A Fiscal Decomposition of Unexpected Inflation: Cross-Country Estimates and Theory*

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Abstract

I estimate a variance decomposition of unexpected inflation for a set of twenty-five countries using a Bayesian-VAR. The decomposition follows from the valuation equation of public debt. Unexpected inflation must be accounted for by news about: surplus-to-GDP ratios, real discounting, GDP growth, future inflation or, in the presence of dollar-denominated debt, real exchange rates and US inflation. Contributions from discounting and output growth are quantitatively large, which implies that unexpected inflation does not need to be accompanied by news of surplus-to-GDP ratios and fiscal policy. Future inflation, which translates the effects of monetary policy on bond prices, reduces unexpected inflation variability in all countries. Building on these results, I explore the sources of unexpected inflation following "aggregate demand" and currency depreciation shocks, and compare empirical findings with an estimated open-economy New-Keynesian model with active fiscal policy.
Keywords: Inflation, Variance Decomposition, New-Keynesian, Fiscal Theory of the Price Level

1. Introduction

In the absence of financial bubbles, the real market value of the stock of public debt equals its intrinsic value, discounted primary surpluses:

Market Value of Debt (**Bond Prices**) / **Price Level** = Intrinsic Value (**Discounting**, **Surpluses**).

This valuation equation leads to a decomposition of unexpected inflation, defined as the difference between its realized and expected values. Surprise movements in the price level must be accounted for by news about bond prices, fiscal surpluses or real discounting. Unexpected inflation can only exist (*i.e.*, have non-zero variance) if it covaries with, and therefore forecasts, one or more of these variables. Which are they, and what are these forecasts? (should I remove this question?)

I estimate a Bayesian-VAR for a set of twenty-five advanced and developing countries and use the estimated models to compute the terms of the decomposition. I then compare my results with those implied by a small-open-economy New-Keynesian (NK) model.

Empirically, I find that real discounting accounts for unexpected inflation as much as, and in many cases more than, surpluses. In addition, surplus contribution often stems from news about economic activity (GDP growth) rather than fiscal policy (surplus-to-GDP ratios).

Given that the valuation equation holds in virtually any micro-founded macroeconomic model that precludes bubbles, these empirical results lead to relevant propositions. That the valuation equation links the price level to public budgets does not imply that unexpected inflation has to follow from news about primary surpluses. Conversely, apparent disconnections between inflation and fiscal policy are not inconsistent with the valuation equation.

News about bond prices also enter the decomposition. Their role depends critically on the currency in which bonds are issued. In my setup, I allow governments to issue nominal, inflation-linked and

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dollar-linked bonds, a generalization not yet explored by previous literature, but necessary to describe the case of many economies, developing ones in particular.

Nominal debt functions as a buffer for unexpected inflation. Central banks react to news of high inflation by raising interest. Lower bond prices reduce the market value of debt and thus absorb part of the inflationary impact of changing discounted surpluses. This pattern is common to all countries in my sample. Nevertheless, for all countries but one, lower bond prices forecast higher *future* inflation. Estimates therefore depict how monetary policy can effectively smooth unexpected inflation over time.

The role of inflation and dollar-linked bond prices is not as clear. Estimated covariances often imply a reduction of unexpected inflation variability. For example, surprise domestic inflation correlates with surprise inflation in the US. Higher US prices devalue the dollar portion of public debt, and thus reduce the required jump in the domestic price level. On the other hand, because domestic inflation can only devalue nominal bonds, the presence of non-nominal debt requires larger variations of the price level to re-establish the equality between market and intrinsic debt values.

By further assuming that governments keep a geometric term structure to public debt, we can express unexpected bond price variation as functions of future inflation (domestic and in the US), real interest and real exchange rates. A second decomposition follows - that is the main decomposition I study. It involves only inflation and real variables, as promised by the abstract of this paper.

Besides accounting for unexpected inflation volatility, I apply the decomposition to study the fiscal sources of inflation following two reduced-form shocks, designed to simulate realistic scenarios: an "aggregate demand" recession shock, and a currency depreciation shock.

In the aggregate demand shock, domestic output and inflation decline. The combination is consistent with the experience of many countries in events like the Great Recession and COVID. So, where does lower inflation come from? For most economies (nine of the eleven advanced ones), I find that lower inflation is entirely accounted for by lower discounting. The effect of surpluses is typically neutral but, perhaps surprisingly, that of surplus-to-GDP ratios is *deflationary* and economically large. This suggests that, should governments react to a recession by credibly reducing current *and future* surplus-to-GDP ratios, the inflationary impact of such policies would be substantial. This is a plausible explanation for the worldwide inflation outbreak in the years following COVID.

The currency depreciation scenario is more inflationary in developing countries, as their governments rely more on dollar-linked debt. As a depreciated exchange rate raises the market value of debt in domestic currency, it counts as an inflationary force. Furthermore, estimated responses resemble the effects of a "sudden stop" of foreign capital flows: growth declines and interest rates increase. I find that emerging markets often manage to revert the inflationary impact of a depreciated currency and lower growth with contractionary fiscal and monetary policies.

In terms of methodology, the analysis of multiple countries is challenging. Each country presents a unique economic experience that can cloud the common patterns we are interested in. In addition, available data is often considerably more limited than United States data. For instance, market-price measures of public debt and real and dollar bond price data are seldom available in a sufficiently large time span. I present an empirical model that partially circumvents these limitations and allows the estimation of the decomposition. The method requires five commonly available macroeconomic time series: output growth, the short-term interest rate, the inflation rate, the real exchange rate and the par-value of public debt. For many countries, these time series go back decades in the past.

Finally, the use of Bayesian regressions ensures that estimated VARs are stationary, as necessary for the decomposition to hold. Because public debt is a highly persistent process, and because in many countries it has increased dramatically in the last fifty years, OLS estimation regularly returns unstable dynamics.[1] By properly tuning the prior distribution's hyperparameters, we can ensure stability at the same time we discipline parameter search by a goodness-of-fit criterion.

[1]See Bohn (1998), Uctum et al. (2006), Yoon (2012), Azzimonti et al. (2014) for analysis and estimates of public debt persistence.

2. Unexpected Inflation Decomposition

2.1. Connection with the Value of Debt

I start by establishing the connection between inflation and the value of debt. Consider an economy with a homogeneous final good which households value. The economy has a government that manages a debt composed by bonds that pay one unit of currency in the next period. Currency is a commodity that only the government can produce, at zero cost. There is nothing special about currency. Households do not value it, and it provides no convenience for trading. Households cannot burn currency.

At the beginning of period t, the face value of debt issued in the previous period is V_{t-1}. The government redeems it for currency, which moves to the hands of households. After redeeming bonds, the government announces that each household i must pay $T_{i,t}$ goods in taxes, payable in currency. It also announces it will sell V_t new bonds and purchase G_t units of the final good at market prices.[2]

Nothing binds the government's choices of $T_{i,t}$ and V_t. Note the difference from the case with bonds payable in goods, in which the government *must* raise tax revenue or issue new debt to redeem old bonds. Additionally, if households accept currency as payment for final goods, nothing binds the government's choice of G_t either.

Let M_t be private holdings of currency at the end of t. As there is no free disposal of currency, the quantity used by the government to redeem $t-1$ bonds and purchase goods is used to either pay taxes, buy new bonds or increase currency holdings:

$$\begin{aligned} V_{t-1} + P_t G_t &= P_t T_t + Q_t V_t + \Delta M_t \\ \implies V_{t-1} &= P_t s_t + Q_t V_t + \Delta M_t \end{aligned} \tag{1}$$

where T_t are aggregate taxes, $s_t = T_t - G_t$ is the primary surplus, P_t is the final good's price and Q_t is the price of new bonds (I state prices in currency units). Equation (1) provides a low of motion for public debt. It holds for all prices and all choices of money holdings, new debt sales, and primary surplus.[3]

If $P_t = 0$, real public debt V_{t-1}/P_t equals infinity. For now, that is a possibility.

Suppose $P_t > 0$. Define $\beta_t \equiv Q_t P_{t+1}/P_t$ as the real discount for public bonds, and $\beta_{t,t+k} \equiv \prod_{\tau=t}^{t+k} \beta_\tau$. Since V satisfies (1), it also satisfies

$$\frac{V_{t-1}}{P_t} = \sum_{i=0}^{k} \beta_{t,t+i-1} \left(s_{t+i} + \Delta M_t \right) + \beta_{t,t+k} \frac{V_{t+k}}{P_{t+k+1}} \qquad \text{for any } k \geq 0 \tag{2}$$

regardless of prices and choices. Expressions (1) and (2) do not represent a constraint on the path of surpluses $\{s_t\}$ and bond sales $\{V_t\}$ the government chooses to follow. They merely express future debt given paths for quantities and prices.

So far, all we did was public finances accounting. I now move to economic behavior. If $P_t = 0$, households demand infinite final goods and there is no equilibrium. Therefore $P_t > 0$.

Given a utility function over consumption paths $U(\{c_t\})$, the optimal consumption-savings choice involves two conditions. First: $\beta_{t,t+k}$ = marginal rate of substitution between time-t and time-$t+k$ consumption. Second, the transversality condition $\lim_{k\to\infty} \beta_{t,t+k} V_{t+k}/P_{t+k+1} \leq 0$. Coupled with a no-Ponzi condition that establishes the reverse inequality, it implies

$$\lim_{k\to\infty} \beta_{t,t+k} \frac{V_{t+k}}{P_{t+k+1}} = 0 \text{ at every period } t. \tag{3}$$

If bonds were redeemable in goods, (3) would represent a debt sustainability condition, as it forces

[2] That the government forces households to pay for taxes and new bonds in currency is not necessary for the argument. All else follows if the government accepts payment in goods but stands ready to exchange these goods for currency at market prices.

[3] Strictly speaking, (1) holds for all *feasible* choices of money holdings, that is, choices that respect households' budget constraint. Otherwise, one could point out that, for $B_{t-1} > 0$, $M_t = M_{t-1}$ and $s_t = B_t = 0$ violates (1). That would nevertheless involve households burning up currency.

the government to eventually raise the resources to pay for past borrowing.

With nominal debt, however, the government has no constraints on its choice of debt and surplus, as already pointed out. Replacing (3) on (2) and taking expectation yields

$$\frac{V_{t-1}}{P_t} = \sum_{i=0}^{\infty} E_t \left[\beta_{t,t+i-1} \left(s_{t+i} + \Delta M_{t+i} \right) \right]. \tag{4}$$

Given the face value of maturing public debt, the relative price of the consumption good is given by the expected β-discounted stream of real primary surpluses. This latter term - the right-hand side of (4) - I call the *real value of public debt*. My definition of debt value follows a "beginning-of-period" convention: it includes current period surplus s_t and starts discounting at $t+1$.

In the case of nominal debt, (4) is a *valuation equation*. Hence, expression (3) implies not a no-default condition, but a no-bubble condition: market prices should reflect fundamental value.

Now, define the inflation rate $\Pi_t = P_t / P_{t-1}$, and take innovations on both sides

$$\frac{V_{t-1}}{P_{t-1}} \Delta E_t \Pi_t^{-1} = \sum_{i=0}^{\infty} \Delta E_t \left[\beta_{t,t+i-1} \left(s_{t+i} + \Delta M_{t+i} \right) \right]. \tag{5}$$

Equations (4) and (5) do not depend on equilibrium selection mechanisms. Both hold in all models in which the transversality condition (3) holds, including the canonical three-equation New-Keynesian model. In a FTPL interpretation, any shock can lead the government to change the conditional distribution of future surpluses (in units of goods) backing the stock of public nominal liabilities. The relative price of bonds in terms of goods $1/P_t$ then adjusts to reflect that change, much in the same way that news of future dividends (in currency units) per stock change the relative price of stocks in terms of currency (Cochrane (2005)). Unexpected inflation $\Delta E_t \Pi_t$ follows. Also like stocks, changes in stochastic discounting β affect fundamental value, and therefore affect prices. In models with "passive" fiscal policy, the government observes unexpected inflation and changes the path of surpluses so that the intrinsic value reflects the new real market value of debt.

2.2. *Inflation Decomposition in the Simplest Environment*

Start by linearizing the law of motion (2).

$$v_t + s_t = \frac{1}{\beta} \left(v_{t-1} + i_{t-1} - \pi_t \right) \tag{6}$$

where v_t is *end-of-period* stock of real debt, $i_t = -\log(Q_t)$ and $\pi_t = \log(\Pi_t)$. I assume $\Delta M_t = 0$ (households do not hold currency). Note that v and s are both in levels - I assume them to be stationary for simplicity. Moreover, I linearize around the point $v = 1$, which I take to be the average real debt level.

The interpretation of (6) is the same as before. The expression on the right is the linearizing beginning-of-period stock of debt, corresponding to V_{t-1}/P_t in the non-linear formulation. Previous period debt accrues by the mean real interest $(1/\beta)$ plus its local variation $i_t - \pi_t$. A 1% higher real interest raises debt by 1% on average because debt is on average one. The government redeems bonds for currency, and then sells new debt v_t and runs a surplus s_t to soak it up.

Repeating the same steps as before, solve (6) forward:

$$\frac{1}{\beta} \left(v_{t-1} + i_{t-1} - \pi_t \right) = \sum_{k=0}^{\infty} \beta^k E_t s_{t+k} - \frac{1}{\beta} \sum_{k=1} \beta^k E_t r_{t+k}$$

where $r_t = i_{t-1} - \pi_t$ is the *ex-post* real interest rate. The expression on the right-hand side is the linearized real value of debt. It includes time-t surplus, and starts discounting at $t+1$. The inflation rate on the left represents the price level equalizing the beginning-of-period of debt to its real value.

Take innovation, and multiply both sides by β to find

$$\Delta E_t \pi_t = -\beta \sum_{k=0}^{\infty} \beta^k \Delta E_t s_{t+k} + \sum_{k=0}^{\infty} \beta^k \Delta E_t r_{t+k}. \tag{7}$$

In this simple environment, unexpected higher inflation is accounted for by news of larger deficits $-s$ or news of higher discounting r, and vice-versa. That is, news about the real value of debt. This decomposition was introduced by Cochrane (2022), and follows similar decompositions for stock returns and price-dividend ratios (Campbell and Shiller (1988), Campbell and Ammer (1993)).

Now, for each term in the equation, take covariance with unexpected inflation. We arrive at an initial decomposition of inflation variance.

$$\text{var}(\Delta E_t \pi_t) = -\text{cov}\left[\Delta E_t \pi_t, \beta \sum_{k=0}^{\infty} \beta^k \Delta E_t s_{t+k}\right] + \text{cov}\left[\Delta E_t \pi_t, \sum_{k=0}^{\infty} \beta^k \Delta E_t r_{t+k}\right]. \tag{8}$$

In the simplest environment, if unexpected inflation "exists", it must forecast deficits or higher discounting.

2.3. VAR-Based Measures

In the next sections, I use models in VAR form $X_t = AX_{t-1} + Ke_t$ to measure (7) and (8). To measure the innovations decomposition (7), suppose we are interested in the response of the system to a shock e_t. Using $\Delta E_t s_{t+k} = \mathbf{1}_s' A^k K e_t$ and similar expressions to the other terms, (7) implies

$$\mathbf{1}_\pi' K e_t = -\beta \mathbf{1}_s' (I - \beta A)^{-1} K e_t + \mathbf{1}_r' (I - \beta A)^{-1} K e_t.$$

More generally, suppose we want to measure the decomposition given a value only to innovations to a subset x_t of X_t. Start by projecting the entire vector of residuals using

$$\begin{aligned} \text{Proj}(e_t | \Delta E_t x_t) &= \text{cov}(e_t, \Delta E_t x_t) \text{var}(\Delta E_t x_t)^{-1} \Delta E_t x_t \\ &= \Omega K' \mathbf{1}_y \left(\mathbf{1}_x' K \Omega K' \mathbf{1}_x\right)^{-1} \Delta E_t x_t, \end{aligned}$$

where $\Omega = \text{cov}(e_t)$ and $\mathbf{1}_y$ are the columns of the identity matrix corresponding to variables in y (for instance, $\mathbf{1}_\pi' K$ selects the row of K corresponding to inflation).[4] We can then use the prior formula to measure the decomposition using $\text{Proj}(e_t)$ in the place of e_t.

- *The variance decomposition (8) is equivalent to the innovations decomposition (7) applied to the shock $Proj(e_t \mid \Delta E_t \pi_t = 1)$.*

To prove the proposition, divide the two sides of (8) by $\text{var}(\Delta E_t \pi_t)$ and replace the formulas implied by the model:

$$\begin{aligned} 1 &= -\beta \overbrace{\mathbf{1}_s' (I - \beta A)^{-1} K \Omega K' \mathbf{1}_\pi}^{\text{cov}\left[\Delta E_t \pi_t, \sum_k \beta^k \Delta E_t s_{t+k}\right]} \overbrace{\left(\mathbf{1}_\pi' K \Omega K' \mathbf{1}_\pi\right)^{-1}}^{\text{var}(\Delta E_t \pi_t)^{-1}} + \mathbf{1}_r' (I - \beta A)^{-1} K \Omega K' \mathbf{1}_\pi \left(\mathbf{1}_\pi' K \Omega K' \mathbf{1}_\pi\right)^{-1} \\ &= -\beta \mathbf{1}_s' (I - \beta A)^{-1} K \text{Proj}(e_t \mid \Delta E_t \pi_t = 1) + \mathbf{1}_r' (I - \beta A)^{-1} K \text{Proj}(e_t \mid \Delta E_t \pi_t = 1) \end{aligned}$$

The left-hand side above is $\Delta E_t \pi_t = 1$, and the right-hand side is the innovations decomposition applied to the projection of the residuals e_t onto $\Delta E_t \pi_t$.

2.4. Generalizing Public Financing Instruments

2.4.1. Currency Denomination

The debt process considered so far is too unrealistic to be taken to the data. I generalize the financing instruments available to the government in two dimensions: currency denomination and maturity

[4] I assume invertibility of $\mathbf{1}_x' K \Omega K' \mathbf{1}_x$.

