Informality and Policy in General Equilibrium

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Abstract

We propose a heterogeneous-agents model of informal labor in which we derive income dynamics endogenously from search frictions. We calibrate the model to replicate features of Brazilian household data, such as the formal-informal earnings differential over the income distribution, and we simulate the economy's transition path following a policy of repressing informal labor. Short and long-run impacts differ and general equilibrium effects matter. In the long run, households' welfare and average firm productivity improve, and unemployment declines. In the short run, reduced aggregate savings lead to higher real interest and unemployment. Realistic variations of a benchmark case shows that some groups, usually the unemployed and households with low wealth, can be net losers.

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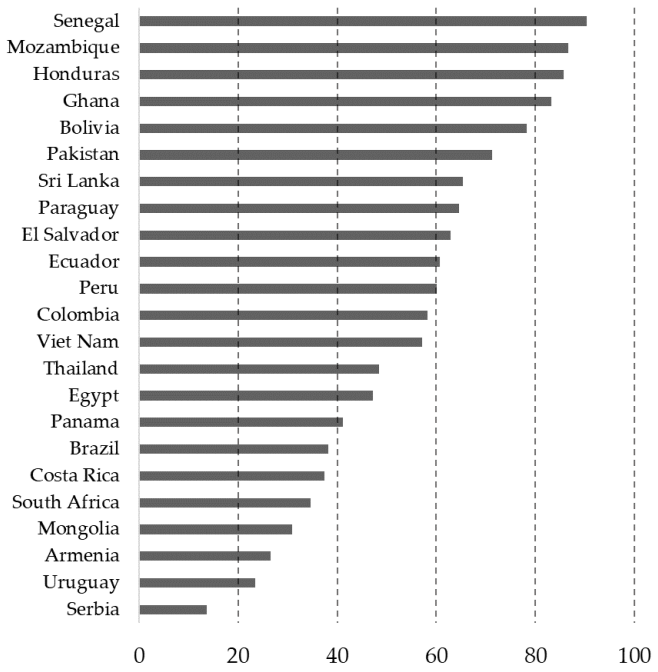


Source: International Labour Organization

Figure 1: Informal employment as share of non-agricultural employment (2015)

1 Introduction

We build a heterogeneous-agent model in which risk-averse households face a stochastic stream of income and can save to smooth consumption. The distribution of income shocks follows from a Diamond-Mortensen-Pissarides (DMP) search model in which firms can hire workers informally. It is thus endogenous and sensitive to policy. The interest rate clears an incomplete capital market, as in Aiyagari (1994).

We use the model to answer the question: should governments repress informal labor activity? We are motivated by the large size of the informal sector in many emerging market countries (see figure 1), as well as the common claim that reducing the size of the informal sector leads to economic gains.[1] Is that the case? If so, what are the mechanisms that lead to such gains? How does the reduction of the informal market affect households in different regions of the wealth and income distributions? Do general equilibrium effects matter? Our study attempts to answer these questions.

The key contribution of this paper is to provide a tractable framework featuring, *simultaneously*, (1) a labor income process sensitive to labor market interventions from the government, and (2) its effect on welfare and decision-making by risk-averse households. We argue risk is critical when considering the effects of informality. The existence of informal labor arrangements affects the rate at which workers transition in and out of unemployment – their worst state in terms of consumption –, and the government's ability to provide insurance to workers in the form of

[1] See, for instance, the International Labour Organization (ILO) Recommendation No. 204 (2015).

unemployment benefits. Hence, households that dislike risk will have different preferences over informality policies than households with linear utility.

We also allow households to save using risk-free bonds, and solve for the interest rate that clears capital markets. The ability to smooth consumption over time matters as wealthier agents, being less risk averse, experience lower welfare losses when their income process becomes riskier.[2] Besides, as marginal propensity to consume changes from household to household, the way in which the government returns extra revenues raised from fighting informality matters for welfare and general equilibrium.

In terms of the mechanics of the labor market, we select model ingredients to reproduce some of the properties of the joint distribution of income and informality in Brazilian household-level micro data. We use the *PNAD* (National Household Sample Survey) dataset, a panel of households, to measure labor income as well as the workers formalization status.

We observe three stylized facts. First, there is a significant presence of informal workers (around 30%) in all regions of the income distribution. Informality is more prevalent at the bottom of the distribution, but also present at the top.[3] Second, the income differential, defined as the difference in average income earned in the formal and informal sectors, is large among low-income workers, in particular those workers with earnings close to the legal minimum wage prevailing in Brazil. However, such differential declines and even vanishes among those with higher income, to which the minimum wage is not a constraint. This pattern is true after controlling using proxies for human capital and the labor market in which these workers are inserted. The third fact is that informal workers transition to unemployment more frequently than formal ones, which builds on the idea that informal jobs are more unstable and, thus, less desirable from the point of view of workers.[4]

Two ingredients are key to our results. Our observation about income differentials (the second stylized fact) suggests that minimum wage laws play an important allocative role. We introduce it in the model by forcing only formal labor agreements to satisfy the minimum wage constraint. Another key ingredient - an innovation of our model - is that we introduce firm heterogeneity in the "ability" to hire informally, which we call *hiding ability*. This new layer of heterogeneity captures different costs of not formalizing labor contracts (proximity to public vigilance, firm size, and so on). Firms with more hiding ability choose to offer informal contracts, even for productive workers. The model thus replicates the low income differential among high-income workers that we find in the data.

We use the model to simulate the repression of informality: firms begin to lose their hiding ability. The economy's response largely underscores the "parasitic" character (as in La Porta and Shleifer (2008)) of informal firms. Lacking the option of circumventing costly regulation, unproductive firms refrain from posting job vacancies and free up resources for more productive ones. Average labor income increases and labor contracts last longer, on average. On the other hand, fewer firms in the market leads to lower job-finding rates. Which effect dominates in the determination of the unemployment rate depends on where we look at in the transition path. In the short run, our simulation points to an increase in the unemployment rate; in the long run, a small decrease.

[2] Formally, when agents have CRRA utility, the coefficient of absolute risk aversion declines as consumption increases.

[3] See La Porta and Shleifer (2008) and Meghir et al. (2015) for additional evidence.

[4] Bosch and Esteban-Pretel (2012) study this pattern in detail using a different Brazilian dataset and find similar conclusions.

These differences along the transition path relate to general equilibrium effects. In our baseline simulation, capital demand increases as firms hire more efficiency units of labor. Aggregate savings *decline* in the initial periods after the repression of informality begins, as consumption-smoothing households decrease savings in anticipation of higher future income. Market clearing then requires a significant increase in interest rates (up to 4% annually) and thus a reduction in firm value. Fewer firms post vacancies and unemployment increases. In the long run, as aggregate savings catch up, the interest rate declines back to levels near the original steady state and unemployment goes down.

In terms of welfare, despite the short-run increase in unemployment, we find large gains for almost all groups of households in our baseline case. These gains, often above 20% of pre-repression lifetime consumption, take place in the short and in the long run. They are smaller for unemployed and low-wealth households, to which negative income shocks (moving to unemployment) represent a larger drop in utility given the reduced capacity to smooth consumption. That is, risk matters.

However, realistic extensions of our benchmark simulation suggests that such positive view of the repression of informality should be taken with caution. In two variations of our benchmark - the lack of endogenous adjustment of income taxation and the anticipation of the repression by private agents - lead to groups of households (usually the poor and unemployed) being welfare losers, respectively in the long and in the short run.

Literature. The existing literature on informality comprises empirical, theoretical and quantitative papers that focus on the role of firms and/or workers. We do not provide a complete review of the literature (see Ulyssea (2020) for that). Instead, we highlight papers that we believe to be more connected to our points.

The work in La Porta and Shleifer (2008) and La Porta and Shleifer (2014) provides an overview of the competing theories of informality: the romantic view (as in De Soto (1989)), the parasite view (supported by more recent paper described below) and the dual view. The authors present evidence to support the dual view. It states that informal firms are less productive (smaller, inefficient, run by poorly educated entrepreneurs), they can't compete with formal ones and they are largely disconnected from the formal economy. Our framework allows us to differentiate between "parasite view" firms and "dual view" firms. Ulyssea (2018) provides a model with intensive and extensive margin of informality that successfully unifies the three theories in a single framework. Meghir et al. (2015) develop a wage-posting model with productivity-heterogeneous firms and homogeneous workers in which informal firms crowd the labor market and make it harder for workers to find better, formal jobs Tighter enforcement leads to a distribution with more productive firms in the new equilibrium and higher welfare. We find a similar pattern but predict a short-run phase of higher unemployment, and identify groups of households that can be harmed.

In recent work, Dix-Carneiro et al. (2019) study the effects of openness to international trade on informality, and find similar predictions in terms of productivity and welfare as informality shrinks. Bosch and Esteban-Pretel (2012) focus on transition rates. They study a search model in which *ex-ante* homogeneous workers and firms decide to sign a formal or informal contract upon a match. In a later article, Bosch and Esteban-Pretel (2015) use a similar model to study the effects of unemployment benefits on formality and unemployment. Granda and Hamann (2015) provide evidence that informal individuals and households in Colombia present a higher savings rate than

formal ones. In their model, labor and capital markets are fully segmented between formals and informals, unlike our model. Albertini and Terriau (2019) provide evidence from Argentina of informality rates, as well as job finding and separation rates, over the life cycle.

2 Income and Informality: Three Empirical Facts

2.1 The Data

We use micro data from the *PNAD contínua*, the main household survey in Brazil. The survey is nationally representative, and each household is interviewed up to five consecutive quarters. We have data from 2012-Q1 to 2020-Q1, but focus on the 2018-Q3 wave, as a quarter we consider to be a good approximation of average conditions in the labor market. The appendix presents evidence from other quarters.

By law, employment contracts in Brazil must be recorded in a labor booklet. PNAD asks employed individuals whether their current labor contract has been recorded.[5] We classify as an "informal worker" any individual who answers no. We classify the remaining private sector workers as "formal workers". All remaining workers we classify as unemployed. We exclude civil servants, members of the military, employers and auxiliary family workers. Unlike Meghir et al. (2015) and most informality papers, we also exclude self-employed individuals from the analysis, as we understand that self-employment is determined by factors not well captured by our search-theoretic approach.

The real monthly income of a worker is his or her labor income at the main job divided by Brazil's consumer price index. All figures correspond to 2018Q3 Brazilian reals. We report values in dollars using a conversion rate of 4 reals per dollars, which is close to the exchange rate average between 2017 and 2019. We exclude workers with reported income above 10,000 Brazilian reals (about 3% of the population) to avoid having sample averages being affected by outliers, as well as individuals outside the 18-68 years of age range.[6] [7]

We group workers according to their *occupational group* and their *educational level*.[8] Besides providing control for personal and firm productivity, grouping is also important to account for the fact that workers participate in different labor markets. Table 1 reports the population shares of each category in the third quarter of 2018. For an individual to have a given educational level, he or

[5] Survey respondents are aware of their formality status because of how the registration process works. Every person who wishes to engage in an official employment relationship must acquire their labor booklet with the Brazilian government. The labor booklet allows workers to claim benefits such as unemployment insurance, disability insurance, and access to retirement benefits. Employers are required to sign the employee's labor booklet when the employment relationship is established. The employer must also register the employment contract in the official registry (a system called *e-Social*).

[6] Setting this threshold to 20,000 Brazilian reals does not significantly alters our results.

[7] Combining the 33 waves of the survey, we have 7,936,590 individual observations. To compute income statistics, we also exclude from the sample employed workers with labor income or labor status data missing. That results in a total of 7,157,681 income observations of employed workers.

[8] The occupational group variable is mainly a description of the nature of the activity performed by the worker. It is related to, but not perfectly, to industry category. For example, machine operators typically work at the manufacturing industry, but managers and directors are more evenly distributed across industries. We only observe occupational groups for employed workers.

Occupational Group	Share (%)	Educational Level	Share (%)
Agriculture, hunting, fishing	5.3	Elementary	40.4
Administrative staff	8.5	High school	41.1
Construction and mechanical	13.7	College and higher	18.5
Commerce, services and retail	23.1		
Elementary occupations	17.6		
Machine and assembling operators	8.8		
Managers and directors	3.9		
Scientists and intellectuals	10.4		
Technicians	7.8		

Table 1: Occupational Groups and Educational Level: Population Shares

she must have completed the degree.

2.2 Facts

Start by defining the *informality share*: the ratio of informal workers to total employed workers. By excluding the unemployed, we avoid additional noise brought by business cycle fluctuations.[9]

Fact 1. *The informality share is economically significant (10-70%) in all regions of the income distributions. It is larger at its bottom.*

Figure 2a provides the evidence for fact 1. Using data from 2018-Q3, we group individuals according to occupation (filled markers) and education (unfilled). The figure plots the informality shares among workers of the corresponding groups, and the x-axis indicates their average income.

Informality is present over the entire income distribution. It starts at 55% (for farming and fishing) and then decays to 15% (for managers and directors). It declines with groups' average income: a linear regression forecasts that, for each additional one hundred dollars of monthly income, the size of the informal sector declines by 3.6%.[10]

Fact 1 does not imply that informal workers that transition to the formal sector get to earn more. We now ask whether there is an *income differential*, defined as the percentage difference between average income among the formal and the informal in a given groups of workers.

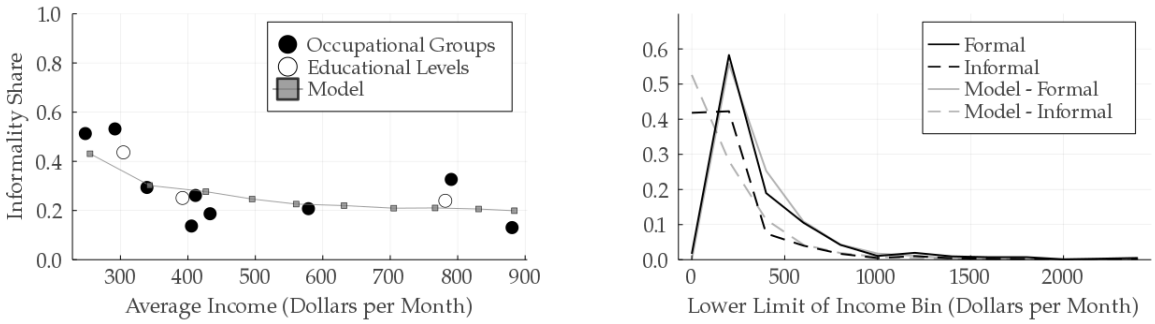
Fact 2. *An income differential exists and is positive among low-income workers. Among higher-income workers, the differential vanishes.*

By low-income workers we mean workers with income close to the legal minimum wage - about US$ 250/month gross or US$ 232 net-of-tax in 2018.[11]. Figure 3 depicts fact 2. It contains two panels, each plotting income differentials: for instance, 25% means the mean income among the

[9] Brazil experienced a sharp recession in 2015-16.

[10] We present more evidence in appendix A. Figure 15 shows informality shares by income quantiles. The message is roughly the same as figure 2a: informality is present over the entire range of the income distribution, and decreases in average income. Figure 16 shows the time series of informality shares for each group. It shows that although informality shares on each group does vary over time, relative to each other it varies less.

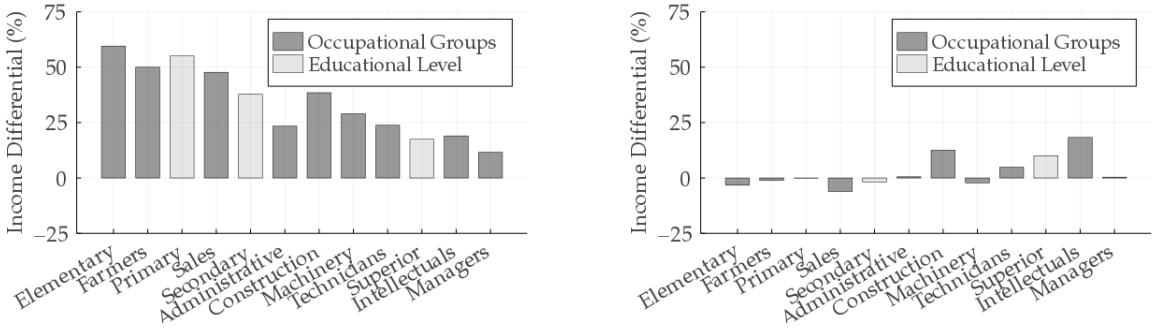
[11] Personal contribution to the social security system for minimum-wage workers was 7%, and the marginal income tax rate in the first income bracket is zero.



(a) Informality Share (b) Income Distribution by Sector

Notes: Panel(a). The circular markers represent worker groups. The x-axis represents the average income of each group in 2018-Q3. The line represents the share predicted by the model, with each marker corresponding to a different worker skill type. Panel(b) Income distribution among formal and informal workers.

Figure 2: Informality and Income



(a) Below US$ 300 (b) Above US$ 300

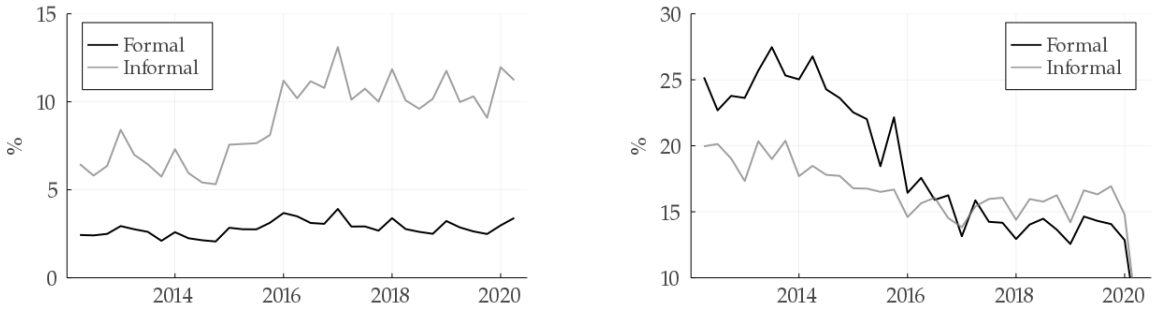
Figure 3: Income Differential

formals is 25% larger than among informals. This time we add another criterium to group workers. Panel 3a plots the differential for workers with after-tax income below US$ 300 per month. We exclude all the other individuals from the subsample. Panel 3b plots the same statistics for workers with income above the same threshold. We order groups in ascending order of mean income in the entire sample, not in each income subsample. Again, we use 2018-Q3 data.[12].

