.00	0
Switzerland	***0.17**	0.02	0	0.21	0.07	-0.13	0
Emerging - 1998 Sample							
Brazil	**0.12**	**-0.03**	***0.02**	0.11	-0.24	0.12	***0.14**
Chile	***0.20**	**0.60**	0.86	-5.19	4.10	-2.32	2.15
Colombia	-0.02	**-0.27**	0.03	-0.27	0.20	0.27	0.03
Czech Republic	***0.24**	**0.40**	**0.04**	**-3.92**	***3.13**	0.60	0
Hungary	0.03	0.04	***-0.23**	**-9.93**	***5.86**	***4.55**	**-0.27**
India	-0.11	-0.32	0.02	**0.95**	**-0.36**	-0.26	***-0.14**
Indonesia	**0.14**	***-0.60**	***-2.13**	1.36	**1.56**	0.50	-0.53
Israel	***0.30**	-0.08	***-0.21**	0.03	**0.51**	0.20	-0.14
Mexico	***0.22**	***-0.32**	**-0.18**	0.46	-0.47	***1.06**	**-0.31**
Poland	0.01	0.01	***-0.11**	0.43	**1.05**	**-1.22**	**-0.14**
Romania	-0.07	-0.02	0.12	-0.48	-0.38	1.08	-0.39
South Africa	**-0.14**	***0.76**	**-0.06**	***-8.71**	**3.20**	***4.57**	0.09
Turkey	***0.41**	-0.03	-0.02	-0.56	**-0.37**	***1.36**	0.03
Ukraine	***0.48**	**-0.41**	0	0.01	**0.44**	***0.45**	0

Notes: I project VAR shocks on $\Delta E_t s_t = -1$. The table shows the estimated terms of decomposition 2 at the posterior distribution's mode. I divide each term by $\delta v/\beta$, so the terms sum up to $\Delta E_t \pi$. I simulate from the posterior to check the statistical significance of the sign of the estimate. Bold indicates 75% of results had the same sign. Bold with an asterisk indicates 90%.

Table 10: Unexpected Inflation Decomposition 2 - Primary Deficit Shock

Country	$\Delta E_t \pi_t =$	Decomposition 2: $\Delta E_t i_t^{US} = \Delta E_t \pi_t^{US} = 1$					
		$-\Delta E_t$(Future Inflation)		$\pm \Delta E_t$(Real Variables)			
		$-d_2(\pi)$	-$d_2(\pi^{US})$	$-d_2(s)$	$-d_2(g)$	$d_2(r)$	$d_2(\Delta h)$
Averages	***0.10**	***-0.54**	***-1.06**	1.16	-0.90	**1.44**	0
Advanced - 1960	***0.40**	***-1.06**	***-0.11**	-0.33	-0.42	***2.26**	***0.07**
Advanced - 1973	***0.18**	***-0.67**	***-0.12**	0.32	-0.53	1.06	**0.11**
Emerging - 1998	**-0.12**	-0.21	***-1.82**	2.12	-1.34	1.19	-0.06
Median	***0.15**	***-0.56**	***-0.27**	0.70	-0.76	**1.57**	0
Advanced - 1960	***0.41**	***-0.92**	***-0.05**	-0.69	-0.76	***2.39**	***0.01**
Advanced - 1973	**0.20**	***-0.65**	***-0.04**	-0.28	-0.03	1.10	0
Emerging - 1998	***-0.21**	**-0.27**	***-1.06**	**2.30**	-1.62	0.12	-0.7
United States	***1**	***-1.54**	-	0.09	0.85	**1.59**	-
Advanced - 1960 Sample							
Canada	***0.46**	**-0.68**	***-0.09**	**-1.54**	0.19	**2.57**	0.01
Denmark	***0.43**	**-0.43**	***-0.35**	-0.94	-0.76	**2.63**	***0.28**
Japan	***0.77**	***-0.99**	0	***1.42**	***-0.77**	***1.12**	0
Norway	***0.15**	***-0.85**	***-0.01**	-0.44	-0.77	**2.21**	***0.01**
Sweden	***0.40**	***-1.36**	***-0.21**	**-1.47**	-0.75	***4.09**	**0.10**
United Kingdom	**0.16**	**-2.05**	0	0.96	0.31	0.93	0
Advanced - 1973 Sample							
Australia	**-0.16**	**-0.56**	***-0.01**	-1.68	0.16	1.93	0
New Zealand	-0.06	**-0.62**	***-0.37**	-1.15	-0.21	1.81	***0.48**
South Korea	***0.49**	**-0.82**	***-0.08**	**3.53**	**-2.50**	0.40	-0.04
Switzerland	***0.45**	***-0.67**	0	0.59	**0.44**	0.10	0
Emerging - 1998 Sample							
Brazil	***-0.48**	***0.12**	***-0.08**	-3.35	-0.24	3.41	***-0.35**
Chile	**-0.18**	-0.27	***-9.08**	-10.71	1.49	**13.60**	**4.79**
Colombia	***-0.27**	0.33	***-1.95**	**6.97**	-3.60	0.32	**-2.33**
Czech Republic	***-0.26**	***-0.65**	***-0.27**	3.90	-3.29	-0.09	0.14
Hungary	***0.22**	**-0.43**	***-0.79**	9.19	-5.47	-2.09	-0.18
India	**-0.23**	-0.24	***-0.24**	0.26	0.17	-0.26	0.08
Indonesia	0.09	**-0.57**	***-4.20**	0.38	**2.31**	**1.57**	0.60
Israel	***-0.46**	-0.01	***-1.58**	0.70	**-2.04**	2.54	-0.06
Mexico	***-0.19**	0.27	***-1.33**	**5.43**	-1.69	-1.94	**-0.93**
Poland	***0.30**	**-0.28**	***-0.73**	1.61	-2.04	**1.82**	-0.07
Romania	***0.68**	**-0.53**	***-2.85**	6.05	0.86	-0.08	**-2.77**
South Africa	***-0.42**	-0.41	***-0.46**	4.60	-1.53	-2.42	-0.20
Turkey	***0.33**	***-1.19**	***-1.88**	2.22	-1.56	**2.32**	0.42
Ukraine	**-0.81**	0.96	0	2.38	-2.09	-2.05	***-0.01**

Notes: I project VAR shocks on $\Delta E_t i_t^{US} = \Delta E_t \pi_t^{US} = 1$. The table shows the estimated terms of decomposition 2 at the posterior distribution's mode. I divide each term by $\delta v/\beta$, so the terms sum up to $\Delta E_t \pi$. I simulate from the posterior to check the statistical significance of the sign of the estimate. Bold indicates 75% of results had the same sign. Bold with an asterisk indicates 90%.

