Title: Something with Unexpected Inflation

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1 Introduction

Under fairly weak assumptions, a variance decomposition of unexpected inflation shocks holds. The decomposition implies that a, say, positive unexpected inflation must follow from either *higher* future fiscal deficits, discounting or real exchange rates, or *lower* future inflation, growth rates or international inflation. The decomposition follows from the fact that, in equilibrium, the real market value of public equals its intrinsic value: discounted surpluses.

I estimate a Bayesian-VAR for a set of twenty-five advanced and developing countries and use the estimated models to calculate the decomposition for each of them. This paper extends Cochrane (2022), who estimates the decomposition for the American economy, in two ways: first, the number of countries in the sample; second, the generality of financing instruments available to the government. To better fit the case of some countries - developing ones mainly - I allow the government to issue inflation-linked and dollar-linked debt. These forms of financing introduce new terms and coefficients to the decomposition (Incomplete)

The analysis of multiple countries is challenging. Each country is a different case, and available data is often considerably more limited than for the United States. For the vast majority of countries, market-price measures of public debt - the theoretically relevant concept - and real and dollar public bond price data are not available in a sufficiently large time span. I present an empirical model that partially circumnvents these limitations and allow the estimation of the decomposition. The method requires five readily available macroeconomic time series: output growth, short-term interest, the inflation rate, the real exchange rate and the par-value of public debt.

Cochrane concludes that discount rate variation accounts about half the variance of unexpected inflation.

Why Bayesian, and not just OLS? First, parameter shrinkage reduces the volatility of estimated coefficients, a valuable property when samples are relatively small. Second, public debt processes are highly persistent.[1] In the period I analyze, it increased significantly in the case of many countries. By properly tuning the parameters of the prior distribution, or hyperparameters, we can ensure stability at the same time we discipline parameter search by a goodness-of-fit criterion.

[1] See Bohn (1998), Uctum et al. (2006) and Yoon (2012).

2 Unexpected Inflation Decomposition

2.1 Connection with the Value of Debt

I start by establishing the connection between inflation and the value of debt. Consider an economy with a homogeneous final good which households value. The economy has a government that manages a debt composed by bonds that pay one unit of currency in the next period. Currency is a commodity that only the government can produce, at zero cost. There is nothing special about currency. Households do not value it, and it provides no convenience for trading. Households cannot burn currency.

At the beginning of period t, the face value of debt issued in the previous period is V_{t-1}. The government redeems it for currency, which moves to the hands of households. After redeeming bonds, the government announces that each household i must pay $T_{i,t}$ goods in taxes, payable in currency. It also announces it will sell V_t new bonds and purchase G_t units of the final good at market prices.[2]

Nothing binds the government's choices of $T_{i,t}$ and V_t. Note the difference from the case with bonds payable in goods, in which the government *must* raise tax revenue or issue new debt to redeem old bonds. Additionally, if households accept currency as payment for final goods, nothing binds the government's choice of G_t either.

Let M_t be private holdings of currency at the end of t. As there is no free disposal of currency, the quantity used by the government to redeem $t-1$ bonds and purchase goods is used to either pay taxes, buy new bonds or increase currency holdings:

$$\begin{aligned} & V_{t-1} + G_t = P_t T_t + Q_t V_t + \Delta M_t \\ \implies & V_{t-1} = P_t s_t + Q_t V_t + \Delta M_t \end{aligned} \tag{1}$$

where T_t are aggregate taxes, $s_t = T_t - G_t$ is the primary surplus, P_t is the final good's price and Q_t is the price of new bonds (I state prices in currency units). Equation (1) provides a low of motion for public debt. It holds for all prices and all choices of money holdings, new debt sales, and primary surplus.[3]

If $P_t = 0$, real public debt V_{t-1}/P_t equals infinity. For now, that is a possiblity.

Suppose $P_t > 0$. Define $\beta_t \equiv Q_t P_{t+1}/P_t$ as the real discount for public bonds, and $\beta_{t,t+k} \equiv \prod_{\tau=t}^{t+k} \beta_\tau$. Since V satisfies (1), it also satisfies

$$\frac{V_{t-1}}{P_t} = \sum_{i=0}^{k} \beta_{t,t+i-1} \left(s_{t+i} + \Delta M_t \right) + \beta_{t,t+k} \frac{V_{t+k}}{P_{t+k+1}} \qquad \text{for any } k \geq 0 \tag{2}$$

regardless of prices and choices. Expressions (1) and (2) do not represent a constraint on the path of surpluses $\{s_t\}$ and bond sales $\{V_t\}$ the government chooses to follow. They merely express future debt given paths for quantities and prices.

[2]That the government forces households to pay for taxes and new bonds in currency is not necessary for the argument. All else follows if the government accepts payment in goods but stands ready to exchange these goods for currency at market prices.

[3]Strictly speaking, (1) holds for all *feasible* choices of money holdings, that is, choices that respect households' budget constraint. Otherwise, one could point out that, for $B_{t-1} > 0$, $M_t = M_{t-1}$ and $s_t = B_t = 0$ violates (1). That would nevertheless involve households burning up currency.

So far, all we did was public finances accounting. I now move to economic behavior. If $P_t = 0$, households demand infinite final goods and there is no equilibrium. Therefore $P_t > 0$.

Given a utility function over consumption paths $U(\{c_t\})$, the optimal consumption-savings choice involves two conditions. First: $\beta_{t,t+k}$ = marginal rate of substitution between time-t and time-$t+k$ consumption. Second, the transversality condition $\lim_{k\to\infty} \beta_{t,t+k} V_{t+k}/P_{t+k+1} \leq 0$. Coupled with a no-Ponzi condition that establishes the reverse inequality, it implies

$$\lim_{k\to\infty} \beta_{t,t+k} \frac{V_{t+k}}{P_{t+k+1}} = 0 \text{ at every period } t. \tag{3}$$

If bonds were redeemable in goods, (3) would represent a debt sustainability condition, as it forces the government to eventually raise the resources to pay for past borrowing.

With nominal debt, however, the government has no constraints on its choice of debt and surplus, as already pointed out. Replacing (3) on (2) and taking expectation yields

$$\frac{V_{t-1}}{P_t} = \sum_{i=0}^{\infty} E_t \left[\beta_{t,t+i-1} \left(s_{t+i} + \Delta M_{t+i} \right) \right]. \tag{4}$$

Given the face value of maturing public debt, the relative price of the consumption good is given by the expected β-discounted stream of real primary surpluses. This latter term - the right-hand side of (4) - I call the *real value of public debt*. My definition of debt value follows a "beginning-of-period" convention: it includes current period surplus s_t and starts discounting at $t+1$.

In the case of nominal debt, (4) is a *valuation equation*. Hence, expression (3) implies not a no-default condition, but a no-bubble condition: market prices should reflect fundamental value.