Differentials are substantially higher among workers with lower income (panel 3a). They exceed 25% for most groups. In addition, the differential declines on the average income of the corresponding group. As for workers with income above the threshold (panel 3b), the estimated differentials are closer to zero. They are all smaller (group by group) than the estimate among workers below the threshold. In addition, the negative relationship between differential and average income is gone. These plots support fact 2: informality has a different impact on the income process of different workers.

The minimum wage is a key factor in accounting for this pattern. Figure 2b plots the labor income distribution (2018-Q3) according to workers' formality status. We group workers in income bins of range US$ 200 each. The x-axis shows the lower limit of the income bin. The absence of

[12]You can find the time series of these statistics in figure 17 in appendix A



(a) Transitions to unemployment

(b) Transitions from unemployment

Figure 4: Transitions Rates in the Data

formal workers in the first bin follows from the US$ 232 legal minimum wage. [13] With forty percent workers in the first income bine, figure 2b also shows that informal agreements do not have to respect the minimum wage rule.

The evidence that the minimum wage is only an actual constraint in the formal sector explains the large income differential among low-income workers. This is precisely the mechanism we apply in our model. It also explains why income differentials decline with productivity (see figure 3a; groups are ordered in increasing order of average income). The average income of informal workers increases with group productivity, while that of formal workers is less responsive given that wages are often set by the legal floor rather than productivity.

Fact 3. *Informal workers transition to unemployment more often than formal workers.*

Facts 1 and 2 relate to employment income. In both cases, we ignore unemployed individuals. So, what about having *vs* not having employment income? Our criterium does not allow us to classify unemployed workers as formal or informal. Also, there are too few interviews to classify them based on their last job.

Instead, we follow our methodology in Maya and Pereira (2020) and use PNAD's panel dimension to investigate the rate at which workers transition to and from unemployment. For each quarter, we restrict the sample to individuals who appear with a valid labor status (employed formal, employed informal and unemployed) on that quarter and the previous one. We then compute the empirical transition matrix across these three states for every period. For example, among workers that report not having a job in quarter t, which share reports a formal job in quarter $t+1$? The two panels in figure 4 plot results for each wave of the survey. Panel 4a plots the time series of transitions to unemployment. Throughout the period range of analysis, informal workers transition to unemployment more often than formal ones, as fact 3 states. Following the 2015-2016 recession, this pattern becomes stronger. Such difference between job ending rates is critical from a welfare standpoint, as losing a job represents the main negative shocks to income experienced by households.

[13] Note that, however reduced, there is a non-zero measure of formal workers in the first bin (reporting to receive less than the minimum wage). This can be attributed either to underreporting or to part-time contracts with reduced workday, for which the labor law accepts payments lower than the legal minimum wage.

Lastly, the evidence on transitions in the opposite directions is not as clear. Panel 4b plots transition rates from unemployment to formal or informal employment. While transitions to formal employment were more frequent before the 2015/16 recession, this pattern reverses in the second half of the sample. Informality increased significantly after the recession. Given the ambiguous evidence, instead of targeting transition rates to employment, we calibrate our model to match unemployment rates and informality, and the laws of motion that govern the flows of workers yield the required job-finding rates in each sector.[14]

3 The Model

We now present a model to reproduce these empirical patterns. The agents are: a one-sized continuum of heterogeneous households, heterogeneous firms, a government and a representative investor (a firm of type II in the language of Ljugqvist and Sargent (2018)). Time is discrete and there is no aggregate uncertainty. The commodities are: a homogeneous final good, physical capital, labor hours and a risk-free bond that promises the owner one unit of final good in the next period. Markets are therefore incomplete. Let q_t be the price of bonds.

Each household contains a worker and a shopper. Shoppers purchase final goods for consumption and bonds to solve a consumption-savings problem. The shopper state is (a, s), where a is beginning-of-period net wealth and s captures variables related to the labor market, over which the shopper has no control. In equilibrium, s follows a Markov chain with transition matrix $g_t(s'|s)$. The labor income is given by $y_t(s)$. Functions g and y follow from the search block of the model.

There is a no-borrowing constraint, which is realistic to emerging market economies. The shopper's utility is separable, with CRRA period utility $u(c) = u^{1-\gamma}/(1-\gamma)$. They solve

$$\begin{aligned} J_t(a,s) = \max_{c,a'} \quad & u(c) + \beta \sum_{s'} g_{t+1}(s'|s) J_{t+1}(a', s') \\ \text{s.t.} \quad & c + q_t a' \leq a + y_t(s), \\ & a' \geq 0. \end{aligned} \tag{1}$$

Value function J is our main measure of welfare.

3.1 The Search Block - General Environment

The labor market has frictions that prevent unmatched firms and workers from matching immediately. These frictions generate unemployment in equilibrium. We allow workers to search on and off the job.[15] Employed workers have no diminished capacity of searching for better positions. The matching function is

$$M(U, F) = \mu U^{\varepsilon} F^{1-\varepsilon} \tag{2}$$

[14] Bosch and Esteban-Pretel (2012) find that transitions from unemployment to informal jobs are more likely than transitions to formal jobs throughout their sample. Importantly, both their sample period and the survey from which they obtain their results are different from ours. Also, they include self-employed individuals in their definition of informal workers.

[15] This is an important assumption to avoid having high-skill workers pass on low or minimum wage offers (in contrast to the data) for reasons similar to the seminal McCall (1970) model.

where U is the measure of searching workers and L is the measure of posted vacancies. The elasticity of job-finding rates to the market tightness (the ratio of vacancies to searching workers), $1-\varepsilon$, is critical as it governs by how job-finding rates drop when less firms are present in the labor market.

Workers have different skill types h, which capture individual productivity. They are constant and with distribution H. Employed workers receive labor income. Unemployed workers derive a leisure utility stated in units of final goods $\zeta_0 + \zeta_1 h$ and an unemployment income ς transferred by the government. Separation between these variables allows us to control the floor on equilibrium wages and unemployment income at the same time.

Each firm employs one worker at most and can direct its search towards workers of a given type. Markets are thus segmented, and we can index them by the corresponding worker type h.

Notation: $\theta_t(h)$ is the market tightness in labor market h and $p_t(h) = M(\theta_t(h)^{-1}, 1)$ is the worker-finding rate for searching firms[16]. The job-finding rate is $p_t(h)\theta_t(h)$ (we often write $p\theta_t(h)$). The cost of posting a vacancy κh is linear on productivity and there is free entry of new firms in each labor market.

Each firm uses capital k and efficiency hours of labor ℓ to produce final goods through a Cobb-Douglas production function $Ak^\alpha \ell^{1-\alpha}$. Let δ be the depreciation rate of capital and r_t its rental price. Then, the choice of rented capital is static and solves $\max Ak^\alpha \ell^{1-\alpha} - (r_t + \delta)k$. The first-order condition $A(k/\ell)^{\alpha-1} = r_t + \delta$ determines the capital-labor ratio k/ℓ as a function of r_t. Let

$$P(r) = A\left(\frac{k}{\ell}(r)\right)^{\alpha-1} - (r+\delta)\frac{k}{\ell}(r). \tag{3}$$

Then, the net-of-capital-cost firm revenue is $P(r_t)\ell$ if the firm uses ℓ efficiency hours of labor.

After paying for its vacancy, the entrant firm draws two individual state variables: $e \in [0,1]$ from a time-t dependent distribution $G_{e,t}$ and $z \in \mathcal{Z}$ from a distribution G_z. The draws are independent. Let $G_t = G_z G_{e,t}$ be the joint distribution. State z represents firm productivity. In hiring a worker of skill type h, the firm has access to zh efficiency hours of labor. The other state variable e is the firm's *hiding ability*, that is, its ability to avoid unmodelled legal and social costs associated to informal activity. To keep the model tractable, we assume hiding costs in the form of reduced efficiency hours applied by the firm. An informal firm with hiding ability e thus uses ezh efficiency hours of labor in production.[17]

The firm state (e, z) is constant over time. After drawing it, the firm can withdraw its vacancy and wait for the next period to draw a new pair.

The timing of the model is the following. The period begins, and firms that paid for a vacancy in the previous period draw a new state and decide whether to search or withdraw. Labor markets open and matches are formed. Workers and firms with new and continuing matches bargain over the wage rage. Then, production takes place, shoppers purchase final goods and bonds, and unmatched firms pay the cost of keeping up their vacancies. New firms might enter the economy;

[16] Note that p_t is *not* the vacancy-filling rate. Employed workers that find a new match can choose to stay in their current job instead of switching.

[17] That specification makes the cost of informality be marginal on productivity. A different specification would have it be a fixed cost. However, such specification would lead informal firms to be, everything else the same, more productive than formal ones, a result at odds with firm-level evidence.

continuing firms might exit. After all agents take their decision, job separations happen at random. The period ends.

The government taxes firms' sales (τ_y) and payroll (τ_w), and workers' income ($\tau_{hh,t}$). All taxes are flat-rate. It also makes one-time unemployment insurance (UI) transfers υ to workers that lose their job and fail to secure a new one; unconditional transfers ς to all unemployed workers, and establishes a minimum wage ω.[18]

After learning their state (e, z), firms decide in which labor market to search and whether to offer a formal or an informal contract. The formality status of a contract cannot be changed.

The government cannot enforce taxation or the minimum wage on informal firms, and informal workers have no access to unemployment insurance. The decision to offer a formal or informal contract takes place *before* the firm finds its match. Therefore, there is no bargaining involved in the determination of the formality status of a contract. Finally, matches end for exogenous reasons. Given the evidence of transitions of employment to unemployment (figure 4a), the separation rate differs between formal λ_f and informal $\lambda_i > \lambda_f$ jobs.

3.2 Value Functions and Decision Sets

The value functions are $V_t^f(z,h)$ (formal) and $V_t^i(ez,h)$ (informal) for matched firms. For workers, the value functions are $W_t^f(z,h)$ (formal), $W_t^i(ez,h)$ (informal) and $W_t^n(h)$ (unemployed). Although firms have two states, e and z, we can state value functions with a single argument. You can think that firms effectively draw two levels of productivity, a "formal" level and an "informal" level, and then decide which one to carry.

To write value functions, additional notation is useful. Conditional on matching with a worker in period t, let $\rho_t^f(z,h)$ be the probability that this worker accepts the job and a new match is formed. Define $\rho_t^i(ez,h)$ analogously. Note that an unmatched firm decides to post a vacancy for a formal contract if $\rho_t^f(z,h)V_t^f(z,h) \geq \max(0, \rho_t^i(z,h)V_t^i(ez,h))$. It posts for an informal contract if $\rho_t^i(z,h)V_t^i(ez,h) > \max(0, \rho_t^f(z,h)V_t^f(z,h))$. If none of the two previous conditions hold, it drops its vacancy from the market.[19] Let

$$\begin{aligned} Z_t^f(h) &= \left\{(e,z) \mid \rho_t^f(z,h)V_t^f(z,h) \geq \max(0, \rho_t^i(ez,h)V_t^i(ez,h))\right\} \\ Z_t^i(h) &= \left\{(e,z) \mid \rho_t^i(ez,h)V_t^i(ez,h) > \max(0, \rho_t^f(z,h)V_t^f(z,h))\right\} \\ Z_t(h) &= Z_t^f(h) \cup Z_t^i(h) \end{aligned} \tag{4}$$

be the sets of firms' states that lead them to offer formal or informal contracts, or drop the vacancy, respectively.

Consider now workers' decisions. A formal worker employed by a state z firm transitions to a

[18] With the one-period installment of unemployment insurance, we calibrate υ to match the expected present discounted value of future flows of benefits. In Brazil, unemployment benefits take the form of five monthly installments that cease as soon as the worker finds a new occupation. Deviations between our model and reality occur as workers find jobs before the completion of the five payment installments. Given the observed job-finding rate (figure 4b) and the possibility of workers not reporting their new jobs, we take the one-period approximation to be sufficiently accurate.

[19] If indifferent, the new firm chooses a formal over an informal contract, and chooses not to post instead of posting an informal contract. These choices are irrelevant for our solution, in any case.

new, formal job if he or she finds a firm that opts for a formal contract vacancy *and* that carries an individual state that leads to a higher value to the worker. That is, the new state (e', z') must satisfy $(e', z') \in Z_t^f(h)$ and $W_t^f(z', h) > W_t^f(z, h)$. Applying this logic we define

$$\begin{aligned}
\Phi_t^f(z,h) &= \left\{(e',z') \in Z_t^f(h) \mid W_t^f(z',h) > W_t^f(z,h)\right\} \\
&\quad \cup \left\{(e',z') \in Z_t^i(h) \mid W_t^i(e'z',h) > W_t^f(z,h)\right\}, \\
\Phi_t^i(ez,h) &= \left\{(e',z') \in Z_t^f(h) \mid W_t^f(z',h) > W_t^i(ez,h)\right\} \\
&\quad \cup \left\{(e',z') \in Z_t^i(h) \mid W_t^i(e'z',h) > W_t^i(ez,h)\right\}.
\end{aligned} \tag{5}$$

as the sets of (firm) states that lead workers to change jobs.[20]

We are ready to state expressions for the value functions. The free entry assumption ensures that the value of unmatched firms equals zero. As for the value of matched firms, we start by stating them as functions of wage rates w as well, $\hat{V}_t^f(w,z,h)$ for example, and then given equilibrium bargained wage rates $w_t^f(z,h)$ and $w_t^i(ez,h)$ we have $V_t^f(z,h) = \hat{V}_t^f(w_t^f(z,h),z,h)$ (same for V_t^i).

$$\begin{aligned}
\hat{V}_t^f(w,z,h) &= (1-\tau_y)P(r_t)zh - (1+\tau_w)w + q_t(1-\lambda_f)\times \\
&\quad \times \left[(1-p\theta_{t+1}(h)) + p\theta_{t+1}(h)\left(1 - \frac{G_{t+1}(\Phi_{t+1}^f(z,h))}{G_{t+1}(Z_{t+1}(h))}\right)\right] V_{t+1}^f(z,h) \\
\hat{V}_t^i(w,ez,h) &= P(r_t)ezh - w + q_t(1-\lambda_i)\times \\
&\quad \times \Bigg[\underbrace{(1-p\theta_{t+1}(h))}_{\substack{\text{Worker finds} \\ \text{no new match}}} + \underbrace{p\theta_{t+1}(h)\left(1 - \frac{G_{t+1}(\Phi_{t+1}^i(ez,h))}{G_{t+1}(Z_{t+1}(h))}\right)}_{\substack{\text{Worker finds new match,} \\ \text{but keeps current one}}}\Bigg] V_{t+1}^i(ez,h)
\end{aligned} \tag{6}$$

A firm keeps its match to the following period if there is no separation shock (probability $1-\lambda$) or if either: 1. the worker fails to find a new offer or 2. the worker finds a new offer, but prefers to stay. The probability of these conditions to apply are in brackets. When losing their employee, the firm returns to the pool of unmatched firms and derives zero value.

To express the worker's value, define the auxiliary function $W_t^s(W, ub, h)$ that returns the value of searching in the labor market with a reservation value W and potential insurance benefit ub in case of no match:

$$\begin{aligned}
W_t^s(W,ub,h) &= (1-p\theta_t(h))(W+ub) + p\theta_t(h)\int_{Z_t^f(h)} \max(W, W_t^f(z,h))\frac{dG_t}{G_t(Z_t(h))} \\
&\quad + p\theta_t(h)\int_{Z_t^i(h)} \max(W, W_t^i(ez,h))\frac{dG_t}{G_t(Z_t(h))}.
\end{aligned}$$

The definition above leaves implied a condition we formalize in the next subsection: the value of a formal or informal contract is independent from the prospective employee's reservation value.[21]

[20] Expressions in (5) imply that workers only decide to switch jobs if they find a match that they *strictly* prefer to their current one.

[21] In the context of our model, this is to say that wages bargained by an employed and an unemployed worker with a

The value of a formal labor contract to the worker is

$$\begin{aligned}\hat{W}_t^f(w,z,h) = (1-\tau_{hh,t})w \\ + q_t\left\{\lambda_f W_{t+1}^s(W_{t+1}^n(h),v,h) + (1-\lambda_f)W_{t+1}^s(W_{t+1}^f(z,h),0,h)\right\}.\end{aligned} \tag{7}$$

The first term in curly brackets captures the probability that the match ends. The worker then re-enters the labor market in the following period and receives UI benefits if he or she finds no other job. The second term represents the chance the match continues and that the workers enters the market with the reservation value equal to the value of the current job. No UI benefit is due in this case, since failure to find a job does not lead to unemployment.

The value of an informal contract is

$$\hat{W}_t^i(w,ez,h) = w + q_t\left\{\lambda_i W_{t+1}^s(W_{t+1}^n(h),0,h) + (1-\lambda_i)W_{t+1}^s(W_{t+1}^i(ez,h),0,h)\right\}. \tag{8}$$

The worker can hide the wage payment from the government and thus pays no income tax. Also, note that transition to unemployment does not lead to UI benefits. Given equilibrium bargained wage functions, we define $W_t^f(z;h) = \hat{W}_t^f(w_t^f(z,h),z,h)$ and the same for W_t^i.

Finally, the worker's value when unemployed is:

$$W_t^n(h) = \varsigma + \zeta_0 + \zeta_1 h + qW_{t+1}^s(W_{t+1}^n(h),0,h). \tag{9}$$

3.3 Wage Setting

We determine wage rates using Nash bargaining subject to minimum wages in the case of formal contracts. Start by defining Nash-bargained wages $\hat{w}$ in the usual way:

$$\begin{aligned}\hat{w}_t^f(z,h) &= \underset{w}{\text{argmax}} \quad \left(\hat{W}_t^f(w,z,h) - W_t^n(h)\right)^{\frac{\eta}{1-\tau_{hh,t}}} \hat{V}_t^f(w,z,h)^{\frac{1-\eta}{1+\tau_w}} \\ \hat{w}_t^i(ez,h) &= \underset{w}{\text{argmax}} \quad \left(\hat{W}_t^i(w,ez,h) - W_t^n(h)\right)^{\eta} \hat{V}_t^i(w,ez,h)^{1-\eta}\end{aligned} \tag{10}$$

First-order conditions for an interior solution indicate a wage rate that is linear on firm's productivity, and leads to the following conditions:

$$\frac{\eta}{1-\eta} = \frac{W_t^f - W_t^n}{V_t^f} = \frac{W_t^i - W_t^n}{V_t^i}. \tag{11}$$

In case there is no wage rate w that guarantees both terms in parenthesis be non-negative, we set $\hat{w}$ to be the worker's reservation wage and the firm then chooses not to offer the contract on that state. [22] The effective wage coincides with the bargained wage in the informal sector, but must respect

new firm are the same. Employed workers can't convert their current position into additional bargaining power. This is a simplifying assumption: it allows us to avoid carrying *three* additional state variables (the two states of the previous employer and whether the previous contract was formal or informal).