Table 11: Unexpected Inflation Decomposition 2 - International Monetary Shock

with x interpreted as an output gap. Add to that the following equations:

$$i_t = i_t^* + \phi(\pi_t - \pi_t^*) \qquad \phi > 1 \tag{ST-1}$$

$$i_t^* - E_t\pi_{t+1}^* = i_t - E_t\pi_{t+1} \tag{ST-2}$$

$$i_t^* \text{ given for all } t \tag{ST-3}$$

$$\pi_t^* \text{ given at } t. \tag{ST-4}$$

Format (ST-1) is due to King and William (1996); i^* is the central bank's desired observed interest rate. The term π_t^* is a stochatic inflation target. Equation (ST-2) asks that the government's choices respect private market conditions and expectations formation. It forces the government to elect *unexpected* inflation only.[1]

Mechanically, one can combine (ST-1) and (ST-2) to find $E_t\pi_{t+1} - E_t\pi_{t+1}^* = \phi(\pi_t - \pi_t^*)$; $\phi > 1$ and Blanchard and Kahn (1980)'s razor then select the unique stationary path $\pi = \pi^*$, $i = i^*$, which form the *observed* equilibrium. Parameter ϕ remains unidentified (Cochrane (2011)).

Researchers have interpreted (ST-1) as a threat of nominal spiral - hence my name choice "spiral threat" selection. Different papers discuss if central banks can indeed rule out nominal spirals, but the key assumptions here do not really relate to what the central bank can do, but what *households believe* it can and would. Indeed, note that there is nothing particularly special about inflation in (ST-1)-(ST-4). One could as well write the whole system using an output target instead:

$$i_t = i_t^* + \phi(y_t - y_t^*) \qquad \phi > 1 \tag{ST-1'}$$

$$i_t^* - E_t y_{t+1}^* = i_t - E_t y_{t+1} \tag{ST-2'}$$

and now the "threat" is not that of a nominal spiral, but of a *real* spiral. Obviously, the central bank cannot trigger a "hyperproduction" (as in hyperinflation) process. Neither could it stop one, say if productivity for some reason started to grow at abnormal rates. But, if the central bank vacuously threatens hyperproduction, and it is the case that agents believe its threat; and if then the central bank vacuously promises to stop the hypothetical hyperproduction it has vacuously threatened to create, and again agents trust its word; then and only then does the Blanchard and Kahn (1980) equilibrium arranged by (ST-1')-(ST-2') arises. The actual powers of the central bank are irrelevant.

E. Deriving the SOE-NK Model

I derive the full model that leads to the linearized equations of section 5. Unless otherwise noted, lowecase letters indicate the log of their uppercase counterpart, *expressed as deviations from their steady state value*. For example, $p_t = \log P_t$. Two exceptions are real value of debt v_t and the primary surplus s_t.

There are two economies: Home and Foreign. Home contains a continuum of size $\tilde{n}$ of households and firms and a government. The foreign economy contains a continuum of size $1 - \tilde{n}$ of households and firms. We will focus on the limit $\tilde{n} \to 0$, which makes the Home economy open but "small", and the Foreign economy "large" and, in practice, closed. Agents in the model trade goods of different varieties (with each firm responsible for the production of a different one), labor hours, and different classes of assets. As currency in Home, agents use the balances in accounts held with the government. The analogous holds in Foreign. The timing of the model is the one described in section 2 of the main text. Households wake up every period and receive currency in exchange for maturing bonds and pother assets. They use it to purchase goods, foreign currency and new assets. Home and Foreign households trade a complete set of stage-contingent securities in addition to risk-free public bonds. Home government issues nominal, real and currency-denominated bonds. Foreign government issues

[1]The attentive eye may have noticed an apparent modelling sin: system (NK), (ST-1)-(ST-4) presents six equations, for only five variables: y, π, π^*, i, i^*. There is no over-identification, nevertheless. Target inflation enters the system both as a static (= forward-looking) variable π_t^* *and* as a state variable, in expected value $E_{t-1}\pi_t^*$. Another way to write (ST-3) would be $E_{t-1}\pi_t = i_t^* - (i_t - E_{t-1}\pi_t)$. It becomes evident then that (ST-4) only really picks the unexpected component of inflation.

only nominal and real debt.

Firms in the Home economy have access to a production function $f_t(N) = \mathcal{A}_t N = \mathcal{T}_t A_t N$; Foreigm firms use $f_t^*(N) = \mathcal{A}_t^* N = \mathcal{T}_t \mathcal{A}_t^* N$. The trend process $\mathcal{T}_t$ is non-stationary: $\log \mathcal{T}_t = \log \mathcal{T}_{t-1} + g + u_{g,t}$. Processes A_t and A_t^* are stationary, and follow the AR(1) motions described in the main text. I use the hat notation to refer to the level of a variable (for example, $\hat{C}_t$ denotes consumption) and omit the hat when referring to its detrended counterpart ($C_t = \hat{C}_t / \mathcal{T}_t$).

The utility function is separable, with period utility being equal to

$$U_t(C, N) = \frac{C^{1-\gamma^{-1}}}{1-\gamma^{-1}} - \frac{N^{1+\varphi}}{1+\varphi} \mathcal{T}_t^{1-\gamma^{-1}},$$

where C aggregates consumption of different varieties as in Dixit and Stiglitz (1977) (see below). The $\mathcal{T}_t$ term multiplying the disutility of labor adjusts for the fact that labor-separable preferences are not homothetic. I opt for this less common specification so that the de-trended linearized model is similar to the most common linearized NK model. Households discount future streams of consumption utility at a rate $\tilde{\beta}$.