Symbol	Description	Nominal Debt	Real Debt	Dollar-Linked Debt
j	Index Symbol	N	R	D
	Notation	δ, ω	δ_R, ω_R	δ_D, ω_D
P_j	Price per Good	P	1	P_t^{US}
$\mathcal{E}_j$	Nominal Exchange Rate	1	P	Dollar NER
H_j	Real Exchange Rate	1	1	Dollar RER
π_j	Log Variation in Price	π	0	π_t^{US}
Δh_j	Log Real Depreciation	0	0	Δh_t

Notes: P = price of consumption basket in domestic currency. P^{US} = price of consumption basket in US dollars. NER = Nominal Exchange Rate. RER = Real Exchange Rate.

Table 1: Public Debt Denomination

structure. I do not consider the case of debt with stage-contingent nominal payoffs.[5]

Starting here, I recycle part of the notation already established. Fix the case of a country's government. Let P_t be the price of the consumption basket in terms of domestic currency.

The payoff of public bonds can be indexed to different currencies, enumerated by j. Let $P_{j,t}$ be the price of the consumer price index in units of currency j. Let $Q^n_{j,t}$ be the discount rate for a zero-coupon public bond paying one unit of currency j after n periods. Without loss of generality, all bonds are zero-coupon. Let $\mathcal{E}_{j,t}$ be the price of currency j in units of domestic currency.

The notation is general enough to accommodate currency-linked bonds *per se*, but also inflation-linked securities such as American TIPS. In this case, one can interpret the "currency" as the final goods basket (so that $P_j = 1$ and $\mathcal{E}_j = P_t$). In the empirical exercises that follow, I consider only nominal bonds ($j = N$), inflation-linked (or real) bonds ($j = R$) and US-dollar- denominated bonds ($j = D$). Table 1 shows the value or interpretation of the variables defined above for these three cases.

I do not assume the price index for government's purchases (denoted P_t^S) and levied aggregates (such as income) to be the same as the price index for households' consumption (P_t). While governments do tend to tax private consumption, they also tax and transfer income, and purchase baskets of goods different than that of households. The fiscal effect of variations in the relative price of government to households' final goods basket must be accounted for.

With these changes, the law of motion of government debt (1) becomes

$$\sum_j \mathcal{E}_{j,t} B^1_{j,t-1} = P^s_t S_t + \sum_j \mathcal{E}_{j,t} \sum_{n=2}^{\infty} Q^{n-1}_{j,t} \left(B^{n-1}_{j,t} - B^n_{j,t-1} \right),$$

where S_t is now the real primary surplus and $B^n_{j,t}$ is the face value of bonds issued in currency j, period t, payable n periods in the future. The term on the left represents the cost of debt in period t; the second term on the right represents proceeds from the selling of new bonds.

Let $\mathcal{V}_{j,t} = \sum_n Q^n_{j,t} B^n_{j,t}$ be the end-of-period market value of nominal debt issued in currency j, $i_{j,t}$ the risk-free rate in bonds issued in currency j and $rx_{j,t} = \mathcal{V}^{-1}_{j,t-1} \sum_{n=1}^{\infty} Q^{n-1}_{j,t} B^n_{j,t-1} - i_{j,t-1}$ the realized excess return on portfolios that mimic the composition of j-currency debt. We can re-write the law of motion in terms of the $\mathcal{V}_j$ and its corresponding returns:

$$\sum_j (1 + rx_{j,t} + i_{j,t-1}) \mathcal{E}_{j,t} \mathcal{V}_{j,t-1} = P^s_t S_t + \sum_j \mathcal{E}_{j,t} \mathcal{V}_{j,t}$$

For empirical work, it is ideal to write the law of motion in terms of real and stationary variables. So, let Y_t be real GDP and let $g_t = Y_t / Y_{t-1} - 1$ be its growth rate. Define the real "exchange rate" $H_{j,t} = \mathcal{E}_{j,t} P_{j,t} / P_t$ and its growth rate $\Delta h_{j,t} = H_{j,t} / H_{j,t-1} - 1$. Define the detrended real value of j-

[5] Publicly-ran social security and pension systems can be modelled as cases of public debt with state-contingent payoffs or, in a fashion compatible with my framework, as an additional layer of uncertainty in the primary surplus process.

indexed debt $V_{j,t} = \mathcal{V}_{j,t}/P_{j,t}Y_t$, the real value of total debt $V_t = \sum_j H_{j,t}V_{j,t}$ and the j-indexed share $\delta_{j,t} = H_{j,t}V_{j,t}/V_t$. I assume a constant currency structure $\delta_{j,t} = \delta_j$.

By properly dividing the whole above equation by P_tY_t, and multiplying and dividing the j sum on the left by $P_{j,t-1}$, $P_{j,t}$, Y_{t-1} and $H_{j,t-1}$, we arrive at a final version for the law of motion of public debt:

$$V_{t-1}\sum_j \frac{(1+rx_{j,t}+i_{j,t-1})(1+\Delta h_{j,t})}{(1+\pi_{j,t})(1+g_t)}\delta_j = \frac{P_t^s}{P_t}s_t + V_t.$$

The law of motion above generalizes (2) for $k=1$. During period t, the government must "pay" V_{t-1} plus realized returns and eventual changes to the relative value of currency j.[6]

I linearize the debt law of motion. Let $\beta_j = \exp\{-(rx_j + i_j - \pi_j - g)\}$ be the steady-state real and growth-adjusted discounting for public debt issued in currency j. I linearize around a steady-state - assumed to exist - with $\beta_j = \beta$ for all j and $P^s = P$. This leads to

$$v_t + s_t + s(p_t^s - p_t) = \frac{v_{t-1}}{\beta} + \frac{v}{\beta}\left[\sum_j \delta_j\left(rx_{j,t} + i_{j,t-1} + \Delta h_{j,t} - \pi_{j,t}\right) - g_t\right], \tag{9}$$

which generalizes (6). Parameter v is the steady-state level of public debt. The right-hand side is the beginning-of-period stock of debt.

Let $r_{j,t} = rx_{j,t} + i_{j,t-1} + \Delta h_{j,t} - \pi_{j,t}$ be the *ex-post* real return on holdings of the j-currency portfolio of public bonds, and define $s_t^p = s_t + s(p_t^s - p_t)$ as the price-adjusted surplus process.

Decomposition 1. *Solve the debt law of motion* (9) *forward and take innovations to arrive at*

$$\frac{v}{\beta}\left[\sum_{j\neq N}\delta_j\Delta E_t r_{j,t} + \delta\left(\Delta E_t rx_t - \Delta E_t\pi_t\right)\right] = \sum_{k=0}^{\infty}\beta^k\Delta E_t s_{t+k}^p + \frac{v}{\beta}\sum_{k=1}^{\infty}\beta^k\Delta E_t g_{t+k}. - \frac{v}{\beta}\sum_{k=1}^{\infty}\sum_j \delta_j\beta^k\Delta E_t r_{j,t+k}.$$

Take covariance with unexpected inflation, and divide both sides by $\delta(v/\beta)$.

$$\begin{aligned} var(\Delta E_t\pi_t) = {} & cov\left[\Delta E_t\pi_t, \delta^{-1}\sum_{j\neq N}\delta_j\Delta E_t r_{j,t}\right] + cov\left(\Delta E_t\pi_t, \Delta E_t rx_t\right) \\ & - cov\left[\Delta E_t\pi_t, \left(\frac{\delta v}{\beta}\right)^{-1}\sum_{k=0}^{\infty}\beta^k\Delta E_t s_{t+k}\right] - cov\left[\Delta E_t\pi_t, \delta^{-1}\sum_{k=0}^{\infty}\beta^k\Delta E_t g_{t+k}\right] \\ & + cov\left[\Delta E_t\pi_t, \delta^{-1}\sum_{k=1}^{\infty}\sum_j\delta_j\beta^k\Delta E_t r_{j,t+k}\right]. \end{aligned} \tag{10}$$

Decomposition 1 generalizes (7). The right-hand side still contains the revision of the value of debt. The left-hand side reveals the new terms Δr_t and Δrx_t containing time-t unexpected jumps in public bond prices, absent in the one-period debt context. Now, *given unexpected variation in the price of long-term debt*, unexpected inflation must be accounted for by news of surpluses or real discounting. Expression (10) converts the innovations decomposition into a decomposition of unexpected inflation.

Compared to the $\delta = 1$ case with nominal debt only, the decomposition contains time-t price-adjustment terms (on the left) and future discounting terms (on the right) related to currency-linked real returns. For countries with dollar-linked debt $\delta_D > 0$, unexpected real exchange depreciation raises the home-currency value of debt and thus acts like an inflationary force (that is the $\Delta Er_{D,t}$ term on the left). News of *future* real exchange depreciation also stimulate inflation by increasing real discounting ($\Delta r_{D,t+k}$ term on the right).

The lower the share of nominal bonds on the stock of δ, the more the price levels must change to deflate total debt and account for innovations on the real value of debt. Expression (10) shows that, all

[6]"Pay" comes in parentheses here because, unlike in (1), the government does not actually redeem the entire term on the left at period t. It only pays for bonds maturing at t.

else the same, lower δ leads to more volatile unexpected inflation.

2.4.2. Geometric Term Structure

I assume the government keeps a geometric maturity structure of its debt. The geometric term structure implies a tractable relationship between short-term interest rate $i_{j,t}$ and the excess returns that I explore to substitute the hard-to-interpret price adjustment terms of decomposition 1.

The term structure is constant over time, but can vary across the different currency portfolios $\{j\}$ of public debt. Specifically, for the slice of public debt linked to currency j, suppose the outstanding volume of bonds decays at a rate ω_j, so that $B^n_{j,t} = \omega_j B^{n-1}_{j,t}$. Define $Q_{j,t} = \sum_{n=1}^{\infty} Q^n_{j,t}\omega_j^{n-1}$ as the weighted-average price of currency j public bonds. Then, $\mathcal{V}_{j,t} = Q_{j,t}B^1_{j,t}$. The total return on currency-j bonds then is $1 + rx_{j,t} + i_{j,t-1} = (1 + \omega_j Q_{j,t})/Q_{j,t-1}$, which I linearize as

$$rx_{j,t} + i_{j,t-1} = \omega_j \beta q_{j,t} - q_{j,t-1} \tag{11}$$

where $q = \log Q$ and I use the log approximation of level returns to effectively re-define $rx_j + i_j$.

Equation (11) above defines the excess return on holdings of the j-currency portfolio of public debt. Given a model for the risk premium $E_t rx_{j,t+1}$, it also defines the price of the debt portfolio as a function of short-term interest:

$$\begin{aligned} q_{j,t} &= \omega_j \beta E_t q_{j,t+1} - E_t rx_{j,t+1} - i_{j,t} \\ &= -\sum_{k=0}^{\infty} (\omega_j \beta)^k E_t \left[rx_{j,t+1+k} + i_{j,t+k} \right]. \end{aligned} \tag{12}$$

The second equation in (12) shows the connection between short-term interest - hence monetary policy - and returns on debt holdings. News of higher interest lower public bond price q and leads to a low excess return.

Lag equation (12) one period and take innovations to find

$$\begin{aligned} \Delta E_t rx_{j,t} &= -\sum_{k=1}^{\infty} (\omega_j \beta)^k \left[\Delta E_t rx_{j,t+k} + \Delta E_t i_{j,t+k-1} \right] \\ &= -\sum_{k=1}^{\infty} (\omega_j \beta)^k \left[\Delta E_t r_{j,t+k} + \Delta E_t \pi_{j,t+k} - \Delta E_t \Delta h_{j,t+k} \right]. \end{aligned} \tag{13}$$

Decomposition 2. *Replace (13) on decomposition 1 and gather terms to find*

$$\begin{aligned} \frac{\delta v}{\beta} \Delta E_t \pi_t = &-\frac{\delta v}{\beta} \sum_{k=1} (\omega \beta)^k \Delta E_t \pi_{t+k} - \sum_{k=0}^{\infty} \beta^k \Delta E_t s^p_{t+k} - \frac{v}{\beta} \sum_{k=1}^{\infty} \beta^k \Delta E_t g_{t+k} \\ &+ \frac{v}{\beta} \sum_{k=1}^{\infty} \sum_j \delta_j \beta^k (1 - \omega_j^k) \Delta E_t r_{t+k} + \frac{\delta_D v}{\beta} \sum_{k=0}^{\infty} (\omega_D \beta)^k \Delta E_t \Delta h_{t+k} - \frac{\delta_D v}{\beta} \sum_{k=0}^{\infty} (\omega_D \beta)^k \Delta E_t \pi^{US}_{t+k}. \end{aligned}$$

Take covariance with unexpected inflation, and divide both sides by $\delta(v/\beta)$.

$$\begin{aligned} var(\Delta E_t \pi_t) = &-cov\left[\Delta E_t \pi_t, \sum_{k=1}^{\infty} (\omega \beta)^k \Delta E_t \pi_{t+k}\right] - cov\left[\Delta E_t \pi_t, \left(\frac{\delta v}{\beta}\right)^{-1} \sum_{k=0}^{\infty} \beta^k \Delta E_t s^p_{t+k}\right] \\ &- cov\left[\Delta E_t \pi_t, \delta^{-1} \sum_{k=1}^{\infty} \beta^k \Delta E_t g_{t+k}\right] + cov\left[\Delta E_t \pi_t, \delta^{-1} \sum_{k=1}^{\infty} \sum_j \delta_j \beta^k (1 - \omega_j^k) \Delta E_t r_{t+k}\right] \\ &+ cov\left[\Delta E_t \pi_t, \frac{\delta_D}{\delta} \sum_{k=0}^{\infty} (\omega_D \beta)^k \Delta E_t \Delta h_{t+k}\right] - cov\left[\Delta E_t \pi_t, \frac{\delta_D}{\delta} \sum_{k=0}^{\infty} (\omega_D \beta)^k \Delta E_t \pi^{US}_{t+k}\right]. \end{aligned} \tag{14}$$

Decomposition 2 says that positive unexpected inflation must forecast either: *lower* future inflation (domestic or foreign), higher deficits, lower growth, higher real discounting or a more depreciated real exchange rate. This is the main decomposition I explore in the empirical exercises.

In it, the ω terms give a clue of which terms derive from the time-t adjustment of bond prices. For example: an interest rate hike $\Delta E_t i_t$ can lead to a fall in nominal bond prices (negative $\Delta E_t rx_t$) and, by decomposition 1, a decline in current inflation. These lower bond prices in turn forecast higher inflation. This is why the decomposition prescribes future inflation as time-t *deflationary force*, like surpluses.[7] Long-term bond prices bring to the current period the fiscal effects of future inflation.

Similar mechanisms apply to the exchange rate and US inflation terms that follow from dollar-linked debt. Lower dollar-bond prices might forecast higher US inflation of lower (appreciated) real exchange in the future, despite the potential opposite effect at time t.

3. Empirical Model and Estimation

The main goal is to estimate the decompositions of unexpected inflation (10) and (14) for a set of twenty-five economies. To do this, I estimate a ten-equation VAR in which the debt law of motion (9) holds by construction. If the VAR is stationary, equation (3) will be satisfied, and the decompositions will hold.

I use annual data. Quarterly data is available, but it often does not go back as many years into the past. This is particularly true for emerging market variables and public debt measures (from all countries). With a focus on long-term debt sustainability, going further into the past is more important than accounting for quarterly dynamics. Additionally, with annual data there is no danger of measurement errors due to seasonality adjustments.

I group countries in four sample categories. The first one contains only the United States. US data covers the period 1945-2019. The second group has six developed economies (you can check the list on table 2). Their sample start in 1960. The third group has four developed economies, with the sample starting in 1973. The last group contains fourteen developing countries. Their sample starts in 1998.

Grouping countries according to sample size helps to account for parameter volatility. Additionally, it provides control for international economic environment and historical events that affect inflation and its fiscal determinants.

I interpret parameters of the VAR as being random and estimate them using Bayesian methods. I establish a prior distribution, and then use data likelihood to compute the posterior.[8] One might ask why do Bayesian and not just OLS. First, parameter shrinkage reduces the volatility of estimated coefficients, an invaluable property when samples are relatively small. Second, as long as the prior distribution leads to a stable VAR, we can calibrate its tightness so as to ensure that the posterior centers around a stable VAR as well. OLS often returns unstable VARs in which the fiscal decomposition does not hold.

I base my prior on OLS-estimated US dynamics. First, because we already have results available in the literature (Cochrane (2022), to the best of my knowledge the decompositions have not been estimated to other countries so far). Second, because the US has the longest sample. Critically, it comprises the repayment of a major public borrowing event - World War II - that renders OLS estimates of the VAR stable and plausible.[9] I estimate the model for the US by OLS and use the resulting VAR to set the mean of the prior for other countries' estimation.

From the ten variables in the VAR, five are observed: the nominal interest (i_t), the inflation rate (π_t), par-value public debt (v_t^b), the real exchange rate to the dollar (Δh_t) and GDP growth (g_t). I select these variables based on (9). Most time series data I collect from the St Louis Fed *FRED* website, the United Nations and the IMF. Details on appendix B.

I convert interest and inflation data to log. The change in dollar exchange rate is the nominal depreciation to the US dollar, plus US inflation minus domestic inflation. In the US case, I use exchange

[7] Of course, higher expected inflation means inflation is expected to grow after time t. Sims (2011) calls that mechanism "stepping on a rake" and argues it was part of the reason why 1970's US inflation kept coming back after temporary declines.

[8] See del Negro and Schorfheide (2011) or Karlsson (2013) for more on Bayesian estimation of VAR models.

[9] Including pre-1950 data in the sample proved necessary. Starting the sample after that leads to an unstable VAR estimate due to the large public debt equation root.

rate to the UK pound, which is available since the 1930s. GDP growth is the log difference of levels data. Public debt is provided as a ratio of GDP by the source (Ali Abbas et al. (2011)), and requires no transformation.

3.1. Public Finances Model

Governments typically report the par or book value of debt, and this is the data I gather. Because theory is based on the *market* value of debt, some adjustment is necessary. The Dallas Fed provides estimates of the market value of debt for the United States following Cox and Hirschhorn (1983) and Cox (1985).[10] I follow a similar methodology.

