Title

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1 Unexpected Inflation in a Benchmark NK Model

1.1 Connection with the Value of Debt

I start by establishing the connection between inflation and the value of debt.

Accounting. Consider a government that manages a public debt composed of one-period bonds, denominated in a specific currency. There is no constraint on the nature of this currency, and the assumption of one-period bonds buys simplicity but is not critical for the argument.

At the beginning of period t, the face value of debt issued in the previous period is V_{t-1}. The government finances the payment of V_{t-1} by either raising new debt maturing in the following period at a price Q_t or by running a (nominal) primary surplus S_t.

$$V_{t-1} = Q_t V_t + S_t \tag{1}$$

Let $P_t > 0$ be the relative price of an arbitrary basket of goods in terms of our selected currency and define $\beta_t \equiv Q_t P_{t+1}/P_t$ and $\beta_{t,t+k} \equiv \prod_{\tau=t}^{t+k} \beta_\tau$. Variable β_t represents a real discount for public bonds relative to the basket of goods. Since V satisfies (1), it also satisfies

$$\frac{V_{t-1}}{P_t} = \sum_{i=0}^{k} \beta_{t,t+i-1} \left(\frac{S_{t+i}}{P_{t+i}} \right) + \beta_{t,t+k} \frac{V_{t+k}}{P_{t+k+1}} \qquad \text{for any } k \geq 0 \tag{2}$$

regardless of the paths of public debt prices and surpluses. Importantly, expressions (1) and (2) do not represent a constraint on the path of nominal surpluses $\{S_t\}$ the government chooses to follow. They merely express the value of future debt given $\{S_t\}$, prices and current face value V_{t-1}.

Economics. I now make two assumptions about agents' behavior.

Assumption 1: At period t economic agents form expectations over the path of future, unknown variables through an operator $\tilde{E}_t$ which satisfies the linearity condition $\tilde{E}_t(X+Y) = \tilde{E}_t(X) + \tilde{E}_t(Y)$ and $\tilde{E}_t(X) = X$ if X is in the information set at t. Such operator can be heterogeneous across agents as long as these two conditions are satisfied.

Assumption 2: $\lim_{k\to\infty} \tilde{E}_t \left(\beta_{t,t+k} \frac{V_{t+k}}{P_{t+k+1}} \right) = 0$ at every period t. Assumption 2 is tipically referred to a no-bubble condition. In most micro-founded models, it is not even an assumption, but a result of optimal intertemporal consumption choice by households.

Assumptions 1 and 2 added to (2) lead to the valuation equation of public debt:

$$\frac{V_{t-1}}{P_t} = \sum_{i=0}^{\infty} \tilde{E}_t \left(\beta_{t,t+i-1} \frac{S_{t+i}}{P_{t+i}} \right). \tag{3}$$

Given the (pre-determined) face value of maturing public debt, the relative price of the basket of goods in terms of debt currency is determined by the expected β-discounted stream of surpluses in terms of the basket of goods. This latter term - the right-hand side of (3) - I call the *real value of public debt*.

Equation (3) provides the connection between fiscal and inflation shocks I explore in the paper. (TODO: Mention importance of revision of public bonds' prices, absent here)

(Incomplete: Maybe include "What about Japan?" footnote)

1.2 The New-Keynesian Model

I start with the two usual equations of the New-Keynesian model. All variables should be interpreted as deviations from a steady-state equilibrium.

$$c_t = E_t c_{t+1} - \sigma \left[i_t - E_t \pi_{t+1} \right] \tag{4}$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa y_t \tag{5}$$

along with an equation for market clearing for goods market:

$$y_t = \gamma c_t + g_t, \tag{6}$$

where y, c, i, π and g represent respectivelly log-output, log-consumption, the interest rate, the inflation rate and government spending in levels.[1]

The stock of real public debt v follows the law of motion (reference to earlier equation, reference to the use of rational expectations - use of E, not $\tilde{E}$, reference to why assumption 2 above holds here)

$$\beta v_t = v_{t-1} + i_{t-1} - \pi_t - s_t \tag{7}$$

where $s_t \equiv \tau_t - g_t$ is the public primary surplus (which does not include interest payments on debt). τ_t are total tax proceeds in levels. In the stationary equilibrium of the NK model assumption 2 above holds, and, hence, v coincides with the real value of public debt.

Policy. *Observed* monetary policy is muted, except for a white-noise shock: $i_t = \epsilon_{i,t}$.

Fiscal policy prescribes the following rules for taxation (which I assume to be entirely *lump-sum*) and public expenditures:

$$\tau_t = \rho_\tau \tau_{t-1} + \alpha_\tau v_t + \epsilon_{\tau,t} \tag{8}$$

$$g_t = \rho_g g_{t-1} - \alpha_g v_t + \epsilon_{g,t}. \tag{9}$$

Stability of public debt requires either $\alpha_\tau > 0$ or $\alpha_g > 0$.

