A Fiscal Decomposition of Unexpected Inflation: Cross-Country Estimates and Theory

Livio C. Maya

November 2022

Introduction: The Fiscal Sources of Unexpected Inflation

- The Valuation Equation of Public Debt

$$\frac{\text{Market Value of Debt } (\textbf{Bond Prices})}{\textbf{Price Level}} = \text{Intrinsic Value } (\textbf{Discounting}, \textbf{Surpluses}).$$

- Unexpected inflation $\Delta E_t \Pi_t$ must accompany news about:
  - Bond prices Q_t
  - Real surpluses $\{s_{t+k}\}$
  - Real discounting $\{R_{t+k}\}$

$$\Delta E_t \Pi_t = \Delta E_t \left[Q_t - \{s_{t+k}\} + \{R_{t+k}\} \right]$$

- If unexpected inflation exists, it must nowcast/forecast some of these variables
- What are these forecasts? How do they change across countries? How can theory explain them?

- **This paper.**
  1. Estimate a Bayesian-VAR to measure the decomposition for 25 countries
     - Variance decomposition: $\text{var}\left[\Delta E\, \Pi\right] = \text{cov}\left[\Delta E\, \Pi, \quad Q + \{-s\} + \{R\}\right]$
     - ”Aggregate demand” shock: recession + low inflation
  2. Estimate a New-Keynesian model to reproduce B-VAR decompositions
- **Motivation.** How do you read Debt/Price = Discounted Surpluses?
  - Active fiscal: *”How does inflation react to changes in discounted surpluses?”*
    - Surpluses x Inflation in a given economy
    - Surpluses x Inflation across countries
    - Role of monetary policy?
    - Theory: which shocks cause inflation surprises?
  - Active monetary: *”How should discounted surpluses adjust to unexpected inflation?”*

Introduction: Exercises, Motivation, Results

- **This paper.**
  1. Estimate a Bayesian-VAR to measure the decomposition for 25 countries
     - Variance decomposition: $\text{var}[\Delta E\,\Pi] = \text{cov}[\Delta E\,\Pi, \quad Q + \{-s\} + \{R\}]$
     - "Aggregate demand" shock: recession + low inflation
  2. Estimate a New-Keynesian model to reproduce B-VAR decompositions
- **Motivation.** How do you read $\text{Debt}/\text{Price} = \text{Discounted Surpluses}$?
  - Active fiscal: *"How does inflation react to changes in discounted surpluses?"*
    - Surpluses x Inflation in a given economy
    - Surpluses x Inflation across countries
    - Role of monetary policy?
    - Theory: which shocks cause inflation surprises?
  - Active monetary: *"How should discounted surpluses adjust to unexpected inflation?"*

Introduction: Preview of Key Results

- The variance of unexpected inflation is accounted for by discounted surpluses (all coutries)

$$\underbrace{\text{var}\left[\Delta E\pi\right]}_{>0} = \underbrace{\text{cov}\left[\Delta E\pi, \quad Q\right]}_{<0} + \underbrace{\text{cov}\left[\Delta E\pi, \quad \{-s\} + \{R\}\right]}_{>0}$$

  - Most relevant terms (Surplus-to-GDP, GDP growth and Real discounting) vary
  - Monetary policy trades current for future unexpected inflation
- Lower inflation in recessions: low discounting + higher subsequent surpluses
- Productivity shocks reproduce findings in NK model
  - Policy shocks do not
- Different policy rules explain differences in variance decomposition across countries

Introduction: Related Literature

- **Fiscal Theory of the Price Level.** Cochrane (2022a) and Cochrane (2022b).
    - Analysis of multiple countries + more general debt instruments
    - NK model estimated to reproduce decompositions + Productivity shocks

  Leeper (1991), Sims (1994), Cochrane (1998), Cochrane (2005), Sims (2011), Leeper and Leith (2016), Bassetto and Cui (2018), Cochrane (2022c), Brunnermeier et al. (2022).

