

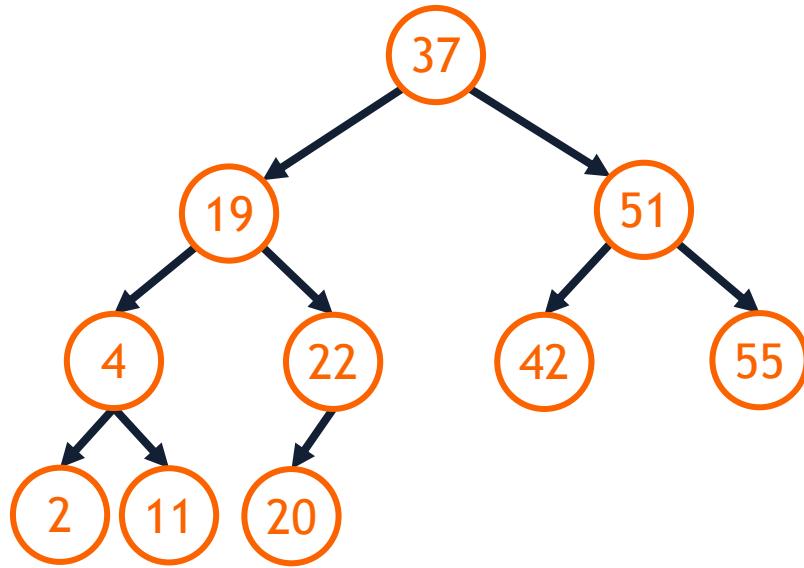
# Binary Search Tree (BST)

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**I** ILLINOIS

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TO THIS FAIRY CHILDREN  
OF THE FUTURE

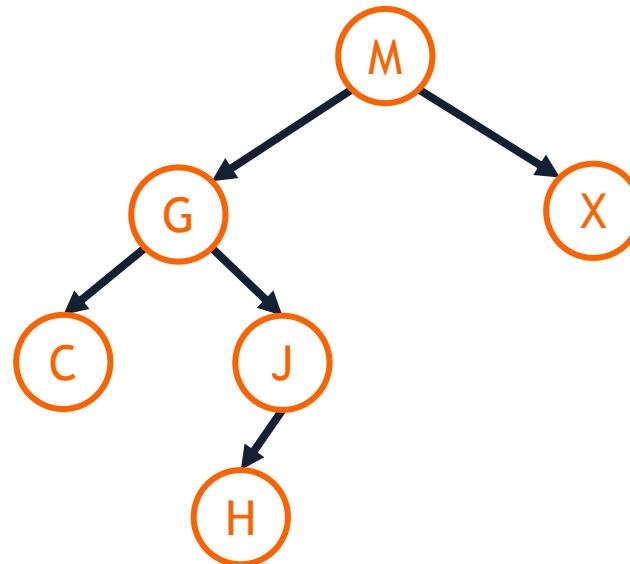
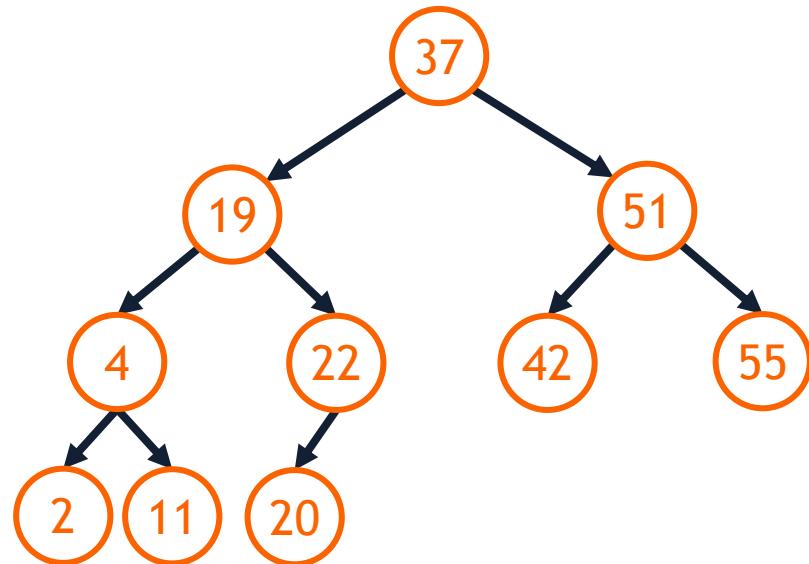
A binary search tree (BST) is an ordered binary tree capable of being used as a search structure:



# BST Order Property

A binary tree is a BST if **for every node in the tree:**

- Nodes in the **left subtree** are **less than itself**.
- Nodes in the **right subtree** are **greater than itself**.