The equations above by themselves do not determine unexpected changes to the real value of public debt, and hence they do not pin down unexpected inflation. The last equation characterizing policy solves that issue:

$$\pi_t = E_{t-1} \pi_t + \eta_t, \tag{10}$$

[1] γ represents the steady-state consumption-to-output ratio. The choice of linearizing equilibrium conditions around the level of government spending and not its log makes the connection with the rest of the paper clearer. I also linearize around an equilibrium with output = real debt.

for an exogenous term η_t. The term $\Delta E_t \pi_t = (E_t - E_{t-1})\pi_t$ I call unexpected current inflation. In this case, unexpected current inflation is given by η_t.

There are two existing selection mechanisms that justify equation (10) and provide an interpretation to it: fiscal selection and the spiral threat selection. Both imply (10) while leaving other equations unchanged (observational equivalence, Cochrane (2011), Cochrane (1998)) and, more importantly, both interpret η as part of public policy, as a government *choice*.

Fiscal selection, or the fiscal theory of the price level, arrives at (10) by means of (3), with causality coming from right to left. Any economic shock can change the conditional distribution of discounted future surpluses (in units of goods) backing the stock of public nominal liabilities. It can thus change its real value. The relative price of public debt in terms of goods $1/P_t$ then adjusts to reflect that change, much in the same way that news of future dividends (in money units) per stock change the relative price of stocks in terms of money (Cochrane (2005)). (Maybe include a reading of 2022 US inflation through the lenses of FS and STS)

Spiral threat selection is the approach that most of the monetary economics literature has adopted so far. It arrives at (10) by means of an explosive root introduced by an interest policy equation of the format $i_t = \phi\pi_t, \phi > 1$. The equation was incorrectly associated to the famous Taylor (1993) rule, for its role in the NK model is by no means to stabilize "demand" shocks via fast, pro-cyclical real interest rates. *Au contraire*, the policy rule here introduces the instability required by the NK model to pin down unexpected inflation. Assuming muted monetary policy $i_t = 0$, the system of equations (4)-(5) (with $c = y$ for simplicity) is "too stable": it contains one explosive eigenvalue for two forward-looking variables. Any choice of unexpected inflation forms a stable equilibrium path that converges to the zero steady state.[2] Equation $i_t = \phi\pi_t$ solves that issue when $\phi > 1$.

Importantly, the *selection of equilibrium* is completely unrelated to the *observed* interest rate. This is why i_t = white noise as above is a perfectly fine specification for *observed* interest. More rigorously, consider the basic NK system (4)-(5), with $c = y$. Add to that the following equations:

$$i_t = i_t^* + \phi(\pi_t - \pi_t^*) \qquad \phi > 1 \tag{ST-1}$$

$$i_t^* - E_t\pi_{t+1}^* = i_t - E_t\pi_{t+1} \tag{ST-2}$$

$$i_t^* \text{ given for all } t \tag{ST-3}$$

$$\pi_t^* \text{ given at } t. \tag{ST-4}$$

Format (ST-1) is due to King and William (1996); i^* is the central bank's desired observed interest rate. The term π_t^* is a stochatic inflation target. Equation (ST-2) asks that the government's choices respect private market conditions and expectations formation. It forces the government to elect *unexpected* inflation only.[3]

Mechanically, one can combine (ST-1) and (ST-2) to find $E_t\pi_{t+1} - E_t\pi_{t+1}^* = \phi(\pi_t - \pi_t^*)$; $\phi > 1$

[2] Economists have interpreted this feature as admissibility of "sunspot" shocks. Without a selection mechanism, (4)-(5) will only determine the unexpected component of one variable, *if* it is fed the unexpected component of the other.

[3] The attentive eye may have noticed an apparent modelling sin: system (4)-(5), (ST-1)-(ST-4) presents six equations, for only five variables: y, π, π^*, i, i^*. There is no over-identification, nevertheless. Target inflation enters the system both as a static (= forward-looking) variable π_t^* *and* as a state variable, in expected value $E_{t-1}\pi_t^*$. Another way to write (ST-3) would be $E_{t-1}\pi_t = i_t^* - (i_t - E_{t-1}\pi_t)$. It becomes evident then that (ST-4) only really picks the unexpected component of inflation.

and Blanchard and Kahn (1980)'s razor then champion the unique stationary path $\pi = \pi^*$, $i = i^*$, which form the *observed* equilibrium. Parameter ϕ remains unidentified (Cochrane (2011)).

