

k-Nearest Neighbor (kNN) exercise

Complete and hand in this completed worksheet (including its outputs and any supporting code outside of the worksheet) with your assignment submission. For more details see the [assignments page](#) on the course website.

The kNN classifier consists of two stages:

- During training, the classifier takes the training data and simply remembers it
- During testing, kNN classifies every test image by comparing to all training images and transferring the labels of the k most similar training examples
- The value of k is cross-validated

In this exercise you will implement these steps and understand the basic Image Classification pipeline, cross-validation, and gain proficiency in writing efficient, vectorized code.

In [1]: # Run some setup code for this notebook.

```
import random
import numpy as np
from cs682.data_utils import load_CIFAR10
import matplotlib.pyplot as plt

from __future__ import print_function

# This is a bit of magic to make matplotlib figures appear inline in the notebook
# rather than in a new window.
%matplotlib inline
plt.rcParams['figure.figsize'] = (10.0, 8.0) # set default size of plots
plt.rcParams['image.interpolation'] = 'nearest'
plt.rcParams['image.cmap'] = 'gray'

# Some more magic so that the notebook will reload external python modules;
# see http://stackoverflow.com/questions/1907993/autoreload-of-modules-in-ipython
%load_ext autoreload
%autoreload 2
```

In [2]: # Load the raw CIFAR-10 data.

```
cifar10_dir = 'cs682/datasets/cifar-10-batches-py'

# Cleaning up variables to prevent loading data multiple times (which may cause memory issue)
try:
    del X_train, y_train
    del X_test, y_test
    print('Clear previously loaded data.')
except:
    pass

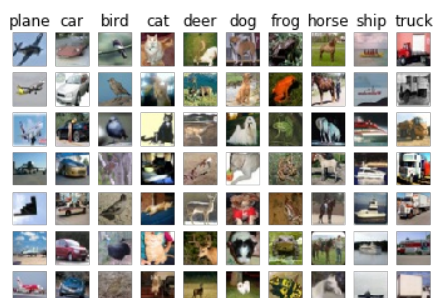
X_train, y_train, X_test, y_test = load_CIFAR10(cifar10_dir)

# As a sanity check, we print out the size of the training and test data.
print('Training data shape: ', X_train.shape)
print('Training labels shape: ', y_train.shape)
print('Test data shape: ', X_test.shape)
print('Test labels shape: ', y_test.shape)
```

```
Training data shape: (50000, 32, 32, 3)
Training labels shape: (50000,)
Test data shape: (10000, 32, 32, 3)
Test labels shape: (10000,)
```

In [3]: # Visualize some examples from the dataset.

```
# We show a few examples of training images from each class.
classes = ['plane', 'car', 'bird', 'cat', 'deer', 'dog', 'frog', 'horse', 'ship', 'truck']
num_classes = len(classes)
samples_per_class = 7
for y, cls in enumerate(classes):
    idxs = np.flatnonzero(y_train == y)
    idxs = np.random.choice(idxs, samples_per_class, replace=False)
    for i, idx in enumerate(idxs):
        plt_idx = i * num_classes + y + 1
        plt.subplot(samples_per_class, num_classes, plt_idx)
        plt.imshow(X_train[idx].astype('uint8'))
        plt.axis('off')
        if i == 0:
            plt.title(cls)
plt.show()
```



```
In [4]: # Subsample the data for more efficient code execution in this exercise
num_training = 5000
mask = list(range(num_training))
X_train = X_train[mask]
y_train = y_train[mask]

num_test = 500
mask = list(range(num_test))
X_test = X_test[mask]
y_test = y_test[mask]
```

```
In [5]: # Reshape the image data into rows
X_train = np.reshape(X_train, (X_train.shape[0], -1))
X_test = np.reshape(X_test, (X_test.shape[0], -1))
print(X_train.shape, X_test.shape)

(5000, 3072) (500, 3072)
```

```
In [6]: from cs682.classifiers import KNearestNeighbor

# Create a kNN classifier instance.
# Remember that training a kNN classifier is a noop:
# the Classifier simply remembers the data and does no further processing
classifier = KNearestNeighbor()
classifier.train(X_train, y_train)
```

We would now like to classify the test data with the kNN classifier. Recall that we can break down this process into two steps:

1. First we must compute the distances between all test examples and all train examples.
2. Given these distances, for each test example we find the k nearest examples and have them vote for the label

Lets begin with computing the distance matrix between all training and test examples. For example, if there are N_{tr} training examples and N_{te} test examples, this stage should result in a $N_{te} \times N_{tr}$ matrix where each element (i,j) is the distance between the i -th test and j -th train example.

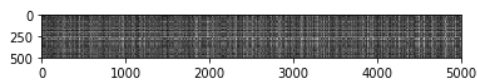
First, open `cs682/classifiers/k_nearest_neighbor.py` and implement the function `compute_distances_two_loops` that uses a (very inefficient) double loop over all pairs of (test, train) examples and computes the distance matrix one element at a time.

```
In [7]: # Open cs682/classifiers/k_nearest_neighbor.py and implement
# compute_distances_two_loops.

# Test your implementation:
dists = classifier.compute_distances_two_loops(X_test)
print(dists.shape)

(500, 5000)
```

```
In [8]: # We can visualize the distance matrix: each row is a single test example and
# its distances to training examples
plt.imshow(dists, interpolation='none')
plt.show()
```



Inline Question #1: Notice the structured patterns in the distance matrix, where some rows or columns are visible brighter. (Note that with the default color scheme black indicates low distances while white indicates high distances.)

- What in the data is the cause behind the distinctly bright rows?
- What causes the columns?

Your Answer: *The differences in test data causes the distinctly bright rows. The differences in training data causes the columns.*

```
In [9]: # Now implement the function predict_labels and run the code below:
# We use k = 1 (which is Nearest Neighbor).
y_test_pred = classifier.predict_labels(dists, k=1)

# Compute and print the fraction of correctly predicted examples
num_correct = np.sum(y_test_pred == y_test)
accuracy = float(num_correct) / num_test
print('Got %d / %d correct => accuracy: %f' % (num_correct, num_test, accuracy))

Got 137 / 500 correct => accuracy: 0.274000
```

You should expect to see approximately 27% accuracy. Now lets try out a larger k , say $k = 5$:

```
In [10]: y_test_pred = classifier.predict_labels(dists, k=5)
num_correct = np.sum(y_test_pred == y_test)
accuracy = float(num_correct) / num_test
print('Got %d / %d correct => accuracy: %f' % (num_correct, num_test, accuracy))

Got 107 / 500 correct => accuracy: 0.214000
```

You should expect to see a slightly better performance than with $k = 1$.

