

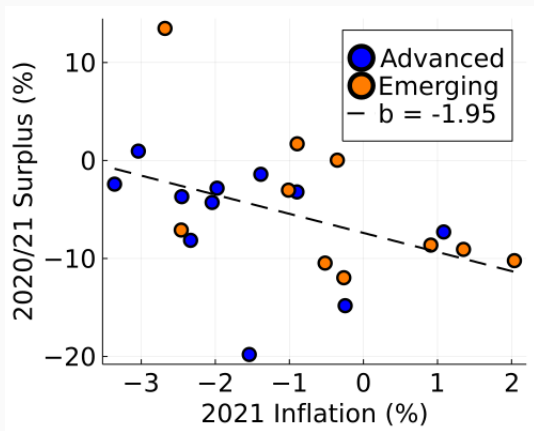
A Fiscal Decomposition of Unexpected Inflation: Cross-Country Estimates and Theory

Livio Maya

Introduction

- What drives innovations to the price level?
- Sources of inflation variation
- Focus on **unexpected inflation** $\Delta E_t \pi_t$
 - Campbell and Ammer (1993)
 - Internal consistency of expectations
- Breakdown of valuation equation of public debt

Fiscal Connection?



COVID Inflation - 21 countries in sample

Valuation Equation of Public Debt

- Stock market - Campbell and Ammer (1993)

Stock price = Discounted Dividends

$$\Delta E_t [\text{Stock price}] = \Delta E_t [\text{Dividends}] - \Delta E_t [\text{Disc Rates}]$$

- Micro-founded monetary models

$$\frac{\text{Bond Prices} \times \text{Bonds}}{\text{Price Level}} = \sum_t \frac{\text{Surpluses}_t}{\text{Discount}_t}$$

$$\Delta E_t [\text{Bond Price}] - \Delta E_t [\text{Price}] = \Delta E_t [\text{Surplus}] - \Delta E_t [\text{Disc}]$$

Exercises

1. Decomposition estimates

- Bayesian VAR for 21 countries
- Inflation shock $\Delta E_t \pi_t = 1$
- Discounted surpluses shock: $\Delta E_t [\text{Disc Surp}] = -1$

2. FTPL, New-Keynesian Model

- Volatile surpluses, no contribution to inflation?
- GMM estimate to reproduce decompositions
- Parametric model of partial debt repayment
- Shocks to long-term growth

Motivation + Results

- Measures not structural
- Stylized facts to be matched by theory
- On average:
 - **Discount rates** → ~80% of total inflation
 - GDP growth → ~20% of total inflation
 - Surplus/GDP → ~0% of total inflation

Inflation has fiscal roots, even if fiscal policy is disconnected from the price level.

Motivation + Results

- Structural interpretation?
- Volatile surpluses, no inflation?
- "Passive" vs "active" fiscal policy
- No debt repayment is inconsistent with decompositions
- On average, 0.78% of 1% GDP deficit is repaid
 - 0.96% in advanced economies
 - 0.59% in developing economies

Discount-driven inflation and realistic surplus process
preclude partial repayment.

Why unexpected inflation, not just inflation?

- New Keynesian theory:
 - Fisher: monetary policy sets expected inflation
 - Fiscal policy sets **unexpected** inflation
- Measures do not depend on state of the economy
- Direct connection with impulse response functions

Literature

- **Monetary-Fiscal Interaction.** Cagan (1956), Sargent and Wallace (1981), Hall and Sargent (1997), Hall and Sargent (2011), Jiang et al. (2019), Corsetti et al. (2019), Sunder-Plassmann (2020), Du et al. (2020), Akhmadieva (2022)
- **Fiscal Theory of the Price Level.** Leeper (1991), Sims (1994), Woodford (1995), Cochrane (1998), Cochrane (2005), Sims (2011), Leeper and Leith (2016), Bassetto and Cui (2018), Cochrane (2022c), Brunnermeier et al. (2022), Cochrane (2022a), Cochrane (2022b)
- **Empirical Finance.** Campbell and Shiller (1988), Cochrane (1992), Campbell and Ammer (1993), Chen and Zhao (2009), Cochrane (2008), Jiang et al. (2019)

Environment

- 1 period = 1 year
- Consumption good price P_t
- Total output Y_t
- Nominal bonds $B_{N,t}^n$, price $Q_{N,t}^n$
 - Pay one unit of currency after n years
- Real bonds $B_{R,t}^n$, price $P_t Q_{R,t}^n$
 - Pay one unit of consumption good after n years
- Primary Surplus $P_t S_t$

Evolution of Public Debt

$$\begin{aligned} & \overbrace{\left[B_{N,t-1}^1 + P_t B_{R,t-1}^1 \right]}^{\text{Issued Currency}} = \Delta M_t \\ & + \underbrace{\left[P_t S_t + \sum_{n=1}^{\infty} Q_{N,t}^n \left(B_{N,t}^n - B_{N,t-1}^{n+1} \right) + P_t \sum_{n=1}^{\infty} Q_{R,t}^n \left(B_{R,t}^n - B_{R,t-1}^{n+1} \right) \right]}_{\text{Retired Currency}} \end{aligned}$$

- This is a **budget constraint**
- Assumption 1: households do not value currency $M_t = 0$

Evolution of Public Debt

- Assumption 1: households do not value currency $M_t = 0$
- End-of-period debt $\mathcal{V}_{N,t}$ and $\mathcal{V}_{R,t}$

$$(1 + r_t^N)\mathcal{V}_{N,t-1} + (1 + r_t^R)(1 + \pi_t)\mathcal{V}_{R,t-1} = P_t S_t + \mathcal{V}_{N,t} + \mathcal{V}_{R,t}$$

- This is an **equilibrium condition**
- Price level adjusts so that

currency issued = currency retired

Evolution of Public Debt

- Constant structure of public debt: $\delta = \mathcal{V}_{N,t} / \mathcal{V}_t$

$$1 + r_t^n = \delta \left[(1 + r_{N,t}) \right] + (1 - \delta) \left[(1 + r_{R,t})(1 + \pi_t) \right]$$

