The Fiscal Decomposition of Unexpected Inflation Volatility: International Estimates

Livio Maya

Abstract

I estimate a variance decomposition of unexpected inflation for a set of twenty-five countries using a Bayesian VAR. The decomposition follows from the valuation equation of public debt. Unexpected inflation volatility must be accounted for by news about: surplus-to-GDP, real discounting, GDP growth, future inflation or, in the presence of dollar-linked debt, real exchange and US inflation. Future inflation, which translates current monetary policy, reduces unexpected inflation volatility in all countries. Contributions from discounting and output growth variation are quantitatively large in the majority of cases: unexpected inflation does not need to be accompanied by news of primary surpluses. I study the decomposition following an "aggregate demand" recession shock and a real depreciation shock; and compare empirical findings with an estimated open-economy New-Keynesian model with active fiscal policy.

1 Introduction

In the absence of financial bubbles, the real market value of the stock of public debt equals its intrinsic value, discounted surpluses:

$$\text{Market Value of Debt (Bond Prices) / Price Level} = \text{Intrinsic Value (Discounting, Surpluses).}$$

Such valuation equation gives rise to decomposition of unexpected inflation, defined as the difference between realized and expected inflation. Surprise movements in the price level must be accounted for by news about bond prices, fiscal surpluses or real discounting. Unexpected inflation can only exist (*i.e.*, have non-zero variance) if it covaries with, and therefore forecasts, one or more of these variables.

I estimate a Bayesian-VAR for a set of twenty-five advanced and developing countries and use the estimated models to compute the terms of the decomposition. I then compare my results with those implied by a small-open-economy New-Keynesian (NK) model.

Empirically, I find that real discounting accounts for unexpected inflation as much as, and in many cases *more than*, surpluses. Higher discounting reduces the intrinsic value of public debt in the same way it reduces the value of other asset prices. In addition, the contribution of surpluses often translate news of GDP growth and economic activity, and not necessarily of surplus-to-GDP ratios and fiscal policy.

What about bond prices? The role of bond price variation in the decomposition depends critically on the currency in which bonds are issued. In my setup, I allow governments to issue nominal, inflation-linked and dollar-linked bonds, a generalization not yet explored by previous literature, but necessary to describe the case of many economies, developing ones in particular.

Nominal debt tends to work as a buffer for unexpected inflation. Central banks react to inflation outbreaks by raising interest. Lower bond prices reduce the market value of debt and thus absorb part of the inflationary impact of changing discounted surpluses. This pattern is common to all countries in the sample. On the other hand, for all countries but one, lower bond prices forecast higher *future* inflation. The evidence then illustrates how monetary policy can efficiently smooth unexpected inflation over time.

The role of inflation and dollar-linked bond prices is not as clear. On the one hand, since inflation cannot devalue real bonds, more inflation is required

Given that the valuation equation of public debt holds in virtually any micro-founded macroeconomic model, including the canonical three-equation NK model, these empirical results lead to an important proposition. That the valuation equation links the price level to public budgets does *not* imply that unexpected inflation has to be observationally associated with news about primary surpluses and fiscal policy.

2 Unexpected Inflation Decomposition

2.1 Connection with the Value of Debt

I start by establishing the connection between inflation and the value of debt. Consider an economy with a homogeneous final good which households value. The economy has a government that manages a debt composed by bonds that pay one unit of currency in the next period. Currency is a commodity that only the government can produce, at zero cost. There is nothing special about currency. Households do not value it, and it provides no convenience for trading. Households cannot burn currency.

At the beginning of period t, the face value of debt issued in the previous period is V_{t-1}. The government redeems it for currency, which moves to the hands of households. After redeeming bonds, the government announces that each household i must pay $T_{i,t}$ goods in taxes, payable in currency. It also announces it will sell V_t new bonds and purchase G_t units of the final good at market prices.[1]

Nothing binds the government's choices of $T_{i,t}$ and V_t. Note the difference from the case with bonds payable in goods, in which the government *must* raise tax revenue or issue new debt to redeem old bonds. Additionally, if households accept currency as payment for final goods, nothing binds the government's choice of G_t either.

Let M_t be private holdings of currency at the end of t. As there is no free disposal of currency, the quantity used by the government to redeem $t-1$ bonds and purchase goods is used to either pay taxes, buy new bonds or increase currency holdings:

$$\begin{aligned} V_{t-1} + P_t G_t &= P_t T_t + Q_t V_t + \Delta M_t \\ \implies V_{t-1} &= P_t s_t + Q_t V_t + \Delta M_t \end{aligned} \tag{1}$$

[1]That the government forces households to pay for taxes and new bonds in currency is not necessary for the argument. All else follows if the government accepts payment in goods but stands ready to exchange these goods for currency at market prices.

where T_t are aggregate taxes, $s_t = T_t - G_t$ is the primary surplus, P_t is the final good's price and Q_t is the price of new bonds (I state prices in currency units). Equation (1) provides a low of motion for public debt. It holds for all prices and all choices of money holdings, new debt sales, and primary surplus.[2]

If $P_t = 0$, real public debt V_{t-1}/P_t equals infinity. For now, that is a possibility.

Suppose $P_t > 0$. Define $\beta_t \equiv Q_t P_{t+1}/P_t$ as the real discount for public bonds, and $\beta_{t,t+k} \equiv \prod_{\tau=t}^{t+k} \beta_\tau$. Since V satisfies (1), it also satisfies

$$\frac{V_{t-1}}{P_t} = \sum_{i=0}^{k} \beta_{t,t+i-1} \left(s_{t+i} + \Delta M_t\right) + \beta_{t,t+k} \frac{V_{t+k}}{P_{t+k+1}} \qquad \text{for any } k \geq 0 \tag{2}$$

regardless of prices and choices. Expressions (1) and (2) do not represent a constraint on the path of surpluses $\{s_t\}$ and bond sales $\{V_t\}$ the government chooses to follow. They merely express future debt given paths for quantities and prices.

So far, all we did was public finances accounting. I now move to economic behavior. If $P_t = 0$, households demand infinite final goods and there is no equilibrium. Therefore $P_t > 0$.

Given a utility function over consumption paths $U(\{c_t\})$, the optimal consumption-savings choice involves two conditions. First: $\beta_{t,t+k} =$ marginal rate of substitution between time-t and time-$t+k$ consumption. Second, the transversality condition $\lim_{k\to\infty} \beta_{t,t+k} V_{t+k}/P_{t+k+1} \leq 0$. Coupled with a no-Ponzi condition that establishes the reverse inequality, it implies

$$\lim_{k\to\infty} \beta_{t,t+k} \frac{V_{t+k}}{P_{t+k+1}} = 0 \text{ at every period } t. \tag{3}$$

If bonds were redeemable in goods, (3) would represent a debt sustainability condition, as it forces the government to eventually raise the resources to pay for past borrowing.

With nominal debt, however, the government has no constraints on its choice of debt and surplus, as already pointed out. Replacing (3) on (2) and taking expectation yields

$$\frac{V_{t-1}}{P_t} = \sum_{i=0}^{\infty} E_t \left[\beta_{t,t+i-1} \left(s_{t+i} + \Delta M_{t+i}\right)\right]. \tag{4}$$

Given the face value of maturing public debt, the relative price of the consumption good is given by the expected β-discounted stream of real primary surpluses. This latter term - the right-hand side of (4) - I call the *real value of public debt*. My definition of debt value follows a "beginning-of-period" convention: it includes current period surplus s_t and starts discounting at $t+1$.

In the case of nominal debt, (4) is a *valuation equation*. Hence, expression (3) implies not a no-default condition, but a no-bubble condition: market prices should reflect fundamental value.

Now, define the inflation rate $\Pi_t = P_t/P_{t-1}$, and take innovations on both sides

$$\frac{V_{t-1}}{P_{t-1}} \Delta E_t \Pi_t^{-1} = \sum_{i=0}^{\infty} \Delta E_t \left[\beta_{t,t+i-1} \left(s_{t+i} + \Delta M_{t+i}\right)\right]. \tag{5}$$

[2] Strictly speaking, (1) holds for all *feasible* choices of money holdings, that is, choices that respect households' budget constraint. Otherwise, one could point out that, for $B_{t-1} > 0$, $M_t = M_{t-1}$ and $s_t = B_t = 0$ violates (1). That would nevertheless involve households burning up currency.

Any shock can lead the government to change the conditional distribution of future surpluses (in units of goods) backing the stock of public nominal liabilities. The relative price of bonds in terms of goods $1/P_t$ then adjusts to reflect that change, much in the same way that news of future dividends (in currency units) per stock change the relative price of stocks in terms of currency (Cochrane (2005)). Unexpected inflation $\Delta E_t \Pi_t$ follows. Also like stocks, changes in stochastic discounting β affect fundamental value, and therefore affect prices.

Importantly, (4) and (5) do not depend to equilibrium selection mechanisms. Both hold on all models in which (3) holds, including the standard New-Keynesian model.

2.2 Inflation Decomposition in the Simplest Environment

Start by linearizing the law of motion (2).

$$v_t + s_t = \frac{1}{\beta}\left(v_{t-1} + i_{t-1} - \pi_t\right) \tag{6}$$

where v_t is *end-of-period* stock of real debt, $i_t = -\log(Q_t)$ and $\pi_t = \log(\Pi_t)$. I assume $\Delta M_t = 0$ (households do not hold currency). Note that v and s are both in levels - I assume them to be stationary for simplicity. Moreover, I linearize around the point $v = 1$, which I take to be the average real debt level.

The interpretation of (6) is the same as before. The expression on the right is the linearizing beginning-of-period stock of debt, corresponding to V_{t-1}/P_t in the non-linear formulation. Previous period debt accrues by the mean real interest $(1/\beta)$ plus its local variation $i_t - \pi_t$. A 1% higher real interest raises debt by 1% on average because debt is on average one. The government redeems bonds for currency, and then sells new debt v_t and runs a surplus s_t to soak it up.

Repeating the same steps as before, solve (6) forward:

$$\frac{1}{\beta}\left(v_{t-1} + i_{t-1} - \pi_t\right) = \sum_{k=0}^{\infty} \beta^k E_t s_{t+k} - \frac{1}{\beta} \sum_{k=1} \beta^k E_t r_{t+k}$$

where $r_t = i_{t-1} - \pi_t$ is the *ex-post* real interest rate. The expression on the right-hand side is the linearized real value of debt. It includes time-t surplus, and starts discounting at $t+1$. The inflation rate on the left represents the price level equalizing the beginning-of-period of debt to its real value.

Take innovation, and multiply both sides by β to find

$$\Delta E_t \pi_t = -\beta \sum_{k=0}^{\infty} \beta^k \Delta E_t s_{t+k} + \sum_{k=0}^{\infty} \beta^k \Delta E_t r_{t+k}. \tag{7}$$

In this simple environment, unexpected higher inflation is accounted for by news of larger deficits $-s$ or news of higher discounting r, and vice-versa. That is, news about the real value of debt. This decomposition was introduced by Cochrane (2022), and follows similar decompositions for stock returns and price-dividend ratios (Campbell and Shiller (1988), Campbell and Ammer (1993)).

Now, for each term in the equation, take covariance with unexpected inflation. We arrive at an

Symbol	Description	Nominal Debt	Real Debt	Dollar-Linked Debt
j	Index Symbol	N	R	D
	Notation	δ, ω	δ_R, ω_R	δ_D, ω_D
P_j	Price per Good	P	1	P_t^{US}
$\mathcal{E}_j$	Nominal Exchange Rate	1	P	Dollar NER
H_j	Real Exchange Rate	1	1	Dollar RER
π_j	Log Variation in Price	π	0	π_t^{US}
Δh_j	Log Real Depreciation	0	0	Δh_t

Notes: P = price of consumption basket in domestic currency. P^{US} = price of consumption basket in US dollars. NER = Nominal Exchange Rate. RER = Real Exchange Rate.

Table 1: Public Debt Denomination

initial decomposition of inflation variance.

$$\text{var}(\Delta E_t \pi_t) = -\text{cov}\left[\Delta E_t \pi_t, \beta \sum_{k=0}^{\infty} \beta^k \Delta E_t s_{t+k}\right] + \text{cov}\left[\Delta E_t \pi_t, \sum_{k=0}^{\infty} \beta^k \Delta E_t r_{t+k}\right]. \tag{8}$$

In the simplest environment, if unexpected inflation "exists", it must forecast deficits or higher discounting.

2.3 Generalizing Public Financing Instruments

2.3.1 Currency Denomination

The debt process considered so far is too unrealistic to be taken to the data. I generalize the financing instruments available to the government in two dimensions: currency denomination and maturity structure. I do not consider the case of debt with stage-contingent nominal payoffs.[3]

Starting here, I recycle part of the notation already established. Fix the case of a country's government. Let P_t be the price of the consumption basket in terms of domestic currency.

