1 Matlab correction

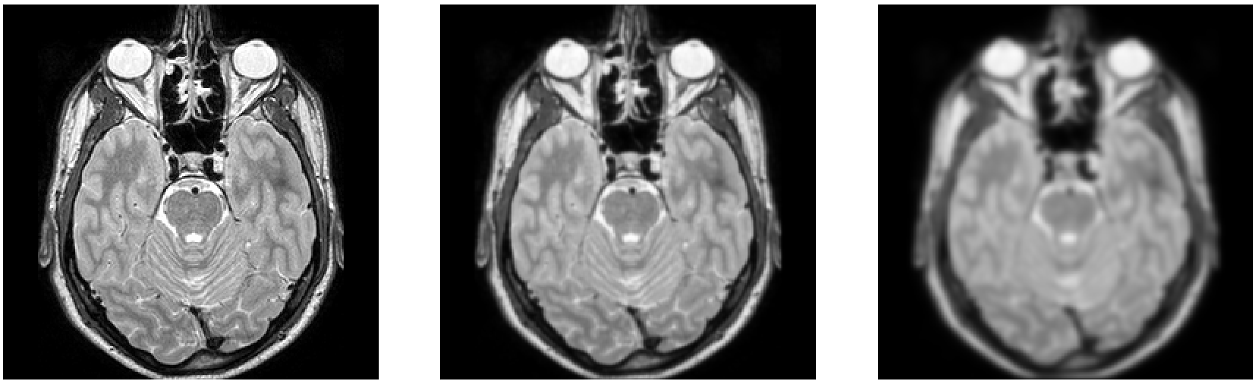
1.1 Linear diffusion

The numerical scheme is coded as follows with matlab:

```matlab
function Z = linearDiffusion(I, nbIter, dt)
% I: Original image
% nbIter: number of iterations
% dt: step time

h = [[0 1 0];[1 -4 1];[0 1 0]];
Z=I; % initialization

for i=1:nbIter;
    Z = Z + dt * conv2(Z, h, 'same');
end;
```

Another way of coding this operator is to compute 4 gradients. This simplifies the formulation of the nonlinear diffusion filter, illustrated in Fig.1.

```matlab
function Z = linearDiffusion(I,nbIter,dt)
% I: Original image
% nbIter: number of iterations
% dt: step time

% masks for gradients computation
hW = [1 -1 0];
hE = [0 -1 1];
hN = hW';
hS = hE';

Z = I; % initialization

for i=1:nbIter;
    % calculate gradient in all directions (N,S,E,W)
    gW = imfilter(Z,hW);
    gE = imfilter(Z,hE);
    gN = imfilter(Z,hN);
    gS = imfilter(Z,hS);

    % next Image
    Z = Z + dt*(gN + gS + gW + gE);
end;
```



(a) Original image.

(b) Linear diffusion filter, with $\alpha = 0.1$, $dt = 0.05$ and 10 iterations.

(c) Linear diffusion filter, with $\alpha = 0.1$, $dt = 0.05$ and 50 iterations.

Figure 1: Linear diffusion filter. Contours are not preserved: this is equivalent to a Gaussian filter.

1.2 Nonlinear diffusion

The nonlinear diffusion is an adaptation of the diffusion to the informations contained in the images (see Fig.2). These informations are high frequency components, evaluated by the gradients in the 4 directions. A specific coefficient is thus applied to each of these directions.

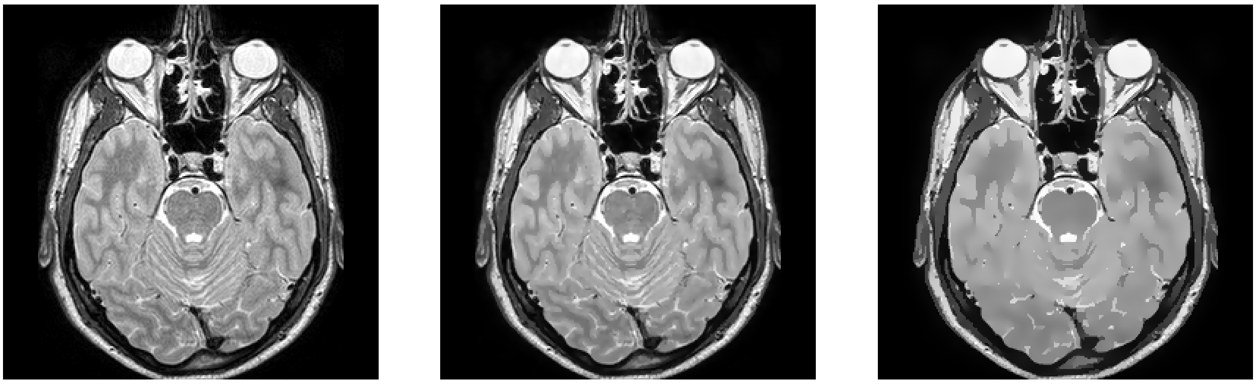
```matlab
function Z = nonlinearDiffusion(I,nbIter,dt,alpha)
% I: Original image
% nbIter: number of iterations
% dt: step time
% alpha: diffusion parameter

hW = [1 -1 0];
hE = [0 -1 1];
hN = hW';
hS = hE';

Z = I;

for i=1:nbIter
    % calculate gradient in all directions (N,S,E,W)
    gW = imfilter(Z,hW);
    gE = imfilter(Z,hE);
    gN = imfilter(Z,hN);
    gS = imfilter(Z,hS);

    % next Image
    Z = Z + dt*(c(gN,alpha).*gN + c(gS,alpha).*gS + c(gW,alpha).*gW + c(gE,alpha).*gE);
end
```

with c defined as follows:

```
function Z = c(I, alpha)
% Perona Malik diffusion coefficient
% I: input image
% alpha: diffusion parameter
%
% absolute value is not necessary in this case
Z = exp(-(I/alpha).^2);
```

Another possibility is:

```
function Z = c2(I, alpha)
% Perona Malik diffusion coefficient
% I: input image
% alpha: diffusion parameter
%
% absolute value is not necessary in this case
Z = 1/(1+(I/alpha).^2);
```



(a) Original image.

