A Fiscal Decomposition of Unexpected Inflation: Cross-Country Estimates and Theory

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Introduction: The Fiscal Sources of Unexpected Inflation

- The Valuation Equation of Public Debt

$$\frac{\text{Market Value of Debt } (\textbf{Bond Prices})}{\textbf{Price Level}} = \text{Intrinsic Value } (\textbf{Discounting}, \textbf{Surpluses}).$$

- Unexpected inflation $\Delta E_t \Pi_t$ must accompany news about:
  - Bond prices Q_t
  - Real surpluses $\{s_{t+k}\}$
  - Real discounting $\{R_{t+k}\}$

$$\Delta E_t \Pi_t = \Delta E_t \left[Q_t - \{s_{t+k}\} + \{R_{t+k}\} \right]$$

- If unexpected inflation exists, it must nowcast/forecast some of these variables
- What are these forecasts? How do they change across countries? How can theory explain them?

- **This paper.**
  1. Estimate a Bayesian-VAR to measure the decomposition for 25 countries
     - Variance decomposition: $\text{var}\left[\Delta E\, \Pi\right] = \text{cov}\left[\Delta E\, \Pi, \quad Q + \{-s\} + \{R\}\right]$
     - ”Aggregate demand” shock: recession + low inflation
  2. Estimate a New-Keynesian model to reproduce B-VAR decompositions
- **Motivation.** How do you read Debt/Price = Discounted Surpluses?
  - Active fiscal: *”How does inflation react to changes in discounted surpluses?”*
    - Surpluses x Inflation in a given economy
    - Surpluses x Inflation across countries
    - Role of monetary policy?
    - Theory: which shocks cause inflation surprises?
  - Active monetary: *”How should discounted surpluses adjust to unexpected inflation?”*

Introduction: Exercises, Motivation, Results

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Introduction: Preview of Key Results

- The variance of unexpected inflation is accounted for by discounted surpluses (all coutries)

$$\underbrace{\text{var}\left[\Delta E\pi\right]}_{>0} = \underbrace{\text{cov}\left[\Delta E\pi, \quad Q\right]}_{<0} + \underbrace{\text{cov}\left[\Delta E\pi, \quad \{-s\} + \{R\}\right]}_{>0}$$

  - Most relevant terms (Surplus-to-GDP, GDP growth and Real discounting) vary
  - Monetary policy trades current for future unexpected inflation
- Lower inflation in recessions: low discounting + higher subsequent surpluses
- Productivity shocks reproduce findings in NK model
  - Policy shocks do not
- Different policy rules explain differences in variance decomposition across countries

Introduction: Related Literature

- **Fiscal Theory of the Price Level.** Cochrane (2022a) and Cochrane (2022b).
    - Analysis of multiple countries + more general debt instruments
    - NK model estimated to reproduce decompositions + Productivity shocks

  Leeper (1991), Sims (1994), Cochrane (1998), Cochrane (2005), Sims (2011), Leeper and Leith (2016), Bassetto and Cui (2018), Cochrane (2022c), Brunnermeier et al. (2022).

- **Monetary-Fiscal Interaction.**
  Cagan (1956), Sargent and Wallace (1981), Hall and Sargent (1997), Hall and Sargent (2011), Jiang et al. (2019), Corsetti et al. (2019), Sunder-Plassmann (2020), Du et al. (2020), Akhmadieva (2022)

- **Empirical Finance** (Decomposition of Returns)
  Campbell and Shiller (1988), Cochrane (1992), Campbell and Ammer (1993), Chen and Zhao (2009).

Introduction: A Map of the Road

1. **Fiscal Decomposition Derivation**
   - Simple environment + General decomposition
2. **Bayesian-VAR**
   - Empirical model + Variance decomposition + ”Aggregate demand” recession
3. **Theory**
   - Closed economy + Productivity shocks + Policy rules + Open economy

