Title: Something with Unexpected Inflation

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1 Introduction

2 Unexpected Inflation and the Value of Debt

2.1 The Valuation Equation

(Include public spending.)

I start by establishing the connection between inflation and the value of debt. Consider an economy with a homogeneous final good which households value. The economy has a government that manages a debt composed by bonds that pay one unit of currency in the next period. Currency is a commodity that only the government can produce, at zero cost. There is nothing special about currency. Households do not value it, and it provides no convenience for trading. Households cannot burn currency.

At the beginning of period t, the face value of debt issued in the previous period is V_{t-1}. The government redeems it for currency, which moves to the hands of households. After redeeming bonds, the government announces that each household i must pay $s_{i,t}$ goods in taxes, payable in currency. It also announces it will sell V_t new bonds.[1]

Nothing binds the government's choices of $s_{i,t}$ and V_t. Note the difference from the case with bonds payable in goods, in which the government *must* raise surpluses or issue new debt to redeem old bonds.

Let M_t be private holdings of currency at the end of t. As there is no free disposal of currency, the volume used to redeem $t-1$ bonds will be used to either pay taxes, buy new bonds or increase currency holdings:

$$V_{t-1} = P_t s_t + Q_t V_t + \Delta M_t \tag{1}$$

where s_t are aggregate taxes, P_t is the consumption good's price and Q_t is the price of new bonds (I state prices in currency units). Equation (1) provides a low of motion for public debt. It holds for all prices and all choices of money holdings, new debt sales, and public taxation.[2]

If $P_t = 0$, real debt V_{t-1}/P_t equals infinity. For now, that is a possibility.

Suppose $P_t > 0$. Define $\beta_t \equiv Q_t P_{t+1}/P_t$ as the real discount for public bonds, and $\beta_{t,t+k} \equiv$

[1] That the government forces households to pay for taxes and new bonds in currency is not necessary for the argument. All else follows if the government accepts payment in goods but stands ready to exchange these goods for currency at market prices.

[2] Strictly speaking, (1) holds for all *feasible* choices of money holdings, that is, choices that respect households' budget constraint. Otherwise, one could point out that $M_t = M_{t-1}$ and $s_t = B_t = 0$ lead to $B_{t-1} = 0$. That would nevertheless involve households burning up currency.

$\prod_{\tau=t}^{t+k} \beta_\tau$. Since V satisfies (1), it also satisfies

$$\frac{V_{t-1}}{P_t} = \sum_{i=0}^{k} \beta_{t,t+i-1} \left(s_{t+i} + \Delta M_t\right) + \beta_{t,t+k} \frac{V_{t+k}}{P_{t+k+1}} \qquad \text{for any } k \geq 0 \tag{2}$$

regardless of prices and choices. Expressions (1) and (2) do not represent a constraint on the path of taxes/surpluses $\{s_t\}$ and bond sales $\{V_t\}$ the government chooses to follow. They merely express future debt given paths $\{s_t\}$ and $\{M_t\}$, prices and the face value of inherited debt V_{t-1}.

So far, there is no economics. Just environment description and public finances accounting. I now make two assumptions about agents' behavior. First, I assume rational expectations, captured by the expectations operator E, which integrates using the actual probability measure. Second:

$$\lim_{k \to \infty} E_t \left(\beta_{t,t+k} \frac{V_{t+k}}{P_{t+k+1}} \right) = 0 \text{ at every period } t. \tag{3}$$

In micro-founded models, the above transversality condition follows from optimal household behavior. It implies that the stock of real debt cannot explode. In particular, it rules out $P_t = 0$ almost surely.

If bonds were real (redeemable in goods), assumption (3) - which I call *debt sustainability* - would effectively represent a no-default condition, as it forces the government to eventually raise the resources (via primary surpluses) to pay for past obligations.

With nominal debt, however, the government has no constraints on its choice of debt and surplus, as pointed out above. Replacing (3) on (2) yields

$$\frac{V_{t-1}}{P_t} = \sum_{i=0}^{\infty} E_t \left[\beta_{t,t+i-1} \left(s_{t+i} + \Delta M_t\right)\right]. \tag{4}$$

Given the face value of maturing public debt, the relative price of the consumption good is given by the expected β-discounted stream of (real) primary surpluses. This latter term - the right-hand side of (4) - I call the *real value of public debt*.

Therefore, in the case of nominal debt, (4) is a *valuation equation*, and debt sustainability implies not a no-default condition, but a no-bubble condition: market prices should reflect fundamental value. It provides the connection between fiscal and inflation shocks I explore in the paper.

(Incomplete: Long-term debt and revision of bonds' prices)

2.2 Partial Debt Repayment in a FTPL Model

2.2.1 Fiscal Selection with Debt Repayment

The environment introduced in the last section nests the basic NK model

$$\begin{aligned} (x_t - \lambda g_t) &= E_t \left(x_{t+1} - \lambda g_{t+1}\right) - \sigma \left[i_t - E_t \pi_{t+1}\right] \\ \pi_t &= \beta E_t \pi_{t+1} + \kappa x_t. \end{aligned} \tag{5}$$

In (5), x is the output gap, π is the inflation rate, g is public spending in levels, i is the nominal interest rate and r_t^n is natural real interest rate (details in appendix D, I ignore productivity shocks). I treat all variables here and below as deviations from the steady state.

By themselves, the two equations do not determine the unexpected componet of inflation. The Phillips curve pins *expected* inflation or, equivalently, the expected change in the inflation rate. Any expectational shock is consistent with a stationary equilibrium. In Blanchard and Kahn (1980) language, the system contains two forward-looking variables to a single explosive root: multiplicity of equilibria ensues.[3]

To solve it, the literature uses either a fiscal selection mechanism or a spiral-threat selection mechanism. In this paper, I favor the fiscal selection mechanism (or the fiscal theory of the price level), and defer to appendix A a detailed discussion of the differences between the two.

The first equation is a Taylor-expasion of the debt law of motion (2). I assume money has no utility to households, so $M_t = 0$.

$$\beta(v_t + s_t) = v_{t-1} + v\left(i_{t-1} - \pi_t\right) \tag{6}$$

In (27), v is the end-of-period real debt in levels and s is the primary surplus. I derive (27) in a more general version in appendix B. The public policy block of the model is:

$$\beta(v_t^* + s_t) = v_{t-1}^* + v\left(i_{t-1} - \pi_t^*\right) \tag{7}$$
$$s_t = \rho s_{t-1} + \alpha v_{t-1}^* + \varepsilon_{s,t} \tag{8}$$
$$\pi_t^* = E_{t-1}\pi_t + \eta_t \tag{9}$$
$$i_t = \phi E_{t-1}\pi_t + \varepsilon_{i,t} \tag{10}$$

where $\alpha > 0$, and $E_{t-1}\varepsilon = 0$ for $\varepsilon \in \{\varepsilon_{s,t}, \varepsilon_{i,t}, \eta_t\}$. This formulation is an adapted version of Cochrane (2022b). The key point of the formulation is to allow *partial* default of nominal debt through unexpected inflation. The portion of new borrowing that is not inflated away, the government can credibly commit to repay with future surpluses.

The mechanics of the solution is simple. From (27) and (7),

$$\beta\left(v_t - v_t^*\right) = \left(v_{t-1} - v_{t-1}^*\right) + v\left(\pi_t - \pi_t^*\right).$$

Since $\beta < 1$, in a stationary solution requires $v_t = v_t^*$ and $\pi_t = \pi_t^*$.

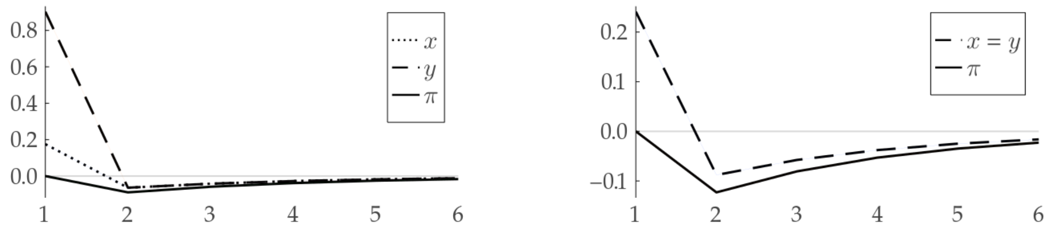
Intuitively, π_t^* works as an inflation target. From (9), it only really determines the unexpected component of inflation, as the expected component is determined by the private sector block (5). By (4), unexpected inflation $\Delta\pi_t^* = \Delta\pi_t = \eta_t$ corresponds to the re-assessment of discounted primary surpluses - the value of debt. Fiscal selection becomes clear: the expression above adds to the usual equations of NK model (5) the missing explosive unit root, and provides the unexpected component of inflation.

Variable v_t^* works as a latent real public debt target, the value of which is not affected by actual inflation, just the government's target. In equilibrium, they are the same, but outside equilibrium they are not. Critically, the surplus process responds *only* to v_t^*, not v_t. That means the government does not commit to repay debt born from arbitrary flucuations in the inflation rate. For example, if a "sunspot shock" leads to a -100% inflation rate - so that public debt roughly doubles on spot - the government does not increase surpluses thereafter in order to validate the initial deflation. That is the essence of the fiscal selection mechanism.[4]

Yet, there *is* debt repayment in the model. Active fiscal policy and public debt repayment are consistent. The positive loading of the primary surplus process on v_{t-1}^* guarantees that higher-than-average debt is eventually met by higher-than-average surpluses. Really, this is why

[3] The lack of determination is most easily seen in a model with flexible prices and constant output, in which the only equilibrium condition is the Fisher equation $i_t = E_t\pi_{t+1}$.

[4] The example illustrates how, contrary to common view, spiral threat selection mechanisms require far more powerful fiscal commitments than the FTPL.



(a) Fiscal Shock

(b) Monetary Shock

Figure 1: NK Model: Expansionary Policy without Unexpected Inflation

Cochrane builds v^* to mimic public debt. To ensure fiscal selection, one could just as well set $v_t^* = 0$, which leads to AR(1) surpluses

$$s_t = \rho s_{t-1} + \epsilon_{s,t}.$$

Then, the law of motion (27) provides the explosive root required for equilibrium determination, and inflation jumps so as to guarantee debt sustainability. An AR(1) model for surpluses, however, introduces counterfactual properties to the model, such as a decline in the market value of debt following a *deficit* (Canzoneri et al. (2001)) and hence high average returns on public bonds (Jiang et al. (2019), this follows from the fact that surpluses tend to be procyclical). As we see next, the (7)-(9) model avoids these inconsistencies by allowing a deficit shock to be followed by surpluses - thus an increase in the value of debt.

In all, the policy block (7)-(10) leads to an intuitive *observed* equilibrium:

$$\begin{aligned} s_t &= \rho s_{t-1} + \alpha v_{t-1} + \varepsilon_{s,t} \\ \Delta E_t \pi_t &= \eta_t \\ i_t &= \phi E_{t-1} \pi_t + \varepsilon_{i,t}, \end{aligned} \tag{11}$$

with a degree of inflation default captured by the second equation and partial repayment captured by surplus' response to debt.

2.2.2 The Ricardian Nature of Policy Shocks

Unexpected inflation is a key component of macroeconomic policy. In the basic NK model, fiscal and monetary stimulus depend on it to generate any inflation *at all*. Indeed, figure 1 shows output and inflation responses to expansionary fiscal (g increases by 1) and monetary (i declines by 1) shocks.[5] I keep $\eta = 0$.

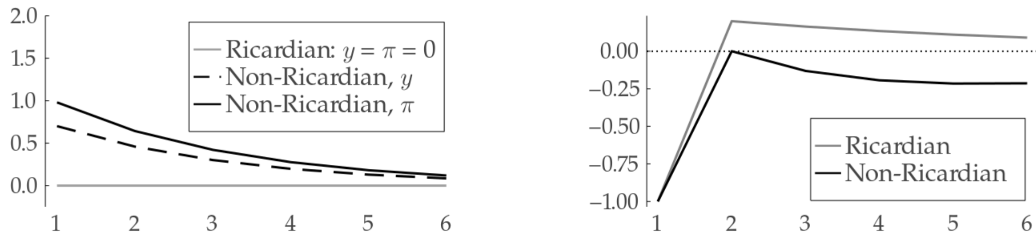
In both cases, expansionary policy leads to above-neutral output on impact. By the Phillips curve, inflation is then expected to *decline*:

$$\Delta \pi_{t+1} \approx \beta E_t \pi_{t+1} - \pi_t = -\kappa x_t$$

Stimulated aggregate demand does *not* lead to a period of high inflation. In both cases, the output gap eventually turns negative, and inflation converges to its steady-state value.

The two shocks depicted by figure 1 do not affect the real value of debt. Eventual borrowing

[5]Public spending follows $g_t = \rho g_t + \varepsilon_{g,t}$. Tax revenue follows $T_t = \rho T_{t-1} + \alpha v_{t-1}^* + \epsilon_{T,t}$, so that (8) holds. Parameters: $\sigma = 0.75 \times 0.5$, $\beta = 0.98$, $v = 1$, $\kappa = 0.5$, $\lambda = \sigma/(1+\sigma)$, $\alpha = 0.2$, $\rho = 0$, $\varphi = 0$.



