Midterm_Project_Report

Risk analysis of interest rates

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1. Introduction

When issuing loans, one of the most important thing a issuer needs to consider is the risk of not getting the money back. If a financial entity can calculate the risk at the point of issuing loans. Then it can adjust the interest rate to mitigate the potential loss.

However, there are some doubts that interest rates actually alter the risk of charged off(see Angbazo 1997), resulting a mismatch between the true risk and the "pre-interest-rate" risk. But how big is the interest-rate risk? How far off are we if the risk is ignored?

To answer this question, we proposed a roadmap to examine the pre- and post-interest-rate risk and a way to exaime the association between the risk of charge-off and the interest-rate.

2. Method

2.1 Dataset

The data used in this project cames from lending club. It included all the issued loans from 2012 to 2015 and detailed information not only the loans themselves also the credit related attributes of the borrowes at the point of issuing.

Originally, there are 42,390 unique issuing record and each has over 150 different variables describing the loan and the borrower. Due to the scope of this project, only 20 variables were picked and eventally, 11 were used to construct the model. The 12 variables are:

- `y`: Binary variables, 1 indicates the loan was determined as a charged off. 0 indicates no charged off
- `fico_10`: Borrower's fico score(high bound) at the point of requesting a loan.
  - This variable is measured at: fico_10 = (fico − $median$(fico))/10
- `inq_last_6_mths`: The borrower's number of inqueries during 6 months before requesting a loan.
- `open_acc`: The number of opening financial accounts of the borrower(bank accounts or fund accounts, etc.) at the time of requesting a loan.
- `dti_scale`: `dti` is the ratio of borrower's total amount of debt to total yearly income, after issuing the loan. this variable was scaled such that it is centered at it's mean and divided by standard deviation estimates.
- `installment_log`: The log of monthly installment of the loan + interest rate, centered at it's mean
- `addr_state`: The borrower's state of residence at the point of requesting the loan.
- `sub_grade`: The borrower's **sub_grade** calculated by lending club at the point of issuing the loan. This grade ranges from A to G with each grade being subdivided into 5 grades from 1 to 5.
- `term`: The expected number of months of paying back the issued loan, for our dataset, only 36 months and 90 months are available.
- `purpose`: the purpose of the loan, categorized by lending club. a total of 14 categories: credit card, car, small business, other, wedding, debt_consolidation, home_improvement, major_purchase, medical, moving, vacation, house, renewable_energy and education.
- `emp_length`: Categorical variable describing the length of employment at current position. from `less than 1 year` to `over 10 years` and `not applicable` a total of 12 levels.
- `delinq_2yrs`: The borrowers occurance of delinquency within last 2 years at the point of loan request.
- `int_rate`: interest rate calculated at the time of issuing the loan.

2.2 Model Selection

The risk of issuing loans are explained as credit risks. It's definition goes: *the possibility of a loss resulting from a borrower's failure to repay a loan or meet contractual obligations* (see Labarre, n.d.)

In this project, we access this risk by calculating the probabily(not a categorical problem) of a charge off(no repay) based on the borrower's information presented at the time of requesting a loan. Natrually, logistic regression is selected as the model. Multi-level logistic regression model was used because the data itself presented a hierarchical structure(for example, `addr_state`, `sub_grades`, `emp_length` are all group level variables).

In the model, 11 variables can be categorized as, outcome, individual level predictors(fixed effect) and group level predictors(random effect). The detailed explanation is listed below:

- outcome: P(`y` = 1)
- Fixed effect:
  - `fico_10`,`inq_last_6mths`,`open_acc`,`dti_scale`,`installment_log`
  - `int_rate`
- Random effect:
  - Random intercept: `addr_state`, `sub_grade`, `term`, `purpose`, `emp_length`
  - Random slope: `int_rate:sub_grade`

The first model will use all the variable except `int_rate` to evaluate the risk, then `int_rate` will be added into the second model and then compare the model fit and it's coefficients.