Fiscal Decomposition: The Valuation Equation

- Environment with discrete time + single good (price P_t) + households and government
- One-period nominal public bonds (price Q_t)
- **In the morning**, the government:
  - redeems bonds B_{t-1} for currency
  - announces real taxes s_t (payable in currency)
  - announces sale of B_t new bonds (payable in currency)
- **In the afternoon**, households trade goods, purchase bonds, pay taxes
- Market clearing + No Money Holding $M = 0$:

$$\boxed{B_{t-1} = P_t s_t + Q_t B_t}$$

Fiscal Decomposition: The Valuation Equation

$$B_{t-1} = P_t s_t + Q_t B_t$$

- *Ex-post* real discounting $\beta_t = Q_t(P_{t+1}/P_t)$ $\beta_{t,t+k} = \prod_{\tau=t}^{t+k} \beta_\tau$
- Iterate law of motion forward:

$$\frac{B_{t-1}}{P_t} = \sum_{k=0}^{p} \beta_{t,t+k-1} s_{t+k} + \beta_{t,t+p} \frac{B_{t+p}}{P_{t+p+1}}$$

- **Key Assumption**: $\lim_{\tau\to\infty} \beta_{t,\tau} \frac{B_\tau}{P_{\tau+1}} = 0$ almost surely (No bubbles)
  - In Macro models: transversality conditions + no Ponzi
- The valuation equation of public debt

$$\frac{B_{t-1}}{P_t} = \sum_{k=0}^{\infty} E_t \left[\beta_{t,t+k-1} s_{t+k}\right]$$

Fiscal Decomposition: In the Simplest Environment

- Nominal rate $1 + i_t = 1/Q_t$ and real interest $r_t = i_t - E_t\pi_{t+1}$
- End-of-period real debt v_t

$$\underbrace{\frac{1}{\beta}v_{t-1} + \frac{v}{\beta}\left(i_{t-1} - \pi_t\right)}_{B_{t-1}/P_t} = s_t + v_t = \sum_{k=0}^{\infty} \beta^k E_t s_{t+k} - \frac{v}{\beta}\sum_{k=1} \beta^k E_t r_{t+k}$$

Linearized valuation equation (all variable in deviations from their mean)

- Innovations $\Delta E_t = E_t - E_{t-1}$ decomposition:

$$\boxed{\Delta E_t \pi_t \;=\; -\frac{\beta}{v}\sum_{k=0}^{\infty} \beta^k \Delta E_t s_{t+k} + \sum_{k=0}^{\infty} \beta^k \Delta E_t r_{t+k}}$$

- Variance decomposition:

$$\text{var}\left[\Delta E_t \pi_t\right] \;=\; -\text{cov}_\pi\left[\frac{\beta}{v}\sum_{k=0}^{\infty} \beta^k \Delta E_t s_{t+k}\right] + \text{cov}_\pi\left[\sum_{k=0}^{\infty} \beta^k \Delta E_t r_{t+k}\right]$$

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Fiscal Decomposition: Currency and Term Structures + Growth

- Real **market value** debt-to-GDP v_t, surplus-to-GDP s_t and GDP growth g_t (stationarity!)
- Bonds (j, n) promisses one unit of currency j after n periods Currencies
  - Nominal bonds
  - Real bonds (currency denomination = final goods)
  - US Dollar bonds

  Constant structure $\{\delta_j\}$, $\{\omega_j^n\}$
- Bond price $Q_{j,t}^n$, excess return $rx_{j,t}$ $\quad 1 + \text{return}_{j,t} = 1 + rx_{j,t} + i_{j,t-1} = \frac{\sum_n Q_{j,t}^{n-1} B_{j,t-1}^n}{\sum_n Q_{j,t-1}^n B_{j,t-1}^n}$

  (one-period bonds $\implies rx = 0$)
- Debt law of motion:

$$\frac{v_{t-1}}{\beta} + \frac{v}{\beta}\left[-g_t + \sum_j \delta_j \left(rx_{j,t} + i_{j,t-1} + \Delta h_{j,t} - \pi_{j,t}\right)\right] = v_t + s_t$$

- *Ex-post* real return $r_{j,t} = rx_{j,t} + i_{j,t-1} + \Delta h_{j,t} - \pi_{j,t}$

$$\Delta E_t \pi_t = \underbrace{\left[\Delta E_t rx_t + \sum_{j \neq N} \frac{\delta_j}{\delta} \Delta E_t r_{j,t} \right]}_{\text{Innovation to Bond Prices}} - \frac{\beta}{\delta v} \underbrace{\left[\sum_{k=0}^{\infty} \beta^k \Delta E_t s_{t+k} + \frac{v}{\beta} \sum_{k=0}^{\infty} \beta^k \Delta E_t g_{t+k} - \frac{v}{\beta} \sum_{k=1}^{\infty} \sum_j \delta_j \beta^k \Delta E_t r_{j,t+k} \right]}_{\text{Innovation to the Intrinsic Value of Debt}}$$

$$\equiv d_1(rx) + d_1(r_0) - d_1(s) - d_1(g) + d_1(r)$$

- Variance decomposition.

