The New-Keynesian Model: Conceptual Underpinnings

Livio Maya

The Price Level

- What determines the price level? Inflation?
- How to model modern institutions? Central banks, interest targets, forward guidance etc?
- Theory accompanies institutional change. Metallic standards, fiduciary money, central banking, credit cards, crypto...
- This presentation: some old theory, but mainly the New-Keynesian Model
  - How does it pin down the price level? Does it indeed?
  - What are its dynamic properties?
  - What story does it tell? What vision of the economy does it translate?

The NK Model: Some Experiments

- Private sector block:

$$
\begin{aligned}
y_t &= E_t y_{t+1} - \gamma \left(i_t - E_t \pi_{t+1} \right) \\
\pi_t &= \beta E_t \pi_{t+1} + \kappa y_t
\end{aligned}
$$

- Add a Taylor Rule:

$$
\begin{aligned}
i_t &= \phi \pi_t + u_t \qquad \phi > 1 \\
u_t &= \rho u_{t-1} + \varepsilon_t
\end{aligned}
$$

- Effect of monetary shock $\varepsilon_0 = 1$
  - Low persistency $\rho = 0.5$

The NK Model: Some Experiments

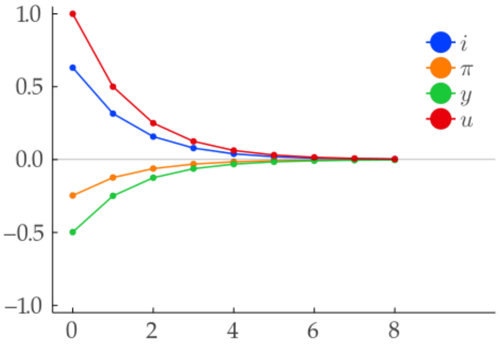
- Private sector block:

$$y_t = E_t y_{t+1} - \gamma \left(i_t - E_t \pi_{t+1} \right)$$
$$\pi_t = \beta E_t \pi_{t+1} + \kappa y_t$$

- Add a Taylor Rule:

$$i_t = \phi \pi_t + u_t \qquad \phi > 1$$
$$u_t = \rho u_{t-1} + \varepsilon_t$$

- Effect of monetary shock $\varepsilon_0 = 1$
  - Low persistency $\rho = 0.5$



The NK Model: Some Experiments

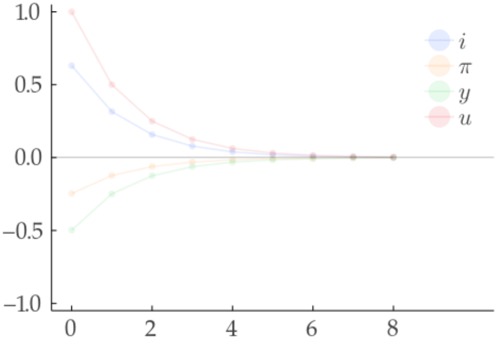
- Private sector block:

$$y_t = E_t y_{t+1} - \gamma \left(i_t - E_t \pi_{t+1} \right)$$
$$\pi_t = \beta E_t \pi_{t+1} + \kappa y_t$$

- Add a Taylor Rule:

$$i_t = \phi \pi_t + u_t \qquad \phi > 1$$
$$u_t = \rho u_{t-1} + \varepsilon_t$$

- Effect of monetary shock $\varepsilon_0 = 1$
  - Low persistency $\rho = 0.5$
  - High persistency $\rho = 0.9$



The NK Model: Some Experiments

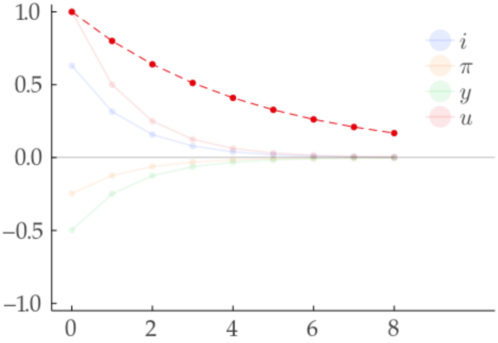
- Private sector block:

$$
\begin{aligned}
y_t &= E_t y_{t+1} - \gamma \left(i_t - E_t \pi_{t+1} \right) \\
\pi_t &= \beta E_t \pi_{t+1} + \kappa y_t
\end{aligned}
$$