Researchers have interpreted (ST-1) as a threat of nominal spiral - hence my name choice "spiral threat" selection. Different papers discuss if central banks can indeed rule out nominal spirals, but the key assumptions here do not really relate to what the central bank can do, but what *households believe* it can and would. Indeed, note that there is nothing particularly special about inflation in (ST-1)-(ST-4). One could as well write the whole system using an output target instead:

$$i_t = i_t^* + \phi(y_t - y_t^*) \qquad \phi > 1 \tag{ST-1'}$$

$$i_t^* - E_t y_{t+1}^* = i_t - E_t y_{t+1} \tag{ST-2'}$$

and now the "threat" is not that of a nominal spiral, but of a *real* spiral. Obviously, the central bank cannot trigger a "hyperproduction" (as in hyperinflation) process. Neither could it stop one, say if productivity for some reason started to grow at exponential rates. But, if the central bank vacuously threatens hyperproduction, and it is the case that agents believe its threat; and if then the central bank vacuously promisses to stop the hypothetical hyperproduction it has vacuously threatened to create, and again agents trust its word; then and only then does the Blanchard and Kahn (1980) equilibrium arranged by (ST-1')-(ST-2') arises. The actual powers of the central bank are irrelevant.

While I favor a fiscal selection interpretation of unexpected inflation throughout the article, the takeaway from this discussion is that both equilibrium selection mechanisms interpret as a government *choice* - even if an indirect one - the determination of unexpected inflation.

1.3 Relationship with Sims (1980) Orthogonalization Method

The view that unexpected inflation is a choice automatically microfounds the orthogonalization process proposed by Sims (1980).

(Incomplete)

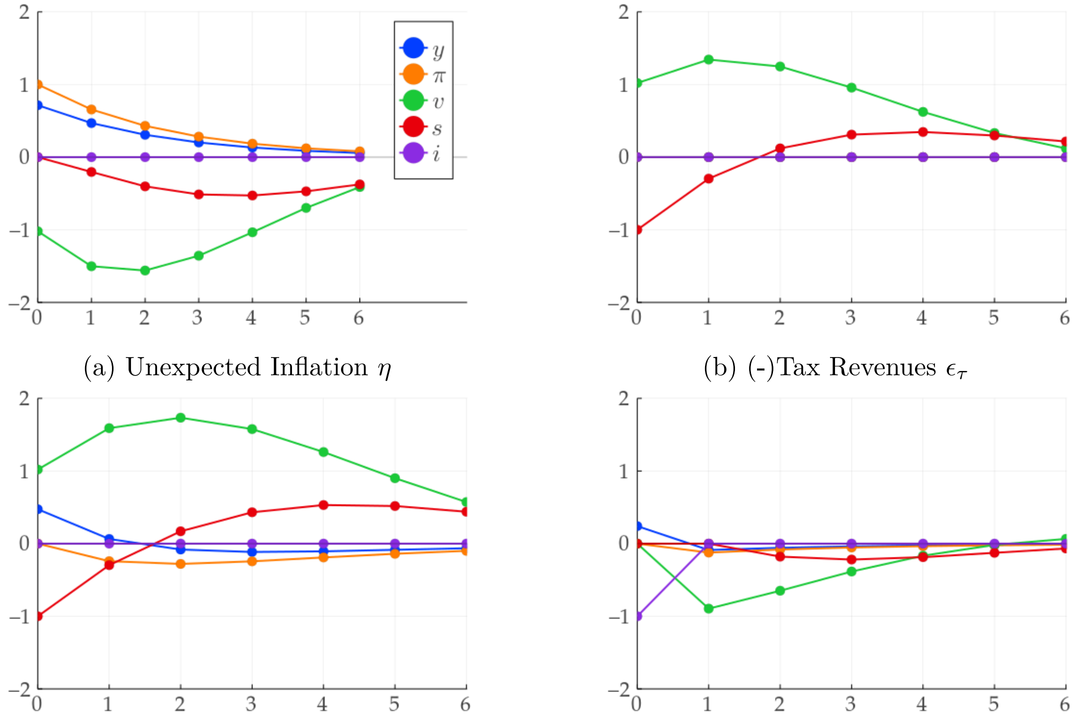
1.4 Unexpected Current Inflation

Consider the response of the New-Keynesian model to η, and ϵ shocks, one at a time, plotted in figure 1. Calibration follows literature standards: $\sigma = 0.5$, $\beta = 0.98$, $\gamma = 0.75$ and $\rho_\tau = \rho_g = 0.5$. Momentarily, I set $\kappa = 0.50$ to make figures pretty. In this benchmark case, I consider fiscal adjustment via taxation only: $\alpha_\tau = 0.2$ and $\alpha_g = 0.0$.

Panel 1a plots the response to the unexpected inflation shock η. Inflation jumps by assumption, the fiscal interpretation being that agents foresee a reduced stream of surpluses. Accordingly, the real value of public debt v jumps down on spot. A lower debt leads taxation τ to decline (not plotted) via the $\alpha_\tau v_{t-1}$ term. The government runs deficits starting in the first period following the shock (I refer to $s < 0$ as a fiscal deficit). These deficits 1. slowly bring v back to zero and 2. validade agents' expectation at period zero of a lower value of public bonds - indeed primary surpluses were lower.

