# Cross-Country Evidence of Discount-Driven Inflation and Debt Repayment *

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**Abstract**

I decompose the valuation equation of public debt (market value of debt/price level = discounted surpluses) and measure the fiscal sources of inflation variation for twenty-one countries using Bayesian vector autoregressions. Innovations to inflation are primarily driven by changes to discount rates. Even using post-COVID data, contributions from surpluses are lower and derive mostly from economic activity (GDP growth) rather than fiscal policy (surplus/GDP ratios). A fiscal theory of the price level, New-Keynesian model with partial debt repayment can reproduce discount-driven inflation and realistic fiscal policy.
**Keywords:** Inflation, Fiscal Theory of the Price Level, Bayesian-VAR, Variance Decomposition, Partial Debt Repayment

## 1. Introduction

The use of the VAR to measure terms of the decomposition implicitly forces *consistency of expectations*: changes to surpluses or discount rates must change the real value of public debt; conversely, innovations to bond prices or the inflation rate must translate changes in expected surpluses or real discounting.

(...) I call that the *marked-to-market decomposition*.

Positive (say) innovations to discount rates reduce discounted surpluses but tend to reduce bond prices too, which partially balances the valuation equation. This observations calls for a new decomposition that internalizes that property: changes to discount rates on the right-hand side automatically change market prices on the left. I set up a model of public finances that links bond price innovations to revisions in the path of real discount rates and inflation. This leads to a second decomposition; one that nets out the effect of discount shocks on discounted surpluses from its effect on market prices. On the left-hand side of the valuation equation, the only term is the change to real bond prices due to revisions in inflation expectations. I call that the *total inflation decomposition*.

Unexpected inflation variation is accounted for, mostly, by discount rate variation.

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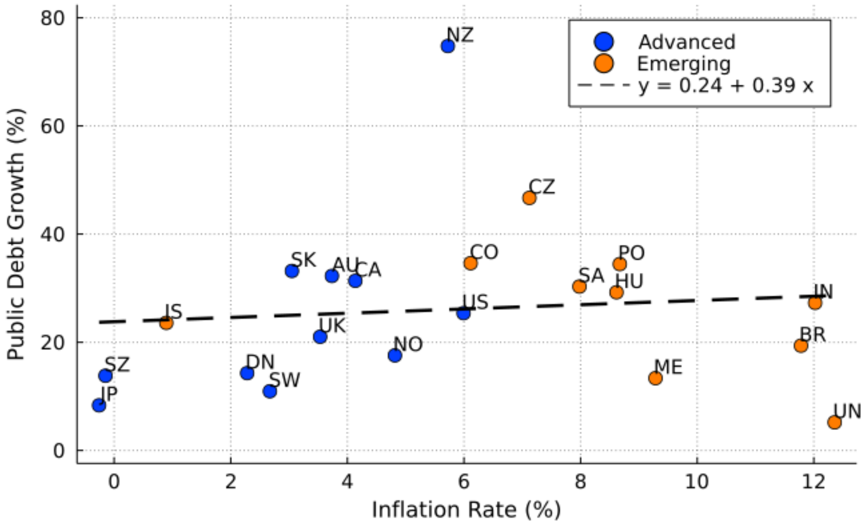



Notes: I compute total debt growth in each year as the percentage increase in the debt-to-GDP ratio $d$ multiplied by the inflation rate $\pi$ and GDP growth $g$ (see data sources in the appendix): $d_t/d_{t-1}(1+g_t)(1+\pi_t)$. I then multiply 2020 and 2021 data points. Table 1 contains the list of countries.

Figure 1: Public Debt Growth vs Price Level Growth in the years 2020-21

The share stemming from surplus variation is attributable mostly to GDP growth rather than the most common gauge of fiscal policy, the surplus-to-GDP ratio. In fact, I estimate that, for fifteen countries in the sample, surpluses-to-GDP *reduce* inflation's unexpected variation.

Furthermore, analysis of the VARs underlying decomposition estimates suggests that the short-term dynamics of both discount rates and primary surpluses are poor indicators of their contribution to the value of debt. Like in the aftermath of the COVID pandemic, inflation outbreaks are typically accompanied by larger deficits (and, in the case of advanced economies, lower real interest). *But deficits can be repaid*, and expectations can form around that belief. Cochrane (2022b) calls that property an "S"-shaped pattern: lower and then higher surpluses-to-GDP. There is nothing really new about it: Hansen et al. (1991) show that some form of "S"-shape pattern must hold for any government that promotes present-value budget balance at all times, a common assumption in non-monetary fiscal models.

## 2. Fiscal Decompositions of Unexpected Inflation

### 2.1. From the Budget Constraint to the Valuation Equation

Consider an economy with a consumption good which households value. There is a government that manages a debt composed by nominal and real bonds, possibly with different maturities. Upon maturing, real bonds deliver one unit of consumption good, while nominal bonds deliver one unit of currency. Currency is a commodity that only the government can produce, at zero cost. Households do not value it and they cannot burn it. The price of the consumption good in terms of currency is $P_t$.

The government brings from period $t-1$ a schedule $\{B^n_{N,t-1}\}$ of nominal bonds and $\{B^n_{R,t-1}\}$ of real bonds, where $n$ denotes maturity. I denote $Q^n_{N,t}$ the price of nominal bonds and $P_t Q^n_{R,t}$ the price of real bonds (I state prices in currency units).

In period $t$, the government pays for maturing debt $B^1_{N,t-1} + P_t B^1_{R,t-1}$ and public spending $P_t G_t$ using currency. It retires currency from circulation by charging taxes $P_t T_t$ and selling new issues of nominal $Q^n_{N,t}(B^n_{N,t} - B^{n+1}_{N,t-1})$ and real $P_t Q^n_{R,t}(B^n_{R,t} - B^{n+1}_{R,t-1})$ bonds. The difference between currency introduced and retired by government trading changes private sector's aggregate holdings of it, $M_t$. Therefore:

$$\overbrace{B^1_{N,t-1} + P_t B^1_{R,t-1} + P_t G_t}^{\text{Currency introduced}} = \overbrace{P_t T_t + \sum_{n=1}^{\infty} Q^n_{N,t}\left(B^n_{N,t} - B^{n+1}_{N,t-1}\right) + P_t \sum_{n=1}^{\infty} Q^n_{R,t}\left(B^n_{R,t} - B^{n+1}_{R,t-1}\right)}^{\text{Currency retired}} + \Delta M_t.$$

The equation above is a *budget constraint* faced by the government. It must be respected for any choice of money holdings by households.

For simplicity and clarity of the argument, I assume that currency does not facilitate trade and, since it does not pay interest, that households do not bring currency from one period to the next: $M_t = 0$.[1] But they do value currency *in a given period*, as they need it to pay taxes and buy public bonds.[2] Our task is to determine that value in terms of consumption goods, i.e. the price level.

With $M = 0$, the budget constraint becomes

$$(1 + r^N_t)\mathcal{V}_{N,t-1} + (1 + r^R_t)(1 + \pi_t)\mathcal{V}_{R,t-1} = P_t S_t + \mathcal{V}_{N,t} + \mathcal{V}_{R,t}$$

where $S_t = T_t - G_t$ is the primary surplus, $1 + \pi_t = P_t/P_{t-1}$ is the inflation rate, $\mathcal{V}_{N,t} = \sum_{n=1}^{\infty} Q^n_{N,t} B^n_{N,t}$ and $\mathcal{V}_{R,t} = P_t \sum_{n=1}^{\infty} Q^n_{R,t} B^n_{R,t}$ are the end-of-period nominal and the real debt, and

$$1 + r^N_t = \frac{\sum_{n=1}^{\infty} Q^{n-1}_{N,t} B^n_{N,t-1}}{\mathcal{V}_{N,t-1}} \quad \text{and} \quad (1 + \pi_t)(1 + r^R_t) = \frac{P_t}{P_{t-1}} \frac{\sum_{n=1}^{\infty} Q^{n-1}_{R,t} B^n_{R,t-1}}{\mathcal{V}_{R,t-1}}$$

are nominal returns on holdings of the basket of nominal and real bonds. The equation above is no longer a budget constraint, but an equilibrium condition.

Let $\mathcal{V}_t = \mathcal{V}_{N,t} + \mathcal{V}_{R,t}$ be end-of-period public debt and $\delta_t = \mathcal{V}_{N,t}/\mathcal{V}_t$ the relative share of nominal debt, both at market prices. The nominal return on the entire basket of public bonds is

$$1 + r^n_t = \delta_t(1 + r^N_t) + (1 - \delta_t)(1 + r^R_t)(1 + \pi_t).$$

Since public debt and surpluses are not stationary in the data, I detrend both using output $Y_t$. Define $V_t = \mathcal{V}_t/(P_t Y_t)$ as the real debt-to-GDP ratio and $s_t = S_t/Y_t$ as the surplus-to-GDP ratio.[3] From the last flow equation for public debt, we get:

$$\frac{1 + r^n_t}{(1 + \pi_t)(1 + g_t)} V_{t-1} = s_t + V_t,$$

where $g_t$ is the growth rate of GDP. The equilibrium condition above provides a law of motion for the real market value of public debt. The left-hand side contains the beginning-of-period (but after

[1] This is not necessary for the arguments of the paper. We could alternatively add money holdings to the surplus definition and the equations of the paper will hold.

[2] The fact that the government charges taxes and sells bonds for currency is not necessary for the argument, as long as it stands ready to exchange currency for consumption goods at market prices. That would equally give value to currency.

[3] If $P_t = 0$, households demand infinite goods and there is no equilibrium.

bond prices change) real market value of debt, which must be "paid for" by primary surpluses or future debt. Define $\rho_t = (1+\pi_t)(1+g_t)/(1+r_t^n)$ as the *ex-post*, growth-adjusted real discount for public bonds, and $\rho_{t,t+j} = \prod_{\tau=t}^{t+j} \rho_\tau$. The following is the key assumption of the paper.

**Assumption 1** (No Bubble): $\lim_{j\to\infty} E_t\, \rho_{t,t+j} V_{t+j} = 0$ in every period $t$.

The interpretation of assumption 1 depends on whether the government uses nominal debt.[1] If all debt is real, it represents a no-default condition. If the limit is positive, there are paths of primary surpluses that lead public debt to explode. The government eventually defaults.

If there is nominal debt (the case of the countries I consider), the government has no constraint on its choice of surpluses, as long as households attribute value to currency in a given period.[2] The zero limit condition becomes a no-bubble condition, which guarantees that the market value of debt equals discounted surpluses (just iterate the flow equation forward):

$$\frac{V_{t-1}}{\rho_t} = \sum_{j=0}^{\infty} E_t \left[\rho_{t+1,t+j} s_{t+j}\right].$$

The equation above is the valuation equation of public debt. This is an *equilibrium condition*, not a budget constraint. It is the condition upon which households accept to hold public bonds and currency. Households redeem bonds for currency and can trade currency for taxes, which have real value. Therefore, the stream of surpluses provides value for the stock of currency and public debt, and determines the price level.[3] A similar equation, stock price = discounted dividends, expresses the condition for households to hold firms' equity shares (Cochrane (2005)).

The valuation equation is a common equilibrium condition in macroeconomic models, as it only depends on assumption 1. It does not depend on equilibrium selection mechanisms (fiscal theory or spiral threat).

### 2.2. Linearization and the Marked-to-Market Decomposition

Linearization of the valuation equation of public debt allows estimation using vector autoregressions. I start with the assumption that the government keeps the denomination structure of public debt constant over time.

**Assumption 2** (Constant Currency Structure): $\delta_t = \delta$ for every $t$.

Then, linearization of the last flow equation for public debt and the definition of nominal return

[1]Typical models of intertemporal household choice do not imply the limit of assumption 1 as a result of the transversality condition, as we use *ex-post* discounting $\rho_{t,t+j}$. They do imply instead that $E_t \Lambda_{t,t+j} V_{t+j}$ converges to zero, where $\Lambda$ is the marginal rate of intertemporal substitution. *Ex-post* real returns and $\Lambda$ coincide when markets are complete. Otherwise, the limit that defines 1 is not necessary for household optimality. See Bohn (1995).

[2]Existence of a positive price level requires the government to ensure $\sum_{j=0}^{\infty} E_t \rho_{t+1,t+j} s_{t+j} > (1+r_t^R)\mathcal{V}_{R,t-1}/Y_t$.

[3]Again, the valuation equation determines the price level provided that $\delta_t > 0$ (some nominal debt). Note that time-$t$ price level only shows up in the denominator of $\mathcal{V}_N$ on the left-hand side of the valuation equation:

$$\frac{V_{t-1}}{\rho_t} = (1+r_t^N)\frac{\mathcal{V}_{N,t-1}}{P_t Y_t} + (1+r_t^R)\frac{\mathcal{V}_{R,t-1}}{Y_t}.$$

yields:

$$\rho\left(v_t + \frac{s_t}{V}\right) = v_{t-1} + r_t^n - \pi_t - g_t \tag{1}$$

$$r_t^n = \delta r_t^N + (1-\delta)\left(r_t^R + \pi_t\right), \tag{2}$$

where $\rho = (1+g)(1+\pi)/(1+r^n)$ and symbols without $t$ subscripts (like $V$) correspond to steady-state values. To save notation, I re-define all variables above as deviations from the steady state. Additionally, I re-define growth rates $r_t^n$, $r_t^N$, $r_t^R$, $\pi_t$ and $g_t$ as log-growth rates. Finally, $v_t = \log(V_t) - \log(V)$.

Like before, I solve the flow equation (1) forward and use assumption 1.

$$\overbrace{v_{t-1} + r_t^n - \pi_t}^{\substack{\text{Real market value}\\ \text{of public debt}}} = \overbrace{\frac{\rho}{V}\sum_{j=0}^{\infty}\rho^j E_t s_{t+j} + \sum_{j=0}^{\infty}\rho^j E_t g_{t+j} - \sum_{j=1}^{\infty}\rho^j E_t(r_{t+j}^n - \pi_{t+j})}^{\text{Discounted surpluses}}$$

The expression above is the linearized valuation equation of public debt.

**Decomposition 1** (Marked-to-Market)**:** Take innovations on the valuation equation of public debt.

$$\boxed{\epsilon_{r^n,t} - \epsilon_{\pi,t} = \epsilon_{s,t} + \epsilon_{g,t} - \epsilon_{r,t}} \tag{3}$$

The terms of the decomposition are $\epsilon_{r^n,t} = \Delta E_t r_t^n$, $\epsilon_{\pi,t} = \Delta E_t \pi_t$, $\epsilon_{s,t} = (\rho/V)\sum_{j=0}^{\infty}\rho^j \Delta E_t s_{t+j}$, $\epsilon_{g,t} = \sum_{j=0}^{\infty}\rho^j \Delta E_t g_{t+j}$ and $\epsilon_{r,t} = \sum_{j=1}^{\infty}\rho^j \Delta E_t(r_{t+j}^n - \pi_{t+j})$.

The right-hand side of (3) contains revisions of discounted surpluses. These are divided between news about surplus-to-GDP ratios $\epsilon_{s,t}$, GDP growth $\epsilon_{g,t}$ and real discount rates $\epsilon_{r,t}$. The left-hand side contains the innovation to the price of public bonds $\epsilon_{r^n,t}$ in real terms. Given bond prices (this is why I call "marked-to-market"), surprise inflation $\epsilon_{\pi,t}$ devalues public debt so that its value coincides once again with discounted surpluses.

The interpretation of (3) is similar to the analogous decomposition for divided-yields in Campbell and Ammer (1993). A decline in real bond prices $\epsilon_{r^n,t} - \epsilon_{\pi,t}$ must correspond to higher expected real interest $\epsilon_{r,t}$ or lower surpluses $\epsilon_{s,t}/V + \epsilon_{g,t}$. To see this, suppose that neither change. If the primary surplus remains fixed, it will then be greater than debt rollover cost, since the value of debt has fallen in period zero and real interest is unchanged. Public debt-to-GDP will then decline, leading to an even lower rollover cost, and so on. Debt will spiral down, which assumption 1 rules out. Therefore, at some point, either bond prices increase to bring the value of debt back to a level consistent with surpluses, or surpluses fall to meet the lower rollover cost.

On the opposite direction, news of future surpluses or discount rates must be matched by unexpected changes to either bond prices or the price level. We can replace equation (2) to highlight that inflation can only devalue the *nominal* portion of public debt:

$$\epsilon_{r^n,t} - \epsilon_{\pi,t} = \delta\left(\Delta E_t r_t^N - \Delta E_t \pi_t\right) + (1-\delta)\Delta E_t r_t^R.$$

A one percentage increase in the price level devalues total debt by $\delta$%. The $1-\delta$ share of real bonds is not devalued because, in currency units, their prices grow along with the price level.

### 2.3. Geometric Term Structure and the Total Inflation Decomposition

Innovations to bond prices ($\epsilon_{r^n,t}$) are informative about the expected future path of nominal interest rates, and thus inflation and real discount rates. For instance, if the yield on two-year Treasury notes falls below the Fed Funds rate, it is reasonable to conjecture that market participants anticipate interest rate cuts by the Fed. The following assumption leads to a tractable relationship between the short-term interest rate, inflation and bond returns, which I then explore to decompose $\epsilon_{r^n,t}$.

**Assumption 3** (Constant Geometric Term Structure)**:**

$$B^n_{N,t} = \omega_N B^{n-1}_{N,t} \quad \text{and} \quad B^n_{R,t} = \omega_R B^{n-1}_{R,t} \qquad \text{in every } t \text{ with } \omega_N, \omega_R \in [0,1].$$

Define $Q_{N,t} = \sum_{n=1}^{\infty} Q^n_{N,t}\omega_N^{n-1}$ and $Q_{R,t} = \sum_{n=1}^{\infty} Q^n_{R,t}\omega_R^{n-1}$ as the weighted-average market price of nominal and real bonds. Then, $\mathcal{V}_{N,t} = Q_{N,t}B^1_{N,t}$ and $\mathcal{V}_{R,t} = P_t Q_{R,t} B^1_{R,t}$. The linearized returns on public bonds are

$$\begin{aligned} r_t^N &= (\omega_N \rho) q_{N,t} - q_{N,t-1} \\ r_t^R &= (\omega_R \rho) q_{R,t} - q_{R,t-1} \end{aligned} \tag{4}$$

where $q_{N,t} = \log(Q_{N,t}/Q_N)$ and the analogous for $q_{R,t}$.[1] Expression (4) defines the return on holdings of public bonds. Note we can also use it to compute the price of the two public debt portfolios given models for expected returns $E_t r_t^N$ and $E_t r_t^R$.

**Assumption 4** (Constant Term Premia)**:** Let $r_t = i_t - E_t \pi_{t+1}$ be the real interest rate.

$$E_t r^N_{t+1} = i_t \quad \text{and} \quad E_t r^R_{t+1} = r_t \quad \text{in every } t.$$

Because variables are stated as deviations of steady state, assumption 4 does not imply the absence of risk premium to bond holdings (the expectations hypothesis), but rather that such premium is constant. Variation in expected nominal returns are only due to changes to expected future nominal interest. It also implies that we can write the $\epsilon_{r,t}$ term of decomposition (3) as $\sum_{j=1}^{\infty} \rho^j \Delta E_t r_{t+j}$.

Move the equations in (4) one period forward and iterate them forward using assumption 4:

$$q_{N,t} = -\sum_{j=0}^{\infty} (\omega_N \rho)^j E_t i_{t+j} \quad \text{and} \quad q_{R,t} = -\sum_{j=0}^{\infty} (\omega_R \rho)^j E_t r_{t+j}. \tag{5}$$

The equations in (5) show the connection between short-term interest (nominal or real) and returns on debt holdings. News of higher interest lower public bond prices and lead to low returns. In fact, (5) implies that we can decompose unexpected real returns on public debt holdings as follows:

$$\epsilon_{r^n,t} - \epsilon_{\pi,t} = \overbrace{-\,\delta \sum_{j=0}^{\infty} (\omega_N \rho)^j \Delta E_t \pi_{t+j}}^{\substack{\text{Effect of inflation over} \\ \text{nominal debt value}}} \overbrace{-\sum_{j=1}^{\infty} \rho^j [\delta \omega_N^j + (1-\delta)\omega_R^j] \Delta E_t r_{t+j}}^{\substack{\text{Effect of discount rates over} \\ \text{nominal and real debt}}}.$$

News of real bond prices must correspond to news about future real interest (which affect the price

[1] In levels, the nominal return is $(B^1_{N,t-1} + \omega_N Q_{N,t} B^1_{N,t-1})/(Q_{N,t-1} B^1_{N,t-1})$. The analogous is true for the real return.

of all bonds) or current/future inflation (which affect the price of nominal bonds only). The $\omega$'s in the sum corresponding to real interest differ it from the $\epsilon_{r,t}$ term in decomposition (3). They govern duration, or the sensitiveness of bond prices to changes in future interest. When $\omega_N = \omega_R = 0$, all bonds have a one-period maturity. Their beginning-of-period nominal values are one (nominal) or $P_t$ (real). They do not depend on future interest. When $\omega_N = \omega_R = 1$, public debt works as if it was constituted only of consols, whose price are most sensitivive to interest rate changes.

**Decomposition 2** (Total Inflation)**:** Replace the decomposition of bond prices on the marked-to-market decomposition.

$$\boxed{-\varepsilon_{\pi,t} = \varepsilon_{s,t} + \varepsilon_{g,t} - \varepsilon_{r,t}} \tag{6}$$

The terms of the decomposition are $\varepsilon_{\pi,t} = \delta \sum_{j=0}^{\infty} (\omega_N \rho)^j \Delta E_t \pi_{t+j}$, $\varepsilon_{s,t} = \epsilon_{s,t}$, $\varepsilon_{g,t} = \epsilon_{g,t}$ and $\varepsilon_{r,t} = \sum_{j=1}^{\infty} \rho^j [1 - (\delta \omega_N^j + (1-\delta)\omega_R^j)] \Delta E_t r_{t+j}$.