E.1. Households

Households in Home solve the following sequential problem in period zero, already stated in its detrended form:

$$\begin{aligned}
\max_{\{C_H(j)\},\{C_F(j)\}} \quad & E \sum_{t=0}^{\infty} \tilde{\beta}^t \mathcal{T}_t^{1-\gamma^{-1}} \left[\frac{C_t^{1-\gamma^{-1}}}{1-\gamma^{-1}} - \frac{N_t^{1+\varphi}}{1+\varphi} \right] \\
\text{s.t.} \quad & C_t = \left[(1-(1-\tilde{n})\alpha)^{\frac{1}{\eta}} C_{H,t}^{\frac{\eta-1}{\eta}} + ((1-\tilde{n})\alpha)^{\frac{1}{\eta}} C_{F,t}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \\
& C_{H,t} = \left(\frac{1}{\tilde{n}} \int_0^{\tilde{n}} C_{H,t}(j)^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}} \\
& C_{F,t} = \left(\frac{1}{1-\tilde{n}} \int_{\tilde{n}}^{1} C_{F,t}(j)^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}} \\
& \int_0^{\tilde{n}} P_{H,t}(j) C_{H,t}(j) dj + \int_{\tilde{n}}^{1} P_{F,t}(j) C_{F,t}(j) dj + E_t \left(M_{t+1} B_{t+1} e^{g+u_{g,t+1}} \right) \leq B_t + W_t N_t - \hat{T}_t \\
& \lim_{t\to\infty} E_t \left(M_{t+1} B_{t+1} e^{g+u_{g,t+1}} \right) \geq 0
\end{aligned} \tag{33}$$

where $C_{H,t}(j)$ indicates consumption of variety j produced at Home, $C_{F,t}(j)$ indicates consumption of variety j produced by at Foreign, B_{t+1} denotes the detrended payoff of the selected portfolio in period $t+1$, M_{t+1} is the state price divided by its conditional probability (so that $E(MB)$ is the end-of-period value of the portfolio), W_t is the detrended wage rate and $\hat{T}$ denotes lump-sum taxes. As the size of the Home economy converges to zero, the weight of Foreign goods on the basket of Home households approaches α. Therefore, $1-\alpha$ is a measure of Home bias.

I define the following price indices, which lead to the minimum cost P_t of purchasing one unit of the aggregate basket of goods C_t.

$$P_{H,t} = \left(\frac{1}{\tilde{n}} \int_0^{\tilde{n}} P_{H,t}(j)^{1-\epsilon} ds \right)^{\frac{1}{1-\epsilon}} \qquad P_{F,t} = \left(\frac{1}{1-\tilde{n}} \int_{\tilde{n}}^{1} P_{F,t}(j)^{1-\epsilon} ds \right)^{\frac{1}{1-\epsilon}}$$

$$P_t = \left[(1-(1-\tilde{n})\alpha) P_{H,t}^{1-\eta} + (1-\tilde{n})\alpha P_{F,t}^{1-\eta} \right]^{\frac{1}{1-\eta}}$$

First-order conditions for $C_{H,t}$, $C_{H,t}(j)$, $C_{F,t}$ and $C_{F,t}(j)$ yield:

$$C_{H,t}(j) = \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\epsilon} C_{H,t} \qquad C_{F,t}(j) = \left(\frac{P_{F,t}(j)}{P_{F,t}}\right)^{-\epsilon} C_{F,t}$$
$$C_{H,t} = (1-(1-\tilde{n})\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t \qquad C_{F,t} = (1-\tilde{n})\alpha\left(\frac{P_{F,t}}{P_t}\right)^{-\eta} C_t.$$

Using the price indices the conditions above imply that $\int_0^{\tilde{n}} P_{H,t}(j)C_{H,t}(j)dj = P_{H,t}C_{H,t}$, $\int_0^{\tilde{n}} P_{F,t}(j)C_{F,t}(j)dj = P_{F,t}C_{F,t}$ and $P_tC_t = P_{H,t}C_{H,t} + P_{F,t}C_{F,t}$. To state the remaining first-order conditions, I use $(1-i_t)^{-1} = E_t(M_{t+1})$ (the price of the risk-free bond is the sum of the prices of the Arrow claims to each state of nature). The remaining first-order conditions are:

$$M_{t+1} = \tilde{\beta}\frac{u'(C_{t+1})}{u'(C_t)}\frac{P_t}{P_{t+1}}e^{-\gamma^{-1}(g+u_{g,t+1})} \implies (1+i_t)^{-1} = \tilde{\beta}E_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma^{-1}} e^{-\gamma^{-1}(g+u_{g,t+1})}\Pi_{t+1}^{-1}\right]$$
$$\frac{W_t}{P_t} = C_t^{\frac{1}{\gamma}} N_t^{\varphi}.$$

In a steady state, the Eules equation implies $\tilde{\beta} = \exp\{-(i-\pi-\gamma^{-1}g)\}$. The conditions linearize to

$$\begin{aligned} c_t &= E_t c_{t+1} - \gamma\left(i_t - E_t\pi_{t+1}\right) + E_t u_{g,t+1} \\ w_t - p_t &= \frac{1}{\gamma}c_t + \varphi n_t \end{aligned} \tag{34}$$

(recall that the lower-case letters denote log deviations from the steady state of each variable, $c_t = \log(C_t/C)$).

Preferences for foreign economy households are analogous to home economy households'. The share of Home goods on Foreign households' basket is $\tilde{n}\alpha$. Therefore, their demand for individuals goods follows

$$C^*_{H,t}(j) = \left(\frac{P^*_{H,t}(j)}{P^*_{H,t}}\right)^{-\epsilon} C^*_{H,t} \qquad C^*_{F,t}(j) = \left(\frac{P^*_{F,t}(j)}{P^*_{F,t}}\right)^{-\epsilon} C_{F,t}$$
$$C^*_{H,t} = \tilde{n}\alpha\left(\frac{P^*_{H,t}}{P^*_t}\right)^{-\eta} C^*_t \qquad C^*_{F,t} = (1-\tilde{n}\alpha)\left(\frac{P^*_{F,t}}{P^*_t}\right)^{-\eta} C^*_t.$$

Foreign price indices P^*_H, P^*_F and P^* are built analogously to Home indices, using the $\tilde{n}\alpha$ and $1-\tilde{n}\alpha$ weights.

The first-order conditions of Foreign households optimization problem also imply the following:

$$\begin{aligned} (1+i^*_t)^{-1} &= \beta E_t\left[\left(\frac{C^*_{t+1}}{C^*_t}\right)^{-\gamma^{-1}} e^{-\gamma^{-1}(g+u^*_{g,t+1})}\Pi^{*-1}_{t+1}\right] \\ \frac{u'(C^*_{t+1})}{u'(C^*_t)}\frac{H_t}{H_{t+1}} &= \frac{u'(C_{t+1})}{u'(C_t)} \\ \frac{W^*_t}{P^*_t} &= C_t^{*\frac{1}{\gamma}} N_t^{*\varphi}. \end{aligned}$$

The second equation above is the Backus and Smith (1993) condition of international risk sharing under complete markets. It implies that $(C_t/C_0) = (C^*_t/C^*_0)(H_t/H_0)^{\gamma}$. As we are expressing variables as deviations from steady state, this condition leads to the linearized form

$$c_t = c^*_t + \gamma h_t. \tag{35}$$

The other two first-order conditions linearize to

$$
\begin{aligned}
c_t^* &= E_t c_{t+1}^* - \gamma \left(i_t^* - E_t \pi_{t+1}^* \right) + E_t u_{g,t+1} \\
w_t^* - p_t^* &= \frac{1}{\gamma} c_t^* + \varphi n_t^*.
\end{aligned}
\tag{36}
$$