$$\text{var}\left[\Delta E_t \pi_t\right] = \text{cov}_\pi \left[d_1(rx) \right] + \text{cov}_\pi \left[d_1(r_0) \right] - \text{cov}_\pi \left[d_1(s) \right] - \text{cov}_\pi \left[d_1(g) \right] + \text{cov}_\pi \left[d_1(r) \right]$$

Bayesian-VAR: Data and Model

- Annual data on observables $\tilde{x}_t$

$$x_t^{OBS} = \begin{bmatrix} i_t & \text{(Nominal Interest)} \\ \pi_t & \text{(CPI Inflation)} \\ v_t^b & \text{(Par-Value Debt-to-GDP)} \\ g_t & \text{(GDP growth)} \\ \Delta h_t & \text{(Chg. Real Exchange Rate)} \end{bmatrix}$$

- 25 countries (samples starting at 1945, 1960, 1975, 1998)
- General VAR

$$X_t = AX_{t-1} + Ke_t$$

- Decompose $X_t' = [x_t^{OBS'} \ x_t^{NOT'}]$

$$X_t = \begin{bmatrix} x_t^{OBS} \\ x_t^{NOT} \end{bmatrix} = \begin{bmatrix} a & 0 \\ b & c \end{bmatrix} \begin{bmatrix} x_{t-1}^{OBS} \\ x_{t-1}^{NOT} \end{bmatrix} + \begin{bmatrix} I \\ k \end{bmatrix} e_t$$

Bayesian-VAR: Empirical Challenges and Solutions

1. Explosive debt dynamics in 1960-2020 implies unstable OLS VARs

$$\tilde{x}_t = \tilde{a}\,\tilde{x}_{t-1} + \tilde{b}\,\tilde{u}_{t-1} + \varepsilon_t$$
$$\tilde{u}_t = \tilde{a}_u\,\tilde{u}_{t-1} + \varepsilon_{u,t}$$

   - Estimate US model ($\tilde{a}_u$) by OLS (stable!)
   - Estimate ($\tilde{a}, \tilde{b}$) with a Bayesian-Regression

$$\tilde{a}^{BAY} = (X'X + \lambda^{-1})^{-1}(X'X\,\tilde{a}^{OLS} + \lambda^{-1}\,\tilde{a}^{PRIOR})$$

   λ maximizes the marginal distribution $p(\text{data})$ and ensures stability

2. Public finance data do not respect law of motion of public debt
   - Define surplus from the law of motion: $s_t = \frac{v_{t-1}}{\beta} - v_t + \frac{v}{\beta}\left[-g_t + \sum_j \delta_j \left(rx_{j,t} + i_{j,t-1} + \Delta h_{j,t} - \pi_{j,t}\right)\right]$
3. No data on the market value of debt, only its par value (v_t^b)
   - Model for market *vs* par value (Cox (1985)): $v_t = v_t^b + \frac{v}{\beta}\sum_j \delta_j\left(rx_{j,t} + i_{j,t-1} - i_{j,t-1}^b\right)$
4. No data on bond returns Geometric Term Structure
   - Geometric maturity structure: $rx_{j,t} + i_{j,t-1} = (\omega_j\beta)q_{j,t} - q_{j,t-1}$

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Bayesian-VAR: Variance Decomposition