The payoff of public bonds can be indexed to different currencies, enumerated by j. Let $P_{j,t}$ be the price of the consumer price index in units of currency j. Let $Q_{j,t}^n$ be the discount rate for a zero-coupon public bond paying one unit of currency j after n periods. Without loss of generality, all bonds are zero-coupon. Let $\mathcal{E}_{j,t}$ be the price of currency j in units of domestic currency.

The notation is general enough to accommodate currency-linked bonds *per se*, but also inflation-linked securities such as American TIPS. In this case, one can interpret the "currency" as the final goods basket (so that $P_j = 1$ and $\mathcal{E}_j = P_t$). In the empirical exercises that follow, I consider only nominal bonds ($j = N$), inflation-linked (or real) bonds ($j = R$) and US-dollar- denominated bonds ($j = D$). Table 1 shows the value or interpretation of the variables defined above for these three cases.

[3]Publicly-ran social security and pension systems can be modelled as cases of public debt with state-contingent payoffs or, in a fashion compatible with my framework, as an additional layer of uncertainty in the primary surplus process.

I do not assume the price index for government's purchases (denoted P_t^S) and levied aggregates (such as income) to be the same as the price index for households' consumption (P_t). While governments do tend to tax private consumption, they also tax and transfer income, and purchase baskets of goods different than that of households. The fiscal effect of variations in the relative price of government to households' final goods basket must be accounted for.

With these changes, the law of motion of government debt (1) becomes

$$\sum_j \mathcal{E}_{j,t} B_{j,t-1}^1 = P_t^s S_t + \sum_j \mathcal{E}_{j,t} \sum_{n=2}^{\infty} Q_{j,t}^{n-1} \left(B_{j,t}^{n-1} - B_{j,t-1}^n \right),$$

where S_t is now the real primary surplus and $B_{j,t}^n$ is the face value of bonds issued in currency j, period t, payable n periods in the future. The term on the left represents the cost of debt in period t; the second term on the right represents proceeds from the selling of new bonds.

Let $\mathcal{V}_{j,t} = \sum_n Q_{j,t}^n B_{j,t}^n$ be the end-of-period market value of nominal debt issued in currency j, $i_{j,t}$ the risk-free rate in bonds issued in currency j and $rx_{j,t} = \mathcal{V}_{j,t-1}^{-1} \sum_{n=1}^{\infty} Q_{j,t}^{n-1} B_{j,t-1}^n - i_{j,t-1}$ the realized excess return on portfolios that mimic the composition of j-currency debt. We can re-write the law of motion in terms of the $\mathcal{V}_j$ and its corresponding returns:

$$\sum_j (1 + rx_{j,t} + i_{j,t-1}) \mathcal{E}_{j,t} \mathcal{V}_{j,t-1} = P_t^s S_t + \sum_j \mathcal{E}_{j,t} \mathcal{V}_{j,t}$$

For empirical work, it is ideal to write the law of motion in terms of real and stationary variables. So, let Y_t be real GDP and let $g_t = Y_t / Y_{t-1} - 1$ be its growth rate. Define the real "exchange rate" $H_{j,t} = \mathcal{E}_{j,t} P_{j,t} / P_t$ and its growth rate $\Delta h_{j,t} = H_{j,t} / H_{j,t-1} - 1$. Define the detrended real value of j-indexed debt $V_{j,t} = \mathcal{V}_{j,t} / P_{j,t} Y_t$, the real value of total debt $V_t = \sum_j H_{j,t} V_{j,t}$ and the j-indexed share $\delta_{j,t} = H_{j,t} V_{j,t} / V_t$. I assume a constant currency structure $\delta_{j,t} = \delta_j$.

By properly dividing the whole above equation by $P_t Y_t$, and multiplying and dividing the j sum on the left by $P_{j,t-1}$, $P_{j,t}$, Y_{t-1} and $H_{j,t-1}$, we arrive at a final version for the law of motion of public debt:

$$V_{t-1} \sum_j \frac{(1 + rx_{j,t} + i_{j,t-1})(1 + \Delta h_{j,t})}{(1 + \pi_{j,t})(1 + g_t)} \delta_j = \frac{P_t^s}{P_t} s_t + V_t.$$

The law of motion above generalizes (2) for $k = 1$. During period t, the government must "pay" V_{t-1} plus realized returns and eventual changes to the relative value of currency j.[4]

I linearize the debt law of motion. Let $\beta_j = \exp\{-(rx_j + i_j - \pi_j - g)\}$ be the steady-state real and growth-adjusted discounting for public debt issued in currency j. I linearize around a steady-state - assumed to exist - with $\beta_j = \beta$ for all j and $P^s = P$. This leads to

$$v_t + s_t + s(p_t^s - p_t) = \frac{v_{t-1}}{\beta} + \frac{v}{\beta} \left[\sum_j \delta_j \left(rx_{j,t} + i_{j,t-1} + \Delta h_{j,t} - \pi_{j,t} \right) - g_t \right], \tag{9}$$

which generalizes (6). Parameter v is the steady-state level of public debt. The right-hand side is the beginning-of-period stock of debt.

[4] "Pay" comes in parentheses here because, unlike in (1), the government does not actually redeem the entire term on the left at period t. It only pays for bonds maturing at t.

Let $r_{j,t} = rx_{j,t} + i_{j,t-1} + \Delta h_{j,t} - \pi_{j,t}$ be the *ex-post* real return on holdings of the j-currency portfolio of public bonds, and define $s^p_t = s_t + s(p^s_t - p_t)$ as the price-adjusted surplus process.

Decomposition 1. *Solve the debt law of motion* (9) *forward and take innovations to arrive at*

$$\frac{v}{\beta}\left[\sum_{j\neq N}\delta_j \Delta E_t r_{j,t} + \delta\Big(\Delta E_t rx_t - \Delta E_t \pi_t\Big)\right] = \sum_{k=0}^{\infty}\beta^k \Delta E_t s^p_{t+k} + \frac{v}{\beta}\sum_{k=1}^{\infty}\beta^k \Delta E_t g_{t+k}. - \frac{v}{\beta}\sum_{k=1}^{\infty}\sum_j \delta_j \beta^k \Delta E_t r_{j,t+k}.$$

Take covariance with unexpected inflation, and divide both sides by $\delta(v/\beta)$.

$$\begin{aligned} var(\Delta E_t \pi_t) = {} & cov\left[\Delta E_t\pi_t, \delta^{-1}\sum_{j\neq N}\delta_j \Delta E_t r_{j,t}\right] + cov\left(\Delta E_t \pi_t, \Delta E_t rx_t\right) \\ & - cov\left[\Delta E_t\pi_t, \left(\frac{\delta v}{\beta}\right)^{-1}\sum_{k=0}^{\infty}\beta^k \Delta E_t s_{t+k}\right] - cov\left[\Delta E_t\pi_t, \delta^{-1}\sum_{k=0}^{\infty}\beta^k \Delta E_t g_{t+k}\right] \\ & + cov\left[\Delta E_t\pi_t, \delta^{-1}\sum_{k=1}^{\infty}\sum_j \delta_j\beta^k \Delta E_t r_{j,t+k}\right]. \end{aligned} \tag{10}$$

Decomposition (1) generalizes (7). The right-hand still contains the revision of the value of debt. The left-hand side reveals the new terms Δr_t and Δrx_t containing time-t unexpected jumps in public bond prices, absent in the one-period debt context. Now, *given unexpected variation in the price of long-term debt*, unexpected inflation must be accounted for by news of surpluses or real discounting. Expression (10) converts the innovations decomposition into a decomposition of unexpected inflation.

Compared to the $\delta = 1$ case with nominal debt only, the decomposition contains time-t price-adjustment terms (on the left) and future discounting terms (on the right) related to currency-linked real returns. For countries with dollar-linked debt $\delta_D > 0$, unexpected real exchange depreciation raises the home-currency value of debt and thus acts like an inflationary force (that is the $\Delta E r_{D,t}$ term on the left). News of *future* real exchange depreciation also stimulate inflation by increasing real discounting ($\Delta r_{D,t+k}$ term on the right).

The lower the share of nominal bonds on the stock of δ, the more the price levels must change to deflate total debt and account for innovations on the real value of debt. Expression (10) shows that, all else the same lower δ leads to more volatile unexpected inflation.

2.3.2 Geometric Term Structure

I assume the government keeps a geometric maturity structure of its debt. The geometric term structure implies a tractable relationship between short-term interest rate $i_{j,t}$ and the excess returns that I explore to substitute the hard-to-interpret price adjustment terms of decomposition 1.

The term structure is constant over time, but can vary across the different currency portfolios $\{j\}$ of public debt. Specifically, for the slice of public debt linked to currency j, suppose the outstanding volume of bonds decays at a rate ω_j, so that $B^n_{j,t} = \omega_j B^{n-1}_{j,t}$. Define $Q_{j,t} = \sum_{n=1}^{\infty} Q^n_{j,t}\omega_j^{n-1}$ as the weighted-average price of currency j public bonds. Then, $\mathcal{V}_{j,t} = Q_{j,t}B^1_{j,t}$. The total return on currency-j bonds then is $1 + rx_{j,t} + i_{j,t-1} = (1+\omega_j Q_{j,t})/Q_{j,t-1}$, which I linearize as

$$rx_{j,t} + i_{j,t-1} = \omega_j \beta q_{j,t} - q_{j,t-1} \tag{11}$$

where $q = \log Q$ and I use the log approximation of level returns to effectively re-define $rx_j + i_j$.

Equation (11) above defines the excess return on holdings of the j-currency portfolio of public debt. Given a model for the risk premium $E_t rx_{j,t+1}$, it also defines the price of the debt portfolio as a function of short-term interest:

$$\begin{aligned} q_{j,t} &= \omega_j \beta E_t q_{j,t+1} - E_t rx_{j,t+1} - i_{j,t} \\ &= -\sum_{k=0}^{\infty} (\omega_j \beta)^k E_t \left[rx_{j,t+1+k} + i_{j,t+k} \right]. \end{aligned} \tag{12}$$

The second equation in (12) shows the connection between short-term interest - hence monetary policy - and returns on debt holdings. News of higher interest lower public bond price q and leads to a low excess return.

Lag equation (12) one period and take innovations to find

$$\begin{aligned} \Delta E_t rx_{j,t} &= -\sum_{k=1}^{\infty} (\omega_j \beta)^k \left[\Delta E_t rx_{j,t+k} + \Delta E_t i_{j,t+k-1} \right] \\ &= -\sum_{k=1}^{\infty} (\omega_j \beta)^k \left[\Delta E_t r_{j,t+k} + \Delta E_t \pi_{j,t+k} - \Delta E_t \Delta h_{j,t+k} \right]. \end{aligned} \tag{13}$$

Decomposition 2. *Replace* (13) *on decomposition 1 and gather terms to find.*

$$\begin{aligned} \frac{\delta v}{\beta} \Delta E_t \pi_t = &-\frac{\delta v}{\beta} \sum_{k=1} (\omega \beta)^k \Delta E_t \pi_{t+k} - \sum_{k=0}^{\infty} \beta^k \Delta E_t s^p_{t+k} - \frac{v}{\beta} \sum_{k=1}^{\infty} \beta^k \Delta E_t g_{t+k} \\ &+ \frac{v}{\beta} \sum_{k=1}^{\infty} \sum_j \delta_j \beta^k (1 - \omega_j^k) \Delta E_t r_{t+k} + \frac{\delta_D v}{\beta} \sum_{k=0}^{\infty} (\omega_D \beta)^k \Delta E_t \Delta h_{t+k} - \frac{\delta_D v}{\beta} \sum_{k=0}^{\infty} (\omega_D \beta)^k \Delta E_t \pi^{US}_{t+k}. \end{aligned}$$

Take covariance with unexpected inflation, and divide both sides by $\delta(v/\beta)$.

$$\begin{aligned} var(\Delta E_t \pi_t) = &-cov\left[\Delta E_t \pi_t, \sum_{k=1}^{\infty} (\omega \beta)^k \Delta E_t \pi_{t+k} \right] - cov\left[\Delta E_t \pi_t, \left(\frac{\delta v}{\beta} \right)^{-1} \sum_{k=0}^{\infty} \beta^k \Delta E_t s^p_{t+k} \right] \\ &- cov\left[\Delta E_t \pi_t, \delta^{-1} \sum_{k=1}^{\infty} \beta^k \Delta E_t g_{t+k} \right] + cov\left[\Delta E_t \pi_t, \delta^{-1} \sum_{k=1}^{\infty} \sum_j \delta_j \beta^k (1 - \omega_j^k) \Delta E_t r_{t+k} \right] \\ &+ cov\left[\Delta E_t \pi_t, \frac{\delta_D}{\delta} \sum_{k=0}^{\infty} (\omega_D \beta)^k \Delta E_t \Delta h_{t+k} \right] - cov\left[\Delta E_t \pi_t, \frac{\delta_D}{\delta} \sum_{k=0}^{\infty} (\omega_D \beta)^k \Delta E_t \pi^{US}_{t+k} \right]. \end{aligned} \tag{14}$$

Decomposition 2 says that positive unexpected inflation must forecast either: *lower* future inflation (domestic or foreign), higher deficits, lower growth, higher real discounting or a more depreciated real exchange rate. This is the main decomposition I explore in the empirical exercises.

