Title

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1 Unexpected Inflation in a Benchmark NK Model

1.1 Connection with the Value of Debt

I start by establishing the connection between inflation and the value of debt.

Accounting. Consider a government that manages a public debt composed of one-period bonds, denominated in a specific currency. There is no constraint on the nature of this currency, and the assumption of one-period bonds buys simplicity but is not critical for the argument.

At the beginning of period t, the face value of debt issued in the previous period is V_{t-1}. The government finances the payment of V_{t-1} by either raising new debt maturing in the following period at a price Q_t or by running a (nominal) primary surplus S_t.

$$V_{t-1} = Q_t V_t + S_t \tag{1}$$

Let $P_t > 0$ be the relative price of an arbitrary basket of goods in terms of our selected currency and define $\beta_t \equiv Q_t P_{t+1}/P_t$ and $\beta_{t,t+k} \equiv \prod_{\tau=t}^{t+k} \beta_\tau$. Variable β_t represents a real discount for public bonds relative to the basket of goods. Since V satisfies (1), it also satisfies

$$\frac{V_{t-1}}{P_t} = \sum_{i=0}^{k} \beta_{t,t+i-1} \left(\frac{S_{t+i}}{P_{t+i}} \right) + \beta_{t,t+k} \frac{V_{t+k}}{P_{t+k+1}} \qquad \text{for any } k \geq 0 \tag{2}$$

regardless of the paths of public debt prices and surpluses. Importantly, expressions (1) and (2) do not represent a constraint on the path of nominal surpluses $\{S_t\}$ the government chooses to follow. They merely express the value of future debt given $\{S_t\}$, prices and current face value V_{t-1}.

Economics. I now make two assumptions about agents' behavior.

Assumption 1: At period t economic agents form expectations over the path of future, unknown variables through an operator $\tilde{E}_t$ which satisfies the linearity condition $\tilde{E}_t(X+Y) = \tilde{E}_t(X) + \tilde{E}_t(Y)$ and $\tilde{E}_t(X) = X$ if X is in the information set at t. Such operator can be heterogeneous across agents as long as these two conditions are satisfied.

Assumption 2: $\lim_{k\to\infty} \tilde{E}_t \left(\beta_{t,t+k} \frac{V_{t+k}}{P_{t+k+1}} \right) = 0$ at every period t. Assumption 2 is tipically referred to a no-bubble condition. In most micro-founded models, it is not even an assumption, but a result of optimal intertemporal consumption choice by households.

Assumptions 1 and 2 added to (2) lead to the valuation equation of public debt:

$$\frac{V_{t-1}}{P_t} = \sum_{i=0}^{\infty} \tilde{E}_t \left(\beta_{t,t+i-1} \frac{S_{t+i}}{P_{t+i}} \right). \tag{3}$$

Given the (pre-determined) face value of maturing public debt, the relative price of the basket of goods in terms of debt currency is determined by the expected β-discounted stream of surpluses in terms of the basket of goods.

Equation (3) provides the connection between fiscal shocks and monetary shocks I explore in the paper. (TODO: Mention importance of revision of public bonds' prices, absent here)

1.2 The New-Keynesian Model

I start with the two usual equations of the New-Keynesian model. All variables should be interpreted as deviations from a steady-state equilibrium.

$$c_t = E_t c_{t+1} - \sigma \left[i_t - E_t \pi_{t+1}\right] \tag{4}$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa y_t \tag{5}$$

along with an equation for market clearing for goods market:

$$y_t = \gamma c_t + g_t, \tag{6}$$

where y, c, i, π and g represent respectivelly log-output, log-consumption, the interest rate, the inflation rate and government spending in levels.[1]

The real value of public debt v follows the law of motion (reference to earlier equation, reference to the use of rational expectations - use of E, not $\tilde{E}$, reference to why assumption 2 above holds here)

$$\beta v_t = v_{t-1} + i_{t-1} - \pi_t - s_t \tag{7}$$

where $s_t \equiv \tau_t - g_t$ is the public primary surplus (which does not include interest payments on debt). τ_t are total tax proceeds in levels.

Policy. *Observed* monetary policy is muted: $i_t = 0$.