- **Proposition.** The variance decomposition

$$1 = \frac{\text{cov}_\pi\left[d_1(rx)\right]}{\text{var}\left[\Delta E_t \pi_t\right]} + \frac{\text{cov}_\pi\left[d_1(r_0)\right]}{\text{var}\left[\Delta E_t \pi_t\right]} - \frac{\text{cov}_\pi\left[d_1(s)\right]}{\text{var}\left[\Delta E_t \pi_t\right]} - \frac{\text{cov}_\pi\left[d_1(g)\right]}{\text{var}\left[\Delta E_t \pi_t\right]} + \frac{\text{cov}_\pi\left[d_1(r)\right]}{\text{var}\left[\Delta E_t \pi_t\right]}$$

  is equivalent to the innovations decomposition applied to VAR shock $\text{Proj}(e \mid \Delta E_t \pi_t = 1)$

$$1 = \Delta E_t \pi_t \equiv d_1(rx) + d_1(r_0) - d_1(s) - d_1(g) + d_1(r)$$

- *"Given 1% unexpected inflation, how do we change our nowcast/forecast of the surplus, discounting and bond prices?"*

Bayesian-VAR: Variance Decomposition

Country	$\Delta E_t \pi_t =$	ΔE_t(Bond Prices)		$-\Delta E_t$(Intrinsic Value of Debt)		
		$d_1(r_0)$	$d_1(rx)$	$-d_1(s)$	$-d_1(g)$	$d_1(r)$
United States	1	*0.03	*-0.78	0.57	0.23	0.96
Advanced - 1960 Sample						
Canada	1	*-0.11	*-1.59	0.62	*1.22	0.86
Denmark	1	*-0.29	-0.30	0.42	-0.04	1.21
Japan	1	0	*-0.52	*1.60	-0.38	0.30
Norway	1	*-0.01	*-0.36	0.60	0.47	0.30
Sweden	1	-0.15	*-0.93	-0.34	*0.98	*1.42
United Kingdom	1	*0.52	*-0.73	*2.89	*0.97	*-2.65
Advanced - 1973 Sample						
Australia	1	*0.07	*-0.76	*2.09	0.66	-1.06
New Zealand	1	-0.10	*-0.86	0.40	*0.87	0.68
South Korea	1	-0.01	*-0.45	*1.91	0.17	-0.62
Switzerland	1	0	*-0.69	0.90	*0.91	-0.12

(a) Advanced Economies

Country	$\Delta E_t \pi_t =$	ΔE_t(Bond Prices)		$-\Delta E_t$(Intrinsic Value of Debt)		
		$d_1(r_0)$	$d_1(rx)$	$-d_1(s)$	$-d_1(g)$	$d_1(r)$
Emerging - 1998 Sample						
Brazil	1	-0.26	*-0.22	-1.46	1.05	1.89
Chile	1	-3.80	-1.33	8.95	-5.71	2.88
Colombia	1	1.51	*-0.96	1.39	-1.09	0.15
Czech Republic	1	*-0.16	*-0.37	-2.31	2.42	1.42
Hungary	1	*-0.57	*-0.93	-0.98	1.60	1.88
India	1	*0.17	*-0.46	1.54	0.05	-0.30
Indonesia	1	*-2.59	*-1.07	1.69	*2.61	0.35
Israel	1	-0.06	*-0.78	-0.55	*1.51	0.88
Mexico	1	-0.02	*-0.74	1.41	0.03	0.32
Poland	1	*-0.45	*-1.15	0.87	-0.39	*2.11
Romania	1	-0.40	*-0.96	2.24	0.42	-0.31
South Africa	1	0.36	*-0.51	1.58	0.25	-0.68
Turkey	1	0.37	*-0.37	-1.18	-0.15	*2.33
Ukraine	1	0	*-0.77	0.65	0.41	*0.70

(b) Emerging Economies

Bayesian-VAR: Variance Decomposition - Takeaways

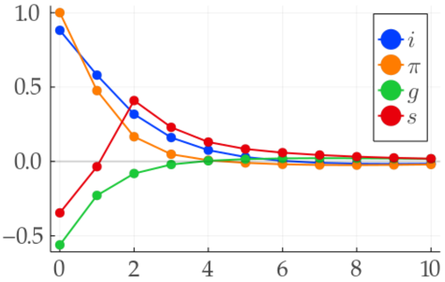

Figure: IRF - Brazil

- Unexpected inflation accounted for by variation in the intrinsic value of debt
- Surplus-to-GDP, GDP growth and real discounting...
  - ...account for unexpected inflation alone in 0/25
  - ...have a positive contribution in 18+/25
- Fiscal roots of inflation do not imply connection between fiscal policy and unexpected inflation
- Nominal bond price dynamics reduce unexpected inflation variance 25/25
  - Effects of monetary policy!