In it, the ω terms give a clue of which terms derive from the time-t adjustment of bond prices. For example: an interest rate hike $\Delta E_t i_t$ can lead to a fall in nominal bond prices (negative $\Delta E_t rx_t$) and, by decomposition 1, a decline in current inflation. These lower bond prices in turn forecast higher inflation. This is why the decomposition prescribes future inflation as time-t *deflationary force*, like surpluses.[5] Long-term bond prices bring to the current period the fiscal effects of future

[5]Of course, higher expected inflation means inflation is expected to grow after time t. Sims (2011) calls that mechanism

inflation.

Similar mechanisms apply to the exchange rate and US inflation terms that follow from dollar-linked debt. Lower dollar-bond prices might forecast higher US inflation of lower (appreciated) real exchange in the future, despite the potential opposite effect at time t.

3 Empirical Model and Estimation

3.1 Overview

The main goal is to estimate the decompositions of unexpected inflation (10) and (14) for a set of twenty-five economies. To do this, I estimate a ten-equation VAR in which the debt law of motion (9) holds by construction. If the VAR is stationary, equation (3) will be satisfied, and the decompositions will hold.

I use annual data. Quarterly data is available, but it often does not go back as many years into the past. This is particularly true for emerging market variables and public debt measures (from all countries). With a focus on long-term debt sustainability, going further into the past is more important than accounting for quarterly dynamics. Additionally, with annual data there is no danger of measurement errors due to seasonality adjustments.

I group countries in four sample categories. The first one contains only the United States. US data covers the period 1945-2019. The second group has six developed economies (you can check the list on table 2). Their sample start in 1960. The third group has four developed economies, with the sample starting in 1973. The last group contains fourteen developing countries. Their sample starts in 1998.

Grouping countries according to sample size helps to account for parameter volatility. Additionally, it provides control for international economic environment and historical events that affect inflation and its fiscal determinants.

I interpret parameters of the VAR as being random and estimate them using Bayesian methods. I establish a prior distribution, and then use data likelihood to compute the posterior.[6]

I base my prior on estimated US dynamics. First, because we already have results available in the literature (Cochrane (2022), to the best of my knowledge the decompositions have not been estimated to other countries so far). Second, the US has the longest sample. Critically, it comprises the repayment of a major public borrowing event - World War II - that renders OLS estimates of the VAR stable and plausible.[7] I estimate the model for the US by OLS and use the resulting VAR as mean of the prior for other countries' estimation.

From the ten variables in the VAR, five are observed: the nominal interest (i_t), the inflation rate (π_t), par-value public debt (v_t^b), the real exchange rate to the dollar (Δh_t) and GDP growth (g_t). I select these variables based on (9). Most time series data I collect from the St Louis Fed *FRED* website, the United Nations and the IMF. Details on appendix B.

I convert interest and inflation data to log. The change in dollar exchange rate is the nominal depreciation to the US dollar, plus US inflation minus domestic inflation. In the US case, I use

"stepping on a rake" and argues it was part of the reason why 1970's US inflation kept coming back after temporary declines.

[6]See del Negro and Schorfheide (2011) or Karlsson (2013) for more on Bayesian estimation of VAR models.

[7]Including pre-1950 data in the sample proved necessary. Starting the sample after that leads to an unstable VAR estimate due to the large public debt equation root.

exchange rate to the UK pound, which is available since the 1930s. GDP growth is the log difference of levels data. Public debt is provided as a ratio of GDP by the source (Ali Abbas et al. (2011)), and requires no transformation.

3.2 Public Finances Model

Governments typically report the par or book value of debt, and this is the data I gather. Because theory is based on the *market* value of debt, some adjustment is necessary. The Dallas Fed provides estimates of the market value of debt for the United States following Cox and Hirschhorn (1983) and Cox (1985).[8] I follow a similar methodology.

Let $\mathcal{V}^b_{j,t}$ be the par value of the j-currency portfolio debt, and let $i^b_{j,t}$ be its average yield-to-maturity by which book values are updated from period to period. The adjustment equation is

$$\mathcal{V}_{j,t} = \mathcal{V}^b_{j,t} \times \frac{\text{market value of bonds}}{\text{book value of bonds}} = \frac{1+\omega_j Q_{j,t}}{(1+i^b_{j,t-1})Q_{j,t-1}} = \frac{1+rx_{j,t}+i_{j,t-1}}{1+i^b_{j,t-1}}.$$

I detrend the $\mathcal{V}$'s, convert to real, sum across portfolios and linearize to arrive at:

$$v_t = v^b_t + \frac{v}{\beta}\left[\sum_j \delta_j \left(rx_{j,t} + i_{j,t-1} - i^b_{j,t-1}\right)\right]. \tag{15}$$

Estimates of the VAR provide an equation for the law of motion of par-value debt. I use (15) to infer a law-of-motion of market-value debt.

The average interest $i^b_{j,t}$ is not observed, so we cannot estimate an equation for it. Instead, I use a model. With a geometric maturity structure of debt and in steady state, every period the government rolls over the volume of bonds coming to maturity (which had maturity $n=1$ in the previous period). That accounts for a share $1-\omega_j$ of total outstanding bonds. In a steady state, the government then issues the same amount of new bonds ($1-\omega_j$ of total debt) at the prevailing interest rate i_t. The average interest therefore satisfies

$$i^b_{j,t} = (1-\omega_j)i_{j,t} + \omega_j i^b_{j,t-1} = (1-\omega_j)\sum_{k=0}^{\infty}\omega_j^k i_{j,t-k} \tag{16}$$

for $j \in \{N, R, D\}$.

3.3 The Bayesian-VAR

Except for exchange rate depreciation, I demean all series. This is to assume that the stationary steady-state (around which I linearize) is given by the sample mean of the series of each country (with the exception of real exchange rate growth, which I assume to be zero).

The functional format of the VAR is

$$x_t = ax_{t-1} + bu_{t-1} + k\,e_t. \tag{17}$$

Both x and u are vectors with ten entries. Five of them are the observed variables enumerated

[8] Web link: https://www.dallasfed.org/research/econdata/govdebt.

above.

Vector u_t groups the same set of variables as x, but for the United States. I often use the "u" notation to refer to the US case. Because the public debt process of each country has a dollar component, and hence depends on dollar interest and inflation, u and ε_u enter the regression of all countries.

There are five shocks hitting the domestic economy: $\varepsilon \sim N(0, \Sigma)$, independent over time. The shocks hitting the US economy are $\varepsilon_u \sim N(0, \Sigma_u)$. I group them in $e_t = [\varepsilon_t' \ \varepsilon_{u,t}']'$. Matrix $k_{10\times 10}$ serves to correctly reproduce the law of motion governing unobserved variables.

The model for US variables is:

$$u_t = a_u u_{t-1} + k_u \, \varepsilon_{u,t} \tag{18}$$

(I use the same notation x to the VAR of all countries and differentiate only in the US case). In (18), k_u is a 10×5 matrix.

Cochrane (2022) imposes the law of motion of public debt in the VAR by using primary surplus "data" calculated from it. Primary deficit equals the change in public debt net of realized return. This procedure bypasses the problem of dealing with accounting conventions in primary surplus reporting that, in most cases, make its data inconsistent with that of public debt. Since the OLS estimator is linear, his VAR satisfies the debt law of motion by construction.

In my VAR, the procedure needs to be adjusted. First, I do not measure excess return, or real interest rates. Second, the debt equation depends on US interest and inflation. Hence, the unrestrictive estimation of (17) spuriously projects these two US variables on domestic ones, which is inconsistent with (18). For these reasons, instead of measuring implied surpluses and then estimating the model, I estimate the model and then fill the equation for primary surpluses that ensures the debt law of motion (9) holds. Before doing that, I also need to include the adjustment equation for market-value debt (15) (the estimated equation is for par-value, not market-value debt!) as well as the three definitions of average interest rates (16) required to do it. These five unobserved variables (surplus s_t^p, market-value debt v_t, and the average interest $\{i_{j,t}^b\}$) complete the ten variables of the VAR.

Note that the estimated equations for par-value and market value of public debt represent their law of motion *after* replacing the equation for primary surpluses, or its equilibrium law of motion.

The estimation has four steps.

Step 1. I estimate the VAR

$$\begin{aligned} \tilde{x}_t &= \tilde{a}\tilde{x}_{t-1} + \tilde{b}\tilde{u}_{t-1} + \varepsilon_t \\ \tilde{u}_t &= \tilde{a}_u \tilde{u}_{t-1} + \varepsilon_{u,t} \end{aligned} \tag{19}$$

where $\tilde{x}$ is a vector with the five observed variables, and $\tilde{u}$ is defined similarly. Matrices $\tilde{a}$, $\tilde{b}$ and $\tilde{a}_u$ are the submatrices of a, b and a_u corresponding to the rows and columns of these observed variables.

I also estimate $\text{cov}(\varepsilon) = \Sigma$ and $\text{cov}(\varepsilon_u) = \Sigma_u$.

Step 2. Stack domestic and US variables $X_t = [x_t' \ u_t']'$. In the United States case, $X_t = u_t$. I assume a constant risk-premium: $E_t rx_{j,t+1} = 0$ for all j. I use the estimated VAR (19) to compute $E_t i_{j,t+i}$ and apply (12) to compute $q_{j,t}$. Equation (11) then yields expressions for excess return of the form

$$rx_{j,t} = \varphi_j' e_t.$$

An equation for real debt is also necessary. I use

$$i_{R,t} = i_t - E_t \pi_{t+1} = \zeta' X_t.$$

under the implied assumption of equal expected real and excess returns between nominal and real debt. In appendix C, I present formulas for the φ's and ζ.

Step 3. Using the estimated model of step 1, I compute the equations for average interest using (16), and fill the corresponding rows of a, b and k (a_u and k_u in the US case). With the equations for average interest filled, I can do the same for the market-price debt using the par-value adjustment equation (15). With the equation for the market-price debt, I use the law of motion (9) and the expressions for excess return and real interest above to fill the equation row for the primary surplus.

This completes the estimation of a, b and k in the general case, a_u and k_u in the US case. For each country, we can stack the equations into a single system for X:

$$X_t = AX_{t-1} + Ke_t. \tag{20}$$

If we order unobserved variables x^o at the top of the x, we can write (20) more explicitly:

$$\begin{pmatrix} x_t \\ u_t \end{pmatrix} = \begin{bmatrix} a & b \\ 0 & a_u \end{bmatrix} \begin{pmatrix} x_{t-1} \\ u_{t-1} \end{pmatrix} + \begin{bmatrix} k \\ 0 \quad k_u \end{bmatrix} \begin{pmatrix} \varepsilon_t \\ \varepsilon_{u,t} \end{pmatrix}$$

$$\text{or yet} \begin{pmatrix} x_t^o \\ \tilde{x}_t \\ u_t^o \\ \tilde{u}_t \end{pmatrix} = \begin{bmatrix} & * & & \\ 0 & \tilde{a} & 0 & \tilde{b} \\ 0 & 0 & * & \\ 0 & 0 & 0 & \tilde{a}_u \end{bmatrix} \begin{pmatrix} x_{t-1}^o \\ \tilde{x}_{t-1} \\ u_{t-1}^o \\ \tilde{u}_{t-1} \end{pmatrix} + \begin{bmatrix} * & \\ I & 0 \\ 0 & * \\ 0 & I \end{bmatrix} \begin{pmatrix} \varepsilon_t \\ \varepsilon_{u,t} \end{pmatrix}.$$

Symbol * indicates the coefficients are filled to ensure that (9), (15) and (16) hold. In appendix C I provide their formulas.

Step 4. I compute sample residuals $\hat{e}$ ($\hat{e}_u$ for the US) from (19), and estimate $\text{cov}(\varepsilon, \varepsilon_u) = \Sigma_{xu} = \sum_i \hat{e}_i \hat{e}_{u,i} / (N-1)$, where N is the sample size. Then:

$$\Omega = \text{cov}(e) = \begin{bmatrix} \Sigma & \Sigma_{xu} \\ \Sigma_{ux} & \Sigma_u \end{bmatrix}.$$

3.4 The Prior Distribution

As the commonly used Litterman (1979) prior, I use a distribution of the Normal-Inverse-Wishart class, with general format

$$\Sigma \sim IW(\Phi; d)$$
$$\theta | \Sigma \sim N(\bar{\theta}, \Sigma \otimes \Omega).$$

where $\theta = [\text{vec}(\tilde{a}')' \ \text{vec}(\tilde{b}')']'$ and vec means stacking the columns. Given the Gaussian likelihood of the model, the posterior distribution is also of the Normal-Inverse-Wishart class. Giannone et al. (2015) provide formulas for the posterior distribution and marginal likelihood.