The marked-to-market decomposition (3) focuses on unexpected changes to current inflation *given* bond prices. Decomposition (6) recognizes that changes to bond prices coalesce from changes to perceived future inflation and real interest. The $\varepsilon_{\pi,t}$ term answers the question: given the path of real discount, how do news about the entire path of inflation affect the market value of debt? This is why I call it the *total inflation* decomposition. Like before, the terms $\varepsilon_{s,t}$ and $\varepsilon_{g,t}$ account for changes in primary surpluses. The $\varepsilon_{r,t}$ term captures the effect of discount rate on discounted surpluses *net of their effect on bond prices*. If discount rates increase, they lower discounted surpluses, which calls for higher inflation. But they also lower bond prices, which reduces the required inflation adjustment. As discussed above, the tuple $(\delta, \omega_N, \omega_R)$ determines by how much prices decline, and therefore the net impact of discount rates on total inflation.

### 2.4. Converting Par to Market-Value Public Debt

Governments report public debt at par value. Because theory is based on market-value debt, some adjustment is necessary. Computing the market value of different bonds separately and adding them up as Cox and Hirschhorn (1983) and Cox (1985) is not feasible because large historical disaggregated data for outstanding bonds and their prices is not available. Instead, I adopt an adjustment model based on the average coupon rates on the basket of nominal and real bonds. I call them $i_t^b$ and $r_t^b$.

The appendix describes the model in detail. In it, the government issues coupon-paying bonds at par, so coupon rates coincide with yields. The model gives an adjustment equation to convert market-value debt-to-GDP $v_t$ to par-value debt-to-GDP $v_t^b$ (both in logs):

$$v_t = v_t^b + q_t + \delta i_t^b + (1-\delta) r_t^b. \tag{7}$$

Par-value debt adds up principal repayments over time; market-value debt adds to that bond-price variation $q_t$ and variation in coupon rates $i_t^b$ and $r_t^b$.

Average coupon rates follow the law of motion below.

$$\begin{aligned} i_t^b &= -(1-\omega_N)^2 q_{N,t} + \omega_N i_{t-1}^b \\ r_t^b &= -(1-\omega_R)^2 q_{R,t} + \omega_R r_{t-1}^b. \end{aligned} \tag{8}$$

Intuitively, the government must roll over a share $1-\omega_N$ of its nominal debt each period. New bonds must keep the geometric structure. Hence, the increment to the average coupon rate is $(1-\omega_N)\sum_{j=0}^{\infty}\omega_N^{j-1}E_t i_{t+j}$, which is approximately $-(1-\omega_N)q_{N,t}$ since $\rho \approx 1$. The analogous holds for real debt.

## 3. Estimates

To estimate the terms of our two decompositions, I estimate a six-equation VAR in which the flow equation of public debt (1) holds by construction. If the estimated VAR systems are stationary, real debt will converge and the decompositions will hold. Keeping the same notation, the vector of variables is

$$x_t = [i_t\ \pi_t\ g_t\ v_t\ r_t^n\ s_t]'.$$

From the six variables in the VAR, three are directly observed: the nominal interest $i_t$, the inflation rate $\pi_t$ and GDP growth $g_t$. I also use data on the par debt-to-GDP $v_t^b$. These four time series are in logs. I demean each, implicitly assuming that historical averages approximate long-term steady states.

Converting par-value to market-value debt requires a series for the nominal return on public bonds $r_t^n$, which is not available. I estimate an auxiliary three-equation VAR $\tilde{x}_t = a\tilde{x}_{t-1} + \tilde{e}_t$ with $\tilde{x}_t = [i_t, \pi_t, g_t]$ and use it to compute the expectations involved in bond price formulas (5), and then a nominal return $r^n$ series using (4) and (2). The resulting series go in equation (7) to convert $v_t^b$ to $v_t$.[1] Lastly, I define the suplus-to-GDP as the residual from the flow equation (1).

Data is annual.[2] Quarterly data is available, but it often does not go back as many years into the past. This is particularly true in the case of emerging market variables and public debt measures. With a focus on long-term debt sustainability, using a large time span of data provides invaluable information regarding variables' covariances and autocovariances that is not worth forgoing to account for quarterly dynamics. Additionally, annual data avoids the danger of measurement errors due to seasonality adjustments.

I group countries in four categories according to when the sample begins: 1951, 1960, 1973 and 1997. The first group contains the United States and the United Kingdom. The next two groups contain developed (or advanced) economies. The 1997 group contains ten developing (or emerging) economies.

### 3.1. The Bayesian VAR

The functional format of the VAR is

$$X_t = AX_{t-1} + e_t \qquad e_t \sim N(0, \Sigma). \tag{9}$$

I assume that the sample averages used to demean data series coincide with their model counterparts, so we can ignore the constant term.

[1]The adjustment equation requires a series for coupon rates $i_t^b$ and $r_t^b$, which I compute using (8). The initial points in each series are respectively $i_t$ and $i_t - E_t\pi_{t+1}$ (with the expected inflation calculated using the three-equation VAR).

[2]Most time series data I collect from the St Louis Fed *FRED* website, the United Nations and the IMF. Details on the appendix.

I interpret VAR parameters $A$ and $\Sigma$ as being random and estimate them using Bayesian regressions. I establish a prior distribution, and then use data likelihood to compute the posterior.[1] I opt to use Bayesian shrinkage as it reduces the volatility of estimated coefficients, an invaluable property when samples are relatively small. In addition, with a prior distribution that leads to a stable VAR, we can calibrate its tightness to ensure that the posterior centers around a stable VAR as well. I base my prior on OLS-estimated US dynamics, which we can directly compare to results available in the literature (Cochrane (2022a)).[2]

The prior distribution belongs to the Normal-Inverse-Wishart (NIW) family. That is, letting $\theta = \text{vec}(A')$, where vec means stack columns,

$$\Sigma \sim IW(\Phi; d)$$
$$\theta|\Sigma \sim N(\bar{\theta}, \Sigma \otimes \bar{\Omega}).$$

With a Gaussian model, the NIW prior distribution is conjugate. Giannone et al. (2015) provide closed-form formulas for the posterior distribution and marginal likelihood.

The mean of the IW distribution is $\Phi/(d - n - 1)$ where $n = 6$ is the dimension of the VAR and larger values of $d$ represent tighter priors. I pick $\Phi$ to be the covariance matrix of OLS residuals in the US regression, and select $d = n + 2 = 8$, the lowest integer that leads to a well-defined distribution mean (which equals $\Phi$).

The prior for $A$ centers around the coefficients estimated for the US via OLS, $\bar{\theta} = \text{vec}(A_{US}^{OLS})$.[3] The conditional covariance between coefficients is:

$$\text{cov}\left(\tilde{a}_{ij}, \tilde{a}_{kl} \mid \Sigma\right) = \begin{cases} \lambda^2 \frac{\Sigma_{ij}}{\Phi_{jj}} & \text{if } j = l \\ 0 & \text{otherwise.} \end{cases}$$

I build $\bar{\Omega}$ to reproduce the covariance structure above. It allows the loadings on a given variable in different equations to be correlated. The different loadings of any single equation are uncorrelated.

Hyperparameter $\lambda$ governs the overall tightness of the prior. For each country, I choose the value of $\lambda$ that maximizes the marginal likelihood of the sample.[4]

The Bayesian procedure breaks the flow equation of public debt (1), which is necessary of the market-to-market decomposition (3) to hold. To restore it when computing the posterior mode and simulation draws, I manually change the loadings of the surplus equation in the VAR along with the covariance structure of the corresponding innovation. The appendix provides details. Finally, because bond returns do not satisfy (4) and (5) under the estimated VAR parameters, the total inflation decomposition (6) does not hold if we compute the $\varepsilon_{r,t}$ term as defined above. Instead, I estimate it using

$$\varepsilon_{r,t} = \epsilon_{r_t} + \epsilon_{r_t^n} + \varepsilon_{\pi,t} - \epsilon_{\pi,t},$$

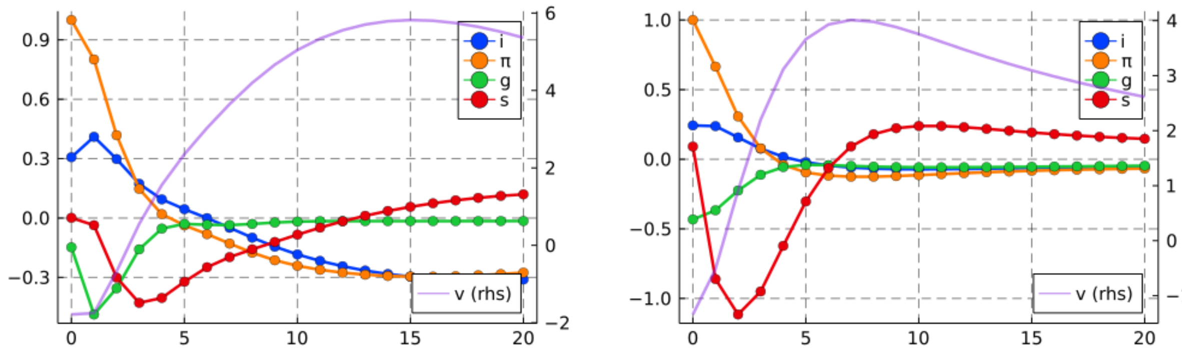
which ensures that (6) holds.

[1]See del Negro and Schorfheide (2011) or Karlsson (2013) for more on Bayesian estimation of VAR models.

[2]To the best of my knowledge the decompositions have not been estimated to other countries so far.

[3]In the US case, this implies that the posterior distribution for $A$ centers around the OLS estimate $A_{US}^{OLS}$ itself.

[4]As Giannone et al. (2015) shows, the likelihood can be decomposed between a term that depends on in-sample model fit and a term that penalizes out-of-sample forecast imprecision, or model complexity.




(a) United States (b) Japan

Figure 2: Two Examples of Responses to the Inflation Shock

### 3.2. The Inflation Shock: Sources of Inflation Variation

I set $\rho = 1$. For each country, $V$ is the average debt-to-GDP ratio in sample; $\delta$ and $\omega$ are based on debt structure data from various sources (see appendix). In the baseline specification, I drop observations from the years 2020 and 2021.

In this paper, I focus on an inflation shock defined as follows.

$$\text{Inflation Shock} = E\left[e \mid e_\pi = 1\right]$$

Inflation unexpectedly jumps by one and the other shocks move contemporaneously exactly as expected, conditional on the inflation change.[1] IRFs to the inflation shock tell us how the expected path of each variable moves given that inflation today is 1% greater than expected. Decompositions (3) and (6) measure which factors account, on average, for such increase from the point of view of the valuation equation of public debt.

Cochrane (2022a) and the appendix prove the following proposition.

**Proposition 1:** Applied to the VAR (9), the marked-to-market decomposition (3) is

$$\frac{\operatorname{cov}(\epsilon_{r^n}, \epsilon_\pi)}{\operatorname{var}(\epsilon_\pi)} - 1 = \frac{\operatorname{cov}(\epsilon_s, \epsilon_\pi)}{\operatorname{var}(\epsilon_\pi)} + \frac{\operatorname{cov}(\epsilon_g, \epsilon_\pi)}{\operatorname{var}(\epsilon_\pi)} - \frac{\operatorname{cov}(\epsilon_r, \epsilon_\pi)}{\operatorname{var}(\epsilon_\pi)}.$$

The analogous is true for the total inflation decomposition (6) (with the $\varepsilon_{\pi,t}$ term containing unexpected future inflation autocovariances).

The main motivation behind the inflation shock is that, when applied to it, the decompositions can be interpreted as *variance decompositions* of unexpected inflation in terms of fiscal factors.

Figure 2 presents the cases of the United States and Japan, which I will later reference. Each graph contains the IRF to the inflation shock computed using posterior mode parameters. The period-zero value of each variable is the inflation shock. By construction, unexpected inflation equals one.

**Results**. Tables 1 and 2 present this paper's main result: the terms of the marked-to-market and total inflation decompositions of the inflation shock. Values printed in red are negative, blue are positive. One asterisk indicates 75% statistical significance, two asterisks 90% (see table footnotes

[1] To calculate projection like the expected value of the inflation shocks, I use $E\left[e \mid Ke = \epsilon\right] = \Sigma K'(K\Sigma K')^{-1}\epsilon$.

for details).

Consider first the marked-to-market decomposition (3) in table 1. In all countries but Czech Republic, the inflation shock calls for a sudden decline in bond prices ($\epsilon_{r^n} < 0$). Central banks react to inflation news by raising nominal interest. Lower bond prices and a higher price level imply a lower real value of public debt. By the valuation equation, lower discounted surpluses. Between "discounted" ($\epsilon_{r,t}$) and "surpluses" ($\epsilon_{s,t} + \epsilon_{g,t}$), the table shows that discounting accounts for the largest share of such drop in 15 of the 21 countries. Term $\epsilon_{r,t}$ is negative in 18. Only in the United Kingdom we find a statistically significant (at 75% confidence) positive contribution. On the cross-country average, 1% unexpected inflation corresponds to a 2% decline in the value of debt, or discounted surpluses. Discount rates account for 2.1% out of this 2% decline, or more than 100% of the total drop. The conclusion is unchanged when we focus on emerging markets only.

The decline in debt value that is not accounted for by discounting must follow from news about primary surpluses. The tables show that contributions from GDP growth to discounted surpluses are usually negative (15/21), although point estimate signals are often not statistically significant. Contributions from surpluses-to-GDP are positive in 15 countries and negative in 6. On average, output growth reduces discounted surpluses (-0.4% of -2%) and therefore contributes positively to inflation variance. The surplus-to-GDP ratio does not.

Table 2 reports the total inflation decomposition. The left-hand term $\varepsilon_{\pi,t}$ represents the change in bond prices net of inflation due to revisions of current and future inflation. On the right-hand side, surplus terms $\varepsilon_{s,t}$ and $\varepsilon_{g,t}$ are the same as in the previous decomposition; the discount term $\varepsilon_{r,t}$ nets out the effect of discount rates on discounted surpluses (the $\epsilon_{r,t}$ of the previous table) from its effect on bond prices.

For most countries, the adjustment above implies $|\varepsilon_{r,t}| < |\epsilon_{r,t}|$. Still, table 2's message is somewhat similar to that of table 1. Given the inflation shock, the contemporaneous jump in "total inflation" is accounted for mostly by discount rates. Term $\varepsilon_{r,t}$ is negative in 18 cases. Only in 12 of 21 is the discount rate the largest devaluing factor on the right-hand side of the decomposition. But, on average, the share of inflation variance it accounts for tends to be larger than that of surpluses. Indeed, in the cross-country average, discounting accounts for 1.8% of the 1.7% average "total inflation" - over 100% of the overall jump (same in emerging markets). Finally, with the exception of the 1951 group, which is heavily influenced by the UK case, averages over subsamples tell a similar story. Inflation variance is accounted for by real discount rate dynamics.

### 3.3. VAR Dynamics: Short vs Long Run and Debt Repayment

Does the proposition that unexpected inflation is accounted for by discount rates imply that it is disconnected from fiscal policy? It did not seem to be in the case of post-COVID inflation. Because $\rho = \exp[-(r-g)]$ is a number close to one, the decompositions are most sensitive to long-term dynamics. Since short-term dynamics can be quite different, tables 1 and 2 are of little help.

Table 3 reports additional information about responses to the inflation shock, focusing on posterior modes. The first two columns contain the change in expected surpluses-to-GDP and GDP growth in the first five years following the shock. Estimates indicate that news of higher inflation tend to accompany news of lower primary surpluses. On average, a 1% sudden rise in the price level forecasts a 1.2% accumulated fiscal deficit in the next five years. From that, 0.7% corresponds to a decline in the surplus-to-GDP ratio and 0.5% to a decline in expected GDP. That

| Country | $\epsilon_{r^n}$ | $-\epsilon_\pi$ | $=$ | $\epsilon_s$ | $+\epsilon_g$ | $-\epsilon_r$ |
|---|---|---|---|---|---|---|
| *Averages* | **-1.0 | **-1 | = | *0.5 | *-0.4 | **-2.1 |
| 1951 (Advanced) | **-0.9 | **-1 | = | -0.2 | **-1.3 | -0.3 |
| 1960 (Advanced) | **-1.2 | **-1 | = | *1.6 | *-0.1 | **-3.6 |
| 1973 (Advanced) | **-1.5 | **-1 | = | -0.2 | -0.4 | **-2.0 |
| 1997 (Emerging) | **-0.8 | **-1 | = | *0.4 | *-0.3 | **-1.8 |
| *1951 Sample (Advanced)* | | | | | | |
| United Kingdom | **-0.8 | **-1 | = | **-2.2 | **-0.9 | *1.3 |
| United States | **-0.9 | **-1 | = | **1.7 | **-1.8 | **-1.9 |
| *1960 Sample (Advanced)* | | | | | | |
| Canada | **-2.9 | **-1 | = | 0.0 | *-1.3 | **-2.6 |
| Denmark | **-1.0 | **-1 | = | 0.4 | -0.2 | **-2.2 |
| Japan | **-0.7 | **-1 | = | **3.5 | **-3.6 | **-1.6 |
| Norway | **-0.7 | **-1 | = | *2.9 | *5.5 | **-10.1 |
| Sweden | **-0.7 | **-1 | = | **1.0 | **-1.0 | **-1.7 |
| *1973 Sample (Advanced)* | | | | | | |
| Australia | **-2.6 | **-1 | = | 0.5 | 0.1 | **-4.2 |
| New Zealand | **-1.0 | **-1 | = | *1.7 | **-1.6 | *-2.1 |
| South Korea | **-0.4 | **-1 | = | *-2.4 | 0.1 | 0.9 |
| Switzerland | **-2.1 | **-1 | = | *-0.6 | -0.1 | **-2.4 |
| *1997 Sample (Emerging)* | | | | | | |
| Brazil | **-0.5 | **-1 | = | **3.4 | 0.5 | **-5.4 |
| Colombia | **-1.4 | **-1 | = | **1.3 | **-1.1 | **-2.8 |
| Czech Republic | *0.2 | **-1 | = | **0.7 | **-1.6 | 0.1 |
| Hungary | **-0.8 | **-1 | = | 0.2 | -0.3 | **-1.7 |
| India | **-0.4 | **-1 | = | **-1.3 | -0.0 | -0.0 |
| Israel | **-0.5 | **-1 | = | *0.7 | 0.1 | **-2.3 |
| Mexico | **-1.3 | **-1 | = | **-1.2 | *0.6 | **-1.7 |
| Poland | **-1.8 | **-1 | = | *0.8 | -0.1 | **-3.4 |
| South Africa | **-1.0 | **-1 | = | 0.2 | *-0.7 | **-1.5 |
| Ukraine | -0.0 | **-1 | = | **-1.2 | *-0.2 | 0.3 |

Notes: The table reports the terms of the fiscal decomposition to the shock $\Delta E_t \pi_t = 1$. VAR coefficients fixed at the posterior distribution's mode. One asterisk indicates that 75% of the values out of 10,000 draws from the posterior had the same sign as the figure reported. Two asterisks indicate 90%.

Table 1: Marked-to-market decomposition of the shock $E\left[e_t \mid e_{\pi,t} = 1\right]$

| Country | $-\varepsilon_\pi$ | = | $\varepsilon_s$ | $+\varepsilon_g$ | $-\varepsilon_r$ |
|---|---|---|---|---|---|
| *Averages* | **-1.7 | = | *0.5 | *-0.4 | **-1.8 |
| 1951 (Advanced) | **-2.3 | = | -0.2 | **-1.3 | *-0.8 |
| 1960 (Advanced) | **-1.9 | = | *1.6 | *-0.1 | **-3.3 |
| 1973 (Advanced) | **-2.3 | = | -0.2 | -0.4 | **-1.7 |
| 1997 (Emerging) | **-1.2 | = | *0.4 | *-0.3 | **-1.2 |
| *1951 Sample (Advanced)* | | | | | |
| United Kingdom | **-3.1 | = | **-2.2 | **-0.9 | -0.0 |
| United States | **-1.6 | = | **1.7 | **-1.8 | **-1.6 |
| *1960 Sample (Advanced)* | | | | | |
| Canada | **-2.5 | = | 0.0 | *-1.3 | *-1.2 |
| Denmark | **-1.6 | = | 0.4 | -0.2 | *-1.8 |
| Japan | **-1.6 | = | **3.5 | **-3.6 | **-1.5 |
| Norway | **-2.0 | = | *2.9 | *5.5 | **-10.4 |
| Sweden | **-1.6 | = | **1.0 | **-1.0 | **-1.6 |
| *1973 Sample (Advanced)* | | | | | |
| Australia | **-3.3 | = | 0.5 | 0.1 | **-3.9 |
| New Zealand | **-2.4 | = | *1.7 | **-1.6 | **-2.4 |
| South Korea | **-1.7 | = | *-2.4 | 0.1 | 0.6 |
| Switzerland | **-1.9 | = | *-0.6 | -0.1 | **-1.2 |
| *1997 Sample (Emerging)* | | | | | |
| Brazil | **-0.7 | = | **3.4 | 0.5 | **-4.6 |
| Colombia | **-0.9 | = | **1.3 | **-1.1 | **-1.1 |
| Czech Republic | **-0.5 | = | **0.7 | **-1.6 | *0.4 |
| Hungary | **-1.8 | = | 0.2 | -0.3 | **-1.6 |
| India | **-1.8 | = | **-1.3 | -0.0 | *-0.4 |
| Israel | **-0.7 | = | *0.7 | 0.1 | **-1.5 |
| Mexico | **-1.4 | = | **-1.2 | *0.6 | *-0.7 |
| Poland | **-1.5 | = | *0.8 | -0.1 | **-2.2 |
| South Africa | **-0.9 | = | 0.2 | *-0.7 | *-0.3 |
| Ukraine | **-1.3 | = | **-1.2 | *-0.2 | 0.0 |

Notes: The table reports the terms of the fiscal decomposition to the shock $\Delta E_t \pi_t = 1$. VAR coefficients fixed at the posterior distribution's mode. One asterisk indicates that 75% of the values out of 10,000 draws from the posterior had the same sign as the figure reported. Two asterisks indicate 90%.