(a) Output and Inflation (b) Tax Revenue (= Primary Surplus)

Figure 2: Ricardian and Non-Ricardian Transfer Shocks

(due to higher spending for instance) is fully repaid via surpluses. That is the meaning of $\eta_t = 0$. As they do not lead to an immediate change in discounted future surpluses, I call them *Ricardian*.

In the model, I treat unexpected inflation as a separate shock η_t but, given the observation above, there is no reason to believe governments do not change their promisses of future surpluses (leading to inflation) *at the same time* they conduct stimulative policy. In that case, I call the policy shock *Non-Ricardian*.

Consider the following example. The US government unexpectedly decides to transfer one dollar to every household in period zero (I normalize population to one). The transfer can but does not have to be accompanied by the implicit promise of a raise in future taxation. I consider the two extremes cases: a complete backing of the current deficit (like in figure 1) and the complete absence of backing.

In the absence of backing, surpluses decline by one in period zero, and are left unaltered thereafter. Households re-evaluate the value of debt, and the price level jumps. That is a Non-Ricardian shock. With full backing, the government implicitly promises to raise taxation in the future so that the real value of debt (discounted surpluses) is unchanged. The price level remains unaltered relative to its expected path. That is a Ricardian shock.

If T_t = total tax revenue, we can translate the two options of fiscal policy as follows:

$$\begin{aligned} \Delta E_t T_t &= \varepsilon_{B,t} + \varepsilon_{NB,t}, \\ \Delta E_t \pi_t &= -\left[v/\beta\right]^{-1} \varepsilon_{NB,t}. \end{aligned} \tag{12}$$

If negative, both $\varepsilon_{B,t}$ and $\varepsilon_{NB,t}$ can capture the one-dollar transfer. But only $\varepsilon_{NB,t}$ leads to unexpected inflation.

Figure 2a plots the responses of inflation and output to $\varepsilon_{B,t} = -1$ and $\varepsilon_{NB,t} = -1$. In the Ricardian case, there is no response of output and inflation, no effects typically associated with "demand" shocks.

If the transfer is not accompanied by the implicit promisse of future surpluses - if the transfer shock is non-Ricardian - inflation jumps on spot and stays high. Lower real interest then leads to increased output. If unexpected inflation was necessary to generate higher inflation in the case of stimulative spending and interest shocks, it is necessary to generate higher inflation *and higher output* in the case of a stimulative transfer shocks (like COVID checks).

Figure 2b shows the primary surplus process in each case. In the Ricardian case, with full debt repayment, period-zero debt jumps by one due to the transfer. Observing larger-than-usual debt, the government increases taxation in the following periods (equation (8), αv^*_{t-1} term), runs surpluses and fulfills its original promisse of full deficit repayment. In the Non-Ricardian case, period-zero inflation $\pi_0 = 1$ impedes real debt from increasing after the announced transfer. The one-dollar transfer is immediately "paid for" by bondholders via inflation.

The $[v/\beta]^{-1}$ coefficient multiplying $\varepsilon_{NB,t}$ ensures that in the example of Non-Ricardian shock, the cost of the transfer is exactly offset by unexpected inflation:

$$\beta \Delta E_t v_t = -\beta \Delta E_t s_t - v \Delta E_t \pi_t = 0.$$

Of course, this does not need to be the case always. Likewise, policy shocks do not have to be always Ricardian or Non-Ricardian. To capture all cases, consider the following reduced-form regression of unexpected inflation on other policy shocks:

$$\begin{aligned}
\Delta E_t \pi_t = \eta_t &= p_T \Delta E_t T_t + p_g \Delta E_t g_t + p_i \Delta E_t i_t \\
&= p_T \varepsilon_{T,t} + p_g \varepsilon_{g,t} + p_i \varepsilon_{i,t} \\
&\equiv -\left(v/\beta\right)^{-1} \left[\psi_T \varepsilon_{T,t} - \psi_g \varepsilon_{g,t}\right] + \beta \psi_i \varepsilon_{i,t}
\end{aligned}$$

The second line uses the fact that shocks ε correspond to the innovation on the corresponding variables (this is why I write (10) with $\phi E_{t-1}\pi_t$ instead of $\phi \pi_t$). The third line defines the ψ's.

If tax collection does not answer to unexpected inflation and all tax shocks are Ricardian, then $\text{cov}(\eta_t, \varepsilon_{T,t}) = 0$ which implies $\psi_T = 0$. If all tax shocks are Non-Ricardian and fiscally neutral, $\eta_t = -(v/\beta)^{-1}\varepsilon_{T,t}$, then $\psi_T = 1$. If $\psi \in (0, 1)$, tax deficits are partially repaid through inflation. If $\psi > 1$, the unexpected inflation that follows a deficit shock reduces the value of debt by a larger amount than the incoming deficit. Lastly, if $\psi < 0$, the real value of debt moves in the *opposite* direction of the policy shock. A dollar transfer raises the value of debt - households believe the government will more than compensate the initial shock with future surpluses.

$$\psi \begin{cases}
> 1 & \text{Non-Ricardian, inflation more than pays for policy shock} \\
= 1 & \text{Non-Ricardian, inflation exactly offsets policy shock} \\
\in (0,1) & \text{Non-Ricardian, inflation partial repayment} \\
= 0 & \text{Ricardian, shock paid via surpluses} \\
< 0 & \text{Non-Ricardian, surpluses more than pay for policy shock}
\end{cases}$$

In the following section, I estimate ψ. Importantly, ψ is a reduced-form parameter in the sense that policy shocks can respond to unexpected inflation too. The most evident case is that of interest rates. In the simplest case, the single-mandate Taylor (1993) rule prescribes $i_t = \phi \pi_t$, which leads to $\Delta E_t i_t = \phi \Delta E_t \pi_t$, or $\psi_i = \beta/\phi > 0$. That does not mean unexpected inflation "pays for" higher interest. We may observe $\psi_i > 0$ simply because, given an inflation spike, the central bank raises interest.

2.2.3 Lower bound on structural ψ?

2.2.4 Fiscal Sources of Unexpected Inflation

2.3 Generalizing Debt Instruments

2.3.1 Currency Denomination

The debt process considered above is too unrealistic to be taken to the data. To better fit the debt profile of different countries, I generalize the financing instruments available to the government in two dimensions: currency denomination and maturity structure. I do not consider the case of

Symbol	Description	Nominal Debt	Inflation-Linked Debt	Dollar-Linked Debt
j	Index Symbol	N	R	D
	Notation	δ, ω	δ_R, ω_R	δ_D, ω_D
P_j	Price per Good	P	1	P_t^{US}
$\mathcal{E}_j$	Nominal Exchange Rate	1	P	Dollar NER
H_j	Real Exchange Rate	1	1	Dollar RER
π_j	Log Variation in Price	π	0	π_t^{US}
Δh_j	Log Real Depreciation	0	0	Δh_t

Notes: P = price of consumption basket in domestic currency. P^{US} = price of consumption basket in US dollars. NER = Nominal Exchange Rate. RER = Real Exchange Rate.

Table 1: Public Debt Denomination

debt with stage-contingent nominal payoffs.[6]

Starting here, I recycle part of the notation already established. Fix the case of a country's government. Let P_t be the price of the consumption basket in terms of domestic currency.

The payoff of public bonds can be indexed to different currencies, enumerated by j. Let $P_{j,t}$ be the price of the consumer price index in units of currency j. Let $Q^n_{j,t}$ be the discount rate for a zero-coupon public bond paying one unit of currency j after n periods. Without loss of generality, all bonds are zero-coupon. Let $\mathcal{E}_{j,t}$ be the price of currency j in units of domestic currency.

The notation is general enough to accomodate currency-linked bonds *per se*, but also inflation-linked securities such as American TIPS. In this case, one can interpret the "currency" as the final goods basket (so that $P_j = 1$ and $\mathcal{E}_j = P_t$). In the empirical exercises that follow, I consider consider domestic currency bonds ($j = N$), inflation-linked (or real) bonds ($j = R$) and US-dollar-denominated bonds ($j = D$). Table 1 shows the value or interpretation of the variables defined above for these three cases.

I do not assume the price index for government's purchases (denoted P_t^S) and levied aggregates (such as income) to be the same as the price index for households' consumption (P_t). While governments do tend to tax private consumption, they also tax and transfer income, and purchase baskets of goods different than that of households (insert reference). The fiscal effect of variations in the relative price of government's to households' final goods basket must be accounted for.

With these changes, the law of motion of government debt (1) becomes

$$\sum_j \mathcal{E}_{j,t} B^1_{j,t-1} = P^s_t S_t + \sum_j \mathcal{E}_{j,t} \sum_{n=2}^{\infty} Q^{n-1}_{j,t} \left(B^{n-1}_{j,t} - B^n_{j,t-1} \right),$$

where S_t is now the real primary surplus and $B^n_{j,t}$ is the face value of bonds issued in currency j, period t payable n periods in the future. The term on the left represents the cost of debt in period t; the second term on the right represents the selling of new bonds of all possible maturities.

Let $\mathcal{V}_{j,t} = \sum_n Q^n_{j,t} B^n_{j,t}$ be the end-of-period market value of nominal debt issued in currency j, $i_{j,t}$ the risk-free rate in bonds issued in currency j and $rx_{j,t} = \mathcal{V}^{-1}_{j,t-1} \sum_{n=1}^{\infty} Q^{n-1}_{j,t} B^n_{j,t-1} - i_{j,t-1}$ the realized excess return on portfolios that mimic the composition of j-currency debt. We can

[6]Publicly-ran social security and pension systems can be modelled as cases of public debt with state-contigent payoffs or, in a fashion compatible with my framework, as an additional layer of uncertainty in the primary surplus process.

re-write the law of motion in terms of the $\mathcal{V}_j$ and its corresponding returns:

$$\sum_j (1 + rx_{j,t} + i_{j,t-1})\mathcal{E}_{j,t}\mathcal{V}_{j,t-1} = P_t^s S_t + \sum_j \mathcal{E}_{j,t}\mathcal{V}_{j,t}$$

For empirical work, it is ideal to write the law of motion in terms of real and stationary variables. So, let Y_t be a trend such that $s_t \equiv S_t/Y_t$ is reasonably stationary, and let $g_{Y,t} = Y_t/Y_{t-1} - 1$ be its growth rate. Define the real "exchange rate" $H_{j,t} = \mathcal{E}_{j,t}P_{j,t}/P_t$ and its growth rate $\Delta h_{j,t} = H_{j,t}/H_{j,t-1} - 1$. Define the de-trended real value of j-indexed debt $V_{j,t} = \mathcal{V}_{j,t}/P_{j,t}Y_t$, the real value of total debt $V_t = \sum_j H_{j,t}V_{j,t}$ and the j-indexed share $\delta_{j,t} = H_{j,t}V_{j,t}/V_t$.

By properly dividing the whole above equation by P_tY_t, and multiplying and dividing the j sum on the left by $P_{j,t-1}$, $P_{j,t}$, Y_{t-1} and $H_{j,t-1}$, we arrive at a final version for the law of motion of public debt:

$$V_{t-1} \sum_j \frac{(1 + rx_{j,t} + i_{j,t-1})(1 + \Delta h_{j,t})}{(1 + \pi_{j,t})(1 + g_{Y,t})} \delta_{j,t-1} = \frac{P_t^s}{P_t} s_t + V_t.$$

Stated now in real quantities, the law of motion above generalizes (1). During period t, the government must "pay" V_{t-1} plus realized returns and eventual changes to the relative value of currency j.[7]

I linearize the debt law of motion. Let $\beta_j = \exp\{-(rx_j + i_j - \pi_j - g_Y)\}$ be the steady-state real and growth-adjusted discounting for public debt issued in currency j. I linearize around a steady-state - assumed to exist - with $\beta_j = \beta$ for all j and $P^s = P$. This leads to

$$\beta\left(v_t + s_t + s(p_t^s - p_t)\right) = v_{t-1} + v\left[\sum_j \delta_j\left(rx_{j,t} + i_{j,t-1} + \Delta h_{j,t} - \pi_{j,t}\right) - g_{Y,t}\right], \qquad (13)$$

which generalizes (27).

2.3.2 Geometric Term Structure

I assume the government keeps a geometric maturity structure of its debt. The geometric term structure implies a tractable relationship between short-term interest rate and the excess return on public bonds.

The term structure is constant over time, but can vary across the different currency portfolios $\{j\}$ of public debt. Specifically, for the slice of public debt linked to currency j, suppose the outstanding volume of bonds decays at a rate ω_j, so that $B_{j,t}^n = \omega_j B_{j,t}^{n-1}$. Define $Q_{j,t} = \sum_{n=1}^{\infty} Q_{j,t}^n \omega_j^{n-1}$ as the weighted-average price of currency j public bonds. Then, $\mathcal{V}_{j,t} = Q_{j,t}B_{j,t}^1$. The total return on currency-j bonds then is $1 + rx_{j,t} + i_{j,t-1} = (1 + \omega_j Q_{j,t})/Q_{j,t-1}$, which I linearize as

$$rx_{j,t} + i_{j,t-1} = \omega_j \beta_j q_{j,t} - q_{j,t-1} \qquad (14)$$

where $q = \log Q$ and I use the log approximation of level returns to effectively re-define $rx_j + i_j$.