In the US case, the prior centers around zero, $\bar{\theta} = 0$, but since it has a very large variance, the posterior centers around the OLS estimate of $\tilde{a}_u$. The estimated VAR for the US is stationary.

In the case of other countries, I center the prior around $\tilde{a} = \tilde{a}_u$, $\tilde{b} = 0$. The economic content of the prior is that the dynamics of the observed variables is the same as that we estimate for the United States. The surpluses process differs from that of the US only to account for the differences in public debt size and term and currency structures.

With $\tilde{b} = 0$, the prior also translates the view that US variables do not affect the dynamics of domestic ones.

The mean of the IW distribution is $\Phi/(d - n - 1)$, where $n = 5$ is the dimension of ε and larger values of d represent tighter priors. I choose Φ to be the identity matrix (one percent standard deviation for all shocks, which are uncorrelated) and select $d = n + 2$, the lowest integer possible that leads to a well-defined distribution mean - which therefore equals Φ.

The conditional covariance between the coefficients in $\tilde{a}$ is

$$\text{cov}\left(\tilde{a}_{ij}, \tilde{a}_{kl} \mid \Sigma\right) = \begin{cases} \lambda^2 \frac{\Sigma_{ij}}{\Phi_{jj}} & \text{if } j = l \\ 0 & \text{otherwise.} \end{cases}$$

The format allows the loadings on the variable in different equations to be correlated. Loadings on the different variables on the same equation are independent. Hyperparameter λ governs the overall tightness of the prior over linear coefficients, with lower values yielding tighter priors.

The conditional covariance of $\tilde{b}$ is

$$\text{cov}\left(\tilde{b}_{ij}, \tilde{b}_{kl} \mid \Sigma\right) = \begin{cases} (\xi\lambda)^2 \frac{\Sigma_{ij}}{\Phi_{u,jj}} & \text{if } j = l \\ 0 & \text{otherwise} \end{cases}$$

where $\Phi_u = \Phi = I$ is the mean of the IW distribution in the US case. Hyperparameter ξ governs the tightness of the prior that US variables do not affect domestic ones more than the mechanical effect of US interest and inflation on domestic public debt. If $\xi = 1$, the prior is just as tight as that of $\tilde{a}$.

Finally, the covariance between $\tilde{a}$ and $\tilde{b}$ is zero.

It is straightforward to build Ω so that the conditional covariance structures above hold.

4 Empirical Results

4.1 Variance Decomposition

In the baseline specification, I calibrate $\beta = 0.98$ for all countries, and set b tightness parameter $\xi = 1/3$. I calibrate parameters δ and ω based on debt structure data gather from various sources (see appendix B). They are reported in Table 2 along with average debt.

The mean debt-to-GDP ratio in the sample was 0.50, with developed countries slightly more indebted on average. Nominal debt tends to account for the bulk of sovereign debt, Chile being a notable exception. Emerging markets' governments tend to rely relatively more on real and especially foreign debt, and issue securities with higher maturity, on average.

To ensure stability of the VAR, I start by finding the hyperparameter λ that maximizes the marginal likelihood.[9] Then, if the mode of the posterior leads to an unstable VAR, I progressively reduce λ in 0.001 steps until it leads to a stable VAR. Given the continuity of the posterior distribution on λ and the fact that $\lambda = 0$ leads to the stable US system, there must exist a non-zero value of λ that leads to a stationary model. You can check the resulting λ's in table 2.

Table 3 contains the estimated terms of decomposition 1, computed at the mode of the posterior distribution. I compute the terms of equation (10) and divide both sides of the equality by $\text{var}(\Delta E\pi_t)$. This leads to:

$$1 = p_1(r_0) + p_1(rx) - p_1(s) - p_1(g) + p_1(r)$$

where r_0 corresponds to the price adjustment term of real and dollar debt (the first term on the right side of equation (10)); the other symbols are self-explanatory.

The p_1's are regression coefficients of the respective decompositions term. For instance

$$p_1(s) = \text{cov}\left[\Delta E_t \pi_t, \left(\frac{\delta v}{\beta}\right)^{-1} \sum_{k=1}^{\infty} \beta^k \Delta E s_{t+k}\right] \text{var}(\Delta E_t \pi_t)^{-1}.$$

These terms indicate how much of inflation variance is explained by the corresponding term. Note that they do not have to be in the $[0, 1]$ interval. It is easy to show that the coefficients also correspond to the (sum of the) impulse response functions of the corresponding variables to a 1% inflation shock, if we let other shocks move contemporaneously according to the projection implied by Ω.

The tables contain cross-country averages and medians, conditional on country group. Highlighted figures indicate the statistical significance of the estimate's *sign*, based on ten-thousand simulations of the posterior distribution.[10] Bold font alone indicates that 75% of the simulated draws have the same sign as the posterior mode. I consider that to indicate a statistically significant result. Bold font with an asterisk indicates 90%.

The first column of the table reports unexpected inflation. Unsurprisingly, unexpected inflation exists, with standard deviations ranging from 1% and 2% in the country group averages. The low figure for emerging markets reminds us of the importance of the sample time period. Emerging markets do not have lower unexpected inflation; we just sampled from a period in which inflation is known to be less volatile in most countries (Stock and Watson (2002), Coibion and Gorodnichenko (2011)).

It is visible from the table that, as sample size shrinks, parameters are estimated less precisely. The evidence is not conclusive to all countries. But a few patterns emerge.

- *In all country groups, concomitant nominal bond price shocks reduce the variance of unexpected inflation: $p_1(rx) < 0$. Monetary policy trades current for future inflation, $-p_2(\pi) < 0$.*

[9] Giannone et al. (2015) show that, in the case of Normal-Inverse-Wishart priors, the marginal likelihood can be decomposed in a goodness-of-fit term and a model-complexity term that penalizes conditional forecast variance. By maximizing the marginal likelihood, we ensure we cannot improve one of these terms without reducing the other. Similar methods for selecting the degree of informativeness of the prior distribution have been used. See, for example, Koop (2013) and Carriero et al. (2015).

[10] I discard draws that lead to unstable VARs.

Country	v (%)	δ_N (%)	δ_R (%)	δ_D (%)	Avg. Term (Years)	λ
Averages	48	74	11	15	6.5	
Advanced - 1960	58	87	8	6	6.4	
Advanced - 1973	32	92	4	4	5.6	
Emerging - 1998	47	63	14	23	6.9	
Median	43	79	5	10	5.6	
Advanced - 1960	53	88	3	2	5.6	
Advanced - 1973	32	94	3	2	5.6	
Emerging - 1998	43	67	6	23	7.6	
United States	60	93	7	0	5	10
Advanced - 1960 Sample						
Canada	71	92	5	3	6.5	0.21
Denmark	37	84	0	16	5.6	0.18
Japan	98	100	0	0	5.5	0.01
Norway	35	99	0	1	3.7	0.19
Sweden	46	69	16	14	4.8	0.16
United Kingdom	61	76	24	0	12.3	0.17
Advanced - 1973 Sample						
Australia	24	90	10	0	7.2	0.18
New Zealand	41	82	6	13	4.3	0.15
South Korea	21	97	0	3	4	0.15
Switzerland	43	100	0	0	6.9	0.23
Emerging - 1998 Sample						
Brazil	70	70	25	5	2.6	0.12
Chile	14	10	57	33	12.8	0.27
Colombia	41	45	23	32	5.6	0.13
Czech Republic	31	91	0	9	5.6	0.15
Hungary	68	76	0	23	4.1	0.14
India	73	90	3	7	10.1	0.25
Indonesia	43	44	0	56	9.2	0.21
Israel	77	43	34	23	6.6	0.13
Mexico	45	65	10	26	5.5	0.15
Poland	47	79	1	20	4.2	0.10
Romania	28	50	0	50	4.8	0.10
South Africa	41	70	20	10	12.9	0.25
Turkey	43	47	23	30	3.6	0.13
Ukraine	43	100	0	0	9.1	0.07

Notes: v is the average public debt-to-GDP, δ_N is the share of nominal debt, δ_R is the share of real or inflation-linked debt, δ_D is the share of dollar-denominated debt (I also include debt denominated in other currencies, but the amounts are negligible in almost all cases). "Avg. Term" is the average maturity across currency portfolios, calculated as $\sum_j \delta_j (1-\omega_j)^{-1}$. λ is the prior tightness parameter.

Table 2: Debt Structure Parameters and Prior Tightness

Country	std($\Delta E\pi$) (%)	Decomposition 1 - Variance decomposition (10)				
		$p_1(r_0)$	$p_1(rx)$	-$p_1(s)$	-$p_1(g)$	$p_1(r)$
Averages	1.6	-0.24	**-0.74***	1.02	**0.36**	0.60
Advanced - 1960	1.6	-0.01	**-0.74***	**0.97**	**0.54***	0.24
Advanced - 1973	1.8	-0.01	**-0.69***	**1.33***	**0.65***	-0.28
Emerging - 1998	1.6	-0.42	**-0.76***	0.99	0.22	0.97
Median	1.3	-0.02	**-0.76***	**0.87***	**0.42***	**0.68***
Advanced - 1960	1.5	**-0.06***	**-0.62***	**0.61***	**0.72***	**0.58**
Advanced - 1973	1.7	**-0.00**	**-0.72***	**1.40***	**0.76***	-0.37
Emerging - 1998	1.2	-0.11	**-0.77***	1.13	**0.33***	**0.79***
United States	1.9	**0.03***	**-0.78***	**0.57**	0.23	**0.96**
Advanced - 1960 Sample						
Canada	1.0	**-0.11***	**-1.59***	0.62	**1.22***	0.86
Denmark	1.2	**-0.29***	**-0.30**	0.42	-0.04	**1.21**
Japan	2.2	0	**-0.52***	**1.60***	-0.38	0.30
Norway	1.5	**-0.01***	**-0.36***	**0.60**	**0.47**	0.30
Sweden	1.5	**-0.15**	**-0.93***	-0.34	**0.98***	**1.42***
United Kingdom	2.0	**0.52***	**-0.73***	**2.89***	**0.97***	**-2.65***
Advanced - 1973 Sample						
Australia	1.5	**0.07***	**-0.76***	**2.09***	**0.66**	**-1.06**
New Zealand	2.0	**-0.10**	**-0.86***	0.40	**0.87***	0.68
South Korea	2.8	-0.01	**-0.45***	**1.91***	0.17	**-0.62**
Switzerland	1.0	0	**-0.69***	**0.90**	**0.91***	-0.12
Emerging - 1998 Sample						
Brazil	1.4	**-0.26**	**-0.22***	**-1.46**	**1.05**	**1.89**
Chile	1.0	-3.80	**-1.33**	8.95	-5.71	2.88
Colombia	0.8	**1.51**	**-0.96***	1.39	-1.09	0.15
Czech Republic	1.1	**-0.16***	**-0.37***	-2.31	**2.42**	**1.42**
Hungary	1.3	**-0.57***	**-0.93***	-0.98	1.60	1.88
India	1.1	**0.17***	**-0.46***	**1.54**	0.05	-0.30
Indonesia	1.2	**-2.59***	**-1.07***	1.69	**2.61***	0.35
Israel	1.3	-0.06	**-0.78***	-0.55	**1.51***	0.88
Mexico	1.0	-0.02	**-0.74***	**1.41**	0.03	0.32
Poland	1.3	**-0.45***	**-1.15***	0.87	-0.39	**2.11***
Romania	1.9	-0.40	**-0.96***	2.24	0.42	-0.31
South Africa	1.0	**0.36**	**-0.51***	1.58	0.25	-0.68
Turkey	2.1	**0.37**	**-0.37***	-1.18	-0.15	**2.33***
Ukraine	5.7	0	**-0.77***	**0.65**	**0.41**	**0.70***

Notes: The table reports the terms of variance decomposition (10) at the posterior distribution's mode. I divide each term by var($\Delta E_t \pi$). I simulate from the posterior to check the statistical significance of the sign of the estimate. Bold indicates 75% of results had the same sign. Bold with an asterisk indicates 90%.