Let $\mathcal{V}^b_{j,t}$ be the par value of the j-currency portfolio debt, and let $i^b_{j,t}$ be its average yield-to-maturity by which book values are updated from period to period. The adjustment equation is

$$\mathcal{V}_{j,t} = \mathcal{V}^b_{j,t} \times \frac{\text{market value of bonds}}{\text{book value of bonds}} = \frac{1+\omega_j Q_{j,t}}{(1+i^b_{j,t-1})Q_{j,t-1}} = \frac{1+rx_{j,t}+i_{j,t-1}}{1+i^b_{j,t-1}}.$$

I detrend the $\mathcal{V}$'s, convert to real, sum across portfolios and linearize to arrive at:

$$v_t = v^b_t + \frac{v}{\beta}\left[\sum_j \delta_j \left(rx_{j,t} + i_{j,t-1} - i^b_{j,t-1}\right)\right]. \tag{15}$$

Estimates of the VAR provide an equation for the law of motion of par-value debt. I use (15) to infer a law-of-motion of market-value debt.

The average interest $i^b_{j,t}$ is not observed, so we cannot estimate an equation for it. Instead, I use a model. With a geometric maturity structure of debt and in steady state, every period the government rolls over the volume of bonds coming to maturity (which had maturity $n = 1$ in the previous period). That accounts for a share $1-\omega_j$ of total outstanding bonds. In a steady state, the government then issues the same amount of new bonds ($1-\omega_j$ of total debt) at the prevailing interest rate i_t. The average interest therefore satisfies

$$i^b_{j,t} = (1-\omega_j)i_{j,t} + \omega_j i^b_{j,t-1} = (1-\omega_j)\sum_{k=0}^{\infty}\omega_j^k i_{j,t-k} \tag{16}$$

for $j \in \{N, R, D\}$.

3.2. The Bayesian-VAR

Except for exchange rate depreciation, I demean all series. This is to assume that the stationary steady-state (around which I linearize) is given by the sample mean of the series of each country (with the exception of real exchange rate growth, which I assume to be zero).

The functional format of the VAR is

$$x_t = a x_{t-1} + b u_{t-1} + k\, e_t. \tag{17}$$

Both x and u are vectors with ten entries. Five of them are the observed variables enumerated above.

Vector u_t groups the same set of variables as x, but for the United States. I often use the "u" notation to refer to the US case. Because the public debt process of each country has a dollar component, and hence depends on dollar interest and inflation, u and ε_u enter the regression of all countries.

There are five shocks hitting the domestic economy: $\varepsilon \sim N(0,\Sigma)$, independent over time. The shocks hitting the US economy are $\varepsilon_u \sim N(0,\Sigma_u)$. I group them in $e_t = [\varepsilon'_t\ \varepsilon'_{u,t}]'$. Matrix $k_{10\times 10}$ serves to correctly reproduce the law of motion governing unobserved variables.

The model for US variables is:

$$u_t = a_u u_{t-1} + k_u\, \varepsilon_{u,t} \tag{18}$$

[10]Web link: https://www.dallasfed.org/research/econdata/govdebt.

(I use the same notation x to the VAR of all countries and differentiate only in the US case). In (18), k_u is a 10×5 matrix.

Cochrane (2022) imposes the law of motion of public debt in the VAR by using primary surplus "data" calculated from it. Primary deficit equals the change in public debt net of realized return. This procedure bypasses the problem of dealing with accounting conventions in primary surplus reporting that, in most cases, make its data inconsistent with that of public debt. Since the OLS estimator is linear, his VAR satisfies the debt law of motion by construction.

In my VAR, the procedure needs to be adjusted. First, I do not measure excess return, or real interest rates. Second, the debt equation depends on US interest and inflation. Hence, the unrestrictive estimation of (17) spuriously projects these two US variables on domestic ones, which is inconsistent with (18). For these reasons, instead of measuring implied surpluses and then estimating the model, I estimate the model and then fill the equation for primary surpluses that ensures the debt law of motion (9) holds. Before doing that, I also need to include the adjustment equation for market-value debt (15) (the estimated equation is for par-value, not market-value debt!) as well as the three definitions of average interest rates (16) required to do it. These five unobserved variables (surplus s_t^p, market-value debt v_t, and the average interest $\{i_{j,t}^b\}$) complete the ten variables of the VAR.

Note that the estimated equations for par-value and market value of public debt represent their law of motion *after* replacing the equation for primary surpluses, or its equilibrium law of motion.

The estimation has four steps.

Step 1. I estimate the VAR

$$\begin{aligned} \tilde{x}_t &= \tilde{a}\tilde{x}_{t-1} + \tilde{b}\tilde{u}_{t-1} + \varepsilon_t \\ \tilde{u}_t &= \tilde{a}_u\tilde{u}_{t-1} + \varepsilon_{u,t} \end{aligned} \tag{19}$$

where $\tilde{x}$ is a vector with the five observed variables, and $\tilde{u}$ is defined similarly. Matrices $\tilde{a}$, $\tilde{b}$ and $\tilde{a}_u$ are the submatrices of a, b and a_u corresponding to the rows and columns of these observed variables.

I also estimate $\mathrm{cov}(\varepsilon) = \Sigma$ and $\mathrm{cov}(\varepsilon_u) = \Sigma_u$.

Step 2. Stack domestic and US variables $X_t = [x_t' \; u_t']'$. In the United States case, $X_t = u_t$. I assume a constant risk-premium: $E_t rx_{j,t+1} = 0$ for all j. I use the estimated VAR (19) to compute $E_t i_{j,t+i}$ and apply (12) to compute $q_{j,t}$. Equation (11) then yields expressions for excess return of the form

$$rx_{j,t} = \varphi_j' e_t.$$

An equation for real debt is also necessary. I use

$$i_{R,t} = i_t - E_t\pi_{t+1} = \zeta' X_t.$$

under the implied assumption of equal expected real and excess returns between nominal and real debt. In appendix C, I present formulas for the φ's and ζ.

Step 3. Using the estimated model of step 1, I compute the equations for average interest using (16), and fill the corresponding rows of a, b and k (a_u and k_u in the US case). With the equations for average interest filled, I can do the same for the market-price debt using the par-value adjustment equation (15). With the equation for the market-price debt, I use the law of motion (9) and the expressions for excess return and real interest above to fill the equation row for the primary surplus.

This completes the estimation of a, b and k in the general case, a_u and k_u in the US case. For each country, we can stack the equations into a single system for X:

$$X_t = AX_{t-1} + Ke_t. \tag{20}$$

If we order unobserved variables x^o at the top of the x, we can write (20) more explicitly:

$$\begin{pmatrix} x_t \\ u_t \end{pmatrix} = \begin{bmatrix} a & b \\ 0 & a_u \end{bmatrix} \begin{pmatrix} x_{t-1} \\ u_{t-1} \end{pmatrix} + \begin{bmatrix} & k \\ 0 & k_u \end{bmatrix} \begin{pmatrix} \varepsilon_t \\ \varepsilon_{u,t} \end{pmatrix}$$

$$\text{or yet} \begin{pmatrix} x_t^o \\ \tilde{x}_t \\ u_t^o \\ \tilde{u}_t \end{pmatrix} = \begin{bmatrix} & & * & \\ 0 & \tilde{a} & 0 & \tilde{b} \\ 0 & 0 & & * \\ 0 & 0 & 0 & \tilde{a}_u \end{bmatrix} \begin{pmatrix} x_{t-1}^o \\ \tilde{x}_{t-1} \\ u_{t-1}^o \\ \tilde{u}_{t-1} \end{pmatrix} + \begin{bmatrix} & * \\ I & 0 \\ 0 & * \\ 0 & I \end{bmatrix} \begin{pmatrix} \varepsilon_t \\ \varepsilon_{u,t} \end{pmatrix}.$$

Symbol * indicates the coefficients are filled to ensure that (9), (15) and (16) hold. In appendix C I provide their formulas.

Step 4. I compute sample residuals $\hat{e}$ ($\hat{e}_u$ for the US) from (19), and estimate $\text{cov}(\varepsilon, \varepsilon_u) = \Sigma_{xu} = \sum_i \hat{e}_i \hat{e}_{u,i}/(N-1)$, where N is the sample size. Then:

$$\Omega = \text{cov}(e) = \begin{bmatrix} \Sigma & \Sigma_{xu} \\ \Sigma_{ux} & \Sigma_u \end{bmatrix}.$$

3.3. The Prior Distribution

As the commonly used Litterman (1979) prior, I use a distribution of the Normal-Inverse-Wishart class, with general format

$$\Sigma \sim IW(\Phi; d)$$
$$\theta|\Sigma \sim N(\bar{\theta}, \Sigma \otimes \bar{\Omega}).$$

where $\theta = [\text{vec}(\tilde{a}')' \ \text{vec}(\tilde{b}')']'$ and vec means stacking the columns. Given the Gaussian likelihood of the model, the posterior distribution is also of the Normal-Inverse-Wishart class. Giannone et al. (2015) provide formulas for the posterior distribution and marginal likelihood.

In the US case, the prior centers around zero, $\bar{\theta} = 0$, but since it has a very large variance, the posterior centers around the OLS estimate of $\tilde{a}_u$. The estimated VAR for the US is stationary.

In the case of other countries, I center the prior around $\tilde{a} = \tilde{a}_u$, $\tilde{b} = 0$. The economic content of the prior is that the dynamics of the observed variables is the same as that we estimate for the United States. The surpluses process differs from that of the US only to account for the differences in public debt size and term and currency structures.

With $\tilde{b} = 0$, the prior also translates the view that US variables do not affect the dynamics of domestic ones.

The mean of the IW distribution is $\Phi/(d-n-1)$, where $n = 5$ is the dimension of ε and larger values of d represent tighter priors. I choose Φ to be the identity matrix (one percent standard deviation for all shocks, which are uncorrelated) and select $d = n + 2$, the lowest integer possible that leads to a well-defined distribution mean - which therefore equals Φ.

The conditional covariance between the coefficients in $\tilde{a}$ is

$$\text{cov}\left(\tilde{a}_{ij}, \tilde{a}_{kl} \mid \Sigma\right) = \begin{cases} \lambda^2 \frac{\Sigma_{ij}}{\Phi_{jj}} & \text{if } j = l \\ 0 & \text{otherwise.} \end{cases}$$

The format allows the loadings on the variable in different equations to be correlated. Loadings on the different variables on the same equation are independent. Hyperparameter λ governs the overall tightness of the prior over linear coefficients, with lower values yielding tighter priors.

The conditional covariance of $\tilde{b}$ is

$$\operatorname{cov}\left(\tilde{b}_{ij}, \tilde{b}_{kl} \mid \Sigma\right) = \begin{cases} (\xi\lambda)^2 \frac{\Sigma_{ij}}{\Phi_{u,jj}} & \text{if } j = l \\ 0 & \text{otherwise} \end{cases}$$

where $\Phi_u = \Phi = I$ is the mean of the IW distribution in the US case. Hyperparameter ξ governs the tightness of the prior that US variables do not affect domestic ones more than the mechanical effect of US interest and inflation on domestic public debt. If $\xi = 1$, the prior is just as tight as that of $\tilde{a}$.

Finally, the covariance between $\tilde{a}$ and $\tilde{b}$ is zero.

It is straightforward to build $\bar{\Omega}$ so that the conditional covariance structures above hold.

4. Empirical Results

4.1. Variance Decomposition

In the baseline specification, I calibrate $\beta = 0.98$ for all countries, and set b tightness parameter $\xi = 1/3$. I calibrate parameters δ and ω based on debt structure data gathered from various sources (see appendix B). They are reported in Table 2 along with average debt-to-output. The mean debt-to-GDP ratio in the sample was 0.50, with developed countries slightly more indebted on average. Nominal debt tends to account for the bulk of sovereign debt, Chile being a notable exception. Emerging markets' governments tend to rely relatively more on real and especially foreign debt, and issue securities with higher maturity, on average.

To ensure stability of the VAR, I start by finding the hyperparameter λ that maximizes the marginal likelihood.[11] Then, if the mode of the posterior leads to an unstable VAR, I progressively reduce λ in 0.001 steps until it leads to a stable VAR. Given the continuity of the posterior distribution on λ and the fact that $\lambda = 0$ leads to the stable US system, there must exist a non-zero value of λ that leads to a stationary model. You can check the resulting λ's in table 2.

Table 3 contains the estimated terms of decomposition 1, computed at the mode of the posterior distribution. I compute the terms of equation (10) and divide both sides of the equality by $\operatorname{var}(\Delta E \pi_t)$. This leads to:

$$1 = p_1(r_0) + p_1(rx) - p_1(s) - p_1(g) + p_1(r)$$

where r_0 corresponds to the price adjustment term of real and dollar debt (the first term on the right side of equation (10)); the other symbols are self-explanatory.

The p_1's are regression coefficients of the respective decompositions term. For instance

$$p_1(s) = \operatorname{cov}\left[\Delta E_t \pi_t, \left(\frac{\delta v}{\beta}\right)^{-1} \sum_{k=1}^{\infty} \beta^k \Delta E s_{t+k}\right] \operatorname{var}(\Delta E_t \pi_t)^{-1}.$$

These terms indicate how much of inflation variance is explained by the corresponding term. Note that they do not have to be in the $[0, 1]$ interval. It is easy to show that the coefficients also correspond to the (sum of the) impulse response functions of the corresponding variables to a 1% inflation shock, if we let other shocks move contemporaneously according to the projection implied by Ω.

The tables contain cross-country averages and medians, conditional on country group. Highlighted figures indicate the statistical significance of the estimate's *sign*, based on ten-thousand simulations of the posterior distribution.[12] Bold font alone indicates that 75% of the simulated draws have the same sign as the posterior mode. I consider that to indicate a statistically significant result. Bold font with an asterisk indicates 90%.

[11] Giannone et al. (2015) show that, in the case of Normal-Inverse-Wishart priors, the marginal likelihood can be decomposed in a goodness-of-fit term and a model-complexity term that penalizes conditional forecast variance. By maximizing the marginal likelihood, we ensure we cannot improve one of these terms without reducing the other. Similar methods for selecting the degree of informativeness of the prior distribution have been used. See, for example, Koop (2013) and Carriero et al. (2015).

[12] I discard draws that lead to unstable VARs.

Country	v (%)	δ_N (%)	δ_R (%)	δ_D (%)	Avg. Term (Years)	λ
Averages	48	74	11	15	6.5	
Advanced - 1960	58	87	8	6	6.4	
Advanced - 1973	32	92	4	4	5.6	
Emerging - 1998	47	63	14	23	6.9	
Median	43	79	5	10	5.6	
Advanced - 1960	53	88	3	2	5.6	
Advanced - 1973	32	94	3	2	5.6	
Emerging - 1998	43	67	6	23	7.6	
United States	60	93	7	0	5	10
Advanced - 1960 Sample						
Canada	71	92	5	3	6.5	0.21
Denmark	37	84	0	16	5.6	0.18
Japan	98	100	0	0	5.5	0.01
Norway	35	99	0	1	3.7	0.19
Sweden	46	69	16	14	4.8	0.16
United Kingdom	61	76	24	0	12.3	0.17
Advanced - 1973 Sample						
Australia	24	90	10	0	7.2	0.18
New Zealand	41	82	6	13	4.3	0.15
South Korea	21	97	0	3	4	0.15
Switzerland	43	100	0	0	6.9	0.23
Emerging - 1998 Sample						
Brazil	70	70	25	5	2.6	0.12
Chile	14	10	57	33	12.8	0.27
Colombia	41	45	23	32	5.6	0.13
Czech Republic	31	91	0	9	5.6	0.15
Hungary	68	76	0	23	4.1	0.14
India	73	90	3	7	10.1	0.25
Indonesia	43	44	0	56	9.2	0.21
Israel	77	43	34	23	6.6	0.13
Mexico	45	65	10	26	5.5	0.15
Poland	47	79	1	20	4.2	0.10
Romania	28	50	0	50	4.8	0.10
South Africa	41	70	20	10	12.9	0.25
Turkey	43	47	23	30	3.6	0.13
Ukraine	43	100	0	0	9.1	0.07

Notes: v is the average public debt-to-GDP, δ_N is the share of nominal debt, δ_R is the share of real or inflation-linked debt, δ_D is the share of dollar-denominated debt (I also include debt denominated in other currencies, but the amounts are negligible in almost all cases). "Avg. Term" is the average maturity across currency portfolios, calculated as $\sum_j \delta_j (1-\omega_j)^{-1}$. λ is the prior tightness parameter.