Equation (14) above defines the excess return on holdings of the j-currency portfolio of public debt. Given a model for the risk premium $E_t rx_{j,t+1}$, it also defines the price of the debt portfolio

[7] "Pay" comes in paranthesis here because, unlike in (1), the government does not actually redeem the entire term on the left at period t. It only pays for bonds maturing at t.

as a function of short-term interest:

$$
\begin{aligned}
q_{j,t} &= \omega_j \beta_j E_t q_{j,t+1} - E_t rx_{j,t+1} - i_{j,t} \\
&= -\sum_{i=0}^{\infty} (\omega_j \beta_j)^i E_t \left[rx_{j,t+1+i} + i_{j,t+i} \right].
\end{aligned}
\tag{15}
$$

The second equation in (15) which clarifies the connection between short-term interest and returns on the market price of debt showing up in (13). Given news of, say, higher interest rates, the discount of public bond increases, and q falls. Equation (14) then prescribes a low excess return on j debt.

3 Cross-Country Estimates of Unexpected Inflation

3.1 The VAR

For each country in the sample, I estimate a seven-equation VAR in which the debt law of motion (13) holds by construction. If the law of motion holds and the VAR is stationary, we can later decompose innovations to the valuation equation.

Selected variables are based on (13): tax revenue, nominal interest, inflation rate, public debt, government spending, real exchange rate and gross domestic product (GDP). From these, only GDP does not show up in (13).

Data is annual, with period ranges varying from country to country. Inflation is the log variation in the consumer price index. The dollar real exchange rate is the nominal exchange rate to the US dollar multiplied by the ratio of US-to-domestic CPI. The nominal interest rate is the log of 1+ interest data. The series for GDP is the log deviation from a log-linear trend.[8] Public spending and public debt data are both divided by the GDP trend.[9] With the exception of exchange rate depreciation, I demean all series.

The functional format of the VAR is

$$
x_t = a(L)x_{t-1} + b(L)u_{t-1} + k\, e_t \tag{16}
$$

where u_t is groups the same set of variables for the United States. The lag matrix polynomial has the format $a(L) = a_1 L + \cdots + a_p L^p$, and the same holds for $b(L)$.

There are six shocks hitting the domestic economy: $\varepsilon \sim N(0, \Sigma)$, independent over time. The shocks hitting the US economy are $\varepsilon_u \sim N(0, \Sigma_u)$. I group them in $e_t = [\varepsilon_t' \; \varepsilon_{u,t}']'$. Matrix $k_{7\times 12}$ incorporates the singularity introduced by the debt law of motion. Because the public debt process of each country has a dollar component, and hence depends on dollar interest and inflation, u and ε_u enter the regression of all countries.

[8] I run OLS on $\log(gdp)_t = c_0 + c_1 t + \nu_t$ using all data available for GDP (which usually covers a longer period than that of the balanced panel) and define $\exp(\hat{c}_0 + \hat{c}_1 t)$ as the economy's trend, or Y_t in the notation of the previous section. The series for GDP is $\hat{\nu}_t$.

[9] Real GDP (constant 2015 prices), GDP deflator, public spending and the nominal exchange rate data come from the United Nations's National Accounts Main Aggregates Database. Consumer price index and and primary surplus data come from the IMF's WEO Database. Public debt (as ratio of GDP) comes Ali Abbas et al. (2011) database, which is kept up-to-date. The sources for interest rate vary from country to country; they are usually the central bank, but also from the IMF's International Financial Statistics database. Appendix C provides further details.

The model for US variables is:

$$u_t = a_u(L)u_{t-1} + k_u\, \varepsilon_{u,t} \tag{17}$$

(I use the same notation x to the VAR of all countries and differentiate only in the US case).

3.2 Debt Law and Excess Return Adjustments

Cochrane (2022a) imposes the law of motion of public debt in the VAR by using primary surplus "data" calculated from it. Primary deficit equals the change in public debt net of realized return. This procedure bypasses the problem of dealing with accounting conventions in primary surplus reporting that, in most cases, make its data inconsistent with that of public debt. Since the OLS estimator is linear, his VAR then satisfies (13).

In our VAR, the procedure needs to be adjusted because we do not measure excess return and, even if we did, the unrestrictive estimation of (16) spuriously projects US interest and inflation on domestic variables, which is inconsistent with (17). Hence, instead of measuring implied surpluses and then estimate the model, I estimate the model and then infer the equation for tax revenues that ensures the debt law of motion (13) holds. The equation for public debt, in turn, represents its law of motion *after* replacing the equation determining tax proceeds.

The estimation has three steps.

Step 1. I estimate the VAR

$$\begin{aligned} \tilde{x}_t &= \tilde{a}(L)\tilde{x}_{t-1} + \tilde{b}(L)\tilde{u}_{t-1} + \varepsilon_t \\ \tilde{u}_t &= \tilde{a}_u(L)\tilde{u}_{t-1} + \varepsilon_{u,t} \end{aligned} \tag{18}$$

where $\tilde{x}$ is a vector with all variables in x except tax revenues, and $\tilde{u}$ is defined similarly. Matrix coefficients $\tilde{a}$, $\tilde{b}$ and $\tilde{a}_u$ exclude the row and column corresponding to tax revenue. I proceed under the assumption that the loadings of all variables in the VAR on previous tax revenue equal zero.

Step 2. I assume a constant risk-premium: $E_t rx_{j,t+1} = 0$ for all j. Then, I use the estimates of (18) to compute $E_t i_{j,t+i}$ and apply (15) to compute $q_{j,t}$. Equation (14) then yields expressions for excess return of the form

$$rx_{j,t} = \varphi_j' e_t.$$

In the case of real debt, I use

$$i_{R,t} = i_{N,t} - E_t\pi_{N,t+1} = \zeta(L)'X_t.$$

where $X_t = [x_t' \; u_t']'$ stacks domestic and US variables. In the United States case, $X_t = u_t$. In appendix E, I present the formulas for the φ's and $\zeta(L)$.

Step 3. I use the debt law of motion (13) and estimates of φ and ζ to compute the equation for tax revenue and its residual as a function of all the other residuals. This completes the estimation of (16) and (17) which we can then stack into a single system for X_t:

$$X_t = A(L)X_{t-1} + Ke_t. \tag{19}$$

More explicitly (and ordering tax revenues at the top of the VAR):

$$\begin{pmatrix} x_t \\ u_t \end{pmatrix} = \begin{bmatrix} a(L) & b(L) \\ 0 & a_u(L) \end{bmatrix} \begin{pmatrix} x_{t-1} \\ u_{t-1} \end{pmatrix} + \begin{bmatrix} \multicolumn{2}{c}{k} \\ 0 & k_u \end{bmatrix} \begin{pmatrix} \varepsilon_t \\ \varepsilon_{u,t} \end{pmatrix}$$

$$\text{or yet} \begin{pmatrix} T_t \\ \tilde{x}_t \\ T_{u,t} \\ \tilde{u}_t \end{pmatrix} = \begin{bmatrix} 0 & \multicolumn{3}{c}{(13)} \\ 0 & \tilde{a}(L) & 0 & \tilde{b}(L) \\ 0 & 0 & 0 & (13) \\ 0 & 0 & 0 & \tilde{a}_u(L) \end{bmatrix} \begin{pmatrix} T_{t-1} \\ \tilde{x}_{t-1} \\ T_{u,t-1} \\ \tilde{u}_{t-1} \end{pmatrix} + \begin{bmatrix} \multicolumn{2}{c}{(13)} \\ I & 0 \\ 0 & (13) \\ 0 & I \end{bmatrix} \begin{pmatrix} \varepsilon_t \\ \varepsilon_{u,t} \end{pmatrix}.$$

In the last equation, I use symbol (13) to indicate that the coefficients are those implied by the debt law of motion. In appendix E, I provide their formulas.

3.3 A Debt-Adjusted Minnesota Prior

3.3.1 Prior for Debt Sustainability

I interpret model parameters $\tilde{a}$ and $\tilde{b}$ as being random, and estimate (18) by esblishing a prior distribution, and then using data likelihood to compute a posterior distribution.

Estimating the model using Bayesian methods has two advantages. First, parameter shrinkage reduces the volatility of estimated coefficients and over-fitting, an invaluable feature when samples contain 20 to 50 observations.[10] Second, the fiscal policy literature estimates highly persistent public debt processes (Bohn (1998), Uctum et al. (2006), Yoon (2012)); in the time period I analyze (1970-2019) the sovereign debt of many economies increased significantly. For these reasons, OLS estimates often estimate explosive dynamics, which is inconsistent with the assumption of debt sustainability. By properly choosing the parameters of the prior distribution, we can ensure stability at the time we search for parameters that provide the best fit.

I specify a prior distribution of the Normal-Inverse-Wishart family, which englobes the commonly used Litterman (1979) (or Minnesota) prior. The Minnesota prior formalizes the view that the variables of interest follow a random walk, $x_t = c + x_{t-1} +$ shock, or a white noise $(1-L)x_t = c+$ shock if differenced.[11] I assume variables of the VAR to be $I(0)$, and adopt a white noise prior.

Nevertheless, the original Minnesota prior center around a parameter vector that leads to an unstable system (16). Indeed, if $\tilde{a}(L) = 0$, the eigenvalues of (18) are zero. The introduction of the debt law of motion (13) then introduces a $1/\beta > 1$ root to the system.

Therefore, to recover stability, I re-center the original distribution around a set of parameters that reproduce the hypothesis that *tax revenues and public spending equally adjust to render debt sustainable.* I believe this prior to be consistent with a more common view that governments do not rely on inflation sparks and monetary policy to ensure real debt repayment. In the equations, given a desired annual rate of tax repayment ρ, the equations have to satisfy

$$\begin{aligned} E_{t-1}T_t &= (\alpha/2)v_{t-1} \\ E_{t-1}G_t &= -(\alpha/2)v_{t-1} \\ E_{t-1}v_t &= \rho v_{t-1}. \end{aligned} \tag{20}$$

[10]Giannone et al. (2015) show that priors of the Normal-Inverse-Wishart family, such as the Minnesota prior, lead to posterior distributions that can be decomposed as posterior = model fit term + expectation volatility term.

[11]The literature about the Minnesota prior is vast. Interested readers can see del Negro and Schorfheide (2011) or Karlsson (2013) for a survey-like approach.

where T, G and v are the (trend-normalized) level of public spending, tax revenues and real debt. In words, each additional unit of debt leads to a $\alpha/2$ increase in taxation and a $\alpha/2$ decline in expenditure one year later. The choice of α must be such that the deviation of real debt to its long-term level decreases at a rate ρ. I set $\rho = 0.955$ so that expectational shocks to debt have a half-life of about fifteen years. By (13),

$$\alpha = \beta^{-1} - \rho.$$

In practice, I change the corresponding elements of $\tilde{a}_1$ (first matrix in the polynominal $\tilde{a}(L)$) from zero to ρ (in the v equation) and $-\alpha/2$ (in the G equation).

3.3.2 Prior Tightness

The prior is of the Normal-Inverse-Wishart distribution family, with general format

$$\begin{aligned}\Sigma &\sim IW(\Phi; d)\\ \theta|\Sigma &\sim N(\bar{\theta}, \Sigma \otimes \Omega).\end{aligned}$$

where $\theta = [\text{vec}(\tilde{a}(L)')' \ \text{vec}(\tilde{b}(L)')']'$ and vec means stacking the columns.

The mean of the IW distribution is $\Phi/(d-n-1)$, where $n = 6$ is the dimension of the square matrices and larger values of d represent tighter priors. I choose Φ to be the identity matrix (one percent standard deviation for all shocks, which are uncorrelated) and select $d = n+2$, the lowest integer possible that leads to a well-defined distribution mean - which therefore equals Φ.

Prior parameters $\bar{\theta}$ and Ω reflect the choices I now describe. Denote i_y the index of variable y. Let $\tilde{a}_{p,ij}$ be the (i,j) element of the p-th matrix in $\tilde{a}(L)$. Its conditional expectation is $E(\tilde{a}_{p,ij} \mid \Sigma) = 0$ unless $P = 1$, $j = i_v$ and $i = i_v$ or i_G, in which case the expectation equals ρ and $-\alpha/2$, respectivelly, as explained above.

The conditional covariance between the coefficients in $\tilde{a}$ is

$$\operatorname{cov}\left(\tilde{a}_{p,ij}, \tilde{a}_{q,kl} \mid \Sigma\right) = \begin{cases} \mu^2 \frac{\lambda^2}{p^2} \frac{\Sigma_{ij}}{\Phi_{jj}} & \text{if } p = q \text{ and } j = l = i_v \\ \frac{\lambda^2}{p^2} \frac{\Sigma_{ij}}{\Phi_{jj}} & \text{if } p = q \text{ and } j = l \neq i_v \\ 0 & \text{otherwise.} \end{cases}$$

The format allows the loadings on the variable in different equations to be correlated. Loadings on the different variables on the same equation are independent. Hyperparameter λ governs the overall tightness of the prior over linear coefficients, with lower values yielding tighter priors. (Provide additional explanation?)