- Debt-to-GDP = $V_t = \mathcal{V}_t / P_t Y_t$
- Surplus-to-GDP = $s_t = S_t / Y_t$

$$\frac{1 + r_t^n}{(1 + \pi_t)(1 + g_t)} V_{t-1} = s_t + V_t$$

Evolution of Public Debt

Linearized equations

$$v_t + \frac{s_t}{V} = \frac{1}{\beta} [v_{t-1} + r_t^n - \pi_t - g_t]$$

$$r_t^n = \delta [r_t^N] + (1 - \delta) [r_t^R + \pi_t]$$

- v_t is log debt-to-GDP
- r_t^n is the nominal return on public debt

Valuation Equation of Public Debt

- Assumption 2: debt does not spiral $\lim_{j \rightarrow \infty} \beta^j v_{t+j} = 0$
- Solve flow equation forward:

$$\underbrace{v_{t-1} + r_t^n - \pi_t}_{\text{Real market value of debt}} = \underbrace{\frac{\beta}{V} \sum_{j=0}^{\infty} \beta^j [E_t s_{t+j} + E_t g_{t+j}] - \sum_{j=1}^{\infty} \beta^j [E_t r_{t+j}^n - E_t \pi_{t+j}]}_{\text{Discounted Surpluses}}$$

Marked-to-Market Decomposition

Take innovation on the valuation equation:

$$\epsilon_{r^n,t} - \epsilon_{\pi,t} = \epsilon_{s,t} + \epsilon_{g,t} - \epsilon_{r,t}$$

Terms:

$$\epsilon_{r^n,t} = \Delta E_t r_t^n$$

$$\epsilon_{\pi,t} = \Delta E_t \pi_t \text{ (current inflation)}$$

$$\epsilon_{s,t} = (\beta/V) \sum_{j=0}^{\infty} \beta^j \Delta E_t s_{t+j}$$

$$\epsilon_{g,t} = \sum_{j=0}^{\infty} \beta^j \Delta E_t g_{t+j}$$

$$\epsilon_{r,t} = \sum_{j=1}^{\infty} \beta^j (\Delta E_t r_{t+j}^n - \Delta E_t \pi_{t+j})$$

Public Finances Model

Why a public finances model?

1. We can do better: bond prices forecast future inflation
2. No historical data for bond price/return r_t^n
3. No data on market value of debt (only book value)

Public Finances Model

Key Assumptions

- Assumption: constant maturity structure
- Decays geometrically at rate ω :

$$B_{N,t}^n = \omega_N B_{N,t}^{n-1}$$

$$B_{R,t}^n = \omega_R B_{R,t}^{n-1}$$

- Assumption: constant (or no) risk premium

$$E_t r_{N,t} = E_t r_{R,t} + E_t \pi_t = i_t$$

Public Finances Model

- Bond prices:

$$q_{N,t} = (\omega_N \beta) E_t q_{N,t} - [i_t]$$

$$q_{R,t} = (\omega_R \beta) E_t q_{R,t} - [i_t - E_t \pi_{t+1}]$$

- Returns:

$$r_{j,t} = (\omega_j \beta) q_{j,t} - q_{j,t-1} \quad j = N, R$$

Break down of bond price variation

Proposition: let $r_t = i_t - E_t \pi_{t+1}$ be the real interest. Then

$$\epsilon_{r^n,t} - \epsilon_{\pi,t} = -\delta \sum_{j=0}^{\infty} (\omega_N \beta)^j \Delta E_t \pi_{t+j} - \sum_{j=1}^{\infty} \beta^j [\delta \omega_N^j + (1 - \delta) \omega_R^j] \Delta E_t r_{t+j}$$

- Higher real discount lowers real and nominal bond prices
- Higher inflation lowers nominal bond prices
- No long-term debt $\omega = 0$:

$$\epsilon_{r^n,t} - \epsilon_{\pi,t} = -\delta \Delta E_t \pi_t$$

Total Inflation Decomposition

Replace bond return decomp on marked-to-market decomp:

$$-\varepsilon_{\pi,t} = \varepsilon_{s,t} + \varepsilon_{g,t} - \varepsilon_{r,t}$$

Terms:

$$\varepsilon_{\pi,t} = \delta \sum_{j=0}^{\infty} (\omega_N \beta)^j \Delta E_t \pi_{t+j} \text{ (current and future inflation)}$$

$$\varepsilon_{s,t} = \epsilon_{s,t} = (\beta/V) \sum_{j=0}^{\infty} \beta^j \Delta E_t s_{t+j}$$

$$\varepsilon_{g,t} = \epsilon_{g,t} = \sum_{j=0}^{\infty} \beta^j \Delta E_t g_{t+j}$$

$$\varepsilon_{r,t} = \sum_{j=1}^{\infty} \beta^j \left[1 - (\delta \omega_N^j + (1 - \delta) \omega_R^j) \right] \Delta E_t r_{t+j}$$

Comparison of Decompositions

- **Marked-to-market:** $\epsilon_{r^n,t} - \epsilon_{\pi,t} = \epsilon_{s,t} + \epsilon_{g,t} - \epsilon_{r,t}$
 - Current inflation given current bond prices
 - Highlights effect of monetary policy
- **Total inflation:** $-\epsilon_{\pi,t} = \epsilon_{s,t} + \epsilon_{g,t} - \epsilon_{r,t}$
 - Path of inflation given path of discount rates
 - Sensitive to future inflation
 - Nets out effect of discount rates on bond prices

Build Market Value of Debt

- Converting par to market value of debt
- Dallas Fed, Cox and Hirschhorn (1983) and Cox (1985)

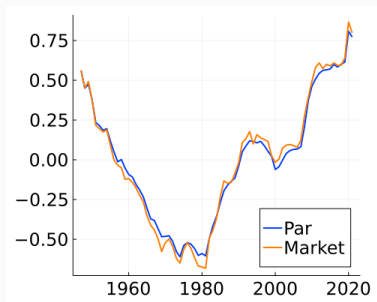
$$\mathcal{V}_{j,t} = \mathcal{V}_{j,t}^b \times \frac{\text{market value of bonds}}{\text{book value of bonds}} = \mathcal{V}_{j,t}^b \frac{Q_{j,t}}{Q_{j,t}^b} \quad \text{for } j = N, R$$

- Book price of bonds evolve according to average interest:

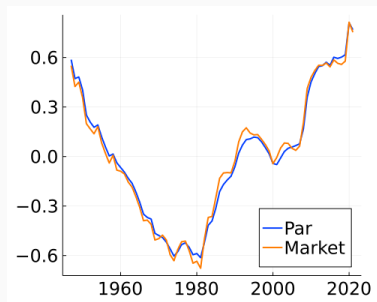
$$i_{N,t}^b = (1 - \omega_N) i_t + \omega_N i_{N,t-1}^b$$

$$i_{R,t}^b = (1 - \omega_R) (i_t - E_t \pi_{t+1}) + \omega_R i_{R,t-1}^b$$

Comparison with Dallas Fed



(a) Dallas Fed



(b) Model

Vector Autoregression

- States X

$$X_t = AX_{t-1} + e_t \quad e_t \sim N(0, \Sigma)$$

- Bayesian estimates of 21 countries
- Samples end in 2019 (no COVID!)
- Prior centered around US OLS estimates

$$\begin{bmatrix} i_t & \text{Nominal Interest} \\ \pi_t & \text{Inflation Rate} \\ g_t & \text{GDP Growth} \\ v_t & \text{Market Value Debt} \\ r_t^n & \text{Bond Return (model built)} \\ s_t & \text{Primary Surplus (model built)} \end{bmatrix}$$

VAR and Decomposition Measures

- VAR uses time series structure to identify revision of expectations

$$\Delta E_t X_{t+j} = A^j X_t$$

$$\sum_{j=0}^{\infty} \beta^j \Delta E_t X_{t+j} = (I - \beta A)^{-1} X_t$$

The Inflation Shock

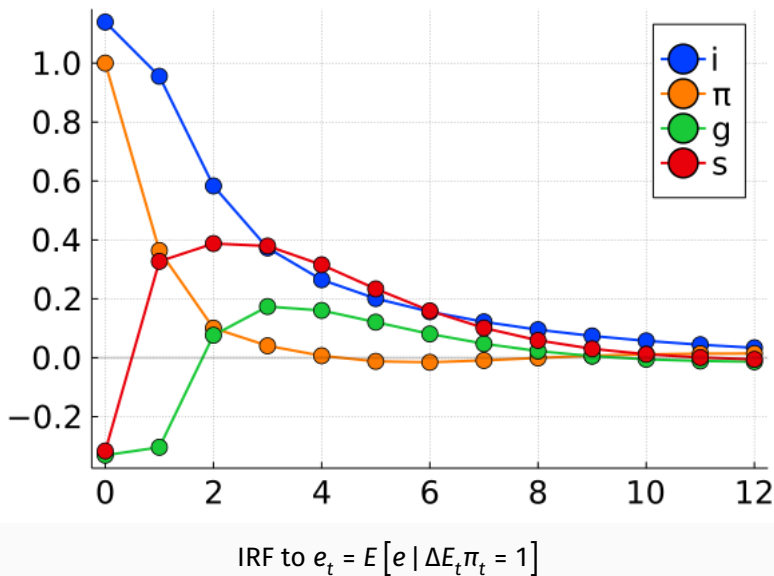
- Source of innovations to inflation $\Delta E_t \pi_t = 1$
- Reduced-form shock $e_t = E[e \mid \Delta E_t \pi_t = 1]$
- Proposition: MtM decomposition

$$\epsilon_{r^n,t} - \epsilon_{\pi,t} = \epsilon_{s,t} + \epsilon_{g,t} - \epsilon_{r,t}$$

same as **variance decomposition**

$$\frac{\text{cov}(\epsilon_{r^n,t}, \epsilon_{\pi,t})}{\text{var}(\epsilon_{\pi,t})} - 1 = \frac{\text{cov}(\epsilon_{s,t}, \epsilon_{\pi,t})}{\text{var}(\epsilon_{\pi,t})} + \frac{\text{cov}(\epsilon_{g,t}, \epsilon_{\pi,t})}{\text{var}(\epsilon_{\pi,t})} - \frac{\text{cov}(\epsilon_{r,t}, \epsilon_{\pi,t})}{\text{var}(\epsilon_{\pi,t})}$$

IRF - Brazil



Inflation Shock - Marked-to-Market

Country	ϵ_{r^n}	$-\epsilon_{\pi}$	=	ϵ_s	$+\epsilon_g$	$-\epsilon_r$
<i>1947 Sample (Advanced)</i>						
United Kingdom	** -0.7	** -1	=	** -2.2	** -0.7	** 1.2
United States	** -0.7	** -1	=	-0.3	** -0.5	** -0.9
<i>1960 Sample (Advanced)</i>						
Canada	** -2.8	** -1	=	0.3	* -1.4	** -2.8
Denmark	** -0.9	** -1	=	0.2	-0.2	** -1.9
Japan	** -0.6	** -1	=	** 2.8	** -3.0	** -1.4
Norway	** -0.7	** -1	=	0.7	* 3.0	** -5.4
Sweden	** -0.6	** -1	=	** 0.9	** -0.9	** -1.6
<i>1973 Sample (Advanced)</i>						
Australia	** -2.2	** -1	=	0.2	0.1	** -3.5
New Zealand	** -1.0	** -1	=	* 1.2	** -1.4	* -1.8
South Korea	** -0.6	** -1	=	** -2.4	0.2	* 0.7
Switzerland	** -2.0	** -1	=	* -0.8	0.1	** -2.3

Inflation Shock: $e_t = E[e \mid \Delta E_t \pi_t = 1]$

Inflation Shock - Marked-to-Market

Country	ϵ_{r^n}	$-\epsilon_{\pi}$	=	ϵ_s	$+\epsilon_g$	$-\epsilon_r$
<i>1997 Sample (Emerging)</i>						
Brazil	** -0.7	** -1	=	** 2.4	-0.1	** -4.0
Colombia	** -1.4	** -1	=	0.2	** -0.7	** -1.9
Czech Republic	* 0.2	** -1	=	* 0.7	** -1.3	-0.2
Hungary	** -0.8	** -1	=	0.0	-0.2	** -1.6
India	* -0.2	** -1	=	** -1.0	-0.1	-0.1
Israel	** -0.4	** -1	=	** 0.8	* -0.4	** -1.8
Mexico	** -1.4	** -1	=	* -1.2	0.0	* -1.3
Poland	** -1.4	** -1	=	** 1.0	* -0.3	** -3.0
South Africa	** -0.6	** -1	=	0.3	** -0.8	** -1.1
Ukraine	** -0.5	** -1	=	** -1.1	0.0	-0.3