Table 2: Debt Structure Parameters and Prior Tightness

Country	std($\Delta E\pi$) (%)	Decomposition 1 - Variance decomposition (10) $p_1(r_0)$	$p_1(rx)$	-$p_1(s)$	-$p_1(g)$	$p_1(r)$
Averages	1.6	-0.24	**-0.74***	1.02	**0.36**	0.60
Advanced - 1960	1.6	-0.01	**-0.74***	**0.97**	**0.54***	0.24
Advanced - 1973	1.8	-0.01	**-0.69***	**1.33***	**0.65***	-0.28
Emerging - 1998	1.6	-0.42	**-0.76***	0.99	0.22	0.97
Median	1.3	-0.02	**-0.76***	**0.87***	**0.42***	**0.68***
Advanced - 1960	1.5	**-0.06***	**-0.62***	**0.61***	**0.72***	**0.58**
Advanced - 1973	1.7	**-0.00**	**-0.72***	**1.40***	**0.76***	-0.37
Emerging - 1998	1.2	-0.11	**-0.77***	1.13	**0.33***	**0.79***
United States	1.9	**0.03***	**-0.78***	**0.57**	0.23	**0.96**
Advanced - 1960 Sample						
Canada	1.0	**-0.11***	**-1.59***	0.62	**1.22***	0.86
Denmark	1.2	**-0.29***	**-0.30**	0.42	-0.04	**1.21**
Japan	2.2	0	**-0.52***	**1.60***	-0.38	0.30
Norway	1.5	**-0.01***	**-0.36***	**0.60**	**0.47**	0.30
Sweden	1.5	**-0.15**	**-0.93***	-0.34	**0.98***	**1.42***
United Kingdom	2.0	**0.52***	**-0.73***	**2.89***	**0.97***	**-2.65***
Advanced - 1973 Sample						
Australia	1.5	**0.07***	**-0.76***	**2.09***	**0.66**	**-1.06**
New Zealand	2.0	**-0.10**	**-0.86***	0.40	**0.87***	0.68
South Korea	2.8	-0.01	**-0.45***	**1.91***	0.17	**-0.62**
Switzerland	1.0	0	**-0.69***	**0.90**	**0.91***	-0.12
Emerging - 1998 Sample						
Brazil	1.4	**-0.26**	**-0.22***	**-1.46**	**1.05**	**1.89**
Chile	1.0	-3.80	**-1.33**	8.95	-5.71	2.88
Colombia	0.8	**1.51**	**-0.96***	1.39	-1.09	0.15
Czech Republic	1.1	**-0.16***	**-0.37***	-2.31	**2.42**	**1.42**
Hungary	1.3	**-0.57***	**-0.93***	-0.98	1.60	1.88
India	1.1	**0.17***	**-0.46***	**1.54**	0.05	-0.30
Indonesia	1.2	**-2.59***	**-1.07***	1.69	**2.61***	0.35
Israel	1.3	-0.06	**-0.78***	-0.55	**1.51***	0.88
Mexico	1.0	-0.02	**-0.74***	**1.41**	0.03	0.32
Poland	1.3	**-0.45***	**-1.15***	0.87	-0.39	**2.11***
Romania	1.9	-0.40	**-0.96***	2.24	0.42	-0.31
South Africa	1.0	**0.36**	**-0.51***	1.58	0.25	-0.68
Turkey	2.1	**0.37**	**-0.37***	-1.18	-0.15	**2.33***
Ukraine	5.7	0	**-0.77***	**0.65**	**0.41**	**0.70***

Notes: The table reports the terms of variance decomposition (10) at the posterior distribution's mode. I divide each term by var($\Delta E_t \pi$). I simulate from the posterior to check the statistical significance of the sign of the estimate. Bold indicates 75% of results had the same sign. Bold with an asterisk indicates 90%.

Table 3: Unexpected Inflation - Variance Decomposition 1

The first column of the table reports unexpected inflation. Unsurprisingly, unexpected inflation exists, with standard deviations ranging from 1% and 2% in the country group averages. The low figure for emerging markets reminds us of the importance of the sample time period. Emerging markets do not have lower unexpected inflation; we just sampled from a period in which inflation is known to be less volatile in most countries (Stock and Watson (2002), Coibion and Gorodnichenko (2011)).

It is visible from the table that, as sample size shrinks, parameters are estimated less precisely. The evidence is not conclusive to all countries. But a few patterns emerge.

- *Unexpected inflation variance is accounted for by the intrinsic value of public debt. No single one of its components - surplus-to-GDP, GDP growth and real discounting - accounts for unexpected inflation variance alone. Nevertheless, each has a positive contribution in at least eighteen of the twenty-five countries.*

The right-hand side of decomposition 1 is the innovation to the real value of debt. That corresponds to the sum $-p_1(s) - p_1(g) + p_1(r)$ in table 3. In *all* countries in the sample, the sum is positive; variation in the real value of debt thus contributes positively to unexpected inflation variance.

No single one of the three terms - surplus-to-GDP $p_1(s)$, growth $p_1(g)$ and real discounting $p_1(r)$ - is clearly more important quantitatively. In seven cases, all three terms are positive, or the surplus and growth terms are positive. In five cases, real discounting and surplus terms, or real discounting and growth terms are positive. Only in the case of Turkey we see two negative terms (surplus and growth). Counting positive contributions, real discounting accounts for a positive share of unexpected inflation variance in eighteen cases. Surplus-to-GDP and GDP growth terms are each positive in twenty cases. Focusing on statistically significant coefficients (at 75% confidence) does not change the overall message. The surplus term is positive and significant in ten cases, the GDP growth term in twelve cases and real discounting in eight cases.

- *In all country groups, concomitant nominal bond price shocks reduce the variance of unexpected inflation: $p_1(rx) < 0$. Monetary policy trades current for future inflation, $-p_2(\pi) < 0$.*

Unexpected inflation tends to come accompanied by a decline in nominal bond prices resulting from monetary policy. Central banks observe inflation and raise interest rates. As bond prices decline, the price level does not have to increase so much to equal market-value debt to its real value.

The covariance between $\Delta E_t rx_t$ and $\Delta E_t \pi_t$ is negative and statistically significant in all countries. They are also economically significant. I estimate $p_1(rx) = -0.78$ for the US. Cochrane (2022) estimates -0.56. The effect is larger in several other cases. They suggest monetary policy is effective in preventing unexpected inflation.

But lower bond prices forecast higher inflation. Decomposition 2 isolates the effect of future inflation. For the US, $p_2(\pi) = -1.12$ (Cochrane estimates -0.59). Inflation forecasts inflation. One can attribute that to price stickiness in the short run and, in the long run, Fisherian effects of monetary policy (Garín et al. (2018), Uribe (2022)). Estimates are similar for other advanced economies. Magnitudes are close -1 and somewhat larger for the United Kingdom. In the case of emerging markets, I also find negative figures, but with smaller magnitudes, as the median shows.

- *In advanced economies, unexpected inflation forecasts deficits, which contributes to its volatility ($-p_1(s) = -p_2(s) > 0$).*

With the exception of Sweden, inflation forecasts deficits (or nowcasts, since the surplus sum in the decompositions includes time t), in advanced economies. In a fiscal-selection reading, unexpected inflation is caused by news of lower primary surpluses.

Magnitudes are economically and, in most cases, statistically significant. I estimate $-p_2(s) = 0.57$ for the US (Cochrane finds -0.06). For other developed countries, it ranges from 0.50 to 3. The large estimates for Australia (2.1) and South Korea (1.9) can be partially attributed to their reduced indebtedness (δv enters the denominator of $p_2(s)$). The United Kingdom (1.89) has a large debt but a low share of

Country	Decomposition 2 - Variance decomposition (14)					
	$-p_2(\pi)$	$-p_2(s)$	$-p_2(g)$	$p_2(r)$	$p_2(\Delta h)$	$-p_2(\pi^{US})$
Averages	**-0.81***	1.02	**0.36**	**0.63**	0.02	**-0.22***
Advanced - 1960	**-1.20***	**0.97**	**0.54***	**0.76**	0	**-0.06***
Advanced - 1973	**-1.01***	**1.33***	**0.65***	0.11	**-0.05**	**-0.03***
Emerging - 1998	**-0.56***	0.99	0.22	0.67	0.06	**-0.37***
Median	**-0.76***	**0.87***	**0.42***	**0.61***	0	**-0.03***
Advanced - 1960	**-1.08***	**0.61***	**0.72***	**0.85***	0	**-0.04***
Advanced - 1973	**-0.91***	**1.40***	**0.76***	-0.14	0	**-0.01***
Emerging - 1998	**-0.61***	1.13	**0.33***	**0.47**	-0.03	**-0.04**
United States	**-1.12***	**0.57**	0.23	**1.32***	0	-
Advanced - 1960 Sample						
Canada	**-1.53***	0.62	**1.22***	**0.78**	**-0.03**	**-0.07***
Denmark	**-0.49***	0.42	-0.04	**1.23**	0.08	**-0.20***
Japan	**-1.14***	**1.60***	-0.38	**0.91***	0	0
Norway	**-0.70***	**0.60**	**0.47**	**0.64**	0	0
Sweden	**-1.02***	-0.34	**0.98***	**1.54***	-0.07	**-0.10**
United Kingdom	**-2.34***	**2.89***	**0.97***	**-0.52**	0	0
Advanced - 1973 Sample						
Australia	**-1.47***	**2.09***	**0.66***	-0.27	0	0
New Zealand	**-1.02***	0.40	**0.87***	**1.04**	**-0.21**	**-0.08***
South Korea	**-0.74***	**1.91***	0.17	-0.33	**0.01**	**-0.03***
Switzerland	**-0.79***	**0.90**	**0.91***	-0.02	0	0
Emerging - 1998 Sample						
Brazil	**-0.11***	**-1.46**	**1.05**	**1.46**	0.07	0
Chile	-0.76	8.95	-5.71	-0.35	1.62	**-2.75**
Colombia	**-0.61***	1.39	-1.09	0.02	**1.34**	-0.04
Czech Republic	-0.02	-2.31	**2.42**	0.98	-0.03	**-0.05**
Hungary	**-0.69***	-0.98	1.60	1.83	**-0.61***	**-0.15***
India	**-1.05***	**1.54**	0.05	0.41	-0.04	**0.09***
Indonesia	**-0.79***	1.69	**2.61***	0.26	**-1.45**	**-1.33***
Israel	**-0.54***	-0.55	**1.51***	0.61	-0.12	0.10
Mexico	**-0.60***	**1.41**	0.03	0.52	**-0.52**	**0.17**
Poland	**-0.59***	0.87	-0.39	**1.43***	-0.11	**-0.21***
Romania	**-1.14***	2.24	0.42	-0.54	0.55	**-0.53***
South Africa	0.05	1.58	0.25	-0.79	-0.07	-0.01
Turkey	**-0.76***	-1.18	-0.15	**3.35***	0.14	**-0.40***
Ukraine	**-0.29**	**0.65**	**0.41***	**0.23**	0	0

Notes: The table reports the estimated terms of variance decomposition (14) at the posterior distribution's mode. I divide each term by $\text{var}(\Delta E_t \pi)$. I simulate from the posterior to check the statistical significance of the sign of the estimate. Bold indicates 75% of results had the same sign. Bold with an asterisk indicates 90%.

Table 4: Unexpected Inflation - Variance Decomposition 2

nominal debt. Sweden is a case of some inflation insulation from fiscal deficits ($p_2(s)$=-0.34) despite the small average debt and low share of nominal bonds.

In the case of developing countries, estimated values are mostly positive like for developed countries, but often not statistically different from zero. I estimate negative p_2 for Brazil certainly due to the large surpluses in the 2000s, when inflation was highest in the time series. Still, I do not draw definitive conclusions in the case of 1998 emerging market sample.

- *In advanced economies, unexpected inflation forecasts a growth decline, which contributes to its volatility ($-p_1(g) = -p_2(g) > 0$). In emerging markets, statistically significant figures indicate the same, but tend to be the exception.*

All countries with statistically non-zero estimates have a positive $-p_1(g) = -p_2(g)$. In these cases, inflation forecasts a GDP growth slowdown, which reduces the real value of debt and causes, in a FTPL interpretation, the unexpected inflation in the first place. The conclusion has more empirical support for developed countries. From them, only for Denmark and Japan I estimate negative (and statistically insignificant) $p_2(g)$.

Estimates are economically relevant. When positive, covariance with inflation accounts from 47% to 122% of total inflation variance. This is one of the differences I find between developed countries in general and the United States, for I find $p_1(g) = 0.23$, not statistically different from zero. Cochrane finds 0.49.

The pattern is similar in emerging markets: statistically significant figures are positive and economically significant. But they tend to be the exception. I estimate large growth contributions to unexpected inflation for Brazil, Czech Republic, Indonesia and Israel. Part of the explanation for the large magnitudes is the low share of nominal debt on the portfolio of developing countries' governments.

- *Long-term bonds often increase the contribution of discounting on unexpected inflation. After accounting for bond price variation, all statistically significant figures indicate such positive contribution: $p_2(r) > 0$. This result is particularly stronger for the United States, developed economies in the 1960 sample and emerging markets.*

Cochrane concludes that discount rate variation accounts for a large share of inflation variance in the United States ($p_2(r) = 1$). I find a similar conclusion for the United States ($p_2(r) = 1.32$) and advanced economies in the 1960 sample (with the exception of the United Kingdom).

Both decompositions 1 and 2 have future real interest terms. Higher discounting lowers the real value of public debt, the right-hand side of decomposition 1. If $p_1 > 0$, it increases unexpected inflation variability; if $p_1 < 0$ it reduces. In either case, if the government finances itself with one-period bonds $\omega_j = 0$, this is the only effect to be considered, and $p_2(r) = p_1(r)$.

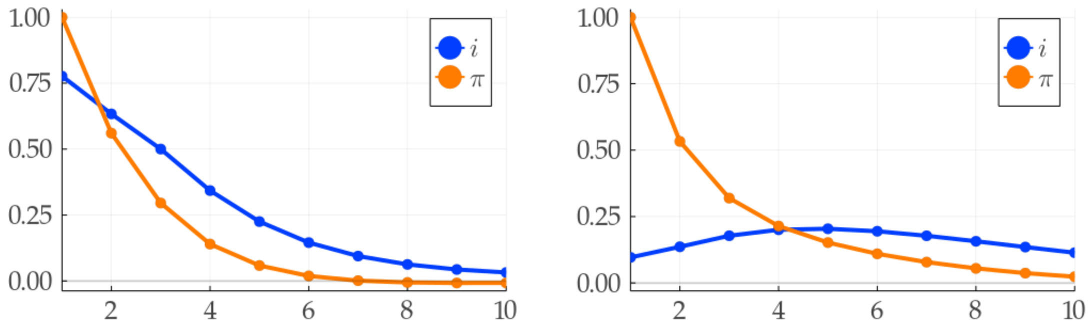
With long-term bonds, (13) shows that higher discounting also leads to lower bond prices, which soak up part of such inflationary effect.[13] If the government borrows using perpetuities ($w_j = 1$), these two forces cancel out and real interest dynamics contributes neither for nor against unexpected inflation: $p_2 = 0$.

The intermediary case $0 < \omega_j < 1$ does *not* lead to the intermediary conclusion. Long-term bonds can *enhance* the impact, positive or negative, of real interest to inflation variance. With $0 < w_j < 1$, long-term bonds reflect (and cancel out) only short-term changes in real interest. If these short-term changes have an opposite signal to the overall covariance, then long-term bonds will insulate inflation from the "wrong" period of varying interest. Mathematically, if $\sum_k (1-\omega^k)\Delta E_t r_k$ and $\sum_k \Delta E_t r_k$ have the same sign, then

$$\left|\sum_k \Delta E_t r_{t+k}\right| > \left|\sum_k (1-\omega^k)\Delta E_t r_{t+k}\right|$$

if and only if $\sum_k \Delta E_t r_k$ and $\sum_k \omega^k \Delta E_t r_k$ also have the same sign.

[13] In the case of nominal and dollar debt, this is *given* the long-term inflation and real interest effects, which I am holding constant here.



(a) Poland: Lower Contribution $p_1 < p_2$

(b) Norway: Larger Contribution $p_2 > p_1$

Figure 1: Long-Term Bonds and the Contribution of Discounting to Unexpected Inflation

In figures 1a and 1b, I plot the IRFs of nominal interest and inflation in the cases of Poland and Norway. In Poland, unexpected inflation leads to a prolonged period of high real interest, a result of high rates in response to inflation in the 2000s. That is an inflationary force. As the inflation shock hits, the price of long-term bond declines - a deflationary force that partially counteracts the former effect. Hence, $p_1(r) < p_2(r)$.

In Norway, unexpected inflation leads a smaller but longer period of higher nominal interest. This is likely a consequence of the delayed response of monetary policy to 1970s inflation. Real interest is at first negative, then turns positive. Overall, the real value of debt declines, $p_1(r) > 0$. But the price of bonds *rises* - which is another inflationary effect - as it responds more to near term real interest.[14] Hence, $p_2(r) > p_1(r)$.

Among advanced economies, in the six cases that long-term bonds enhance the contribution of real interest $\|p_2\| > \|p_1\|$, they contribute to make real interest dynamics more *inflationary*. Cochrane finds a similar result for the US, but in a much weaker magnitude: $p_1(r) = 1.0$, $p_2(r) = 1.04$. I find $p_1(r) = 0.96$, $p_2(r) = 1.32$.

In all countries of the 1960 sample, with exception of the UK, unexpected inflation forecasts real interest, which accounts for 0.5 to 1.5 times unexpected inflation variance. Like for the US, *discounting matters*.

In the 1973 sample, I estimate real interest to reduce inflation variance, probably due to delayed responses of monetary policy to 1970s (which affect results more than in the 1960 sample). New Zealand, the single country in the group with a positive $p_2(r)$ raised interest in the 70s as inflation was high but stable, and increased interest swiftly in 1985 concomitantly to a new inflation spike.

As for emerging markets, I again estimate economically significant, but statistically insignificant coefficients. In all but three cases (two being the volatile Chile and South Africa cases) they are positive. The four statistically significant coefficients are large and positive. In all, discounting and growth dynamics look more important drivers of unexpected inflation in emerging markets then surplus-to-GDP ratios.

- *For countries with dollar debt, US inflation variation contributes to reduce the volatility of unexpected inflation ($p_2(\pi^{US}) < 0$). The effect is stronger in emerging markets, since their governments tend to issue more dollar-linked bonds.*

Decomposition 2 isolates the contributions of real exchange rate and US inflation. They are only non-zero for countries with dollar debt. In all but three countries with dollar debt (India, Israel and Mexico), US inflation works as a deflationary force. Unexpected domestic inflation forecasts/nowcasts US inflation, which devalues dollar-linked bonds.

[14] The more precise statement is: the effect of real interest is a rise in bond prices. Here, I am holding the effect of constant the effect of inflation, which is another term in the decomposition, $p_2(\pi)$. Nominal and dollar bond prices do not necessarily rise. They do not in figure 1b.

Magnitudes are larger in the 1998 emerging markets sample, typically in the interval -0.10 to -0.60 (-2.75 for Chile derives from its large reliance on dollar debt, $\delta_D = 0.33$). This likely due to negative inflation shocks in 2008, as inflation declined in the US with the Great Recession. Indeed, in Mexico inflation does not fall significantly in 2008/2009.

Among advanced economies, conclusions are similar in both samples, but covariances are considerably smaller.

As for real exchange rate, in only four of the thirteen developing countries with dollar debt I estimate statistically significant contributions to unexpected inflation variance. Signs are mixed, so there are no obvious conclusions to be made.