Given λ, hyperparameter μ governs the tightness of loadings on previous period debt v_{t-1}. As $\mu \to 0$, the VAR gets closer to reproducing (20), and variables other than v_t and G_t respond less to v_{t-1}.

The conditional mean of $\tilde{b}(L)$ is zero. Its conditional covariance is

$$\operatorname{cov}\left(\tilde{b}_{p,ij}, \tilde{b}_{q,kl} \mid \Sigma\right) = \begin{cases} \xi^2 \frac{\lambda^2}{p^2} \frac{\Sigma_{ij}}{\Phi_{u,jj}} & \text{if } p = q \text{ and } j = l \\ 0 & \text{otherwise} \end{cases}$$

where $\Phi_u = \Phi = I$ is the mean of the IW distribution in the US case. Hyperparameter ξ governs the tightness of the prior that US variables do not affect domestic ones more than the mechanical effect of US interest and inflation on domestic public debt.

Finally, the covariance between $\tilde{a}$ and $\tilde{b}$ is zero.

It is straightforward to set Ω so that the conditional covariance structures above hold.

3.4 Unexpected Inflation

3.4.1 Baseline Specification

In the baseline specification, I assume a single lag on $a(L)$, $\mu = 1$ (no tighter prior for loadings on previous-period debt v_{t-1}) and $\xi \approx 0$ (which sets $\tilde{b} = 0$; US variables impact the domestic economy only through their direct effect on public debt). I calibrate $\beta = 0.98$, $v =$ corresponding country's average public debt (as ratio to GDP trend) and $\rho = 0.955$. Paramaters δ and ω I calibrate based on debt structure data (see appendix C).

Priors of the Normal-Inverse-Wishart class are conjugate and admit closed-form solutions for both the posterior distribution and the marginal likelihood. I set λ so as to maximize the marginal likelihood, which, as Giannone et al. (2015) show, can be decomposed in a model fit term and a model complexity term.

Table 2 reports estimated statistics related to the reduced-form residuals ε. I am interested in the stochastic properties of unexpected inflation $\Delta E_t \pi_t = \varepsilon_{\pi,t}$. Statistics are calculated at the mode of the posterior distribution. The asterisk indicates that the sign of the reported value is statistically significant at the 10% confidence level. I perform inference by simulating a ten thousand sample for each country.

The first column reports the standard deviation of ε_π. The remaining five columns report the statistic $p(\pi, c) \equiv \Sigma_{\pi,c}/\Sigma_c$, which is the projection coefficient of ε_π on ε_c. Divided by 100, The value of $p(\pi, y)$ answers the question what do we project unexpected inflation to be if we observe variable y to be 1% greater than expected in the previous period (and nothing else).

I find average standard deviation of unexpected inflation to be 1.56%; the median equals 1.30%. The real value of debt varies. All advanced economies have below-average inflation volatility. Only in the cases of Australia, Sweden and the United States I estimate values above one. On the other hand, all developing countries have standard deviation above one; in four cases, above two. In all, I estimate standard deviation amoung developing countries to be roughly twice that of advanced ones: 0.80% median vs 1.61%.

3.4.2 Fiscal Policy Shocks

Moving to shock correlation, the first part of table 2 shows the results we would expect to observe in a some fiscal/monetary policy benchmarks.[12] The values I report do not consider potential general equilibrium effects (for example, how a fiscal shock would affect discounting and, through discounting, the value of debt and unexpected inflation). The simplification clarifies the connection between policy description and shock correlation.

The first two cases refer to the Ricardian character of fiscal policy shocks. By "Ricardian" I mean whether they affect the (beginning-of-period) real value of debt. If the government implicitly promisses to revert changes to taxation or public spending in such a way that the value of its debt stays the same, then, by (4), there should be no unexpected inflation. Hence $p = 0$. If, on the contrary, news of taxation/spending leave the distribution of future surpluses unaltered, then each additional dollar of tax revenue raises the real value of debt by the exact same amount (and the analogous for spending). Only the first term of the sum in (4) changes. Hence the real

[12] All the benchmarks consider the case of short-term, nominal debt.

Country	$\sigma(\pi)$	$p(\pi, T)$	$p(\pi, G)$	$p(\pi, i)$	$p(\pi, \Delta h)$	$p(\pi, y)$
Benchmarks						
Ricardian Shocks		0	0			
Non-Ricardian Shocks		-1	1			
Taylor Rule $i_t = \phi\pi_t$				ϕ^{-1}		
Pegged Currency				0	-1	
Cross-Section Aggregates						
Average	1.56*	-0.20*	-0.12	0.53*	0.02*	-0.05*
Median	1.28*	-0.15*	0.07	0.50*	-0.01	-0.02
Advanced	0.80*	-0.04*	-0.05	0.37*	-0.02	-0.01
Developing	1.61*	-0.29*	0.36	0.64*	0.02	-0.03
Advanced						
Australia	1.36*	-0.30*	-0.49	0.48*	-0.01	-0.11
Canada	0.76*	0.04	0.11	0.26*	-0.10*	0.01
Denmark	0.79*	-0.11*	-0.05	0.34*	-0.02	0.10
Germany (Euro)	0.50*	-0.04	-0.35	0.31*	-0.02	0.14*
Iceland	0.96*	-0.03*	0.04	0.27	0.10*	-0.12*
Japan	0.64*	-0.02	-1.02*	0.56*	0.03*	0.16*
New Zealand	0.68*	-0.13*	0.21	0.44*	-0.02	-0.11
Norway	0.95*	-0.04	-0.87*	0.37*	-0.03*	-0.01
Republic of Korea	0.90*	0.01	0.49*	0.48*	0.06*	-0.10*
Sweden	1.15*	-0.03	0.45*	-0.12	0.02	0.15*
Switzerland	0.59*	-0.10*	-0.15	0.46*	-0.01	0.23*
United Kingdom	0.80*	-0.09	0.18	0.25*	-0.03*	-0.08
United States	1.27*	-0.20*	-0.32	0.51*	-0.05	-0.02
Developing						
Brazil	1.29*	-0.19*	0.39	0.51*	0.02	-0.08
Chile	1.09*	-0.18*	-0.45	0.90*	0.04	0.05
Colombia	1.37*	-0.29*	-1.32*	0.63*	0.00	0.40*
Czech Republic	1.19*	-0.36*	-0.44	0.79*	-0.03	0.06
Hungary	1.61*	-0.29*	0.10	0.83*	-0.05	-0.08
India	1.63*	-0.35*	-0.33	0.56*	0.01	-0.12
Indonesia	3.87*	-0.73*	-1.91	1.40*	0.26*	-1.35*
Israel	1.44*	-0.27*	1.15*	0.95*	-0.10*	0.11
Mexico	1.47*	-0.37*	0.97	0.53*	0.02	0.02
Poland	1.68*	-0.55*	0.36	0.80*	-0.02	-0.19
Romania	2.37*	0.32	2.92*	0.74*	-0.03	0.01
Russia	1.68*	0.09	0.85*	0.34	0.07*	-0.03
South Africa	1.37*	-0.11	0.49	0.64*	0.05*	0.02
Turkey	4.74*	-0.89*	-6.24*	0.19*	0.19*	-0.04
Ukraine	5.49*	-0.24*	1.77	0.48*	0.12	-0.29*

Notes: First column in percentages. T = tax revenues, G = public spending, i = nominal interest, Δh = real exchange depreciation, y = gross domestic product. Asterisks indicate significance at a 10% confidence level, based on 10,000 simulations of the posterior distribution.

Table 2: BVAR Estimation (optimal λ, domestic variables): Unexpected Inflation

value of debt and thus inflation move exactly by the same amount: $p = 1$. Is it worth to show the algebra here?

(A note on methodology: the benchmark values justify why I work with detrended tax revenues and public spending. If instead I linearized around log deviations from the mean, the "Non-Ricardian" benchmarks above would be country-dependent, equal to their unconditional mean in levels.)

My estimates suggest that taxation (T) shocks tend to fall between these two cases, with an average inflation projection of -0.20%. Estimates have a negative sign to all but four countries, two advanced and two developing. Nevertheless, the magnitude of the coefficients is significantly larger in the case of developing economies (-0.30% median vs -0.04%). Tax shocks more often translate into inflation shocks in emerging markets. The signs are statistically significant in all but three countries, two of them being the cases of positive projections (Romania and Russia). The uniformity of the projections is also remarkable, with nine of the fifteen cases falling in the interval -0.19% to -0.37%.

As for advanced markets, estimates are much closer to zero, and we more often fail to reject $p = 0$. For developed countries, tax revenue shocks are more "Ricardian", a potential explanation for the quiet inflation since the 90s. Only the projections of Australia and the United States fall in the $(-0.19\%, -0.37\%)$ interval that characterizes developing countries.

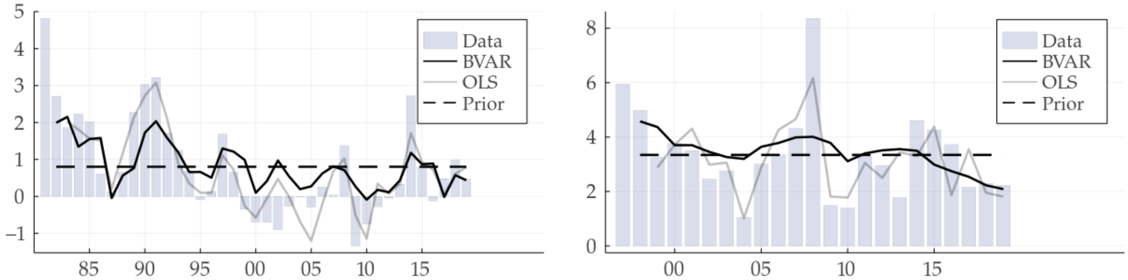
In the case of spending (G) shocks, the evidence is not as clear. Estimated coefficients are wildly heterogeneous. Thirteen of the twenty-eight estimated figures - and the unconditional average - are *negative*, meaning that unexpectedly *large* spending tends to be accompanied by unexpectedly *lower* inflation. There is no discernible pattern differencing advanced and emerging markets. The 0.36% median among developing countries is encouraging, but not sufficiently reproducible in the simulations of the posterior distribution. Focusing only on statistically significant projections also does not help: from the nine significant estimates, four are negative.

3.4.3 Monetary Policy Shocks

Consider now interactions with monetary policy. If the central bank follows a single-mandate Taylor rule $i_t = \phi\pi_t$, and we observe interest rates unexpectedly increase by 1%, our best guess for unexpected inflation is $(1/\phi_t)\%$. Another explanation for positive coefficients would be a higher-than-annual frequency Fisherian effect, as the literature has been recently considering (Garín et al. (2018), Uribe (2022)). (So far, there was no explanation of Neo-Fisherian effect. Introduce it here?) The present exercise cannot distinguish between the two possibilities.

The evidence for the projection of unexpected inflation on unexpected interest (i) is unequivocal: (reduced-form) interest rate shocks correlate with higher inflation. A 1% higher interest "nowcasts" a 0.5% higher inflation, on average. With the exception of Sweden, all estimated coefficients are positive. Three (including Sweden) are not statistically significant. All but that of Indonesia are lower than one. The projection is considerably weaker in advanced economies (0.37% median vs 0.64% in developing countries).

Because central banks do not follow strict policy rules, we cannot interpret coefficients in the $[0, 1]$ range as evidence in favor of the Taylor principle $\phi > 1$. Neither can we conclude in favor of Neo-Fisherian effects (higher interest $\implies$ higher inflation) since central banks do often increase interest in response to inflation. But I conclude that either of these effects explains the cross-sectional evidence: either central banks in emerging markets respond less strongly to inflation (lower ϕ), or fiscal policy is less capable from insulating from the effects of interest on public debt.



(a) Japan (b) Chile

Notes: Bars are inflation data, lines are the previous-period expected value.

Figure 3: Inflation Fit with Different Priors

The next column shows the projection of unexpected inflation on real exchange rate (Δh) shocks. Since $\Delta h_t = \Delta e_t + \pi_{u,t} - \pi_t$, if a country pegs it nominal exchange rate e, we should observe $p = -1$ (absent foreign shocks to which domestic inflation answer). The evidence overwhelmingly rejects $p = -1$, however, which should be expected as no country in the sample pegs their currency value. Most estimates have an absolute value lower or equal to 0.10%. Those that don't - Indonesia, Turkey and Ukraine - are positive.

Lastly, I ask if unexpected inflation correlates with the changes to the business cycle, as captured by unexpected gross domestic product (y). The last column of table 2 shows no clear pattern. In fifteen cases, the projection is negative, in thirteen it is positive, as the roughly zero medians show. Among statistically significant coefficients, five are positive. The statistically significant average is probably the result of the negative Indonesia and Ukraine negative outliers. Surprisingly perhaps, output shocks do not appear to affect - on average - the real value of debt.

3.4.4 Robustness

Are the empirical findings robust? I consider alternative specifications for the model and its estimation procedure. Table 3 shows all results; I focus on medians. Panel (a) repeats the baseline findings.

In panel (b) I set $\mu \approx 0$, which in practice fixes the response of the VAR to previous debt. The system reproduces (20). Then, starting at the optimal λ that maximizes the marginal likelihood, I reduce λ until I find a value that leads to a stable VAR.[13]

In panel (c) I set $\lambda \approx 0$, which means running the model using prior mode loadings of $\tilde{a}$ and $\tilde{b}$. In the case of inflation, that amounts to $E_{t-1}\pi_t = 0$, or expected inflation = mean inflation.