E.2. The Real Exchange Rate

Let $\mathcal{E}_t$ be the nominal exchange rate: the price of one unit foreign currency in units of home currency. In the language of table 2, $\mathcal{E} = \mathcal{E}_D$. The law of one price $P_{i,t}(j) = \mathcal{E}_t P_{i,t}^*(j)$, $i = H, F$ holds for each individual good. This leads to $P_{H,t} = \mathcal{E}_t P_{H,t}^*$ and $P_{F,t} = \mathcal{E}_t P_{F,t}^*$. However, due to home bias, $P_t \neq \mathcal{E}_T P_t^*$, and so the real exchange $H_t = \mathcal{E}_t P_t^* / P_t$ fluctuates. The foreign economy's price level is $P_t^* = \alpha \tilde{n} P_{H,t}^* + (1 - \alpha \tilde{n}) P_{F,t}^*$. As the size of the home economy $\tilde{n}$ converges to zero, we have $P_t^* = P_{F,t}^* = P_{F,t} / \mathcal{E}_t$. Therefore, in this limit we have $H_t = P_{F,t} / P_t$.

Define Home's terms of trade as $Z_t = P_{F,t} / P_{H,t}$. Linearize the definition of P and take the limit $\tilde{n} \to 0$ to arrive at the expression

$$
p_t = p_{H,t} + \alpha z_t, \tag{37}
$$

the first difference of which leads to (29). Linearization of H_t gives $h_t = p_{F,t} - p_t$, which combines with (37) yields (28).

E.3. Price-Setting and Firm Behavior

Prices are sticky like in Calvo (1983) and firms operate in the usual monopolistic competition setting of NK models. The demand for a home firm's variety sold at price p in period t is

$$
\begin{aligned}
Y_t^p &= \left(\frac{p}{P_{H,t}} \right)^{-\epsilon} \left(\frac{P_{H,t}}{P_t} \right)^{-\eta} \left[(1 - (1 - \tilde{n})\alpha) C_t + \tilde{n} \alpha \left(\frac{1 - \tilde{n}}{\tilde{n}} \right) \left(\frac{1}{H_t} \right)^{-\eta} C_t^* \right] \\
&\equiv \left(\frac{p}{P_{H,t}} \right)^{-\epsilon} \chi_t,
\end{aligned}
\tag{38}
$$

where χ_t is the demand for a variety whose price equals the average price level in the home economy. The $(1 - \tilde{n}) / \tilde{n}$ term multiplying foreign demand adjusts for the number of Foreign households $(1 - \tilde{n})$ and total number of Home firms $\tilde{n}$.

Firms readjust their price with probability θ. The resetting firm chooses the new price level to solve the optimization (de-trended) problem[1]

$$
\begin{aligned}
\max_p \quad & \sum_{i=0}^{\infty} (\theta \tilde{\beta})^i \mathcal{T}_{t+i} E_t \left[\frac{u'(C_{t+1})}{u'(C_t)} \frac{P_t}{P_{t+i}} \left(p Y_{t+i}^p - \Psi_{t+i}(Y_{t+i}^p) \right) \right] \\
\text{s.t.} \quad & Y_{t+i}^p = \left(\frac{p}{P_{H,t+i}} \right)^{-\epsilon} \chi_{t+i}
\end{aligned}
$$

where $\Psi_t(y) = y \hat{W}_t / \mathcal{A}_t = y W_t / A_t$ is the nominal cost function. The first-order condition is

$$
\sum_{i=0}^{\infty} (\theta \tilde{\beta})^i \mathcal{T}_{t+i} E_t \left[\frac{u'(C_{t+1})}{u'(C_t)} \frac{P_t}{P_{t+i}} Y_{t+i}^{\hat{p}} \left(\hat{p} - \frac{\epsilon}{\epsilon - 1} \frac{W_{t+i}}{A_{t+i}} \right) \right] = 0
$$

Linearized, the first-order conditions above leads to

$$
(1 - \beta\theta)^{-1} \left(\hat{p} - \mathcal{M} \right) = \sum_{i=0}^{\infty} (\beta\theta)^i E_t \left(w_{t+i} - a_{t+i} \right) \tag{39}
$$

[1] I am ignoring the existence of any non-degenerate stochastic discount factor. To a first-order approximation, such factor would be irrelevant to the optiomal choice of price. Moreover, firms in the model are owned by the risk-neutral financial intermediary.

where $\mathcal{M} = \log(\epsilon/(\epsilon-1))$ is the desired markup over marginal costs. I define $\beta \equiv \tilde{\beta}e^{-g}$. Note that in section 2 of the main text, I defined $\beta_j = \exp\{rx_j + i_j - \pi_j - g\}$ and $\beta_j = \beta$. The definitions are not exactly the same but differ by little, so I use the same value. An expression entirely analogous to (39) holds for firms of the Foreign economy.

E.4. Equilibrium

In equilibrium, the supply of each variety equals households' demand. In the Foreign economy, as $\tilde{n} \to 0$, we have $C^*_{F,t} = C^*_t = Y^*_t$. Take logs and use (35) to arrive at equation (30) of the main text, the equilibrium risk-sharing condition. Furthermore, $C^*_t = Y^*_t$ and the Euler equation in (36) leads to the Foreign IS curve (first equation in (21)). In the Home economy, the market-clearing condition in the goods market implies, from (38):

$$Y_t(j) = \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\epsilon} \chi_t.$$

$Y(j)$ refers to Home firms' production. Define the aggregate output $Y_t = \left(\frac{1}{\tilde{n}}\int_0^{\tilde{n}} Y_t(j)^{\frac{\epsilon-1}{\epsilon}} dj\right)^{\frac{\epsilon}{\epsilon-1}}$. Integrate the market-clearing condition above leads to

$$Y_t = \chi_t = \left(\frac{P_{H,t}}{P_t}\right)^{-\eta}\left[(1-(1-\tilde{n})\alpha)C_t + \tilde{n}\alpha\left(\frac{1-n}{n}\right)\left(\frac{1}{H_t}\right)^{-\eta} C^*_t\right], \tag{40}$$

which implies $Y_t(i) = (P_{H,t}(i)/P_{H,t})^{-\epsilon} Y_t$. I take the limit $\tilde{n} \to 0$ and log linearize (40) around a symmetric steady state, meaning a steady state in which $Q = 1$ and $C = C^*$.