Inflation Shock: $e_t = E[e \mid \Delta E_t \pi_t = 1]$

Inflation Shock - Total Inflation

Country	$-\varepsilon_{\pi}$	=	ε_s	$+\varepsilon_g$	$-\varepsilon_r$
<i>1947 Sample (Advanced)</i>					
United Kingdom	** -2.8	=	** -2.2	** -0.7	0.1
United States	** -1.5	=	-0.3	** -0.5	** -0.7
<i>1960 Sample (Advanced)</i>					
Canada	** -2.6	=	0.3	* -1.4	** -1.5
Denmark	** -1.6	=	0.2	-0.2	** -1.6
Japan	** -1.5	=	** 2.8	** -3.0	** -1.3
Norway	** -2.0	=	0.7	* 3.0	** -5.7
Sweden	** -1.6	=	** 0.9	** -0.9	** -1.5
<i>1973 Sample (Advanced)</i>					
Australia	** -3.1	=	0.2	0.1	** -3.4
New Zealand	** -2.3	=	* 1.2	** -1.4	** -2.1
South Korea	** -2.0	=	** -2.4	0.2	0.2
Switzerland	** -2.0	=	* -0.8	0.1	** -1.3

Inflation Shock: $e_t = E[e \mid \Delta E_t \pi_t = 1]$

Inflation Shock - Total Inflation

Country	$-\varepsilon_{\pi}$	=	ε_s	$+\varepsilon_g$	$-\varepsilon_r$
<i>1997 Sample (Emerging)</i>					
Brazil	** -0.8	=	** 2.4	-0.1	** -3.1
Colombia	** -0.7	=	0.2	** -0.7	-0.2
Czech Republic	** -0.5	=	* 0.7	** -1.3	0.1
Hungary	** -1.4	=	0.0	-0.2	** -1.3
India	** -1.4	=	** -1.0	-0.1	* -0.4
Israel	** -0.6	=	** 0.8	* -0.4	** -1.0
Mexico	** -1.4	=	* -1.2	0.0	-0.3
Poland	** -1.4	=	** 1.0	* -0.3	** -2.1
South Africa	** -0.8	=	0.3	** -0.8	* -0.3
Ukraine	** -1.2	=	** -1.1	0.0	-0.1

Inflation Shock: $e_t = E[e \mid \Delta E_t \pi_t = 1]$

Inflation Shock - Averages

Country	ϵ_{r^n}	$-\epsilon_{\pi}$	=	ϵ_s	$+\epsilon_g$	$-\epsilon_r$
<i>Averages</i>	** -1.0	** -1	=	0.1	** -0.4	** -1.7
1947 (Advanced)	** -0.7	** -1	=	** -1.2	** -0.6	0.1
1960 (Advanced)	** -1.1	** -1	=	* 1.0	* -0.5	** -2.6
1973 (Advanced)	** -1.4	** -1	=	-0.4	-0.3	** -1.7
1997 (Emerging)	** -0.7	** -1	=	0.2	** -0.4	** -1.5

Marked-to-Market

Country	$-\epsilon_{\pi}$	=	ϵ_s	$+\epsilon_g$	$-\epsilon_r$
<i>Averages</i>	** -1.6	=	0.1	** -0.4	** -1.3
1947 (Advanced)	** -2.2	=	** -1.2	** -0.6	-0.3
1960 (Advanced)	** -1.9	=	* 1.0	* -0.5	** -2.3
1973 (Advanced)	** -2.3	=	-0.4	-0.3	** -1.6
1997 (Emerging)	** -1.0	=	0.2	** -0.4	** -0.9

Total Inflation

Inflation Shock - Takeaways

- Discount rates: 80% of inflation variation (average)
- The rest comes mostly from GDP growth
- Positive contribution of surplus-to-GDP 7/21
- Monetary policy reduces inflation in 20/21

Inflation has fiscal roots, even if fiscal policy is disconnected from the price level.

Discounted Surpluses Shock

- Discount rates drive innovations to inflation
- What drives discounted surpluses?
- Put differently: what drives unexpected returns on the **basket** of public bonds?

$$\begin{aligned}e_t &= E[e \mid \Delta E_t(\text{Disc Surpl}) = -1] \\&= E[e \mid \Delta E_t(\epsilon_{s,t} + \epsilon_{g,t} - \epsilon_{r,t}) = -1]\end{aligned}$$

Discounted Surpluses Shock - Marked-to-Market

Country	ϵ_{r^n}	$-\epsilon_{\pi}$	=	ϵ_s	$+\epsilon_g$	$-\epsilon_r$
<i>1947 Sample (Advanced)</i>						
United Kingdom	** -0.8	** -0.2	=	** -0.5	-0.1	* -0.4
United States	** -0.7	** -0.3	=	0.0	** 0.2	** -1.2
<i>1960 Sample (Advanced)</i>						
Canada	** -0.8	** -0.2	=	* 0.2	-0.1	** -1.1
Denmark	** -0.8	** -0.2	=	* 0.6	* 0.5	** -2.0
Japan	** -0.6	** -0.4	=	0.0	-0.2	** -0.8
Norway	** -0.6	** -0.4	=	* 1.0	* 1.9	** -3.9
Sweden	** -0.6	** -0.4	=	** 0.7	-0.2	** -1.5
<i>1973 Sample (Advanced)</i>						
Australia	** -0.8	** -0.2	=	* 0.5	* 0.2	** -1.7
New Zealand	** -0.6	** -0.4	=	** 0.8	** -0.5	** -1.3
South Korea	** -0.6	** -0.4	=	** -2.4	** 1.3	0.2
Switzerland	** -0.8	** -0.2	=	-0.1	* 0.2	** -1.1

Discounted Surpluses Shock: $E[e \mid \Delta E_t(\epsilon_{s,t} + \epsilon_{g,t} - \epsilon_{r,t}) = -1]$