Table 2: Total inflation decomposition of the shock $E\left[e_t \mid e_{\pi,t} = 1\right]$

| Country | $\sum_{t=0}^{5} s_t$ | $\sum_{t=0}^{5} g_t$ | $\sum_{t=0}^{5} r_t$ | $v_{10} - v_0$ | $\frac{10 \times A_{sv}}{V}$ |
|---|---|---|---|---|---|
| *Averages* | **-0.7 | **-0.5 | **0.4 | **2.8 | **1.3 |
| 1951 (Advanced) | **-1.3 | **-1.0 | **-1.0 | **3.5 | 0.0 |
| 1960 (Advanced) | **-1.6 | **-0.7 | -0.1 | **4.8 | **1.2 |
| 1973 (Advanced) | **-0.8 | **-0.6 | -0.2 | **2.5 | **0.7 |
| 1997 (Emerging) | -0.0 | **-0.3 | **1.3 | **1.8 | **1.9 |
| *1951 Sample (Advanced)* | | | | | |
| United Kingdom | **-1.1 | **-0.7 | **-2.0 | 0.1 | **0.4 |
| United States | **-1.5 | **-1.2 | 0.1 | **6.8 | **-0.3 |
| *1960 Sample (Advanced)* | | | | | |
| Canada | 0.1 | **-1.2 | 0.3 | **4.3 | **1.0 |
| Denmark | *-1.7 | -0.2 | -0.1 | **5.8 | **0.3 |
| Japan | **-3.8 | **-1.2 | -0.1 | **5.1 | **0.4 |
| Norway | **-0.7 | -0.1 | *-0.4 | **2.7 | **4.1 |
| Sweden | **-1.9 | **-0.9 | -0.2 | **6.3 | **0.3 |
| *1973 Sample (Advanced)* | | | | | |
| Australia | *-0.6 | *-0.2 | *-0.6 | 0.4 | **0.7 |
| New Zealand | **-1.4 | **-1.0 | **-0.9 | **3.2 | **1.1 |
| South Korea | **-0.5 | *-0.4 | **-0.5 | 0.3 | **0.5 |
| Switzerland | **-0.9 | **-0.7 | **1.1 | **6.3 | **0.6 |
| *1997 Sample (Emerging)* | | | | | |
| Brazil | **1.4 | -0.1 | **3.8 | **1.9 | **2.0 |
| Colombia | *0.4 | **-1.0 | **2.3 | **4.6 | **1.6 |
| Czech Republic | -0.2 | **-1.6 | *0.4 | *-1.6 | **2.2 |
| Hungary | -0.2 | -0.2 | *0.8 | **2.5 | **2.1 |
| India | **-0.8 | 0.1 | *-0.6 | *1.1 | **0.8 |
| Israel | **-0.9 | -0.1 | **1.3 | **1.2 | **2.0 |
| Mexico | -0.2 | *0.6 | **1.4 | *1.7 | **1.4 |
| Poland | **0.4 | -0.1 | **2.3 | **4.0 | **4.0 |
| South Africa | -0.0 | **-0.8 | **1.1 | **2.4 | *0.5 |
| Ukraine | **-0.4 | *-0.2 | -0.0 | 0.2 | **2.4 |

Notes: The first four columns report variable's path in the impulse-response function to the inflation shock, which hits at $t = 0$. $A_{sv}/V$ is the loading of $v$ on the $s$ equation, divided by average debt-to-GDP. Values computed at posterior distribution's mode. One asterisk indicates that 75% of the values out of 10,000 draws from the posterior had the same sign as the figure reported. Two asterisks indicate 90%.

Table 3: VAR Dynamics and the Response to the Inflation Shock

tends to be the case country-by-country too. In 18 of 21, primary surpluses are expected to decline. In 17, surpluses-to-GDP decline. In 19, GDP growth does.

However, the VARs indicate that the initial deficits are eventually repaid by higher surpluses. More specifically, surpluses-to-GDP rise partly in response to larger debt. The fourth column of table 3 shows that public debt increases over the IRF relative to the initial fall provoked by the unexpected higher price level.[1] The last column reports $10 \times A_{s,v}/V$, with $A_{s,t}$ being the loading of debt $v$ on the equation for surpluses-to-GDP $s$. These estimates answer the question: if this year's debt-to-GDP is 10% higher than expected, how do we change our forecast of next year's surplus-to-GDP as a share of average debt? On average: 1.3%, an economically large amount consistent with Bohn (1998).

Put together, these observations imply that surpluses-to-GDP tend to follow a pattern of deficits in the first years of the transition, followed by a period when they turn positive. The inflation shock hits as surpluses (in levels) decline. Accumulated deficits translate into larger debt, which forecasts higher surpluses later. The cases of the United States and Japan in figure 2 provide clear examples. I follow Cochrane (2022b) and call that an "S"-shaped surplus process.

The fact that debt is estimated to be highly persistent (average autocorrelation = 0.91, lowest = 0.79) suggests that it is a critical state variable, largely accounting for the "S"-shaped pattern. It also suggests that surpluses-to-GDP adjustment is small (otherwise debt would not be so persistent), which may render it hard to detect in empirical work!

Lastly, table 2 also reports the sum of real interest paid by the government in the first five years of the transition to the inflation shock. While governments of developing countries pay higher interest on average in the short run, those of advanced markets pay *lower* interest. The effect of tighter monetary policy over discount rates - the quantity of interest for inflation variation - is not immediate. In summary, short-term dynamics of both discount rates and primary surpluses are often poor indicators of the fiscal sources of inflation variation.

### 3.4. Robustness Checks

In the baseline exercise, we find that unexpected inflation is driven mainly by discount rate variation. I check if this conclusion is robust to modelling choices made along the way. Table 4 reports cross-country averages of the total inflation decomposition (6) to the inflation shock. To facilitate comparison, the top panel re-prints baseline averages.

The first check is to include data for the years 2020 and 2021. These are the first years of the the worldwide inflation outbreak following the COVID pandemic. Because inflation and fiscal deficits show up in the data at the same time, our estimates of $\varepsilon_{s,t}$ tend to be lower. Surpluses-to-GDP become a more important factor to account for inflation variation. The average contribution changes from 0.5 to 0.1; it declines for most countries. Despite the huge recessions, we do not verify the output growth factor $\varepsilon_{g,t}$ gain importance. The reason for this is that, in most countries, inflation only arrives in 2021 when their economies re-open and, for the same reason, GDP growth *recovers*. In any case, the estimates continue to support the conclusion of inflation being discount-driven on average and for most countries.[2]

[1] Czech Republic is not really an exception. Its debt process recovers and increases relative to time zero in a shorter time span, peaking only a few periods after the inflation shock.

[2] The discount term $\varepsilon_{r,t}$ is negative in the case of 19 countries.

| Country Group | $-\varepsilon_\pi$ | $=$ | $\varepsilon_s$ | $+\varepsilon_g$ | $-\varepsilon_r$ |
|---|---|---|---|---|---|
| *Averages - Baseline* | **-1.7 | = | *0.5 | *-0.4 | **-1.8 |
| 1951 (Advanced) | **-2.3 | = | -0.2 | **-1.3 | *-0.8 |
| 1960 (Advanced) | **-1.9 | = | *1.6 | *-0.1 | **-3.3 |
| 1973 (Advanced) | **-2.3 | = | -0.2 | -0.4 | **-1.7 |
| 1997 (Emerging) | **-1.2 | = | *0.4 | *-0.3 | **-1.2 |
| *2020/21 data included* | **-1.7 | = | 0.1 | -0.2 | **-1.6 |
| 1951 (Advanced) | **-2.5 | = | *-0.7 | **-1.3 | *-0.5 |
| 1960 (Advanced) | **-1.9 | = | *1.2 | -0.6 | **-2.5 |
| 1973 (Advanced) | **-2.4 | = | -0.4 | -0.3 | **-1.7 |
| 1997 (Emerging) | **-1.2 | = | -0.2 | 0.3 | **-1.3 |
| *Lower debt duration* | **-1.3 | = | **0.9 | *-0.4 | **-1.8 |
| 1951 (Advanced) | **-2.2 | = | **-0.7 | **-1.3 | -0.1 |
| 1960 (Advanced) | **-1.5 | = | *1.7 | *-0.4 | **-2.8 |
| 1973 (Advanced) | **-1.8 | = | 0.7 | -0.4 | **-2.1 |
| 1997 (Emerging) | **-0.9 | = | **0.8 | *-0.3 | **-1.5 |
| *Higher discount $r-g$ ($\rho = 0.97$)* | **-1.6 | = | -0.1 | **-0.5 | **-1.1 |
| 1951 (Advanced) | **-2.2 | = | **-0.5 | **-1.1 | *-0.6 |
| 1960 (Advanced) | **-1.8 | = | 0.2 | **-0.6 | **-1.4 |
| 1973 (Advanced) | **-2.2 | = | *-0.8 | *-0.4 | **-1.0 |
| 1997 (Emerging) | **-1.1 | = | 0.2 | *-0.3 | **-1.0 |
| *Minnesota Prior ($\lambda = 0.10$)* | **-1.5 | = | *-0.4 | *-0.3 | **-0.8 |
| 1951 (Advanced) | **-2.0 | = | **-1.2 | **-0.7 | -0.2 |
| 1960 (Advanced) | **-1.7 | = | -0.4 | -0.3 | **-1.1 |
| 1973 (Advanced) | **-2.0 | = | *-1.1 | *-0.3 | -0.5 |
| 1997 (Emerging) | **-1.0 | = | 0.1 | *-0.2 | **-0.9 |
| *OLS estimates* | **-1.6 | = | *0.5 | *-0.3 | **-1.9 |
| 1951 (Advanced) | **-2.3 | = | -0.2 | **-1.3 | -0.8 |
| 1960 (Advanced) | **-1.9 | = | *1.6 | *-0.1 | **-3.3 |
| 1973 (Advanced) | **-2.3 | = | -0.2 | -0.4 | *-1.7 |
| 1997 (Emerging) | **-1.1 | = | *0.5 | -0.2 | **-1.4 |

Notes: The table reports cross-country averages of the terms of the fiscal decomposition to the shock $\Delta E_t \pi_t = 1$. VAR coefficients fixed at the posterior distribution's mode. One asterisk indicates that 75% of the values out of 10,000 draws from the posterior had the same sign as the figure reported. Two asterisks indicate 90%.

Table 4: Robustness Checks - Total inflation decomposition of the shock $E\left[e_t \mid e_{\pi,t} = 1\right]$

The next robustness check reduces term strucure parameters $\omega_N$ and $\omega_R$ ("Lower debt duration" panel). Comparison between our measure of the market value of US debt with estimates provided by the Dallas Fed reveals large differences (of up to 15% in log units), particularly in the 1980s (plot in the appendix). Reducing the average duration of the US debt from 5 to 2.5 years significantly reduces that difference.[1] I make the same adjustment to all countries and re-run results. Lower $\omega$'s reduce the volatility of the proxy series for $r^n$ and thus increase the volatility of the surplus-to-GDP series $s_t$ (computed as the residual of the flow equation). The $\varepsilon_{s,t}$ terms grow in size but become more positive, which reinforces our key findings.

One might be worried that no discounting $\rho = 1$ is inappropriate, particularly for emerging markets. The "Higher discounting" panel reduces $\rho$ from 1 to 0.97.[2] Discount rates continue to dominate the decomposition, but this change does increase the importance of surpluses in explaining unexpected inflation variance (lower $\varepsilon_{s,t}$). Having seen how short and long-run dynamics are the opposite of each other in many countries, this result is somewhat mechanical. Intuitively, with higher average interest, the same change in expected future surplus/return calls for a smaller adjustment in current real bond prices, as the resulting wedge in the value of debt compounds at a higher rate. Hence, 1% unexpected inflation will tend to be relatively more associated with changing expectations in the near future which, as we saw, involve lower surpluses and, in the case of advanced economies, lower discount rates. Note that the average decomposition of emerging countries are the least changed: in their case, real interest grows shortly after the inflation shock hits (third column of table 3).

The last two panels change the baseline prior distribution. First, I consider the so-called Minnesota prior. Introduced by Litterman (1979), it captures the belief that variables in levels follow a random walk. Since we have a VAR in difference, the Minnesota prior amounts to centering the distribution around $A = 0$. I also set $\Phi =$ indentity and $\lambda = 0.1$. The latter makes the prior considerably tighter than in the baseline.[3] The effect on estimates is similar to augmenting the discount rate. Strong shrinkage forces IRFs to stabilize faster. "S"-shaped surpluses become more like AR(1)s and future discount rates are not as large. While discount rates are still the main force accounting for unexpected total inflation (about half, on average), its importance declines substantially. I take such tight version of the Minnesota prior as a poor model of prior beliefs, especially if we consider the low-frequency nature of the exercise. Most series display a very large persistence in a 30 to 50-year time span. One can even argue $x_t = x_{t-1}+$ shock to be a more parsimonious prior model than $x_t =$ shock.

The final panel simply estimates matrix $A$ by OLS, which coincides with $\lambda = \infty$.[4] We want to answer: How much of our results follow from this particular prior choice? Can we say that estimated "S"-shaped surpluses, higher future discounting, etc on most countries follow from a US prior centered around these features? OLS results indicate that we cannot. If anything, inflation is even more discount-driven in the 1997 group.

[1] Parameters $\omega_N = \omega_R$ go from 0.8 to 0.6. Cochrane (2022a) sets $\omega = 0.69$.

[2] Recall that $1 - \rho$ is approximately $r - g$ in steady state, not just $r$. Therefore, the change to 0.97 amounts to a highly contrasting check relative to the baseline $\rho = 1$.

[3] The lowest value of $\lambda$ in the baseline is 0.12, for India.

[4] I actually continue to use Bayesian regressions, but center each country's prior on its own OLS estimate, which guarantees that the mode of the posterior distribution coincides with it. Hyperparameter $\Phi$ I set to be the identity matrix.

## 4. Discount-Driven Inflation in the New-Keynesian Framework

In any monetary model in which assumption 1 holds, structural shocks will also lead to their versions of the decompositions. The finding of discount-driven inflation shocks (as defined previously) begs the question: how far do we need to go to find discount-driven inflation following structural shocks?

In this section, I consider the class of linear models. Assuptions 1-4 continue to hold. I also use symbols $\epsilon$ and $\varepsilon$ to refer to the terms of decompositions (3) and (6), respectively.

Let $e_t$ be a vector of shocks and $s_t = \mathbf{s}(L)e_t$ be the moving average representation of a model of surpluses-to-GDP, where $\mathbf{s}(L) = \mathbf{s}_0 + \mathbf{s}_1 L + \mathbf{s}_2 L^2 + \dots$ is a vector of lag polynomials, one for each shock in vector $e_t$ (to save space I denote $\mathbf{s}(L)e_t$ the inner product). Note that

$$\varepsilon_{s,t} = \epsilon_{s,t} = \left(\frac{\rho}{V}\right)\sum_{j=0}^{\infty}\rho^j \Delta E_t s_{t+j} = \left(\frac{\rho}{V}\right)\sum_{j=0}^{\infty}\rho^j \mathbf{s}_j e_t = \left(\frac{\rho}{V}\right)\mathbf{s}(\rho)e_t.$$

Analogously, let $g_t = \mathbf{g}(L)e_t$ be the model for GDP growth. Then:

$$\varepsilon_{g,t} = \epsilon_{g,t} = \mathbf{g}(\rho)e_t.$$

The next proposition is a corollary of decompositions (3) and (6).

**Proposition 2:** Suppose $\rho = 1$. If:

1. the government fully repays deficits: $\mathbf{s}(\rho) = 0$, and
2. GDP is trend-stationary: $\mathbf{g}(1) = 0$,

then unexpected inflation is accounted for by discount rates: $\varepsilon_{\pi,t} = -\varepsilon_{r,t}$. Informally, if $\rho \approx 1$, $\varepsilon_{\pi,t} \approx -\varepsilon_{r,t}$.

The first condition $\mathbf{s}(\rho) = 0$ calls the government to credibly promise the repayment of any surprise deficit $\mathbf{s}_0 e_t < 0$, plus its average interest cost, with future surpluses $[\mathbf{s}_1 + \rho\mathbf{s}_2 + \dots]e_t > 0$. As Cochrane (2022b) argues, $\mathbf{s}(\rho) = 0$ is most cleanly represented by the S-shape surplus process discussed in the previous section. This condition is common to non-monetary models or models in which the government does not issue its own currency. In these models, $\mathbf{s}(\rho) \neq 0$ implies that debt spirals away in response to at least one shock (see Hansen et al. (1991)).

The second condition $\mathbf{g}(1) = 0$ implies that GDP is trend stationary.[1] Shocks do not cause permanent deviations from its long-term deterministic trend. (I state the proposition in terms of vectors $\mathbf{g}(L)$ and $\mathbf{s}(L)$, but conclusions obviously hold for each individual shock in $e_t$ and the corresponding lag polynomials. Even if output is not stationary, shocks with a temporary effect on GDP can only lead to surplus-driven inflation if $\mathbf{s}(\rho) \neq 0$.)

Proposition 2 offers sufficient conditions for discount-driven inflation. How $\varepsilon_{\pi,t} = -\varepsilon_{r,t}$ forms in equilibrium depends on the particular model. I present next the case of a fiscal theory of the price level (FTPL), New-Keynesian (NK) model. Cochrane (2022b) discusses extensively the decomposition of monetary and fiscal disturbances. I focus on productivity shocks, which previous literature (Smets and Wouters (2007) and similars) suggests are the main drivers of business cycle

[1] To see this, note that the long-term response of output $Y_t$ to a shock $e_t$ is $\Delta E_t Y_{t+\infty} = [\mathbf{g}_0 + \mathbf{g}_1 + \mathbf{g}_2 + \dots]e_t = \mathbf{g}(1)e_t$, where $\mathbf{g}_1, \mathbf{g}_2, \dots$ are the coefficients of the lag polynomial $\mathbf{g}$. Output is trend-stationary when innovations do not change the long-term forecast, therefore $\mathbf{g}(1) = 0$.

fluctuations.

In the model, I also consider a fiscal policy rule that allows partial debt repayment, and comports $\mathbf{s}(\rho) = 1$ as a particular case. In the next part of the paper, I estimate the degree of debt repayment for each country.

Lastly, I do not test for the presence of unit roots in GDP, but I do exemplify in the model how permanent disturbances can lead to discount-driven positive unexpected inflation combined with *higher* GDP growth. Conditions for proposition 2 are sufficient, but not necessary.

### 4.1. Model Equations

The model contains four independent AR(1) disturbances:

$$u_{j,t} = \rho_j u_{j,t-1} + e_{j,t}, \qquad e_{j,t} \sim N(0, \sigma_j^2), \quad \text{for } j = a, g, i, s.$$

Their interpretation is the following: $u_{a,t}$ and $u_{g,t}$ are respectively temporary and permanent disturbances to productivity; $u_{i,t}$ and $u_{s,t}$ are disturbances to the monetary and fiscal policy rules.[1]

The equations characterizing household and firm behavior are the typical intertemporal IS and forward-looking Phillips curve:

$$\begin{aligned} E_t g_{t+1} &= \gamma(i_t - E_t \pi_{t+1}) = \gamma r_t \\ \pi_t &= \rho E_t \pi_{t+1} + \kappa y_t - \kappa_a u_{a,t} \\ g_t &= y_t - y_{t-1} + u_{g,t}, \end{aligned} \tag{10}$$

where $y_t$ is the output's deviation from the technology trend.[2] I avoid working with the more usual output gap concept to bring to light the role of productivity disturbances $u_{a,t}$ and $u_{g,t}$.

Let $\mathbf{g}_j$ be the row of $\mathbf{g}$ corresponding of $e_j$. Disturbance $u_{a,t}$ is temporary, and therefore $\mathbf{g}_a(1) = 0$. To see this, note that, when $u_{g,t} = 0$, $g_t = y_t - y_{t-1}$. Hence, in a stationary solution for $y_t$, $g_t$ must integrate to zero. On the other hand, disturbances $u_{g,t}$ are permanent in the sense that $\mathbf{g}_g(1) \neq 0$.

