A Fiscal Decomposition of Unexpected Inflation: Cross-Country Estimates and Theory

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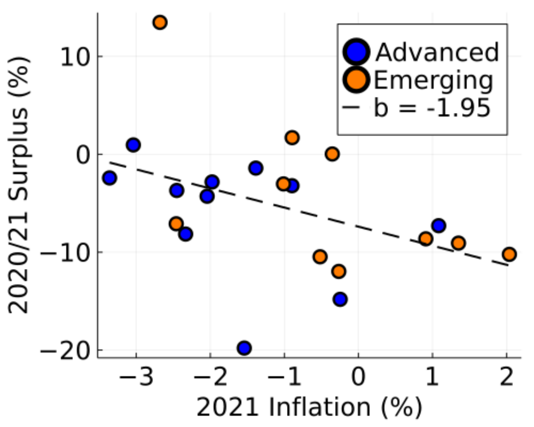
Fiscal Connection?



COVID Inflation - 21 countries in sample

Introduction

- Sources of inflation variation
- What drives innovations to the price level?
- Breakdown of valuation equation of public debt

$$\frac{\textbf{Bond Prices} \times \text{Bonds}}{\textbf{Price Level}} = E \sum_t \frac{\textbf{Surpluses}_t}{\textbf{Discount}_t}$$

- Common backing condition in macroeconomic models

Valuation Equation of Public Debt

- Campbell and Ammer (1993): Stock Prices

$$\textbf{Stock Price} \times \text{Shares} = E \sum_t \frac{\textbf{Dividends}_t}{\textbf{Discount}_t}$$

- Cochrane (2022c), this paper: Public Nominal Liabilities

$$\frac{\textbf{Bond Prices} \times \text{Bonds}}{\textbf{Price Level}} = E \sum_t \frac{\textbf{Surpluses}_t}{\textbf{Discount}_t}$$

Exercises

1. Decomposition estimates
   - Bayesian VAR for 21 countries
   - Inflation shock $\Delta E_t \pi_t = 1$
   - Discounted surpluses shock: $\Delta E_t[\text{Disc Surp}] = -1$
2. FTPL, New-Keynesian Model
   - Volatile surpluses, no contribution to inflation?
   - Parametric model of partial debt repayment
   - GMM estimate to reproduce decompositions

Motivation + Results

- Measures not structural
- Stylized facts to be matched by theory
- On average:
    - Discount rates → ~80% of total inflation
    - GDP growth → ~20% of total inflation
    - Surplus/GDP → ~0% of total inflation

Inflation has fiscal roots, even if fiscal policy is disconnected from the price level.

Motivation + Results

- Structural interpretation?
- Model with partial debt repayment
- Volatile surpluses, no inflation?
- On average, 0.78% of 1% GDP deficit is repaid
  - 0.96% in advanced economies
  - 0.59% in developing economies

Discount-driven inflation and realistic surplus process preclude partial repayment.

Why unexpected inflation, not just inflation?

- New Keynesian theory:
  - Fisher: monetary policy sets expected inflation
  - Fiscal policy sets unexpected inflation
- Measures do not depend on state of the economy
- Direct connection with impulse response functions

Literature

- **Monetary-Fiscal Interaction.** Cagan (1956), Sargent and Wallace (1981), Hall and Sargent (1997), Hall and Sargent (2011), Jiang et al. (2019), Corsetti et al. (2019), Sunder-Plassmann (2020), Du et al. (2020), Akhmadieva (2022)
- **Fiscal Theory of the Price Level.** Leeper (1991), Sims (1994), Woodford (1995), Cochrane (1998), Cochrane (2005), Sims (2011), Leeper and Leith (2016), Bassetto and Cui (2018), Cochrane (2022c), Brunnermeier et al. (2022), Cochrane (2022a), Cochrane (2022b)
- **Empirical Finance.** Campbell and Shiller (1988), Cochrane (1992), Campbell and Ammer (1993), Chen and Zhao (2009), Cochrane (2008), Jiang et al. (2019)

Environment

- 1 period = 1 year
- Consumption good price P_t
- Total output Y_t
- Nominal bonds $B^n_{N,t}$, price $Q^n_{N,t}$
  - Pay one unit of currency after *n* years
- Real bonds $B^n_{R,t}$, price $P_t Q^n_{R,t}$
  - Pay one unit of consumption good after *n* years
- Primary Surplus $P_t S_t$

Evolution of Public Debt

$$\overbrace{\left[B^1_{N,t-1} + P_t B^1_{R,t-1}\right]}^{\text{Issued Currency}} = \Delta M_t$$
$$\underbrace{+ \left[P_t S_t + \sum_{n=1}^{\infty} Q^n_{N,t}\left(B^n_{N,t} - B^{n+1}_{N,t-1}\right) + P_t \sum_{n=1}^{\infty} Q^n_{R,t}\left(B^n_{R,t} - B^{n+1}_{R,t-1}\right)\right]}_{\text{Retired Currency}}$$

- This is a budget constraint
- Assumption 1: households do not value currency $M_t = 0$

Evolution of Public Debt

- Assumption 1: households do not value currency $M_t = 0$
- End-of-period debt $\mathscr{V}_{N,t}$ and $\mathscr{V}_{R,t}$

$$(1 + r_t^N)\mathscr{V}_{N,t-1} + (1 + r_t^R)(1 + \pi_t)\mathscr{V}_{R,t-1} = P_t S_t + \mathscr{V}_{N,t} + \mathscr{V}_{R,t}$$

- This is an equilibrium condition
- Price level adjusts so that

currency issued = currency retired

Evolution of Public Debt

- Constant structure of public debt: $\delta = \mathscr{V}_{N,t} / \mathscr{V}_t$

$$1 + r_t^n = \delta \left[(1 + r_{N,t}) \right] + (1 - \delta) \left[(1 + r_{R,t})(1 + \pi_t) \right]$$

- Debt-to-GDP = $V_t = \mathscr{V}_t / P_t Y_t$
- Surplus-to-GDP = $s_t = S_t / Y_t$

$$\frac{1 + r_t^n}{(1 + \pi_t)(1 + g_t)} V_{t-1} = s_t + V_t$$

Evolution of Public Debt

Linearized equations

$$v_t + \frac{s_t}{V} = \frac{1}{\beta}\left[v_{t-1} + r_t^n - \pi_t - g_t\right]$$

$$r_t^n = \delta\left[r_t^N\right] + (1-\delta)\left[r_t^R + \pi_t\right]$$

- v_t is log debt-to-GDP
- r_t^n is the nominal return on public debt

Valuation Equation of Public Debt

- Assumption 2: debt does not spiral $\lim_{j\to\infty} \beta^j v_{t+j} = 0$
- Solve flow equation forward:

$$\overbrace{v_{t-1} + r_t^n - \pi_t}^{\text{Real market value of debt}} = \underbrace{\frac{\beta}{V} \sum_{j=0}^{\infty} \beta^j \left[E_t s_{t+j} + E_t g_{t+j}\right] - \sum_{j=1}^{\infty} \beta^j \left[E_t r_{t+j}^n - E_t \pi_{t+j}\right]}_{\text{Discounted Surpluses}}$$

Now take innovations $\Delta E_t = E_t - E_{t-1}$

Marked-to-Market Decomposition

Take innovation on the valuation equation:

$$\boxed{\epsilon_{r^n,t} - \epsilon_{\pi,t} = \epsilon_{s,t} + \epsilon_{g,t} - \epsilon_{r,t}}$$

Terms:

$\epsilon_{r^n,t} = \Delta E_t r_t^n$

$\epsilon_{\pi,t} = \Delta E_t \pi_t$ (current inflation)

$\epsilon_{s,t} = (\beta/V)\sum_{j=0}^{\infty} \beta^j \Delta E_t s_{t+j}$

$\epsilon_{g,t} = \sum_{j=0}^{\infty} \beta^j \Delta E_t g_{t+j}$

$\epsilon_{r,t} = \sum_{j=1}^{\infty} \beta^j (\Delta E_t r_{t+j}^n - \Delta E_t \pi_{t+j})$

Public Finances Model

Why a public finances model?