Panel (d) sets $\xi = 1$. In the baseline case, I set $\xi \approx 0$, so that $\tilde{b}(L) = 0$. I implicitly assume that the effect of US variables over domestic ones can be successfully modelled by the white noise shocks ε_t; US variables themselves do not need to be included in the VAR, except for their impact on public debt. But past literature has argued that the opposed is true, which motivates $\xi = 1$ (I then re-optimize λ).[14]

The next two panels (e) and (f) report results using OLS estimates. This is easily done by setting shrinkage to zero, or $\lambda \approx +\infty$. In panel (f) I allow lag polynominal to have two non-zero lags.

[13]Simply ensuring (20) does not guarantee a stable system, since data other than public debt can lead to explosive estimated eigenvalues.

[14]For example, Neumeyer and Perri (2005) and Mendoza (2010) consider the role of international credit and interest rates; Fernández et al. (2020) study the quantitative importance of commodity price variation.

Figure 3 shows the effect of shrinkage in cases of Japan and Chile. Bars correspond to observed inflation, and each curve depicts its expected value in the previous period (the difference to the bar corresponds to unexpected inflation). The curves correspond to the stable BVAR (panels (b)), prior distribution loadings (panel (c)) and OLS with two lags (panel (f)).

In general, the different model specifications lead to the same qualitative conclusions as the baseline. Tighter priors (panels (b) and (c)) lead to more volatile unexpected inflation, as expected. Shocks in emerging markets are more volatile in all cases. Estimated projections are also similar to the baseline case. Estimates using the prior distribution's loadings - panel (c) - indicate positive projection of unexpected inflation given unexpected spending, a result that is closer to our Non-Ricardian benchmark.

Finally, the last part of table 3 presents two alternative calibrations. Panel (g) considers a substantially higher discounting $\beta = 0.90$, as one could argue $\beta = 0.98$ is unrealistic for emerging markets. Panel (h) reduces debt persistence parameter ρ of the prior distribution from 0.955 to 0.75. In both cases, I re-optimize tightness parameter λ. Neither modification significantly changes baseline estimates.

4 Debt Sustainability and Unexpected Total Inflation

4.1 Decomposition of the Debt Valuation Equation

Along with a fiscal-theory equilibrium selection mechanism, equation (4) shows that the unexpected inflation measured in the previous section must be the result of the revision of expectations of discounted future surpluses. Higher-than-expected inflation devalues nominal debt so as to equalize to it to its revised real value. In the simplified environment of section 2, that means unexpected inflation derives from lower-than-anticipated surpluses or higher-than anticipated discount rates.

Cochrane (2022a) applies an expected return decomposition, as in Campbell and Ammer (1993), to the United States. The decomposition separates how each component of the real value of debt changes so as to lead to a unexpected inflation show. I mimic his decomposition now in the more general environment with currency-linked debt.

To find the decomposition, solve (13) forward and apply condition $\lim_{i\to\infty} \beta^i v_{t-1+i} = 0$ (which is the linearized version of assumption **??**). After that, take innovations to arrive at

$$\frac{v}{\beta}\left[\sum_{j\neq N}\delta_j \Delta E_t r_{j,t} + \delta\Big(\Delta E_t rx_t - \underbrace{\Delta E_t \pi_t}_{\substack{\text{Unexpected}\\ \text{Inflation}}}\Big)\right] = \sum_{i=0}^{\infty}\beta^i \Delta E_t s^p_{t+i} - \frac{v}{\beta}\sum_{i=1}^{\infty}\sum_{j}\delta_j \beta^i \Delta E_t r_{j,t+i} \quad (21)$$

In (30), $s^p = s_t + v(p^s_t - p_t)$ is the price adjusted surplus and $r_{j,t} = rx_{j,t} + i_{j,t-1} + \Delta h_{j,t} - \pi_{j,t}$ is the *ex-post* real return on holdings of the j-currency portfolio of public debt, in domestic currency. Note that I apply assumptions $g_{Y,t} = 0$ for all t and $E_t rx_{j,T} = 0$ for $T > t$. I also simplify notation with $rx_{N,t} = rx_t$, $i_{N,t} = i_t$ and $\pi_{N,t} = \pi_t$.

The right-hand side of (30) contains the revision of expectations over future surpluses - including time t - and over future real returns on all currency portfolios of public debt - starting at $t+1$. This amounts to the *change in the real value of debt* in period t. The left-hand side contains period-t changes to debt *prices*. We can re-write it as

$$\frac{v}{\beta}\Delta E_t\left[\delta_D\left(rx_{D,t} + \Delta h_t - \pi_{US,t}\right) + \delta_R\left(rx_{R,t}\right) + \delta\left(rx_t - \pi_t\right)\right].$$

Median	$\sigma(\pi)$	$p(\pi,T)$	$p(\pi,G)$	$p(\pi,i)$	$p(\pi,\Delta h)$	$p(\pi,gdp)$
(a) Baseline: BVAR, domestic variables ($\xi=0$)						
All	1.28*	-0.15*	0.07	0.50*	-0.01	-0.02
Advanced	0.80*	-0.04*	-0.05	0.37*	-0.02	-0.01
Developing	1.61*	-0.29*	0.36	0.64*	0.02	-0.03
Alternative Model Specifications						
(b) BVAR, domestic, stable λ, $\mu=0$						
All	1.33*	-0.13*	0.01	0.49*	0.00	-0.00
Advanced	0.84*	-0.08*	-0.08	0.46*	-0.02	-0.02
Developing	1.57*	-0.22*	0.20	0.70*	0.02	0.01
(c) BVAR, domestic, $\lambda=0$ ($E_{t-1}\pi_t=0$)						
All	2.17*	-0.14*	0.43*	0.47*	-0.01	0.01
Advanced	1.56*	-0.10*	0.45*	0.37*	-0.02*	0.03
Developing	3.65*	-0.38*	0.26	0.62*	0.02	-0.11*
(d) BVAR, with US variables ($\xi=1$)						
All	1.12*	-0.16*	0.07	0.49*	0.00	0.01
Advanced	0.71*	-0.05*	-0.11	0.32*	-0.03	-0.01
Developing	1.27*	-0.28*	0.15	0.62*	0.02*	0.03
(e) OLS ($\lambda=\infty$), domestic						
All	1.16*	-0.14*	-0.23	0.57*	0.01	-0.01
Advanced	0.77*	-0.06*	-0.19	0.43*	-0.02	0.02
Developing	1.29*	-0.27*	-0.48	0.75*	0.04	-0.05
(f) OLS ($\lambda=\infty$), domestic, $p=2$						
All	0.84*	-0.18*	-0.30	0.53*	-0.00	-0.05
Advanced	0.68*	-0.13*	-0.06	0.40*	-0.01	0.04
Developing	0.99*	-0.26*	-0.60	0.71*	0.01	-0.11
Alternative Calibration						
(g) Higher Discounting $\beta=0.90$						
All	1.28*	-0.15*	0.07	0.50*	-0.01	-0.02
Advanced	0.80*	-0.06*	-0.06	0.37*	-0.02	-0.01
Developing	1.59*	-0.27*	0.39	0.64*	0.02	-0.04
(h) Lower Debt Persistence $\rho=0.75$						
All	1.27*	-0.16*	0.05	0.51*	-0.01	-0.01
Advanced	0.80*	-0.05*	-0.07	0.38*	-0.02	-0.01
Developing	1.54*	-0.29*	0.39	0.64*	0.02	-0.04

Notes: First column in percentages. T = tax revenues, G = public spending, i = nominal interest, Δh = real exchange depreciation, y = gross domestic product. Asterisks indicate significance at a 10% confidence level, based on 10,000 simulations from posterior distribution.

Table 3: BVAR Estimation of Unexpected Inflation: Robustness (Cross-Country Medians)

In generalizing debt instruments to include currency-linked and long-term bonds, we break the direct connection between unexpected inflation and the real value of debt established by (4). Indeed, note that if set $\delta_D = \delta_R = 0$ and $rx_t = 0$, the left-hand side of (30) simplifies to $(v/\beta)\Delta E_t \pi_t$.

In the more general case, we need to account for changes in exchange rates Δh_t and long-term bond prices, captured by excess returns rx_t. These additional terms might not be negligible: in his study of US inflation in the 1970s, Sims (2011)'s exemplifies, in a fiscal-selection model, how contractionary monetary policy can lead to a decline in short-run inflation *via* a reduction in the market price of long-term bonds.

In all, decomposition (4) says that unexpected inflation derives from changes in the real value of debt, net of variation in other debt prices.

4.2 Geometric Maturity Structure and Unexpected Total Inflation

4.3 Decomposition of the Debt Valuation Equation

Economic news lead not only to the revision of expectations of current inflation, but also of its entire future path. As Sims (2011) exemplifies with his study of the effects of monetary policy in US inflation in the 1970s, the short-run innovation can be remarkably different than the one long-run one.

(Incomplete)

We can solve the linearized law of motion of public debt (27) forward and apply assumption (**??**) to arrive at

$$v_{t-1} = \sum_{i=0}^{\infty} \beta^i E_t s_{t+i} - \sum_{i=0}^{\infty} \beta^i E_t i_{t-1+i} + \sum_{i=0}^{\infty} \beta^i E_t \pi_{t+i},$$

which generalizes (4). The inflation revaluation of public debt term (the denominator on the left side of (4)) corresponds to the first term of the inflation sum on the right-hand side. Taking innovations:

$$0 = \sum_{i=0}^{\infty} \beta^i \Delta E_t s_{t+i} - \sum_{i=0}^{\infty} \beta^i \Delta E_t i_{t-1+i} + \underbrace{\sum_{i=0}^{\infty} \beta^i \Delta E_t \pi_{t+i}}_{\substack{\text{Unexpected Total} \\ \text{Inflation}}} . \tag{22}$$

Expression (22) is from Cochrane (2022a). The sum $\sum \beta^i \Delta E_t \pi_{t+i}$ I call *unexpected total inflation*, which is really shorthand for revision of expectations over the discounted inflation path. I also refer to unexpected future surpluses and interest in a similar manner.

Decomposition (22) simply states that surprises to the relative price of public liabilities in terms of goods, captured by unexpected current inflation $\Delta E_t \pi_{t+1}$, reflect changes in the real value of public debt. Re-writting it as

$$- \underbrace{\Delta E_t \pi_t}_{\substack{\text{Unexpected Current} \\ \text{Inflation}}} = \underbrace{\sum_{i=0}^{\infty} \beta^i \Delta E_t s_{t+i} - \sum_{i=0}^{\infty} \beta^i \Delta E_t i_{t-1+i} + \overbrace{\sum_{i=1}^{\infty} \beta^i \Delta E_t \pi_{t+i}}^{\substack{\text{Unexpected Future} \\ \text{Inflation}}}}_{\text{Innovation to the Real Value of Public Debt}} . \tag{23}$$

makes that point clear. It also reminds us that equation (22) continues to be a debt valuation equation, not a budget constraint. Yet, the use of language such as "higher total inflation *pays* for lower total surpluses" can often simplify the exposition.

Policy Shock	$\sum \beta^i \Delta E_t \pi_{t+i}$	$\sum \beta^i \Delta E_t s_{t+i}$	$-\sum \beta^t \Delta E_t i_{t-1+i}$
Fiscal Adjustment via Taxes ($\alpha_\tau > 0$, $\alpha_g = 0$)			
Unexpected inflation η_t	1.46	-1.46	0
(-) Tax revenue ϵ_τ	0	0	0
Government spending ϵ_g	-2.06	2.06	0
(-) Monetary policy ϵ_i	-0.66	-0.32	0.98
Fiscal Adjustment via Spending ($\alpha_\tau = 0$, $\alpha_g > 0$)			
Unexpected inflation η_t	1.46	-1.46	0
(-) Tax revenue ϵ_τ	-0.02	0.02	0
Government spending ϵ_g	-2.17	2.17	0
(-) Monetary policy ϵ_i	-0.66	-0.32	0.98
Fiscal Adjustment via Taxes + Taylor Rule ($i_t = \phi\pi_t$)			
Unexpected inflation η_t	2.49	-1.27	-1.22
(-) Tax revenue ϵ_τ	0	0	0
Government spending ϵ_g	-3.9	2	1.92
(-) Monetary policy ϵ_i	-1.24	-0.35	1.59

Table 4: Unexpected Total Inflation

Table 4 shows the decomposition for the policy shocks of NK model. Note the minus in front of the unexpected total interest; each row sums to zero. I re-calibrate $\kappa = 3.8$ to a more realistic value, since the quantitative aspect is more relevant now.[15]

The first panel corresponds to the case of figure (5). Taxes respond to real debt variation, not spending. The 1% unexpected current inflation shock leads to a roughly 1.5% increase in unexpected total inflation. Bondholders pay for the unexpected decline in total surpluses.

