Theoretical Framework for Prior Knowledge Transfer in Deep Learning

Thesis Defense Presentation

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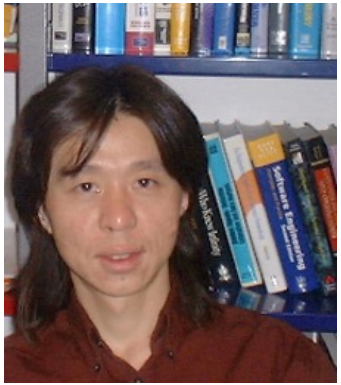

Prof. Yongyi Mao
Expert in
information theory,
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Prof. Pascal Germain
Expert in
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reinforcement learning,
bandits,
and AI4Health

Prof. Mario Marchand
Expert in
learning theory,
PAC Bayes, author
of “set covering machine”

The Decade of Deep Learning

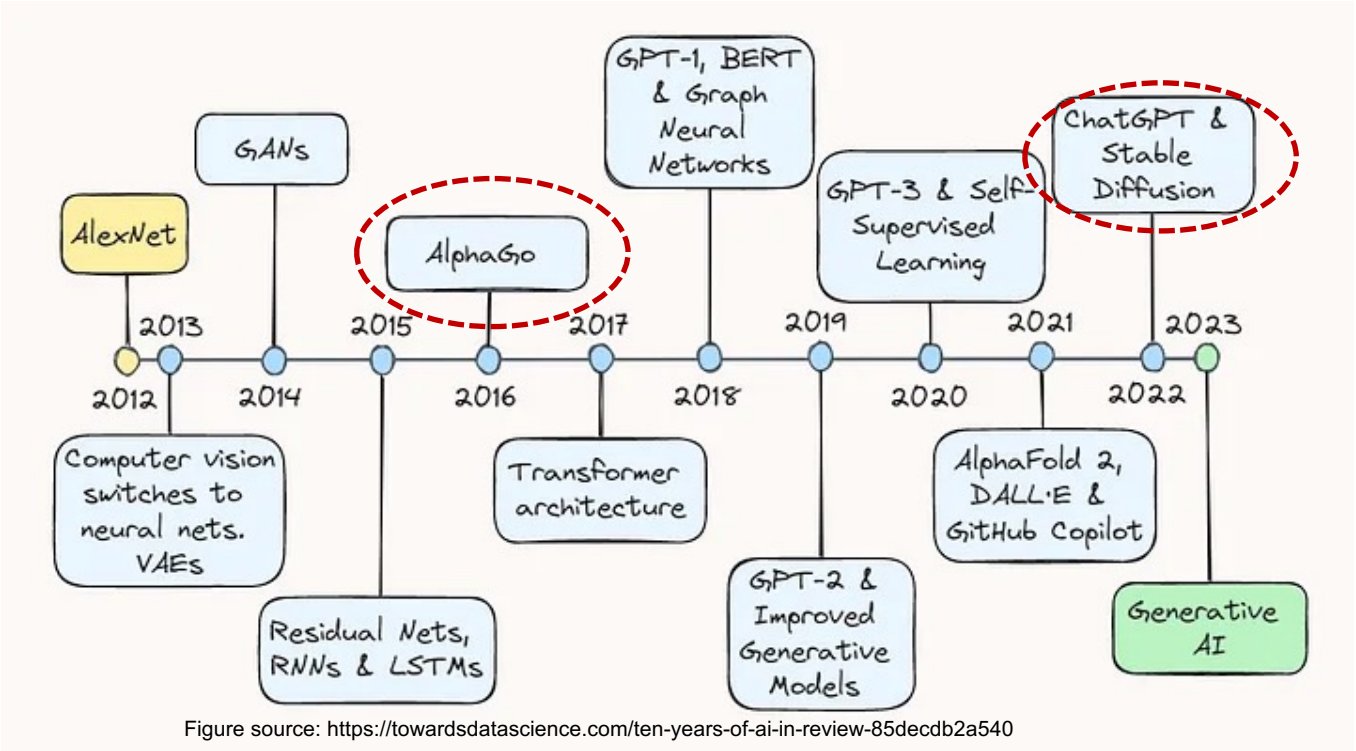


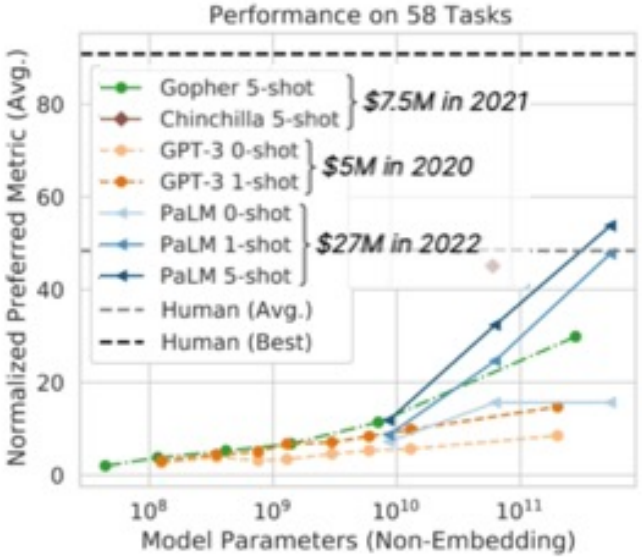
Figure source: https://towardsdatascience.com/ten-years-of-ai-in-review-85decdb2a540

Large Model Scale

Training from scratch becomes infeasible for most researchers:

Limited data + high computational costs

Need prior knowledge transfer!!!!



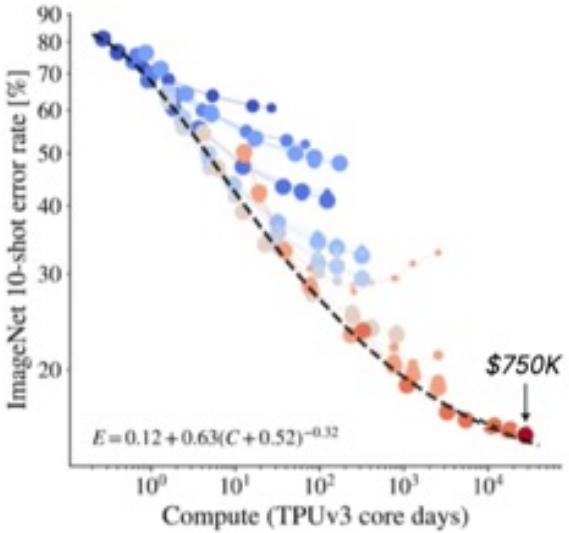


Figure source: http://colinraffel.com/talks/faculty2023collaborative.pdf

Prior Knowledge Transfer has been Ubiquitous

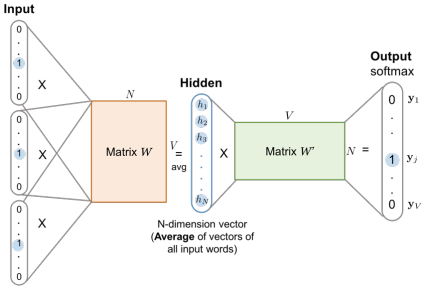
Computer vision downstream tasks:

Finetune on the pre-trained model for ImageNet

NLP downstream tasks:

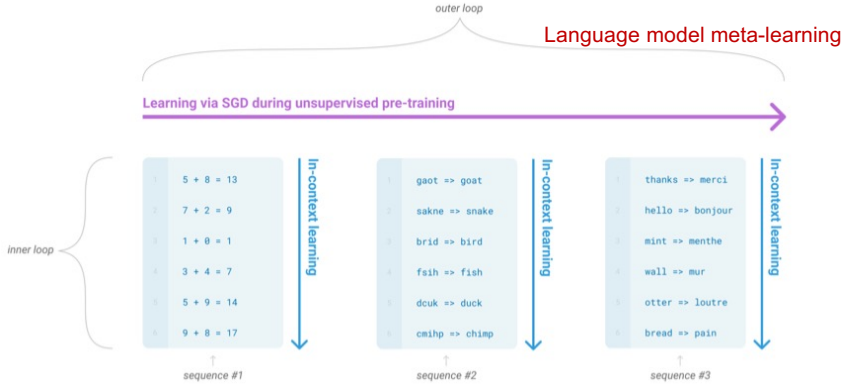
Transfer unsupervised or self-supervised learned feature representation (word embedding)

CBOW

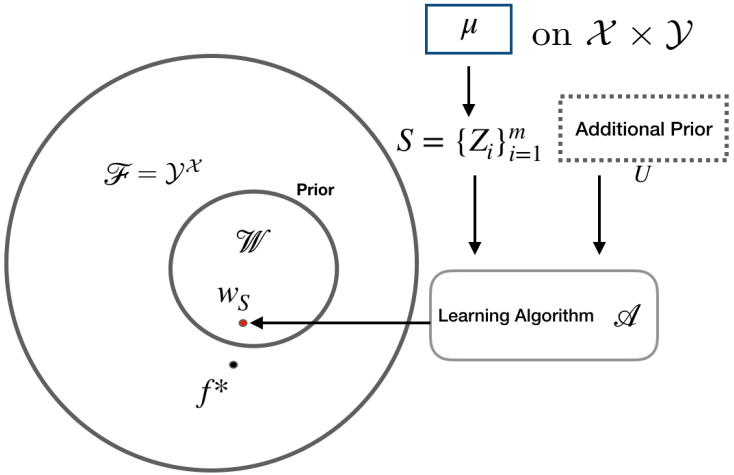


In-context learning:

Transfer knowledge from LLM



Prior Knowledge for Learning A Single Task



$$R_\mu(w) = \mathbb{E}_{Z\sim\mu}\ell(w, Z), R_S(w) = \frac{1}{m}\sum_{i=1}^m \ell(w, Z_i)$$

$$W_S = \mathcal{A}(U, S)$$

- **Objective of Learning**

$$\text{Small } R_\mu(w_S) - R^* = \underbrace{R_\mu(w_S) - \inf_{w\in\mathcal{W}} R_\mu(w)}_{\text{estimation error – complexity}} + \underbrace{\inf_{w\in\mathcal{W}} R_\mu(w) - R^*}_{\text{approximation error – bias}}$$

Prior knowledge:

1. Selection of $\mathcal{W}$
2. Hyperparameters of $\mathcal{A} - U$

Contributions

Theoretical Frameworks for…

- Domain Adaptation

  Algorithm-Dependent Bounds for Representation Learning of Multi-Source Domain Adaptation. AISTATS 2023.

- Meta-learning

  Generalization Bounds for Meta-Learning: An Information-Theoretic Analysis. NeurIPS 2021.

