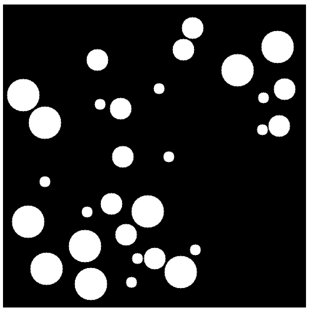
# Tutorial: Granulometry

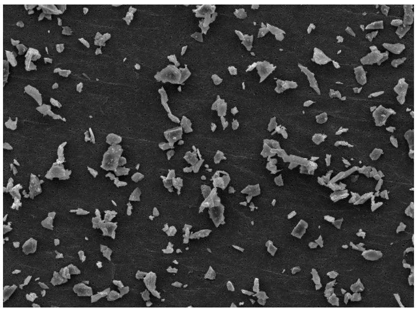
**Note**

This tutorial aims to compute specific image measurements on digital images. It consists in determining the size distribution of the 'particles' included in the image to be analyzed.

The image measurements will be realized on a simulated image of disks and an image of silicon carbide powder acquired by Scanning Electron Microscopy (SEM, Fig. 1).



(a) Simulated image.



(b) Real image of powder, by SEM.

Figure 1: These images are used for computing a granulometry of objects by mathematical morphology.

## 1 Morphological granulometry

### 1.1 Introduction to mathematical morphology

The mathematical morphology operations are based upon the Minkowski addition, denoted $\oplus$. If $A$ and $S$ are two sets of an euclidean space $\mathcal{E}$, the Minkowski addition of $A$ and $S$ is the set resulting of the addition of all pairs of elements of $A$ and $S$ (Eq. 1).

$$\forall (A,S) \in \mathcal{E}^2, A \oplus S \quad = \{a+b | a \in A, b \in S\} = \bigcup_{a\in A, b\in S} \{a+b\} \tag{1}$$
$$\forall (A,S) \in \mathcal{E}^2, A \ominus S \quad = \{a-b | a \in A, b \in S\} = \bigcup_{a\in A, b\in S} \{a-b\} \tag{2}$$

The operations $\delta_S(A)$ and $\varepsilon_S(A)$ respectiveley denote the dilation and the erosion of the set $A$ by the structuring element $S$.

$$\delta_S(A) \quad = A \oplus S$$
$$\varepsilon_S(A) \quad = A \ominus \check{S}$$

The morphological opening $\phi_S$ and closing $\delta_S$ are respectively defined by:

$$\begin{aligned} \phi_S &= \varepsilon_S \circ \delta_S \\ \gamma_S &= \delta_S \circ \varepsilon_S \end{aligned}$$

These operations are widely used, especially when dealing with the shape of objects, as well as their size. More informations can be found in [1].

With Matlab, the useful functions are imopen for the morphological opening, imreconstruct for the geodesic reconstruction. For counting the number of objects, one can use bweuler is the case of objects with no hole, or bwlabel for a labelling and counting algorithm.

With Python, the useful functions are binary_opening of the module scipy.ndimage for the morphological opening, binary_propagation for the geodesic reconstruction. For counting the number of objects, one can use measurements.label for a labelling and counting algorithm.

### 1.2 Granulometry

- Load and visualize the image 'simulation' Fig.1.
- Generate $n$ images by applying morphological openings (with reconstruction) to the binary image with the use of structuring elements of increasing size: $1 < \cdots < n$.
- Compute the morphological granulometry expressed in terms of surface area density. It consists in calculating the specific surface area of the grains in relation with the size $n$ of the structuring element.
- Compute the morphological granulometry expressed in terms of number density. It consists in calculating the specific number of grains in relation with the size $n$ of the structuring element.
- Compare and discuss.

## 2 Real application

- Load and visualize the image 'powder' of Fig. 1b.
- Realize the image segmentation step. I can simply consist in a binarization by a global threshold, followed by hole filling. Noise can also be removed my

mathematical morphology opening.

- Compute the morphological granulometry (surface area, number).
- What can you conclude?

## References

[1] Pierre Soille. *Morphological Image Analysis: Principles and Applications*. Springer-Verlag New York, Inc., Secaucus, NJ, USA, 2nd edition, 2003. 2