- Add a Taylor Rule:

$$
\begin{aligned}
i_t &= \phi \pi_t + u_t \qquad \phi > 1 \\
u_t &= \rho u_{t-1} + \varepsilon_t
\end{aligned}
$$

- Effect of monetary shock $\varepsilon_0 = 1$
  - Low persistency $\rho = 0.5$
  - High persistency $\rho = 0.9$



The NK Model: Some Experiments

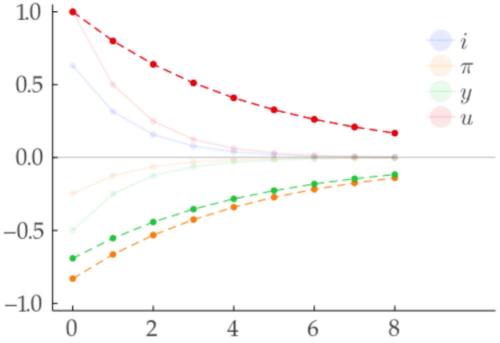
- Private sector block:

$$y_t = E_t y_{t+1} - \gamma \left(i_t - E_t \pi_{t+1} \right)$$
$$\pi_t = \beta E_t \pi_{t+1} + \kappa y_t$$

- Add a Taylor Rule:

$$i_t = \phi \pi_t + u_t \qquad \phi > 1$$
$$u_t = \rho u_{t-1} + \varepsilon_t$$

- Effect of monetary shock $\varepsilon_0 = 1$
  - Low persistency $\rho = 0.5$
  - High persistency $\rho = 0.9$



The NK Model: Some Experiments

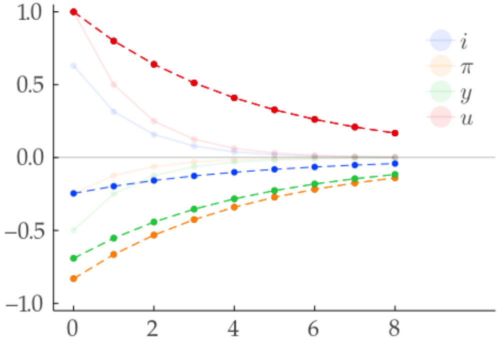
- Private sector block:

$$
\begin{aligned}
y_t &= E_t y_{t+1} - \gamma \left(i_t - E_t \pi_{t+1} \right) \\
\pi_t &= \beta E_t \pi_{t+1} + \kappa y_t
\end{aligned}
$$

- Add a Taylor Rule:

$$
\begin{aligned}
i_t &= \phi \pi_t + u_t \qquad \phi > 1 \\
u_t &= \rho u_{t-1} + \varepsilon_t
\end{aligned}
$$

- Effect of monetary shock $\varepsilon_0 = 1$
  - Low persistency $\rho = 0.5$
  - High persistency $\rho = 0.9$
- Role of interest? CB magical powers?

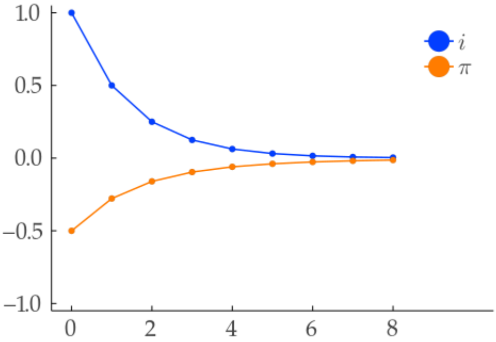


The NK Model: Some Experiments

- Private sector block:

$$y_t = E_t y_{t+1} - \gamma \left(i_t - E_t \pi_{t+1} \right)$$
$$\pi_t = \beta E_t \pi_{t+1} + \kappa y_t$$

- $i_t = \phi \pi_t + u_t$, different u_t (more later)

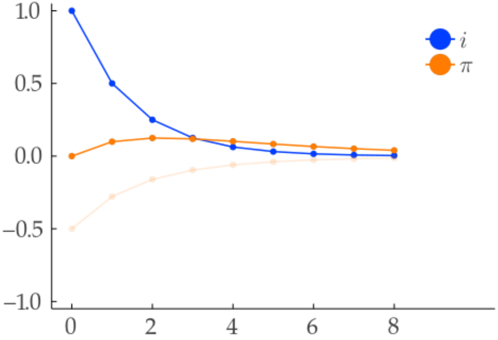


The NK Model: Some Experiments

- Private sector block:

$$
\begin{aligned}
y_t &= E_t y_{t+1} - \gamma \left(i_t - E_t \pi_{t+1} \right) \\
\pi_t &= \beta E_t \pi_{t+1} + \kappa y_t
\end{aligned}
$$