1. Decompose bond price term
2. No r_t^n data: use proxy
3. No data on market value of debt (only book value)

Public Finances Model

Key Assumptions

- **Assumption**: constant maturity structure
- Decays geometrically at rate ω:

$$B^{n}_{N,t} = \omega_N \, B^{n-1}_{N,t}$$
$$B^{n}_{R,t} = \omega_R \, B^{n-1}_{R,t}$$

- **Assumption**: constant risk premium

$$E_t r_{N,t} = E_t r_{R,t} + E_t \pi_t = i_t$$

Public Finances Model

- Bond prices:

$$q_{N,t} = (\omega_N \beta) E_t q_{N,t} - \left[i_t \right]$$

$$q_{R,t} = (\omega_R \beta) E_t q_{R,t} - \left[i_t - E_t \pi_{t+1} \right]$$

- Returns:

$$r_{j,t} = (\omega_j \beta) q_{j,t} - q_{j,t-1} \qquad j = N, R$$

Break down of bond price variation

Proposition: let $r_t = i_t - E_t\pi_{t+1}$ be the real interest. Then

$$\epsilon_{r^n,t} - \epsilon_{\pi,t} = -\delta \sum_{j=0}^{\infty} (\omega_N \beta)^j \Delta E_t \pi_{t+j} - \sum_{j=1}^{\infty} \beta^j [\delta \omega_N^j + (1-\delta)\omega_R^j] \Delta E_t r_{t+j}$$

- Discounting affects real and nominal bond prices
- Inflation affects nominal bond prices

Total Inflation Decomposition

Replace bond return decomp on marked-to-market decomp:

$$\boxed{-\varepsilon_{\pi,t} = \varepsilon_{s,t} + \varepsilon_{g,t} - \varepsilon_{r,t}}$$

Terms:

$\varepsilon_{\pi,t} = \delta \sum_{j=0}^{\infty} (\omega_N \beta)^j \Delta E_t \pi_{t+j}$ (current and future inflation)

$\varepsilon_{s,t} = \epsilon_{s,t} = (\beta / V) \sum_{j=0}^{\infty} \beta^j \Delta E_t s_{t+j}$

$\varepsilon_{g,t} = \epsilon_{g,t} = \sum_{j=0}^{\infty} \beta^j \Delta E_t g_{t+j}$

$\varepsilon_{r,t} = \sum_{j=1}^{\infty} \beta^j \left[1 - (\delta \omega_N^j + (1 - \delta) \omega_R^j) \right] \Delta E_t r_{t+j}$

Comparison of Decompositions

- **Marked-to-market**: $\boxed{\epsilon_{r^n,t} - \epsilon_{\pi,t} = \epsilon_{s,t} + \epsilon_{g,t} - \epsilon_{r,t}}$
  - Current inflation given current bond prices
  - Highlights effect of monetary policy
- **Total inflation**: $\boxed{-\varepsilon_{\pi,t} = \varepsilon_{s,t} + \varepsilon_{g,t} - \varepsilon_{r,t}}$
  - Path of inflation given path of discount rates
  - Sensitive to future inflation
  - Nets out effect of discount rates on bond prices

Build Market Value of Debt

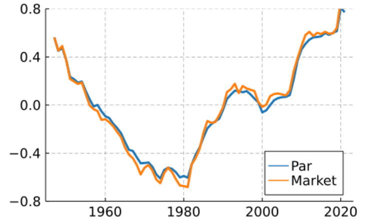
- Converting par to market value of debt
- Dallas Fed, Cox and Hirschhorn (1983) and Cox (1985)

$$\mathscr{V}_{j,t} = \mathscr{V}_{j,t}^{b} \times \frac{\text{market value of bonds}}{\text{book value of bonds}} = \mathscr{V}_{j,t}^{b} \frac{Q_{j,t}}{Q_{j,t}^{b}} \quad \text{for } j = N, R$$

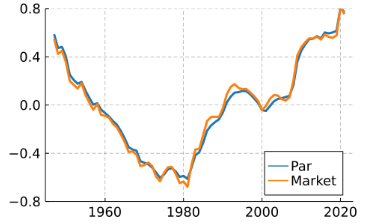
- Book price of bonds evolve according to average interest:

$$\begin{aligned} i_{N,t}^{b} &= (1 - \omega_N) i_t + \omega_N i_{N,t-1}^{b} \\ i_{R,t}^{b} &= (1 - \omega_R)(i_t - E_t \pi_{t+1}) + \omega_R i_{R,t-1}^{b} \end{aligned}$$

Comparison with Dallas Fed



(a) Dallas Fed



(b) Model

Vector Autoregression

- States X

$$X_t = AX_{t-1} + e_t \qquad e_t \sim N(0, \Sigma)$$

- Bayesian estimates of 21 countries
- Samples end in 2019 (no COVID!)
- Prior centered around US OLS estimates

$$\begin{bmatrix} i_t \text{ Nominal Interest} \\ \pi_t \text{ Inflation Rate} \\ g_t \text{ GDP Growth} \\ v_t \text{ Market Value Debt} \\ r_t^n \text{ Bond Return (model built)} \\ s_t \text{ Primary Surplus (model built)} \end{bmatrix}$$

VAR and Decomposition Measures

- VAR uses time series structure to identify revision of expectations

$$\Delta E_t X_{t+j} = A^j X_t$$

$$\sum_{j=0}^{\infty} \beta^j \Delta E_t X_{t+j} = (I - \beta A)^{-1} X_t$$

The Inflation Shock

- Inflation unexpectedly jumps: $\Delta E_t \pi_t = 1$
- Other shocks allowed to jump as well

$$\textbf{(Inflation Shock)} \quad e_t = E\left[e \mid \Delta E_t \pi_t = 1\right]$$