[22] If we set $\hat{w}$ to be the firm's reservation wage, workers in turn would reject the contract.

the minimum wage constraint in the formal one:

$$\begin{aligned} w_t^f(z,h) &= \max(\omega, \hat{w}_t^f(z,h)), \\ w_t^i(ez,h) &= \hat{w}_t^i(ez,h). \end{aligned} \tag{12}$$

See section B of the Appendix for more discussion about the wage determination mechanism.

3.4 Free Entry and Transition Rates

The condition of free entry determines the number of firms paying for vacancies. It implies that, for each labor market h, the expected value of drawing a new pair (e,z) and having the opportunity of searching in the market equals the cost of doing so:

$$p_{t+1}(h)q_t \int \max\left\{0, \rho_{t+1}^f(z,h)V_{t+1}^f(z,h), \rho_{t+1}^i(ez,h)V_{t+1}^i(ez,h)\right\} dG_{t+1} = \kappa h. \tag{13}$$

In practice, condition (13) determines the worker-finding rate $p_t(h)$ compatible with free entry. The matching function then gives the equilibrium market tightness:

$$p_t(h) = M(\theta_t(h)^{-1}, 1). \tag{14}$$

Since employed workers search with full efficiency in the model, the market tightness coincides with the number of searching firms per worker:

$$\theta_t(h) = G_t(Z_t(h))F_{t-1}(h). \tag{15}$$

Lastly, to state ρ_t^f and ρ_t^i (the probabilities that a firm holds the worker conditional on a match), we need to reference the distribution of workers. Let $U_t(h)$ be the share of unemployed workers of type h. Let $E_t^f(\{e,z\};h)$ and $E_t^i(\{e,z\};h)$ be the shares of employed formal and informal workers. These three measures sum to one. Let $\Psi_{i,t}^f(z,h) = \{(e',z') \ : \ V_t^i(e'z',h) < V_t^f(z,h)\}$ be the set of firm states such that a type h informal worker employed by it decides to switch jobs to a state z formal firm. Define $\Psi_{f,t}^f(z,h)$, $\Psi_{f,t}^i(ez,h)$ and $\Psi_{i,t}^i(ez,h)$ analogously. Then:[23]

$$\begin{aligned} \rho_t^f(z) &= U_{t-1}(h) + \lambda_f E_{t-1}^f(\mathcal{E}\times\mathcal{Z};h) + \lambda_i E_{t-1}^i(\mathcal{E}\times\mathcal{Z};h) + \\ &\quad + (1-\lambda_f)E_{t-1}^f(\Psi_{f,t}^f(z,h);h) + (1-\lambda_i)E_{t-1}^i(\Psi_{i,t}^f(z,h);h) \\ \rho_t^i(z) &= U_{t-1}(h) + \lambda_f E_{t-1}^f(\mathcal{E}\times\mathcal{Z};h) + \lambda_i E_{t-1}^i(\mathcal{E}\times\mathcal{Z};h) + \\ &\quad + (1-\lambda_f)E_{t-1}^f(\Psi_{f,t}^i(z,h);h) + (1-\lambda_i)E_{t-1}^i(\Psi_{i,t}^i(z,h);h) \end{aligned} \tag{16}$$

3.5 The Government

The government manages a public debt D. Fiscal policy is neutral, in that public debt is constant. Let $C_{gov,t}$ be public spending, which has no impact on households' utility. The budget constraint of

[23] Because employed workers suffer no loss of search efficiency, the normalized measure of searching workers each period equals one, and so the expressions in (16) require no denominator.

the government is

$$\begin{aligned} C_{gov,t} + v \int U_{UI,t}(h)dH + \varsigma \int U_t(h)dH + D = \\ \tau_y \int \int zhdE_t^f(.;h)dH + (\tau_w + \tau_{hh,t}) \int \int w_t^f(z,h)dE_t^f(.;h)dH + q_t D, \end{aligned} \tag{17}$$

where $U_{UI,t}(h)$ is the share of type h workers that receive the one-time UI transfer.

In our simulations, we allow the government to set one of two variables only, public expenditure $C_{gov,t}$ or the income tax rate $\tau_{hh,t}$. The government sets the other one so that public policy is feasible (in the sense that the budget constraint (17) holds in all periods).

3.6 The Financial Intermediary

The financial intermediary operates the technology that converts final goods into physical capital and vice-versa, and stores capital from one period to the next. It invests in capital and equity of existing firms, and posts new vacancies in the market. To finance these purchases, the intermediary issues one-period bonds to households.

The financial intermediary is a risk-neutral agent which, for simplicity, we allow to consume negative amounts. We defer to the section D of the Appendix a complete description of its optimization problem and solution. For now, all we need is the following no-arbitrage condition:

$$q_t = \frac{1}{1 + r_{t+1}}. \tag{18}$$

3.7 The Shopper's State

The shopper's exogenous state is $s = (d, e, z; h)$. State d represents labor market status: employed formal, employed informal, unemployed and unemployed with UI benefits. The pair (e, z) is the employer state. The income function y is

$$y_t(s) = \begin{cases} (1 - \tau_{hh,t}) w_t^f(z,h) & \text{if } d = \text{employed, formal} \\ w_t^i(ez,h) & \text{if } d = \text{employed, informal} \\ \varsigma & \text{if } d = \text{unemployed withouth UI} \\ \varsigma + v & \text{if } d = \text{unemployed with UI.} \end{cases} \tag{19}$$

With λ_f, λ_i, $p_t(h)\theta_t(h)$ and all the value functions, we have all we need to calculate the transition matrix g_t (section C of the Appendix). Given g_t, the distribution of households across states (a, s), denoted $x_t(a, s)$, satisfies

$$\begin{aligned} x_t(\{a'\}, \{s'\}) = \int_{\{(a,s) \mid a'(a,s) \in \{a'\}\}} g_t(\{s'\}|s) dx_{t-1} \\ \int_{\{h=\bar{h}\}} dx_t = H(\bar{h}) \end{aligned} \tag{20}$$

where $a'(a,s)$ solves problem (1). The distribution of workers of a given type over different labor market status E_t^f, E_t^i and U_t follow from x_t.[24]

3.8 General Equilibrium

We are ready to define general equilibrium paths. The set of aggregate state variables in period t include the distribution of households x_{t-1} (along with the distribution of workers E_{t-1}^f, E_{t-1}^i and U_{t-1}), the measure of firms that paid for a vacancy in the previous period F_{t-1} and the stock of capital supplied by the investor K_t (a predetermined variable).

Definition 1 (General Equilibrium)**.** *Given an initial aggregate state x_0, F_0 and K_1, a feasible public policy sequence $\{C_{gov,t}, \tau_{hh,t}\}$ and a path for the distribution of hiding abilities $G_{e,t}$, an equilibrium consists on a path of: value functions for firms and workers V_t^f, V_t^i, W_t^f, W_t^i, W_t^n; wage rates w_t^f, w_t^i; market tightness θ_t; discount q_t and interest r_t rates; firm measures F_t; household measures x_t; income processes y_t; transition matrices g_t; and consumption/savings policies c_t, a_t' such that:*

1. *worker and firms' value follow (6), (7), (8) and (9);*
2. *wage rates are determined by Nash bargaining (subject to the minimum wage in the case of formal contracts);*
3. *the free entry condition (13) determines the number of searching firms F_t, and market tightness follows from (15)*
4. *discount rates reflects capital rental cost (equation (18) holds);*
5. *the measure of households follows from g (equation (20));*
6. *the transition matrix and the income process follow from optimal worker and firm decisions;*
7. *consumption/savings policies solve the searcher's problem (1) given y_t and g_{t+1};*
8. *the capital market clears:*

$$K_{t+1} = q_t \left[\int a_t'(a,s) dx_t - Q_{t+1} - D \right]. \tag{21}$$

By Walras' law, clearing of the capital market implies clearing in the final goods market:

$$Y_t = C_t + C_{gov,t} + Inv_t + VC_t$$

where Y_t is aggregate output and $Inv_t = K_{t+1} - (1-\delta)K_t$ is investment in physical capital.

Definition 2 (Steady State)**.** *Given a feasible public policy pair $\{C_{gov}, \tau_{hh}\}$ and a distribution for hiding abilities G_e, a steady state equilibrium is an equilibrium in the sense of definition 1 in which the variables of the path do not depend on time, the initial conditions for the aggregate state variables coincide with their constant path values and the public policy and hiding ability distribution sequences are defined by $\{C_{gov}, \tau_{hh}\}$ and $\{G_e\}$ for all periods.*

[24] For example, $E_t^f(\{z\};h) = x_t(\mathscr{A}, \text{formal}, \mathscr{E}, \{z\}, h)H(h)$, where $\mathscr{A}$ is the set of possible net wealth positions.

Symbol	Interpretation	Source/Target	Value
Preferences			
β	Intertemporal discounting	$r = 15\%$ per year	0.95458
γ	Relative risk aversion	Literature standard	3
Technology			
A	Model scale	$P(r) = 1$	0.7062
α	Capital income share	Literature standard	0.33
δ	Rate of capital depreciation	$\delta K/Y = 0.1$	0.01546
Labor Market			
λ_f	Separation rate (formal)	Transition unemployment	0.04
λ_i	Separation rate (informal)	Transition unemployment	0.098
κ	Vacancy cost	Market tightness = 1	0.064
μ	Matching function constant	Unemployment rate = 16%	0.255
$1-\varepsilon$	Job-finding rate elasticity	Literature average	0.375
η	Worker's "bargaining power"	Hosios condition	0.625
ζ_0	Leisure	Income differential (low inc.)	0.02
ζ_1	Leisure	Income differential (low inc.)	0.16
Public Policy			
τ_y	Sales tax rate	Tax law	0.272
τ_w	Payroll tax rate	Tax and labor law	0.1648
τ_{hh}	Household income tax rate	Tax law	0.09
ω	Minimum wage	Labor law	0.0998
ς	Unemployment income	*Bolsa Família* program	0.01
υ	Unemployment insurance	Social security	0.1833
D	Public debt	Debt-to-GDP = 42%	0.6583
Distributions			
ξ	Skill distribution parameter	Formal income histogram	0.61
σ_z	Productivity dispersion	Income variance and differential	100%
ϕ	Probability of $e = 0$	Informality share	0.75
ν	Hiding ability distribution	Income differential (high inc.)	9

Table 2: Baseline Calibration

There is no aggregate uncertainty. In our simulations, we start with a baseline steady state equilibrium that gives the initial values for the aggregate state. The economy then faces an "MIT" shock: a sudden, unanticipated changes in the paths of the hiding ability distribution $G_{e,t}$ and public policy. We calculate the equilibrium associated with these new paths.

4 Calibration

Table 2 presents our baseline calibration. Some parameters are literature standards, some are disciplined by 2018 Brazilian law, and some we choose to match data targets, in particular facts 1-3. One model period = one quarter; one model final good = US$ 2,500.

Starting with preference parameters, we choose a coefficient of relative risk-aversion equal to 3. To calibrate the intertemporal discounting β, we start by taking from the Brazilian Central Bank website data on corporate credit interest (*ICC* - Credit Cost Indicator). After discounting for annual

CPI inflation and averaging on the Jan-2012 to Dec-2019 period, we find a 16.1% average real interest rate. We calibrate β so that the model generates a slightly lower value of 15% annualized interest rate (discount $q \approx 0.96566$).[25]

Moving to technology parameters, we set A so that $P(r) = 1$. The share of capital income α is 0.33. The rate of capital depreciation we set so that, on the aggregate, capital investment is ten percent of aggregate output. Added to investment in vacancy costs, total investment equals 16.8% of output, close to Brazilian average since 1996.

Parameters related to public policy are entirely calibrated based on Brazilian law and actual policy. The statutory tax rate on profits is 9%. To ease notation, we did not define a profit tax. Instead, we adjust the sales and payroll tax rates accordingly. The payroll tax rate $\tau_w = (1.28) \times (0.91) - 1 = 0.1648$ corresponds to the sum of the mandatory employer contributions to the social security (20%) and to the worker's severance fund (8%).[26] The sales tax is harder to calibrate due to the myriad of different forms of taxation established by the federal and local governments. In a calibration of the Brazilian economy, Ulyssea (2018) uses a rate of 29.25%, calculated by the combination of taxes over manufactured products (*IPI*) and additional contributions to social security charged on revenue (*PIS/COFINS*). We opt for a lower rate (20%) as the model can't capture firms' ability to pass taxes over to consumers. After correcting for profit taxation, however, we end up with a figure similar to Ulyssea, $\tau_y = 1 - (0.8) \times (0.91) = 0.272$. The income tax rate $\tau_{hh} = 0.09$ is based on the worker's (also mandatory) contribution to the social security system.[27]

The minimum wage $\omega = 249.5/2,500 = 0.0998$ and UI benefits $v = 0.1833$ follow the labor law.[28] We calibrate unemployment income ς to be BR$ 100 = US$ 25.[29]

Moving on to labor market parameters, we choose rates of exogenous separation λ_f and λ_i to target observed employment-to-unemployment transition rates as in figure 4a (3% for formal jobs and 7.5% for informal ones). The vacancy cost parameter κ and the matching function constant μ target respectively an average market tightness of one and an average unemployment rate of 16%.[30] The exponent in the matching function ε governs the elasticity of the job-finding rate to market tightness. As we lack the empirical evidence of the number of vacancies in Brazil, we can't disciple our choice by the data. Instead, we take a median point of the literature and set $\varepsilon = 0.625$ (Shimer

[25] This is a large value for the average yield of household investment. We stick to it as our main concert is the correct discounting of future streams of profits by firms. Such discounting is critical for their formalization choice.

[26] The law requires employers to contribute towards a severance fund (*FGTS*), with monthly payments set at 8% of worker's wage rate. It is financed by taxes charged on the firms' payroll as well. Workers can't withdraw money from their account except when losing their job without "just cause", retiring and a few other specific contingencies.

[27] This choice of τ_{hh} assumes no personal income taxation, a choice we make based on the observation that most workers locate in the first bracket of the progressive income taxation scheme, so they pay no income taxes.

[28] Public UI benefits in Brazil depend on the worker's income in his or her previous job. We opt for a constant value because the lower and upper limits on the value of the benefits are close to each other. The lower limit is set at the minimum wage BR$ 998 = US$ 249.5 per month and the upper limit in 2018 was BR$ 1,677 = US$ 419. Given the higher concentration of workers in the lower end of the income distribution we assume a benefit of BR$ 1,100 = US$ 275. Workers receive five monthly installments, so we set $v = (5/3) \times 275/2,500 = 0.1833$.

[29] This target is motivated by the *Bolsa Familia* program, a basic income program that provides financial assistance to poor households in Brazil. The value of the monthly transfer for standard participation in the program was 85 Brazilian reals in 2018 and the average benefit per household (not individual) was 187 reals, so we take our calibration to be conservative in the sense that it is more generous to households than the actual program.

[30] We calculate these averages through the formulas $\int \theta(h)dH$ and $\int U(h)dH$. Note that the average market tightness across types h is also the unconditional market tightness (total firms searching divided by total workers searching), since all workers search in the market every period.

(2005) sets $\varepsilon = 0.72$, Bosch and Esteban-Pretel (2012) set $\varepsilon = 0.5$; see Petrongolo and Pissarides (2001) for a survey). We set the worker's "bargaining power" η to satistfy the Hosios (1990) efficiency condition. The two parameters governing the worker's private value of leisure ζ_0 and ζ_1 we set to calibrate the level and slope of income differential among low-income workers.

We now turn to the parameterization of the distributions of the model, H, G_z and G_e. We discretize skill levels with ten equally-spaced values $\underline{h} = h_1 < h_2 < \cdots < h_{10} = \bar{h}$, with a distribution H such that $H(h_{t+1}) = \xi H(h_t)$. We choose a value of ξ to target the share of formal workers receiving a labor income in the US$200-400 income range. Bounds $\underline{h} = 0.03$ and $\bar{h} = 0.185$ are so that the ten worker groups cover the range of average income observed among different occupational groups and educational levels (between US$250 and US$900, monthly). As for productivity draws, we assume $G_z \sim \text{LogNormal}(-\sigma_z^2/2, \sigma_z^2)$. We set $\sigma_z = 100\%$, a value that generates enough variance in the log income of various worker groups and that keeps the income differential away from the minimum wage low.

Government Surveillance. We model the repression of informality by changes in the distribution of hiding abilities e. Tighter enforcement corresponds to a lower probability that a firm draws a large hiding ability. Specifically, the distribution $G_{e,t}$ is such that draws of e are determined by the following mixture:

$$e = (1 - \text{Dummy}_{Surveillance,t}) \times \text{Beta}[\nu, 1].$$

$\text{Dummy}_{Surveillance,t}$ captures the probability that a firm draws $e = 0$, which increases as the government tightens its control over labor contracts. It takes two values:

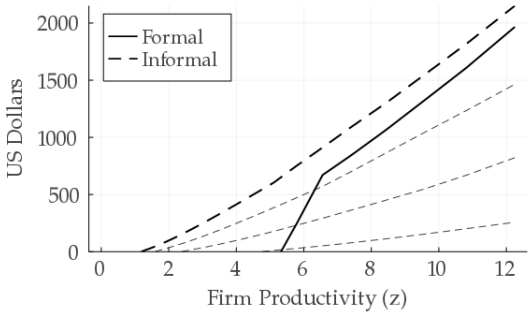
$$\text{Dummy}_{Surveillance,t} = \begin{cases} 1 & \text{with probability } \phi_t \\ 0 & \text{with probability } 1 - \phi_t. \end{cases}$$

Parameter ϕ_t captures the measure of contracts audited by the government. Our simulation of the repression of informal activity consists of increasing ϕ_t over time.

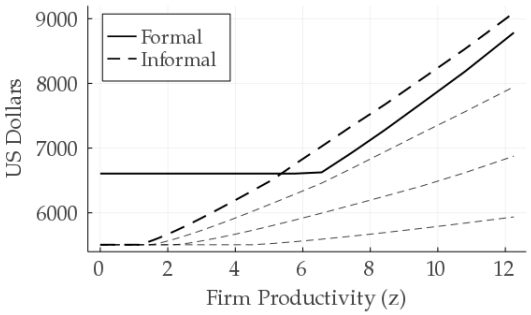
The other term of the mixture is a Beta distribution, a natural choice given its $[0, 1]$ support. Shape parameter ν governs the extent to which the distribution concentrates mass close to the upper bound 1. Higher values of ν yield higher average hiding ability. There is a tension in the calibration of ν and ϕ as both parameters have a significant impact on the size of the informal sector. In the baseline calibration, we set $\nu = 9$ to target the low income differential among high-income workers. This implies that the Beta distribution has an average of 90%. We then set $\phi_t = 0.75$ to match informality shares.