The taxation shock leads to a "zero-zero-zero" decomposition as inflation and interest are unchanged, and future taxes pay for the current negative shock. A 1% increase in government spending leads to a *positive* unexpected total surplus of about 2%, which pay for the unexpected total *deflation*. Finally, a 1% unexpected decline in interest creates fiscal space consumed by lower total inflation and surpluses in a two-to-one ratio. The reported measures quantify how the three expansionary policy shocks, which fail to create unexpected current inflation by assumption, actually create unexpected total *disinflation*, by result.

The second panel considers the case of debt stabilization via changing expenditure g. I set $\alpha_g = 0.07$, so that the decomposition of the unexpected inflation shock is about the same. Switching the variable of adjustment does little to change the decomposition of the other shocks.

The third panel returns to tax adjustment with the same $\alpha_\tau = 0.2$, but includes a more realistic Taylor rule to monetary policy $i_t = \phi\pi_t$. I use $\phi = 0.50$. Active monetary policy leads to larger reactions of each term of the decomposition to our policy shocks in comparison with the baseline case. Results also reveal the Fisherian character of the NK model. Unexpected total inflation and unexpected total interest have the same signal in all cases.

[15] I calibrate $\kappa = (1-\theta)(1-\beta\theta)/\theta$ using the price rigidity parameter $\theta = 0.65^4$ estimated by Smets and Wouters (2007), adjusted for annual frequency.

5 Theoretical Benchmarks: The SOE-NK Model

5.1 Model Equations and Monetary Policy

5.2 Optimal Unexpected Inflation

5.3 Two Extensions

6 Sensitivity to Policy Rules

6.1 Rules and Optimal Inflation

6.2 A Ricardian Equivalence Result for Unexpected Total Inflation

As (re-)stated by Barro (1974), the Ricardian Equivalence theorem says that different taxation plans fail to change households' perception of their own wealth, or the real value of debt, which is pinned down by the stock of bonds inherited from the previous period and the assumption of debt sustainability.[16] Thus they do not affect households' consumption path. The value of debt cannot jump unexpectedly at the beginning of any period.

In the environment with nominal debt, that is not case: the real value of public debt suddenly changes, and equation (23) shows that such change is given by unexpected current inflation. Therefore, in the NK model, that aspect of Ricardian Equivalence fails to hold, which is why early studies of the fiscal selection mechanism referred to it as being "Non-Ricardian" (although, again, (4) and (23) do not depend on the selection mechanism).

However, under one key condition, the second and often more celebrated aspect of Ricardian Equivalence does hold in the NK model. (For the following statements, we can generalize the interest rate policy to a Taylor Rule $i_t = \phi_1 \pi_t + \phi_2 y_t + \epsilon_{i,t}$.)

Proposition 1. *Given any exogenous innovation vector to the NK model, if $\alpha_g = 0$, the equilibrium paths of consumption, inflation, output and interest rates do not depend on ρ_g, ρ_τ or α_τ. The result only holds for combinations of parameters the lead to unique, stationary equilibria of the NK model.*

Corollary 1 (Ricardian Equivalence of Total Inflation)**.** *Under the assumptions of proposition 1, the innovation terms of the debt value decompositions* (22) *and* (23) *do not depend on ρ_g, ρ_τ or α_τ.*

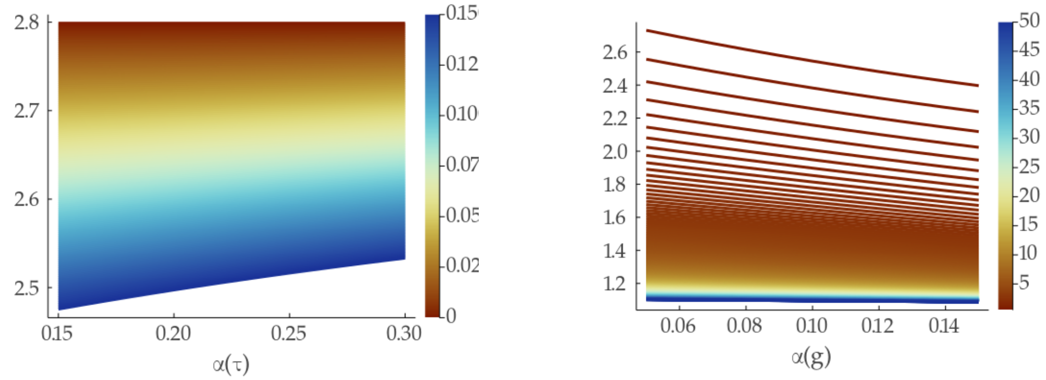
The proof of proposition 1 is straightforward: if $\alpha_g = 0$, the system of equations (24)-(26), (29) plus the interest rule, by itself, has a unique, stationary solution. The solution of the overall system is unique by assumption, so they must coincide.

In addition, since any equilibrium path of the linearized NK model is given by an initial condition plus the responses of each sequence of innovations (the model is linear), proposition 1 essentially says that equilibrium paths do not depend on ρ_g, ρ_τ or α_τ. So, even if the value of public debt fluctuates in the NK model: 1. unexpected current inflation is a sufficient statistic for all other changes of behavior due to re-valuation of public debt; and 2. the conclusion of the Ricardian Equivalence theorem still applies: parameters of debt-repayment timing ρ_g, ρ_τ, α_τ do not affect households' consumption decision.

The key condition that proposition 1 asks is that $\alpha_g = 0$.

(Incomplete)

[16] In the environemnt where Ricardian Equivalency is usually stated, debt is real. Thus, debt sustainability means a no-default condition.



(a) UTI as function of α_τ for different α_g

(b) UTI as function of α_g for growing κ

Figure 4: Ricardian Equivalence of Unexpected Total Inflation

For this reason, when considering only stable, unique solutions, as $\kappa \to \infty$, the terms of the inflation decomposition become less sensitive to *all* parameters governing tax and spending (including α_g).

(Incomplete)a

7 Conclusion

Path forward:

- Consider variation in risk-premia, particularly important for emerging markets

References

Ali Abbas, S. M., Belhocine, N., El-Ganainy, A., and Horton, M. (2011). Historical Patterns and Dynamics of Public Debt—Evidence From a New Database. *IMF Economic Review*, 59(4):717–742.

Andrieu, C. and Thoms, J. (2008). A tutorial on adaptive MCMC. *Statistics and Computing*, 18(4):343–373.

Barro, R. J. (1974). Are Government Bonds Net Wealth? *Journal of Political Economy*, 82(6):1095–1117.

Blanchard, O. J. and Kahn, C. M. (1980). The Solution of Linear Difference Models under Rational Expectations. *Econometrica*, 48(5):1305–1311.

Bohn, H. (1998). The Behavior of U. S. Public Debt and Deficits. *The Quarterly Journal of Economics*, 113(3):949–963.

Campbell, J. Y. and Ammer, J. (1993). What Moves the Stock and Bond Markets? A Variance Decomposition for Long-Term Asset Returns. *The Journal of Finance*, 48(1):3–37.

Canzoneri, M. B., Cumby, R. E., and Diba, B. T. (2001). Is the Price Level Determined by the Needs of Fiscal Solvency? *American Economic Review*, 91(5):1221–1238.

Cochrane, J. H. (1998). A Frictionless View of U.S. Inflation. *NBER Macroeconomics Annual*, 13:323–384.

Cochrane, J. H. (2005). Money as stock. *Journal of Monetary Economics*, 52(3):501–528.

Cochrane, J. H. (2011). Determinacy and Identification with Taylor Rules. *Journal of Political Economy*, 119(3):565–615.

Cochrane, J. H. (2022a). The fiscal roots of inflation. *Review of Economic Dynamics*, 45:22–40.

Cochrane, J. H. (2022b). A fiscal theory of monetary policy with partially-repaid long-term debt. *Review of Economic Dynamics*, 45:1–21.

Cox, W. M. (1985). The behavior of treasury securities monthly, 1942–1984. *Journal of Monetary Economics*, 16(2):227–250.

Cox, W. M. and Hirschhorn, E. (1983). The market value of U.S. government debt; Monthly, 1942–1980. *Journal of Monetary Economics*, 11(2):261–272.

del Negro, M. and Schorfheide, F. (2011). Bayesian Macroeconometrics. In Geweke, J., Koop, G., and Van Dijk, H., editors, *The Oxford Handbook of Bayesian Econometrics*, pages 292–389. Oxford University Press.

Fernández, A., Schmitt-Grohé, S., and Uribe, M. (2020). Does the Commodity Super Cycle Matter? Technical Report w27589, National Bureau of Economic Research, Cambridge, MA.

Garín, J., Lester, R., and Sims, E. (2018). Raise Rates to Raise Inflation? Neo-Fisherianism in the New Keynesian Model. *Journal of Money, Credit and Banking*, 50(1):243–259.

Giannone, D., Lenza, M., and Primiceri, G. (2015). Prior Selection for Vector Autoregressions. *The Review of Economics and Statistics*, 97(2):436–451.

Jiang, Z., Lustig, H., Van Nieuwerburgh, S., and Xiaolan, M. (2019). The U.S. Public Debt Valuation Puzzle. Technical Report w26583, National Bureau of Economic Research, Cambridge, MA.

Karlsson, S. (2013). Forecasting with Bayesian Vector Autoregression. In *Handbook of Economic Forecasting*, volume 2, pages 791–897. Elsevier.

King, R. G. and William, K. (1996). Limits on Interest Rate Rules in the IS Model. *FRB Richmond Economic Quarterly*, 82(2):47–75.

Klein, P. (2000). Using the generalized Schur form to solve a multivariate linear rational expectations model. *Journal of Economic Dynamics and Control*, 24(10):1405–1423.

Litterman, R. (1979). Techniques of forecasting using Vector Auto Regression. *Federal Reserve Bank of Minneapolis Working Paper*, 115.

Mendoza, E. G. (2010). Sudden Stops, Financial Crises, and Leverage. *The American Economic Review*, 100(5):1941–1966.

Neumeyer, P. A. and Perri, F. (2005). Business cycles in emerging economies: The role of interest rates. *Journal of Monetary Economics*, 52(2):345–380.

Sims, C. A. (1980). Macroeconomics and Reality. *Econometrica*, 48(1):1–48.

Sims, C. A. (2011). Stepping on a rake: The role of fiscal policy in the inflation of the 1970s. *European Economic Review*, 55(1):48–56.

Smets, F. and Wouters, R. (2007). Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach. *The American Economic Review*, 97(3):586–606.

Taylor, J. B. (1993). Discretion versus policy rules in practice. *Carnegie-Rochester Conference Series on Public Policy*, 39:195–214.

Uctum, M., Thurston, T., and Uctum, R. (2006). Public Debt, the Unit Root Hypothesis and Structural Breaks: A Multi-Country Analysis. *Economica*, 73(289):129–156.

Uribe, M. (2022). The Neo-Fisher Effect: Econometric Evidence from Empirical and Optimizing Models. *American Economic Journal: Macroeconomics*, 14(3):133–162.

Yoon, G. (2012). War and peace: Explosive U.S. public debt, 1791–2009. *Economics Letters*, 115(1):1–3.

8 Unexpected Inflation in a Benchmark NK Model

8.1 The New-Keynesian Model

I start with the two usual equations of the New-Keynesian model. All variables should be interpreted as deviations from a steady-state equilibrium.

$$c_t = E_t c_{t+1} - \sigma \left[i_t - E_t \pi_{t+1} \right] \tag{24}$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa y_t \tag{25}$$

along with an equation for market clearing for goods market:

$$y_t = \gamma c_t + g_t, \tag{26}$$

where y, c, i, π and g represent respectivelly log-output, log-consumption, the interest rate, the inflation rate and government spending in levels.[17]

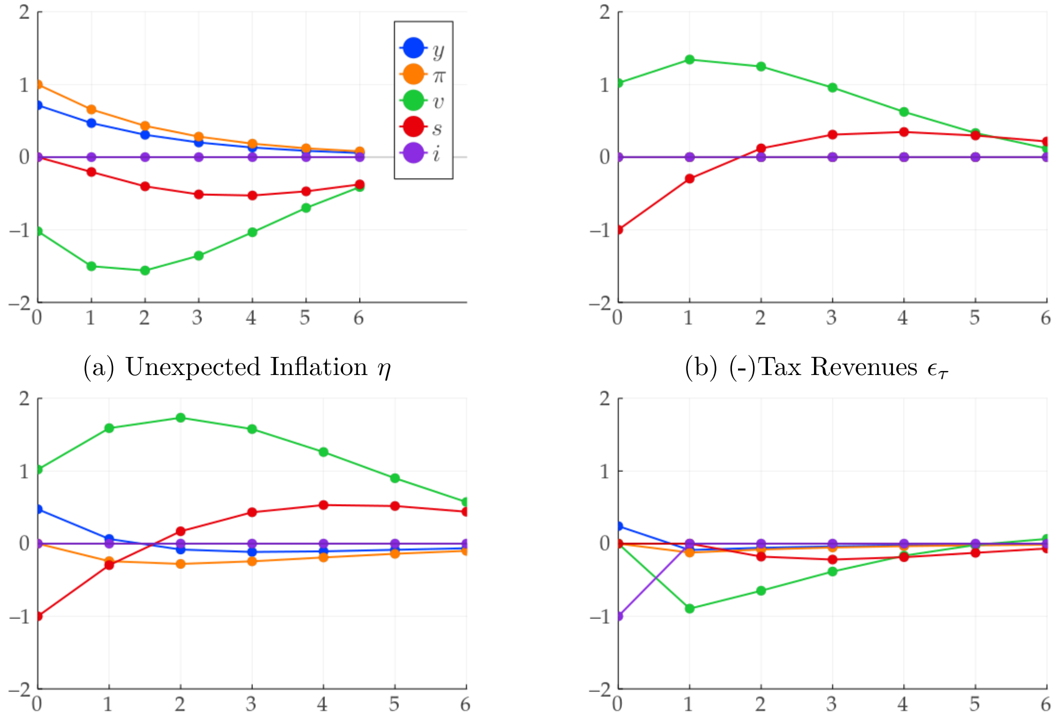
The stock of real public debt v follows the linearized version of the law of motion (2) (for $k = 0$): (reference to the use of rational expectations - use of E, not $\tilde{E}$, reference to why assumption **??** above holds here)

$$\beta v_t = v_{t-1} + i_{t-1} - \pi_t - s_t \tag{27}$$

where $s_t \equiv \tau_t - g_t$ is the public primary surplus (which does not include interest payments on debt). τ_t are total tax proceeds in levels. In the stationary equilibrium of the NK model assumption **??** above holds, and, hence, v coincides with the real value of public debt.