The central bank fixes nominal interest according to a demeaned Taylor (1993) rule:

$$i_t = \phi \pi_t + u_{i,t}.$$

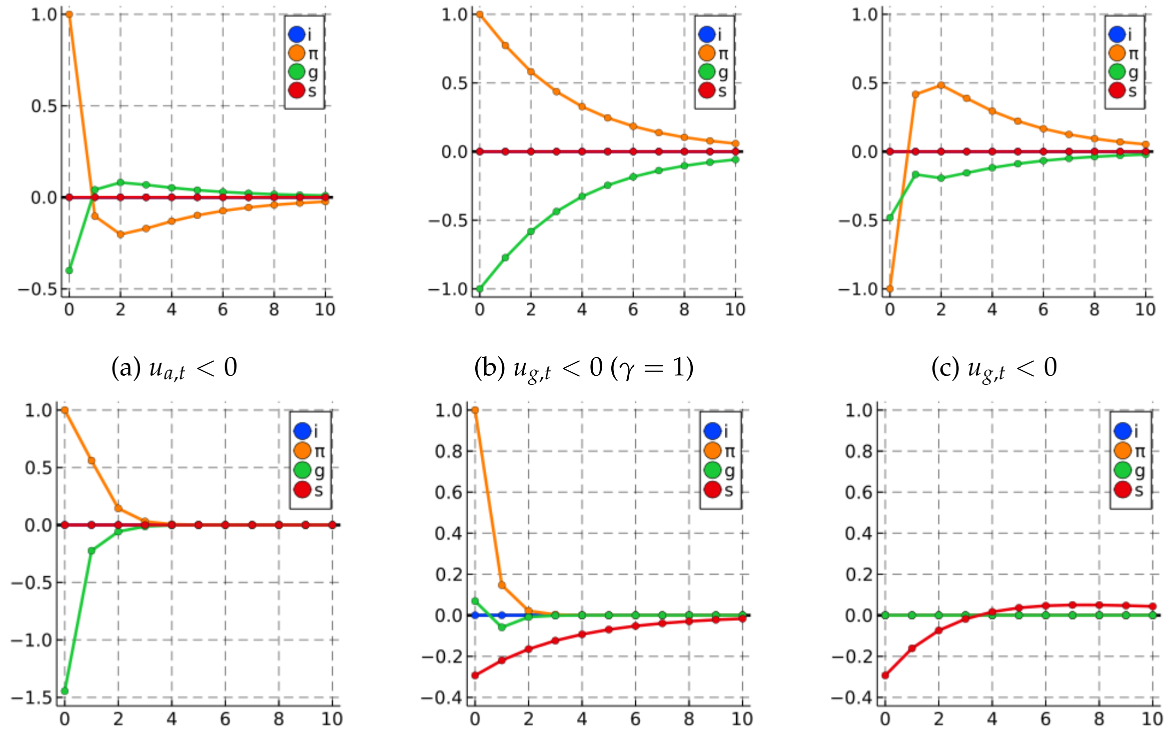
I compute IRFs using $\phi = 0$ to isolate other model forces from the effect of interest rate changes. I also start with $s = 0$ and focus on fiscal policy later.

By itself, system (10) does not determine the unexpected component of the inflation rate. (Canzoneri et al. (2001), Cochrane (2011). There are three forward-looking variables but only two explosive roots.) In FTPL models, the indeterminacy issue is solved by means of the valuation equation of public debt. Unexpected inflation is the only value consistent with (10) and the marked-to-market decomposition (3) at the same time. Additionally, in both decompositions we read causality from right to left: private agents observe discounted surpluses and attribute value to public debt in light of it.

Note that FTPL is not a necessary argument, but rather a choice of interpretation. All conclusions

[1] In the full model (see appendix), the production function is $\mathcal{A}_t N_t = \mathcal{T}_t A_t N_t$, where $N_t$ are hours of labor employed, $\log \mathcal{A}_t = u_{a,t}$ and $\log \mathcal{T}_t = \log \mathcal{T}_{t-1} + g + u_{g,t}$.

[2] The slopes of the Phillips curve are $\kappa = \lambda(\gamma^{-1} + \varphi)$ and $\kappa_a = \lambda(1 + \varphi)$, where $\varphi$ is the Frish elasticity of labor and $\lambda = (1 - \rho\theta)(1 - \theta)/\theta$ is a function of price reset rate $\theta$.




(a) $u_{a,t} < 0$ (b) $u_{g,t} < 0$ ($\gamma = 1$) (c) $u_{g,t} < 0$

(d) $u_{g,t} < 0$ ($\rho_g = 0.15$) (e) $u_{s,t} < 0$ (f) $u_{s,t} < 0$ ($\nu = 1$)

Notes: Public finance parameters approximate the US case: $\rho = V = \delta = 1$ and $\omega_N = 0.8$. Elasticity parameters are literature standards: $\gamma = 0.4$, $\varphi^{-1} = 1/3$ (Hall (2009)). Price rigidity $\theta = 0.25$ follows Kehoe and Midrigan (2015). Policy parameters are $\phi = \tau = 0$ and $\alpha = 0.2$. Persistence parameters are $\rho_a = \rho_g = \rho_i = \rho_s = 0.75$.

Figure 3: Basic NK model: IRF to structural shocks

are robust to the selection mechanism of choice. Observational equivalence between fiscal theory, $MV = PY$ and the more common spiral threat model implies that any time-0 inflation selected by one theory can be generated by the other two. See Cochrane (2023), chapter 22.

### 4.2. Productivity Shocks and Discount-Driven Inflation

How does discount-driven inflation come about in the NK model? Figure 3a considers a temporary productivity shock $e_{a,t} < 0$ normalized to generate inflation = 1 in period zero. I calibrate parameters for an annual model (see the figure's footnotes), so quantities are meaningful. The large degree of price flexibility $\theta = 0.25$ implies a relatively flat Phillips curve, which is not unreasonable for an annual model. For now, I pick persistence parameters $\rho_j$ to make the figures look pretty.

In response to the decline in produtivity, we see higher inflation in period zero. Since there is no permanent change in output and the surplus-to-GDP is constant, proposition 2 applies, and inflation is fiscally accounted by discount rates. Indeed, the model forecasts a protracted period of elevated real interest. The underlying mechanism is simply intertemporal substitution and market clearing. Lower productivity leads to a sudden decline in output and consumption. Households understand the disturbance is temporary, and that consumption will grow back to its long-term level. As they attempt to smooth it over time by selling assets, bond prices decline and real interest increases to clear the capital market. The intrinsic value of public debt declines along with that of other assets, which results in a time-0 increase in the price level. In the equations: since $\mathbf{g}_a(1) = 0$,

| Marked-to-Market Decomposition | $\epsilon_{r^n}$ | $-\epsilon_\pi$ | $=$ | $\epsilon_s$ | $+\epsilon_g$ | $-\epsilon_r$ |
|---|---|---|---|---|---|---|
| $u_{a,t}$ | 0 | -1 | $=$ | 0 | 0 | -1 |
| $u_{g,t}$ $(\gamma = 1, \rho = 0.75)$ | 0 | -1 | $=$ | 0 | -4.1 | 3.1 |
| $u_{g,t}$ $(\gamma = 0.4, \rho = 0.75)$ | 0 | -1 | $=$ | 0 | 1.5 | -2.5 |
| $u_{g,t}$ $(\gamma = 0.4, \rho = 0.15)$ | 0 | -1 | $=$ | 0 | -1.7 | 0.7 |
| $u_{s,t}$ | 0 | -1 | $=$ | -1.2 | 0 | 0.2 |
| **Total Inflation Decomposition** | | $-\varepsilon_\pi$ | $=$ | $\varepsilon_s$ | $+\varepsilon_g$ | $-\varepsilon_r$ |
| $u_{a,t}$ | | -0.6 | $=$ | 0 | 0 | -0.6 |
| $u_{g,t}$ $(\gamma = 1, \rho = 0.75)$ | | -2.5 | $=$ | 0 | -4.1 | 1.6 |
| $u_{g,t}$ $(\gamma = 0.4, \rho = 0.75)$ | | 0.1 | $=$ | 0 | 1.5 | -1.3 |
| $u_{g,t}$ $(\gamma = 0.4, \rho = 0.15)$ | | -1.6 | $=$ | 0 | -1.7 | 0.2 |
| $u_{s,t}$ | | -1.2 | $=$ | -1.2 | 0 | 0 |

Notes: Fiscal decompositions of the structural shocks in the NK model. I normalize shocks to $\varepsilon_{\pi,t} = 1$.

Table 5: Fiscal Decompositions in the New-Keynesian Model

the initial decline in output precedes a period of above-average growth $g$. The IS equation in (10) calls for higher $r$ during the transition. The decomposition then implies $\Delta E_t \pi_t > 0$ in period zero. That completes the argument.

**Remark:** In the NK model, a temporary decline in productivity leads to discount-driven inflation as households demand higher real interest rates during the recovery period.

In the example of figure 3a, nominal interest is constant. Higher real interest is entirely accounted by low inflation. A positive feedback in the policy rule $\phi > 0$ does not change the fact that output declines in response to the productivity shock, or the IS equation. Therefore, it does not reverse the conclusion of discount-driven inflation.

Table 5 reports the marked-to-market and total inflation decompositions following each of the shocks in figure (3). I normalize their signs and sizes so that they imply unexpected inflation $\Delta\pi_t = \epsilon_{\pi,t} = 1$. The first row indicates that, in the case of temporary productivity shock, it is accounted for by higher discount rates, as discussed.

Next, we look at permanent disturbances to technology $u_{g,t}$. By construction, permanent shocks to output have long-term effects, which allows $\mathbf{g}_g(1) \neq 0$ and hence $\varepsilon_{g,t} \neq 0$.

One might guess that a permanent decline in output would lead to unexpectedly higher inflation accounted for by $\varepsilon_{g,t} < 0$. Figures 3b and 3c show that might not be the case. They plot responses to two negative $e_{g,t}$ shocks. Plot 3b shows the more intuitive case. In it, I use log utility: $\gamma = 1$. Output declines in period zero, and growth continues to be negative in the following periods. Households foresee the decline in consumption and demand more assets, driving down real interest. Nominal interest is fixed, so lower real interest comes in the form of high expected inflation in $t \geq 1$. While lower discount rates are deflationary, surpluses decline along with GDP (the surplus-to-GDP ratio is unchanged). Discounted surpluses fall in $t = 0$, which leads to an increase in the price level as households run away from currency. The fiscal decompositions (second row of table 5) show a jump in inflation that is accounted for by lower output growth, undeterred by more favourable discount rates.

The fiscal decompositions (second row of table 5) show a jump in inflation that is accounted by lower output growth, undeterred by more favourable discount rates. Given the empirical finding of discount-driven inflation, this might suggest that permanent technology disturbances like $u_{g,t}$ cannot be the main drivers of business cycles at annual frequencies. But this conclusion is innacurate.

Nevertheless, permanent growth disturbances can cause discount-driven inflation as well. Figure 3c changes intertemporal elasticity to the more empirically plausible $\gamma = 0.4$ (see Hall (1988) and Campbell and Mankiw (1989)). I continue to consider a negative shock $u_{g,t} < 0$, but now the shock is accompanied by a time-0 *drop* in the inflation rate, which I normalize to -1. As the third row of table 5 confirms, higher prices in this case precede *higher* growth and lower real interest.

The explanation for the difference between figures 3b and 3c is directly related to the fiscal decompositions of unexpected inflation, and how they pin down inflation at $t = 0$. As we lower the elasticity of substitution $\gamma$, the decline in real interest resulting from any fixed path of low output growth is more pronounced. (With $\phi = 0$, the IRFs reproduce lower real interest through higher inflation rates in $t \geq 1$.) Therefore, in the marked-to-market decomposition, $\epsilon_{r,t}$ grows in size relative to $\epsilon_{g,t}$. Visually in the IRF, note that in the $\gamma = 1$ case the paths of $g$ and $\pi$ are equally distant from the zero line. In the $\gamma = 0.4$ case, $g$ is closer to zero. Hence, if we reduce $\gamma$ sufficiently, $\epsilon_{\pi,t}$ (unexpected inflation) turns negative. This is the case in figure 3c.

**Remark:** In the NK model, permanent shocks to productivity can lead to both discount-driven and surplus-driven unexpected inflation, depending on the anticipated real interest.

Experimenting with different disturbances verifies the arbitrary character of the fiscal decomposition. In the example of figure 3c, inflation increases after period zero. Although unexpected inflation is negative, total inflation in the sense of decomposition (6) is positive, as table 5 shows. This does not have to be the case. By increasing $\rho_g$ from 0.75 (figure baseline) to 0.9, total inflation remains negative. On the other hand, by reducing $\rho_g$ to 0.15 (figure 3d), we go back to surplus-driven inflation, as in the log utility case.

### 4.3. Partial Debt Repayment

So far, I have assumed $s_t = 0$. To test the first condition of proposition 2, we obviously need a model that does not assume it hold by construction. I start with a countercyclical policy rule subject to AR(1) disturbances:

$$h_t = \tau g_t + u_{s,t}.$$

The model $s_t = h_t$ allows time-varying surpluses-to-GDP but does not allow debt repayment. Condition $\mathbf{s}(\rho) = 0$ fails by construction. (To see this, set $\tau = 0$; the moving average representation of the AR(1) is $u_{s,t} = (1 - \rho_s L)^{-1} e_t$. Therefore, we get $\mathbf{s}_s(\rho) = (1 - \rho_s \rho)^{-1} > 1$, where $\mathbf{s}_s$ is the lag polynomial corresponding to shocks $\varepsilon_{s,t}$. With an AR(1), unexpected deficits are followed by deficits, not surpluses that repay them. See Cochrane (2022b).)

The following specification allows full debt repayment (as in $s_t = 0$), while having power against the alternative of partial or no repayment (as in $s_t = h_t$).

$$\begin{aligned} s_t &= s_t^* + (1 - \nu) h_t \\ s_t^* &= \alpha v_{t-1}^* + \nu h_t \\ \rho \left( v_t^* + \frac{s_t^*}{V} \right) &= v_{t-1}^*, \end{aligned} \tag{11}$$

with $\alpha > 0$. The following proposition establishes $\nu$ as the degree of debt repayment given a general exogenous process $h_t = \mathbf{h}(L)e_t$.

**Proposition 3** (Surplus Model Solution)**:** Let $\mu = (1/\rho - \alpha/V)$ and suppose $|\mu| < 1$. The solution to the surplus-to-GDP ratio model is

$$s_t = s(L)e_t \equiv \left\{1 - \underbrace{\left[\frac{\alpha\nu}{V}(1-\mu L)^{-1}L\right]}_{\text{Repayment Term}}\right\}h(L)e_t$$

It satisfies $\mathbf{s}(\rho) = (1-\nu)\mathbf{h}(\rho)$.

The solution for $s_t$ indicates that the model can generate the S-shape surplus process when $\nu > 0$. This is most easily seen for the white noise model $h_t = e_{s,t}$. The solution becomes

$$\left\{1 - \frac{\alpha\nu}{V}\left[L + \mu L^2 + \mu^2 L^3 + \dots\right]\right\}e_{s,t}$$

Note the negative sign in front of the bracket term: a deficit shock $e_{s,t} < 0$ is followed by a stream of surpluses starting in $t+1$. If $\alpha$ is small, this repayment term is close to zero, implying that period-by-period repayments are small. On the other hand, if $\alpha$ is smaller, $\mu < 1$ is larger; hence, the process of debt repayment lasts longer.

The point of the proposition is that a single parameter $\nu$ governs how much of an innovation to the surplus process is promised by the government to be accompanied by a future adjustment in the opposite direction. When $\nu = 1$, we get "full" repayment in the sense that $\varepsilon_{s,t} = (\rho/V)\mathbf{s}(\rho)e_t = 0$.

To understand how policy rule (11) yields the solution above, start by replacing the first equation on the second:

$$s_t = \alpha v^*_{t-1} + h_t. \tag{12}$$

The surplus-to-GDP ratio follows the exogenous rule $h_t$ plus a term that depends positively on $v^*_{t-1}$. In turn, $v^*_{t-1}$ is a state variable that accumulates the slice $s^*_t$ of past deficits and accrues at the average growth-adjusted interest $1/\rho$:

$$v^*_t = \left(\frac{1}{\rho}\right)v^*_{t-1} - \frac{s^*_t}{V} \qquad = \underbrace{\left(\frac{1}{\rho} - \frac{\alpha}{V}\right)}_{\mu}v^*_{t-1} - \frac{\nu}{V}h_t$$

Any deficit innovation $\Delta E_t s^*_t < 0$ raises $v^*_t$ in $t$, and drifts actual surpluses upward in $t+1$ through the term $\alpha v^*$. By the expression above, if $|\mu| < 1$, then $v^*_t$ is stationary. This implies that the government always adjusts $s^*_t$ to repay itself. To see this, solve $v^*_t$ forward and take innovations:

$$v^*_{t-1} = \left(\frac{\rho}{V}\right)\sum_{j=0}^{\infty}\rho^j E_t s^*_{t+j} \quad \Longrightarrow \quad 0 = \left(\frac{\rho}{V}\right)\sum_{j=0}^{\infty}\rho^j \Delta E_t s^*_{t+j} = \left(\frac{\rho}{V}\right)\mathbf{s}^*(\rho)e_t \quad \Longrightarrow \quad \mathbf{s}^*(\rho) = 0.$$

(This trick only works if $v^*_t$ is stationary.) To prove the proposition, solve $v^*_t$ backward:

$$v^*_t = -\left(\frac{\nu}{V}\right)(1-\mu L)^{-1}h_t$$

and replace that on (12). Straight computation of $\mathbf{s}(\rho)$ proves the last claim of the proposition.

Another way to see it is to observe the following:

$$0 = \left(\frac{\rho}{V}\right) \sum_{j=0}^{\infty} \rho^j \Delta E_t \underbrace{\left[s_{t+j} - (1-\nu)h_{t+j}\right]}_{s^*_{t+j}} \quad \Longrightarrow \quad \epsilon_{s,t} = \left(\frac{\rho}{V}\right) \mathbf{s}(\rho) e_t = (1-\nu) \left(\frac{\rho}{V}\right) \mathbf{h}(\rho) e_t.$$

The last equality shows we can control the size of the surspls-to-GDP terms of the decompositions $\varepsilon_{s,t}$. Intuitively, since only a share $\nu$ of surplus innovations enter $s^*_t$, the model implies that this share $\nu$ of fiscal deficits are repaid. When $\nu = 0$, there is no debt repayment: $v^*_t = s^*_t = 0$ and $s_t = h_t$. When $\nu = 1$, all debt is eventually repaid: $s_t = s^*_t$ thus $\mathbf{s}(\rho) = 0$. A more sophisticated model would allow different repayment rates for different sources of deficits. Jacobson et al. (2019), for instance, consider a model that separates between "regular", backed spending and "emergency", unbacked spending.

Figures 3e and 3f show the responses of the NK model to negative fiscal policy shocks $u_{s,t} < 0$ in the cases $\nu = 0$ and $\nu = 1$, respectively. I shut down feedback responses to surpluses: $\tau = 0$.

When $\nu = 0$, households do not expect the government to raise future surpluses after the initial deficit. The stream of deficits puts real public debt on an explosive path unless a higher price level reduces its size in period zero. Assumption 1 rules out the former, so we get time-0 inflation. Nominal bondholders effectively pay for the stream of fiscal deficits.

When $\nu = 1$, households expect initial deficits to be paid through above-average surpluses. The real value of debt grows in the initial periods of the IRFs (not shown), but higher surpluses eventually stabilize it. The valuation equation of public debt does not call for any unexpected change in the price level.

### 4.4. GMM Estimates

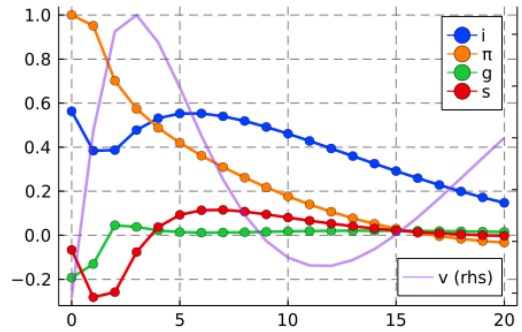
## 5. Data, Events, and Interpretation

In this section I present the dataset used in the estimation and discuss some economic events faced by the countries considered in the paper. Figures **??**-**??** present, for each case: the surplus-to-GDP and debt (top graph, debt is out of scale to fit the figure), GDP growth (second from the top), and nominal interest and inflation rates (both in the third). I also report the estimated series for inflation disturbances $e_\pi$, or reduced-form inflation shock, in the bottom graph. I do not use the term "inflation shock" to avoid confusion with the term defined in section 3.

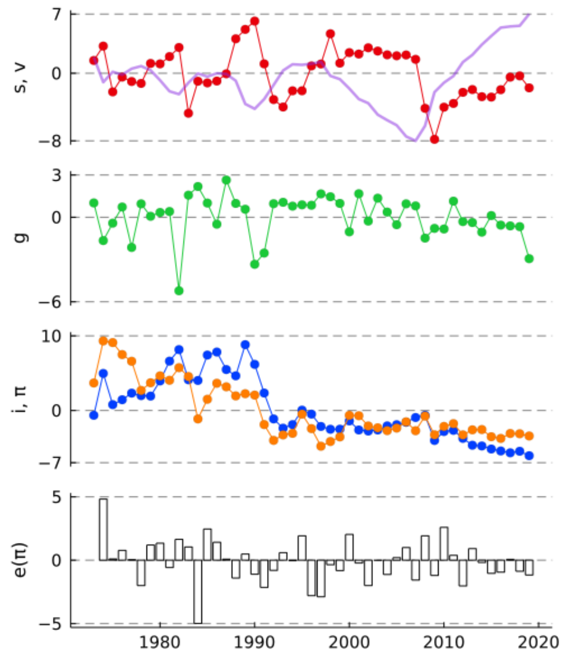
### 5.1. Australia

The large inflation in the 1970s preceeds a brief period of below-average real interest and roughly fifteen years of above-average ex-post real interest. The IRF to the inflation shock precisely replicates this event, and yields $\varepsilon_{r,t} > 0$. Consistent with the prediction of the New-Keynesian model, the increase in real interest coincides with the recovery of GDP growth, from a 2.6% average in the 1974-1980 period to 4.1% in 1983-1989. The sharp 1982 recession was hardly forecastable, as it coincided with the severe Eastern Australian drought. The neutral growth term $\varepsilon_{g,t} = 0.1 \approx 0.0$ follows.