Discounted Surpluses Shock - Marked-to-Market

Country	ϵ_{r^n}	$-\epsilon_{\pi}$	=	ϵ_s	$+\epsilon_g$	$-\epsilon_r$
<i>1997 Sample (Emerging)</i>						
Brazil	** -0.5	** -0.5	=	** 1.4	0.1	** -2.6
Colombia	** -0.6	** -0.4	=	0.0	** -0.3	** -0.8
Czech Republic	** -0.4	** -0.6	=	-0.1	-0.3	** -0.6
Hungary	** -0.6	** -0.4	=	* 0.4	-0.3	** -1.2
India	** -0.5	** -0.5	=	-0.1	* -0.2	** -0.7
Israel	** -0.7	** -0.3	=	** 0.6	-0.1	** -1.5
Mexico	** -0.6	** -0.4	=	** -0.6	0.1	* -0.6
Poland	** -0.7	** -0.3	=	** 0.5	-0.1	** -1.4
South Africa	** -0.7	** -0.3	=	* -0.2	0.0	** -0.8
Ukraine	** -0.5	** -0.5	=	** -0.4	* -0.1	** -0.6

Discounted Surpluses Shock: $E[e \mid \Delta E_t(\epsilon_{s,t} + \epsilon_{g,t} - \epsilon_{r,t}) = -1]$

Discounted Surpluses Shock - Total Inflation

Country	$-\epsilon_{\pi}$	=	ϵ_s	$+\epsilon_g$	$-\epsilon_r$
<i>1947 Sample (Advanced)</i>					
United Kingdom	** -0.9	=	** -0.5	-0.1	* -0.3
United States	** -0.5	=	0.0	** 0.2	** -0.7
<i>1960 Sample (Advanced)</i>					
Canada	** -0.5	=	* 0.2	-0.1	** -0.6
Denmark	** -0.6	=	* 0.6	* 0.5	** -1.6
Japan	** -0.7	=	0.0	-0.2	** -0.5
Norway	** -0.9	=	* 1.0	* 1.9	** -3.8
Sweden	** -0.8	=	** 0.7	-0.2	** -1.2
<i>1973 Sample (Advanced)</i>					
Australia	** -0.6	=	* 0.5	* 0.2	** -1.3
New Zealand	** -0.8	=	** 0.8	** -0.5	** -1.2
South Korea	** -1.2	=	** -2.4	** 1.3	0.0
Switzerland	** -0.5	=	-0.1	* 0.2	** -0.6

Discounted Surpluses Shock: $E[e \mid \Delta E_t(\epsilon_{s,t} + \epsilon_{g,t} - \epsilon_{r,t}) = -1]$

Discounted Surpluses Shock - Total Inflation

Country	$-\epsilon_{\pi}$	=	ϵ_s	$+\epsilon_g$	$-\epsilon_r$
<i>1997 Sample (Emerging)</i>					
Brazil	** -0.3	=	** 1.4	0.1	** -1.9
Colombia	** -0.3	=	0.0	** -0.3	-0.1
Czech Republic	** -0.5	=	-0.1	-0.3	-0.2
Hungary	** -0.6	=	* 0.4	-0.3	** -0.8
India	** -0.6	=	-0.1	* -0.2	** -0.3
Israel	** -0.2	=	** 0.6	-0.1	** -0.7
Mexico	** -0.6	=	** -0.6	0.1	-0.1
Poland	** -0.5	=	** 0.5	-0.1	** -0.9
South Africa	** -0.3	=	* -0.2	0.0	* -0.1
Ukraine	** -0.6	=	** -0.4	* -0.1	** -0.1

Discounted Surpluses Shock: $E[e \mid \Delta E_t(\epsilon_{s,t} + \epsilon_{g,t} - \epsilon_{r,t}) = -1]$

Discounted Surpluses Shock - Averages

Country	ϵ_{r^n}	$-\epsilon_{\pi}$	=	ϵ_s	$+\epsilon_g$	$-\epsilon_r$
<i>Averages</i>	** -0.6	** -0.4	=	0.1	0.1	** -1.2
1947 (Advanced)	** -0.8	** -0.2	=	* -0.2	0.1	** -0.8
1960 (Advanced)	** -0.7	** -0.3	=	* 0.5	0.4	** -1.9
1973 (Advanced)	** -0.7	** -0.3	=	-0.3	0.3	** -1.0
1997 (Emerging)	** -0.6	** -0.4	=	* 0.2	* -0.1	** -1.1

Marked-to-Market

Country	$-\epsilon_{\pi}$	=	ϵ_s	$+\epsilon_g$	$-\epsilon_r$
<i>Averages</i>	** -0.6	=	0.1	0.1	** -0.8
1947 (Advanced)	** -0.7	=	* -0.2	0.1	** -0.5
1960 (Advanced)	** -0.7	=	* 0.5	0.4	** -1.6
1973 (Advanced)	** -0.8	=	-0.3	0.3	** -0.8
1997 (Emerging)	** -0.4	=	* 0.2	* -0.1	** -0.5

Total Inflation

Model Overview

- How hard to reproduce empirical findings? Interpretation?
- Fiscal theory of the price level, New-Keynesian model
- **Partial debt repayment** (but still FTPL!)
- Trend shocks

Model Equations

- Private sector

$$y_t = E_t y_{t+1} - \gamma(i_t - E_t \pi_{t+1}) + \rho_g u_{g,t}$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa y_t - \kappa_a a_t$$

$$g_t = y_t - y_{t-1} - u_{g,t}$$

- Central bank

$$i_t = \phi_\pi \pi_t + \phi_g g_t + u_{i,t}$$

Why trend shocks?