- $i_t = \phi \pi_t + u_t$, different u_t (more later)

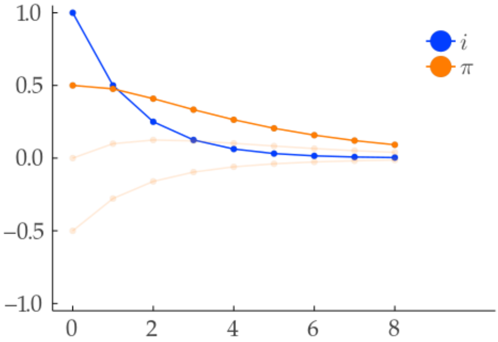


The NK Model: Some Experiments

- Private sector block:

$$y_t = E_t y_{t+1} - \gamma \left(i_t - E_t \pi_{t+1} \right)$$
$$\pi_t = \beta E_t \pi_{t+1} + \kappa y_t$$

- $i_t = \phi \pi_t + u_t$, different u_t (more later)

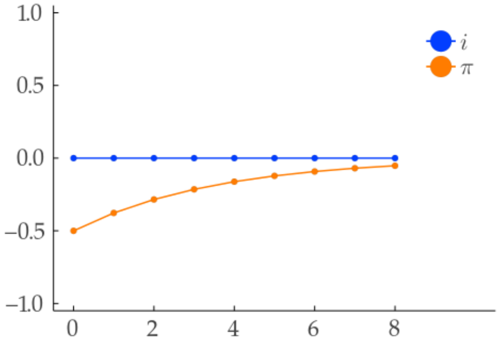


The NK Model: Some Experiments

- Private sector block:

$$y_t = E_t y_{t+1} - \gamma \left(i_t - E_t \pi_{t+1} \right)$$
$$\pi_t = \beta E_t \pi_{t+1} + \kappa y_t$$

- $i_t = \phi \pi_t + u_t$, different u_t (more later)
- Where is inflation coming from?



Some Monetary Theory History

How did we get here? Theories of the Price Level:

- Commodity Money (*"I value because I can eat"*)
- Commodity Standard (*"I value because I can trade for something I can eat"*)
- The Quantity Theory (*"I value because it is convenient"*)
  - Fisher, Pigou
  - $MV = PY$
  - $i_t = r_t + E_t \pi_{t+1}$

Some Monetary Theory History

How did we get here? Theories of the Price Level:

- Original Keynesianism (*"It is not about money"*)
  - Wage price spirals, unions, bargaining power, NRA...
  - Static Phillips curve in the 60s:
$$\pi_t = \kappa y_t$$
- Monetarism (*"It is all about money; and who controls it"*)
  - Central banks at the center stage
  - Fed action causes business cycles, inflation: 4% rule
  - Friedman (1968): long-run neutrality + "central banks can't peg interest rate"

Interest Targeting

Criticism of interest pegs

- **Instability** (Friedman, Bernanke, Krugman...)
  - Unstable equilibria: interest pegs lead to spirals
  - Adaptive expectations and Old-Keynesian models
- **Indeterminacy** (Sargent and Wallace (1975))
  - Frictionless model with constant output: $i_t = E_t \pi_{t+1}$

$$i_t = 0 \implies E_t \pi_{t+1} = 0$$

  - What about unexpected inflation $\Delta E_t \pi_t = (E_t - E_{t-1})\pi_t$?
  - Rational expectations and New-Keynesian models

Interest Targeting

- Original system

$$y_t = y^e_{t+1} - \gamma \left(i_t - \pi^e_{t+1} \right)$$
$$\pi_t = \beta \pi^e_{t+1} + \kappa y_t$$

- Static IS, $\beta = 1$, Taylor rule

$$y_t = -\gamma \left(i_t - \pi^e_{t+1} \right)$$
$$\pi_t = \pi^e_{t+1} + \kappa y_t$$
$$i_t = \phi \pi_t + u_t$$