Policy. *Observed* monetary policy is muted, except for a white-noise shock: $i_t = \epsilon_{i,t}$.

[17] γ represents the steady-state consumption-to-output ratio. The choice of linearizing equilibrium conditions around the level of government spending and not its log makes the connection with the rest of the paper clearer. I also linearize around an equilibrium with output = real debt.



(a) Unexpected Inflation η

(b) (-)Tax Revenues ϵ_τ

(c) Government Spending ϵ_g

(d) (-)Nominal Interest ϵ_i

Figure 5: NK Model - Impulse-Response Function to Policy Shocks

Fiscal policy prescribes the following rules for taxation (which I assume to be entirely *lump-sum*) and public expenditures:

$$\tau_t = \rho_\tau \tau_{t-1} + \alpha_\tau v_t + \epsilon_{\tau,t} \tag{28}$$

$$g_t = \rho_g g_{t-1} - \alpha_g v_t + \epsilon_{g,t}. \tag{29}$$

Stability of public debt requires either $\alpha_\tau > 0$ or $\alpha_g > 0$.

8.2 Unexpected Current Inflation

Consider the response of the New-Keynesian model to η, and ϵ shocks, one at a time, plotted in figure 5. Calibration follows literature standards: $\sigma = 0.5$, $\beta = 0.98$, $\gamma = 0.75$ and $\rho_\tau = \rho_g = 0.5$. Momentarily, I set $\kappa = 0.50$ to make figures pretty. In this benchmark case, I consider fiscal adjustment via taxation only: $\alpha_\tau = 0.2$ and $\alpha_g = 0$.

Panel 5a plots the response to the unexpected inflation shock η. Inflation jumps by assumption, the fiscal interpretation being that agents foresee a reduced stream of surpluses. Accordingly, the real value of public debt v jumps down on spot. A lower debt leads taxation τ to decline (not plotted) via the $\alpha_\tau v_{t-1}$ term. The government runs deficits starting in the first period following the shock (I refer to $s < 0$ as a fiscal deficit). These deficits 1. slowly bring v back to zero and 2. validade agents' expectation at period zero of a lower value of public bonds - indeed primary surpluses were lower.

The impact on economic activity resembles the typical Keynesian "demand" shock, combining an increase in inflation and output at the same time. Positive inflation in period zero leads to a negative real interest rate; the IS curve (24) then implies output larger than future output -

output is large and declining.[18] Large output implies large marginal costs, and, by (25), inflation greater than future inflation - inflation is thus positive and declining.

Panels 5b and 5c show that, in the absence of unexpected inflation in period zero, expansionary fiscal policy fails to generate inflation at all in the basic NK model. A negative shock to taxation - the model version of COVID checks - simply leads to an increase in public debt, subsequently paid through taxes that turn positive in period two. Households are unconstrained and have zero marginal propensity to consume out of their checks. Output thus stays put, which implies $\pi_t = E_t \pi_{t+1}$ by the Phillips curve. $\pi_0 = 0$ follows from the absence of unexpected inflation.

A positive shock to public spending g does affect output and inflation, as the government directly purchases goods from firms (equation (26)). We can think of government spending as a transfer to a fictional "public household" with constant marginal propensity to consume equal to one. Output increases in period zero. The Phillips curve then says that current inflation must be greater than future. But since current inflation is zero (no unexpected jump by assumption), that means inflation *declines* from period zero to one. In the absence of unexpected inflation, the NK model predicts below-average inflation, or even deflation, as a consequence of increased public expenditure.

Lastly, panel 5d corresponds to an expansionary monetary policy shock. Without unexpected inflation, the effect of a monetary policy shock is purely Fisherian (references of Fisherian interest shock): lower interest forecasts lower inflation. Stimulative interest does stimulate output, albeit for a single period, as low inflation produces a contractionary effect thereafter.

Figure 5 assumes fiscal adjustment is carried out entirely through tax instead of spending adjustments ($\alpha_\tau > 0$, $\alpha_g = 0$). The symmetric opposite assumption little changes the predictions of the model. The only qualitative change happens in the case of the tax reduction shock ϵ_τ. Since government spending changes with debt, there are small output effects that lead to lower inflation in the transition - again the "wrong" sign. The quantitative effects are nevertheless small, and I leave the IRF figures to the appendix.

8.3 Unexpected Inflation as a Choice

Section will probably be cut

8.3.1 Relationship with Sims (1980) Orthogonalization

The view that unexpected inflation is a choice automatically microfounds the orthogonalization process proposed by Sims (1980). Inflation should come first in the VAR, since the government need not react to other structural shocks.

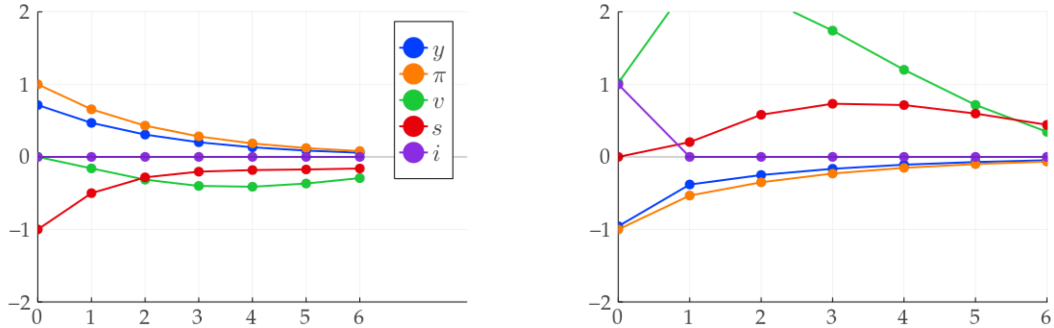
(Incomplete)

8.3.2 Combining Policy Shocks

The understanding of unexpected inflation as a integrand part of public policy opens the question of how it relates to other policy choices. The correlation with other policy shocks (the ϵ's) is particularly important in the NK model.

The IRF and the decomposition of the debt valuation equation show that having policy shocks be accompanied by unexpected current inflation is critical for the NK model to deliver

[18] The apparently small response of output follows from the choice of κ. Values that are lower than my choice lead to more pronounced responses of equal sign.



(a) ...COVID checks and inflation (b) ...and the monetary-fiscal effort to fight it

Figure 6: Combining shocks in the NK Model successfully reproduces...

responses consistent with most economists' view of the effects of "stimulative" policy on the price level and empirical evidence. (PAPERS WITH IDENTIFIED FISCAL SHOCKS)

(Incomplete)

In light of the connection between inflation and the value of debt, one might even expect η and the ϵ's to be correlated. Say, unexpectedly large spending leads to lower surpluses, hence a lower value of debt, hence unexpected inflation. However intuitive, the claims requires empirical inquiry. This is where we go next.

9 Empirical Models

I study two Bayesian estimation that differ in the information contained in the choice of prior. (Incomplete)

9.0.1 Unexpected Current Inflation

9.1 A Tighter Prior: Estimates of Unexpected Total Inflation

In general, public debt data will not respect the laws of motion (**??**) and (**??**) for a few reasons. Government agencies in the vast majorities of cases report data on the book value of debt, while the theoretical law of motion of public debt (**??**) leads to expressions that involve the *market value* of debt. Moreover, measurement errors related to changing accounting conventions, applicability of different rates to the accrual of debt than the short-term interest rate we use, incorrect specification of the term and currency strucuture of debt and so on.

Yet, we can use knowledge of the dynamics of public debt to refine our parameter search by asking that it implies *debt sustainability*. More deeply, nothing guarantees that the estimated VAR (**??**) leads to a stable path for a variable with an explosive eigenvalue as introduced by (**??**). Any model that predicts an explosive debt dynamics while theoretically possible, does not satisfy the basic assumption **??** I make in this paper. Automatically, the decomposition of the valuation equation does not hold as in (22): the three terms do not necessarily sum to zero.

Therefore, at this point, I explicitly assume that assumption **??** holds for all economies considered in this empirical exercise. In practice, that means we can change our prior to filter out of the estimation combinations of parameters that do not lead to a stable debt dynamics. In the second empirical exercise, I pursue this variation of the estimation procedure.

Sadly, in breaking the VAR format (**??**), we can no longer take advantage of the convenience provided by the conjugate Minnesota prior and its closed-form solution.

Debt Value Decomposition.. The generalized law of motion of public debt leads to a different decomposition of the valuation equation (22). Solve (**??**) for forward, apply assumption **??** and take innovations to arrive at the following expression.

$$
\begin{aligned}
0 = &\sum_{i=0}^{\infty} \beta^i \Delta E_t s_{t+i}^p \\
&- \frac{v}{\beta}\left[\sum_{i=0}^{\infty}\sum_{j\neq N} \beta^i \delta_j \Delta E_t r_{j,t+i} + \delta\left(\Delta E_t rx_t + \sum_{i=0}^{\infty}\beta^i \Delta E_t i_{t-1+i} - \underbrace{\sum_{i=0}^{\infty}\beta^i \Delta E_t \pi_{t+i}}_{\text{Unexpected Total Inflation}}\right)\right]
\end{aligned} \tag{30}
$$

In (30), $s_t^p = s_t + v(p_t^s - p_t)$ is the price-adjusted surplus deviation and $r_{j,t} = rx_{j,t} + i_{j,t-1} + \Delta h_{j,t} - \pi_{j,t}$ is the *ex-post* real return on holdings of the j-currency portfolio, in domestic currency.[19] I simplify notation by setting, $rx_t = rx_{1,t}$ $i_t = i_{1,t}$, $\pi_t = \pi_{1,t}$.

Again, we can highlight the innovation to the real value of public debt at the beginning of period t.

$$
\begin{aligned}
\frac{v}{\beta}\left[\sum_{j\neq N}\delta_j \Delta E_t r_{j,t} + \delta\left(\Delta E_t rx_t - \overbrace{\Delta E_t \pi_t}^{\text{Unexpected Current Inflation}}\right)\right] = &\sum_{i=0}^{\infty}\beta^i \Delta E_t s_{t+i}^p \\
&+ \frac{v}{\beta}\left[\sum_{i=1}^{\infty}\sum_{j\neq N}\delta_j \beta^i \Delta E_t r_{j,t+i} + \delta\left(\sum_{i=1}^{\infty}\beta^i \Delta E_t i_{t-1+i} - \underbrace{\sum_{i=1}^{\infty}\beta^i \Delta E_t \pi_{t+i}}_{\text{Unexpected Future Inflation}}\right)\right]
\end{aligned} \tag{31}
$$

Currency-linked and long-term maturity debt breaks the equality (23) between unexpected current inflation and the change in the real value of debt (right-hand side of the expression above). The price of the public nominal liabilities is no longer the only variable that can translate debt sustainability. News of lower discounted surpluses can be met with lower nominal ($\Delta E_t rx_t$) or real ($\Delta E_t r_{j,t}$) bond prices. This is a key mechanism explored by Sims (2011) and Cochrane (2022b) to generate a negative response of inflation to interest rates.

[19] I also apply the assumption of this particular application to save space: $g_{Y,t} = 0$ for all t and $\Delta E_t rx_T$ for $T > t$.

Appendices

All the sections of the Appendix are unfinished.

A Equilibrium Selection in the NK Model

The environment introduced in the last subsection nests the basic NK model (26) and (25). By themselves, these two equations do not determine *unexpected inflation*:

$$\Delta E_t \pi_t = E_t \pi_t - E_{t-1} \pi_t. \tag{32}$$

There are two existing selection mechanisms that justify equation (32) and provide an interpretation to it: fiscal selection and the spiral threat selection. Both imply (32) while leaving other equations unchanged (observational equivalence, Cochrane (2011), Cochrane (1998)) and, more importantly, both interpret η as part of public policy, as a government *choice*.