Fiscal consolidation and the recovery of surpluses-to-GDP in the mid-80s coincides with the large, negative inflation residual in 1984. This explains the negative contemporaneous response of




| | $i$ | $\pi$ | $g$ | $v$ | $r^n$ | $s$ |
|---|---|---|---|---|---|---|
| $i$ | 0.81* | 0.19* | 0.84* | 0.0 | 0.03 | 0.05 |
| $\pi$ | 0.05 | 0.85* | 0.24 | 0.01 | −0.06 | 0.11 |
| $g$ | 0.01 | −0.02 | 0.1 | −0.0 | 0.04 | −0.1 |
| $v$ | 0.9 | −1.21 | −6.1* | 0.91* | −0.24 | −3.79* |
| $r^n$ | 0.92* | 0.07 | −0.2 | −0.02 | −0.11 | −0.06 |
| $s$ | −0.01 | 0.09 | 1.08* | 0.01 | 0.03 | 0.72* |




Notes: All results evaluated at posterior mode. Top left: IRF to the inflation shock. Bottom left: autoregressive matrix $A$. Right: Residuals to the inflation equation (bottom); demeaned data for debt and surpluses-to-GDP (top), GDP growth (second from top), nominal interest and inflation (third from top).

**Australia**: IRF to inflation shock (top left), VAR (bottom left) and demeaned data (right)

$s_t$ to the inflation shock. The "S"-shape pattern in its IRF, with higher debt eventually leading to higher surpluses-to-GDP is consistent with fiscal policy experience in Australia. In response to growing debt, the Australian government commits to deficit ceilings in the 1985-86 (the "trilogy" commitment) and 1993-94 budgets (see Gruen and Sayegh (2005)). It also approves the Charter of Budget Honesty in 1998, which sets budget balance (on average) as a primary objective of fiscal policy. On the other hand and more recently, growing debt (even prior to COVID) has not been met by a fiscal adjustment that succeeds in stabilizing it.[1]

### 5.2. Brazil

### 5.3. Canada

### 5.4. Colombia

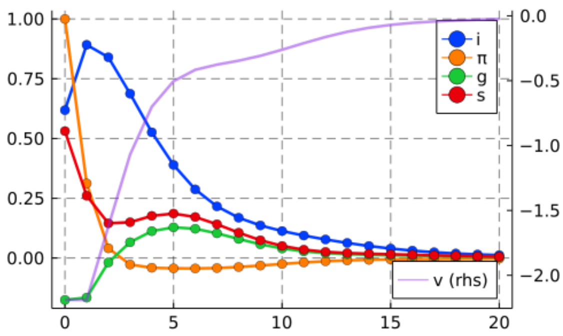
### 5.5. Czech Republic

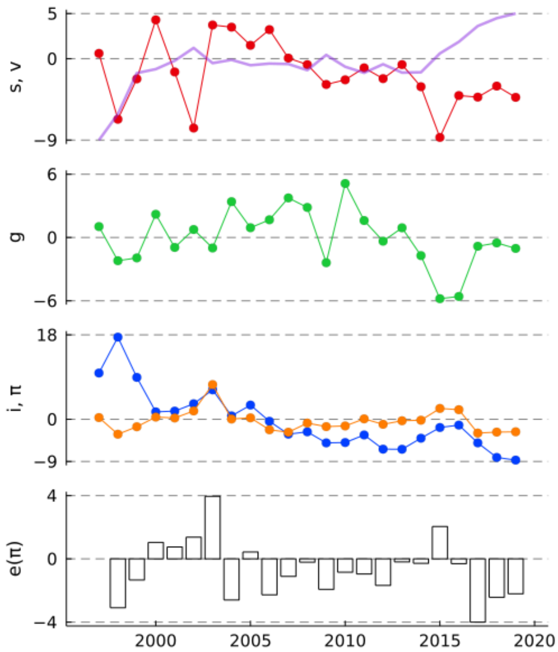
### 5.6. Denmark

### 5.7. Hungary

The prolongued period of high real interest following the inflation shock ($\varepsilon_{r,t} > 0$) follows mainly from the near-zero policy set by the Hungarian National Bank in the 2010s, particularly when inflation rises in the late 2016. Re-running results with data ending in 2015 changes $\varepsilon_{r,t}$ from -1.5 to -0.7. The surplus-to-GDP response involves a positive contemporaneous jump, and then the

[1] The Australian government enacts a debt ceiling rule in 2007. After multiple raises of the statutory debt limit, the ceiling was repealed in 2013. Two subsequent motions to re-introduced it - in 2017 and 2018 - were not approved.

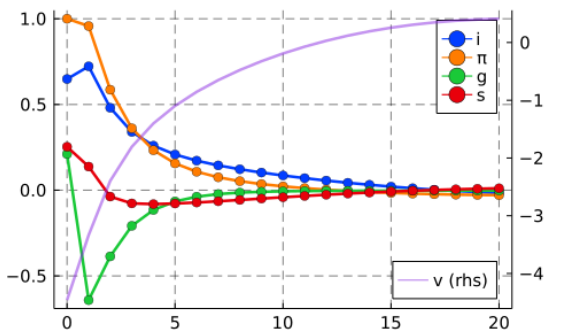


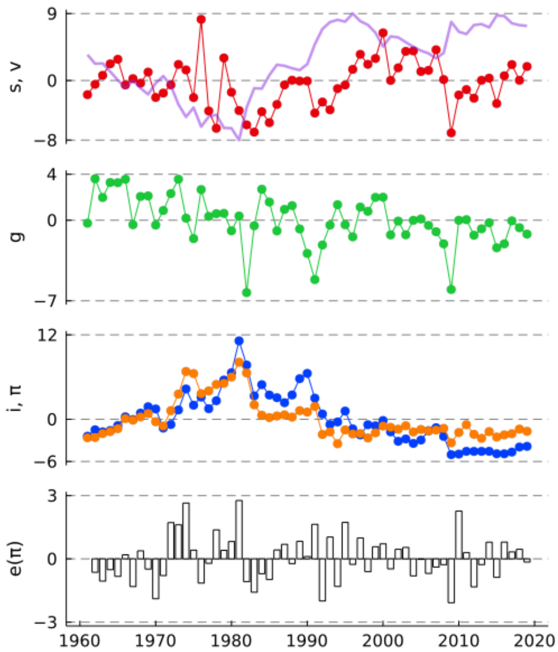




| | $i$ | $\pi$ | $g$ | $v$ | $r^n$ | $s$ |
|---|---|---|---|---|---|---|
| $i$ | 0.53* | 0.4 | 0.24 | −0.13* | 0.04 | −0.12 |
| $\pi$ | 0.02 | 0.51* | 0.23 | 0.01 | −0.0 | −0.28* |
| $g$ | 0.06 | −0.13 | 0.16 | 0.06 | 0.1 | 0.24 |
| $v$ | 0.54 | −0.47 | −0.55 | 0.85* | −0.06 | −0.61 |
| $r^n$ | 1.26* | 0.03 | 0.25 | 0.11* | −0.05 | 0.01 |
| $s$ | 0.41* | 0.08 | 0.26 | 0.13* | −0.06 | 0.42 |

Notes: All results evaluated at posterior mode. Top left: IRF to the inflation shock. Bottom left: autoregressive matrix $A$. Right: Residuals to the inflation equation (bottom); demeaned data for debt and surpluses-to-GDP (top), GDP growth (second from top), nominal interest and inflation (third from top).

**Brazil**: IRF to inflation shock (top left), VAR (bottom left) and demeaned data (right)

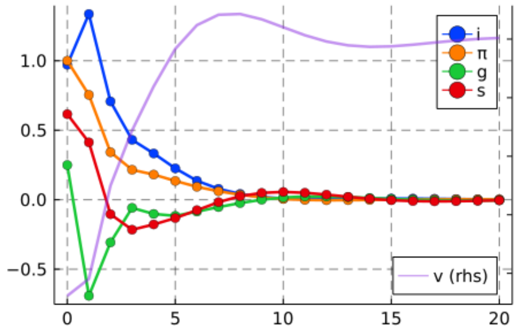


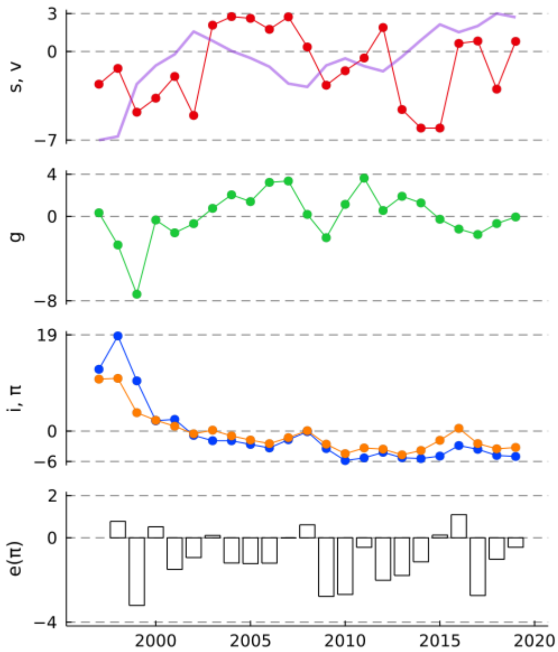




| | $i$ | $\pi$ | $g$ | $v$ | $r^n$ | $s$ |
|---|---|---|---|---|---|---|
| $i$ | 0.73* | 0.22 | 0.4* | −0.0 | 0.01 | −0.12 |
| $\pi$ | −0.03 | 0.68* | 0.06 | −0.03* | −0.06 | 0.01 |
| $g$ | −0.29* | −0.03 | 0.81* | −0.01 | 0.21* | −0.1 |
| $v$ | 2.09* | −1.3 | −2.24* | 0.99* | −0.5 | 0.1 |
| $r^n$ | 1.24* | 0.43 | −0.37 | 0.05 | −0.21 | 0.34 |
| $s$ | −0.37* | 0.76* | 0.72* | 0.07* | 0.1 | 0.23 |

Notes: All results evaluated at posterior mode. Top left: IRF to the inflation shock. Bottom left: autoregressive matrix $A$. Right: Residuals to the inflation equation (bottom); demeaned data for debt and surpluses-to-GDP (top), GDP growth (second from top), nominal interest and inflation (third from top).

**Canada**: IRF to inflation shock (top left), VAR (bottom left) and demeaned data (right)

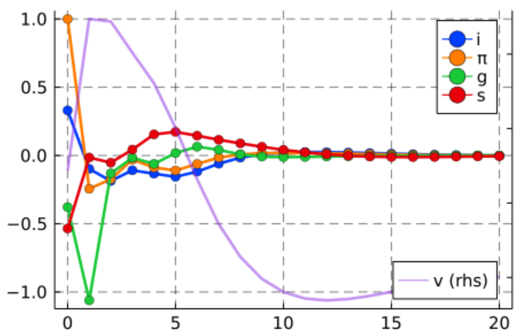


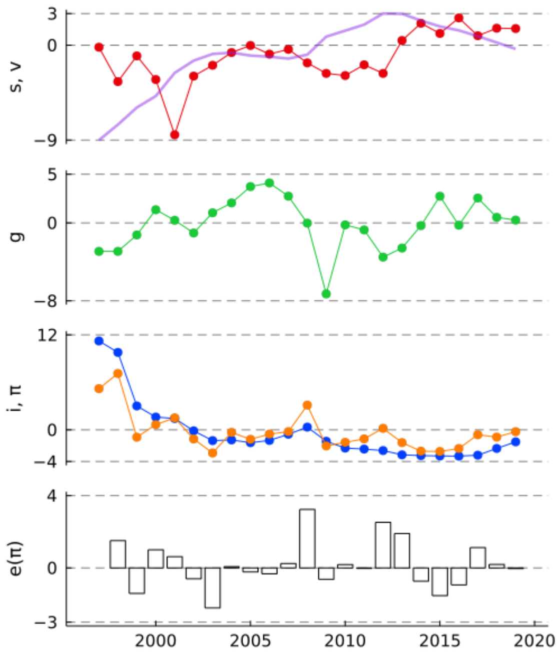




| | $i$ | $\pi$ | $g$ | $v$ | $r^n$ | $s$ |
|---|---|---|---|---|---|---|
| $i$ | −0.14 | 1.37* | 0.57* | −0.04 | 0.1 | −0.09 |
| $\pi$ | −0.22 | 1.02* | 0.26 | −0.01 | 0.05 | −0.14 |
| $g$ | −0.16 | −0.4* | 0.33* | 0.0 | 0.24* | 0.26* |
| $v$ | 2.59 | −2.57 | −1.65 | 0.89* | −0.7 | −1.37 |
| $r^n$ | 1.4* | −0.15 | −0.12 | 0.04 | −0.24 | 0.04 |
| $s$ | −0.32 | 0.7 | 0.37 | 0.06 | 0.07 | 0.51* |

Notes: All results evaluated at posterior mode. Top left: IRF to the inflation shock. Bottom left: autoregressive matrix $A$. Right: Residuals to the inflation equation (bottom); demeaned data for debt and surpluses-to-GDP (top), GDP growth (second from top), nominal interest and inflation (third from top).

**Colombia**: IRF to inflation shock (top left), VAR (bottom left) and demeaned data (right)

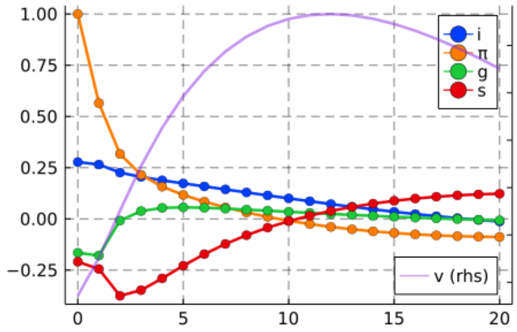


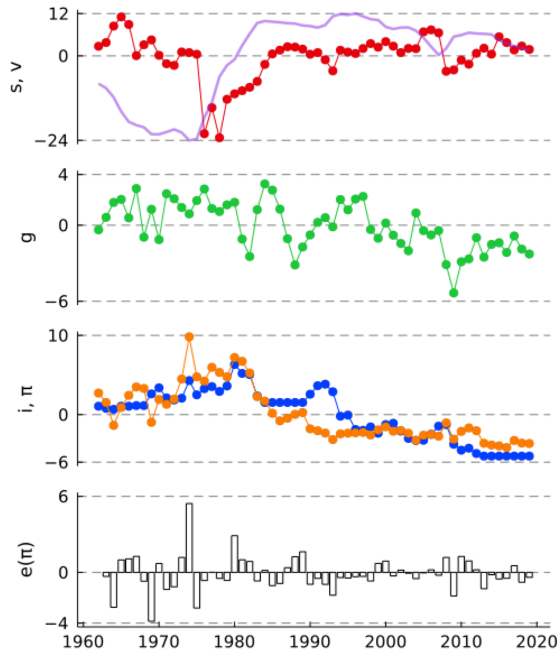




| | $i$ | $\pi$ | $g$ | $v$ | $r^n$ | $s$ |
|---|---|---|---|---|---|---|
| $i$ | 0.15 | −0.04 | −0.17* | −0.06* | −0.21* | 0.06 |
| $\pi$ | −0.16 | −0.08 | −0.16 | −0.06* | −0.24* | 0.05 |
| $g$ | 0.2 | −0.93* | 0.32 | −0.01 | 0.08 | 0.15 |
| $v$ | 1.12 | 0.3 | 0.18 | 0.99* | 0.48 | −2.49* |
| $r^n$ | 2.87* | −0.3 | 0.99* | 0.14* | 0.38* | −0.22 |
| $s$ | 0.5 | 0.12 | 0.19 | 0.06 | 0.02 | 0.6* |

Notes: All results evaluated at posterior mode. Top left: IRF to the inflation shock. Bottom left: autoregressive matrix $A$. Right: Residuals to the inflation equation (bottom); demeaned data for debt and surpluses-to-GDP (top), GDP growth (second from top), nominal interest and inflation (third from top).

**Czech Republic**: IRF to inflation shock (top left), VAR (bottom left) and demeaned data (right)

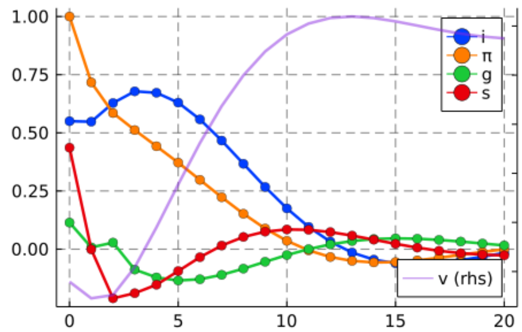


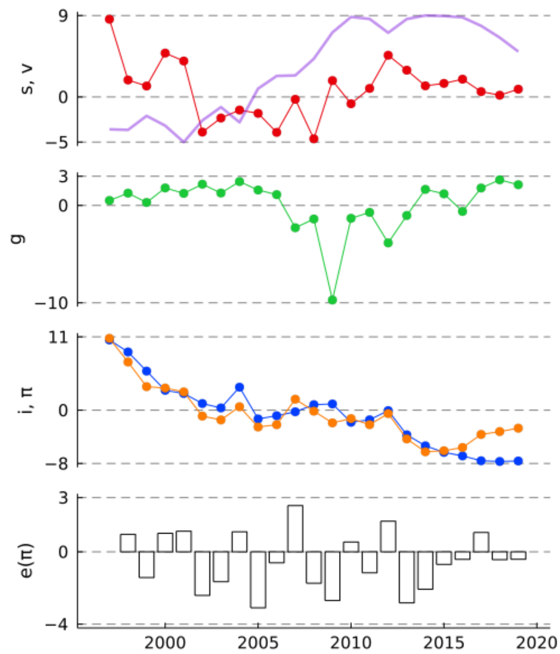




| | $i$ | $\pi$ | $g$ | $v$ | $r^n$ | $s$ |
|---|---|---|---|---|---|---|
| $i$ | 0.89* | 0.04 | 0.16* | −0.0 | −0.0 | 0.02 |
| $\pi$ | 0.1 | 0.55* | 0.22 | −0.01* | −0.0 | −0.07* |
| $g$ | 0.06 | 0.01 | 0.5* | −0.0 | 0.12* | 0.02 |
| $v$ | 1.32 | −0.12 | −1.38 | 0.99* | 0.05 | −1.76* |
| $r^n$ | 0.97* | 0.18 | 0.08 | 0.01 | −0.1 | −0.05 |
| $s$ | −0.19 | −0.1 | 0.28 | 0.01 | −0.1 | 0.66* |

Notes: All results evaluated at posterior mode. Top left: IRF to the inflation shock. Bottom left: autoregressive matrix $A$. Right: Residuals to the inflation equation (bottom); demeaned data for debt and surpluses-to-GDP (top), GDP growth (second from top), nominal interest and inflation (third from top).

**Denmark**: IRF to inflation shock (top left), VAR (bottom left) and demeaned data (right)

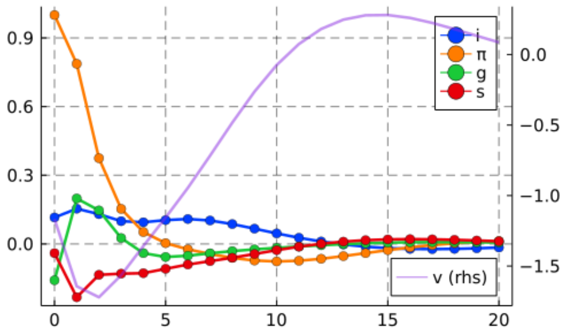


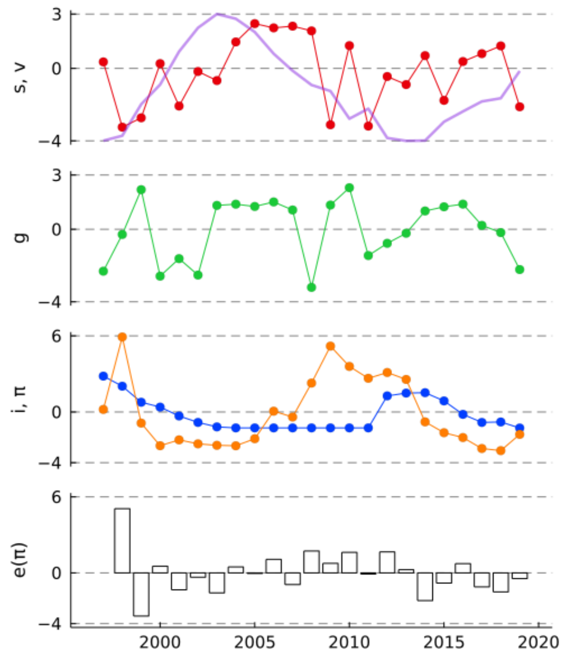




| | $i$ | $\pi$ | $g$ | $v$ | $r^n$ | $s$ |
|---|---|---|---|---|---|---|
| $i$ | 0.55* | 0.41* | −0.01 | −0.01 | 0.04 | −0.32* |
| $\pi$ | −0.11 | 0.83* | −0.06 | −0.02 | −0.0 | −0.22 |
| $g$ | −0.38 | −0.22 | 0.13 | −0.12* | 0.1 | 0.43* |
| $v$ | 1.03 | −0.74 | −0.34 | 0.98* | −0.11 | −0.24 |
| $r^n$ | 1.37* | −0.36 | 0.31 | 0.05 | −0.07 | 0.42* |
| $s$ | 0.55* | −0.15 | 0.39* | 0.14* | −0.04 | 0.3 |

Notes: All results evaluated at posterior mode. Top left: IRF to the inflation shock. Bottom left: autoregressive matrix $A$. Right: Residuals to the inflation equation (bottom); demeaned data for debt and surpluses-to-GDP (top), GDP growth (second from top), nominal interest and inflation (third from top).