Fiscal policy prescribes the following rules for taxation (which I assume to be entirely *lump-sum*) and public expenditures:

$$\tau_t = \rho_\tau \tau_{t-1} + \alpha_\tau v_t + \epsilon_{\tau,t} \tag{8}$$

$$g_t = \rho_g g_{t-1} - \alpha_g v_t + \epsilon_{g,t}. \tag{9}$$

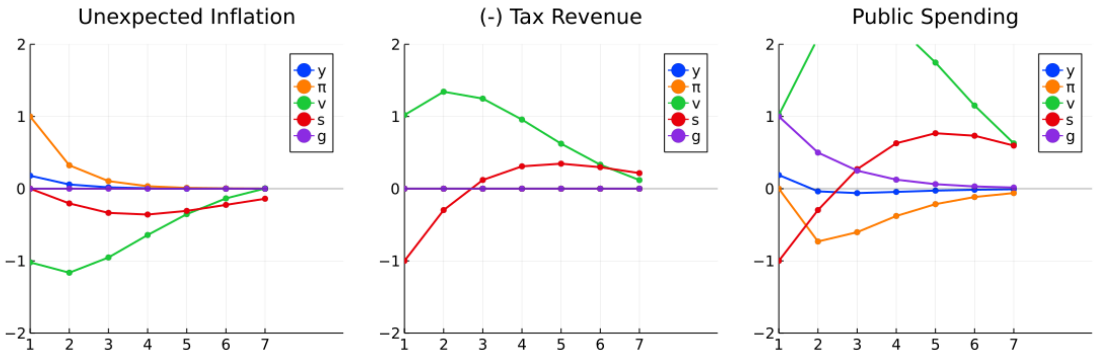
Stability of public debt requires either $\alpha_\tau > 0$ or $\alpha_g > 0$.

The equations above by themselves do not determine unexpected changes to the real value of public debt, and hence they do not pin down unexpected inflation. The last equation characterizing policy solves that issue:

$$\pi_t = E_{t-1}\pi_t + \eta_t, \tag{10}$$

for an exogenous term η_t. The term $\Delta E_t \pi_t = (E_t - E_{t-1})\pi_t$ I call unexpected current inflation. In this case, unexpected current inflation is given by η_t.

[1] γ represents the steady-state consumption-to-output ratio. The choice of linearizing equilibrium conditions around the level of government spending and not its log makes the connection with the rest of the paper clearer. I also linearize around an equilibrium with output = real debt.



(a) Unexpected Inflation η (b) Tax Revenues ϵ_1 (c) Government Spending ϵ_2

Figure 1: NK Model - Impulse-Response Function to Fiscal Shocks

There are two existing selection mechanisms that justify equation (10) and provide an interpretation to it: fiscal selection and the spiral threat selection. Both imply (10) while leaving other equations unchanged (observational equivalence, Cochrane (2011), Cochrane (1998)) and, more importantly, both interpret η as part of public policy, as a government *choice*.

Fiscal selection, or the fiscal theory of the price level, arrives at (10) by means of (3), with causality coming from right to left. Any economic shock changes the conditional distribution of discounted future surpluses in terms of goods - the real value of public debt. The relative price of public debt in terms of goods ($1/P_t$) adjusts to reflect such change, much in the same way that news of future dividends change the relative price of claims to those dividends (stock prices). (Maybe include a reading of US inflation 2022 through the lenses of FS and STS)

Spiral selection arrives at (10) by means of an explosive root introduced by an equation of the type $i_t = \phi \pi_t$ with $\phi > 1$. This is the approach most of the monetary literature has adopted.

1.3 Unexpected Current and Future Inflation

Current Inflation. To have fiscal shocks - the ϵ's in (8) and (9) - be accompanied by unexpected current inflation η_t is critical for the New-Keynesian model to deliver responses to fiscal policy consistent with the view of most economists and empirical evidence (PAPERS WITH IDENTIFIED FISCAL SHOCKS).

In light of the connection between inflation and the value of debt, one might even expect η and the ϵ's to be correlated. Say, unexpectedly large spending leads to lower surpluses, hence a lower value of debt, hence unexpected inflation. However intuitive, the proposition requires empirical verification. This is where we go in the next section.

For now, consider the response of the New-Keynesian model to η, ϵ_1 and ϵ_2 shocks, one at a time, plotted in figure (1). I base my calibration on standard choices of the macroeconomic literature, adapted for annual periods. I use $\sigma = 0.5$, $\beta = 0.98$, $\kappa = 3.8$, $\gamma = 0.75$ and $\rho_\tau = \rho_g = 0.5$.[2] Importantly, I start by considering fiscal adjustment via taxation only: $\alpha_\tau = 0.2$ and $\alpha_g = 0.0$.