$$\begin{aligned} y_t &= -\eta\left(p_{H,t} - p_t\right) + (1-\alpha)c_t + \alpha y^*_t + \alpha\eta h_t \\ &= \alpha\eta z_t + (1-\alpha)c_t + \alpha c_t - \alpha\gamma h_t + \alpha\eta h_t \\ &= c_t + \alpha\left[\eta + (1-\alpha)(\eta-\gamma)\right] z_t \\ &\equiv c_t + \alpha\gamma\bar{\omega} z_t \end{aligned} \tag{41}$$

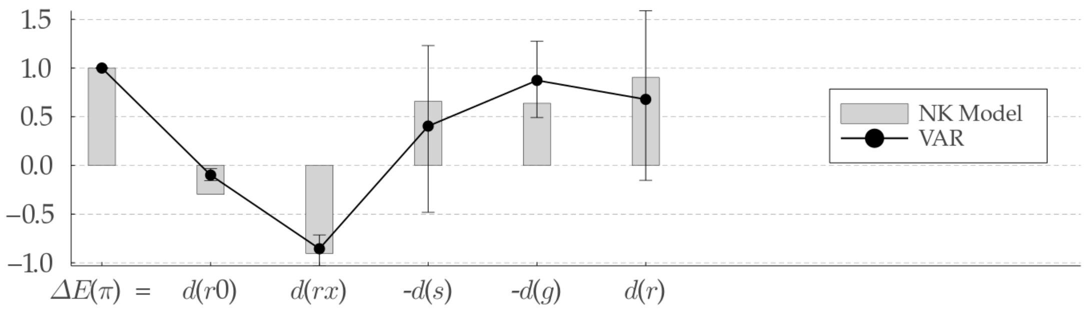
In the second equality, I used (37) and (30). The last row defines $\gamma\bar{\omega} = \eta + (1-\alpha)(\eta-\gamma)$, which governs the elasticity of Home goods demand to terms of trade variation, *given domestic consumption*. The term $\eta + (1-\alpha)\eta$ captures the price effect: depreciated terms of trade means relatively cheaper Home goods to Home and Foreign households. The $-(1-\alpha)\gamma$ term captures the fact that, given Home consumption, a terms of trade variation implies a real exchange variation which implies variation in Foreign output and thus Foreign demand. For instance, if Home terms of trade depreciates (z_t increases), the risk-sharing condition implies that Foreign output is declining (equation (35) with fixed c_t). Note that the last equality in (41) re-states equation (31). Replace it on the Euler equation in (34) to arrive at (24).

The only equations left to derive are the New-Keynesian Phillips curves of each country. In Foreign, let $\hat{p}^*_t$ be the optimal choice of price for firms re-optimizing in period t. The law of motion for the the price level is $p^*_t = \theta\hat{p}^*_t + (1-\theta)p_{t-1}$, since $p^*_{H,t} = p^*_t$ in the $\tilde{n} \to 0$ limit. Combine that with the expression for $\hat{p}^*_t$, the analogous of (39), to arrive at

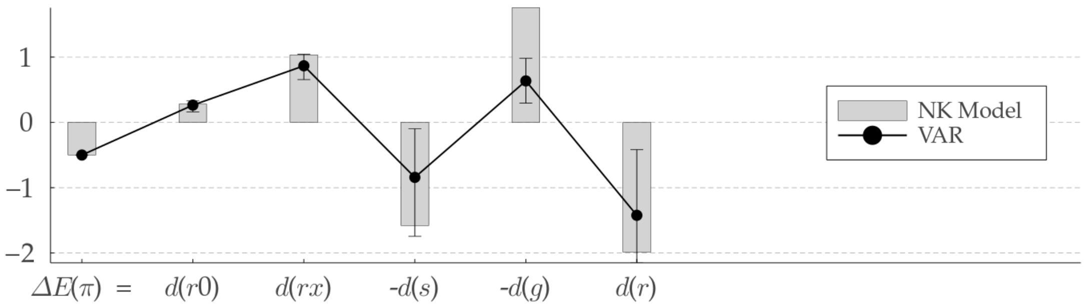
$$\begin{aligned} \pi^*_t &= \lambda(w^*_t - p^*_t - a^*_t) + \beta E_t \pi^*_{t+1} \\ &= \lambda(\gamma^{-1} + \varphi)y_t - \lambda(1+\varphi)a_t + \beta E_t \pi^*_{t+1}, \end{aligned}$$

which corresponds to Foreign's Phillips curve in (21). In the second line, we use households' condition for intratemporal optimality - second condition in (36), and $y_t = a_t + n_t$. Do the same for Home, using $\hat{p}_t$ (optimal price for its re-setting firms) and the law of motion $p_{H,t} = \theta\hat{p}^*_t + (1-\theta)p_{H,t-1}$ to arrive at

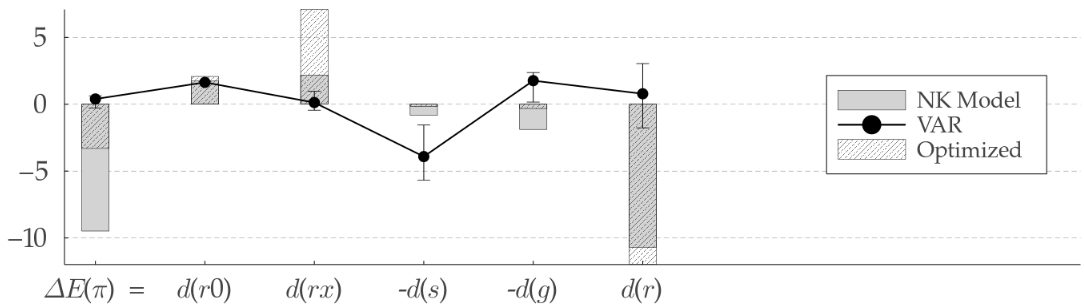
$$\begin{aligned} \pi_{H,t} &= \lambda(w_t - p_{H,t} - a_t) + \beta E_t \pi_{H,t+1} \\ &= \lambda(\gamma^{-1} + \varphi)y_t - \lambda(1+\varphi)a_t - \lambda\alpha(\omega-1)z_t + \beta E_t \pi_{H,t+1}. \end{aligned}$$



(a) Variance Decomposition $\Delta E_t \pi_t = 1$



(b) Recession Scenario $\Delta E_t \pi_t = -0.5$, $\Delta E_t g_t = -1$



Notes: The graphs depict the fiscal decompositions 1 in the model's Home economy (SOE-NK). The hatched bars in plot 8c correspond to the model optimized to replicate the real depreciation scenario and nothing else ($w = 1$ in the objective function (23)). The ribbons correspond to the 25 and 75 percentiles of the posterior distribution.