Table 3: Unexpected Inflation - Variance Decomposition 1

Country	Decomposition 2 - Variance decomposition (14)					
	$-p_2(\pi)$	$-p_2(s)$	$-p_2(g)$	$p_2(r)$	$p_2(\Delta h)$	-$p_2(\pi^{US})$
Averages	**-0.81***	1.02	**0.36**	**0.63**	0.02	**-0.22***
Advanced - 1960	**-1.20***	**0.97**	**0.54***	**0.76**	0	**-0.06***
Advanced - 1973	**-1.01***	**1.33***	**0.65***	0.11	**-0.05**	**-0.03***
Emerging - 1998	**-0.56***	0.99	0.22	0.67	0.06	**-0.37***
Median	**-0.76***	**0.87***	**0.42***	**0.61***	0	**-0.03***
Advanced - 1960	**-1.08***	**0.61***	**0.72***	**0.85***	0	**-0.04***
Advanced - 1973	**-0.91***	**1.40***	**0.76***	-0.14	0	**-0.01***
Emerging - 1998	**-0.61***	1.13	**0.33***	**0.47**	-0.03	**-0.04**
United States	**-1.12***	**0.57**	0.23	**1.32***	0	-
Advanced - 1960 Sample						
Canada	**-1.53***	0.62	**1.22***	**0.78**	**-0.03**	**-0.07***
Denmark	**-0.49***	0.42	-0.04	**1.23**	0.08	**-0.20***
Japan	**-1.14***	**1.60***	-0.38	**0.91***	0	0
Norway	**-0.70***	**0.60**	**0.47**	**0.64**	0	0
Sweden	**-1.02***	-0.34	**0.98***	**1.54***	-0.07	**-0.10**
United Kingdom	**-2.34***	**2.89***	**0.97***	**-0.52**	0	0
Advanced - 1973 Sample						
Australia	**-1.47***	**2.09***	**0.66***	-0.27	0	0
New Zealand	**-1.02***	0.40	**0.87***	**1.04**	**-0.21**	**-0.08***
South Korea	**-0.74***	**1.91***	0.17	-0.33	**0.01**	**-0.03***
Switzerland	**-0.79***	**0.90**	**0.91***	-0.02	0	0
Emerging - 1998 Sample						
Brazil	**-0.11***	**-1.46**	**1.05**	**1.46**	0.07	0
Chile	-0.76	8.95	-5.71	-0.35	1.62	**-2.75**
Colombia	**-0.61***	1.39	-1.09	0.02	**1.34**	-0.04
Czech Republic	-0.02	-2.31	**2.42**	0.98	-0.03	**-0.05**
Hungary	**-0.69***	-0.98	1.60	1.83	**-0.61***	**-0.15***
India	**-1.05***	**1.54**	0.05	0.41	-0.04	**0.09***
Indonesia	**-0.79***	1.69	**2.61***	0.26	**-1.45**	**-1.33***
Israel	**-0.54***	-0.55	**1.51***	0.61	-0.12	0.10
Mexico	**-0.60***	**1.41**	0.03	0.52	**-0.52**	**0.17**
Poland	**-0.59***	0.87	-0.39	**1.43***	-0.11	**-0.21***
Romania	**-1.14***	2.24	0.42	-0.54	0.55	**-0.53***
South Africa	0.05	1.58	0.25	-0.79	-0.07	-0.01
Turkey	**-0.76***	-1.18	-0.15	**3.35***	0.14	**-0.40***
Ukraine	**-0.29**	**0.65**	**0.41***	**0.23**	0	0

Notes: The table reports the estimated terms of variance decomposition (14) at the posterior distribution's mode. I divide each term by var$(\Delta E_t \pi)$. I simulate from the posterior to check the statistical significance of the sign of the estimate. Bold indicates 75% of results had the same sign. Bold with an asterisk indicates 90%.

Table 4: Unexpected Inflation - Variance Decomposition 2

Unexpected inflation tends to come accompanied by a decline in nominal bond prices resulting from monetary policy. Central banks observe inflation and raise interest rates. As bond prices decline, the price level does not have to increase so much to equal market-value debt to its real value.

The covariance between $\Delta E_t rx_t$ and $\Delta E_t \pi_t$ is negative and statistically significant in all countries. They are also economically significant. I estimate $p_1(rx) = -0.78$ for the US. Cochrane (2022) estimates -0.56. The effect is larger in several other cases. They suggest monetary policy is effective in preventing unexpected inflation.

But lower bond prices forecast higher inflation. Decomposition 2 isolates the effect of future inflation. For the US, $p_2(\pi) = -1.12$ (Cochrane estimates -0.59). Inflation forecasts inflation. One can attribute that to price stickiness in the short run and, in the long run, Fisherian effects of monetary policy (Garín et al. (2018), Uribe (2022)). Estimates are similar for other advanced economies. Magnitudes are close -1 and somewhat larger for the United Kingdom. In the case of emerging markets, I also find negative figures, but with smaller magnitudes, as the median shows.

- *In advanced economies, unexpected inflation forecasts deficits, which contributes to its volatility ($-p_1(s) = -p_2(s) > 0$).*

With the exception of Sweden, inflation forecasts deficits (or nowcasts, since the surplus sum in the decompositions includes time t), in advanced economies. In a fiscal-selection reading, unexpected inflation is caused by news of lower primary surpluses.

Magnitudes are economically and, in most cases, statistically significant. I estimate $-p_2(s) = 0.57$ for the US (Cochrane finds -0.06). For other developed countries, it ranges from 0.50 to 3. The large estimates for Australia (2.1) and South Korea (1.9) can be partially attributed to their reduced indebtedness (δv enters the denominator of $p_2(s)$). The United Kingdom (1.89) has a large debt but a low share of nominal debt. Sweden is a case of some inflation insulation from fiscal deficits ($p_2(s)$=-0.34) despite the small average debt and low share of nominal bonds.

In the case of developing countries, estimated values are mostly positive like for developed countries, but often not statistically different from zero. I estimate negative p_2 for Brazil certainly due to the large surpluses in the 2000s, when inflation was highest in the time series. Still, I do not draw definitive conclusions in the case of 1998 emerging market sample.

- *In advanced economies, unexpected inflation forecasts a growth decline, which contributes to its volatility ($-p_1(g) = -p_2(g) > 0$). In emerging markets, statistically significant figures indicate the same, but tend to be the exception.*

All countries with statistically non-zero estimates have a positive $-p_1(g) = -p_2(g)$. In these cases, inflation forecasts a GDP growth slowdown, which reduces the real value of debt and causes, in a FTPL interpretation, the unexpected inflation in the first place. The conclusion has more empirical support for developed countries. From them, only for Denmark and Japan I estimate negative (and statistically insignificant) $p_2(g)$.

Estimates are economically relevant. When positive, covariance with inflation accounts from 47% to 122% of total inflation variance. This is one of the differences I find between developed countries in general and the United States, for I find $p_1(g) = 0.23$, not statistically different from zero. Cochrane finds 0.49.

The pattern is similar in emerging markets: statistically significant figures are positive and economically significant. But they tend to be the exception. I estimate large growth contributions to unexpected inflation for Brazil, Czech Republic, Indonesia and Israel. Part of the explanation for the large magnitudes is the low share of nominal debt on the portfolio of developing countries' governments.

- *Long-term bonds often increase the contribution of discounting on unexpected inflation. After accounting for bond price variation, all statistically significant figures indicate such positive contribution: $p_2(r) > 0$. This result is particularly stronger for the United States, developed economies in the 1960 sample and emerging markets.*

Cochrane concludes that discount rate variation accounts for a large share of inflation variance in the United States ($p_2(r) = 1$). I find a similar conclusion for the United States ($p_2(r) = 1.32$) and advanced economies in the 1960 sample (with the exception of the United Kingdom).

Both decompositions 1 and 2 have future real interest terms. Higher discounting lowers the real value of public debt, the right-hand side of decomposition 1. If the $p_1 > 0$, it increases unexpected inflation variability; if $p_1 < 0$ it reduces. In either case, if the government finances itself with one-period bonds $\omega_j = 0$, this is the only effect to be considered, and $p_2(r) = p_1(r)$.

With long-term bonds, (13) shows that higher discounting also leads to lower bond prices, which soak up part of such inflationary effect.[11] If the government borrows using perpetuities ($w_j = 1$), these two forces cancel out and real interest dynamics contributes neither for nor against unexpected inflation: $p_2 = 0$.

The intermediary case $0 < \omega_j < 1$ does *not* lead to the intermediary conclusion. Long-term bonds can *enhance* the impact, positive or negative, of real interest to inflation variance. With $0 < w_j < 1$, long-term bonds reflect (and cancel out) only short-term changes in real interest. If these short-term changes have an opposite signal to the overall covariance, then long-term bonds will insulate inflation from the "wrong" period of varying interest. Mathematically, if $\sum_k (1-\omega^k)\Delta E_t r_k$ and $\sum_k \Delta E_t r_k$ have the same sign, then

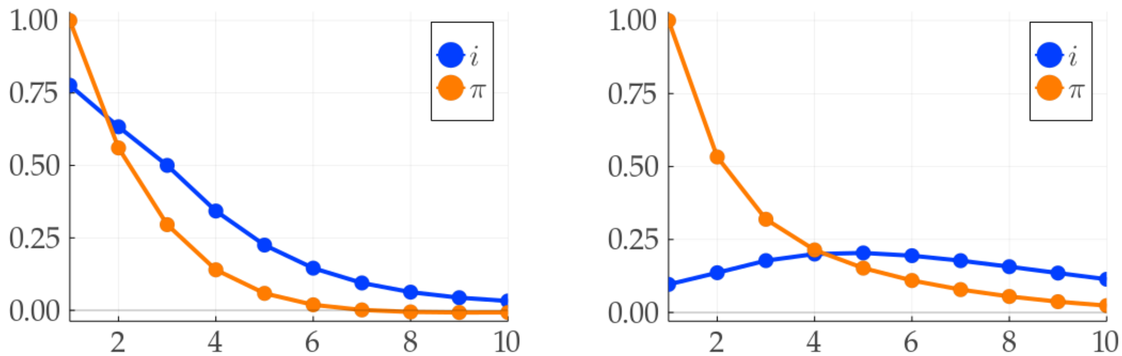
$$\left| \sum_k \Delta E_t r_{t+k} \right| > \left| \sum_k (1-\omega^k) \Delta E_t r_{t+k} \right|$$

if and only if $\sum_k \Delta E_t r_k$ and $\sum_k \omega^k \Delta E_t r_k$ also have the same sign.

In figures 1a and 1b, I plot the IRFs of nominal interest and inflation in the cases of Poland and Norway. In Poland, unexpected inflation leads to a prolonged period of high real interest, a result of high rates in response to inflation in the 2000s. That is an inflationary force. As the inflation shock hits, the price of long-term bond declines - a deflationary force that partially counteracts the former effect. Hence, $p_1(r) < p_2(r)$.

In Norway, unexpected inflation leads a smaller but longer period of higher nominal interest. This is likely a consequence of the delayed response of monetary policy to 1970s inflation. Real interest is at first negative, then turns positive. Overall, the real value of debt declines, $p_1(r) > 0$. But the price of bonds *rises* - which is another inflationary effect - as it responds more to near term

[11] In the case of nominal and dollar debt, this is *given* the long-term inflation and real interest effects, which I am holding constant here.



(a) Poland: Lower Contribution $p_1 < p_2$ (b) Norway: Larger Contribution $p_2 > p_1$

Figure 1: Long-Term Bonds and the Contribution of Discounting to Unexpected Inflation

real interest.[12] Hence, $p_2(r) > p_1(r)$.

Among advanced economies, in the six cases that long-term bonds enhance the contribution of real interest $\|p_2\| > \|p_1\|$, they contribute to make real interest dynamics more *inflationary*. Cochrane finds a similar result for the US, but in a much weaker magnitude: $p_1(r) = 1.0$, $p_2(r) = 1.04$. I find $p_1(r) = 0.96$, $p_2(r) = 1.32$.

In all countries of the 1960 sample, with exception of the UK, unexpected inflation forecasts real interest, which accounts for 0.5 to 1.5 times unexpected inflation variance. Like for the US, *discounting matters*.

In the 1973 sample, I estimate real interest to reduce inflation variance, probably due to delayed responses of monetary policy to 1970s (which affect results more than in the 1960 sample). New Zealand, the single country in the group with a positive $p_2(r)$ raised interest in the 70s as inflation was high but stable, and increased interest swiftly in 1985 concomitantly to a new inflation spike.

As for emerging markets, I again estimate economically significant, but statistically insignificant coefficients. In all but three cases (two being the volatile Chile and South Africa cases) they are positive. The four statistically significant coefficients are large and positive. In all, discounting and growth dynamics look more important drivers of unexpected inflation in emerging markets then surplus-to-GDP ratios.

- *For countries with dollar debt, US inflation variation contributes to reduce the volatility of unexpected inflation ($p_2(\pi^{US}) < 0$). The effect is stronger in emerging markets, since their governments tend to issue more dollar-linked bonds.*

Decomposition 2 isolates the contributions of real exchange rate and US inflation. They are only non-zero for countries with dollar debt. In all but three countries with dollar debt (India, Israel and Mexico), US inflation works as a deflationary force. Unexpected domestic inflation forecasts/nowcasts US inflation, which devalues dollar-linked bonds.

