Tutorial: Shape Diagrams

The objective is to study some shape diagrams and the possibility to define properties that may be useful in order to distinguish the different objects.

Shape diagrams are representations of single shapes (connected compact sets, see [2, 3, 4]) as points in the 2-D unit square plane. They are based on inequalities between 6 geometrical measurements: area A, perimeter P, radius of the inscribed circle r, radius of the circumscribed circle R, minimum Feret diameter ω and maximum Feret diameter d. In this way, the morphometrical functionals used in the different shape diagrams are normalized ratios of such geometrical functionals. The following table shows the morphometrical functionals for non-convex sets:

Geometrical functionals	Inequalities	Morphological functionals
r, R	$r \leq R$	r/R
ω, R	$\omega \leq 2R$	$\omega/2R$
A, R	$A \leq \pi R^2$	$A/\pi R^2$
d, R	$d \leq 2R$	$d/2R$
r, d	$2r \leq d$	$2r/d$
ω, d	$\omega \leq d$	ω/d
A, d	$4A \leq \pi d^2$	$4A/\pi d^2$
R, d	$\sqrt{3}R \leq d$	$\sqrt{3}R/d$
r, P	$2\pi r \leq P$	$2\pi r/P$
ω, P	$\pi\omega \leq P$	$\pi\omega/P$
A, P	$4\pi A \leq P^2$	$4\pi A/P^2$
d, P	$2d \leq P$	$2d/P$
R, P	$4R \leq P$	$4R/P$
r, A	$\pi r^2 \leq A$	$\pi r^2/A$
r, ω	$2r \leq \omega$	$2r/\omega$

Table 1: Morphometrical functionals.

1 Geometrical functionals

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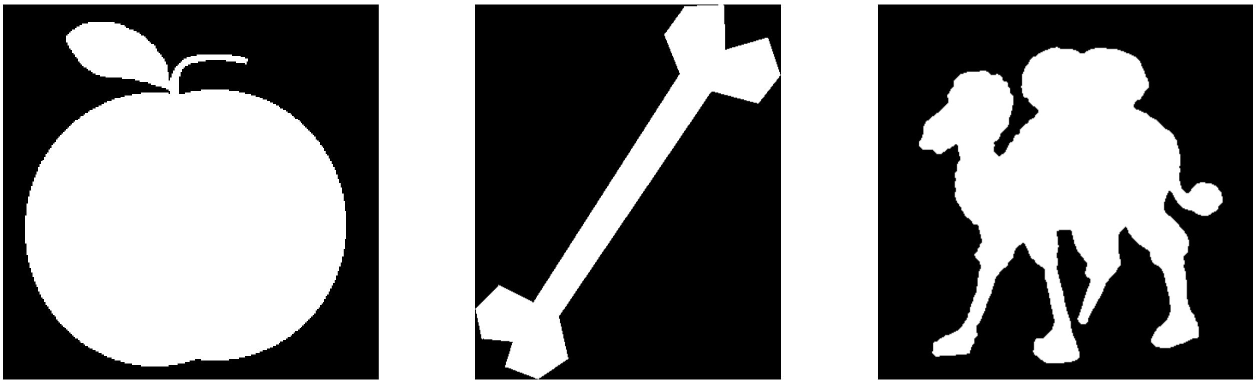
Code functions in order to evaluate the different parameters:

- the area, Crofton perimeter and Feret diameters have been already presented in the tutorial about integral geometry.



(a) Apple. (b) Bone. (c) Camel.

Figure 1: The different processes will be applied on images from the Kimia database [1, 5].

- the radius of the inscribed circle can be defined from the ultimate erosion of a set.

The function bwdist computes the distance map of a binary image.

The function scipy.ndimage.morphology.distance_transform_cdt computes the distance map of a binary image (chamfer distance transform).

2 Morphometrical functionals

Code and evaluate some of the morphometrical functionals listed in the table 1. Note that each of them has a physical meaning, e.g. $\frac{4\pi A}{P^2}$ (circularity), $\frac{4A}{\pi d^2}$ (roundness), $2\omega/P$ (thinness).

3 Shape diagrams

- Visualize the different shape diagrams for all the images (from the Kimia database) within the three classes 'apple', 'bone' and 'camel'. The Fig.2 illustrates the result for the shape diagram ($x = 2r/d, y = P/\pi d$).
- Which shape diagram is the most appropriate for the discrimination of such objects?

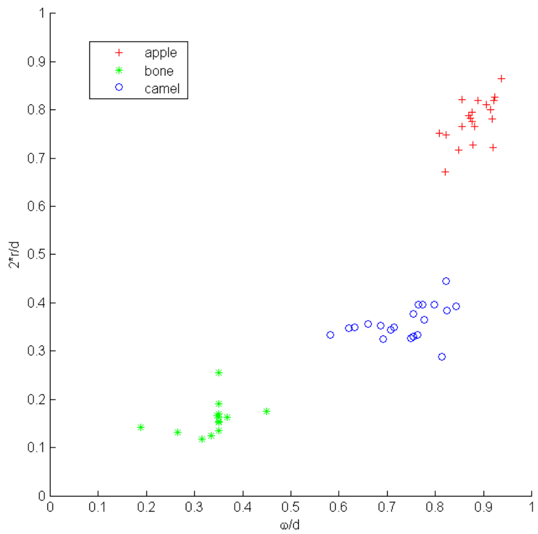


Figure 2: Example of a shape diagram.

4 Shape classification

- Use a K-means clustering method for automatic classification of such shapes.
- Propose a method to quantify the classification accuracy for each shape diagram.

References

[1] http://vision.lems.brown.edu/content/available-software-and-databases. 2

[2] Séverine Rivollier, Johan Debayle, and Jean-Charles Pinoli. Shape diagrams for 2d compact sets-part i: analytic convex sets. australian journal of. *The Australian Journal of Mathematical Analysis and applications*, 7(2), 2010. 1

[3] Séverine Rivollier, Johan Debayle, and Jean-Charles Pinoli. Shape diagrams for 2d compact sets-part ii: analytic simply connected sets. *The Australian Journal of Mathematical Analysis and applications*, 7(2), 2010. 1

[4] Séverine Rivollier, Johan Debayle, and Jean-Charles Pinoli. Shape diagrams for 2d compact sets-part iii: convexity discrimination for analytic and discretized simply connected sets. *The Australian Journal of Mathematical Analysis and applications*, 7(2), 2010. 1

[5] Daniel Sharvit, Jacky Chan, Huseyin Tek, and Benjamin B Kimia. Symmetry-based indexing of image databases. In *Content-Based Access of Image and Video Libraries, 1998. Proceedings. IEEE Workshop on*, pages 56–62. IEEE, 1998. 2