Fiscal Sources of Inflation Variation: International Estimates *

Livio Maya

Abstract

I decompose the valuation equation of public debt (market value of debt/price level = discounted surpluses) and measure the fiscal sources of inflation variation for twenty-one countries using Bayesian vector autoregressions. Innovations to inflation are primarily driven by changes to discount rates. Even using post-COVID data, contributions from surpluses are lower and derive mostly from economic activity (GDP growth) rather than fiscal policy (surplus/GDP ratios). A fiscal theory of the price level, New-Keynesian model with partial debt repayment can reproduce discount-driven inflation and realistic fiscal policy.
Keywords: Inflation, Fiscal Theory of the Price Level, Bayesian-VAR, Variance Decomposition, Partial Debt Repayment

1. Introduction

The use of the VAR to measure terms of the decomposition implicitly forces *consistency of expectations*: changes to surpluses or discount rates must change the real value of public debt; conversely, innovations to bond prices or the inflation rate must translate changes in expected surpluses or real discounting.

(...) I call that the *marked-to-market decomposition*.

Positive (say) innovations to discount rates reduce discounted surpluses but tend to reduce bond prices too, which partially balances the valuation equation. This observations calls for a new decomposition that internalizes that property: changes to discount rates on the right-hand side automatically change market prices on the left. I set up a model of public finances that links bond price innovations to revisions in the path of real discount rates and inflation. This leads to a second decomposition; one that nets out the effect of discount shocks on discounted surpluses from its effect on market prices. On the left-hand side of the valuation equation, the only term is the change to real bond prices due to revisions in inflation expectations. I call that the *total inflation decomposition*.

2. Fiscal Decompositions of Unexpected Inflation

*Paper previously circulated with the title "A Fiscal Decomposition of Unexpected Inflation: Cross-Country Estimates and Theory". I thank John Cochrane, Miguel Bandeira, Gustavo Perfeira, Daniel Cuzzi, Lucas Lima, Diogo Guillen and many seminar participants at PUC-Rio, FGV EESP and the University of São Paulo for useful comments and suggestions.

2.1. From the Budget Constraint to the Valuation Equation

Consider an economy with a consumption good which households value. There is a government that manages a debt composed by nominal and real bonds, possibly with different maturities. Upon maturing, real bonds deliver one unit of consumption good, while nominal bonds deliver one unit of currency. Currency is a commodity that only the government can produce, at zero cost. Households do not value it and they cannot burn it. The price of the consumption good in terms of currency is P_t.

The government brings from period $t-1$ a schedule $\{B^n_{N,t-1}\}$ of nominal bonds and $\{B^n_{R,t-1}\}$ of real bonds, where n denotes maturity. I denote $Q^n_{N,t}$ the price of nominal bonds and $P_t Q^n_{R,t}$ the price of real bonds (I state prices in currency units).

In period t, the government pays for maturing debt $B^1_{N,t-1} + P_t B^1_{R,t-1}$ and public spending $P_t G_t$ using currency. It retires currency from circulation by charging taxes $P_t T_t$ and selling new issues of nominal $Q^n_{N,t}(B^n_{N,t} - B^{n+1}_{N,t-1})$ and real $P_t Q^n_{R,t}(B^n_{R,t} - B^{n+1}_{R,t-1})$ bonds. The difference between currency introduced and retired by government trading changes private sector's aggregate holdings of it, M_t. Therefore:

$$\overbrace{B^1_{N,t-1} + P_t B^1_{R,t-1} + P_t G_t}^{\text{Currency introduced}} = \overbrace{P_t T_t + \sum_{n=1}^{\infty} Q^n_{N,t}\left(B^n_{N,t} - B^{n+1}_{N,t-1}\right) + P_t \sum_{n=1}^{\infty} Q^n_{R,t}\left(B^n_{R,t} - B^{n+1}_{R,t-1}\right)}^{\text{Currency retired}} + \Delta M_t.$$

The equation above is a *budget constraint* faced by the government. It must be respected for any choice of money holdings by households.

For simplicity and clarity of the argument, I assume that currency does not facilitate trade and, since it does not pay interest, that households do not bring currency from one period to the next: $M_t = 0$.[1] But they do value currency *in a given period*, as they need it to pay taxes and buy public bonds.[2] Our task is to determine that value in terms of consumption goods, i.e. the price level.

With $M = 0$, the budget constraint becomes

$$(1 + r^N_t)\mathcal{V}_{N,t-1} + (1 + r^R_t)(1 + \pi_t)\mathcal{V}_{R,t-1} = P_t S_t + \mathcal{V}_{N,t} + \mathcal{V}_{R,t}$$

where $S_t = T_t - G_t$ is the primary surplus, $1 + \pi_t = P_t / P_{t-1}$ is the inflation rate, $\mathcal{V}_{N,t} = \sum_{n=1}^{\infty} Q^n_{N,t} B^n_{N,t}$ and $\mathcal{V}_{R,t} = P_t \sum_{n=1}^{\infty} Q^n_{R,t} B^n_{R,t}$ are the end-of-period nominal and the real debt, and

$$1 + r^N_t = \frac{\sum_{n=1}^{\infty} Q^{n-1}_{N,t} B^n_{N,t-1}}{\mathcal{V}_{N,t-1}} \quad \text{and} \quad (1 + \pi_t)(1 + r^R_t) = \frac{P_t}{P_{t-1}} \frac{\sum_{n=1}^{\infty} Q^{n-1}_{R,t} B^n_{R,t-1}}{\mathcal{V}_{R,t-1}}$$

are nominal returns on holdings of the basket of nominal and real bonds. The equation above is no longer a budget constraint, but an equilibrium condition.

Let $\mathcal{V}_t = \mathcal{V}_{N,t} + \mathcal{V}_{R,t}$ be end-of-period public debt and $\delta_t = \mathcal{V}_{N,t} / \mathcal{V}_t$ the relative share of nominal

[1] This is not necessary for the arguments of the paper. We could alternatively add money holdings to the surplus definition and the equations of the paper will hold.

[2] The fact that the government charges taxes and sells bonds for currency is not necessary for the argument, as long as it stands ready to exchange currency for consumption goods at market prices. That would equally give value to currency.

debt, both at market prices. The nominal return on the entire basket of public bonds is

$$1 + r_t^n = \delta_t(1 + r_t^N) + (1 - \delta_t)(1 + r_t^R)(1 + \pi_t).$$

Since public debt and surpluses are not stationary in the data, I detrend both using output Y_t. Define $V_t = \mathcal{V}_t/(P_t Y_t)$ as the real debt-to-GDP ratio and $s_t = S_t/Y_t$ as the surplus-to-GDP ratio.[1] From the last flow equation for public debt, we get:

$$\frac{1 + r_t^n}{(1 + \pi_t)(1 + g_t)} V_{t-1} = s_t + V_t,$$

where g_t is the growth rate of GDP. The equilibrium condition above provides a law of motion for the real market value of public debt. The left-hand side contains the beginning-of-period (but after bond prices change) real market value of debt, which must be "paid for" by primary surpluses or future debt. Define $\rho_t = (1 + \pi_t)(1 + g_t)/(1 + r_t^n)$ as the *ex-post*, growth-adjusted real discount for public bonds, and $\rho_{t,t+j} = \prod_{\tau=t}^{t+j} \rho_\tau$. The following is the key assumption of the paper.

Assumption 1 (No Bubble): $\lim_{j\to\infty} E_t\, \rho_{t,t+j} V_{t+j} = 0$ in every period t.

The interpretation of assumption 1 depends on whether the government uses nominal debt.[2] If all debt is real, it represents a no-default condition. If the limit is positive, there are paths of primary surpluses that lead public debt to explode. The government eventually defaults.

If there is nominal debt (the case of the countries I consider), the government has no constraint on its choice of surpluses, as long as households attribute value to currency in a given period.[3] The zero limit condition becomes a no-bubble condition, which guarantees that the market value of debt equals discounted surpluses (just iterate the flow equation forward):

$$\frac{V_{t-1}}{\rho_t} = \sum_{j=0}^{\infty} E_t \left[\rho_{t+1,t+j} s_{t+j}\right].$$

The equation above is the valuation equation of public debt. This is an *equilibrium condition*, not a budget constraint. It is the condition upon which households accept to hold public bonds and currency. Households redeem bonds for currency and can trade currency for taxes, which have real value. Therefore, the stream of surpluses provides value for currency and the public debt, and determines the price level.[4] A similar equation, stock price = discounted dividends, expresses the condition for households to hold firms' equity shares (Cochrane (2005)).

The valuation equation is a common equilibrium condition in macroeconomic models, as it only depends on assumption 1. It does not depend on equilibrium selection mechanisms (fiscal

[1] If $P_t = 0$, households demand infinite goods and there is no equilibrium.

[2] Typical models of intertemporal household choice do not imply the limit of assumption 1 as a result of the transversality condition, as we use *ex-post* discounting $\rho_{t,t+j}$. They do imply instead that $E_t\Lambda_{t,t+j}V_{t+j}$ converges to zero, where Λ is the marginal rate of intertemporal substitution. *Ex-post* real returns and Λ coincide when markets are complete. Otherwise, the limit that defines 1 is not necessary for household optimality. See Bohn (1995).

[3] Existence of a positive price level requires the government to ensure $\sum_{j=0}^{\infty} E_t\rho_{t+1,t+j}s_{t+j} > (1 + r_t^R)\mathcal{V}_{R,t-1}/Y_t$.

[4] Again, the valuation equation determines the price level provided that $\delta_t > 0$ (some nominal debt). Note that time-t price level only shows up in the denominator of $\mathcal{V}_N$ on the left-hand side of the valuation equation:

$$\frac{V_{t-1}}{\rho_t} = (1 + r_t^N)\frac{\mathcal{V}_{N,t-1}}{P_t Y_t} + (1 + r_t^R)\frac{\mathcal{V}_{R,t-1}}{Y_t}.$$

theory or spiral threat).

2.2. Linearization and the Marked-to-Market Decomposition

Linearization of the valuation equation of public debt allows estimation using vector autoregressions. I start with the assumption that the government keeps the denomination structure of public debt constant over time.

Assumption 2 (Constant Currency Structure)**:** $\delta_t = \delta$ for every t.

Then, linearization of the last flow equation for public debt and the definition of nominal return yields:

$$\rho\left(v_t + \frac{s_t}{V}\right) = v_{t-1} + r_t^n - \pi_t - g_t \tag{1}$$

$$r_t^n = \delta r_t^N + (1-\delta)\left(r_t^R + \pi_t\right), \tag{2}$$

where $\rho = (1+g)(1+\pi)/(1+r^n)$ and symbols without t subscripts (like V) correspond to steady-state values. To save notation, I re-define all variables above as deviations from the steady state. Additionally, I re-define growth rates r_t^n, r_t^N, r_t^R, π_t and g_t as log-growth rates. Finally, $v_t = \log(V_t) - \log(V)$.

Like before, I solve the flow equation (1) forward and use assumption 1.

$$\overbrace{v_{t-1} + r_t^n - \pi_t}^{\substack{\text{Real market value}\\ \text{of public debt}}} = \overbrace{\frac{\rho}{V}\sum_{j=0}^{\infty}\rho^j E_t s_{t+j} + \sum_{j=0}^{\infty}\rho^j E_t g_{t+j} - \sum_{j=1}^{\infty}\rho^j E_t(r_{t+j}^n - \pi_{t+j})}^{\text{Discounted surpluses}}$$

The expression above is the linearized valuation equation of public debt.

Decomposition 1 (Marked-to-Market)**:** Take innovations on the valuation equation of public debt.

$$\boxed{\epsilon_{r^n,t} - \epsilon_{\pi,t} = \epsilon_{s,t} + \epsilon_{g,t} - \epsilon_{r,t}} \tag{3}$$

The terms of the decomposition are $\epsilon_{r^n,t} = \Delta E_t r_t^n$, $\epsilon_{\pi,t} = \Delta E_t \pi_t$, $\epsilon_{s,t} = (\rho/V)\sum_{j=0}^{\infty}\rho^j \Delta E_t s_{t+j}$, $\epsilon_{g,t} = \sum_{j=0}^{\infty}\rho^j \Delta E_t g_{t+j}$ and $\epsilon_{r,t} = \sum_{j=1}^{\infty}\rho^j \Delta E_t(r_{t+j}^n - \pi_{t+j})$.