- Continual Meta-learning

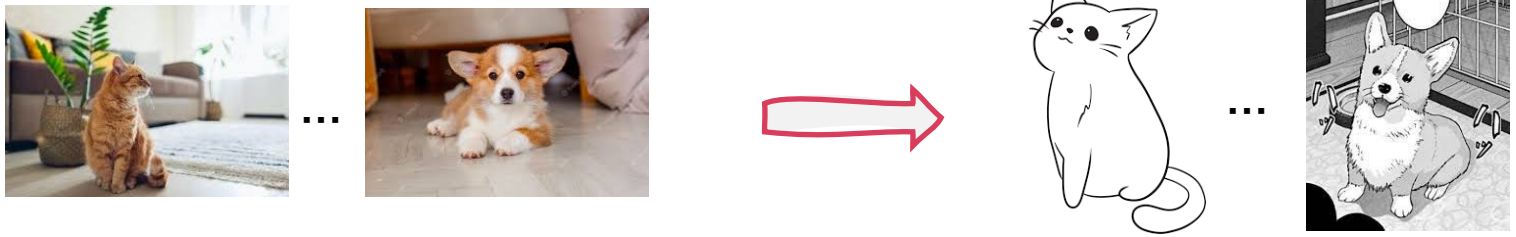
  On the Stability-Plasticity Dilemma in Continual Meta-Learning: Theory and Algorithm. NeurIPS 2023.

Start from a simple prior knowledge transfer concept: Domain Adaptation

Domain Adaptation (DA):

Infer Three Other Related Aspects from One Concept（举一反三）

Source Domain $\mathcal{S}$ $\neq$ $\mathcal{T}$ **Target Domain**



- Supervised DA (SDA)

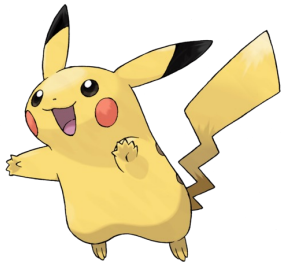
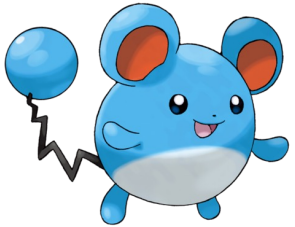
$$S = \{(X_j^s, Y_j^s)\}_{j=1}^{m_s}, (X_j^s, Y_j^s) \sim \mathcal{S} \quad + \quad T = \{(X_j^t, Y_j^t)\}_{j=1}^{m_t}, (X_j^t, Y_j^t) \sim \mathcal{T}$$

- Unsupervised DA (UDA)

$$S = \{(X_j^s, Y_j^s)\}_{j=1}^{m_s}, (X_j^s, Y_j^s) \sim \mathcal{S} \quad + \quad T_X = \{X_j^t\}_{j=1}^{m_t}, X_j^t \sim \mathcal{T}(X)$$

Similarity



 # Q1 How to measure the similarity between domains?

d: $\mathcal{H}$-divergence, f-divergence, IPM, discrepancy …

Conditions for successful DA

 # Q2 Under what conditions can we achieve successful DA?

- Supervised DA

$$R_{\mathcal{T}}(h) \leq \underbrace{R_{\mathcal{S}}(h) + d(\mathcal{S}, \mathcal{T})}$$

All the terms are small: a sufficient condition

- Unsupervised DA

$$R_{\mathcal{T}}(h) \leq \overbrace{R_{\mathcal{S}}(h) + d(\mathcal{S}(X), \mathcal{T}(X)) + \lambda^*} \quad \text{Ideal joint error}$$

Covariate shift $\mathcal{S}(Y|X) = \mathcal{T}(Y|X), \mathcal{S}(X) \neq \mathcal{T}(X)$ $\not\Leftarrow$ $\lambda^* = \min_{h \in \mathcal{H}}[R_{\mathcal{S}}(h) + R_{\mathcal{T}}(h)]$ is small

(necessary in the context of UDA)

$\Updownarrow$

$f_s = f_t$

$f_s \in \mathcal{H}_s, f_t \in \mathcal{H}_t, f_s \neq f_t$

$\not\Rightarrow$

$f_s, f_t \notin \mathcal{H}$

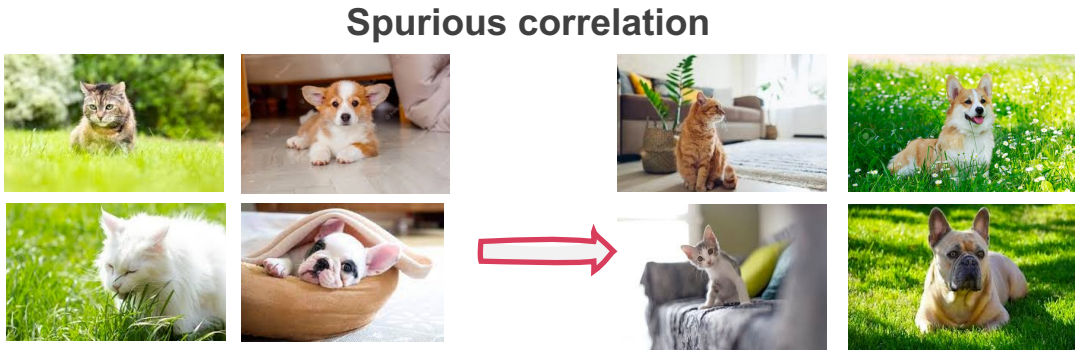
$\Updownarrow$

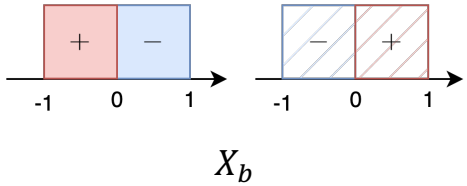
$\mathcal{H}_s \cap \mathcal{H}_t \neq \emptyset$

$\mathcal{H}_s := \{h \in \mathcal{H} : R_{\mathcal{S}}(h) \leq \gamma\}, \mathcal{H}_t := \{h \in \mathcal{H} : R_{\mathcal{T}}(h) \leq \gamma\}$

[1] David, Shai Ben, et al. "Impossibility theorems for domain adaptation." AISTATS 2010.

Invariant Representation Learning





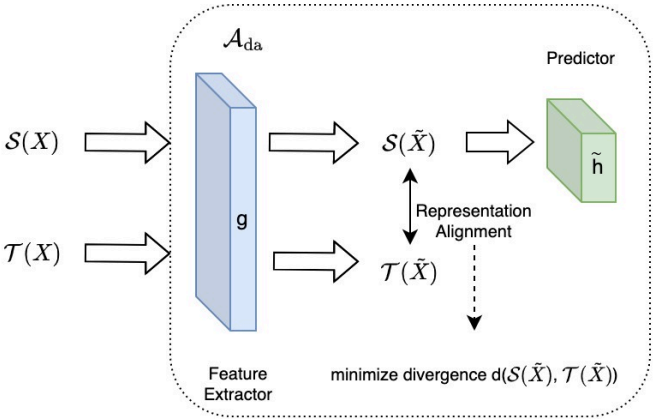
Y={+ (cat), - (dog)} X_b– background, $X_{\setminus b}$ –other features

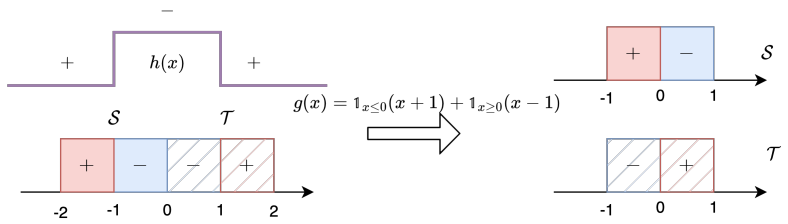
$$\mathcal{X} \xrightarrow{g} \tilde{\mathcal{X}} \xrightarrow{\tilde{h}} \mathcal{Y}$$

$$\exists\, g(X) = g(X_b, X_{\setminus b}) = X_{\setminus b} \quad \text{s.t. } R_{\mathcal{S}}(h) + R_{\mathcal{T}}(h) \leq \gamma, h = \tilde{h} \circ g \Rightarrow \lambda^* \leq \gamma$$

 # Q3 Is representation learning always beneficial for DA? No

Fail of Marginal Representation Alignment





Similar pattern to spurious correlation

After marginal representation alignment,

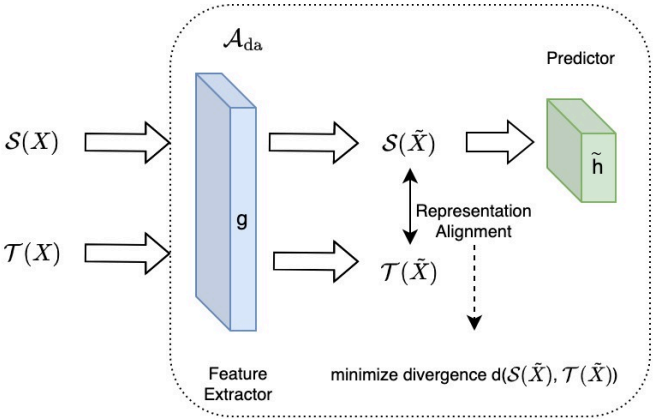
$$\nexists \tilde{h} \circ g \text{ s.t. } R_{\mathcal{T}}(\tilde{h} \circ g) \leq \gamma$$
$$\text{even } R_{\mathcal{S}}(\tilde{h} \circ g) < \gamma, d(\mathcal{S}(\tilde{X}), \mathcal{T}(\tilde{X})) = 0, \lambda^* = 0$$

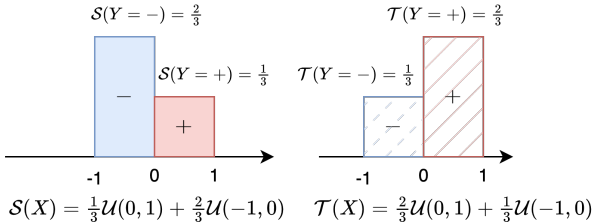
#P1 Previous bounds (e.g. [1-4]) cannot be directly used (No representation analysis) !!!

#P2 Marginal alignment cannot remove spurious correlation!!!