Magnitudes are larger in the 1998 emerging markets sample, typically in the interval -0.10 to -0.60 (-2.75 for Chile derives from its large reliance on dollar debt, $\delta_D = 0.33$). This likely due to

[12] The more precise statement is: the effect of real interest is a rise in bond prices. Here, I am holding the effect of constant the effect of inflation, which is another term in the decomposition, $p_2(\pi)$. Nominal and dollar bond prices do not necessarily rise. They do not in figure 1b.

negative inflation shocks in 2008, as inflation declined in the US with the Great Recession. Indeed, in Mexico inflation does not fall significantly in 2008/2009.

Among advanced economies, conclusions are similar in both samples, but covariances are considerably smaller.

As for real exchange rate, in only four of the thirteen developing countries with dollar debt I estimate statistically significant contributions to unexpected inflation variance. Signs are mixed, so there are no obvious conclusions to be made.

4.2 Response to Reduced-Form Shocks

4.2.1 "Aggregate Demand" Shock

The variance decompositions of inflation (10) and (14) are algebraically equivalent to the main decompositions 1 and 2, applied to the shock

$$\Delta E_t \pi = e_\pi = 1\% \qquad \text{and} \qquad e_{-\pi} = \Omega_{-\pi,\pi}\Omega_\pi^{-1} e_\pi,$$

where $e_{-\pi}$ piles all shocks but e_π, Ω_π is inflation variance and $\Omega_{-\pi,\pi}$ is covariance vector between $e_{-\pi}$ and e_π.

I now generalize the procedure to other innovations. Given a set of shocks x and their value e_x, I project the response of the other shocks:

$$e_{-x} = \Omega_{-x,x}\Omega_x^{-1} e_x$$

and calculate the terms in decompositions 1 and 2 to $\Delta E_t X_t = K e_t$.

Because I do not orthogonalize or attempt to build structural shocks, the decompositions do not tell a causal story and do not provide a structural interpretation. But, like the variance decompositions above, they provide clues about inflation dynamics to support model building.

I begin with an "aggregate demand" shock, captured by a negative surprise in GDP growth and in the inflation rate: $\Delta E_t g_t = -1$ and $\Delta E_t \pi_t = -0.5$. I choose a lower inflation shock based on the change in US inflation in the 2008 recession. The model is linear, so you can interpret values in percentage points too.

To save space, I report only decomposition 2, in table 5. The first column reports unexpected inflation. The other columns report the six terms of the decomposition. I divide the whole expression by $\delta v/\beta$ so that the terms combine to $dE\pi_t$:

$$\Delta E_t \pi_t = -d_\pi - d_s - d_g + d_r + d_{\Delta h} - d_{\pi^{US}},$$

where for example $d_g = \delta^{-1} \sum_{k=1}^{\infty} \beta^k \Delta E_t g_{t+k}$.

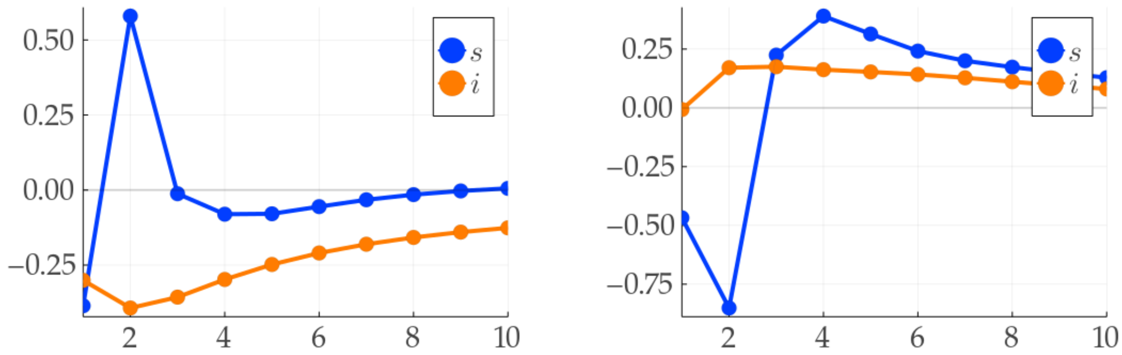
Given the aggregate demand shock, unexpected inflation equals -0.5 by construction. But where does it come from?

- *Lower inflation in the low aggregate demand scenario follows from lower real interest and larger surpluses. Many countries, like the US, react by raising deficits, but higher subsequent surpluses revert its inflationary effect. Lower growth, stimulative monetary policy and, for emerging markets, real exchange are inflationary.*

Country	$\Delta E_t \pi_t$	Decomposition 2					
		$-d_\pi$	$-d_s$	$-d_g$	d_r	$d_{\Delta h}$	$-d_\pi us$
Averages	-0.50	**0.78***	**-4.23***	**2.71***	-0.88	**0.59***	**0.54***
Advanced - 1960	-0.50	**1.24***	**-1.47***	**1.33***	**-1.70***	**0.04**	**0.05***
Advanced - 1973	-0.50	**0.83***	**-1.60***	**0.74***	**-0.56**	**0.06**	**0.02***
Emerging - 1998	-0.50	**0.58***	**-6.41**	**3.96***	-0.56	**1.01***	**0.93***
Median	-0.50	**0.64***	**-1.46***	**1.30***	**-1.23***	**0.18***	**0.11***
Advanced - 1960	-0.50	**1.13***	**-1.43***	**1.25***	**-1.78***	0	**0.03***
Advanced - 1973	-0.50	**0.82***	**-1.19***	**0.65**	**-0.43**	**0.03***	**0.01***
Emerging - 1998	-0.50	**0.48***	**-3.20***	**1.36***	-0.58	**0.51***	**0.61***
United States	-0.50	**0.57***	**-0.65**	**1.32***	**-1.75***	0	-
Advanced - 1960 Sample							
Canada	-0.50	**1.38***	-0.45	0.30	**-1.78***	-0.00	**0.05***
Denmark	-0.50	**1.06***	**-2.64**	**2.75***	**-1.78**	-0.04	**0.15***
Japan	-0.50	**0.60***	**-1.51***	**1.64***	**-1.23***	0	0
Norway	-0.50	**0.99***	**-1.36**	**1.72***	**-1.86***	0	0
Sweden	-0.50	**1.19***	-0.65	**0.87**	**-2.31***	**0.30***	**0.11***
United Kingdom	-0.50	**2.19***	**-2.20**	**0.73**	**-1.22**	0	0
Advanced - 1973 Sample							
Australia	-0.50	**0.96***	**-1.46**	**0.66**	-0.67	0	0
New Zealand	-0.50	**0.68***	**-0.84**	**0.63**	**-1.24**	**0.19**	**0.07***
South Korea	-0.50	**1.06***	**-3.17***	**1.74***	-0.20	**0.05***	**0.02***
Switzerland	-0.50	**0.64***	**-0.93***	-0.07	-0.13	0	0
Emerging - 1998 Sample							
Brazil	-0.50	**0.08**	1.87	0.13	**-2.85**	**0.23***	**0.04***
Chile	-0.50	**2.10***	-30.50	**30.54**	-7.76	-0.63	**5.74***
Colombia	-0.50	**0.49**	**-10.90**	**7.57***	-0.07	1.16	**1.26***
Czech Republic	-0.50	**0.51***	-0.07	**0.25**	**-1.61**	**0.27***	**0.14***
Hungary	-0.50	**0.64***	10.82	-5.29	**-7.91**	**0.70***	**0.54***
India	-0.50	0.45	**-1.16**	**0.71**	-0.44	-0.02	-0.05
Indonesia	-0.50	0.02	**-11.24***	**1.42**	0.76	**6.82***	**1.73**
Israel	-0.50	**0.47***	**-3.18**	**1.17**	-0.73	**0.98***	**0.79***
Mexico	-0.50	**0.48***	**-4.56***	**1.94***	0.04	**0.92***	**0.68***
Poland	-0.50	**0.50***	-0.14	**1.30**	**-2.90**	0.32	**0.42***
Romania	-0.50	**0.36**	**-8.16***	**2.05**	2.15	**2.33***	**0.77***
South Africa	-0.50	**1.59***	**-30.02***	**11.15***	**15.60***	**0.87**	**0.30***
Turkey	-0.50	**0.73***	0.64	**0.52**	**-3.31***	0.18	**0.73***
Ukraine	-0.50	-0.33	**-3.22**	**1.92***	**1.13**	0	0

Notes: I set $\varepsilon_g = -1$ and $\varepsilon_\pi = -0.5$, and regress the values of other shocks on it. The table shows the estimated terms of decomposition 2 at the posterior distribution's mode. I divide each term by $\delta v/\beta$, so the terms sum up to $\Delta E_t \pi$. I simulate from the posterior to check the statistical significance of the sign of the estimate. Bold indicates 75% of results had the same sign. Bold with an asterisk indicates 90%.

Table 5: Unexpected Inflation Decomposition 2 - Recession Shock



(a) United States, $d_s = 0.65$

(b) Ukraine, $d_s = 3.22$

Figure 2: Fiscal Shocks in Recessions: Deficits then Surpluses

For all developed economies, the discounting effect contributes to the low inflation, although the result is stronger in the 1960 sample. Reported figures already incorporate the effect of higher long-term bond prices, but inspection of decomposition 1 (not reported) reveals that they are not driving the sign of the coefficients. Lower real interest raises the real value of debt - discounted surpluses - as it raises the value of other assets. The effect on the price level is negative. Magnitudes are quantitatively significant, often larger than -1.5 (for a -0.5 inflation shock). Cochrane (2022) finds the this conclusion for the United States. It holds for other countries too.

In the case of emerging markets, like before, the conclusion is similar, but estimates are more often statistically insignificant or too volatile (Chile and South Africa are notorious). But most coefficients, significant or not, are negative. Monetary policy in South Africa and Ukraine - the two positive and significant coefficients - did not react strongly to recent recession shocks (2009 for both and, in the Ukrainian case, 2014/15 following the Russian invasion of Crimea, see figure 2b).

To all countries except Ukraine, the table suggests a strong response of monetary policy through lower interest. Higher bond prices forecast lower future inflation, which corresponds to a time-t *inflationary* force in the decomposition, $d_\pi < 0$. Lower growth is also inflationary, as it raises the relative size of public debt.

Results indicate that fiscal policy, on the other hand, is *deflationary*, in that

$$\Delta E_t s_t + \sum_{k=1}^{\infty} \beta^k \Delta E_t s_{t+k} < 0.$$

This is not an indication that governments do not incur in deficits to "stimulate demand", although that is the case for many of them. To others, like that of the United States and Ukraine depicted in figure 2, a deficit at the time recession hits is followed by future surpluses. For the real value of debt, what matters is discounted surpluses, not current surpluses. Hence the deflationary effect $-d_s < 0$.

Again results are economically and statistically significant. Most estimates are larger than -1. In a causal reading, they suggest that, should governments manage to convince the public that large recession deficits would *not* be followed by future surpluses, the impact on unexpected inflation would be large. One can interpret the post-COVID, worldwide surge in inflation as an example of that (I do not include 2021-2022 data in the estimation).

The result of deflationary fiscal policy is common to all samples, but stronger in emerging markets, *despite* their lower share of nominal debt (recall that δ enters in the denominator of d_s). More procyclical fiscal policy in developing countries has been previously identified by the macroeconomic literature. See, for instance Kaminsky et al. (2004), Alesina et al. (2008) and Ilzetzki (2011).

Finally, I find for emerging markets an inflationary effect stemming from real exchange rate depreciation. I interpret these figures as capital exit and flight-to-quality movements in exchange rates (Jiang et al. (2020), Kekre and Lenel (2021)).

4.2.2 Exchange Rate Depreciation Shock

I next consider episodes of real exchange rate depreciation. I consider a 10% shock to Δh. Such episodes can be due to shocks in the international economy, like Global Recessions, or shocks in the domestic economy, like sudden stops. I am going to focus on the latter, so I prevent US variables from jumping in the VAR:

$$e_{\Delta h} = 10, \qquad \varepsilon_u = 0.$$

US variables are unaffected by domestic dynamics, so they do not respond to the shock at all. Table 6 reports results.

- *In emerging markets, real exchange depreciation shocks forecast low growth and contractionary monetary and fiscal policy. Higher nominal interest trades current for future inflation, $-d_\pi < 0$; higher surpluses are deflationary. Lower growth and the depreciated currency are inflationary. Unexpected inflation is statistically zero in most cases.*

In emerging markets, real exchange rate shocks resemble "sudden stops", events in which foreign households abruptly sell holdings of domestic assets.[13] Nominal exchange depreciates and raises the in-domestic-currency value of public debt, which counts as an inflationary force by our decompositions. Its magnitude depends on the size of the dollar-linked portfolio of public debt but is positive and statistically significant to all countries.