2.3 Experimental Design

The whole dataset was divided into training set and validation set. The model was trained on training set data and it's performance was validated in the testing set to avoid over training. The acquisition of the training set and the validation set is done in the following manner:

- The ratio of training vs testing was set to 7:3.
- Separate the original dataset according to `y` into `y = 0` and `y = 1`
- Within the two table, randomly draw 30% of the rows and combine them as validation set.
- Combine the remaining rows of the two tables as training set

The resulting training dataset has 29,637 records and the validation set has 12,717 records

2.4 Result Validation and Inference

There are different ways to access a logistic regression model's fit. Since the purpose of this project is to study the probability of charge off conditioned on the borrower's credit history. It is un-necessary to make categorical predictions, meaning setting threshold to predict 0 response or 1 response. In fact during the model validation process, it is impossible to make such predictions because the overall probability estimated of charge off is quite low.

Instead of categorical predictive power, the more adequate validation method is to use the model to simulate the original dataset, then examine the simulated distribution of charge off rate compared to the original data.

When the model is deemed to be valid, statistical inference will be drawn from the coef estimates. especially, the two models mentioned above will be compared to explore the association of `int_rate` and the risks as well as it's potential impact.

2.5 Model Estimation Package

For this project, `glmer` function with `bimonial` family and `logit` link function in `lmer` package was used. Alternatively, `winBUGS` and `rstanarm` packages can also be used to evaluate the model coefficients.

2.6 Limitations

There are 3 major limitations associated with the project.

1. One of the biggest assumptions associated with this project comes with the limitation of the dataset. In the real-world situation, one is able to apply for more than one loans. However, because the dataset masked all the member identification info, the assumption is that: when conditioned at all the credit information at the point of applying a loan, the risk is independent of the member id variable.
2. The second biggest limitation of this model is predictive power into the future. because the estimation only took into account the fisical year, then it is in theory not adequate to access the change of risk due to time. In other words, the model assumes the risk is independent of time.
3. Because of the nature of the `lmer` package, coefficients and standard errors estimated is not the most accurate. To achieve more accruate results, one can use `stan_glmer` from `rstanarm` instead of `glmer` and set the `intercept_prior = NULL`. Because of the computing cost of this approach, this project will not include this approach into the analysis.

However, this does not affect our research goal if we are only examine the inference as a year average from 2012 to 2015. The deviation of this projects findings to the future true effects depends on the risk's rate of change with time which can be estimated in future works.

3. EDA and Model Building

3.1 Data Wrangling and EDA

During the data wrangling and EDA process, all 150+ variables were examined and filtered based on the completeness, informativeness and relavence. Generally, variables contain over 70% `NA` values are dropped. It is not adviced to do so in a more serious settings, but for the scope of this project, unless the model fit is too poor, these variables will be ignored. Variables providing the same information in different coding or context will be discarded while keeping only one(for example, variables like `purpose` and `purpose_detail`, unless estimating the difference within each purpose is desired, the detailed info will be discarded unless doing so resulting a poor model fit).

After selecting the relevant variables, EDA was performed to get a feel of the overall data. Mosaic plot and histogram was used to exaimine the distribution of each variable by its own or conditioned by the outcome.

The source code can be found in the following files:

- Data wrangling: `./Data_Wrangling_&_EDA/Lendingclubreaddata.R`
- EDA: `./Data_Wrangling_&_EDA/EDA.R`

3.2 Model Building

After EDA, 18 predictors was selected and cleaned(includes scaling, taking log, etc.). the full list of variables is shown in `Table. 1`:

Using the above variables, 2 models were constructed.

The 1st model building is constructed of the following steps:

1. Further drop un-informative variables using no-pooling logistic regression's AIC.
2. Use the model selected by AIC and turn it into a multi-level model as described in previous sections.
3. Examine the model fit via binned residual plot and the conditional distribution of the simulated "fake original dataset"

Table 1: Variable used in this project

variables
term
grade
sub_grade
emp_length
verification_status
purpose
addr_state
delinq_2yrs
inq_last_6mths
open_acc
chargeoff_within_12_mths
delinq_amnt
loan_amnt_log
cr_length_log
fico_10
installment_log
dti_scale
int_rate_scale

The 2nd model building is constructed of the following steps:

1. Use the formula of model 1, add `int_rate` as fixed effect and it's interaction with `sub_grade` as random effect.
2. Access model fit using the same method and model 1.

After model validation, two models were compared and it's inference were drawn. Then, the effect of `int_rate` was also examined.