- **Monetary-Fiscal Interaction.**
  Cagan (1956), Sargent and Wallace (1981), Hall and Sargent (1997), Hall and Sargent (2011), Jiang et al. (2019), Corsetti et al. (2019), Sunder-Plassmann (2020), Du et al. (2020), Akhmadieva (2022)

- **Empirical Finance** (Decomposition of Returns)
  Campbell and Shiller (1988), Cochrane (1992), Campbell and Ammer (1993), Chen and Zhao (2009).

Introduction: A Map of the Road

1. **Fiscal Decomposition Derivation**
   - Simple environment + General decomposition
2. **Bayesian-VAR**
   - Empirical model + Variance decomposition + ”Aggregate demand” recession
3. **Theory**
   - Closed economy + Productivity shocks + Policy rules + Open economy

Fiscal Decomposition: The Valuation Equation

- Environment with discrete time + single good (price P_t) + households and government
- One-period nominal public bonds (price Q_t)
- **In the morning**, the government:
  - redeems bonds B_{t-1} for currency
  - announces real taxes s_t, payable in currency
  - announces sale of B_t new bonds, payable in currency
- **In the afternoon**, households trade goods, purchase bonds, pay taxes
- Let's count currency:

$$\Delta M_t = B_{t-1} - P_t s_t - Q_t B_t$$

- Assume money has no form of value: $M = 0$

$$\boxed{B_{t-1} = P_t s_t + Q_t B_t}$$

Fiscal Decomposition: The Valuation Equation

$$B_{t-1} = P_t s_t + Q_t B_t$$

- *Ex-post* real discounting $\beta_t = Q_t(P_{t+1}/P_t)$ $\beta_{t,t+k} = \prod_{\tau=t}^{t+k} \beta_\tau$
- Iterate law of motion forward:

$$\frac{B_{t-1}}{P_t} = \sum_{k=0}^{p} \beta_{t,t+k-1} s_{t+k} + \beta_{t,t+p} \frac{B_{t+p}}{P_{t+p+1}}$$

- **Key Assumption**: $\lim_{\tau\to\infty} \beta_{t,\tau} \frac{B_\tau}{P_{\tau+1}} = 0$ almost surely (No bubbles)
  - In Macro models: transversality conditions + no Ponzi
- The valuation equation of public debt

$$\frac{B_{t-1}}{P_t} = \sum_{k=0}^{\infty} E_t \left[\beta_{t,t+k-1} s_{t+k}\right]$$

Fiscal Decomposition: In the Simplest Environment

- Nominal rate $i_t = Q_t^{-1} - 1$ and real interest $r_t = i_t - E_t \pi_{t+1}$
- End-of-period real debt v_t

$$\underbrace{\frac{1}{\beta} v_{t-1} + \frac{v}{\beta}\left(i_{t-1} - \pi_t\right)}_{B_{t-1}/P_t} = v_t + s_t = \sum_{k=0}^{\infty} \beta^k E_t s_{t+k} - \frac{v}{\beta} \sum_{k=1} \beta^k E_t r_{t+k}$$

Linearized valuation equation (all variable in deviations from their mean)

- Innovations ΔE_t decomposition:

$$\boxed{\Delta E_t \pi_t = -\frac{\beta}{v} \sum_{k=0}^{\infty} \beta^k \Delta E_t s_{t+k} + \sum_{k=0}^{\infty} \beta^k \Delta E_t r_{t+k}}$$

- Variance decomposition:

$$\mathrm{var}\left[\Delta E_t \pi_t\right] = -\mathrm{cov}_\pi \left[\frac{\beta}{v} \sum_{k=0}^{\infty} \beta^k \Delta E_t s_{t+k}\right] + \mathrm{cov}_\pi \left[\sum_{k=0}^{\infty} \beta^k \Delta E_t r_{t+k}\right]$$