Inline Question 2 We can also other distance metrics such as L1 distance. The performance of a Nearest Neighbor classifier that uses L1 distance will not change if (Select all that apply.):

1. The data is preprocessed by subtracting the mean.
2. The data is preprocessed by subtracting the mean and dividing by the standard deviation.
3. The coordinate axes for the data are rotated.

4. None of the above.

Your Answer: 3.

Your explanation: Rotating the coordinate axes for the data will not change the performance of a Nearest Neighbor classifier that uses L1 distance. Because when the coordinate axes are rotated, the absolute value of difference between two pixels keeps the same.

```
In [11]: # Now lets speed up distance matrix computation by using partial vectorization
# with one loop. Implement the function compute_distances_one_loop and run the
# code below:
dists_one = classifier.compute_distances_one_loop(X_test)

# To ensure that our vectorized implementation is correct, we make sure that it
# agrees with the naive implementation. There are many ways to decide whether
# two matrices are similar; one of the simplest is the Frobenius norm. In case
# you haven't seen it before, the Frobenius norm of two matrices is the square
# root of the squared sum of differences of all elements; in other words, reshape
# the matrices into vectors and compute the Euclidean distance between them.
difference = np.linalg.norm(dists - dists_one, ord='fro')
print('Difference was: %f' % (difference, ))
if difference < 0.001:
    print('Good! The distance matrices are the same')
else:
    print('Uh-oh! The distance matrices are different')
```

Difference was: 0.000000
Good! The distance matrices are the same

```
In [12]: # Now implement the fully vectorized version inside compute_distances_no_loops
# and run the code
dists_two = classifier.compute_distances_no_loops(X_test)

# check that the distance matrix agrees with the one we computed before:
difference = np.linalg.norm(dists - dists_two, ord='fro')
print('Difference was: %f' % (difference, ))
if difference < 0.001:
    print('Good! The distance matrices are the same')
else:
    print('Uh-oh! The distance matrices are different')
```

Difference was: 0.000000
Good! The distance matrices are the same

```
In [13]: # Let's compare how fast the implementations are
def time_function(f, *args):
    """
    Call a function f with args and return the time (in seconds) that it took to execute.
    """
    import time
    tic = time.time()
    f(*args)
    toc = time.time()
    return toc - tic

two_loop_time = time_function(classifier.compute_distances_two_loops, X_test)
print('Two loop version took %f seconds' % two_loop_time)

one_loop_time = time_function(classifier.compute_distances_one_loop, X_test)
print('One loop version took %f seconds' % one_loop_time)

no_loop_time = time_function(classifier.compute_distances_no_loops, X_test)
print('No loop version took %f seconds' % no_loop_time)

# you should see significantly faster performance with the fully vectorized implementation
```

Two loop version took 68.097904 seconds
One loop version took 117.312169 seconds
No loop version took 0.518309 seconds

Cross-validation

We have implemented the k-Nearest Neighbor classifier but we set the value $k = 5$ arbitrarily. We will now determine the best value of this hyperparameter with cross-validation.

```
In [14]: num_folds = 5
k_choices = [1, 3, 5, 8, 10, 12, 15, 20, 50, 100]

X_train_folds = []
y_train_folds = []
#####
# TODO:
# Split up the training data into folds. After splitting, X_train_folds and
# y_train_folds should each be lists of length num_folds, where
# y_train_folds[i] is the label vector for the points in X_train_folds[i].
# Hint: Look up the numpy array_split function.
#####
X_train_folds = np.array_split(X_train, num_folds)
y_train_folds = np.array_split(y_train, num_folds)
#####
#
# END OF YOUR CODE
#####
# A dictionary holding the accuracies for different values of k that we find
# when running cross-validation. After running cross-validation,
# k_to_accuracies[k] should be a list of length num_folds giving the different
# accuracy values that we found when using that value of k.
k_to_accuracies = {}
```

```

k_to_accuracies = {}

#####
# TODO:
# Perform k-fold cross validation to find the best value of k. For each
# possible value of k, run the k-nearest-neighbor algorithm num_folds times,
# where in each case you use all but one of the folds as training data and the
# last fold as a validation set. Store the accuracies for all fold and all
# values of k in the k_to_accuracies dictionary.
#####
for k in k_choices:
    accuracies = []
    for i in range(num_folds):
        current_X_train = np.concatenate(X_train_folds[0:i] + X_train_folds[i+1:])
        current_y_train = np.concatenate(y_train_folds[0:i] + y_train_folds[i+1:])
        X_value = X_train_folds[i]
        y_value = y_train_folds[i]

        classifier.train(current_X_train, current_y_train)
        dists = classifier.compute_distances_no_loops(X_value)
        y_value_pred = classifier.predict_labels(dists, k=k)

        num_correct = np.sum(y_value_pred == y_value)
        accuracy = float(num_correct) / len(y_value)
        accuracies.append(accuracy)

    k_to_accuracies[k] = accuracies
#####
#                                     END OF YOUR CODE                                     #
#####

# Print out the computed accuracies
for k in sorted(k_to_accuracies):
    for accuracy in k_to_accuracies[k]:
        print('k = %d, accuracy = %f' % (k, accuracy))

```

```

k = 1, accuracy = 0.263000
k = 1, accuracy = 0.257000
k = 1, accuracy = 0.264000
k = 1, accuracy = 0.278000
k = 1, accuracy = 0.266000
k = 3, accuracy = 0.215000
k = 3, accuracy = 0.198000
k = 3, accuracy = 0.207000
k = 3, accuracy = 0.215000
k = 3, accuracy = 0.217000
k = 5, accuracy = 0.201000
k = 5, accuracy = 0.214000
k = 5, accuracy = 0.223000
k = 5, accuracy = 0.226000
k = 5, accuracy = 0.229000
k = 8, accuracy = 0.205000
k = 8, accuracy = 0.228000
k = 8, accuracy = 0.220000
k = 8, accuracy = 0.235000
k = 8, accuracy = 0.227000
k = 10, accuracy = 0.210000
k = 10, accuracy = 0.232000
k = 10, accuracy = 0.235000
k = 10, accuracy = 0.237000
k = 10, accuracy = 0.230000
k = 12, accuracy = 0.218000
k = 12, accuracy = 0.248000
k = 12, accuracy = 0.240000
k = 12, accuracy = 0.239000
k = 12, accuracy = 0.233000
k = 15, accuracy = 0.207000
k = 15, accuracy = 0.231000
k = 15, accuracy = 0.246000
k = 15, accuracy = 0.240000
k = 15, accuracy = 0.227000
k = 20, accuracy = 0.220000
k = 20, accuracy = 0.220000
k = 20, accuracy = 0.257000
k = 20, accuracy = 0.232000
k = 20, accuracy = 0.237000
k = 50, accuracy = 0.221000
k = 50, accuracy = 0.238000
k = 50, accuracy = 0.250000
k = 50, accuracy = 0.222000
k = 50, accuracy = 0.253000
k = 100, accuracy = 0.219000
k = 100, accuracy = 0.240000
k = 100, accuracy = 0.228000
k = 100, accuracy = 0.231000
k = 100, accuracy = 0.231000