Now, define the inflation rate $\Pi_t = P_t/P_{t-1}$, and take innovations on both sides

$$\frac{V_{t-1}}{P_{t-1}} \Delta E_t \Pi_t^{-1} = \sum_{i=0}^{\infty} \Delta E_t \left[\beta_{t,t+i-1} \left(s_{t+i} + \Delta M_{t+i} \right) \right]. \tag{5}$$

Any shock can lead the government to change the conditional distribution of future surpluses (in units of goods) backing the stock of public nominal liabilities. The relative price of bonds in terms of goods $1/P_t$ then adjusts to reflect that change, much in the same way that news of future dividends (in currency units) per stock change the relative price of stocks in terms of currency (Cochrane (2005)). Unexpected inflation $\Delta E_t \Pi_t$ follows. Also like stocks, changes in stochastic discounting β also affect fundamental value, and thus affect prices.

Importantly, (4) and (5) do not depend to equilibrium selection mechanisms. Both hold on all models in which (3) holds, inclusing the standard New-Keynesian model.

2.2 Inflation Decomposition in the Simplest Environment

Start by linearizing the law of motion (2).

$$v_t + s_t = \frac{1}{\beta} \left(v_{t-1} + i_{t-1} - \pi_t \right) \tag{6}$$

where v_t is *end-of-period* stock of real debt, $i_t = -\log(Q_t)$ and $\pi_t = \log(\Pi_t)$. I assume $\Delta M_t = 0$ (households do not hold currency). Note that v and s are both in levels - I assume them to be stationary for simplicity. Moreover, I linearize around the point $v = 1$, which I take to be the average real debt level.

The interpretation of (6) is the same as before. The expression on the right is the linearizing beginning-of-period stock of debt, corresponding to V_{t-1}/P_t in the non-linear formulation. Previous period debt accrues by the mean real interest $(1/\beta)$ plus its local variation $i_t - \pi_t$. A 1% higher real interest raises debt by 1% on average because debt is on average one. The government redeems bonds for currency, and then sells new debt v_t and runs a surplus s_t to soak it up.

Repeating the same steps as before, solve (6) forward:

$$\frac{1}{\beta}\left(v_{t-1} + i_{t-1} - \pi_t\right) = \sum_{k=0}^{\infty} \beta^k E_t s_{t+k} - \frac{1}{\beta} \sum_{k=1} \beta^k E_t r_{t+k}$$

where $r_t = i_{t-1} - \pi_t$ is the *ex-post* real interest rate. The expression on the right-hand side is the linearized real value of debt. It includes time-t surplus, and starts discounting at $t+1$. The inflation rate on the left represents the price level equalizing the beginning-of-period of debt to its real value.

Take innovation, and multiply both sides by β to find

$$\Delta E_t \pi_t = -\beta \sum_{k=0}^{\infty} \beta^k \Delta E_t s_{t+k} + \sum_{k=0}^{\infty} \beta^k \Delta E_t r_{t+k}. \tag{7}$$

In this simple environment, unexpected higher inflation is accounted for by news of larger deficits $-s$ or news of higher discounting r, and vice-versa. That is, news about the real value of debt. This decomposition was introduced by Cochrane (2022), and follows similar decompositions for stock returns and price-dividend ratios (Campbell and Shiller (1988), Campbell and Ammer (1993)).

Now, for each term in the equation, take covariance with unexpected inflation. We arrive at an initial decomposition of inflation variance.

$$\operatorname{var}(\Delta E_t \pi_t) = -\operatorname{cov}\left[\Delta E_t \pi_t, \beta \sum_{k=0}^{\infty} \beta^k \Delta E_t s_{t+k}\right] + \operatorname{cov}\left[\Delta E_t \pi_t, \sum_{k=0}^{\infty} \beta^k \Delta E_t r_{t+k}\right]. \tag{8}$$

In the simplest environment, if unexpected inflation "exists", it must forecast deficits or higher discounting.

2.3 Generalizing Public Financing Instruments

2.3.1 Currency Denomination

The debt process considered so far is too unrealistic to be taken to the data. I generalize the financing instruments available to the government in two dimensions: currency denomination and maturity structure. I do not consider the case of debt with stage-contingent nominal payoffs.[4]

Starting here, I recycle part of the notation already established. Fix the case of a country's government. Let P_t be the price of the consumption basket in terms of domestic currency.

The payoff of public bonds can be indexed to different currencies, enumerated by j. Let $P_{j,t}$

[4] Publicly-ran social security and pension systems can be modelled as cases of public debt with state-contigent payoffs or, in a fashion compatible with my framework, as an additional layer of uncertainty in the primary surplus process.

Symbol	Description	Nominal Debt	Real Debt	Dollar-Linked Debt
j	Index Symbol	N	R	D
	Notation	δ, ω	δ_R, ω_R	δ_D, ω_D
P_j	Price per Good	P	1	P_t^{US}
$\mathcal{E}_j$	Nominal Exchange Rate	1	P	Dollar NER
H_j	Real Exchange Rate	1	1	Dollar RER
π_j	Log Variation in Price	π	0	π_t^{US}
Δh_j	Log Real Depreciation	0	0	Δh_t

Notes: P = price of consumption basket in domestic currency. P^{US} = price of consumption basket in US dollars. NER = Nominal Exchange Rate. RER = Real Exchange Rate.

Table 1: Public Debt Denomination

be the price of the consumer price index in units of currency j. Let $Q_{j,t}^n$ be the discount rate for a zero-coupon public bond paying one unit of currency j after n periods. Without loss of generality, all bonds are zero-coupon. Let $\mathcal{E}_{j,t}$ be the price of currency j in units of domestic currency.

The notation is general enough to accomodate currency-linked bonds *per se*, but also inflation-linked securities such as American TIPS. In this case, one can interpret the "currency" as the final goods basket (so that $P_j = 1$ and $\mathcal{E}_j = P_t$). In the empirical exercises that follow, I consider only nominal bonds ($j = N$), inflation-linked (or real) bonds ($j = R$) and US-dollar- denominated bonds ($j = D$). Table 1 shows the value or interpretation of the variables defined above for these three cases.

I do not assume the price index for government's purchases (denoted P_t^S) and levied aggregates (such as income) to be the same as the price index for households' consumption (P_t). While governments do tend to tax private consumption, they also tax and transfer income, and purchase baskets of goods different than that of households. The fiscal effect of variations in the relative price of government's to households' final goods basket must be accounted for.

With these changes, the law of motion of government debt (1) becomes

$$\sum_j \mathcal{E}_{j,t} B_{j,t-1}^1 = P_t^s S_t + \sum_j \mathcal{E}_{j,t} \sum_{n=2}^{\infty} Q_{j,t}^{n-1} \left(B_{j,t}^{n-1} - B_{j,t-1}^n \right),$$

where S_t is now the real primary surplus and $B_{j,t}^n$ is the face value of bonds issued in currency j, period t, payable n periods in the future. The term on the left represents the cost of debt in period t; the second term on the right represents proceeds from the selling of new bonds.

