The Repression of Informal Labor: Aggregate Effects and Transition Dynamics

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Abstract

This paper studies the effects of public policies designed to fight informal labor activity. We propose a general equilibrium model with heterogeneous agents in which the income process follows from a search model. We calibrate it to generate stylized income/informality facts from Brazilian household-level data. Firms opt between offering formal or informal contracts and have heterogeneous ability to operate informally. Such heterogeneity leads some productive firms to choose informal contracts. It allows the model to produce similar income averages among high-income workers in the formal and informal sector, a property we find in the data. We then use the model to simulate the economy's response to the repression of informal labor activity by the government. Our simulation suggests that short and long-run impacts differ. General equilibrium effects matter for both. In the long run, households' welfare and average firm productivity improve, and unemployment decreases. However, in the short run, reduced aggregate savings leads to a 4% increase in interest rates and a 2.5% increase in the unemployment rate. We also show that if the government fails to transfer back to households the additional tax revenue, these effects hold in the long run as well. In addition, if the policy is anticipated by economic agents, then output declines and informality increases prior to implementation. In all cases, households with greater wealth experience larger welfare gains.

1 Introduction

In its Recommendation No. 204 (2015), the International Labour Organization (ILO) defines the informal economy as "economic activities by workers and economic units that are (...) not

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covered or insufficiently covered by formal arrangements and does not cover illicit activities (...)" (ILO (2015)). The definition highlights the main property of informal activity: not being registered with legal authorities. Follows from such property the fact that agents involved in informal activity have increased ability to avoid different forms of regulation[1], and thus face an economic environment significantly affected by it. Workers have access to job opportunities in firms that might not exist if forced to formalize, at the cost of reduced participation in social security programs that require formal employment for benefit entitlement. Firms can avoid production costs stemming from taxation and regulation (minimum wage, severance payments). Governments have their ability to raise tax revenues compromised as businesses can opt for hiding their operation and not paying taxes.

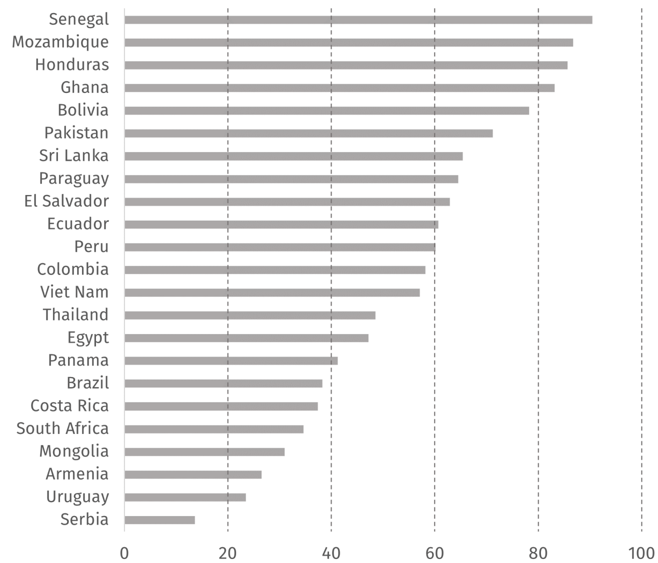
The ILO's Recommendation provides not only a definition of informal activity but guidance to member countries to promote and facilitate the transition of the informal economy into a formal economy, under the justification of potential economic gains of such a transition. We can often find similar propositions debated among policymaking circles, usually under arguments, among others, of protection of workers' rights and gains in productivity. These arguments coincide with the "parasite view" of informality, the idea that informal firms crowd the labor market with low-quality jobs and prevent workers from finding better ones in the formal sector. The parasite view has also gained recent support in the academic literature, in research papers such as Meghir et al. (2015) and Dix-Carneiro et al. (2019).

Motivated by the significant size of the informal sector in many emerging market countries - see figure 1 for ILO evidence - and the potential consequences for households' welfare and public finances, in this paper we examine the economic effects of public policies that succeed in repressing informal activity. We build a heterogeneous-agent macro model with incomplete markets in which households choose consumption and savings levels, and face a stochastic stream of income. Interest rates are endogenous and clear the capital market, as in Aiyagari (1994).

The distribution of income shocks is not exogenous in the model. It follows from a standard Diamond-Mortensen-Pissarides (DMP) search model in which firms can opt between hiring workers formally and informally. We select model ingredients so that it can reproduce some of the properties of the joint distribution of income and informality in Brazilian household-level micro data. We use the *PNAD* (National Household Sample Survey) dataset, a panel of households that offers enough observables for us to measure labor income as well as the formalization status of the sampled individuals. We observe three facts from the data. First, there is a significant presence of informal workers (about 30%) in all regions of the income distribution. While informality is more prevalent at the bottom of the distribution (low income workers), it is also present at economically significant levels among high-income workers[2].

[1] See, for example, Almeida & Carneiro (2012).

[2] See La Porta & Shleifer (2008) and Meghir et al. (2015) for additional evidence.



Source: International Labour Organization

Figure 1: Informal employment as share of non-agricultural employment (2015)

Second, the income differential, defined as the difference in average income earned in the formal and informal sectors, is large among low-income workers, in particular those workers with earnings close to the legal minimum wage prevailing in Brazil. However, such differential declines and even vanishes among those with higher income, to which the minimum wage is not a constraint. This pattern is true after controlling using proxies for human capital and the labor market in which these workers are inserted. The third fact is that informal workers transition to unemployment more frequently than formal ones, which builds on the idea that informal jobs are more unstable and, thus, less desirable from the point of view of workers. Bosch & Esteban-Pretel (2012) study this pattern in detail using a different Brazilian dataset and find similar conclusions.

Our observation about income differentials - the second fact above - suggests, first, that minimum wage laws and/or personal reservation wages play a significant allocative role in the labor market and, second, that either the inverse relationship between firm productivity and informality (as documented in La Porta & Shleifer (2008) and La Porta & Shleifer (2014)) is less than perfect or the relationship between firm productivity and worker's wage is less than perfect. These two ideas underlie some of the main assumptions in our search model. In it, formal contracts must observe a minimum wage, common across all groups of firms and workers. The minimum wage introduces a critical non-linearity in human capital: doubling everyone's productivity does not result in doubled labor income and equal unemployment. We also allow workers to search for new hiring opportunities on-the-job, an important tool to have individuals of higher skill level to accept minimum wage offers - another pattern observed in the data. A key innovation of our search model, designed to generate the same pattern of income differential observed, is that we introduce firm heterogeneity in their “ability” to hire informaly, or *hiding ability*. This layer of heterogeneity captures different potential costs of not formalizing labor contracts. The source of these differences might include (but are not limited to) geographical location, proximity to public vigilance, firm size and reputational costs. In the model, firms with higher hiding ability choose to offer informal contracts even for highly productive workers, which helps to generate low income gaps, while firms with lower hiding ability and/or productivity keep the size of informality in check.

We make assumptions that render the equilibrium in the search block of the model uniquely determined given a path for the interest rate. The search block then yields a distribution for the households’ income process. With it, we can calculate the solution to the households’ optimization problem, which we solve using standard dynamic programming techniques. Households’ utility then provides a measure of personal economic welfare sensitive to *income risk*, which plays a significant role in our results and represents a departure from most informality literature. Search models in their majority, and ours is not an exception, assume a *risk-neutral worker*. His or her value function amounts, in all, to the present discounted value of the stream of current and future income. This is a convenient assumption that not only delivers tractability, but also allows for proper use of intuitive bargaining schemes, such as the so-called Nash bargaining solution. However, as a measure of actual economic welfare, it ignores the element of risk, for changes in the income distribution that result on the same *average* income lead the model to predict identical levels of welfare. For this reason, we do make use of the risk-neutral worker’s value to base his or her decisions in the search model, which keeps tractability and comparability with previous research, but we do not make welfare inferences based on such measure.

We calibrate our model so that in the baseline equilibrium we observe the same facts observed in the data. We then proceed to simulate the repression of informality by the government. We model such fight by changing the distribution of hiding abilities: new firms lose any ability to operate informally. To a large extent, the economy’s response to the policy change underscores the "parasite" character of informality and thus agrees with the recent literature. Lacking the option of circumventing costly regulation, unproductive firms refrain from posting job vacancies and open space in the labor market for more productive ones. Average labor income increases. In addition, as formal jobs last longer, the average duration of employment spells increases. On the other hand, the value of vacancies decline, less firms populate the labor market and job-finding rates decline. Which effect dominates for determination of the unemployment rate depends on where we look at in the transition path. In the short run, our simulation points to an increase in the unemployment rate; in the long run, a decrease.

Such differences along the transition path relate to general equilibrium effects, in particular equilibrium in the capital market, often ignored in the analysis of informality. In our baseline simulation, capital demand increases as firms hire more efficiency units of labor. Aggregate savings do not and, in fact, *decline* in the initial periods following the beginning of the fight against informality, as consumption-smoothing households decrease savings in anticipation of higher future income and reduced marginal value of wealth. Market clearing then requires a significant increase in interest rates (up to 4% annually) in the short run and therefore a reduction in the discounted present value of firms' profits. By free entry, less firms opt to post vacancies and we observe higher unemployment. In the long run, as aggregate savings catch up, the interest rate declines back to levels near the original steady state and unemployment decreases.

We also look at welfare. Despite these undesirable short-run general equilibrium effects, we find significant welfare gains for almost all groups of households in our baseline exercise. These gains - often surpassing the mark of 20% of pre-repression lifetime consumption - take place in the short as well as in the long run. They are smaller, however, among unemployed workers and households with low wealth, a results that lines with the fact that negative income shocks (another expression for unemployment) become more traumatic events, given that returning to employment is less likely.

Having a baseline result, we look at two similar policies again designed to repel informality. In the first, government adjusts its budget constraint not by reducing income taxation - as it is the case in the baseline simulation - but simply by increase public consumption. We show that, in this case, interest and unemployment rates increase in the short *and* in the long run. Aggregate savings do not recover as before and the effect on interest rate persists. This contributes to a more pronounced decline of job-finding rates and, consequently, an increase the unemployment rate. The result holds even with the assumption that formal jobs are inherently of longer duration. In terms of welfare, gains are not as pronounced as before, seldom exceeding 15% of lifetime consumption, but remain positive for most groups of households. We do observe however that, in the long run, unemployed households with low wealth are worse off.

In the second policy sensitivity study, we consider the case that economic agents anticipate the repression of informality in five years. The endogeous selection of productive firms does not start immediately, as entrant firms still have the possibility of hiring informally. However, interest and unemployment increase starting at the announcement of the policy, for the same reasons described above. In fact, we observe an output contraction before policy implementation along with an *increase* of informal activity that follows from higher income taxes. These higher taxes also contribute to a short-run reduction in average household income. In terms of welfare, these effects lead to more losers from the policy in the short run (a little less than 20% of the population). Losing households, once again, tend to be those with low net wealth, thus reduced capacity of self insuring, at the time of the policy announcement.

1.1 Literature

Informal labor is gaining increasing attention from researchers. The existing literature comprises empirical, theoretical and quantitative papers that focus on the role of firms and/or workers. We do not provide a complete review of the literature (see Ulyssea (2020) for that). Instead, we highlight some the papers that we believe to be more connected to the points we make in this paper.

The work in La Porta & Shleifer (2008) and La Porta & Shleifer (2014) provides an overview of the competing theories of informality: the romantic view (as in De Soto (1989)), the parasite view (supported by more recent paper described below) and the dual view. The authors present evidence to support the dual view. It states that informal firms are less productive (smaller, inefficient, run by poorly educated entrepreneurs), they can't compete with formal ones and they are largely disconnected from the formal economy. Our framework allows us to differentiate between "parasite view" firms and "dual view" firms. Ulyssea (2018) provides a model with intensive and extensive margin of informality (firms size *vs* formal status composition) that successfully unifies the three theories in a single framework. Due to government auditing, the marginal cost of labor of informal firms grows more rapidly than that of formal ones. This feature links firm size to labor and firm formality.

Meghir, Narita & Robin (2015) develop a wage-posting model with productivity-heterogeneous firms and homogeneous workers in which informal firms crowd the labor market and make it harder for workers to find the better, formal jobs. The model reproduces the overlapp of productivity levels in the formal and informal sectors seen in Brazilian firm-level data. Given productivity, informal jobs pay higher wages to compensate for lost benefits. Tighter enforcement leads to a distribution with more productive firms in the new equilibrium and higher welfare. We find the exact same pattern, but predict a short-run phase of higher unemployment that follows from general equilibrium effects in the capital market. We also identify groups of households that can be harmed by tighter enforcement. In recent work, Dix-Carneiro, Goldberg, Meghir & Ulyssea (2019) study the effects of openness to international trade on informality, and find similar predictions in terms of productivity and welfare as informality shrinks. Bosch & Esteban-Pretel (2012) focus on transition rates. They study a search model in which *ex-ante* homogeneous workers and firms decide to sign a formal or informal contract upon a match. They estimate the model to replicate Brazilian data job-finding and job-destruction rates, and show that it can successfully reproduce the main cyclicality patterns of unemployment, formality share and transition rates. In a later article, Bosch & Esteban-Pretel (2015) use a similar model to study the effects of unemployment benefits on formality and unemployment. Granda & Hamann (2015) provide evidence that informal individuals and households in Colombia present a higher savings rate than formal ones. In their model, labor and capital markets are fully segmented between formals and informals, unlike our model. Albertini & Terriau (2019) provide evidence from Argentina of informality rates, as well as job finding and separation rates, over the life cycle. They use a model with endogenous educational level choice - an important absence of our framework in which human capital is fixed - and a household free choice condition between formal and informal labor markets (equal expected value between sectors) to reproduce the stylized facts.

2 Income and Informality: Three Empirical Facts

Income dynamics represents the main link between informality and households' welfare and choices. For this reason, it is critical that our model reproduces an empirically accurate joint distribution of informality and income. In this section, we present three empirical facts we find using Brazilian data. As we design some key ingredients of our model to reproduce these facts, presenting them prior to the model facilitates its exposition in section 3.

2.1 The Data

We use the Continuous National Household Sample Survey (PNAD), an open dataset consisting on the answers of interviewed households to socio-economic questions. The survey is representative of the entire Brazilian population. Each household is interviewed up to five consecutive quarters, and the survey provides sampling weights that we use to compute all results throughout the paper. The first wave was interviewed during the first quarter of 2012, which is when our data starts. The last wave in our dataset is that of the first quarter of 2020. We often use statistics of the sample interview in the third quarter of 2018.

According to Brazilian labor law, every employment contract must be officially registered. The PNAD survey asks if individuals who report to be employed in the reference period have such official registration of their employment contract [3]. Therefore, we classify as an "informal worker" any individual who reports to have a job in the private sector without an official registration[4]. All remaining workers that report being employed in the private sector we classify as "formal workers". We remove from the sample public servants, military employees, employers and auxiliary family workers. Unlike Meghir, Narita & Robin (2015) and most research of informality, we also exclude self-employed individuals from our analysis. The reason for this is that we do not model self-employment explicitly, and we believe the

[3]Survey respondents are aware of their formality status because of how the registration process works. Every person who wishes to engage in an official employment relationship must acquire a so-called "labor booklet" with the Brazilian government. The labor booklet allows workers to claim benefits such as unemployment insurance, disability insurance, and access to retirement benefits. Employers are required to sign the employee's labor booklet when the employment relationship is established. The employer must also register the employment contract in the official registry (a system called *e-Social*).

[4]We also try further restricting the definition of an informal worker by requiring, in addition to the lack of an official registration, that the worker reports not having contributed to a pension system institute (the law requires that all workers contribute to the public pension system). Results remain almost identical.

Occupational Group	Share (%)	Educational Level	Share (%)
Agriculture, hunting, fishing	5.32	Elementary	40.38
Administrative staff	8.45	High school	41.11
Construction and mechanical	13.71	Superior	18.51
Commerce, services and retail	23.09		
Elementary occupations	17.59		
Machine and assembling operators	8.78		
Managers and directors	3.93		
Scientists and intellectuals	10.41		
Technicians	7.80		

Table 1: Occupational Groups and Educational Level: Population Shares

dynamics of self-employment are determined by factors that are not captured by our search-theoretic approach. Finally, all remaining workers report not having a job. We classify them as unemployed regardless of whether they report to be searching for a new one.

We compute real monthly income in the sample by dividing reported effective labor income at the main job by the official consumer price index, so that all figures correspond to 2018Q3 Brazilian currency (Reals). We then exclude from our sample all individuals whose reported income exceeds 10,000 Brazilian reals. That corresponds to the 97th percentile of income, and thus leaves out about 3% of the population. We exclude high income levels to avoid heavy distortions to income averages provoked by sample outliers[5]. In addition, we exclude from the sample individuals with the age strictly below 18 years of age and strictly above 68 years of age. After these filters, we end up with 7,936,590 (unweighted) individual observations over 33 quarters. To compute income statistics, we also exclude from the sample employed workers with labor income or labor status data missing. That results in a total of 7,157,681 income observations of employed workers.

As we are interested in the effects of informality on households with different personal productivity levels, we establish two proxies for it in the data. We group workers according to their *occupational group* or their *educational level* (we do not interact these two variables to avoid small sample issues)[6]. Besides providing some control for households human capital level, these groups should also control for the labor markets workers participate and search for jobs in. Table 1 reports the population shares of each category in the third quarter of 2018. The educational level can take three values: primary, secondary or superior education. For an individual to belong to a certain group, he or she must have completed the required degree. The occupational group variable is mainly a description of the nature of the activity performed by the worker. It is related to, but not perfectly, to industry category. For example, machine operators typically work at the manufacturing industry, but managers and directors are more evenly distributed across industries. We opt for using the occupational group instead

[5]Setting this threshold to 20,000 Brazilian reals does not significantly alters our results.
[6]Note we only observe occupational groups for employed workers.

of industry to have better control over the labor market in which a worker participates, and not the goods market in which his or her employer does. As opposed to educational achievement, the use of occupational groups has the advantage of yielding more groups of households and, arguably, offering a more precise control of the labor market in which they are inserted.

Finally, because most readers are used to dollar units, we convert Brazilian reals to US dollars using a conversion rate of 4 Brazilian reals per dollar, a conveniently round number close to the average exchange rate during the 2017-2020 period, and report statistics stated in currency units using dollars.

2.2 Facts

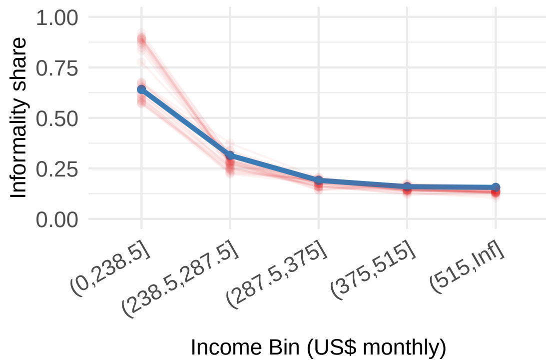
We start by defining the *informality share*: the ratio of informal workers to total employed workers. By excluding the unemployed, we avoid additional noise brought by business cycle fluctuations[7].

Fact 1. *The informality share is large among workers in all regions of the income distribution. It's larger at the bottom, but also significant at the top.*

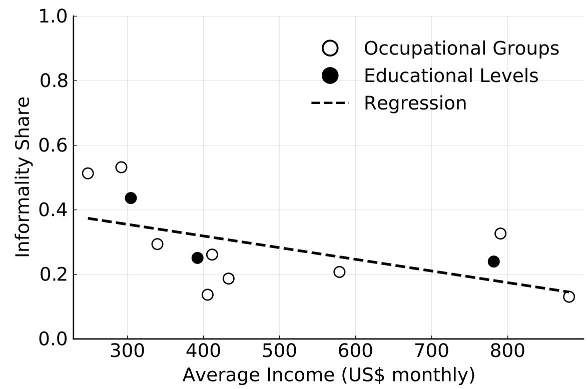
Figure 2 provides the evidence for fact 1. Panel 2a shows the observed informality shares, conditional on income quintiles. We re-calculate the limits of each income bin quarter by quarter. The labels on the x-axis fo the figure show the limits for 2018Q3. The different curves refer to data from different sample waves, with the thicker curve again corresponding to 2018Q3. The numbers reveals that informality is present over the entire income distribution, but is more prevalent among poorer workers (about 63% in the first income bin). In addition, the relationship is non-linear: the shares rapidly decline with income at the bottom part of the distribution, and later stabilize at about 13% of employed workers, a still significant share of the employed labor force, as stated by fact 1. These patterns obtain consistently over all sample periods.

A similar conclusion holds if we condition workers on their educational level and their occupational group, as shown in panel 2b. Each circular marker corresponds to an occupational group (filled) or an educational level (unfilled). The x-axis position indicates the average income among employed workers of the corresponding group, and the y-axis its informality share. Both statistics use data from the 2018Q3 wave. You can find the time series of informality shares for each group in figure 18 of appendix A. It shows that although informality shares on each group does vary over time, relative to each other it varies less. Back to figure 2b, we also show a linear regression forecast curve, calculated using only occupational group data. The linear coefficient predicts a reduction on the share of informal workers of about 3.6% for each additional one hundred dollars of average income. The figure reveals that the informality share

[7]Brazil experienced a sharp recession in 2015-16.



(a) Income Bins



(b) Worker Groups

Notes: Panel(a). Points in the x-axis define income quintiles. Each line represents the share of informal workers for different income quintiles. Each quarter in the sample (2012Q1 - 2020Q1) is associated to a separate line. The thicker line represents 2018Q3. Income quintiles are recomputed for each quarter, but the labels correspond to the 2018Q3 quintile. Panel (b). The circular markers represent worker groups. The x-axis represents the average income of each group in 2018Q3. The dashed line plots the forecast of a linear regression of the informality share on average income.

Figure 2: Informality Share *vs* Income

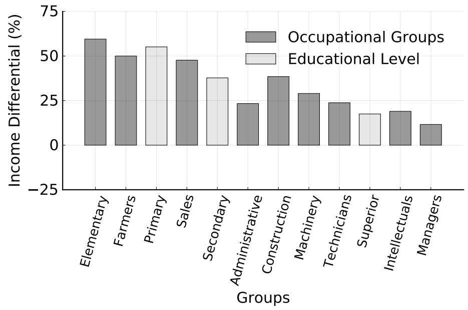
is highly heterogeneous across occupational groups, but never negligible. It varies between about 55% (for farming and fishing) and 15% (for managers and directors).

Obviously, fact 1 by itself does not imply that informal workers that transition to the formal sector should expect labor income gains. Proceeding from panel 2b, we now ask whether there is an *income differential*, defined as the percentage difference between average income among formals and informals in a given groups of workers, and if there is, what is the size of such differential.