- Otherwise, output stationary $\implies \epsilon_{g,t} \approx 0$
- Model solution: $X_t = a(L)e_t$ for finite $a(1)$
- Growth term of decomposition:

$$\epsilon_{g,t} = \mathbf{1}'_g a(\beta) e_t$$

- In the absence of trend shocks:

$$\begin{aligned} g_t &= \mathbf{1}'_g a(L) e_t = \mathbf{1}'_y (1 - L) a(L) e_t \\ \mathbf{1}'_g a(L) &= \mathbf{1}'_y (1 - L) a(L) \end{aligned}$$

- Therefore $\epsilon_{g,t} = \mathbf{1}'_g a(\beta) e_t \approx \mathbf{1}'_g a(1) e_t = 0$

Model Equations

- Flow of debt

$$v_t + \frac{S_t}{V} = \frac{1}{\beta} [v_{t-1} + r_t^n - \pi_t - g_t]$$
$$r_t^n = \delta [r_t^N] + (1 - \delta) [r_t^R + \pi_t]$$

- Bond prices and return

$$q_{N,t} = (\omega_N \beta) E_t q_{N,t} - [i_t]$$
$$q_{R,t} = (\omega_R \beta) E_t q_{R,t} - [i_t - E_t \pi_{t+1}]$$
$$r_{j,t} = (\omega_j \beta) q_{j,t} - q_{j,t-1} \quad j = N, R$$

Fiscal Policy

- Surpluses could follow

$$h_t = \tau(\pi_t + g_t) + u_{s,t}$$

where $u_{s,t}$ is a standard AR(1)

- No debt repayment
- News about surpluses **always** met by unexpected inflation

Fiscal Policy

- Surplus process

$$s_t = s_t^* + (1 - v) h_t$$

$$s_t^* = \alpha v_{t-1}^* + v h_t$$

$$v_{t-1}^* = \beta (v_t^* + s_t^*)$$

- s_t and s_t^* respond to "debt value target" v^*

$$s_t = \alpha v_{t-1}^* + h_t$$

but **not** to actual debt v_t (or arbitrary $\Delta E_t \pi_t$)

Fiscal Policy

- What is the role of v_t^* ?

$$s_t = s_t^* + (1 - v) h_t \quad (1)$$

$$s_t^* = \alpha v_{t-1}^* + v h_t \quad (2)$$

$$v_{t-1}^* = \beta (v_t^* + s_t^*) \quad (3)$$

- (2) and (3): v^* is stationary
- Solve (3) forward:

$$v_{t-1}^* = \beta \sum_{j=0}^{\infty} \beta^j \Delta E_t s_{t+j}^* = \beta \sum_{j=0}^{\infty} \beta^j \left[E_t s_{t+j} - (1 - v) E_t h_{t+j} \right]$$

Fiscal Policy

$$v_{t-1}^* = \beta \sum_{j=0}^{\infty} \beta^j \left[E_t s_{t+j} - (1 - v) E_t h_{t+j} \right]$$

- Take innovations $\Delta E_t = E_t - E_{t-1}$

$$\beta \sum_{j=0}^{\infty} \beta^j \Delta E_t s_{t+j} = (1 - v) \sum_{j=0}^{\infty} \beta^j \Delta E_t h_{t+j}$$

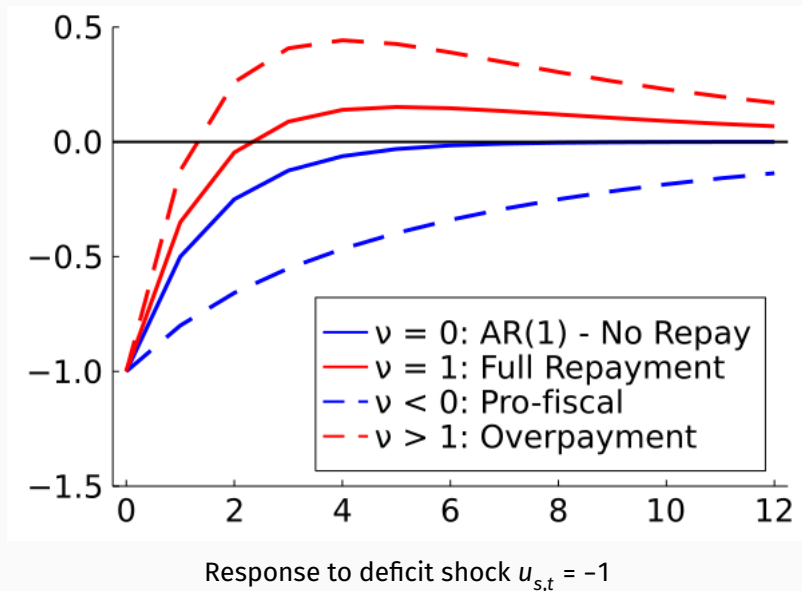
$$\epsilon_{s,t} = (1 - v) \sum_{j=0}^{\infty} \beta^j \Delta E_t h_{t+j}$$

- v governs debt repayment

Partial debt repayment

- $v = 0$ No debt repayment: $\epsilon_{s,t} = \sum_{j=0}^{\infty} \beta^j \Delta E_t h_{t+j}$
 - $s_t = h_t$ (standard AR(1))
- $v = 1$ Full debt repayment: $\epsilon_{s,t} = 0$
 - $s_t = s_t^* = \alpha v_t^* + h_t$
- $v < 0$ "Pro-fiscal" surplus: $\epsilon_{s,t} / \sum_{j=0}^{\infty} \beta^j \Delta E_t h_{t+j} > 1$
- $v > 1$ "Overpayment": $\epsilon_{s,t} / \sum_{j=0}^{\infty} \beta^j \Delta E_t h_{t+j} < 0$

Partial debt repayment - Cases



GMM Estimation

- Method of moments:

$$\text{Min}_{\theta} \quad w \left\| \mathcal{D}_{VAR} - \mathcal{D}_{NK}(\theta) \right\| + (1-w) \left\| \mathcal{M} - \mathcal{M}_{NK}(\theta) \right\|$$

- \mathcal{D} contains MtM decomposition for inflation shock
- \mathcal{M} contains second moments
- Estimates for the **United States**

GMM Estimation

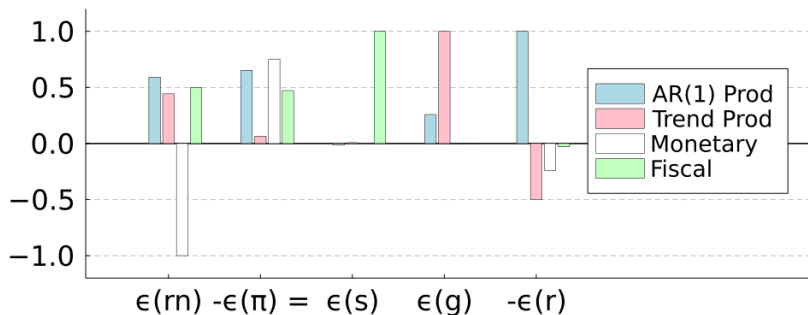
United States Estimates

Fixed		Estimated	
Parameter	Value	Parameter	Value
β	0.99	ρ_a	0.98
γ	0.4	ρ_g	0.23
φ	3	ρ_i	0.00
θ	0.25	ρ_s	0.72
$\bar{\omega}$	γ^{-1}	ϕ_π	0.68
σ_a	1	ϕ_g	0.00
		τ	-0.06
		ν	0.89
		α	0.01
		σ_g	1.21
		σ_g	0.53
		σ_g	1.07

