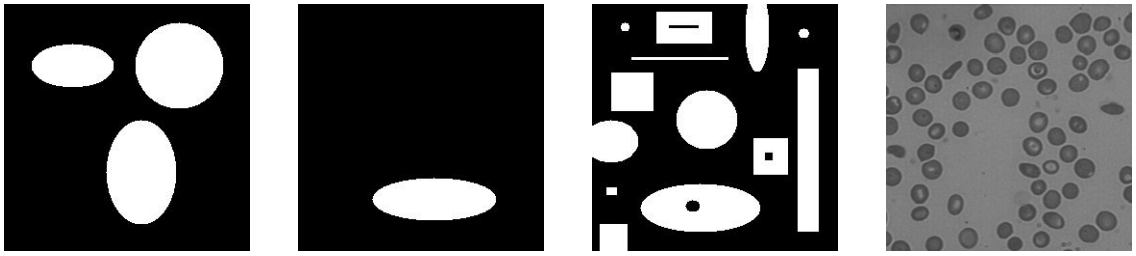
Tutorial: Binary Mathematical Morphology

The objective of this tutorial is to process binary images with the elementary operators of mathematical morphology. More particularly, different image transformations, based on the morphological reconstruction, will be studied (closing holes, removing small objects...).

The different transformations will be applied on the following images Fig. 1:



(a) I_1. (b) M. (c) I_2. (d) Cells.

Figure 1: Images to use for this tutorial.

1 Introduction to mathematical morphology

Mathematical morphology started in the 1960s with Serra and Matheron [1]. It is based on Minkowski addition of sets. The main operators are erosion and dilation, and by composition, opening and closing. More informations can be found in [2].

The erosion of a binary set A by the structuring element B is defined by:

$$\varepsilon_B(A) = A \ominus B = \{z \in A | B_z \subseteq A\} \tag{1}$$

where, $B_z = \{b + z | b \in B\}$.

The dilation can by obtained by:

$$\delta_B(A) = A \oplus B = \{z \in A | (B^s)_z \cap A \neq \varnothing\} \tag{2}$$

where $B^s = \{x \in E | -x \in B\}$ is the symmetric of B.

By composition, the opening is defined as: $A \circ B = (A \ominus B) \oplus B$. The closing is defined as: $A \bullet B = (A \oplus B) \ominus B$.

2 Elementary operators

Test the functions of dilation, erosion, opening and closing on the image I_2 by varying:

1. the shape of the structuring element,
2. the size of the structuring element.

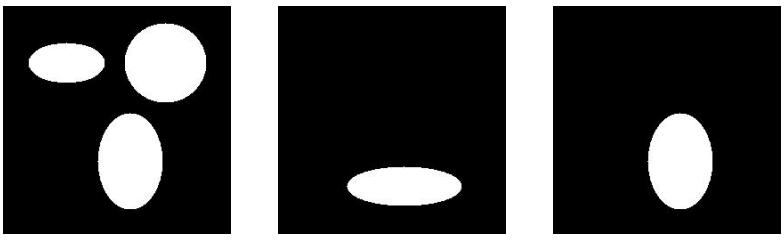
The MATLAB® functions are imdilate, imerode, imopen and imclose. The function strel creates a structuring element.

The python functions come from the python module scipy.ndimage.morphology. Useful functions are binary_dilation, binary_erosion, binary_opening and binary_closing. The function ndimage.generate_binary_structure creates a structuring element.

3 Morphological reconstruction

The operator of morphological reconstruction ρ is very powerful and largely used for practical applications. The principle is very simple. We consider two binary images: I_1 (the studied binary image) and M (the marker image). The objective is to reconstruct the elements of I_1 marked by M as illustrated in the Fig. 2.

To do this, we iteratively dilate the marker M while being included in I_1 (see Eq.3). In order to guarantee this inclusion in A, we keep from each dilated set its intersection with I_1. The algorithm is stopped when the process dilation-intersection is equal to the identity transformation (convergence).



(a) I_1 (b) M (c) $rec(I_1, M)$

Figure 2: Illustration of morphological reconstruction of I_1 by M.

Let $\delta_I^c(M) = \delta_{B_1}(M) \cap I$ be the dilation of the marker set M constrained to the set I. Then, the morphological reconstruction is defined as:

$$\rho_I(M) = \lim_{n \to \infty} \underbrace{\delta_I \circ \ldots \circ \delta_I(M)}_{n \text{ times}} \tag{3}$$

The algorithm of this morphological reconstruction is here detailed:
Data: image I and marker M
Result: reconstructed image $rec(I, M)$
$r = area(M)$;
$s = 0$;
while $r \neq s$ **do**
  $s = r$;
  $M = I \cap (M \oplus B_1)$;
  $r = area(M)$;
end
$rec(I, M) = M$;

1. Implement the algorithm.
2. Test this operator with the images I_1 and M.

The function bwarea evaluates the area of a 2D binary object.

Evaluate the area of a set may be done by counting the number of its pixels.

4 Operators by reconstruction

Using the reconstruction operator, implement the 3 following transformations:

1. removing the border objects,
2. removing the small objects,
3. closing the object holes.

Test these operators on the image I_2.

The morphological reconstruction function to use is imreconstruct.

The morphological reconstruction function to use is binary_propagation in the module ndimage.morphology.

5 Cleaning of the image of cells

1. Threshold the image of cells (Fig. 1d).
2. Process the resulting binary image with the 3 cleaning processes of the previous question.

References

[1] Jean Serra. *Image Analysis and Mathematical Morphology.* London: academic press, 1982. 1

[2] Pierre Soille. *Morphological Image Analysis: Principles and Applications*. Springer-Verlag New York, Inc., Secaucus, NJ, USA, 2nd edition, 2003. 1