- When necessary, $\phi = 0$ captures interest peg
- Replace and re-organize:

$$(1 + \kappa\gamma\phi)\pi_t = (1 + \kappa\gamma)\pi^e_{t+1} - \kappa\gamma u_t$$

Old Keynesian Models

$$(1 + \kappa\gamma\phi)\pi_t = (1 + \kappa\gamma)\pi^e_{t+1} - \kappa\gamma u_t$$

- Adaptive expectations: $\pi^e_{t+1} = \pi_{t-1}$

$$\pi_t = \frac{1 + \kappa\gamma}{1 + \kappa\gamma\phi}\pi_{t-1} - cu_t$$

Old Keynesian Models

$$(1+\kappa\gamma\phi)\pi_t = (1+\kappa\gamma)\pi^e_{t+1} - \kappa\gamma u_t$$

- Adaptive expectations: $\pi^e_{t+1} = \pi_{t-1}$

$$\pi_t = \frac{1+\kappa\gamma}{1+\kappa\gamma\phi}\pi_{t-1} - cu_t$$

- Interest peg $\phi = 0$ is unstable:

$$\pi_t = (1+\kappa\gamma)\pi_{t-1} - cu_t$$

Old Keynesian Models

$$(1+\kappa\gamma\phi)\pi_t = (1+\kappa\gamma)\pi^e_{t+1} - \kappa\gamma u_t$$

- Adaptive expectations: $\pi^e_{t+1} = \pi_{t-1}$

$$\pi_t = \frac{1+\kappa\gamma}{1+\kappa\gamma\phi}\pi_{t-1} - cu_t$$

- Interest peg $\phi = 0$ is unstable:

$$\pi_t = (1+\kappa\gamma)\pi_{t-1} - cu_t$$

- **Taylor Principle**: $\phi > 1$ in $i_t = \phi\pi_t + u_t$
  - $\uparrow$ Interest $\implies$ $\downarrow$ "Aggregate Demand" $\implies$ $\downarrow$ Inflation
  - Feedback rules stabilizes an unstable steady state

Old Keynesian Models

$$(1+\kappa\gamma\phi)\pi_t = (1+\kappa\gamma)\pi^e_{t+1} - \kappa\gamma u_t$$

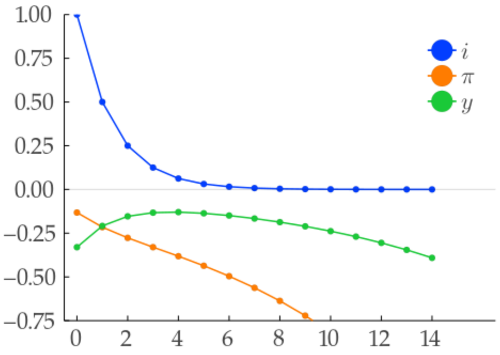
- Adaptive expectations: $\pi^e_{t+1} = \pi_{t-1}$

$$\pi_t = \frac{1+\kappa\gamma}{1+\kappa\gamma\phi}\pi_{t-1} - cu_t$$

- Interest peg $\phi = 0$ is unstable:

$$\pi_t = (1+\kappa\gamma)\pi_{t-1} - cu_t$$

- **Taylor Principle**: $\phi > 1$ in $i_t = \phi\pi_t + u_t$
  - $\uparrow$ Interest $\implies \downarrow$ "Aggregate Demand" $\implies \downarrow$ Inflation
  - Feedback rules stabilizes an unstable steady state



Old Keynesian Models

$$(1+\kappa\gamma\phi)\pi_t = (1+\kappa\gamma)\pi^e_{t+1} - \kappa\gamma u_t$$

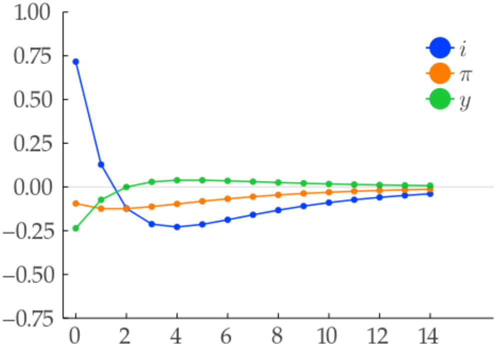
- Adaptive expectations: $\pi^e_{t+1} = \pi_{t-1}$

$$\pi_t = \frac{1+\kappa\gamma}{1+\kappa\gamma\phi}\pi_{t-1} - cu_t$$

- Interest peg $\phi = 0$ is unstable:

$$\pi_t = (1+\kappa\gamma)\pi_{t-1} - cu_t$$

- **Taylor Principle**: $\phi > 1$ in $i_t = \phi\pi_t + u_t$
  - $\uparrow$ Interest $\implies$ $\downarrow$ "Aggregate Demand" $\implies$ $\downarrow$ Inflation
  - Feedback rules stabilizes an unstable steady state