The right-hand side of (3) contains revisions of discounted surpluses. These are divided between news about surplus-to-GDP ratios $\epsilon_{s,t}$, GDP growth $\epsilon_{g,t}$ and real discount rates $\epsilon_{r,t}$. The left-hand side contains the innovation to the price of public bonds $\epsilon_{r^n,t}$ in real terms. Given bond prices (this is why I call "marked-to-market"), surprise inflation $\epsilon_{\pi,t}$ devalues public debt so that its value coincides once again with discounted surpluses. We can replace equation (2) to highlight that inflation can only devalue the *nominal* portion of public debt:

$$\epsilon_{r^n,t} - \epsilon_{\pi,t} = \delta\left(\Delta E_t r_t^N - \Delta E_t \pi_t\right) + (1-\delta)\Delta E_t r_t^R.$$

A one percentage increase in the price level devalues total debt by δ%. The $1-\delta$ share of real bonds is not devalued because, in currency units, their prices grow along with the price level.

2.3. Geometric Term Structure and the Total Inflation Decomposition

Innovations to bond prices ($\epsilon_{r^n,t}$) are informative about the expected future path of nominal interest rates, and thus inflation and real discount rates. For instance, if the yield on two-year Treasury notes falls below the Fed Funds rate, it is reasonable to conjecture that market participants anticipate interest rate cuts by the Fed. The following assumption leads to a tractable relationship between the short-term interest rate, inflation and bond returns, which I then explore to decompose $\epsilon_{r^n,t}$.

Assumption 3 (Constant Geometric Term Structure)**:**

$$B^n_{N,t} = \omega_N B^{n-1}_{N,t} \quad \text{and} \quad B^n_{R,t} = \omega_R B^{n-1}_{R,t} \qquad \text{in every } t \text{ with } \omega_N, \omega_R \in [0,1].$$

Define $Q_{N,t} = \sum_{n=1}^{\infty} Q^n_{N,t}\omega_N^{n-1}$ and $Q_{R,t} = \sum_{n=1}^{\infty} Q^n_{R,t}\omega_R^{n-1}$ as the weighted-average market price of nominal and real bonds. Then, $\mathcal{V}_{N,t} = Q_{N,t}B^1_{N,t}$ and $\mathcal{V}_{R,t} = P_t Q_{R,t} B^1_{R,t}$. The linearized returns on public bonds are

$$\begin{aligned} r^N_t &= (\omega_N \rho) q_{N,t} - q_{N,t-1} \\ r^R_t &= (\omega_R \rho) q_{R,t} - q_{R,t-1} \end{aligned} \tag{4}$$

where $q_{N,t} = \log(Q_{N,t}/Q_N)$ and the analogous for $q_{R,t}$.[1] Expression (4) defines the return on holdings of public bonds. Note we can also use it to compute the price of the two public debt portfolios given models for expected returns $E_t r^N_t$ and $E_t r^R_t$.

Assumption 4 (Constant Term Premia)**:** Let $r_t = i_t - E_t\pi_{t+1}$ be the real interest rate.

$$E_t r^N_{t+1} = i_t \quad \text{and} \quad E_t r^R_{t+1} = r_t \quad \text{in every } t.$$

Because variables are stated as deviations of steady state, assumption 4 does not imply the absence of risk premium to bond holdings (the expectations hypothesis), but rather that such premium is constant. Variation in expected nominal returns are only due to changes to expected future nominal interest. It also implies that we can write the $\epsilon_{r,t}$ term of decomposition (3) as $\sum_{j=1}^{\infty} \rho^j \Delta E_t r_{t+j}$.

Move the equations in (4) one period forward and iterate them forward using assumption 4:

$$q_{N,t} = -\sum_{j=0}^{\infty} (\omega_N \rho)^j E_t i_{t+j} \quad \text{and} \quad q_{R,t} = -\sum_{j=0}^{\infty} (\omega_R \rho)^j E_t r_{t+j}. \tag{5}$$

The equations in (5) show the connection between short-term interest (nominal or real) and returns on debt holdings. News of higher interest lower public bond prices and lead to low returns. In fact, (5) implies that we can decompose unexpected real returns on public debt holdings as follows:

$$\epsilon_{r^n,t} - \epsilon_{\pi,t} = -\,\delta \overbrace{\sum_{j=0}^{\infty} (\omega_N \rho)^j \Delta E_t \pi_{t+j}}^{\substack{\text{Effect of inflation over} \\ \text{nominal debt value}}} - \overbrace{\sum_{j=1}^{\infty} \rho^j [\delta \omega_N^j + (1-\delta)\omega_R^j] \Delta E_t r_{t+j}}^{\substack{\text{Effect of discount rates over} \\ \text{nominal and real debt}}}.$$

News of real bond prices must correspond to news about future real interest (which affect the price

[1] In levels, the nominal return is $(B^1_{N,t-1} + \omega_N Q_{N,t} B^1_{N,t-1})/(Q_{N,t-1} B^1_{N,t-1})$. The analogous is true for the real return.

of all bonds) or current/future inflation (which affect the price of nominal bonds only). The ω's in the sum corresponding to real interest differ it from the $\epsilon_{r,t}$ term in decomposition (3). They govern duration, or the sensitiveness of bond prices to changes in future interest. When $\omega_N = \omega_R = 0$, all bonds have a one-period maturity. Their beginning-of-period nominal values are one (nominal) or P_t (real). They do not depend on future interest. When $\omega_N = \omega_R = 1$, public debt works as if it was constituted only of consols, whose price are most sensitivive to interest rate changes.

Decomposition 2 (Total Inflation)**:** Replace the decomposition of bond prices on the marked-to-market decomposition.

$$\boxed{-\varepsilon_{\pi,t} = \varepsilon_{s,t} + \varepsilon_{g,t} - \varepsilon_{r,t}} \tag{6}$$

The terms of the decomposition are $\varepsilon_{\pi,t} = \delta \sum_{j=0}^{\infty} (\omega_N \rho)^j \Delta E_t \pi_{t+j}$, $\varepsilon_{s,t} = \epsilon_{s,t}$, $\varepsilon_{g,t} = \epsilon_{g,t}$ and $\varepsilon_{r,t} = \sum_{j=1}^{\infty} \rho^j [1 - (\delta \omega_N^j + (1-\delta)\omega_R^j)] \Delta E_t r_{t+j}$.

The marked-to-market decomposition (3) focuses on unexpected changes to current inflation *given* bond prices. Decomposition (6) recognizes that changes to bond prices coalesce from changes to perceived future inflation and real interest. The $\varepsilon_{\pi,t}$ term answers the question: given the path of real discount, how do news about the entire path of inflation affect the market value of debt? This is why I call it the *total inflation* decomposition. Like before, the terms $\varepsilon_{s,t}$ and $\varepsilon_{g,t}$ account for changes in primary surpluses. The $\varepsilon_{r,t}$ term captures the effect of discount rate on discounted surpluses *net of their effect on bond prices*. If discount rates increase, they lower discounted surpluses, which calls for higher inflation. But they also lower bond prices, which reduces the required inflation adjustment. As discussed above, the tuple $(\delta, \omega_N, \omega_R)$ determines by how much prices decline, and therefore the net impact of discount rates on total inflation.

2.4. Converting Par to Market-Value Public Debt

Governments report public debt at par value. Because theory is based on market-value debt, some adjustment is necessary. Computing the market value of different bonds separately and adding them up as Cox and Hirschhorn (1983) and Cox (1985) is not feasible because large historical disaggregated data for outstanding bonds and their prices is not available. Instead, I adopt an adjustment model based on the average coupon rates on the basket of nominal and real basket of public debt. I call them i_t^b and r_t^b.

I present the complete model in the appendix. The government issues coupon-paying bonds at par, so coupon rates coincide with yields. The model gives an adjustment equation to convert market-value debt-to-GDP v_t to par-value debt-to-GDP v_t^b (both in logs):

$$v_t = v_t^b + q_t + \delta i_t^b + (1-\delta) r_t^b. \tag{7}$$

Par-value debt adds up the principal repayments over time; market-value debt adds to that bond-price variation q_t and variation in coupon rates i_t^b and r_t^b.

Average coupon rates follow the law of motion below.

$$\begin{aligned} i_t^b &= -(1-\omega_N)^2 q_{N,t} + \omega_N i_{t-1}^b \\ r_t^b &= -(1-\omega_R)^2 q_{R,t} + \omega_R r_{t-1}^b. \end{aligned} \tag{8}$$

Intuitively, the government must roll over a share $1-\omega_N$ of its nominal debt each period. New bonds must keep the geometric structure. Hence, the increment to the average coupon rate is $(1-\omega_N)\sum_{j=0}^{\infty}\omega_N^{j-1}E_t i_{t+j}$, which is approximately $-(1-\omega_N)q_{N,t}$ since $\rho\approx 1$. The analogous holds for real debt.

3. Estimates

I measure the terms of the marked-to-market and total inflation decompositions for different combinations of shocks. To do this, I estimate a six-equation VAR in which the debt law of motion (1) holds by construction. If the estimated VAR systems are stationary, real debt will converge and the decompositions will hold. Keeping the same notation, the vector of variables is

$$x_t = [i_t\ \pi_t\ g_t\ v_t\ r_t^n\ s_t]'.$$

From the six variables in the VAR, three are directly observed: the nominal interest i_t, the inflation rate π_t and GDP growth g_t. I also use data on the par debt-to-GDP v_t^b. These four time series are in logs. I demean each, implicitly assuming that historical averages approximate long-term steady states.

Converting par-value to market-value debt requires a series for nominal return on public bonds r_t^n, which is not available. I estimate an auxiliary three-equation VAR $\tilde{x}_t = a\tilde{x}_{t-1}+\tilde{e}_t$ with $\tilde{x}_t=[i_t,\pi_t,g_t]$ and use it to compute the expectations involved in bond price formulas (5), and then a nominal return r^n series using (4) and (2). The resulting series go in equation (7) to convert v_t^b to v_t.[1] Lastly, I define the suplus-to-GDP as the residual from the flow equation (1).

Data is annual.[2] Quarterly data is available, but it often does not go back as many years into the past. This is particularly true in the case of emerging market variables and public debt measures from all countries. With a focus on long-term debt sustainability, using a large time span of data provides invaluable information regarding variables' covariances and autocovariances that is not worth forgoing to account for quarterly dynamics. Additionally, with annual data there is no danger of measurement errors due to seasonality adjustments.

I group countries in four categories according to when the sample begins: 1947, 1960, 1973 and 1997. The first group contains the United States and the United Kingdom. The next two groups contain developed economies. The last group (1997 sample) contains ten developing countries.

3.1. The Bayesian VAR

The functional format of the VAR is

$$X_t = AX_{t-1}+e_t \qquad e_t\sim N(0,\Sigma). \tag{9}$$

I assume that the sample averages used to demean data series coincide with their model counterparts, so we can ignore the constant term.

[1]The adjustment equation requires a series for coupon rates i_t^b and r_t^b, which I compute using (8). The initial point in each series are respectively i_t and $i_t - E_t\pi_{t+1}$ (with the expected inflation calculated using the three-equation VAR).

[2]Most time series data I collect from the St Louis Fed *FRED* website, the United Nations and the IMF. Details on the appendix.

I interpret VAR parameters A and Σ as being random and estimate them using Bayesian regressions. I establish a prior distribution, and then use data likelihood to compute the posterior.[1] I opt to use Bayesian shrinkage as it reduces the volatility of estimated coefficients, an invaluable property when samples are relatively small. In addition, with a prior distribution that leads to a stable VAR, we can calibrate its tightness to ensure that the posterior centers around a stable VAR as well. I base my prior on OLS-estimated US dynamics, which we can directly compare to results available in the literature (Cochrane (2022)).[2]

The prior distribution belongs to the Normal-Inverse-Wishart (NIW) family. That is, letting $\theta = \text{vec}(A')$, where vec means stack columns,

$$\Sigma \sim IW(\Phi; d)$$
$$\theta|\Sigma \sim N(\bar{\theta}, \Sigma \otimes \bar{\Omega}).$$

With a Gaussian model, the NIW prior distribution is conjugate. Giannone et al. (2015) provide closed-form formulas for the posterior distribution and marginal likelihood.