4.1 Mechanics and Model Performance

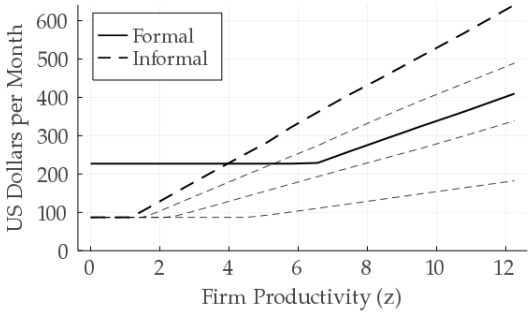
Figures 5a to 5c plot the main functions of the search block: firms' value V, workers' value W and net wage rates $(1 - \tau_{hh})w^f, w^i$ respectively. Taking figure 5a, the solid curve corresponds to the formal firm $V^f(z, h)$; the thick dashed curve corresponds to the informal firm $V^i(ez, h)$ with hiding ability $e = 1$; the other three lighter dashed curves correspond to lower levels of hiding ability, $e = 0.75, 0.5, 0.25$, from the top to the bottom. The interpretation is the same for the other two plots. All figures focus on $h = \underline{h}$, but other choices lead to similar patterns.



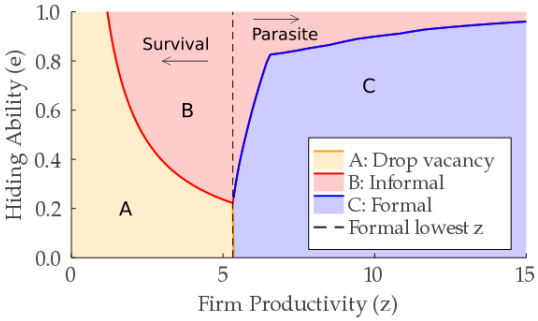
(a) Firm Value



(b) Worker Value



(c) Net-of-Tax Wage Rates



(d) Regions of Production

Notes: Panel (a). Firm value functions. Panel (b). Worker value functions. Panel (c). Net-of-tax income functions. In all panels, x-axis indicates firm productivity z. Panel (d). Unmatched firm decision as a function of its state.

Figure 5: Equilibrium Functions - Lowest Skill Type

These functions capture the decisions of agents in the labor market. Firms draw their state (e, z) and then decide whether to post a vacancy and, if they do, whether to offer a formal or an informal contract, all according to which curve is above the other (and the zero line) in figure 5a. For large z, the marginal value of productivity is higher in the case of a formal contract. The formal value function has greater slope. For any level of hiding ability e, there is always a corresponding $z^*(e)$ such that firms with productivities $z > z^*(e)$ prefer to offer a formal agreement. This is a consequence of a lower exogenous separation rate in the formal sector.

In the case of workers (figure 5b), the lower bound on the value of employment corresponds to the unemployment value W^n (not plotted). Workers do not accept an offer with value lower than W^n. The kink in the formal value of both firms and workers (the solid curves) marks the productivity level $\underline{z}(h)$ below which the minimum wage constraint binds. When $z > \underline{z}(h)$ worker and firm agree on the preferred type of contract (equilibrium condition (11)). When $z < \underline{z}(h)$, they disagree for most combinations of e and h, with firms offering informal agreements.

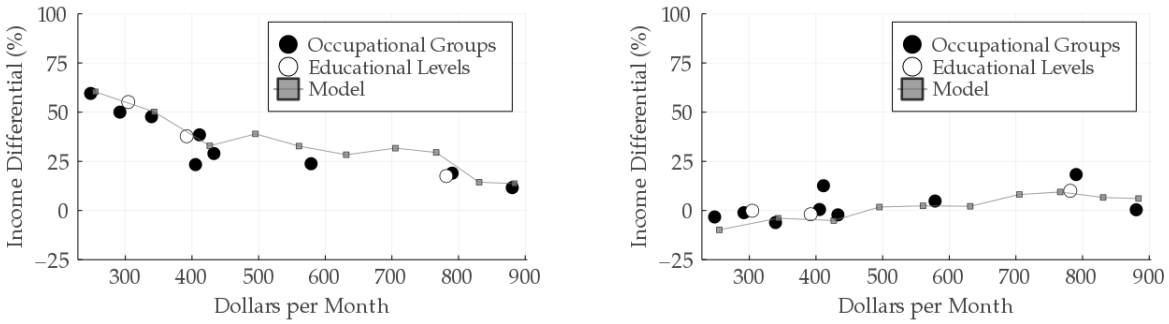
But firms can also offer informal agreements when z is large and they are productive. The value functions show that, in this case, workers will typically agree on the informal condition of the contract. The wage function 5c depicts the reason: greater payment compensates the worker for losing the benefits of being hired formally (*compensating differentials*, as in Meghir et al. (2015)). This mechanism allows the model to replicate fact 2. In turn, the inverse relationship between income and informality share (fact 1) is a byproduct not of higher wages in the formal sector, but of the different productivity *compositions* of each sector. The greater slope of formal firm's value means that the formal sector concentrates firms and workers with superior productivity on average.[31]

Figure 5d makes this point more explicit. It shows the decision of the unmatched firm as a function of its productivity (x-axis) and hiding ability (y-axis). Regions A, B and C correspond to dropping the vacancy, offering an informal contract, and offering a formal contract, respectively. The dashed line is the lowest firm productivity $\underline{z}^f$ compatible with a formal contract (that is, $V^f(\underline{z}^f, \underline{h}) = 0$).

The regions allow us to reconciliate the model with two classic views of informality: the *parasite* view and the *survival* view. One can call "parasites" informal firms with productivity levels superior to $\underline{z}^f$ (region B, right of the dashed line), since the contract would be profitable in the formal sector. Informal firms with productivity levels lower than $\underline{z}^f$ (region B, left of the dashed line) also choose an informal contract for profit maximization, but in their case hiring the worker formally would not be economically viable. They are "survival" firms, as their existence depends on the possibility of circumventing laws and regulations.

These two categories of informal firms represent conflicting forces in the simulation of the repression of informality. Ignoring other general equilibrium effects, without the option of the informal agreement, parasite firms simply migrate to the formal sector, which leads to a *decrease* in the average productivity of searching firms and no change in the number of firms searching in the labor market. Survival firms, on the other hand, exit the market. This leads to an *increase* in the

[31] The value functions of both firms and workers have a slight convexity in z. This is explained by on-the-job search: as productivity increases, not only does the discounted sum of dividends (linearly) increases, but also the probability that the worker leaves the job by finding a better one declines (sets Φ^f and Φ^i shrink), which leads to greater continuation value to the firm. Nash bargaining then distributes the value gain to the worker *via* larger bargained wages, which justifies the convexity of the worker's value as well.



(a) Income lower than US$ 300

(b) Income greater than US$ 300

Notes: The plots show the income differentials. Panel (a) considers only workers receiving less than *US*$300 income. Panel (b) considers only workers receiving more than *US*$300. In both plots, the x-axis indicate the *unconditional* average of each group as a proxy for productivity.

Figure 6: Income Differentials - Data and Baseline Model

average productivity of searching firms and a *decline* in the total number of vacancies.

Performance. Can the model replicate the stylized facts of informality of section 2?

Figures 2a and 2b contain the model counterparts of the data they depict, with each square marker representing a different skill type h.

Figures 6a and 6b repeat figures 3a and 3b, but, now, the x-axis contains the *unconditional* average income of each group rather than their name. "Unconditional" means I take averages on the whole sample of workers, not just the ones with income above or below the US$ 300 threshold. For example, managers and directors (the right-most marker in the figures) have an unconditional average income of US$ 880. The income differential is 11% among those with income lower than US$ 300, and 0% among those with income greater than US$ 300. The solid curves again represent model predictions.

The model succeedes in replicating the patterns of the data. The share of informal workers declines as we move from low to high skill types, but remains economically significant. In terms of income distribution, the model does a fine job in replicating the empirical evidence, especially considering the single parameter to target it, ξ.[32]

The model also successfully generates the large differential among low-income workers and the reduced differential among high-income workers. The minimum wage is key for this result. In shutting down the minimum wage (set $\omega = 0$), the differential below the threshold got to zero, for all skill types. As for the differential above the threshold, the key feature of the model to generate it is the heterogeneity in hiding abilities e, which allows the calibration to generate productive informal firms paying high wages, while keeping the size of the informal sector in check.

Finally, table 3 reports some additional targeted and non-targeted moments. We can match targeted moments. Most importantly, the patterns related to transitions to unemployment (fact 3).

As for the untargeted moments, direct transitions between formal and informal employment

[32]One caveat is that, in the data, more informal workers locate in the second income bin (US$ 200 - US$ 400) then in our model. One potential explanation is the existence of informal firms that choose to respect the minimum wage law, a force we do not capture.

Variable	Data/Target	Model
Targeted		
Unemployment rate	0.16	0.160
Market tightness	1	1.003
Informality share	0.3	0.326
Transition formal-unemployment	0.03	0.030
Transition informal-unemployment	0.075	0.075
Non-Targeted		
Transition unemployment-employment	0.2 - 0.3	0.233
Transition formal-informal	0.045	0.014
Transition informal-formal	0.18	0.057
Variance of log income	0.2 - 2.0	0.242

Table 3: Targeted and Non-targeted Moments

appear to be underrepresented in the model, as we do not allow firms and workers to re-negotiate formality status. As for the variance of income (calculated in logs, and conditional on the occupational group in the data or the skill type in the model), we observe in the data large as well as small values of such variance over the different survey waves and occupational groups. Values vary between 0.22 and 1.96. In our calibration, increasing the dispersion σ_z of productivity draws generates additional volatility, but hinders our ability to keep income differentials among high income workers low. Lastly, the model predicts a transition rate to employment inside the range observed in the data between 2012 and 2020.

5 The Repression of Informality: A Simulation

We simulate the repression of informal labor activity by changing the distribution of hiding abilities. We start with a benchmark case and different specifications later.

In period zero, the economy is in the baseline steady state, with surveillance rate $\phi_0 = 0.75$. From period one onward, we set $\phi_t = 1$, $t = 1, 2, 3, \ldots$ All unmatched firms draw $e = 0$ with probability one, which effectively removes their option of offering an informal contract in the market.

We calculate an equilibrium path in the sense of definition 1, which requires a description of public policy $\{C_{gov,t}, \tau_{hh,t}\}$. In our benchmark simulation, we fix public expenditure C_{gov}. The government meets its budget constraint (17) by changing income tax $\tau_{hh,t}$.

We simulate the transition path for four hundred quarters.[33]

5.1 Aggregates and the Income Process

We first focus on prices and quantities. Table 4 summarizes the transition. Each column corresponds to a different period of the transition path. The symbol "%Δ" means percentage deviation from

[33] We also experiment with five hundred quarters and find similar results.

Variables	Baseline $t = 0$	One year $t = 4$	Ten years $t = 40$	Forty years $t = 80$	Long run $t = \infty$
Interest rate (%)	15	16.8	17.8	16	15
Income tax rate (%)	9	0.2	-12.4	-16.7	-19.1
Informality share (%)	32.9	13	0	0	0
Unemployment rate (%)	16	18.5	16.8	15.9	15.5
Market tightness (%)	100.3	55.5	51	59	62.7
Job-finding rate (%)	23.4	17.2	16.2	17.1	17.5
Transition unemployment (%)	4.5	4	3.3	3.2	3.2
Firm revenue - avg (%Δ)	0	6	17.4	21.6	24.5
Firm revenue - entry (%Δ)	0	56.7	53.6	54.4	57.5
Labor income - avg (%Δ)	0	12.1	34.3	43.2	49
Labor income - entry (%Δ)	0	51.5	64.9	71.3	76.7
Aggregates					
Output (%Δ)	0	6.3	23.1	28.9	32.7
Capital (%Δ)	0	-1.3	9.5	23.5	32.5
Efficiency Hours (%Δ)	0	10.2	30.3	31.5	33.7
Savings (%Δ)	0	-1.5	11.4	22.3	29.3
Sales tax revenue (%Δ)	0	18	46.1	53	57.6
Income tax revenue ($)	24.1	0.6	-46.8	-65.6	-77

Notes: Endogenous variables in the benchmark exercise. "%Δ" means percentage deviation from its baseline steady state value.

Table 4: Simulating the Repression of Informal Labor

baseline steady state.

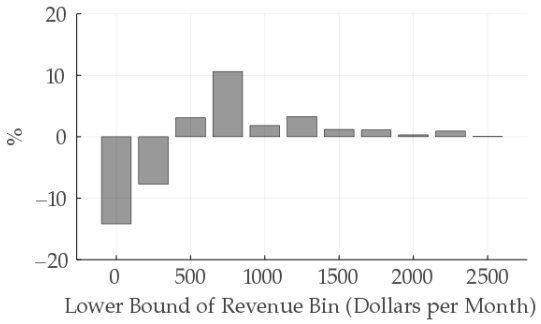
5.1.1 The Long-Run

Consider first what happens in the long run. As expected, $\phi = 1$ banishes informal activity. Note that, while interest rates are roughly unchanged, the income tax rate changes significantly (from 9% to -19.3%).[34]

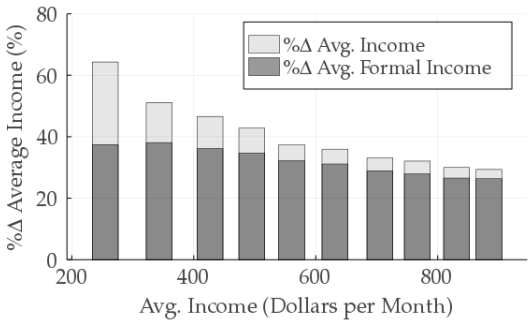
The unemployment rate in the new steady state falls slightly to 15.5% from 16% in the baseline equilibrium. This follows from two opposing forces: a decline in the rate workers transition to unemployment (from 4.5 to 3.2%) and a decline in the rate workers transition to employment (job-finding rates, from 23.4% to 17.5%). Workers transition to unemployment more rarely due to the lower separation rates in the formal sector. Without informality, matches have a higher average duration. However, finding a job becomes rarer too, as market tightness drops from 1 to 0.63. Less firms populate the job market.

Like in Meghir et al. (2015), the repression of informality causes an increase in the average firm productivity: average firm revenue increases by 25%. The average productivity of firms employing workers coming out of unemployment (which we indicate by "entry") increases by 58%. Figure 7a depicts the gain in average productivity. We build US$200 bins of firm gross revenue and show how the number of jobs in each bin varies in the new steady state. The plot clarifies that unproductive firms leave the economy. The exercise gives practical sense to the "survival" character of these

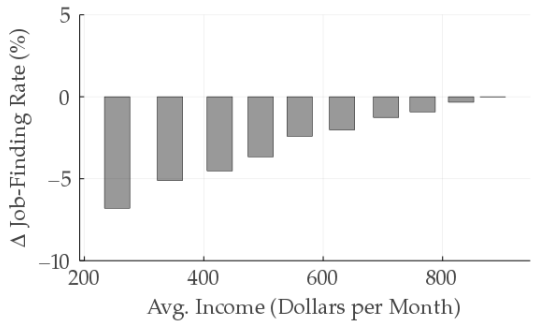
[34] We do not regard this as a problematic result of the model. In comparison to other countries, the Brazilian system imposes large taxes on goods and services, and lower taxes on income. Besides, we ignore in the model the progressive nature of income taxation, which thus causes understatement of income tax proceeds.



(a) Change in Firm Revenue Histogram



(b) Labor Income



(c) Job-Finding Rate

Notes: All panels refer to the long-run effects of the repression of informality. Panel (a). We show the change in the number of firms that locate in each $250 gross revenue bin. The x-axis corresponds to the lower bound on the revenue bin. Panel (b). Percentage change in average labor income for workers of each skill type h. Panel (c). Change in the job-finding rate for workers of each skill type h.

Figure 7: Long-Run Effects of the Repression of Informality

firms, as discussed in section 4, except that we now consider general equilibrium effects. With more productive firms and lower taxes, labor income increases by 77% for entry-level jobs and 49% overall.

Turning to aggregates, with a greater supply of labor, and applied on more productive employment, total efficiency hours of labor employed in production increases and, with it, so does the demand for physical capital by firms. The relatively stable interest rate indicates that such increased demand is met by additional aggregate savings. Total output increases by about 33%. Sales tax revenue increase by 58%, since firms can no longer avoid taxes and their revenues increase.

How do the effects above change between workers of different skill types? Figures 7b and 7c help with this question. For each skill level h, figure 7c plots the change in job-finding rates, and 7b plots the percentage change in average labor income and average *formal* labor income (the shaded component). The x-axis contains the average income of each skill group (in the baseline steady state).

Job-finding rates decline more in the case of low-skill workers. This is due to the minimum wage, which prices low-skill workers out of formality more often. As informal contracts cease from being an option to the firm, the set of productivity states that justify the creation of a job vacancy becomes smaller.

Figure 7b indicates a considerable increase in average labor income for workers of all skill types. For formal workers, such increase is not too different across skill types (25% and 40%). Such impact is not an obvious prediction of the model. The static impact of moving "parasite" informal firms into formality is a *decline* of unmatched firms average productivity.[35]

As for gains in average labor income overall, they are much less homogeneous, ranging between 29% and 64%. Low skill workers observe the largest gains. As the model replicates 2, this is expected. Groups of low-skill workers concentrate more minimum-wage-bind agreements. As informal jobs vanish, employment opportunities always entail payment of the legal floor, which lifts the average labor income.

5.1.2 Does the transition path matter?

It does. Besides table 4, we show in figures 8 and 9 the path of selected macroeconomic aggregates. The solid curves correspond to the benchmark equilibrium we discuss now.

The unemployment rate (figure 9b) sharply increases in the first half of the transition path, reaching 18.5% in $t = 5$. The prediction of lower unemployment in the long run does not extend to the equilibrium transition. On the contrary, the economy experiences a long period - about twenty year - of higher-than-initial unemployment, a result that steady-state models would fail to capture.