Bayesian-VAR: ”Aggregate Demand” Recession

- ”Aggregate demand” recessions (Great Recession in 2008) feature:
  - Low inflation
  - Low growth
  - Fiscal deficits (often)
- Does that deny the fiscal sources of inflation?
- Where does unexpected (dis)inflation come from?
- Scenario:

$$\Delta E_t g_t = -1 \qquad \Delta E_t \pi_t = -0.5$$

VAR Shock: $\text{Proj}(e \mid \Delta E_t g_t = -1,\ \Delta E_t \pi_t = -0.5)$

Bayesian-VAR: ”Aggregate Demand” Recession

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Advanced - 1960 Sample						
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Denmark	-0.50	***0.20**	***0.86**	**-2.64**	***2.75**	**-1.67**
Japan	-0.50	0	***0.83**	***-1.51**	***1.64**	***-1.46**
Norway	-0.50	0	***0.63**	**-1.36**	***1.72**	**-1.49**
Sweden	-0.50	***0.41**	***1.22**	-0.65	**0.87**	***-2.35**
United Kingdom	-0.50	0.11	***2.54**	**-2.20**	**0.73**	-1.68
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Switzerland	-0.50	0	***1.18**	***-0.93**	-0.07	**-0.67**

(a) Advanced Economies

Country	$\Delta E_t \pi_t =$	ΔE_t(Bond Prices)		$-\Delta E_t$(Intrinsic Value of Debt)		
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Emerging - 1998 Sample						
Brazil	-0.50	***0.37**	**0.06**	1.87	0.13	**-2.39**
Chile	-0.50	***15.78**	***2.94**	-30.50	**30.54**	-19.26
Colombia	-0.50	**1.86**	***0.67**	**-10.90**	***7.57**	0.31
Czech Republic	-0.50	***0.37**	***0.61**	-0.07	**0.25**	**-1.65**
Hungary	-0.50	***0.99**	***0.60**	10.82	-5.29	**-7.63**
India	-0.50	-0.03	0.13	**-1.16**	**0.71**	-0.15
Indonesia	-0.50	***8.23**	**-0.55**	***-11.24**	**1.42**	1.64
Israel	-0.50	***1.79**	**0.37**	**-3.18**	**1.17**	-0.65
Mexico	-0.50	***1.69**	***0.81**	***-4.56**	***1.94**	-0.38
Poland	-0.50	***0.87**	***1.00**	-0.14	**1.30**	***-3.53**
Romania	-0.50	***2.08**	**0.21**	***-8.16**	**2.05**	**3.31**
South Africa	-0.50	-0.10	0.35	***-30.02**	***11.15**	***18.13**
Turkey	-0.50	***0.99**	***0.23**	0.64	**0.52**	***-2.88**
Ukraine	-0.50	0	**-0.68**	**-3.22**	***1.92**	**1.48**

(b) Emerging Economies

Bayesian-VAR: "Aggregate Demand" Recession - Takeaways

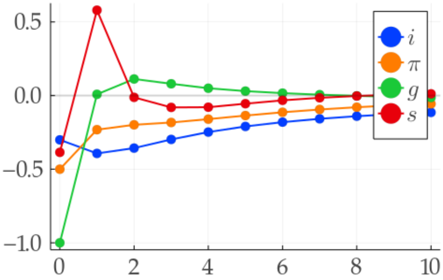

Figure: IRF - United States

- Lower inflation follows from...
  - lower discounting (monetary policy) in 19/25
  - larger surplus-GDP **ratios**, current or in the future in 22/25
- COVID: what if governments reacted to a recession by credibly reducing $\{s\}$ permanently?
- Direction of causality?