The mean of the IW distribution is $\Phi/(d-n-1)$ where $n = 6$ is the dimension of the VAR and larger values of d represent tighter priors. I pick Φ to be the identity matrix (uncorrelated shocks, with a standard deviation of one percent) and select $d = n + 2 = 8$, the lowest integer that leads to a well-defined distribution mean (which equals Φ).

The prior for A centers around the coefficients estimated for the US via OLS, $\bar{\theta} = \text{vec}(A_{US}^{OLS})$.[3] The conditional covariance between coefficients is:

$$\text{cov}\left(\tilde{a}_{ij}, \tilde{a}_{kl} \mid \Sigma\right) = \begin{cases} \lambda^2 \frac{\Sigma_{ij}}{\Phi_{jj}} & \text{if } j = l \\ 0 & \text{otherwise.} \end{cases}$$

I build $\bar{\Omega}$ to reproduce the covariance structure above. It allows the loadings on a given variable in different equations to be correlated. The different loadings of any single equation are uncorrelated.

Hyperparameter λ governs the overall tightness of the prior. For each country, I choose the value of λ that maximizes the marginal likelihood of the sample.[4]

Finally, the Bayesian procedure breaks the flow equation of public debt (1), as it linearly combines the equation for the US with that of the estimated country. To restore it when computing the posterior mode and simulation draws, I manually change the loadings of the surplus equation in the VAR along with the covariance structure of its corresponding shock. The appendix provides details.

3.2. The Inflation Shock - Sources of Inflation Variation

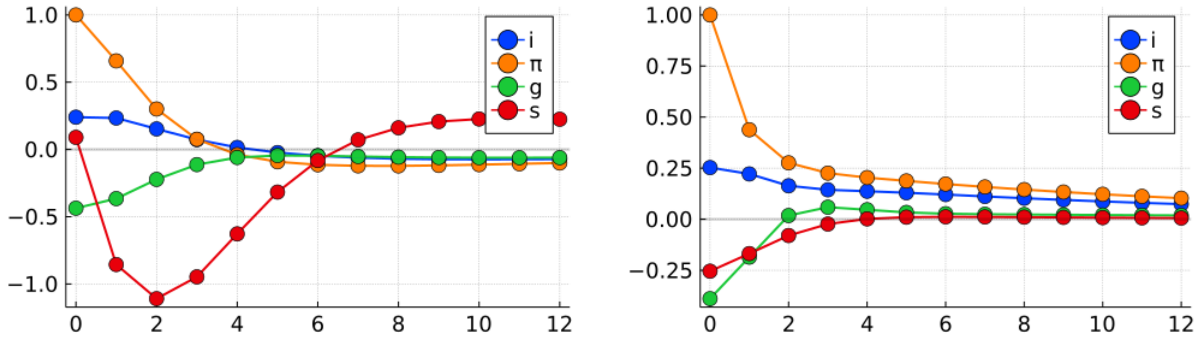
I set $\rho = 1$. For each country, V is the average debt-to-GDP ratio in sample; δ and ω are based on debt structure data from various sources (see appendix). In the baseline specification, I drop observations from the years 2020 and 2021.

[1] See del Negro and Schorfheide (2011) or Karlsson (2013) for more on Bayesian estimation of VAR models.

[2] To the best of my knowledge the decompositions have not been estimated to other countries so far.

[3] In the US case, this implies that the posterior distribution for A centers around the OLS estimate A_{US}^{OLS} itself.

[4] As Giannone et al. (2015) shows, the likelihood can be decomposed between a term that depends on in-sample model fit and a term that penalizes out-of-sample forecast imprecision, or model complexity.



(a) Japan (b) Comparison to Dallas Fed Estimates

Notes: Each figure plots the impulse response function to the inflatio shock, calculated using posterior mode parameters.

Figure 1: US Par and Market Value of Public Debt

In this paper, I focus on an inflation shock defined as follows.

$$\text{Inflation Shock} = E\left[e \mid e_\pi = 1\right]$$

Inflation unexpectedly jumps by one and the other shocks move contemporaneously exactly as expected, conditional on the inflation change.[1] IRFs to the inflation shock tell us how the expected path of each variable moves given that inflation today is 1% greater than expected. Decompositions (3) and (6) measure which factors account, on average, for such increase from the point of view of the valuation equation of public debt.

As shown by Cochrane (2022) and the appendix, the main motivation behind the inflation shock is that, when applied to it, the decompositions can be interpreted as *variance decompositions* of unexpected inflation. Specifically, the terms of the marked-to-market decomposition for example are:

Cochrane (2022) and the appendix prove the following proposition.

Proposition: Applied to the VAR (9), the marked-to-market decomposition (3) is

$$\frac{\text{cov}(\epsilon_{r^n}, \epsilon_\pi)}{\text{var}(\epsilon_\pi)} - 1 = \frac{\text{cov}(\epsilon_s, \epsilon_\pi)}{\text{var}(\epsilon_\pi)} + \frac{\text{cov}(\epsilon_g, \epsilon_\pi)}{\text{var}(\epsilon_\pi)} - \frac{\text{cov}(\epsilon_r, \epsilon_\pi)}{\text{var}(\epsilon_\pi)}.$$

The analogous is true for the total inflation decomposition (6) (with the $\varepsilon_{\pi,t}$ term containing unexpected future inflation autocovariances).

The main motivation behind the inflation shock is that, when applied to it, the decompositions can be interpreted as *variance decompositions* of unexpected inflation. They tell us the fiscal sources of unexpected inflation.

Figure (1) presents the cases of Japan and South Korea, which I will later use as reference. Each graph contains the IRF to the inflation shock computed using posterior mode parameters. The initial value of each variable is the inflation shock. By construction, unexpected inflation equals one.

[1] To calculate projection like the expected value of the inflation shocks, I use $E\left[e \mid Ke = \epsilon\right] = \Sigma K'(K\Sigma K')^{-1}\epsilon$.

Results. Tables 1 and 2 present my main result: the terms of the marked-to-market and total inflation decompositions. Values printed in red are negative, blue are positive. One asterisk indicates 75% statistical significance, two asterisks 90% (see table footnotes for details).

Consider first the marked-to-market decomposition (3) in table 1. In all countries but Czech Republic, the inflation shock calls for a sudden decline in bond prices ($\epsilon_{r^n} < 0$). Central banks react to inflation news by raising nominal interest. Lower bond prices and a higher price level imply a lower real value of public debt. By the valuation equation, lower discounted surpluses. Between "discounted" ($\epsilon_{\pi,t}$) and "surpluses" ($\epsilon_{s,t} + \epsilon_{g,t}$), the table shows that discounting accounts for the largest share of such drop in 14 of the 21 countries. Term $\epsilon_{\pi,t}$ is negative in 18. Only in the United Kingdom we find a statistically significant (at 75% confidence) positive contribution. On the cross-country average, 1% unexpected inflation corresponds to a 2% decline in the value of debt, or discounted surpluses. Discount rates account for 1.9% out of this 2% decline, or 95% of the total drop. In the case of emerging markets, that figure drops to 82%.

The decline in debt value that is not accounted for by discounting must follow from news about primary surpluses. The tables show that contributions from GDP growth to discounted surpluses are usually negative (16/21), although point estimate signals are often not statistically significant. Contributions from surplus/GDP are positive in 13 countries and negative in 8. On average, output growth reduces discounted surpluses (-0.3% of -2%) and therefore contributes positively to inflation variance. The surplus-to-GDP ratio does not.

Table 2 reports the total inflation decomposition. The left-hand term ε_π represents the change in bond prices net of inflation due to revisions of current and future inflation. On the right-hand side, surplus terms ε_s and ε_g are unchanged; the discount term ε_r nets out the effect of discount rates on discounted surpluses (the $\epsilon_{\pi,t}$ of the previous table) from its effect on bond prices.

For most countries, the adjustment above implies $\varepsilon_r < \epsilon_r$. Still, table 2's message is somewhat similar to that of table 1. Given the inflation shock, the contemporaneous jump in "total inflation" is mostly accounted for by discount rates. Term $\varepsilon_{\pi,t}$ is negative in 17 cases. Only in about half (11/21), the discount rate is the largest devaluing factor on the right-hand side of the decomposition. But, on average, the share of inflation it accounts for tends to be larger than that of surpluses. Indeed, in the cross-country average, discounting accounts for 1.5% of the 1.6% average "total inflation" - over 93% of the overall jump (72% in the case of emerging markets). The remaining 7% or so comes from GDP growth innovations. With the exception of the 1947 sample, which is heavily influenced by the UK case, averages over subsamples tell a similar story.

3.3. Robustness Checks

In the baseline exercise, we find that unexpected inflation is driven mainly by discount rate variation. I check if this conclusion is robust to modelling choices made along the way. Table (3) reports cross-country averages of the total inflation decomposition (6) to the inflation shock. To facilitate comparison, the top panel repeats the averages in the baseline case.

The first check is to include data for the years 2020 and 2021. These are the first years of the the worldwide inflation outbreak following the COVID pandemic. Because inflation and fiscal deficits show up in the data at the same time, our estimates of $\epsilon_{s,t}$ tend to be lower. Surpluses-to-GDP become a more important factor to account for inflation variation. The average contribution changes from 0.2 to -0.1; it declines in all but three countries. Despite the huge recessions, we

Country	ϵ_{r^n}	$-\epsilon_\pi$	=	ϵ_s	$+\epsilon_g$	$-\epsilon_r$
Averages	**-1.0	**-1	=	0.2	*-0.3	**-1.9
1947 (Advanced)	**-0.8	**-1	=	**-1.4	**-0.4	0.0
1960 (Advanced)	**-1.2	**-1	=	*1.5	*-0.2	**-3.5
1973 (Advanced)	**-1.5	**-1	=	-0.2	-0.4	**-1.9
1997 (Emerging)	**-0.7	**-1	=	*0.0	*-0.3	**-1.4
1947 Sample (Advanced)						
United Kingdom	**-0.8	**-1	=	**-2.0	**-0.8	*0.9
United States	**-0.7	**-1	=	**-0.8	-0.1	**-0.9
1960 Sample (Advanced)						
Canada	**-2.9	**-1	=	0.0	*-1.3	**-2.5
Denmark	**-1.0	**-1	=	0.4	-0.3	**-2.1
Japan	**-0.7	**-1	=	**3.7	**-3.7	**-1.6
Norway	**-0.7	**-1	=	*2.7	*5.1	**-9.5
Sweden	**-0.7	**-1	=	**1.0	**-1.0	**-1.7
1973 Sample (Advanced)						
Australia	**-2.6	**-1	=	0.4	0.1	**-4.1
New Zealand	**-1.0	**-1	=	*1.7	**-1.6	*-2.1
South Korea	**-0.4	**-1	=	*-2.4	0.1	0.8
Switzerland	**-2.1	**-1	=	*-0.6	-0.1	**-2.4
1997 Sample (Emerging)						
Brazil	**-0.5	**-1	=	**3.0	0.4	**-4.9
Colombia	**-1.4	**-1	=	**1.5	**-1.1	**-2.9
Czech Republic	*0.2	**-1	=	*0.6	**-1.4	-0.0
Hungary	**-0.8	**-1	=	0.6	*-0.7	**-1.7
India	**-0.3	**-1	=	**-1.1	-0.0	-0.1
Israel	**-0.5	**-1	=	**0.7	-0.0	**-2.2
Mexico	**-1.4	**-1	=	**-4.9	0.5	2.0
Poland	**-1.6	**-1	=	*0.9	-0.1	**-3.4
South Africa	**-0.9	**-1	=	0.3	*-0.7	**-1.5
Ukraine	-0.0	**-1	=	**-1.1	*-0.2	0.3

Notes: The table reports the terms of the fiscal decomposition to the shock $\Delta E_t \pi_t = 1$. VAR coefficients fixed at the posterior distribution's mode. One asterisk indicates that 75% of the values out of 10,000 draws from the posterior had the same sign as the figure reported. Two asterisks indicate 90%.