(c) Currency Depreciation Scenario $\Delta E_t h_t = 10$, $\Delta E_t i_t^* = \Delta E_t \pi_t^* = \Delta E_t g_t^* = 0$

Figure 10: Fiscal Decomposition 1 - Open NK Model with Trend Shocks

which is Home's Phillips curve (25). This completes the derivation of the equations of the model that characterize the private sector.

F. Additional Plots and Tables

I devote this section to report tables and graphs that would take too much space in the main text.

Country	$\Delta E_t \pi_t =$	Decomposition 1: $\Delta E_t \pi_t = -0.5$, $\Delta E_t g_t = -1$				
		ΔE_t(Bond Prices)		$-\Delta E_t$(Intrinsic Value of Debt)		
		$d_1(r_0)$	$d_1(rx)$	-$d_1(s)$	-$d_1(g)$	$d_1(r)$
Averages	-0.50	***1.45**	***0.81**	***-4.23**	***2.71**	-1.24
Advanced - 1960	-0.50	***0.15**	***1.38**	***-1.47**	***1.33**	***-1.89**
Advanced - 1973	-0.50	***0.11**	***1.07**	***-1.60**	***0.74**	**-0.82**
Emerging - 1998	-0.50	***2.49**	***0.48**	**-6.41**	***3.96**	-1.01
Median	-0.50	***0.26**	***0.70**	***-1.46**	***1.30**	***-1.46**
Advanced - 1960	-0.50	***0.13**	***1.04**	***-1.43**	***1.25**	***-1.68**
Advanced - 1973	-0.50	***0.08**	***1.02**	***-1.19**	**0.65**	**-0.99**
Emerging - 1998	-0.50	***0.99**	***0.36**	***-3.20**	***1.36**	-0.52
United States	-0.50	***0.03**	***1.00**	**-0.65**	***1.32**	***-2.21**
Advanced - 1960 Sample						
Canada	-0.50	***0.14**	***2.21**	-0.45	0.30	***-2.70**
Denmark	-0.50	***0.20**	***0.86**	**-2.64**	***2.75**	**-1.67**
Japan	-0.50	0	***0.83**	***-1.51**	***1.64**	***-1.46**
Norway	-0.50	0	***0.63**	**-1.36**	***1.72**	**-1.49**
Sweden	-0.50	***0.41**	***1.22**	-0.65	**0.87**	***-2.35**
United Kingdom	-0.50	0.11	***2.54**	**-2.20**	**0.73**	-1.68
Advanced - 1973 Sample						
Australia	-0.50	**0.06**	***1.54**	**-1.46**	**0.66**	-1.31
New Zealand	-0.50	***0.26**	***0.87**	**-0.84**	**0.63**	**-1.42**
South Korea	-0.50	***0.10**	***0.70**	***-3.17**	***1.74**	0.14
Switzerland	-0.50	0	***1.18**	***-0.93**	-0.07	**-0.67**
Emerging - 1998 Sample						
Brazil	-0.50	***0.37**	**0.06**	1.87	0.13	**-2.39**
Chile	-0.50	***15.78**	***2.94**	-30.50	**30.54**	-19.26
Colombia	-0.50	**1.86**	***0.67**	**-10.90**	***7.57**	0.31
Czech Republic	-0.50	***0.37**	***0.61**	-0.07	**0.25**	**-1.65**
Hungary	-0.50	***0.99**	***0.60**	10.82	-5.29	**-7.63**
India	-0.50	-0.03	0.13	**-1.16**	**0.71**	-0.15
Indonesia	-0.50	***8.23**	**-0.55**	***-11.24**	**1.42**	1.64
Israel	-0.50	***1.79**	**0.37**	**-3.18**	**1.17**	-0.65
Mexico	-0.50	***1.69**	***0.81**	***-4.56**	***1.94**	-0.38
Poland	-0.50	***0.87**	***1.00**	-0.14	**1.30**	***-3.53**
Romania	-0.50	***2.08**	**0.21**	***-8.16**	**2.05**	**3.31**
South Africa	-0.50	-0.10	0.35	***-30.02**	***11.15**	***18.13**
Turkey	-0.50	***0.99**	***0.23**	0.64	**0.52**	***-2.88**
Ukraine	-0.50	0	**-0.68**	**-3.22**	***1.92**	**1.48**

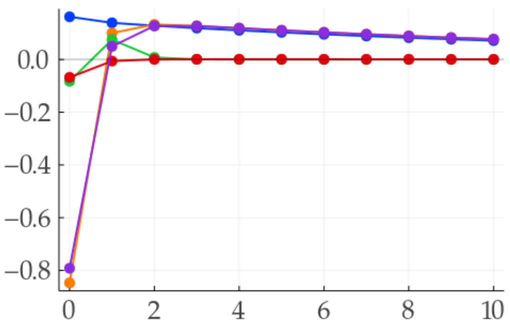
Notes: I project VAR shocks on $\Delta E_t g_t = -1$ and $\Delta E_t \pi_t = -0.5$. The table shows the estimated terms of decomposition 1 at the posterior distribution's mode. I divide each term by var($\Delta E_t \pi$). I simulate from the posterior to check the statistical significance of the sign of the estimate. Bold indicates 75% of results had the same sign. Bold with an asterisk indicates 90%.