New Keynesian Models

$$(1+\kappa\gamma\phi)\pi_t = (1+\kappa\gamma)\pi^e_{t+1} - \kappa\gamma u_t$$

- Rational Expectations: $\pi^e_{t+1} = E_t\pi_{t+1}$

$$E_t\pi_{t+1} = \frac{1+\kappa\gamma\phi}{1+\kappa\gamma}\pi_t - cu_t$$

- Interest peg $\phi = 0$ is stable, but indeterminate (unexpected inflation?): $E_t\pi_{t+1} = \left(\frac{1}{1+\kappa\gamma}\right)\pi_t - cu_t$

New Keynesian Models

$$(1+\kappa\gamma\phi)\pi_t = (1+\kappa\gamma)\pi^e_{t+1} - \kappa\gamma u_t$$

- Rational Expectations: $\pi^e_{t+1} = E_t\pi_{t+1}$

$$E_t\pi_{t+1} = \frac{1+\kappa\gamma\phi}{1+\kappa\gamma}\pi_t - cu_t$$

- Interest peg $\phi = 0$ is stable, but indeterminate (unexpected inflation?): $E_t\pi_{t+1} = \left(\frac{1}{1+\kappa\gamma}\right)\pi_t - cu_t$
- $\phi > 1$ is unstable. Solve forward (present as function of future)

$$\pi_t = \alpha E_t\pi_{t+1} + u_t \qquad |\alpha| < 1$$

$$= \sum_{i=0}^{\infty}\alpha^i E_t u_{t+i} + \lim_{i\to\infty}\alpha^i E_t\pi_{t+i}$$

Pruning Equilibria

- Can we do $\lim_{i\to\infty} \alpha^i E_t \pi_{t+i} = 0$?

Pruning Equilibria

- Can we do $\lim_{i\to\infty} \alpha^i E_t \pi_{t+i} = 0$?
- Non-linear deterministic model:

$$1 + i_t = (1 + r)\Phi(\Pi_t)$$
$$\Pi_{t+1} = \beta(1 + i_t)$$

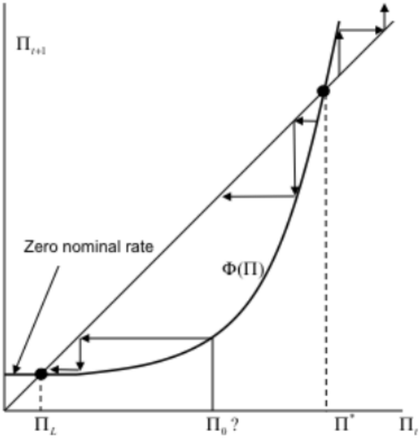
- Equilibrium: $\beta = (1 + r)$, $\Pi_{t+1} = \Phi(\Pi_t)$

Pruning Equilibria

- Can we do $\lim_{i \to \infty} \alpha^i E_t \pi_{t+i} = 0$?
- Non-linear deterministic model:

$$\begin{aligned} 1 + i_t &= (1 + r)\Phi(\Pi_t) \\ \Pi_{t+1} &= \beta(1 + i_t) \end{aligned}$$

- Equilibrium: $\beta = (1 + r)$, $\Pi_{t+1} = \Phi(\Pi_t)$
- "Good" steady state: $\Phi'(\Pi^*) > 1$
- Rule out explosiveness?
  - "Unreasonable"
  - CB "intervention"
  - Blow up the world