Let $\mathcal{V}_{j,t} = \sum_n Q_{j,t}^n B_{j,t}^n$ be the end-of-period market value of nominal debt issued in currency j, $i_{j,t}$ the risk-free rate in bonds issued in currency j and $rx_{j,t} = \mathcal{V}_{j,t-1}^{-1} \sum_{n=1}^{\infty} Q_{j,t}^{n-1} B_{j,t-1}^n - i_{j,t-1}$ the realized excess return on portfolios that mimic the composition of j-currency debt. We can re-write the law of motion in terms of the $\mathcal{V}_j$ and its corresponding returns:

$$\sum_j (1 + rx_{j,t} + i_{j,t-1}) \mathcal{E}_{j,t} \mathcal{V}_{j,t-1} = P_t^s S_t + \sum_j \mathcal{E}_{j,t} \mathcal{V}_{j,t}$$

For empirical work, it is ideal to write the law of motion in terms of real and stationary variables.

So, let Y_t be real GDP and let $g_t = Y_t/Y_{t-1} - 1$ be its growth rate. Define the real "exchange rate" $H_{j,t} = \mathcal{E}_{j,t}P_{j,t}/P_t$ and its growth rate $\Delta h_{j,t} = H_{j,t}/H_{j,t-1} - 1$. Define the detrended real value of j-indexed debt $V_{j,t} = \mathcal{V}_{j,t}/P_{j,t}Y_t$, the real value of total debt $V_t = \sum_j H_{j,t}V_{j,t}$ and the j-indexed share $\delta_{j,t} = H_{j,t}V_{j,t}/V_t$.

By properly dividing the whole above equation by P_tY_t, and multiplying and dividing the j sum on the left by $P_{j,t-1}$, $P_{j,t}$, Y_{t-1} and $H_{j,t-1}$, we arrive at a final version for the law of motion of public debt:

$$V_{t-1}\sum_j \frac{(1 + rx_{j,t} + i_{j,t-1})(1 + \Delta h_{j,t})}{(1 + \pi_{j,t})(1 + g_t)}\delta_{j,t-1} = \frac{P^s_t}{P_t}s_t + V_t.$$

The law of motion above generalizes (2) for $k = 1$. During period t, the government must "pay" V_{t-1} plus realized returns and eventual changes to the relative value of currency j.[5]

I linearize the debt law of motion. Let $\beta_j = \exp\{-(rx_j + i_j - \pi_j - g)\}$ be the steady-state real and growth-adjusted discounting for public debt issued in currency j. I linearize around a steady-state - assumed to exist - with $\beta_j = \beta$ for all j and $P^s = P$. This leads to

$$v_t + s_t + s(p^s_t - p_t) = \frac{v_{t-1}}{\beta} + \frac{v}{\beta}\left[\sum_j \delta_j\left(rx_{j,t} + i_{j,t-1} + \Delta h_{j,t} - \pi_{j,t}\right) - g_t\right], \tag{9}$$

which generalizes (6). Parameter v is the steady-state level of public debt. The right-hand side is the beginning-of-period stock of debt.

Let $r_{j,t} + i_{j,t-1} + \Delta h_{j,t} - \pi_{j,t}$ be the *ex-post* real return on holdings of the j-currency portfolio of public bonds, and define $s^p_t = s_t + s(p^s_t - p_t)$ as the price-adjusted surplus process.

Decomposition 1. *Solve the debt law of motion* (9) *forward and take innovations to arrive at*

$$\frac{v}{\beta}\left[\sum_{j\neq N}\delta_j\Delta E_t r_{j,t} + \delta\Big(\Delta E_t rx_t - \Delta E_t\pi_t\Big)\right] = \sum_{k=0}^{\infty}\beta^k\Delta E_t s^p_{t+k} + \frac{v}{\beta}\sum_{k=1}^{\infty}\beta^k\Delta E_t g_{t+k}. - \frac{v}{\beta}\sum_{k=1}^{\infty}\sum_j \delta_j\beta^k\Delta E_t r_{j,t+k}. \tag{10}$$

Take covariance with unexpected inflation, and divide both sides by $\delta(v/\beta)$.

$$\begin{aligned}
var(\Delta E_t\pi_t) = {} & cov\left[\Delta E_t\pi_t, \delta^{-1}\sum_{j\neq N}\delta_j\Delta E_t r_{j,t}\right] + cov\left(\Delta E_t\pi_t, \Delta E_t rx_t\right) \\
& - cov\left[\Delta E_t\pi_t, \left(\frac{\delta v}{\beta}\right)^{-1}\sum_{k=0}^{\infty}\beta^k\Delta E_t s_{t+k}\right] - cov\left[\Delta E_t\pi_t, \delta^{-1}\sum_{k=0}^{\infty}\beta^k\Delta E_t g_{t+k}\right] \\
& + cov\left[\Delta E_t\pi_t, \delta^{-1}\sum_{k=1}^{\infty}\sum_j \delta_j\beta^k\Delta E_t r_{j,t+k}\right].
\end{aligned} \tag{11}$$

Equation (10) generalizes (7). The right-hand still contains the revision of the value of debt. The left-hand side reveals the new terms Δr_t and Δrx_t containing time-t unexpected jumps in public bond prices, absent in the one-period debt context. Now, *given unexpected variation in the price of long-term debt*, unexpected inflation must be accounted for by news of surpluses or real

[5] "Pay" comes in paranthesis here because, unlike in (1), the government does not actually redeem the entire term on the left at period t. It only pays for bonds maturing at t.

discounting. Expression (11) converts the innovations decomposition into a decomposition of unexpected inflation.

Compared to the $\delta = 1$ case with nominal debt only, the decomposition contains time-t price-adjustment terms (on the left) and future discounting terms (on the right) related to currency-linked real returns. For countries with dollar-linked debt $\delta_D > 0$, unexpected real exchange depreciation raises the home-currency value of debt and thus acts like an inflationary force (that is the $\Delta E r_{D,t}$ term on the left). News of *future* real exchange depreciation also stimulate inflation by increasing real discounting ($\Delta r_{D,t+k}$ term on the right).

The lower the share of nominal bonds on the stock of δ, the more the price levels must change to deflate total debt and account for innovations on the real value of debt. Expression (11) shows that, all else the same lower δ leads to more volatile unexpected inflation.

2.3.2 Geometric Term Structure

I assume the government keeps a geometric maturity structure of its debt. The geometric term structure implies a tractable relationship between short-term interest rate $i_{j,t}$ and the excess returns that I explore to substitute the hard-to-interpret price adjustment terms of decomposition (10).

The term structure is constant over time, but can vary across the different currency portfolios $\{j\}$ of public debt. Specifically, for the slice of public debt linked to currency j, suppose the outstanding volume of bonds decays at a rate ω_j, so that $B^n_{j,t} = \omega_j B^{n-1}_{j,t}$. Define $Q_{j,t} = \sum_{n=1}^{\infty} Q^n_{j,t}\omega_j^{n-1}$ as the weighted-average price of currency j public bonds. Then, $\mathcal{V}_{j,t} = Q_{j,t}B^1_{j,t}$. The total return on currency-j bonds then is $1 + rx_{j,t} + i_{j,t-1} = (1 + \omega_j Q_{j,t})/Q_{j,t-1}$, which I linearize as

$$rx_{j,t} + i_{j,t-1} = \omega_j \beta q_{j,t} - q_{j,t-1} \tag{12}$$

where $q = \log Q$ and I use the log approximation of level returns to effectively re-define $rx_j + i_j$.