Emerging markets respond to the depreciation shock by raising interest (unreported figures). Lower bond prices forecast future inflation by (13). Fiscal policy response is contractionary, $-d_s < 0$, with statistically and economically significant coefficients.[14] Median coefficients of growth and discounting effects indicate that both are inflationary, but the growth effect is more widespread in the sample (nine of the fourteen countries). Indeed, sudden stops are characterized by output drops (Calvo et al. (2006)).

Results suggest that policy efforts are often successful in mitigating unexpected inflation (although often at the cost of future inflation, $d_\pi > 0$). From the fourteen cases, I estimate statistically significant $dE\pi > 0$ in two.

The evidence for advanced economies is far less conclusive. From the set of developed countries, Canada, Denmark, Sweden, New Zealand and South Korea have a share of dollar debt greater than

[13] The literature on sudden stops is vast. See, for example, Calvo et al. (2006) and Verner and Gyöngyösi (2020) for empirical analysis, and Chari et al. (2005) and Mendoza (2010) for theoretical and quantitative accounts.

[14] Hungary and Ukraine are exceptions, although Hungarian estimates look suspiciously volatile for a country with low dollar debt, $\delta_D = 0.23$. Ukrainian result is driven by the 30% surge in inflation following the 2014 invasion of Crimea, which corresponds to an estimated fiscal *deficit* at the same time its currency depreciated.

Country		Decomposition 2					
	$\Delta E_t \pi_t$	$-d_\pi$	$-d_s$	$-d_g$	d_r	$d_{\Delta h}$	$-d_{\pi^{US}}$
Averages	0.16	-0.27	**-4.48***	0.03	0.36	**4.53***	0
Advanced - 1960	**-0.39**	0.21	**-1.78**	-0.90	**1.29**	**0.80***	0
Advanced - 1973	**0.41**	0.15	**-1.41**	0.95	0.12	**0.61***	0
Emerging - 1998	0.29	**-0.57**	**-6.82**	0.03	-0.07	**7.57***	0
Median	0.02	**-0.16**	**-1.23***	-0.34	**0.37**	**1.53***	0
Advanced - 1960	**-0.69***	0.11	**-1.51**	**-1.29**	**1.90**	**0.27***	0
Advanced - 1973	**0.36**	0.11	-1.50	0.49	-0.09	**0.18***	0
Emerging - 1998	0.05	**-0.55***	**-2.90***	**1.29**	**0.49**	**3.25***	0
United States	**0.56***	**-0.75**	-0.19	**1.78**	-0.28	0	-
Advanced - 1960 Sample							
Canada	**-0.68***	0.81	**-2.64**	**-1.15**	1.82	**0.48***	0
Denmark	**-0.81**	0.32	**-3.87**	**-2.54**	**3.09**	**2.19***	0
Japan	**-0.36**	-0.10	**-1.80***	**2.15***	-0.61	0	0
Norway	**-1.29***	-0.20	-1.23	**-2.10**	**2.18**	**0.06***	0
Sweden	1.52	-0.83	-0.28	-1.43	1.98	**2.09***	0
United Kingdom	-0.69	1.24	-0.85	-0.34	-0.75	0	0
Advanced - 1973 Sample							
Australia	-0.10	**1.21**	-0.25	**-0.86**	-0.21	**0.03***	0
New Zealand	0.39	-0.84	**-3.91**	**1.76**	1.32	**2.07***	0
South Korea	**1.00**	0.40	**-2.74**	**3.67***	-0.66	**0.33***	0
Switzerland	**0.33**	-0.18	**1.25**	**-0.78**	0.02	0	0
Emerging - 1998 Sample							
Brazil	0.02	**-0.16***	-0.76	**1.43**	**-1.24**	**0.74***	0
Chile	0.41	**1.90**	**-87.99**	**29.66**	4.07	**52.78***	0
Colombia	-0.02	**-1.00***	**-11.74***	**3.06**	0.65	**9.00***	0
Czech Republic	0.08	-0.06	-6.64	3.88	1.36	**1.53***	0
Hungary	-0.85	0.03	**39.37**	**-29.48**	**-12.43**	**1.67**	0
India	-0.62	**3.87***	**-4.65**	-0.36	-0.02	**0.54***	0
Indonesia	-0.28	**-0.93***	**-12.74***	**2.34***	0.35	**10.71***	0
Israel	0.25	0.51	**-13.85***	**3.35**	**4.61***	**5.63***	0
Mexico	-0.69	**-0.94**	4.77	-7.65	-1.25	**4.39***	0
Poland	**-0.65**	0.26	**-0.08**	-3.56	0.62	**2.11***	0
Romania	2.58	**-3.18**	**-7.36**	**2.15**	0.37	**10.61***	0
South Africa	0.20	**-1.50***	3.52	-1.06	-2.10	**1.35***	0
Turkey	**2.26***	**-2.29***	-1.15	**-4.42**	**5.18***	**4.93***	0
Ukraine	**1.34**	**-4.45***	**3.79**	**1.16**	0.81	**0.02***	0

Notes: I set $\varepsilon_{\Delta h} = 10$, and regress the values of other shocks on it. The table shows the estimated terms of decomposition 2 at the posterior distribution's mode. I divide each term by $\delta v/\beta$, so the terms sum up to $\Delta E_t \pi$. I simulate from the posterior to check the statistical significance of the sign of the estimate. Bold indicates 75% of results had the same sign. Bold with an asterisk indicates 90%.

Table 6: Unexpected Inflation Decomposition 2 - Real Exchange Depreciation

1%. These five economies respond to the depreciation event by raising surpluses, like emerging markets.

But depreciation shocks do not necessarily lead to a drop in output. They do the opposite in the 1960 sample (except for Japan), and the sign of coefficients split two and two in the 1973 sample. These episodes do not look like "sudden stops" in emerging markets. The case of South Korea is an exception. Its large output coefficient $-d_g > 0$ is driven by the 1998 recession following the Asian financial crisis.

The contribution of monetary policy is also ambiguous, as is the overall effect on unexpected inflation.

5 New-Keynesian Model Benchmarks

This section is incomplete.

5.1 Overview

The variance decomposition is not structural, it is simply a measure of which variables move along with unexpected inflation to account for it. The same is true for the other reduced-form shocks I analyzed.

How do these measures compare with theory? In this section, I contrast empirical estimates with the decomposition resulting from a simple open-economy New-Keynesian model. The private sector block of the model is inspired on Galí and Monacelli (2005). Public policy is characterized by a Taylor (1993) rule and an "active" fiscal policy, as in the FTPL.

To give the simple model the best change to reproduce empirical patterns, I estimate its parameters by a method of moments to approximate the decomposition. I use as moments targets the values computed for New Zealand. I choose New Zealand first for its reliance on real and dollar-debt and, second, because its decomposition seldom deviates from the broader results we find for developed countries, even in the 1960 sample.

5.2 Model Equations

5.2.1 Private Sector and Market Clearing

The general environment is similar to Galí and Monacelli (2005). The model is at this point well known in the macroeconomic literature, so I state only its linearized version. Appendix F presents a quick derivation.

The Home economy is open to international trade and capital markets. The world is populated by other economies and its households, and the model takes the limit $n \to 0$, where n is the relative size of the domestic economy in terms of households and firms. Therefore, domestic variables do not affect world consumption = output y_t^*, interest i_t^* and inflation π_t^*.

The Home economy features a representative household that consumes and saves, and firms that operate under monopolistic competition and Calvo price rigidity. Goods are differentiated, and households imperfectly substitute one for the other. International capital markets are complete, and there is perfect risk sharing.

The private sector + equilibrium equations of the model are

$$y_t = E_t y_{t+1} - \frac{1}{\sigma}\left[i_t - E_t \pi_{t+1} + \alpha\bar{\omega} E_t \Delta z_{t+1}\right] \tag{21}$$

$$\pi_{H,t} = \beta E_t \pi_{H,t+1} + \kappa y_t - \kappa_\alpha y_t^* - \kappa_a a_t \tag{22}$$

$$y_t = y_t^* + \frac{1}{\sigma_\alpha} z_t \tag{23}$$

$$h_t = (1-\alpha) z_t \tag{24}$$

$$\pi_t = \pi_{H,t} + \alpha \Delta z_t \tag{25}$$

$$a_t = \rho_a a_{t-1} + \varepsilon_{a,t} \tag{26}$$

The model contains two key international prices: terms of trade z_t and the real exchange rate h_t (these are averages of their bilateral counterparts). Domestic output prices $p_{H,t}$ and the consumer price index p_t are related by $p_t = p_{H,t} + \alpha z_t$, where α is the weight of foreign goods on the Home economy basket. If $\alpha < 1$, there is home bias in consumption. Equation (25) follows. Price variables are in logs.

Equation (24) follows from a decomposition of real exchange rate: foreign-to-consumer price ratio (h_t) equals foreign-to-domestic-output price ratio (z_t) times (plus, in logs) domestic-output-to-consumer price ratio ($-\alpha z_t$). With complete markets, the Backus and Smith (1993) condition holds: domestic consumption rises relative to foreign consumption when h depreciates:

$$c_t = y_t^* + \frac{1}{\sigma} h_t. \tag{27}$$

Equation (21) is the intertemporal IS; it follows from households' Euler equation, added to

$$y_t = c_t + \frac{\alpha\bar{\omega}}{\sigma} z_t, \tag{28}$$

which states that demand for home goods equals domestic consumption plus a term that adjusts for relative price variation of Home goods. Parameter $\bar{\omega}$ can be greater or lower than zero.[15] Depreciated terms of trade reduce the relative price for Home goods, but also correspond to lower foreign output due to the international risk-sharing rule (27). The net result on aggregate demand for domestic output can go either way.

Equation (23) follows from (24), (27) and (28). Parameter $\sigma_a = \sigma(1 - \alpha + \alpha\bar{\omega})^{-1}$ adjusts intertemporal substitution for the presence of home bias and imperfect substitution different varieties of goods.

Lastly, the forward-looking Phillips Curve follows from optimal price-setting behavior. Its parameters are $\kappa = \lambda(\sigma_\alpha + \varphi) > 0$, $\kappa_a = \lambda(1+\varphi) > 0$ and $\kappa_\alpha = \lambda(\alpha(1-\bar{\omega})\sigma_\alpha)$. Parameter φ is the Frisch elasticity of labor supply, and $\lambda = (1-\beta\theta)(1-\theta)/\theta$ is a function of intertemporal discounting β and the price-resetting rate $1-\theta$.

[15] I use the bar notation on $\bar{\omega}$ to differentiate it from the geometric term structure parameter of public debt.

5.2.2 Model Solution and Indeterminacy

5.2.3 Public Policy and Equilibrium Selection

The public policy block of the model contains the additional explosive root required by equilibrium selection.

$$i_t = \phi \pi_t + \varepsilon_{i,t} \tag{29}$$

$$\beta \left[v_{l,t} + s_{l,t} - \alpha s_l z_t \right] = v_{l,t-1} + v_l \left[\sum_j \delta_j \left(rx_{j,t} + i_{j,t-1} + \Delta h_{j,t} - \pi_{j,t} \right) - g_t \right] \qquad l = 1,2 \tag{30}$$

$$s_{1,t} = \rho s_{1,t-1} + \tau_1 y_t + \psi v_{1,t-1} + \varepsilon_{1,t} \tag{31}$$

$$s_{2,t} = \rho s_{2,t-1} + \tau_2 y_t + \varepsilon_{2,t} \tag{32}$$

The interest rate follows a single-mandate Taylor rule. Fiscal policy is active, but accommodates partial debt repayment. Specifically, there are two debt processes, $\{v_{l,t}\}$, and two surplus processes $\{s_{l,t}\}$, indexed by $l = 1,2$.

The government keeps the same term and currency structure on the two debt processes, so they accrue interest at the same rate (the term in brackets on the right-hand side of (30) does not depend on l). Since v_l and s_l are conveniently stated in levels, $v_t = v_{1,t} + v_{2,t}$, and equation (9) holds with $s_t = s_{1,t} + s_{2,t}$ and $p^s_t = p_{H,t}$.

The setup is inspired by Jacobson et al. (2019).

5.2.4 A Representative "Foreign" Economy

$$y^*_t = E_t y^*_{t+1} - \sigma^{-1} \left[i^*_t - E_t \pi^*_{t+1} \right] \tag{33}$$

$$\pi^*_t = \beta E_t \pi^*_{t+1} + \kappa^* y^*_t - \kappa^*_a a^*_t \tag{34}$$

$$i^*_t = \phi^* \pi^*_t + \varepsilon^*_{i,t} \tag{35}$$

$$\Delta E_t \pi^*_t = \varepsilon^*_{\pi,t} + \mu^*_i \varepsilon^*_{i,t} + + \mu^*_a \varepsilon^*_{a,t} \tag{36}$$

$$a^*_t = \rho^*_a a^*_{t-1} + \varepsilon^*_{a,t} \tag{37}$$

Calibrated for the United States. All parameter are specific, except for β and σ.

