

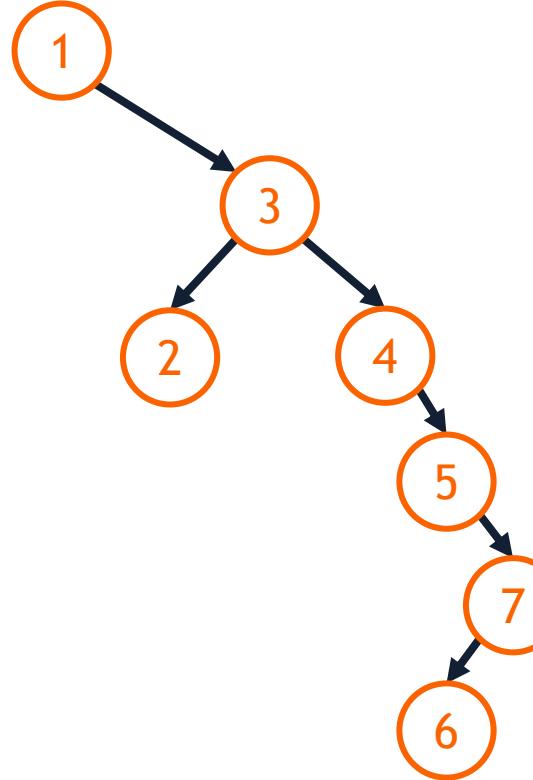
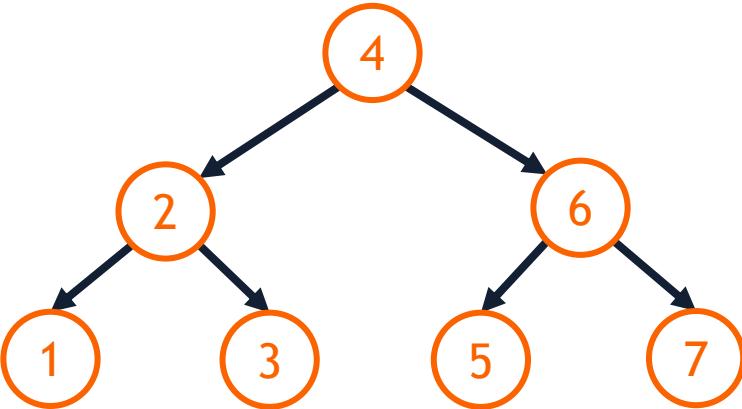
BST Analysis

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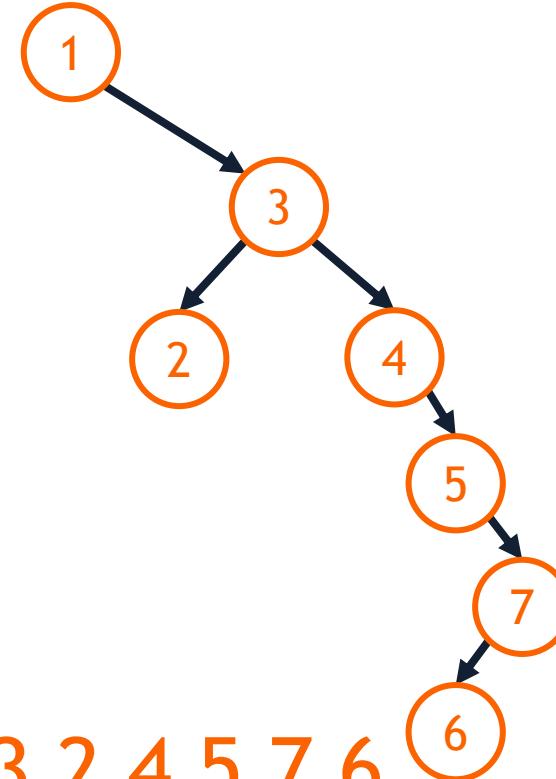
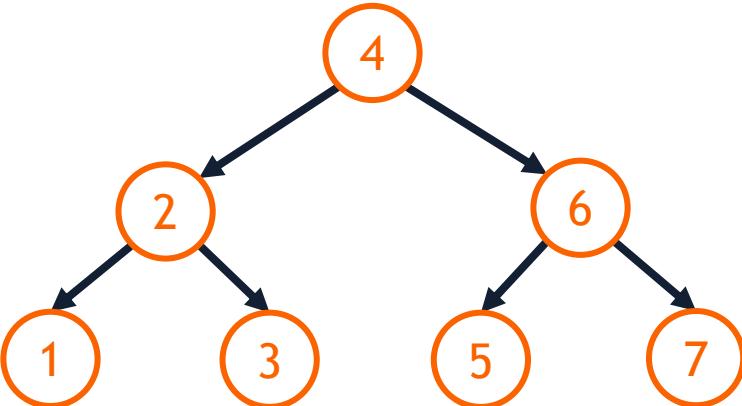
ALMA MATER
TO THIS FAIRY CHILDREN
OF THE FUTURE

Binary Search Trees (BSTs) can take on many forms and structures, even containing the same data:



BST Forms

Both trees contain the values {1, 2, 3, 4, 5, 6, 7}:



Insert Order: 4 2 3 6 7 1 5 vs.

