Tutorial: Morphological skeletonization

Note

This tutorial aims to skeletonize objects with specific tools from mathematical morphology (thinning, maximum ball...).

The different processes will be applied on the following image:

1 Hit-or-miss transform

The hit-or-miss transformation enables specific pixel configurations to be detected. Based on a pair of disjoint structuring elements $T = (T^1, T^2)$, this transformation is defined as:

$$\eta_T(X) = \{x, T^1_x \subseteq X, T^2_x \subseteq X^c\} \quad (1)$$
$$= \epsilon_{T^1}(X) \cap \epsilon_{T^2}(X^c) \quad (2)$$

where $\epsilon_B(X)$ denotes the erosion of X using the structuring element B.

1. Implement the hit-or-miss transform.
2. Test this operator with the following pair of disjoint structuring elements:

$$\begin{array}{|c|c|c|}\hline +1 & +1 & +1 \\ \hline 0 & +1 & 0 \\ \hline -1 & -1 & -1 \\ \hline \end{array} \qquad \left(T^1 = \begin{array}{|c|c|c|}\hline 1 & 1 & 1 \\ \hline 0 & 1 & 0 \\ \hline 0 & 0 & 0 \\ \hline \end{array}, \quad T^2 = \begin{array}{|c|c|c|}\hline 0 & 0 & 0 \\ \hline 0 & 0 & 0 \\ \hline 1 & 1 & 1 \\ \hline \end{array} \right)$$

where the points with $+1$ (resp. -1) belong to T^1 (resp. T^2).

2 Thinning and thickening

Using the hit-or-miss transform, it is possible to make a thinning or thickening of a binary object in the following way:

$$\begin{aligned} \theta_T(X) &= X \backslash \eta_T(X) \quad (3) \\ \chi_T(X) &= X \cup \eta_T(X^c) \quad (4) \end{aligned}$$

These two operators are dual in the sense that $\theta_T(X) = (\chi_T(X^c))^c$.

1. Implement these two transformations.
2. Test these operators with the previous pair of structuring elements.

3 Topological skeleton

By using the two following pairs of structuring elements with their rotations (90°) in an iterative way (8 configurations are thus defined), the thinning process converges to a resulting object which is homothetic (topologically equivalent) to the initial object.

$+1$	$+1$	$+1$
0	$+1$	0
-1	-1	-1

0	$+1$	0
$+1$	$+1$	-1
0	-1	-1

1. Implement this transformation (the convergence has to be satisfied).
2. Test this operator and comment.

4 Morphological skeleton

A ball $B_n(x)$ with center x and radius n is maximum with respect to the set X if there exists neither indice k nor centre y such that:

$$B_n(x) \subseteq B_k(y) \subseteq X$$

In this way, the morphological skeleton of a set X is constituted by all the centers of maximum balls. Mathematically, it is defined as:

$$S(X) = \bigcup_r \epsilon_{B_r(0)}(X) \backslash \gamma_{B_1(0)}(\epsilon_{B_r(0)}(X)) \quad (5)$$

1. Implement this transformation.
2. Test this operator and compare it with the topological skeleton.