The impact on economic activity resembles the typical Keynesian "demand" shock, combining an increase in inflation and output at the same time. Positive inflation in period zero leads to a negative real interest rate; the IS curve (4) then implies output larger than future output - output is large and declining.[4] Large output implies large marginal costs, and, by (5), inflation greater than

[4] The apparently small response of output follows from the choice of κ. Values that are lower than my choice lead



(a) Unexpected Inflation η

(b) (-)Tax Revenues ϵ_τ

(c) Government Spending ϵ_g

(d) (-)Nominal Interest ϵ_i

Figure 1: NK Model - Impulse-Response Function to Policy Shocks

future inflation - inflation is thus positive and declining.

Panels 1b and 1c show that, in the absence of unexpected inflation in period zero, expansionary fiscal policy fails to generate inflation at all in the basic NK model. A negative shock to taxation - the model version of COVID checks - simply leads to an increase in public debt, subsequently paid through taxes that turn positive in period two. Households are unconstrained and have zero marginal propensity to consume out of their checks. Output thus stays put, which implies $\pi_t = E_t\pi_{t+1}$ by the Phillips curve. $\pi_0 = 0$ follows from the absence of unexpected inflation.

A positive shock to public spending g does affect output and inflation, as the government directly purchases goods from firms (equation (6)). We can think of government spending as a transfer to a fictional "public household" with constant marginal propensity to consume equal to one. Output increases in period zero. The Phillips curve then says that current inflation must be greater than future. But since current inflation is zero (no unexpected jump by assumption), that means inflation *declines* from period zero to one. In the absence of unexpected inflation, the NK model predicts below-average inflation, or even deflation, as a consequence of increased public expenditure.

Lastly, panel 1d corresponds to an expansionary monetary policy shock. Without unexpected inflation, the effect of a monetary policy shock is purely Fisherian (references of Fisherian interest shock): lower interest forecasts lower inflation. Stimulative interest does stimulate output, albeit for a single period, as low inflation produces a contractionary effect thereafter.

Figure 1 assumes fiscal adjustment is carried out entirely through tax instead of spending adjustments ($\alpha_\tau > 0$, $\alpha_g = 0$). The symmetric opposite assumption little changes the predictions of

to more pronounced responses of equal sign.

the model. The only qualitative change happens in the case of the tax reduction shock ϵ_τ. Since government spending changes with debt, there are small output effects that lead to lower inflation in the transition - again the "wrong" sign. The quantitative effects are nevertheless small, and I leave the IRF figures to the appendix.

Combined Effects. To have fiscal shocks - the ϵ's in (8) and (9) - be accompanied by unexpected current inflation η_t is critical for the New-Keynesian model to deliver responses to fiscal policy consistent with the view of most economists and empirical evidence (PAPERS WITH IDENTIFIED FISCAL SHOCKS).

In light of the connection between inflation and the value of debt, one might even expect η and the ϵ's to be correlated. Say, unexpectedly large spending leads to lower surpluses, hence a lower value of debt, hence unexpected inflation. However intuitive, the proposition requires empirical verification. This is where we go in the next section.

1.5 Unexpected Total Inflation

Economic news lead not only to the revision of expectations of current inflation, but also of its entire future path. As Sims (2011) exemplifies with his study of the effects of monetary policy in US inflation in the 1970s, the short-run innovation can be remarkably different than the one long-run one.

(Incomplete)

We can solve the linearized law of motion of public debt (7) forward and apply assumptions 1 and 2 to arrive at

$$v_{t-1} = \sum_{i=0}^{\infty} \beta^i E_t s_{t+i} - \sum_{i=0}^{\infty} \beta^i E_t i_{t-1+i} + \sum_{i=0}^{\infty} \beta^i E_t \pi_{t+i},$$

which generalizes (3). The inflation revaluation of public debt term (the denominator on the left side of (3)) corresponds to the first term of the inflation sum on the right-hand side. Taking innovations:

$$0 = \sum_{i=0}^{\infty} \beta^i \Delta E_t s_{t+i} - \sum_{i=0}^{\infty} \beta^i \Delta E_t i_{t-1+i} + \underbrace{\sum_{i=0}^{\infty} \beta^i \Delta E_t \pi_{t+i}}_{\substack{\text{Unexpected Total} \\ \text{Inflation}}}. \tag{11}$$

Expression (11) is from Cochrane (2022). The sum $\sum \beta^i \Delta E_t \pi_{t+i}$ I call *unexpected total inflation*, which is really shorthand for revision of expectations over the discounted inflation path. I also refer to unexpected future surpluses and interest in a similar manner.