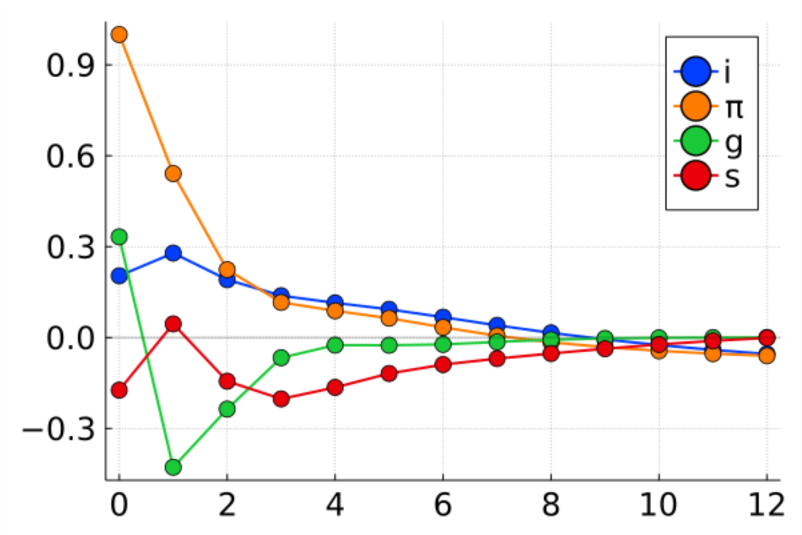
- Proposition: MtM decomposition

$$\epsilon_{r^n,t} - \epsilon_{\pi,t} = \epsilon_{s,t} + \epsilon_{g,t} - \epsilon_{r,t}$$

same as variance decomposition

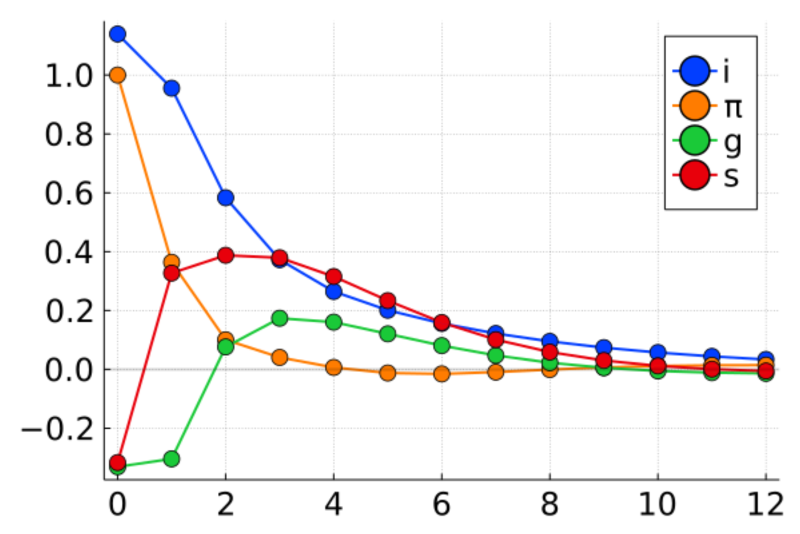
$$\frac{\text{cov}(\epsilon_{r^n,t}, \epsilon_{\pi,t})}{\text{var}(\epsilon_{\pi,t})} - 1 = \frac{\text{cov}(\epsilon_{s,t}, \epsilon_{\pi,t})}{\text{var}(\epsilon_{\pi,t})} + \frac{\text{cov}(\epsilon_{g,t}, \epsilon_{\pi,t})}{\text{var}(\epsilon_{\pi,t})} - \frac{\text{cov}(\epsilon_{r,t}, \epsilon_{\pi,t})}{\text{var}(\epsilon_{\pi,t})}$$

IRF - United States



IRF to $e_t = E\left[e \mid \Delta E_t \pi_t = 1\right]$

IRF - Brazil



IRF to $e_t = E\left[e \mid \Delta E_t \pi_t = 1\right]$

Inflation Shock - Marked-to-Market

Country	ϵ_{r^n}	$-\epsilon_\pi$	=	ϵ_s	$+\epsilon_g$	$-\epsilon_r$
1947 Sample (Advanced)						
United Kingdom	**-0.7	**-1	=	**-2.2	**-0.7	**1.2
United States	**-0.7	**-1	=	-0.3	**-0.5	**-0.9
1960 Sample (Advanced)						
Canada	**-2.8	**-1	=	0.3	*-1.4	**-2.8
Denmark	**-0.9	**-1	=	0.2	-0.2	**-1.9
Japan	**-0.6	**-1	=	**2.8	**-3.0	**-1.4
Norway	**-0.7	**-1	=	0.7	*3.0	**-5.4
Sweden	**-0.6	**-1	=	**0.9	**-0.9	**-1.6
1973 Sample (Advanced)						
Australia	**-2.2	**-1	=	0.2	0.1	**-3.5
New Zealand	**-1.0	**-1	=	*1.2	**-1.4	*-1.8
South Korea	**-0.6	**-1	=	**-2.4	0.2	*0.7
Switzerland	**-2.0	**-1	=	*-0.8	0.1	**-2.3

Inflation Shock: $e_t = E[e \mid \Delta E_t \pi_t = 1]$

Inflation Shock - Marked-to-Market

Country	ϵ_{r^n}	$-\epsilon_\pi$	=	ϵ_s	$+\epsilon_g$	$-\epsilon_r$
1997 Sample (Emerging)						
Brazil	**-0.7	**-1	=	**2.4	-0.1	**-4.0
Colombia	**-1.4	**-1	=	0.2	**-0.7	**-1.9
Czech Republic	*0.2	**-1	=	*0.7	**-1.3	-0.2
Hungary	**-0.8	**-1	=	0.0	-0.2	**-1.6
India	*-0.2	**-1	=	**-1.0	-0.1	-0.1
Israel	**-0.4	**-1	=	**0.8	*-0.4	**-1.8
Mexico	**-1.4	**-1	=	*-1.2	0.0	*-1.3
Poland	**-1.4	**-1	=	**1.0	*-0.3	**-3.0
South Africa	**-0.6	**-1	=	0.3	**-0.8	**-1.1
Ukraine	**-0.5	**-1	=	**-1.1	0.0	-0.3

Inflation Shock: $e_t = E[e \mid \Delta E_t \pi_t = 1]$

Inflation Shock - Total Inflation

Country	$-\varepsilon_\pi$	=	ε_s	$+\varepsilon_g$	$-\varepsilon_r$
1947 Sample (Advanced)					
United Kingdom	**-2.8	=	**-2.2	**-0.7	0.1
United States	**-1.5	=	-0.3	**-0.5	**-0.7
1960 Sample (Advanced)					
Canada	**-2.6	=	0.3	*-1.4	**-1.5
Denmark	**-1.6	=	0.2	-0.2	**-1.6
Japan	**-1.5	=	**2.8	**-3.0	**-1.3
Norway	**-2.0	=	0.7	*3.0	**-5.7
Sweden	**-1.6	=	**0.9	**-0.9	**-1.5
1973 Sample (Advanced)					
Australia	**-3.1	=	0.2	0.1	**-3.4
New Zealand	**-2.3	=	*1.2	**-1.4	**-2.1
South Korea	**-2.0	=	**-2.4	0.2	0.2
Switzerland	**-2.0	=	*-0.8	0.1	**-1.3

Inflation Shock: $e_t = E[e \mid \Delta E_t \pi_t = 1]$

Inflation Shock - Total Inflation

Country	$-\varepsilon_\pi$	=	ε_s	$+\varepsilon_g$	$-\varepsilon_r$
1997 Sample (Emerging)					
Brazil	**-0.8	=	**2.4	-0.1	**-3.1
Colombia	**-0.7	=	0.2	**-0.7	-0.2
Czech Republic	**-0.5	=	*0.7	**-1.3	0.1
Hungary	**-1.4	=	0.0	-0.2	**-1.3
India	**-1.4	=	**-1.0	-0.1	*-0.4
Israel	**-0.6	=	**0.8	*-0.4	**-1.0
Mexico	**-1.4	=	*-1.2	0.0	-0.3
Poland	**-1.4	=	**1.0	*-0.3	**-2.1
South Africa	**-0.8	=	0.3	**-0.8	*-0.3
Ukraine	**-1.2	=	**-1.1	0.0	-0.1