Equation (12) above defines the excess return on holdings of the j-currency portfolio of public debt. Given a model for the risk premium $E_t rx_{j,t+1}$, it also defines the price of the debt portfolio as a function of short-term interest:

$$\begin{aligned} q_{j,t} &= \omega_j \beta E_t q_{j,t+1} - E_t rx_{j,t+1} - i_{j,t} \\ &= -\sum_{k=0}^{\infty} (\omega_j \beta)^k E_t \left[rx_{j,t+1+k} + i_{j,t+k} \right]. \end{aligned} \tag{13}$$

The second equation in (13) shows the connection between short-term interest - hence monetary policy - and returns on debt holdings. News of higher interest lower public bond price q and leads to a low excess return.

Lag equation (13) one period and take innovations to find

$$\begin{aligned} \Delta E_t rx_{j,t} &= -\sum_{k=1}^{\infty} (\omega_j \beta)^k \left[\Delta E_t rx_{j,t+k} + \Delta E_t i_{j,t+k-1} \right] \\ &= -\sum_{k=1}^{\infty} (\omega_j \beta)^k \left[\Delta E_t r_{j,t+k} + \Delta E_t \pi_{j,t+k} - \Delta E_t \Delta h_{j,t+k} \right]. \end{aligned} \tag{14}$$

Decomposition 2. *Replace* (14) *on decomposition 1 and gather terms to find.*

$$\frac{\delta v}{\beta}\Delta E_t \pi_t = -\frac{\delta v}{\beta}\sum_{k=1}(\omega\beta)^k \Delta E_t \pi_{t+k} - \sum_{k=0}^{\infty}\beta^k \Delta E_t s^p_{t+k} - \frac{v}{\beta}\sum_{k=1}^{\infty}\beta^k \Delta E_t g_{t+k}$$
$$+\frac{v}{\beta}\sum_{k=1}^{\infty}\sum_j \delta_j \beta^k (1-\omega^i_j)\Delta E_t r_{t+k} + \frac{\delta_D v}{\beta}\sum_{k=1}^{\infty}(\omega_D\beta)^k \Delta E_t \Delta h_{t+k} - \frac{\delta_D v}{\beta}\sum_{k=1}^{\infty}(\omega_D\beta)^k \Delta E_t \pi^{US}_{t+k}. \tag{15}$$

Take covariance with unexpected inflation, and divide both sides by $\delta(v/\beta)$.

$$\begin{aligned} var(\Delta E_t\pi_t) &= -cov\left[\Delta E_t\pi_t, \sum_{k=1}^{\infty}(\omega\beta)^k\Delta E_t\pi_{t+k}\right] - cov\left[\Delta E_t\pi_t, \left(\frac{\delta v}{\beta}\right)^{-1}\sum_{k=0}^{\infty}\beta^k\Delta E_t s^p_{t+k}\right] \\ &- cov\left[\Delta E_t\pi_t, \delta^{-1}\sum_{k=1}^{\infty}\beta^k\Delta E_t g_{t+k}\right] + cov\left[\Delta E_t\pi_t, \delta^{-1}\sum_{k=1}^{\infty}\sum_j \delta_j\beta^k(1-\omega^k_j)\Delta E_t r_{t+k}\right] \\ &+ cov\left[\Delta E_t\pi_t, \frac{\delta_D}{\delta}\sum_{k=1}^{\infty}(\omega_D\beta)^k\Delta E_t\Delta h_{t+k}\right] - cov\left[\Delta E_t\pi_t, \frac{\delta_D}{\delta}\sum_{k=1}^{\infty}(\omega_D\beta)^k\Delta E_t\pi^{US}_{t+k}\right]. \end{aligned} \tag{16}$$

Decomposition 2 says that positive unexpected inflation must forecast either: *lower* future inflation (domestic or foreign), higher deficits, lower growth, higher real discounting or a more depreciated real exchange rate. This is the main decomposition I explore in the empirical exercises.

In (15), the ω terms give a clue of which terms derive from the time-t adjustment of bond prices. For example: an interest rate hike $\Delta E_t i_t$ can lead to a fall in nominal bond prices (negative $\Delta E_t rx_t$) and, by decomposition 1, a decline in current inflation. These lower bond prices in turn forecast higher inflation. This is why the decomposition prescribes future inflation as time-t *deflationary force*, like surpluses.[6] Long-term bond prices bring to the current period the fiscal effects of future inflation.

Similar mechanisms apply to the exchange rate and US inflation terms that follow from dollar-linked debt. Lower dollar-bond prices might forecast higher US inflation of lower (appreciated) real exchange in the future, despite the potential opposite effect at time t.

3 Empirical Model and Estimation

3.1 Overview

The main goal is to estimate the decompositions of unexpected inflation (11) and (16) for a set of twenty-five economies. To do this, I estimate a ten-equation VAR in which the debt law of motion (9) holds by construction. If the VAR is stationary, equation (3) will be satisfied, and the decompositions will hold.

I use annual data. Quarterly data is available, but it often does not go back as many years into the past. This is particularly true for emerging market variables and public debt measures

[6] Of course, higher expected inflation means inflation is expected to grow after time t. Sims (2011) calls that mechanism "stepping on a rake" and argues it was part of the reason why 1970's US inflation kept coming back after temporary declines.

(from all countries). With a focus on long-term debt sustainability, going further into the past is more important than accounting for quarterly dynamics. Additionally, with annual data there is no danger of measurement errors due to seasonality adjustments.

I group countries in four sample categories. The first one contains only the United States. US data covers the period 1939-2019. The second group has six developed economies (you can check the list on table (2)). Their sample start in 1960. The third group has four developed economies, with the sample starting in 1973. The last group contains fourteen developing countries. Their sample starts in 1998.

Grouping countries according to sample size helps to account for parameter volatility. Additionaly, it provides control for international economic nevironment and historical events that affect inflation and its fiscal determinants.

I interpret parameters of the VAR as being random, and estimate them using Bayesian methods. I establish a prior distribution, and then use data likelihood to compute the posterior.[7]

I base my prior on estimated US dynamics. First, because we already have results available in the literature (Cochrane (2022), to the best of my knowledge the decompositions have not been estimated to other countries so far). Second, the US has the longest sample. Critically, it comprises the repayment of a major public borrowing event - World War II - that renders OLS estimates of the VAR stable and plausible.[8] I estimate the model for the US by OLS and use the resulting VAR as mean of the prior for other countries' estimation.

From the ten variables in the VAR, five are observed: the nominal interest (i_t), the inflation rate (π_t), par-value public debt (v^b), the real exchange rate to the dollar (Δh) and GDP growth (g). I select these variables based on (9). Most time series data I collect from the St Louis Fed *FRED* website, the United Nations and the IMF. Details on appendix B.