```

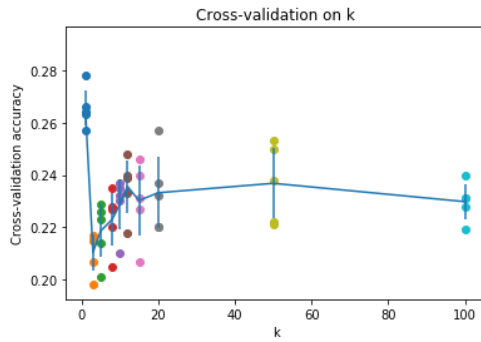
```

In [15]: # plot the raw observations
for k in k_choices:
    accuracies = k_to_accuracies[k]
    plt.scatter([k] * len(accuracies), accuracies)

# plot the trend line with error bars that correspond to standard deviation
accuracies_mean = np.array([np.mean(v) for k,v in sorted(k_to_accuracies.items())])
accuracies_std = np.array([np.std(v) for k,v in sorted(k_to_accuracies.items())])
plt.errorbar(k_choices, accuracies_mean, yerr=accuracies_std)
plt.title('Cross-validation on k')
plt.xlabel('k')

```

```
plt.ylabel('Cross-validation accuracy')
plt.show()
```



```
In [16]: # Based on the cross-validation results above, choose the best value for k,
# retrain the classifier using all the training data, and test it on the test
# data. You should be able to get above 28% accuracy on the test data.
best_k = 1

classifier = KNearestNeighbor()
classifier.train(X_train, y_train)
y_test_pred = classifier.predict(X_test, k=best_k)

# Compute and display the accuracy
num_correct = np.sum(y_test_pred == y_test)
accuracy = float(num_correct) / num_test
print('Got %d / %d correct => accuracy: %f' % (num_correct, num_test, accuracy))

Got 137 / 500 correct => accuracy: 0.274000
```

Inline Question 3 Which of the following statements about k -Nearest Neighbor (k -NN) are true in a classification setting, and for all k ? Select all that apply.

1. The training error of a 1-NN will always be better than that of 5-NN.
2. The test error of a 1-NN will always be better than that of a 5-NN.
3. The decision boundary of the k -NN classifier is linear.
4. The time needed to classify a test example with the k -NN classifier grows with the size of the training set.
5. None of the above.

Your Answer: 5.

Your explanation: In practice, the usage of KNN is very complicated. It is hard to say whether the training/test error would increase when k is increasing. In KNN, we care about test more than training. The decision boundary looks like linear. The image classification pipeline is: Input --> Learning --> Evaluation. We only train the classifier to remember the data. The time needed to classify a test example is up to the size of testing data set.

Multiclass Support Vector Machine exercise

Complete and hand in this completed worksheet (including its outputs and any supporting code outside of the worksheet) with your assignment submission. For more details see the [assignments page](#) on the course website.

In this exercise you will:

- implement a fully-vectorized **loss function** for the SVM
- implement the fully-vectorized expression for its **analytic gradient**
- **check your implementation** using numerical gradient
- use a validation set to **tune the learning rate and regularization strength**
- **optimize** the loss function with **SGD**
- **visualize** the final learned weights

```
In [1]: # Run some setup code for this notebook.

import random
import numpy as np
from cs682.data_utils import load_CIFAR10
import matplotlib.pyplot as plt

from __future__ import print_function

# This is a bit of magic to make matplotlib figures appear inline in the
# notebook rather than in a new window.
%matplotlib inline
plt.rcParams['figure.figsize'] = (10.0, 8.0) # set default size of plots
plt.rcParams['image.interpolation'] = 'nearest'
plt.rcParams['image.cmap'] = 'gray'

# Some more magic so that the notebook will reload external python modules;
# see http://stackoverflow.com/questions/1907993/autoreload-of-modules-in-ipython
%load_ext autoreload
%autoreload 2
```

CIFAR-10 Data Loading and Preprocessing

```
In [2]: # Load the raw CIFAR-10 data.
cifar10_dir = 'cs682/datasets/cifar-10-batches-py'

# Cleaning up variables to prevent loading data multiple times (which may cause memory issue)
try:
    del X_train, y_train
    del X_test, y_test
    print('Clear previously loaded data.')
except:
    pass

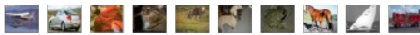
X_train, y_train, X_test, y_test = load_CIFAR10(cifar10_dir)

# As a sanity check, we print out the size of the training and test data.
print('Training data shape: ', X_train.shape)
print('Training labels shape: ', y_train.shape)
print('Test data shape: ', X_test.shape)
print('Test labels shape: ', y_test.shape)

Training data shape: (50000, 32, 32, 3)
Training labels shape: (50000,)
Test data shape: (10000, 32, 32, 3)
Test labels shape: (10000,)
```

```
In [3]: # Visualize some examples from the dataset.
# We show a few examples of training images from each class.
classes = ['plane', 'car', 'bird', 'cat', 'deer', 'dog', 'frog', 'horse', 'ship', 'truck']
num_classes = len(classes)
samples_per_class = 7
for y, cls in enumerate(classes):
    idxs = np.flatnonzero(y_train == y)
    idxs = np.random.choice(idxs, samples_per_class, replace=False)
    for i, idx in enumerate(idxs):
        plt_idx = i * num_classes + y + 1
        plt.subplot(samples_per_class, num_classes, plt_idx)
        plt.imshow(X_train[idx].astype('uint8'))
        plt.axis('off')
        if i == 0:
            plt.title(cls)
plt.show()
```





```
In [4]: # Split the data into train, val, and test sets. In addition we will
# create a small development set as a subset of the training data;
# we can use this for development so our code runs faster.
num_training = 49000
num_validation = 1000
num_test = 1000
num_dev = 500

# Our validation set will be num_validation points from the original
# training set.
mask = range(num_training, num_training + num_validation)
X_val = X_train[mask]
y_val = y_train[mask]

# Our training set will be the first num_train points from the original
# training set.
mask = range(num_training)
X_train = X_train[mask]
y_train = y_train[mask]

# We will also make a development set, which is a small subset of
# the training set.
mask = np.random.choice(num_training, num_dev, replace=False)
X_dev = X_train[mask]
y_dev = y_train[mask]

# We use the first num_test points of the original test set as our
# test set.
mask = range(num_test)
X_test = X_test[mask]
y_test = y_test[mask]

print('Train data shape: ', X_train.shape)
print('Train labels shape: ', y_train.shape)
print('Validation data shape: ', X_val.shape)
print('Validation labels shape: ', y_val.shape)
print('Test data shape: ', X_test.shape)
print('Test labels shape: ', y_test.shape)

Train data shape: (49000, 32, 32, 3)
Train labels shape: (49000,)
Validation data shape: (1000, 32, 32, 3)
Validation labels shape: (1000,)
Test data shape: (1000, 32, 32, 3)
Test labels shape: (1000,)
```