4. Results

4.1 Model 1

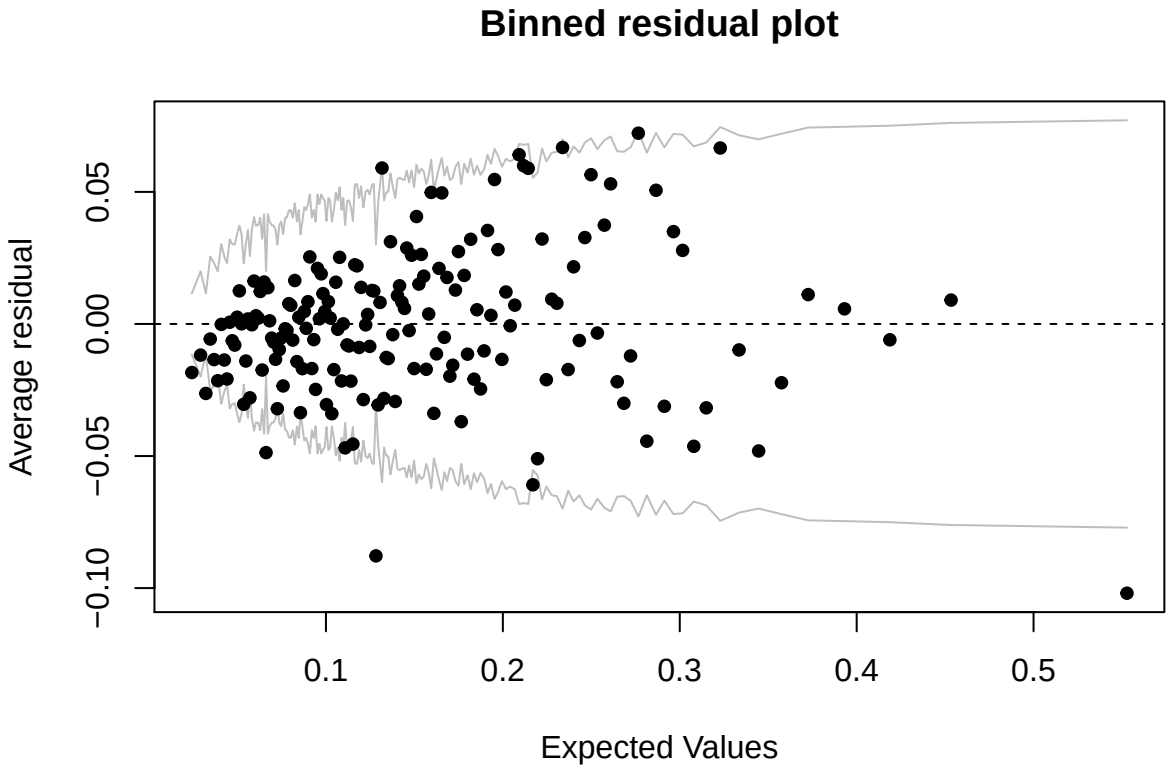
4.1.1 Model Description and Validation

Following the described steps the first model was built. The results are shown below:

```
## glmer(formula = y ~ fico_10 + inq_last_6mths + open_acc + dti_scale +
##     installment_log + delinq_2yrs + (1 | addr_state) + (1 | sub_grade) +
##     (1 | term) + (1 | purpose) + (1 | emp_length), data = lendingclub_model_train,
##     family = binomial)
##                 coef.est coef.se
## (Intercept)     -1.53     0.27
## fico_10         -0.07     0.01
## inq_last_6mths   0.13     0.01
## open_acc        -0.01     0.00
## dti_scale        0.09     0.02
## installment_log -0.02     0.03
## delinq_2yrs     -0.07     0.03
##
## Error terms:
```

```
##  Groups     Name        Std.Dev.
##  addr_state (Intercept) 0.14
##  sub_grade  (Intercept) 0.31
##  purpose    (Intercept) 0.33
##  emp_length (Intercept) 0.21
##  term       (Intercept) 0.33
##  Residual               1.00
## ---
## number of obs: 29673, groups: addr_state, 50; sub_grade, 35; purpose, 14; emp_length, 12; term, 2
## AIC = 23544.1, DIC = 23025.4
## deviance = 23272.7
```