The New-Keynesian Model: Setup

- B-VAR decompositions not structural: all shocks are reduced-form
- Theory: How far do we need to go? Not much!
- Two-country NK model:
  - Home economy with $n \to 0$ households and firms (small and open)
  - Foreign economy with $1 - n \to 1$ households and firms (large and "closed")
- **The Standard.** Intertemporal substitution + Calvo rigidity
- **The New.** Production function $\mathcal{A}_t N = \mathcal{T}_t \, A_t \, N$ (Home), $\mathcal{A}^*_t N = \mathcal{T}_t \, A^*_t \, N$ (Foreign)

$$\begin{aligned}
&\text{(Trend component)} \quad && \log \mathcal{T}_t = \log \mathcal{T}_{t-1} + u_{g,t} \\
&\text{(AR(1) component)} \quad && a_t = \rho_a a_{t-1} + \varepsilon_{a,t} \\
& && a^*_t = \rho_a a^*_{t-1} + \varepsilon^*_{a,t}
\end{aligned}$$

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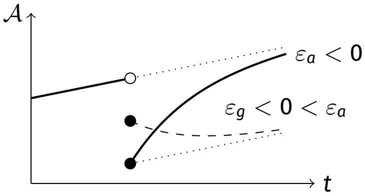


The New-Keynesian Model: The Foreign, Closed Economy

- Private Sector

$$
\begin{aligned}
y_t^* &= E_t y_{t+1}^* - \gamma \; [i_t^* - E_t \pi_{t+1}^*] + E_t u_{g,t+1} \\
\pi_t^* &= \beta \; E_t \pi_{t+1}^* + \kappa \; y_t^* - \kappa_a \; a_t^* \\
g_t^* &= y_t^* - y_{t-1}^* + u_{g,t}
\end{aligned}
$$

Why Trend? Growth

- Unexpected inflation indeterminacy? FTPL.
- Monetary and Fiscal Policy

$$
\begin{aligned}
i_t^* &= \phi_\pi \; \pi_t^* + \phi_g \; g_t^* + \varepsilon_{i,t}^* \\
s_t^* &= \rho_s \; s_{t-1}^* + \tau_\pi \; \pi_t^* + \tau_g \; g_t^* + \varepsilon_{s,t}^*
\end{aligned}
$$

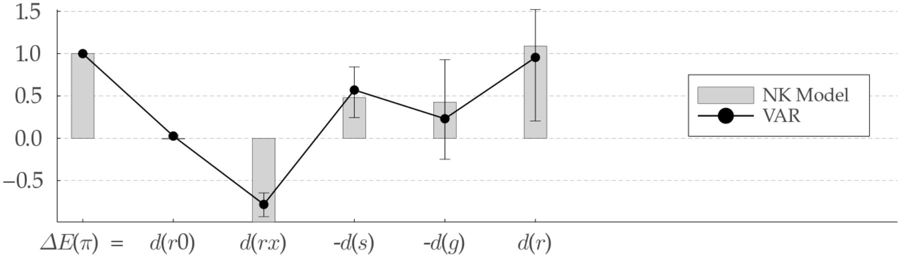
- GMM for US moments

$$
\text{Min}_\Psi \quad \alpha_1 \, \|\mathcal{D}_{VAR} - \mathcal{D}_{NK}(\Psi)\| + \alpha_2 \, \|\mathcal{M} - \mathcal{M}_{NK}(\Psi)\| \qquad \text{s.t. } \Psi \in \Theta
$$

Parameters

The New-Keynesian Model: Reproducing the Variance Decomposition

- **Result.** AR(1) productivity shocks $\varepsilon_{a,t}$ alone reproduce the variance decomposition with positive contributions from surplus-to-output, growth and real interest terms



Target: United States

Moments

The New-Keynesian Model: Reproducing the Variance Decomposition

$$\boxed{1 = \Delta E_t \pi_t = d(rx) - d(s) - d(g) + d(r)}$$

- Key Ingredients
  - Persistent shock: $\rho_a = 0.98$
  - Strong Taylor: $\phi = 0.8$
  - Countercyclical deficits: $\tau_g = 0.7$
- What is the story?
  - Low productivity leads to a recession
    $$d(g) < 0$$
  - Government raises deficit to fight recession
    $$d(s) < 0$$
  - Monetary policy raises nominal interest
    $$d(rx) < 0$$
    $$d(r) > 0$$

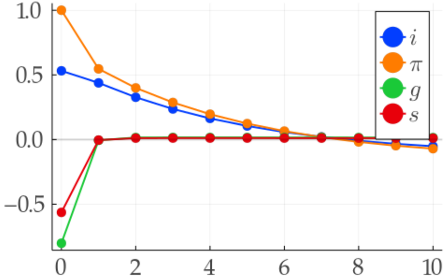


Figure: IRF to $\Delta E_t \pi_t = 1$ ($\varepsilon_{a,t} = -1.15$)