Panel (1a) plots the response to the unexpected inflation shock η. Inflation jumps by assumption, the fiscal interpretation being that agents foresee a reduced stream of surpluses. Accordingly, the real

[2] To calibrate the Phillips curve steepness $\kappa = (1-\theta)(1-\beta\theta)/\theta$, I use the price rigidity parameter $\theta = 0.65$[4] estimated by Smets and Wouters (2007).

j		P_j	$\mathcal{E}_j$	H_j
1	Nominal Debt	P	1	1
2	Inflation-Linked Debt	1	P	1
3	Dollar-Denominated Debt	P_t^{US}	Dollar NER	Dollar RER

Notes: P = price of consumption basket in domestic currency. P^{US} = price of consumption basket in US dollars. NER = Nominal Exchange Rate. RER = Real Exchange Rate.

Table 1: Public Debt Denomination

value of public debt v jumps down on spot. A lower debt leads taxation τ to decline (not plotted) via the $\alpha_\tau v_{t-1}$ term. The government runs deficits starting in the first period following the shock (I refer to $s < 0$ as a fiscal deficit). These deficits 1. slowly bring v back to zero and 2. validade agents' expectation at period zero of a lower value of public bonds - indeed primary surpluses were lower.

The impact on economic activity resembles the typical Keynesian "demand" shock, combining an increase in inflation and output at the same time. Positive inflation in period zero leads to a negative real interest rate; the IS curve (4) then implies output larger than future output - output is large and declining.[3] Large output implies large marginal costs, and, by (5), inflation greater than future inflation - inflation is thus positive and declining.

Panels (1b) and (1c) show that, in the absence of unexpected inflation in period zero, expansionary fiscal policy fails to generate inflation at all in the basic NK model. A negative shock to taxation - the model version of the government sending checks to households - simply leads

2 Empirical Models

I solve two empirical models.

2.1 A Bayesian VAR: Stochastic Properties of Unexpected Inflation

The first model is a standard VAR:

$$X_t = \mu + \Psi(L)X_{t-1} + \Lambda w_{t-1} + \eta_t, \tag{11}$$

which must include inflation as one of the variables in X. The term w_t is a forcing process that I use to model the impact of international variables over domestic ones. I assume $\eta \sim N(0, \Sigma)$ and independent over time.

I base the decision of which variables to include in the VAR on a more general law of motion for public debt than (reference to earlier) which, it turns out, involves several variables economists usually include in macroeconometric research.

2.1.1 Generalizing Public Debt Instruments

Fix the case of a country and its government. Let P_t be the price of the final goods basket in terms of the country's domestic currency.

[3] The apparently small response of output follows from the choice of κ. Values that are lower than my choice lead to more pronounced responses of equal sign.

I generalize the class of financing instruments available to the government in two dimensions: currency denomination and maturity structure. I do not consider the case of debt with stage-contingent nominal payoffs.[4]

The value of public bonds can be linked to different currencies, enumerated by j. Let $P_{j,t}$ be the price of the consumer price index in units of currency j. Let $Q^n_{j,t}$ be the discount rate for a zero-coupon public bond paying one unit of currency j after n periods. Without loss of generality, all bonds are zero-coupon. Let $\mathcal{E}_{j,t}$ be the price of currency j in units of domestic currency.

The notation is general enough to accomodate currency-linked bonds *per se*, but also inflation-linked securities such as American TIPS. In this case, one can interpret the "currency" as the final goods basket (so that $P_j = 1$ and $\mathcal{E}_j = P_t$). In the empirical exercises that follow, I consider consider domestic currency bonds ($j = 1$), inflation-linked (or real) bonds ($j = 2$) and US-dollar-denominated bonds ($j = 3$). Table 1 shows the value or interpretation of the variables defined above for these three cases.

I do not assume the price index for government's purchases (denoted P^S_t) and levied aggregates (such as income) to be the same as the price index for households' consumption (P_t). While governments do tend to tax private consumption, they also tax and transfer income, and purchase baskets of goods different than that of households (insert reference). The fiscal effect of variations in the relative price of government's to households' final goods basket must be accounted for.