6 Robustness

7 Conclusion

References

Alesina, A., Campante, F. R., and Tabellini, G. (2008). Why is Fiscal Policy Often Procyclical? *Journal of the European Economic Association*, 6(5):1006–1036.

Ali Abbas, S. M., Belhocine, N., El-Ganainy, A., and Horton, M. (2011). Historical Patterns and Dynamics of Public Debt—Evidence From a New Database. *IMF Economic Review*, 59(4):717–742.

Backus, D. K. and Smith, G. W. (1993). Consumption and real exchange rates in dynamic economies with non-traded goods. *Journal of International Economics*, 35(3-4):297–316.

Calvo, G. A., Izquierdo, A., and Talvi, E. (2006). Sudden Stops and Phoenix Miracles in Emerging Markets. *The American Economic Review*, 96(2):405–410.

Campbell, J. Y. and Ammer, J. (1993). What Moves the Stock and Bond Markets? A Variance Decomposition for Long-Term Asset Returns. *The Journal of Finance*, 48(1):3–37.

Campbell, J. Y. and Shiller, R. J. (1988). The Dividend-Price Ratio and Expectations of Future Dividends and Discount Factors. *The Review of Financial Studies*, 1(3):195–228.

Carriero, A., Clark, T. E., and Marcellino, M. (2015). Bayesian VARs: Specification Choices and Forecast Accuracy. *Journal of Applied Econometrics*, 30(1):46–73.

Chari, V. V., Kehoe, P. J., and McGrattan, E. R. (2005). Sudden Stops and Output Drops. *American Economic Review*, 95(2):381–387.

Cochrane, J. H. (2005). Money as stock. *Journal of Monetary Economics*, 52(3):501–528.

Cochrane, J. H. (2022). The fiscal roots of inflation. *Review of Economic Dynamics*, 45:22–40.

Coibion, O. and Gorodnichenko, Y. (2011). Monetary Policy, Trend Inflation, and the Great Moderation: An Alternative Interpretation. *American Economic Review*, 101(1):341–370.

Cox, W. M. (1985). The behavior of treasury securities monthly, 1942–1984. *Journal of Monetary Economics*, 16(2):227–250.

Cox, W. M. and Hirschhorn, E. (1983). The market value of U.S. government debt; Monthly, 1942–1980. *Journal of Monetary Economics*, 11(2):261–272.

del Negro, M. and Schorfheide, F. (2011). Bayesian Macroeconometrics. In Geweke, J., Koop, G., and Van Dijk, H., editors, *The Oxford Handbook of Bayesian Econometrics*, pages 292–389. Oxford University Press.

Eitrheim, Ø., Klovland, J. T., and Qvigstad, J. F. (2007). Historical Monetary Statistics for Norway - Part II. *Norges Bank Occasional Papers*, 38.

Galí, J. and Monacelli, T. (2005). Monetary Policy and Exchange Rate Volatility in a Small Open Economy. *The Review of Economic Studies*, 72.

Garín, J., Lester, R., and Sims, E. (2018). Raise Rates to Raise Inflation? Neo-Fisherianism in the New Keynesian Model. *Journal of Money, Credit and Banking*, 50(1):243–259.

Giannone, D., Lenza, M., and Primiceri, G. (2015). Prior Selection for Vector Autoregressions. *The Review of Economics and Statistics*, 97(2):436–451.

Ilzetzki, E. (2011). Rent-seeking distortions and fiscal procyclicality. *Journal of Development Economics*, 96(1):30–46.

Jacobson, M., Leeper, E., and Preston, B. (2019). Recovery of 1933. Technical Report w25629, National Bureau of Economic Research, Cambridge, MA.

Jiang, Z., Krishnamurthy, A., and Lustig, H. (2020). Dollar Safety and the Global Financial Cycle. Technical Report w27682, National Bureau of Economic Research, Cambridge, MA.

Kaminsky, G. L., Reinhart, C. M., and Végh, C. A. (2004). When It Rains, It Pours: Procyclical Capital Flows and Macroeconomic Policies. *NBER Macroeconomics Annual*, 19:11–53.

Karlsson, S. (2013). Forecasting with Bayesian Vector Autoregression. In *Handbook of Economic Forecasting*, volume 2, pages 791–897. Elsevier.

Kekre, R. and Lenel, M. (2021). The Flight to Safety and International Risk Sharing. Technical Report w29238, National Bureau of Economic Research, Cambridge, MA.

Koop, G. M. (2013). Forecasting with Medium and Large Bayesian VARS. *Journal of Applied Econometrics*, 28(2):177–203.

Litterman, R. (1979). Techniques of forecasting using Vector Auto Regression. *Federal Reserve Bank of Minneapolis Working Paper*, 115.

Mendoza, E. G. (2010). Sudden Stops, Financial Crises, and Leverage. *The American Economic Review*, 100(5):1941–1966.

Sims, C. A. (2011). Stepping on a rake: The role of fiscal policy in the inflation of the 1970s. *European Economic Review*, 55(1):48–56.

Stock, J. H. and Watson, M. W. (2002). Has the Business Cycle Changed and Why? *NBER Macroeconomics Annual*, 17:159–218.

Taylor, J. B. (1993). Discretion versus policy rules in practice. *Carnegie-Rochester Conference Series on Public Policy*, 39:195–214.

Uribe, M. (2022). The Neo-Fisher Effect: Econometric Evidence from Empirical and Optimizing Models. *American Economic Journal: Macroeconomics*, 14(3):133–162.

Verner, E. and Gyöngyösi, G. (2020). Household Debt Revaluation and the Real Economy: Evidence from a Foreign Currency Debt Crisis. *American Economic Review*, 110(9):2667–2702.

A Linearization

B Data Sources and Treatment

B.1 Sources

I collect a significant share of the data from the St. Louis Fed's *FRED* website. In the case of countries with sample starting after 1970 I get data from the United Nations's National Accounts Main Aggregates Database. Their database also contains exchange rate data, which I use only in the case of emerging markets (with sample starting after 1998).

Whenever omitted in the list below, the source for interest rate data is the FRED; and the source of debt structure data is the OECD's Central Government Debt database. Finally, unless otherwise noted, public debt data I get from the database from Ali Abbas et al. (2011), which is kept up-to-date.

Australia 1973-2021. All except GDP and public debt from FRED.

Brazil 1998-2021. Debt structure data I collect from the Brazilian Central Bank.

Canada 1960-2021. All except public debt from FRED.

Chile 1998-2021.

Colombia 1998-2021. Debt structure data I collect from the Internal Debt Profile report, available at the Investor Relations Colombia webpage.

Czech Republic 1998-2021.

Denmark 1960-2021. All except public debt from FRED.

Hungary 1998-2021.

India 1998-2021. Debt structure data collect from the Status Paper on Government Debt report, 2019-2020, available at the Department of Economic Affairs.

Indonesia 1998-2021. Debt structure data I gather from the 2014 "Central Government Debt Profile" report and the 2018 "Government Securities Management" report, both from the Ministry of Finance.

Israel 1998-2021.

Japan 1960-2021. All except public debt from FRED.

Mexico 1998-2021.

Norway 1960-2021. All except public debt and interest rates from FRED. I interpolate the debt data for the year 1966. FRED interest data goes back to 1979, I splice it with historical data from Eitrheim et al. (2007), available at the website of the Norges Bank.

New Zealand 1973-2021. All except GDP and public debt from FRED.

Poland 1998-2021.

Romania 1998-2021. Interest rate is the deposit rate series from IMF's International Finance Statistics. Debt structure data I collect from the 2018 "Flash Report on the Romanian Public Debt" and the 2019-2021 and 2021-2023 "Government Debt Management Strategy" report, all from the Treasury and Public Debt Deparment (Ministry of Public Finance).

South Africa 1998-2021. Debt structure data from the 2020/2021 Debt Management Report, from the National Treasury Department.

South Korea 1973-2021. All except GDP and public debt from FRED.

Sweden 1960-2021. All except public debt from FRED. I interpolate the debt data for the year 1965 and 1966.

Switzerland 1973-2021. Interest, CPI and exchange rate from FRED.

Turkey 1998-2021.

Ukraine 1998-2021. Interest rate data from National Bank of Ukraine (NBU Key Policy Rate). Debt structure data I collect from "Ukraine's Public Debt Performance in 2021 and Local Market Update", from the Ministry of Finance of Ukraine.

United Kingdom 1960-2021. Interest data from the Bank of England (Base Rate); I splice it with discount rate data from the FRED. Inflation and GDP data from FRED.

United States 1950-2021. All data collected from the FRED. I use real exchange rate to the United Kingdom, since nominal exchange rate is available since before 1950.

C Additional Details of the BVAR Estimation

D Alternative Reduced-Form Shocks

D.1 Primary Deficit Shock

E Equilibrium Selection in the NK Model

F Deriving the SOE-NK Model

Country	$\Delta E_t \pi_t$	Decomposition 2					
		$-d_\pi$	$-d_s$	$-d_g$	d_r	$d_{\Delta h}$	-$d_\pi us$
Averages	**0.13***	-0.02	-0.89	**0.92***	0.18	0.02	**-0.08***
Advanced - 1960	**0.02**	**0.12**	-0.13	**0.31**	-0.28	**0.02**	**-0.01***
Advanced - 1973	**0.33***	**-0.27***	1.04	0.39	-0.79	**-0.02**	**-0.01***
Emerging - 1998	**0.13***	-0.02	**-1.84**	**1.30**	**0.78**	0.04	**-0.13**
Median	**0.08***	-0.02	0.01	**0.41***	0.16	0	0
Advanced - 1960	0	**0.08***	**-0.19**	**0.27**	-0.16	**0.01***	0
Advanced - 1973	**0.29***	-0.31	0.66	0.35	-0.57	0	**-0.01***
Emerging - 1998	**0.13***	-0.03	-0.13	**0.47**	**0.47**	-0.07	-0.01
United States	**-0.11***	**0.10***	0.09	**1.24***	**-1.55***	-	-
Advanced - 1960 Sample							
Canada	0.01	**0.32***	-0.47	0.26	-0.16	**0.05***	0
Denmark	**0.12***	**0.29***	**-0.72**	**0.41**	0.16	0.02	**-0.04***
Japan	0.04	-0.01	**-0.30**	**1.27***	**-0.92***	0	0
Norway	0	**0.08**	-0.04	0.12	-0.16	0	0
Sweden	-0.03	**0.07**	-0.09	**-0.45***	**0.44**	0.03	**-0.03***
United Kingdom	0	-0.06	**0.83**	0.27	**-1.04**	0	0
Advanced - 1973 Sample							
Australia	**0.08**	**-0.51**	**2.80**	0.63	**-2.84**	0	0
New Zealand	**0.41***	**-0.40***	0.05	0.06	**0.81**	**-0.10**	**-0.03***
South Korea	**0.65***	-0.22	1.10	0.79	-1.00	0	**-0.01***
Switzerland	**0.17***	0.02	0.21	0.07	-0.13	0	0
Emerging - 1998 Sample							
Brazil	**0.12**	**-0.03**	0.11	-0.24	0.12	**0.14***	**0.02***
Chile	**0.20***	**0.60**	-5.19	4.10	-2.32	2.15	0.86
Colombia	-0.02	**-0.27**	-0.27	0.20	0.27	0.03	0.03
Czech Republic	**0.24***	**0.40**	**-3.92**	**3.13***	0.60	0	**0.04**
Hungary	0.03	0.04	**-9.93**	**5.86***	**4.55***	**-0.27**	**-0.23***
India	-0.11	-0.32	**0.95**	**-0.36**	-0.26	**-0.14***	0.02
Indonesia	**0.14**	**-0.60***	1.36	**1.56**	0.50	-0.53	**-2.13***
Israel	**0.30***	-0.08	0.03	**0.51**	0.20	-0.14	**-0.21***
Mexico	**0.22***	**-0.32***	0.46	-0.47	**1.06***	**-0.31**	**-0.18**
Poland	0.01	0.01	0.43	**1.05**	**-1.22**	**-0.14**	**-0.11***
Romania	-0.07	-0.02	-0.48	-0.38	1.08	-0.39	0.12
South Africa	**-0.14**	**0.76***	**-8.71***	**3.20**	**4.57***	0.09	**-0.06**
Turkey	**0.41***	-0.03	-0.56	**-0.37**	**1.36***	0.03	-0.02
Ukraine	**0.48***	**-0.41**	0.01	**0.44**	**0.45***	0	0

Notes: I set ε_{v^b} so that the innovation to primary surpluses equals -1, and regress the values of other shocks on it. The table shows the estimated terms of decomposition 2 at the posterior distribution's mode. I divide each term by $\delta v/\beta$, so the terms sum up to $\Delta E_t \pi$. I simulate from the posterior to check the statistical significance of the sign of the estimate. Bold indicates 75% of results had the same sign. Bold with an asterisk indicates 90%.

Table 7: Unexpected Inflation Decomposition 2 - Primary Deficit Shock