Table 1: Marked-to-market decomposition of the shock $E\left[e_t \mid e_{\pi,t} = 1\right]$

Country	$-\varepsilon_\pi$	=	ε_s	$+\varepsilon_g$	$-\varepsilon_r$
Averages	**-1.6	=	0.2	*-0.3	**-1.5
1947 (Advanced)	**-2.3	=	**-1.4	**-0.4	*-0.5
1960 (Advanced)	**-1.9	=	*1.5	*-0.2	**-3.2
1973 (Advanced)	**-2.3	=	-0.2	-0.4	*-1.7
1997 (Emerging)	**-1.1	=	*0.0	*-0.3	**-0.8
1947 Sample (Advanced)					
United Kingdom	**-2.9	=	**-2.0	**-0.8	-0.2
United States	**-1.6	=	**-0.8	-0.1	**-0.7
1960 Sample (Advanced)					
Canada	**-2.5	=	0.0	*-1.3	*-1.1
Denmark	**-1.6	=	0.4	-0.3	*-1.8
Japan	**-1.5	=	**3.7	**-3.7	**-1.5
Norway	**-2.0	=	*2.7	*5.1	**-9.8
Sweden	**-1.6	=	**1.0	**-1.0	**-1.6
1973 Sample (Advanced)					
Australia	**-3.3	=	0.4	0.1	**-3.9
New Zealand	**-2.3	=	*1.7	**-1.6	**-2.4
South Korea	**-1.7	=	*-2.4	0.1	0.6
Switzerland	**-1.9	=	*-0.6	-0.1	**-1.2
1997 Sample (Emerging)					
Brazil	**-0.7	=	**3.0	0.4	**-4.1
Colombia	**-0.9	=	**1.5	**-1.1	**-1.4
Czech Republic	**-0.5	=	*0.6	**-1.4	*0.3
Hungary	**-1.6	=	0.6	*-0.7	**-1.5
India	**-1.6	=	**-1.1	-0.0	*-0.4
Israel	**-0.7	=	**0.7	-0.0	**-1.4
Mexico	**-1.5	=	**-4.9	0.5	2.9
Poland	**-1.5	=	*0.9	-0.1	**-2.3
South Africa	**-0.9	=	0.3	*-0.7	*-0.5
Ukraine	**-1.3	=	**-1.1	*-0.2	0.0

Notes: The table reports the terms of the fiscal decomposition to the shock $\Delta E_t \pi_t = 1$. VAR coefficients fixed at the posterior distribution's mode. One asterisk indicates that 75% of the values out of 10,000 draws from the posterior had the same sign as the figure reported. Two asterisks indicate 90%.

Table 2: Total inflation decomposition of the shock $E\left[e_t \mid e_{\pi,t} = 1\right]$

do not verify the output growth factor $\varepsilon_{s,t}$ gain importance. The reason for this is that, in most countries, inflation only arrived in 2021, when their economies re-opened and, for the same reason, GDP growth *recovered*. In any case, the estimates continue to support the conclusion of inflation being discount-driven on average and for most countries.[1]

The next robustness check reduces term strucure parameters ω_N and ω_R ("Lowered debt duration" panel). Comparison between our measure of the market value of US debt with estimates provided by the Dallas Fed reveals a large differences (of up to 15% in log units), particularly in the 1980s (plot in the appendix). Reducing the average duration of the US debt from 5 to 2.5 years significantly reduces that difference.[2] I make the same adjustment to all countries and re-run results. Lower ω's reduce the volatility of the proxy series for r^n and thus increase the volatility of the surplus-to-GDP series s_t (computed as the residual of the flow equation). The $\epsilon_{\pi,t}$ terms grow in size but become more positive, which reinforces our key findings.

The next panel, "Higher discounting", simply reduce ρ from 1 to 0.97.[3] Discount rates continue to dominate the decomposition, but this change does increase the importance of surpluses in explaining unexpected inflation variance (lower $\varepsilon_{\pi,t}$), especially for advanced economies. The explanation is that, for them, surpluses-to-GDP often display the "s"-shape response to the inflation shock: the fall initially, and turn positive later. Higher discounting reduces the effect of this recovery in the decompositions.

The last two panels change the prior I adopted in the baseline. I first change the prior to be centered around $A = 0$. Since variables are stated in difference, this amounts to the well-known Litterman (1979) (or Minnesota) prior. I set $\lambda = 0.10$, which makes, for most countries, the prior distributions considerably tighter than in the baseline.[4] The other panel simply estimates matrix A by OLS.[5] These two cases test - in opposite directions - if the choices related to our Baysesian regressions can be blamed for the conclusion of discount-drive inflation. The table strongly suggests that they can't.

4. A New-Keynesian Model with Partial Debt Repayment

4.1. Model Equations

Fiscal policy:

$$\begin{aligned} h_t &= \tau\,(g_t + \pi_t) + u_{s,t} \\ u_{s,t} &= \rho_s u_{s,t-1} + w_{s,t} \qquad w_{s,t} \sim N(0, \sigma_s) \end{aligned} \tag{10}$$

Process h_t is an AR(1) process, except for a feedback contemporaneous dependency on output growth and inflation. Importantly, h_t does not depend on debt v_t.

[1] The discount term $\epsilon_{\pi,t}$ is negative in the case of 19 countries.

[2] Parameters $\omega_N = \omega_R$ go from 0.8 to 0.6. Cochrane (2022) sets $\omega = 0.69$.

[3] Recall that $1 - \rho$ is approximately $r - g$ in steady state, not just r. Therefore, the change to 0.97 is not simplistic.

[4] The lowest value of λ in the baseline is 0.12, for India.

[5] I actually continue to use Bayesian regressions, but center each country's prior on its own OLS estimate, which guarantees that the mode of the posterior distribution coincides with it. Hyperparameter Φ continues to be the identity matrix.

Country Group	$-\varepsilon_\pi$	$=$	ε_s	$+\varepsilon_g$	$-\varepsilon_r$
Averages - Baseline	**-1.6	=	0.2	*-0.3	**-1.5
1947 (Advanced)	**-2.3	=	**-1.4	**-0.4	*-0.5
1960 (Advanced)	**-1.9	=	*1.5	*-0.2	**-3.2
1973 (Advanced)	**-2.3	=	-0.2	-0.4	*-1.7
1997 (Emerging)	**-1.1	=	*0.0	*-0.3	**-0.8
COVID - 2020/21 data added	**-1.7	=	-0.1	-0.1	**-1.5
1947 (Advanced)	**-2.3	=	**-1.4	*-0.3	*-0.6
1960 (Advanced)	**-1.9	=	*1.2	-0.6	**-2.5
1973 (Advanced)	**-2.4	=	-0.4	-0.3	*-1.7
1997 (Emerging)	**-1.2	=	-0.4	0.3	**-1.1
Lowered debt duration	**-1.3	=	**0.8	*-0.4	**-1.7
1947 (Advanced)	**-2.1	=	**-1.3	**-0.5	-0.3
1960 (Advanced)	**-1.5	=	**1.7	*-0.4	**-2.8
1973 (Advanced)	**-1.8	=	0.7	-0.4	**-2.0
1997 (Emerging)	**-0.9	=	*0.8	*-0.4	**-1.3
Higher discount $r-g$ ($\rho = 0.97$)	**-1.6	=	-0.2	**-0.4	**-0.9
1947 (Advanced)	**-2.1	=	**-1.4	**-0.4	*-0.3
1960 (Advanced)	**-1.8	=	0.2	**-0.6	**-1.4
1973 (Advanced)	**-2.2	=	*-0.8	*-0.4	*-1.0
1997 (Emerging)	**-1.1	=	0.0	**-0.3	**-0.8
Minnesota Prior with $\lambda = 0.10$	**-1.4	=	-0.2	*-0.3	**-0.9
1947 (Advanced)	**-2.0	=	**-1.3	**-0.5	-0.3
1960 (Advanced)	**-1.6	=	0.1	*-0.6	**-1.1
1973 (Advanced)	**-2.0	=	*-1.2	*-0.3	*-0.5
1997 (Emerging)	**-1.0	=	0.2	*-0.2	**-1.0
OLS estimates	**-1.6	=	*0.4	*-0.3	**-1.8
1947 (Advanced)	**-2.3	=	**-1.4	**-0.4	*-0.5
1960 (Advanced)	**-1.9	=	*1.6	*-0.1	**-3.3
1973 (Advanced)	**-2.3	=	-0.2	-0.4	*-1.7
1997 (Emerging)	**-1.1	=	*0.5	*-0.2	**-1.3

Notes: The table reports cross-country averages of the terms of the fiscal decomposition to the shock $\Delta E_t \pi_t = 1$. VAR coefficients fixed at the posterior distribution's mode. One asterisk indicates that 75% of the values out of 10,000 draws from the posterior had the same sign as the figure reported. Two asterisks indicate 90%.

Table 3: Robustness Checks - Total inflation decomposition of the shock $E\left[e_t \mid e_{\pi,t} = 1\right]$

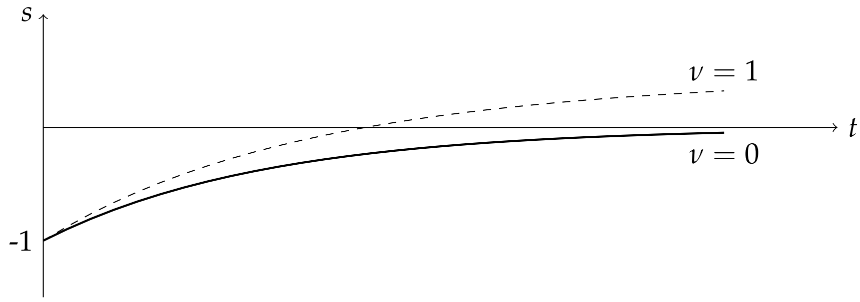


Figure 2: Surplus IRF with and without repayment

Actual surplus:

$$
\begin{aligned}
s_t &= s_t^* + (1-\nu)h_t \\
s_t^* &= \alpha v_t^* + \nu h_t \\
\rho\left(v_t^* + \frac{s_t}{V}\right) &= v_{t-1}^*
\end{aligned}
\tag{11}
$$

ν is debt-repayment.

Surplus term of decompositions (3) and (6):

$$
\epsilon_{s,t} = \varepsilon_{s,t} = (1-\nu)\left(\frac{\rho}{V}\right)\sum_{j=0}^{\infty}\rho^j \Delta E_t h_{t+j}
$$

$\nu = 0$: no debt repayment. $\nu = 1$: full debt repayment.

4.2. GMM Estimates

5. Data, Events, and Interpretation

In this section I present the dataset used in the estimation and discuss some economic events faced by the countries considered in the paper. Figures 3-6 present, for each case: the surplus-to-GDP (top graph), GDP growth (second from the top), and nominal interest and inflation rates (both in the third). I also report the estimated series for inflation disturbances e_π, or reduced-form inflation shock, in the bottom graph. I do not use the term "inflation shock" to avoid confusion with the term defined in section 3.

Korea. The estimated VAR reflects the events of the mid-1970s and early 1980s. Korea experiences a strong inflation surge in 1974/1975 with little interest rate response. The inflation disturbance hits as the strong pre-1973 output growth fades following the oil crisis, and fiscal deficits grow to pay for the Heavy Chemical Industrialization Plan (the "Big Push", see Collins and Park (1989)). A large negative reduced-form inflation shock in 1977 coincides with the recovery of GDP growth and the surplus-to-GDP. A new pair of positive-negative inflation disturbance hit in 1980 and 1982, and the GDP growth and surplus-to-GDP movements are the same: both fall and recover. In these episodes the Korean interest rate does follow inflation more closely, but not strongly or persistently enough to change the message of our estimated VARs. It is worth pointing out that, in the 1960s, interest rates did increase persistently to an inflation surge but, sadly, they do not enter the dataset since debt data is missing.

Mexico. The Mexican economy presents declining inflation and growing surplus-to-GDP ratios in the early 2000s. Nominal interest falls faster than inflation until 2005. GDP growth alternates good and bad years, with a small recession in 2001/2002. The IRF to the inflation shock replicates these patterns. The decompositions speak out the lasting responses of surplus-to-GDP and real interest. If we used $\rho = 0.975$, the discounting component of the total inflation decomposition $\varepsilon_{\pi,t}$ declines from 2.9 to (Incomplete). Using a Minnesota prior, it becomes (Incomplete). Another reduced-form inflation shock hits in 2017. The inflation hike as widely associated to trade disputes with the US and the lifting of energy price subsidies. From a fiscal accounting perspective, it looks to be discount-driven, as nominal interest grows faster than inflation in the following years, while the surplus-to-GDP ratio *increases*.

6. Conclusion

References

Ali Abbas, S. M., Belhocine, N., El-Ganainy, A., and Horton, M. (2011). Historical Patterns and Dynamics of Public Debt—Evidence From a New Database. *IMF Economic Review*, 59(4):717–742.