Fiscal Decomposition: In the Simplest Environment

- Nominal rate $i_t = Q_t^{-1} - 1$ and real interest $r_t = i_t - E_t \pi_{t+1}$
- End-of-period real debt v_t

$$\underbrace{\frac{1}{\beta} v_{t-1} + \frac{v}{\beta}\left(i_{t-1} - \pi_t\right)}_{B_{t-1}/P_t} = v_t + s_t = \sum_{k=0}^{\infty} \beta^k E_t s_{t+k} - \frac{v}{\beta} \sum_{k=1} \beta^k E_t r_{t+k}$$

Linearized valuation equation (all variable in deviations from their mean)

- Innovations ΔE_t decomposition:

$$\boxed{\Delta E_t \pi_t \; = \; -\frac{\beta}{v} \sum_{k=0}^{\infty} \beta^k \Delta E_t s_{t+k} \; + \sum_{k=0}^{\infty} \beta^k \Delta E_t r_{t+k}}$$

- Variance decomposition:

$$\operatorname{var}\left[\Delta E_t \pi_t\right] \; = \; -\operatorname{cov}_\pi \left[\frac{\beta}{v} \sum_{k=0}^{\infty} \beta^k \Delta E_t s_{t+k}\right] \; + \operatorname{cov}_\pi \left[\sum_{k=0}^{\infty} \beta^k \Delta E_t r_{t+k}\right]$$

Fiscal Decomposition: In the Simplest Environment

- Nominal rate $i_t = Q_t^{-1} - 1$ and real interest $r_t = i_t - E_t \pi_{t+1}$
- End-of-period real debt v_t

$$\underbrace{\frac{1}{\beta} v_{t-1} + \frac{v}{\beta}\left(i_{t-1} - \pi_t\right)}_{B_{t-1}/P_t} = v_t + s_t = \sum_{k=0}^{\infty} \beta^k E_t s_{t+k} - \frac{v}{\beta} \sum_{k=1} \beta^k E_t r_{t+k}$$

Linearized valuation equation (all variable in deviations from their mean)

- Innovations ΔE_t decomposition:

$$\boxed{\Delta E_t \pi_t \; = \; -\frac{\beta}{v} \sum_{k=0}^{\infty} \beta^k \Delta E_t s_{t+k} \; + \sum_{k=0}^{\infty} \beta^k \Delta E_t r_{t+k}}$$

- Variance decomposition:

$$\operatorname{var}\left[\Delta E_t \pi_t\right] \; = \; -\operatorname{cov}_\pi \left[\frac{\beta}{v} \sum_{k=0}^{\infty} \beta^k \Delta E_t s_{t+k}\right] \; + \operatorname{cov}_\pi \left[\sum_{k=0}^{\infty} \beta^k \Delta E_t r_{t+k}\right]$$

Fiscal Decomposition: Currency and Term Structures + Growth

- Real **market value** debt-to-GDP v_t, surplus-to-GDP s_t and GDP growth g_t (stationarity!)
- Bonds (j, n) promisses one unit of currency j after n periods Currencies
  - Nominal bonds
  - Real bonds (currency denomination = final goods)
  - US Dollar bonds

  Constant structure $\{\delta_j\}$, $\{\omega_j^n\}$

- Bond price $Q_{j,t}^n$, excess return $rx_{j,t}$ $\quad 1 + \text{return}_{j,t} = 1 + rx_{j,t} + i_{j,t-1} = \frac{\sum_n Q_{j,t}^{n-1} B_{j,t-1}^n}{\sum_n Q_{j,t-1}^n B_{j,t-1}^n}$

  (one-period bonds $\implies$ $rx = 0$)
- Debt law of motion:

$$\frac{v_{t-1}}{\beta} + \frac{v}{\beta}\left[-g_t + \sum_j \delta_j \left(rx_{j,t} + i_{j,t-1} + \Delta h_{j,t} - \pi_{j,t}\right)\right] = v_t + s_t$$

