Tutorial: Multiscale Analysis

Note

This tutorial aims to study some multiscale image processing ans analysis methods, based on pyramidal and scale-space representations.

The different processes will be applied on the following MR image.

Figure 1: Brain MR image.

1 Pyramidal decomposition and reconstruction

1.1 Decomposition

The algorithm of the pyramidal decomposition is given below:

Data: image A_0
Result: pyramid of approximations $\{A_i\}_i$, pyramid of details$\{D_i\}_i$
for *i=1 to 3* **do**
  filtering: $F = filt(A_{i-1})$;
  subsampling: $A_i = ech(F, 0.5)$;
  details: $D_i = A_{i-1} - ech(A_i, 2)$;
end

The initial image (with the highest resolution), denoted A_0, represents the level 0 of the pyramid. To build the pyramid, we successively carry out the following steps:

1. a Gaussian filtering,
2. a subsampling,
3. a calculation of the details (residues).

Make a pyramidal decomposition with 4 levels of the image 'brain'.

Informations

You can use the matlab command `cell` to store the images of the different levels of the pyramid. `imfilter` and `fspecial` will be used to perform the filtering process, `imresize` is used to down- or over-sampling the image.

1.2 Reconstruction

To reconstruct the original image, we carry out the following steps at each level of the pyramid:

1. an oversampling,
2. an addition of the details.

The algorithm of the pyramidal reconstruction is given below:

Data: image A_3, pyramid of details$\{D_i\}_i$
Result: reconstructed pyramid $\{B_i\}_i$
initialization : $B_3 = A_3$;
for *i=3 to 1* **do**
1 | oversampling: $R = ech(B_i, 2)$;
2 | adding details: $B_{i-1} = R + D_i$
end

1. Reconstruct the original image from the last level of the pyramid.
2. Make the same reconstruction without adding the details.
3. Calculate the resulting error between the reconstruted image and the original image.

2 Scale-space decomposition and multiscale filtering

We are going to decompose (without any sampling) the image with the highest resolution with a morphological operator, dilation or erosion. The resulting images will have the same size.

2.1 Morphological multiscale decomposition

Build the two scale-space decompositions of the 'brain' image, with a disk of increasing radius as a structuring element

Informations

See the matlab functions imdilate and imerode.

Informations

See the functions morphology.erosion and morphology.dilation from skimage.

2.2 Kramer and Bruckner multiscale decomposition

Now, we are going to use the iterative filter of Kramer and Bruckner [1] defined as:

$$MK_B^n(f) = K_B(MK_B^{n-1}(f)) \tag{1}$$

where B denotes a disk of a fixed radius r, and:

$$K_B(f)(x) = \begin{cases} D_B(f)(x) & \text{if } D_B(f)(x) - f \leq f - E_B(f)(x) \\ E_B(f)(x) & \text{otherwise} \end{cases} \tag{2}$$

where $D_B(f), E_B(f)$ represent respectively the dilation and the erosion of the image f with the structuring element B.

Implement and test this filter on the image 'brain' for different values of n.

References

[1] Henry P. Kramer and Judith B. Bruckner. Iterations of a non-linear transformation for enhancement of digital images. *Pattern Recognition*, 7(1):53 – 58, 1975.