I convert interest and inflation data to log. The change in dollar exchange rate is the nominal depreciation to the US dollar, plus US inflation minus domestic inflation. In the US case, I use exchange rate to the UK pound, which is available since the 1930s. GDP growth is the log difference of levels data. Public debt is provided as a ratio of GDP by the source (Ali Abbas et al. (2011)), and requires no transformation.

3.2 Public Finances Model

Governments typically report the par or book value of debt, and this is the data I gather. Because theory is based on the *market* value of debt, some adjustment is necessary. The Dallas Fed provides estimates of the market value of debt for the United States following Cox and Hirschhorn (1983) and Cox (1985).[9] I follow a similar methodology.

Let $\mathcal{V}^b_{j,t}$ be the par value of the j-currency portfolio debt, and let $i^b_{j,t}$ be its average yield-to-maturity by which book values are updated from period to period. The adjustment equation is

$$\mathcal{V}_{j,t} = \mathcal{V}^b_{j,t} \times \frac{\text{market value of bonds}}{\text{book value of bonds}} = \frac{1 + \omega_j Q_{j,t}}{(1 + i^b_{j,t-1})Q_{j,t-1}} = \frac{1 + rx_{j,t} + i_{j,t-1}}{1 + i^b_{t-1}}.$$

[7]See del Negro and Schorfheide (2011) or Karlsson (2013) for more on Bayesian estimation of VAR models.

[8]Including WWII data in the sample proved necessary. Starting the sample in 1950 leads to an unstable VAR estimate due to the large public debt equation root.

[9]Web link: https://www.dallasfed.org/research/econdata/govdebt.

I detrend the $\mathcal{V}$'s, convert to real, sum across portfolios and linearize to arrive at:

$$v_t = v_t^b + \frac{v}{b}\left[\sum_j \delta_j \left(rx_{j,t} + i_{j,t-1} - i_{j,t-1}^b\right)\right]. \tag{17}$$

Estimates of the VAR provide an equation for the law of motion of par-value debt. I use (17) to infer a law-of-motion of market-value debt.

The average interest $i_{j,t}^b$ is not observed, so we cannot estimate an equation for it. Instead, I use a model. With a geometric maturity structure of debt and in steady state, every period the government rolls over the volume of bonds coming to maturity (which had maturity $n = 1$ in the previous period). That accounts for a share $1 - \omega$ of total outstanding bonds. In a steady state, the government then issues the same amount of new bonds ($1 - \omega$ of total debt) at the prevailing interest rate i_t. The average interest therefore satisfies

$$i_{j,t}^b = (1-\omega)i_{j,t} + \omega_j i_{j,t-1}^b = (1-\omega_j)\sum_{k=0}^{\infty} \omega_j^k i_{j,t-k} \tag{18}$$

for $j \in \{N, R, D\}$.

3.3 The Bayesian-VAR

With the exception of exchange rate depreciation, I demean all series. This is to assume that the stationary steady-state (around which I linearize) is given by the sample mean of the series of each country (with the exception of real exchange rate growth, which I assume to be zero).

The functional format of the VAR is

$$x_t = ax_{t-1} + bu_{t-1} + k\, e_t. \tag{19}$$

Both x and u are vectors with ten entries. Five of them are the observed variables enumerated above.

Vector u_t groups the same set of variables as x, but for the United States. I often use the "u" notation to refer to the US case. Because the public debt process of each country has a dollar component, and hence depends on dollar interest and inflation, u and ε_u enter the regression of all countries.

There are five shocks hitting the domestic economy: $\varepsilon \sim N(0, \Sigma)$, independent over time. The shocks hitting the US economy are $\varepsilon_u \sim N(0, \Sigma_u)$. I group them in $e_t = [\varepsilon_t' \ \varepsilon_{u,t}']'$. Matrix $k_{10\times 10}$ serves to correctly reproduce the law of motion governing unobserved variables.

The model for US variables is:

$$u_t = a_u u_{t-1} + k_u\, \varepsilon_{u,t} \tag{20}$$

(I use the same notation x to the VAR of all countries and differentiate only in the US case). In (20), k_u is a 10×5 matrix.

Cochrane (2022) imposes the law of motion of public debt in the VAR by using primary surplus "data" calculated from it. Primary deficit equals the change in public debt net of realized return. This procedure bypasses the problem of dealing with accounting conventions in primary surplus reporting that, in most cases, make its data inconsistent with that of public debt. Since the OLS estimator is linear, his VAR satisfies the debt law of motion by construction.

In my VAR, the procedure needs to be adjusted. First, I do not measure excess return, or real interest rates. Second, the debt equation depends on US interest and inflation. Hence, the unrestrictive estimation of (19) spuriously projects these two US variables on domestic ones, which is inconsistent with (20). For these reasons, instead of measuring implied surpluses and then estimating the model, I estimate the model and then fill the equation for primary surpluses that ensures the debt law of motion (9) holds. Before doing that, I also need to include the adjustment equation for market-value debt (17) (the estimated equation is for par-value, not market-value debt!) as well as the three definitions of average interest rates (18) required to do it. These five unobserved variables (surplus s_t^p, market-value debt v_t, and the average interest $\{i_{j,t}^b\}$) complete the ten variables of the VAR.

Note that the estimated equations for par-value and market value of public debt represent their law of motion *after* replacing the equation for primary surpluses, or its equilibrium law of motion.

The estimation has four steps.

Step 1. I estimate the VAR

$$\begin{aligned} \tilde{x}_t &= \tilde{a}\tilde{x}_{t-1} + \tilde{b}\tilde{u}_{t-1} + \varepsilon_t \\ \tilde{u}_t &= \tilde{a}_u\tilde{u}_{t-1} + \varepsilon_{u,t} \end{aligned} \tag{21}$$

where $\tilde{x}$ is a vector with the five observed variables, and $\tilde{u}$ is defined similarly. Matrices $\tilde{a}$, $\tilde{b}$ and $\tilde{a}_u$ are the submatrices of a, b and a_u corresponding to the rows and columns of these observed variables.

I also estimate $\text{cov}(\varepsilon) = \Sigma$ and $\text{cov}(\varepsilon_u) = \Sigma_u$.

Step 2. Stack domestic and US variables $X_t = [x_t' \; u_t']'$. In the United States case, $X_t = u_t$. I assume a constant risk-premium: $E_t rx_{j,t+1} = 0$ for all j. I use the estimated VAR (21) to compute $E_t i_{j,t+i}$ and apply (13) to compute $q_{j,t}$. Equation (12) then yields expressions for excess return of the form

$$rx_{j,t} = \varphi_j' e_t.$$

An equation for real debt is also necessary. I use

$$i_{R,t} = i_t - E_t\pi_{t+1} = \zeta' X_t.$$

under the implied assumption of equal expected real and excess returns between nominal and real debt. In appendix C, I present formulas for the φ's and ζ.