US Model Parameters

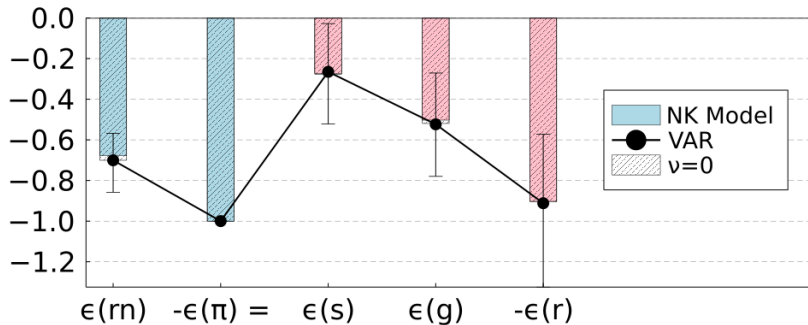
GMM Estimation

United States Estimates



Fiscal decomposition of structural shocks

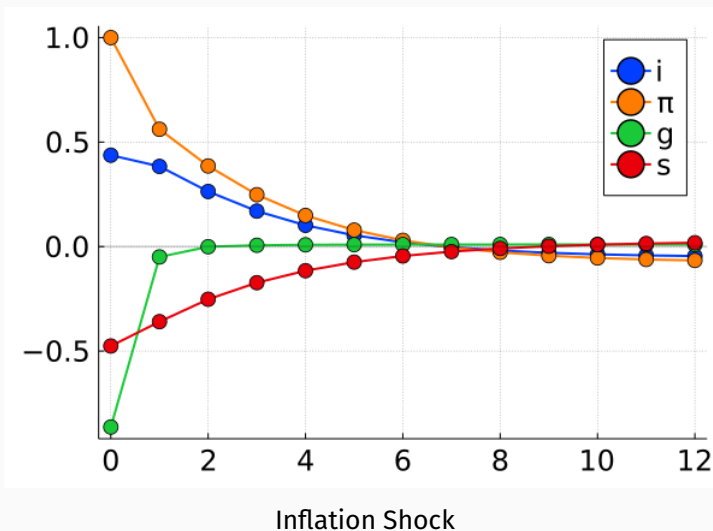
Is AR(1) surplus a good model?



MtM decomposition of Inflation Shock $e_t = E[e \mid \Delta E_t \pi_t = 1]$

Is AR(1) surplus a good model?

Structural shocks: $\varepsilon_a = -1$, $\varepsilon_g = -0.2$, $\varepsilon_i = -0.3$, $\varepsilon_s = -0.5$



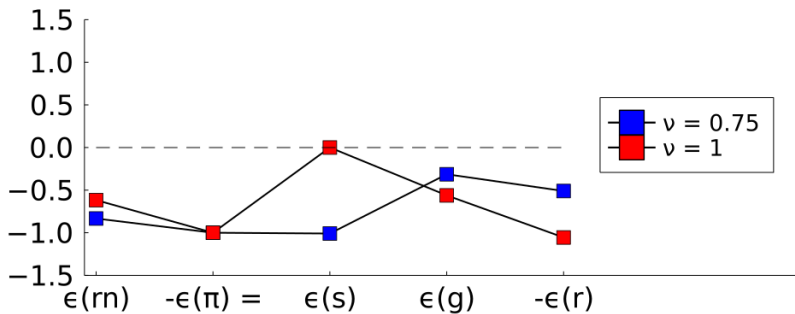
Is AR(1) surplus a good model?

- $\nu = 0$ precludes realistic fiscal policy and discount-driven inflation at the same time

	Data	$\nu = 0.9$	$\nu = 0$		Data	$\nu = 0.9$	$\nu = 0$
σ_i/σ_g	1.29	0.77	1.25	$\text{cor}(\pi, i)$	0.70	0.88	0.89
σ_π/σ_g	1.20	1.10	1.56	$\text{cor}(\pi, g)$	-0.11	-0.35	-0.40
σ_s/σ_g	1.08	1.09	0.45	$\text{cor}(g, i)$	0.04	-0.35	-0.04
$\text{acor}(i)$	0.91	0.75	0.87	$\text{cor}(i, s)$	-0.26	-0.28	-0.46
$\text{acor}(\pi)$	0.69	0.72	0.81	$\text{cor}(\pi, s)$	-0.28	-0.29	-0.41
$\text{acor}(g)$	0.14	0.14	0.16	$\text{cor}(g, s)$	0.01	-0.04	-0.05
$\text{acor}(s)$	0.64	0.72	0.27				

Second Moment Fit

Is AR(1) surplus a good model?



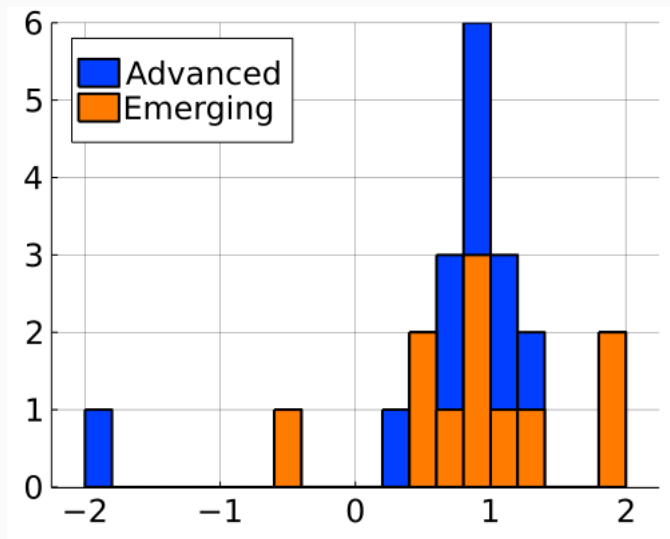
Comparative Statics for v

Cross-Country Estimates

$$\text{Min}_{\theta} \quad w \left\| \mathcal{D}_{VAR} - \mathcal{D}_{NK}(\theta) \right\| + (1-w) \left\| \mathcal{M} - \mathcal{M}_{NK}(\theta) \right\|$$