# Dictionary

A dictionary associates a **key** with **data**:

Your login email → Your profile data

Your phone number → Your phone record

A website URL → The webpage

Your street address → Your home

# Dictionary ADT

**find** → Finds the data associated with a key in the dictionary

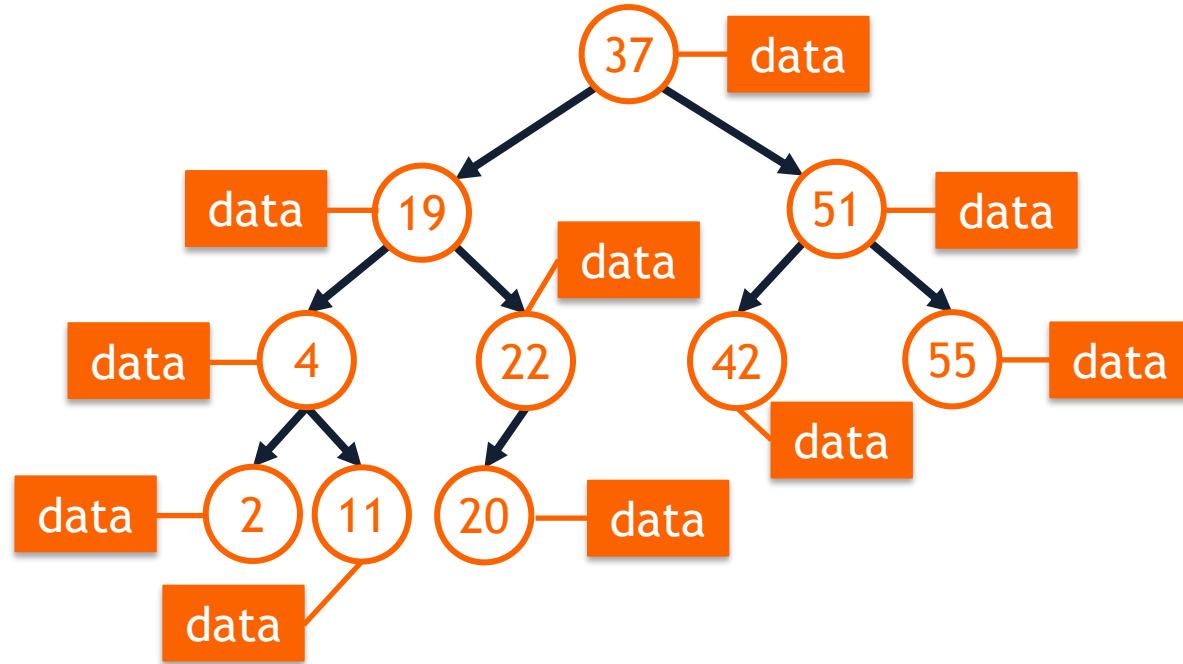
**insert** → Adds a key/data pair to the dictionary

**remove** → Removes a key from the dictionary

**empty** → Returns true if the dictionary is empty

# BST-Based Dictionary

A BST used to implement a dictionary will store both a key and data at every node:



# bst/Dictionary.h

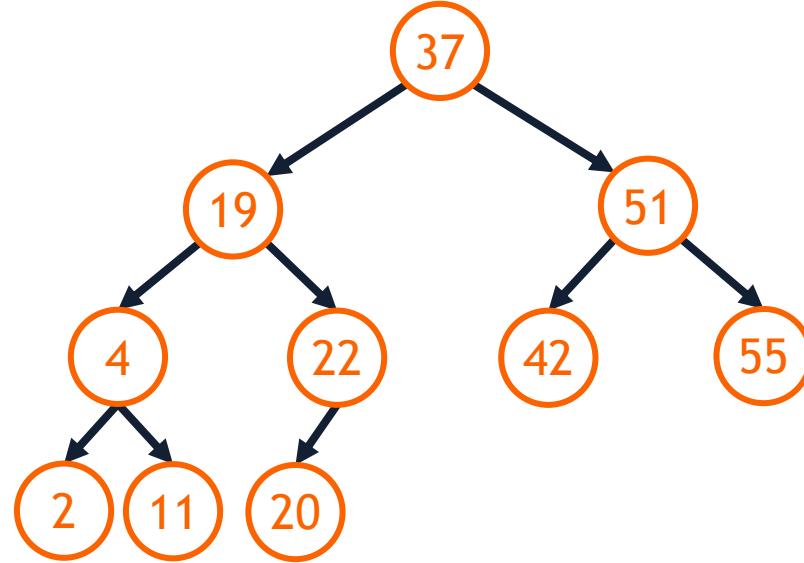
```
10 template <typename K, typename D>
11 class Dictionary {
12     public:
13         Dictionary();
14         const D & find(const K & key);
15         void insert(const K & key, const D & data);
16         const D & remove(const K & key);
17
18     private:
19         class TreeNode {
20             public:
21                 const K & key;
22                 const D & data;
23                 TreeNode *left, *right;
24                 TreeNode(const K & key, const D & data)
25                     : key(key), data(data), left(nullptr), right(nullptr) { }
26
27         TreeNode *head_;
```

# Example: Finding in a BST

find(42)

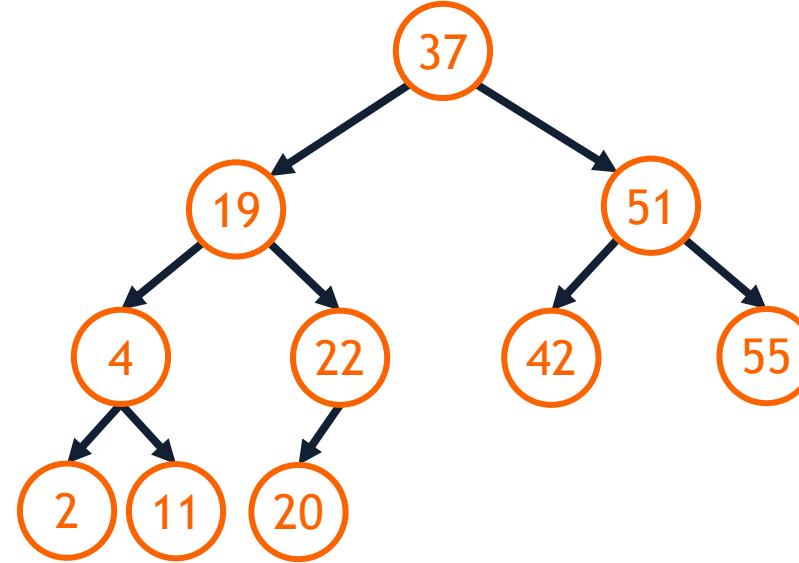
find(11)

find(17)



# Runtime of BST::find

Worst-case outcome of find:



## bst/Dictionary.cpp

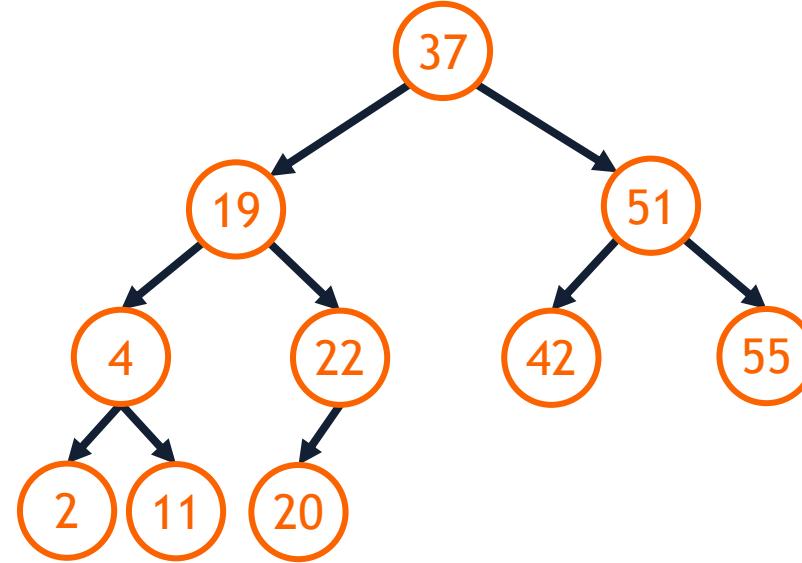
```
18 template <typename K, typename D>
19 const D & Dictionary<K, D>::find(const K & key) {
20     TreeNode *& node = _find(key, head_);
21     if (node == nullptr) { throw std::runtime_error("key not found"); }
22     return node->data;
23 }

...
...

60 template <typename K, typename D>
61 typename Dictionary<K, D>::TreeNode *& Dictionary<K, D>::_find(
62     const K & key, TreeNode *& cur) const {
63     if (cur == nullptr) { return cur; }
64     else if (key == cur->key) { return cur; }
65     else if (key < cur->key) { return _find(key, cur->left); }
66     else { return _find(key, cur->right); }
```

# Insert into a BST

insert(17)

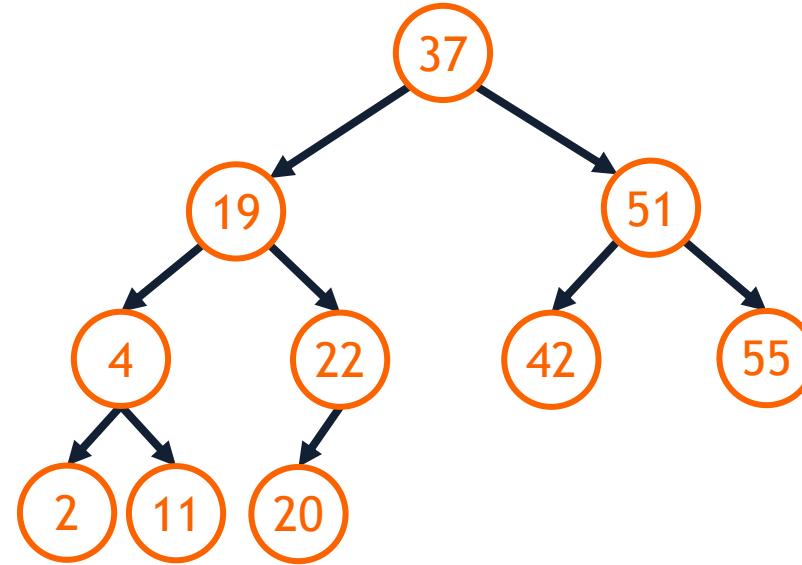


## bst/Dictionary.cpp

```
26 /**
27 * insert()
28 * Inserts `key` and associated `data` into the Dictionary.
29 */
30 template <typename K, typename D>
31 void Dictionary<K, D>::insert(const K & key, const D & data) {
32     TreeNode *& node = _find(key, head_);
33     node = new TreeNode(key, data);
34 }
```

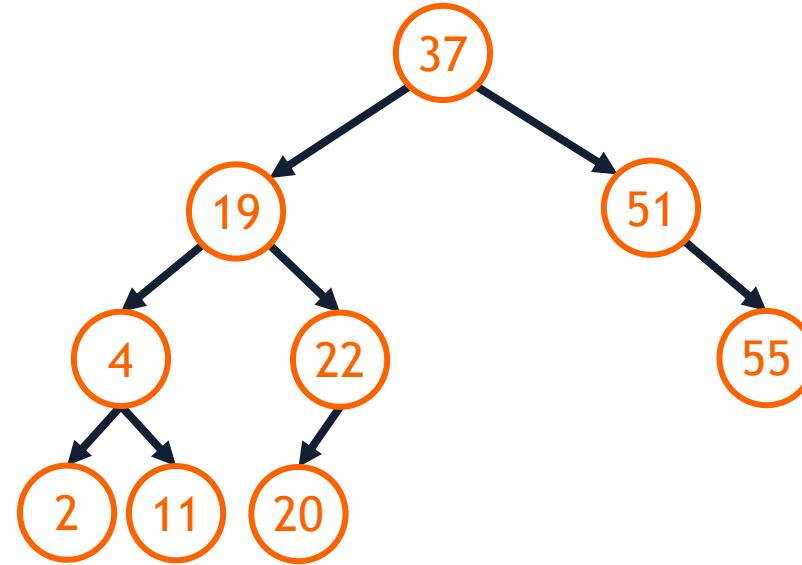
# Remove from a BST

remove(42)