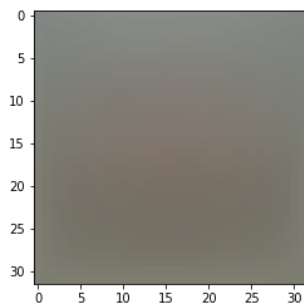
```
In [5]: # Preprocessing: reshape the image data into rows
X_train = np.reshape(X_train, (X_train.shape[0], -1))
X_val = np.reshape(X_val, (X_val.shape[0], -1))
X_test = np.reshape(X_test, (X_test.shape[0], -1))
X_dev = np.reshape(X_dev, (X_dev.shape[0], -1))

# As a sanity check, print out the shapes of the data
print('Training data shape: ', X_train.shape)
print('Validation data shape: ', X_val.shape)
print('Test data shape: ', X_test.shape)
print('dev data shape: ', X_dev.shape)

Training data shape: (49000, 3072)
Validation data shape: (1000, 3072)
Test data shape: (1000, 3072)
dev data shape: (500, 3072)
```

```
In [6]: # Preprocessing: subtract the mean image
# first: compute the image mean based on the training data
mean_image = np.mean(X_train, axis=0)
print(mean_image[:10]) # print a few of the elements
plt.figure(figsize=(4,4))
plt.imshow(mean_image.reshape((32,32,3)).astype('uint8')) # visualize the mean image
plt.show()
```

```
[130.64189796 135.98173469 132.47391837 130.05569388 135.34804082
131.75402041 130.96055102 136.14328571 132.47636735 131.48467347]
```



```
In [7]: # second: subtract the mean image from train and test data
X_train -= mean_image
X_val -= mean_image
X_test -= mean_image
X_dev -= mean_image
```

```
In [8]: # third: append the bias dimension of ones (i.e. bias trick) so that our SVM
# only has to worry about optimizing a single weight matrix W.
X_train = np.hstack([X_train, np.ones((X_train.shape[0], 1))])
X_val = np.hstack([X_val, np.ones((X_val.shape[0], 1))])
X_test = np.hstack([X_test, np.ones((X_test.shape[0], 1))])
X_dev = np.hstack([X_dev, np.ones((X_dev.shape[0], 1))])

print(X_train.shape, X_val.shape, X_test.shape, X_dev.shape)

(49000, 3073) (1000, 3073) (1000, 3073) (500, 3073)
```

SVM Classifier

Your code for this section will all be written inside `cs682/classifiers/linear_svm.py`.

As you can see, we have prefilled the function `svm_loss_naive` which uses for loops to evaluate the multiclass SVM loss function.

```
In [9]: # Evaluate the naive implementation of the loss we provided for you:
from cs682.classifiers.linear_svm import svm_loss_naive
import time

# generate a random SVM weight matrix of small numbers
W = np.random.randn(3073, 10) * 0.0001

loss, grad = svm_loss_naive(W, X_dev, y_dev, 0.000005)
print('loss: %f' % (loss, ))

loss: 9.350899
```

The `grad` returned from the function above is right now all zero. Derive and implement the gradient for the SVM cost function and implement it inline inside the function `svm_loss_naive`. You will find it helpful to interleave your new code inside the existing function.

To check that you have correctly implemented the gradient correctly, you can numerically estimate the gradient of the loss function and compare the numeric estimate to the gradient that you computed. We have provided code that does this for you:

```
In [10]: # Once you've implemented the gradient, recompute it with the code below
# and gradient check it with the function we provided for you

# Compute the loss and its gradient at W.
loss, grad = svm_loss_naive(W, X_dev, y_dev, 0.0)

# Numerically compute the gradient along several randomly chosen dimensions, and
# compare them with your analytically computed gradient. The numbers should match
# almost exactly along all dimensions.
from cs682.gradient_check import grad_check_sparse
f = lambda w: svm_loss_naive(w, X_dev, y_dev, 0.0)[0]
grad_numerical = grad_check_sparse(f, W, grad)

# do the gradient check once again with regularization turned on
# you didn't forget the regularization gradient did you?
loss, grad = svm_loss_naive(W, X_dev, y_dev, 5e1)
f = lambda w: svm_loss_naive(w, X_dev, y_dev, 5e1)[0]
grad_numerical = grad_check_sparse(f, W, grad)

numerical: 8.151000 analytic: 3.719634, relative error: 3.733049e-01
numerical: 0.503656 analytic: -0.883742, relative error: 1.000000e+00
numerical: 14.490215 analytic: 5.285488, relative error: 4.654564e-01
numerical: 15.765072 analytic: 4.796724, relative error: 5.334334e-01
numerical: -7.813754 analytic: -2.365843, relative error: 5.351794e-01
numerical: 1.005929 analytic: 2.539933, relative error: 4.326182e-01
numerical: -11.088588 analytic: 3.771159, relative error: 1.000000e+00
numerical: 0.439496 analytic: -1.477330, relative error: 1.000000e+00
numerical: 11.794324 analytic: 3.043169, relative error: 5.898001e-01
numerical: 7.037745 analytic: -0.342501, relative error: 1.000000e+00
numerical: -9.746163 analytic: 1.475139, relative error: 1.000000e+00
numerical: 12.868116 analytic: 3.156257, relative error: 6.060679e-01
numerical: -11.277165 analytic: 2.296155, relative error: 1.000000e+00
numerical: -14.807696 analytic: 1.873974, relative error: 1.000000e+00
numerical: 27.171786 analytic: 0.532211, relative error: 9.615787e-01
numerical: 8.812894 analytic: 0.478996, relative error: 8.969001e-01
numerical: -1.855186 analytic: -0.971226, relative error: 3.127500e-01
numerical: 2.726997 analytic: -2.664449, relative error: 1.000000e+00
numerical: -7.393182 analytic: -0.235937, relative error: 9.381482e-01
numerical: 14.496352 analytic: 3.890544, relative error: 5.768134e-01
```

