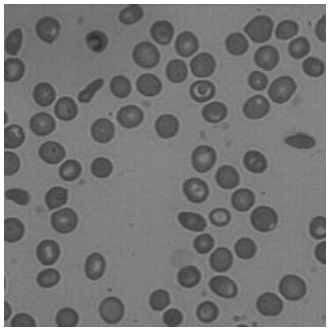
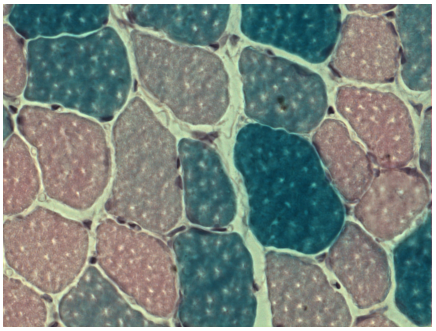
Tutorial: Histogram-based image segmentation

This tutorial aims to implement some image segmentation methods based on histograms (thresholding and "k-means" clustering).

The different processes will be applied on the following images:



(a) Cells.



(b) Muscle cells, author: Damien Freyssenet, University Jean Monnet, Saint-Etienne, France.

1 Manual thresholding

The most simple segmentation method is thresholding.

- Visualize the histogram of the grayscale image 'cells'.
- Make the segmentation with a threshold value determined from the image histogram.

2 k-means clustering

Let $X = \{x_i\}_{i\in[1;n],n\in\mathbb{N}}$ be a set of observations (the points) in $\mathbb{R}^d$. The k-means clustering consists in partitioning X in $k\,(k < n)$ disjoint subsets $\tilde{S}$ such that:

$$\tilde{S} = \arg\min_{S=\{S_i\}_{i\leq k}} \sum_{i=1}^{k} \sum_{x_j\in S_i} \|x_j - \mu_i\|^2 \tag{1}$$

where μ_i is the mean value of the elements in S_i. The k-means algorithm is iterative. From a set of k initial elements $\{m_i^{(1)}\}_{i\in[1;k]}$ (randomly selected), the algorithm iterates the following (t) steps:

- Each element of X is associated to an element m_i according to a distance criterion (computation of a Voronoi partition):

$$S_i^{(t)} = \left\{ \mathbf{x}_j : \left\| \mathbf{x}_j - \mathbf{m}_i^{(t)} \right\| \leq \left\| \mathbf{x}_j - \mathbf{m}_{i^*}^{(t)} \right\|, \forall i^* \in [1;k] \right\} \tag{2}$$

- Computation of the new mean values for each class:

$$\mathbf{m}_i^{(t+1)} = \frac{1}{|S_i^{(t)}|} \sum_{\mathbf{x}_j \in S_i^{(t)}} \mathbf{x}_j \tag{3}$$

  where $|S_i^{(t)}|$ is the number of elements of $S_i^{(t)}$.

3 Grayscale image, $k = 2$ in one dimension

The objective is to binarize image image 'cells', which is a grayscale image. The set X is defined by $X = \{I(p)\}$, for p being the pixels of the image I.

- Implement the algorithm proposed below (Alg. 1).
- Test this operator on the image 'cells'.
- Test another method of automatic thresholding (defined by Otsu in [1]).
- Compare the values of the thresholds (manual and automatic).

Data: Original image A
Data: Stop condition ε
Result: thresholded image
Initialize T_0, for example at $\frac{1}{2}(\max(A) + \min(A))$;
$done \leftarrow False$;
while *NOT done* **do**
  Segment the image A with the threshold value T;
  it generates two classes G_1 (intensities $\geq T$) and G_2 (intensities $< T$);
  Compute the mean values, denoted μ_1, μ_2, of the two classes G_1, G_2, respectively;
  Compute the new threshold value $T_i = \frac{1}{2}(\mu_1 + \mu_2)$;
  if $|T_i - T_{i-1}| < \varepsilon$ **then**
    $done \leftarrow True$
  end
end
Segment the image with the estimated threshold value.

Algorithm 1: Algorithm for automatic threshold computation.

The Matlab function `graythresh` computes the automatic treshold by the method of Otsu.

For use with python, use the module skimage. filter and the function threshold_otsu.

4 Simulation example, $k = 3$ in two dimensions

The objective is to generate a set of 2-D random points (within $k = 3$ distinct classes) and to apply the k-means clustering for separating the points and evaluating the method (the classes are known!).

Write a function for generating a set of n random points around a point with coordinates (x, y).

We will use the Matlab function randn.

We will use the python function randn from the module numpy.random.

- Use this function to generate 3 set of points (in a unique matrix) around the points $(0, 0)$, $(3, 4)$ and $(-5, -3)$.
- Use the Matlab function kmeans for separating the points. The result is presented in Fig. 1.
- Use the python function sklearn.cluster.KMeans for separating the points. The result is presented in Fig.1.

Verify the utility of the option replicate.

Verify the utility of the option n_init=10.

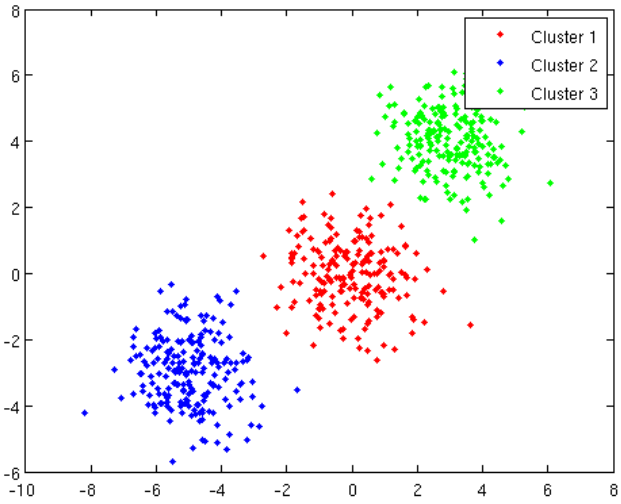


Figure 1: Resulting clustering of random points.

5 Color image segmentation using K-means: $k = 3$ in 3D

The k-means clustering is now used for segmenting the color image representing the muscle cells 'Tv16.png'.

Which points have to be separated? Transform the original image into a vector of size $N \times 3$ (where N is the number of pixels) which represent the 3 components R, G and B of each image pixel.

The MATLAB® function reshape can perform the transformation.

The python function numpy.reshape does the same transformation.

- Visualize the 3-D map (histogram) of all these color intensities.
- Make the clustering of this 3-D map by using the K-means method.
- Visualize the corresponding segmented image.

References

[1] Nobuyuki Otsu. A Threshold Selection Method from Gray-Level Histograms. *Systems, Man and Cybernetics, IEEE Transactions on*, 9(1):62–66, Jan 1979. 2