- Repeat procedure for each country in the sample
- Use corresponding debt profile $(\delta, \omega_N, \omega_R)$

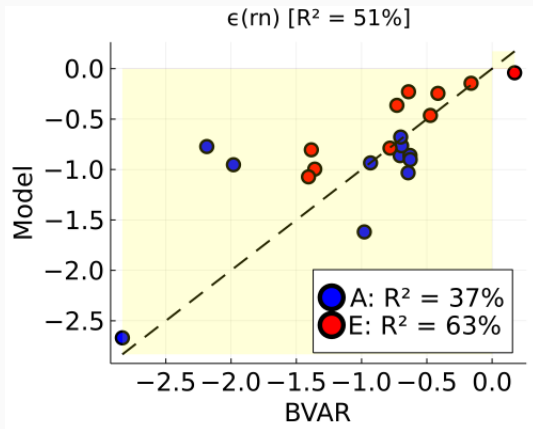
Cross-Country Estimates of Debt Repayment v



Histogram of v estimates

Cross-Country Fit of Fiscal Decomposition

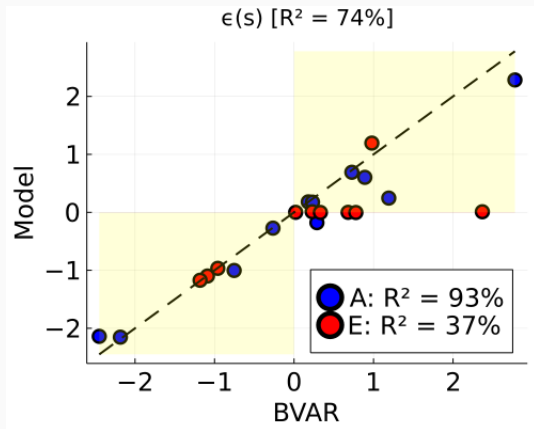
$$\epsilon_{r^n,t} - \epsilon_{\pi,t} = \epsilon_{s,t} + \epsilon_{g,t} - \epsilon_{r,t}$$



Fit of Bond Price Term $\epsilon_{r^n,t}$

Cross-Country Fit of Fiscal Decomposition

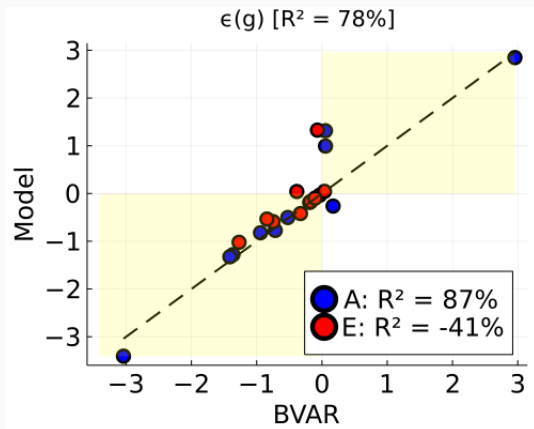
$$\epsilon_{r^n,t} - \epsilon_{\pi,t} = \epsilon_{s,t} + \epsilon_{g,t} - \epsilon_{r,t}$$



Fit of Surplus Term $\epsilon_{s,t}$

Cross-Country Fit of Fiscal Decomposition

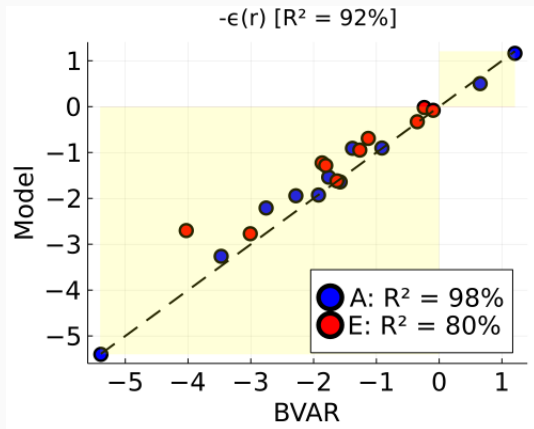
$$\epsilon_{r^n,t} - \epsilon_{\pi,t} = \epsilon_{s,t} + \epsilon_{g,t} - \epsilon_{r,t}$$



Fit of Growth Term $\epsilon_{g,t}$

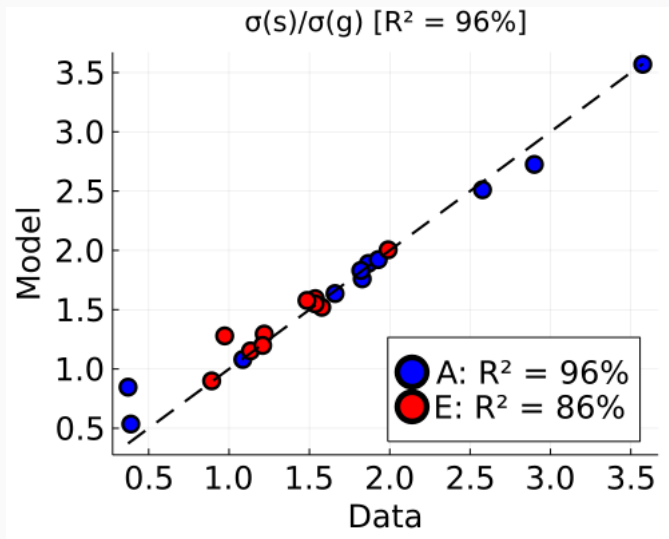
Cross-Country Fit of Fiscal Decomposition

$$\epsilon_{r^n,t} - \epsilon_{\pi,t} = \epsilon_{s,t} + \epsilon_{g,t} - \epsilon_{r,t}$$



Fit of Discount Term $\epsilon_{r,t}$

Cross-Country Fit of Fiscal Policy Volatility



Fit of Fiscal Policy Volatility

Frametitle



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