Inline Question 1:

It is possible that once in a while a dimension in the gradcheck will not match exactly. What could such a discrepancy be caused by? Is it a reason for concern? What is a simple example in one dimension where a gradient check could fail? How would change the margin affect of the frequency of this happening? *Hint: the SVM loss function is not strictly speaking differentiable*

Your Answer: When the weight is about zero, discrepancies could be caused by the discontinuity in the first derivation of the loss function at the point of zero.

```
In [11]: # Next implement the function svm_loss_vectorized; for now only compute the loss;
# we will implement the gradient in a moment.
tic = time.time()
loss_naive, grad_naive = svm_loss_naive(W, X_dev, y_dev, 0.000005)
toc = time.time()
print('Naive loss: %e computed in %fs' % (loss_naive, toc - tic))

from cs682.classifiers.linear_svm import svm_loss_vectorized
tic = time.time()
loss_vectorized = svm_loss_vectorized(W, X_dev, y_dev, 0.000005)
```



```

loss_vectorized, _ = svm_loss_vectorized(W, X_dev, y_dev, 0.000005)
toc = time.time()
print('Vectorized loss: %e computed in %fs' % (loss_vectorized, toc - tic))

# The losses should match but your vectorized implementation should be much faster.
print('difference: %f' % (loss_naive - loss_vectorized))

Naive loss: 9.350899e+00 computed in 0.111435s
Vectorized loss: 9.350899e+00 computed in 0.000996s
difference: -0.000000

```

```

In [12]: # Complete the implementation of svm_loss_vectorized, and compute the gradient
# of the loss function in a vectorized way.

# The naive implementation and the vectorized implementation should match, but
# the vectorized version should still be much faster.
tic = time.time()
_, grad_naive = svm_loss_naive(W, X_dev, y_dev, 0.000005)
toc = time.time()
print('Naive loss and gradient: computed in %fs' % (toc - tic))

tic = time.time()
_, grad_vectorized = svm_loss_vectorized(W, X_dev, y_dev, 0.000005)
toc = time.time()
print('Vectorized loss and gradient: computed in %fs' % (toc - tic))

# The loss is a single number, so it is easy to compare the values computed
# by the two implementations. The gradient on the other hand is a matrix, so
# we use the Frobenius norm to compare them.
difference = np.linalg.norm(grad_naive - grad_vectorized, ord='fro')
print('difference: %f' % difference)

Naive loss and gradient: computed in 0.118952s
Vectorized loss and gradient: computed in 0.000000s
difference: 2409.615770

```

Stochastic Gradient Descent

We now have vectorized and efficient expressions for the loss, the gradient and our gradient matches the numerical gradient. We are therefore ready to do SGD to minimize the loss.

```

In [13]: # In the file linear_classifier.py, implement SGD in the function
# LinearClassifier.train() and then run it with the code below.
from cs682.classifiers import LinearSVM
svm = LinearSVM()
tic = time.time()
loss_hist = svm.train(X_train, y_train, learning_rate=1e-7, reg=2.5e4,
                      num_iters=1500, verbose=True)
toc = time.time()
print('That took %fs' % (toc - tic))

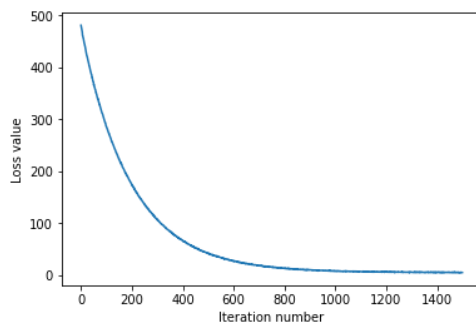
iteration 0 / 1500: loss 481.071436
iteration 100 / 1500: loss 286.060557
iteration 200 / 1500: loss 173.165237
iteration 300 / 1500: loss 107.563805
iteration 400 / 1500: loss 65.847510
iteration 500 / 1500: loss 41.814827
iteration 600 / 1500: loss 26.684772
iteration 700 / 1500: loss 18.804308
iteration 800 / 1500: loss 12.823496
iteration 900 / 1500: loss 10.423532
iteration 1000 / 1500: loss 8.292271
iteration 1100 / 1500: loss 7.071681
iteration 1200 / 1500: loss 5.888292
iteration 1300 / 1500: loss 5.651718
iteration 1400 / 1500: loss 5.305108
That took 11.087382s

```

```

In [14]: # A useful debugging strategy is to plot the loss as a function of
# iteration number:
plt.plot(loss_hist)
plt.xlabel('Iteration number')
plt.ylabel('Loss value')
plt.show()

```



```

In [20]: # Write the LinearSVM.predict function and evaluate the performance on both the
# training and validation set
y_train_pred = svm.predict(X_train)
print('training accuracy: %f' % (np.mean(y_train == y_train_pred), ))
y_val_pred = svm.predict(X_val)
print('validation accuracy: %f' % (np.mean(y_val == y_val_pred), ))