Equilibrium in the capital market, another aspect often ignored by the informality literature, provides the key mechanism leading to higher unemployment rate. Figure 8e shows that the interest rate increases in the first 20-30 quarters of the transition. It reaches almost 19% per year, 4% more than the original level.

How do higher interest rates explain the surge in unemployment? To firms, the existence of search frictions turns labor contracts into assets. They pay an upfront cost κh and collect dividends from production later, if they find an employee.[36] The effect of an interest rate hike on the value of labor in this environment is thus similar to the effect on the value of any other asset: V^f and V^i decline and, with it, the value of posting a vacancy (left-hand side of (13)).[37]

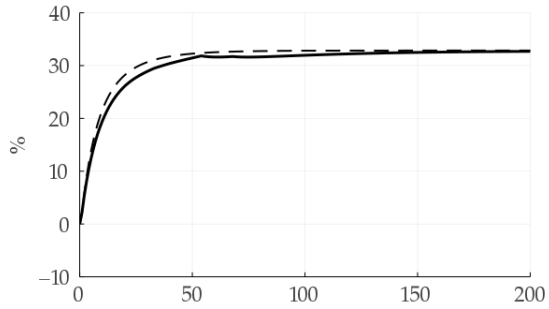
By the free entry condition, the lower value of vacancies leads to a reduction in the number of searching firms. The job-finding rate goes down, about 16% ten years after the repression of informality begins. During the initial years of the transition, when the job market faces this slowdown, some informal firms still exist in the economy (figure 9a). Therefore, while the job-finding rates undershoot their long-run values (figure 9c), transition rates to unemployment remain high relative to the long-run. The end result is the increase in the unemployment.

And why do interest rates increase in the first place? Figures 8a and 8b show that aggregate labor and average income increase throughout the transition path (again due to growing firm productivity, see "Firm Revenue" in table 4). The increase in efficiency hours applied by production

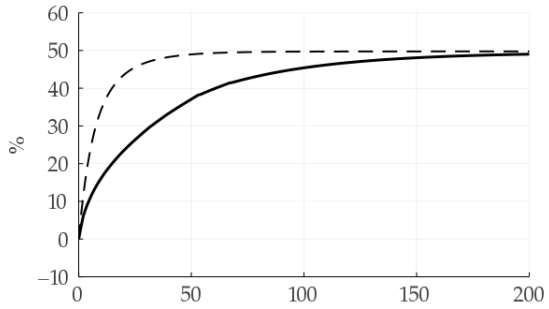
[35] In fact, it is indeed the case that, on average, unemployed workers match with less productive formal firms in the new equilibrium.

[36] Search frictions would also make the contract an asset to the worker, if we considered any cost for the worker to search: an utility or monetary cost, or yet a lost in searching ability in the case of employed individuals.

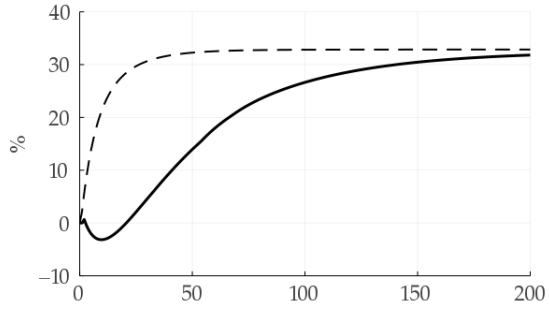
[37] See Hall (2017), Di Tella and Hall (2020) and Kehoe et al. (2022) for models in which variation in stochastic discounting account for business cycle fluctuations of unemployment. See Maya and Pereira (2020) for a model of stochastic discounting and the fluctuations of informal labor.



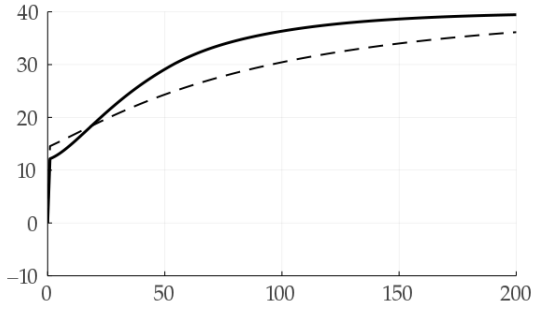
(a) %Δ Efficiency Hours of Labor



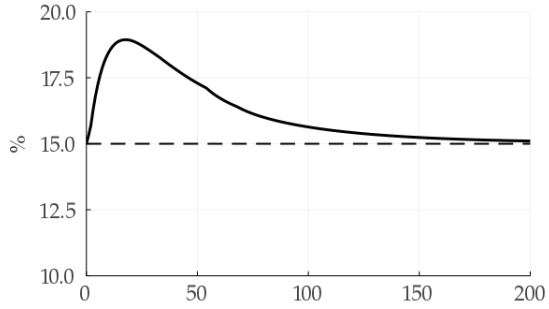
(b) %Δ Avg. After-Tax Income



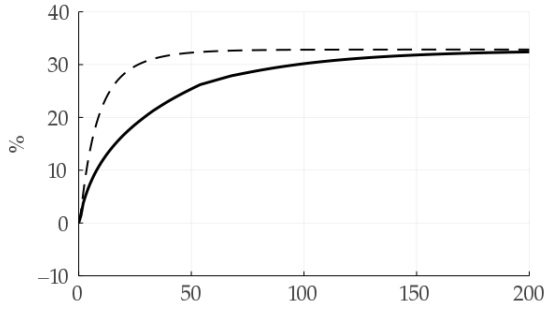
(c) %Δ Stock of Capital



(d) %Δ Aggregate Consumption



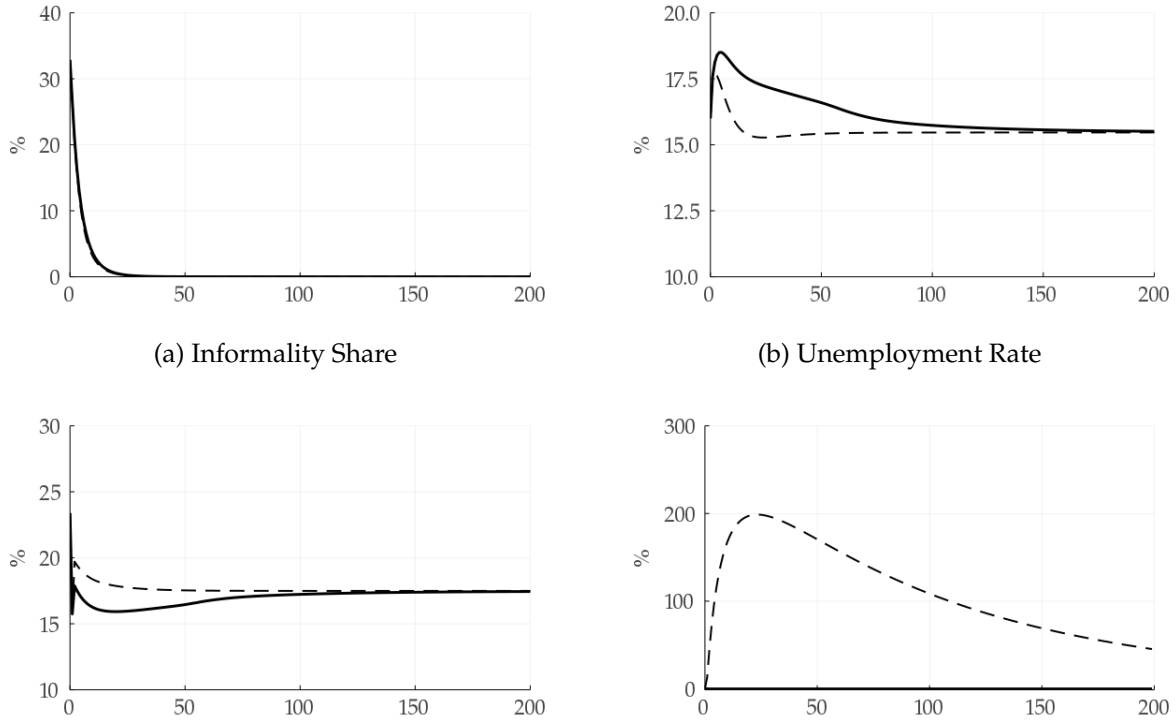
(e) Interest Rates



(f) %Δ Aggregate Output

Notes: The figures show the equilibrium transition paths of selected aggregates. The symbol %Δ indicates percentage deviation from the baseline steady state. The dashed lines correspond to an open-economy equilibrium, in which the interest rate is exogenously determined.

Figure 8: Transition Dynamics (Dashed = Open Economy, no GE)



(a) Informality Share

(b) Unemployment Rate

(c) Job-Finding Rate

(d) Foreign Debt to (annual) GDP

Notes: The figures show the equilibrium transition paths of selected aggregates. The symbol %Δ indicates percentage deviation from the baseline steady state. The dashed lines correspond to an open-economy equilibrium, in which the interest rate is exogenously determined.

Figure 9: Transition Dynamics (Dashed = Open Economy, no GE)

drives up the marginal productivity of capital as soon as the transition begins. Firms demand more capital, and the financial intermediary increases the supply of bonds.

However, households' aggregate savings fail to match the increase in capital demand. In fact, savings decline for the first few periods of the transition. This is due to intertemporal consumption smoothing. Optimizing shoppers choose a pair of consumption and savings to equalize marginal utility. Anticipating higher after-tax income *in the future*, they increase consumption *in advance*, in the first few periods of the transition, and lower their demand for bonds. Figure 8d shows the 11% jump in aggregate consumption upon announcement of the repression of informal labor.

These two effects combine to produce the increase in interest rates that characterize the economy in the early periods following the end of informality.

As we move along the transition path, the increase in average income realizes and aggregate household savings approach its long-run value. The capital supply curve shifts and its rental price declines steadily. Unemployment declines and the economy converges to its new long-run equilibrium, which differs significantly from the path that leads to it.

5.1.3 The open economy: does general equilibrium matter?

Compared to most version of the DMP model, variation in the real rate of interest is the central implication of forcing general equilibrium. We now consider the outcome of our exercise when we shut down this channel. This is similar to assuming an open economy with perfect capital mobility and financial integration.[38]

In the framework, we drop the last equilibrium condition in definition 1. The capital market does not necessarily clear anymore; households and the intermediary trade bonds with the rest of the world at an internationally determined interest rate of 15% per year, like before. We also keep the rest of the calibration of section 4 unchanged. Thus, foreign debt

$$\text{Foreign debt}_{t+1} = K_{t+1} - q_t \left(\int a'_t(a,s) dx_t - Q_{t+1} - D \right)$$

is zero in the steady state, but fluctuates in the transition path.

The dashed lines in figures 8 and 9 represent the open economy case. You can find the analogous of table 4 in the appendix.

In the benchmark case with fluctuating interest, the repression of informality leads to an almost unchanged rate of interest in the long run. Accordingly, the figures show that, for most variables, the long-run equilibrium with an open economy is very similar to the benchmark. But the economy's transition differs.

Without a surge in real interest, the initial increase in unemployment rapidly converges to its long-run value. The model no longer predicts the protracted period of high unemployment, and job finding rates are bigger than in the benchmark. This result highlights the importance of considering general equilibrium effects as we do in this paper.

The repression of informality still leads to more productive firms in the market. With more

[38] Having a closed economy as our default assumption follows from evidence of weaker financial integration of developing countries with the international markets (Bai and Zhang (2012)).

efficiency-hours of labor employed in production, aggregate output and capital demand increase. This time, however, the additional capital is financed not by domestic savings, but by an increase in foreign debt (figure 9d). Finally, despite the apparently lower cost of repressing informality in terms of unemployment, aggregate consumption follows a path that, except for the first few years, is below that of the benchmark. Households consume less as we shut down the wealth effect of higher interest rates.

5.2 Welfare

The households' value J (equation (1)) is our measure of welfare. To assess the welfare impact of the repression of informal activity, we calculate the *consumption equivalent welfare gain*, as in Lucas (1987). It is the value of m that solves

$$E\sum_t \beta^t u((1+m)c_t^0) = E\sum_t \beta^t u(c_t^1),$$

where superscript 0 indicates the consumption process in the baseline equilibrium, and superscript 1 indicates the after-policy equilibrium.

The consumption equivalent m answers the question: by how much should you increase a household's lifetime consumption to compensate it if informality was *not* repressed? Positive values of m mean that the household requires an increase in consumption. Therefore, it would be better off if informality was repressed. Negative values mean the household would be worse off.

In our transition paths, we compute consumption equivalents using

$$m_t(a,s) = \left[\frac{J_t(a,s)}{J_0(a,s)}\right]^{\frac{1}{1-\gamma}} - 1, \tag{22}$$

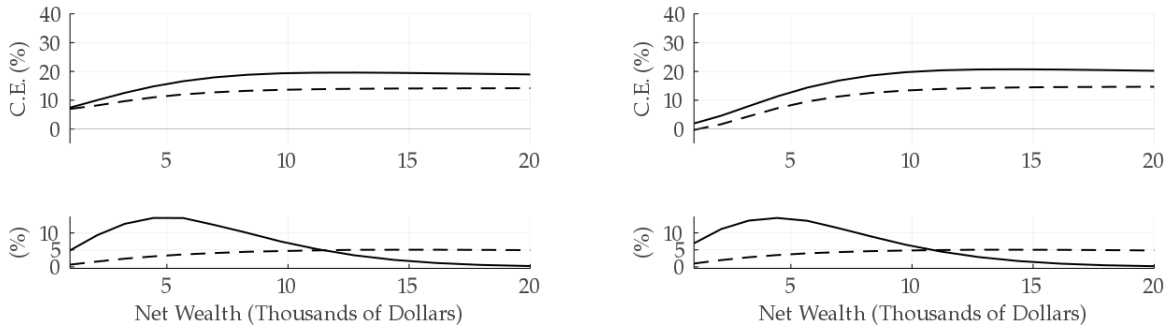
where J_0 is the value function in the baseline steady state.

To state results, we fix net wealth positions a, a skill type - we focus on the lowest $\underline{h}$ and highest $\bar{h}$ - and aggregate consumption equivalents $\int m_t(a,s)dx_t$ on the remaining states in s.

Figure 10 shows results. Each individual plot contains two graphs, with the axis representing net wealth a. The upper graph plots consumption equivalents, the lower panel plots the distribution of a. Figures 10a and 10b focus on the short run ($t = 1$); the other two consider the long run. Figures 10b and 10d look only at households with unemployed workers; the other two integrate across households in all labor-market states. Finally, solid and dashed lines represent the lowest skill type and highest skill type, respectively.

Almost all households are better off with the repression of informality, both in the short and in the long run. Consumption equivalents often exceed 20% of pre-repression lifetime consumption. Higher welfare follows from the increase in expected lifetime income.

The combination of lower job-finding rates and risk aversion should reduce welfare. Risk-averse households care relatively more about low-income states of nature, which tend to be unemployment states. As informality vanishes, the occurrence of these states becomes more harmful as returning to employment is less likely. Thus, the income process becomes riskier.



(a) Short-Run, All Households

(b) Short-Run, Unemployed

(c) Long-Run, All Households

(d) Long-Run, Unemployed

Notes: Each figure contains two panel; the upper panel plots consumption equivalents aggregated across worker states s, the lower panel plots the distribution of household wealth. Solid and dashes lines refer to lowest and highest skill types, $\underline{h}$ and $\bar{h}$, respectively.

Figure 10: Consumption Equivalents and Wealth Distributions

The pictures show this through two patterns. First, the welfare gains of the unemployed are in all cases lower than the average.[39] Second, welfare gains are lower among households with low wealth, and thus reduced capacity of self-insuring against negative income shocks.

Other forces also affect welfare, like changes in interest rates and access to UI benefits. In the case of our measure of welfare $\int m_t dx_t$, the distribution of households x_t matters too. For example, lower consumption equivalents in the long run (despite the negative equilibrium effects of the short run) can be attributed to the fact that informal workers receive no weight in the long run measure (there are none left!). Short-run averages, on the other hand, place significant weight on informal workers. In fact, this also accounts for the fact that low-skill households gain more in the short run, while high-skill ones gain more in the long run.

The main message of figure 10 is that, despite the pervasive labor market effects that follow the repression of informality in the short run, most households experience a welfare improvement in either term horizon.

6 Policy Sensitivity

Our benchmark simulation points out to large economic gains form the government's fight against informal labor. We now consider two alternative policy specifications in an attempt to challenge this conclusion. In both cases, our baseline calibration continues to be as in section 4.

6.1 No Tax Adjusment

Perhaps the strongest assumption from our first policy exercise is that the government will fully rebate to households the proceeds from the end of informality. That resulted in a negative income tax rate $\tau_{hh,t}$. In this subsection, we fix the income tax rate, and instead adjust the government's budget constraint by changing public spending $C_{gov,t}$.[40]

Figures 12 and 13 present the paths of the main endogenous variables.[41] The solid curves reproduce the benchmark simulation for comparison, and the dashed ones correspond to the case of no tax adjustment.

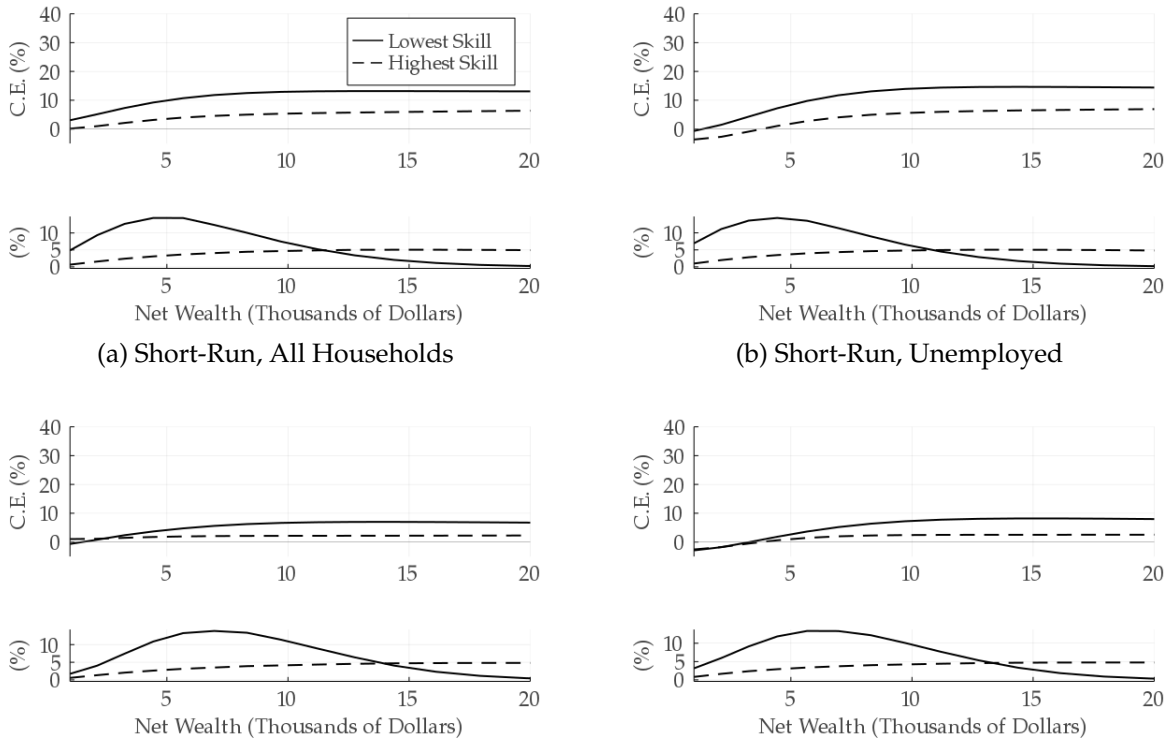
Contrary to the benchmark case, in the absence of lower income taxes, interest and unemployment rates increase by 0.8% and 1.2% *in the long term*, compared to the baseline equilibrium. Like before, changes in the average firm revenue and aggregate efficiency hours show that the repression of informality drives unproductive firms out of the market, but the resulting effect on labor income is not so pronounced (16% growth compared to 49% benchmark).