The binned residual plot is shown below



Under current model, we have the following performance:

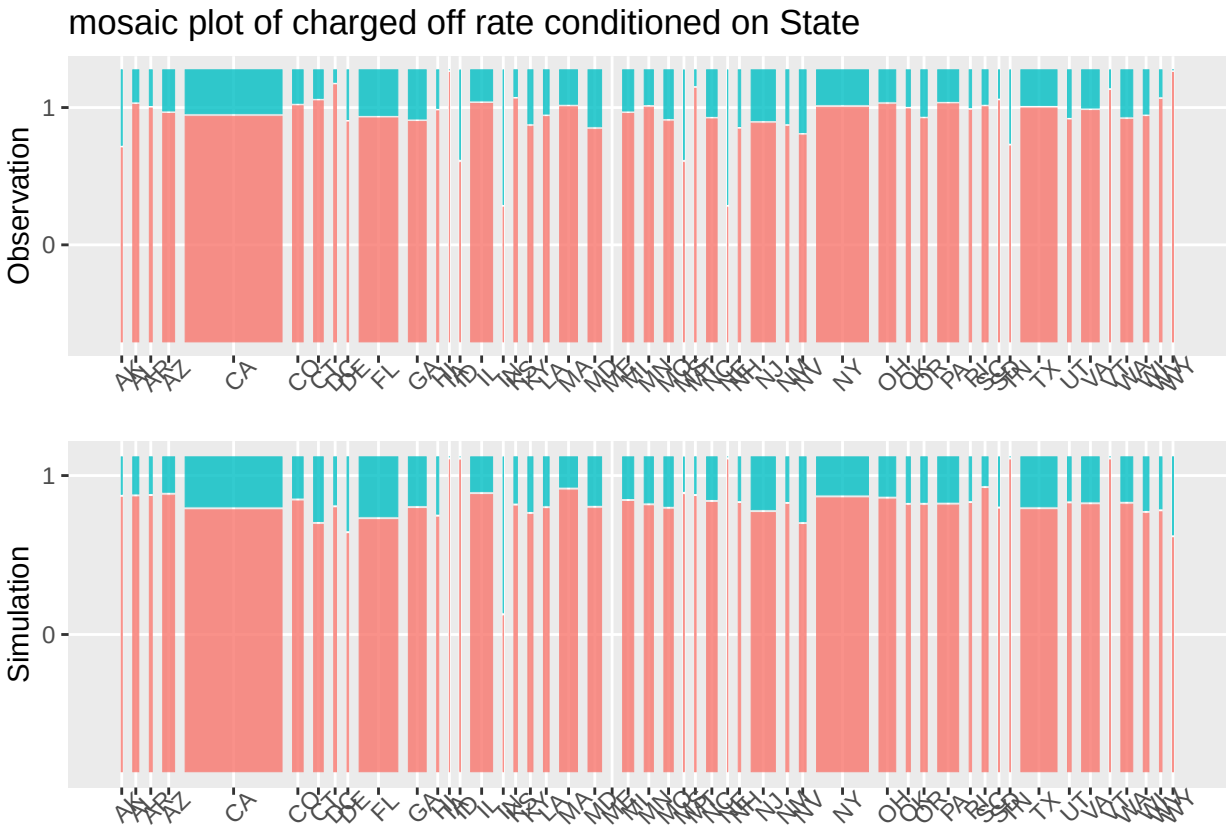
```
## expected occurence of charged of under model 1 is 1893
```

```
## true observed occurence of charged off in validation set is 1921
```

The predicted occurence is really close to the original dataset. After examine the distribution using mosaic plot, the model is able to capture the overall trend of the data. One example is shown below:

Table 2: Fixed Effect of Model 1

	Coef	SE
(Intercept)	-1.533	0.271
fico_10	-0.068	0.012
inq_last_6mths	0.132	0.011
open_acc	-0.013	0.004
dti_scale	0.091	0.019
installment_log	-0.018	0.031
delinq_2yrs	-0.074	0.032



4.1.2 Inference

For model 1, shown in `Table. 2`, the fixed effect can be interpret as:

- `Intercept`: The intercept is hard to interpret, it associated with the probability of charged off for a person with median fico score, 0 inquiry in last 6 months,0 open account,0 delinquency in 2 years, mean dti and mean installment. But without any random effect information.
- `fico_10` is -0.068 means that on average, with each 10 points higher in fico score were 1.7% less likely to have a charged off when controlling for other fixed effect
- `inq_last_6_mths` is 0.132 means that on average, 1 more inquiry in the last 6 months is associated with 3.3% increase in probability of charged off when controlling for other fixed effect
- The relative large standard error estimated for `intallment_log` and `delinq_2yrs` indicates that the data does not have enough sample to have a good estimation of the coefficient.

- the other fixed effect coefficients and be interpret in very similar manner.

The result of the random effect is not discussed because it is not relavent to what is interested in this project.

4.2 Model 2

4.2.1 Model Description and Validation

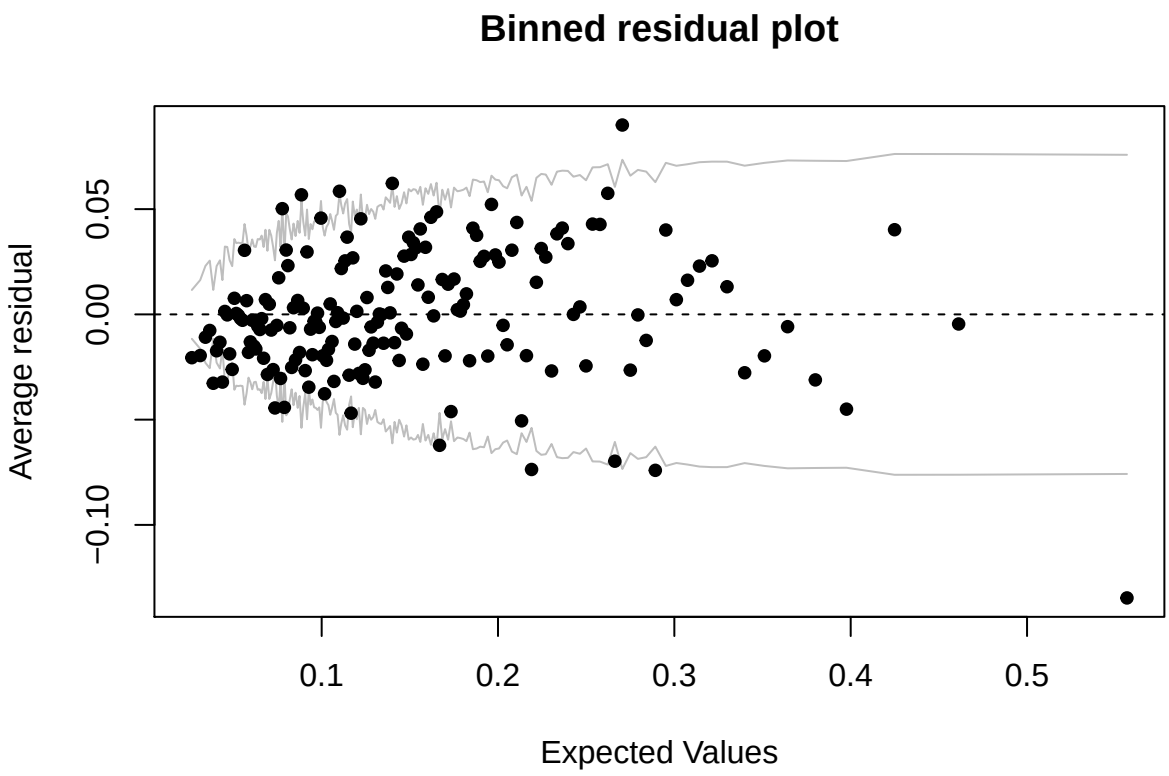
The second model is evaluated as:

```
## glmer(formula = y ~ int_rate_scale + fico_10 + inq_last_6mths +
##     open_acc + dti_scale + installment_log + delinq_2yrs + (1 |
##     addr_state) + (1 + int_rate_scale | sub_grade) + (1 | term) +
##     (1 | purpose) + (1 | emp_length), data = lendingclub_model_train_2,
##     family = binomial)
##                 coef.est coef.se
## (Intercept)     -2.25     0.24
## int_rate_scale   0.08     0.01
## fico_10         -0.05     0.01
## inq_last_6mths   0.13     0.01
## open_acc        -0.01     0.00
## dti_scale        0.10     0.02
## installment_log -0.07     0.03
## delinq_2yrs     -0.07     0.03
##
## Error terms:
##  Groups     Name           Std.Dev. Corr
##  addr_state (Intercept)    0.14
##  sub_grade  (Intercept)    0.16
##             int_rate_scale 0.02     -1.00
##  purpose    (Intercept)    0.32
##  emp_length (Intercept)    0.21
##  term       (Intercept)    0.26
##  Residual                  1.00
## ---
## number of obs: 29673, groups: addr_state, 50; sub_grade, 35; purpose, 14; emp_length, 12; term, 2
## AIC = 23481.2, DIC = 23115.6
## deviance = 23283.4
```