Bohn, H. (1995). The Sustainability of Budget Deficits in a Stochastic Economy. *Journal of Money, Credit and Banking*, 27(1):257.

Cochrane, J. H. (2005). Money as stock. *Journal of Monetary Economics*, 52(3):501–528.

Cochrane, J. H. (2022). The fiscal roots of inflation. *Review of Economic Dynamics*, 45:22–40.

Collins, S. M. and Park, W.-A. (1989). External Debt and Macroeconomic Performance in South Korea. In Sachs, J. D., editor, *Developing Country Debt and the World Economy*, pages 121–140. University of Chicago Press, Chicago.

Cox, W. M. (1985). The behavior of treasury securities monthly, 1942–1984. *Journal of Monetary Economics*, 16(2):227–250.

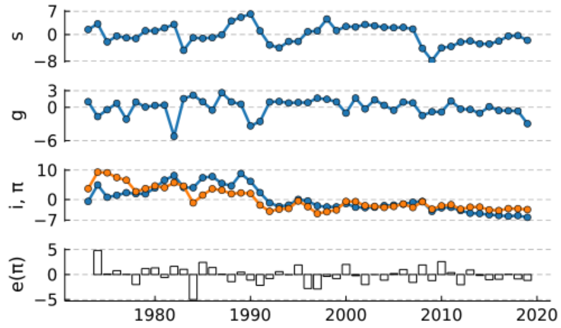
Cox, W. M. and Hirschhorn, E. (1983). The market value of U.S. government debt; Monthly, 1942–1980. *Journal of Monetary Economics*, 11(2):261–272.

del Negro, M. and Schorfheide, F. (2011). Bayesian Macroeconometrics. In Geweke, J., Koop, G., and Van Dijk, H., editors, *The Oxford Handbook of Bayesian Econometrics*, pages 292–389. Oxford University Press.

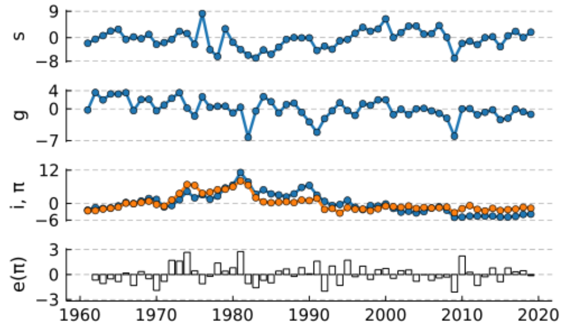
Eitrheim, Ø., Klovland, J. T., and Qvigstad, J. F. (2007). Historical Monetary Statistics for Norway - Part II. *Norges Bank Occasional Papers*, 38.

Giannone, D., Lenza, M., and Primiceri, G. (2015). Prior Selection for Vector Autoregressions. *The Review of Economics and Statistics*, 97(2):436–451.

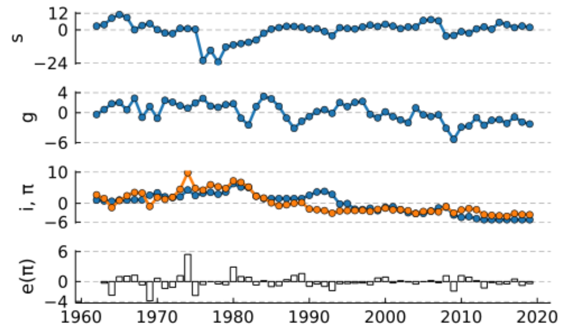
Hall, G. J. and Sargent, T. J. (2015). A History of US Debt Limits. Technical report, National Bureau of Economic Research.



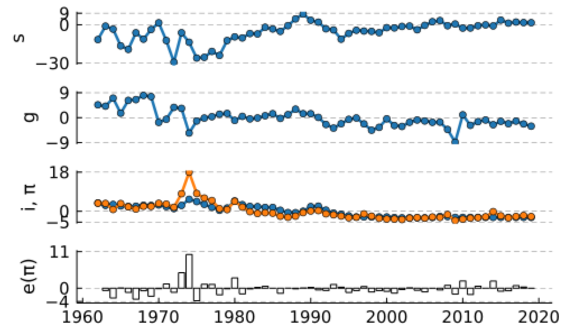
(a) Australia



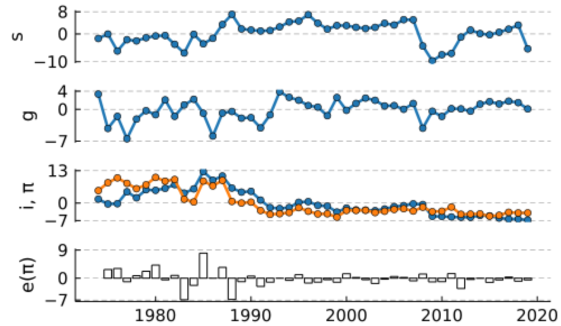
(b) Canada



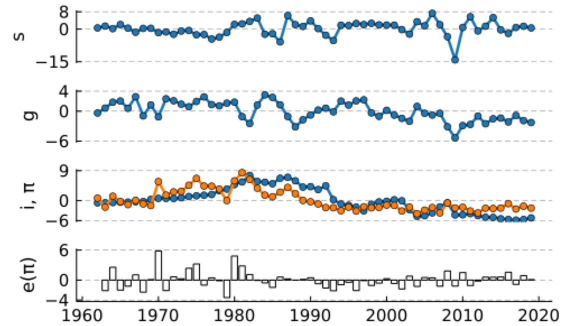
(c) Denmark



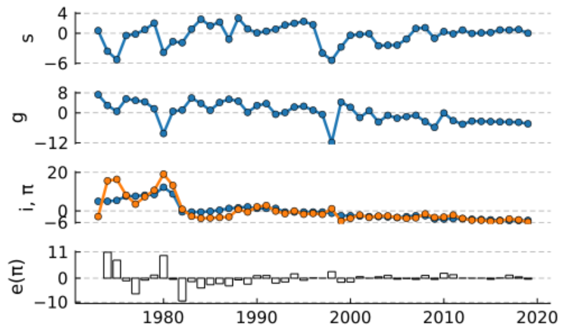
(d) Japan



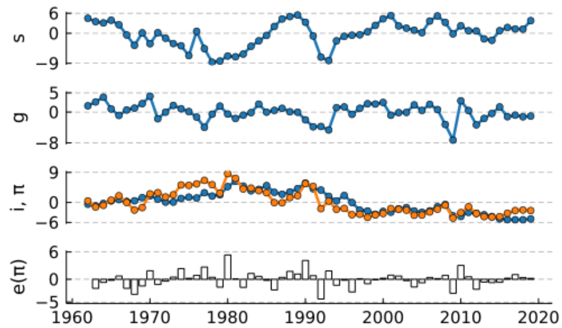
(e) New Zealand



(f) Norway



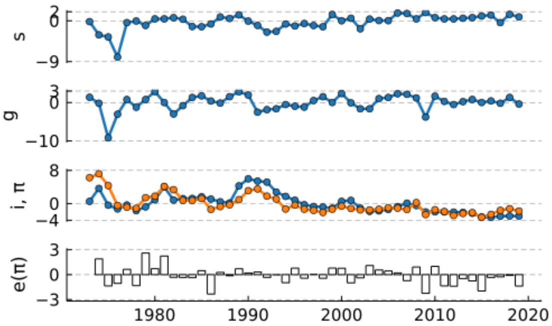
(g) South Korea



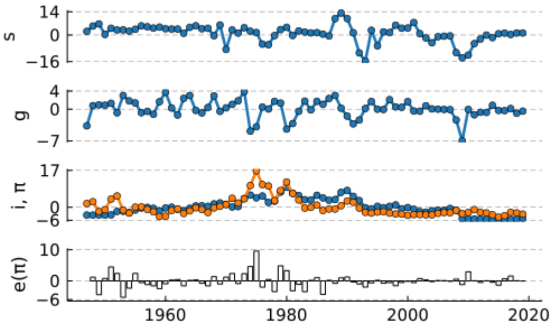
(h) Sweden

Notes: Bottom figure plots the residual of the inflation equation of the VAR (9), calculated using posterior mode parameters. The three plots above plot demeaned data used in the Bayesian regression: surplus-to-GDP and GDP growth in the top two plots, interest and inflation (red) in the third one. In these plots, variables are demeaned, in that zero corresponds to the sample average.

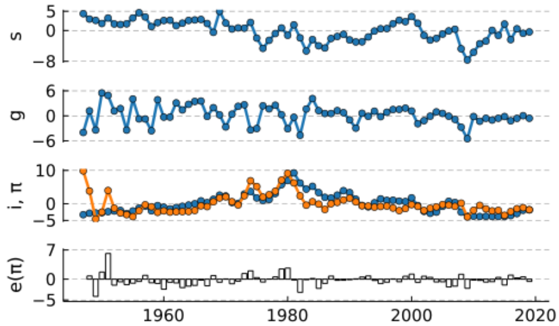
Figure 3: Inflation Residuals and Fiscal Factors (Advanced Economies)



(a) Switzerland



(b) United Kingdom



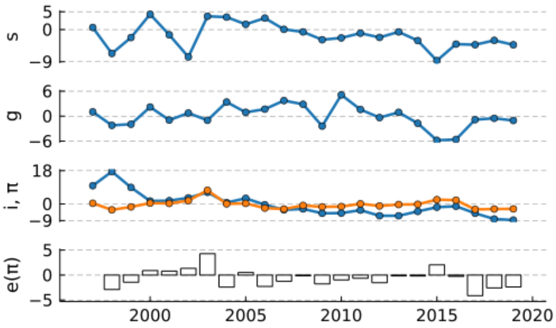
(c) United States

Notes: see notes to figure 3.

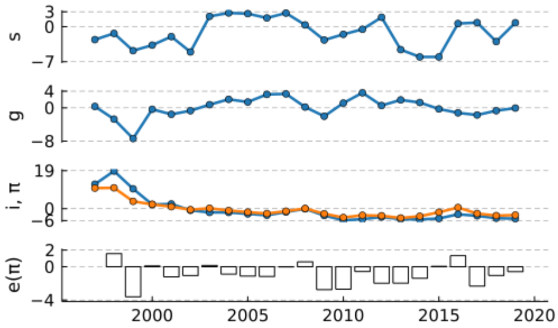
Figure 4: Inflation Residuals and Fiscal Factors (Advanced Economies, Continued)

Karlsson, S. (2013). Forecasting with Bayesian Vector Autoregression. In *Handbook of Economic Forecasting*, volume 2, pages 791–897. Elsevier.

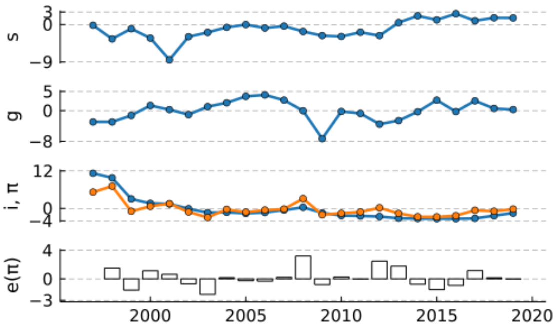
Litterman, R. (1979). Techniques of forecasting using Vector Auto Regression. *Federal Reserve Bank of Minneapolis Working Paper*, 115.