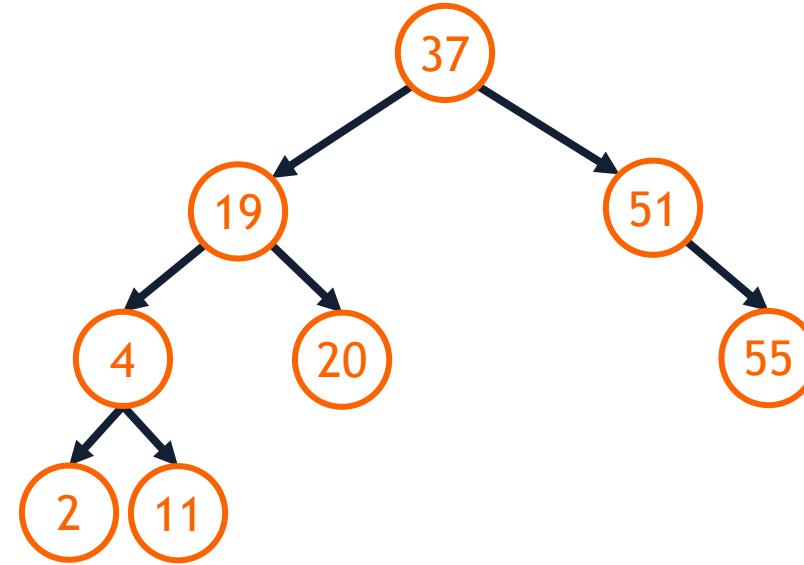
# Remove from a BST

remove(22)



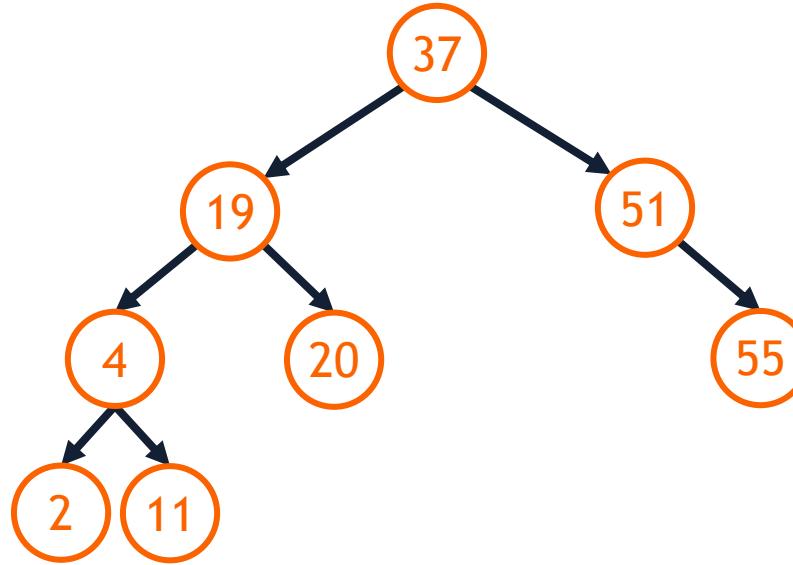
# Remove from a BST

remove(37)



# In-Order Predecessor (IOP)

The in-order predecessor is the previous node in an in-order traversal of a BST.