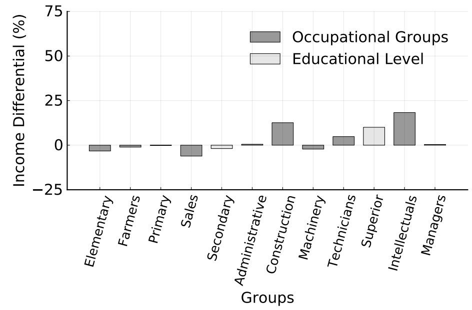
Fact 2. *An income differential exists and is positive among low-income workers. Among higher-income workers, the differential vanishes.*

More specifically, by low-income workers we mean workers with income close to the legal minimum wage. In 2018, Brazilian law established a minimum wage of 998 Brazilian reals per month, or US$ 249.5 using our conversion rate. Net of taxes, the amount reduces to about US$ 232[8]. Figure 3 depicts fact 2. It contains two panels. Each panel plots income differentials, expressed here as a percentage of the corresponding average income in the informal sector (for instance, 25% means that the average labor income among formals is 25% larger than among informals). We again group individuals in the sample according to their occupational groups (dark bars) and their educational level (light bars), but this time we add another criterium for grouping them. Panel 3a plots the differential for workers with income below the threshold of US$ 300 (recall the minimum wage of US$ 232, net of tax). We exclude all the other individuals from the subsample. Panel 3b plots the same statistics for workers with

[8]Personal contribution to the social security system for minimum-wage workers was 7%, and the marginal income tax rate in the first income bracket is zero.



(a) Below US$ 300



(b) Above US$ 300

Figure 3: Income Differential

income above the same threshold. We order groups in ascending order of productivity, proxied by average income (in the entire group sample, not in each income subsample). Again, we use data from the third quarter of 2018 as a good representation of the pattern we find for other quarters. You can find the time series of these statistics in figure 19 in appendix A. To facilitate comparison, we set the limits of the y-axis to be same in both pictures.

The two plots reveal a staggering difference in the income differentials across the two subsamples. Differentials are substantially higher among workers with lower income (panel 3a). Focusing on these low-income individuals, the differential exceeds twenty five percentage points for most groups. In addition, the differential declines on the average income of the corresponding group. As for workers with income above the threshold (panel 3b), the estimated differentials locate closer to zero. In fact, they are all smaller (group by group) than the estimate among workers below the threshold. More than that, most groups present a number *below* zero, which indicates that, on average, informal workers are paid more than formal ones. In addition, the negative relationship between differential and average income (interpretable as productivity) is no longer there. These plots support fact 2 and provide evidence that informality (or formality) has a different impact on the income process of different workers.

We believe the minimum wage to be a key factor in accounting for this pattern. Our next figure, numbered 4, plots the histogram of labor income distribution of workers according to their formality status. We again focus on the 2018Q3 wave. To build the figure, we group workers in income bins of size 200 dollars each. The x-axis shows the lower limit of the income bin (for example, the first point corresponds to the bin of workers with monthly income between zero and two hundred dollars). The relative absence of formal workers in the first bin is not surprising. As we mentioned, in 2018 Brazilian law established an after-tax minimum wage of US$ 232, which stays above the upper limit of the first income bin. The minimum wage is therefore a possible explanation as to why we observe few formal workers reporting labor income gains below the mark of two hundred dollars[9]. Figure 4 also appears to reject -

[9]Note that, however reduced, there is a non-zero measure of formal workers in the first bin (reporting to

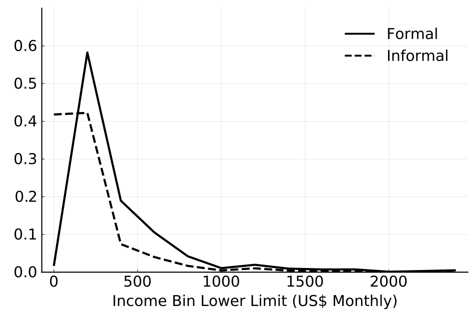


Figure 4: Income Distribution of Formal and Informal Workers

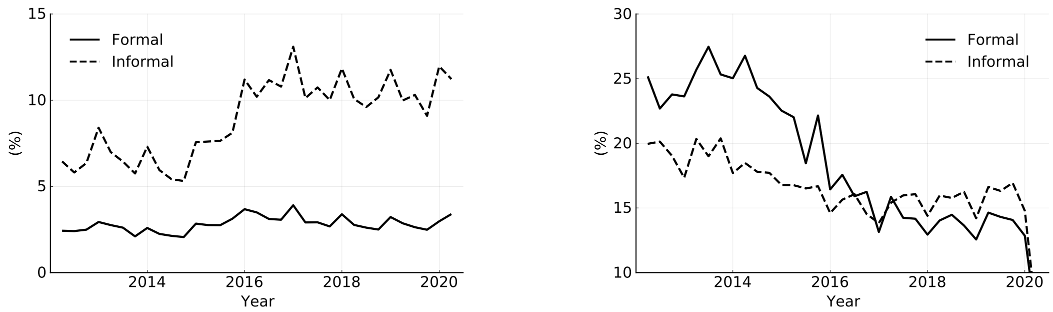
with over forty percent of informal workers in the 0-200 hundred dollar bin - the hypothesis that informal agreements in general respect the minimum wage rule. In principle, we could point out to alternative explanations such as underreporting, but it is difficult to argue why they would not apply to formal workers as well.

The existence of the legal minimum wage and the evidence that it is an actual constraint only in the formal sector provide an explanation for the large income differential among low income workers. This is the exact mechanism we apply in our model to generate such large differentials. It also provides an explanation as to why they decline with productivity (see figure 3a; groups are ordered in increasing order of average income). The average income of informal workers increases with group productivity, while that of formal workers is less responsive given that wages are often set by the legal floor rather than actual firm/job productivity.

Fact 3. *Informal workers transition to unemployment more often than formal workers.*

Facts 1 and 2 relate to employment income. In both cases, we ignore unemployed individuals. So, what about having *vs* not having employment income? Unfortunately, our criterium does not allow us to classify unemployed workers as formal or informal. The small number of interviews given by each family (5 at the maximum, less than that in most cases) also does not allow us to build a large enough stock of unemployed individuals that we can call formal or informal based on their last job. Instead, we follow our methodology in Maya & Pereira (2020) and make use of PNAD's panel dimension to investigate the rate at which workers transition to and from unemployment. That way we can always classify observations as formal or informal. For each quarter, we restrict the sample to individuals who appear with a valid labor status (employed formal, employed informal and unemployed, as previously defined) on that quarter and the previous one. We then compute the empirical transition matrix across these three states for every period. For example, among workers that report not having a job in quarter t, which share reports a formal job in quarter $t+1$? The two panels in figure 5

receive less than the minimum wage). This can be attributed either to underreporting or to part-time contracts with reduced workday, for which the labor law accepts payments lower than the legal minimum wage.



(a) Transitions to unemployment

(b) Transitions from unemployment

Figure 5: Transitions Rates in the Data

show the numbers. Panel 5a plots the time series of transitions to unemployment. Throughout the period range of analysis, informal workers transition to unemployment more often than formal ones, as fact 3 states. Following the 2015-2016 recession, this pattern becomes stronger. Such difference between job ending rates is critical from a welfare standpoint, given that the loss of labor income potentially represents one the main negative shocks to overall income experienced by households.

Unfortunately, the evidence on transitions in the opposite directions is not as clear. Panel 5b plots transition rates from unemployment to formal or informal employment. While transitions to formal employment were more frequent before the 2015-2016 recession, this pattern reverses in the second half of the sample. Informality increased significantly after the recession. Given the ambiguous evidence, instead of targetting transition rates to employment, we calibrate our model to match unemployment rates and informality, and the laws of motion that govern the flows of workers yield the required job-finding rates in each sector[10].

3 Theoretical Framework

Having studied the properties of income dynamics and informality in the data, our next step is to develop a quantitative model capable of reproducing these properties. In this section, we present such model. The agents are: a one-sized continuum of heterogeneous households, heterogeneous firms, a government and a representative investor (a firm of type II in the sense of Ljugqvist & Sargent (2018)). Time is discrete and there is no aggregate uncertainty. The commodities of the model are: a homogeneous final good, physical capital, labor hours and a one-period, risk-free bond that promisses the payment of one unit of final good in the following period. Markets are therefore incomplete. Let q_t be the price of bonds.

[10]Bosch & Esteban-Pretel (2012) find that transitions from unemployment to informal jobs are more likely than transitions to formal jobs throughout their sample. Importantly, both their sample period and the survey from which they obtain their results are different from ours. Also, they include self-employed individuals in their definition of informal workers.

Each household contains a worker and a shopper. Shoppers purchase final goods for consumption and bonds as a form of saving. They make their choice to solve a standard consumption-savings problem, which we present directly in recursive form for brevity. The shopper state is (a, s), where a is beginning-of-period net wealth and s captures variables related to the labor market over which the shopper has no control. In equilibrium, s follows a Markov chain with transition matrix $g_t(s'|s)$. The labor income function is $y_t(s)$. The search model we develop determines both g and y functions.

The shopper's utility is separable, with CRRA period utility $u(c) = u^{1-\gamma}/(1-\gamma)$. There is a no-borrowing constraint, which we take to be a realistic assumption to emerging market economies. They then solve

$$\begin{aligned} J_t(a,s) = \max_{c,a'} \quad & u(c) + \beta \sum_{s'} g_{t+1}(s'|s) J_{t+1}(a',s') \\ \text{s.t.} \quad & c + q_t a' \leq a + y_t(s), \\ & a' \geq 0. \end{aligned} \tag{1}$$

Value function J as defined in equation (1) consists on our main measure of welfare. The shopper takes as given the evolution of the state s in problem (1).

3.1 The Search Block - General Environment

The labor market in the economy contains DMP-style frictions that prevent firms willing to hire and workers searching for jobs from immediately matching. Such frictions generate unemployment in equilibrium. We allow workers to search on and off the job, an important assumption to avoid having high-skill workers pass on low or minimum wage offers (in constrast to the data) for reasons similar to the seminal McCall (1970) model. For simplicity, employed workers have no diminished capacity of searching for jobs. The matching function has a standard Cobb-Douglas form:

$$M(U, F) = \mu U^{\varepsilon} F^{1-\varepsilon} \tag{2}$$

where U is the measure of searching workers and L is the measure of posted vacancies. Parameter ε is of critical importance. The elasticity of job-finding rates to the market tightness (the ratio of vacancies to searching workers in the market) is given by $1 - \varepsilon$. In reducing or outright shutting down informal activity, we reduce the firm's value of having a posted vacancy (the choice of an informal contract is no longer there!). As a consequence, fewer firms participate in the labor market and so tightness declines. A higher elasticity (lower value of ε) then implies a sharper drop in job-finding rates - a critical variable in the matter of income risk and unemployment.

Workers have different productivity levels h, a variable we refer to as *skill type*, or simply type. These productivity levels are constant over time and the distribution of types over the population is H. Employed workers receive labor income paid by the firm. Unemployed workers derive a private utility of leisure (denominated in units of final good) $\zeta_0 + \zeta_1 h$ and an unemployment income ς transferred by the government. The distinction between these two variables allows us to govern the floor on paid wages while leaving the unemployment income - a critical variable! - open in the calibration so we can match actual public transfer programs in Brazil.

Each firm in the model corresponds to a single vacancy and a single worker. When posting vacancies, we assume firms can direct their search towards workers of any given type h. Thus, each skill type h effectively defines a segmented labor market in the model. The economic environments of workers of different types therefore are not linked to each other. We follow standard notation of the search literature: $\theta_t(h)$ is the market tightness in labor market h (that is, corresponding to workers of skill type h) and $p_t(h) = M(\theta_t(h)^{-1}, 1)$ is the worker-finding rate for searching firms[11]. The job-finding rate $p_t(h)\theta_t(h)$ then follows - we often write $p\theta_t(h)$ instead to save space. The cost of posting a vacancy κh is linear on productivity, as a way of preventing the model from artificially generating a job-finding rate that is increasing on households' types. Finally, we assume free-entry of new firms in the market.

Each firm uses capital k and *efficiency* hours of labor ℓ to produce final goods according to a Cobb-Douglas production function $Ak^\alpha \ell^{1-\alpha}$. Let δ be the depreciation rate of capital and r_t its rental price in period t. Then, the choice of rented capital is static and solves $\max\ Ak^\alpha \ell^{1-\alpha} - (r_t + \delta)k$. The first-order condition $A(k/\ell)^{\alpha-1} = r_t + \delta$ determines the capital-labor ratio k/ℓ as a function of r_t. Let

$$P(r) = A\left(\frac{k}{\ell}(r_t)\right)^{\alpha-1} - (r+\delta)\frac{k}{\ell}(r). \tag{3}$$

Then, the net-of-capital-cost firm revenue is $P(r_t)\ell$ if the firm applies ℓ efficiency hours labor in its production.

After paying for its vacancy, the entrant firm draws two individual state variables: e from a time-t dependent distribution $G_{e,t}$ and z from a distribution G_z. The support of the two distributions are respectivelly $\mathcal{E} = [0, 1]$ and $\mathcal{Z}$. The draws of e and z are independent, but we use the joint distribution $G_t = G_z G_{e,t}$ in the notation to save space. State z represents firm productivity. In hiring a worker of skill type h, the firm has access to zh efficiency hours of labor until the match ends. Like type h, productivity z is constant over time. As for the other state variable e, we refer to it as a firm's *hiding ability*. It captures unmodelled differences in the economic environment of different business that lead to dissimilar capacities of avoiding legal, financial or social punishment for establishing labor contracts outside legal standards. Small street stores on the countryside may not face the same punishment from law enforcement

[11]Note that p_t is *not* the vacancy-filling rate. Employed workers that find a new match can opt for staying in their current job instead of switching.

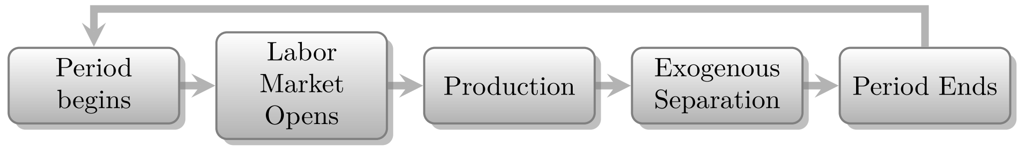


Figure 6: Timing of the Model

or from public opinion as multinational companies eager to keep positive images of their brand, or large retailers located on Times Square. To keep the model tractable, we assume hiding costs in the form of reduced efficiency hours applied by the firm. An informal firm with hiding ability e uses ezh efficiency hours of labor in production, which justifies $e \in [0, 1]$ almost surely. That specification makes the cost of informality be marginal on productivity. A different specification would have it be a fixed cost. However, such specification would lead informal firms to be, everything else the same (including hiding costs), more productive than formal ones, a result at odds with firm-level evidence. Like states h and z, hiding costs e are constant over time. After drawing its individual state (e, z), the firm can withdraw its vacancy and wait for the next period to draw a new pair.

Figure 6 illustrates the timing of the model. The period begins and the labor market opens. Firms that paid for a vacancy in the previous period draw a new state (e, z), start their search and potentially match with a worker. We allow them to drop their vacancies if they draw an unfavourable state. Bargaininig over the wage rate takes place for both new and continuing matches. Then, production takes place, shoppers purchase final goods and bonds, and unmatched firms pay the cost of keeping up their vacancies. New (unmatched) firms might enter the economy or continuing ones might exit. Before the period ends, separations between workers and firms occur for exogenous reasons, and then the period ends. Note that shoppers make their decision without knowing if their households' workers keep their job.

The government taxes economic agents, make unconditional transfers to unemployed workers, establishes a minimum wage and pays one-time unemployment insurance (UI) benefits to laid off workers. To capture the relevant margins for Brazilian firms and workers, we assume four different forms of taxation, all charged at flat rates. Three of them are sales tax τ_y, payroll tax τ_w and worker income tax $\tau_{hh,t}$. The final tax is profit taxation, but to avoid unnecessary notation, we embed profit taxation on the sales and payroll taxes. The minimum wage is ω, the same for all firms and markets, and the UI benefit is a one-period transfer of υ final goods extended only to workers that lose their job *and* fail to find a new job at the beginning of the following period (that is, employed workers that transition to unemployment). Although having unemployment insurance payments extend for multiple periods be a more realistic assumption, the one-period specification offers more tractability at a small cost of realism[12].

[12]With the one-period convention we calibrate υ to match the expected present discounted value of future

After learning their state (e, z) at the beginning of a period, firms decide in which labor market to search and also whether to offer a formal or an informal labor contract. The formality status of a contract can't change thereafter. The government can't enforce taxation or the minimum wage on informal firms. Moreover, informal workers have no access to unemployment benefits in case they lose their job, a realistic and relevant feature of the model. Regardless of previous job formality status, all unemployed workers have access to the government's unconditional transfer ς. Not being subject to government norms and regulation reduces costs to the firm, but hiding its activities also come at a cost for the firm, captured by their hiding ability state e. The decision to offer a formal or informal contract takes place *before* the firm finds its match. Therefore, there is no bargaining involved in the determination of the formality status of a contract: firms choose the format that maximizes their value. Finally, matches end for exogenous reasons. In light of the evidence of transitions of employment to unemployment (figure 5a), we assume the separation rate differs between formal (λ_f) and informal (λ_i) jobs. In particular, informal workers lose their job more often: $\lambda_i > \lambda_f$.

3.2 Value Functions and Decision Sets

The value functions are $V_t^f(z, h)$ (formal) and $V_t^i(ez, h)$ (informal) for firms with matched worker of skill type h. For workers, the value functions are $W_t^f(z, h)$ for employed formal workers, $W_t^i(ez, h)$ for employed informal workers and $W_t^n(h)$ for unemployed workers. Although each firm has two individual states, z and e, we can state value functions with a single argument for both since hiding ability affects efficiency hours in the same way productivity does. One can think that firms effectively draw two levels of productivity, one corresponding to formality and the other to informality, and then decide which one to carry[13].

To write the expressions of the value functions, it is useful to establish additional notation. Conditional on matching with a worker in the labor market in period t, let $\rho_t^f(z, h)$ be the probability that this worker accepts the job and a new match is formed. Define $\rho_t^i(ez, h)$ analogously. Note that an unmatched firm decides to post a vacancy for a formal contract if $\rho_t^f(z, h)V_t^f(z, h) \geq \max(0, \rho_t^i(z, h)V_t^i(ez, h))$. It posts for an informal contract if $\rho_t^i(z, h)V_t^i(ez, h) > \max(0, \rho_t^f(z, h)V_t^f(z, h))$. If none of the two previous conditions hold, it drops its vacancy from the market[14]. So, define the correspondences $Z_t^f(h)$ and $Z_t^i(h)$ that return the set of firms' states that lead them to opt for a formal and informal contract, respectivelly. We also define the set of states $Z(h)$ that lead the unmatched firm *not* to

flows of benefits. In Brazil, unemployment benefits take the form of five monthly installments that cease as soon as the worker finds a new occupation. Deviations between our model and reality occur as workers find jobs before the completion of the five payment installments. Given the observed job-finding rate (figure 5b) and the possibility of workers not reporting their new jobs, we take the one-period approximation to be sufficiently accurate.

[13] This is how we solve the model numerically. See section D of the appendix for details.

[14] If indifferent, the new firm chooses a formal over an informal contract, and chooses not to post instead of posting an informal contract. These choices are irrelevant for our solution, in any case.

withdraw its vacancy.

$$\begin{aligned} Z_t^f(h) &= \left\{(e,z) \mid \rho_t^f(z,h)V_t^f(z,h) \geq \max(0, \rho_t^i(ez,h)V_t^i(ez,h))\right\} \\ Z_t^i(h) &= \left\{(e,z) \mid \rho_t^i(ez,h)V_t^i(ez,h) > \max(0, \rho_t^f(z,h)V_t^f(z,h))\right\} \\ Z_t(h) &= Z_t^f(h) \cup Z_t^i(h) \end{aligned} \tag{4}$$

It is also useful to define the set of firm states that lead an employed worker to change jobs. For example, a formal worker employed by a state z firm transitions to a new, formal job if he or she finds a firm that opts for a formal contract vacancy *and* that carries an individual state that leads to a higher value to the worker. That is, the new state (e', z') must satisfy $(e', z') \in Z_t^f(h)$ and $W_t^f(z', h) > W_t^f(z, h)$. Applying this logic we define our correspondences[15]:

$$\begin{aligned} \Phi_t^f(z,h) = &\left\{(e',z') \in Z_t^f(h) \mid W_t^f(z',h) > W_t^f(z,h)\right\} \\ &\cup \left\{(e',z') \in Z_t^i(h) \mid W_t^i(e'z',h) > W_t^f(z,h)\right\}, \\ \Phi_t^i(ez,h) = &\left\{(e',z') \in Z_t^f(h) \mid W_t^f(z',h) > W_t^i(ez,h)\right\} \\ &\cup \left\{(e',z') \in Z_t^i(h) \mid W_t^i(e'z',h) > W_t^i(ez,h)\right\}. \end{aligned} \tag{5}$$

We are now ready to define our value functions. The free entry assumption ensures that the value of unmatched firms equals zero. As for the value of matched firms, we start by stating them as functions of wage rates w as well, $\hat{V}_t^f(w, z, h)$ for example, and then given equilibrium bargained wage rates $w_t^f(z, h)$ and $w_t^i(ez, h)$ we have $V_t^f(z, h) = \hat{V}_t^f(w_t^f(z, h), z, h)$ (the same for V_t^i).

$$\begin{aligned} \hat{V}_t^f(w,z,h) = &(1-\tau_y)P(r_t)zh - (1+\tau_w)w + q_t(1-\lambda_f)\times \\ &\times \left[(1-p\theta_{t+1}(h)) + p\theta_{t+1}(h)\left(1 - \frac{G_{t+1}(\Phi_{t+1}^f(z,h))}{G_{t+1}(Z_{t+1}(h))}\right)\right] V_{t+1}^f(z,h) \\ \hat{V}_t^i(w,ez,h) = &P(r_t)ezh - w + q_t(1-\lambda_i)\times \\ &\times \left[(1-p\theta_{t+1}(h)) + p\theta_{t+1}(h)\left(1 - \frac{G_{t+1}(\Phi_{t+1}^i(ez,h))}{G_{t+1}(Z_{t+1}(h))}\right)\right] V_{t+1}^i(ez,h) \end{aligned} \tag{6}$$

A firm keeps its match to the following period if there is no separation shock (probability $1-\lambda$) or if one of two conditions apply: 1. the worker fails to find a new offer or 2. the worker finds a new offer, but prefers to stay in his or her current contract. The probability of either these conditions apply accounts for the term in brackets in equations (6). When losing their employee for any reason, the firm returns to the pool of unmatched firms and derives zero

[15] Expressions in (5) imply that workers only decide to switch jobs if they find a match that they *strictly* prefer to their current one.

value. Another observation of (6) is that it leaves it implied that wages are re-negotiated every period - otherwise continuation values would also be functions of w.

The value of workers is more cumbersome to state, since we also need to care for the value derived when the match ends. It is useful to first define the auxiliary function $W_t^s(W, ub, h)$ that returns the value of searching (hence the superscript "s") in the labor market with a reservation value W and potential insurance benefit ub in case of no match:

$$W_t^s(W, ub, h) = (1 - p\theta_t(h))(W + ub) + p\theta_t(h) \int_{Z_t^f(h)} \max(W, W_t^f(z, h)) \frac{dG_t}{G_t(Z_t(h))} \\ + p\theta_t(h) \int_{Z_t^i(h)} \max(W, W_t^i(ez, h)) \frac{dG_t}{G_t(Z_t(h))}.$$

The definition above leaves implied a condition we formalize in the next subsection: the value of a formal or informal contract is independent from the prospective employee's reservation value. In the context of our model, this amounts to say that wages bargained by an employed and an unemployed worker with a new firm are the same. Employed workers can't convert their current position into additional bargaining power. This is a simplifying assumption: it allows us to avoid carrying *three* additional state variables (the two states of the previous employer and whether the previous contract was formal or informal).