4.2. *Response to Reduced-Form Shocks*

4.2.1. *"Aggregate Demand" Recession*

As shown before, the variance decomposition is similar to the innovations decomposition applied to $\Delta E_t \pi_t = 1$, after projecting the remaining shocks of the system. I now study the case of two other innovations. Because I do not orthogonalize or attempt to build structural shocks, the decompositions do not tell a causal story and do not provide a structural interpretation. But, like the variance decompositions above, they provide clues about inflation dynamics to support model building.

I begin with an "aggregate demand" shock, captured by a negative surprise in GDP growth and in the inflation rate: $\Delta E_t g_t = -1$ and $\Delta E_t \pi_t = -0.5$. I choose a lower inflation shock based on the change in US inflation in the 2008 recession. The model is linear, so you can interpret values in percentage points too.

To save space, I report only decomposition 2, in table 5. The first column reports unexpected inflation. The other columns report the six terms of the decomposition. I divide the whole expression by $\delta v / \beta$ so that the terms combine to $dE\pi_t$:

$$\Delta E_t \pi_t = -d_\pi - d_s - d_g + d_r + d_{\Delta h} - d_{\pi^{US}},$$

where for example $d_g = \delta^{-1} \sum_{k=1}^{\infty} \beta^k \Delta E_t g_{t+k}$.

Given the aggregate demand shock, unexpected inflation equals -0.5 by construction. But where does it come from?

- *Lower inflation in the low aggregate demand scenario follows from lower real interest and larger surpluses. Many countries, like the US, react by raising deficits, but higher subsequent surpluses revert its inflationary effect. Lower growth, stimulative monetary policy and, for emerging markets, real exchange are inflationary.*

For all developed economies, the discounting effect contributes to the low inflation, although the result is stronger in the 1960 sample. Reported figures already incorporate the effect of higher long-term bond prices, but inspection of decomposition 1 (not reported) reveals that they are not driving the sign of the coefficients. Lower real interest raises the real value of debt - discounted surpluses - as it raises the value of other assets. The effect on the price level is negative. Magnitudes are quantitatively significant, often larger than -1.5 (for a -0.5 inflation shock). Cochrane (2022) finds the this conclusion for the United States. It holds for other countries too.

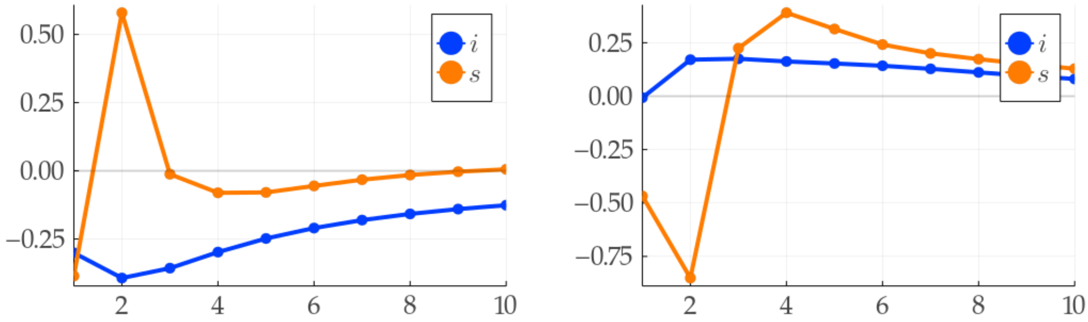
In the case of emerging markets, like before, the conclusion is similar, but estimates are more often statistically insignificant or too volatile (Chile and South Africa are notorious). But most coefficients, significant or not, are negative. Monetary policy in South Africa and Ukraine - the two positive and significant coefficients - did not react strongly to recent recession shocks (2009 for both and, in the Ukrainian case, 2014/15 following the Russian invasion of Crimea, see figure 2b).

To all countries except Ukraine, the table suggests a strong response of monetary policy through lower interest. Higher bond prices forecast lower future inflation, which corresponds to a time-t *inflationary* force in the decomposition, $d_\pi < 0$. Lower growth is also inflationary, as it raises the relative size of public debt.

Country	$\Delta E_t \pi_t$	Decomposition 2					
		$-d_\pi$	$-d_s$	$-d_g$	d_r	$d_{\Delta h}$	$-d_{\pi^{US}}$
Averages	-0.50	**0.78***	**-4.23***	**2.71***	-0.88	**0.59***	**0.54***
Advanced - 1960	-0.50	**1.24***	**-1.47***	**1.33***	**-1.70***	**0.04**	**0.05***
Advanced - 1973	-0.50	**0.83***	**-1.60***	**0.74***	**-0.56**	**0.06**	**0.02***
Emerging - 1998	-0.50	**0.58***	**-6.41**	**3.96***	-0.56	**1.01***	**0.93***
Median	-0.50	**0.64***	**-1.46***	**1.30***	**-1.23***	**0.18***	**0.11***
Advanced - 1960	-0.50	**1.13***	**-1.43***	**1.25***	**-1.78***	0	**0.03***
Advanced - 1973	-0.50	**0.82***	**-1.19***	**0.65**	**-0.43**	**0.03***	**0.01***
Emerging - 1998	-0.50	**0.48***	**-3.20***	**1.36***	-0.58	**0.51***	**0.61***
United States	-0.50	**0.57***	**-0.65**	**1.32***	**-1.75***	0	-
Advanced - 1960 Sample							
Canada	-0.50	**1.38***	-0.45	0.30	**-1.78***	-0.00	**0.05***
Denmark	-0.50	**1.06***	**-2.64**	**2.75***	**-1.78**	-0.04	**0.15***
Japan	-0.50	**0.60***	**-1.51***	**1.64***	**-1.23***	0	0
Norway	-0.50	**0.99***	**-1.36**	**1.72***	**-1.86***	0	0
Sweden	-0.50	**1.19***	-0.65	**0.87**	**-2.31***	**0.30***	**0.11***
United Kingdom	-0.50	**2.19***	**-2.20**	**0.73**	**-1.22**	0	0
Advanced - 1973 Sample							
Australia	-0.50	**0.96***	**-1.46**	**0.66**	-0.67	0	0
New Zealand	-0.50	**0.68***	**-0.84**	**0.63**	**-1.24**	**0.19**	**0.07***
South Korea	-0.50	**1.06***	**-3.17***	**1.74***	-0.20	**0.05***	**0.02***
Switzerland	-0.50	**0.64***	**-0.93***	-0.07	-0.13	0	0
Emerging - 1998 Sample							
Brazil	-0.50	**0.08**	1.87	0.13	**-2.85**	**0.23***	**0.04***
Chile	-0.50	**2.10***	-30.50	**30.54**	-7.76	-0.63	**5.74***
Colombia	-0.50	**0.49**	**-10.90**	**7.57***	-0.07	1.16	**1.26***
Czech Republic	-0.50	**0.51***	-0.07	**0.25**	**-1.61**	**0.27***	**0.14***
Hungary	-0.50	**0.64***	10.82	-5.29	**-7.91**	**0.70***	**0.54***
India	-0.50	0.45	**-1.16**	**0.71**	-0.44	-0.02	-0.05
Indonesia	-0.50	0.02	**-11.24***	**1.42**	0.76	**6.82***	**1.73**
Israel	-0.50	**0.47***	**-3.18**	**1.17**	-0.73	**0.98***	**0.79***
Mexico	-0.50	**0.48***	**-4.56***	**1.94***	0.04	**0.92***	**0.68***
Poland	-0.50	**0.50***	-0.14	**1.30**	**-2.90**	0.32	**0.42***
Romania	-0.50	**0.36**	**-8.16***	**2.05**	2.15	**2.33***	**0.77***
South Africa	-0.50	**1.59***	**-30.02***	**11.15***	**15.60***	**0.87**	**0.30***
Turkey	-0.50	**0.73***	0.64	**0.52**	**-3.31***	0.18	**0.73***
Ukraine	-0.50	-0.33	**-3.22**	**1.92***	**1.13**	0	0

Notes: I set $\varepsilon_g = -1$ and $\varepsilon_\pi = -0.5$, and regress the values of other shocks on it. The table shows the estimated terms of decomposition 2 at the posterior distribution's mode. I divide each term by $\delta v / \beta$, so the terms sum up to $\Delta E_t \pi$. I simulate from the posterior to check the statistical significance of the sign of the estimate. Bold indicates 75% of results had the same sign. Bold with an asterisk indicates 90%.

Table 5: Unexpected Inflation Decomposition 2 - Recession Shock



(a) United States, $d_s = 0.65$ (b) Ukraine, $d_s = 3.22$

Figure 2: Fiscal Shocks in Recessions: Deficits then Surpluses

Results indicate that fiscal policy, on the other hand, is *deflationary*, in that

$$\Delta E_t s_t + \sum_{k=1}^{\infty} \beta^k \Delta E_t s_{t+k} < 0.$$

This is not an indication that governments do not incur in deficits to "stimulate demand", although that is the case for many of them. To others, like that of the United States and Ukraine depicted in figure 2, a deficit at the time recession hits is followed by future surpluses. For the real value of debt, what matters is discounted surpluses, not current surpluses. Hence the deflationary effect $-d_s < 0$.

Again results are economically and statistically significant. Most estimates are larger than -1. In a causal reading, they suggest that, should governments manage to convince the public that large recession deficits would *not* be followed by future surpluses, the impact on unexpected inflation would be large. One can interpret the post-COVID, worldwide surge in inflation as an example of that (I do not include 2021-2022 data in the estimation).

The result of deflationary fiscal policy is common to all samples, but stronger in emerging markets, *despite* their lower share of nominal debt (recall that δ enters in the denominator of d_s). More procyclical fiscal policy in developing countries has been previously identified by the macroeconomic literature. See, for instance Kaminsky et al. (2004), Alesina et al. (2008) and Ilzetzki (2011).

Finally, I find for emerging markets an inflationary effect stemming from real exchange rate depreciation. I interpret these figures as capital exit and flight-to-quality movements in exchange rates (Jiang et al. (2020), Kekre and Lenel (2021)).

4.2.2. *Exchange Rate Depreciation Shock*

I next consider episodes of real exchange rate depreciation. I consider a 10% shock to Δh. Such episodes can be due to shocks in the international economy, like Global Recessions, or shocks in the domestic economy, like sudden stops. I am going to focus on the latter, so I prevent US variables from jumping in the VAR:

$$e_{\Delta h} = 10, \qquad \varepsilon_u = 0.$$

US variables are unaffected by domestic dynamics, so they do not respond to the shock at all. Table 6 reports results.

- *In emerging markets, real exchange depreciation shocks forecast low growth and contractionary monetary and fiscal policy. Higher nominal interest trades current for future inflation, $-d_\pi < 0$; higher surpluses are deflationary. Lower growth and the depreciated currency are inflationary. Unexpected inflation is statistically zero in most cases.*

In emerging markets, real exchange rate shocks resemble "sudden stops", events in which foreign

Country	$\Delta E_t \pi_t$	Decomposition 2					
		$-d_\pi$	$-d_s$	$-d_g$	d_r	$d_{\Delta h}$	-$d_{\pi^{US}}$
Averages	0.16	-0.27	**-4.48***	0.03	0.36	**4.53***	0
Advanced - 1960	**-0.39**	0.21	**-1.78**	-0.90	**1.29**	**0.80***	0
Advanced - 1973	**0.41**	0.15	**-1.41**	0.95	0.12	**0.61***	0
Emerging - 1998	0.29	**-0.57**	**-6.82**	0.03	-0.07	**7.57***	0
Median	0.02	**-0.16**	**-1.23***	-0.34	**0.37**	**1.53***	0
Advanced - 1960	**-0.69***	0.11	**-1.51**	**-1.29**	**1.90**	**0.27***	0
Advanced - 1973	**0.36**	0.11	-1.50	0.49	-0.09	**0.18***	0
Emerging - 1998	0.05	**-0.55***	**-2.90***	**1.29**	**0.49**	**3.25***	0
United States	**0.56***	**-0.75**	-0.19	**1.78**	-0.28	0	-
Advanced - 1960 Sample							
Canada	**-0.68***	0.81	**-2.64**	**-1.15**	1.82	**0.48***	0
Denmark	**-0.81**	0.32	**-3.87**	**-2.54**	**3.09**	**2.19***	0
Japan	**-0.36**	-0.10	**-1.80***	**2.15***	-0.61	0	0
Norway	**-1.29***	-0.20	-1.23	**-2.10**	**2.18**	**0.06***	0
Sweden	1.52	-0.83	-0.28	-1.43	1.98	**2.09***	0
United Kingdom	-0.69	1.24	-0.85	-0.34	-0.75	0	0
Advanced - 1973 Sample							
Australia	-0.10	**1.21**	-0.25	**-0.86**	-0.21	**0.03***	0
New Zealand	0.39	-0.84	**-3.91**	**1.76**	1.32	**2.07***	0
South Korea	**1.00**	0.40	**-2.74**	**3.67***	-0.66	**0.33***	0
Switzerland	**0.33**	-0.18	**1.25**	**-0.78**	0.02	0	0
Emerging - 1998 Sample							
Brazil	0.02	**-0.16***	-0.76	**1.43**	**-1.24**	**0.74***	0
Chile	0.41	**1.90**	**-87.99**	**29.66**	4.07	**52.78***	0
Colombia	-0.02	**-1.00***	**-11.74***	**3.06**	0.65	**9.00***	0
Czech Republic	0.08	-0.06	-6.64	3.88	1.36	**1.53***	0
Hungary	-0.85	0.03	**39.37**	**-29.48**	**-12.43**	**1.67**	0
India	-0.62	**3.87***	**-4.65**	-0.36	-0.02	**0.54***	0
Indonesia	-0.28	**-0.93***	**-12.74***	**2.34***	0.35	**10.71***	0
Israel	0.25	0.51	**-13.85***	**3.35**	**4.61***	**5.63***	0
Mexico	-0.69	**-0.94**	4.77	-7.65	-1.25	**4.39***	0
Poland	**-0.65**	0.26	**-0.08**	-3.56	0.62	**2.11***	0
Romania	2.58	**-3.18**	**-7.36**	**2.15**	0.37	**10.61***	0
South Africa	0.20	**-1.50***	3.52	-1.06	-2.10	**1.35***	0
Turkey	**2.26***	**-2.29***	-1.15	**-4.42**	**5.18***	**4.93***	0
Ukraine	**1.34**	**-4.45***	**3.79**	**1.16**	0.81	**0.02***	0

Notes: I set $\varepsilon_{\Delta h} = 10$, and regress the values of other shocks on it. The table shows the estimated terms of decomposition 2 at the posterior distribution's mode. I divide each term by $\delta v/\beta$, so the terms sum up to $\Delta E_t \pi$. I simulate from the posterior to check the statistical significance of the sign of the estimate. Bold indicates 75% of results had the same sign. Bold with an asterisk indicates 90%.

Table 6: Unexpected Inflation Decomposition 2 - Real Exchange Depreciation

households abruptly sell holdings of domestic assets.[15] Nominal exchange depreciates and raises the in-domestic-currency value of public debt, which counts as an inflationary force by our decompositions. Its magnitude depends on the size of the dollar-linked portfolio of public debt but is positive and statistically significant to all countries.

Emerging markets respond to the depreciation shock by raising interest (unreported figures). Lower bond prices forecast future inflation by (13). Fiscal policy response is contractionary, $-d_s < 0$, with statistically and economically significant coefficients.[16] Median coefficients of growth and discounting effects indicate that both are inflationary, but the growth effect is more widespread in the sample (nine of the fourteen countries). Indeed, sudden stops are characterized by output drops (Calvo et al. (2006)).

Results suggest that policy efforts are often successful in mitigating unexpected inflation (although often at the cost of future inflation, $d_\pi > 0$). From the fourteen cases, I estimate statistically significant $dE\pi > 0$ in two.

The evidence for advanced economies is far less conclusive. From the set of developed countries, Canada, Denmark, Sweden, New Zealand and South Korea have a share of dollar debt greater than 1%. These five economies respond to the depreciation event by raising surpluses, like emerging markets.

But depreciation shocks do not necessarily lead to a drop in output. They do the opposite in the 1960 sample (except for Japan), and the sign of coefficients split two and two in the 1973 sample. These episodes do not look like "sudden stops" in emerging markets. The case of South Korea is an exception. Its large output coefficient $-d_g > 0$ is driven by the 1998 recession following the Asian financial crisis.

The contribution of monetary policy is also ambiguous, as is the overall effect on unexpected inflation.

5. The Decomposition in a New-Keynesian Framework

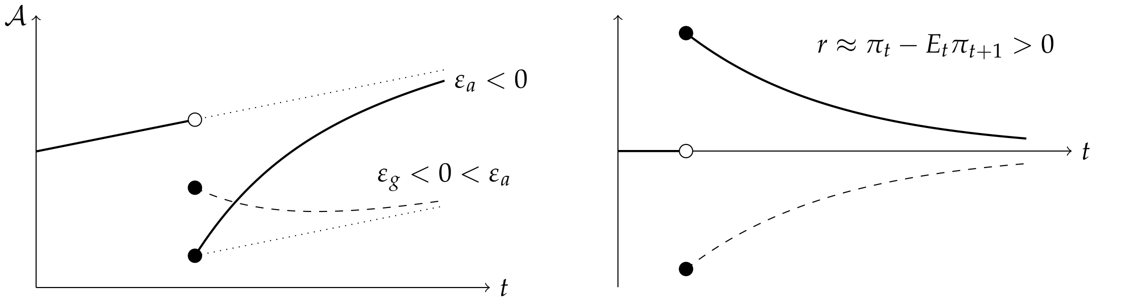
The estimates of the last section correspond to decompositions implied by reduced-form, not structural shocks. Yet, in any theoretical model in which the transversality condition (3) holds, structural shocks will also lead to their own decompositions. In this section, I ask the question: how far do we need to go to find a model that can reproduce - with structural shocks - the measures implied by the estimated VARs?