(b) Non linear diffusion filter, with $\alpha = 0.1$, $dt = 0.05$ and 10 iterations.

(c) Non linear diffusion filter, with $\alpha = 0.1$, $dt = 0.05$ and 100 iterations.

Figure 2: Nonlinear diffusion filter. Contours are preserved.

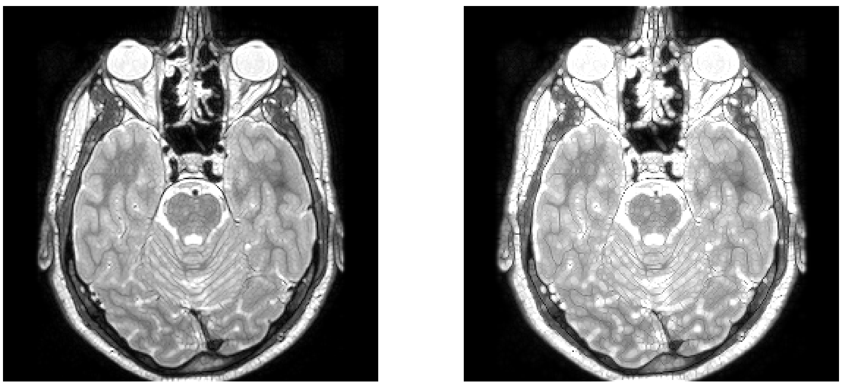
1.3 Degenerate diffusion

Erosion and dilation can theoretically be coded with this numerical scheme. However, some numerical problems appear in the first version (Fig.3), which are corrected in the second version (Fig.4).

1.3.1 First version

This first version is the direct transcription of the numerical scheme. As observed in Fig.3, shocks (peaks) appear after a few iterations.

```
function [Zerosion,Zdilation] = morphologicalDiffusion2(I,nbIter,dt)
% I: original image
% nbIter: number of iterations
% dt: time increment
h = [-1 0 1];

Zerosion = I;
Zdilation = I;

for i=1:nbIter
    % calculate gradient in vertical V and horizontal H directions, for
    % dilation
    gH = imfilter(Zdilation, h);
    gV = imfilter(Zdilation, h');

    % same computation, for erosion
    jH = imfilter(Zerosion, h);
    jV = imfilter(Zerosion, h');

    % next step
     Zdilation  = Zdilation + dt * sqrt(gV.^2 +gH.^2);
     Zerosion = Zerosion - dt * sqrt(jV.^2 +jH.^2);

end
```



(a) Dilation by diffusion, for $dt = 0.02$ and nbIter=20.

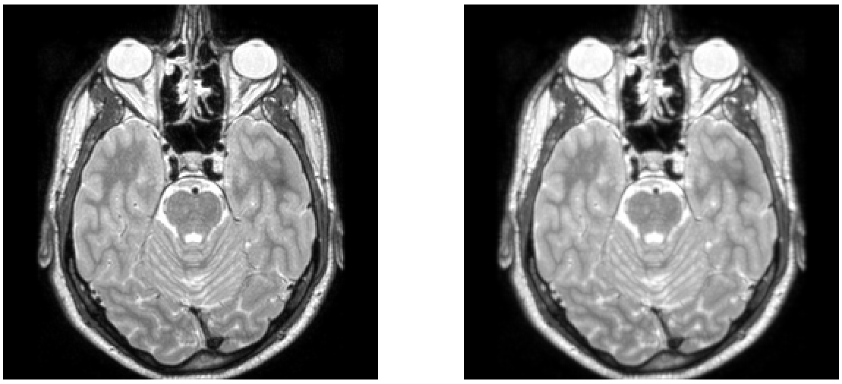
(b) Dilation by diffusion, for $dt = 0.02$ and nbIter=50.

Figure 3: Mathematical morphology operations by diffusion.

1.3.2 More sophisticated version

Another scheme, more numerically stable, can be preferred to the previous one. Results are presented in Fig. 4.

```matlab
function [Zerosion,Zdilation] = morphologicalDiffusion(I,nbIter,dt)
% I: original image
% nbIter: number or iterations
% dt: time increment
hW = [1 -1 0];
hE = [0 -1 1];
hN = hW';
hS = hE';

Zerosion = I;
Zdilation = I;

for i=1:nbIter;
    % calculate gradient in all directions (N,S,E,W)
    gW = imfilter(Zdilation,hW);
    gE = imfilter(Zdilation,hE);
    gN = imfilter(Zdilation,hN);
    gS = imfilter(Zdilation,hS);

    jW = imfilter(Zerosion,hW);
    jE = imfilter(Zerosion,hE);
    jN = imfilter(Zerosion,hN);
    jS = imfilter(Zerosion,hS);

    % next step
    g = sqrt( min(0,-gW).^2 + max(0,gE).^2 + min(0,-gN).^2 + max(0,gS).^2
        ↪ );
    j = sqrt( max(0,-jW).^2 + min(0,jE).^2 + max(0,-jN).^2 + min(0,jS).^2
        ↪ );
    Zdilation = Zdilation + dt * g;
    Zerosion = Zerosion - dt * j;

end;
```



(a) Dilation by diffusion, for $dt = 0.02$ and nbIter=20.

(b) Dilation by diffusion, for $dt = 0.02$ and nbIter=50.

Figure 4: Mathematical morphology operations by diffusion, sophisticated version.