Assignment 1

Deadline: Wednesday, 26^{th} of April 2023

Upload your solutions at: https://tinyurl.com/AML-2023-ASSIGNMENT1

1. **(0.75 points)** Give examples for:

a) a finite hypothesis class $\mathcal{H}$ with VCdim($\mathcal{H}$) = 2023. Justify your choice. **(0.25 points)**

b) an infinite hypothesis class $\mathcal{H}$ with VCdim($\mathcal{H}$) = 2023. Justify your choice. **(0.25 points)**

c) an infinite hypothesis class $\mathcal{H}$ with VCdim($\mathcal{H}$) = ∞. Justify your choice. **(0.25 points)**

2. **(0.75 points)** Consider $\mathcal{H}$ to be the following hypothesis class:

$$\mathcal{H} = \{h_a : \mathbb{R}^3 \to \{0,1\} \mid h_a(\mathbf{x}) = \mathbf{1}_{[\|x\|_2 \leq a]}(\mathbf{x}), \mathbf{x} = (x_1, x_2, x_3) \in \mathbb{R}^3, \|x\|_2 = \sqrt{x_1^2 + x_2^2 + x_3^2}\}$$

Consider the realizability assumption.

a) show that $\mathcal{H}$ can be (ϵ,δ)-PAC learned by giving an algorithm A and determining the sample complexity $m_{\mathcal{H}}(\epsilon, \delta)$ such that the definition of PAC learnability is satisfied. **(0.5 points)**

b) compute VCdim($\mathcal{H}$). **(0.25 points)**

3. **(1 point)** Compute VCdim($\mathcal{H}$) where $\mathcal{H}$ is the following hypothesis class:

$$\mathcal{H} = \{h_{\theta_1,\theta_2} : \mathbb{R}^2 \to \{0,1\} \mid h_{\theta_1,\theta_2}(\mathbf{x}) = h_{\theta_1,\theta_2}(x_1, x_2) = \mathbf{1}_{[\theta_1 + x_1 \cdot sin(\theta_2) + x_2 \cdot cos(\theta_2) > 0]}, \theta_1, \theta_2 \in \mathbb{R}, \mathbf{x} \in \mathbb{R}^2\}.$$

4. **(1.5 points)** Consider $\mathcal{H}_\alpha$ to be the class of axis aligned rectangles with fixed aspect-ratio α, where $\alpha \in \mathbb{R}$ and $\alpha > 0$:

$$\mathcal{H}_\alpha = \{h_{a,b,c,d} : \mathbb{R}^2 \to \{0,1\} | h_{a,b,c,d}(\mathbf{x}) = \mathbf{1}_{[a,b]\times[c,d]}(\mathbf{x}), a, b, c, d \in \mathbb{R}, a < b, c < d, \frac{d-c}{b-a} = \alpha\}$$

Consider the realizability assumption.

a) give a learning algorithm A that returns a hypothesis h_S from $\mathcal{H}_\alpha$, $h_S = A(S)$ consistent with the training set S (h_S has empirical risk 0 on S). **(0.75 points)**

b) find the sample complexity $m_{\mathcal{H}_\alpha}(\epsilon, \delta)$ in order to show that $\mathcal{H}_\alpha$ is PAC – learnable. **(0.75 points)**

5. **(1 point)** Compute VCdim($\mathcal{H}$) where $\mathcal{H}$ is the following hypothesis class:

$$\mathcal{H} = \{h_\theta : \mathbb{R} \to \{0,1\} | h_\theta(x) = \mathbf{1}_{[\theta,\theta+1]\cup[\theta+2,\theta+4]\cup[\theta+6,\theta+9]}(x), \theta \in \mathbb{R}\}.$$

Ex-officio: 0.5 points