Step 3. Using the estimated model of step 1, I compute the equations for average interest using (18), and fill the corresponding rows of a, b and k (a_u and k_u in the US case). With the equations for average interest filled, I can do the same for the market-price debt using the par-value adjustment equation (17). With the equation for the market-price debt, I use the law of motion (9) and the expressions for excess return and real interest above to fill the equation row for the primary surplus.

This completes the estimation of a, b and k in the general case, a_u and k_u in the US case. For each country, we can stack the equations into a single system for X:

$$X_t = AX_{t-1} + Ke_t. \tag{22}$$

If we order unobserved variables x^o at the top of the x, we can write (22) more explicitly:

$$\begin{pmatrix} x_t \\ u_t \end{pmatrix} = \begin{bmatrix} a & b \\ 0 & a_u \end{bmatrix} \begin{pmatrix} x_{t-1} \\ u_{t-1} \end{pmatrix} + \begin{bmatrix} & k \\ 0 & k_u \end{bmatrix} \begin{pmatrix} \varepsilon_t \\ \varepsilon_{u,t} \end{pmatrix}$$

$$\text{or yet} \begin{pmatrix} x_t^o \\ \tilde{x}_t \\ u_t^o \\ \tilde{u}_t \end{pmatrix} = \begin{bmatrix} & & * & \\ 0 & \tilde{a} & 0 & \tilde{b} \\ 0 & 0 & & * \\ 0 & 0 & 0 & \tilde{a}_u \end{bmatrix} \begin{pmatrix} x_{t-1}^o \\ \tilde{x}_{t-1} \\ u_{t-1}^o \\ \tilde{u}_{t-1} \end{pmatrix} + \begin{bmatrix} & * \\ I & 0 \\ 0 & * \\ 0 & I \end{bmatrix} \begin{pmatrix} \varepsilon_t \\ \varepsilon_{u,t} \end{pmatrix}.$$

Symbol * indicates the coefficients are filled to ensure that (9), (17) and (18) hold. In appendix C I provide their formulas.

Step 4. I compute sample residuals $\hat{e}$ ($\hat{e}_u$ for the US) from (21), and estimate $\text{cov}(\varepsilon, \varepsilon_u) = \Sigma_{xu} = \sum_i \hat{e}_i \hat{e}_{u,i}/(N-1)$, where N is the sample size. Then:

$$\Omega = \text{cov}(e) = \begin{bmatrix} \Sigma & \Sigma_{xu} \\ \Sigma_{ux} & \Sigma_u \end{bmatrix}.$$

3.4 The Prior Distribution

As the commonly used Litterman (1979) prior, I use a distribution of the Normal-Inverse-Wishart class, with general format

$$\Sigma \sim IW(\Phi; d)$$
$$\theta|\Sigma \sim N(\bar{\theta}, \Sigma \otimes \Omega).$$

where $\theta = [\text{vec}(\tilde{a}')' \ \text{vec}(\tilde{b}')']'$ and vec means stacking the columns. Given the Gaussian likelihood of the model, the posterior distribution is also of the Normal-Inverse-Wishart class. Giannone et al. (2015) provide formulas for the posterior distribution and marginal likelihood.

In the US case, the prior centers around zero, $\bar{\theta} = 0$, but since it has a very large variance, the posterior centers around the OLS estimate of $\tilde{a}_u$. The estimated VAR for the US is stationary.

In the case of other countries, I center the prior around $\tilde{a} = \tilde{a}_u$, $\tilde{b} = 0$. The economic content of the prior is that the dynamics of the observed variables is the same as that we estimate for the United States. The supluses process differs from that of the US only to account for the differences in public debt size and term and currency structures.

With $\tilde{b} = 0$, the prior also translates the view that US variables do not affect the dynamics of domestic ones.

The mean of the IW distribution is $\Phi/(d-n-1)$, where $n = 5$ is the dimension of ε and larger values of d represent tighter priors. I choose Φ to be the identity matrix (one percent standard deviation for all shocks, which are uncorrelated) and select $d = n+2$, the lowest integer possible that leads to a well-defined distribution mean - which therefore equals Φ.

The conditional covariance between the coefficients in $\tilde{a}$ is

$$\text{cov}\left(\tilde{a}_{ij}, \tilde{a}_{kl} \mid \Sigma\right) = \begin{cases} \lambda^2 \frac{\Sigma_{ij}}{\Phi_{jj}} & \text{if } j = l \\ 0 & \text{otherwise.} \end{cases}$$

The format allows the loadings on the variable in different equations to be correlated. Loadings on the different variables on the same equation are independent. Hyperparameter λ governs the overall tightness of the prior over linear coefficients, with lower values yielding tighter priors.

The conditional covariance of $\tilde{b}$ is

$$\operatorname{cov}\left(\tilde{b}_{ij}, \tilde{b}_{kl} \mid \Sigma\right) = \begin{cases} (\xi\lambda)^2 \frac{\Sigma_{ij}}{\Phi_{u,jj}} & \text{if } j = l \\ 0 & \text{otherwise} \end{cases}$$

where $\Phi_u = \Phi = I$ is the mean of the IW distribution in the US case. Hyperparameter ξ governs the tightness of the prior that US variables do not affect domestic ones more than the mechanical effect of US interest and inflation on domestic public debt. If $\xi = 1$, the prior is just as tight as that of $\tilde{a}$.

Finally, the covariance between $\tilde{a}$ and $\tilde{b}$ is zero.

It is straightforward to build Ω so that the conditional covariance structures above hold.

4 Empirical Results

4.1 Variance Decomposition

In the baseline specification, I calibrate $\beta = 0.98$ for all countries, and set b tightness parameter $\xi = 1/3$. I calibrate parameters δ and ω based on debt structure data gather from various sources (see appendix B). They are reported in Table 2 along with average debt.

The mean debt-to-GDP ratio in the sample was 0.50, with developed countries slightly more indebted on average. Nominal debt tends to account for the bulk of sovereign debt, Chile being a notable exception. Emerging markets' governments tend to rely relatively more on real and especially foreign debt, and issue securities with higher maturity, on average.

To ensure stability of the VAR, I start by finding the hyperparameter λ that maximizes the marginal likelihood.[10] Then, if the mode of the posterior leads to an unstable VAR, I progressively reduce λ in 0.001 steps until it leads to a stable VAR. Given the continuity of the posterior distribution on λ and the fact that $\lambda = 0$ leads to the stable US system, there must exist a non-zero value of λ that leads to a stationary model. You can check the resulting λ's in table 2.

Table (3) contains the estimated terms of decomposition 1, computed at the mode of the posterior distribution. I compute the terms of equation (11) and divide both sides of the equality by $\operatorname{var}(\Delta E\pi_t)$. This leads to:

$$1 = p_1(r_0) + p_1(rx) - p_1(s) - p_1(g) + p_1(r)$$

where r_0 corresponds to the price adjustment term of real and dollar debt (the first term on the right side of equation (11)); the other symbols are self-explanatory.

