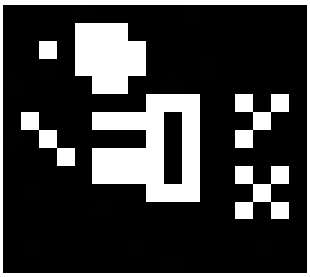
Tutorial: Topological Description

Note

The objective of this tutorial is to classify all foreground points of a binary image according to their topological signification: interior, isolated, border.... The reader can refer to [1] for more details.

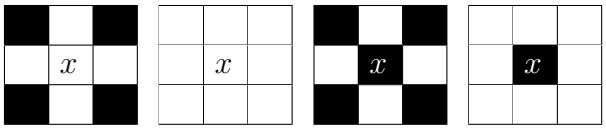
The different processes will be realized on the following binary image.



(a) test

Preliminary definitions:

- y is 4-adjacent to x if $|y_1 - x_1| + |y_2 - x_2| \leq 1$.
- y is 8-adjacent to x if $\max(|y_1 - x_1|, |y_2 - x_2|) \leq 1$.
- $V_4(x) = \{y : y \text{ is 4-adjacent to } x\}$; $V_4^*(x) = V_4(x) \backslash \{x\}$.
- $V_8(x) = \{y : y \text{ is 8-adjacent to } x\}$; $V_8^*(x) = V_8(x) \backslash \{x\}$.
- a n-path is a point sequence $(x_0, \ldots, x_k)$ with x_j n-adjacent to x_{j-1} for $j = 1, \ldots, k$.
- two points $x, y \in X$ are n-connected in X if there exists a n-path $(x = x_0, \ldots, x_k = y)$ such that $x_j \in X$. It defines an equivalence relation.
- the equivalence classes of the previous binary relation are the n-components of X.



(a) $V_4(x)$ (b) $V_8(x)$ (c) $V_4^*(x)$ (d) $V_8^*(x)$

Figure 1: Different neighborhoods. By convention, pixels in white are of value 1, in black of value 0.

1 Connectivity numbers

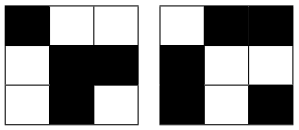
Let $Comp_n(X)$ be the number of n-components ($n = 4$ or $n = 8$ within the selected topology V_4 or V_8) of the set X of foreground points (object). We define the following set:

$$CAdj_n(x, X) = \{C \in Comp_n(X) : C \text{ is n-adjacent to } x\}$$

We select the 8-connectivity for the set X of foreground points (object) and the 4-connectivity for the complementary $\overline{X}$.

Note

Warning: the definition of $CAdj_n$ introduces the n-adjacency to the central pixel x. In the case of the following configuration (Fig.2), $T_8 = 2$, $\bar{T}_8 = 2$ and $TT_8 = 3$.



(a) X (b) $V_8^*(x) \cap X$

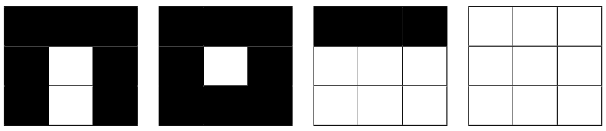
Figure 2: The pixel in the bottom right corner is not C-adjacent-4 to x.

1. Create a function for determining the connectivity number: $T_8(x, X) = \#CAdj_8(x, V_8^*(x) \cap X)$,
2. Create a function for determining the connectivity number: $\overline{T}_8(x, X) = \#CAdj_4(x, V_8^*(x) \cap \overline{X})$,
3. Create a function for determining the number: $TT_8(x, X) = \#(V_8^*(x) \cap X)$.

Test these functions on some foreground points of the image 'test'.

Use the functions bwlabel and bwlabeln.

Use scipy.ndimage.measurements.



(a) end (b) isolated (c) border (d) interior

Figure 3: Points configurations.

2 Topological classification of binary points

From the connectivity numbers $T_8(x, X)$, $\overline{T}_8(x, X)$ and $TT_8(x, X)$, it is possible to classify a foreground point x within the binary image X according to its topological signification:

$T_8(x, X)$	$\overline{T}_8(x, X)$	$TT_8(x, X)$	Type
0	1		isolated point
1	0		interior point
1	1	> 1	border point
1	1	1	end point
2	2		2-junction point
3	3		3-junction point
4	4		4-junction point

The following Fig. 3 shows the classification of 4 points.

With the help of this table, classify the points of the image 'test'.

References

[1] Michel Couprie and Gilles Bertrand. Discrete topological transformations for image processing. In *Digital Geometry Algorithms*, pages 73–107. Springer, 2012. 1