The lower income growth is due not only to the constant (not lower) income tax rate, but also to the equilibrium effect of lower job-finding rates and higher unemployment. Workers spend less time employed and, when employed, bargain lower wages. With less income, households increase

[39] We take the change in welfare among households in unemployment states to be of particular relevance, since new generations of workers enter the labor market unemployed.

[40] As motivation, in reality widespread surveillance of firms might be far from free. In addition, the bureaucracy associated with registering and covering more workers with the social security system might require application of more public funds.

[41] You can find a table analogous to 4 in the appendix.



(a) Short-Run, All Households

(b) Short-Run, Unemployed

(c) Long-Run, All Households

(d) Long-Run, Unemployed

Notes: Each figure contains two panel; the upper panel plots consumption equivalents aggregated across worker states s, the lower panel plots the distribution of household wealth. Solid and dashes lines refer to lowest and highest skill types, $\underline{h}$ and $\bar{h}$, respectively.

Figure 11: Consumption Equivalents and Wealth Distributions - No Adjustment of Tax Income

their savings by a lower amount (17% *vs* 29%). Not as funded, the financial intermediary supplies less capital to firms that, again, demand more of it. Market clearing thus requires a higher level of interest rate in the long run. By the same mechanism as before, comparatively higher interest diminishes the value of workers to firms. Less of them pay for the vacancy, and the job-finding rate declines.

Transition effects differ from long-term ones in the same way as in the benchmark. Aggregate savings take longer to pick up than capital demand, and the interest rate and the unemployment rate overshoot. However, in the case of the interest rate the size of the overshooting is not as pronounced, as households - facing not such a large gain in income - do not increase their consumption (and lower savings) as much as before.

We conclude the lack of tax adjustments makes the fight against informality be a more disruptive to the labor market.

Is that enought to revert our welfare results? In a important way, yes. Figure 11 is the counterpart of figure 10. At the time of policy implementation, most households still favor it. However, now we can identify groups that don't. Unemployed, low-wealth households are net losers. They experience an immediate decline in job-finding rate that can't be countered by a cushion of savings, and is not made up for by higher expected future income.

In the long run, gains are smaller compared to short run as we have less informal and less employed workers. The average of consumption equivalents becomes negative for zero-wealth households even when we take it across all low-skill households (not just the unemployed). As for the unemployed, we find declining welfare for workers of all skill levels, and for wealth levels above the zero limit.

In all, with no tax adjustment, most households continue to favor the repression of informality. But now we can say that a larger group of households - mostly poor, unemployed ones - would, in the long run, prefer to go back to the original equilibrium with informal activity. This exercise highlights the importance of fiscal adjustment that comes with the repression of informal labor.

6.2 Anticipated Policy

A large policy enterprise such as the prohibition of a labor category would likely be preceded by public debates or other signals that it is coming. In that sense, we should question the assumption of the benchmark simulation that the repression is not anticipated.

Our next simulation changes this. At $t = 0$, the economy is in the initial steady state. At $t = 1$, the government announces that it will start to repress informal contracts starting at $t = 20$ (five years following the announcement).[42] For fiscal adjustment, we return to the original assumption of changes to the income tax rate.

The long-run effects are the same as before. The dotted curves of figures 12 and 13 describe the effects in the transition.[43]

The dynamics following the start of the repression - year five - resemble that of the benchmark simulation. But the early years, after announcement and prior to implementation, look different from before. During these initial years, aggregate efficiency hours and average income *decline*. Households still anticipate future income growth and increase consumption immediately. The interest rate therefore starts to rise upon announcement (at a lower rate, since capital demand is going down). It then has the same effect as in all our previous exercises: lower job-finding rates and higher unemployment.

However, unlike our baseline simulation, there is no repression in the early years. In fact, the informality share *increases* before repression starts. Hence, low duration matches still exist. The combination of lower job-finding rates and the yet high average separation rates result in an increasing unemployment rate.

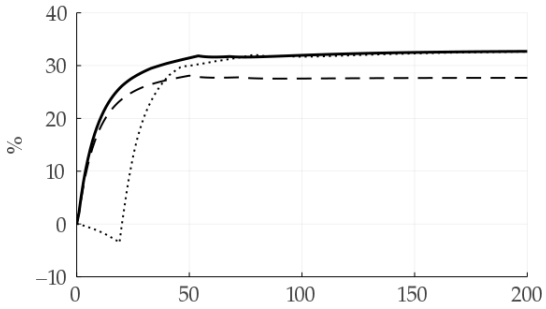
In addition, the absence of repression in the initial periods mutes the endogenous selection of productive firms. This explains why aggregate efficiency hours and output decline. Finally, the government increases income tax rate in the short run to over 15%, which contributes to the increase in informality shares and the decline in household income.[44]

How about welfare? Figure 14 plots the consumption equivalent diagrams for the anticipated repression. We again consider the short run, specifically the time of announcement ($t = 1$).

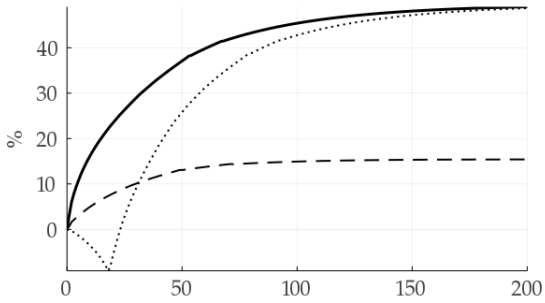
[42] Formally, we set $\phi_t = 1$ for $t \geq 20$, and calculate the general equilibrium paths.

[43] You can check the counterpart of table 4 for the anticipated policy in the appendix.

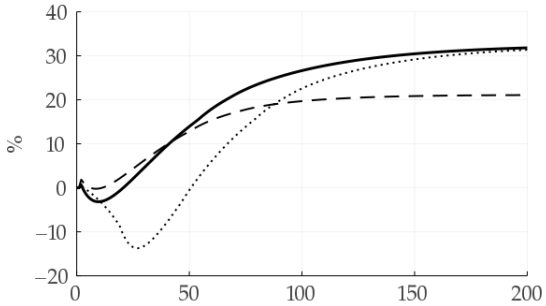
[44] The additional tax revenues compensate for the losses in sales/payroll taxes (the formal sector shrinks) and the additional expenditure with unconditional transfers (unemployment increases).



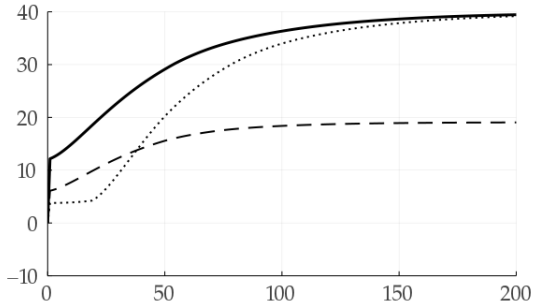
(a) %Δ Efficiency Hours of Labor



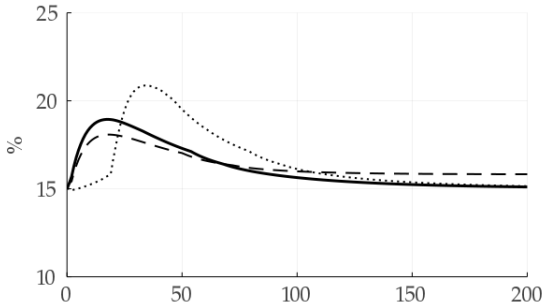
(b) %Δ Avg. After-Tax Income



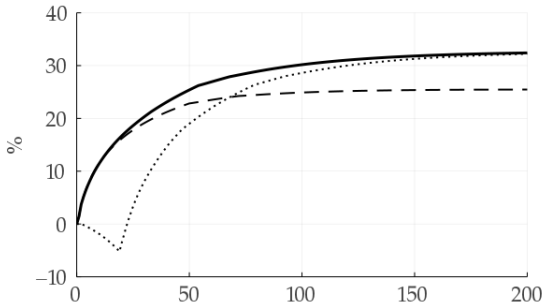
(c) %Δ Stock of Capital



(d) %Δ Aggregate Consumption



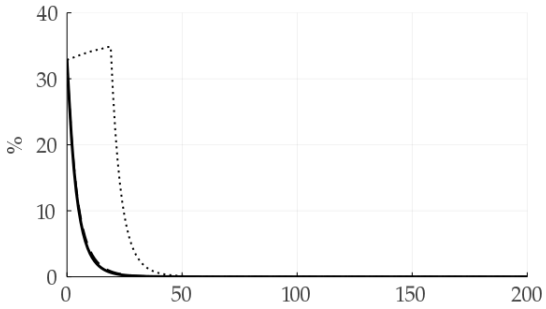
(e) Interest Rates



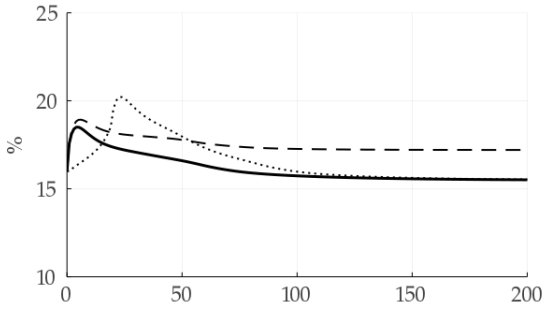
(f) %Δ Aggregate Output

Notes: The figures show the equilibrium transition paths of selected aggregates. The symbol %Δ indicates percentage deviation from the baseline steady state. The dashed lines correspond to a government that adjusts its budget constraint using public spending, not taxes. The dotted lines correspond to the case the repression of informality is anticipated by private agents.

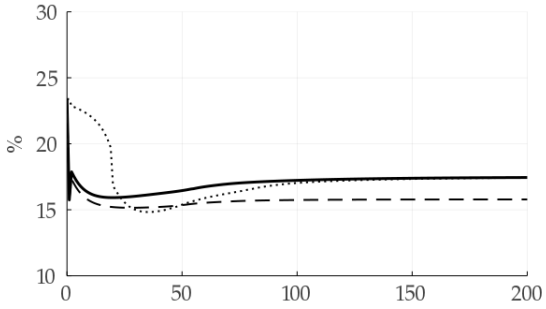
Figure 12: Transition Dynamics (Dashed = No Tax Adjustment, Dot = Anticipated Policy)



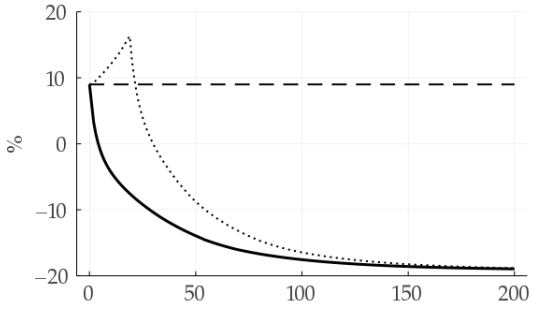
(a) Informality Share



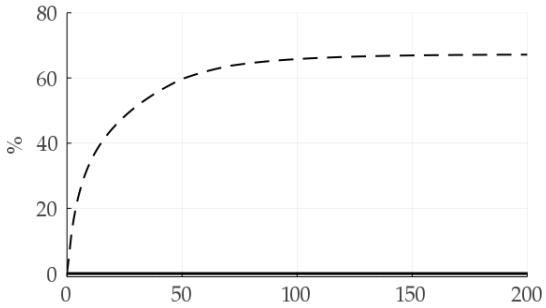
(b) Unemployment Rate



(c) Job-Finding Rate



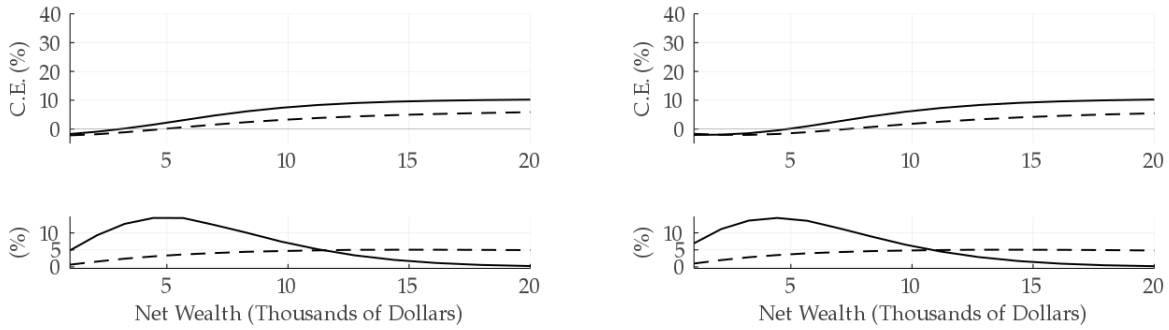
(d) Income Tax Rate



(e) %Δ Government Consumption

Notes: The figures show the equilibrium transition paths of selected aggregates. The symbol %Δ indicates percentage deviation from the baseline steady state. The dashed lines correspond to a government that adjusts its budget constraint using public spending, not taxes. The dotted lines correspond to the case the repression of informality is anticipated by private agents.

Figure 13: Transition Dynamics (Dashed = No Tax Adjustment, Dot = Anticipated Policy)



(a) Short-Run, All Households (b) Short-Run, Unemployed

Notes: Each figure contains two panel; the upper panel plots consumption equivalents aggregated across worker states s, the lower panel plots the distribution of household wealth. Solid and dashes lines refer to lowest and highest skill types, $\underline{h}$ and $\bar{h}$, respectively.

Figure 14: Consumption Equivalents and Wealth Distributions - Anticipated Policy

Once more, most households favor the repression. But now we see a negative average consumption equivalent for low-wealth households of *all* types. To put it simply, with if the repression is anticipated, the policy is welfare-reducing for low wealth households and welfare-enhancing for high wealth households. In all, 19.7% of households oppose the repression of informality.

Another result we hadn't find before is that the large group of welfare-losers *in the short run*. In the previous exercise (no tax adjustment), we only observed average negative consumption equivalents in the long-term steady state.

The key insight of the exercise is that, if the fight against informal contracts is anticipated, the equilibrium "costs" (higher unemployment, lower job-finding rates) arrive prior to policy implementation. The main "benefit" (the change in the average productivity of firms) only kicks off upon actual policy implementation.

7 Concluding Remarks

Informality is a remarkable trait of labor markets, especially in developing countries. In this paper, we present a model - calibrated to reproduce basic informality facts - to evaluate if public policy should fight against it. We integrate to the analysis important elements that previous research had ignored, such as risk-aversion, general equilibrium and transition dynamics. We show that these elements can profoundly change the predictions of models that focus on the labor market only, and thus should not be ignored. Our results provide researchers that do not focus on the macroeconomics the necessary *ad-hoc* robustness tests to be verified - the most important one being discount rate variation.

Like part of the existing literature, we find that the existence of a large number of "parasite" firms, which do not need informality to be solvent, makes the repression of informal markets desirable for many households. 30% informal workers do not translate into 30% more unemployed following the repression. On the other hand, realistic variations of the benchmark simulation suggest that some groups of individuals, in particular the poor and unemployed, can experience welfare losses from the resulting changes to the income process, which becomes riskier.

There are many other avenues for future informality research. Considering the intersection with macroeconomics that we have studied, two variations in the framework call our attention. First, the fact that formal jobs have higher average duration is likely to be endogenous to potentially distortionary regulation. Relaxing the assumption of exogenous separation rates should thus be an interesting extension. Second, formal and informal workers tend to work in industries that produce different goods. Reduced substitutability among these goods could also be key to better capture the extent to which the formal sector can successfully absorb informal workers.

Acknowledgements

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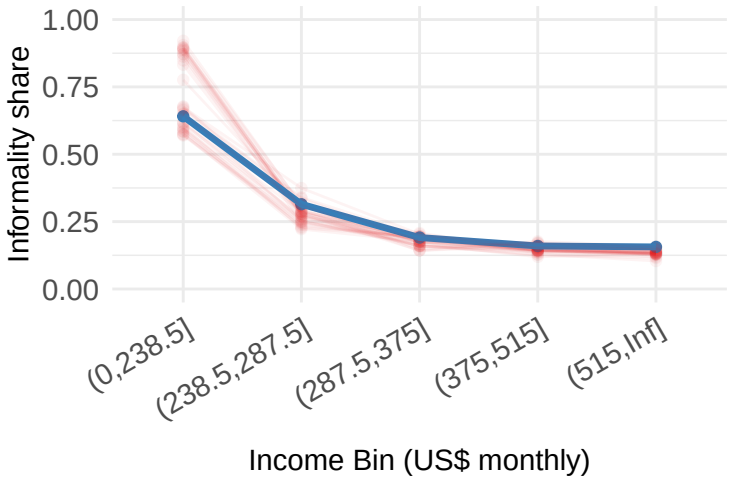

Notes: Points in the x-axis define income quintiles. Each line represents the share of informal workers for different income quintiles. Each quarter in the sample (2012-Q1 - 2020-Q1) is associated to a separate line. The thicker line represents 2018-Q3. Income quintiles are recomputed for each quarter, but the labels correspond to the 2018-Q3 quintile.

