

$$\begin{array}{ccccccc} c_0 & c_1 & c_2 & c_3 & c_4 & c_5 & c_6 & c_7 \\ \hline \Delta^1 c_0 & \Delta^1 c_1 & \Delta^1 c_2 & \Delta^1 c_3 & \Delta^1 c_4 & \Delta^1 c_5 & \Delta^1 c_6 \\ \hline \Delta^2 c_0 & \Delta^2 c_1 & \Delta^2 c_2 & \Delta^2 c_3 & \Delta^2 c_4 & \Delta^2 c_5 \\ \hline \Delta^3 c_0 & \Delta^3 c_1 & \Delta^3 c_2 & \Delta^3 c_3 & \Delta^3 c_4 \end{array}$$

$$P^{(r)}(t) = \frac{h!}{(h-r)!} \sum_{i=0}^{n-r} \Delta^r c_i B_i^{n-r}(t)$$

$$t = T \cdot s \Rightarrow \frac{dP}{dt} = \frac{dP}{ds} \cdot \frac{ds}{dt} = \frac{dP}{ds} \cdot \frac{1}{T}$$

$$B_i^{n-r}(t) = \binom{n-r}{i} x^i (1-x)^{n-r-i}$$

and so

$$\begin{aligned} P(t) &= \sum_{i=0}^n c_i \binom{n}{i} \sum_{j=0}^{n-i} \binom{n-i}{j} (-1)^j t^{i+j} \\ &= \sum_{i=0}^n c_i \binom{n}{i} \sum_{j=i}^n \binom{n-i}{j-i} (-1)^{j-i} t^j, \\ &= \sum_{j=0}^n \sum_{i=0}^j c_i \binom{n}{i} \binom{n-i}{j-i} (-1)^{j-i} t^j, \end{aligned}$$

and it follows that

$$a_j = \sum_{i=0}^j \binom{n}{i} \binom{n-i}{j-i} (-1)^{j-i} c_i.$$

This can alternatively be written as

$$a_j = \sum_{i=0}^j \binom{n}{j} \binom{j}{i} (-1)^{j-i} c_i = \binom{n}{j} \Delta^j c_0.$$

$$f(1-t)^n$$

$$\begin{array}{ccccccc} c_0 & c_1 & c_2 & c_3 & c_4 & c_5 & c_6 & c_7 & c_8 & c_9 \\ \hline \Delta^1 c_0 & \Delta^1 c_1 & \Delta^1 c_2 & \Delta^1 c_3 & \Delta^1 c_4 & \Delta^1 c_5 & \Delta^1 c_6 & \Delta^1 c_7 & \Delta^1 c_8 \\ \hline \Delta^2 c_0 & \Delta^2 c_1 & \Delta^2 c_2 & \Delta^2 c_3 & \Delta^2 c_4 & \Delta^2 c_5 & \Delta^2 c_6 & \Delta^2 c_7 \\ \hline \Delta^3 c_0 & \Delta^3 c_1 & \Delta^3 c_2 & \Delta^3 c_3 & \Delta^3 c_4 & \Delta^3 c_5 & \Delta^3 c_6 \\ \hline \Delta^4 c_0 & \Delta^4 c_1 & \Delta^4 c_2 & \Delta^4 c_3 & \Delta^4 c_4 & \Delta^4 c_5 \end{array}$$

$$\begin{cases} \Delta^r c_0 = \frac{(h-r)!}{h!} P^{(r)}(0) \\ \Delta^r c_{n-r} = \frac{(h-r)!}{h!} P^{(r)}(1) \end{cases}$$

$$P^{(r)}(t) = \frac{h!}{(h-r)!} \sum_{i=0}^{n-r} \Delta^r c_i B_i^{n-r}(t)$$

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$$P(s), s \in [0,1]$$

$$t = T \cdot s$$

$$\frac{dP}{dt} = \frac{dP}{ds} \cdot \frac{ds}{dt} = \frac{dP}{ds} \cdot \frac{1}{T}$$