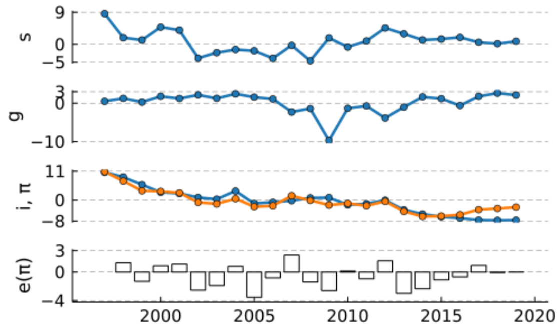
(a) Brazil



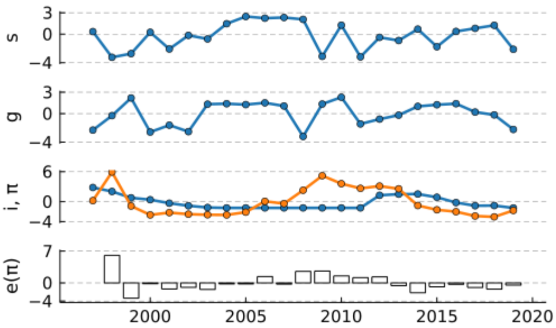
(b) Colombia



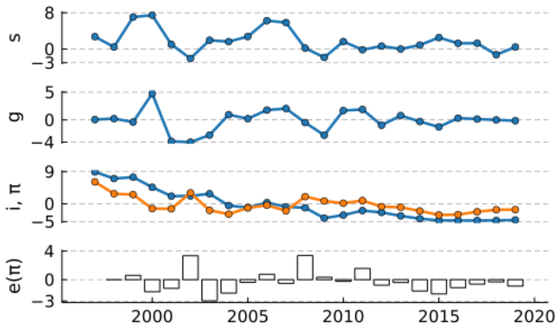
(c) Czech Republic



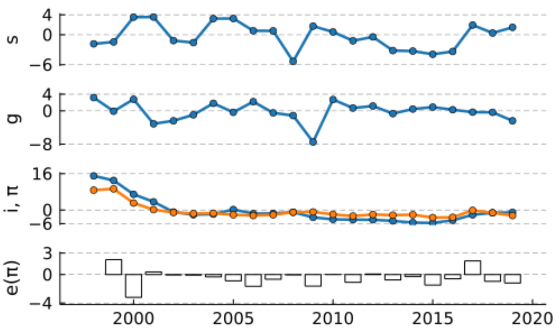
(d) Hungary



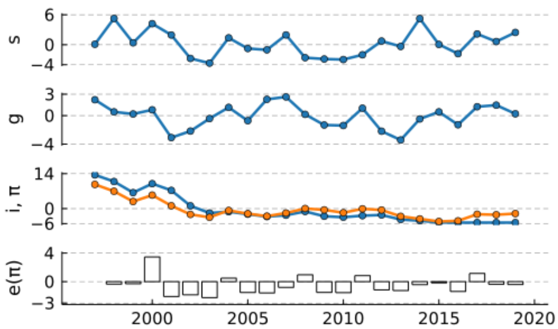
(e) India



(f) Israel



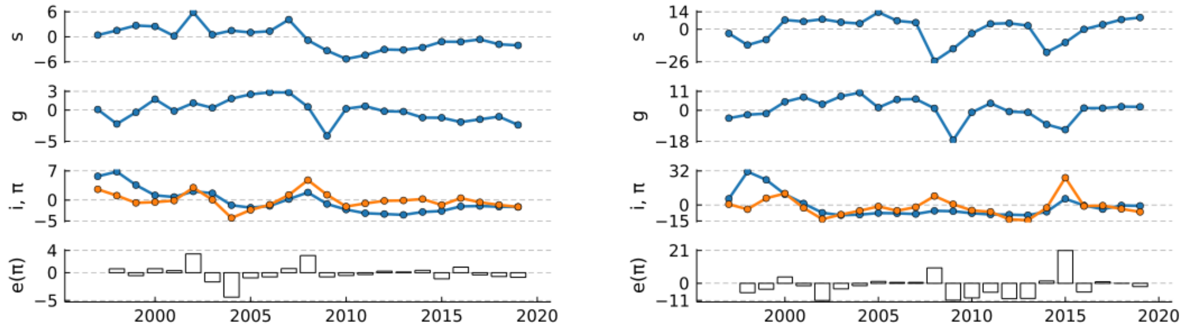
(g) Mexico



(h) Poland

Notes: Bottom figure plots the residual of the inflation equation of the VAR (9), calculated using posterior mode parameters. The three plots above plot demeaned data used in the Bayesian regression: surplus-to-GDP and GDP growth in the top two plots, interest and inflation (red) in the third one.

Figure 5: Inflation Residuals and Fiscal Factors (Emerging Economies)



(a) South Africa

(b) Ukraine

Notes: see notes to figure 5.

Figure 6: Inflation Residuals and Fiscal Factors (Emerging Economies, Continued)

A. Data Sources and Treatment

I collect a significant share of the data from the St. Louis Fed's *FRED* website. In the case of countries with sample starting after 1970 I get data from the United Nations's National Accounts Main Aggregates Database.

Whenever omitted in the list below, the source for interest rate data is the FRED; and the source of debt structure data is the OECD's Central Government Debt database. Finally, unless otherwise noted, public debt data I get from the database from Ali Abbas et al. (2011), which is kept up-to-date (Correct this sentence).

Australia 1973-2021. All except GDP and public debt from FRED.

Brazil 1998-2021. Debt structure data I collect from the Brazilian Central Bank.

Canada 1960-2021. All except public debt from FRED.

Chile 1998-2021.

Colombia 1998-2021. Debt structure data I collect from the Internal Debt Profile report, available at the Investor Relations Colombia webpage.

Czech Republic 1998-2021.

Denmark 1960-2021. All except public debt from FRED.

Hungary 1998-2021.

India 1998-2021. Debt structure data collect from the Status Paper on Government Debt report, 2019-2020, available at the Department of Economic Affairs.

Indonesia 1998-2021. Debt structure data I gather from the 2014 "Central Government Debt Profile" report and the 2018 "Government Securities Management" report, both from the Ministry of Finance.

Israel 1998-2021.

Japan 1960-2021. All except public debt from FRED.

Mexico 1998-2021.

Norway 1960-2021. All except public debt and interest rates from FRED. I interpolate the debt data for the year 1966. FRED interest data goes back to 1979, I splice it with historical data from Eitrheim et al. (2007), available at the website of the Norges Bank.

New Zealand 1973-2021. All except GDP and public debt from FRED.

Poland 1998-2021.

Romania 1998-2021. Interest rate is the deposit rate series from IMF's International Finance Statistics. Debt structure data I collect from the 2018 "Flash Report on the Romanian Public Debt" and the 2019-2021 and 2021-2023 "Government Debt Management Strategy" report, all from the Treasury and Public Debt Deparment (Ministry of Public Finance).

South Africa 1998-2021. Debt structure data from the 2020/2021 Debt Management Report, from the National Treasury Department.

South Korea 1973-2021. All except GDP and public debt from FRED. Interest rate: INTDSRKRM193N. Price level: KORCPIALLMINMEI.

Sweden 1960-2021. All except public debt from FRED. I interpolate the debt data for the year 1965 and 1966.

Switzerland 1973-2021. Interest, CPI and exchange rate from FRED.

Turkey 1998-2021.

Ukraine 1998-2021. Interest rate data from National Bank of Ukraine (NBU Key Policy Rate). Debt structure data I collect from "Ukraine's Public Debt Performance in 2021 and Local Market Update", from the Ministry of Finance of Ukraine.

United Kingdom 1960-2021. Interest data from the Bank of England (Base Rate); I splice it with discount rate data from the FRED. Inflation and GDP data from FRED.

United States 1950-2021. All data collected from the FRED. I use real exchange rate to the United Kingdom, since nominal exchange rate is available since before 1950.

B. A Model of the Par Value of Public Debt

Keeping the assumption of a geometric term structure, suppose the government sells bonds with coupons at par, as they usually do. Let i_t^b be the average coupon rate of nominal bonds and r_t^b the same for real bonds. Let $\mathcal{V}_{N,t}^{b,n}$ and $\mathcal{V}_{R,t}^{b,n}$ be the principal payment due n periods ahead, in dollars. After n periods, the government must pay $\mathcal{V}_{N,t}^{b,n}$ (principal) + $i_t^b \mathcal{V}_{N,t}^{b,n}$ (coupon) dollars; and $\mathcal{V}_{R,t}^{b,n}(1+r_t^b)/P_{t+n}$ consumption goods. These correspond to the quantities $B_{N,t}^n$ and $B_{R,t}^n$ defined earlier. Using the geometric term structure assumption 3, we get

$$\mathcal{V}_{N,t}^{b,n} = \omega_N \mathcal{V}_{N,t}^{b,n-1} \quad \text{and} \quad (\mathcal{V}_{R,t}^{b,n}/P_{t+n}) = \omega_R(\mathcal{V}_{R,t}^{b,n-1}/P_{t+n-1}).$$

The market-value of public debt at the beginning of period t corresponds to the sum of the market-value of each principal + coupon payment:

$$
\begin{aligned}
\overbrace{\mathcal{V}_{N,t-1}(1+r_t^N)}^{\substack{\text{Market Value,}\\ \text{Beginning of Period}}} &= \overbrace{\left[\mathcal{V}_{N,t-1}^{b,1}(1+i_{t-1}^b) + \mathcal{V}_{N,t-1}^{b,2}(1+i_{t-1}^b)Q_{N,t}^1 + \mathcal{V}_{N,t-1}^{b,3}(1+i_{t-1}^b)Q_{N,t}^2 + \dots\right]}^{\text{Market value of principal + coupon payments}} \\
&= \mathcal{V}_{N,t-1}^{b,1}(1+i_{t-1}^b)\left[1+\omega_N Q_{N,t}^1 + \omega_N^2 Q_{N,t}^2 + \dots\right] \\
&= \mathcal{V}_{N,t-1}^{b,1}(1+i_{t-1}^b)\left(1+\omega_N Q_{N,t}\right) \\
\mathcal{V}_{R,t-1}(1+r_t^R)(1+\pi_t) &= (\mathcal{V}_{N,t-1}^{b,1}/P_{t-1})(1+r_{t-1}^b)\left[P_t + \omega_R P_t Q_{N,t}^1 + \omega_N^2 P_t Q_{N,t}^2 + \dots\right] \\
&= \mathcal{V}_{R,t-1}^{b,1}(1+r_{t-1}^b)(1+\pi_t)
\end{aligned}
$$

Since bonds are issued at par, the par-value of public debt is just the sum of principals: $\mathcal{V}_{N,t-1}^b = \sum_{n=1}^{\infty}\mathcal{V}_{N,t-1}^{b,n}$, $\mathcal{V}_{R,t-1}^b = \sum_{n=1}^{\infty}\mathcal{V}_{R,t-1}^{b,n}$, and $\mathcal{V}_t^b = \mathcal{V}_{N,t}^b + \mathcal{V}_{R,t}^b$.

Next, I linearize. Let $V_{N,t} = \mathcal{V}_{N,t}/(P_t Y_t)$ and $V_{R,t} = \mathcal{V}_{R,t}/(P_t Y_t)$ be debt-to-GDP ratios, and $v_{N,t} = \log(V_{N,t}/V_N)$ and $v_{R,t} = \log(V_{R,t}/V_N)$ be their log deviations from steady state. Linearization of the equations above yields

$$
\begin{aligned}
v_{N,t-1} + r_t^N &= v_{N,t-1}^b + i_{t-1}^b + \rho\omega_N q_{N,t} \\
v_{R,t-1} + r_t^R &= v_{R,t-1}^b + r_{t-1}^b + \pi_t + \rho\omega_R q_{R,t}
\end{aligned}
$$

(I have redefined i_t^b and r_t^b to be log-return as deviation from the steady state). Up to a first-order approximation, $v_t = \delta v_{N,t} + (1-\delta)v_{R,t}$. Combining the equations above, we get

$$
v_{t-1} + r_t^n = v_{t-1}^b + r_t^{n,b} + \rho\left[\delta\omega_N q_{N,t} + (1-\delta)\omega_R q_{R,t}\right] \tag{12}
$$

$$
r_t^{n,b} = \delta i_{t-1}^b + (1-\delta)\omega_R\left(r_{t-1}^b + \pi_t\right). \tag{13}
$$

Expressions (12) and (13) have clear interpretations. Equation (13) defines the current-period coupon payment $r_t^{n,b}$. It only depends on time-t information through the inflation rate, as real bond coupons vary with the price level. Equation (12) says that the beginning-of-period market-value debt equals previous-period par-value debt + coupon payments + variation in the price of long-term bonds.[1] Replacing (2) for r_t^n leads to the adjustment equation:

$$
v_t = v_t^b + q_t + \delta i_t^b + (1-\delta)r_t^b. \tag{7}
$$

Replacing equation (12) in the flow equation of public debt (1) yields

$$
\rho\left(v_t - \left[\delta\omega_N q_{N,t} + (1-\delta)\omega_R q_{R,t}\right] + \frac{s_t}{V}\right) = v_{t-1}^b + r_t^{n,b} - \pi_t - g_t. \tag{14}
$$

This expression is similar to Hall and Sargent (2015) (equation 8 of their paper and first expression in page 11, not numbered). The par value of public debt v_{t-1}^b accrued by period coupons $r_t^{n,b}$ yields the new value for the market value of debt v_t netted out of long-term bond price variation (approximated by the term in brackets on the left).

