Exam: Geometry

Dure : 1h30 - 20 points

1 Geometrical characterization

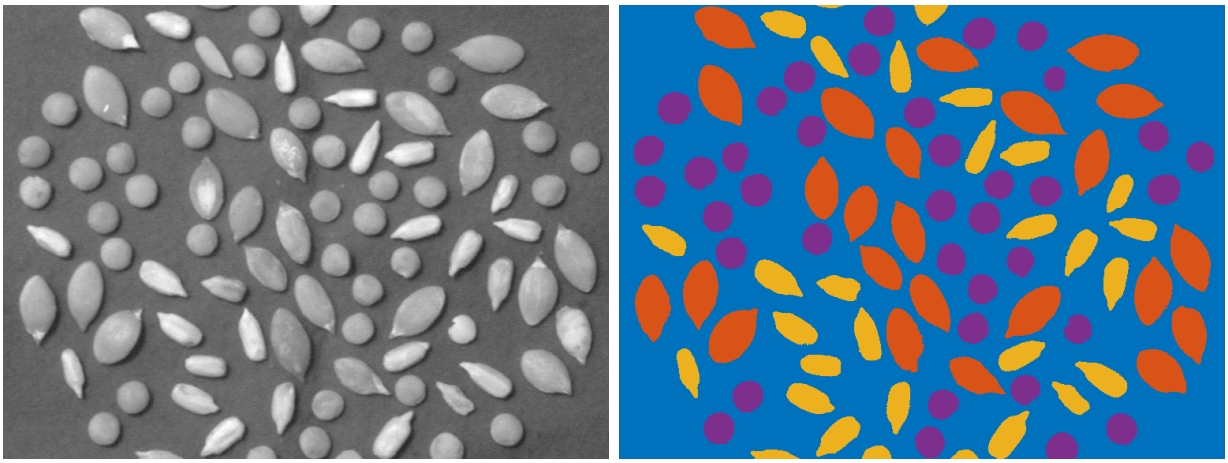

Figure 1: This image contains grains with different shapes. 3 different types of grains can be observed. An idea of the segmentation result is proposed.

With the grayscale image of Fig.1, propose a method to count the number of grains of each type.

Informations

The function ind2rgb can convert a labelled array into a RGB image with the following syntax: ind2rgb(L, lines(4));, with lines(4) defining a colormap with 4 colors. It has been used to generate the segmented image.

2 Stochastic geometry

Let X be a random set defined as a rectangle with sides a, b and an orientation θ in relation to the x-axis. a and b are independant random variables. a follows a Normal law with parameters (μ_a, σ_a). b follows a Normal law with parameters (μ_b, σ_b). θ follows a uniform distribution between 0 and 2π.

- Show different realizations of the random set X with the parameters $\mu_a = 50, \sigma_a = 10, \mu_b = 100, \sigma_b = 30$.
- Calculate (from an analytical point of view) the expectation of the area and of the squared area of this random set, denoted $\mathbb{E}[A(X)]$ and $\mathbb{E}[A^2(X)]$ respectively, as a function of $\mu_a, \mu_b, \sigma_a, \sigma_b$.
- From $N = 1000$ realizations of this random set (using the parameters given in question 1), compute $\mathbb{E}[A(X)]$ and $\mathbb{E}[A^2(X)]$. Deduce and compare the estimated parameters of the Normal laws to the real ones.
- Show (in a graph) the resulting error of the estimation as a function of the number of realizations N.

Informations

You can use the Matlab function randn to generate some realizations of a Normal distribution.