[1] Ben-David, Shai, et al. "A theory of learning from different domains." (*ML 2010*)
[2] Zhao, et al. "On learning invariant representations for domain adaptation." (ICML 2019)
[3] Zhang, Yuchen, et al. "Bridging theory and algorithm for domain adaptation." *(ICML 2019)*
[4] Acuna, David, et al. "f-domain adversarial learning: Theory and algorithms." (ICML 2021)

Fail of Marginal Representation Alignment





$$\mathcal{S}(Y) \neq \mathcal{T}(Y)$$

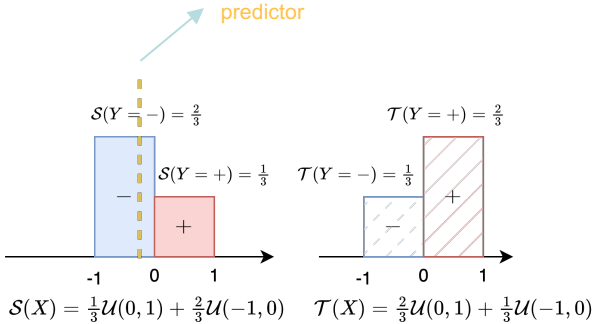
#P3 Under large target shift, $\downarrow d(\mathcal{S}(\tilde{X}), \mathcal{T}(\tilde{X}))$, Negative transfer !!!

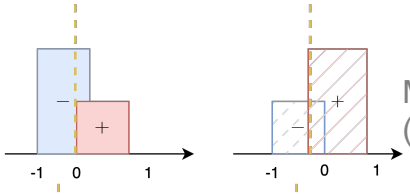
Lower bound ([1,2]):

$$R_{\mathcal{T}}(\tilde{h} \circ g) \geq \frac{1}{2}\left(\sqrt{D_{JS}(\mathcal{S}(Y)\|\mathcal{T}(Y))} - \sqrt{D_{JS}(\mathcal{S}(\tilde{X})\|\mathcal{T}(\tilde{X}))}\right)^2 - R_{\mathcal{S}}(\tilde{h} \circ g) \quad \text{when} \quad \sqrt{D_{JS}(\mathcal{S}(Y)\|\mathcal{T}(Y))} \geq \sqrt{D_{JS}(\mathcal{S}(\tilde{X})\|\mathcal{T}(\tilde{X})}$$

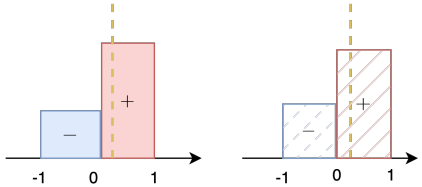
[1] Zhao, et al. "On learning invariant representations for domain adaptation." (ICML 19)
[2] Combes, et al. "Domain adaptation with conditional distribution matching and generalized label shift." (NeurIPS 20)

Joint Representation Alignment





Marginal Alignment
(most previous works)



Label ratio correction ([1, 2])

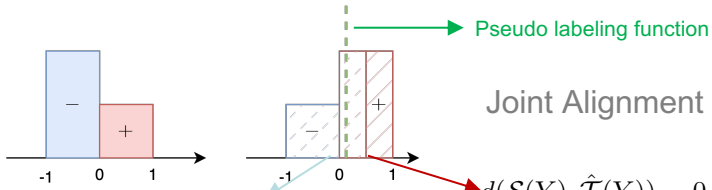
Use $\hat{\mathcal{T}}$ multiple times, unstable for large shift

Assume $\mathcal{S}(X|Y)=\mathcal{T}(X|Y)$, estimate label ratio $r(y)=\frac{\hat{\mathcal{T}}(y)}{\mathcal{S}(y)}$

Minimize $d(\mathcal{S}^r,\hat{\mathcal{T}})$ via optimal transport [1]

Minimize $\sum_y\left[r(y)R_{\mathcal{S}(X,Y=y)}(h)+\hat{\mathcal{T}}(y)d(\mathcal{S}(X|Y=y),\hat{\mathcal{T}}(X|Y=y))\right]$ [2]

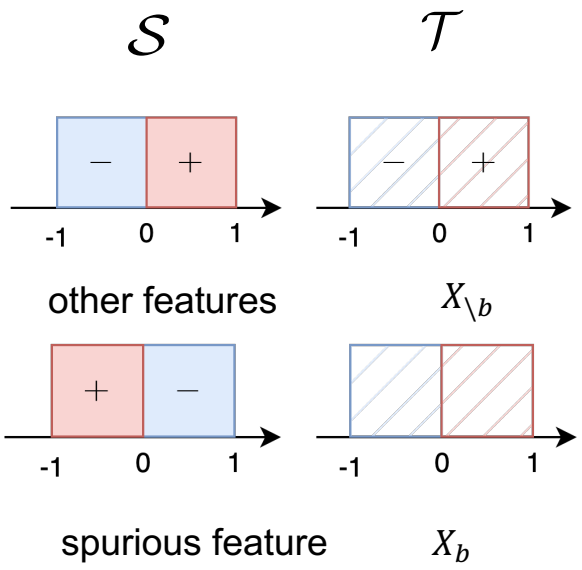
More stable to target shift



Joint Alignment (Ours)

[1] Ievgen Redko et al. “Optimal transport for multi-source domain adaptation under target shift”. (AISTATS 19)
[2] Changjian Shui et al. “Aggregating from multiple target-shifted sources”. In: International Conference on Machine Learning.” (ICML 21)

Joint Representation Alignment



$$d(\mathcal{S}(X_{\setminus b}, Y), \hat{\mathcal{T}}(X_{\setminus b}, Y)) = d(\mathcal{S}(X_{\setminus b}, Y), \mathcal{T}(X_{\setminus b}, Y)) = 0$$

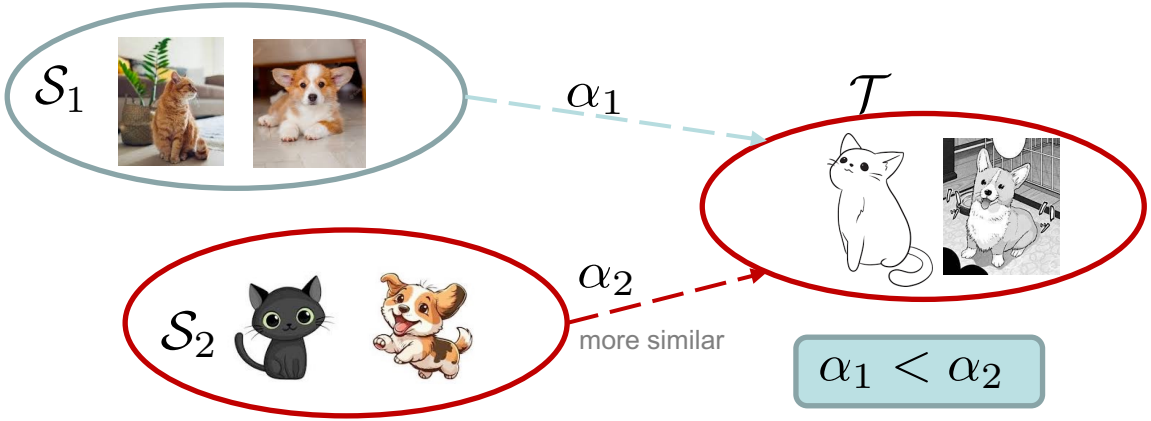
Pseudo label is credible

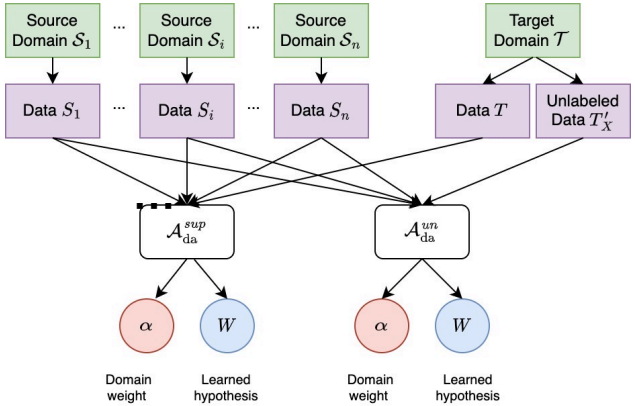
$$d(\mathcal{S}(X_b), \mathcal{T}(X_b)) = 0$$
$$d(\mathcal{S}(X_b, Y), \hat{\mathcal{T}}(X_b, Y)) \text{ is large}$$

Joint alignment can remove spurious feature

Multi-source Domain Adaptation (MDA)

Select most relevant sources via similarity





Combined source distribution:

$\mathcal{S}^{\boldsymbol{\alpha}} := \sum_{i=1}^{N} \alpha_i \mathcal{S}_i,\ \Delta_N := \{\boldsymbol{\alpha} : \alpha_i \geq 0, \sum_{i=1}^{N} \alpha_i = 1\}$

Target and pseudo target distributions:

$\mathcal{T}$ and $\hat{\mathcal{T}}$

Joint alignment for MDA $\min_{\boldsymbol{\alpha}} d(\mathcal{S}^{\boldsymbol{\alpha}}, \hat{\mathcal{T}})$

Algorithm-Dependent Bounds for Representation Learning of Multi-Source Domain Adaptation. AISTATS 2023.

Previous works:

$\sum_{i=1}^{N} \alpha_i \left[R_{\mathcal{S}_i}(h) + d(\mathcal{S}_i(X), \mathcal{T}(X))\right]$ Zhao (2018), Wen (2020) …

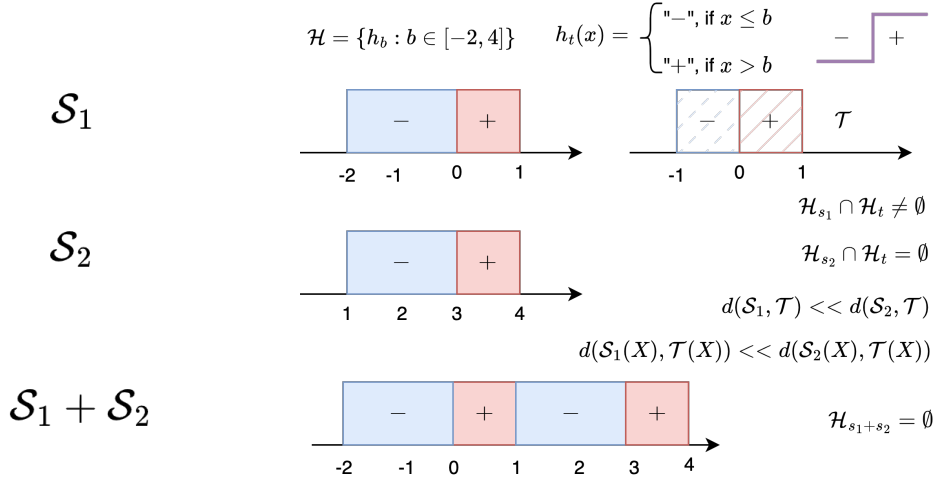
$\sum_{i=1}^{N} \alpha_i \left[\sum_y r_i(y) R_{\mathcal{S}_i(X,Y=y)}(h) + \hat{\mathcal{T}}(y) d(\mathcal{S}_i(X|Y=y), \mathcal{T}(X|Y=y))\right]$ Shui (2021)