Inflation Shock: $e_t = E[e \mid \Delta E_t \pi_t = 1]$

Inflation Shock - Averages

Country	ϵ_{r^n}	$-\epsilon_\pi$	=	ϵ_s	$+\epsilon_g$	$-\epsilon_r$
Averages	**-1.0	**-1	=	0.1	**-0.4	**-1.7
1947 (Advanced)	**-0.7	**-1	=	**-1.2	**-0.6	0.1
1960 (Advanced)	**-1.1	**-1	=	*1.0	*-0.5	**-2.6
1973 (Advanced)	**-1.4	**-1	=	-0.4	-0.3	**-1.7
1997 (Emerging)	**-0.7	**-1	=	0.2	**-0.4	**-1.5

Marked-to-Market

Country	$-\varepsilon_\pi$	=	ε_s	$+\varepsilon_g$	$-\varepsilon_r$
Averages	**-1.6	=	0.1	**-0.4	**-1.3
1947 (Advanced)	**-2.2	=	**-1.2	**-0.6	-0.3
1960 (Advanced)	**-1.9	=	*1.0	*-0.5	**-2.3
1973 (Advanced)	**-2.3	=	-0.4	-0.3	**-1.6
1997 (Emerging)	**-1.0	=	0.2	**-0.4	**-0.9

Total Inflation

Inflation Shock - Takeaways

- Discount rates: 80% of inflation variation (average)
- The rest comes mostly from GDP growth
- Positive contribution of surplus-to-GDP 7/21
- Monetary policy reduces inflation in 20/21

Inflation has fiscal roots, even if fiscal policy is disconnected from the price level.

Robustness - OLS Estimates

Country	ϵ_{r^n}	$-\epsilon_\pi$	=	ϵ_s	$+\epsilon_g$	$-\epsilon_r$
Averages	**-1.0	**-1	=	0.2	**-0.4	**-1.8
1947 (Advanced)	**-0.7	**-1	=	**-1.2	**-0.6	0.2
1960 (Advanced)	**-1.2	**-1	=	*1.0	*-0.5	**-2.6
1973 (Advanced)	**-1.4	**-1	=	-0.4	-0.3	**-1.7
1997 (Emerging)	**-0.7	**-1	=	0.4	*-0.3	**-1.8

Marked-to-Market

Country	$-\varepsilon_\pi$	=	ε_s	$+\varepsilon_g$	$-\varepsilon_r$
Averages	**-1.6	=	0.2	**-0.4	**-1.4
1947 (Advanced)	**-2.2	=	**-1.2	**-0.6	-0.3
1960 (Advanced)	**-1.9	=	*1.0	*-0.5	**-2.4
1973 (Advanced)	**-2.4	=	-0.4	-0.3	**-1.6
1997 (Emerging)	**-1.0	=	0.4	*-0.3	**-1.1

Total Inflation

Robustness - Minnesota Prior

Country	ϵ_{r^n}	$-\epsilon_\pi$	=	ϵ_s	$+\epsilon_g$	$-\epsilon_r$
Averages	**-1.0	**-1	=	0.2	**-0.4	**-1.8
1947 (Advanced)	**-0.7	**-1	=	**-1.2	**-0.6	0.2
1960 (Advanced)	**-1.1	**-1	=	*1.0	*-0.5	**-2.6
1973 (Advanced)	**-1.4	**-1	=	-0.5	-0.3	**-1.7
1997 (Emerging)	**-0.7	**-1	=	0.3	*-0.3	**-1.8

Marked-to-Market

Country	$-\varepsilon_\pi$	=	ε_s	$+\varepsilon_g$	$-\varepsilon_r$
Averages	**-1.6	=	0.2	**-0.4	**-1.4
1947 (Advanced)	**-2.2	=	**-1.2	**-0.6	-0.3
1960 (Advanced)	**-1.9	=	*1.0	*-0.5	**-2.4
1973 (Advanced)	**-2.3	=	-0.5	-0.3	**-1.6
1997 (Emerging)	**-1.1	=	0.3	*-0.3	**-1.1

Total Inflation

Robustness - 2021 Sample

Country	ϵ_{r^n}	$-\epsilon_\pi$	=	ϵ_s	$+\epsilon_g$	$-\epsilon_r$
Averages	**-1.0	**-1	=	-0.1	*-0.2	**-1.7
1947 (Advanced)	**-0.8	**-1	=	**-1.2	**-0.5	0.0
1960 (Advanced)	**-1.2	**-1	=	*0.8	*-0.6	**-2.4
1973 (Advanced)	**-1.4	**-1	=	-0.6	-0.2	**-1.6
1997 (Emerging)	**-0.8	**-1	=	-0.1	0.0	**-1.7

Marked-to-Market

Country	$-\varepsilon_\pi$	=	ε_s	$+\varepsilon_g$	$-\varepsilon_r$
Averages	**-1.6	=	-0.1	*-0.2	**-1.3
1947 (Advanced)	**-2.2	=	**-1.2	**-0.5	*-0.4
1960 (Advanced)	**-1.9	=	*0.8	*-0.6	**-2.1
1973 (Advanced)	**-2.4	=	-0.6	-0.2	**-1.5
1997 (Emerging)	**-1.1	=	-0.1	0.0	**-1.0

Total Inflation

Discounted Surpluses Shock

- Discount rates drive innovations to inflation
- What drives discounted surpluses?
- Put differently: what drives unexpected returns on the basket of public bonds?

$$
\begin{aligned}
e_t &= E[e \mid \Delta E_t(\text{Disc Surpl}) = -1] \\
&= E[e \mid \Delta E_t(\epsilon_{s,t} + \epsilon_{g,t} - \epsilon_{r,t}) = -1]
\end{aligned}
$$

Discounted Surpluses Shock - Marked-to-Market

Country	ϵ_{r^n}	$-\epsilon_\pi$	=	ϵ_s	$+\epsilon_g$	$-\epsilon_r$
1947 Sample (Advanced)						
United Kingdom	**-0.8	**-0.2	=	**-0.5	-0.1	*-0.4
United States	**-0.7	**-0.3	=	0.0	**0.2	**-1.2
1960 Sample (Advanced)						
Canada	**-0.8	**-0.2	=	*0.2	-0.1	**-1.1
Denmark	**-0.8	**-0.2	=	*0.6	*0.5	**-2.0
Japan	**-0.6	**-0.4	=	0.0	-0.2	**-0.8
Norway	**-0.6	**-0.4	=	*1.0	*1.9	**-3.9
Sweden	**-0.6	**-0.4	=	**0.7	-0.2	**-1.5
1973 Sample (Advanced)						
Australia	**-0.8	**-0.2	=	*0.5	*0.2	**-1.7
New Zealand	**-0.6	**-0.4	=	**0.8	**-0.5	**-1.3
South Korea	**-0.6	**-0.4	=	**-2.4	**1.3	0.2
Switzerland	**-0.8	**-0.2	=	-0.1	*0.2	**-1.1

Discounted Surpluses Shock: $E[e \mid \Delta E_t(\epsilon_{s,t} + \epsilon_{g,t} - \epsilon_{r,t}) = -1]$