**Hungary**: IRF to inflation shock (top left), VAR (bottom left) and demeaned data (right)



| | $i$ | $\pi$ | $g$ | $v$ | $r^n$ | $s$ |
|---|---|---|---|---|---|---|
| $i$ | 0.61* | −0.01 | −0.03 | −0.03 | −0.13* | −0.13 |
| $\pi$ | −0.3 | 0.74* | −0.45* | −0.03 | 0.0 | 0.5* |
| $g$ | 0.37 | 0.25 | 0.11 | 0.07 | −0.02 | 0.03 |
| $v$ | 1.39 | −0.51 | −0.39 | 0.97* | 0.61 | −0.08 |
| $r^n$ | 1.44* | 0.25 | 0.04 | 0.09* | 0.26 | 0.26 |
| $s$ | −0.01 | −0.17 | 0.56* | 0.06 | −0.24 | −0.13 |

Notes: All results evaluated at posterior mode. Top left: IRF to the inflation shock. Bottom left: autoregressive matrix $A$. Right: Residuals to the inflation equation (bottom); demeaned data for debt and surpluses-to-GDP (top), GDP growth (second from top), nominal interest and inflation (third from top).

**India**: IRF to inflation shock (top left), VAR (bottom left) and demeaned data (right)

typical "s"-shaped pattern of deficits followed by surpluses. In the data, the combination of large deficits and *negative* inflation residuals starting in 2002 explains the positive initial response. Public debt starts to rise around that period, implying that households expected higher surpluses and/or lower discount rates. In 2008, Hungary enacts the Fiscal Responsibility Act, aiming long-term debt sustainability and establishing the Fiscal Council.[1] Surpluses-to-GDP have improved ever since. The VAR reproduces that event through $A_{s,v} = 0.12 > 0$. In the IRF, debt eventually rises and is fully repaid by resulting surpluses-to-GDP, which cannot account for the inflation shock ($\epsilon_{s,t} > 0$).

### 5.8. India

### 5.9. Israel

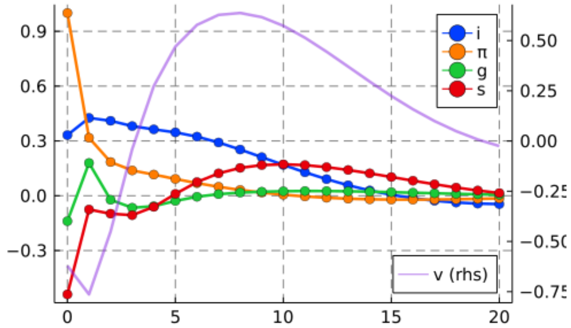
### 5.10. Japan

Economists often cite Japan as a counterexample for fiscal-inflation connections. How can dormant inflation and large deficits co-exist? While the inflation shock cannot answer why inflation does not follow unexpected deficits, it does answer what follows inflation when it *does* show up. It therefore provides a clue about the underlying reasons for the lack of price level surprises since the 90s.

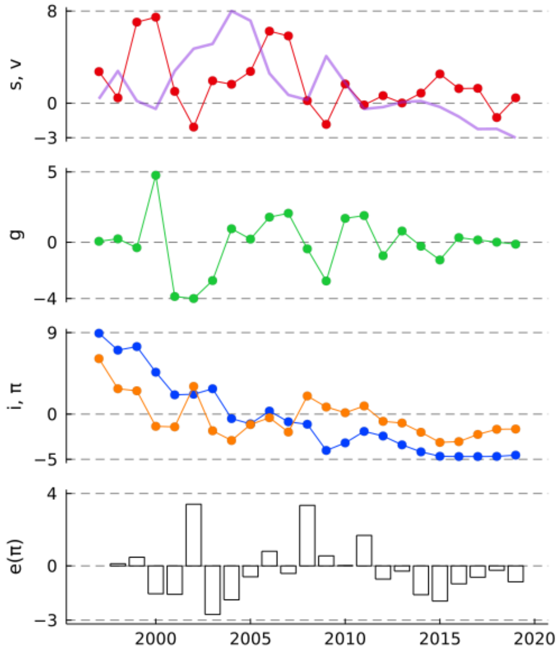
The early response of the VAR reinforces the puzzle: inflation forecasts higher deficits due to both lower $g_t$ and lower $s_t$. It resembles the 1973/74 inflationary episode - often associated with the oil crisis and/or central bank inaction (Ito (2013)). GDP growth falls by 10% in 1973, and surpluses-to-GDP fall sharply in the following years.

However, the decompositions are far more sensitive to long-term dynamics. Inflation variation

[1]See Kopits (2011) and Kopits and Romhanyi (2013) for discussions.

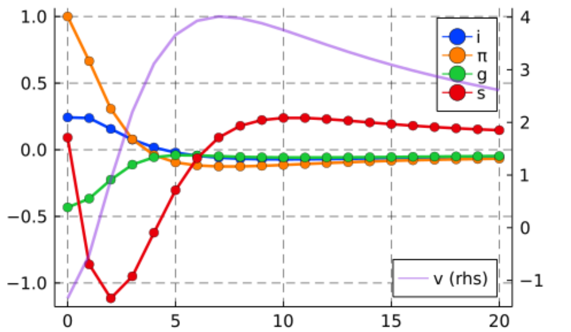



| | $i$ | $\pi$ | $g$ | $v$ | $r^n$ | $s$ |
|---|---|---|---|---|---|---|
| $i$ | $0.92^*$ | $0.01$ | $0.14$ | $0.03$ | $-0.02$ | $-0.26^*$ |
| $\pi$ | $0.12$ | $0.21$ | $0.12$ | $-0.01$ | $0.08$ | $-0.21$ |
| $g$ | $-0.02$ | $0.25$ | $0.04$ | $0.05$ | $-0.17$ | $0.19$ |
| $v$ | $0.67$ | $-0.61$ | $-0.48$ | $0.69^*$ | $0.23$ | $-0.19$ |
| $r^n$ | $0.72^*$ | $0.26$ | $-0.35$ | $-0.07$ | $0.05$ | $0.57^*$ |
| $s$ | $-0.04$ | $0.31$ | $-0.02$ | $0.15^*$ | $-0.07$ | $0.58^*$ |

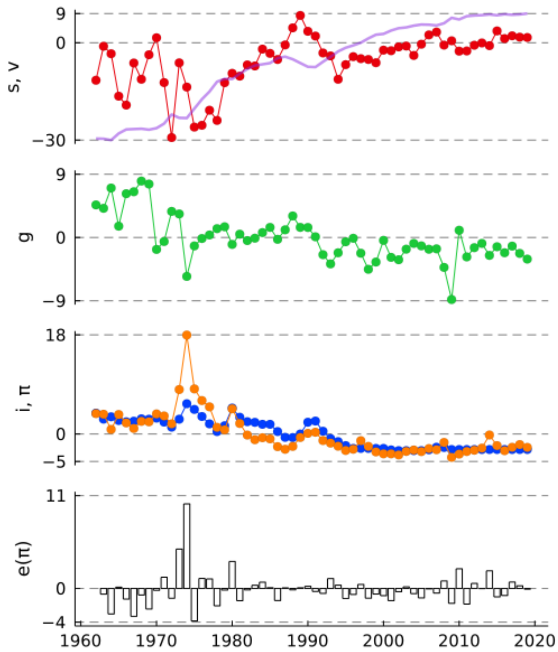



Notes: All results evaluated at posterior mode. Top left: IRF to the inflation shock. Bottom left: autoregressive matrix $A$. Right: Residuals to the inflation equation (bottom); demeaned data for debt and surpluses-to-GDP (top), GDP growth (second from top), nominal interest and inflation (third from top).

**Israel**: IRF to inflation shock (top left), VAR (bottom left) and demeaned data (right)

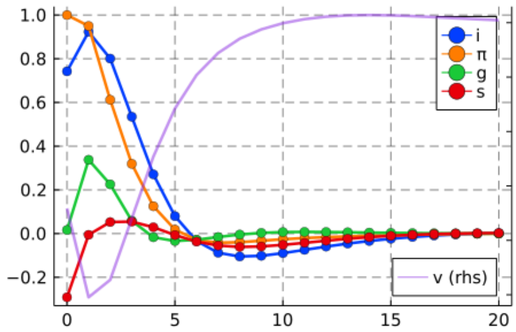



| | $i$ | $\pi$ | $g$ | $v$ | $r^n$ | $s$ |
|---|---|---|---|---|---|---|
| $i$ | $0.72^*$ | $0.06$ | $0.1$ | $-0.0$ | $-0.06$ | $0.01$ |
| $\pi$ | $-0.25$ | $0.7^*$ | $0.22$ | $-0.01$ | $-0.16$ | $-0.02$ |
| $g$ | $0.07$ | $-0.34^*$ | $0.13$ | $-0.02^*$ | $0.02$ | $-0.02$ |
| $v$ | $1.46$ | $0.1$ | $-1.25^*$ | $1.02^*$ | $0.13$ | $-0.61^*$ |
| $r^n$ | $1.29^*$ | $0.26$ | $0.39$ | $0.03^*$ | $0.19$ | $-0.0$ |
| $s$ | $0.02$ | $-0.19$ | $1.27^*$ | $0.04^*$ | $0.19$ | $0.64^*$ |

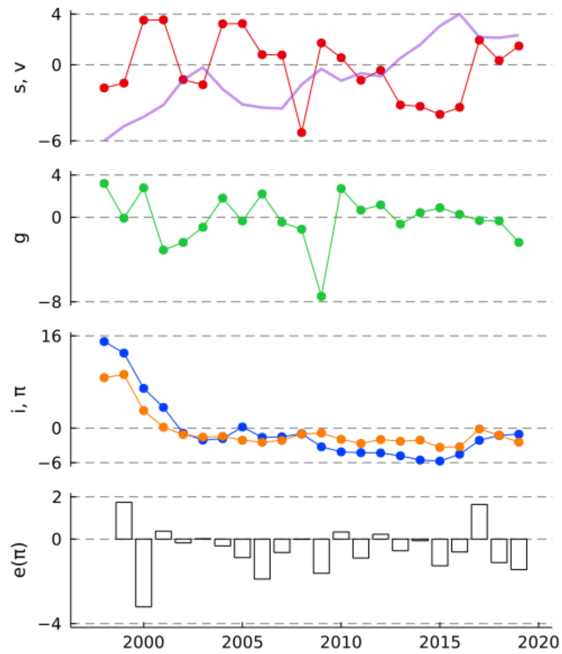



Notes: All results evaluated at posterior mode. Top left: IRF to the inflation shock. Bottom left: autoregressive matrix $A$. Right: Residuals to the inflation equation (bottom); demeaned data for debt and surpluses-to-GDP (top), GDP growth (second from top), nominal interest and inflation (third from top).

**Japan**: IRF to inflation shock (top left), VAR (bottom left) and demeaned data (right)




| | $i$ | $\pi$ | $g$ | $v$ | $r^n$ | $s$ |
|---|---|---|---|---|---|---|
| $i$ | $0.94^*$ | $0.04$ | $0.17$ | $0.03$ | $-0.2^*$ | $0.12$ |
| $\pi$ | $0.4^*$ | $0.33$ | $-0.03$ | $-0.01$ | $-0.19^*$ | $-0.17$ |
| $g$ | $-0.35$ | $0.6$ | $-0.05$ | $-0.0$ | $-0.03$ | $0.19$ |
| $v$ | $-0.36$ | $-0.24$ | $0.31$ | $0.87^*$ | $0.58$ | $-0.81$ |
| $r^n$ | $0.46$ | $0.48$ | $0.23$ | $-0.01$ | $0.3^*$ | $-0.1$ |
| $s$ | $0.34$ | $-0.09$ | $0.0$ | $0.06$ | $-0.02$ | $0.3$ |




Notes: All results evaluated at posterior mode. Top left: IRF to the inflation shock. Bottom left: autoregressive matrix $A$. Right: Residuals to the inflation equation (bottom); demeaned data for debt and surpluses-to-GDP (top), GDP growth (second from top), nominal interest and inflation (third from top).

**Mexico**: IRF to inflation shock (top left), VAR (bottom left) and demeaned data (right)

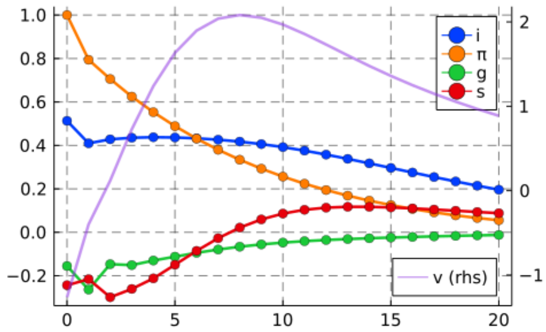
is accounted for, mostly, by output growth. In the data, growth never recovers from the 1973 recession, and the IRF reproduces such protracted decline with precision. Thus, the lack of persistent surprises to GDP growth explains, to a great extent, dormant inflation in Japan.

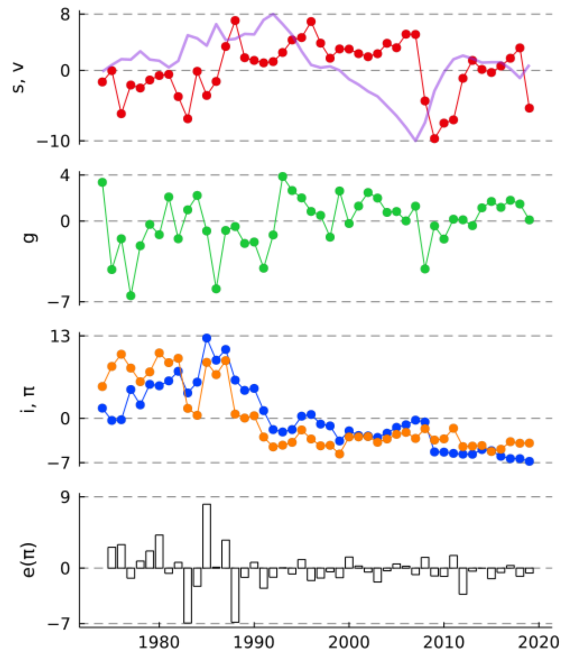
As for fiscal policy, we estimate a small but positive feedback of $v_t$ on $s_t$ ($A_{s,v} = 0.04$), resulting from the increase of surpluses-to-GDP along with public debt in the last forty years. They recover in the 80s, following public efforts of fiscal consolidation (Miyazaki (2007)). The "s"-shape response of the surplus-to-GDP ratio captures that experience, and suggests that the co-existence of deficits and quiet inflation in Japan follows from the expectation of future debt repayments. In fact, surpluses-to-GDP appear to be the main factor backing the rise of market-value (= discounted surpluses) since the 80s.

Lastly, the positive contribution of discount rates to inflation variance is consistent with the rise of nominal interest relative to inflation in the 80s and early 90s.

### 5.11. Mexico

The Mexican economy presents declining inflation and growing surplus-to-GDP ratios in the early 2000s. Nominal interest falls faster than inflation until 2005. GDP growth alternates good and bad years, with a small recession in 2001/2002. The IRF to the inflation shock replicates these patterns. The decompositions speak out the lasting responses of surplus-to-GDP and real interest. If we used $\rho = 0.97$, the discounting component of the total inflation decomposition $\varepsilon_{r,t}$ declines from 2.9 to 1.3. Using the tight Minnesota prior as in table 2, it becomes -1.9, statistically significant at 90% confidence. Another reduced-form inflation shock hits in 2017. The inflation hike as widely associated to trade disputes with the US and the lifting of energy price subsidies. From a




| | $i$ | $\pi$ | $g$ | $v$ | $r^n$ | $s$ |
|---|---|---|---|---|---|---|
| $i$ | 0.77* | 0.18 | 0.45* | 0.01 | 0.09 | −0.03 |
| $\pi$ | −0.06 | 0.94* | 0.32 | 0.0 | 0.07 | −0.04 |
| $g$ | −0.16 | −0.09 | 0.1 | 0.01 | 0.04 | 0.07 |
| $v$ | 1.24* | −0.68 | −1.11 | 0.85* | −0.16 | −1.54* |
| $r^n$ | 1.18* | −0.05 | −0.23 | −0.03 | −0.22 | 0.02 |
| $s$ | 0.06 | −0.09 | 0.18 | 0.04* | −0.07 | 0.6* |

Notes: All results evaluated at posterior mode. Top left: IRF to the inflation shock. Bottom left: autoregressive matrix $A$. Right: Residuals to the inflation equation (bottom); demeaned data for debt and surpluses-to-GDP (top), GDP growth (second from top), nominal interest and inflation (third from top).

**New Zealand**: IRF to inflation shock (top left), VAR (bottom left) and demeaned data (right)

fiscal accounting perspective, it looks to be discount-driven, as nominal interest grows faster than inflation in the following years, while the surplus-to-GDP ratio *increases*.

### 5.12. New Zealand

### 5.13. Norway

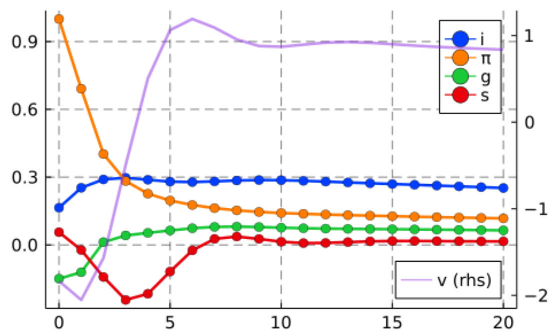
### 5.14. Poland

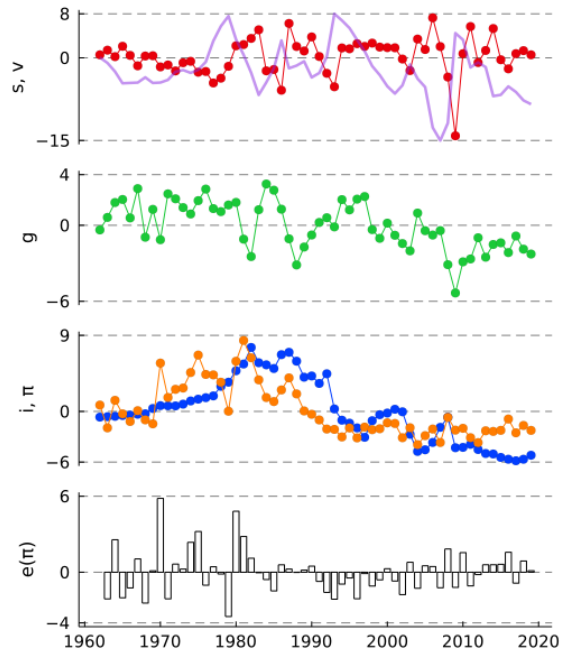
### 5.15. South Africa

### 5.16. South Korea

The estimated VAR reflects the events of the mid-1970s and early 1980s. Korea experiences a strong inflation surge in 1974/1975 with little interest rate response. The inflation disturbance hits as the strong pre-1973 output growth fades following the oil crisis, and fiscal deficits grow to pay for the Heavy Chemical Industrialization Plan (the "Big Push", see Collins and Park (1989)). A large negative reduced-form inflation shock in 1977 coincides with the recovery of GDP growth and the surplus-to-GDP. A new pair of positive-negative inflation disturbances hit in 1980 and 1982, with similar responses from GDP growth and surpluses-to-GDP. In the events of the 80s, nominal interest does react to inflation more strongly, but not to the point of changing the message of the VAR.[1]

[1] In the 1960s, Korean interest rates does increase persistently to an inflation surge, but, sadly, they do not enter the dataset since debt data is missing.

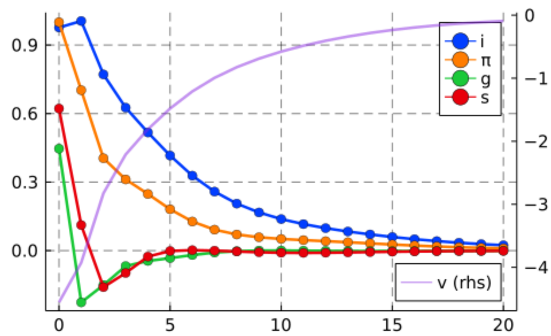


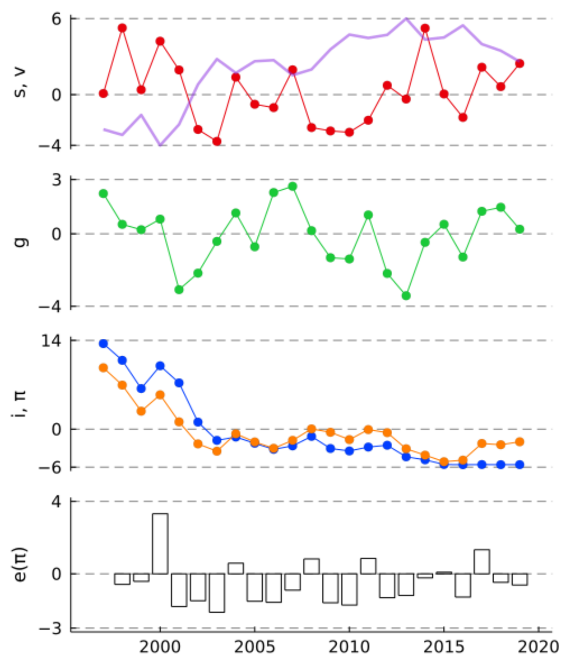




| | $i$ | $\pi$ | $g$ | $v$ | $r^n$ | $s$ |
|---|---|---|---|---|---|---|
| $i$ | 0.82* | 0.16* | 0.22* | 0.0 | 0.01 | 0.06 |
| $\pi$ | 0.19 | 0.63* | 0.34* | 0.01 | −0.13 | −0.09 |
| $g$ | −0.19* | 0.15 | 0.61* | 0.01 | 0.19* | 0.07 |
| $v$ | 2.15* | −1.28 | 0.74 | 0.56* | −0.18 | −2.19* |
| $r^n$ | 1.13* | −0.04 | −0.02 | −0.02 | −0.11 | −0.12 |
| $s$ | −0.23 | 0.1 | −0.39 | 0.09* | 0.0 | 0.47* |

Notes: All results evaluated at posterior mode. Top left: IRF to the inflation shock. Bottom left: autoregressive matrix $A$. Right: Residuals to the inflation equation (bottom); demeaned data for debt and surpluses-to-GDP (top), GDP growth (second from top), nominal interest and inflation (third from top).