1 3 2 4 5 7 6

Puzzle

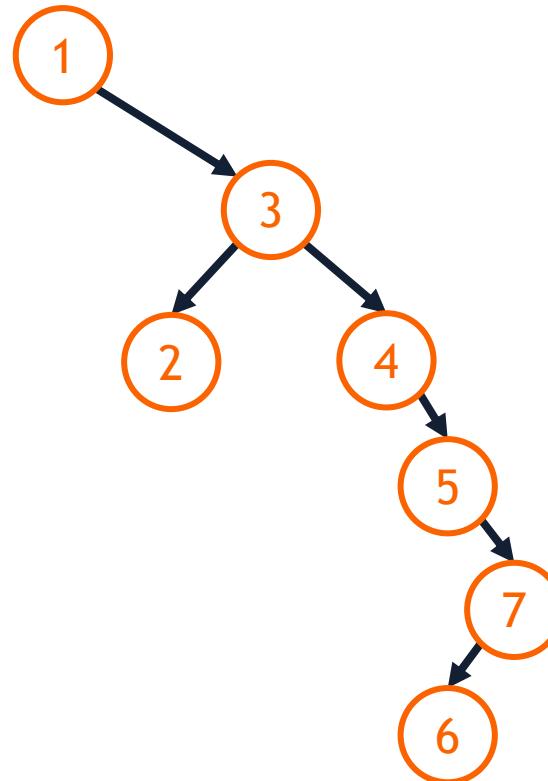
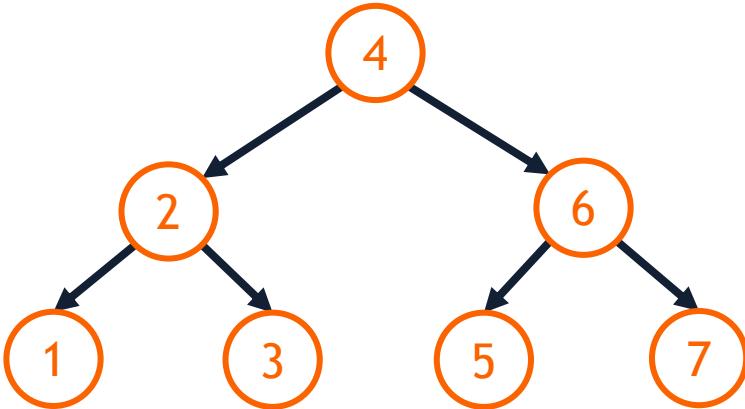
How many possible ways can we insert the same data into a BST?

Puzzle

Operation	BST Average Case	BST Worst Case	Sorted Array	Sorted List
find	$O(\lg n)$	$O(n)$	$O(\lg n)$	$O(n)$
insert			$O(n)$	
remove			$O(n)$	

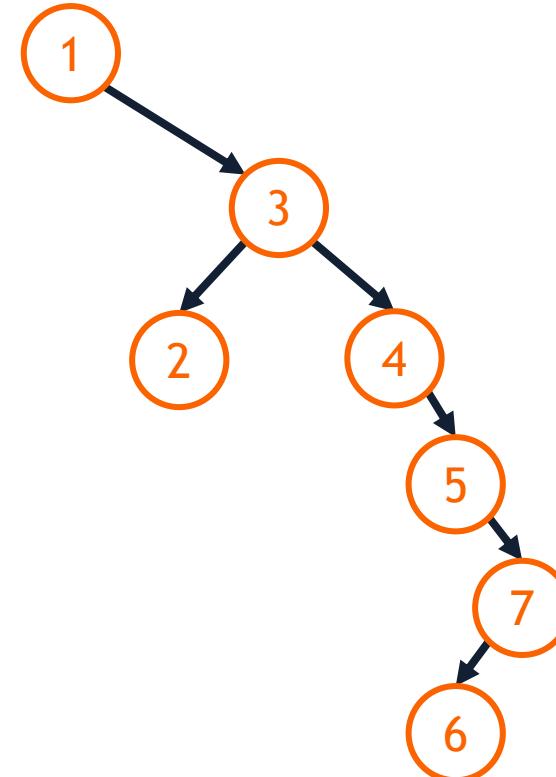
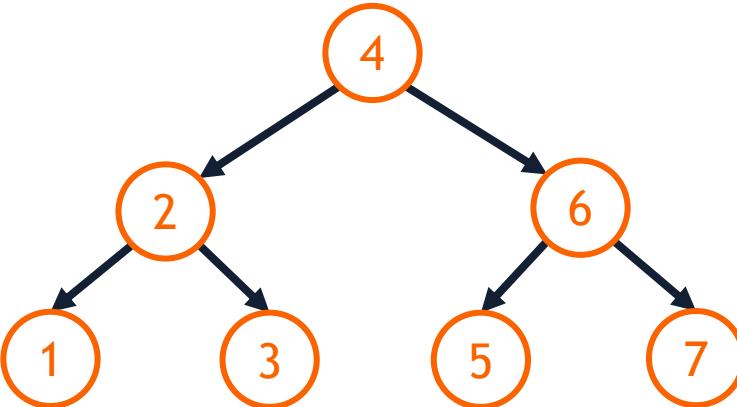
Height Balance Factor

The height balance factor (b) of a node is the difference in height between its two subtrees:



Balanced BST

A balanced BST is a BST where **every node's balance factor has a magnitude of 0 or 1**:



BST Analysis

- There are $n!$ different ways to create BSTs with the same data.
 - The worst-case BST will have a height proportional to the number of nodes.
 - An average BST will have a height proportional to the logarithm of the number of nodes.
- Using a height balance factor, we can formalize if a given BST is balanced.