Table 12: Unexpected Inflation Decomposition 1 - Recession Scenario

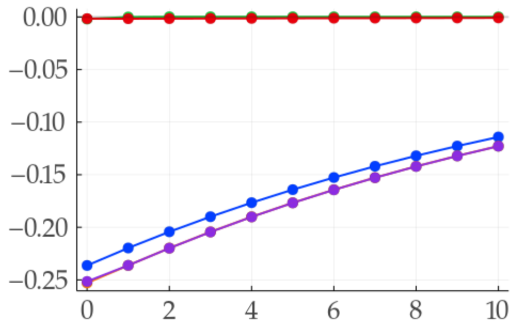
Country	$\Delta E_t \pi_t =$	Decomposition 1: $\Delta E_t \Delta h_t = 10$, $\Delta E_t u_t = 0$				
		ΔE_t(Bond Prices)		$-\Delta E_t$(Intrinsic Value of Debt)		
		$d_1(r_0)$	$d_1(rx)$	-$d_1(s)$	-$d_1(g)$	$d_1(r)$
Averages	0.16	***3.92**	-0.46	***-4.48**	0.03	1.15
Advanced - 1960	**-0.39**	***0.77**	-0.32	**-1.78**	-0.90	1.84
Advanced - 1973	**0.41**	***0.50**	0.16	**-1.41**	0.95	0.22
Emerging - 1998	0.29	***6.53**	**-0.71**	**-6.82**	0.03	1.26
Median	0.02	***1.64**	***-0.37**	***-1.23**	-0.34	**0.73**
Advanced - 1960	***-0.69**	***0.21**	-0.43	**-1.51**	**-1.29**	**1.82**
Advanced - 1973	**0.36**	***0.18**	0.07	-1.50	0.49	0.23
Emerging - 1998	0.05	***3.19**	***-0.65**	***-2.90**	**1.29**	0.62
United States	***0.56**	0.03	-0.37	-0.19	**1.78**	-0.69
Advanced - 1960 Sample						
Canada	***-0.68**	***0.26**	-0.13	**-2.64**	**-1.15**	**2.98**
Denmark	**-0.81**	***1.84**	**-1.54**	**-3.87**	**-2.54**	**5.30**
Japan	**-0.36**	***-0.01**	***-1.29**	***-1.80**	***2.15**	**0.58**
Norway	***-1.29**	***0.06**	-0.73	-1.23	**-2.10**	**2.72**
Sweden	1.52	***2.29**	0.03	-0.28	-1.43	0.92
United Kingdom	-0.69	0.17	1.76	-0.85	-0.34	-1.43
Advanced - 1973 Sample						
Australia	-0.10	0.04	**1.36**	-0.25	**-0.86**	-0.37
New Zealand	0.39	***1.64**	0.12	**-3.91**	**1.76**	0.78
South Korea	**1.00**	***0.32**	0.02	**-2.74**	***3.67**	-0.27
Switzerland	**0.33**	0	**-0.89**	**1.25**	**-0.78**	**0.73**
Emerging - 1998 Sample						
Brazil	0.02	***0.96**	0.03	-0.76	**1.43**	**-1.65**
Chile	0.41	***36.81**	**2.32**	**-87.99**	**29.66**	19.61
Colombia	-0.02	***7.55**	-0.06	***-11.74**	**3.06**	1.16
Czech Republic	0.08	***1.01**	-0.46	-6.64	3.88	2.28
Hungary	-0.85	***3.07**	-1.10	**39.37**	**-29.48**	**-12.71**
India	-0.62	***0.58**	**-0.91**	**-4.65**	-0.36	***4.74**
Indonesia	-0.28	***12.48**	***-1.06**	***-12.74**	***2.34**	-1.30
Israel	0.25	***3.32**	**-1.96**	***-13.85**	**3.35**	***9.40**
Mexico	-0.69	***4.08**	-0.02	4.77	-7.65	-1.87
Poland	**-0.65**	***2.46**	0.08	**-0.08**	-3.56	0.46
Romania	2.58	***10.07**	**-3.05**	**-7.36**	**2.15**	0.78
South Africa	0.20	***1.95**	0.18	3.52	-1.06	**-4.39**
Turkey	***2.26**	***7.03**	***-0.85**	-1.15	**-4.42**	1.63
Ukraine	**1.34**	***0.02**	**-3.08**	**3.79**	**1.16**	-0.55

Notes: I project VAR shocks on $\Delta E_t \Delta h_t = 10$ and $\Delta E_t u_t = 0$. The table shows the estimated terms of decomposition 1 at the posterior distribution's mode. I divide each term by var($\Delta E_t \pi$). I simulate from the posterior to check the statistical significance of the sign of the estimate. Bold indicates 75% of results had the same sign. Bold with an asterisk indicates 90%.

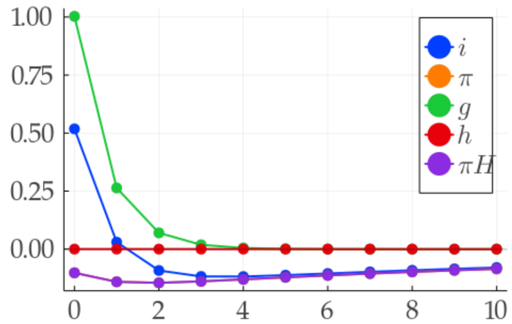
Table 13: Unexpected Inflation Decomposition 1 - Real Exchange Depreciation



(a) Home Interest Shock $\varepsilon_{i,t} = 1$



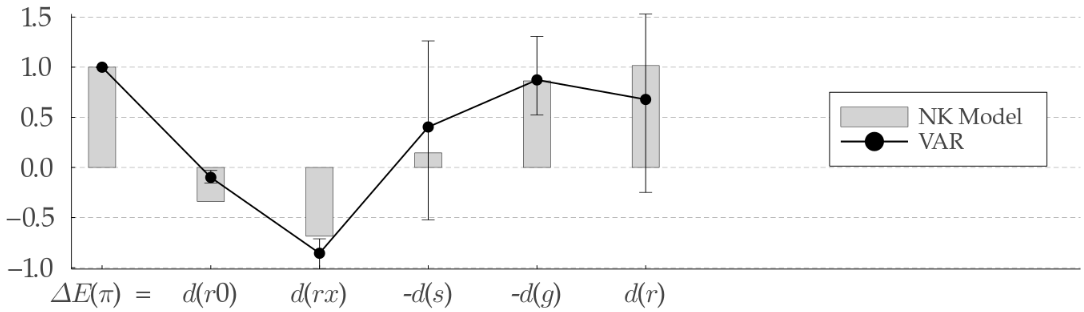
(b) Home Surplus Shock $\varepsilon_{s,t} = 1$



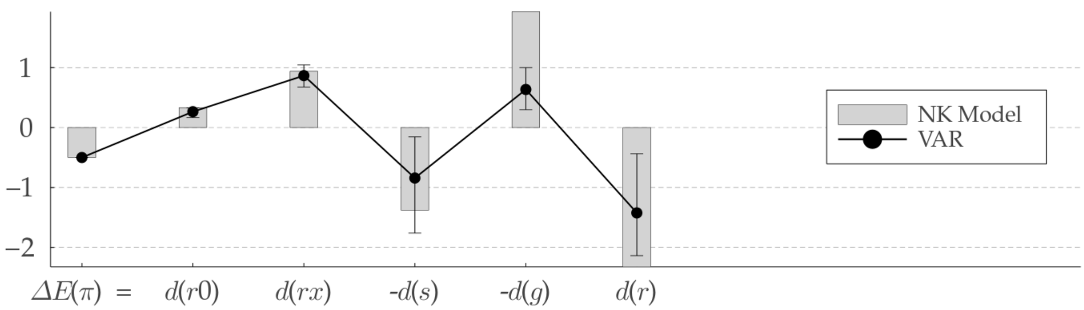
(c) Trend Shock $\varepsilon_{g,t} = 1$

Notes: IRFs of the Home country in the NK model, which works as an open economy (SOE-NK). The plots contain responses to domestic shocks. Figure 9 of the main text shows responses to shocks in the Foreign economy.