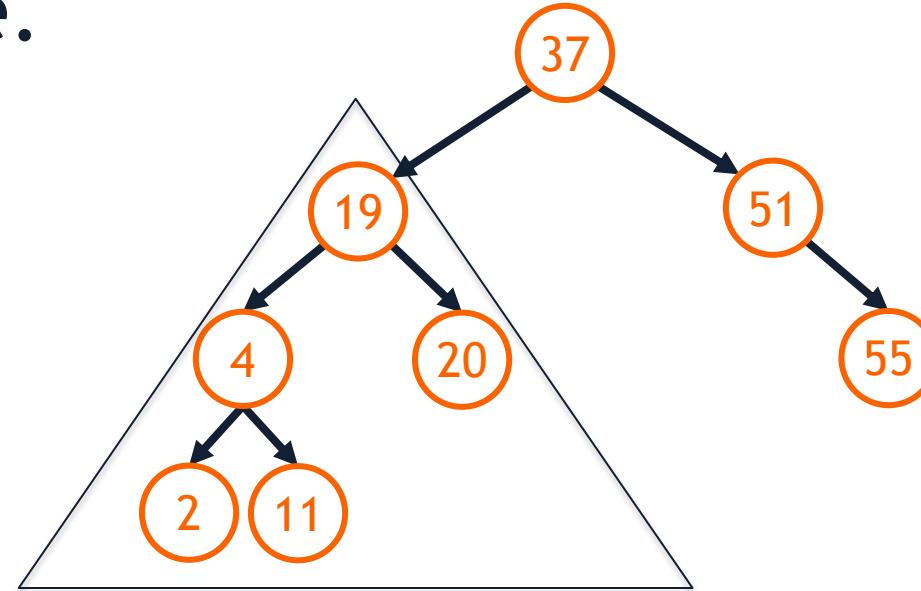


In-Order Traversal: 2 4 11 19 20 37 51 55

# In-Order Predecessor (IOP)

The IOP of a node will always be the right-most node in the node's left sub-tree.

IOP(37)



# BST::remove

**Zero children:** Simple, delete the node.

**One child:** Simple, works like a linked list.

**Two children:**

- Find the IOP of the node to be removed.
- Swap with the IOP.
- Remove the node in it's new position.

## bst/Dictionary.cpp

```
37 /**
38 * remove()
39 * Removes `key` from the Dictionary. Returns the associated data.
40 */
41 template <typename K, typename D>
42 const D & Dictionary<K, D>::remove(const K & key) {
43     TreeNode *& node = _find(key, head_);
44     return _remove(node);
45 }
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101
102 template <typename K, typename D>
103 const D & Dictionary<K, D>::_remove(TreeNode *& node) {
104     // Zero child remove:
105     if (node->left == nullptr && node->right == nullptr) {
106         const D & data = node->data;
107         delete(node);
108         node = nullptr;
109         return data;
110     }
111     ...
112 }
```

## bst/Dictionary.cpp

```
112 // One-child (left) remove
113 else if (node->left != nullptr && node->right == nullptr) {
114     const D & data = node->data;
115     TreeNode *temp = node;
116     node = node->left;
117     delete(temp);
118     return data;
119 }
120
121 // One-child (right) remove
122 else if (node->left == nullptr && node->right != nullptr) {
123     const D & data = node->data;
124     TreeNode *temp = node;
125     node = node->right;
126     delete(temp);
127     return data;
128 }
```

## bst/Dictionary.cpp

```
130 // Two-child remove:  
131 else {  
132     // Find the IOP  
133     TreeNode *& iop = _iop( node->left );  
134  
135     // Swap the node to remove and the IOP  
136     _swap( node, iop );  
137  
138     // Remove the new IOP node that was swapped  
139     return _remove( node );  
140 }  
141 }
```

## bst/main.cpp

```
15 int main() {
16     Dictionary<int, std::string> t;
17     t.insert(37, "thirty four");
18     t.insert(19, "nineteen");
19     t.insert(51, "fifty one");
20     t.insert(55, "fifty five");
21     t.insert(4, "four");
22     t.insert(11, "eleven");
23     t.insert(20, "twenty");
24     t.insert(2, "two");
25
26     cout << "t.find(51): " << t.find(51) << endl;
27
28     cout << "t.remove(11): " << t.remove(11) << " (zero child remove)" << endl;
29     cout << "t.remove(51): " << t.remove(51) << " (one child remove)" << endl;
30     cout << "t.remove(19): " << t.remove(19) << " (two child remove)" << endl;
31
32     cout << "t.find(51): " << t.find(51) << endl;
33     return 0;
34 }
```

# BST

- A binary search tree (BST) is an ordered binary tree.
- The use of the ordered property allows us to create an efficient dictionary structure.