The value of a formal labor contract to the worker (again as a function of the wage rate w) is

$$\hat{W}_t^f(w, z, h) = (1 - \tau_{hh,t})w \\ + q_t \left\{ \lambda_f W_{t+1}^s(W_{t+1}^n(h), \upsilon, h) + (1 - \lambda_f) W_{t+1}^s(W_{t+1}^f(z, h), 0, h) \right\}. \tag{7}$$

The dividend flow is simply the net-of-income-tax wage rate. The continuation value is discounted by the price of the risk-free bond q_t. One can argue that we should discount using preference intertemporal discounting β, but results are not very affected by changing it, so we opt for the literature standard. The first term in curly brackets captures the possibility the match ends. In that case, the worker re-enters the labor market at the beginning of the following period. Not finding a new match then means he or she transitions to unemployment *and* receives unemployment benefits υ. The second term in curly brackets represents the chance the worker keeps his or her job and enters the job market with the reservation value equal to the value of the current job. In this case, no unemployment benefit is due in case the worker can't find a new match, as he or she does not transition to unemployment.

The value of an informal contract is

$$\hat{W}_t^i(w, ez, h) = w + q_t \left\{ \lambda_i W_{t+1}^s(W_{t+1}^n(h), 0, h) + (1 - \lambda_i) W_{t+1}^s(W_{t+1}^i(ez, h), 0, h) \right\}. \tag{8}$$

The worker can hide the wage payment from the government and thus pays no income tax. Also, note that transition to unemployment does not lead to unemployment insurance benefits. Given an equilibrium bargained wage function $w_t^f(z;h)$ and $w_t^i(ez;h)$, we have $W_t^f(z;h) = \hat{W}_t^f(w_t^f(z,h), z, h)$ and the same for W_t^i.

Finally, the worker's value at unemployment is:

$$W_t^n(h) = \varsigma + \zeta_0 + \zeta_1 h + qW_{t+1}^s(W_{t+1}^n(h), 0, h). \tag{9}$$

3.3 Wage Setting

We determine wage rates using Nash bargaining subject to minimum wages (in the case of formal contracts). Start by defining Nash-bargained wages $\hat{w}$ in the usual way:

$$\begin{aligned} \hat{w}_t^f(z,h) &= \underset{w}{\operatorname{argmax}} \quad \left(\hat{W}_t^f(w,z,h) - W_t^n(h)\right)^{\frac{\eta}{1-\tau_{h h,t}}} \hat{V}_t^f(w,z,h)^{\frac{1-\eta}{1+\tau_w}} \\ \hat{w}_t^i(ez,h) &= \underset{w}{\operatorname{argmax}} \quad \left(\hat{W}_t^i(w,ez,h) - W_t^n(h)\right)^{\eta} \hat{V}_t^i(w,ez,h)^{1-\eta} \end{aligned} \tag{10}$$

The first-order conditions for an interior solution to the optimization problems indicate a wage rate that is linear on firm's productivity, and that leads to the following conditions (we omit value functions' arguments):

$$\frac{\eta}{1-\eta} = \frac{W_t^f - W_t^n}{V_t^f} = \frac{W_t^i - W_t^n}{V_t^i}. \tag{11}$$

In case there is no wage rate w that guarantees both terms in parenthesis be non-negative, we set $\hat{w}$ to be the worker's reservation wage and the firm then chooses not to offer the contract on such state. If we set $\hat{w}$ to be the firm's reservation wage, workers in turn would reject the contract. The bargained wage $\hat{w}$ coincides with the effective wage rate in the case of an informal contract: $w_t^i(ez,h) = \hat{w}_t^i(ez,h)$. In the case of the formal contract, the minimum wage rate must be obliged, so the effective wage is $w_t^f(z,h) = \max(\omega, \hat{w}_t^f(z,h))$. For future reference:

$$\begin{aligned} w_t^f(z,h) &= \max(\omega, \hat{w}_t^f(z,h)), \\ w_t^i(ez,h) &= \hat{w}_t^i(ez,h). \end{aligned} \tag{12}$$

This wage-setting mechanism deserves a few observations. First, the unemployed household is always weakly (in practice, strictly) better off accepting a job than staying unemployed. Searching workers without a job always transition to employment in case they find a match. Thus $p\theta$ represents not only the job-finding rate but also the transition rate from unemployment to employment[16]. Second, using (10) and (12) to set wage rates for all instances of bargaining

[16] Another assumption leading to this is that firms observe their type at the beginning of the period, *before* finding a worker in the labor market.

meets the requirement needed for us to state value functions simply as (6), (7) and (8). The assumption we make is that the outside option used in the bargaining process is always that of unemployment, *even if the worker is employed*. We believe such simplification to be of little consequence to our results for two reasons. First, in equilibrium it is probably the case that workers transition from less productive to more productive matches. Allowing for the "correct" outside options would just lead to higher wage rates in general, with little allocative effect that we can't control by properly setting bargaining power η. Second, and perhaps more importantly, given that wage rates are negotiated on a period-by-period basis, the differentiated wages resulting from better outside options would prevail for a single period only.

Another observation of the Nash bargaining procedure is that we adjust the bargaining power in the case of a formal contract to reflect the incidence of taxation (meaning we divide η by $1-\tau_{hh}$ and $1-\eta$ by $1+\tau_w$), a necessary adjustment if we are to observe condition (11) holding. The two equations in (11) imply that whenever the effective wage coincides with the interior solution to the Nash problem $\hat{w}$, firms and workers agree on the preferred type of contract. In equilibrium, we *only* fail to observe (11) when the minimum wage impedes the theoretical bargained wage to prevail. In that case, firm and worker might disagree on their preferred contract only if the firm offers an informal one[17].

One final observation that we already commented on, and that relates to the previous two observations, is that we do not allow formal workers to use potential unemployment benefit payments to bargain higher wages. There is no υ summing the outside option W^n in the first equation in (10). We choose not to include it for several reasons. First, as we just said, unemployment benefits are only *potential*. Workers coming out of unemployment that fail to agree on a wage rate with the firm have no access to the benefit, as they wouldn't first transition to employment. Even if we consider continuing formal employees, whether they would receive the benefit depends on the assumption that the government pays insurance benefits to workers that become unemployed for not being able to agree with the firm on a wage rate - an assumption we need not make as this is an off-equilibrium outcome. A second reason for leaving out the unemployment benefit threat is our empirical observation of section 2 of the low income differential between formal and informal workers (fact 2), a pattern that suggests a low or inexistent additional bargaining power owned by formal workers. Another reason is that including υ as an ouside value would break condition (11), and so new workers (as well continuing workers, depending on the off-equilibrium assumption) could disagree with the formality choice made by the firm, a property of the model we do not wish to break. Finally, including the benefit outside option to continuing workers but not to new ones

[17]To see this claim, consider a firm drawing a state (e, z) that chooses to post a formal contract offer. We know then that the firm anticipates (correctly, in equilibrium) $V^f(z,h) > V^i(ez,h)$. If the minimum wage constraint binds, the first equality in (11) fails to hold; instead, we have $(W^f - W^n)/V^f > \eta/(1-\eta) = (W^i - W^n)/V^i > (W^i - W^n)/V^f$. It follows that $W^f - W^n > W^i - W^n$. Firm and worker agree.

would again involve defining additional state variables to keep track of previous employment positions, a complication we prefer to avoid.

3.4 Free Entry and Transition Rates

The condition of free entry determines the number of firms paying for vacancies each period. At the beginning of each period, firms that paid for the vacancy in the previous period draw a pair (e, z), decide if they keep their vacancy and, if they do, if they offer a formal or an informal labor contract. Free entry implies that, for each labor market h, the expected value of drawing a new pair (e, z) and having the opportunity of searching in the market equals the cost of doing so, κh:

$$p_{t+1}(h)q_t \int \max\left\{0, \rho^f_{t+1}(z,h)V^f_{t+1}(z,h), \rho^i_{t+1}(ez,h)V^i_{t+1}(ez,h)\right\} dG_{t+1} = \kappa h. \tag{13}$$

Like most search models, condition (13) in practice determines the worker-finding rate $p_t(h)$ compatible with free entry. By standard arguments, the matching function then gives the equilibrium market tightness:

$$p_t(h) = M(\theta_t(h)^{-1}, 1). \tag{14}$$

Since employed workers search with full efficiency in the model, the market tightness coincides with the number of searching firms in any labor market h (per worker of type h). Therefore, tightness $\theta_t(h)$ and the number of firms that paid for a vacancy in the previous period $F_{t-1}(h)$ are linked through the following equation:

$$\theta_t(h) = G_t(Z_t(h))F_{t-1}(h). \tag{15}$$

Lastly, to state ρ^f_t and ρ^i_t (the probabilities that a firm holds the worker conditional on a match), we need to reference the distribution of workers in each labor market h. Given a skill level h, let $U_t(h)$ be the share of unemployed workers of type h that are unemployed. Let $E^f_t(\{e,z\};h)$ and $E^i_t(\{e,z\};h)$ be the measures of employed formal and informal workers normalized by the measure of type h workers, $H(h)$. Therefore $U_t(h) + E^f_t(\mathcal{E}\times\mathcal{Z};h) + E^i_t(\mathcal{E}\times\mathcal{Z};h) = 1$ for any skill type. Let $\Psi^f_{i,t}(z,h) = \{(e',z') \ : \ V^i_t(e'z',h) < V^f_t(z,h)\}$ be the set of firm states such that a type h informal worker employed by it decides to switch jobs to a state z formal firm. Define $\Psi^f_{f,t}(z,h)$, $\Psi^i_{f,t}(ez,h)$ and $\Psi^i_{i,t}(ez,h)$ analogously. We have:

$$\begin{aligned}
\rho^f_t(z) &= U_{t-1}(h) + \lambda_f E^f_{t-1}(\mathcal{E}\times\mathcal{Z};h) + \lambda_i E^i_{t-1}(\mathcal{E}\times\mathcal{Z};h)+ \\
&\quad + (1-\lambda_f)E^f_{t-1}(\Psi^f_{f,t}(z,h);h) + (1-\lambda_i)E^i_{t-1}(\Psi^f_{i,t}(z,h);h) \\
\rho^i_t(z) &= U_{t-1}(h) + \lambda_f E^f_{t-1}(\mathcal{E}\times\mathcal{Z};h) + \lambda_i E^i_{t-1}(\mathcal{E}\times\mathcal{Z};h)+ \\
&\quad + (1-\lambda_f)E^f_{t-1}(\Psi^i_{f,t}(z,h);h) + (1-\lambda_i)E^i_{t-1}(\Psi^i_{i,t}(z,h);h)
\end{aligned} \tag{16}$$

Because employed workers suffer no loss of search efficiency, the normalized measure of searching workers each period equals one, and so the expressions above require no denominator.

3.5 The Government

To add realism to the model, the government manages a public debt D. We use no time subscript because all our simulations assume fiscal neutral policies that allow the government to keep the stock of public debt constant in all periods. Taxes represent the public source of revenue. Expenditures comprise UI payments $TR_{UI,t}$, unconditional transfers $TR_{unc,t}$ and public consumption $C_{gov,t}$. Public consumption has no impact on households' utility. The government's budget constraint is

$$\begin{gathered} C_{gov,t} + \upsilon \int U_{UI,t}(h)dH + \varsigma \int U_t(h)dH + D = \\ \tau_y \int \int zhdE_t^f(.;h)dH + (\tau_w + \tau_{hh,t}) \int \int w_t^f(z,h)dE_t^f(.;h)dH + q_t D, \end{gathered} \tag{17}$$

where $U_{UI,t}(h)$ is the share of type h workers that receive the one-time UI transfer. The left-hand side groups expenses, the right-hand side groups sources of revenue.

In our simulations, we allow the government to set one of two variables only, public expenditure $C_{gov,t}$ or the income tax rate $\tau_{hh,t}$. The government sets the other one so that public policy is feasible (in the sense that the budget constraint (17) holds in all periods).

3.6 The Representative Investor

The representative investor operates the technology that converts final goods into physical capital and vice-versa, and stores capital from one period to the next. He or she invests not only in capital, but also holds the equity of existing firms and posts new vacancies in the market. To finance these purchases, the investor issues one-period bonds to households. The representative investor is a risk-neutral agent which, for simplicity, we allow to consume negative amounts.

Let DIV_t be aggregate dividends and Q_t the value of existing firms. The following proposition captures the behavior of the representative investor and its consequences for equilibrium prices and quantities.

Proposition 1. *If the representative investor stores a positive and finite amount of capital K_{t+1}, then*

$$q_t = \frac{1}{1 + r_{t+1}} \tag{18}$$

holds, and the representative investor is indifferent between equity and capital investment.

Period $t+1$ consumption is then given by

$$(1+r_{t+1})K_{t+1}+Q_{t+1}-\frac{1}{q_t}\left[K_{t+1}+Q_t-DIV_t\right]$$

and the investor's balance sheet equality is

$$K_{t+1}+q_tQ_{t+1}=\textit{Value of bonds sold to households.}$$

We provide in section C of the appendix the proof for proposition 1 and the details of the representative investor's problem.

3.7 The Shopper's State

Usually, descriptions of search models involve equations describing the evolution of the measure of workers in and out of jobs (in steady-state models, the equation simply says that the number of workers transitioning to employment coincides with that of workers transitioning to unemployment). For our model, it is easier first to describe the shopper's exogenous state s and calculate its transition matrix g. Then, measures of employed and unemployed workers as well as the distribution between formal and informal contracts evolve according to g.

The shopper's exogenous state is $s=(d,e,z;h)$. State d represents labor market status: whether the worker is employed and, if so, whether the contract is formal or informal. The pair (e,z) represents the productivity and hiding ability of the employer. Of course, e is irrelevant if the labor contract is formal, and both (e,z) are irrelevant if d indicates that the worker is unemployed. Having the period income function $y(s)$ be a function of current but not previous state requires that we define an additional labor market status: unemployment insurance receiver. The income function y then is

$$y_t(s)=\begin{cases}(1-\tau_{hh,t})w^f(z,h) & \text{if } d=\text{employed, formal}\\ w_t^i(ez,h) & \text{if } d=\text{employed, informal}\\ \varsigma & \text{if } d=\text{unemployed (no insurance)}\\ \varsigma+\upsilon & \text{if } d=\text{unemployed (with insurance).}\end{cases} \tag{19}$$

Having separation rates λ_f, λ_i and the transition-to-employment rate $p_t(h)\theta_t(h)$, we can calculate the probability of workers to transition to and from unemployment. Having firms' value V_t^f and V_t^i, we can calculate their choices between formal and informal contracts each period. Having the workers' value W_t^f and W_t^i, we can calculate the probability of workers to switch jobs. Therefore, we have all we need to calculate the entire transition matrix g. The computation is tedious and offers little additional intuition of the model, so we leave it to section B of the appendix. Given the transition matrix g, the distribution of households across states (a, s), denoted $x_t(a, s)$, satisfies the following equations:

$$\begin{aligned} x_t(\{a'\}, \{s'\}) &= \int_{\{(a,s)\ \mid\ a'(a,s) \in \{a'\}\}} g_t(\{s'\}|s) dx_{t-1} \\ \int_{\{h=\bar{h}\}} dx_t &= H(\bar{h}) \end{aligned} \tag{20}$$

where $a'(a, s)$ solves problem (1). Of course, the distribution of workers of a given type over different labor market status E_t^f, E_t^i and U_t follow from x_t[18].

3.8 General Equilibrium

We are ready to define general equilibrium paths. The set of aggregate state variables of model in period t include the distribution of households x_{t-1} (along with the corresponding distribution of workers in the labor market E_{t-1}^f, E_{t-1}^i and U_{t-1}), the measure of firms that paid for a vacancy in the previous period F_{t-1} and the stock of capital supplied by the investor K_t (a predetermined variable).

Definition 1 (General Equilibrium)**.** *Given an initial aggregate state x_0, F_0 and K_1, a feasible public policy sequence $\{C_{gov,t}, \tau_{hh,t}\}$ and a path for the distribution of hiding abilities $G_{e,t}$, an equilibrium consists on a path of: value functions for firms and workers V_t^f, V_t^i, W_t^f, W_t^i, W_t^n; wage rates w_t^f, w_t^i; market tightness θ_t; discount q_t and interest r_t rates; firm measures F_t; household measures x_t; income processes y_t; transition matrices g_t; and consumption/savings policies c_t, a_t' such that:*

1. *worker and firms' value follow* (6)*,* (7)*,* (8) *and* (9)*;*

2. *wage rates are determined by Nash bargaining (subject to the minimum wage in the case of formal contracts);*

3. *the free entry condition* (13) *determines the number of searching firms F_t, and market tightness follows from* (15)

4. *discount rates reflects capital rental cost (equation* (18) *holds);*

5. *the measure of households follows from g (equation* (20)*);*

6. *the transition matrix and the income process follow from optimal worker and firm decisions;*

7. *consumption/savings policies solve the searcher's problem* (1) *given y_t and g_{t+1};*

[18] For example, $E_t^f(\{z\}; h) = x_t(\mathscr{A}, \text{formal}, \mathscr{E}, \{z\}, h)H(h)$, where $\mathscr{A}$ is the set of possible net wealth positions.

8. *the capital market clears:*

$$K_{t+1} = q_t \left[\int a_t'(a,s)dx_t - Q_{t+1} - D \right]. \tag{21}$$

By Walras' law, clearing of the capital market implies clearing in the final goods market:

$$Y_t = C_t + C_{got,t} + Inv_t + VC_t$$

where Y_t is aggregate output and $Inv_t = K_{t+1} - (1-\delta)K_t$ is investment in physical capital.

Definition 2 (Steady State). *Given a feasible public policy pair $\{C_{gov}, \tau_{hh}\}$ and a distribution for hiding abilities G_e, a steady state equilibrium is an equilibrium in the sense of definition 1 in which the variables of the path do not depend on time, the initial conditions for the aggregate state variables coincide with their constant path values and the public policy and hiding ability distribution sequences are defined by $\{C_{gov}, \tau_{hh}\}$ and $\{G_e\}$ for all periods.*

The model does not include aggregate uncertainty. In our simulations, we start with a baseline steady state equilibrium. It provides the initial values for the aggregate state required by definition 1. The economy then faces an "MIT" shock: sudden, unanticipated changes in the paths of the hiding ability distribution $G_{e,t}$ (which captures the repression of informal labor) and public policy (either government consumption or the income tax). We calculate the equilibrium associated with these new paths. Our baseline steady state, which we describe in the following section, also provides benchmarks for prices, aggregates and welfare.

4 Calibration

We calibrate parameters with a mix of standard choices in the Labor and Macroeconomics literature, parameters disciplined by the Brazilian labor, tax and social security law as of 2018, and of course parameters chosen to have the model match specific targets observed in the data, in particular those associated with facts 1 through 3. To facilitate comparison, we assume one model period corresponds to one quarter. Our main goal is to replicate the patterns of income and informality we find in our analysis of Brazilian data in section 2, while sticking to reasonable values for the parameters. The scale of the model is 1 unit of model good = 10,000 Brazilian reals (or US$ 2,500 using our conversion rate). Table 2 presents our baseline calibration.

Starting with preference parameters, we choose a coefficient of relative risk-aversion equal to 3, a standard value in the Macroeconomics literature. To calibrate the intertemporal discounting β, we start by taking from the Brazilian Central Bank website data on corporate credit interest (*ICC* - Credit Cost Indicator). After discounting for annual CPI inflation and

Symbol	Interpretation	Source/Target	Value
Preferences			
β	Intertemporal discounting	$r = 15\%$ per year	0.95458
γ	Relative risk aversion	Literature standard	3
Technology			
A	Model scale	$P(r) = 1$	0.7062
α	Capital income share	Literature standard	0.33
δ	Rate of capital depreciation	$\delta K/Y = 0.1$	0.01546
Labor Market			
λ_f	Separation rate (formal)	Transition unemployment	0.04
λ_i	Separation rate (informal)	Transition unemployment	0.098
κ	Vacancy cost	Market tightness = 1	0.064
μ	Matching function constant	Unemployment rate = 16%	0.255
$1 - \varepsilon$	Job-finding rate elasticity	Literature average	0.375
η	Worker's "bargaining power"	Hosios condition	0.625
ζ_0	Leisure	Income differential (low inc.)	0.02
ζ_1	Leisure	Income differential (low inc.)	0.16
Public Policy			
τ_y	Sales tax rate	Tax law	0.272
τ_w	Payroll tax rate	Tax and labor law	0.1648
τ_{hh}	Household income tax rate	Tax law	0.09
ω	Minimum wage	Labor law	0.0998
ς	Unemployment income	*Bolsa Família* program	0.01
υ	Unemployment insurance	Social security	0.1833
D	Public debt	Debt-to-GDP = 42%	0.6583
Distributions			
ξ	Skill distribution parameter	Formal income histogram	0.61
σ_z	Productivity dispersion	Income variance and differential	100%
ϕ	Probability of $e = 0$	Informality share	0.75
ν	Hiding ability distribution	Income differential (high inc.)	9

Table 2: Baseline Calibration

averaging on the Jan-2012 to Dec-2019 period, we find a 16.1% average real interest rate. We calibrate β so that the model generates a slightly lower value of 15% annualized interest rate (discount $q \approx 0.96566$). This is a large value for the average yield of household investment. We stick to it as our main concert is the correct discounting of future streams of profits by firms. Such discounting is critical for their formalization choice.

Moving to technology parameters, we set A so that $P(r) = 1$. The share of capital income α is set at 0.33. The rate of capital depreciation we set so that, on the aggregate, capital investment is ten percent of aggregate output. Added to investment in vacancy costs, total investment equals 16.8% of output, close to Brazilian average since 1996.

Parameters related to public policy are entirely calibrated based on Brazilian law and actual policy. The statutory tax rate on profits is 9%. To ease notation, we did not define a profit tax. Instead, we adjust the sales and payroll tax rates accordingly. The payroll tax rate $\tau_w = (1.28) \times (0.91) - 1 = 0.1648$ corresponds to the sum of the mandatory employer contributions to the social security (20%) and to the worker's severance fund (8%)[19]. The sales tax is harder to calibrate due to the myriad of different forms of taxation established by the federal and local governments. In a calibration of the Brazilian economy, Ulyssea (2018) uses a rate of 29.25%, calculated by the combination of taxes over manufactured products (*IPI*) and additional contributions to social security charged on revenue (*PIS/COFINS*). We opt for a lower rate (20%) as the model can't capture firms' ability to pass taxes over to consumers. After correcting for profit taxation, however, we end up with a figure similar to Ulyssea, $\tau_y = 1 - (0.8) \times (0.91) = 0.272$. The income tax rate $\tau_{hh} = 0.09$ is based on the worker's (also mandatory) contribution to the social security system. This choice of τ_{hh} assumes no personal income taxation, a choice we make based on the observation that most workers locate in the first bracket of the progressive income taxation scheme, so they pay no income taxes. The minimum wage $\omega = 249.5/2,500 = 0.0998$ follows the labor law. We calibrate unemployment income ς to be BR\$ 100 = US\$ 25. This target is motivated by the *Bolsa Familia* program, a basic income program that provides financial assistance to poor households in Brazil. The value of the monthly transfer for standard participation in the program was 85 Brazilian reals in 2018 and the average benefit per household (not individual) was 187 reals, so we take our calibration to be conservative in the sense that it is more generous to households than the actual program. Finally, public UI benefits in Brazil depend on the worker's income in his or her previous job. We opt for a constant value because the lower and upper limits on the value of the benefits are close to each other. The lower limit is set at the minimum wage BR\$ 998 = US\$ 249.5 per month and the upper limit in 2018 was BR\$ 1,677 = US\$ 419. Given the higher concentration of workers in the lower end of the income distribution we

[19]The law requires employers to contribute towards a severance fund (*FGTS*), with monthly payments set at 8% of worker's wage rate. It is financed by taxes charged on the firms' payroll as well. Workers can't withdraw money from their account except when losing their job without "just cause", retiring and a few other specific contingencies.

assume a benefit of BR\$ 1,100 = US\$ 275. Workers receive five monthly installments, so we set $\upsilon = (5/3) \times 275/2,500 = 0.1833$.