Discounted Surpluses Shock - Marked-to-Market

Country	ϵ_{r^n}	$-\epsilon_\pi$	=	ϵ_s	$+\epsilon_g$	$-\epsilon_r$
1997 Sample (Emerging)						
Brazil	**-0.5	**-0.5	=	**1.4	0.1	**-2.6
Colombia	**-0.6	**-0.4	=	0.0	**-0.3	**-0.8
Czech Republic	**-0.4	**-0.6	=	-0.1	-0.3	**-0.6
Hungary	**-0.6	**-0.4	=	*0.4	-0.3	**-1.2
India	**-0.5	**-0.5	=	-0.1	*-0.2	**-0.7
Israel	**-0.7	**-0.3	=	**0.6	-0.1	**-1.5
Mexico	**-0.6	**-0.4	=	**-0.6	0.1	*-0.6
Poland	**-0.7	**-0.3	=	**0.5	-0.1	**-1.4
South Africa	**-0.7	**-0.3	=	*-0.2	0.0	**-0.8
Ukraine	**-0.5	**-0.5	=	**-0.4	*-0.1	**-0.6

Discounted Surpluses Shock: $E[e \mid \Delta E_t(\epsilon_{s,t} + \epsilon_{g,t} - \epsilon_{r,t}) = -1]$

Discounted Surpluses Shock - Total Inflation

Country	$-\varepsilon_\pi$	=	ε_s	$+\varepsilon_g$	$-\varepsilon_r$
1947 Sample (Advanced)					
United Kingdom	**-0.9	=	**-0.5	-0.1	*-0.3
United States	**-0.5	=	0.0	**0.2	**-0.7
1960 Sample (Advanced)					
Canada	**-0.5	=	*0.2	-0.1	**-0.6
Denmark	**-0.6	=	*0.6	*0.5	**-1.6
Japan	**-0.7	=	0.0	-0.2	**-0.5
Norway	**-0.9	=	*1.0	*1.9	**-3.8
Sweden	**-0.8	=	**0.7	-0.2	**-1.2
1973 Sample (Advanced)					
Australia	**-0.6	=	*0.5	*0.2	**-1.3
New Zealand	**-0.8	=	**0.8	**-0.5	**-1.2
South Korea	**-1.2	=	**-2.4	**1.3	0.0
Switzerland	**-0.5	=	-0.1	*0.2	**-0.6

Discounted Surpluses Shock: $E[e \mid \Delta E_t(\epsilon_{s,t} + \epsilon_{g,t} - \epsilon_{r,t}) = -1]$

Discounted Surpluses Shock - Total Inflation

Country	$-\varepsilon_\pi$	=	ε_s	$+\varepsilon_g$	$-\varepsilon_r$
1997 Sample (Emerging)					
Brazil	**-0.3	=	**1.4	0.1	**-1.9
Colombia	**-0.3	=	0.0	**-0.3	-0.1
Czech Republic	**-0.5	=	-0.1	-0.3	-0.2
Hungary	**-0.6	=	*0.4	-0.3	**-0.8
India	**-0.6	=	-0.1	*-0.2	**-0.3
Israel	**-0.2	=	**0.6	-0.1	**-0.7
Mexico	**-0.6	=	**-0.6	0.1	-0.1
Poland	**-0.5	=	**0.5	-0.1	**-0.9
South Africa	**-0.3	=	*-0.2	0.0	*-0.1
Ukraine	**-0.6	=	**-0.4	*-0.1	**-0.1

Discounted Surpluses Shock: $E[e \mid \Delta E_t(\epsilon_{s,t} + \epsilon_{g,t} - \epsilon_{r,t}) = -1]$

Discounted Surpluses Shock - Averages

Country	ϵ_{r^n}	$-\epsilon_\pi$	=	ϵ_s	$+\epsilon_g$	$-\epsilon_r$
Averages	**-0.6	**-0.4	=	0.1	0.1	**-1.2
1947 (Advanced)	**-0.8	**-0.2	=	*-0.2	0.1	**-0.8
1960 (Advanced)	**-0.7	**-0.3	=	*0.5	0.4	**-1.9
1973 (Advanced)	**-0.7	**-0.3	=	-0.3	0.3	**-1.0
1997 (Emerging)	**-0.6	**-0.4	=	*0.2	*-0.1	**-1.1

Marked-to-Market

Country	$-\varepsilon_\pi$	=	ε_s	$+\varepsilon_g$	$-\varepsilon_r$
Averages	**-0.6	=	0.1	0.1	**-0.8
1947 (Advanced)	**-0.7	=	*-0.2	0.1	**-0.5
1960 (Advanced)	**-0.7	=	*0.5	0.4	**-1.6
1973 (Advanced)	**-0.8	=	-0.3	0.3	**-0.8
1997 (Emerging)	**-0.4	=	*0.2	*-0.1	**-0.5

Total Inflation

Model Overview

- How hard to reproduce empirical findings? Interpretation?
- Fiscal theory of the price level, New-Keynesian model
- Partial debt repayment (but still FTPL!)
- Trend shocks

Model Equations

- Private sector

$$y_t = E_t y_{t+1} - \gamma(i_t - E_t \pi_{t+1}) + \rho_g u_{g,t}$$
$$\pi_t = \beta E_t \pi_{t+1} + \kappa y_t - \kappa_a a_t$$
$$g_t = y_t - y_{t-1} - u_{g,t}$$

- Central bank

$$i_t = \phi_\pi \pi_t + \phi_g g_t + u_{i,t}$$

Why trend shocks?

- Otherwise, output stationary $\implies \varepsilon_{g,t} \approx 0$
- Model solution: $X_t = a(L)e_t$ for finite $a(1)$
- Growth term of decomposition:

$$\epsilon_{g,t} = 1_g' a(\beta) e_t$$

- In the absence of trend shocks:

$$\begin{aligned} g_t = 1_g' a(L) e_t &= 1_y' (1 - L) a(L) e_t \\ 1_g' a(L) &= 1_y' (1 - L) a(L) \end{aligned}$$

- Therefore $\epsilon_{g,t} = 1_g' a(\beta) e_t \approx 1_g' a(1) e_t = 0$

Model Equations

- Flow of debt

$$v_t + \frac{s_t}{V} = \frac{1}{\beta}\left[v_{t-1} + r_t^n - \pi_t - g_t\right]$$

$$r_t^n = \delta\left[r_t^N\right] + (1-\delta)\left[r_t^R + \pi_t\right]$$

- Bond prices and return

$$q_{N,t} = (\omega_N \beta) E_t q_{N,t} - \left[i_t\right]$$

$$q_{R,t} = (\omega_R \beta) E_t q_{R,t} - \left[i_t - E_t \pi_{t+1}\right]$$

$$r_{j,t} = (\omega_j \beta) q_{j,t} - q_{j,t-1} \qquad j = N, R$$

Fiscal Policy

- Surplus-to-GDP could follow

$$s_t = h_t = \tau(\pi_t + g_t) + u_{s,t}$$

  where $u_{s,t} \sim \text{AR}(1)$

- No debt repayment:

$$s_t \downarrow \implies \sum_\tau \beta^\tau s_\tau \downarrow \implies \pi_t \uparrow$$

Fiscal Policy

$h \sim$ AR(1)

- v = debt repayment parameter
- Surplus-to-GDP process

$$s_t = s_t^* + (1 - v)\, h_t$$
$$s_t^* = \alpha v_{t-1}^* + v\, h_t$$
$$v_t^* = (1/\beta)\, v_{t-1}^* - s_t^*$$

- s_t and s_t^* respond to debt target v^*

Fiscal Policy

- What is the role of v_t^*?

$$s_t = s_t^* + (1 - v)\, h_t \tag{1}$$
$$s_t^* = \alpha v_{t-1}^* + v\, h_t \tag{2}$$
$$v_t^* = (1/\beta)\, v_{t-1}^* - s_t^* \tag{3}$$