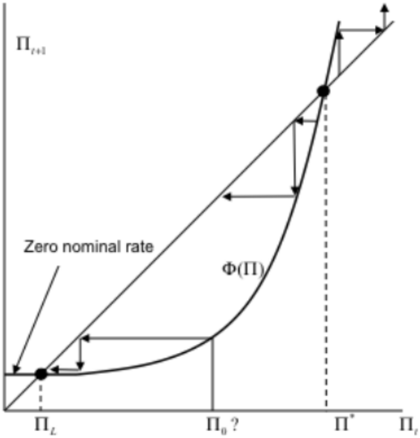


Pruning Equilibria

- Can we do $\lim_{i \to \infty} \alpha^i E_t \pi_{t+i} = 0$?
- Non-linear deterministic model:

$$1 + i_t = (1+r)\Phi(\Pi_t)$$
$$\Pi_{t+1} = \beta(1 + i_t)$$

- Equilibrium: $\beta = (1+r)$, $\Pi_{t+1} = \Phi(\Pi_t)$
- "Good" steady state: $\Phi'(\Pi^*) > 1$
- Rule out explosiveness?
  - "Unreasonable"
  - CB "intervention"
  - Blow up the world
- "Bad" equilibrium $\Phi'(\Pi_L) < 1$?



Interpreting the NK Model - Frictionless Case

- Anomalies still haunt
- Frictionless case $i_t = E_t \pi_{t+1}$ with peg $i_t = 0$:

$$E_t \pi_{t+1} = 0$$

  1 stable root to 1 forward-looking $\implies$ indeterminacy

- $\phi > 1$ introduces an unstable root and yields determinacy if spirals ruled out

$$E_t \pi_{t+1} = \phi \pi_t \implies \pi_t = 0$$

Interpreting the NK Model - Frictionless Case

$$i_t = E_t \pi_{t+1}$$

- Concept generalizes to stochastic target π_t^*

Interpreting the NK Model - Frictionless Case

$$i_t = E_t \pi_{t+1}$$

- Concept generalizes to stochastic target π_t^*
- Just set up the right interest rule!

$$\begin{aligned} i_t &= E_t \pi_{t+1}^* + \phi\left(\pi_t - \pi_t^*\right) \\ \implies E_t\left(\pi_{t+1} - \pi_{t+1}^*\right) &= \phi\left(\pi_t - \pi_t^*\right) \end{aligned}$$

Interpreting the NK Model - Frictionless Case

$$i_t = E_t \pi_{t+1}$$

- Concept generalizes to stochastic target π_t^*
- Just set up the right interest rule!

$$\begin{aligned} i_t &= E_t \pi_{t+1}^* + \phi \left(\pi_t - \pi_t^* \right) \\ \implies E_t \left(\pi_{t+1} - \pi_{t+1}^* \right) &= \phi \left(\pi_t - \pi_t^* \right) \end{aligned}$$

- $\phi > 1 \implies$ unstable $\implies \pi_t = \pi_t^*$

Interpreting the NK Model - Frictionless Case

$$i_t = E_t \pi_{t+1}$$

- Concept generalizes to stochastic target π_t^*
- Just set up the right interest rule!

$$\begin{aligned} i_t &= E_t \pi_{t+1}^* + \phi \left(\pi_t - \pi_t^*\right) \\ \implies E_t \left(\pi_{t+1} - \pi_{t+1}^*\right) &= \phi \left(\pi_t - \pi_t^*\right) \end{aligned}$$

- $\phi > 1 \implies$ unstable $\implies \pi_t = \pi_t^*$
- Now, define $u_t = E_t \pi_{t+1}^* - \phi \pi_t^*$:

$$i_t = \phi \pi_t + u_t$$

  This is where we started! (But not AR(1))

Interpreting the NK Model - Frictionless Case

Summary and Interpretation:

- The model does not pin down unexpected inflation $\Delta E_t \pi_t$
  - 1 forward-looking variable, 0 explosive roots
- $\phi > 1$ provides unstable root, selects $\Delta E_t \pi_t$. How?
  - Via interest rule, central bank threats spiral $|\pi_t| \rightarrow \infty$
  - Agents abominate spirals, jump to π_t^* (problem is here)
- The central bank chooses $\Delta E_t \pi_t$
- No "stimulate demand" (no Phillips curve, constant y)

Interpreting the NK Model

$$y_t = E_t y_{t+1} - \gamma \left(i_t - E_t \pi_{t+1} \right)$$
$$\pi_t = \beta E_t \pi_{t+1} + \kappa y_t$$

- 2 forward-looking variables, 1 explosive root (indeterminacy)
- Model pins down $\Delta E_t y_t$ or $\Delta E_t \pi_t$, but not both
- Suppose $E_t \pi_{t+1}$ given. Choose $\{i_t^*\}$ stable, stochastic target π_t^*

$$i_t^* - E_t \pi_{t+1}^* = i_t - E_t \pi_{t+1} \qquad \text{(Private Sector)}$$
$$i_t = i_t^* + \phi \left(\pi_t - \pi_t^* \right) \qquad \text{(Policy Rule)}$$