Moving on to labor market parameters, we choose rates of exogenous separation λ_f and λ_i to target observed transition rates from formal and informal employment to unemployment, as shown in figure 5a. We target pre-recession rates, 3% for formal jobs and 7.5% for informal ones. The vacancy cost parameter κ and the matching function constant μ target respectivelly an average market tightness of one and an average unemployment rate of 16%. We calculate these averages through the formulas $\int \theta(h)dH$ and $\int U(h)dH$. Note that the average market tightness across types h is also the unconditional market tightness (total firms searching divided by total workers searching), since all workers search in the market every period. The exponent in the matching function ε governs the elasticity of the job-finding rate to market tightness. As we lack the empirical evidence of the number of vacancies in Brazil, we can't disciple our choice by the data. Instead, we take a median point of the literature and set $\varepsilon = 0.625$ (Shimer (2005) sets $\varepsilon = 0.72$, Bosch & Esteban-Pretel (2012) set $\varepsilon = 0.5$; see Petrongolo & Pissarides (2001) for a survey). We set the worker's "bargaining power" η to satistfy the Hosios (1990) efficiency condition. The two parameters governing the worker's private value of leisure ζ_0 and ζ_1 we set to calibrate the level and slope of income differential among low-income workers (this is similar to targetting the average wage rates among informal workers).

We now turn to the parameterization of the distributions of the model, H, G_z and G_e. We discretize skill levels with ten equally-spaced values $\underline{h} = h_1 < h_2 < \cdots < h_{10} = \bar{h}$, with a distribution H such that $H(h_{t+1}) = \xi H(h_t)$. We choose a value of ξ to target the share of formal workers receiving a labor income in the US\$200-US\$400 income range. Bounds $\underline{h} = 0.03$ and $\bar{h} = 0.185$ are so that the ten worker groups cover the range of average income observed among different occupational groups and educational levels (between US\$250 and US\$900, monthly). As for productivity draws, we assume $G_z \sim \text{LogNormal}(-\sigma_z^2/2, \sigma_z^2)$. We set $\sigma_z = 100\%$, a value that generates enough variance in the log income of various worker groups and that keep the income differential away from the minimum wage low, as we saw to be the case in the data.

4.1 Hiding Ability and Government Surveillance

Given that we lack a structural model of government surveillance of informal contracts, we capture the repression of informality by changes in the distribution of hiding abilities. The tightening of enforcement leads to lower probabilities that firms draw high values of hiding ability. In particular, the distribution $G_{e,t}$ is such that draws of e are determined by the following mixture:

$$e = (1 - \text{Dummy}_{Surveillance,t}) \times \text{Beta}[\nu, 1].$$

$\text{Dummy}_{Surveillance,t}$ captures the probability that a firm draws $e = 0$, which increases as the government tightens its control over labor contracts. It can only take two values:

$$\text{Dummy}_{Surveillance,t} = \begin{cases} 1 & \text{with probability } \phi_t \\ 0 & \text{with probability } 1 - \phi_t. \end{cases}$$

Parameter ϕ_t captures the measure of firms/jobs that face conditions that make it unfeasible under any circumnstances to hire informally. This would be the case if they were audited by the government. Thus, our simulation of the repression of informal activity consists on increases of ϕ_t over time.

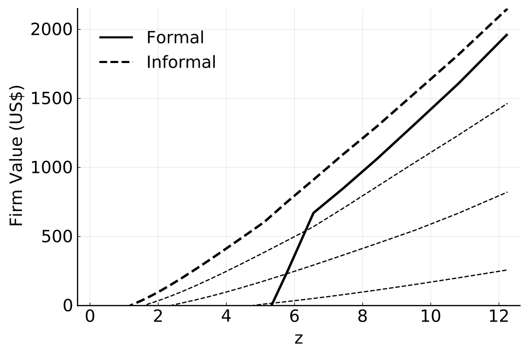
The other term of the mixture is a Beta distribution, a natural choice given that its support is the $[0, 1]$ inverval, as required by the model. Shape parameter ν governs the extent to which the distribution concentrates mass close to the upper bound 1. Thus, *higher* values of ν indicate *lower* average hiding costs and *greater* capacity of firms to hire informally. There is a tension in the calibration of ν and ϕ as both parameters have a significant impact on the size of the informal sector. In the baseline calibration, we set $\nu = 9$ to target the low income differential among high income workers. This implies that the Beta distribution has an average of 90%. That choice requires a somewhat high value of ϕ, which we set at 75% to generate a realistically small informal sector.

4.2 Numerical computation

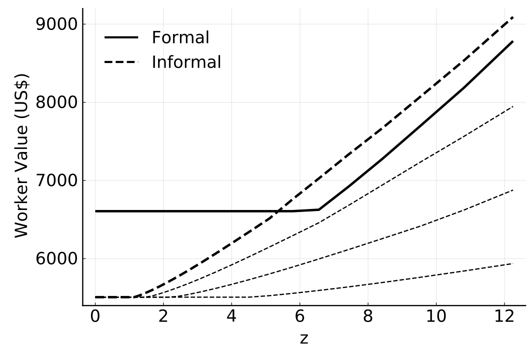
The numerical procedure we use to approximate the equilibria we study is, in general, standard in the Macroeconomics literature. We defer details to section D of the appendix. We discretize the distribution of (e, z) with 50 productivity values, 49 positive ones calculated using Tauchen's method (which also discretizes G_z), and zero. To account for the G_e distribution, we adjust the probability of each productivity draw, in that each new firm effectively draws a pair (z, ez) in the discretized grid. See the appendix for the details. For each skill level h, the space of exogenous states s therefore contains 102 states: fifty points of formal employment, fifty of informal employment, unemployment and unemployment with UI benefits. We discretize the asset grid with a fifty-point grid, and evaluate asset levels out of the grid using linear interpolation. We compute the solution to the shopper's problem (1) using the endogenous grid method (Carroll (2006)).

4.3 Baseline Equilibria and Model Performance

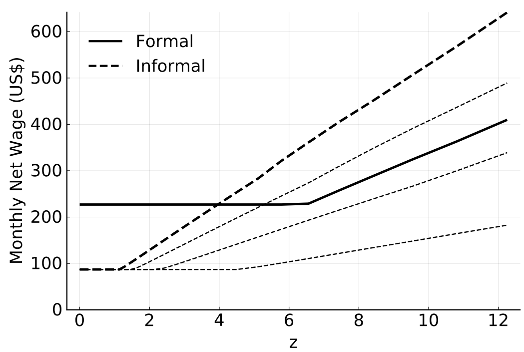
Before proceding to our simulations, we explore the model's mechanics as well as its performance in reproducing the three facts of section 2. Figure 7 plots the main functions of the search equilibrium: firms' value V in panel 7a, workers' value W in panel 7b and (net)



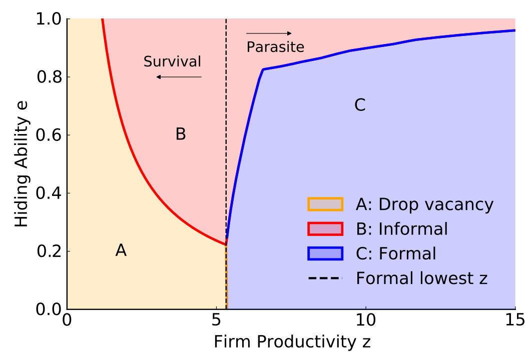
(a) Firm Value



(b) Worker Value



(c) Wage Rates



(d) Regions of Production

Notes: Panel (a). Firm value functions $V^f(z,h)$ (solid) and $V^i(ez,h)$ (dashed) for $\underline{h}$, the lowest skill type. The thick dashed curve corresponds to $e=1$. The remaining three dashed curves correspond, respectively, to $e=0.75$, $e=0.50$ and $e=0.25$ from the top to the bottom. Panel (b). Worker value functions $W^f(z,h)$ (solid) and $V^i(ez,h)$ (dashed) for $\underline{h}$, the lowest skill type. Dashed curves have the same interpretation as panel (a). Panel (c). Net-of-tax income functions $(1-\tau_{hh})w^f(z,h)$ (solid) and $w^f(z,h)$ (dashed) for $\underline{h}$. Dashed line indicates net-of-tax minimum wage. In all panels, x-axis indicates firm productivity z. Panel (d). We divide the firm's state space into three regions according to the decision of an unmatched firm of whether and which type of contract to post in the labor market.

Figure 7: Equilibrium Functions - Lowest Skill Type

wage rates $(1-\tau_{hh})w^f, w^i$ in panel 7c. We also depict in panel 7d the choice of searching firms of whether and which type of contract to post in the labor market. Taking panel 7a, the solid curve corresponds to the formal firm $V^f(z,h)$; the thick dashed curve corresponds to the informal firm $V^i(ez,h)$ with hiding ability $e=1$; the other three thin dashed curves correspond to lower levels of hiding ability, $e=0.75, 0.5, 0.25$, from the top to the bottom. The interpretation of the solid and dashed curves is similar for the other two function plots. All curves plot the functions evaluated at $\underline{h}$, which is the lowest skill level in the calibration. The same is true for the production regions. There is nothing special about this choice of skill level for the plots - we choose it to make figures prettier.

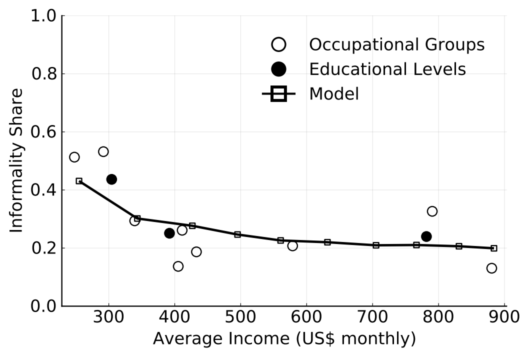
In the case of panel 7a, the x-axis crosses the y-axis at zero. Entrant firms that draw a state associated with a negative match value immediatly leave the market, and so we have no interest on the shape of the value function in those states. In the case of panel 7b, with the value of workers, the lower bound on the employment value of any worker corresponds to the unemployment value W^n (not plotted). The kink in the formal value of both firms and workers (the solid curves) marks the productivity level below which the minimum wage constraint binds (approximately 6.5 in this case). The constraint does not apply for informal contracts. Still focusing on the panels with value functions, notice their slight convexity on z. It is explained by on-the-job search: as productivity increases, not only does the discounted sum of dividends (linearly) increases, but also the probability that the worker leaves the job by finding a better one declines (sets Φ^f and Φ^i shrink), which leads to greater continuation value to the firm. Nash bargaining then distributes the value gain to the worker *via* larger bargained wages, which justifies the convexity of the worker's value as well.

Comparing panels 7a and 7b, we note a feature of the model we emphasized in subsection 3.3. Whenever the minimum wage is not binding, worker and firm agree on the preferred type of contract (equilibrium condition (11)). When the minimum wage does bind, it tends to be the case that workers prefer formal contracts, but the set of productivity states that are high enough to justify formal firm activity is somewhat reduced. Note, in addition, that the slope of the formal firm's value (and hence of the formal worker's value too) is greater than that of the informal firm *even with* $e = 1$. Therefore, if we make the productivity grid sufficiently large, then for each value of hiding ability e, there would be a productivity level $z^*(e)$ such that the unmatched firm opts for a formal contract whenever it draws z *greater* or equal to $z^*(e)$. The wage function in panel 7c suggests this is a key mechanism of the model. Given a productivity z, it is *not* the case that formal firms always pay higher wages. The answer to that question depends on the hiding ability of the informal firm against which we compare the salary received in the formal sector. Indeed, assuming the minimum wage contraint doesn't bind, informal workers hired by firms with perfect hiding ability ($e = 1$) tend to receive wages significantly higher than their formal counterparts hired by firms with similar productivity. Therefore, the inverse relationship between income and informality share, as stated by fact 1 of section 2, is a byproduct not of higher wages in the formal sector per se, but of the different productivity *compositions* of each sector: the greater slope of formal firm's value means that the formal sector concentrates firms and workers with superior productivity, on average.

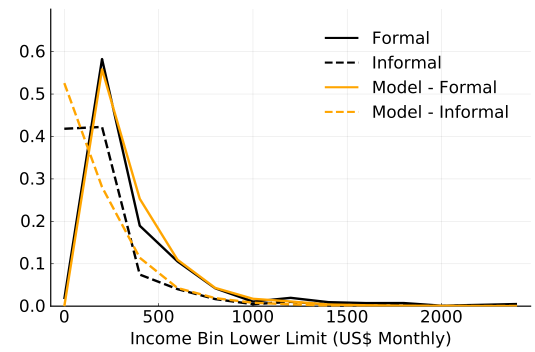
In fact, we can see this more clearly by looking at the decision of searching firms as shown by panel 7d. The x-axis contains different firm productivity levels z and the y-axis contains hiding ability levels e. There are three regions in figure: A (yellow region), B (red) and C (blue). An unmatched firm that at the beginning of a period draws a pair (e, z) in region A drops its vacancy from the market, as the value of formal and informal workers is negative. Pairs in region B lead the firm to offer an informal contract and pairs in region C lead the firm to offer a formal contract. We also draw with a dashed line the lowest firm productivity compatible with a formal contract $\underline{z}^f$ (that is, $\underline{z}$ such that $V^f(\underline{z}^f, \underline{h}) = 0$). Such level of productivity is of particular interest to us, for two reasons. First, it reconciliates our model with two classic visions of informality present in the literature: the "parasite" view and the "survival" view. We can call informal firms with productivity levels superior to $\underline{z}^f$ (region B, to the right of the dashed line) parasites in the sense that their economic activity would be viable in the formal sector. They opt for an informal contract only for profit maximization reasons. On the other hand, informal firms with productivity levels lower than $\underline{z}^f$ (region B, to the left of the dashed line) also choose an informal contract for profit maximization, but in their case hiring the worker formally would not be economically viable. In that sense, their *survival* as firms (or jobs) depends on the possibility of operating informally. Second, and more importantly, these two "types" of informal firms lead to conflicting forces in the simulation of the repression of informality. Ignoring other general equilibrium effects, without the option of the informal agreement, "parasite firms" (to the right of $\underline{z}^f$) simply migrate to the formal sector, which leads to a *decrease* in the average productivity of searching firms and no change in the number of firms searching in the labor market. "Survival firms", on the contrary, exit the market, as they can't survive by offering formal contracts. Such exit leads to an *increase* in the average productivity of searching firms (by definition, their productivity is always lower than that of formal firms) and a *decline* in the total number of firms searching in the market. In our simulations the latter force dominates the former.

Figure 8 presents four panels with figures that illustrate the model's performance in the dimensions we care about the most. We already presented all these plots in section 2, but now we also show the predictions of the model. Panels 8a, 8c and 8d show, respectively, the informality share, the income differential among workers earning less than US$ 300 and the income differential among workers earning more than that. In each of the plots, the filled circles correspond to occupational groups (as defined in section 2) in the data, the unfilled circles correspond to educational levels and the squares linked by the black curve correspond to the skill types in the model. Again, the x-axis shows our proxy for group productivity, the average income of the corresponding group (*not* conditional on excluding workers above and below the threshold). In the case of panels 8c and 8d, the lack of smoothness in the model curves follows from the discretization process. Setting an income threshold to calculate a statistic leaves results rather sensitive to the position of the productivity points in the grid. Panel 8b shows the histogram of income. Like we do in section 2, we group workers in US$200 income bins and plot the histogram in the data (black curves) and in the model (red/lighter curves). The solid curves indicate the income of formal workers, the dashed curves indicate that of informal ones.

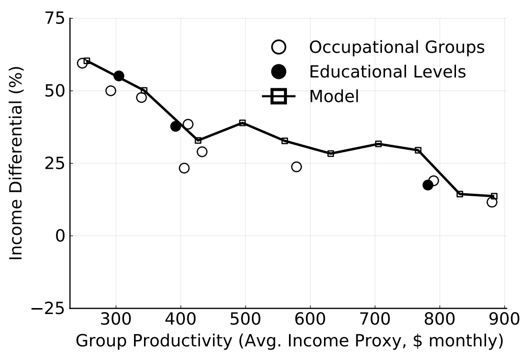
The model succedes in replicating the patterns of the data the figures convey, in particular facts 1 and 2. The share of informal workers declines as we move from low to high skill types, but remains economically significant. As worker skill increases, the probability of productivity draws (by new firms) leading to binding minimum wage decreases and, with it, so does the probability that the firm chooses to post informal contract vacancies to avoid it. In addition,



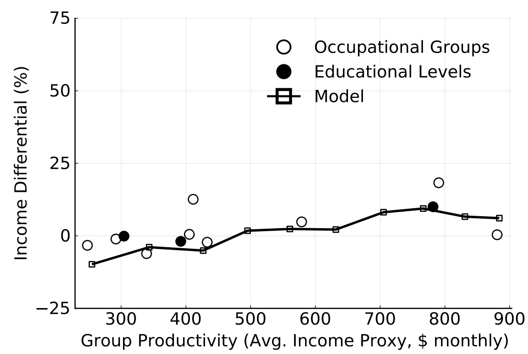
(a) Informality Share



(b) Income histogram



(c) Income differential (below US$ 300)



(d) Income differential (above US$ 300)

Notes: Panel (a). Informality share in the data (filled and unfilled circles) and the model (linked squares circles). Each square circle corresponds to a different skill level h. The x-axis indicates our proxy for group productivity, the average income of each group among all workers. Panel (b). Income histogram in the data (balck curves) and in the model (lighter curves). Each income bin has a size of US$200 (per month). Panels (c) and (d). The plots show the income differential: the difference of the average income among formal and informal workers, expressed as a percentage of the average income among informal ones. Panel (c) considers only workers receiving less than US300 income. Panel (d) considers only workers receiving more than US300. In both plots, the x-axis continues to indicate the *unconditional* average of each group as a proxy for productivity.

Figure 8: Informality and Income - Data and Baseline Calibration

the difference in the slopes of the formal and informal firm value functions - as we verify in plot 7a - also implies that firms opt for a formal contract more often as h increases. Thus, the existence of a minimum wage is not the only driver of informality shares decreasing in average income. In terms of income distribution, we also take that the model does a fine job in replicating the empirical evidence, especially taking into consideration that we devote a single parameter to target it (ξ, the skill distribution parameter). As expected, we observe no formal worker earning labor income below US$ 200 per month - a consequence of the minimum wage constraint. One caveat is that, in the data, more informal workers locate in the second income bin (US$ 200 - US$ 400) then in our model. One potential explanation is the existence of informal firms that choose to respect the minimum wage law, a force we don't capture with our analytical framework. The minimum wage assumption also plays a key role in generating realistic income differential patterns. Panels 8c and 8d show that the model successfully generates the large differential among low-income workers and the reduced differential among high-income workers. In shutting down the minimum wage (set $\omega = 0$), the differential below the threshold immediately retreats to levels near zero, for all skill types. As for the low income differential above the threshold, the key feature of the model to generate it is the heterogeneity in hiding abilities e, which allows the calibration to generate productive informal firms paying high wages, while keeping the size of the informal sector as a whole in check.

Finally, table 3 reports some additional targeted and non-targeted moments. The calibration is successful in having the model replicate the targeted moments, in particular the patterns related to transitions to unemployment. As we see in 3 of section 2, it is the case that in the model informal workers transition more often to unemployment, a mere consequence of our choice of calibration of λ_f and λ_i. As for the untargeted moments, direct transitions between formal and informal employment appear to be underrepresented in the model, probably a consequence of the fact that we don't allow firms and workers to re-negotiate the formality status of the labor contract. Given the relatively low frequency with which we observe such transitions in the data, we do not take this failure to be too problematic. As for the variance of income (calculated in logs, and conditional on the occupational group in the data or the skill type in the model), we observe in the data large as well as small values of such variance over the different survey waves and occupational groups. Values vary between 0.22 and 1.96. In our calibration, increasing the dispersion σ_z of productivity draws generates additional volatility, but hinders our ability to keep income differentials among high income workers low. Lastly, the model predicts a transition rate to employment inside the range observed in the data between 2012 and 2020.

Variable	Data/Target	Model
Targeted		
Unemployment rate	0.16	0.160
Market tightness	1	1.003
Informality share	0.3	0.326
Transition formal-unemployment	0.03	0.030
Transition informal-unemployment	0.075	0.075
Non-Targeted		
Transition unemployment-employment	0.2 - 0.3	0.233
Transition formal-informal	0.045	0.014
Transition informal-formal	0.18	0.057
Variance of log income	0.2 - 2.0	0.242

Table 3: Targeted and Non-targeted Moments

5 The Repression of Informality: A Simulation

We simulate the repression of informal labor activity by changing the distribution of hiding abilities. In period zero, the economy is in the baseline steady state as calibrated in the previous section. In particular, $\phi_0 = 0.75$ (searching firms draw $e = 0$ with a probability of 75%). From period one onward, we set $\phi_t = 1$, $t = 1, 2, 3, \ldots$. Therefore, starting in period one, all unmatched firms draw $e = 0$ with probability one. Obviously, these firms no longer opt to offer informal contracts, as they result in no efficiency hours employed in production and thus no revenue. Instead, they decide between posting a vacancy for a formal job or leaving the market without posting a vacancy and waiting for the next period. Formally, we calculate an equilibrium path in the sense of definition 1 in which the initial values of the aggregate state variables are those from our baseline steady state. Calculation of the path also requires us to take a stance on public policy. In our baseline simulation, we fix public expenditure C_{gov} at its baseline value and change the income tax rate $\tau_{hh,t}$ so that the budget constraint (17) holds every period. This makes the policy change to be fiscally neutral. In section 6 we examine the opposite case. We simulate the transition path for four hundred quarters (in addition to period zero), or one hundred years. We also experiment with five hundred quarters and find similar results.