[1] The ωq terms scale the variation in bond price by ω, the share of long-term bonds.

We can replace (7) again on the left-hand side of (14) to arrive at a flow equation for v_t^b:

$$\rho\left(v_t^b + \underbrace{[\delta(i_t^b + (1-\omega_N)q_{N,t}) + (1-\delta)(r_t^b + (1-\omega_R)q_{R,t})]}_{\text{"Revenue" effect of changing average coupon rates}} + \frac{s_t}{V}\right) = v_{t-1}^b + r_t^{n,b} - \pi_t - g_t. \quad (15)$$

The evolution of par-value debt differs from that of market-value debt in two respects. First, instead of nominal return accrual, it pays the average coupon rate. In the equation, that means replacing r_t^n by $r_t^{n,b}$. Second, since par-value debt only sums up principal payments, a new term (bracket, left-hand side) must be added to account for changes in the average coupon rate. Intuitively, if the government sells, for each existing bond, a new one with same principal payment but higher coupon rate, it will raise enough revenue to retire all existing bonds while leaving the par value of public debt v_t^b unchanged (the sum of principal payments will be the same). Of course, this will also cause later increases to coupon payment disbursements $r_t^{n,b}$ (see (13)). Since bonds are sold at par, the "revenue"-generating effect of changing coupon rates must be computed relative to actual average discount rates, which are the $(1-\omega)q_t$ terms, as explained below.

Average Coupon Rates. To keep a geometric term structure, every period the government must roll over a share of $1-\omega_N$ of nominal and $1-\omega_R$ of real debt. It then issues debt for all future maturities keeping the same geometric structure. Since bonds are sold at par by assumption, the coupon rate corresponds to the yield to maturity. In light of the constant term premium assumption 3, the increment in the average coupon rate is $(1-\omega_N)\sum(\omega_N\rho)E_t i_{t+n} = -(1-\omega_N)q_{N,t}$ for nominal bonds, and $-(1-\omega_R)q_{R,t}$ for real bonds.[1] Therefore, the law of motion to the average coupon rates are

$$\begin{aligned} i_t^b &= -(1-\omega_N)^2 q_{N,t} + \omega_N i_{t-1}^b \\ r_t^b &= -(1-\omega_R)^2 q_{R,t} + \omega_R r_{t-1}^b. \end{aligned} \quad (8)$$

Average (nominal or real) coupon rates are ω-weighted (since only a share $1-\omega$ of debt is rolled over) moving averages of ω-weighted (since the geometric term structure must be kept) averages of expected future short-term interest. We can re-writte the left-hand side of (15):

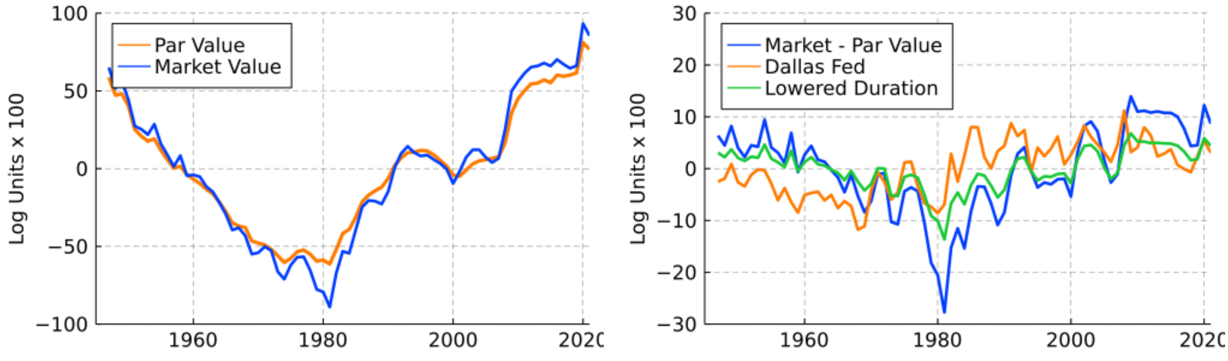
$$\rho\left(v_t^b + \underbrace{\left[\delta\left(i_t^b - (1-\omega_N)\sum_{j=0}^{\infty}(\omega_N^j\rho)E_t i_{t+j}\right) + (1-\delta)\left(r_t^b - (1-\omega_R)\sum_{j=0}^{\infty}(\omega_N^j\rho)E_t r_{t+j}\right)\right]}_{\text{"Revenue" effect of changing average coupon rates}} + \frac{s_t}{V}\right).$$

Since i_t^b and r_t^b move slower than interest rates, the revenue effect of changing coupon rates will tend to be negative when they grow.

Limit cases. If $\omega_N = \omega_R = 0$, the government does not issue long-term bonds. (8) implies $i_t^b = -q_{N,t} = i_t$ (and $r_t^b = r_t$). The nominal return on the stock of debt and coupon payment flows coincide, $r_t^n = r_t^{n,b}$, and, by (12), $v_t = v_t^{n,b}$.

The case $\omega_N = \omega_R = 1$ is analogous to the government financing itself using perpetuities only. Coupon rates become invariant to interest rate variation ($i_t^b = r_t^b = 0$). This implies $v_t = v_t^b + q_t$. On the other hand, bond prices q_t become more volatile.

[1] The ρ should not enter the sum. Since it is a number close to one, I introduce it to arrive at the convenient simplification wth $q_{N,t}$ and $q_{R,t}$.



(a) Par and Market Value of Debt

(b) Comparison to Dallas Fed Estimates

Notes: US data. The left figure plots our data for par-value of public debt (log, then demeaned) and our baseline calculation of the market-value of public debt. The blue line on the right-hand figure corresponds to the difference between these two series. I also plot the difference using the Dallas Fed's estimates of the market value of debt, as well as my measure after reducing the average duration of US debt (both nominal and real) from five to two and a half years.

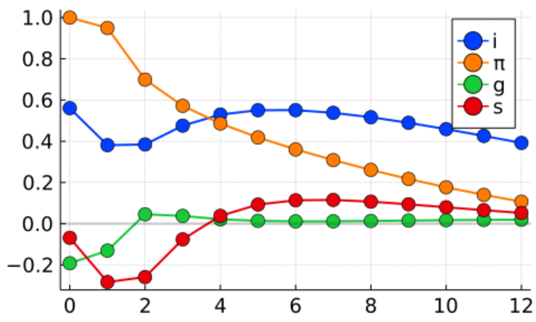
Figure 7: US Par and Market Value of Public Debt

In both of these limit cases, the law of motion for par-value debt satisfies the intuitive flow equation

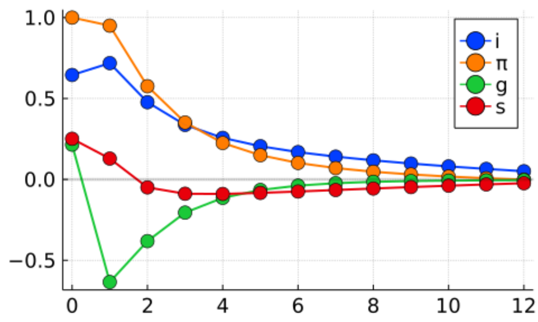
$$\rho\left(v_t^b + \frac{s_t}{V}\right) = v_{t-1}^b + r_t^{n,b} - \pi_t - g_t.$$

C. Additional Tables and Graphs

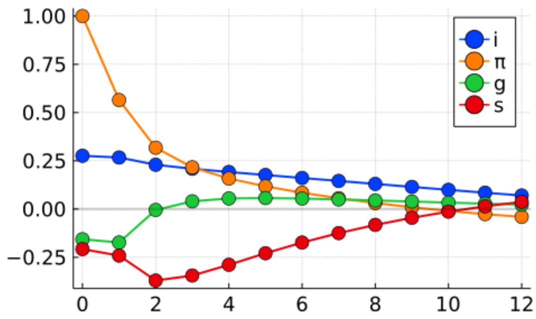
D. Restoring the Flow Equation in the VAR



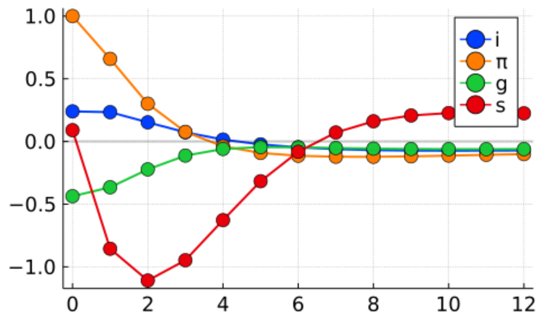
(a) Australia



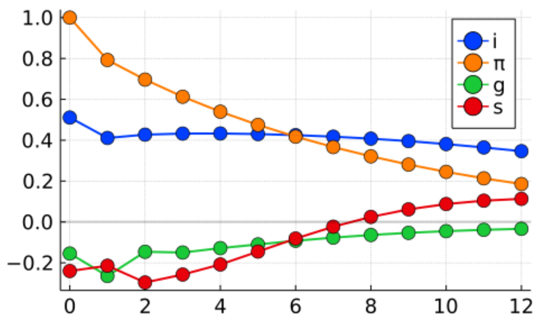
(b) Canada



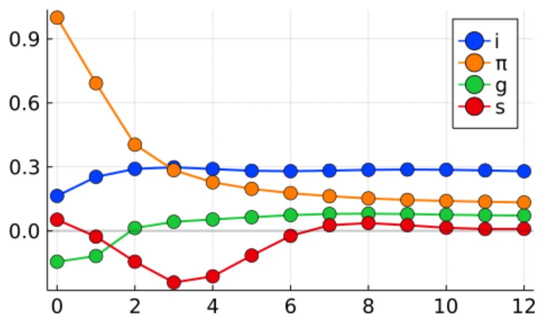
(c) Denmark



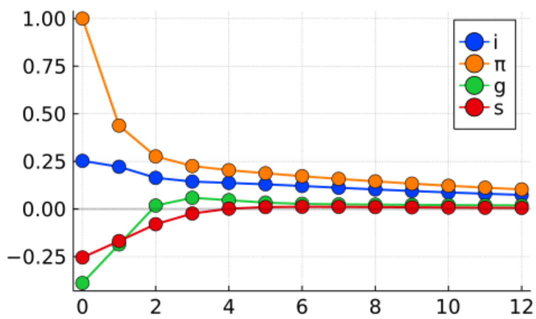
(d) Japan



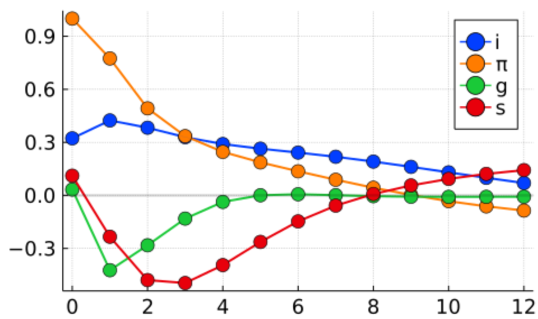
(e) New Zealand



(f) Norway



(g) South Korea



(h) Sweden

Notes: Each figure plots the impulse response function to the inflation shock, calculated using posterior mode parameters.