Fiscal Decomposition of Unexpected Inflation

- *Ex-post* real return $r_{j,t} = rx_{j,t} + i_{j,t-1} + \Delta\pi_{j,t} - \pi_{j,t}$

$$\Delta E_t\pi_t = \underbrace{\left[\Delta E_t rx_t + \sum_{j\neq N}\frac{\delta_j}{\delta}\Delta E_t r_{j,t}\right]}_{\text{Innovation to Bond Prices}} - \frac{\beta}{\delta v}\underbrace{\left[\sum_{k=0}^{\infty}\beta^k\Delta E_t s_{t+k} + \frac{v}{\beta}\sum_{k=0}^{\infty}\beta^k\Delta E_t g_{t+k} - \frac{v}{\beta}\sum_{k=1}^{\infty}\sum_{j}\delta_j\beta^k\Delta E_t r_{j,t+k}\right]}_{\text{Innovation to the Intrinsic Value of Debt}}$$

$$\equiv d_1(rx) + d_1(r_0) - d_1(s) - d_1(g) + d_1(r)$$

- Variance decomposition.

$$\text{var}\left[\Delta E_t\pi_t\right] = \text{cov}_\pi\left[d_1(rx)\right] + \text{cov}_\pi\left[d_1(r_0)\right] - \text{cov}_\pi\left[d_1(s)\right] - \text{cov}_\pi\left[d_1(g)\right] + \text{cov}_\pi\left[d_1(r)\right]$$

Bayesian-VAR: Data and Model

- Annual data on observables $\tilde{x}_t$

$$\tilde{x}_t = \begin{bmatrix} i_t & \text{(Nominal Interest)} \\ \pi_t & \text{(CPI Inflation)} \\ v_t^b & \text{(Par-Value Debt-to-GDP)} \\ g_t & \text{(GDP growth)} \\ \Delta h_t & \text{(Chg. Real Exchange Rate)} \end{bmatrix}$$

- 25 countries (samples starting at 1945, 1960, 1975, 1998)
- General VAR

$$X_t = AX_{t-1} + Ke_t$$

- Decompose $X'_t = [x'_t \; u'_t]$

$$x_t = a\, x_{t-1} + b\, u_{t-1} + k\, \varepsilon_t$$
$$u_t = a_u\, u_{t-1} + k_u\, \varepsilon_{u,t}$$

Bayesian-VAR: Empirical Challenges and Solutions

1. Explosive debt dynamics in 1960-2020 implies unstable OLS VARs

$$\tilde{x}_t = \tilde{a}\,\tilde{x}_{t-1} + \tilde{b}\,\tilde{u}_{t-1} + \varepsilon_t$$
$$\tilde{u}_t = \tilde{a}_u\,\tilde{u}_{t-1} + \varepsilon_{u,t}$$

   - Estimate US model ($\tilde{a}_u$) by OLS (stable!)
   - Estimate ($\tilde{a}$, $\tilde{b}$) with a Bayesian-Regression

$$\tilde{a}^{BAY} = (X'X + \lambda^{-1})^{-1}(X'X\,\tilde{a}^{OLS} + \lambda^{-1}\,\tilde{a}^{PRIOR})$$

   λ maximizes the marginal distribution $p(\text{data})$ and ensures stability

2. No data on bond returns Geometric Term Structure
   - Geometric maturity structure: $rx_{j,t} + i_{j,t-1} = (\omega_j \beta) q_{j,t} - q_{j,t-1}$
3. No data on the market value of debt, only its par value (v_t^b)
   - Model for market *vs* par value (Cox (1985)): $v_t = v_t^b + \frac{v}{\beta}\sum_j \delta_j \left(rx_{j,t} + i_{j,t-1} - i^b_{j,t-1} \right)$
4. Public finance data do not respect law of motion of public debt
   - Define surplus from the law of motion: $s_t = \frac{v_{t-1}}{\beta} - v_t + \ldots$