The intent of the exercise is to simulation the *repression* of informality in the sense of the word. The means by which the government pursues such a policy in reality are irrelevant as long as: (1) the actions are strong enough to enforce formalization of labor relations throughout the economy; (2) the policy does not involve changes in the economic margins related to formal contracts (this involves unaltered taxes for firms and households, and unaltered unemployment

Variables	Baseline $t=0$	One year $t=4$	Ten years $t=40$	Forty years $t=80$	Long run $t=\infty$
Interest rate (%)	15	16.8	17.9	16	15
Income tax rate (%)	9	0.2	-12.5	-16.8	-19.3
Informality share (%)	32.9	13	0	0	0
Unemployment rate (%)	16	18.5	16.8	15.9	15.5
Market tightness (%)	100.1	55.3	51	59	62.7
Job-finding rate (%)	23.4	17.2	16.2	17.1	17.5
Transition unemployment (%)	4.5	4.0	3.3	3.2	3.2
Firm revenue - avg (%Δ)	0	6	17.5	21.7	24.7
Firm revenue - entry (%Δ)	0	56.7	53.7	54.5	57.7
Labor income - avg (%Δ)	0	12.2	34.5	43.5	49.3
Labor income - entry (%Δ)	0	51.5	65	71.5	76.9
Aggregates					
Output (%Δ)	0	6.3	23.2	29.1	32.9
Capital (%Δ)	0	-1.3	9.7	23.9	32.9
Efficiency Hours (%Δ)	0	10.7	30.8	32	33.2
Savings (%Δ)	0	-1.5	11.6	22.6	29.6
Sales tax revenue (%Δ)	0	17.9	46.3	53.2	57.8
Income tax revenue ($)	24	0.5	-47.1	-66.1	-77.6

Notes: The table shows the value of some endogenous variables and statistics in the transition path. In $t=0$, the economy is in the baseline steady state. In $t=1$, the government starts the repression of informality: the value of ϕ_t changes to 1, where it stays for good. Throughout the transition, the government adjusts the income tax rate $\tau_{hh,t}$ so that its budget constraint (17) holds with the baseline equilibrium value of government consumption C_{gov}. The symbol "%Δ" means that the variable is represented as percentage deviation from its baseline steady state value.

Table 4: Simulation of Informality Repression

benefits[20]); and (3) the economic possibilities of workers during unemployment that are not related to securing a new job also remain the same. Condition (2) does *not* say that taxes have to be constant, only that the specific *policy* adopted by the government to repress informality does not rely on such taxes. We do consider changes to taxation that follow from general equilibrium effects and fiscal neutrality. Condition (3) above rules out changes to unemployment income ς, which we calibrate to replicate Brazilian social security transfer programs.

5.1 Aggregates and the Income Process

We first focus on the effects of the repression of informality on macroeconomic variables, prices and the income process. In the next subsection we discuss welfare effects. Table 4 summarizes the transition with some of the main endogenous variables and statistics. The first column indicates the variable (the symbol "%Δ" means that the variable is represented as percentage deviation from its baseline steady state value). The other columns correspond each to one point in time during the transition.

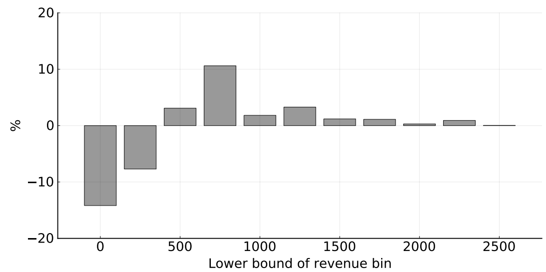
[20] In the simulation we do alter income taxation, but as a response to the endogenous changes provoked by a separate policy - movements in ϕ - that, in itself, do not rely on income taxation.

It proves useful to first focus on long run effects (last column of table 4). Removing informality from the labor market entails significant changes to the economic environment of the model. We see that $\phi = 1$, as expected, is sufficient to drive informal activity out of the labor market. All employed workers have a formal contract in the new steady state. Note that, while interest rates are roughly unchanged, the income tax rate changes significantly (from 9% to -19.3%). Since government's revenues come mostly from sales taxation[21], the required change in income taxation is significant. The unemployment rate in the new steady state falls slightly to 15.5% from 16% in the baseline equilibrium. This is the end result of two opposing forces: a significant decline in the rate workers transition to unemployment (from 4.5 to 3.2%) and an also significant decline in job-finding rates (= transition to employment rates), from 23.4% to 17.5%. Workers transition to unemployment more rarely as a direct consequence of our assumption that the separation rate of formal jobs is smaller than that of informal jobs. The end of informality thus implies that matches have a higher duration, on average. At the same time, finding a job becomes a less frequent event as well, as the relative number of firms posting vacancies is smaller (lower market tightness). Contrary to standard search models, we allow workers to search in the labor market every period, employed or not, with full efficiency. Therefore, the measure of searching workers in the model is the same in any equilibrium, and changes to job-finding rates are entirely accounted for by fluctuations in the measure of new firms.

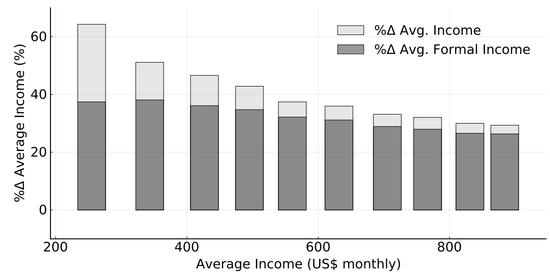
We can also see from table 4 the effects on firm productivity and wages. The repression of informal work causes an increase in the average firm productivity (a similar prediction to Meghir et al. (2015)). We can see this by the 25% increase in average firm revenue. The average productivity of firms employing workers coming out of unemployment (which we indicate by "entry") increases by 58%. Panel 9a of figure 9 depicts the gain in average productivity. We build $200 bins of firm gross revenue and show how the number of jobs in each bin varies from steady state to steady state. The negative values for low revenue bins reveal that unproductive firms leave the economy. For the most part, these are the informal firms we referred to as "survival firms" in section 4, that depend on the possibility of avoiding regulation (taxes and especially the minimum wage) to operate. As they no longer participate in the economy, congestion in the labor market reduces and, although finding a job becomes harder, workers improve their chance of finding a better one in case they do. These productivity gains, paired with lower income taxes, translate into higher labor income for workers, as paid wages are increasing in firm productivity (figure 7c). As expected, gains are again more pronounced for workers coming out of unemployment: 77% against the overall 49% increase.

Finally, we analyze how aggregates react. As households are working more time (the unemployment rate drops) and on more productive jobs, total efficiency hours of labor

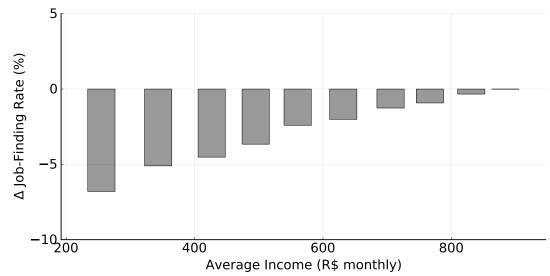
[21]We do not regard this as a problematic result of the model. In comparison to other countries, the Brazilian system imposes large taxes on goods and services, and lower taxes on income. Besides, we ignore in the model the progressive nature of income taxation, which thus causes understatement of income tax proceeds.



(a) Change in Firm Revenue Histogram



(b) Labor Income



(c) Job-Finding Rate

Notes: All panels refer to the long-run effects of the repression of informality. Panel (a). We show the change in the number of firms that locate in each \$250 gross revenue bin. The x-axis corresponds to the lower bound on the revenue bin. Negative values mean that there are less firms with gross revenue in range corresponding to the bin. Panel (b). Percentage change in average labor income for workers of each skill type h. The x-axis shows the average income of workers with the corresponding type in the baseline steady state. Light bars correspond to overall labor income; dark bars correspond only to formal labor income. Panel (c). Change in the job-finding rate for workers of each skill type h. The x-axis has the same interpretation as panel (b).

Figure 9: Long-Run Effects of the Repression of Informality

employed in production increases and, with it, the demand for physical capital by firms. The relatively stable interest rate indicates that such increased demand is met by additional aggregate savings as well. With more efficiency hours of labor and capital employed, total output increases by about 33%. Sales tax revenue increase by 58%, since firms no longer have the option of avoiding taxes and their average revenue improves. The additional tax proceeds allows the government to reduce the income tax rate into negative territory. The aggregate deficit of \$77.6 per month with income tranfers coincides with the average transfer, given that the economy features a unity measure of households.

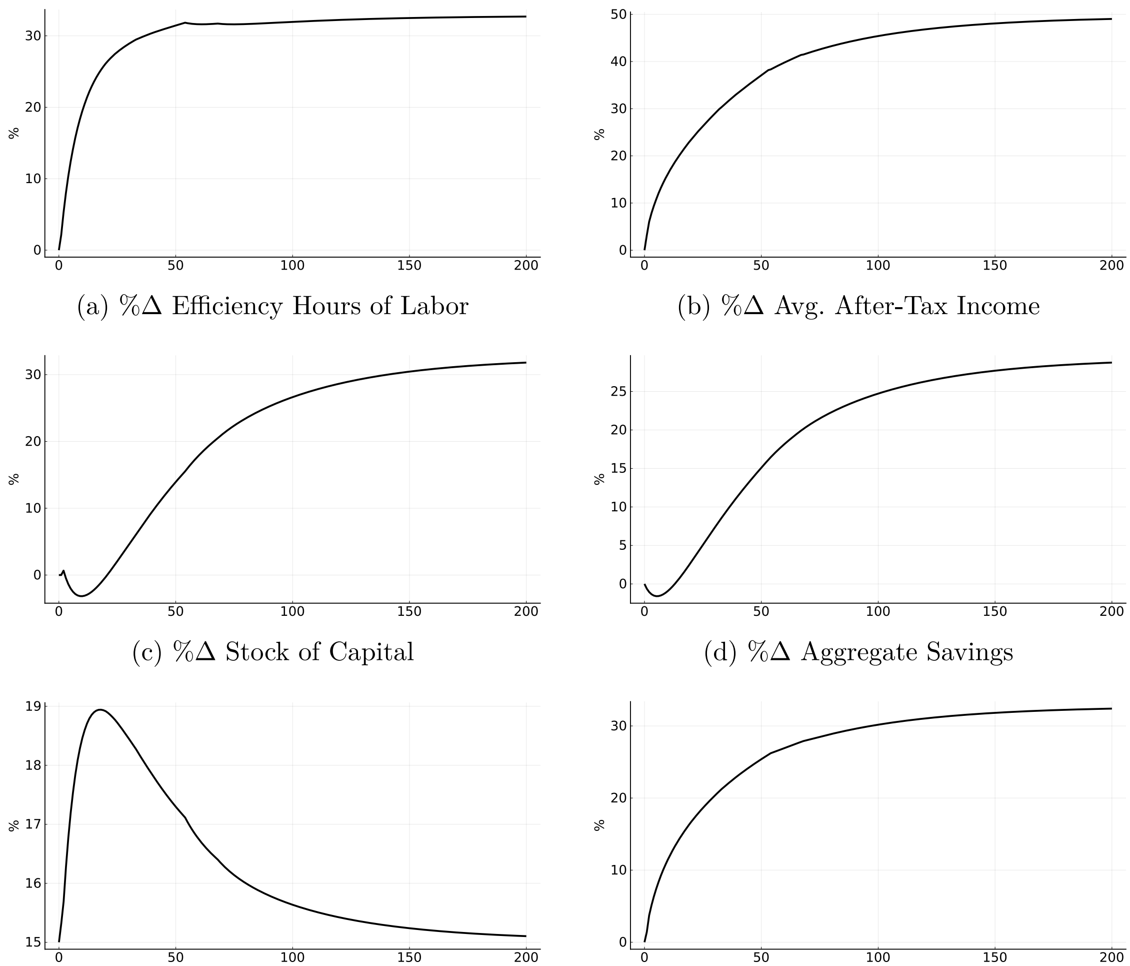
We calculate the numbers presented in table 4 by looking at the whole population. But how do the effects change between workers of different skill types? Panels 9b and 9c of figure 9 help with this question. For each skill level h, panel 9c plots the change in job-finding rates among the corresponding group of workers and 9b plots the percentage change in average labor income (light bars) and average *formal* labor income (darker bars). For both plots, the x-axis contain the average income of each skill group in the baseline steady state as a proxy of group productivity.

Panel 9c shows that job-finding rates decline more in the case of low-skill workers. The underlying reason is the existence of a minimum wage, which price low-skill workers out of formality more often then high-skill ones. Thus, as informal contracts cease from being an option to the firm, the set of productivity states that justify the creation of a job vacancy becomes smaller, and such effect is magnified in job markets with low-skill workers. As for panel 9b, it indicates a significant increase in average labor income for workers of all skill levels. As commented previously, this is to a large extent a consequence of the change in the income tax rate and gains in firm productivity shown in panel 9a. Looking first at the increases in average labor income among formal workers, we see that they are not very different across skill types. They fall between 26% and 37%. Note that the impact of changes in firm productivity on formal income is not straightforward. Recall from figure 7d that the static impact of the inclusion of "parasite" informal firms into formality is a *decline* of average productivity among unmatched firms. In fact, it is indeed the case that, on average, unemployed workers match with less productive formal firms in the equilibrium without informality. As for gains in average labor income *overall* (as opposed to only that of formal workers), they are much less homogeneous, ranging between 29% and 64%. Low skill workers observe the largest gains. Given fact 2 of section 2, which our model replicates, this is not very surprising. Groups of low skill workers concentrate more low-income earners, to which the minimum wage binds more often. As informal jobs vanish, employment opportunities - however rare - always entail payment of the legal floor, which lifts the average labor income.

Having seen what the long run looks like, we now pay attention to what transition dynamics look like. Table 4 again provides a picture of these dynamics, but figures 10 and 11 facilitate visualization. They show the paths of some of the main endogenous variables. The horizontal axis always shows time t. For many of the variables, specifically those with caption that reads "$\%\Delta$", we show the percentage difference between their value in period t and their baseline value in period zero. For example, panel 10c plots $K_t/K_0 - 1$. We choose the order of the plots to facilitate our exposition.

Focusing first on figure 10, we have in panel 10a the path of aggregate efficiency hours of labor used in production. Starting from the first period of the transition, it increases until it stabilizes at its long-run level, 33% higher than the initial equilibrium. The intuition is the same as before: average firm productivity increases as the end of informality drives unproductive jobs out of the market. In fact, we can also see this by noting that, for the first fifteen years of the transition, the unemployment rate is *higher* than in the baseline equilibrium (panel 11b of figure 11), which means that the total number of hours of labor declines. Aggregate efficiency hours can only increase if the average efficiency hours of each job increases. As before, gains in firm productivity and lower income taxes lead to higher average after-tax income, panel 10b.

As we argue to be the case in the long-run equilibrium, the increase in efficiency hours used in production drives up the capital demanded by firms as soon as the transition begins.



(a) %Δ Efficiency Hours of Labor

(b) %Δ Avg. After-Tax Income

(c) %Δ Stock of Capital

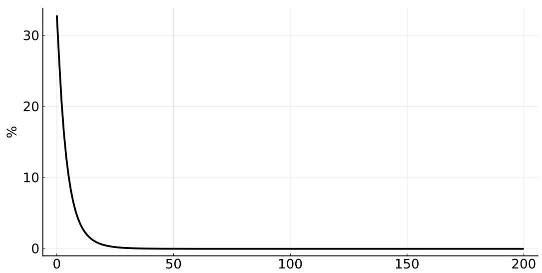
(d) %Δ Aggregate Savings

(e) Interest Rates

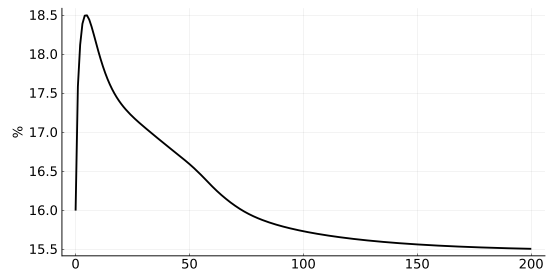
(f) %Δ Aggregate Output

Notes: The figures show the equilibrium paths of different variables in the model. With the exception of panel 10e, the curves draw percentage difference between the value of each variable in time t and their baseline value. For example, panel 10c plots $K_t/K_0 - 1$. The economy starts at the baseline steady state in period 0. In period one, ϕ (the probability that unmatched firms have of drawing zero hiding ability) unexpectedly changes from 0.75 to 1, where it stays thereafter. The transition takes 400 periods to reach the new steady state. Panel 10b shows the average after tax and transfers income. It includes unemployed workers. Panel 10e plots *annualized* interest rates.

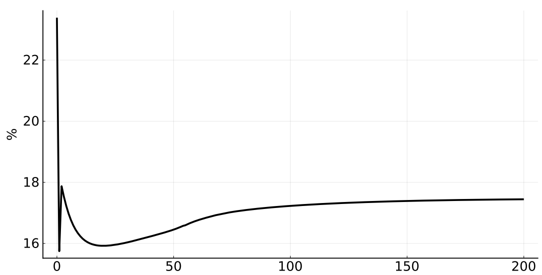
Figure 10: Repression of Informality - Transition Dynamics



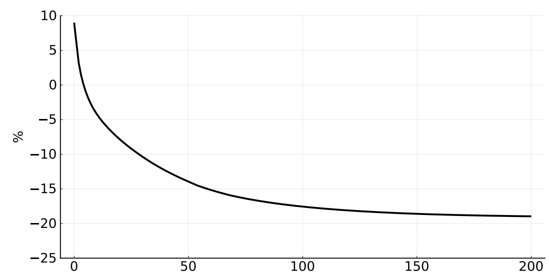
(a) Informality Share



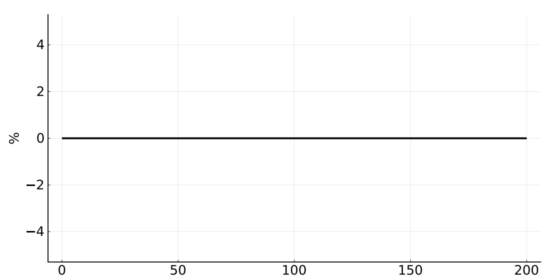
(b) Unemployment Rate



(c) Job-Finding Rate



(d) Income Tax Rate



(e) %Δ Government Consumption

Notes: The figures show the equilibrium paths of different variables in the model. The economy starts at the baseline steady state in period 0. In period one, ϕ (the probability that unmatched firms have of drawing zero hiding ability) unexpectedly changes from 0.75 to 1, where it stays thereafter. The transition takes 400 periods to reach the new steady state.

Figure 11: Repression of Informality - Transition Dynamics

However, during the transition aggregate savings (panel 10d) does *not* increase from the initial period. In fact, they decline for the first few periods and only catch up as time goes by. The underlying reason for this is consumption smoothing. Optimizing shoppers choose a pair of consumption and savings to equalize marginal utility. Anticipating higher after-tax income, as shown by panel 10b, they increase consumption in the first few periods of the transition and lower the stock of bonds. In turn, the representative investor builds and supplies less capital. Additional capital demand and reduced capital supplied combine to produce a large, short-run increase in the rental price of capital, or the interest rate of the economy (panel 10e). It reaches 19% per year, four percent more than its long-run value. This is a general equilibrium effect usually ignored by models that highlight the parasite character of informal firms.

By equation (18), such increase in interest rate translates into lower bond prices and lower discounting. As a result, the value of labor contracts to firms (V^f, V^i) declines in the short run, and with it the value of posting vacancies (the left-hand side of (13)). By the free entry condition (13), this leads to a reduction in the number of firms choosing to enter the market and an undershooting of the job-finding rate, which reaches 16.2% ten years after the repression of informality begins, as stated by table 4, and even drops below 16% (panel 11c). During the initial years of the transition, when the job market faces this slowdown, some informal firms still exist in the economy (panel 11a) - recall that the government only impedes firms from forming *new* informal contracts. *Existing* informal matches do not end in period $t = 1$. Therefore, at the same time that job-finding rates undershoot their long-run values, transition rates to unemployment remain high relative to the long-run. The end result is a short-run increase in the unemployment rate (panel 11b) that peaks at 18.5% (2.5 percentage points in excess of the initial steady state) in $t = 5$, less than two years following the beginning of the repression.

As we move along the transition path, the increase in average income realizes and aggregate household savings approach its long-run value. The capital supply curve shifts and its rental price declines steadily. As discounting increases, so does the discounted present value of firms' profits. More firms enter the market (relative to the short run slowdown) and job-finding rates rebound (although they don't achieve their pre-repression values). In addition, with an ever decreasing measure of informal matches in the economy, average job duration increases. Increasing job-finding rates and match duration lead to lower unemployment, which stabilizes at a level 0.5% lower than the pre-repression level. Finally, panels 11d and 11e show the path of fiscal policy variables. The income tax rate declines as less informal firms exist and the government expands its tax base, also enlarged by the higher average firm revenue. Public consumption remains unaltered by construction.

5.2 Welfare

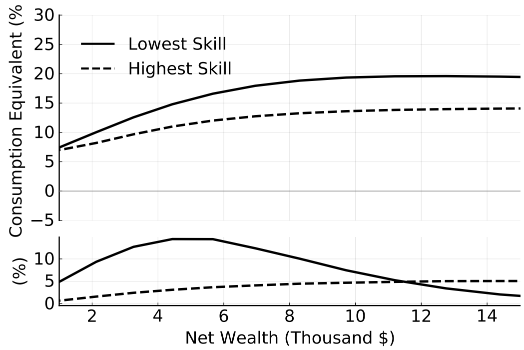
As defined by equation (1), the households' value J provides our measure of welfare. To compare welfare in the baseline economy with welfare after the repression of informal activity, we make use of the concept of *consumption equivalent welfare gain*, as used by Lucas (1987), or simply consumption equivalent. It is the value of m that solves the equation

$$E \sum \beta^t u((1 - m_t)c_t^0) = E \sum \beta^t u(c_t^1),$$

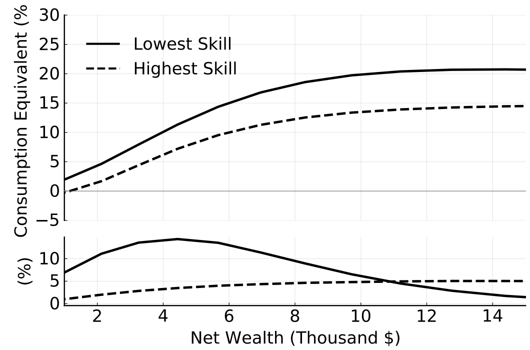
where superscript 0 indicates the baseline equilibrium consumption process, and superscript 1 indicates the after-policy equilibrium. Positive values of m mean that the household is better off after the implementation of the policy studied. The main advantage of looking at consumption equivalent measures of welfare instead of direct comparison of value functions is that it provides an interpretation of welfare change in terms lifetime consumption. In practice, we calculate it using our value function J and the functional form of the utility function:

$$m_t(a, s) = \left[\frac{J_t(a, s)}{J_0(a, s)} \right]^{\frac{1}{1-\gamma}} - 1. \tag{22}$$

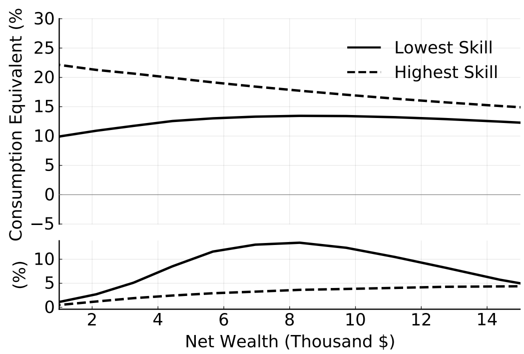
To state our results, we fix net wealth positions a and aggregate consumption equivalents $\int m_t(a, s) dx_t$. Figure 12 depicts our results. It contains four panels, each with two diagrams that share a common horizontal axis representing different net wealth states a. The upper figure plots consumption equivalents. The lower figure plots the distribution of households according to their net wealth. The distinction between each panel lies in the time t we use to calculate the averages and the group of households across which we aggregate. For example, panel 12a focus on period $t = 1$ (the short run) and households in all states, while panel 12b considers only households at the unemployment states. The two lower panels follow the same logic, but calculate consumption equivalents and distributions in the long-run steady state. Finally, in all cases, solid lines represent averages among households with workers of the lowest skill type, whereas dashed lines refer to households with workers of the highest skill type. We opt for looking only at workers with the most extreme levels of skill to avoid polluting the pictures with too many curves. Inspection of consumption equivalents quickly reveals that almost all households are better off with the repression of informality, both in the short and in the long run. Consumption equivalents are positive and economically significant. They often exceed 20% of pre-repression lifetime consumption. This follows from the increase in expected lifetime income resulting from the fight against informal activity. The main force in the model going against these welfare gains is the combination of lower job-finding rates (especially in the short run) and risk aversion. Risk averse households put more weight on bad states of nature, namely unemployment, in which income drastically falls. As informality vanishes, the occurrence of these states becomes more harmful as returning to employment is



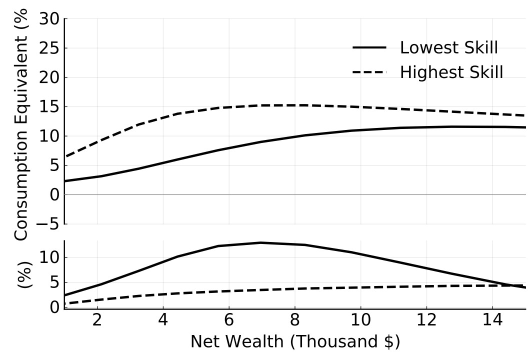
(a) Short-Run, All Households



(b) Short-Run, Unemployed



(c) Long-Run, All Households



(d) Long-Run, Unemployed

Notes: Each panel contains two figures. The upper figure plots consumption equivalents, as defined by equation (22), aggregated across a given group of households. The lower figure plots the histogram of households' wealth. The x-axis is common to the upper and lower figures and contains net wealth positions a. In all plots, the solid line refers to aggregates acroos households of the lowest skill type $\underline{h}$ and the dashed line refers to aggregates across the highest skill type $\bar{h}$. Panel (a). Consumption equivalents and distribution as of period $t = 1$, calculated across households in all states s. Panel (b). Consumption equivalents and distribution as of period $t = 1$, calculated across households in unemployment states. Panel (c). Consumption equivalents and distribution in the long run, calculated across households in all states s. Panel (d). Consumption equivalents and distribution in the long run, calculated across households in unemployment states.