With these changes, the law of motion of government debt (1) becomes

$$\sum_j \mathcal{E}_{j,t} B^1_{j,t-1} = P^s_t S_t + \sum_j \mathcal{E}_{j,t} \sum_{n=2}^{\infty} Q^{n-1}_{j,t} \left(B^{n-1}_{j,t} - B^n_{j,t-1} \right), \tag{12}$$

where S_t is now the real primary surplus and $B^n_{j,t}$ is the face value of bonds issued in currency j, period t payable n periods in the future. The term on the left represents the cost of debt in period t; the second term on the right represents the selling of new bonds of all possible maturities.

Let $\mathcal{V}_{j,t} = \sum_n Q^n_{j,t} B^n_{j,t}$ be the end-of-period market value of nominal debt issued in currency j, $i_{j,t}$ the risk-free rate in bonds issued in currency j and $rx_{j,t} = \mathcal{V}^{-1}_{j,t-1} \sum_{n=1}^{\infty} Q^{n-1}_{j,t} B^n_{j,t-1} - i_{j,t-1}$ the realized excess return on portfolios that mimic the composition of j-currency debt. We can re-write (12) in terms of the $\mathcal{V}_j$ and its corresponding returns:

$$\sum_j (1 + rx_{j,t} + i_{j,t-1}) \mathcal{E}_{j,t} \mathcal{V}_{j,t-1} = P^s_t S_t + \sum_j \mathcal{E}_{j,t} \mathcal{V}_{j,t}$$

For empirical work, it is ideal to write the law of motion in terms of real and stationary variables. So, let Y_t be a trend such that $s_t \equiv S_t/Y_t$ is reasonably stationary, and let $g_{Y,t} = Y_t/Y_{t-1} - 1$ be its growth rate. Define the real "exchange rate" $H_{j,t} = \mathcal{E}_{j,t} P_{j,t}/P_t$ and its growth rate $\Delta h_{j,t} = H_{j,t}/H_{j,t-1} - 1$. Define the de-trended real value of j-indexed debt $V_{j,t} = \mathcal{V}_{j,t}/P_{j,t} Y_t$, the real value of total debt $V_t = \sum_j H_{j,t} V_{j,t}$ and the j-indexed share $\delta_{j,t} = H_{j,t} V_{j,t}/V_t$.

By properly dividing the whole above equation by $P_t Y_t$, and multiplying and dividing the j sum on the left by $P_{j,t-1}$, $P_{j,t}$, Y_{t-1} and $H_{j,t-1}$, we arrive at a final version for the law of motion of

[4] Publicly-ran social security and pension systems can be modelled as cases of public debt with state-contigent payoffs or, in a fashion compatible with my framework, as an additional layer of uncertainty in the primary surplus process.

public debt:

$$V_{t-1}\sum_j \frac{(1+rx_{j,t}+i_{j,t-1})(1+\Delta h_{j,t})}{(1+\pi_{j,t})(1+g_{Y,t})}\delta_{j,t-1} = \frac{P_t^s}{P_t}s_t + V_t. \tag{13}$$

Stated now in real quantities, (13) generalizes (1). During period t, the government must "pay" V_{t-1} plus realized returns and eventual changes to the relative value of currency j.[5]

2.1.2 Data Sources and Treatment

2.2 Unexpected Inflation

2.3 A Tighter Prior: Long-Term Debt Repayment

Let $\beta_j = \exp\{-(rx_j + i_j - \pi_j - g_Y)\}$ be the steady-state real and growth-adjusted discounting for public debt issued in currency j. I linearize around a steady-state - assumed to exist - with $\beta_j = \beta$ for all j and $P^s = P$. This leads to

$$\beta\left(v_t + s_t + s(p_t^s - p_t)\right) = v_{t-1} + v\left[\sum_j \delta_j\left(rx_{j,t} + i_{j,t-1} + \Delta h_{j,t} - \pi_{j,t}\right) - g_t\right], \tag{14}$$

which generalizes (7).

References

Cochrane, J. H. (1998). A Frictionless View of U.S. Inflation. *NBER Macroeconomics Annual*, 13:323–384.

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Smets, F. and Wouters, R. (2007). Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach. *The American Economic Review*, 97(3):586–606.

[5] "Pay" comes in paranthesis here because, unlike in (1), the government does not actually redeem the entire term on the left at period t. It only pays for bonds maturing at t.