Bayesian-VAR: Empirical Challenges and Solutions

1. Explosive debt dynamics in 1960-2020 implies unstable OLS VARs

$$\tilde{x}_t = \tilde{a}\,\tilde{x}_{t-1} + \tilde{b}\,\tilde{u}_{t-1} + \varepsilon_t$$
$$\tilde{u}_t = \tilde{a}_u\,\tilde{u}_{t-1} + \varepsilon_{u,t}$$

   - Estimate US model ($\tilde{a}_u$) by OLS (stable!)
   - Estimate ($\tilde{a}, \tilde{b}$) with a Bayesian-Regression

$$\tilde{a}^{BAY} = (X'X + \lambda^{-1})^{-1}(X'X\,\tilde{a}^{OLS} + \lambda^{-1}\,\tilde{a}^{PRIOR})$$

   λ maximizes the marginal distribution $p(\text{data})$ and ensures stability

2. No data on bond returns Geometric Term Structure
   - Geometric maturity structure: $rx_{j,t} + i_{j,t-1} = (\omega_j\beta)q_{j,t} - q_{j,t-1}$

3. No data on the market value of debt, only its par value (v_t^b)
   - Model for market *vs* par value (Cox (1985)): $v_t = v_t^b + \frac{v}{\beta}\sum_j \delta_j \left(rx_{j,t} + i_{j,t-1} - i^b_{j,t-1}\right)$

4. Public finance data do not respect law of motion of public debt
   - Define surplus from the law of motion: $s_t = \frac{v_{t-1}}{\beta} - v_t + \ldots$

Bayesian-VAR: Empirical Challenges and Solutions

1. Explosive debt dynamics in 1960-2020 implies unstable OLS VARs

$$\tilde{x}_t = \tilde{a}\,\tilde{x}_{t-1} + \tilde{b}\,\tilde{u}_{t-1} + \varepsilon_t$$
$$\tilde{u}_t = \tilde{a}_u\,\tilde{u}_{t-1} + \varepsilon_{u,t}$$

   - Estimate US model ($\tilde{a}_u$) by OLS (stable!)
   - Estimate ($\tilde{a}$, $\tilde{b}$) with a Bayesian-Regression

$$\tilde{a}^{BAY} = (X'X + \lambda^{-1})^{-1}(X'X\,\tilde{a}^{OLS} + \lambda^{-1}\,\tilde{a}^{PRIOR})$$

   λ maximizes the marginal distribution $p(\text{data})$ and ensures stability

2. No data on bond returns Geometric Term Structure
   - Geometric maturity structure: $rx_{j,t} + i_{j,t-1} = (\omega_j \beta) q_{j,t} - q_{j,t-1}$
3. No data on the market value of debt, only its par value (v_t^b)
   - Model for market *vs* par value (Cox (1985)): $v_t = v_t^b + \frac{v}{\beta}\sum_j \delta_j \left(rx_{j,t} + i_{j,t-1} - i^b_{j,t-1} \right)$
4. Public finance data do not respect law of motion of public debt
   - Define surplus from the law of motion: $s_t = \frac{v_{t-1}}{\beta} - v_t + \dots$

Bayesian-VAR: Empirical Challenges and Solutions

1. Explosive debt dynamics in 1960-2020 implies unstable OLS VARs

$$\tilde{x}_t = \tilde{a}\,\tilde{x}_{t-1} + \tilde{b}\,\tilde{u}_{t-1} + \varepsilon_t$$
$$\tilde{u}_t = \tilde{a}_u\,\tilde{u}_{t-1} + \varepsilon_{u,t}$$

   - Estimate US model ($\tilde{a}_u$) by OLS (stable!)
   - Estimate ($\tilde{a}$, $\tilde{b}$) with a Bayesian-Regression

$$\tilde{a}^{BAY} = (X'X + \lambda^{-1})^{-1}(X'X\,\tilde{a}^{OLS} + \lambda^{-1}\,\tilde{a}^{PRIOR})$$