**Norway**: IRF to inflation shock (top left), VAR (bottom left) and demeaned data (right)

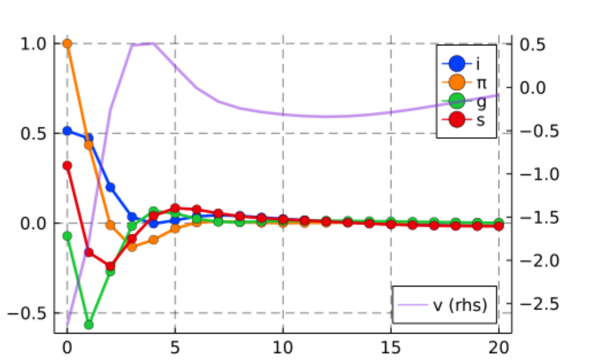


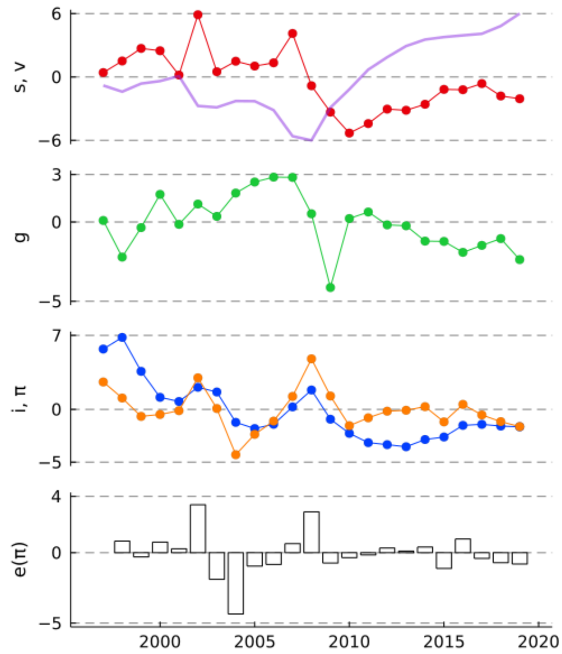




| | $i$ | $\pi$ | $g$ | $v$ | $r^n$ | $s$ |
|---|---|---|---|---|---|---|
| $i$ | 0.64* | 0.2 | 0.36 | −0.03 | −0.02 | −0.23 |
| $\pi$ | −0.07 | 0.71* | 0.38 | −0.03 | −0.0 | −0.38* |
| $g$ | −0.05 | −0.16 | 0.5* | 0.0 | 0.11 | −0.06 |
| $v$ | −0.54 | 0.01 | −2.0 | 0.67* | −0.13 | 0.5 |
| $r^n$ | 1.05 | 0.21 | −0.05 | 0.05 | −0.04 | 0.44 |
| $s$ | 0.8 | −0.16 | 0.5 | 0.19* | −0.01 | 0.18 |

Notes: All results evaluated at posterior mode. Top left: IRF to the inflation shock. Bottom left: autoregressive matrix $A$. Right: Residuals to the inflation equation (bottom); demeaned data for debt and surpluses-to-GDP (top), GDP growth (second from top), nominal interest and inflation (third from top).

**Poland**: IRF to inflation shock (top left), VAR (bottom left) and demeaned data (right)

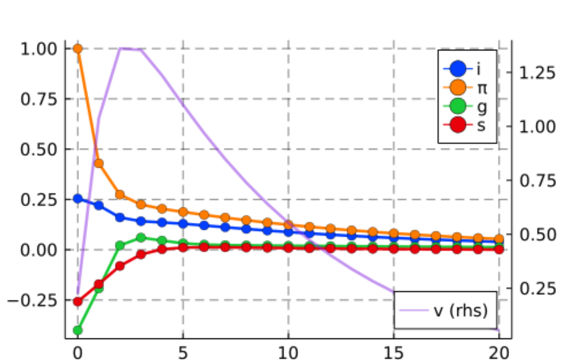


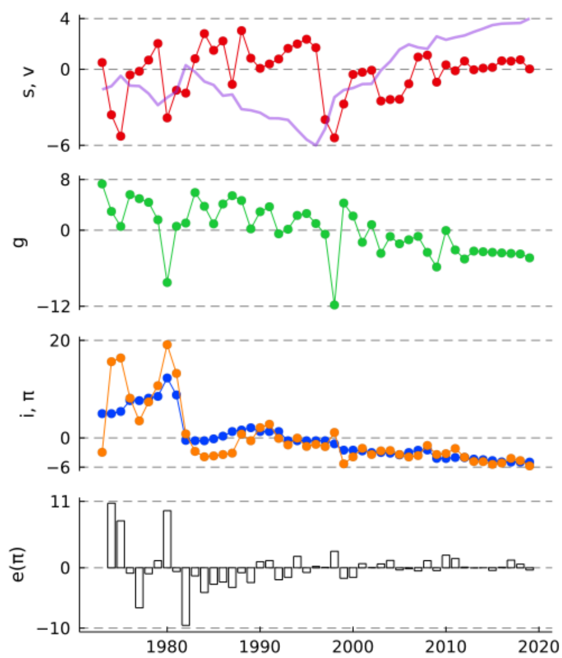




| | $i$ | $\pi$ | $g$ | $v$ | $r^n$ | $s$ |
|---|---|---|---|---|---|---|
| $i$ | 0.78* | 0.05 | 0.21 | 0.02 | −0.05 | 0.12 |
| $\pi$ | −0.1 | 0.39 | 0.19 | −0.01 | −0.07 | 0.08 |
| $g$ | −0.06 | −0.63* | 0.03 | −0.03* | 0.04 | 0.16 |
| $v$ | −0.04 | 1.66 | −0.57 | 0.95* | 0.27 | −1.83* |
| $r^n$ | 1.13* | 0.0 | 0.23 | −0.04 | 0.12 | −0.5 |
| $s$ | 0.51* | −0.54* | 0.22 | 0.02 | −0.05 | 0.41* |

Notes: All results evaluated at posterior mode. Top left: IRF to the inflation shock. Bottom left: autoregressive matrix $A$. Right: Residuals to the inflation equation (bottom); demeaned data for debt and surpluses-to-GDP (top), GDP growth (second from top), nominal interest and inflation (third from top).

**South Africa**: IRF to inflation shock (top left), VAR (bottom left) and demeaned data (right)

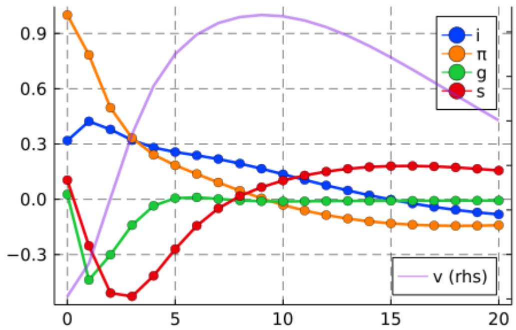



| | $i$ | $\pi$ | $g$ | $v$ | $r^n$ | $s$ |
|---|---|---|---|---|---|---|
| $i$ | 0.83* | 0.07 | 0.17* | 0.0 | −0.05 | 0.05 |
| $\pi$ | 0.93* | 0.23 | 0.21 | 0.01 | −0.06 | −0.08 |
| $g$ | 0.26 | −0.16 | 0.29 | −0.01 | 0.16 | −0.34 |
| $v$ | −0.27 | 0.13 | −0.79 | 0.94* | −0.13 | −1.52 |
| $r^n$ | 0.98* | −0.01 | −0.01 | −0.0 | 0.06 | 0.01 |
| $s$ | 0.01 | −0.04 | 0.06 | 0.01 | 0.02 | 0.39* |

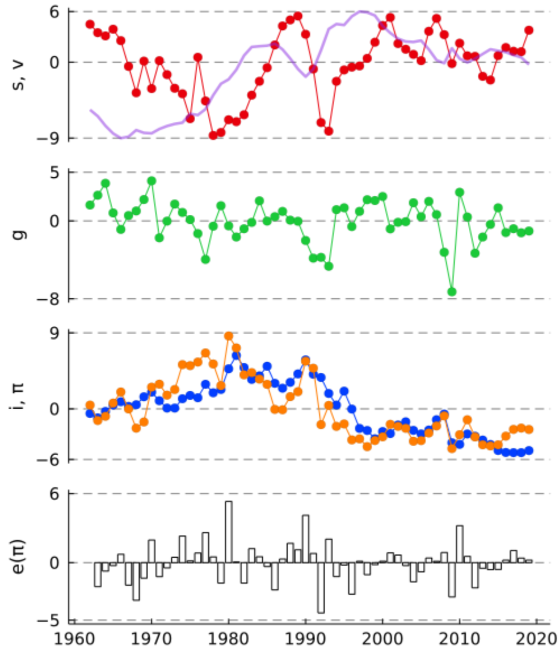
Notes: All results evaluated at posterior mode. Top left: IRF to the inflation shock. Bottom left: autoregressive matrix $A$. Right: Residuals to the inflation equation (bottom); demeaned data for debt and surpluses-to-GDP (top), GDP growth (second from top), nominal interest and inflation (third from top).

**South Korea**: IRF to inflation shock (top left), VAR (bottom left) and demeaned data (right)





| | $i$ | $\pi$ | $g$ | $v$ | $r^n$ | $s$ |
|---|---|---|---|---|---|---|
| $i$ | 0.82* | 0.16* | 0.22* | −0.0 | 0.01 | 0.02 |
| $\pi$ | 0.14 | 0.69* | 0.24 | −0.01 | −0.05 | −0.07 |
| $g$ | −0.14 | −0.23 | 0.52* | −0.01 | 0.26* | −0.13 |
| $v$ | 1.26* | 0.05 | −2.03* | 0.99* | −0.66* | −1.25* |
| $r^n$ | 1.09* | 0.04 | −0.14 | 0.0 | −0.17 | 0.02 |
| $s$ | −0.07 | −0.2 | 0.48* | 0.01 | 0.12 | 0.62* |





Notes: All results evaluated at posterior mode. Top left: IRF to the inflation shock. Bottom left: autoregressive matrix $A$. Right: Residuals to the inflation equation (bottom); demeaned data for debt and surpluses-to-GDP (top), GDP growth (second from top), nominal interest and inflation (third from top).

**Sweden**: IRF to inflation shock (top left), VAR (bottom left) and demeaned data (right)

### 5.17. Sweden

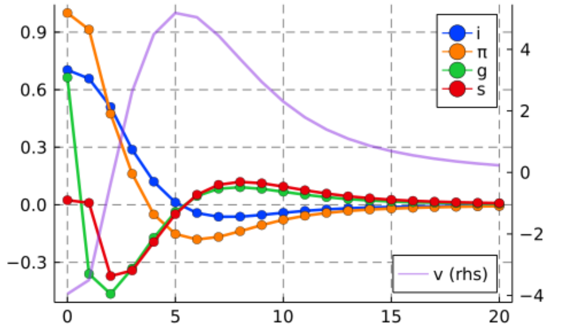
### 5.18. Switzerland

### 5.19. United Kingdom

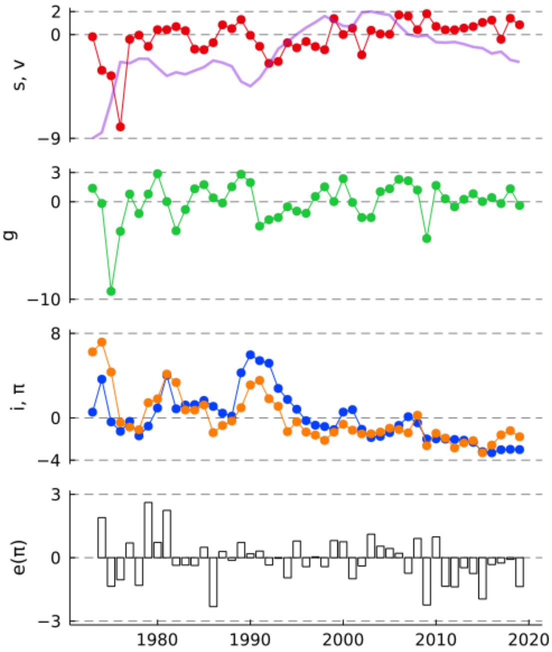
### 5.20. United States

### 5.21. Ukraine

## 6. Conclusion

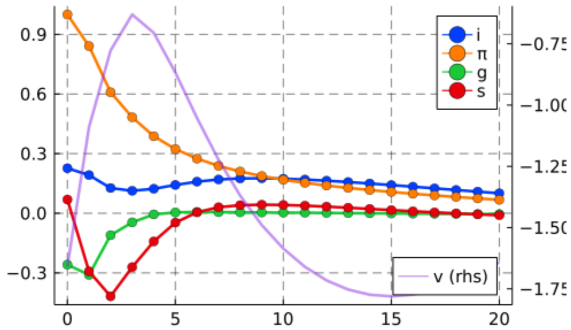



| | $i$ | $\pi$ | $g$ | $v$ | $r^n$ | $s$ |
|---|---|---|---|---|---|---|
| $i$ | 0.72* | 0.05 | 0.33 | −0.01 | 0.07 | 0.03 |
| $\pi$ | 0.14 | 0.5* | −0.02 | −0.01 | −0.12 | 0.18 |
| $g$ | −0.08 | −0.4 | 0.1 | −0.0 | −0.01 | 0.12 |
| $v$ | 1.24 | 1.37 | −2.3 | 0.97* | 0.15 | −1.41 |
| $r^n$ | 0.86* | 0.39 | −0.02 | 0.02 | −0.08 | −0.19 |
| $s$ | −0.07 | −0.17 | 0.34* | 0.01 | −0.02 | 0.14 |

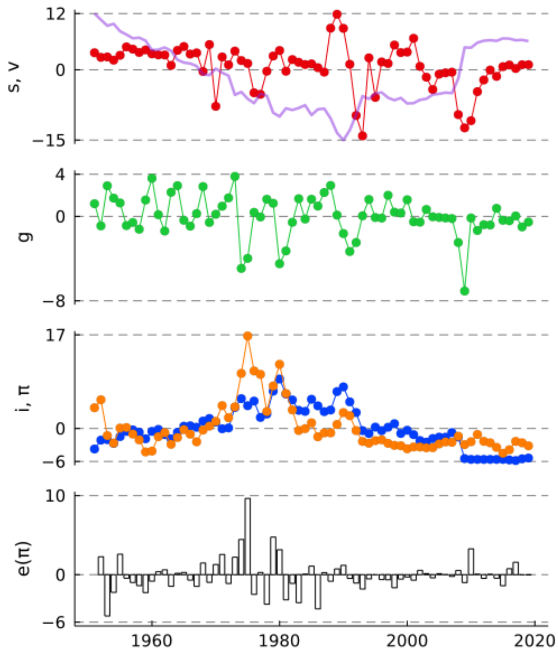



Notes: All results evaluated at posterior mode. Top left: IRF to the inflation shock. Bottom left: autoregressive matrix $A$. Right: Residuals to the inflation equation (bottom); demeaned data for debt and surpluses-to-GDP (top), GDP growth (second from top), nominal interest and inflation (third from top).

**Switzerland**: IRF to inflation shock (top left), VAR (bottom left) and demeaned data (right)

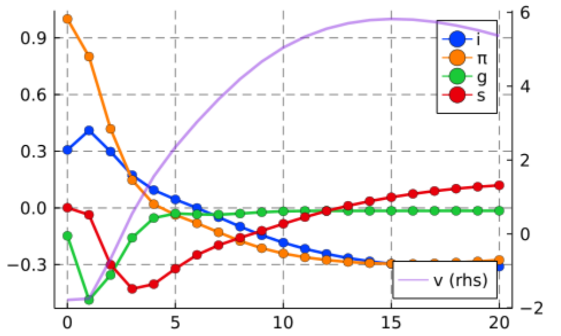


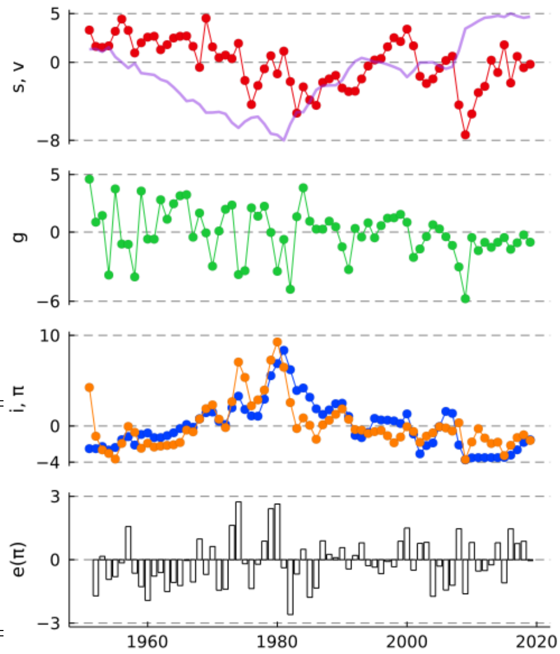

| | $i$ | $\pi$ | $g$ | $v$ | $r^n$ | $s$ |
|---|---|---|---|---|---|---|
| $i$ | 0.73* | 0.15* | 0.46* | −0.01* | 0.03 | 0.08* |
| $\pi$ | 0.24 | 0.77* | 0.14 | 0.01 | −0.07 | −0.04 |
| $g$ | −0.08 | 0.02 | 0.67* | 0.01 | 0.15* | −0.0 |
| $v$ | 0.78 | −0.93 | −3.35* | 0.96* | −0.54* | −0.77* |
| $r^n$ | 1.44* | −0.2 | −0.96 | 0.01 | −0.4* | −0.26 |
| $s$ | 0.34 | −0.04 | 1.09* | 0.02 | 0.04 | 0.38* |




Notes: All results evaluated at posterior mode. Top left: IRF to the inflation shock. Bottom left: autoregressive matrix $A$. Right: Residuals to the inflation equation (bottom); demeaned data for debt and surpluses-to-GDP (top), GDP growth (second from top), nominal interest and inflation (third from top).

**United Kingdom**: IRF to inflation shock (top left), VAR (bottom left) and demeaned data (right)

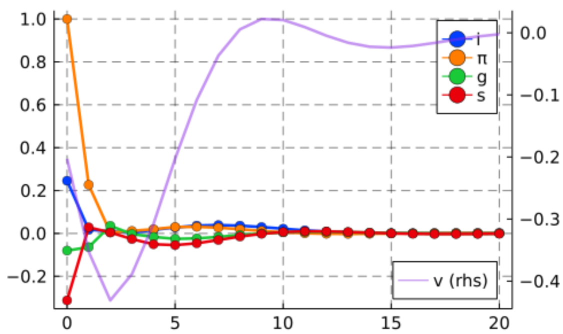


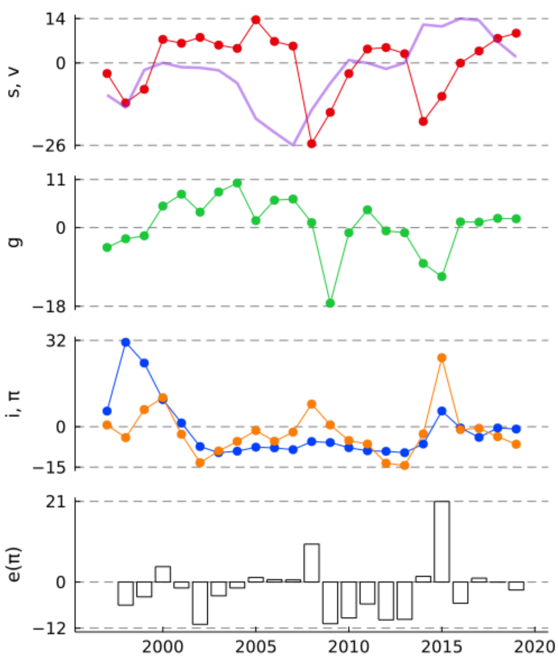




| | $i$ | $\pi$ | $g$ | $v$ | $r^n$ | $s$ |
|---|---|---|---|---|---|---|
| $i$ | 0.7* | 0.03* | 0.03* | −0.02* | −0.14* | −0.12* |
| $\pi$ | −0.04* | 0.47* | −0.14* | −0.04* | −0.28* | −0.21* |
| $g$ | −0.29* | −0.31* | 0.16* | −0.03* | 0.13* | −0.05* |
| $v$ | 1.99* | 0.54* | 0.32* | 1.16* | 0.85* | −0.32* |
| $r^n$ | 1.43* | 0.36* | 0.46* | 0.07* | 0.35* | 0.36* |
| $s$ | −0.14* | −0.2* | 0.07* | −0.02* | −0.2* | 0.54* |

Notes: All results evaluated at posterior mode. Top left: IRF to the inflation shock. Bottom left: autoregressive matrix $A$. Right: Residuals to the inflation equation (bottom); demeaned data for debt and surpluses-to-GDP (top), GDP growth (second from top), nominal interest and inflation (third from top).

