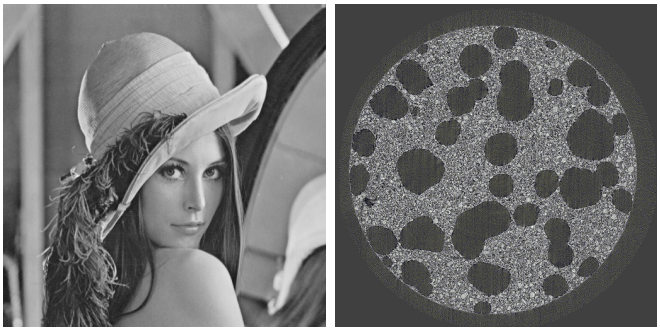
Tutorial: Morphological Geodesic Filtering

Note

This tutorial aims to test different morphological filters and particularly the geodesic ones (using reconstruction) for gray-level images.

The different processes will be applied on the following images:



(a) Lena (b) 2-D section of a cement paste (X-ray tomography)

1 Morphological centre

The morphological centre is an auto-dual filter using a family of operators $\{\psi_i\}_i$:

$$C(f) = (f \vee (\wedge\{\psi_i(f)\})) \wedge (\vee\{\psi_i(f)\}) \tag{1}$$

- Implement this transformation with the family $\{\gamma\phi\gamma, \phi\gamma\phi\}$ where γ denotes the morphological opening and ϕ the morphological closing.
- Test this operator by varying the size of the structuring element.
- Add a 'salt and pepper' noise to the 'Lena' image and compare the morphological center with the median filtering.
- Try to reduce the noisy image 'cement paste'.

2 Alternate sequential filters (ASF)

ASF can be defined from a family of openings and closings:

$$N_i(f) = \gamma_i\phi_i \circ \gamma_{i-1}\phi_{i-1} \ldots \gamma_2\phi_2 \circ \gamma_1\phi_1(f) \quad (2)$$
$$M_i(f) = \phi_i\gamma_i \circ \phi_{i-1}\gamma_{i-1} \ldots \phi_2\gamma_2 \circ \phi_1\gamma_1(f) \quad (3)$$

where γ_k (resp. ϕ_k) denotes the opening (resp. closing) with a structuring element of size k.

- Implement these two operators.
- Test these transformations on the noisy image by varying the parameter i of the filter.

3 Reconstruction filters

The geodesic dilation of size 1 and n are respectively defined as:

$$\delta_f(g) = \wedge(\delta_{B_1}(g), f) \quad (4)$$
$$\delta_f^n(g) = \delta_f(\delta_f \ldots (\delta_f(g))) \quad (5)$$

where δ_{B_1} denotes the morphological dilation with a disk of radius 1 as structuring element. The opening ($\gamma_k^{rec}(f)$) and closing ($\phi_k^{rec}(f)$) by reconstruction are then defined as:

$$\gamma_k^{rec}(f) = \vee\{\delta_f^n(\epsilon_{B_k}(f)), n > 0\} \quad (6)$$
$$\phi_k^{rec}(f) = M - \gamma_k^{rec}(M - f) \quad (7)$$

where ϵ_{B_k} denotes the morphological erosion with a disk B of radius k as structuring element.

- Implement these two operators $\gamma_k^{rec}(f)$ and $\phi_k^{rec}(f)$.
- Test these transformations by varying the parameter k of the filter.
- Implement and test (on the noisy image) the filters of morphological center and ASF using the geodesic operators. Compare with the classical ones.