the binned residual plot is:

Table 3: Fixed Effect of Model 2

	Coef	SE
(Intercept)	-2.252	0.237
int_rate_scale	0.083	0.010
fico_10	-0.047	0.009
inq_last_6mths	0.132	0.010
open_acc	-0.012	0.004
dti_scale	0.096	0.019
installment_log	-0.070	0.029
delinq_2yrs	-0.073	0.032



under this model the fake data generating result is:

```
## expected occurence of charged of under model 1 is 1889
```

```
## true observed occurence of charged off in validation set is 1921
```

The overall distribution of predicted charged off conditioning on the variables are also very similar to the original data.

4.2.2 Inference

The coefficients are shown in `Table. 3`. The same as the 1st model, the intercept value is still very difficult to interpret. It is related to the probability at merely the same condition but with interest rate at the lowest.

One interesting finding is that, by adding interest rate, the coefficient of fico score, installment changed dramatically. This is a sign indicating that this three variables are confounding covariates(correlation shown in `Table.4`). The coefficient of interest rate indicates that each 1% increase of the issuing interest rate is

Table 4: Correlation among confounding covariates

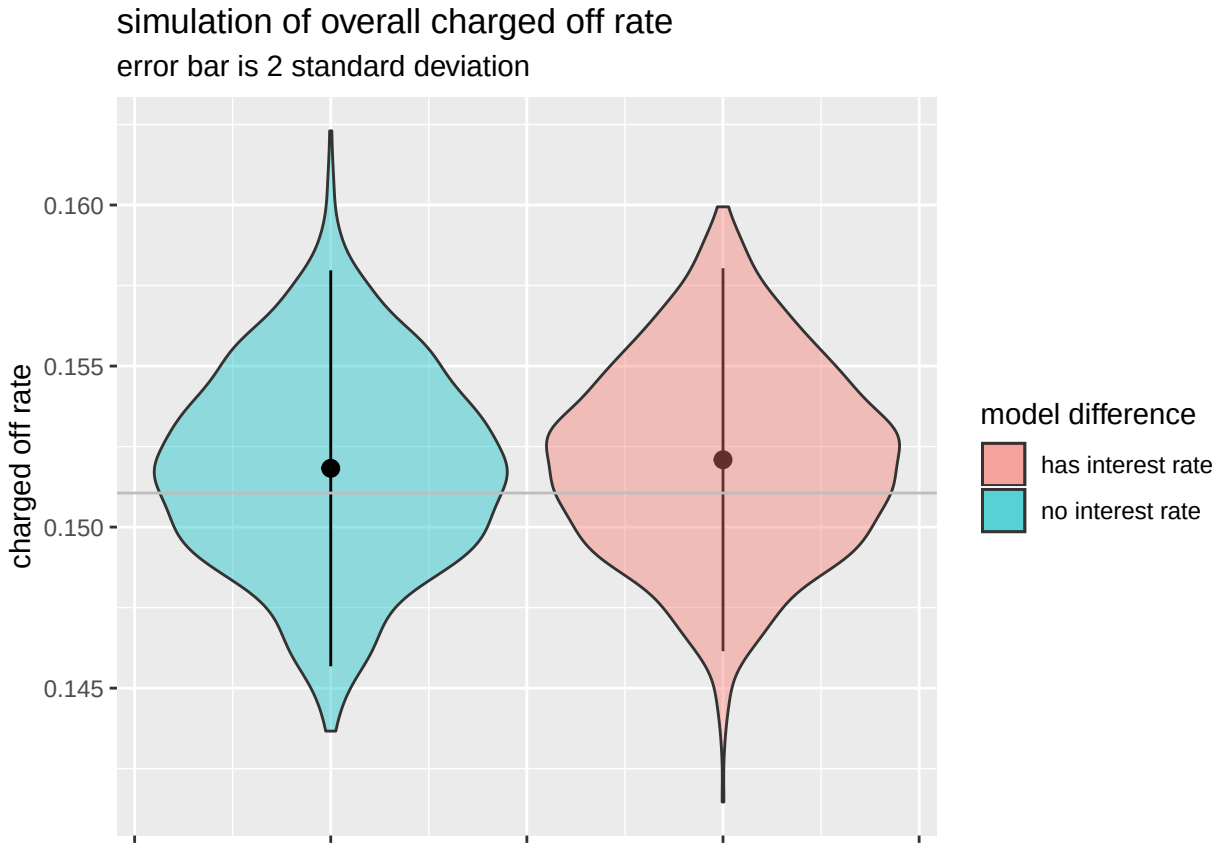
	interest rate	fico score	installment
interest rate	1.000	-0.703	0.235
fico score	-0.703	1.000	0.073
installment	0.235	0.073	1.000