[10]Giannone et al. (2015) show that, in the case of Normal-Inverse-Wishart priors, the marginal likelihood can be decomposed in a goodness-of-fit term and a model-complexity term that penalizes conditional forecast variance. By maximizing the marginal likelihood, we ensure we cannot improve one of these terms without reducing the other. Similar methods for selecting the degree of informativeness of the prior distribution have been used. See, for example, Koop (2013) and Carriero et al. (2015).

Country	v (%)	δ_N (%)	δ_R (%)	δ_D (%)	Avg. Term (Years)	λ
Averages	48	74	11	15	6.5	
Advanced - 1960	58	87	8	6	6.4	
Advanced - 1973	32	92	4	4	5.6	
Emerging - 1998	47	63	14	23	6.9	
Median	43	79	5	10	5.6	
Advanced - 1960	53	88	3	2	5.6	
Advanced - 1973	32	94	3	2	5.6	
Emerging - 1998	43	67	6	23	7.6	
United States	60	93	7	0	5	10
Advanced - 1960 Sample						
Canada	71	92	5	3	6.5	0.21
Denmark	37	84	0	16	5.6	0.18
Japan	98	100	0	0	5.5	0.01
Norway	35	99	0	1	3.7	0.19
Sweden	46	69	16	14	4.8	0.16
United Kingdom	61	76	24	0	12.3	0.17
Advanced - 1973 Sample						
Australia	24	90	10	0	7.2	0.18
New Zealand	41	82	6	13	4.3	0.15
South Korea	21	97	0	3	4	0.15
Switzerland	43	100	0	0	6.9	0.23
Emerging - 1998 Sample						
Brazil	70	70	25	5	2.6	0.12
Chile	14	10	57	33	12.8	0.27
Colombia	41	45	23	32	5.6	0.13
Czech Republic	31	91	0	9	5.6	0.15
Hungary	68	76	0	23	4.1	0.14
India	73	90	3	7	10.1	0.25
Indonesia	43	44	0	56	9.2	0.21
Israel	77	43	34	23	6.6	0.13
Mexico	45	65	10	26	5.5	0.15
Poland	47	79	1	20	4.2	0.10
Romania	28	50	0	50	4.8	0.10
South Africa	41	70	20	10	12.9	0.25
Turkey	43	47	23	30	3.6	0.13
Ukraine	43	100	0	0	9.1	0.07

Notes: v is the average public debt-to-GDP, δ_N is the share of nominal debt, δ_R is the share of real or inflation-linked debt, δ_D is the share of dollar-denominated debt (I also include debt denominated in other currencies, but the amounts are negligible in almost all cases). "Avg. Term" is the average maturity across currency portfolios, calculated as $\sum_j \delta_j(1-\omega_j)^{-1}$. λ is the prior tightness parameter.

Table 2: Debt Structure Parameters and Prior Tightness

Country	std($\Delta E\pi$)	Decomposition 1				
	(%)	$p_1(r)$	$p_1(rx)$	-$p_1(s)$	-$p_1(g)$	$p_1(r)$
Averages	1.6	-0.11	**-0.69**	-0.94	**1.87**	0.87
Advanced - 1960	1.5	0.00	**-0.78**	**1.05**	-0.03	0.76
Advanced - 1973	1.8	-0.01	**-0.71**	*1.52*	**-0.51**	-0.32
Emerging - 1998	1.6	-0.19	**-0.66**	-2.65	**3.23**	1.27
Median	1.3	-0.01	**-0.74**	**0.97**	**0.66**	*0.52*
Advanced - 1960	1.5	**-0.06**	**-0.80**	**0.82**	*0.12*	1.21
Advanced - 1973	1.7	*-0.01*	**-0.73**	*1.61*	**0.87**	*-0.27*
Emerging - 1998	1.2	-0.19	**-0.66**	0.70	**1.02**	*0.30*
United States	2.1	**0.03**	**-0.53**	*1.22*	-0.30	0.59
Advanced - 1960 Sample						
Canada	1.0	**-0.11**	**-1.61**	0.52	-0.05	*2.26*
Denmark	1.2	**-0.28**	-0.19	1.49	-0.99	0.97
Japan	2.1	0	**-0.81**	**1.07**	*-0.87*	**1.60**
Norway	1.5	**-0.01**	**-0.37**	*0.57*	0.29	0.52
Sweden	1.5	*-0.13*	**-0.90**	-0.08	*0.66*	*1.45*
United Kingdom	2.0	**0.52**	**-0.79**	**2.74**	**0.75**	**-2.22**
Advanced - 1973 Sample						
Australia	1.4	**0.08**	**-0.78**	**2.70**	*0.78*	*-1.77*
New Zealand	2.0	*-0.09*	**-0.90**	-0.22	**1.18**	*1.04*
South Korea	2.9	-0.01	**-0.46**	**3.08**	-0.86	*-0.75*
Switzerland	1.0	0	**-0.69**	*0.52*	**0.96**	0.21
Emerging - 1998 Sample						
Brazil	1.3	*-0.16*	**-0.23**	3.27	*1.37*	-3.25
Chile	1.0	-2.76	-0.32	*-17.60*	*18.61*	3.07
Colombia	0.8	**2.54**	**-0.89**	-5.97	*4.92*	0.40
Czech Republic	1.1	**-0.12**	**-0.29**	1.63	*0.48*	-0.70
Hungary	1.2	**-0.54**	**-0.88**	-4.67	3.54	3.56
India	1.1	**0.17**	**-0.51**	0.97	0.21	0.17
Indonesia	1.1	*-2.33*	**-1.07**	1.21	*3.00*	0.20
Israel	1.3	0.29	**-0.58**	4.98	*2.16*	-5.85
Mexico	1.0	0.12	**-0.74**	*1.06*	0.67	-0.12
Poland	1.2	*-0.38*	**-1.23**	0.39	-0.35	**2.57**
Romania	1.9	-0.36	**-0.87**	*2.47*	0.51	-0.75
South Africa	1.0	*0.34*	**-0.44**	**-24.42**	**9.96**	**15.56**
Turkey	2.1	**0.51**	**-0.36**	-0.90	*-0.44*	**2.19**
Ukraine	5.7	0	**-0.78**	0.44	**0.57**	*0.77*

Notes: v is the average public debt-to-GDP, δ_N is the share of nominal debt, δ_R is the share of real or inflation-linked debt, δ_D is the share of dollar-denominated debt (I also include debt denominated in other currencies, but the amounts are negligible in almost all cases). "Avg. Term" is the average maturity across currency portfolios, calculated as $\sum_j \delta_j(1-\omega_j)^{-1}$. λ is the prior tightness parameter.

Table 3: Unexpected Inflation Volatility Decomposition

The p_1's are regression coefficients of the respective decompositions term. For instance

$$p_1(s) = \text{cov}\left[\Delta E_t \pi_t, \left(\frac{\delta v}{\beta}\right)^{-1} \sum_{k=1}^{\infty} \beta^k \Delta E s_{t+k}\right].$$

These terms indicate how much of inflation variance is explained by the corresponding term. Note that they do not have to be in the $[0, 1]$ interval.

