Completion of Occluded Shapes using Symmetry

H. Zabrodsky[1] S. Peleg[1] D. Avnir[2]

[1]Institute of Computer Science
The Hebrew University of Jerusalem
91904 Jerusalem, Israel
gigi@cs.huji.ac.il

[2]Department of Organic Chemistry
The Hebrew University of Jerusalem
91904 Jerusalem, Israel

Abstract

Symmetry is usually viewed as a discrete feature: an object is either symmetric or non-symmetric. Following the view that symmetry is a continuous feature, a Continuous Symmetry Measure (CSM) has been developed to evaluate symmetries of shapes and objects. In this paper we extend the symmetry measure to evaluate the symmetry of occluded shapes. Additionally, using the symmetry measure, we reconstruct occluded shapes by locating the center of symmetry of the shape.

1 Introduction

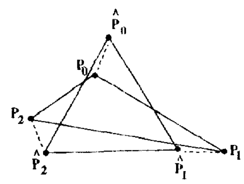
The exact mathematical definition of symmetry [1, 2] is inadequate to describe and quantify the symmetries found in the natural world nor those found in the visual world (a classic example is that of faces). Furthermore, even perfectly symmetric objects loose their exact symmetry when projected onto an image plane or retina due to occlusion, self-occlusion, digitization, etc. Previous work [4, 3] introduced a symmetry measure to quantify the deviation of shapes and objects from perfect symmetry. In this paper we deal with evaluating the deviation from perfect symmetry of incomplete data such as occluded shapes.

2 A Symmetry Measure

The **Continuous Symmetry Measure** (CSM) described in [4, 3, 5] quantifies the minimum effort necessary to turn a given shape into a symmetric shape. This effort is measured by the sum of the square distances moved by points of the shape.

A shape P is represented by a sequence of n points $\{P_i\}_{i=0}^{n-1}$. The CSM of the shape $P = \{P_i\}_{i=0}^{n-1}$ is evaluated by finding closest symmetric shape $\hat{P}$ of P (Fig.1) and computing: $s(P) = \frac{1}{n}\Sigma_{i=0}^{n-1}\|P_i - \hat{P}_i\|^2$. A geometrical algorithm for deriving the symmetry transform of a shape P was given in [4, 3, 5].

Figure 1:
The symmetric shape closest to $\{P_0, P_1, P_2\}$ is $\{\hat{P}_0, \hat{P}_1, \hat{P}_2\}$.
CSM $= \frac{1}{3}\Sigma_{i=0}^{2}\|P_i - \hat{P}_i\|^2$



C_3- Symmetry = 12.80

Representing a shape by points must precede the evaluation of symmetry (see [4]). One method represents a shape by points selected along its contour at equal angular intervals about its centroid (Fig. 2).

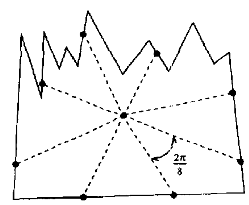


Figure 2:
Points are distributed along the contour at regular angular intervals around the centroid.

3 Symmetry of Occluded Shapes

When a symmetric object is partially occluded, we use the symmetry measure to evaluate the symmetry of the occluded shapes, locate the center of symmetry and reconstruct the symmetric shape most similar to the unoccluded original.

Angular selection of points about a point other than the centroid will give a different symmetry measure value. We define the **center of symmetry** of a shape as that point about which angular selection gives the minimum symmetry measure value. When a symmetric shape is not occluded the center of symmetry aligns with the centroid of the shape. However, the center of symmetry of truncated or occluded objects does not align with its centroid but aligns with the (unknown)

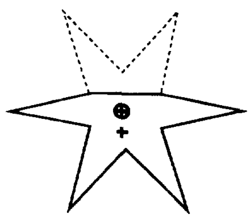


Figure 3: The CSM indicates greater symmetry when point selection is about the center of symmetry (marked by $\oplus$) than about the centroid (marked by $+$).

centroid of the unoccluded shape. Thus the center of symmetry of a shape is robust under truncation and occlusion.