Decomposition (11) simply states that surprises to the relative price of public liabilities in terms of goods, captured by unexpected current inflation $\Delta E_t \pi_{t+1}$, reflect changes in the real value of public debt. Re-writting it as

$$- \underbrace{\Delta E_t \pi_t}_{\substack{\text{Unexpected Current} \\ \text{Inflation}}} = \underbrace{\sum_{i=0}^{\infty} \beta^i \Delta E_t s_{t+i} - \sum_{i=0}^{\infty} \beta^i \Delta E_t i_{t-1+i} + \overbrace{\sum_{i=1}^{\infty} \beta^i \Delta E_t \pi_{t+i}}^{\substack{\text{Unexpected Future} \\ \text{Inflation}}}}_{\text{Innovation to the Real Value of Public Debt}}.$$

makes that point clear. It also reminds us that equation (11) continues to be a debt valuation

Policy Shock	$\sum \beta^i \Delta E_t \pi_{t+i}$	$\sum \beta^i \Delta E_t s_{t+i}$	$-\sum \beta^t \Delta E_t i_{t-1+i}$
Fiscal Adjustment via Taxes ($\alpha_\tau > 0$, $\alpha_g = 0$)			
Unexpected inflation η_t	1.46	-1.46	0
(-) Tax revenue ϵ_τ	0	0	0
Government spending ϵ_g	-2.06	2.06	0
(-) Monetary policy ϵ_i	-0.66	-0.32	0.98
Fiscal Adjustment via Spending ($\alpha_\tau = 0$, $\alpha_g > 0$)			
Unexpected inflation η_t	1.46	-1.46	0
(-) Tax revenue ϵ_τ	-0.02	0.02	0
Government spending ϵ_g	-2.17	2.17	0
(-) Monetary policy ϵ_i	-0.66	-0.32	0.98
Fiscal Adjustment via Taxes + Taylor Rule ($i_t = \phi \pi_t$)			
Unexpected inflation η_t	2.49	-1.27	-1.22
(-) Tax revenue ϵ_τ	0	0	0
Government spending ϵ_g	-3.9	2	1.92
(-) Monetary policy ϵ_i	-1.24	-0.35	1.59

Table 1: Unexpected Total Inflation

equation, not a budget constraint. Yet, the use of language such as "higher total inflation *pays* for lower total surpluses" can often simplify the exposition.

The β discounting is the linearized version of the $\beta_{t,t+k}$ discounting term of equations (2) and (3). It means that we mark-to-(steady-state-)market each revision of expectation, so that we may interpret each sum in terms of current-period market value.

Table 1 shows the decomposition for the policy shocks of NK model. Note the minus in front of the unexpected total interest; each row sums to zero. I re-calibrate $\kappa = 3.8$ to a more realistic value, since the quantitative aspect is more relevant now.[5]

The first panel corresponds to the case of figure (1). Taxes respond to real debt variation, not spending. The 1% unexpected current inflation shock leads to a roughly 1.5% increase in unexpected total inflation. Bondholders pay for the unexpected decline in total surpluses.

The taxation shock leads to a "zero-zero-zero" decomposition as inflation and interest are unchanged, and future taxes pay for the current negative shock. A 1% increase in government spending leads to a *positive* unexpected total surplus of about 2%, which pay for the unexpected total *deflation*. Finally, a 1% unexpected decline in interest creates fiscal space consumed by lower total inflation and surpluses in a two-to-one ratio. The reported measures quantify how the three expansionary policy shocks, which fail to create unexpected current inflation by assumption, actually create unexpected total *disinflation*, by result.

The second panel considers the case of debt stabilization via changing expenditure g. I set $\alpha_g = 0.07$, so that the decomposition of the unexpected inflation shock is about the same. Switching the variable of adjustment does little to change the decomposition of the other shocks.

The third panel returns to tax adjustment with the same $\alpha_\tau = 0.2$, but includes a more realistic

[5] I calibrate $\kappa = (1-\theta)(1-\beta\theta)/\theta$ using the price rigidity parameter $\theta = 0.65^4$ estimated by Smets and Wouters (2007), adjusted for annual frequency.

Taylor rule to monetary policy $i_t = \phi\pi_t$. I use $\phi = 0.50$. Active monetary policy leads to larger reactions of each term of the decomposition to our policy shocks in comparison with the baseline case. Results also reveal the Fisherian character of the NK model. Unexpected total inflation and unexpected total interest have the same signal in all cases.

1.6 A Ricardian Equivalence Result for Inflation Decomposition

2 Empirical Models

I study two Bayesian estimation that differ in the information contained in the choice of prior. (Incomplete)

I estimate the model for a set of 24 economies, listed in table (REFERENCE).

2.1 A Bayesian VAR: Stochastic Properties of Unexpected Inflation

The first model is a Bayesian VAR. The autoregression is

$$X_t = \mu + \Psi(L)X_{t-1} + \Lambda w_{t-1} + \varepsilon_t. \tag{12}$$

Of course, domestic inflation is one of the variables in X, and the corresponding residual is a measure of unexpected current inflation. The term w_t is a forcing process that I use to model the impact of international variables over domestic ones. I assume $\varepsilon \sim N(0, \Sigma)$ and independent over time.