I report cross-country averages and medians, conditional on country group. Highlighted figures indicate the statistical significance of the estimate's *sign*, based on ten-thousand simulations of the posterior distribution.[11] Italic font indicates that 75% of the simulated draws have the same sign as the posterior mode. Bold font indicates 90%.

The first column of the table reports unexpected inflation. Unsurprisingly, unexpected inflation exists, with standard deviations ranging from 1% and 2% in the country group averages. The low figure for emerging markets reminds us of the importance of the sample time period. Emerging markets do not have lower unexpected inflation; we just sampled from a period in which inflation is known to be less volatile in most countries (Stock and Watson (2002), Coibion and Gorodnichenko (2011)).

4.2 Response to Reduced-Form Shocks

5 New-Keynesian Model Benchmarks

6 Robustness

7 Conclusion

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[11] I discard draws that lead to unstable VARs.

Country	Decomposition 2					
	$-p_2(\pi)$	$-p_2(s)$	$-p_2(g)$	$p_2(r)$	$p_2(\Delta h)$	$p_2(\pi^{US})$
Averages	**-0.74**	-0.94	**1.87**	*0.83*	*0.27*	**-0.28**
Advanced - 1960	**-1.21**	**1.05**	-0.03	**1.26**	0	**-0.06**
Advanced - 1973	**-1.04**	*1.52*	**0.51**	0.08	*-0.05*	**-0.03**
Emerging - 1998	**-0.43**	-2.65	**3.23**	0.84	*0.49*	**-0.47**
Median	**-0.72**	**0.97**	**0.66**	**0.61**	0	**-0.05**
Advanced - 1960	**-1.05**	**0.82**	*0.12*	**1.33**	0	**-0.03**
Advanced - 1973	**-0.93**	*1.61*	**0.87**	-0.07	0	**-0.01**
Emerging - 1998	**-0.64**	0.70	**1.02**	*0.31*	0.01	**-0.12**
United States	**-0.96**	*1.22*	-0.30	*1.05*	0	0
Advanced - 1960 Sample						
Canada	**-1.56**	0.52	-0.05	*2.19*	*-0.03*	**-0.06**
Denmark	**-0.48**	1.49	-0.99	1.09	0.08	**-0.19**
Japan	**-1.10**	**1.07**	*-0.87*	**1.90**	0	0
Norway	**-0.72**	0.57	0.29	*0.87*	0	0
Sweden	**-1.00**	-0.08	*0.66*	**1.58**	-0.05	*-0.10*
United Kingdom	**-2.42**	**2.74**	**0.75**	-0.07	0	0
Advanced - 1973 Sample						
Australia	**-1.53**	**2.70**	**0.78**	-0.95	0	0
New Zealand	**-1.08**	-0.22	**1.18**	*1.41*	*-0.21*	**-0.08**
South Korea	**-0.79**	**3.08**	-0.86	-0.42	0.01	**-0.02**
Switzerland	**-0.76**	*0.52*	**-0.96**	0.28	0	0.21
Emerging - 1998 Sample						
Brazil	**-0.12**	3.27	*1.37*	-3.63	*0.11*	0
Chile	0.31	*-17.60*	**18.61**	-2.07	*5.34*	**-3.60**
Colombia	**-0.68**	-5.97	*4.92*	0.61	*2.39*	-0.27
Czech Republic	0.05	1.63	*0.48*	-1.11	0.03	**-0.07**
Hungary	**-0.67**	-4.67	3.54	3.56	**-0.59**	**-0.17**
India	**-0.92**	0.97	0.21	0.65	-0.01	**0.10**
Indonesia	**-0.68**	1.21	*3.00*	0.10	*-1.03*	**-1.59**
Israel	**-0.37**	4.98	*2.16*	*-5.82*	-0.01	0.06
Mexico	**-0.66**	*1.06*	0.67	0.31	*-0.53*	*0.14*
Poland	**-0.63**	0.39	-0.35	**1.89**	-0.10	**-0.20**
Romania	**-1.14**	*2.47*	0.51	-0.90	0.63	**-0.57**
South Africa	0.56	**-24.42**	**9.96**	**14.56**	0.39	-0.05
Turkey	-0.80	-0.90	*-0.44*	**3.31**	*0.26*	**-0.43**
Ukraine	*-0.31*	0.44	**0.57**	*0.30*	0	0

Notes: v is the average public debt-to-GDP, δ_N is the share of nominal debt, δ_R is the share of real or inflation-linked debt, δ_D is the share of dollar-denominated debt (I also include debt denominated in other currencies, but the amounts are negligible in almost all cases). "Avg. Term" is the average maturity across currency portfolios, calculated as $\sum_j \delta_j (1-\omega_j)^{-1}$. λ is the prior tightness parameter.

Table 4: Unexpected Inflation Volatility Decomposition

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A Linearization

B Data Sources and Treatment

B.1 Sources

I collect a significant share of the data from the St. Louis Fed's *FRED* website. In the case of countries with sample starting after 1970 I get data from the United Nations's National Accounts Main Aggregates Database. Their database also contains exchange rate data, which I use only in the case of emerging markets (with sample starting after 1998).

Whenever omitted in the list below, the source for interest rate data is the FRED. Finally, unless otherwise noted, public debt data I get from the database from Ali Abbas et al. (2011), which is kept up-to-date.

Australia 1973-2021. All except GDP and public debt from FRED.

Brazil 1998-2021.

Canada 1960-2021. All except public debt from FRED.

Chile 1998-2021.

Colombia 1998-2021.

Czech Republic 1998-2021.

Denmark 1960-2021. All except public debt from FRED.

Hungary 1998-2021.

India 1998-2021.

Indonesia 1998-2021.

Israel 1998-2021.

Japan 1960-2021. All except public debt from FRED.

Mexico 1998-2021.

Norway 1960-2021. All except public debt and interest rates from FRED. I interpolate the debt data for the year 1966. FRED interest data goes back to 1979, I splice it with historical data from Eitrheim et al. (2007), available at the website of the Norges Bank.

New Zealand 1973-2021. All except GDP and public debt from FRED.

Poland 1998-2021.

Romania 1998-2021. Interest rate is the deposit rate series from IMF's International Finance Statistics.

South Africa 1998-2021.

South Korea 1973-2021. All except GDP and public debt from FRED.

Sweden 1960-2021. All except public debt from FRED. I interpolate the debt data for the year 1965 and 1966.

Switzerland 1973-2021. Interest, CPI and exchange rate from FRED.

Turkey 1998-2021.

Ukraine 1998-2021. Interest rate data from National Bank of Ukraine (NBU Key Policy Rate).

United Kingdom 1960-2021. Interest data from the Bank of England (Base Rate); I splice it with discount rate data from the FRED. Inflation and GDP data from FRED.

United States 1950-2021. All data collected from the FRED. I use real exchange rate to the United Kingdom, since nominal exchange rate is available since before 1950.

C Additional Details of the BVAR Estimation

D Equilibrium Selection in the NK Model

E Deriving the SOE-NK Model