Figure 12: Consumption Equivalents and Wealth Distributions

less likely. In this particular sense, the income distribution becomes riskier to households. The pictures show this through two noticeable patterns. First, the welfare gains of the unemployed are in all cases lower than the average taken across all households[22]. Second, welfare gains tend to be relatively lower among households with low levels of net wealth, that is, households with reduced capacity of (self) insuring against negative income shocks.

Of course, other forces play a role in determining welfare variation, such as changes in interest rates, access to unemployment benefits and the distribution of households itself. For example, the lower consumption equivalents in the long run (despite the negative general equilibrium effects of the short run) can in part be attributed to the fact that informal workers receive no weight in our averages (there are no informal workers left!). Short-run averages, on the other hand, place significant weight on informal workers existing in the economy when the repression of informality starts, which translates into high average consumption equivalents. In fact, this also accounts for the fact that low skill households gain more in the short run, while high skill households gain more in the long run, another interesting result.

In all, the main message of figure 12 is that, despite some perhaps undesirable general equilibrium effects that follow the repression of informality through the transition, as well as the elevated income risk we highlight, most households experience a significant welfare *improvement* as a result of the repression of informal labor activity.

5.3 The Open Economy Case

In this subsection, we consider the open economy case. Our choice of assuming a closed economy in our baseline simulation follows from the evidence of weaker financial integration of emerging market countries - where informality is more prevalent - with the international capital market (see, for example, Bai & Zhang (2012)). Altering the framework to an open econommy model simply amounts to dropping the last equilibrium condition in definition 1. The capital market does not necessarily clear anymore. Instead, households and the representative investor trade bonds with the rest of the world. The domestic economy is small in the sense that prevailing interest rates are exogenously determined by the international market.

We stick to our baseline calibration discussed in section 4. We assume the interest rate is 15% yearly, following our target for the closed-economy case. Hence, the calibration leads to a stationary equilibrium in which foreign debt

$$\text{Foreign debt}_{t+1} = K_{t+1} - q_t \left(\int a'_t(a,s) dx_t - Q_{t+1} - D \right) \tag{23}$$

[22]We take the change in welfare among households in unemployment states to be of particular relevance, since new generations of workers enter the labor market unemployed. Of course, we can't translate that in our framework, as we have no model equivalent of "new generations of workers".

is zero[23]. However, changes in economic policy now lead to fluctuations in foreign debt. Table 4 shows that the repression of informal activity does not affect interest rates too much in the long run - additional capital demand is met by additional savings from households with more income. This suggests that the open economy will not affect the long-term predictions of the model too much. Short-term predictions however change considerably, as we shut down one of the major general equilibrium channels. Figure 13 shows selected paths of the transition. To facilitate comparison, we also plot the paths of our closed-economy benchmark (dashed lines). For this open-economy case we increase the length of the transition path to 800 periods. In section A of the appendix you can find a table similar to 4 for this open economy simulation.

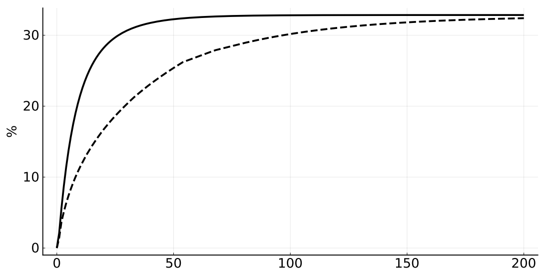
We can see from diagrams 13a and 13b that, without the sharp increase in interest rates from the baseline simulation, the initial increase in unemployment rapidly converges to its long-run value. With additional hours of labor employed in production, aggregate output follows a path above the one we observe in the closed-economy case. Moving to the next two plots, despite the reduced rate of unemployment and increased future income, aggregate consumption stays *below* the baseline path. To a large extent, this is due to the lower returns on bond purchases (again, interest do not increase in the short run), which also explains why aggregate savings do not increase as much as in the closed-economy case. We again observe an increase in capital demand, as firms apply more efficiency hours of labor. This time, however, it is met not only by domestic savings but also by increased foreign debt (panel 13e), that reaches fifty percent of annual output.

In the long run, most variables experience changes similar to the closed-economy case. This is expected, since the interest rate is roughly unchanged in the long-run equilibrium.

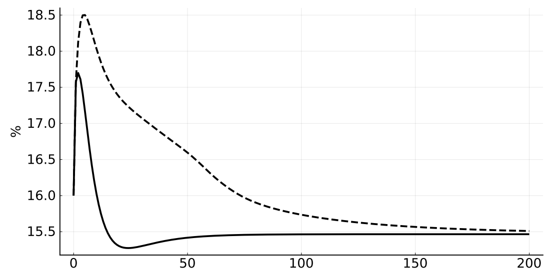
6 Policy Sensitivity

One of the key, and perhaps surprising, takeaways from our baseline simulation is that, despite a considerable increase in unemployment rates and decrease in job-finding rates in the short run, households are, for the most part, in favor of the repression of informality. In this section, we explore how sensitive these predictions of the model are to different policies, all still aimed at reducing the size of the informal sector to zero. Specifically, we consider two new policies. In the first on, the government opts *not* to adjust the income tax rate along the transition path. In the second one, we consider the effects of a partially anticipated policy: the government *announces* at $t = 1$ that it will start the repression of informality at $t = 20$ (that is, after five years). Our baseline calibration continues to be as in section 4.

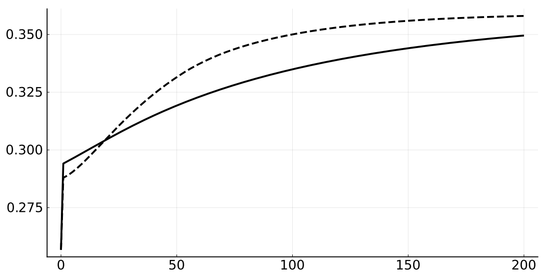
[23]Note that foreign debt is a predetermined variable. Therefore, (23) only holds *after* policy implementation. In period $t = 1$, when the government announces the new policy, the value of foreign debt coincides with its initial steady state level.



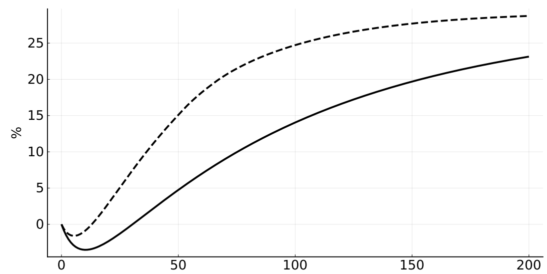
(a) %Δ Output



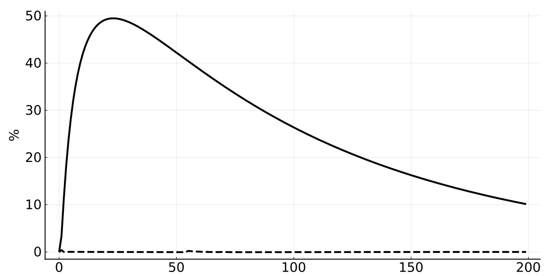
(b) Unemployment Rate



(c) %Δ Consumption



(d) %Δ Aggregate Savings



(e) Foreign Debt to (annual) GDP

Notes: The figures show the equilibrium paths of different variables in the model, in the open economy case. We also plot the path of the variables in the closed economy benchmark (same as in figures 10 and 11) The economy starts at the baseline steady state in period 0. In period one, ϕ (the probability that unmatched firms have of drawing zero hiding ability) unexpectedly changes from 0.75 to 1, where it stays thereafter. The transition takes 400 periods to reach the new steady state. To calculate foreign debt to GDP ratio (panel 13e), we multiply output by four, so that the ratio is annualized.

Figure 13: Repression of Informality - Open Economy (Dashed = Closed Economy Benchmark)

Variables	Baseline $t = 0$	One year $t = 4$	Ten years $t = 40$	Forty years $t = 80$	Long run $t = \infty$
Interest rate (%)	15	16.3	17.4	16.2	15.9
Income tax rate (%)	9	9	9	9	9
Informality share (%)	32.9	13.7	0	0	0
Unemployment rate (%)	16	18.9	17.9	17.4	17.2
Market tightness (%)	100.1	50.5	42.5	46.2	47
Job-finding rate (%)	23.4	16.7	15.2	15.7	15.8
Transition unemployment (%)	4.5	4.1	3.3	3.3	3.3
Firm revenue - avg (%Δ)	0	6.7	17.3	19.6	20.2
Firm revenue - entry (%Δ)	0	58.3	55.2	54.2	55.2
Labor income - avg (%Δ)	0	5.3	13.2	15.5	16
Labor income - entry (%Δ)	0	40.7	38.7	38.3	38.9
Aggregates					
Output (%Δ)	0	6.4	21.3	24.7	25.4
Capital (%Δ)	0	0.7	10.1	18.7	21
Efficiency Hours (%Δ)	0	9.5	27.7	28.1	28.1
Savings (%Δ)	0	-0.7	9.1	15.4	17
Sales tax revenue (%Δ)	0	17.6	44	48	48.9
Income tax revenue ($)	24	28.7	34.9	35.9	36.1

Notes: The table shows the value of some endogenous variables and statistics in the transition path. In $t = 0$, the economy is in the baseline steady state. In $t = 1$, the government starts the repression of informality: the value of ϕ_t changes to 1, where it stays for good. Throughout the transition, the government adjusts its consumption level $C_{gov,t}$ so that its budget constraint (17) holds with the income tax rate $\tau_{hh,t}$ of the original calibration. The symbol "%Δ" means that the variable is represented as percentage deviation from its baseline steady state value.

Table 5: Simulation of Informality Repression: No Adjustment of Tax Income

6.1 Absence of Tax Adjusment

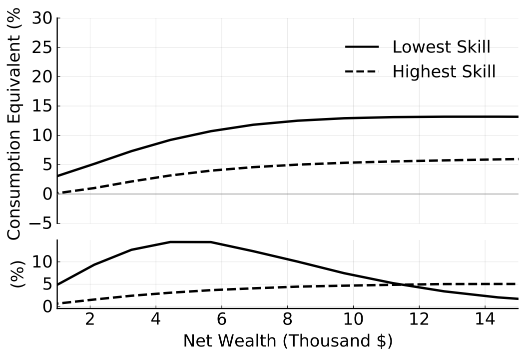
In our simulations of section 5, we assume that the government incurs in no additional costs when tightening the enforcement of formal contracts. In reality, effective and widespread surveillance might be far from free. In addition, the bureaucracy associated with registering and covering more workers with the social security system might require application of more public funds. Even if we ignore these costs, one might argue that political reasons prevent the additional tax revenue accruing from the expansion of the formal sector from converting into lower income tax rates, as in panel 11d. Given these considerations, this subsection presents the results from the repression of informality in a context where the government adjusts its budget constraint (17) not by changing the income tax rate $\tau_{hh,t}$ (which we keep in its baseline value of 9%), but rather by adjusting its consumption of final goods $C_{gov,t}$.

Table 5 shows the results. You can also check transition paths in section A of the appendix. The last column of the table reveals that some of the transitory general equilibrium effects we see in the baseline simulation now become persistent. In the long run, we have 0.9% higher interest rates and 1.2% higher unemployment rate. Changes in average firm revenue and aggregate efficiency hours reveal that the repression of informality drives unproductive firms out of the market, like before. Nonetheless, the resulting effect on labor income is not so big anymore. Average labor income still increases by 16% in the long run, a smaller variation than the one from table 4, 49%. This is due not only to the mechanical effect of a constant (as opposed to lower) income tax rate, but also due to the equilibrium effect of lower job-finding rates and higher unemployment. Workers spend less time employed and, when employed, earn less on average. With less income than in the baseline simulation, households increase their savings by a lower amount (17% *vs* 30%). Not as funded, the representative investor then supplies less capital to firms that, like before, demand more of it. Market clearing thus requires a higher level of interest rate in the long run. Essentially, public spending crowds out private investment and consumption. Finally, we need to explain why less firms enter the market. Such reduction follows from the fact that, without the lower taxes, worker and firm now split a smaller pie. For the same state (e, z), the value of the matched firm is now smaller. The increased interest rate also serves to reduce the present discouted value of firms' profits V^f. These changes reduce the value of vacancies and, by free entry, the number of firms that search in the labor market.

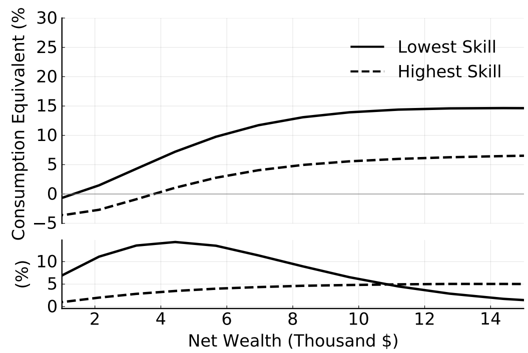
Transition effects differ from long term ones in the same way as before. As aggregate savings take longer to pick up than capital demand, the interest rate and the unemployment rate overshoot their long-run values. However, in the case of the interest rate the size of the overshooting is not as pronounced. One year after repression starts it increased by 1.3%, compared to 1.8% in the baseline simulation. The cause of the difference is in the fact that savings (hence capital supply) do not decrease as much as: -0.7% here, -1.5% in the baseline. Households do not increase consumption as much because they do not antecipate as big of an increase in expected income as before.

What about welfare? Figure 14 is the counterpart of figure 12 for the case without tax adjustment. It plots consumption equivalents (upper diagrams) and wealth distributions (lower diagrams). It is still the case that, at the time of policy implementation, most households favor it. However, now we can identify groups that don't. Unemployed, low-wealth households are net losers. They experience an immediate decline in job-finding rate that can't be countered by a cushion of savings. The same was true when the government adjusted down the income tax rate, but now future gains fail to make up for the immediate losses. Just like our baseline case, short-term gains are decreasing in worker skill level.

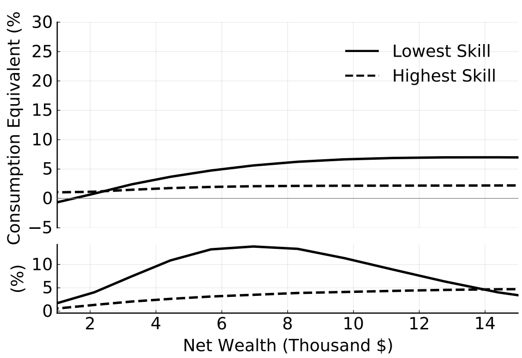
In the long run, gains are smaller compared to short run as we have less informal workers *and less employed workers* (unlike the baseline). In fact, the average of consumption equivalents becomes negative for zero-wealth households even when we take it across all low-skill households (not only unemployed). As for the unemployed, we find negative averages for workers of all skill levels and for wealth levels above the zero bound.



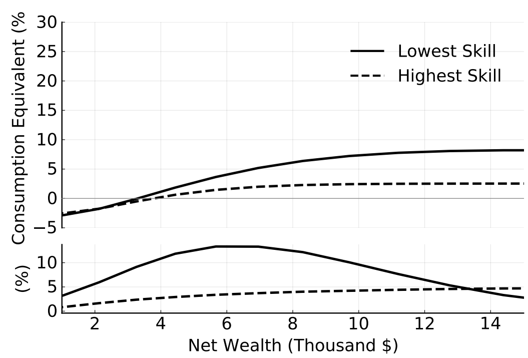
(a) Short-Run, All Households



(b) Short-Run, Unemployed



(c) Long-Run, All Households



(d) Long-Run, Unemployed

Notes: Each panel contains two figures. The upper figure plots consumption equivalents, as defined by equation (22), aggregated across a given group of households. The lower figure plots the histogram of households' wealth. The x-axis is common to the upper and lower figures and contains net wealth positions a. In all plots, the solid line refers to aggregates acroos households of the lowest skill type $\underline{h}$ and the dashed line refers to aggregates across the highest skill type $\bar{h}$. Panel (a). Consumption equivalents and distribution as of period $t = 1$, calculated across households in all states s. Panel (b). Consumption equivalents and distribution as of period $t = 1$, calculated across households in unemployment states. Panel (c). Consumption equivalents and distribution in the long run, calculated across households in all states s. Panel (d). Consumption equivalents and distribution in the long run, calculated across households in unemployment states.

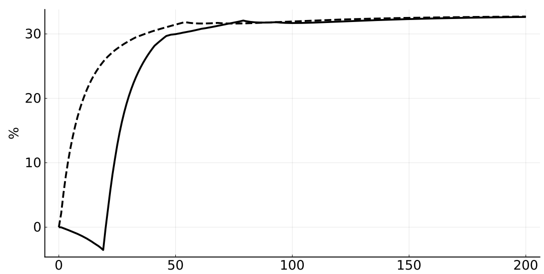
Figure 14: Consumption Equivalents and Wealth Distributions - No Adjustment of Tax Income

6.2 Anticipated Policy

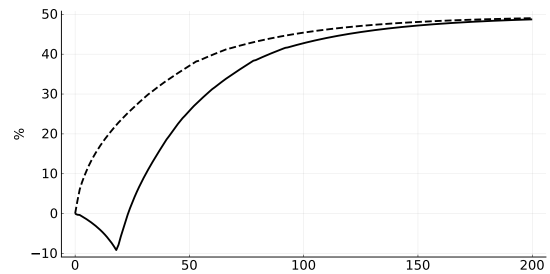
Our next simulation changes the timing of the model. A large-scale public program of targetting informal contracts - as the one we simulate - is unlikely not to be, at least to some extent, anticipated by economic agents. Our baseline exercise assumes a fully unanticipated policy change. Here, we change that assumption. At $t = 0$, the economy is in the initial steady state. At $t = 1$, the government announces that it will start to repress informal contracts starting at $t = 20$ (five years following the announcement). Formally, we set $\phi_t = 1$ for $t \geq 20$, and calculate the general equilibrium paths. In terms of fiscal adjustment, we return to the original assumption of changes to the income tax rate that hold constant government consumption. Because we know that the long-run steady state is the same one as the baseline, in the main text we only show the transition paths, figures 15 and 16. You can check the counterpart of table 4 for the anticipated policy in the appendix.

The figures show that the dynamics that follow the start of the fight against informality - period $t = 20$ - resamble the baseline simulation. But the early years of the transition, after announcement and prior to implementation, look different from before. During these initial five years, aggregate efficiency hours and average after tax and transfers income decline (panels 15a and 15b). It is still true that households anticipate future income gains and reduce their saving level immediately. The interest rate therefore starts to rise upon announcement (at a lower rate, since capital demand is going down). Higher interest rates, like before, diminish the value of hired workers to new firms. The free entry condition then leads to fewer firms entering the market and lower job-finding rates. Exactly like the baseline case. However, unlike our baseline simulation, there is no repression taking place immediately after the announcement. In fact, panel 16a shows that the informality share *increases* before repression starts. Thus, low duration matches still populate the job market. The combination of lower job-finding rates and the yet high average separation rates (due to high informality) results in an increasing unemployment rate (panel 16b). In addition, the absence of repression in the initial periods mutes the endogenous selection of productive firms that characterizes the long run. This provides further explanation as to why aggregate efficiency hours declines. Such decline also leads to an output contraction, something none of the previous two simulations presented. Finally, the government increases income tax rate in the short run to over 15%, which contributes to the increase in informality shares, contributes to the decline in household income and reinforces the mechanism above. The additional tax revenues compensate for the losses in sales/payoll taxes (the formal sector shrinks) and the additional expenditure with unconditional transfers (unemployment increases).