I set a two-country New-Keynesian model and study how the terms of the decompositions behave in response to monetary, fiscal and technology shocks. The private sector framework is most similar to De Paoli (2009). There is a Foreign and a Home economy. The size of the Home economy converges to zero. In the limit, the Foreign economy behaves exactly as if it was closed, and the Home economy is "small". Firms produce differentiated goods using labor in a linear production function, and operate in a monopolistic competition environment. Prices are Calvo (1983)-sticky in the currency of the producing firm (producer-currency pricing), but not in the currency of the other country. The weight of Home goods in the basket of Foreign households converges to zero. But Home households have a bias towards consumption of Home goods (you can interpret that as consumption of services and other non-tradable commodities). As consumption baskets differ, the real exchange rate varies. Financial markets are complete, as in Galí and Monacelli (2005).

A theoretical model opens the door to causal interpretations of decompositions 1 and 2. Should we read them as "how the price level must respond to news of debt value" or "how the debt value must respond to news of the price level"? I assume a FTPL setup of public policy, which points to the former. Households observe the path of surpluses and attribute value to public debt in light of it. Surprise inflation or deflation ensues. Monetary policy is characterized by a Taylor (1993) rule.

[15] The literature on sudden stops is vast. See, for example, Calvo et al. (2006) and Verner and Gyöngyösi (2020) for empirical analysis, and Chari et al. (2005) and Mendoza (2010) for theoretical and quantitative accounts.

[16] Hungary and Ukraine are exceptions, although Hungarian estimates look suspiciously volatile for a country with low dollar debt, $\delta_D = 0.23$. Ukrainian result is driven by the 30% surge in inflation following the 2014 invasion of Crimea, which corresponds to an estimated fiscal *deficit* at the same time its currency depreciated.



(a) Productivity Path $\mathcal{A}_t$

(b) $-a$ (= Mg. Cost $\approx -E_t\Delta\pi_t$ at constant wages)

Figure 3: Two Negative Technological Shocks

5.1. Technology and Trend Shocks

For reasons that will become clear, the economy is not stationary. The production function is $\mathcal{A}_t N = \mathcal{T}_t A_t N$ in the Home country and $\mathcal{A}_t^* N = \mathcal{T}_t A_t^* N$ in Foreign, where N is the number of hours of labor employed. Process $\mathcal{T}_t$ is the trend of the productivity process, common to both Home and Foreign. It introduces a unit root to productivity:

$$\log \mathcal{T}_t = \log \mathcal{T}_{t-1} + g + u_{g,t}$$
$$u_{g,t} = \rho_g u_{g,t-1} + \varepsilon_{g,t}.$$

A vast literature followed Nelson and Plosser (1982), who first tested and failed to reject the presence of a unit root in US GNP. Although a definitive conclusion has not been reached - testing for unit roots is hard! - I do not regard the assumption of trend shocks as unreasonable.

Processes A_t and A_t^* are stationary. They capture temporary fluctuations of productivity around the trend. The laws of motion for $a_t = \log A_t$ and $a_t^* = \log A_t^*$ are

$$a_t = \rho_a a_t + \varepsilon_{a,t}$$
$$a_t^* = \rho_a a_t^* + \varepsilon_{a,t}^*.$$

By themselves, $\mathcal{T}$ and A (or A^*) have a clear interpretation. Shock $e_{g,t}$ captures changes to long-term productivity; $e_{a,t}$ captures short-term deviations from the long term. Combinations of $e_{g,t}$ and $e_{a,t}$ are also "structural": like the individual shocks, all they do is change the path of productivity. In particular, by combining $e_{g,t}$ and $e_{a,t}$ we can generate permanent disturbances that are more general than the random walk $\log \mathcal{T}_t$. Such generality will prove critical. Figure 3a sketches two relevant examples, which I will refer to later. It plots the productivity process $\mathcal{A}_t$. The solid curve corresponds to a temporary negative shock $e_a < 0$. Productivity declines and then slowly grows as it reverts back to trend. The dashed curve depicts the combination $e_g < 0$, $0 < e_a < -e_g$. Productivity drops in period zero, and continues to decline until the total effect of the technological shock on productivity reaches e_g. Both disturbances will lead to recessions in the model, but the dynamic properties of marginal costs will imply different paths for inflation and real interest rates.

5.2. The Foreign, "Closed" Economy

The key ingredients of the model are well known in the macroeconomic literature, so I state only its linearized version. Appendix F presents a full derivation. I consider first the Foreign economy. As the measure of Home households and firms approaches zero, prices and quantities in the Foreign economy respond only to demand and supply of its own households and firms. Therefore, we can view the case of the Foreign country as the study of a closed economy - the equations are exactly the same. The private sector block is

$$
\begin{aligned}
y_t^* &= E_t y_{t+1}^* - \gamma \left[i_t^* - E_t \pi_{t+1}^* \right] + E_t u_{g,t+1} \\
\pi_t^* &= \beta E_t \pi_{t+1}^* + \kappa^* y_t^* - \kappa_a a_t^* \\
g_t^* &= y_t^* - y_{t-1}^* + u_{g,t},
\end{aligned}
\tag{21}
$$

where y_t^* is log detrended output, π_t^* is log inflation and g_t^* is the growth rate of GDP. The asterisk indicates the variable corresponds to the Foreign economy. The top equation is the dynamic IS, which follows from households' intertemporal optimization and output market clearing. The second equation is the forward-looking Phillips curve, which follows from optimal price-setting behavior by firms. Its parameters are $\kappa^* = \lambda(\gamma^{-1} + \varphi) > 0$ and $\kappa_a = \lambda(1 + \varphi) > 0$. Parameter φ is the Frisch elasticity of labor supply, and $\lambda = (1 - \beta\theta)(1 - \theta)/\theta$ is a function of intertemporal discounting β and the price-resetting rate $1 - \theta$.

By themselves, the IS and Phillips curve equations in (21) do not determine unexpected inflation. The Phillips curve pins *expected* inflation, or the expected change in the inflation rate. Any expectational shock is consistent with a stationary equilibrium. In Blanchard and Kahn (1980) language, the system contains two forward-looking variables (y^* and π^*) for a single explosive root: multiplicity of equilibrium ensues.[17] To solve it, the literature uses either a fiscal selection mechanism (the FTPL) or a spiral-threat selection mechanism. I use the former, and defer to appendix E a discussion of the difference between the two.

The public policy block of the Foreign economy is[18]

$$
\begin{aligned}
i_t^* &= \phi_\pi \pi_t^* + \phi_g g_t^* + \varepsilon_{i,t}^* \\
s_t^* &= s_t^{p*} = \rho_s s_{t-1}^* + \tau_\pi \pi_t^* + \tau_g g_t^* + \varepsilon_{s,t}^*.
\end{aligned}
\tag{22}
$$

The law of motion for public debt v^* is given by (9), except that all variables carry an asterisk. Leeper (1991) was the first to observe that such law of motion can provide the additional unstable root that the model requires for equilibrium uniqueness. That can only happen if surpluses do not respond strongly enough to debt. Note that is the case here: public debt v^* does not enter the equation for surpluses s^*. If it did, the government would adjust primary surpluses to stabilize real debt *for any possible value of the price level*, which would hence remain indeterminate. With a fiscal-active model such as ours, that is not the case. Agents observe discounted surpluses and value public debt accordingly. Inflation jumps as a result of such re-evaluation. Therefore, in a FTPL model like this one, we read decompositions 1 and 2 with causality running from right to left. News about the intrinsic value of public debt or bond prices *cause* unexpected inflation.

The equations in (21) and (22) form the canonical closed-economy New-Keynesian model with active fiscal policy, and augmented for non-stationary technology.[19] I compare the fiscal decompositions implied by it with VAR estimates for the United States. A subset of structural parameters I calibrate - see table 7. Parameters β, δ and ω are the same as the ones I used in the estimation of the VAR for the US. In particular, $\delta_D = 0$ implies that the real exchange rate and foreign inflation terms of decomposition 2 equal zero. Elasticities $\gamma = 0.4$ and $\varphi = 3$ follow literature standards. Price rigidity $\theta = 0.25$ follows the low-frequency estimate in Kehoe and Midrigan (2015) and the macro-estimate in Smets and Wouters (2007).

Let σ_a be the standard deviation of $\varepsilon_{a,t}$, σ_a^* the standard deviation of $\varepsilon_{a,t}^*$, and so on. I group all such parameters in a single vector σ. The ε and ε^* shocks are uncorrelated. The remaining parameters of the model $\Psi = (\rho_a, \rho_g, \phi_\pi, \phi_g, \rho, \tau_\pi, \tau_g, \sigma')$ I estimate by a method of moments procedure. The set of moments contains the terms of different decompositions we saw so far - I denote them D. To

[17] The lack of determination is most easily seen in a model with flexible prices and constant output, in which the only equilibrium condition is the Fisher equation $i_t = E_t \pi_{t+1}$. An interest rate peg $i_{t-1} = 0$ leads to $E_{t-1}\pi_t = 0$. One cannot determine $\Delta E_t \pi_t$.

[18] By assumption public basket of goods is the same as that of households, so real surpluses s_t coincide with price-adjusted surpluses s^{p*}.

[19] I solve the model by solving forward-looking variables forward, using Klein (2000) method.

Parameter	Interpretation	Optim. Interval	Value No Trend	Value Full
A. Calibrated Parameters				
β	Intertemporal discount		0.98	
v	Average debt-to-GDP		country dependent	
δ	Currency structure of debt		country dependent	
ω	Term structure of debt		country dependent	
γ	Intertemporal elasticity		0.4	
ϕ^{-1}	Frisch elasticity of labor supply		1/3	
θ	Price rigidity		0.25	
α	Share Foreign goods in Home basket		0.45	
$\bar{\omega}$	Demand elasticity to terms of trade		γ^{-1}	
B. Estimated Parameters				
ρ_a	Temporary productivity persistence	$[0,1]$	0.974	0.84[a]
ρ_g	Trend disturbance persistence	$[0,1]$	-	0.26
ϕ_π	Interest sensitivity to inflation	$[0.3,1]$	0.810	0.94
ϕ_g	Interest sensitivity to GDP growth	$[0,1]$	0.355	0.60
ρ_s	Surplus persistence	$[0,1]$	0[a]	0.002
τ_π	Surplus sensitivity to inflation		0[a]	0.20
τ_g	Surplus sensitivity to GDP growth		0.850	0.09
σ_i^*, σ_i	Interest shock std	$(0,\infty)$	0[a]	0.001
σ_s^*, σ_s	Surplus shock std	$(0,\infty)$	0[a]	0.006
σ_a^*, σ_a	Temporary technology shock std	$(0,\infty)$	-	0.21
σ_g	Technology trend shock std	$(0,\infty)$	0[a]	0.39

[a] The values indicated were fixed, not estimated.

Table 7: List of Model Parameters

avoid having the model yield completely irrealistic dynamics, I also include as targeted moments: the standard deviations of interest i_t and inflation π_t, both expressed as ratios of GDP growth g_t standard deviation, as well as the three correlations between these variables. These five moments are calculated from the ergodic distribution of the system and grouped in vector $\mathcal{M}$. The optimization problem has the format

$$\text{Min}_{\Psi} \quad \frac{w}{n_{\mathcal{D}}} \left\| \mathcal{D}_{VAR} - \mathcal{D}_{NK}(\Psi) \right\| + \frac{1-w}{n_{\mathcal{M}}} \left\| \mathcal{M} - \mathcal{M}_{NK}(\Psi) \right\| \qquad \text{s.t. } \Psi \in \Theta \tag{23}$$

where $\|.\|$ is the Euclidian norm, w is a scalar governing the relative weight we give to decomposition moments (I find that $w = 0.9$ works well), and $n_{\mathcal{D}}$ and $n_{\mathcal{M}}$ are the sizes of vectors $\mathcal{D}$ and $\mathcal{M}$; D_{VAR} denotes VAR-implied decomposition terms, M denotes data moments and NK subscript denotes New-Keynesian model values. I restrict parameters to a subset Θ which limits them to reasonable values. Table 7 reports the list of parameters, estimated values and constraints.

- *Temporary technology shocks $e_{a,t}$ reproduce the decomposition of unexpected inflation variance with positive contributions from surplus-to-output, growth and real interest terms.*

For this first proposition, I turn off all shocks in the model, except for temporary technology shocks $\varepsilon_{a,t}$: $\sigma_g^* = \sigma_i^* = \sigma_s^* = 0$. The remaining parameters solve (23). Targeted decompositions include only decompositions 1 and 2 of the variance of unexpected inflation variance (tables 3 and 4); vectors D thus contain thirteen entries.

Figure 4 displays results: bars correspond to the model, circles correspond to the VAR. Marker ribbons indicate percentiles 25 and 75 from the posterior distribution. The model successfully replicates the VAR decompositions.

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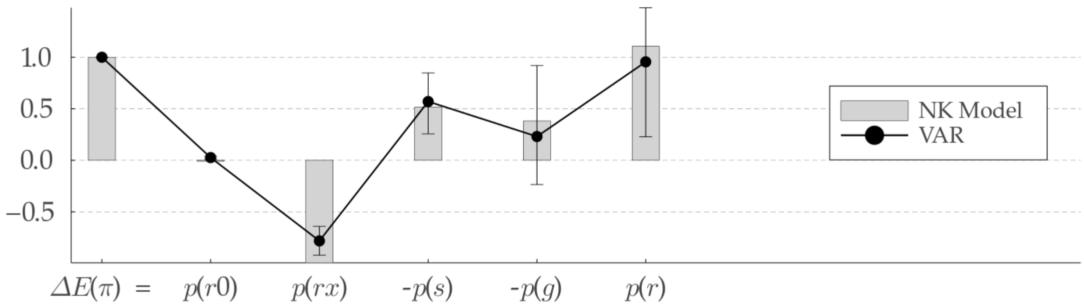
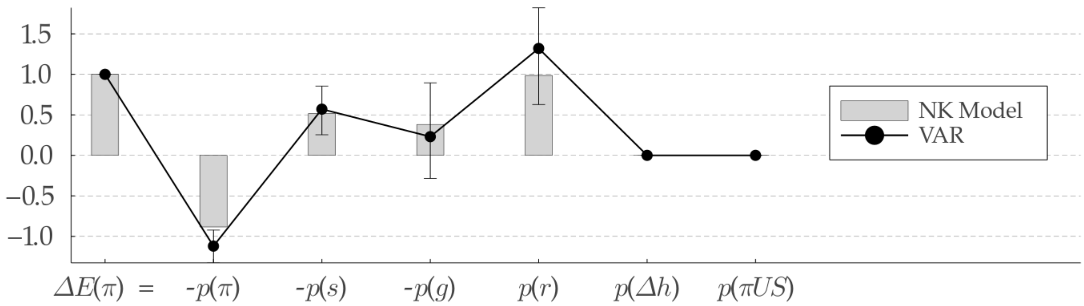


(a) Variance Decomposition 1 - Equation (10)



(b) Variance Decomposition 2 - Equation (14)

Figure 4: Decomposition of Unexpected Inflation - NK Model, Temporary Technology Shocks

	σ_i/σ_g	σ_π/σ_g	cor(π, i)	cor(π, g)	cor(i, g)
United States					
Data	1.00	1.01	0.54	-0.24	0.16
Model (No Trend)	1.35	1.94	0.98	-0.70	-0.55
Model (Full)	1.00	1.08	0.82	-0.32	0.28
New Zealand (Not Targeted)					
Data	2.04	2.17	0.77	-0.39	-0.35
Model	0.90	0.88	0.76	-0.23	0.45

Table 8: Estimated Second Moments

The large interest rule parameters ϕ_π and ϕ_g matter to reproduce the negative contribution of excess returns $p_1(rx)$ and the positive contribution of real return $p_1(r)$ and $p_2(r)$. Figure 5a shows the response function to $\varepsilon = -1.10$, which yields the decomposition of unexpected variance. The solid curves correspond to the estimated $\phi_\pi = 0.81$. As inflation jumps, the central bank raises interest by the Taylor rule. Bond prices fall, so $p_1(rx) < 0$. As inflation starts to decline, real interest rates becomes positive (figure 5b), so $p_1(r) > 0$, $p_2(r) > 0$. The dashed lines in these two graphs represent the response to the same shock if we set $\phi_\pi = 0.3$. The initial jump in nominal interest is not as strong, so the model fails to generate large $p_1(rx)$. The real interest rate jumps down on spot and stays lower than the baseline for three periods, which precludes the model from generating large $p_1(r)$ and $p_2(rx)$. Note how the weaker response of nominal interest leads to higher unexpected inflation, but lower *future* inflation.

Finally, the optimization entails countercyclical fiscal policy (deficits in recessions), as the IRF shows. Negative growth in period one leads to a large deficit, which drives the positive contribution of discounted surpluses to unexpected inflation like the VAR. Despite being able to replicate the empirical decompositions, this first simplified version of the NK model fails to generate realistic second moments, as table 8 shows. Growth is not volatile enough, and interest rates are countercyclical, contrary to the data.

- *In the absence of trend shocks, the NK model fails to replicate the decompositions of the "aggregate demand" shock, even with fiscal and monetary shocks. The model with trend shocks accomplishes that by generating a recession in which marginal costs decline over time. Policy shocks are not necessary.*

I bring back to the model interest rate, surplus and productivity trend shocks. I re-estimate parameters, but now I include as a target in problem (23) the decompositions of unexpected inflation in the "aggregate demand" recession scenario, considered in table 5 (with $\Delta E_t \pi_t = -0.5$ and $\Delta E_t \pi_t = -1$). Vector $\mathcal{D}$ now contains twenty-six entries. To properly characterize a_t as a measure of temporary shocks, I restrict $\rho_a = 0.84$. This leads a_t disturbances to have a half-life of about four years. The gray bars in the four panels in figure 6 depict the fit of the model, which I refer to as the model with trend shocks. The model does a good job of reproducing the decompositions estimated for the United States. In the same panels, the hatched bars correspond to the same optimization, but under the constraint $\sigma_g = 0$ (no trend shocks). Even with disturbances to monetary and fiscal policy, it is clear that the NK model loses the ability to reproduce the two empirical decompositions of unexpected inflation at the same time.