Marginal Costs vs B-VAR IRF

The New-Keynesian Model: Reproducing the Variance Decomposition

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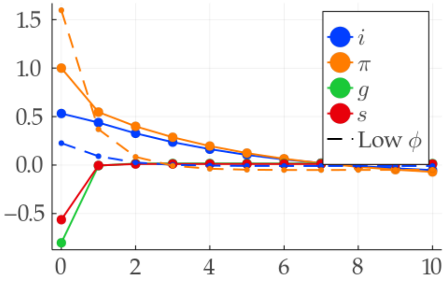

Figure: IRF to $\Delta E_t \pi_t = 1$ ($\varepsilon_{a,t} = -1.15$)

Marginal Costs
vs B-VAR IRF

Frame title

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Appendix: Public Debt Currency Denomination

Return

Symbol	Description	Nominal Debt	Real Debt	Dollar Debt
j	Index Symbol	N	R	D
	Notation	δ, ω	δ_R, ω_R	δ_D, ω_D
P_j	Price per Good	P	1	P_t^{US}
$\mathcal{E}_j$	Nominal Exchange Rate	1	P	Dollar NER
H_j	Real Exchange Rate	1	1	Dollar RER
π_j	Log Variation in Price	π	0	π_t^{US}
Δh_j	Log Real Depreciation	0	0	Δh_t

Table: Public Debt Denomination

Appendix: Geometric Term Structure

Return

- To each currency portfolio j, fixed geometric maturity structure:

$$B^n_{j,t} = \omega_j B^{n-1}_{j,t}$$

- Total return on currency-j portfolio:

$$1 + rx_{j,t} + i_{j,t-1} = \frac{1 + \omega_j Q_{j,t}}{Q_{j,t-1}} \quad \Longrightarrow \quad \boxed{rx_{j,t} + i_{j,t-1} = (\omega_j \beta) q_{j,t} - q_{j,t-1}}$$

- Assume **constant risk premia** $E_t rx_{j,t+1} = 0$

$$\boxed{q_{j,t} = (\omega_j \beta) E_t q_{j,t+1} - i_{j,t}} = -\sum_{k=0}^{\infty} (\omega_j \beta)^k E_t i_{j,t+k}$$

Appendix: NK Model Parameters

Return

Appendix: Why Trend Shocks? The Growth Component

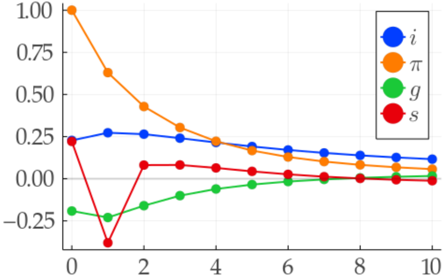
Return

Appendix: Estimated Moments

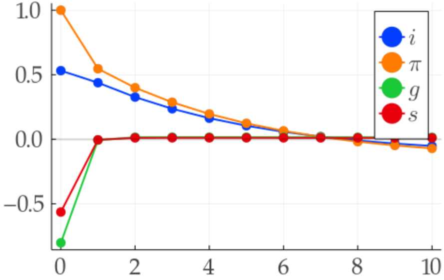
Return

Appendix: Simple Model - US Data vs Model

NK Simple



(a) B-VAR



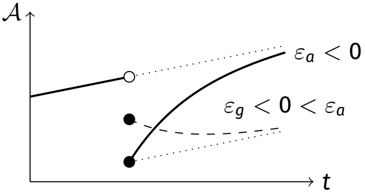
(b) NK Model (Only Prod. Shocks)

Appendix: Marginal Costs

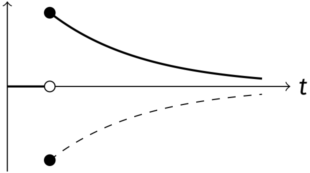
NK Simple

- AR(1) Productivity Shock
  - High marginal costs + strong Taylor rule ($\phi_\pi \approx 1$):

$$i_t \approx \underbrace{\pi_t > E_t\pi_{t+1}}_{mc_t>0} \implies r_t = i_t - E_t\pi_{t+1} > 0$$



(a) Productivity Path $\mathcal{A}_t$



(b) $-a_t$ or Mg. Cost at fixed wages