- (2) and (3): v^* is stationary
- Solve (3) forward:

$$v_{t-1}^* = \beta \sum_{j=0}^{\infty} \beta^j E_t s_{t+j}^* = \beta \sum_{j=0}^{\infty} \beta^j \left[E_t s_{t+j} - (1 - v) E_t h_{t+j} \right]$$

Fiscal Policy

$$v^*_{t-1} = \beta \sum_{j=0}^{\infty} \beta^j \left[E_t s_{t+j} - (1 - v) E_t h_{t+j} \right]$$

- Take innovations $\Delta E_t = E_t - E_{t-1}$

$$\beta \sum_{j=0}^{\infty} \beta^j \Delta E_t s_{t+j} = (1 - v) \sum_{j=0}^{\infty} \beta^j \Delta E_t h_{t+j}$$

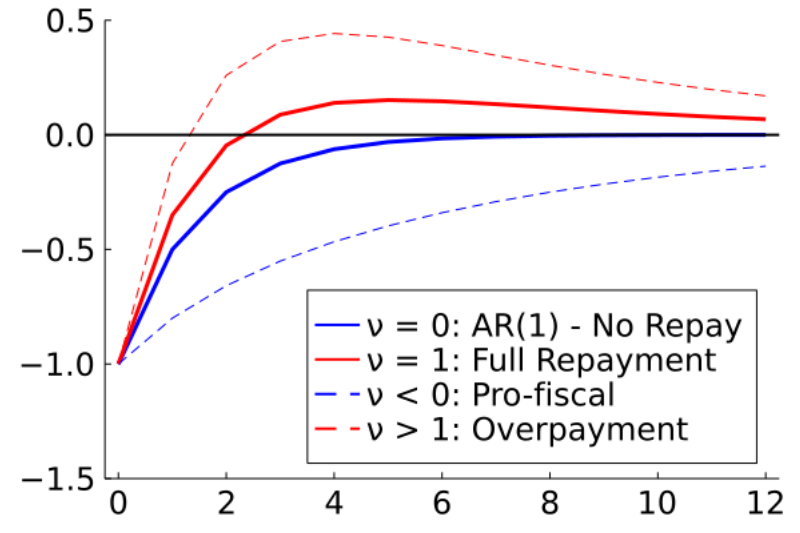
$$\boxed{\epsilon_{s,t} = (1 - v) \sum_{j=0}^{\infty} \beta^j \Delta E_t h_{t+j}}$$

- v governs debt repayment

Partial debt repayment

- $\boxed{v = 0}$ No debt repayment: $\epsilon_{s,t} = \sum_{j=0}^{\infty} \beta^j \Delta E_t h_{t+j}$
  - $s_t = h_t$ (standard AR(1))
- $\boxed{v = 1}$ Full debt repayment: $\epsilon_{s,t} = 0$
  - $s_t = s_t^* = \alpha v_t^* + h_t$
- $\boxed{v < 0}$ "Pro-fiscal" surplus: $\epsilon_{s,t} / \sum_{j=0}^{\infty} \beta^j \Delta E_t h_{t+j} > 1$
- $\boxed{v > 1}$ "Overpayment": $\epsilon_{s,t} / \sum_{j=0}^{\infty} \beta^j \Delta E_t h_{t+j} < 0$

Partial debt repayment - Cases



Response to deficit shock $u_{s,t} = -1$

GMM Estimation

- Method of moments:

$$\text{Min}_{\theta} \quad {\scriptstyle w} \left\| \mathscr{D}_{VAR} - \mathscr{D}_{NK}(\theta) \right\| + {\scriptstyle (1-w)} \left\| \mathscr{M} - \mathscr{M}_{NK}(\theta) \right\|$$

- $\mathscr{D}$ contains MtM decomposition for inflation shock
- $\mathscr{M}$ contains second moments
- Estimates for the United States

GMM Estimation

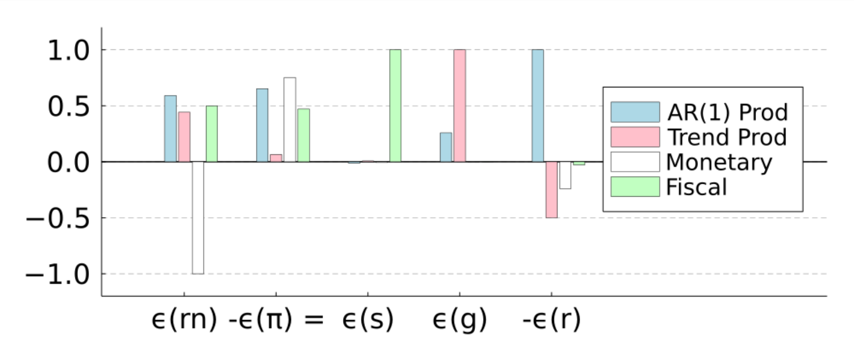
United States Estimates

Fixed		Estimated	
Parameter	Value	Paramater	Value
β	0.99	ρ_a	0.98
γ	0.4	ρ_g	0.23
φ	3	ρ_i	0.00
θ	0.25	ρ_s	0.72
$\bar{\omega}$	γ^{-1}	ϕ_π	0.68
σ_a	1	ϕ_g	0.00
		τ	-0.06
		v	0.89
		α	0.01
		σ_g	1.21
		σ_g	0.53
		σ_g	1.07

US Model Parameters

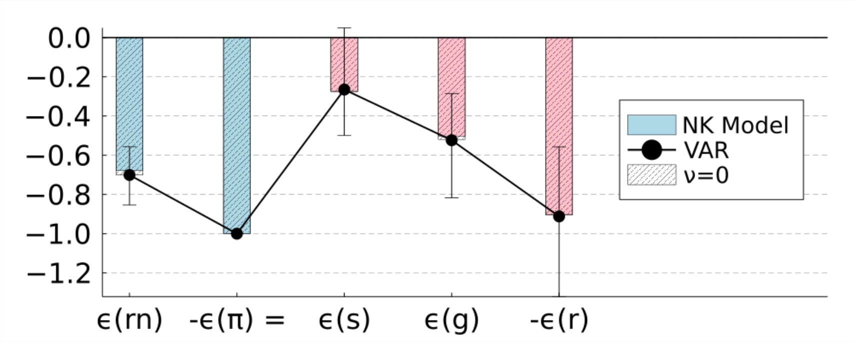
GMM Estimation

United States Estimates



Fiscal decomposition of structural shocks

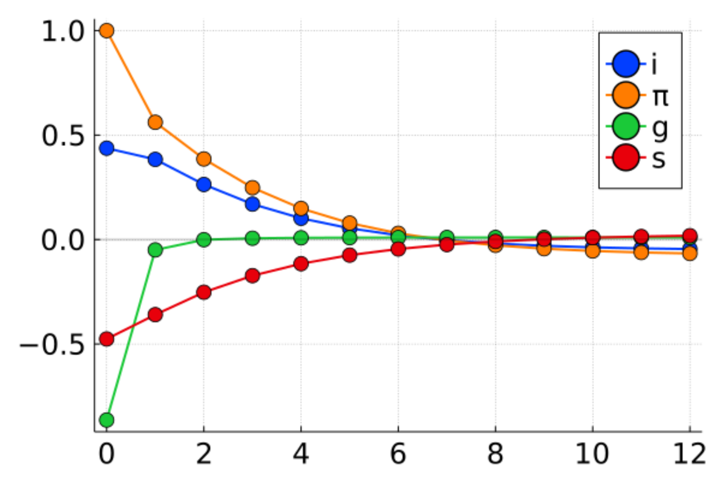
Is AR(1) surplus a good model?