To help understand these results, the two panels in figure 7 plot the terms of the decompositions in response to the four structural shocks. I normalize the size of the shocks so that the largest decomposition term equals one, and its sign so that the resulting unexpected inflation $\Delta E_t \pi_t$ is positive. The minus sign "-" in the legend of the plots indicate that the four shocks must be negative. Figures 5e-5f contain the IRFs to each of them. We saw in table 5 that the unexpected deflation $\Delta E_t \pi_t = -0.5$ characterizing the recession scenario followed from lower surplus-to-GDP ratios and lower real interest. In the absence of trend shocks, I argue that New-Keynesian model struggles in delivering the latter.

Consider first the effect of policy shocks. Inspection of the structural shocks' decompositions in figure 7 reveals that neither fiscal nor monetary policy shocks can lead to the correct pattern of real interest. An expansionary monetary policy shock (lower i_t, graph 5g) successfully yields lower real interest, but only through positive unexpected inflation - the recession scenario, on the contrary, asks for negative $\Delta E_t \pi_t$. The positive effect on inflation follows from the increase in bond prices, which elevates the market price of public bonds. To restore the valuation equation (4), inflation jumps. An expansionary fiscal policy also yields positive real interest. However, as plot 5h shows, real interest is too small period by period, due to the strong Taylor rule coefficient $\varphi_\pi = 0.94$.

Moving to technology shocks, the model cannot rely on temporary disturbances $e^*_{a,t}$ alone to replicate both the variance decompositions *and* the recession decompositions, since unexpected inflation responds to the underlying shocks in opposite directions. By adding trend shocks, the model reproduces both decompositions. Plots 5c and 5d show the economy's impulse response. The response to $\Delta E_t \pi_t = 1$ - the decomposition of unexpected inflation - is similar to that of the stationary model, figure (5a), except that the primary surplus rises in the first period and only turns negative afterwards. The structural shocks that lead to it are

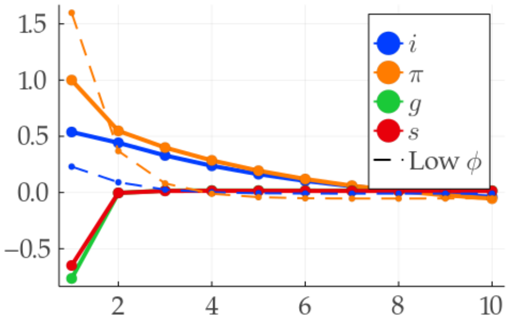
$$\varepsilon^*_{i,t} = \varepsilon^*_{s,t} \approx 0 \qquad \varepsilon^*_{a,t} = -0.74 \qquad \varepsilon_{g,t} = -0.24.$$

By comparing figures 5c and 5e, it is easy to see that productivity shocks justify the decomposition of unexpected inflation variance almost by itself.

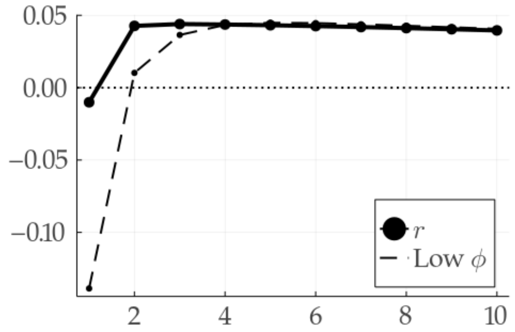
However, trend shocks are necessary to replicate the fiscal decomposition of recessions shocks ($\Delta E_t \pi_t = -0.5, \Delta E_t g_t = -1$). Figure 5d plots the economy's response. The underlying structural shocks are

$$\varepsilon^*_{i,t} = \varepsilon^*_{s,t} \approx 0 \qquad \varepsilon^*_{a,t} = 0.51 \qquad \varepsilon_{g,t} = -1.34.$$

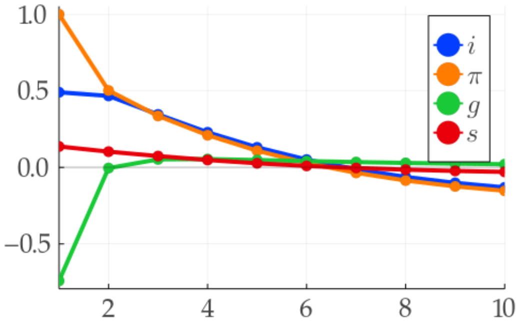
The combination of these structural shocks lead to a productivity path similar to that depicted in plot 3a. Productivity falls in the initial period and, contrary to temporary shock $e^*_{a,t}$, *continues to decline.* That leads to the critical feature of this particular productivity path, illustrated in figure 3b: the recession coincides with a period of relatively low *detrended* marginal costs. Actual marginal costs do increase since productivity falls, but *relative to trend* they are low - that is the role of the positive $\varepsilon^*_{a,t}$ disturbance. In turn, by the Phillips curve, firms react to low marginal costs by setting current prices at relatively



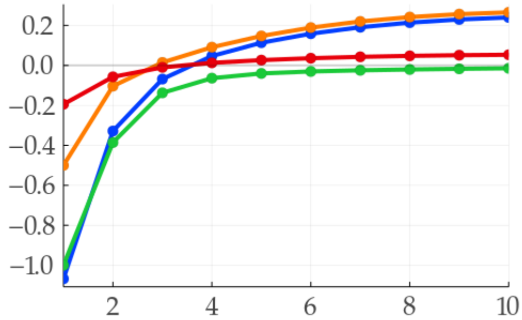
(a) No Trend Shocks, $\Delta E_t \pi_t = 1$



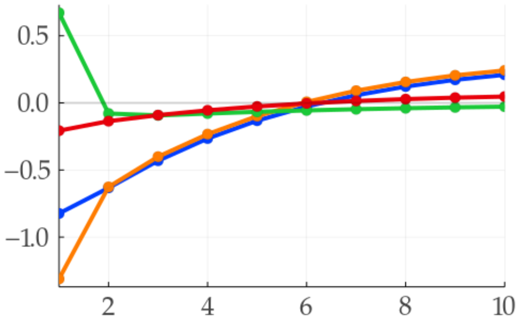
(b) No Trend Shocks, $\Delta E_t \pi_t = 1$, Real Interest



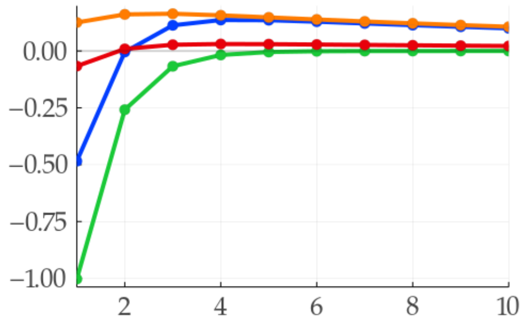
(c) Full Model, $\Delta E_t \pi_t = 1$



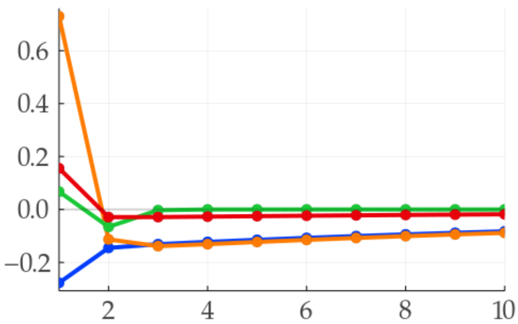
(d) Full Model, $\Delta E_t \pi_t = -0.5$, $\Delta E_t g_t = -1$



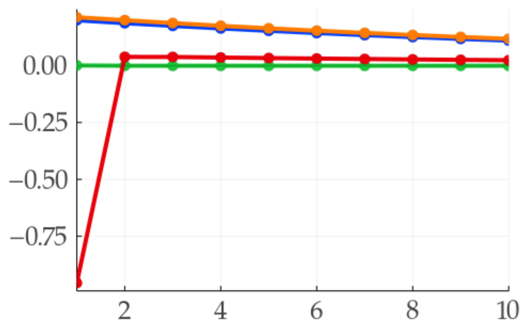
(e) Full Model, Productivity Shock $\varepsilon^*_{a,t} = 1$



(f) Full Model, Trend Shock $\varepsilon_{g,t} = -1$

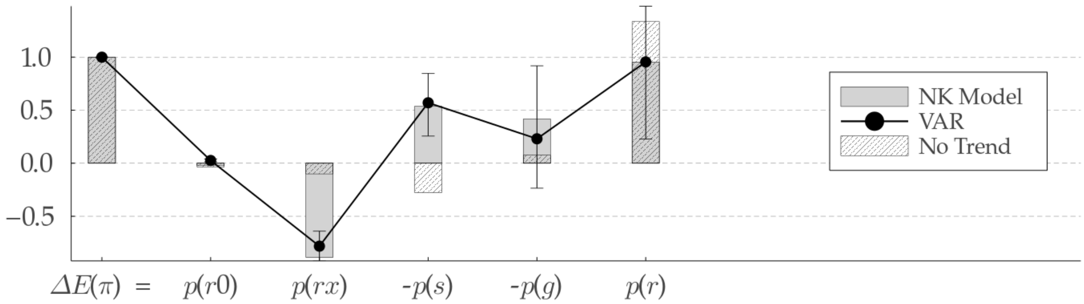


(g) Full Model, Interest Shock $\varepsilon^*_{ir,t} = -1$

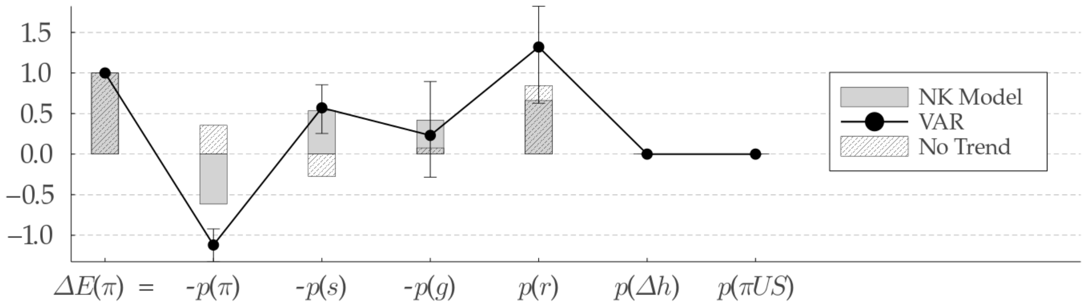


(h) Full Model, Surplus Shock $\varepsilon^*_{s,t} = -1$

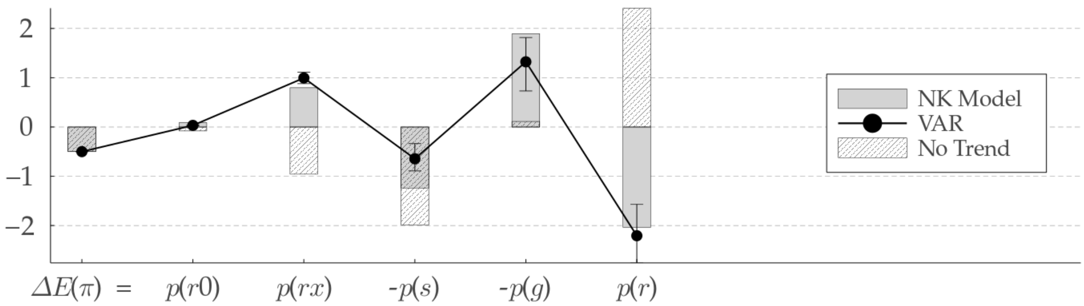
Figure 5: IRFs for the Foreign, "Closed" Economy



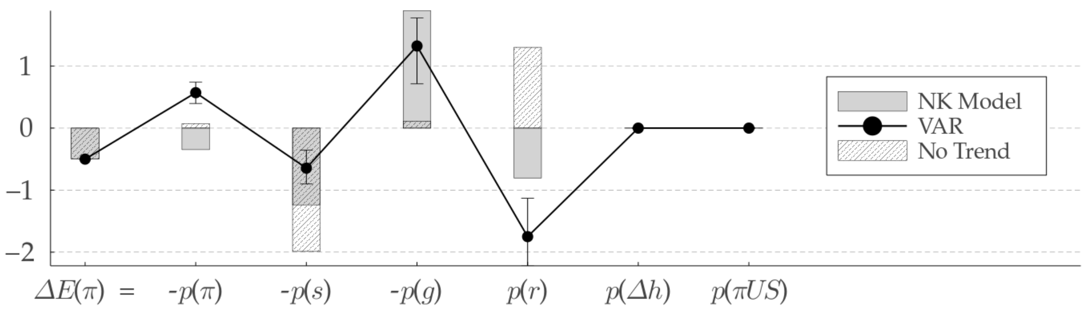
(a) Decomposition 1 - Variance Decomposition $\Delta E_t \pi = 1$



(b) Decomposition 2 - Variance Decomposition $\Delta E_t \pi = 1$

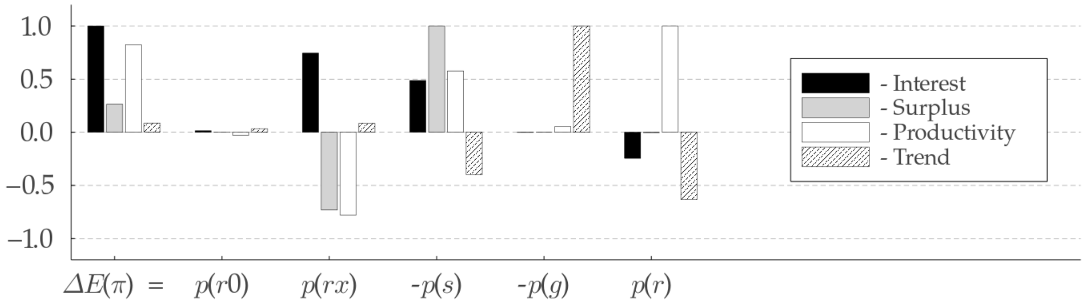


(c) Decomposition 1 - Recession Scenario $\Delta E_t \pi = -0.5$, $\Delta E_t g = -1$

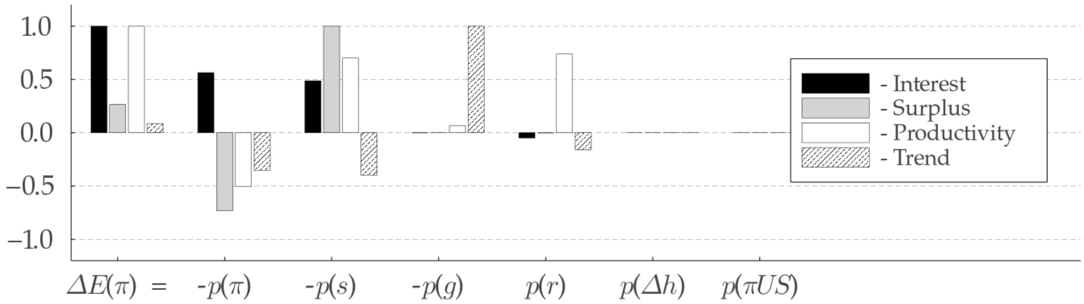


(d) Decomposition 2 - Recession Scenario $\Delta E_t \pi = -0.5$, $\Delta E_t g = -1$

Figure 6: Fiscal Decomposition - NK Model with Trend Shocks



(a) Decomposition 1



(b) Decomposition 2

Figure 7: Fiscal Decomposition of Structural Shocks

lower levels than future ones: inflation grows over time in expected value. The strong Taylor rule ensures $i_t \approx \pi_t$ and completes the argument, which I summarize below.

$$\begin{aligned} & E_t\pi_{t+1} \approx \beta E_t\pi_{t+1} > \pi_t \qquad \text{(by the Phillips curve, } \kappa^* y_t - \kappa_a a_t < 0) \\ \implies & E_t\pi_{t+1} > \phi_\pi^{-1}(i_t - \phi_g g_t) \approx i_t \qquad \text{(by the strong Taylor rule, } \phi_\pi \approx 1) \\ \implies & r_t < 0 \end{aligned}$$

In the absence of trend shocks or some other productivity process that can generate a recession with relatively low marginal utility, the basic NK model fails to replicate the negative contribution of real interest to the deflation that characterizes the "aggregate demand" recession scenario. Trend shocks also help us through the IS equation. Since it has some persistence ($\rho_g = 0.26$), $\varepsilon_{g,t} < 0$ leads to multiple periods of negative growth. By the IS equation, that implies multiple periods of negative real interest, which is what we need.

The combination of structural shocks that lead to the two decompositions (variance and recession) require no policy shocks. As table 7 suggests with low values of σ_i^* and σ_s^*, policy shocks are also not critical for the model to reproduce empirical second moments. We see in table 8, row "Full", that it does so relatively well, although the correlation between nominal interest and inflation is somewhat larger than the evidence. Interest and surplus rules (22) are enough to ensure the required dynamics. Table 7 reports even larger estimated Taylor rule parameters ϕ_π and ϕ_g than in the simplified stationary model. It also requires a surplus process that reponds more to inflation than to output growth.

5.3. The Home, Open Economy

I consider now the case of the Home country, which represents that of a small, open economy. I proceed under the restriction that structural parameters (γ, φ, θ, ...) in Home are the same as those in Foreign; it turns out these parameters need not be re-estimated. Parameters specific to the open economy case I calibrate based on the experience of New Zealand, which is the country I compare results with. My choice for New Zealand follows from the observation that its VAR decompositions are often close to the average of developed countries. Additionally, New Zealand public debt has a significant share of dollar debt, which will be important when we discuss the real depreciation shock. Given Foreign prices and quantities, the equilibrium conditions in the private sector of Home are:

$$\begin{aligned} y_t &= E_t y_{t+1} - \gamma \left[i_t - E_t \pi_{t+1} + \alpha\bar{\omega} E_t \Delta z_{t+1} \right] + E_t u_{g,t+1} && (24)\\ \pi_{H,t} &= \beta E_t \pi_{H,t+1} + \kappa y_t - \kappa_\alpha y_t^* - \kappa_a a_t && (25)\\ g_t &= y_t - y_{t-1} + u_{g,t} && (26)\\ y_t &= y_t^* + \gamma_\alpha z_t && (27)\\ h_t &= (1-\alpha) z_t && (28)\\ \pi_t &= \pi_{H,t} + \alpha \Delta z_t && (29) \end{aligned}$$

The model contains two key international prices: terms of trade z_t and the real exchange rate h_t. Domestic output prices $p_{H,t}$ and the consumer price index p_t are related by $p_t = p_{H,t} + \alpha z_t$, where α is the weight of foreign goods on the Home economy basket. Equation (29) follows. I set $\alpha = 0.45$ following the Reserve Bank of New Zealand's KITT model (Beneš et al. (2009)). Therefore, there is home bias in consumption. Price variables are in logs.