Figure 11: IRFs for the Home Economy



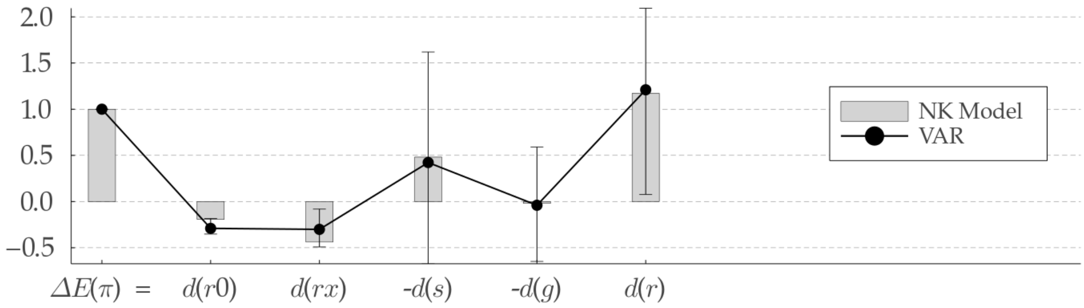
(a) Variance Decomposition $\Delta E_t \pi_t = 1$



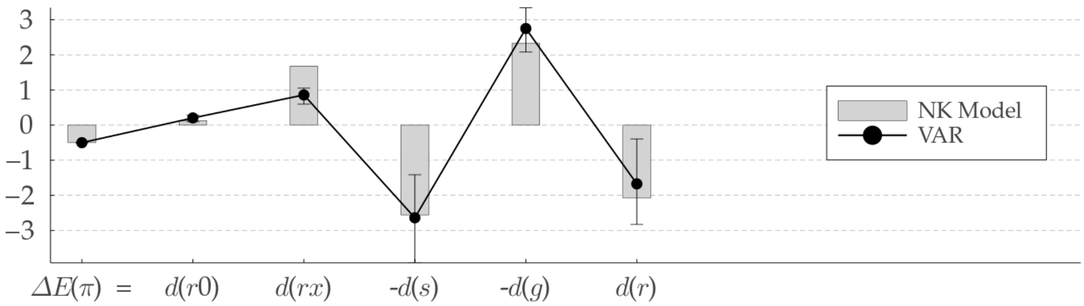
(b) Recession Scenario $\Delta E_t \pi_t = -0.5$, $\Delta E_t g_t = -1$

Notes: The graphs depict the decomposition 1, applied to unexpected inflation variance and the recession scenario, in the model's Home, "open" economy. I solve problem (23) by varying only policy parameters to match Sweden's VAR decomposition and second moments. The ribbons correspond to the 25 and 75 percentiles of the posterior distribution.

Figure 12: Fiscal Decomposition 1 - SOE-NK Model Calibrated for **Sweden**



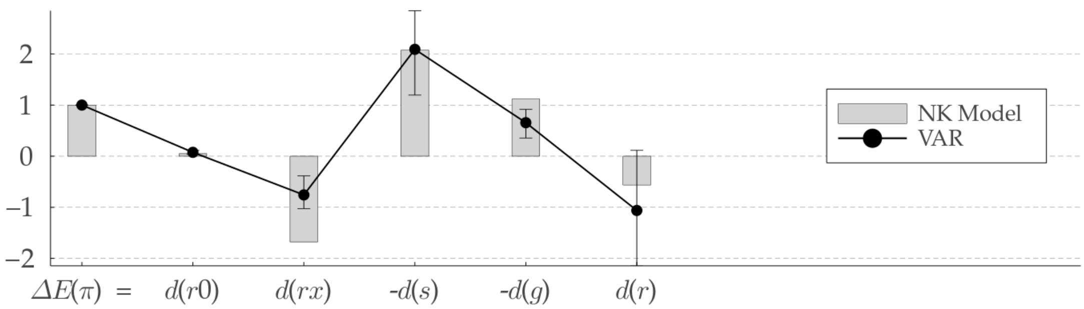
(a) Variance Decomposition $\Delta E_t \pi_t = 1$



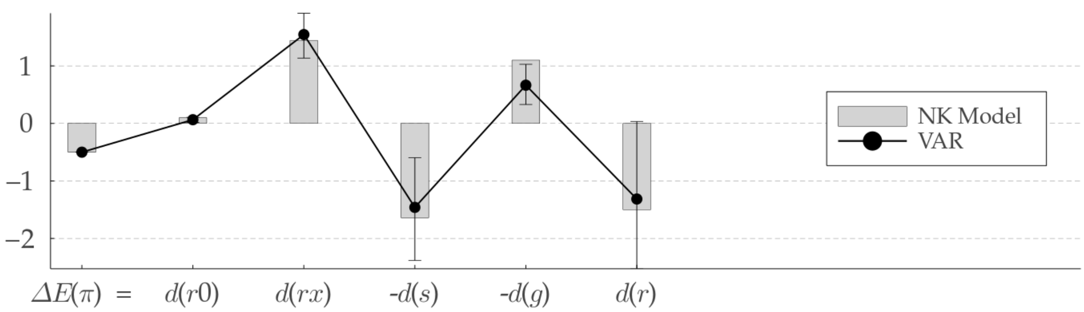
(b) Recession Scenario $\Delta E_t \pi_t = -0.5$, $\Delta E_t g_t = -1$

Notes: The graphs depict the decomposition 1, applied to unexpected inflation variance and the recession scenario, in the model's Home, "open" economy. I solve problem (23) by varying only policy parameters to match Denmark's VAR decomposition and second moments. The ribbons correspond to the 25 and 75 percentiles of the posterior distribution.

Figure 13: Fiscal Decomposition 1 - SOE-NK Model Calibrated for **Denmark**



(a) Variance Decomposition $\Delta E_t \pi_t = 1$



(b) Recession Scenario $\Delta E_t \pi_t = -0.5$, $\Delta E_t g_t = -1$

Notes: The graphs depict the decomposition 1, applied to unexpected inflation variance and the recession scenario, in the model's Home, "open" economy. I solve problem (23) by varying only policy parameters to match Australia's VAR decomposition and second moments. The ribbons correspond to the 25 and 75 percentiles of the posterior distribution.

Figure 14: Fiscal Decomposition 1 - SOE-NK Model Calibrated for **Australia**