associated with 2% increase of charged off risk when controlling for other fixed effect.

5. Conclusions

5.1 Risk Estimation

The overall risk evaluation of both models gives a really close estimation on the validation set. 1893/1921 for the 1st model and 1889/1921. But this results is get only with one simulation. If we repeat the simulation for 1000 times.



The grey horizontal line is the true observation and as we can see from the graph, our estimation is really close to the true observationand the model which has interest rate considered showed an slightly higher estimation. They both estimates the risks as around 15.2% with 95% creditable interval from 14.6% to 15.8%.

5.2 `int_rate` Association Estimation

The interest rate although has a statistical significant coeficient value, however, the actual value is estimated at 2% increase in risk for every 1% increase in interest rate. However, the actual impact on the model is strongly determined by its variability(meaning the difference between max interest rate and min interest rate).

Table 5: estimate power of association

	power
int_rate_scale	1.245
fico_10	-0.658
inq_last_6mths	0.792
open_acc	-0.216
dti_scale	0.384
installment_log	-0.210
delinq_2yrs	-0.146

The power in `table.5` is calculated as the $coef \cdot 4 \cdot se$, if we consider the variability of interest rate(which most), the total association of interest rate is the highest among all the fixed effects. It's related to merely 30% changes of the probability in the estimated model.

However, such association can not be interpreted as casual relation. The direction of the effect can be explained in both ways, higher risk estimated by lending club resulting a higher interest rate or higher interest rate causing borrower more likely to have a charged off. To settle this question, one needs to design an experiment which has samples of very similar risks estimated and loan requested with different interest rate. Without such experiment, the interest-rate risk can not be percisely estimated.

5.3 Conclusion and Future Directions

The estimated model shows that the interest rate is highly associated with the risk of charged off. However, it is not possible to percisely estimate the true risk of charged off imposed by interest rate. However, even without the interest rate, the model had already provide very close estimation of the risks. This in a sense indecates that ignoring the interest rate risk is not likely to cause a bad risk estimation.

Future directions:

1. Use `stan_glmer` to get better parameter estimation.
2. Compare distribution generated by model with the original observed outcome using chi-square test of independence.
3. Including the amount owed at the time of charge off into the data, a model can be built to estimate the actual expected loss of money thus improve the risk analysis model to a dollar amount losses.

Reference

Angbazo, Lazarus. 1997. "Commercial Bank Net Interest Margins, Default Risk, Interest-Rate Risk, and Off-Balance Sheet Banking." *Journal of Banking & Finance Volume 21, Issue 1, January 1997, Pages 55-87 Journal of Banking & Finance* 21 (1): 55–87.

Labarre, Olivia. n.d. "Credit Risk." *Https://Www.investopedia.com/Terms/c/Creditrisk.asp*.