MtM decomposition of Inflation Shock $e_t = E\left[e \mid \Delta E_t \pi_t = 1\right]$

Is AR(1) surplus a good model?

Structural shocks: $\varepsilon_a = -1$, $\varepsilon_g = -0.2$, $\varepsilon_i = -0.3$, $\varepsilon_s = -0.5$



Inflation Shock

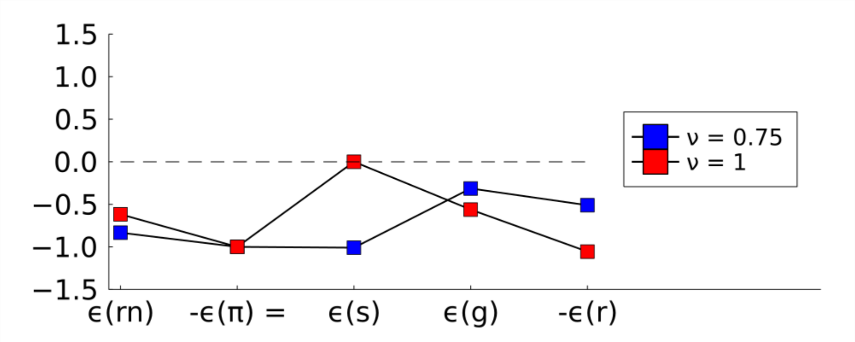
Is AR(1) surplus a good model?

- $v = 0$ precludes realistic fiscal policy and discount-driven inflation at the same time

	Data	$v = 0.9$	$v = 0$		Data	$v = 0.9$	$v = 0$
σ_i / σ_g	1.29	0.77	1.25	$\text{cor}(\pi, i)$	0.70	0.88	0.89
σ_π / σ_g	1.20	1.10	1.56	$\text{cor}(\pi, g)$	-0.11	-0.35	-0.40
σ_s / σ_g	1.08	1.09	0.45	$\text{cor}(g, i)$	0.04	-0.35	-0.04
acor(i)	0.91	0.75	0.87	$\text{cor}(i, s)$	-0.26	-0.28	-0.46
acor(π)	0.69	0.72	0.81	$\text{cor}(\pi, s)$	-0.28	-0.29	-0.41
acor(g)	0.14	0.14	0.16	$\text{cor}(g, s)$	0.01	-0.04	-0.05
acor(s)	0.64	0.72	0.27				

Second Moment Fit

Is AR(1) surplus a good model?



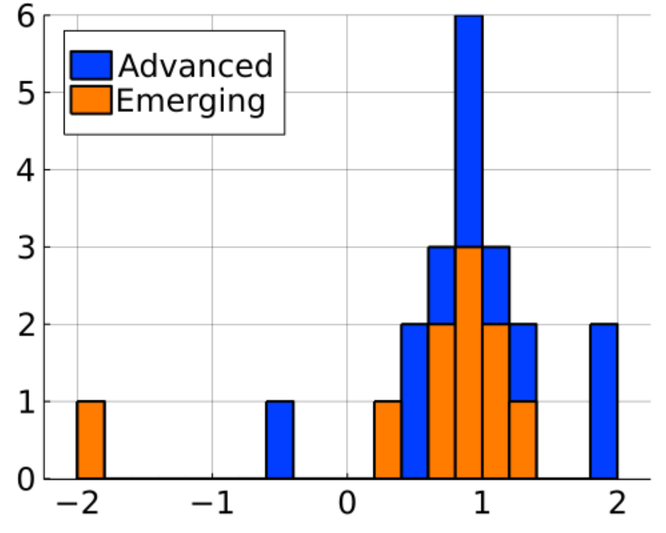
Comparative Statics for ν

Cross-Country Estimates

$$\text{Min}_{\theta} \quad w \left\| \mathscr{D}_{VAR} - \mathscr{D}_{NK}(\theta) \right\| + (1-w) \left\| \mathscr{M} - \mathscr{M}_{NK}(\theta) \right\|$$

- Repeat procedure for each country in the sample
- Use corresponding debt profile $(\delta, \omega_N, \omega_R)$

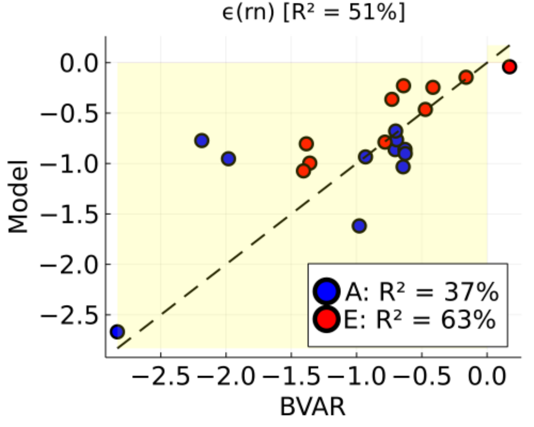
Cross-Country Estimates of Debt Repayment v



Histogram of v estimates

Cross-Country Fit of Fiscal Decomposition

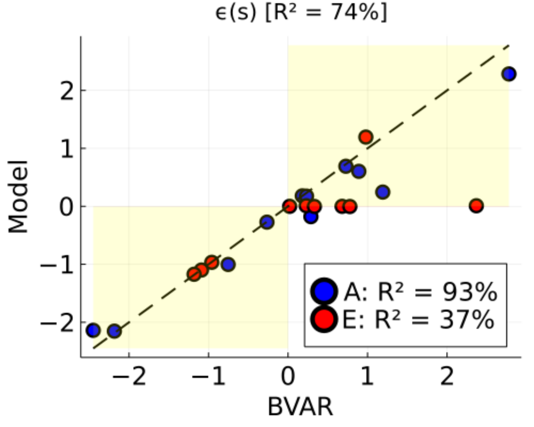
$$\boxed{\color{red}{\epsilon_{r^n,t}} - \epsilon_{\pi,t} = \epsilon_{s,t} + \epsilon_{g,t} - \epsilon_{r,t}}$$



Fit of Bond Price Term $\epsilon_{r^n,t}$

Cross-Country Fit of Fiscal Decomposition

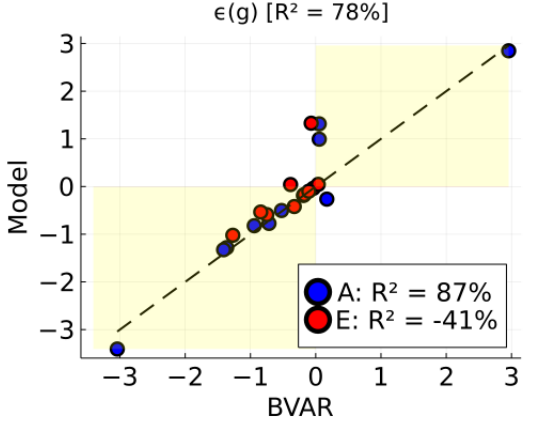
$$\boxed{\epsilon_{r^n,t} - \epsilon_{\pi,t} = \epsilon_{s,t} + \epsilon_{g,t} - \epsilon_{r,t}}$$