Multi-source Domain Adaptation (MDA)

 # Q4 Why is domain weight relevant for MDA?

 # Q5 Why not adapt from an equally mixed source?

Let $\mathcal{H}_{s_1} := \{h \in \mathcal{H} : R_{\mathcal{S}_1}(h) \leq \gamma\}, \mathcal{H}_{s_2} := \{h \in \mathcal{H} : R_{\mathcal{S}_2}(h) \leq \gamma\}$
and $\mathcal{H}_t := \{h \in \mathcal{H} : R_{\mathcal{T}}(h) \leq \gamma\}$



Successful adaptation with

$$\alpha_1 = 1, \alpha_2 = 0$$

We can also construct

$\mathcal{H}_{s_1} \cap \mathcal{H}_t \neq \emptyset$, $\mathcal{H}_{s_2} \cap \mathcal{H}_t \neq \emptyset$
However, $\mathcal{H}_{s_1} \cap \mathcal{H}_{s_2} \cap \mathcal{H}_t = \emptyset$

Theoretical Results (Simplified): Unsupervised MDA

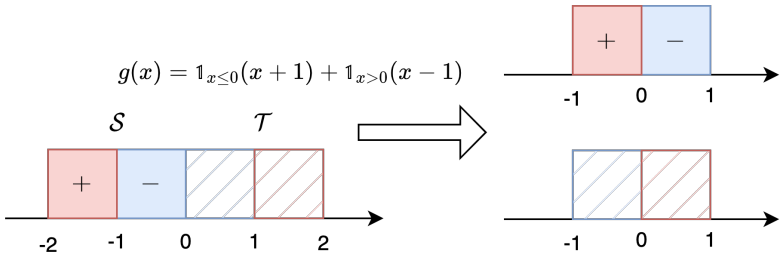
$h \circ g$ pseudo labeling function (parametrized by u, v)

$$R_{\mathcal{T}}(u,v) \leq \mathbf{W}_1(\tilde{\mathcal{T}}_{u,v}, \tilde{\mathcal{S}}^{\boldsymbol{\alpha}}_u) + R^*_{rep}(u,v) + R^*$$

$$R^* = R_{\mathcal{S}^{\boldsymbol{\alpha}}}(u^*, v^*) + R_{\mathcal{T}}(u^*, v^*)$$
$$u^*, v^* = \arg\min_{u,v} R_{\mathcal{S}^{\boldsymbol{\alpha}}}(u,v) + R_{\mathcal{T}}(u,v)$$

$$R^*_{rep}(u,v) \overset{\text{def}}{=} LM \int_{\mathcal{Z}\times\mathcal{Z}} \left[\rho_{\tilde{x}}(g(u^*,x), g(u^*,x')) - \rho_{\tilde{x}}(g(u,x), g(u,x'))\right] d\gamma^*_{u,v}(\hat{z}, z')$$

Lipschitz constants



$$g^*(x) = \mathbb{1}_{x\leq 0}(x+1) + \mathbb{1}_{x>0}(1-x)$$

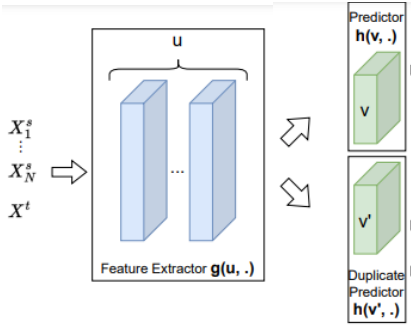
$$h^*(x) = \begin{cases} \text{"+", if } x \leq 0 \\ \text{"−", if } x > 0 \end{cases}$$

$$R_{\mathcal{T}}(h^* \circ g^*) = 0 = R^*_{rep}(h^* \circ g^*)$$

$$R_{\mathcal{T}}(h^* \circ g) = 1 \leq R^*_{rep}(h^* \circ g) \leq LM$$

$R^* = 0, W_1 = 0$

Theoretical Results (Simplified): Unsupervised MDA



$$\hat{R}_{\mathcal{S}^{\boldsymbol{\alpha}}}(u, v') \overset{\text{def}}{=} \sum_{i=1}^{N} \frac{\alpha_i}{m_i} \sum_{j=1}^{m_i} \ell(h(v', g(u, X_{i,j}^s)), Y_{i,j}^s)$$

$$\hat{R}_{\mathcal{T}_{u,v}}(u, v, v') \overset{\text{def}}{=} \frac{1}{m_t} \sum_{j=1}^{m_t} \ell(h(v', g(u, X_j^t)), h(v, g(u, X_j^t)))$$

Disagreement

$$U, V = \mathcal{A}_{un}(S^{\boldsymbol{\alpha}}, T_X) \quad \boldsymbol{\alpha} \text{ will be optimized}$$

KR duality

$$\max_{v'} [\hat{R}_{\mathcal{T}_{u,v}}(u, v, v') - \hat{R}_{\mathcal{S}^{\boldsymbol{\alpha}}}(u, v')]$$

$$D_{KL}(P_{U,V|S^{\boldsymbol{\alpha}},T'_X} \| P_{U,V})$$

Generalization gap

Expected target true risk

$$\mathbb{E}_{U,V,S^{\boldsymbol{\alpha}},T}[R_{\mathcal{T}}(U,V)] \leq \mathbb{E}_{U,V,S^{\boldsymbol{\alpha}},T}\left[\hat{\mathbf{W}}_1(\tilde{\mathcal{T}}_{U,V}, \tilde{\mathcal{S}}_U^{\boldsymbol{\alpha}})\right] + \sqrt{2\sigma'^2\left(\sum_{i=1}^{N} \frac{\alpha_i^2}{m_i} + \frac{1}{m'_t}\right) I(U,V; S^{\boldsymbol{\alpha}}, T'_X)} + \mathbb{E}_{U,V} R^*_{rep}(U,V) + R^*$$

- Mitigate target shift #P3
- Remove spurious correlation #P2
- Generalization for Few-shot Adaptation
  - Equal importance $\alpha_i = \frac{1}{N}$, sample size $m_i = m$
  - $\frac{1}{m'_t} \ll \sum_{i=1}^{N} \frac{\alpha_i^2}{m_i}$, $\mathcal{O}(\sqrt{\sum_{i=1}^{N} \frac{\alpha_i^2}{m_i}}) = \mathcal{O}(\frac{1}{\sqrt{Nm}})$
  - few-shot adaptation with $N \to \infty$
- Theoretic gap: suboptimal representation #P1

IMDA Algorithm (part for unsupervised MDA)

1. fix $\boldsymbol{\alpha}$, $\min_{u,v} \max_{v'} [\hat{R}_{\mathcal{T}_{u,v}}(u, v, v') - \hat{R}_{\mathcal{S}^{\boldsymbol{\alpha}}}(u, v')]$

2. fix u, v optimize the domain weights $\boldsymbol{\alpha}$, $\forall C_0, C_1 > 0$:

$$\min_{\boldsymbol{\alpha}} \left(C_0 \hat{R}_{\mathcal{S}^{\boldsymbol{\alpha}}}(u, v) - \hat{R}_{\mathcal{S}^{\boldsymbol{\alpha}}}(u, v') + C_1 \sqrt{\delta_u + \delta_v} R(\boldsymbol{\alpha}) \right) ,$$

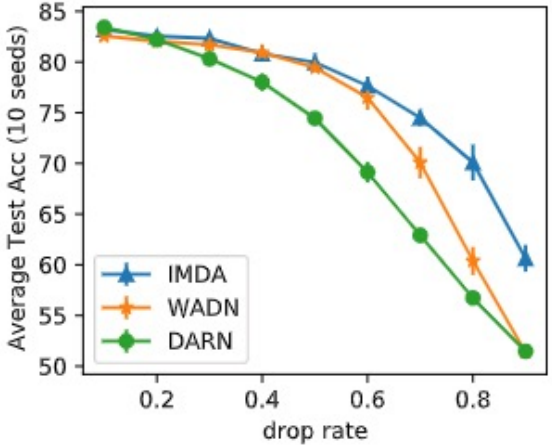
$$R(\boldsymbol{\alpha}) = \sqrt{\sum_{i=1}^{N} \frac{\alpha_i^2}{m_i}}, \text{ s.t.} \forall i \in [N], \alpha_i \geq 0, \sum_{i=1}^{N} \alpha_i = 1 ,$$

Accumulate gradient norm for u, v
(gradient norm estimation of mutual information term for SGLD, omitted in presentation)

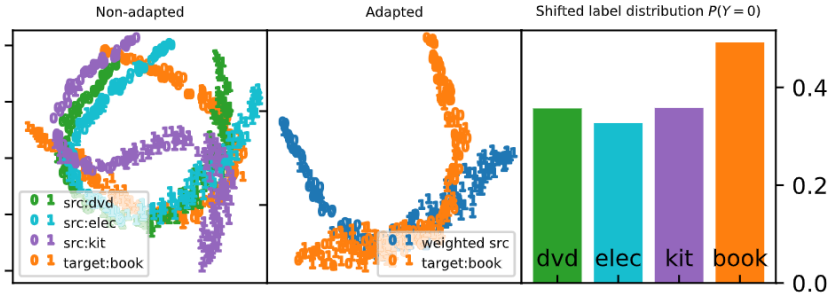
Approximation of $R_{\mathcal{S}^{\boldsymbol{\alpha}}}(u^*, v^*)$

Experimental Results (Amazon review)

IMDA(Ours, joint alignment)
WADN(label ratio correction)
DARN(marginal alignment)



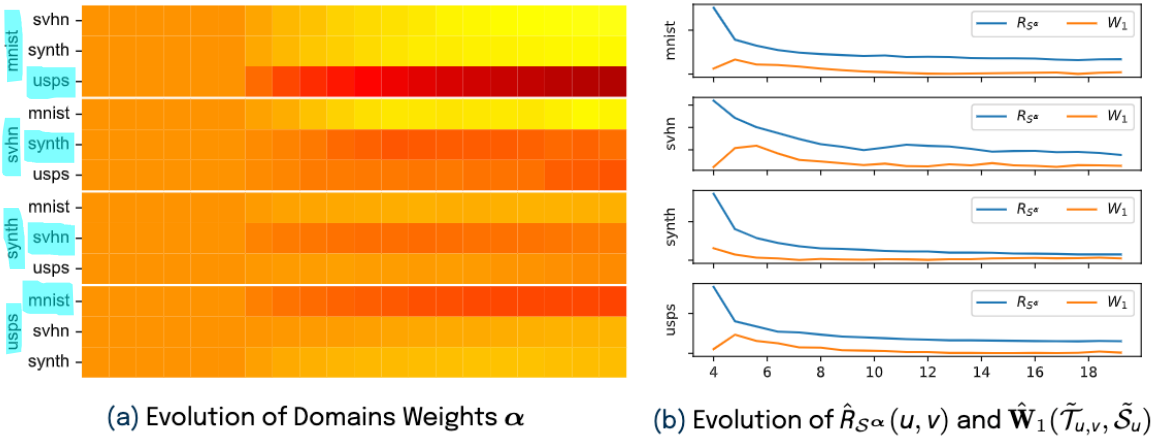
Randomly drop (p)% negative reviews (Y=0) in source domain