Estimating (12) using Bayesian methods has two advantages for us. First, we can interpret different specifications of the prior distribution over random parameters μ, Ψ, Λ and Σ as different views of the economy by the economic agents. Second, parameter shrinkage (*i.e.*, reducing the dependency of estimated parameters on the data) allows the econometrician to reduce the imprecision of out-of-sample forecasts in detriment of model fit, a good deal to analysts less concerned in explaining GDP forty years in the past and more interested in anticipating interest rates and inflation one year in the future. That latter point, and the fact that it has been most commonly adopted distribution in recent BVAR models - so it is a natural starting point -, justifies my choice for the Litterman (1979) (or Minnesota) prior.[6]

Minnesota Prior. The prior formalizes the view that the variables of interest follow a random walk, or a white-noise process if we difference them.[7] The distribution is part of the Normal-Inverse-Wishart family, in that the prior has the general format

$$\Sigma \sim IW(\Phi; d)$$
$$\theta|\Sigma \sim N(b, \Sigma \otimes \Omega).$$

where $\theta = (\mu'\text{vec}(\Psi')'\text{vec}(\Lambda')')'$ and vec means stacking the columns.

The mean of the IW distribution is $\Phi/(d - N - 1)$, where N is the dimension of the square matrices and larger values of d represent tighter priors. I choose Φ to be the identity matrix (one

[6] Giannone et al. (2015) show that priors of the Normal-Inverse-Wishart family, such as the Minnesota prior, lead to posterior distributions that can be decomposed as posterior = model fit term + penalty for model complexity.

[7] The literature about the Minnesota prior is vast. Interested readers can see del Negro and Schorfheide (2011) or Karlsson (2013) for a survey-like approach.

j		P_j	$\mathcal{E}_j$	H_j
1	Nominal Debt	P	1	1
2	Inflation-Linked Debt	1	P	1
3	Dollar-Denominated Debt	P_t^{US}	Dollar NER	Dollar RER

Notes: P = price of consumption basket in domestic currency. P^{US} = price of consumption basket in US dollars. NER = Nominal Exchange Rate. RER = Real Exchange Rate.

Table 2: Public Debt Denomination

percent standard deviation for all shocks, which are uncorrelated) and select $d = N + 2$, the lowest integer possible that leads to a well-defined distribution mean - which therefore equals Φ.

Prior parameters b and Ω reflect the choices that follow. Given Σ, parameter μ has zero mean, 10^6 variance and is uncorrelated with Ψ or Λ. Let $\Psi_{p,ij}$ be the (i, j) element of the p-th matrix in $\Psi(L)$. The conditional expectation is $E(\Psi_{p,ij} \mid \Sigma) = 0$ if $p > 1$ or if $i \neq j$.

In the baseline specification, I assume variables are $I(0)$. Hence, $E(\Psi_{1,ij} \mid \Sigma) = 0$ even when $i = j$. When I later test priors that prescribe some variables to have unit roots, I set $E(\Psi_{1,ii} \mid \Sigma) = 1$ for their corresponding indexes.

The conditional covariance between the coefficients in Ψ is

$$\operatorname{cov}\left(\Psi_{p,ij}, \Psi_{q,kl} \mid \Sigma\right) = \begin{cases} \frac{\lambda^2}{p^2}\frac{\Sigma_{ij}}{\Phi_{jj}} & \text{if } p = q \text{ and } j = l \\ 0 & \text{otherwise.} \end{cases}$$

The conditional mean of Λ is zero. Its conditional covariance is

$$\operatorname{cov}\left(\Lambda_{ij}, \Lambda_{kl} \mid \Sigma\right) = \begin{cases} \lambda^2\Sigma_{ij} & \text{if } j = l \\ 0 & \text{otherwise} \end{cases}$$

and zero with the elements of Ψ.

I base the decision of which variables to include in the VAR on a more general law of motion for public debt than (1) which, it turns out, involves several variables economists usually include in macroeconometric research.

2.1.1 Generalizing Public Debt Instruments

Starting here, I recycle all notation established in section 1.

Fix the case of a country and its government. Let P_t be the price of the final goods basket in terms of the country's domestic currency.

I generalize the class of financing instruments available to the government in two dimensions: currency denomination and maturity structure. I do not consider the case of debt with stage-contingent nominal payoffs.[8]

The value of public bonds can be linked to different currencies, enumerated by j. Let $P_{j,t}$ be the price of the consumer price index in units of currency j. Let $Q_{j,t}^n$ be the discount rate for a

[8] Publicly-ran social security and pension systems can be modelled as cases of public debt with state-contigent payoffs or, in a fashion compatible with my framework, as an additional layer of uncertainty in the primary surplus process.

zero-coupon public bond paying one unit of currency j after n periods. Without loss of generality, all bonds are zero-coupon. Let $\mathcal{E}_{j,t}$ be the price of currency j in units of domestic currency.