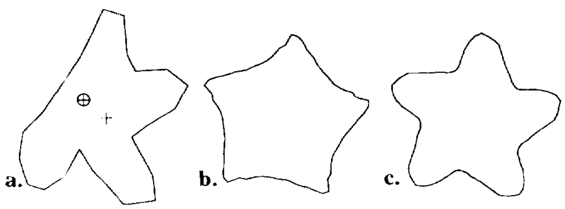


Figure 4: a) Original occluded shape, its centroid ($+$) and its center of symmetry($\oplus$). b,c) The closest C_5-symmetric shapes following angular selection about the centroid (b) and about the center of symmetry (c).

To find the center of symmetry, we use an iterative gradient descent procedure to converge from the centroid of the occluded shape to its center of symmetry.

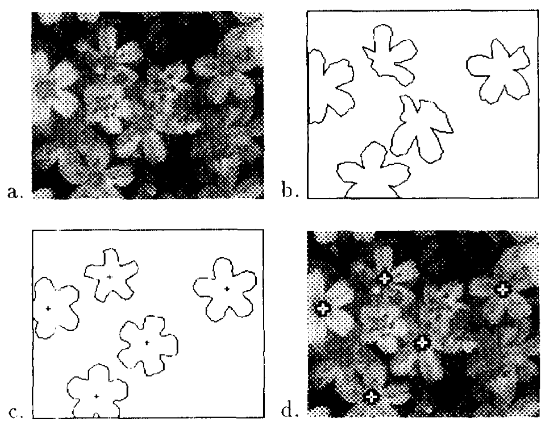


Figure 5: a-b) A collection of occluded asymmetric flowers. c) The closest symmetric shapes and their center of symmetry. d) The center of symmetry of the occluded flowers are marked by 'x'.

The closest symmetric shape obtained by angular selection about the center of symmetry is visually more similar to the original than that obtained by angular selection about the centroid of the occluded shape (Fig. 4). In Fig. 5 the center of symmetry and the closest symmetric shapes were found for several occluded flowers.

The process of reconstructing the occluded shape can be improved by replacing the averaging stage of the completion process with a robust clustering method. The improvement in reconstruction of an occluded shape is shown in Figure 6. This method improves both the shape and the localization of the reconstruction.

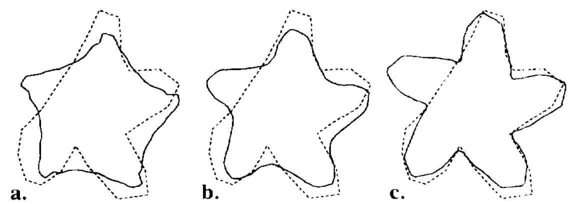


Figure 6: Reconstruction of an occluded almost symmetric shape. The original shape (dashed line) and the reconstructed shape (solid line).
The closest symmetric shape following angular selection about: a) the centroid, b) the center of symmetry, c) the center of symmetry using clustering in the completion.

References

[1] W. Miller. *Symmetry Groups and their Applications.* Academic Press, 1972.

[2] H. Weyl. *Symmetry.* Princeton Univ. Press, 1952.

[3] H. Zabrodsky, S. Peleg, and D. Avnir. Hierarchical symmetry. In *International Conference on Pattern Recognition*, volume C: Image, Speech, and Signal Analysis, pages 9–12, The Hague, August-September 1992.

[4] H. Zabrodsky, S. Peleg, and D. Avnir. A measure of symmetry based on shape similarity. In *IEEE Conference on Computer Vision and Pattern Recognition*, pages 703–706, Champaign, June 1992.

[5] H. Zabrodsky, S. Peleg, and D. Avnir. Continuous symmetry measures. *J. Am. Chem. Soc.*, 114:7843–7851, Sept 1992.