T-SNE, drop rate p=0.5

$\mathcal{Y}$={0, 1}

Experimental Results (Digits)



(a) Evolution of Domains Weights α

(b) Evolution of $\hat{R}_{S^\alpha}(u, v)$ and $\hat{\mathbf{W}}_1(\tilde{\mathcal{T}}_{u,v}, \tilde{\mathcal{S}}_u)$

Summary of Contribution

- Unified approach for both supervised and unsupervised MDA
- Formal theoretical analysis on representation learning of MDA

  In contrast to [1], we provide explicit bounds.
- Mitigate target shift through joint alignment
- Memory efficient algorithm

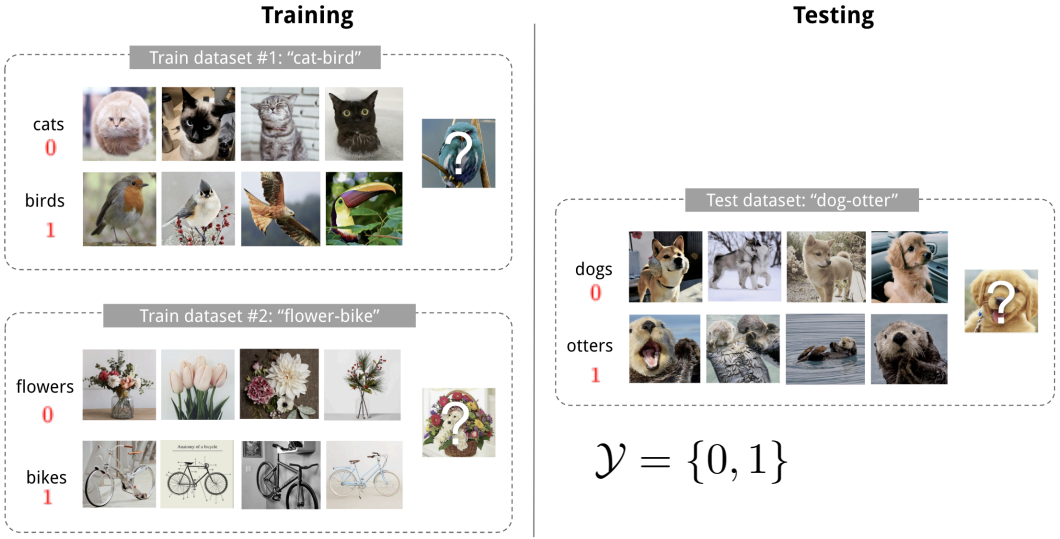
  1 duplicate predictor VS N domain discriminator and $N|\mathcal{Y}|$ centroids [2]
- Fully algorithm-dependent bounds via information-theoretic learning

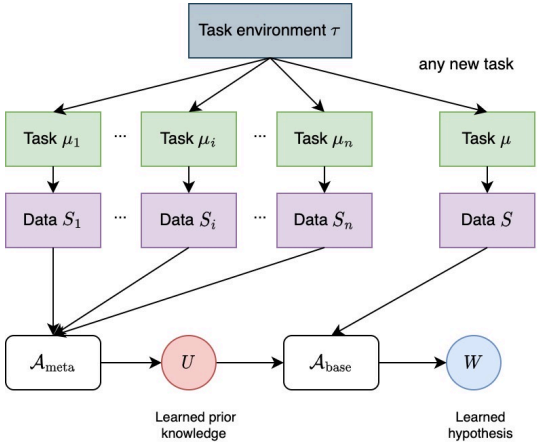
  The bounds in [3,4,5] contain non-optimizable KL divergence, where the generalization gap of KL is not algorithm-dependent.

[1] Zhao, et al. "On learning invariant representations for domain adaptation." (ICML 2019)
[2] Changjian Shui et al. “Aggregating from multiple target-shifted sources”. In: International Conference on Machine Learning.” (ICML 21)
[3] Wu, Xuetong, Jonathan H Manton, Uwe Aickelin, and Jingge Zhu. “An Information-Theoretic Analysis for Transfer Learning: Error Bounds and Applications” [2022]
[4] Wu, Xuetong, Jonathan H Manton, Uwe Aickelin, and Jingge Zhu. “Information-theoretic analysis for transfer learning”. [ISIT 2020]
[5] Wang, Ziqiao, and Yongyi Mao. "Information-Theoretic Analysis of Unsupervised Domain Adaptation." [ICLR 2023]

Closer to real intelligence: Learning to Learn (Meta-Learning)

Meta-learning / Learning to Learn





Problem formulation

Figure source: https://lilianweng.github.io/posts/2018-11-30-meta-learning/

Learn higher level knowledge: how to classify objects

Joint-training

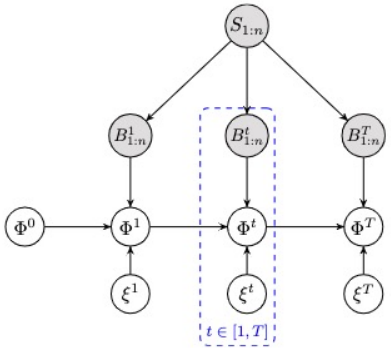
- Meta learner $\mathcal{A}_{\text{meta}}$: $U = \mathcal{A}_{\text{meta}}(S_{1:n}) \sim P_{U|S_{1:n}}$
- Base learner $\mathcal{A}_{\text{base}}$: $W = \mathcal{A}_{\text{base}}(U, S) \sim P_{W|U,S}$

True meta risk:

$$R_\tau(U) \stackrel{\text{def}}{=} \mathbb{E}_{S\sim\mu_{m,\tau}}\mathbb{E}_{W\sim P_{W|S,U}}[R_\mu(W)] = \mathbb{E}_{\mu\sim\tau}\mathbb{E}_{S|\mu\sim\mu^m}\mathbb{E}_{W\sim P_{W|S,U}}[R_\mu(W)]$$

Empirical meta risk:

$$R_{S_{1:n}}(U) \stackrel{\text{def}}{=} \frac{1}{n}\sum_{i=1}^{n}\mathbb{E}_{W_i\sim P_{W_i|S_i,U}}[R_{S_i}(W_i)]$$



Jointly optimize $\Phi = (U, W_{1:n})$ [1]

Meta generalization gap:

$$|\text{gen}^{\text{joi}}_{\text{meta}}(\tau, \mathcal{A}_{\text{meta}}, \mathcal{A}_{\text{base}})| \stackrel{\text{def}}{=} \mathbb{E}_{U,S_{1:n}}[R_\tau(U) - R_{S_{1:n}}(U)] \quad \text{(environment-level uncertainty)}$$

$$\leq \sqrt{\frac{2\sigma^2}{nm} I(U, W_{1:n}; S_{1:n})} \leq \underbrace{\sqrt{\frac{2\sigma^2}{mn} I(U; S_{1:n})}}_{T_1} + \underbrace{\sqrt{\frac{2\sigma^2}{mn}\sum_{i=1}^{n} I(W_i; S_i|U)}}_{T_2} \quad \text{(task-level uncertainty)}$$

Assume bounded MI, improved rate over [1]

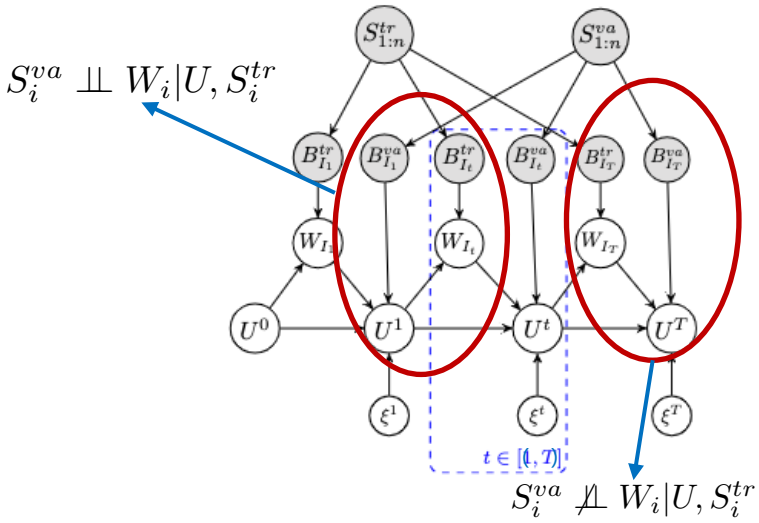
$$\mathcal{O}\left(\frac{1}{\sqrt{nm}} + \frac{1}{\sqrt{m}}\right) \text{ VS. } \mathcal{O}\left(\frac{1}{\sqrt{n}} + \frac{1}{\sqrt{m}}\right)$$

- Few-shot learning $n \to \infty$, m small: $T_1 \to 0$, $T_2 > 0$ Biased estimation, consistent with [2]
- No need of transfer $m \to \infty$, n is finite: $T_1 \to 0$, $T_2 \to 0$

[1] Amit, Ron and Ron Meir. "Meta-learning by adjusting priors based on extended PAC-Bayes theory" (ICML 2018)
[2] Bai, Yu, Minshuo Chen, Pan Zhou, Tuo Zhao, Jason Lee, Sham Kakade, Huan Wang, and Caiming Xiong. "How Important is the Train-Validation Split in Meta-Learning?" (ICML 2021)

Alternate-training

$S_{1:n} = (S^{tr}_{1:n}, S^{va}_{1:n})$, $S^{tr}_i \perp\!\!\!\perp S^{va}_i$, $m = m_{tr} + m_{va}$



- Meta learner $\mathcal{A}_{\text{meta}}$: $U = \mathcal{A}_{\text{meta}}(S_{1:n}) \sim P_{U|S_{1:n}}$
- Base learner $\mathcal{A}_{\text{base}}$: $W = \mathcal{A}_{\text{base}}(U, S^{tr}) \sim P_{W|U,S^{tr}}$