The notation is general enough to accomodate currency-linked bonds *per se*, but also inflation-linked securities such as American TIPS. In this case, one can interpret the "currency" as the final goods basket (so that $P_j = 1$ and $\mathcal{E}_j = P_t$). In the empirical exercises that follow, I consider consider domestic currency bonds ($j = 1$), inflation-linked (or real) bonds ($j = 2$) and US-dollar-denominated bonds ($j = 3$). Table 2 shows the value or interpretation of the variables defined above for these three cases.

I do not assume the price index for government's purchases (denoted P_t^S) and levied aggregates (such as income) to be the same as the price index for households' consumption (P_t). While governments do tend to tax private consumption, they also tax and transfer income, and purchase baskets of goods different than that of households (insert reference). The fiscal effect of variations in the relative price of government's to households' final goods basket must be accounted for.

With these changes, the law of motion of government debt (1) becomes

$$\sum_j \mathcal{E}_{j,t} B^1_{j,t-1} = P_t^s S_t + \sum_j \mathcal{E}_{j,t} \sum_{n=2}^{\infty} Q^{n-1}_{j,t} \left(B^{n-1}_{j,t} - B^n_{j,t-1} \right), \tag{13}$$

where S_t is now the real primary surplus and $B^n_{j,t}$ is the face value of bonds issued in currency j, period t payable n periods in the future. The term on the left represents the cost of debt in period t; the second term on the right represents the selling of new bonds of all possible maturities.

Let $\mathcal{V}_{j,t} = \sum_n Q^n_{j,t} B^n_{j,t}$ be the end-of-period market value of nominal debt issued in currency j, $i_{j,t}$ the risk-free rate in bonds issued in currency j and $rx_{j,t} = \mathcal{V}^{-1}_{j,t-1} \sum_{n=1}^{\infty} Q^{n-1}_{j,t} B^n_{j,t-1} - i_{j,t-1}$ the realized excess return on portfolios that mimic the composition of j-currency debt. We can re-write (13) in terms of the $\mathcal{V}_j$ and its corresponding returns:

$$\sum_j (1 + rx_{j,t} + i_{j,t-1}) \mathcal{E}_{j,t} \mathcal{V}_{j,t-1} = P_t^s S_t + \sum_j \mathcal{E}_{j,t} \mathcal{V}_{j,t}$$

For empirical work, it is ideal to write the law of motion in terms of real and stationary variables. So, let Y_t be a trend such that $s_t \equiv S_t/Y_t$ is reasonably stationary, and let $g_{Y,t} = Y_t/Y_{t-1} - 1$ be its growth rate. Define the real "exchange rate" $H_{j,t} = \mathcal{E}_{j,t} P_{j,t}/P_t$ and its growth rate $\Delta h_{j,t} = H_{j,t}/H_{j,t-1} - 1$. Define the de-trended real value of j-indexed debt $V_{j,t} = \mathcal{V}_{j,t}/P_{j,t}Y_t$, the real value of total debt $V_t = \sum_j H_{j,t} V_{j,t}$ and the j-indexed share $\delta_{j,t} = H_{j,t} V_{j,t}/V_t$.

By properly dividing the whole above equation by $P_t Y_t$, and multiplying and dividing the j sum on the left by $P_{j,t-1}$, $P_{j,t}$, Y_{t-1} and $H_{j,t-1}$, we arrive at a final version for the law of motion of public debt:

$$V_{t-1} \sum_j \frac{(1 + rx_{j,t} + i_{j,t-1})(1 + \Delta h_{j,t})}{(1 + \pi_{j,t})(1 + g_{Y,t})} \delta_{j,t-1} = \frac{P_t^s}{P_t} s_t + V_t. \tag{14}$$

Stated now in real quantities, (14) generalizes (1). During period t, the government must "pay" V_{t-1} plus realized returns and eventual changes to the relative value of currency j.[9]

[9] "Pay" comes in paranthesis here because, unlike in (1), the government does not actually redeem the entire term on the left at period t. It only pays for bonds maturing at t.

2.1.2 Variable Selection and Data Treatment

Based on the law of motion (14) I select eight variables for the VAR: public debt, inflation, nominal interest, real exchange rate, tax proceeds, government spending, relative price of public good basket and gross domestic product (GDP). From these, only GDP does not show up in (14). Conversely, I do not include data on the excess return of the public debt portfolios rx_j as it is not available to the majority of countries in the sample. Under the assumption of an exponential maturity structure for public debt, unexpected excess return is determined by unexpected interest rate movements. (Show that.) I also de-trend non-stationary variables using a constant linear trend of log-GDP - so I assume a constant log-linear trend - which then implies $g_Y = 0$.

