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HW4 Chapter 6

Solution 1:

The Heavy Pill:

We know the "expected" weight of a bunch of pills. The difference between the expected weight and the actual weight will indicate which bottle contributed the heavier pill(s), assumed we select a difference number of pill(s) from each bottle.

Therefore, we would want to take 1 pill from bottle #1, 2 pills from bottle #2, 3 pills from bottle #3, and so on (20 pills from bottle #20). If all pills were 1g each, we would have (1g + 2g + … + 20g) = 210g in total. Any special pill(s) would come from the extra 0.1g pill(s).

Thus, we will use (actualWeight-210g)/0.1g = Heavy Pill Bottle Number

Solution 2:

Basketball:

P (winning game #1) = p

P (winning game #2) = $[p*p*(1-p) + p*(1-p)*p + (1-p)*p*p] + p^3$

$= 3(1-p)*p^2 + p^3$

$= 3p^2-2p^3$

Assume both winning game #1 and game #2 shares the SAME probability

=> $p = 3p^2-2p^3$ => $(2p-1)(p-1) = 0$

If $0 < p < 0.5$, we should play game #1,

Else if $0.5 < p < 1$, we should play game #2,

Else ($p = 0$ or 0.5 or 1), playing either game will result the same.

Solution 3:

Dominos:

The answer to the question is NO. Since the (8x8) chessboard initially has 32 black squares and 32 white squares. By removing the two diagonally opposite corners, we will yield 30 pieces of one color squares and 32 pieces of another color squares. Assume we have 30 black squares and 32 white squares left from the "cut".

Each domino we place on the chessboard would have to take up exactly 1 black square and 1 white square. However, since have 31 dominos but only have 30 black squares and 32 white squares left from the board. Thus covering the rest of the chessboard (after the "cut") with 31 dominos is impossible.

Solution 4:

Ants on a Triangle:

The only way for all ants NOT to collide from each other is that every ant has to walk in the same direction (wither clockwise or counterclockwise).

(a) *3-ant problem*

P (all 3 ants move in the same direction)
= P (all 3 ants move in clockwise) + P (all 3 ants move in counterclockwise)
= $0.5^3+0.5^3 = 0.25$

Hence, P (chance of one ant collide with another ant)
= 1 – P (all 3 ants move in the same direction) = 0.75

(b) *N-ant problem*

P (all N ants move in the same direction)
= P (all N ants move in clockwise) + P (all N ants move in counterclockwise)
= $0.5^N+0.5^N = 0.5^{N-1}$

Hence, P (chance of one ant collide with another ant)
= 1 – P (all N ants move in the same direction) = $1 - 0.5^{N-1}$

Solution 5:

Jugs of Water:

Steps:

#1 fill the 5-quart jug with water fully

#2 pour water from 5-quart jug into and fill the 3-quart jug fully

#3 clear the 3-quart jug and now there should be 2 quarts of water remains in the 5-quart jug

#4 pour the remaining 2 quarters of water into the 3-quart jug

#5 fill the 5-quart jug with water fully again

#6 pour water from 5-quart jug into and fill the (partially pre-filled) 3-quart jug fully

#7 Finally, the current amount of water remains in the 5-quart jug is exactly 4 quarters of water

Solution 6:

Blue-eyed Island:

Assuming that there are N people on the island and C people with blue eyes (C > 0). Also assumes that all N people would be able to look at everyone's eyes before 8:00pm of that day and make reasonable judgment.

If C = 1, the person with blue eyes with left on the 1st day after checking everyone's eye color and deduced that he is the one with blue eyes (as C > 0).

If C = 2, after the 1st day, both person with blue eyes should think that if C=1, then the person he/she sees on day 1 should be gone. If both people see another person with blue eyes still appears on the island on the 2nd day, therefore deduced that there must be 2 people with blue eyes. Thus both people with blue eyes will be gone by 2nd day.

To generalize this as C people with blue eyes, each person should be able to deduce that there are either C-1 of people with blue eyes or C people with blue eyes. So, if everyone still sees C-1 people with blue eyes on (C-1)th day, then all C people with blue eyes will be gone by day C.

Hence, with N people on the island and C people with blue eyes (C > 0), it will takes up to C days for every person with blue eyes to left the island.

Solution 7:

The Apocalypse:

The gender ratio of the new world will (also) be approximately 50% to 50%.

Logically, think of the world's populate is a gigantic string of "B" and "G" which "B" stands for boy and "G" stands for girl. Since the odds of having a male body is the same as having a female baby (both 50%), this means, by constructing the gigantic string (or aka constructing the entire population), the next new baby that is a girl or boy shares equal probability. Therefore, at the end of the "baby production process", there should be approximately the same number of "B"s and "G"s that builds the string.

Finally, this approach also make sense because if you break the string into pieces that ends with "G", it essentially presents a single (random) family's children structure if you take this from a population perspective.

As a result, the reasoning approach aligns with the question perfectly and hence we will be having approximately 50% to 50% male and female gender ratio.

<u>Please refer to *Solution7* for the simulation part of this question.</u>

Solution 8:

The Egg Drop Problem:

Start with the brute force approach. Drop Egg1 from floor #1 and up, stop until Egg1 breaks, and hence we find the threshold floor.

However, we didn't use Egg2 in this approach and the overall steps used to figure out the threshold floor are not minimized.

Next, we start with, say drop Egg1 from the 1^{st} floor. If not break, we add 10 floors (11st floor) and drop Egg1 again. Repeat this process until Egg1 breaks. Then we go to the previous selected floor that Egg1 dropped but didn't break. We then start dropping the Egg2 from ("that floor"+1 floor), and increment this process linearly until Egg2 breaks. Thus we find the threshold floor.

Ultimately, we discover that it is unavoidable to preform the linear increment test when we find that Egg1 breaks at some floor. But since the floor numbers are getting higher and higher, the chances that Egg1 will breaks will be higher for upper floor than lower floor, but we still have the same number of possible tries (linear increments) when figuring out the exact threshold floor using Egg2. This is the place which we can improve our algorithm.

We will start drop Egg1 at X floor, and then X-1 incremented floor if not breaks. (In this way, we will be preforming X+(X-1)+(X-2)+…+1 = 100 number of tries)

=> X = 13.65 ≈14

If Egg1 breaks, we then need to go back to the "previous selected floor"+1 floor and start drop Egg2 until we find out the threshold floor. In this we, as the total number of floors accumulates, the number of linear tries that Egg2 need to perform after Egg1 breaks at some floor is "minimized globally".

Solution 9:

100 Lockers:

The locker that will be left open after 100 "rounds" (start from all open locker scenario) is that its locker (number) has odd number of pair factors. Essentially, these lockers (number) are "perfect squares".

For example, locker #36 will be touched (open/close) from round (1,36), (2, 18), (3, 12), (4, 9) and (5, 5). Since (5, 5) only counts round #5, hence locker #36 will be visited 9 times, starting in open status and thus end in open status.

In conclusion, after 100 rounds, only locker #1, #4, #9, #16, #25, #36, #47, #64, #81, #100 will be left open.

Solution 10:

Poison:

What we can do is to use the test strip as a binary indicator for poisoned or un-poisoned. We'll need to take each bottle number and look at its binary representation. If there is a 1 in the X^{th} digit, then we will add a drop of this bottle's contents to test strip X. Finally, we wait 7 days to reveal the results, if test strip X is positive, then set bit X of the result value. Reading all the test strips will give us the ID of the pensioned bottle.

[Follow Up]
Please refer to *Solution10* (and relevant classes) for the simulation part of this question.