True meta risk:

$$R_\tau(U) \overset{\text{def}}{=} \mathbb{E}_{S\sim\mu_{m,\tau}} \mathbb{E}_{W\sim P_{W|S^{tr},U}}[R_\mu(W)] = \mathbb{E}_{\mu\sim\tau} \mathbb{E}_{S|\mu\sim\mu^m} \mathbb{E}_{W\sim P_{W|S^{tr},U}}[R_\mu(W)]$$

Empirical meta risk:

$$\tilde{R}_{S_{1:n}}(U) \overset{\text{def}}{=} \frac{1}{n}\sum_{i=1}^{n} \mathbb{E}_{W_i\sim P_{W_i|S^{tr}_i,U}}[R_{S^{va}_i}(W_i)]$$

Meta generalization gap:

$$\text{gen}^{\text{alt}}_{\text{meta}}(\tau, \mathcal{A}_{\text{meta}}, \mathcal{A}_{\text{base}}) \overset{\text{def}}{=} \mathbb{E}_{U,S_{1:n}}[R_\tau(U) - \tilde{R}_{S_{1:n}}(U)] \leq \sqrt{\frac{2\sigma^2}{nm^{va}} \underbrace{I(U, W_{1:n}; S^{va}_{1:n} | S^{tr}_{1:n})}}$$

$$\leq \underbrace{\mathbb{E}_{S_{1:n}} D_{\text{KL}}(P_{U|S_{1:n}} || P_{U|S^{\text{tr}}_{1:n}})}_{T_1} + \underbrace{\mathbb{E}_{U,S_{1:n}} \sum_{i=1}^{n} D_{\text{KL}}(P_{W_i|S_i,U} || P_{W_i|S^{\text{tr}}_i,U})}_{T_2}$$

environment-level uncertainty

task-level uncertainty

Train-validation split trade-off, $\uparrow m_{va}$ <=>$\downarrow m_{tr}$:

- $\downarrow \frac{2\sigma^2}{nm_{va}}$
- $\uparrow D_{\text{KL}}(P_{W_i|S_i,U} || P_{W_i|S^{\text{tr}}_i,U})$ Similar form to chaser loss [1]
- $\uparrow D_{\text{KL}}(P_{U|S_{1:n}} || P_{U|S^{\text{tr}}_{1:n}})$

=0 if S^{va}_i has been used to update U

[1] Yoon, Jaesik, Taesup Kim, Ousmane Dia, Sungwoong Kim, Yoshua Bengio, and Sungjin Ahn. “Bayesian model-agnostic meta-learning”. (NeurIPS 2018)

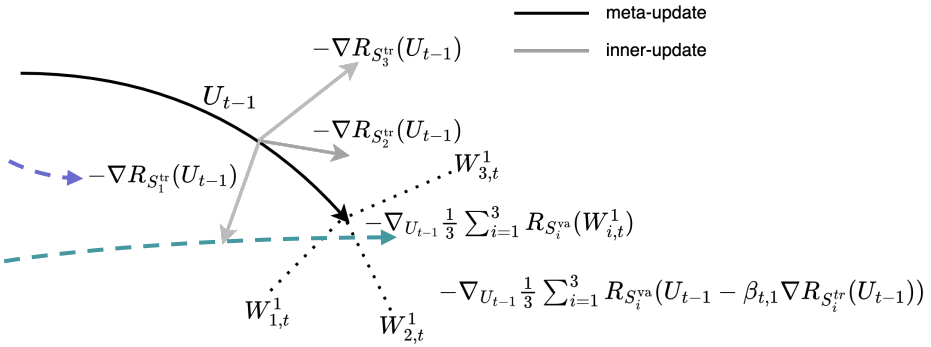
Non-vacuous bound for few-shot learning

Inner update:

$$W_{i,t}^0 = U^{t-1}$$
$$W_{i,t}^k = W_{i,t}^{k-1} - \beta_{t,k}\nabla R_{B_{i,t,k}^{\mathrm{tr}}}(W_{i,t}^{k-1}) + \zeta^{t,k}, \zeta^{t,k} \sim \mathcal{N}(\mathbf{0}, \tfrac{2\beta_{t,k}}{\gamma_{t,k}}\mathbb{I}_d) \quad \texttt{-- SGLD}$$

meta update:

$$U^t = U^{t-1} - \eta_t \nabla \tilde{R}_{B_{I_t}^{\mathrm{va}}}(U^{t-1}) + \xi_t, \xi^t \sim \mathcal{N}(\mathbf{0}, \tfrac{2\eta_t}{\gamma_t}\mathbb{I}_d)$$



MAML + noise -> meta-SGLD

Bound through Gradient Estimation:

$$|\mathrm{gen}_{\mathrm{meta}}^{\mathrm{alt}}(\tau, \mathrm{SGLD}, \mathrm{SGLD})| \le \sqrt{\frac{2\sigma^2 I(U, W_{1:n}; S_{1:n}^{va} | S_{1:n}^{tr})}{n m_{\mathrm{va}}}} \le \frac{\sigma}{\sqrt{n m_{\mathrm{va}}}} \sqrt{\sum_{t=1}^{T} \mathbb{E}_{B_{I_t}^{va}, B_{I_t}^{tr}, W_{I_t}, U^{t-1}} \frac{\eta_t \gamma_t \|\epsilon_t^u\|_2^2}{2} + \sum_{t=1}^{T}\sum_{i=1}^{|I_t|}\sum_{k=1}^{K} \mathbb{E}_{B_{i,t,k}^{va}, B_{i,t,k}^{tr}, W_{i,t}^{k-1}} \frac{\beta_{t,k}\gamma_{t,k}\|\epsilon_{t,i,k}^w\|_2^2}{2}}$$

Train + validation

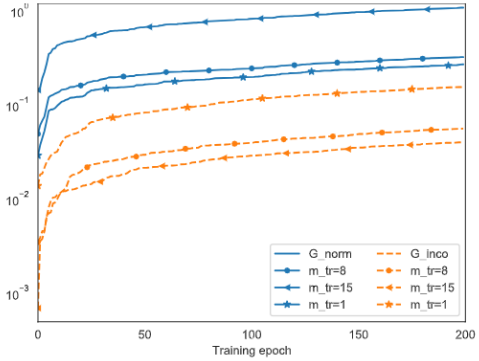
Gradient incoherence:

$$\epsilon_t^u \overset{\mathrm{def}}{=} \nabla \tilde{R}_{B_{I_t}}(U^{t-1}) - \nabla \tilde{R}_{B_{I_t}^{\mathrm{tr}}}(U^{t-1}) \qquad \epsilon_{t,i,k}^w \overset{\mathrm{def}}{=} \nabla R_{B_{i,t,k}}(W_{i,t}^{k-1}) - \nabla R_{B_{i,t,k}^{\mathrm{tr}}}(W_{i,t}^{k-1})$$

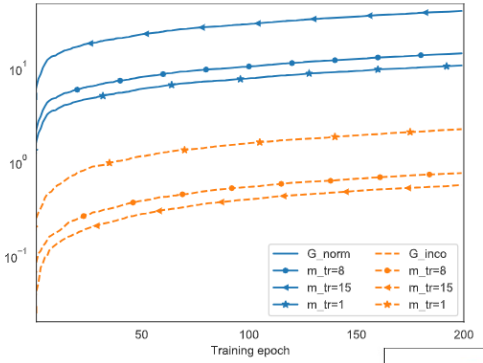
Gradient norm:

$$\epsilon_t^u = \nabla \tilde{R}_{B_{I_t}^{\mathrm{va}}}(U^{t-1}) \qquad \epsilon_{t,i,k}^w = \nabla R_{B_{i,t,k}^{\mathrm{tr}}}(W_{i,t}^{k-1})$$

Non-vacuous bound for few-shot learning



(a) Bound of U



(b) Bound of W

← 2D Gaussian

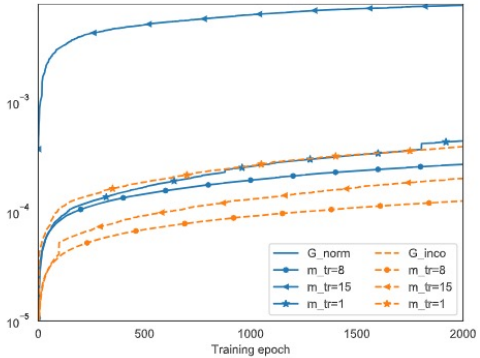
Consistent with [1]

- $m_{\text{tr}} = 15$: tightest bound, largest G_norm/ G_inco gap, optimal split
- $m_{\text{tr}} = 1$ (1-shot learning): poorest estimation, largest G_inco bound,

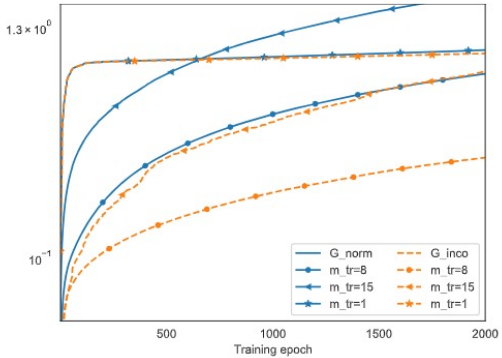
$$m_{tr} + m_{va} = 16$$

Omniglot →

- $m_{\text{tr}} = 15$, G_norm $\gg$ G_inco
- $m_{\text{tr}} = 8$, tightest bound among three settings, optimal split
- $m_{\text{tr}} = 1$, G_norm $\approx$ G_inco



(a) Bound of U



(b) Bound of W

[1] Denevi, Giulia, Carlo Ciliberto, Dimitris Stamos, and Massimiliano Pontil. “Learning to learn around a common mean”. (NeurIPS 2018)

Summary of Contribution

- Unified theoretical analysis
  - analysis for both joint training and alternate training
  - interpretable theoretical results
- Flexible bounds
  - data-dependent, algorithm-dependent
  - valid for non-convex loss, not constrained in linear models
- Non-vacuous bound for gradient-based few-shot learning
  - orders of magnitude tighter in most situations
  - empirically validated the train-validation split trade-off