Figure 15: Informality Share by Income Bins

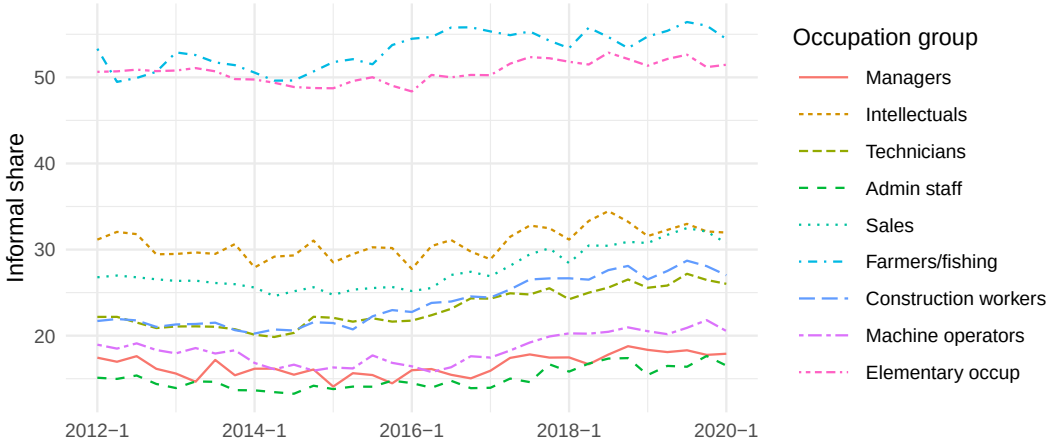

Figure 16: Informality over time by occupational group

Appendices

A Additional Plots and Tables

In this section of the appendix we show additional plots and tables of the data and model.

B Discussion of the Wage Determination Mechanism

In section 3.3 we present the wage determination mechanism of our model.

This mechanism deserves a few observations. First, the unemployed household is always weakly (in practice, strictly) better off accepting a job than staying unemployed. Searching workers without a job always transition to employment in

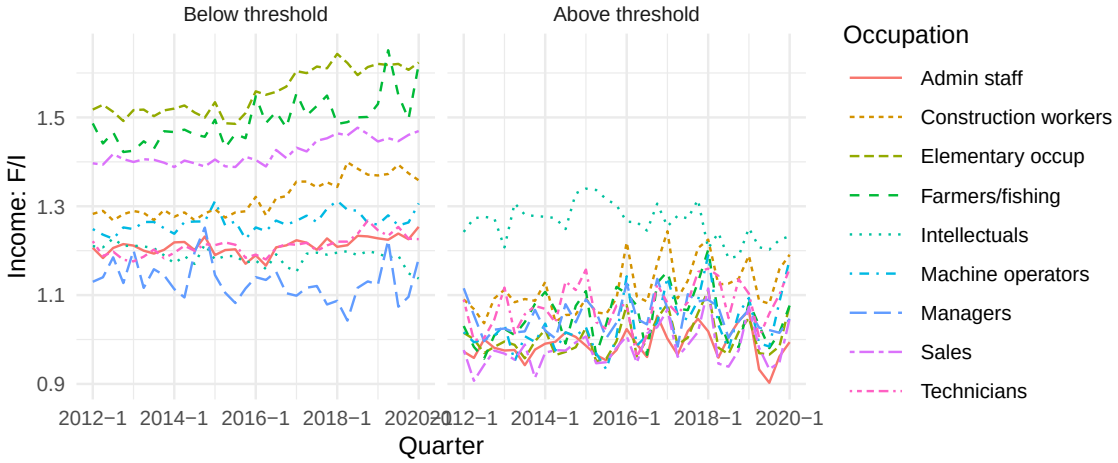

Figure 17: Earnings differential by occupation over time

Variables	Baseline $t = 0$	One year $t = 4$	Ten years $t = 40$	Forty years $t = 80$	Long run $t = \infty$
Interest rate (%)	15	15	15	15	15
Income tax rate (%)	9	-5.7	-18.8	-19.2	-19.2
Informality share (%)	32.9	12.2	0	0	0
Unemployment rate (%)	16	17.4	15.4	15.5	15.5
Market tightness (%)	100.3	74	63.4	62.9	62.8
Job-finding rate (%)	23.4	19.2	17.6	17.5	17.5
Transition unemployment (%)	4.5	3.9	3.2	3.2	3.2
Firm revenue - avg (%Δ)	0	9.5	23.5	24.5	24.6
Firm revenue - entry (%Δ)	0	57.5	57.7	57.7	57.7
Labor income - avg %Δ)	0	21.8	47.5	49	49.2
Labor income - entry (%Δ)	0	61	76.4	76.8	76.9
Aggregates					
Output (%Δ)	0	11.4	31.7	32.7	32.9
Capital (%Δ)	0	11.4	31.7	32.7	32.9
Efficiency Hours (%Δ)	0	11.4	31.7	32.7	32.9
Savings (%Δ)	0	-2.5	2.4	10.9	28.7
Sales tax revenue (%Δ)	0	24	56.4	57.6	57.8
Income tax revenue ($)	24.1	-18.8	-75.1	-77.3	-77.5

Notes: The table shows the value of some endogenous variables and statistics in the transition path, in the open economy case. In $t = 0$, the economy is in the baseline steady state. In $t = 1$, the government starts the repression of informality: the value of ϕ_t changes to 1, where it stays for good. Throughout the transition, the government adjusts its consumption level $C_{gov,t}$ so that its budget constraint (17) holds with the income tax rate $\tau_{hh,t}$ of the original calibration. The symbol "%Δ" means that the variable is represented as percentage deviation from its baseline steady state value.

Table 5: Simulation of Informality Repression: Open Economy

Variables	Baseline $t=0$	One year $t=4$	Ten years $t=40$	Forty years $t=80$	Long run $t=\infty$
Interest rate (%)	15	16.3	17.4	16.2	15.8
Income tax rate (%)	9	9	9	9	9
Informality share (%)	32.9	13.7	0	0	0
Unemployment rate (%)	16	18.8	17.9	17.3	17.2
Market tightness (%)	100.3	50.6	42.6	46.2	47.1
Job-finding rate (%)	23.4	16.7	15.2	15.7	15.8
Transition unemployment (%)	4.5	4.1	3.3	3.3	3.3
Firm revenue - avg (%Δ)	0	6.7	17.1	19.4	20.2
Firm revenue - entry (%Δ)	0	58.3	55.1	54.1	55.2
Labor income - avg (%Δ)	0	5.2	13.1	15.3	16
Labor income - entry (%Δ)	0	40.8	38.7	38.3	39
Aggregates					
Output (%Δ)	0	6.4	21.2	24.5	25.4
Capital (%Δ)	0	0.7	9.8	18.3	21
Efficiency Hours (%Δ)	0	9.1	27.2	27.2	27.6
Savings (%Δ)	0	-0.7	8.9	15.2	17
Sales tax revenue (%Δ)	0	17.6	43.9	47.8	49
Income tax revenue ($)	24.1	28.7	35	35.9	36.2

Notes: The table shows the value of some endogenous variables and statistics in the transition path. In $t=0$, the economy is in the baseline steady state. In $t=1$, the government starts the repression of informality: the value of ϕ_t changes to 1, where it stays for good. Throughout the transition, the government adjusts its consumption level $C_{gov,t}$ so that its budget constraint (17) holds with the income tax rate $\tau_{hh,t}$ of the original calibration. The symbol "%Δ" means that the variable is represented as percentage deviation from its baseline steady state value.

Table 6: Simulation of Informality Repression: No Adjustment of Tax Income

case they find a match. Thus $p\theta$ represents not only the job-finding rate but also the transition rate from unemployment to employment.[45] Second, using (10) and (12) to set wage rates for all instances of bargaining meets the requirement needed for us to state value functions simply as (6), (7) and (8). The assumption we make is that the outside option used in the bargaining process is always that of unemployment, *even if the worker is employed*. We believe such simplification to be of little consequence to our results for two reasons. First, in equilibrium it is probably the case that workers transition from less productive to more productive matches. Allowing for the "correct" outside options would just lead to higher wage rates in general, with little allocative effect that we can't control by properly setting bargaining power η. Second, and perhaps more importantly, given that wage rates are negotiated on a period-by-period basis, the differentiated wages resulting from better outside options would prevail for a single period only.

Another observation of the Nash bargaining procedure is that we adjust the bargaining power in the case of a formal contract to reflect the incidence of taxation (meaning we divide η by $1-\tau_{hh}$ and $1-\eta$ by $1+\tau_w$), a necessary adjustment if we are to observe condition (11) holding. The two equations in (11) imply that whenever the effective wage coincides with the interior solution to the Nash problem $\hat{w}$, firms and workers agree on the preferred type of contract. In equilibrium, we *only* fail to observe (11) when the minimum wage impedes the theoretical bargained wage to prevail. In that case, firm and worker might disagree on their preferred contract only if the firm offers an informal one[46].

One final observation that we already commented on, and that relates to the previous two observations, is that we do not allow formal workers to use potential unemployment benefit payments to bargain higher wages. There is no v summing the outside option W^n in the first equation in (10). We choose not to include it for several reasons. First,

[45] Another assumption leading to this is that firms observe their type at the beginning of the period, *before* finding a worker in the labor market.

[46] To see this claim, consider a firm drawing a state (e,z) that chooses to post a formal contract offer. We know then that the firm anticipates (correctly, in equilibrium) $V^f(z,h) > V^i(ez,h)$. If the minimum wage constraint binds, the first equality in (11) fails to hold; instead, we have $(W^f - W^n)/V^f > \eta/(1-\eta) = (W^i - W^n)/V^i > (W^i - W^n)/V^f$. It follows that $W^f - W^n > W^i - W^n$. Firm and worker agree.

Variables	Baseline $t=0$	One year $t=4$	Ten years $t=40$	Forty years $t=80$	Long run $t=\infty$
Interest rate (%)	15	15	20.6	17.1	15
Income tax rate (%)	9	9.9	-5.1	-14.7	-19.1
Informality share (%)	32.9	33.4	0.6	0	0
Unemployment rate (%)	16	16.3	18.6	16.5	15.5
Market tightness (%)	100.3	93.8	41.6	54.1	62.7
Job-finding rate (%)	23.4	22.7	14.9	16.6	17.5
Transition unemployment (%)	4.5	4.5	3.3	3.2	3.2
Firm revenue - avg (%Δ)	0	0.1	12	20.2	24.5
Firm revenue - entry (%Δ)	0	0.4	55.9	52.1	57.5
Labor income - avg (%Δ)	0	-0.7	21.5	39.5	49.0
Labor income - entry (%Δ)	0	0	56.2	67.1	76.7
Aggregates					
Output (%Δ)	0	-0.4	14.6	26.5	32.7
Capital (%Δ)	0	-0.3	-7.9	16	32.5
Efficiency Hours (%Δ)	0	-0.8	27.6	31.9	32.7
Savings (%Δ)	0	-1.3	-2.4	16.9	29.3
Sales tax revenue (%Δ)	0	-1	35.5	50.2	57.6
Income tax revenue ($)	24.1	26.2	-18.2	-56.8	-77

Notes: The table shows the value of some endogenous variables and statistics in the transition path. In $t=0$, the economy is in the baseline steady state. In $t=1$, the government announces that, starting at period $t=20$, it will repression informality: the value of ϕ_t changes to 1, where it stays for good. Throughout the transition, the government adjusts the income tax rate $\tau_{hh,t}$ so that its budget constraint (17) holds with the baseline equilibrium value of government consumption C_{gov}. The symbol "%Δ" means that the variable is represented as percentage deviation from its baseline steady state value.

Table 7: Simulation of Informality Repression: Anticipated Policy

as we just said, unemployment benefits are only *potential*. Workers coming out of unemployment that fail to agree on a wage rate with the firm have no access to the benefit, as they wouldn't first transition to employment. Even if we consider continuing formal employees, whether they would receive the benefit depends on the assumption that the government pays insurance benefits to workers that become unemployed for not being able to agree with the firm on a wage rate - an assumption we need not make as this is an off-equilibrium outcome. A second reason for leaving out the unemployment benefit threat is our empirical observation of section 2 of the low income differential between formal and informal workers (fact 2), a pattern that suggests a low or inexistent additional bargaining power owned by formal workers. Another reason is that including v as an ouside value would break condition (11), and so new workers (as well continuing workers, depending on the off-equilibrium assumption) could disagree with the formality choice made by the firm, a property of the model we do not wish to break. Finally, including the benefit outside option to continuing workers but not to new ones would again involve defining additional state variables to keep track of previous employment positions, a complication we prefer to avoid.

C Transition matrix of household state

In this section of the appendix, we show how to calculate the transition matrix $g(s'|s)$ for the shopper's exogenous state $s=(d,e,z;h)$, given a steady-state equilibrium in the labor market. So, assume we have in hands a pair of firm value functions V_t^f and V_t^i, workers' value function W_t^f, W_t^i, W_t^n as well as the market tightness $\theta_t(h)$ and the associated vacancy-filling rate $p_t(h)$.

We assume a discrete support for the distributions of hiding ability e and productivity z. Because the worker's type is constant, we start by setting $g_t((.,h')|(.,h))=0$ whenever $h'\neq h$. To keep notation light, we omit type h hereafter. We also establish an easier notation for the remaining states. Let $i(e,z)$ denote the state in which the worker is informally employed by a firm with hiding ability/productivity pair (e,z); let $f(z)$ denote the state in which the worker is formally

employed by a firm with productivity z; let u denote unemployment without benefits and ub unemployment receiving the insurance payment.

We start by noticing that the unemployment insurance benefit lasts a single period: receivers can't transition to another period of unemployment insurance payment: $g_t(ub|ub) = g_t(ub|u) = 0$. Any unemployed worker transition to the unemployment without benefit state if he or she fails to find a match when the labor market opens (recall that the job-finding rate coincides with the rate at which workers transition from unemployment to employment):

$$g_t(u|ub) = g_t(u|u) = 1 - p\theta_t.$$

Transition to employment depends on the choice of new firms on whether to offer formal or informal contracts on each state (e, z).

$$g_t(f(z)|u) = g_t(f(z)|ub) = p\theta_t \mathbf{1}_{(e,z)\in Z_t^f} \frac{G_t(e,z)}{G_t(Z)}$$
$$g_t(i(e,z)|u) = g_t(i(e,z)|ub) = p\theta_t \mathbf{1}_{(e,z)\in Z_t^i} \frac{G_t(e,z)}{G_t(Z_t)}$$

This completes the statement of transitions from unemployment states.

Transitions from employment are less straighforward as we need to take into account the worker's chance of transitioning to a new job from on-the-job search. We start by noting that $g_t(.|u)$ gives not only the probability distribution of states to workers coming from unemployment but also of workers that lose their job at the end of a period and enter the market in the next period. We can thus recycle the use of $g_t(.|u)$. For instance, formal workers transition to the unemployment with benefit state ub if they lose their job (probability λ_f) and fail to find a new one (probability $g_t(u|u)$):

$$g_t(ub|f(z)) = \lambda_f g_t(u|u)$$
$$g_t(u|i(e,z)) = \lambda_i g_t(u|u)$$

(a similar reasoning stands for informal workers transitioning to unemployment without benefit state u). Also, $g_t(u|f(z)) = g_t(ub|i(e,z)) = 0$.

Consider now the probability that the employed worker remains on his or her current state, say $f(z)$ (the case $i(e,z)$ is analogous). As a convention, we assume that an employed worker finding a competing job offer decides to switch only if he or she *strictly* prefers the new offer. Therefore, in matching with a firm at the exact same state as his or her current employer, the workers opts to stay in the current position (this choice is obviously inconsequential for the model). To transition from $f(z)$ to the same state $f(z)$, the worker can then either lose the current job but find a similar one in the following period or simply stay at the current position (if no better match is found):

$$\begin{aligned} g_t(f(z)|f(z)) = {} & \lambda_f g_t(f(z)|u) + (1-\lambda_f)(1-p\theta_t) \\ & + (1-\lambda_f)\frac{p\theta_t}{G_t(Z_t)} G_t(\{(e',z') \in Z_t^f \mid W_t^f(z) \geq W_t^f(z')\}) \\ & + (1-\lambda_f)\frac{p\theta_t}{G_t(Z_t)} G_t(\{(e',z') \in Z_t^i \mid W_t^f(z) \geq W_t^i(e',z')\}). \end{aligned}$$

Our worker at $f(z)$ can also transition to a new formal job $f(z')$. That can happen either with an exogenous separation in between or through on-the-job search. The transition from employment requires $W^f(z') > W^f(z)$:

$$g_t(f(z' \neq z)|f(z)) = \lambda_f g_t(f(z')|u) + (1-\lambda_f)p\theta_t \sum_{e'} \frac{G_t(e',z')}{G_t(Z_t)} \mathbf{1}_{(e',z')\in Z_t^f} \mathbf{1}_{W_t^f(z') > W_t^f(z)}$$

The last case we need to consider is the transition from a formal job $f(z)$ to an informal job $i(e,z)$. The reasoning is the same as the previous case.

$$g_t(i(e',z')|f(z)) = \lambda_f g_t(i(e',z')|u) + (1-\lambda_f)p\theta_t \frac{G_t(e',z')}{G_t(Z_t)} \mathbf{1}_{(e',z')\in Z_t^i} \mathbf{1}_{W_t^i(e',z') > W_t^f(z)}.$$

We are done. The transition matrix for a worker employed with an informal contract is analogous to that of a worker with a formal contract. Notice that, for all s, $\sum_{s'} g_t(s'|s) = 1$.

D The Intermediary's Problem

In this section, we describe the financial intermediary's problem in detail.

Let DIV_t be aggregate dividends and Q_t the value of existing firms. The following proposition captures the behavior of the representative investor and its consequences for equilibrium prices and quantities.

Proposition 1. *If the representative investor stores a positive and finite amount of capital K_{t+1}, then*

$$q_t = \frac{1}{1 + r_{t+1}}$$

holds, and the representative investor is indifferent between equity and capital investment. Period $t+1$ consumption is then given by

$$(1 + r_{t+1})K_{t+1} + Q_{t+1} - \frac{1}{q_t}\left[K_{t+1} + Q_t - DIV_t\right]$$

and the investor's balance sheet equality is

$$K_{t+1} + q_t Q_{t+1} = \textit{Value of bonds sold to households}.$$

To keep matters simple, we assume the equity of existing firms is packaged by a fund of firms responsible for posting all vacancies in the economy. The investor purchases shares of the fund of firms. The supply of such shares is one in all periods. Shareholders receive as dividends the sum of matched firms' profits discounted by the cost of posting new vacancies VC_t.