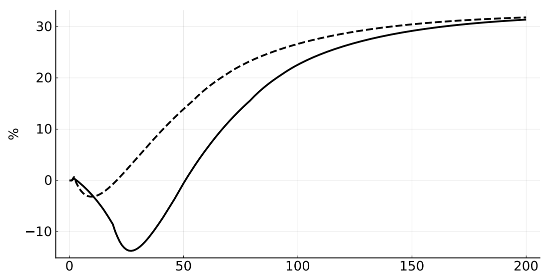
Figure 17 plots the consumption equivalent diagrams for the anticipated policy. The long-run equilibrium is the same, so we only show the numbers in the short run. At the time of the announcement, most households favor the repression, just like the baseline simulation and



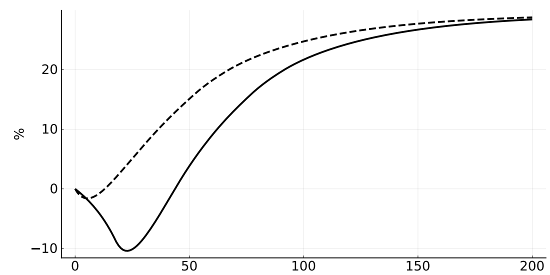
(a) %Δ Efficiency Hours of Labor



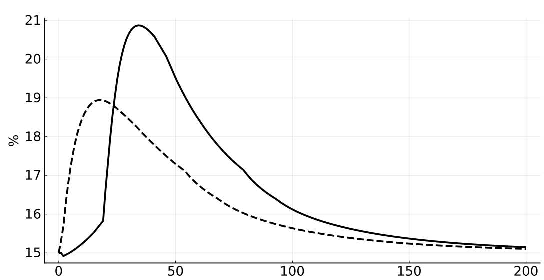
(b) %Δ Avg. After-Tax Income



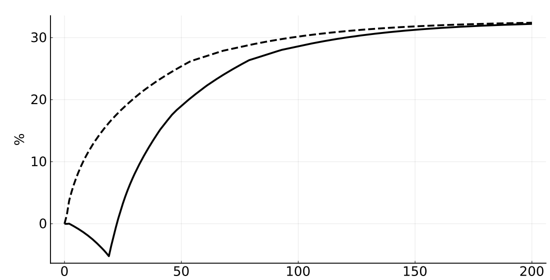
(c) %Δ Stock of Capital



(d) %Δ Aggregate Savings



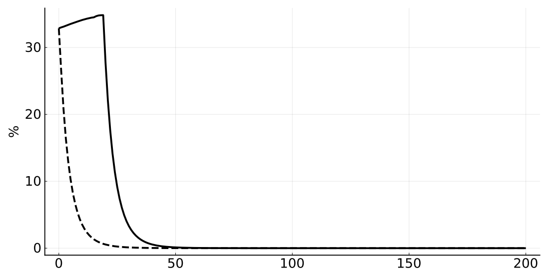
(e) Interest Rates



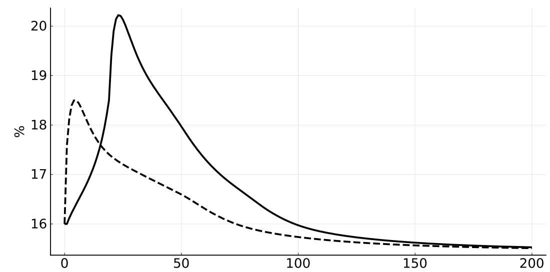
(f) %Δ Aggregate Output

Notes: The figures show the equilibrium paths of different variables in the model. With the exception of panel 15e, the curves draw percentage difference between the value of each variable in time t and their baseline value. For example, panel 15c plots $K_t/K_0 - 1$. The economy starts at the baseline steady state in period 0. In $t = 1$, the government announces that, starting at period $t = 20$, it will repression informality: the value of ϕ_t changes to 1, where it stays for good. The transition takes 400 periods to reach the new steady state. Panel 15b shows the average after tax and transfers income. It includes unemployed workers. Panel 15e plots *annualized* interest rates.

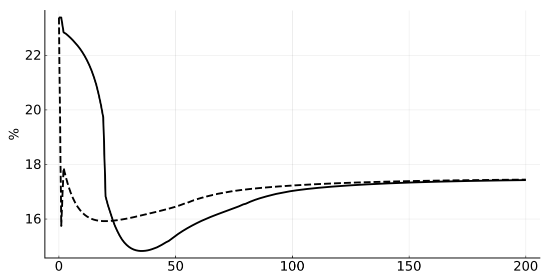
Figure 15: Repression of Informality - Anticipated Policy



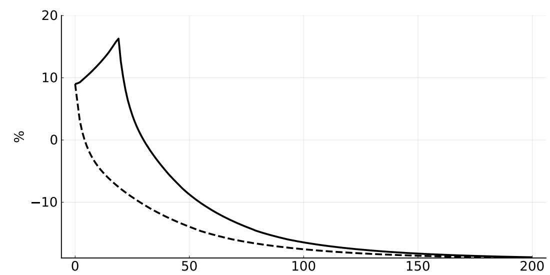
(a) Informality Share



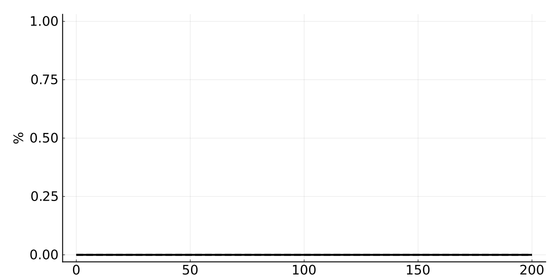
(b) Unemployment Rate



(c) Job-Finding Rate



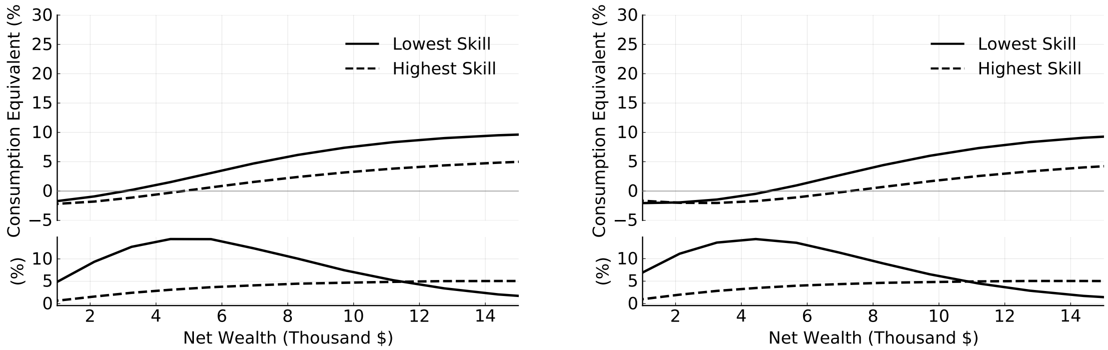
(d) Income Tax Rate



(e) %Δ Government Consumption

Notes: The figures show the equilibrium paths of different variables in the model. In this simulation, the government does not adjust income taxes in response to additional revenues raised from the end of informality. The economy starts at the baseline steady state in period 0. In $t = 1$, the government announces that, starting at period $t = 20$, it will repression informality: the value of ϕ_t changes to 1, where it stays for good. The transition takes 400 periods to reach the new steady state.

Figure 16: Repression of Informality - Anticipated Policy



(a) Short-Run, All Households
(b) Short-Run, Unemployed

Notes: Each panel contains two figures. The upper figure plots consumption equivalents, as defined by equation (22), aggregated across a given group of households. The lower figure plots the histogram of households' wealth. The x-axis is common to the upper and lower figures and contains net wealth positions a. In all figures, the solid line refers to aggregates acroos households of the lowest skill type $\underline{h}$ and the dashed line refers to aggregates across the highest skill type $\bar{h}$. Panel (a). Consumption equivalents and distribution as of period $t = 1$, calculated across households in all states s. Panel (b). Consumption equivalents and distribution as of period $t = 1$, calculated across households in unemployment states.

Figure 17: Consumption Equivalents and Wealth Distributions - Anticipated Policy

the case of no tax adjustment. But now, we have a negative average consumption equivalent for low-wealth households of *all* types. Like before, there are more net losers among the unemployed, but the average consumption equivalent is already negative if we look at the population average. Basically, with the anticipated repression of informality, the policy is welfare-reducing for low wealth households and welfare-enhancing for high wealth households. In all, 19.7% of households oppose the repression of informality. Comparison betwen these numbers and consumption equivalents in the unanticipated case highlights the importance (for welfare) of transition dynamics in the period between announcement and implementation. Such period features the main general equilibrium "costs" of the repression of informality (higher unemployment, lower job-finding rates) that our paper highlights. At the same time, the main benefit from the fight against informality, the change in the profile of firm productivity, does not kick off until the policy is actually implemented.

7 Conclusion

Informality is arguably a rather impactful economic phenomenon, especially among developing countries. Supported by various arguments, directions to curb it often fill the discourse of policymakers and professional economists alike. We believe that systematic studies of how such policies would impact the economy should accompany the public debate. The main contribution of this paper is to provide one instance of such studies. Our focus is on the examination of the general equilibrium effects of an eventual repression of informal activity by the government, as well the differences between the long and short run. We adapt the standard search-frictional labor market environment - the workhorse model of modern macro-labor analysis - to reproduce three empirical facts we identify from Brazilian household-level data. Perhaps the most striking of these facts being that informal workers do not earn less than formal ones, if we look away from the region of income close to the legal minimum wage. The main innovation of our search model we design to address this fact. We allow firms to have heterogeneous abilities to hide from the government and operate informally. This layer of heterogeneity allows the model to generate productive informal firms in equilibrium while keeping the size of the informal sector empirically accurate.

The search model yields an endogenous, structural income process that we then feed into an otherwise standard Aiyagari (1994) model of heterogeneous households and incomplete markets that permits proper computation of general equilibrium effects, especially those that relate to interest rates and the capital market. Many results emerge from the simulation. To a large extent, our model agrees with the recent literature studying informality on its parasitic character. Less informal firms also mean more productive firms, on average, and thus higher labor income. However, we also observe a significant increase in interest rates in the short run, caused by a combination of elevated capital demand and reduced supply. Both of these movements are directly connected to the change in average firm productivity stemming from the repression of informal activity. Paired with the increase in interest rate comes also a period of elevated unemployment. While one can point out at these two effects as making the fight against informality too costly from a polical standpoint or even undesirable, our welfare analysis suggests otherwise. In fact, we find that most households *gain* from the repression, despite the increased risk that unemployment shocks represent.

We also examine two alternative policies, also aimed at removing informal activity from the economy. In the first, the government fails to transfer back to households the additional tax proceeds accruing from the enlarged formal sector. We show that, in this case, the increase in interest and unemployment rates occur not only in the short but also in the long run. In the second alternative policy, economic agents anticipate the start of the fight against informality in five years. Unemployment starts to increase upon announcement, and since unproductive firms continue to exist in the labor market, output declines prior to actual policy implementation. While it is still true that most households benefit from the repression of informality with these two alternative policies, the share of households that lose from it increases. They are often the low-wealth unemployed, of any skill type, which highlights the role of income risk in the determination of welfare. In the case of the anticipated policy, 20% of households are net losers and therefore oppose the repression in the short run.

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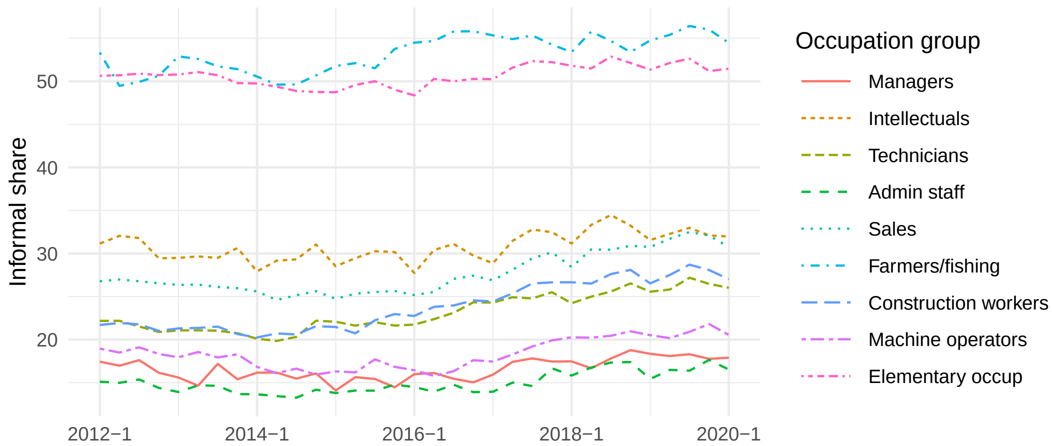


Figure 18: Informality over time by occupational group

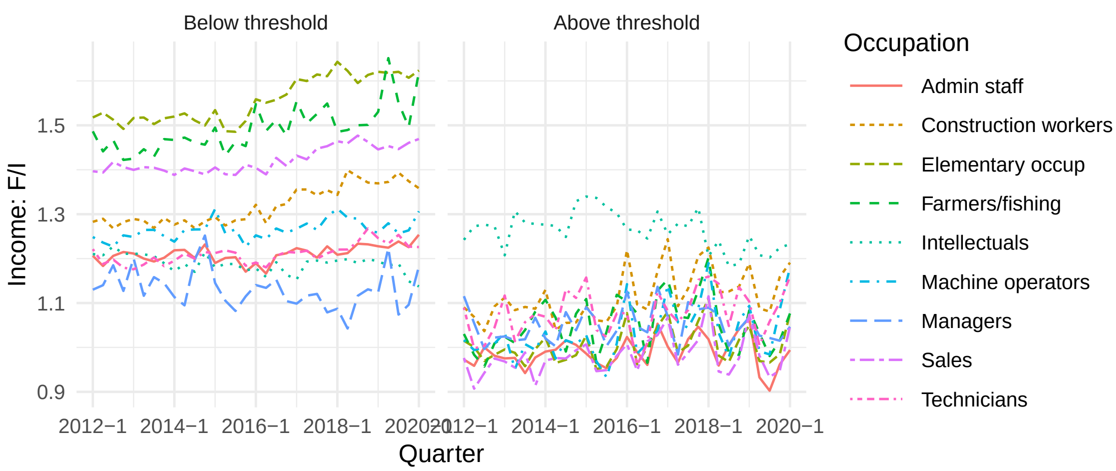


Figure 19: Earnings differential by occupation over time

Appendices

A Additional Plots and Tables

In this section of the appendix we show additional plots and tables of the data and model.

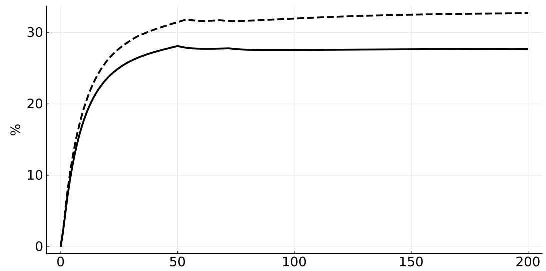
B Transition matrix of household state

In this section of the appendix, we show how to calculate the transition matrix $g(s'|s)$ for the shopper's exogenous state $s = (d, e, z; h)$, given a steady-state equilibrium in the labor market. So, assume we have in hands a pair of firm value functions V_t^f and V_t^i, workers' value function W_t^f, W_t^i, W_t^n as well as the market tightness $\theta_t(h)$ and the associated vacancy-filling rate $p_t(h)$.

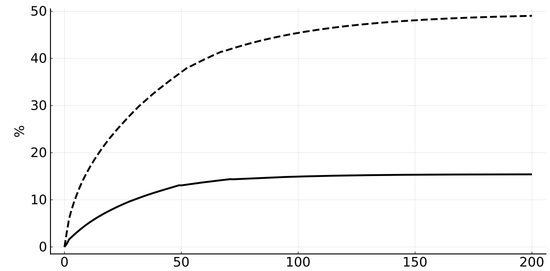
Variables	Baseline $t=0$	One year $t=4$	Ten years $t=40$	Forty years $t=80$	Long run $t=\infty$
Interest rate (%)	15	15	15	15	15
Income tax rate (%)	9	-5.7	-18.8	-19.2	-19.3
Informality share (%)	32.9	12.2	0	0	0
Unemployment rate (%)	16	17.4	15.4	15.5	15.4
Market tightness (%)	100.1	73.8	63.3	62.7	62.8
Job-finding rate (%)	23.4	19.2	17.6	17.5	17.5
Transition unemployment (%)	4.5	3.9	3.2	3.2	3.2
Firm revenue - avg (%Δ)	0	9.5	23.5	24.6	24.8
Firm revenue - entry (%Δ)	0	57.5	57.7	57.7	57.7
Labor income - avg %Δ)	0	21.8	47.6	49.1	49.3
Labor income - entry (%Δ)	0	61	76.4	76.9	77
Aggregates					
Output (%Δ)	0	11.4	32	32.8	33
Capital (%Δ)	0	11.4	32	32.8	33
Efficiency Hours (%Δ)	0	11.5	32	32.8	33
Savings (%Δ)	0	-2.5	2.5	11.2	29.7
Sales tax revenue (%Δ)	0	24	56.4	57.7	57.9
Income tax revenue ($)	24	-18.7	-75.1	-77.3	-77.7

Notes: The table shows the value of some endogenous variables and statistics in the transition path, in the open economy case. In $t = 0$, the economy is in the baseline steady state. In $t = 1$, the government starts the repression of informality: the value of ϕ_t changes to 1, where it stays for good. Throughout the transition, the government adjusts its consumption level $C_{gov,t}$ so that its budget constraint (17) holds with the income tax rate $\tau_{hh,t}$ of the original calibration. The symbol "%Δ" means that the variable is represented as percentage deviation from its baseline steady state value.

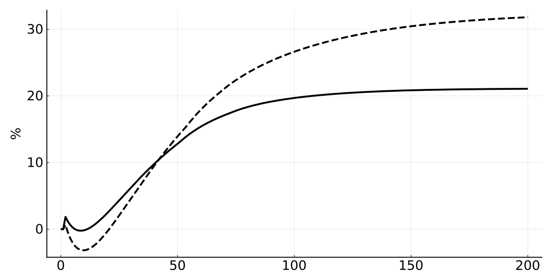
Table 6: Simulation of Informality Repression: Open Economy



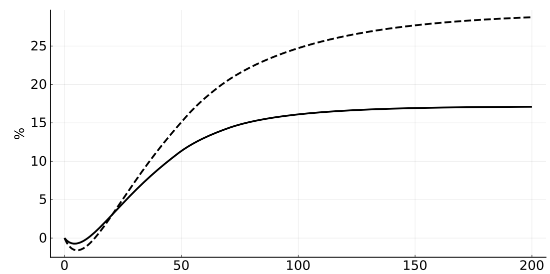
(a) %Δ Efficiency Hours of Labor



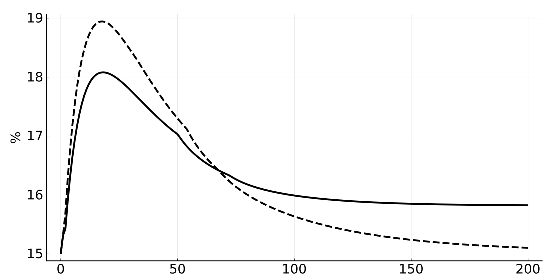
(b) %Δ Avg. After-Tax Income



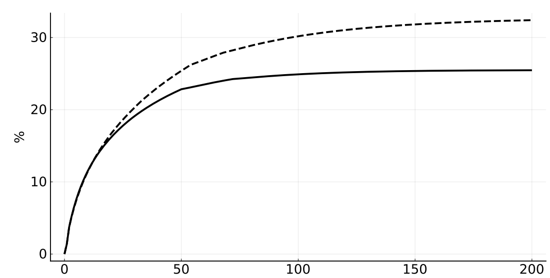
(c) %Δ Stock of Capital



(d) %Δ Aggregate Savings



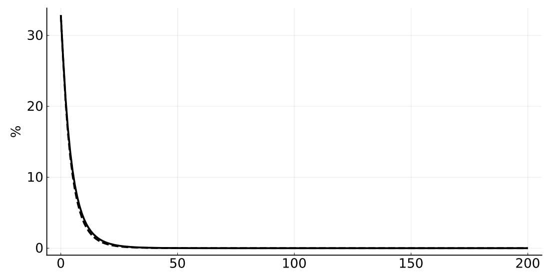
(e) Interest Rates



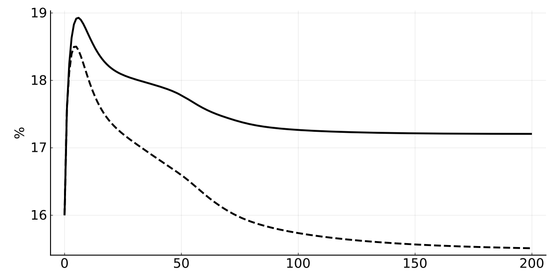
(f) %Δ Aggregate Output

Notes: The figures show the equilibrium paths of different variables in the model. In this simulation, the government does not adjust income taxes in response to additional revenues raised from the end of informality. With the exception of panel 20e, the curves draw percentage difference between the value of each variable in time t and their baseline value. For example, panel 20c plots $K_t/K_0 - 1$. The economy starts at the baseline steady state in period 0. In period one, ϕ (the probability that unmatched firms have of drawing zero hiding ability) unexpectedly changes from 0.75 to 1, where it stays thereafter. The transition takes 400 periods to reach the new steady state. Panel 20b shows the average after tax and transfers income. It includes unemployed workers. Panel 20e plots *annualized* interest rates.

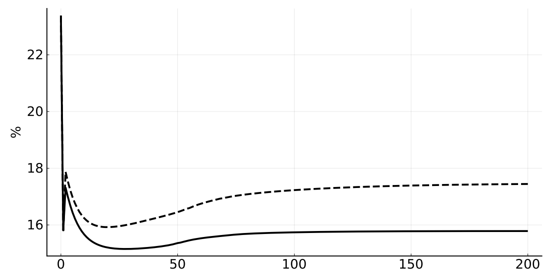
Figure 20: Repression of Informality - No Adjustment of Tax Rate



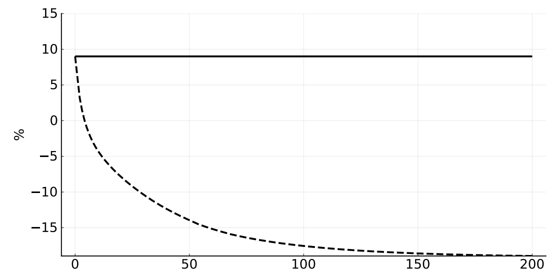
(a) Informality Share



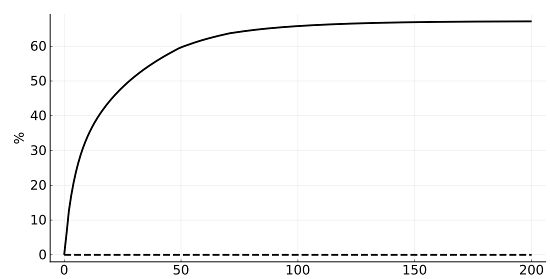
(b) Unemployment Rate



(c) Job-Finding Rate



(d) Income Tax Rate



(e) %Δ Government Consumption

Notes: The figures show the equilibrium paths of different variables in the model. In this simulation, the government does not adjust income taxes in response to additional revenues raised from the end of informality. The economy starts at the baseline steady state in period 0. In period one, ϕ (the probability that unmatched firms have of drawing zero hiding ability) unexpectedly changes from 0.75 to 1, where it stays thereafter. The transition takes 400 periods to reach the new steady state.

Figure 21: Repression of Informality - No Adjustment of Tax Rate

Variables	Baseline $t = 0$	One year $t = 4$	Ten years $t = 40$	Forty years $t = 80$	Long run $t = \infty$
Interest rate (%)	15	15	20.7	17.1	15
Income tax rate (%)	9	9.9	-5.2	-14.8	-19.3
Informality share (%)	32.9	33.4	0.6	0	0
Unemployment rate (%)	16	16.3	18.6	16.5	15.5
Market tightness (%)	100.1	93.6	41.6	54.1	62.7
Job-finding rate (%)	23.4	22.7	14.9	16.6	17.5
Transition unemployment (%)	4.5	4.5	3.3	3.2	3.2
Firm revenue - avg (%Δ)	0	0.2	12	20.4	24.7
Firm revenue - entry (%Δ)	0	0.4	56	55.8	57.7
Labor income - avg (%Δ)	0	-0.7	21.6	39.9	49.3
Labor income - entry (%Δ)	0	0	56.3	69.6	76.9
Aggregates					
Output (%Δ)	0	-0.4	14.7	26.8	32.9
Capital (%Δ)	0	-0.3	-7.7	16.3	32.9
Efficiency Hours (%Δ)	0	-0.4	28.1	32.4	33.2
Savings (%Δ)	0	-1.3	-2.3	17.1	29.6
Sales tax revenue (%Δ)	0	-1	35.7	50.5	57.8
Income tax revenue ($)	24	26.1	-18.4	-57.3	-77.6

Notes: The table shows the value of some endogenous variables and statistics in the transition path. In $t = 0$, the economy is in the baseline steady state. In $t = 1$, the government announces that, starting at period $t = 20$, it will repression informality: the value of ϕ_t changes to 1, where it stays for good. Throughout the transition, the government adjusts the income tax rate $\tau_{hh,t}$ so that its budget constraint (17) holds with the baseline equilibrium value of government consumption C_{gov}. The symbol "%Δ" means that the variable is represented as percentage deviation from its baseline steady state value.