A More Challenging Scenario: Continual Meta-Learning

The World is Changing…

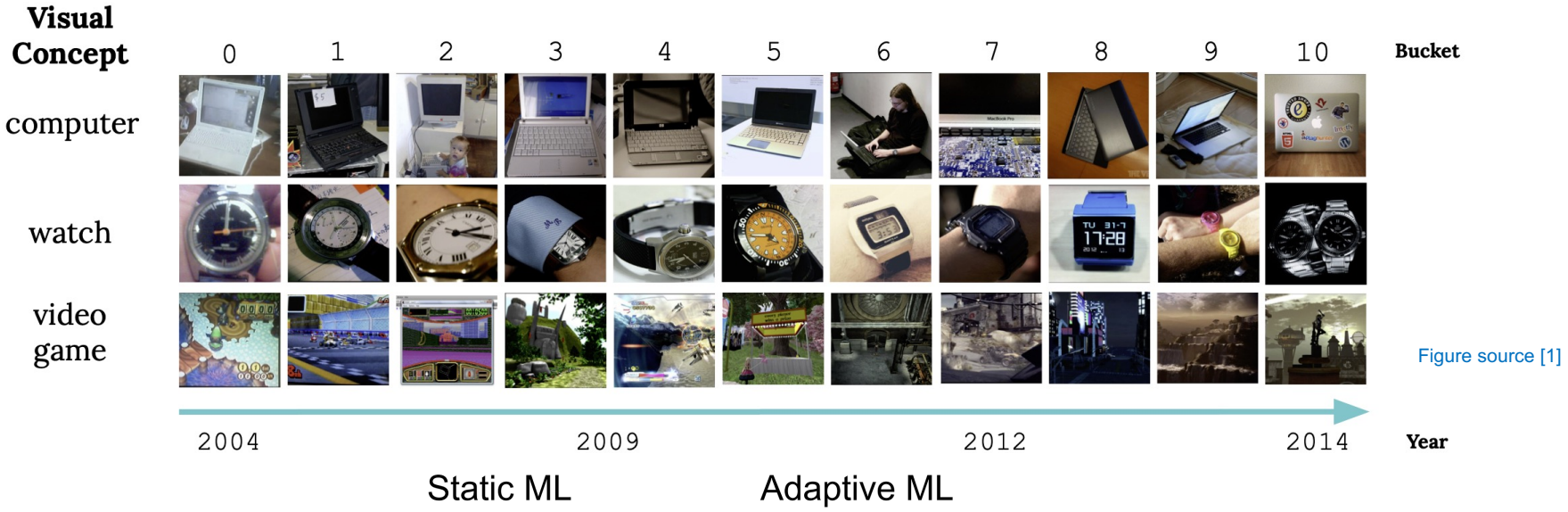


Figure source [1]

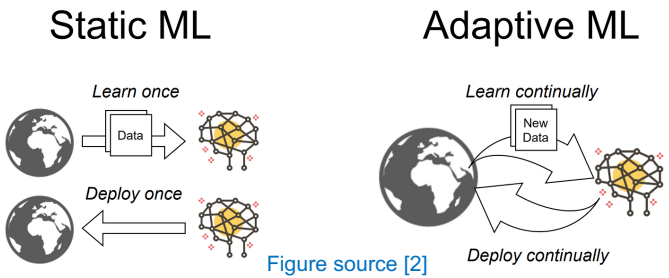
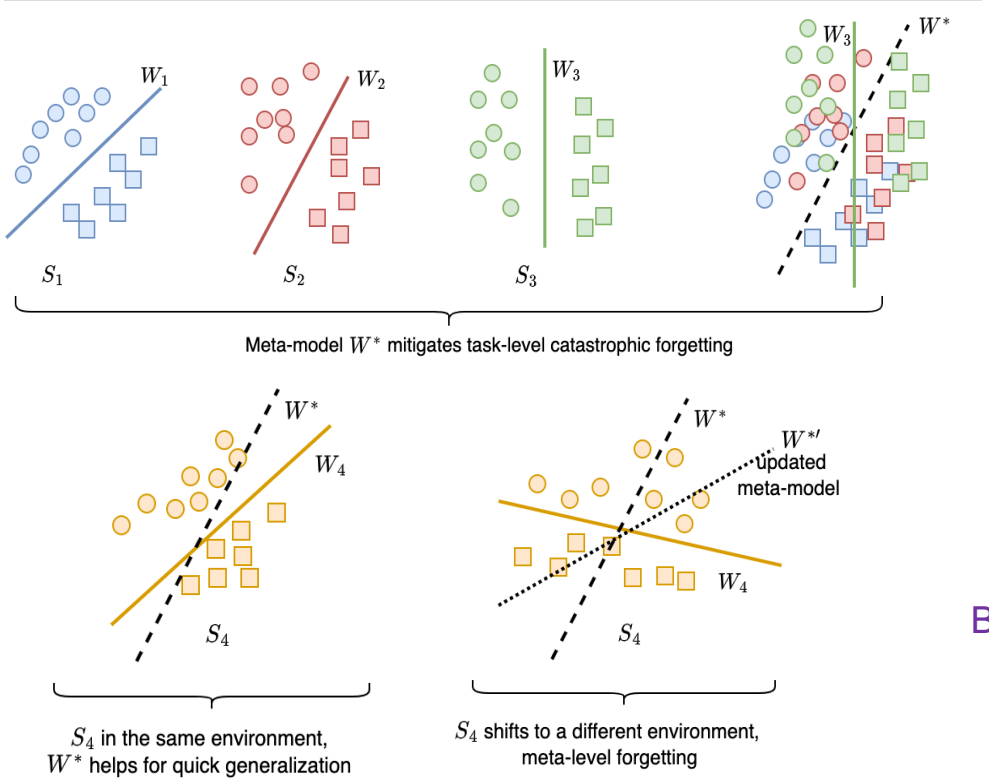


Figure source [2]

[1] Lin, Zhiqiu, et al. "The clear benchmark: Continual learning on real-world imagery." NeurIPS dataset (2021).
[2] https://ai.kuleuven.be/stories/post/2021-05-10-continual-learning/

Bi-level Learning-forgetting Trade-off

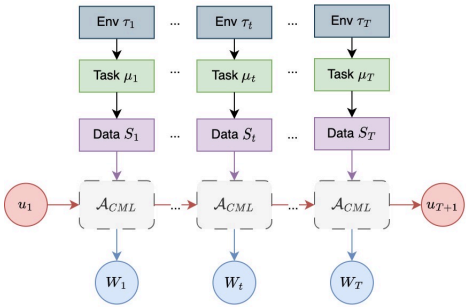


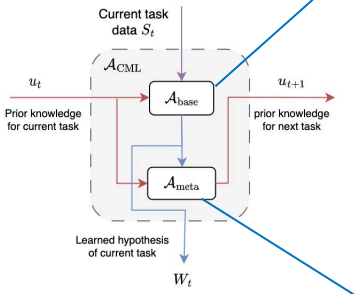
Catastrophic forgetting
- negative backward transfer

Quick generalization
- positive forward transfer

Bi-level learning-forgetting trade-off…

Continual Meta-Learning (CML)





Base learner – batch learning algorithms, $W_t = \mathcal{A}(u_t, S_t), u_t = (\beta_t, \phi_t)$

- Excess Risk:

$$R_{\text{excess}}(\mathcal{A}, u_t) \stackrel{\text{def}}{=} \mathbb{E}_{S_t}\mathbb{E}_{W_t \sim P_{W_t|S_t,u_t}}\left[\mathcal{L}_{\mu_t}(W_t) - \mathcal{L}_{\mu_t}(w_t^*)\right], w_t^* = \arg\min_{w\in\mathcal{W}} \mathcal{L}_{\mu_t}(w)$$

- Assume $\mathcal{L}_{\mu_t}(\cdot)$ has quadratic growth => unified excess risk upper bound:

$$f_t(u_t) = \kappa_t\Big(a\beta_t + \frac{b\|\phi_t - w_t\|^2 + \epsilon_t + \epsilon_0}{\beta_t} + \Delta_t\Big), \kappa_t, \epsilon_t, \beta_t, \Delta_t, a, b, \epsilon_0 \in \mathbb{R}^+, \forall t \in [T]$$

- f_t is **convex**, valide for $\mathcal{A} \in \{\text{Gibbs, RLM, SGD, SGLD}\}$

Meta learner — online learning algorithms

- dynamic regret for N static slots

$$R_T^{\text{dynamic}}(u_{1:N}^*) \stackrel{\text{def}}{=} \sum_{n=1}^{N}\sum_{k=1}^{M_n}\left[f_{n,k}(u_{n,k}) - f_{n,k}(u_n^*)\right], u_n^* \stackrel{\text{def}}{=} \arg\min_u \frac{1}{M_n}\sum_{k=1}^{M_n} f_{n,k}(u)$$

Continual meta-learning objective

— select $u_{1:T}$ to minimize the Average Excess Risk (AER):

$$\text{AER}_{\mathcal{A}}^T \stackrel{\text{def}}{=} \frac{1}{T}\sum_{t=1}^{T} R_{\text{excess}}(\mathcal{A}, u_t) \le \frac{1}{T} R_T^{\text{dynamic}}(u_{1:N}^*) + \frac{1}{T}\sum_{n=1}^{N}\sum_{k=1}^{M_n} f_{n,k}(u_n^*).$$

Dynamic CML (DCML)

K-step SGD

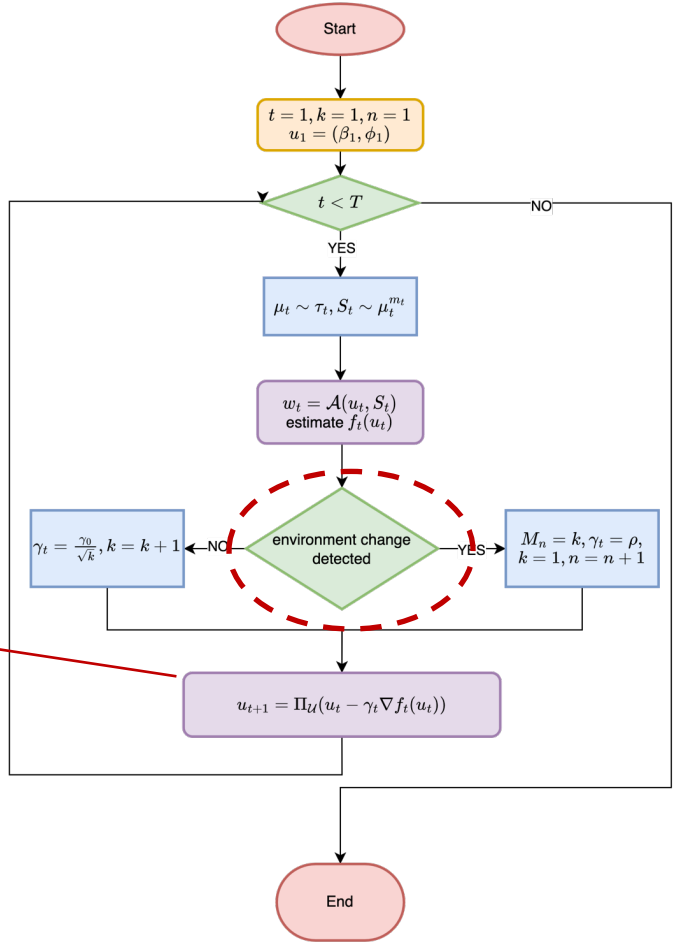
$$\kappa_t = \sqrt{\frac{1}{K\alpha} + \frac{2}{m_t}\alpha L}$$

control meta level trade-off

$$\phi_{t+1} = (1 - \frac{2b\kappa_t\gamma_t}{\beta_t})\phi_t + \frac{2b\kappa_t\gamma_t}{\beta_t} w_t$$

$$\beta_{t+1} = \beta_t - \gamma_t(a\kappa_t - \frac{\kappa_t(b\|\phi_t - w_t\|^2 + \epsilon_t + \epsilon_0)}{{\beta_t}^2})$$

control task level trade-off



AER Bound for DCML

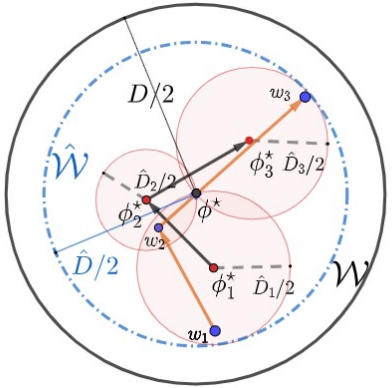
Meta-param (k-th task in n-th slot): $u_{n,k} = (\beta_{n,k}, \phi_{n,k}) \in \hat{\mathcal{U}}_n \stackrel{\text{def}}{=} \hat{\mathcal{W}}_n \times \hat{\mathcal{B}}_n$