Data is annual, with period ranges varying from country to country (Table with sample size). Inflation is the log variation in the consumer price index. The dollar real exchange rate is the nominal exchange rate to the US dollar multiplied by the ratio of US-to-domestic CPI. The nominal interest rate is the log of 1+ interest data. I take the price index for public spending and tax revenue to be the GDP deflator, since income taxation accounts for the bulk of tax proceeds for most countries. I normalize the average log relative price of the public basket to zero. The series for GDP is the log deviation from trend.[10] Public spending and public debt data are both divided by the GDP trend described in the appendix. Finally, I use primary surplus data, divided by the GDP deflator, to build a tax revenue series as government spending plus primary surplus.[11]

2.1.3 Current Unexpected Inflation

(Calculate statistical significance of averages and means)

2.2 A Tighter Prior: Long-Term Debt Repayment

Let $\beta_j = \exp\{-(rx_j + i_j - \pi_j - g_Y)\}$ be the steady-state real and growth-adjusted discounting for public debt issued in currency j. I linearize around a steady-state - assumed to exist - with $\beta_j = \beta$ for all j and $P^s = P$. This leads to

$$\beta \left(v_t + s_t + s(p_t^s - p_t)\right) = v_{t-1} + v \left[\sum_j \delta_j \left(rx_{j,t} + i_{j,t-1} + \Delta h_{j,t} - \pi_{j,t} \right) - g_t \right], \tag{15}$$

which generalizes (7).

[10] I run OLS on $\log(gdp)_t = c_0 + c_1 t + \nu_t$ using all data available for GDP (which usually covers a longer period than that of the balanced panel) and define $\exp(\hat{c}_0 + \hat{c}_1 t)$ as the economy's trend, or Y_t in the notation of the previous subsection. The series for GDP is $\hat{\nu}_t$.

[11] Real GDP (constant 2015 prices), GDP deflator, public spending and the nominal exchange rate data come from the United Nations's National Accounts Main Aggregates Database. Consumer price index and and primary surplus data come from the IMF's WEO Database. Public debt (as ratio of GDP) comes Ali Abbas et al. (2011) database, which is kept up-to-date. The sources for interest rate vary from country to country; they are usually the central bank, but also from the IMF's International Financial Statistics database.

Country	Std(π)	$b(\pi, Tx)$	$b(\pi, G)$	$b(\pi, i)$	$b(\pi, gdp)$	$b(\pi, p^s - p)$
Advanced						
Denmark	0.63*	-0.08	0.08	0.10	0.04	-0.36
Norway	0.82*	0.01	-0.71*	0.34*	0.02	-0.00
Sweden	0.98*	0.10	0.42	-0.23*	0.09	-0.42*
Switzerland	0.54*	0.31*	-0.35	0.46*	0.26*	-0.36*
United Kingdom	0.71*	0.04	0.29	0.36*	-0.03	-0.11
Iceland	0.73*	-0.11*	-0.05	-0.08	-0.06	-0.17
Canada	0.67*	0.08	-0.07	0.24	0.05	0.10
United States	0.88*	0.06	-1.32*	0.34*	0.11	-1.05*
Australia	0.68*	0.13	-0.50	0.69*	-0.26*	-0.20*
Japan	0.57*	0.15*	-0.88*	0.48*	0.15*	-0.47*
Republic of Korea	0.82*	-0.57*	-0.24	0.54*	-0.28*	-0.27*
Developing						
Hungary	0.89*	0.24*	-0.26	0.57*	-0.08	-0.17
Poland	0.90*	-0.14	0.24	0.57*	0.00	-0.24
Ukraine	4.96*	1.98*	2.09	0.53*	-0.38*	-0.82*
Romania	10.84*	5.03*	9.19*	2.47*	-1.87*	-2.03*
Turkey	1.57*	0.29	-0.48	0.49*	0.21*	0.33
Russia	1.07*	0.10	-0.75	0.79*	-0.04	-0.02
Brazil	0.88*	-0.29*	-0.18	0.52*	-0.14	-0.49*
Colombia	1.13*	0.31	-1.10*	0.70*	0.44*	-0.78*
Chile	0.70*	-0.09	-0.73	0.55*	0.05	-0.22*
Mexico	0.94*	-0.43*	0.43	0.40*	0.03	-0.12
South Africa	0.71*	0.05	-0.08	1.12*	0.20	-0.55*
India	0.96*	-0.41*	0.29	0.16	0.20*	-0.26*
Indonesia	3.11*	0.23	-1.67	1.36*	-1.70*	0.60*
Average						
All	1.53	0.29	0.15	0.56	-0.12	-0.34
Advanced	0.73	0.01	-0.30	0.29	0.01	-0.30
Developing	2.20	0.53	0.54	0.79	-0.24	-0.37
Median						
All	0.88	0.07	-0.21	0.50	0.02	-0.25
Advanced	0.71	0.06	-0.24	0.34	0.04	-0.27
Developing	0.96	0.10	-0.18	0.57	0.00	-0.24

Table 3: BVAR Estimation: Current Unexpected Inflation

2.2.1 Geometric Maturity Structure of Public Debt

2.2.2 Debt Decomposition

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Appendices

A Additional Plots of the NK Model

B Data Treatment

Calulation of GDP trend

Table with data sources for each country.