Fit of Surplus Term $\epsilon_{s,t}$

Cross-Country Fit of Fiscal Decomposition

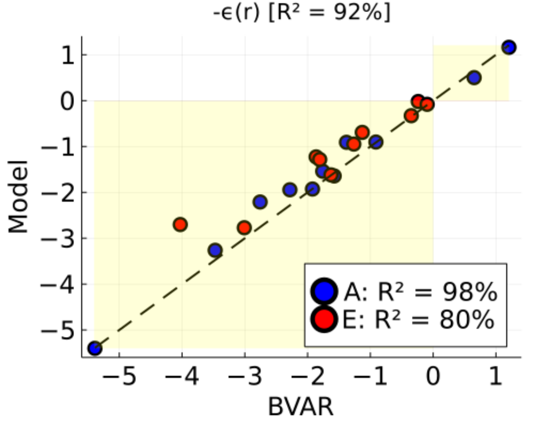
$$\boxed{\epsilon_{r^n,t} - \epsilon_{\pi,t} = \epsilon_{s,t} + \epsilon_{g,t} - \epsilon_{r,t}}$$



Fit of Growth Term $\epsilon_{g,t}$

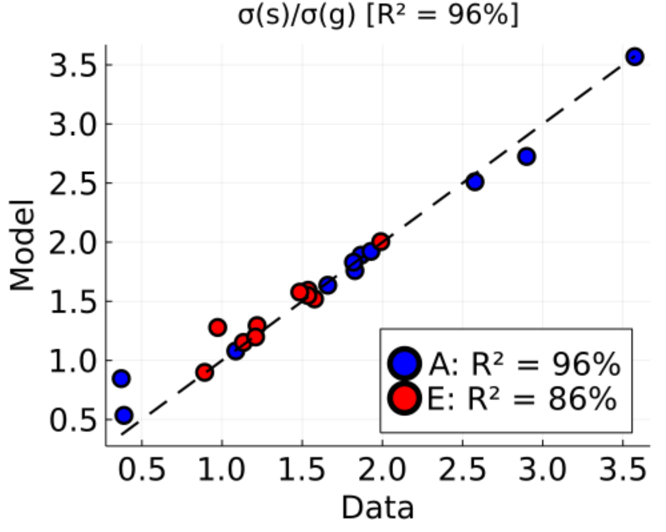
Cross-Country Fit of Fiscal Decomposition

$$\boxed{\epsilon_{r^n,t} - \epsilon_{\pi,t} = \epsilon_{s,t} + \epsilon_{g,t} - \epsilon_{r,t}}$$



Fit of Discount Term $\epsilon_{r,t}$

Cross-Country Fit of Fiscal Policy Volatility



Fit of Fiscal Policy Volatility

Conclusion

- Innovations to inflation driven mostly by discount rates
- Monetary-fiscal models require partial debt repayment (80-100%)
- Fiscal decomposition as useful moment to identify debt repayment

Frametitle

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References I

Akhmadieva, V. (2022). Fiscal adjustment in a panel of countries 1870–2016. *Journal of Comparative Economics*, 50(2):555–568.

Bassetto, M. and Cui, W. (2018). The fiscal theory of the price level in a world of low interest rates. *Journal of Economic Dynamics and Control*, 89:5–22.

Brunnermeier, M. K., Merkel, S., and Sannikov, Y. (2022). The Fiscal Theory of the Price Level With a Bubble.

Cagan, P. (1956). The Monetary Dynamics of Hyperinflation. In *Studies in the Quantity Theory of Money*, pages 25–117. University of Chicago Press, milton friedman edition.

Campbell, J. Y. and Ammer, J. (1993). What Moves the Stock and Bond Markets? A Variance Decomposition for Long-Term Asset Returns. *The Journal of Finance*, 48(1):3–37.

References II

Campbell, J. Y. and Shiller, R. J. (1988). The Dividend-Price Ratio and Expectations of Future Dividends and Discount Factors. *The Review of Financial Studies*, 1(3):195–228.

Chen, L. and Zhao, X. (2009). Return Decomposition. *Review of Financial Studies*, 22(12):5213–5249.

Cochrane, J. H. (1992). Explaining the Variance of Price–Dividend Ratios. *The Review of Financial Studies*, 5(2):243–280.

Cochrane, J. H. (1998). A Frictionless View of U.S. Inflation. *NBER Macroeconomics Annual*, 13:323–384.

Cochrane, J. H. (2005). Money as stock. *Journal of Monetary Economics*, 52(3):501–528.

Cochrane, J. H. (2008). The Dog That Did Not Bark: A Defense of Return Predictability. *The Review of Financial Studies*, 21(4):1533–1575.

References III

Cochrane, J. H. (2022a). The fiscal roots of inflation. *Review of Economic Dynamics*, 45:22–40.

Cochrane, J. H. (2022b). A fiscal theory of monetary policy with partially-repaid long-term debt. *Review of Economic Dynamics*, 45:1–21.

Cochrane, J. H. (2022c). The Fiscal Theory of the Price Level.

Corsetti, G., Dedola, L., Jarociński, M., Maćkowiak, B., and Schmidt, S. (2019). Macroeconomic stabilization, monetary-fiscal interactions, and Europe's monetary union. *European Journal of Political Economy*, 57:22–33.

Cox, W. M. (1985). The behavior of treasury securities monthly, 1942–1984. *Journal of Monetary Economics*, 16(2):227–250.

Cox, W. M. and Hirschhorn, E. (1983). The market value of U.S. government debt; Monthly, 1942–1980. *Journal of Monetary Economics*, 11(2):261–272.

References IV

Du, W., Plueger, C. E., and Schreger, J. (2020). Sovereign Debt Portfolios, Bond Risks, and the Credibility of Monetary Policy. *The Journal of Finance*, 75(6):3097–3138.

Hall, G. J. and Sargent, T. J. (1997). Accounting for the federal government's cost of funds. *Economic Perspectives*, 21(4).

Hall, G. J. and Sargent, T. J. (2011). Interest Rate Risk and Other Determinants of Post-WWII US Government Debt/GDP Dynamics. *American Economic Journal: Macroeconomics*, 3(3):192–214.

Jiang, Z., Lustig, H., Van Nieuwerburgh, S., and Xiaolan, M. (2019). The U.S. Public Debt Valuation Puzzle. Technical Report w26583, National Bureau of Economic Research, Cambridge, MA.

Leeper, E. and Leith, C. (2016). Chapter 30 - Understanding Inflation as a Joint Monetary–Fiscal Phenomenon. In *Handbook of Macroeconomics*, volume 2, pages 2305–2415. Elsevier.

References V

Leeper, E. M. (1991). Equilibria under ‘active’ and ‘passive’ monetary and fiscal policies. *Journal of Monetary Economics*, 27(1):129–147.

Sargent, T. J. and Wallace, N. (1981). Some Unpleasant Monetarist Arithmetic. *Quarterly Review*, 5.

Sims, C. A. (1994). A simple model for study of the determination of the price level and the interaction of monetary and fiscal policy. *Economic Theory*, 4(3):381–399.

Sims, C. A. (2011). Stepping on a rake: The role of fiscal policy in the inflation of the 1970s. *European Economic Review*, 55(1):48–56.

Sunder-Plassmann, L. (2020). Inflation, default and sovereign debt: The role of denomination and ownership. *Journal of International Economics*, 127:103393.

Woodford, M. (1995). Price-level determinacy without control of a monetary aggregate. *Carnegie-Rochester Conference Series on Public Policy*, 43:1–46.