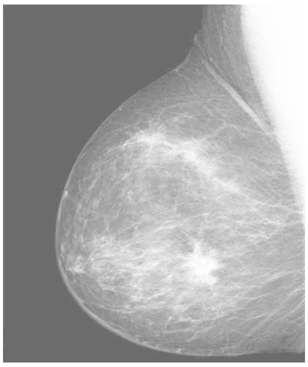
Tutorial: Logarithmic Image Processing (LIP)

> **Note**
>
> This tutorial aims to study the LIP model, which is a vector space of gray-level images, consistent with the physical laws of Weber and Fechner as well with the visual perception laws. More particularly, the LIP dynamic expansion, used for image enhancement, will be implemented as well as the Sobel LIP filter for edge detection.

The different processes will be applied on a mammographic image.



(a) breast

1 Introduction

The LIP model (Logarithmic Image processing) has been introduced in the mid 1980s. It defines a mathematical framework for image processing in a bounded interval. It is mathematically rigorous and physically justified. For more informations, refer to [3, 1, 4].

An image is represented by its gray-tone function f, defined on a spatial domain $D \subset \mathbb{R}^2$, and with values into $[0, M[$, where $M > 0$. For all gray-tone functions S defined on D, a vector space is defined by the operations of addition $\triangle\!\!\!\!\!+$ and multiplication $\triangle\!\!\!\!\!\times$, and by extension with the operations of subtraction $\triangle\!\!\!\!\!-$ and negation:

$$\forall f, g \in S,\ f \mathbin{\triangle\!\!\!\!\!+} g = f + g - \frac{fg}{M} \tag{1}$$

$$\forall f \in S, \forall \lambda \in \mathbb{R},\ \lambda \mathbin{\triangle\!\!\!\!\!\times} f = M - M\left(1 - \frac{f}{M}\right)^{\lambda} \tag{2}$$

$$\forall f \in S,\ \ \mathbin{\triangle\!\!\!\!\!-} f = \frac{-Mf}{M - f} \tag{3}$$

$$\forall f, g \in S,\ f \mathbin{\triangle\!\!\!\!\!-} g = M\frac{f - g}{M - g} \tag{4}$$

The vector space S of gray-tone functions is algebrically and topologically isomorph to the classical vector space, defined by the isomorphism φ:

$$\forall f \in S,\ \varphi(f) = -M \ln\left(1 - \frac{f}{M}\right) \tag{5}$$

The inverse isomorphism φ^{-1} is then defined by:

$$f = \varphi^{-1}(\varphi(f)) = M\left(1 - \exp\left(-\frac{\varphi(f)}{M}\right)\right) \tag{6}$$

This fundamental isomorphism allows the definition of the following operations on gray-tone functions:

- Dot product: $\forall f, g \in S,\ \ < f, g >_{\triangle} = \int \varphi(f)\varphi(g)$
- Euclidean norm: $\forall f \in S,\ \ \parallel f \parallel_{\triangle} = |\varphi(f)|_{\mathbb{R}}$, with $|\cdot|_{\mathbb{R}}$ is the classical absolute value.

2 Elementary LIP operations

Classically, gray-tones are coded with 8 bits, and thus one can choose $M = 256$. Be careful that there might exist sampling issues, and it might be required to considere images in the LIP space with a double precision.

1. Implement the elementary LIP operations $\mathbin{\triangle\!\!\!\!\!+}$, $\mathbin{\triangle\!\!\!\!\!\times}$ and $\mathbin{\triangle\!\!\!\!\!-}$.
2. Test these operators on the image 'breast'.

3 LIP dynamic expansion

The LIP model enables to define an image transformation that expanses, in an optimal way, the overall dynamic range (gray-tones) while preserving a physical or visual sense. Let a graytone function denoted f be defined on the spatial support $D \subset \mathbb{R}^2$. Its upper and lower bounds are denoted $f_u = \sup_{x \in D} f(x)$ and $f_l = \inf_{x \in D} f(x)$, with $0 < f_l < f_u < M_0$.

The dynamic range of f on D is defined by $R(f) = f_u - f_l$. The LIP scalar multiplication of f by a real number $\lambda > 0$ yields to the following dynamic :

$$R(\lambda \triangle\!\!\!\times f) = \lambda \triangle\!\!\!\times f_u - \lambda \triangle\!\!\!\times f_l \tag{7}$$

It has been shown [2] that there exists an optimal value $\lambda_0(f)$ that maximizes the dynamic range, i.e.:

$$R(\lambda_0(f) \triangle\!\!\!\times f) = \max_{\lambda > 0} R(\lambda \triangle\!\!\!\times f). \tag{8}$$

?

1. Determine the explicit (analytic) expression of the parameter $\lambda_0(f)$.
2. Calculate the value of the parameter $\lambda_0(f)$ for the image 'breast'.
3. Enhance the image and compare the result with the histogram equalization.

Histogram equalization in MATLAB® is performed with the histeq function.

In python, one can use the scikit-image module, and the skimage.exposure.equalize_hist function.

4 LIP edge detection

?

1. Implement the Sobel filter within the LIP framework.
2. Compare the results with the classical Sobel filter.

References

[1] M. Jourlin and J. C. Pinoli. Logarithmic image processing. *Acta Stereologica*, 6:651–656, 1987. 1

[2] Michel Jourlin and Jean-Charles Pinoli. Image dynamic range enhancement and stabilization in the context of the logarithmic image processing model. *Signal Process.*, 41(2):225–237, January 1995. 3

[3] J. C. Pinoli. *Contribution à la modélisation, au traitement et à l'analyse d'image*. PhD thesis, Département de Mathématiques, Université de Saint-Etienne, France, 1987. 1

[4] J. C. Pinoli. The logarithmic image processing model: Connections with human brightness perception and contrast estimators. *Journal of Mathematical Imaging and Vision*, 7(4):341–358, October 1997. 1