Equation (28) follows from a decomposition of real exchange rate: Foreign-to-Home consumer price ratio (h_t) equals Foreign-to-Home output price ratio (z_t) times (plus, in logs) Home output-to-consumer price ratio ($-\alpha z_t$). With complete markets, the Backus and Smith (1993) condition holds: domestic consumption rises relative to foreign consumption when h depreciates:

$$c_t = y_t^* + \gamma h_t. \tag{30}$$

Equation (24) is the intertemporal IS; it follows from households' Euler equation, added to the market-clearing condition

$$y_t = c_t + \alpha\bar{\omega}\gamma z_t, \tag{31}$$

which states that demand for home goods equals domestic consumption plus a term that adjusts for relative price variation of Home goods. Parameter $\bar{\omega}$ can be greater or lower than zero.[20] Depreciated terms of trade reduce the relative price for Home goods, which increases foreign demand. But they also correspond to lower foreign output due to the international risk-sharing rule (30). The net result on aggregate demand for domestic output can go either way. I set $\bar{\omega} = \gamma^{-1}$, which leads to zero net exports every period. Equation (27) follows from (28), (30) and (31). Parameter $\gamma_a = (1 - \alpha + \alpha\bar{\omega})\gamma$ adjusts intertemporal substitution γ for the presence of home bias and imperfect substitution between different varieties of goods. Lastly, the parameters of the Phillips curve (25) are $\kappa = \lambda(\gamma_\alpha^{-1} + \varphi) > 0$, κ_a (same as Foreign) and $\kappa_\alpha = \lambda\alpha(1-\bar{\omega})\gamma_\alpha^{-1}$.

The problem of unexpected inflation determinacy that characterizes the closed-economy NK model is also present in this open-economy version. To see this, replace (27), (28) and (29) in the Phillips curve to reduce the system to

$$\begin{aligned} y_t &= E_t y_{t+1} - \gamma_\alpha \left[i_t - E_t \pi_{H,t+1} \right] + \alpha(\bar{\omega} - 1) E_t \Delta y_{t+1}^* + E_t u_{g,t+1} \\ \pi_{H,t} &= \beta E_t \pi_{H,t+1} + \kappa y_t - \kappa_\alpha y_t^* - \kappa_a a_t. \end{aligned}$$

These two equations determine the distribution of y_t and $\pi_{H,t}$. Note the similarity with their closed-economy counterpart (21). Since trend growth $u_{g,t}$ and Foreign output-trend ratio y^* are determined elsewhere, they do not affect the determinacy/stability properties of the system, which again requires an additional unstable root.

[20] I use the bar notation on $\bar{\omega}$ to differentiate it from the geometric term structure parameter of public debt.

The public policy block of the Home economy is

$$\begin{aligned} i_t &= \phi_\pi \pi_t + \phi_g g_t + \varepsilon_{i,t} \\ s_t &= \rho_s s_{t-1} + \tau_\pi \pi_t + \tau_g g_t + \varepsilon_{s,t} \\ s_t^p &= s_t - \alpha s z_t. \end{aligned} \tag{32}$$

The equations are similar to those of the Foreign economy, except that surplus s_t and price-adjusted surplus s_t^p differ. I assume the government taxes domestic production, so the price index for its surplus is $p_{H,t}$, not p_t. I again prevent surpluses from stabilizing public debt for arbitrary values of unexpected inflation. Fiscal policy is therefore active, and provides the additional unstable root to determine unexpected inflation.

5.4. Extensions

5.4.1. Countercyclical Fiscal Policy and Partial Debt Repayment

$$i_t = \phi \pi_t + \varepsilon_{i,t} \tag{33}$$

$$\beta \left[v_{l,t} + s_{l,t} - \alpha s_l z_t \right] = v_{l,t-1} + v_l \left[\sum_j \delta_j \left(rx_{j,t} + i_{j,t-1} + \Delta h_{j,t} - \pi_{j,t} \right) - g_t \right] \qquad l = 1,2 \tag{34}$$

$$s_{1,t} = \rho s_{1,t-1} + \tau_1 y_t + \psi v_{1,t-1} + \varepsilon_{1,t} \tag{35}$$

$$s_{2,t} = \rho s_{2,t-1} + \tau_2 y_t + \varepsilon_{2,t} \tag{36}$$

The interest rate follows a single-mandate Taylor rule. Fiscal policy is active, but accommodates partial debt repayment. Specifically, there are two debt processes, $\{v_{l,t}\}$, and two surplus processes $\{s_{l,t}\}$, indexed by $l = 1, 2$.

The government keeps the same term and currency structure on the two debt processes, so they accrue interest at the same rate (the term in brackets on the right-hand side of (34) does not depend on l). Since v_l and s_l are conveniently stated in levels, $v_t = v_{1,t} + v_{2,t}$, and equation (9) holds with $s_t = s_{1,t} + s_{2,t}$ and $p_t^s = p_{H,t}$.

The setup is inspired by Jacobson et al. (2019).

6. Robustness

7. Conclusion

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A. Linearization

B. Data Sources and Treatment

B.1. Sources

I collect a significant share of the data from the St. Louis Fed's *FRED* website. In the case of countries with sample starting after 1970 I get data from the United Nations's National Accounts Main Aggregates Database. Their database also contains exchange rate data, which I use only in the case of emerging markets (with sample starting after 1998).

Whenever omitted in the list below, the source for interest rate data is the FRED; and the source of debt structure data is the OECD's Central Government Debt database. Finally, unless otherwise noted, public debt data I get from the database from Ali Abbas et al. (2011), which is kept up-to-date.

Australia 1973-2021. All except GDP and public debt from FRED.

Brazil 1998-2021. Debt structure data I collect from the Brazilian Central Bank.

Canada 1960-2021. All except public debt from FRED.

Chile 1998-2021.

Colombia 1998-2021. Debt structure data I collect from the Internal Debt Profile report, available at the Investor Relations Colombia webpage.

Czech Republic 1998-2021.

Denmark 1960-2021. All except public debt from FRED.

Hungary 1998-2021.

India 1998-2021. Debt structure data collect from the Status Paper on Government Debt report, 2019-2020, available at the Department of Economic Affairs.

Indonesia 1998-2021. Debt structure data I gather from the 2014 "Central Government Debt Profile" report and the 2018 "Government Securities Management" report, both from the Ministry of Finance.

Israel 1998-2021.

Japan 1960-2021. All except public debt from FRED.

Mexico 1998-2021.

Norway 1960-2021. All except public debt and interest rates from FRED. I interpolate the debt data for the year 1966. FRED interest data goes back to 1979, I splice it with historical data from Eitrheim et al. (2007), available at the website of the Norges Bank.

New Zealand 1973-2021. All except GDP and public debt from FRED.

Poland 1998-2021.

Romania 1998-2021. Interest rate is the deposit rate series from IMF's International Finance Statistics. Debt structure data I collect from the 2018 "Flash Report on the Romanian Public Debt" and the 2019-2021 and 2021-2023 "Government Debt Management Strategy" report, all from the Treasury and Public Debt Deparment (Ministry of Public Finance).

South Africa 1998-2021. Debt structure data from the 2020/2021 Debt Management Report, from the National Treasury Department.

South Korea 1973-2021. All except GDP and public debt from FRED.

Sweden 1960-2021. All except public debt from FRED. I interpolate the debt data for the year 1965 and 1966.

Switzerland 1973-2021. Interest, CPI and exchange rate from FRED.

Turkey 1998-2021.

Ukraine 1998-2021. Interest rate data from National Bank of Ukraine (NBU Key Policy Rate). Debt structure data I collect from "Ukraine's Public Debt Performance in 2021 and Local Market Update", from the Ministry of Finance of Ukraine.

United Kingdom 1960-2021. Interest data from the Bank of England (Base Rate); I splice it with discount rate data from the FRED. Inflation and GDP data from FRED.

United States 1950-2021. All data collected from the FRED. I use real exchange rate to the United Kingdom, since nominal exchange rate is available since before 1950.

C. Additional Details of the BVAR Estimation

D. Alternative Reduced-Form Shocks

D.1. Primary Deficit Shock

E. Equilibrium Selection in the NK Model

The New-Keynesian model (21) does not determine unexpected inflation $\Delta E_t \pi_t$. Any value of unexpected inflation is consistent with a stationary equilibrium. There are two existing selection mechanisms: fiscal selection and spiral threat selection. Both pin down $\Delta E_t \pi_t$ while leaving other equations unchanged. This feature characterizes the *observational equivalence proposition* (Cochrane (2011), Cochrane (1998)), which states that, in the absence of further assumptions, one cannot use data to test selection mechanisms.

Fiscal Selection. Fiscal selection, or the fiscal theory of the price level, determines unexpected inflation by means of (5), with causality running from right to left. Any economic shock can change the conditional distribution of discounted future surpluses (in units of goods) backing the stock of public nominal liabilities. It can thus change its real value. The relative price of public debt in terms of goods $1/P_t$ then adjusts to reflect that change, much in the same way that news of future dividends (in currency units) per stock change the relative price of stocks in terms of currency (Cochrane (2005)).

Spiral-Threat Selection. Spiral threat selection is the approach that most of the monetary economics literature has adopted so far. It starts by attributing causality in equation (5) from left to right: to any price level, no matter how large or small, the government alters its surplus choice to reflect the new value of public debt. It then pins down unexpected inflation by means of an explosive root introduced by an interest policy equation of the format $i_t = \phi \pi_t, \phi > 1$. The equation was associated to the celebrated Taylor (1993) rule, but its role in the NK model is not to stabilize "demand" shocks via rapidly-adjusted, pro-cyclical real interest rates. On the contrary, the policy rule here introduces the instability required by the NK model to pin down unexpected inflation. Assuming muted monetary policy $i_t = 0$, the system of equations (21) is "too stable": it contains one explosive eigenvalue for two forward-looking variables. Any choice of unexpected inflation forms a stable equilibrium path that converges to the zero steady state.[21] Equation $i_t = \phi \pi_t$ solves that issue when $\phi > 1$.

Importantly, the *selection of equilibrium* is completely unrelated to the *observed* interest rate. For instance, $i_t =$ white noise would be a perfectly valid specification for *observed* interest. More rigorously,

[21] Economists have interpreted this feature as admissibility of "sunspot" shocks. Without a selection mechanism, (21) will only determine the unexpected component of one variable, *if* it is fed the unexpected component of the other.

Country	$\Delta E_t \pi_t$	Decomposition 2					
		$-d_\pi$	$-d_s$	$-d_g$	d_r	$d_{\Delta h}$	$-d_{\pi^{us}}$
Averages	**0.13***	-0.02	-0.89	**0.92***	0.18	0.02	**-0.08***
Advanced - 1960	**0.02**	**0.12**	-0.13	**0.31**	-0.28	**0.02**	**-0.01***
Advanced - 1973	**0.33***	**-0.27***	1.04	0.39	-0.79	**-0.02**	**-0.01***
Emerging - 1998	**0.13***	-0.02	**-1.84**	**1.30**	**0.78**	0.04	**-0.13**
Median	**0.08***	-0.02	0.01	**0.41***	0.16	0	0
Advanced - 1960	0	**0.08***	**-0.19**	**0.27**	-0.16	**0.01***	0
Advanced - 1973	**0.29***	-0.31	0.66	0.35	-0.57	0	**-0.01***
Emerging - 1998	**0.13***	-0.03	-0.13	**0.47**	**0.47**	-0.07	-0.01
United States	**-0.11***	**0.10***	0.09	**1.24***	**-1.55***	-	-
Advanced - 1960 Sample							
Canada	0.01	**0.32***	-0.47	0.26	-0.16	**0.05***	0
Denmark	**0.12***	**0.29***	**-0.72**	**0.41**	0.16	0.02	**-0.04***
Japan	0.04	-0.01	**-0.30**	**1.27***	**-0.92***	0	0
Norway	0	**0.08**	-0.04	0.12	-0.16	0	0
Sweden	-0.03	**0.07**	-0.09	**-0.45***	**0.44**	0.03	**-0.03***
United Kingdom	0	-0.06	**0.83**	0.27	**-1.04**	0	0
Advanced - 1973 Sample							
Australia	**0.08**	**-0.51**	**2.80**	0.63	**-2.84**	0	0
New Zealand	**0.41***	**-0.40***	0.05	0.06	**0.81**	**-0.10**	**-0.03***
South Korea	**0.65***	-0.22	1.10	0.79	-1.00	0	**-0.01***
Switzerland	**0.17***	0.02	0.21	0.07	-0.13	0	0
Emerging - 1998 Sample							
Brazil	**0.12**	**-0.03**	0.11	-0.24	0.12	**0.14***	**0.02***
Chile	**0.20***	**0.60**	-5.19	4.10	-2.32	2.15	0.86
Colombia	-0.02	**-0.27**	-0.27	0.20	0.27	0.03	0.03
Czech Republic	**0.24***	**0.40**	**-3.92**	**3.13***	0.60	0	**0.04**
Hungary	0.03	0.04	**-9.93**	**5.86***	**4.55***	**-0.27**	**-0.23***
India	-0.11	-0.32	**0.95**	**-0.36**	-0.26	**-0.14***	0.02
Indonesia	**0.14**	**-0.60***	1.36	**1.56**	0.50	-0.53	**-2.13***
Israel	**0.30***	-0.08	0.03	**0.51**	0.20	-0.14	**-0.21***
Mexico	**0.22***	**-0.32***	0.46	-0.47	**1.06***	**-0.31**	**-0.18**
Poland	0.01	0.01	0.43	**1.05**	**-1.22**	**-0.14**	**-0.11***
Romania	-0.07	-0.02	-0.48	-0.38	1.08	-0.39	0.12
South Africa	**-0.14**	**0.76***	**-8.71***	**3.20**	**4.57***	0.09	**-0.06**
Turkey	**0.41***	-0.03	-0.56	**-0.37**	**1.36***	0.03	-0.02
Ukraine	**0.48***	**-0.41**	0.01	**0.44**	**0.45***	0	0

Notes: I set ε_{v^b} so that the innovation to primary surpluses equals -1, and regress the values of other shocks on it. The table shows the estimated terms of decomposition 2 at the posterior distribution's mode. I divide each term by $\delta v/\beta$, so the terms sum up to $\Delta E_t \pi$. I simulate from the posterior to check the statistical significance of the sign of the estimate. Bold indicates 75% of results had the same sign. Bold with an asterisk indicates 90%.

Table 9: Unexpected Inflation Decomposition 2 - Primary Deficit Shock

consider the simplest possible NK model

$$x_t = E_t y_{t+1} - \gamma(i_t - E_t \pi_{t+1}) \qquad \text{(NK)}$$
$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t,$$

with x interpreted as an output gap. Add to that the following equations:

$$i_t = i_t^* + \phi(\pi_t - \pi_t^*) \qquad \phi > 1 \qquad \text{(ST-1)}$$
$$i_t^* - E_t \pi_{t+1}^* = i_t - E_t \pi_{t+1} \qquad \text{(ST-2)}$$
$$i_t^* \text{ given for all } t \qquad \text{(ST-3)}$$
$$\pi_t^* \text{ given at } t. \qquad \text{(ST-4)}$$

Format (ST-1) is due to King and William (1996); i^* is the central bank's desired observed interest rate. The term π_t^* is a stochatic inflation target. Equation (ST-2) asks that the government's choices respect private market conditions and expectations formation. It forces the government to elect *unexpected* inflation only.[22]

Mechanically, one can combine (ST-1) and (ST-2) to find $E_t \pi_{t+1} - E_t \pi_{t+1}^* = \phi(\pi_t - \pi_t^*)$; $\phi > 1$ and Blanchard and Kahn (1980)'s razor then select the unique stationary path $\pi = \pi^*$, $i = i^*$, which form the *observed* equilibrium. Parameter ϕ remains unidentified (Cochrane (2011)).

Researchers have interpreted (ST-1) as a threat of nominal spiral - hence my name choice "spiral threat" selection. Different papers discuss if central banks can indeed rule out nominal spirals, but the key assumptions here do not really relate to what the central bank can do, but what *households believe* it can and would. Indeed, note that there is nothing particularly special about inflation in (ST-1)-(ST-4). One could as well write the whole system using an output target instead:

$$i_t = i_t^* + \phi(y_t - y_t^*) \qquad \phi > 1 \qquad \text{(ST-1')}$$
$$i_t^* - E_t y_{t+1}^* = i_t - E_t y_{t+1} \qquad \text{(ST-2')}$$

and now the "threat" is not that of a nominal spiral, but of a *real* spiral. Obviously, the central bank cannot trigger a "hyperproduction" (as in hyperinflation) process. Neither could it stop one, say if productivity for some reason started to grow at abnormal rates. But, if the central bank vacuously threatens hyperproduction, and it is the case that agents believe its threat; and if then the central bank vacuously promisses to stop the hypothetical hyperproduction it has vacuously threatened to create, and again agents trust its word; then and only then does the Blanchard and Kahn (1980) equilibrium arranged by (ST-1')-(ST-2') arises. The actual powers of the central bank are irrelevant.

F. Deriving the SOE-NK Model

[22] The attentive eye may have noticed an apparent modelling sin: system (NK), (ST-1)-(ST-4) presents six equations, for only five variables: y, π, π^*, i, i^*. There is no over-identification, nevertheless. Target inflation enters the system both as a static (= forward-looking) variable π_t^* *and* as a state variable, in expected value $E_{t-1}\pi_t^*$. Another way to write (ST-3) would be $E_{t-1}\pi_t = i_t^* - (i_t - E_{t-1}\pi_t)$. It becomes evident then that (ST-4) only really picks the unexpected component of inflation.