Table 7: Simulation of Informality Repression: Anticipated Policy

We assume a discrete support for the distributions of hiding ability e and productivity z. Because the worker's type is constant, we start by setting $g_t((.,h')|(.,h)) = 0$ whenever $h' \neq h$. To keep notation light, we omit type h hereafter. We also establish an easier notation for the remaining states. Let $i(e,z)$ denote the state in which the worker is informally employed by a firm with hiding ability/productivity pair (e,z); let $f(z)$ denote the state in which the worker is formally employed by a firm with productivity z; let u denote unemployment without benefits and ub unemployment receiving the insurance payment.

We start by noticing that the unemployment insurance benefit lasts a single period: receivers can't transition to another period of unemployment insurance payment: $g_t(ub|ub) = g_t(ub|u) = 0$. Any unemployed worker transition to the unemployment without benefit state if he or she fails to find a match when the labor market opens (recall that the job-finding rate coincides with the rate at which workers transition from unemployment to employment):

$$g_t(u|ub) = g_t(u|u) = 1 - p\theta_t.$$

Transition to employment depends on the choice of new firms on whether to offer formal or informal contracts on each state (e,z).

$$g_t(f(z)|u) = g_t(f(z)|ub) = p\theta_t \mathbf{1}_{(e,z)\in Z_t^f} \frac{G_t(e,z)}{G_t(Z)}$$
$$g_t(i(e,z)|u) = g_t(i(e,z)|ub) = p\theta_t \mathbf{1}_{(e,z)\in Z_t^i} \frac{G_t(e,z)}{G_t(Z_t)}$$

This completes the statement of transitions from unemployment states.

Transitions from employment are less straighforward as we need to take into account the worker's chance of transitioning to a new job from on-the-job search. We start by noting that $g_t(.|u)$ gives not only the probability distribution of states to workers coming from unemployment but also of workers that lose their job at the end of a period and enter the market in the next period. We can thus recycle the use of $g_t(.|u)$. For instance, formal workers transition to the unemployment with benefit state ub if they lose their job (probability λ_f) and fail to find a new one (probability $g_t(u|u)$):

$$g_t(ub|f(z)) = \lambda_f g_t(u|u)$$
$$g_t(u|i(e,z)) = \lambda_i g_t(u|u)$$

(a similar reasoning stands for informal workers transitioning to unemployment without benefit state u). Also, $g_t(u|f(z)) = g_t(ub|i(e,z)) = 0$.

Consider now the probability that the employed worker remains on his or her current state, say $f(z)$ (the case $i(e,z)$ is analogous). As a convention, we assume that an employed worker finding a competing job offer decides to switch only if he or she *strictly* prefers the new offer. Therefore, in matching with a firm at the exact same state as his or her current employer, the workers opts to stay in the current position (this choice is obviously inconsequential for the model). To transition from $f(z)$ to the same state $f(z)$, the worker can then either lose the current job but find a similar one in the following period or simply stay at the current position (if no better match is found):

$$\begin{aligned} g_t(f(z)|f(z)) = \lambda_f g_t(f(z)|u) &+ (1-\lambda_f)(1-p\theta_t) \\ &+ (1-\lambda_f)\frac{p\theta_t}{G_t(Z_t)} G_t(\{(e',z') \in Z_t^f \mid W_t^f(z) \geq W_t^f(z')\}) \\ &+ (1-\lambda_f)\frac{p\theta_t}{G_t(Z_t)} G_t(\{(e',z') \in Z_t^i \mid W_t^f(z) \geq W_t^i(e',z')\}). \end{aligned}$$

Our worker at $f(z)$ can also transition to a new formal job $f(z')$. That can happen either with an exogenous separation in between or through on-the-job search. The transition from employment requires $W^f(z') > W^f(z)$:

$$g_t(f(z' \neq z)|f(z)) = \lambda_f g_t(f(z')|u) + (1-\lambda_f)p\theta_t \sum_{e'} \frac{G_t(e',z')}{G_t(Z_t)} \mathbf{1}_{(e',z')\in Z_t^f} \; \mathbf{1}_{W_t^f(z') > W_t^f(z)}$$

The last case we need to consider is the transition from a formal job $f(z)$ to an informal job $i(e,z)$. The reasoning is the same as the previous case.

$$g_t(i(e',z')|f(z)) = \lambda_f g_t(i(e',z')|u) + (1-\lambda_f)p\theta_t \frac{G_t(e',z')}{G_t(Z_t)} \mathbf{1}_{(e',z')\in Z_t^i} \; \mathbf{1}_{W_t^i(e',z') > W_t^f(z)}.$$

We are done. The transition matrix for a worker employed with an informal contract is analogous to that of a worker with a formal contract. Notice that, for all s, $\sum_{s'} g_t(s'|s) = 1$.

C The Investor's Problem

In this section we describe formally the investor's environment and provide a proof for proposition 1. To simplify things, we assume the equity of existing firms is packaged by a fund of firms responsible for posting all vacancies in the economy. The investor purchases shares of the fund of firms. The supply of such shares is one in all periods. Shareholders receive as dividends the sum of matched firms' profits discounted by the cost of posting new vacancies VC_t.

In any given period t, the investor sells B_t bonds to households at a price q_t and uses the proceeds to purchase X_{t+1} shares of the fund of firms, each at a price $Q_t - DIV_t$ (that is the value of future stream of payments), or K_{t+1} units of capital. Note that X_{t+1} and K_{t+1} are both predetermined. Therefore, the investor's balance sheet constraint is

$$K_{t+1} + X_{t+1}\left(Q_t - DIV_t\right) = q_t B_t. \tag{24}$$

In the following period $t+1$, the investor supplies capital to firms, and receives rental payment $(r_t+\delta)K_{t+1}$ plus the now depreciated capital $(1-\delta)K_{t+1}$, which amounts to $(1+r_{t+1})K_{t+1}$. Investment in shares of the fund of firms yields dividends $X_{t+1}DIV_{t+1}$ plus the $t+1$ value of the shares, $X_{t+1}\left(Q_{t+1} - DIV_{t+1}\right)$. They sum up to $X_{t+1}Q_{t+1}$. Finally, the investor must repay period t debt B_t. The difference between investment payoffs and debt repayment is consumed away:

$$C_{inv,t+1} = (1+r_{t+1})K_{t+1} + X_{t+1}Q_{t+1} - B_t. \tag{25}$$

The representative investor's problem then is to maximize (25) by choosing a portfolio (K_{t+1}, X_{t+1}, B_t) that satisfies the balance sheet constraint (24).

The following proposition is critical in solving the investor's problem.

Proposition 2. *Aggregate dividends DIV_t satisfy the following equation:*

$$Q_t = DIV_t + q_t Q_{t+1} \tag{26}$$

Proof. We prove that aggregate dividends in a single market h satisfy

$$\begin{aligned} DIV_t(h) = &\int V_t^f(z,h)dE_t^f(z,h) + \int V_t^i(ez,h)dE_t^i(ez,h) \\ &- q_t\left[\int V_{t+1}^f(z,h)dE_{t+1}^f(z,h) + \int V_{t+1}^i(ez;h)dE_{t+1}^i(ez;h)\right]. \end{aligned}$$

. The proposition then follows. Let $\bar{E}^f_t = E^f_t(\mathcal{Z}; h)$ be the share of h-type workers in the formal sector, and define $\bar{E}^i$ similarly. To simplify notation, we fix h and omit it from the proof hereafter. The construction of the transition matrix in section B of this appendix implies that the following relationship holds:

$$
\begin{aligned}
dE^f_t(z) &= dE^f_{t-1}(z) - \left[\lambda_f + (1-\lambda_f)p\theta_t \frac{G(\Phi^f_t(z))}{G_t(Z_t)}\right] dE^f_{t-1}(z) \\
&\quad + p\theta_t \mathbf{1}_{(e,z)\in Z^f_t} \frac{dG_t(e,z)}{G_t(Z_t)} \left[U_{t-1} + \lambda_f \bar{E}^f_{t-1} + \lambda_i \bar{E}^i_{t-1} + (1-\lambda_f)E^f(\Psi^f_{f,t-1}(z)) + (1-\lambda_i)E^i(\Psi^f_{i,t-1}(z))\right] \\
dE^i_t(ez) &= dE^i_{t-1}(ez) - \left[\lambda_i + (1-\lambda_i)p\theta_t \frac{G(\Phi^i_t(z))}{G_t(Z_t)}\right] dE^i_{t-1}(ez) \\
&\quad + p\theta_t \mathbf{1}_{(e,z)\in Z^i_t} \frac{dG_t(e,z)}{G_t(Z_t)} \left[U_{t-1} + \lambda_f \bar{E}^f_{t-1} + \lambda_i \bar{E}^i_{t-1} + (1-\lambda_f)E^f(\Psi^f_{i,t-1}(z)) + (1-\lambda_i)E^i(\Psi^i_{i,t-1}(z))\right]
\end{aligned}
\tag{27}
$$

The two equations simply contain the law of motion for the measure of workers in each employment state.

Total dividends are the difference between total period profits acquired by matched firms DIV^F and total cost with new vacancies DIV^V. We first calculate the former. Let us call $\pi^f(z)$ the period profit of a formal firm at state z, and the same be for $\pi^i(ez)$. Firm value function (6) implies that

$$
\pi^f_t(z) = V^f_t(z) - q_t \left[1 - \left(\lambda_f + (1-\lambda_f)p\theta_{t+1} \frac{G_{t+1}(\Phi^f_{t+1}(z))}{G_{t+1}(Z_{t+1})}\right)\right] V^f_{t+1}(z)
$$

and the analogous expression for $\pi^i(ez)$. Then, total period profits is given by

$$
\begin{aligned}
DIV^F_t &= \int \pi^f_t(z) dE^f_t + \int \pi^i_t(ez) dE^i_t \\
&= \int \left\{ V^f_t(z) - q_t \left[1 - \left(\lambda_f + (1-\lambda_f)p\theta_{t+1} \frac{G_{t+1}(\Phi^f_{t+1}(z))}{G_{t+1}(Z_{t+1})}\right)\right] V^f_{t+1}(z) \right\} dE^f_t \\
&+ \int \left\{ V^i_t(ez) - q_t \left[1 - \left(\lambda_i + (1-\lambda_i)p\theta_{t+1} \frac{G_{t+1}(\Phi^i_{t+1}(ez))}{G_{t+1}(Z_{t+1})}\right)\right] V^i_{t+1}(ez) \right\} dE^i_t
\end{aligned}
$$

To calculate aggregate spending with vacancy costs, we first re-state the free-entry condition (13):

$$
\kappa h = q_t \left\{ \int_{Z^f_{t+1}} \rho^f_{t+1}(z) V^f_{t+1}(z) dG_{t+1} + \int_{Z^i_{t+1}} \rho^i_{t+1}(ez) V^i_{t+1}(ez) dG_{t+1} \right\}.
$$

The aggregate spending with vacancy costs then satisfies the following equations:

$$
\begin{aligned}
DIV_t^V &= \kappa h F_t \\
&= \kappa h \frac{\theta_{t+1}}{G_{t+1}(Z_{t+1})} \\
&= q_t \left\{ \int_{Z_{t+1}^f} \rho_{t+1}^f(z) \frac{\theta_{t+1}}{G_{t+1}(Z_{t+1})} V_{t+1}^f(z) dG_{t+1} + \int_{Z_{t+1}^i} \rho_{t+1}^i(ez) \frac{\theta_{t+1}}{G_{t+1}(Z_{t+1})} V_{t+1}^i(ez) dG_{t+1} \right\} \\
&= q_t \int_{Z_{t+1}^f} p\theta_{t+1} \left[U_t + \lambda_f \bar{E}_t^f + \lambda_i \bar{E}_t^i + (1-\lambda_f) E_t^f(\Psi_{f,t+1}^f(z)) + (1-\lambda_i) E_t^i(\Psi_{i,t+1}^f(z)) \right] V_{t+1}^f(z) \frac{dG_{t+1}}{G_{t+1}(Z_{t+1})} \\
&+ q_t \int_{Z_{t+1}^i} p\theta_{t+1} \left[U_t + \lambda_f \bar{E}_t^f + \lambda_i \bar{E}_t^i + (1-\lambda_f) E_t^f(\Psi_{f,t+1}^i(ez)) + (1-\lambda_i) E_t^i(\Psi_{i,t+1}^i(ez)) \right] V_{t+1}^i(ez) \frac{dG_{t+1}}{G_{t+1}(Z_{t+1})} \\
&= q_t \int V_{t+1}^f dE_{t+1}^f - q_t \int \left[1 - \left(\lambda_f + (1-\lambda_f) p\theta_{t+1} \frac{G(\Phi_{t+1}^f(z))}{G_{t+1}(Z_{t+1})} \right) \right] V_t^f(z) dE_t^f \\
&+ q_t \int V_{t+1}^i dE_{t+1}^i - q_t \int \left[1 - \left(\lambda_i + (1-\lambda_i) p\theta_{t+1} \frac{G(\Phi_{t+1}^i(z))}{G_{t+1}(Z_{t+1})} \right) \right] V_t^i(z) dE_t^i
\end{aligned}
$$

The first equality is simply the definition of aggregate spending in vacancy costs. The second equality uses (15). The third one replaces the free-entry condition. The fourth equality replaces the definition (16) of ρ^f and ρ^i. The last equality replaces the integrated version of equation (27).

After cancelling out the repeated terms, the result follows:

$$
\begin{aligned}
DIV_t(h) &= DIV_t^F(h) - DIV_t^V(h) \\
&= \int V_t^f(z,h) dE_t^f(z,h) + \int V_t^i(ez,h) dE_t^i(ez,h) \\
&- q_t \left[\int V_{t+1}^f(z,h) dE_{t+1}^f(z,h) \int V^i(ez;h) dE^i(ez;h) \right].
\end{aligned}
$$

□

Note that proposition 2 guarantees that, *ex-ante*, equity investment at period t guarantees a return of $1/q_t$. In the period of an MIT shock, the *ex-post* return might differ as Q_{t+1} jumps to different level.

By replacing (24) and (26) in the objective (25), we find a new expression

$$
\begin{aligned}
C_{inv,t+1} &= (1 + r_{t+1}) K_{t+1} + X_{t+1} Q_{t+1} - \frac{1}{q_t} \left[K_{t+1} + q_t X_{t+1} Q_{t+1} \right] \\
&= (1 + r_{t+1}) K_{t+1} - \frac{1}{q_t} K_{t+1}
\end{aligned} \tag{28}
$$

If $q_t < (1 + r_{t+1})^{-1}$, the investors would not invest in capital (that is, transform final goods acquired from selling bonds to the households into capital). If $q_t > (1 + r_{t+1})^{-1}$, the investor would supply an infinite amount of bonds in period t and supply an infinite amount of capital in $t+1$. If condition (18) holds, the investor becomes indifferent between equity and capital investment, as both yield the same return. This proves the first claim in proposition 1.

In being indifferent with respect to the holdings of fund of firms shares, the investor chooses $X_t = 1$, so that market clears. Replacing it in equation (25) together with constraint (24) yields the consumption expression found in the text of the proposition:

$$
C_{int,t+1} = (1 + r_{t+1}) K_{t+1} + Q_{t+1} - \frac{1}{q_t} \left[K_{t+1} + Q_t - DIV_t \right]
$$

Unlike (28), the expression above holds even in initial MIT shock periods.

Finally, replacing $X_t = 1$ and (26) in (24) yields the proposition's equation for the investor's balance sheet

$$K_{t+1} + q_t Q_{t+1} = q_t B_t.$$

Note that this expression only holds in non-MIT shock periods. If anticipated and realized Q_{t+1} differ, the expression above only holds if we use anticipated Q_{t+1}.

D Details of the numerical solution

We present the discretization process as well as the algorithms we use to generate our results. All numerical computations we perform using the *Julia* programming language. For data manipulation, we use *R*.

D.1 Grids

We discretize firm productivity with $n_z = 50$ grid points. We start by using Tauchen's method to discretize a normal distribution with mean zero and standard deviation σ_z, with $n_z - 1$ points. We use three standard deviations to each side of the distribution. We take the exponential of the $n_z - 1$ grid points and add zero to the grid, thus forming a new grid with a total of n_z points. Tauchen's method gives the probability of each of the $nz - 1$ original points. The discrete probability of $z = 0$ equals zero. Finally, we normalize the values of the grid so that

$$\sum_{i_z=1}^{n_z} z_i G_z(z_i) = 1,$$

where $\mathcal{Z} = \{z_1 = 0, z_2, \ldots, z_{n_z}\}$ is our productivity grid.

Next, we discretize G_e. The first step is to discretize a Beta distribution. We start with 200 equally-spaced points between zero and 0.9999. To each point we calculate the corresponding quintiles of a Beta distribution with parameters ν and one. These 200 quintiles yield 199 mid points (the points halfway between each two quintiles). To each of these points we assign a probability of 1/199. The second step performs the mixture. We start by adding the zero to the 199 points, and end with a 200-point grid $\mathcal{E}$ for hiding abilities. The probability of zero is ϕ_t. The probability of any other point is $(1 - \phi_t)(1/199)$.

In the numerical solution, we do not work with the distribution G for the pair (e, z). Instead, we use an equivalent distribution $\hat{G}$ for the pair (z, ez). The discretized grid for both entries of the tuple is the productivity grid $\mathcal{Z}$. So, we start with $\hat{G}(e, ez) = 0$, and do the following iteration. For each pair of $z_i \in \mathcal{Z}$ and $e_j \in \mathcal{E}$, we update $\hat{G}$:

$$\begin{aligned}
G(z_i, \underline{z}(e_j z_i)) &= G(z_i, \underline{z}(e_j z_i)) + (1 - \omega(e_j z_i)) G_z(z_i) G_e(e_j) \\
G(z_i, \bar{z}(e_j z_i)) &= G(z_i, \bar{z}(e_j z_i)) + \omega(e_j z_i) G_z(z_i) G_e(e_j)
\end{aligned}$$

where we choose $\underline{z}(e_j z_i) \in \mathcal{Z}$ and $\bar{z}(e_j z_i) \in \mathcal{Z}$ such that they are consecutive to each other and $\underline{z}(e_j z_i) \leq e_j z_i \leq \bar{z}(e_j z_i)$. The weight $\omega(e_j z_i)$ guarantees that the we distribute the probability mass proportinally between the two bounds:

$$e_j z_i = (1 - \omega(e_j z_i)) \underline{z}(e_j z_i) + \omega(e_j z_i) \bar{z}(e_j z_i).$$

For the asset grid, we start with a choice of grid size $n_a = 50$, an upper bound $\bar{a} = 1000$ and a parameter of non-linearity $nl \geq 1$. The first point of the grid is $a_1 = 0$ (following the no-borrowing constraint). We place the following points of the grid using the formula

$$a_i = a_{i-1} + \frac{\bar{a} - a_{i-1}}{(n_a - i + 1)^n l}.$$

This guarantees $a_{n_a} = \bar{a}$. Higher values of the non-linearity parameter nl concentrate more points on the lower end of the asset grid. We set $nl = 2$.

D.2 The Search Block

The search block contains three main algorithms: *search backward iteration*, *search state update* and *fiscal update*.

Search Backward Iteration Given next-period variables V^f_{t+1}, V^i_{t+1}, W^f_{t+1}, W^i_{t+1}, W^n_{t+1}, θ_{t+1}, r_{t+1}, ϕ_{t+1}, a current state E^f_{t-1}, E^i_{t-1}, U_{t-1}, F_{t-1} as well as r_t and public policy variables ϕ_t, $\tau_{hh,t}$:

1. Compute discounting q_t using (18);
2. Calculate wages w^f_t and w^i_t using (11) and (12);
3. Calculate period t values functions using (6), (7), (8) and (9);
4. Calculate period t market tightness using (15) and the job finding rate using (13);
5. Calculate ρ^f_t and ρ^i_t through (16);
6. Compute y_t using (19) and the transition matrix $g_t(s'|s)$.

Search State Update Given next-period variables ρ^f_{t+1}, ρ^i_{t+1}, V^f_{t+1}, V^i_{t+1}, r_{t+1}, ϕ_{t+1}, the current state variables E^f_{t-1}, E^i_{t-1}, U_{t-1} as well as the transition matrix g_t:

1. Update the distribution of workers E^f_t, E^i_t and U_t using the transition matrix g_t and the current distribution E^f_{t-1}, E^i_{t-1} and U_{t-1};
2. Calculate the anticipated next-period market tightness θ_{t+1} using the free entry condition (13) and the associated measure of new firms F_t using (15).

Fiscal Update Given current period wage rates w^f_t, w^i_t, distribution of workers E^f_t, E^i_t, U_t and price of bonds q_t, either:

- Given government consumption $C_{gov,t}$, update the income tax rate $\tau_{hh,t}$ using (17); or
- Given the income tax rate $\tau_{hh,t}$, update government consumption $C_{gov,t}$ using (17).

The difference between the choices in the fiscal update algorithm defines whether government adjusts income taxation (section 5) or public consumption (subsection 6.1).

D.3 The Household Block

The household block contains three main algorithms: *household backward iteration* and *household state update.*

Household Backward Iteration Given next-period variables J_{t+1} and g_{t+1}, the current period income process y_t and bond price q_t, solve the household's problem by solving the Euler Equation

$$q_t u'(c) = \beta \sum_{s'} \frac{\partial J_{t+1}(a', s')}{\partial a} g_{t+1}(s'|s).$$

We use the endogenous-grid method in this step.

Household State Update Given the state distribution x_{t-1} and households' next-period asset choice a'_t, compute x_t using (20).

D.4 Steady States and Transitions

To find a steady state, we start with a guess for the value functions, states variables, market tightness and interest rate. We run the five algorithms above in the order they are presented. After each iteration i, we calculate capital supply K_i using (21). We can also calculate aggregate hours of production L_i using the distribution of workers. The first-order condition for capital demanded by firms $A(k/\ell)^{\alpha-1} = r + \delta$ then provides the interest rate for the next iteration r_{i+1}, as well as value functions, states, and market tightness. Our tolerance for the approximation error is of four decimal digits.

To compute the transition path, we start with an initial steady state, which holds at $t = 0$, a transition length T and a final steady state which the economy reaches by period T. We then iterate the following three algorithms.

Backward Iteration Starting from the final steady state at T, use the *search backward iteration* and *household backward iteration* algorithms to find the path for value functions, wages and market tightness from $t = T - 1$ to $t = 1$.

State Update Starting with the aggregate state from the initial steady state, update from $t = 1$ to $t = T - 1$ using the *search state update*.

Update Fiscal/Interest From $t = 1$ to $t = T - 1$, update the public policy parameter of choice using the *fiscal update* algorithm and the interest rate path as in the case of a steady state.

We iterate the three algorithms above in the space of sequences until convergence of the aggregate state x_t, F_t.