Figure 8: Impulse Response Function - Inflation Shock (Advanced Economies)

Country	ϵ_{r^n}	$-\epsilon_\pi$	=	ϵ_s	$+\epsilon_g$	$-\epsilon_r$
Averages	**-0.6	**-0.4	=	0.1	0.1	**-1.2
1947 (Advanced)	**-0.8	**-0.2	=	*-0.2	0.1	**-0.8
1960 (Advanced)	**-0.7	**-0.3	=	*0.5	0.4	**-1.9
1973 (Advanced)	**-0.7	**-0.3	=	-0.3	0.3	**-1.0
1997 (Emerging)	**-0.6	**-0.4	=	*0.2	*-0.1	**-1.1
1947 Sample (Advanced)						
United Kingdom	**-0.8	**-0.2	=	**-0.5	-0.1	*-0.4
United States	**-0.7	**-0.3	=	0.0	**0.2	**-1.2
1960 Sample (Advanced)						
Canada	**-0.8	**-0.2	=	*0.2	-0.1	**-1.1
Denmark	**-0.8	**-0.2	=	*0.6	*0.5	**-2.0
Japan	**-0.6	**-0.4	=	0.0	-0.2	**-0.8
Norway	**-0.6	**-0.4	=	*1.0	*1.9	**-3.9
Sweden	**-0.6	**-0.4	=	**0.7	-0.2	**-1.5
1973 Sample (Advanced)						
Australia	**-0.8	**-0.2	=	*0.5	*0.2	**-1.7
New Zealand	**-0.6	**-0.4	=	**0.8	**-0.5	**-1.3
South Korea	**-0.6	**-0.4	=	**-2.4	**1.3	0.2
Switzerland	**-0.8	**-0.2	=	-0.1	*0.2	**-1.1
1997 Sample (Emerging)						
Brazil	**-0.5	**-0.5	=	**1.4	0.1	**-2.6
Colombia	**-0.6	**-0.4	=	0.0	**-0.3	**-0.8
Czech Republic	**-0.4	**-0.6	=	-0.1	-0.3	**-0.6
Hungary	**-0.6	**-0.4	=	*0.4	-0.3	**-1.2
India	**-0.5	**-0.5	=	-0.1	*-0.2	**-0.7
Israel	**-0.7	**-0.3	=	**0.6	-0.1	**-1.5
Mexico	**-0.6	**-0.4	=	**-0.6	0.1	*-0.6
Poland	**-0.7	**-0.3	=	**0.5	-0.1	**-1.4
South Africa	**-0.7	**-0.3	=	*-0.2	0.0	**-0.8
Ukraine	**-0.5	**-0.5	=	**-0.4	*-0.1	**-0.6

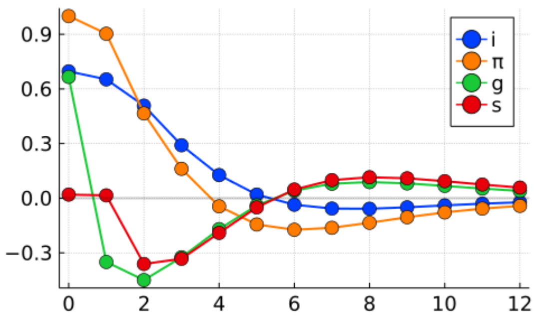
Notes: The table reports the terms of the fiscal decomposition to the shock $\Delta E_t(\text{Disc Surpluses}) = -1$. VAR coefficients fixed at the posterior distribution's mode. One asterisk indicates that 75% of the values out of 10,000 draws from the posterior had the same sign as the figure reported. Two asterisks indicate 90%.

Table 4: Marked-to-market decomposition of the shock
$E\left[e_t \mid \Delta E_t(\text{Disc Surpluses}) = -1\right]$

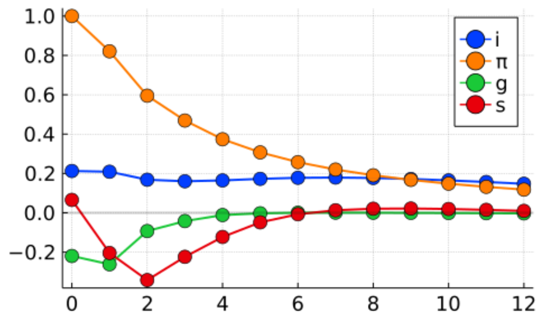
Country	$-\varepsilon_\pi$	=	ε_s	$+\varepsilon_g$	$-\varepsilon_r$
Averages	**-0.6	=	0.1	0.1	**-0.8
1947 (Advanced)	**-0.7	=	*-0.2	0.1	**-0.5
1960 (Advanced)	**-0.7	=	*0.5	0.4	**-1.6
1973 (Advanced)	**-0.8	=	-0.3	0.3	**-0.8
1997 (Emerging)	**-0.4	=	*0.2	*-0.1	**-0.5
1947 Sample (Advanced)					
United Kingdom	**-0.9	=	**-0.5	-0.1	*-0.3
United States	**-0.5	=	0.0	**0.2	**-0.7
1960 Sample (Advanced)					
Canada	**-0.5	=	*0.2	-0.1	**-0.6
Denmark	**-0.6	=	*0.6	*0.5	**-1.6
Japan	**-0.7	=	0.0	-0.2	**-0.5
Norway	**-0.9	=	*1.0	*1.9	**-3.8
Sweden	**-0.8	=	**0.7	-0.2	**-1.2
1973 Sample (Advanced)					
Australia	**-0.6	=	*0.5	*0.2	**-1.3
New Zealand	**-0.8	=	**0.8	**-0.5	**-1.2
South Korea	**-1.2	=	**-2.4	**1.3	0.0
Switzerland	**-0.5	=	-0.1	*0.2	**-0.6
1997 Sample (Emerging)					
Brazil	**-0.3	=	**1.4	0.1	**-1.9
Colombia	**-0.3	=	0.0	**-0.3	-0.1
Czech Republic	**-0.5	=	-0.1	-0.3	-0.2
Hungary	**-0.6	=	*0.4	-0.3	**-0.8
India	**-0.6	=	-0.1	*-0.2	**-0.3
Israel	**-0.2	=	**0.6	-0.1	**-0.7
Mexico	**-0.6	=	**-0.6	0.1	-0.1
Poland	**-0.5	=	**0.5	-0.1	**-0.9
South Africa	**-0.3	=	*-0.2	0.0	*-0.1
Ukraine	**-0.6	=	**-0.4	*-0.1	**-0.1

Notes: The table reports the terms of the fiscal decomposition to the shock $\Delta E_t(\text{Disc Surpluses}) = -1$. VAR coefficients fixed at the posterior distribution's mode. One asterisk indicates that 75% of the values out of 10,000 draws from the posterior had the same sign as the figure reported. Two asterisks indicate 90%.

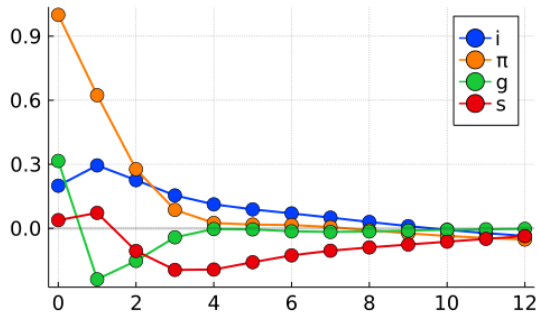
Table 5: Total inflation decomposition of the shock
$E\left[e_t \mid \Delta E_t(\text{Disc Surpluses}) = -1\right]$



(a) Switzerland



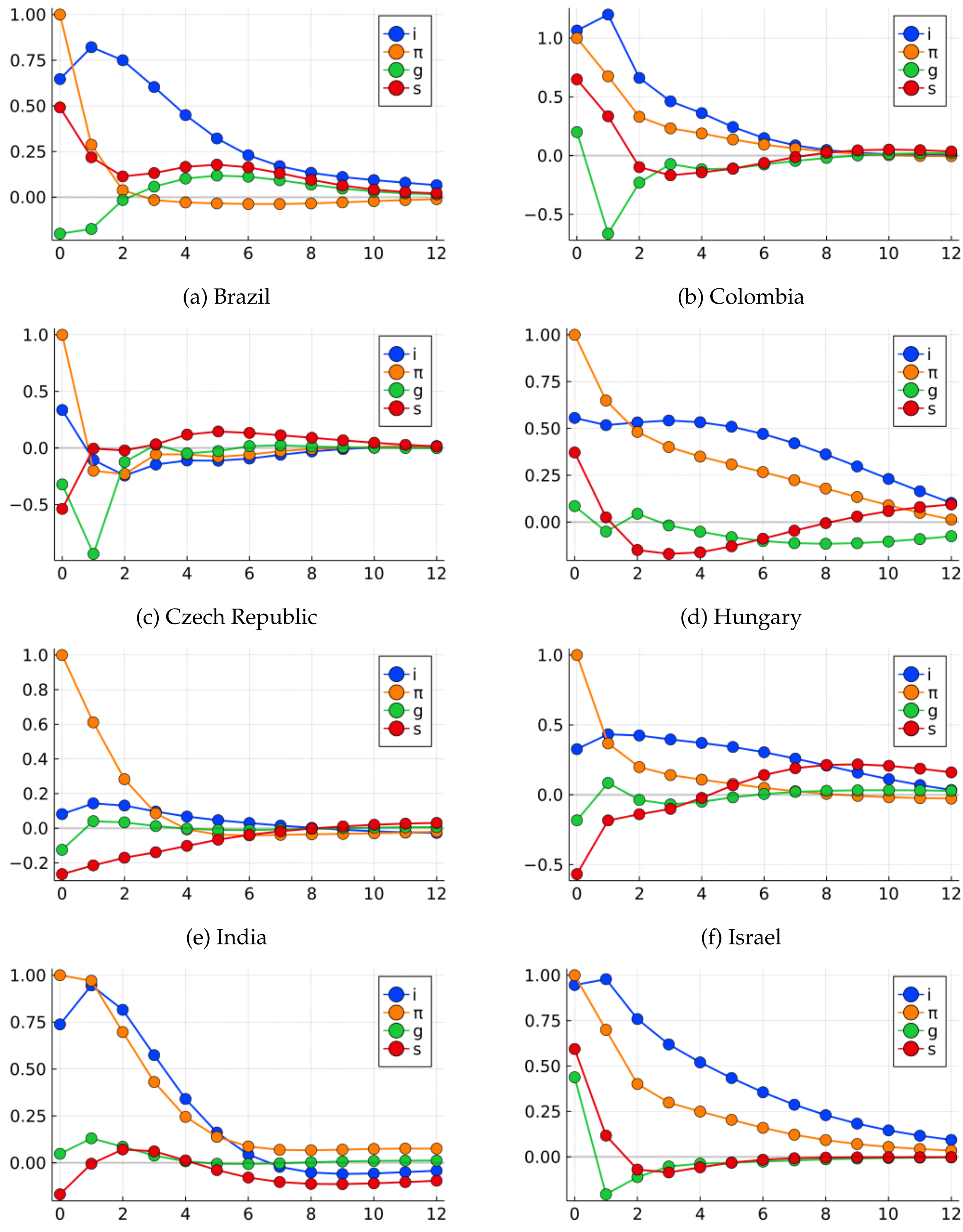
(b) United Kingdom



(c) United States

Notes: see notes to figure 8.

Figure 9: Impulse Response Function - Inflation Shock (Advanced Economies, Continued)



(a) Brazil (b) Colombia

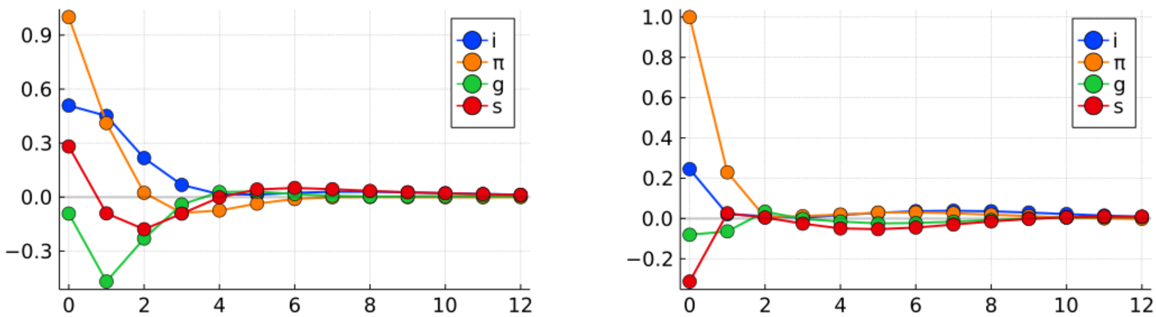
(c) Czech Republic (d) Hungary

(e) India (f) Israel

(g) Mexico (h) Poland

Notes: Each figure plots the impulse response function to the inflation shock, calculated using posterior mode parameters.

Figure 10: Impulse Response Function - Inflation Shock (Emerging Economies)



(a) South Africa

(b) Ukraine

Notes: see notes to figure 10.

Figure 11: Impulse Response Function - Inflation Shock (Emerging Economies, Continued)