Slot optimal $\phi_n^* = \sum_{k=1}^{M_n} \frac{\kappa_{n,k}}{\kappa_n} w_{n,k}$

Slot variance $V_n^2 = \sum_{k=1}^{M_n} \frac{\kappa_{n,k}}{\kappa_n} \|\phi_n^* - w_{n,k}\|_2^2$

max gradient norm of f in n-th slot

$P^* = \sum_{n=1}^{N-1} \|u_n^* - u_{n+1}^*\| + 1$



path length

$$\text{AER}_{\mathcal{A}}^T \le \underbrace{\frac{2}{T}\sum_{n=1}^{N}\sqrt{a(bV_n^2+\epsilon_n+\epsilon_0)}\kappa_n + \frac{\Delta_n}{2}}_{\text{optimal trade-off in hindsight}} + \underbrace{\frac{3}{2T}\sum_{n=1}^{N}\tilde{D}_n G_n\sqrt{M_n-1}}_{\text{average regret over slots}} + \underbrace{\frac{\tilde{D}_{\max}}{T}\sqrt{2P^*\sum_{n=1}^{N}G_n^2}}_{\text{regret w.r.t environment shift}}$$

diameter of $\hat{\mathcal{U}}_n$

Intra-slot task similarity

Inter-slot task (environment) similarity + non-stationarity

AER Bound for DCML

Gibbs base learner:

$$\mathrm{AER}^T_{\mathrm{Gibbs}} \leq \mathcal{O}\left(1 + \bar{V} + \frac{\sqrt{MN} + \sqrt{P^*}}{M\sqrt{N}}\right)\frac{1}{m^{\frac{1}{4}}}, \bar{V} \stackrel{\mathrm{def}}{=} \frac{1}{N}\sum_{n=1}^{N} V_n \,.$$

- Single-task learning $\mathcal{O}\left((D+1)m^{-1/4}\right)$
- Static environments $\mathcal{O}((V+1)m^{-1/4})$ with rate $\mathcal{O}(1/\sqrt{T})$,
- Shifting environments $\mathcal{O}((\bar{V}+1)m^{-1/4})$ with rate $\mathcal{O}(1/\sqrt{M})$
- $\bar{V} \leq V \leq \hat{D} \leq D$

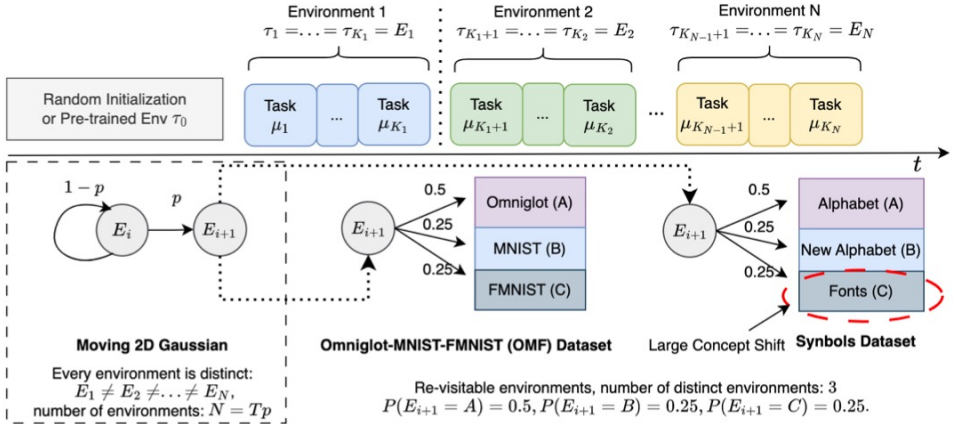
SGD base learner:

$$\mathrm{AER}^T_{\mathrm{SGD}} \leq \mathcal{O}\left(\bar{V} + \frac{\sqrt{MN} + \sqrt{P^*}}{M\sqrt{N}}\right)\sqrt{\frac{1}{K} + \frac{1}{m}}, \bar{V} \stackrel{\mathrm{def}}{=} \frac{1}{N}\sum_{n=1}^{N} V_n \,.$$

- Static Environment
  - $N = 1,\ P^* = 1, M = T$
  - recover static regret $\mathcal{O}(V + \frac{1}{\sqrt{T}})\sqrt{\frac{1}{K} + \frac{1}{m}}$
- Shifting Environment
  - When N is small and P^* is large
  - better than $\mathcal{O}(\bar{V} + \frac{1}{\sqrt{M}} + \sqrt{\frac{P^*}{NM}})$ [1]

[1] Khodak et.al . “Adaptive gradient-based meta-learning methods”. (NeurIPS 2019.)

Experimental Setting



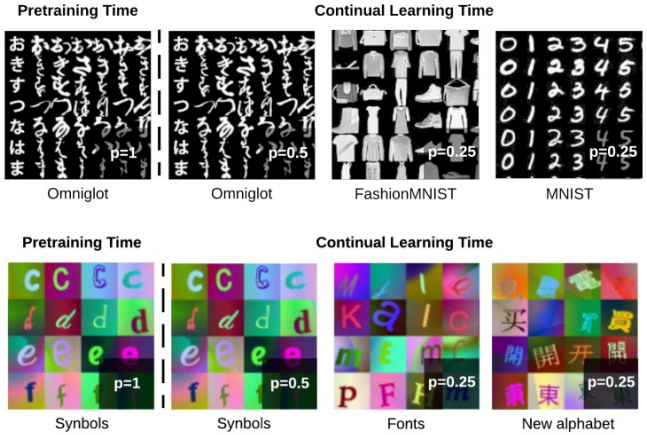
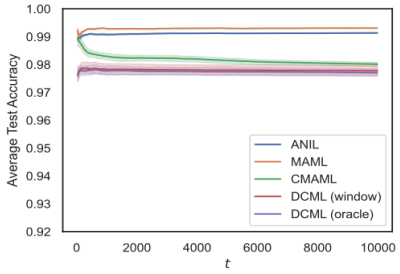


Figure source [1]

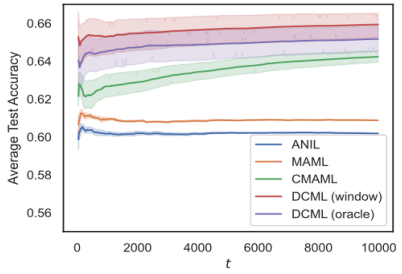
[1] Caccia, Massimo, et al. “Online fast adaptation and knowledge accumulation (osaka): a new approach to continual learning”. NeurIPS (2020)

Experimental Results

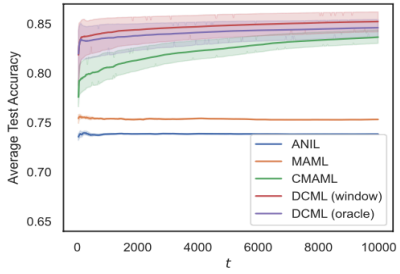
- better overall learning-forgetting trade-off
- stable to different levels of non-stationarity



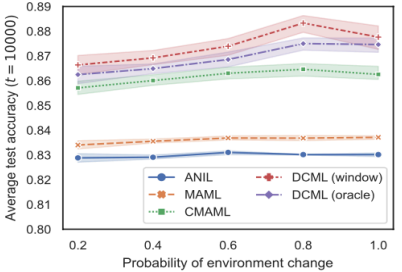
(a) Omniglot, p=0.2



(b) FMNIST, p=0.2



(c) MNIST, p=0.2



(d) All envs

Model	Synbols, $p = 0.2$			
	All Env.	Alpha.	New Alpha.	Font
Fine Tuning	25.3± 0.7	25.4± 0.9	25.1± 0.4	25.2± 0.5
MetaCOG [8]	25.3± 0.8	25.7± 0.9	25.0± 0.8	25.1± 1.0
MetaBGD [8]	29.9± 6.9	31.8± 9.6	28.1± 4.8	27.5± 3.6
ANIL [47]	60.5± 0.4	77.6± 0.6	53.6± 0.2	32.7± 0.4
MAML [4]	76.7± 0.4	**96.8± 0.1**	73.1± 0.1	40.5± 0.7
CMAML [13]	61.9± 2.5	76.2± 2.1	56.8± 3.2	37.9± 2.4
DCML(oracle)	77.2± 0.5	96.0± 0.3	73.4± 0.1	**42.4± 0.6**
DCML(window)	**77.4± 0.4**	96.2± 0.2	**73.8± 0.2**	42.4± 0.7

catastrophic forgetting

use variance to control forgetting

no learning, no forgetting

tend to track change

better trade-off

Large concept shift

Summary of Contribution

- Unified theoretical framework for continual meta-learning
  valid for both static and shifting task environments
- Formal analysis of the bi-level learning-forgetting trade-off
- Theoretically grounded algorithm
  improved rate in bounds
  improved empirical performance on real datasets

Conclusion

Limitation and Future Work

- Limitation
  1. For MDA, the pseudo-labeling process will fail under a large concept shift, and there possibly exists error accumulation.
  2. For Meta-learning, we only analyzed the generalization gap, not excess risk.
  3. For CML, only a single meta-model was considered.
- Future work
  1. Investigate more on pseudo labeling and its relation to noisy labels.
  2. Prove general excess risk bound for meta-learning with non-convex loss.
  3. Explore the theoretical limits of memory-based CML.

Thanks!