**United States**: IRF to inflation shock (top left), VAR (bottom left) and demeaned data (right)




| | $i$ | $\pi$ | $g$ | $v$ | $r^n$ | $s$ |
|---|---|---|---|---|---|---|
| $i$ | 0.74* | −0.24 | −0.47 | −0.04 | 0.14 | −0.12 |
| $\pi$ | 0.14 | 0.06 | 0.1 | −0.02 | −0.0 | −0.43* |
| $g$ | 0.15 | 0.06 | −0.07 | 0.01 | 0.03 | 0.52* |
| $v$ | 0.39 | −0.63 | −0.69 | 0.85* | −0.17 | −0.93 |
| $r^n$ | 1.0* | −0.01 | 0.07 | 0.08 | −0.27 | 0.44 |
| $s$ | 0.13 | 0.2 | 0.3 | 0.1* | −0.05 | 0.53* |

Notes: All results evaluated at posterior mode. Top left: IRF to the inflation shock. Bottom left: autoregressive matrix $A$. Right: Residuals to the inflation equation (bottom); demeaned data for debt and surpluses-to-GDP (top), GDP growth (second from top), nominal interest and inflation (third from top).

**Ukraine**: IRF to inflation shock (top left), VAR (bottom left) and demeaned data (right)

## References


Ali Abbas, S. M., Belhocine, N., El-Ganainy, A., and Horton, M. (2011). Historical Patterns and Dynamics of Public Debt—Evidence From a New Database. *IMF Economic Review*, 59(4):717–742.

Bohn, H. (1995). The Sustainability of Budget Deficits in a Stochastic Economy. *Journal of Money, Credit and Banking*, 27(1):257.

Bohn, H. (1998). The Behavior of U. S. Public Debt and Deficits. *The Quarterly Journal of Economics*, 113(3):949–963.

Campbell, J. Y. and Ammer, J. (1993). What Moves the Stock and Bond Markets? A Variance Decomposition for Long-Term Asset Returns. *The Journal of Finance*, 48(1):3–37.

Campbell, J. Y. and Mankiw, N. G. (1989). Consumption, Income, and Interest Rates: Reinterpreting the Time Series Evidence. *NBER Macroeconomics Annual*, 4:185–216.

Canzoneri, M. B., Cumby, R. E., and Diba, B. T. (2001). Is the Price Level Determined by the Needs of Fiscal Solvency? *American Economic Review*, 91(5):1221–1238.

Cochrane, J. H. (2005). Money as stock. *Journal of Monetary Economics*, 52(3):501–528.

Cochrane, J. H. (2011). Determinacy and Identification with Taylor Rules. *Journal of Political Economy*, 119(3):565–615.

Cochrane, J. H. (2022a). The fiscal roots of inflation. *Review of Economic Dynamics*, 45:22–40.

Cochrane, J. H. (2022b). A fiscal theory of monetary policy with partially-repaid long-term debt. *Review of Economic Dynamics*, 45:1–21.

Cochrane, J. H. (2023). *The Fiscal Theory of the Price Level*. Princeton University Press.

Collins, S. M. and Park, W.-A. (1989). External Debt and Macroeconomic Performance in South Korea. In Sachs, J. D., editor, *Developing Country Debt and the World Economy*, pages 121–140. University of Chicago Press, Chicago.

Cox, W. M. (1985). The behavior of treasury securities monthly, 1942–1984. *Journal of Monetary Economics*, 16(2):227–250.

Cox, W. M. and Hirschhorn, E. (1983). The market value of U.S. government debt; Monthly, 1942–1980. *Journal of Monetary Economics*, 11(2):261–272.

del Negro, M. and Schorfheide, F. (2011). Bayesian Macroeconometrics. In Geweke, J., Koop, G., and Van Dijk, H., editors, *The Oxford Handbook of Bayesian Econometrics*, pages 292–389. Oxford University Press.

Eitrheim, Ø., Klovland, J. T., and Qvigstad, J. F. (2007). Historical Monetary Statistics for Norway - Part II. *Norges Bank Occasional Papers*, 38.

Giannone, D., Lenza, M., and Primiceri, G. (2015). Prior Selection for Vector Autoregressions. *The Review of Economics and Statistics*, 97(2):436–451.

Gruen, D. and Sayegh, A. (2005). The Evolution of Fiscal Policy in Australia. *Oxford Review of Economic Policy*, 21(4):618–635.

Hall, G. J. and Sargent, T. J. (2015). A History of US Debt Limits. Technical report, National Bureau of Economic Research.

Hall, R. E. (1988). Intertemporal Substitution in Consumption. *Journal of Political Economy*, 96(2):339–357.

Hall, R. E. (2009). Reconciling Cyclical Movements in the Marginal Value of Time and the Marginal Product of Labor. *Journal of Political Economy*, 117(2):281–323.

Hansen, L. P., Roberds, W., and Sargent, T. J. (1991). Time Series Implications of Present Value Budget Balance and of Martingale Models of Consumption and Taxes. In Hansen, L. P. and Sargent, T. J., editors, *Rational Expectations Econometrics*. London and Boulder, Co.: Westview Press.

Ito, T. (2013). Great Inflation and Central Bank Independence in Japan. In Bordo, M. and Orphanides, A., editors, *The Great Inflation: The Rebirth of Modern Central Banking*. University of Chicago Press, Chicago.

Jacobson, M., Leeper, E., and Preston, B. (2019). Recovery of 1933. Technical Report w25629, National Bureau of Economic Research, Cambridge, MA.

Karlsson, S. (2013). Forecasting with Bayesian Vector Autoregression. In *Handbook of Economic Forecasting*, volume 2, pages 791–897. Elsevier.

Kehoe, P. and Midrigan, V. (2015). Prices are sticky after all. *Journal of Monetary Economics*, 75:35–53.

Kopits, G. (2011). Independent Fiscal Institutions: Developing Good Practices. *OECD Journal on Budgeting*, 11(3):1–18.

Kopits, G. and Romhanyi, B. (2013). Hungary: A Short-Lived Fiscal Watchdog. In Kopits, G., editor, *Restoring Public Debt Sustainability*, pages 212–233. Oxford University Press.

Litterman, R. (1979). Techniques of forecasting using Vector Auto Regression. *Federal Reserve Bank of Minneapolis Working Paper*, 115.

Miyazaki, M. (2007). Framework for Fiscal Consolidation: Successes and Failures in Japan. *OECD Journal on Budgeting*, 6(4):123–148.

Smets, F. and Wouters, R. (2007). Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach. *The American Economic Review*, 97(3):586–606.

Taylor, J. B. (1993). Discretion versus policy rules in practice. *Carnegie-Rochester Conference Series on Public Policy*, 39:195–214.

## A. Data Sources and Treatment

I collect a significant share of the data from the St. Louis Fed's *FRED* website, and the consumer price index series of many countries from the IMF's World Economic Outlook database. In the case of countries with sample starting after 1970, I get real GDP data from the United Nations's National Accounts Main Aggregates Database.

Whenever omitted in the list below, the source for interest rate data is the FRED; and the source of debt structure data is the OECD's Central Government Debt database. Finally, unless otherwise noted, public debt data I get from the database from Ali Abbas et al. (2011), which is kept up-to-date (Correct this sentence).

**Australia** 1973-2021. Real GDP from the UN. Public debt from the IMF. Interest rate from FRED: IR3TBB01AUM156N. Price index from FRED: AUSCPIALLQINMEI.

**Brazil** 1997-2021. Debt structure data I collect from the Brazilian Central Bank.

**Canada** 1960-2021. All except public debt from FRED.

**Chile** 1997-2021.

**Colombia** 1997-2021. Debt structure data I collect from the Internal Debt Profile report, available at the Investor Relations Colombia webpage.

**Czech Republic** 1997-2021.

**Denmark** 1960-2021. All except public debt from FRED.

**Hungary** 1997-2021. Inflation and debt from the IMF. GDP from the UN. Interest rate from FRED: IR3TIB01HUM156N.

**India** 1997-2021. Debt structure data collect from the Status Paper on Government Debt report, 2019-2020, available at the Department of Economic Affairs.

**Indonesia** 1997-2021. Debt structure data I gather from the 2014 "Central Government Debt Profile" report and the 2018 "Government Securities Management" report, both from the Ministry of Finance.

**Israel** 1997-2021.

**Japan** 1960-2021. Public debt from the IMF. The other series from the FRED. Real GDP: NYGDPPCAPKDJPN. Interest rate: IRSTCB01JPM156N. Price index: JPNCPIALLMINMEI.

**Mexico** 1997-2021. Real GDP from the UN. Price index and public debt from the IMF. Interest rate from FRED: IR3TIB01MXM156N.

**Norway** 1960-2021. All except public debt and interest rates from FRED. I interpolate the debt data for the year 1966. FRED interest data goes back to 1979, I splice it with historical data from Eitrheim et al. (2007), available at the website of the Norges Bank.

**New Zealand** 1973-2021. All except GDP and public debt from FRED.

**Poland** 1997-2021.

**South Africa** 1997-2021. Debt structure data from the 2020/2021 Debt Management Report, from the National Treasury Department.

**South Korea** 1973-2021. Real GDP from the UN and public debt from the IMF. The other two series from the FRED. Interest rate: INTDSRKRM193N. Price index: KORCPIALLMINMEI.

**Sweden** 1960-2021. All except public debt from FRED. I interpolate the debt data for the year 1965 and 1966.

**Switzerland** 1973-2021. Interest, CPI and exchange rate from FRED.

**Turkey** 1997-2021.

**Ukraine** 1997-2021. Interest rate data from National Bank of Ukraine (NBU Key Policy Rate). Debt structure data I collect from "Ukraine's Public Debt Performance in 2021 and Local Market Update", from the Ministry of Finance of Ukraine.

**United Kingdom** 1960-2021. Interest data from the Bank of England (Base Rate); I splice it with discount rate data from the FRED. Inflation and GDP data from FRED.

**United States** 1950-2021. All data collected from the FRED. I use real exchange rate to the United Kingdom, since nominal exchange rate is available since before 1950.

## B. A Model of the Par Value of Public Debt

Keeping the assumption of a geometric term structure, suppose the government sells bonds with coupons at par, as they usually do. Let $i_t^b$ be the average coupon rate of nominal bonds and $r_t^b$ the same for real bonds. Let $\mathcal{V}_{N,t}^{b,n}$ and $\mathcal{V}_{R,t}^{b,n}$ be the principal payment due $n$ periods ahead, in dollars. After $n$ periods, the government must pay $\mathcal{V}_{N,t}^{b,n}$ (principal) + $i_t^b \mathcal{V}_{N,t}^{b,n}$ (coupon) dollars; and $\mathcal{V}_{R,t}^{b,n}(1+r_t^b)/P_{t+n}$ consumption goods. These correspond to the quantities $B_{N,t}^n$ and $B_{R,t}^n$ defined earlier. Using the geometric term structure assumption 3, we get

$$\mathcal{V}_{N,t}^{b,n} = \omega_N \mathcal{V}_{N,t}^{b,n-1} \quad \text{and} \quad (\mathcal{V}_{R,t}^{b,n}/P_{t+n}) = \omega_R(\mathcal{V}_{R,t}^{b,n-1}/P_{t+n-1}).$$

The market-value of public debt at the beginning of period $t$ corresponds to the sum of the market-value of each principal + coupon payment:

$$\begin{aligned}
\overbrace{\mathcal{V}_{N,t-1}(1+r_t^N)}^{\substack{\text{Market Value,}\\ \text{Beginning of Period}}} &= \overbrace{\left[\mathcal{V}_{N,t-1}^{b,1}(1+i_{t-1}^b) + \mathcal{V}_{N,t-1}^{b,2}(1+i_{t-1}^b)Q_{N,t}^1 + \mathcal{V}_{N,t-1}^{b,3}(1+i_{t-1}^b)Q_{N,t}^2 + \dots\right]}^{\text{Market value of principal + coupon payments}} \\
&= \mathcal{V}_{N,t-1}^{b,1}(1+i_{t-1}^b)\left[1 + \omega_N Q_{N,t}^1 + \omega_N^2 Q_{N,t}^2 + \dots\right] \\
&= \mathcal{V}_{N,t-1}^{b,1}(1+i_{t-1}^b)\,(1+\omega_N Q_{N,t}) \\
\mathcal{V}_{R,t-1}(1+r_t^R)(1+\pi_t) &= (\mathcal{V}_{N,t-1}^{b,1}/P_{t-1})(1+r_{t-1}^b)\left[P_t + \omega_R P_t Q_{N,t}^1 + \omega_N^2 P_t Q_{N,t}^2 + \dots\right] \\
&= \mathcal{V}_{R,t-1}^{b,1}(1+r_{t-1}^b)(1+\pi_t)
\end{aligned}$$

Since bonds are issued at par, the par-value of public debt is just the sum of principals: $\mathcal{V}^b_{N,t-1} = \sum_{n=1}^{\infty} \mathcal{V}^{b,n}_{N,t-1}$, $\mathcal{V}^b_{R,t-1} = \sum_{n=1}^{\infty} \mathcal{V}^{b,n}_{R,t-1}$, and $\mathcal{V}^b_t = \mathcal{V}^b_{N,t} + \mathcal{V}^b_{R,t}$.

Next, I linearize. Let $V_{N,t} = \mathcal{V}_{N,t}/(P_t Y_t)$ and $V_{R,t} = \mathcal{V}_{R,t}/(P_t Y_t)$ be debt-to-GDP ratios, and $v_{N,t} = \log(V_{N,t}/V_N)$ and $v_{R,t} = \log(V_{R,t}/V_N)$ be their log deviations from steady state. Linearization of the equations above yields

$$\begin{aligned} v_{N,t-1} + r^N_t &= v^b_{N,t-1} + i^b_{t-1} + \rho\omega_N q_{N,t} \\ v_{R,t-1} + r^R_t &= v^b_{R,t-1} + r^b_{t-1} + \pi_t + \rho\omega_R q_{R,t} \end{aligned}$$

(I have redefined $i^b_t$ and $r^b_t$ to be log-return as deviation from the steady state). Up to a first-order approximation, $v_t = \delta v_{N,t} + (1-\delta)v_{R,t}$. Combining the equations above, we get

$$v_{t-1} + r^n_t = v^b_{t-1} + r^{n,b}_t + \rho\left[\delta\omega_N q_{N,t} + (1-\delta)\omega_R q_{R,t}\right] \tag{13}$$

$$r^{n,b}_t = \delta i^b_{t-1} + (1-\delta)\omega_R\left(r^b_{t-1} + \pi_t\right). \tag{14}$$

Expressions (13) and (14) have clear interpretations. Equation (14) defines the current-period coupon payment $r^{n,b}_t$. It only depends on time-$t$ information through the inflation rate, as real bond coupons vary with the price level. Equation (13) says that the beginning-of-period market-value debt equals previous-period par-value debt + coupon payments + variation in the price of long-term bonds.[1] Replacing (2) for $r^n_t$ leads to the adjustment equation:

$$v_t = v^b_t + q_t + \delta i^b_t + (1-\delta)r^b_t. \tag{7}$$

Replacing equation (13) in the flow equation of public debt (1) yields

$$\rho\left(v_t - \left[\delta\omega_N q_{N,t} + (1-\delta)\omega_R q_{R,t}\right] + \frac{s_t}{V}\right) = v^b_{t-1} + r^{n,b}_t - \pi_t - g_t. \tag{15}$$

This expression is similar to Hall and Sargent (2015) (equation 8 of their paper and first expression in page 11, not numbered). The par value of public debt $v^b_{t-1}$ accrued by period coupons $r^{n,b}_t$ yields the new value for the market value of debt $v_t$ netted out of long-term bond price variation (approximated by the term in brackets on the left).

We can replace (7) again on the left-hand side of (15) to arrive at a flow equation for $v^b_t$:

$$\rho\left(v^b_t + \underbrace{\left[\delta(i^b_t + (1-\omega_N)q_{N,t}) + (1-\delta)(r^b_t + (1-\omega_R)q_{R,t})\right]}_{\text{"Revenue" effect of changing average coupon rates}} + \frac{s_t}{V}\right) = v^b_{t-1} + r^{n,b}_t - \pi_t - g_t. \tag{16}$$

The evolution of par-value debt differs from that of market-value debt in two respects. First, instead of nominal return accrual, it pays the average coupon rate. In the equation, that means replacing $r^n_t$ by $r^{n,b}_t$. Second, since par-value debt only sums up principal payments, a new term (bracket, left-hand side) must be added to account for changes in the average coupon rate. Intuitively, if the government sells, for each existing bond, a new one with same principal payment but higher coupon rate, it will raise enough revenue to retire all existing bonds while leaving the par value of public debt $v^b_t$ unchanged (the sum of principal payments will be the same). Of course, this will also cause later increases to coupon payment disbursements $r^{n,b}_t$ (see (14)). Since bonds are sold at

[1] The $\omega q$ terms scale the variation in bond price by $\omega$, the share of long-term bonds.

par, the "revenue"-generating effect of changing coupon rates must be computed relative to actual average discount rates, which are the $(1-\omega)q_t$ terms, as explained below.

**Average Coupon Rates**. To keep a geometric term structure, every period the government must roll over a share of $1-\omega_N$ of nominal and $1-\omega_R$ of real debt. It then issues debt for all future maturities keeping the same geometric structure. Since bonds are sold at par by assumption, the coupon rate corresponds to the yield to maturity. In light of the constant term premium assumption 3, the increment in the average coupon rate is $(1-\omega_N)\sum(\omega_N\rho)E_t i_{t+n} = -(1-\omega_N)q_{N,t}$ for nominal bonds, and $-(1-\omega_R)q_{R,t}$ for real bonds.[1] Therefore, the law of motion to the average coupon rates are

$$\begin{aligned} i_t^b &= -(1-\omega_N)^2 q_{N,t} + \omega_N i_{t-1}^b \\ r_t^b &= -(1-\omega_R)^2 q_{R,t} + \omega_R r_{t-1}^b. \end{aligned} \tag{8}$$

Average (nominal or real) coupon rates are $\omega$-weighted (since only a share $1-\omega$ of debt is rolled over) moving averages of $\omega$-weighted (since the geometric term structure must be kept) averages of expected future short-term interest. We can re-writte the left-hand side of (16):

$$\rho\left(v_t^b + \underbrace{\left[\delta\left(i_t^b - (1-\omega_N)\sum_{j=0}^{\infty}(\omega_N^j\rho)E_t i_{t+j}\right) + (1-\delta)\left(r_t^b - (1-\omega_R)\sum_{j=0}^{\infty}(\omega_N^j\rho)E_t r_{t+j}\right)\right]}_{\text{"Revenue" effect of changing average coupon rates}} + \frac{s_t}{V}\right).$$

Since $i_t^b$ and $r_t^b$ move slower than interest rates, the revenue effect of changing coupon rates will tend to be negative when they grow.

**Limit cases**. If $\omega_N = \omega_R = 0$, the government does not issue long-term bonds. (8) implies $i_t^b = -q_{N,t} = i_t$ (and $r_t^b = r_t$). The nominal return on the stock of debt and coupon payment flows coincide, $r_t^n = r_t^{n,b}$, and, by (13), $v_t = v_t^{n,b}$.

The case $\omega_N = \omega_R = 1$ is analogous to the government financing itself using perpetuities only. Coupon rates become invariant to interest rate variation ($i_t^b = r_t^b = 0$). This implies $v_t = v_t^b + q_t$. On the other hand, bond prices $q_t$ become more volatile.

In both of these limit cases, the law of motion for par-value debt satisfies the intuitive flow equation

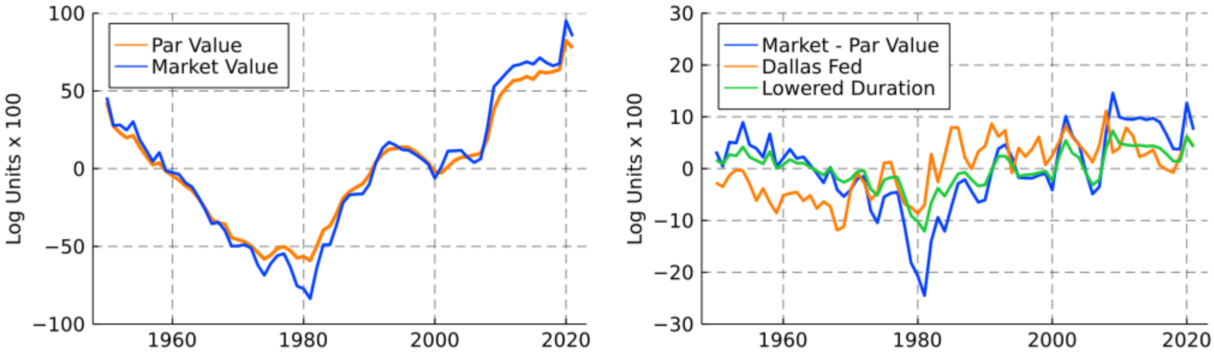
$$\rho\left(v_t^b + \frac{s_t}{V}\right) = v_{t-1}^b + r_t^{n,b} - \pi_t - g_t.$$

## C. Additional Tables and Graphs

## D. Restoring the Flow Equation in the VAR

## E. Derivation of the NK Model with Trend Shocks

[1]The $\rho$ should not enter the sum. Since it is a number close to one, I introduce it to arrive at the convenient simplification wth $q_{N,t}$ and $q_{R,t}$.




(a) Par and Market Value of Debt

(b) Comparison to Dallas Fed Estimates

Notes: US data. The left figure plots our data for par-value of public debt (log, then demeaned) and our baseline calculation of the market-value of public debt. The blue line on the right-hand figure corresponds to the difference between these two series. I also plot the difference using the Dallas Fed's estimates of the market value of debt, as well as my measure after reducing the average duration of US debt (both nominal and real) from five to two and a half years.

Figure 4: US Par and Market Value of Public Debt