Fiscal Selection. Fiscal selection, or the fiscal theory of the price level, arrives at (32) by means of (4), with causality coming from right to left. Any economic shock can change the conditional distribution of discounted future surpluses (in units of goods) backing the stock of public nominal liabilities. It can thus change its real value. The relative price of public debt in terms of goods $1/P_t$ then adjusts to reflect that change, much in the same way that news of future dividends (in currency units) per stock change the relative price of stocks in terms of currency (Cochrane (2005)). (Maybe include a reading of 2022 US inflation through the lenses of FS and STS)

Spiral-Threat Selection. Spiral threat selection is the approach that most of the monetary economics literature has adopted so far. It starts by attributing causality in equation (4) from left to right: to any price level, no matter how large or small, the government alters its surplus choice to reflect the new value of public debt. It then arrives at (32) by means of an explosive root introduced by an interest policy equation of the format $i_t = \phi \pi_t, \phi > 1$. The equation was incorrectly associated to the famous Taylor (1993) rule, for its role in the NK model is by no means to stabilize "demand" shocks via fast, pro-cyclical real interest rates. On the contrary, the policy rule here introduces the instability required by the NK model to pin down unexpected inflation. Assuming muted monetary policy $i_t = 0$, the system of equations (24)-(25) (with $c = y$ for simplicity) is "too stable": it contains one explosive eigenvalue for two forward-looking variables. Any choice of unexpected inflation forms a stable equilibrium path that converges to the zero steady state.[20] Equation $i_t = \phi \pi_t$ solves that issue when $\phi > 1$.

Importantly, the *selection of equilibrium* is completely unrelated to the *observed* interest rate. This is why $i_t =$ white noise as above is a perfectly fine specification for *observed* interest. More rigorously, consider the basic NK system (24)-(25), with $c = y$. Add to that the following equations:

$$i_t = i_t^* + \phi(\pi_t - \pi_t^*) \qquad \phi > 1 \tag{ST-1}$$

$$i_t^* - E_t \pi_{t+1}^* = i_t - E_t \pi_{t+1} \tag{ST-2}$$

$$i_t^* \text{ given for all } t \tag{ST-3}$$

$$\pi_t^* \text{ given at } t. \tag{ST-4}$$

Format (ST-1) is due to King and William (1996); i^* is the central bank's desired observed interest rate. The term π_t^* is a stochatic inflation target. Equation (ST-2) asks that the government's choices respect private market conditions and expectations formation. It forces the government to elect *unexpected* inflation only.[21]

[20] Economists have interpreted this feature as admissibility of "sunspot" shocks. Without a selection mechanism, (24)-(25) will only determine the unexpected component of one variable, *if* it is fed the unexpected component of the other.

[21] The attentive eye may have noticed an apparent modelling sin: system (24)-(25), (ST-1)-(ST-4) presents six equations, for only five variables: y, π, π^*, i, i^*. There is no over-identification, nevertheless. Target inflation enters

Mechanically, one can combine (ST-1) and (ST-2) to find $E_t\pi_{t+1} - E_t\pi^*_{t+1} = \phi(\pi_t - \pi^*_t)$; $\phi > 1$ and Blanchard and Kahn (1980)'s razor then champion the unique stationary path $\pi = \pi^*$, $i = i^*$, which form the *observed* equilibrium. Parameter ϕ remains unidentified (Cochrane (2011)).

Researchers have interpreted (ST-1) as a threat of nominal spiral - hence my name choice "spiral threat" selection. Different papers discuss if central banks can indeed rule out nominal spirals, but the key assumptions here do not really relate to what the central bank can do, but what *households believe* it can and would. Indeed, note that there is nothing particularly special about inflation in (ST-1)-(ST-4). One could as well write the whole system using an output target instead:

$$i_t = i^*_t + \phi(y_t - y^*_t) \qquad \phi > 1 \tag{ST-1'}$$

$$i^*_t - E_t y^*_{t+1} = i_t - E_t y_{t+1} \tag{ST-2'}$$

and now the "threat" is not that of a nominal spiral, but of a *real* spiral. Obviously, the central bank cannot trigger a "hyperproduction" (as in hyperinflation) process. Neither could it stop one, say if productivity for some reason started to grow at abnormal rates. But, if the central bank vacuously threatens hyperproduction, and it is the case that agents believe its threat; and if then the central bank vacuously promisses to stop the hypothetical hyperproduction it has vacuously threatened to create, and again agents trust its word; then and only then does the Blanchard and Kahn (1980) equilibrium arranged by (ST-1')-(ST-2') arises. The actual powers of the central bank are irrelevant.

While I favor a fiscal selection interpretation of unexpected inflation throughout the article, the takeaway from this discussion is that both equilibrium selection mechanisms interpret as a government *choice* - even if an indirect one - the determination of unexpected inflation.

B Linearization

C Data Sources and Treatment

Calulation of GDP trend.

Table with data sources for each country.

Report parameters of public debt structure. List of sources for public debt structure.

D Deriving the SOE-NK Model

E Additional Details of the BVAR Estimation

Show the expressions for $\varphi_{j,0}$, $\varphi_{j,1}$ and ζ. Closed-form solution of the BVAR posterior and marginal likelihood. Show the decomposition of the marginal distribution by Giannone et al. (2015).

F A Rational-Expectations Model with Observed Surpluses

OLS Regression on the wedge between book and market value of debt, to estimate ζ, ρ and σ_v.

Figure with data and conditional expectation for a few countries.

Details of the Metropolis sample.

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the system both as a static (= forward-looking) variable π^*_t *and* as a state variable, in expected value $E_{t-1}\pi^*_t$. Another way to write (ST-3) would be $E_{t-1}\pi_t = i^*_t - (i_t - E_{t-1}\pi_t)$. It becomes evident then that (ST-4) only really picks the unexpected component of inflation.

F.1 The Rational-Expectations Model

In the second empirical exercise, I keep the equations in (**??**) except for the debt equation. In its place, I introduce the law of motion (**??**) for the market value of debt and the equations

$$
\begin{aligned}
v_t^b &= v_t + u_t \\
u_t &= \rho u_{t-1} + \zeta v \delta \left(i_t - i_{t-1}\right) + \epsilon_{v,t}
\end{aligned}
\tag{33}
$$

for the observed book value of debt v_t^b. The market value of debt v_t is not observed. The second equation is motivated by the fact that the market and book value of debt tend to differ in periods of changing nominal interest.

I assume a constant risk premium for all debt portfolios: $E_t \pi_{j,t+1} = 0$, and let equation (15) determine its price. Equation (14) defines the excess return.

I assume the government issues real bonds with interest rates consistent with agents' expectations: $i_{2,t} = i_t - E_t \pi_{t+1}$. Finally, we need a law of motion for the dollar interest-inflation pair $x_t^{US} = [i_t^{US}, \pi_t^{US}]$. I use a simple two-equation VAR $x_t^{US} = \psi x_{t-1}^{US} + \epsilon_{US,t}$.

In total, the rational-expectations model contains twenty-two equations and ten shocks.[22] The ten shocks in $\epsilon = [\epsilon_{-v,t}, \epsilon_{v,t}, \epsilon_{US,t}]$ contain: the shocks to the seven reduced-form equations ($\epsilon_{-v,t}$), the shock to the book value of debt ($\epsilon_{v,t}$) and the two shocks to US variables ($\epsilon_{US,t}$).

Given a set of parameters that lead to a unique and stationary equilibrium, I find the solution in state-space representation

$$
x_t = \Phi x_{t-1} + \Gamma \epsilon_t
$$

using Klein (2000)'s method.

With a solution, I compute the data likelihood using the Kalman filter. The dataset is the same as in the no-intercept BVAR of the previous section.[23]

US case. (Incomplete)

F.2 Steady State and Fixed Parameters

I do not use an intercept in the estimation. Instead, I transform all variables into deviations from a steady state.

In the steady state, public debt corresponds to the average public debt in the dataset (country by country), the same is true for government spending, the relative price of public basket, interest, inflation and gross domestic product. The steady-state surplus must be consistent with public debt, so I set $s = (1-\beta)v/\beta$. Steady-state taxation follows: $T = G + s$. Finally, the steady-state variation in real exchange rate is zero.

A subset of the model's parameters are fixed. Steady-state real discouting β I fix at 0.98. The currency and term structure parameters of public debt (the δ's and ω's) for each country I collect from a myriad of sources compising OECD panel data, official websites and individual government reports. All sources are listed in the appendix.

The parameters of equation (33), $\rho \approx 0.76$, $\zeta \approx 0.23$, $\text{std}(\epsilon_v) \approx 1.17$, I estimate using US data, a case in which both book and market values of debt available.[24]

[22] The seven reduced-form equations (for inflation, interest, the relative price of the public basket, GDP, currency depreciation, tax proceeds and public spending), the law of motion for debt, the two equations in (33) for the book value of debt, equations (15), (14) and $E_t rx_{j,t+1} = 0$ for the three portfolios, the short-term real interest $i_{2,t}$ definition, and the two-equation VAR for US interest and inflation. In practice, the solution contains a few more auxiliary variables to lag q and π, which show up with subscripts t and $t-1$ in the model.

[23] The Gaussian distribution for the initial state of the chain is centered around zero, with the covariance matrix equal to the unconditional covariance matrix of the system (hence dependent on estimated parameters).

[24] The Dallas Fed provides estimates of the market value of debt at https://www.dallasfed.org/research/econdata/govdebt. They are calculated by Jonah Danziger and Tyler Atkinson, using the methodology in Cox and Hirschhorn (1983) and Cox (1985).

The parameter of the US VAR ψ and $\text{cov}(\epsilon_{US})$ I estimate by OLS. They are the same for all countries, with the exception on the US itself.

F.3 Adjusting the Prior Distribution

The new format of the model requires a few adaptations to the Minnesota Prior.

First, there is no reason to believe that the ϵ_v which affects the discrepancy between book and market value of debt *after* controlling for interest variation has any correlation with the other shocks of the model. Sadly, the Inverse-Wishart distribution does not offer enough flexibility to control the variance of each element of the square matrix individually. For this reason, in the new prior distribution $\epsilon_{-v,t}$ continues to follow a IW distribution, centered around the identity as before, with $d = N + 2$ and $N = 7$; and the correlation between shocks in $\epsilon_{-v,t}$ and $\epsilon_{v,t}$ is zero. The prior for the correlation between $\epsilon_{-v,t}$ and $\epsilon_{US,t}$ is a uniform with limits plus and minus one.

The prior distribution for the autoregressive parameters of the seven variables in the reduced-form VAR is the same as before, but slightly altered to ensure that its mode leads to a stable model. I set the loading of tax proceeds on public debt to 0.025; and set the loading of public spending to -0.025. The tightness parameter λ is the one that maximizes the marginal likelihood of the BVAR model for steady-state deviation (the same I use in the "no-intercept" case in table 3).

Lastly, I attribute zero density to parameters that lead to solutions with unrealistically large term in decomposition (30). Specifically, the terms of the decomposition are $C(I - \beta\Phi)^{-1}\Gamma\epsilon_t$ for a properly specified C (see the appendix). For any ϵ in the unit circle, I require $\|C(I - \beta\Phi)^{-1}\Gamma\epsilon\|_2 < M$, where $\|.\|_2$ is the Euclidian norm.[25] I set $M = 10$, which binds the estimation in the case of six countries.

F.4 Unexpected Total Inflation

For each country, I use a Metropolis-type adaptive sampelr to draw from the posterior distribution. Following Andrieu and Thoms (2008), the algorithm randomly alternates between three different update procedures of the parameter vector. All increments are symmetric and centered around the previous draw. They can update the entire parameter vector, a single entry or a given *direction* determined by principal component decomposition. The covariance matrices are updated after each new draw, so that the average acceptance rate is 20%. To ensure convergence to the asymptotic distribution, adaptation eventually vanishes. I provide details of the algorithm in the appendix.

[25] The condition can be summarized by requiring that the matrix norm induced by the vector 2-norm of $C(I - \beta\Phi)^{-1}\Gamma$ is bounded by M.

Median	Inflation	Primary Surplus	Nominal Interest	Excess Return	Real Debt
Unexpected Current Inflation ($\Delta E_t \pi_t = \epsilon_{\pi,t} = 1$)					
All	0.06	-0.38	0.56	-0.21	0.01
Advanced	0.34	-0.66	0.56	-0.22	0.02
Developing	-0.19	-0.21	0.68	-0.21	0
Lower Taxation ($\epsilon_{T,t} = -1$)					
All	0.39	0.18	-0.64	0	-0.03
Advanced	0.48	0.25	-1.10	0.15	0
Developing	0.07	-0.04	-0.11	-0.05	-0.15
Higher Public Spending ($\epsilon_{G,t} = 1$)					
All	0.30	-0.41	-0.98	0.23	0
Advanced	0.67	-0.17	-1.68	0.78	-0.13
Developing	0.10	-0.53	0.26	-0.15	0.18
Looser Monetary Policy ($\epsilon_{i,t} = -1$)					
All	-0.04	0.18	0.15	-0.34	0
Advanced	0.35	0.16	0.09	-0.54	0
Developing	-0.05	0.21	0.20	-0.20	0
Recession ($\epsilon_{gdp,t} = -1$)					
All	0	0.15	-0.16	-0.01	-0.01
Advanced	-0.28	0.24	0.22	-0.09	-0.01
Developing	0.58	0.10	-0.59	0	-0.02

Table 5: Debt Law Prior and Value Decomposition (Cross-Country Medians)