- Equilibrium:

$$i_t = i_t^*$$
$$\pi_t = E_{t-1} \pi_t + \Delta E_t \pi_t^* = \pi_t^*$$

Interpreting the NK Model

- ϕ is unidentified, interest equation reads $i_t = i_t^*$
- $\phi > 1$ is not exactly the Taylor Principle / Rule (threat, not action)
  - Taylor Rule would be $i_t^* = \phi \pi_t$
- Unexpected inflation is *not* the result of "aggregate demand"
  - Recall pricing condition in the NK model: $p = (1 - \beta\theta) \sum_i (\beta\theta)^i (\mu + w_t)$
- Blanchard-Khan is a tool, not a first-order condition
- Zero Lower Bound: what if we pin inflation *in the future*? Forward-Guidance Puzzle

Interest and Inflation in the Data

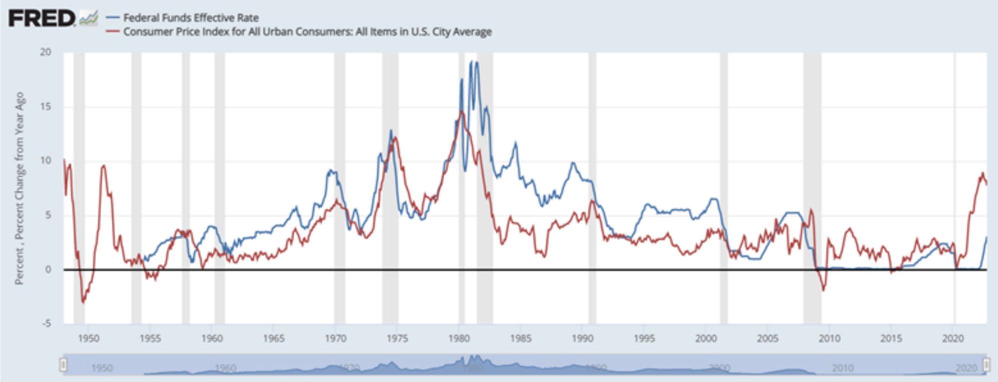


Figure: Interest and Inflation in the US

NK: The List of Issues

Problems with the New-Keynesian Model

- Central banks do not threat spirals
- First-order conditions do not rule out spirals
- No model of unexpected inflation determination; AR(1) $=$ blind selection
- Inconsistent with experience in the Zero Lower Bound period
- Forward Guidance Puzzle

and ...

NK: The List of Issues

Problems with the New-Keynesian Model

- Central banks do not threat spirals
- First-order conditions do not rule out spirals
- No model of unexpected inflation determination; AR(1) $=$ blind selection
- Inconsistent with experience in the Zero Lower Bound period
- Forward Guidance Puzzle

and ...

- **Assumes irrealistic commitment of fiscal policy**

Appendix: Neutrality and the Long-Term

- Fisher equation:

$$r_t = i_t - E_t \pi_{t+1}$$

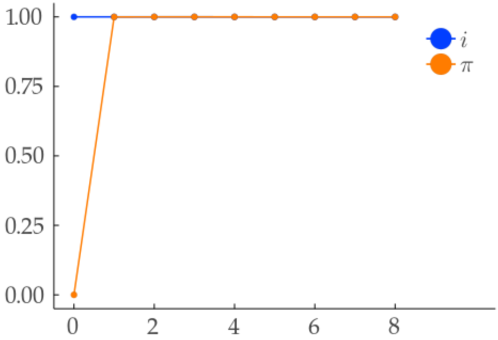
- In the long-run, either:
  - +1 nominal interest $\implies$ +1 inflation
  - Monetary policy not neutral

Appendix: Neutrality and the Long-Term

- Fisher equation:

$$r_t = i_t - E_t \pi_{t+1}$$

- In the long-run, either:
  - +1 nominal interest $\implies$ +1 inflation
  - Monetary policy not neutral



Appendix: Neutrality and the Long-Term

- Fisher equation:

$$r_t = i_t - E_t \pi_{t+1}$$

- In the long-run, either:
  - +1 nominal interest $\implies$ +1 inflation
  - Monetary policy not neutral