In any given period t, the investor sells B_t bonds to households at a price q_t and uses the proceeds to purchase X_{t+1} shares of the fund of firms, each at a price $Q_t - DIV_t$ (that is the value of future stream of payments), or K_{t+1} units of capital. Note that X_{t+1} and K_{t+1} are both predetermined. Therefore, the investor's balance sheet constraint is

$$K_{t+1} + X_{t+1}\left(Q_t - DIV_t\right) = q_t B_t. \tag{23}$$

In the following period $t+1$, the investor supplies capital to firms, and receives rental payment $(r_t + \delta)K_{t+1}$ plus the now depreciated capital $(1-\delta)K_{t+1}$, which amounts to $(1 + r_{t+1})K_{t+1}$. Investment in shares of the fund of firms yields dividends $X_{t+1}DIV_{t+1}$ plus the $t+1$ value of the shares, $X_{t+1}\left(Q_{t+1} - DIV_{t+1}\right)$. They sum up to $X_{t+1}Q_{t+1}$. Finally, the investor must repay period t debt B_t. The difference between investment payoffs and debt repayment is consumed away:

$$C_{inv,t+1} = (1 + r_{t+1})K_{t+1} + X_{t+1}Q_{t+1} - B_t. \tag{24}$$

The representative investor's problem then is to maximize (24) by choosing a portfolio (K_{t+1}, X_{t+1}, B_t) that satisfies the balance sheet constraint (23).

The following proposition is critical in solving the investor's problem.

Proposition 2. *Aggregate dividends DIV_t satisfy the following equation:*

$$Q_t = DIV_t + q_t Q_{t+1} \tag{25}$$

Proof. We prove that aggregate dividends in a single market h satisfy

$$\begin{aligned} DIV_t(h) = &\int V_t^f(z,h)dE_t^f(z,h) + \int V_t^i(ez,h)dE_t^i(ez,h) \\ &- q_t\left[\int V_{t+1}^f(z,h)dE_{t+1}^f(z,h) + \int V_{t+1}^i(ez;h)dE_{t+1}^i(ez;h)\right]. \end{aligned}$$

. The proposition then follows. Let $\bar{E}_t^f = E_t^f(\mathcal{Z};h)$ be the share of h-type workers in the formal sector, and define $\bar{E}^i$ similarly. To simplify notation, we fix h and omit it from the proof hereafter. The construction of the transition matrix in

section C of this appendix implies that the following relationship holds:

$$\begin{aligned}
dE_t^f(z) &= dE_{t-1}^f(z) - \left[\lambda_f + (1-\lambda_f)p\theta_t \frac{G(\Phi_t^f(z))}{G_t(Z_t)}\right] dE_{t-1}^f(z) \\
&\quad + p\theta_t \mathbf{1}_{(e,z)\in Z_t^f} \frac{dG_t(e,z)}{G_t(Z_t)} \left[U_{t-1} + \lambda_f \bar{E}_{t-1}^f + \lambda_i \bar{E}_{t-1}^i + (1-\lambda_f)E^f(\Psi_{f,t-1}^f(z)) + (1-\lambda_i)E^i(\Psi_{i,t-1}^f(z))\right] \\
dE_t^i(ez) &= dE_{t-1}^i(ez) - \left[\lambda_i + (1-\lambda_i)p\theta_t \frac{G(\Phi_t^i(z))}{G_t(Z_t)}\right] dE_{t-1}^i(ez) \\
&\quad + p\theta_t \mathbf{1}_{(e,z)\in Z_t^i} \frac{dG_t(e,z)}{G_t(Z_t)} \left[U_{t-1} + \lambda_f \bar{E}_{t-1}^f + \lambda_i \bar{E}_{t-1}^i + (1-\lambda_f)E^f(\Psi_{i,t-1}^f(z)) + (1-\lambda_i)E^i(\Psi_{i,t-1}^i(z))\right]
\end{aligned} \tag{26}$$

The two equations simply contain the law of motion for the measure of workers in each employment state.

Total dividends are the difference between total period profits acquired by matched firms DIV^F and total cost with new vacancies DIV^V. We first calculate the former. Let us call $\pi^f(z)$ the period profit of a formal firm at state z, and the same be for $\pi^i(ez)$. Firm value function (6) implies that

$$\pi_t^f(z) = V_t^f(z) - q_t \left[1 - \left(\lambda_f + (1-\lambda_f)p\theta_{t+1} \frac{G_{t+1}(\Phi_{t+1}^f(z))}{G_{t+1}(Z_{t+1})}\right)\right] V_{t+1}^f(z)$$

and the analogous expression for $\pi^i(ez)$. Then, total period profits is given by

$$\begin{aligned}
DIV_t^F &= \int \pi_t^f(z) dE_t^f + \int \pi_t^i(ez) dE_t^i \\
&= \int \left\{ V_t^f(z) - q_t \left[1 - \left(\lambda_f + (1-\lambda_f)p\theta_{t+1} \frac{G_{t+1}(\Phi_{t+1}^f(z))}{G_{t+1}(Z_{t+1})}\right)\right] V_{t+1}^f(z) \right\} dE_t^f \\
&\quad + \int \left\{ V_t^i(ez) - q_t \left[1 - \left(\lambda_i + (1-\lambda_i)p\theta_{t+1} \frac{G_{t+1}(\Phi_{t+1}^i(ez))}{G_{t+1}(Z_{t+1})}\right)\right] V_{t+1}^i(ez) \right\} dE_t^i
\end{aligned}$$

To calculate aggregate spending with vacancy costs, we first re-state the free-entry condition (13):

$$\kappa h = q_t \left\{ \int_{Z_{t+1}^f} \rho_{t+1}^f(z) V_{t+1}^f(z) dG_{t+1} + \int_{Z_{t+1}^i} \rho_{t+1}^i(ez) V_{t+1}^i(ez) dG_{t+1} \right\}.$$

The aggregate spending with vacancy costs then satisfies the following equations:

$$\begin{aligned}
DIV_t^V &= \kappa h F_t \\
&= \kappa h \frac{\theta_{t+1}}{G_{t+1}(Z_{t+1})} \\
&= q_t \left\{ \int_{Z_{t+1}^f} \rho_{t+1}^f(z) \frac{\theta_{t+1}}{G_{t+1}(Z_{t+1})} V_{t+1}^f(z) dG_{t+1} + \int_{Z_{t+1}^i} \rho_{t+1}^i(ez) \frac{\theta_{t+1}}{G_{t+1}(Z_{t+1})} V_{t+1}^i(ez) dG_{t+1} \right\} \\
&= q_t \int_{Z_{t+1}^f} p\theta_{t+1} \left[U_t + \lambda_f \bar{E}_t^f + \lambda_i \bar{E}_t^i + (1-\lambda_f)E_t^f(\Psi_{f,t+1}^f(z)) + (1-\lambda_i)E_t^i(\Psi_{i,t+1}^f(z))\right] V_{t+1}^f(z) \frac{dG_{t+1}}{G_{t+1}(Z_{t+1})} \\
&\quad + q_t \int_{Z_{t+1}^i} p\theta_{t+1} \left[U_t + \lambda_f \bar{E}_t^f + \lambda_i \bar{E}_t^i + (1-\lambda_f)E_t^f(\Psi_{f,t+1}^i(ez)) + (1-\lambda_i)E_t^i(\Psi_{i,t+1}^i(ez))\right] V_{t+1}^i(ez) \frac{dG_{t+1}}{G_{t+1}(Z_{t+1})} \\
&= q_t \int V_{t+1}^f dE_{t+1}^f - q_t \int \left[1 - \left(\lambda_f + (1-\lambda_f)p\theta_{t+1} \frac{G(\Phi_{t+1}^f(z))}{G_{t+1}(Z_{t+1})}\right)\right] V_t^f(z) dE_t^f \\
&\quad + q_t \int V_{t+1}^i dE_{t+1}^i - q_t \int \left[1 - \left(\lambda_i + (1-\lambda_i)p\theta_{t+1} \frac{G(\Phi_{t+1}^i(z))}{G_{t+1}(Z_{t+1})}\right)\right] V_t^i(z) dE_t^i
\end{aligned}$$

The first equality is simply the definition of aggregate spending in vacancy costs. The second equality uses (15). The third one replaces the free-entry condition. The fourth equality replaces the definition (16) of ρ^f and ρ^i. The last equality

replaces the integrated version of equation (26).

After cancelling out the repeated terms, the result follows:

$$\begin{aligned} DIV_t(h) &= DIV_t^F(h) - DIV_t^V(h) \\ &= \int V_t^f(z,h)dE_t^f(z,h) + \int V_t^i(ez,h)dE_t^i(ez,h) \\ &- q_t \left[\int V_{t+1}^f(z,h)dE_{t+1}^f(z,h) \int V^i(ez;h)dE^i(ez;h) \right]. \end{aligned}$$

□

Note that proposition 2 guarantees that, *ex-ante*, equity investment at period t guarantees a return of $1/q_t$. In the period of an MIT shock, the *ex-post* return might differ as Q_{t+1} jumps to different level.

By replacing (23) and (25) in the objective (24), we find a new expression

$$\begin{aligned} C_{inv,t+1} &= (1+r_{t+1})K_{t+1} + X_{t+1}Q_{t+1} - \frac{1}{q_t}\left[K_{t+1} + q_t X_{t+1}Q_{t+1}\right] \\ &= (1+r_{t+1})K_{t+1} - \frac{1}{q_t}K_{t+1} \end{aligned} \tag{27}$$

If $q_t < (1+r_{t+1})^{-1}$, the investors would not invest in capital (that is, transform final goods acquired from selling bonds to the households into capital). If $q_t > (1+r_{t+1})^{-1}$, the investor would supply an infinite amount of bonds in period t and supply an infinite amount of capital in $t+1$. If condition (18) holds, the investor becomes indifferent between equity and capital investment, as both yield the same return. This proves the first claim in proposition 1.

In being indifferent with respect to the holdings of fund of firms shares, the investor chooses $X_t = 1$, so that market clears. Replacing it in equation (24) together with constraint (23) yields the consumption expression found in the text of the proposition:

$$C_{int,t+1} = (1+r_{t+1})K_{t+1} + Q_{t+1} - \frac{1}{q_t}\left[K_{t+1} + Q_t - DIV_t\right]$$

Unlike (27), the expression above holds even in initial MIT shock periods.

Finally, replacing $X_t = 1$ and (25) in (23) yields the proposition's equation for the investor's balance sheet

$$K_{t+1} + q_t Q_{t+1} = q_t B_t.$$

Note that this expression only holds in non-MIT shock periods. If anticipated and realized Q_{t+1} differ, the expression above only holds if we use anticipated Q_{t+1}.

E Details of the numerical solution

In this section, we present the numerical procedure we use to solve the model in detail. On a high level, the algorithms are standard from the Macroeconomics literature.

We discretize the distribution of (e,z) with 50 productivity values, 49 positive ones calculated using Tauchen's method (which also discretizes G_z), and zero. To account for the G_e distribution, we adjust the probability of each productivity draw, in that each new firm effectively draws a pair (z,ez) in the discretized grid. More details below. For each skill level h, the space of exogenous states s therefore contains 102 states: fifty points of formal employment, fifty of informal employment, unemployment and unemployment with UI benefits. We discretize the asset grid with a fifty-point grid, and evaluate asset levels out of the grid using linear interpolation. We compute the solution to the shopper's problem (1) using the endogenous grid method (Carroll (2006)).

All numerical computations we perform using the *Julia* programming language. For data manipulation, we use *R*.

E.1 Grids

We discretize firm productivity with $n_z = 50$ grid points. We start by using Tauchen's method to discretize a normal distribution with mean zero and standard deviation σ_z, with $n_z - 1$ points. We use three standard deviations to each side of the distribution. We take the exponential of the $n_z - 1$ grid points and add zero to the grid, thus forming a new grid with a total of n_z points. Tauchen's method gives the probability of each of the $nz - 1$ original points. The discrete probability of $z = 0$ equals zero. Finally, we normalize the values of the grid so that

$$\sum_{i_z=1}^{n_z} z_i G_z(z_i) = 1,$$

where $\mathcal{Z} = \{z_1 = 0, z_2, \ldots, z_{n_z}\}$ is our productivity grid.

Next, we discretize G_e. The first step is to discretize a Beta distribution. We start with 200 equally-spaced points between zero and 0.9999. To each point we calculate the corresponding quintiles of a Beta distribution with parameters ν and one. These 200 quintiles yield 199 mid points (the points halfway between each two quintiles). To each of these points we assign a probability of $1/199$. The second step performs the mixture. We start by adding the zero to the 199 points, and end with a 200-point grid $\mathcal{E}$ for hiding abilities. The probability of zero is ϕ_t. The probability of any other point is $(1 - \phi_t)(1/199)$.

In the numerical solution, we do not work with the distribution G for the pair (e, z). Instead, we use an equivalent distribution $\hat{G}$ for the pair (z, ez). The discretized grid for both entries of the tuple is the productivity grid $\mathcal{Z}$. So, we start with $\hat{G}(e, ez) = 0$, and do the following iteration. For each pair of $z_i \in \mathcal{Z}$ and $e_j \in \mathcal{E}$, we update $\hat{G}$:

$$\begin{aligned} G(z_i, \underline{z}(e_j z_i)) &= G(z_i, \underline{z}(e_j z_i)) + (1 - \omega(e_j z_i)) G_z(z_i) G_e(e_j) \\ G(z_i, \bar{z}(e_j z_i)) &= G(z_i, \bar{z}(e_j z_i)) + \omega(e_j z_i) G_z(z_i) G_e(e_j) \end{aligned}$$

where we choose $\underline{z}(e_j z_i) \in \mathcal{Z}$ and $\bar{z}(e_j z_i) \in \mathcal{Z}$ such that they are consecutive to each other and $\underline{z}(e_j z_i) \leq e_j z_i \leq \bar{z}(e_j z_i)$. The weight $\omega(e_j z_i)$ guarantees that the we distribute the probability mass proportinally between the two bounds:

$$e_j z_i = (1 - \omega(e_j z_i)) \underline{z}(e_j z_i) + \omega(e_j z_i) \bar{z}(e_j z_i).$$

For the asset grid, we start with a choice of grid size $n_a = 50$, an upper bound $\bar{a} = 1000$ and a parameter of non-linearity $nl \geq 1$. The first point of the grid is $a_1 = 0$ (following the no-borrowing constraint). We place the following points of the grid using the formula

$$a_i = a_{i-1} + \frac{\bar{a} - a_{i-1}}{(n_a - i + 1)^{nl}}.$$

This guarantees $a_{n_a} = \bar{a}$. Higher values of the non-linearity parameter nl concentrate more points on the lower end of the asset grid. We set $nl = 2$.

E.2 The Search Block

The search block contains three main algorithms: *search backward iteration*, *search state update* and *fiscal update*.

Search Backward Iteration Given next-period variables V^f_{t+1}, V^i_{t+1}, W^f_{t+1}, W^i_{t+1}, W^n_{t+1}, θ_{t+1}, r_{t+1}, ϕ_{t+1}, a current state E^f_{t-1}, E^i_{t-1}, U_{t-1}, F_{t-1} as well as r_t and public policy variables ϕ_t, $\tau_{hh,t}$:

1. Compute discounting q_t using (18);
2. Calculate wages w^f_t and w^i_t using (11) and (12);
3. Calculate period t values functions using (6), (7), (8) and (9);
4. Calculate period t market tightness using (15) and the job finding rate using (13);
5. Calculate ρ^f_t and ρ^i_t through (16);
6. Compute y_t using (19) and the transition matrix $g_t(s'|s)$.

Search State Update Given next-period variables ρ^f_{t+1}, ρ^i_{t+1}, V^f_{t+1}, V^i_{t+1}, r_{t+1}, ϕ_{t+1}, the current state variables E^f_{t-1}, E^i_{t-1}, U_{t-1} as well as the transition matrix g_t:

1. Update the distribution of workers E_t^f, E_t^i and U_t using the transition matrix g_t and the current distribution E_{t-1}^f, E_{t-1}^i and U_{t-1};
2. Calculate the anticipated next-period market tightness θ_{t+1} using the free entry condition (13) and the associated measure of new firms F_t using (15).

Fiscal Update Given current period wage rates w_t^f, w_t^i, distribution of workers E_t^f, E_t^i, U_t and price of bonds q_t, either:

- Given government consumption $C_{gov,t}$, update the income tax rate $\tau_{hh,t}$ using (17); or
- Given the income tax rate $\tau_{hh,t}$, update government consumption $C_{gov,t}$ using (17).

The difference between the choices in the fiscal update algorithm defines whether government adjusts income taxation (section 5) or public consumption (subsection 6.1).

E.3 The Household Block

The household block contains three main algorithms: *household backward iteration* and *household state update*.

Household Backward Iteration Given next-period variables J_{t+1} and g_{t+1}, the current period income process y_t and bond price q_t, solve the household's problem by solving the Euler Equation

$$q_t u'(c) = \beta \sum_{s'} \frac{\partial J_{t+1}(a', s')}{\partial a} g_{t+1}(s'|s).$$

We use the endogenous-grid method in this step.

Household State Update Given the state distribution x_{t-1} and households' next-period asset choice a'_t, compute x_t using (20).

E.4 Steady States and Transitions

To find a steady state, we start with a guess for the value functions, states variables, market tightness and interest rate. We run the five algorithms above in the order they are presented. After each iteration i, we calculate capital supply K_i using (21). We can also calculate aggregate hours of production L_i using the distribution of workers. The first-order condition for capital demanded by firms $A(k/\ell)^{\alpha-1} = r + \delta$ then provides the interest rate for the next iteration r_{i+1}, as well as value functions, states, and market tightness. Our tolerance for the approximation error is of four decimal digits.

To compute the transition path, we start with an initial steady state, which holds at $t = 0$, a transition length T and a final steady state which the economy reaches by period T. We then iterate the following three algorithms.

Backward Iteration Starting from the final steady state at T, use the *search backward iteration* and *household backward iteration* algorithms to find the path for value functions, wages and market tightness from $t = T - 1$ to $t = 1$.

State Update Starting with the aggregate state from the initial steady state, update from $t = 1$ to $t = T - 1$ using the *search state update*.

Update Fiscal/Interest From $t = 1$ to $t = T - 1$, update the public policy parameter of choice using the *fiscal update* algorithm and the interest rate path as in the case of a steady state.

We iterate the three algorithms above in the space of sequences until convergence of the aggregate state x_t, F_t.