   λ maximizes the marginal distribution $p(\text{data})$ and ensures stability

2. No data on bond returns Geometric Term Structure
   - Geometric maturity structure: $rx_{j,t} + i_{j,t-1} = (\omega_j \beta) q_{j,t} - q_{j,t-1}$
3. No data on the market value of debt, only its par value (v_t^b)
   - Model for market *vs* par value (Cox (1985)): $v_t = v_t^b + \frac{v}{\beta} \sum_j \delta_j \left(rx_{j,t} + i_{j,t-1} - i^b_{j,t-1} \right)$
4. Public finance data do not respect law of motion of public debt
   - Define surplus from the law of motion: $s_t = \frac{v_{t-1}}{\beta} - v_t + \ldots$

Frame title

References

Akhmadieva, V. (2022). Fiscal adjustment in a panel of countries 1870–2016. *Journal of Comparative Economics*, 50(2):555–568.

Bassetto, M. and Cui, W. (2018). The fiscal theory of the price level in a world of low interest rates. *Journal of Economic Dynamics and Control*, 89:5–22.

Brunnermeier, M. K., Merkel, S., and Sannikov, Y. (2022). The Fiscal Theory of the Price Level With a Bubble.

Cagan, P. (1956). The Monetary Dynamics of Hyperinflation. In *Studies in the Quantity Theory of Money*, pages 25–117. University of Chicago Press, milton friedman edition.

Campbell, J. Y. and Ammer, J. (1993). What Moves the Stock and Bond Markets? A Variance Decomposition for Long-Term Asset Returns. *The Journal of Finance*, 48(1):3–37.

Campbell, J. Y. and Shiller, R. J. (1988). The Dividend-Price Ratio and Expectations of Future Dividends and Discount Factors. *The Review of Financial Studies*, 1(3):195–228.

Chen, L. and Zhao, X. (2009). Return Decomposition. *Review of Financial Studies*, 22(12):5213–5249.

Cochrane, J. H. (1992). Explaining the Variance of Price–Dividend Ratios. *The Review of Financial Studies*, 5(2):243–280.

Cochrane, J. H. (1998). A Frictionless View of U.S. Inflation. *NBER Macroeconomics Annual*, 13:323–384.

Cochrane, J. H. (2005). Money as stock. *Journal of Monetary Economics*, 52(3):501–528.

Cochrane, J. H. (2022a). The fiscal roots of inflation. *Review of Economic Dynamics*, 45:22–40.

Cochrane, J. H. (2022b). A fiscal theory of monetary policy with partially-repaid long-term debt. *Review of Economic Dynamics*, 45:1–21.

Cochrane, J. H. (2022c). The Fiscal Theory of the Price Level.

Appendix: Public Debt Currency Denomination

Return

Symbol	Description	Nominal Debt	Real Debt	Dollar Debt
j	Index Symbol	N	R	D
	Notation	δ, ω	δ_R, ω_R	δ_D, ω_D
P_j	Price per Good	P	1	P_t^{US}
$\mathcal{E}_j$	Nominal Exchange Rate	1	P	Dollar NER
H_j	Real Exchange Rate	1	1	Dollar RER
π_j	Log Variation in Price	π	0	π_t^{US}
Δh_j	Log Real Depreciation	0	0	Δh_t

Table: Public Debt Denomination

Appendix: Geometric Term Structure

Return

- To each currency portfolio j, fixed geometric maturity structure:

$$B_{j,t}^{n} = \omega_j B_{j,t}^{n-1}$$

- Total return on currency-j portfolio:

$$1 + rx_{j,t} + i_{j,t-1} = \frac{1 + \omega_j Q_{j,t}}{Q_{j,t-1}} \quad \Longrightarrow \quad \boxed{rx_{j,t} + i_{j,t-1} = (\omega_j \beta) q_{j,t} - q_{j,t-1}}$$

- Assume **constant risk premia** $E_t rx_{j,t+1} = 0$

$$\boxed{q_{j,t} = (\omega_j \beta) E_t q_{j,t+1} - i_{j,t}} = -\sum_{k=0}^{\infty} (\omega_j \beta)^k E_t i_{j,t+k}$$