```

```
training accuracy: 0.140408
validation accuracy: 0.130000
```

```
In [42]: # Use the validation set to tune hyperparameters (regularization strength and
# learning rate). You should experiment with different ranges for the learning
# rates and regularization strengths; if you are careful you should be able to
# get a classification accuracy of about 0.4 on the validation set.
learning_rates = [0.9e-7, 1.1e-7]
regularization_strengths = [2.6e4, 5.2e4]

# results is dictionary mapping tuples of the form
# (learning_rate, regularization_strength) to tuples of the form
# (training_accuracy, validation_accuracy). The accuracy is simply the fraction
# of data points that are correctly classified.
results = {}
best_val = -1 # The highest validation accuracy that we have seen so far.
best_svm = None # The LinearSVM object that achieved the highest validation rate.

#####
# TODO:
# Write code that chooses the best hyperparameters by tuning on the validation
# set. For each combination of hyperparameters, train a linear SVM on the
# training set, compute its accuracy on the training and validation sets, and
# store these numbers in the results dictionary. In addition, store the best
# validation accuracy in best_val and the LinearSVM object that achieves this
# accuracy in best_svm.
#
# Hint: You should use a small value for num_iters as you develop your
# validation code so that the SVMs don't take much time to train; once you are
# confident that your validation code works, you should rerun the validation
# code with a larger value for num_iters.
#####
for lr in learning_rates:
    for reg in regularization_strengths:
        svm = LinearSVM()
        svm.train(X_train, y_train, learning_rate = lr, reg = reg, num_iters = 3000, batch_size = 200)
        y_train_pred = svm.predict(X_train)
        train_accuracy = np.mean(y_train == y_train_pred)
        y_val_pred = svm.predict(X_val)
        val_accuracy = np.mean(y_val == y_val_pred)
        results[(lr, reg)] = (train_accuracy, val_accuracy)
        if val_accuracy > best_val:
            best_val = val_accuracy
            best_svm = svm

#####
# END OF YOUR CODE
#####

# Print out results.
for lr, reg in sorted(results):
    train_accuracy, val_accuracy = results[(lr, reg)]
    print('lr %e reg %e train accuracy: %f val accuracy: %f' % (
        lr, reg, train_accuracy, val_accuracy))

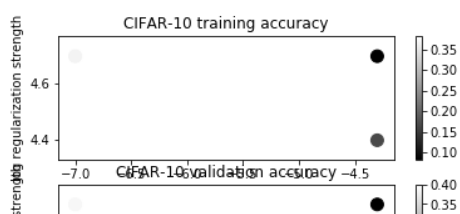
print('best validation accuracy achieved during cross-validation: %f' % best_val)

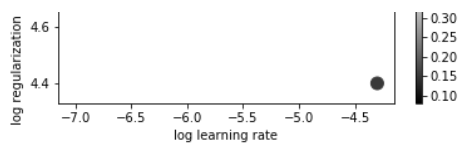
lr 9.000000e-08 reg 2.600000e+04 train accuracy: 0.384714 val accuracy: 0.386000
lr 9.000000e-08 reg 5.200000e+04 train accuracy: 0.365327 val accuracy: 0.378000
lr 1.100000e-07 reg 2.600000e+04 train accuracy: 0.380796 val accuracy: 0.376000
lr 1.100000e-07 reg 5.200000e+04 train accuracy: 0.369531 val accuracy: 0.377000
best validation accuracy achieved during cross-validation: 0.386000
```

```
In [27]: # Visualize the cross-validation results
import math
x_scatter = [math.log10(x[0]) for x in results]
y_scatter = [math.log10(x[1]) for x in results]

# plot training accuracy
marker_size = 100
colors = [results[x][0] for x in results]
plt.subplot(2, 1, 1)
plt.scatter(x_scatter, y_scatter, marker_size, c=colors)
plt.colorbar()
plt.xlabel('log learning rate')
plt.ylabel('log regularization strength')
plt.title('CIFAR-10 training accuracy')

# plot validation accuracy
colors = [results[x][1] for x in results] # default size of markers is 20
plt.subplot(2, 1, 2)
plt.scatter(x_scatter, y_scatter, marker_size, c=colors)
plt.colorbar()
plt.xlabel('log learning rate')
plt.ylabel('log regularization strength')
plt.title('CIFAR-10 validation accuracy')
plt.show()
```





```
In [28]: # Evaluate the best svm on test set
y_test_predict = best_svm.predict(X_test)
test_accuracy = np.mean(y_test == y_test_predict)
print('linear SVM on raw pixels final test set accuracy: %f' % test_accuracy)
```

linear SVM on raw pixels final test set accuracy: 0.362000

```
In [29]: # Visualize the learned weights for each class.
# Depending on your choice of learning rate and regularization strength, these may
# or may not be nice to look at.
w = best_svm.W[:-1,:] # strip out the bias
w = w.reshape(32, 32, 3, 10)
w_min, w_max = np.min(w), np.max(w)
classes = ['plane', 'car', 'bird', 'cat', 'deer', 'dog', 'frog', 'horse', 'ship', 'truck']
for i in range(10):
    plt.subplot(2, 5, i + 1)

    # Rescale the weights to be between 0 and 255
    wimg = 255.0 * (w[:, :, :, i].squeeze() - w_min) / (w_max - w_min)
    plt.imshow(wimg.astype('uint8'))
    plt.axis('off')
    plt.title(classes[i])
```



Inline question 2:

Describe what your visualized SVM weights look like, and offer a brief explanation for why they look the way that they do.

Your answer: The pictures are vague. In my opinion, the reason is that the classifier is plane.

Softmax exercise

Complete and hand in this completed worksheet (including its outputs and any supporting code outside of the worksheet) with your assignment submission. For more details see the [assignments page](#) on the course website.

This exercise is analogous to the SVM exercise. You will:

- implement a fully-vectorized **loss function** for the Softmax classifier
- implement the fully-vectorized expression for its **analytic gradient**
- **check your implementation** with numerical gradient
- use a validation set to **tune the learning rate and regularization strength**
- **optimize** the loss function with **SGD**
- **visualize** the final learned weights

```
In [1]: import random
import numpy as np
from cs682.data_utils import load_CIFAR10
import matplotlib.pyplot as plt

from __future__ import print_function

%matplotlib inline
plt.rcParams['figure.figsize'] = (10.0, 8.0) # set default size of plots
plt.rcParams['image.interpolation'] = 'nearest'
plt.rcParams['image.cmap'] = 'gray'

# for auto-reloading extenrnal modules
# see http://stackoverflow.com/questions/1907993/autoreload-of-modules-in-ipython
%load_ext autoreload
%autoreload 2

In [2]: def get_CIFAR10_data(num_training=49000, num_validation=1000, num_test=1000, num_dev=500):
    """
    Load the CIFAR-10 dataset from disk and perform preprocessing to prepare
    it for the linear classifier. These are the same steps as we used for the
    SVM, but condensed to a single function.
    """
    # Load the raw CIFAR-10 data
    cifar10_dir = 'cs682/datasets/cifar-10-batches-py'

    X_train, y_train, X_test, y_test = load_CIFAR10(cifar10_dir)

    # subsample the data
    mask = list(range(num_training, num_training + num_validation))
    X_val = X_train[mask]
    y_val = y_train[mask]
    mask = list(range(num_training))
    X_train = X_train[mask]
    y_train = y_train[mask]
    mask = list(range(num_test))
    X_test = X_test[mask]
    y_test = y_test[mask]
    mask = np.random.choice(num_training, num_dev, replace=False)
    X_dev = X_train[mask]
    y_dev = y_train[mask]

    # Preprocessing: reshape the image data into rows
    X_train = np.reshape(X_train, (X_train.shape[0], -1))
    X_val = np.reshape(X_val, (X_val.shape[0], -1))
    X_test = np.reshape(X_test, (X_test.shape[0], -1))
    X_dev = np.reshape(X_dev, (X_dev.shape[0], -1))

    # Normalize the data: subtract the mean image
    mean_image = np.mean(X_train, axis = 0)
    X_train -= mean_image
    X_val -= mean_image
    X_test -= mean_image
    X_dev -= mean_image

    # add bias dimension and transform into columns
    X_train = np.hstack([X_train, np.ones((X_train.shape[0], 1))])
    X_val = np.hstack([X_val, np.ones((X_val.shape[0], 1))])
    X_test = np.hstack([X_test, np.ones((X_test.shape[0], 1))])
    X_dev = np.hstack([X_dev, np.ones((X_dev.shape[0], 1))])

    return X_train, y_train, X_val, y_val, X_test, y_test, X_dev, y_dev

# Cleaning up variables to prevent loading data multiple times (which may cause memory issue)
try:
    del X_train, y_train
    del X_test, y_test
    print('Clear previously loaded data.')
except:
    pass

# Invoke the above function to get our data.
X_train, y_train, X_val, y_val, X_test, y_test, X_dev, y_dev = get_CIFAR10_data()
print('Train data shape: ', X_train.shape)
print('Train labels shape: ', y_train.shape)
print('Validation data shape: ', X_val.shape)
print('Validation labels shape: ', y_val.shape)
```

```
print('Test data shape: ', X_test.shape)
print('Test labels shape: ', y_test.shape)
print('dev data shape: ', X_dev.shape)
print('dev labels shape: ', y_dev.shape)
```

```
Train data shape: (49000, 3073)
Train labels shape: (49000,)
Validation data shape: (1000, 3073)
Validation labels shape: (1000,)
Test data shape: (1000, 3073)
Test labels shape: (1000,)
dev data shape: (500, 3073)
dev labels shape: (500,)
```

Softmax Classifier

Your code for this section will all be written inside `cs682/classifiers/softmax.py`.

```
In [3]: # First implement the naive softmax loss function with nested loops.
# Open the file cs682/classifiers/softmax.py and implement the
# softmax_loss_naive function.

from cs682.classifiers.softmax import softmax_loss_naive
import time

# Generate a random softmax weight matrix and use it to compute the loss.
W = np.random.randn(3073, 10) * 0.0001
loss, grad = softmax_loss_naive(W, X_dev, y_dev, 0.0)

# As a rough sanity check, our loss should be something close to -log(0.1).
print('loss: %f' % loss)
print('sanity check: %f' % (-np.log(0.1)))

loss: 2.314992
sanity check: 2.302585
```

Inline Question 1:

Why do we expect our loss to be close to $-\log(0.1)$? Explain briefly.**

Your answer: In math, $-\log(0.1) = \log 10$. In this case, we have 10 classes and 0.1 chance (probability) in each class. Our loss should be close to $\log 10$ at the start to make the numerator is almost equal to denominator.

```
In [9]: # Complete the implementation of softmax_loss_naive and implement a (naive)
# version of the gradient that uses nested loops.
loss, grad = softmax_loss_naive(W, X_dev, y_dev, 0.0)

# As we did for the SVM, use numeric gradient checking as a debugging tool.
# The numeric gradient should be close to the analytic gradient.
from cs682.gradient_check import grad_check_sparse
f = lambda w: softmax_loss_naive(w, X_dev, y_dev, 0.0)[0]
grad_numerical = grad_check_sparse(f, W, grad, 10)

# similar to SVM case, do another gradient check with regularization
loss, grad = softmax_loss_naive(W, X_dev, y_dev, 5e1)
f = lambda w: softmax_loss_naive(w, X_dev, y_dev, 5e1)[0]
grad_numerical = grad_check_sparse(f, W, grad, 10)

numerical: -0.448141 analytic: 0.002775, relative error: 1.000000e+00
numerical: -0.108030 analytic: 0.000770, relative error: 1.000000e+00
numerical: -0.528329 analytic: -0.002990, relative error: 9.887463e-01
numerical: 1.133781 analytic: 0.001170, relative error: 9.979376e-01
numerical: -0.274670 analytic: -0.004633, relative error: 9.668242e-01
numerical: -2.289194 analytic: -0.004039, relative error: 9.964775e-01
numerical: -1.944561 analytic: -0.002969, relative error: 9.969512e-01
numerical: -0.674844 analytic: -0.007922, relative error: 9.767932e-01
numerical: 1.956750 analytic: 0.007300, relative error: 9.925667e-01
numerical: -0.036928 analytic: -0.007613, relative error: 6.581466e-01
numerical: 4.158921 analytic: 0.140521, relative error: 9.346329e-01
numerical: -1.756923 analytic: -0.001391, relative error: 9.984182e-01
numerical: 0.946373 analytic: 0.000012, relative error: 9.999743e-01
numerical: -2.655656 analytic: -0.006999, relative error: 9.947427e-01
numerical: -1.176206 analytic: -0.002758, relative error: 9.953216e-01
numerical: 0.273545 analytic: 0.004587, relative error: 9.670149e-01
numerical: 1.443569 analytic: -0.004716, relative error: 1.000000e+00
numerical: 0.029298 analytic: -0.006343, relative error: 1.000000e+00
numerical: 0.673433 analytic: 0.001845, relative error: 9.945349e-01
numerical: 0.470728 analytic: 0.090837, relative error: 6.764852e-01
```

```
In [10]: # Now that we have a naive implementation of the softmax loss function and its gradient,
# implement a vectorized version in softmax_loss_vectorized.
# The two versions should compute the same results, but the vectorized version should be
# much faster.
tic = time.time()
loss_naive, grad_naive = softmax_loss_naive(W, X_dev, y_dev, 0.000005)
toc = time.time()
print('naive loss: %e computed in %fs' % (loss_naive, toc - tic))

from cs682.classifiers.softmax import softmax_loss_vectorized
tic = time.time()
loss_vectorized, grad_vectorized = softmax_loss_vectorized(W, X_dev, y_dev, 0.000005)
toc = time.time()
print('vectorized loss: %e computed in %fs' % (loss_vectorized, toc - tic))

# As we did for the SVM, we use the Frobenius norm to compare the two versions
```

```
# or the gradient.
grad_difference = np.linalg.norm(grad_naive - grad_vectorized, ord='fro')
print('Loss difference: %f' % np.abs(loss_naive - loss_vectorized))
print('Gradient difference: %f' % grad_difference)

naive loss: 2.314992e+00 computed in 0.020558s
vectorized loss: 2.314992e+00 computed in 0.005029s
Loss difference: 0.000000
Gradient difference: 294.529737
```

```
In [11]: # Use the validation set to tune hyperparameters (regularization strength and
# learning rate). You should experiment with different ranges for the learning
# rates and regularization strengths; if you are careful you should be able to
# get a classification accuracy of over 0.35 on the validation set.
from cs682.classifiers import Softmax
results = {}
best_val = -1
best_softmax = None
learning_rates = [1e-7, 5e-7]
regularization_strengths = [2.6e4, 5e4]

#####
# TODO:
# Use the validation set to set the learning rate and regularization strength. #
# This should be identical to the validation that you did for the SVM; save   #
# the best trained softmax classifier in best_softmax.                         #
#####
for lr in learning_rates:
    for reg in regularization_strengths:
        softmax = Softmax()
        softmax.train(X_train, y_train, learning_rate = lr, reg = reg, num_iters = 5000, batch_size = 200)
        y_train_pred = softmax.predict(X_train)
        train_accuracy = np.mean(y_train == y_train_pred)
        y_val_pred = softmax.predict(X_val)
        val_accuracy = np.mean(y_val == y_val_pred)
        results[(lr, reg)] = (train_accuracy, val_accuracy)
        if val_accuracy > best_val:
            best_val = val_accuracy
            best_softmax = softmax
#####
#                                     END OF YOUR CODE
#####

# Print out results.
for lr, reg in sorted(results):
    train_accuracy, val_accuracy = results[(lr, reg)]
    print('lr %e reg %e train accuracy: %f val accuracy: %f' % (
        lr, reg, train_accuracy, val_accuracy))

print('best validation accuracy achieved during cross-validation: %f' % best_val)

lr 1.000000e-07 reg 2.600000e+04 train accuracy: 0.369265 val accuracy: 0.384000
lr 1.000000e-07 reg 5.000000e+04 train accuracy: 0.347673 val accuracy: 0.365000
lr 5.000000e-07 reg 2.600000e+04 train accuracy: 0.367143 val accuracy: 0.387000
lr 5.000000e-07 reg 5.000000e+04 train accuracy: 0.337633 val accuracy: 0.358000
best validation accuracy achieved during cross-validation: 0.387000
```

```
In [12]: # evaluate on test set
# Evaluate the best softmax on test set
y_test_pred = best_softmax.predict(X_test)
test_accuracy = np.mean(y_test == y_test_pred)
print('softmax on raw pixels final test set accuracy: %f' % (test_accuracy, ))

softmax on raw pixels final test set accuracy: 0.364000
```

Inline Question - True or False

It's possible to add a new datapoint to a training set that would leave the SVM loss unchanged, but this is not the case with the Softmax classifier loss.

Your answer: True.

Your explanation: In SVM, we choose the maximum value in loss function. In Softmax, the score will change if we add a new datapoint. Because the denominator in loss function expression changes.

```
In [13]: # Visualize the learned weights for each class
w = best_softmax.W[:-1, :] # strip out the bias
w = w.reshape(32, 32, 3, 10)

w_min, w_max = np.min(w), np.max(w)

classes = ['plane', 'car', 'bird', 'cat', 'deer', 'dog', 'frog', 'horse', 'ship', 'truck']
for i in range(10):
    plt.subplot(2, 5, i + 1)

    # Rescale the weights to be between 0 and 255
    wimg = 255.0 * (w[:, :, :, i].squeeze() - w_min) / (w_max - w_min)
    plt.imshow(wimg.astype('uint8'))
    plt.axis('off')
    plt.title(classes[i])
```





Implementing a Neural Network

In this exercise we will develop a neural network with fully-connected layers to perform classification, and test it out on the CIFAR-10 dataset.

```
In [1]: # A bit of setup

import numpy as np
import matplotlib.pyplot as plt

from cs682.classifiers.neural_net import TwoLayerNet

from __future__ import print_function

%matplotlib inline
plt.rcParams['figure.figsize'] = (10.0, 8.0) # set default size of plots
plt.rcParams['image.interpolation'] = 'nearest'
plt.rcParams['image.cmap'] = 'gray'

# for auto-reloading external modules
# see http://stackoverflow.com/questions/1907993/autoreload-of-modules-in-ipython
%load_ext autoreload
%autoreload 2

def rel_error(x, y):
    """ returns relative error """
    return np.max(np.abs(x - y) / (np.maximum(1e-8, np.abs(x) + np.abs(y))))
```

We will use the class `TwoLayerNet` in the file `cs682/classifiers/neural_net.py` to represent instances of our network. The network parameters are stored in the instance variable `self.params` where keys are string parameter names and values are numpy arrays. Below, we initialize toy data and a toy model that we will use to develop your implementation.

```
In [2]: # Create a small net and some toy data to check your implementations.
# Note that we set the random seed for repeatable experiments.

input_size = 4
hidden_size = 10
num_classes = 3
num_inputs = 5

def init_toy_model():
    np.random.seed(0)
    return TwoLayerNet(input_size, hidden_size, num_classes, std=1e-1)

def init_toy_data():
    np.random.seed(1)
    X = 10 * np.random.randn(num_inputs, input_size)
    y = np.array([0, 1, 2, 2, 1])
    return X, y

net = init_toy_model()
X, y = init_toy_data()
```

Forward pass: compute scores

Open the file `cs682/classifiers/neural_net.py` and look at the method `TwoLayerNet.loss`. This function is very similar to the loss functions you have written for the SVM and Softmax exercises: It takes the data and weights and computes the class scores, the loss, and the gradients on the parameters.

Implement the first part of the forward pass which uses the weights and biases to compute the scores for all inputs.

```
In [3]: scores = net.loss(X)
print('Your scores:')
print(scores)
print()
print('correct scores:')
correct_scores = np.asarray([
    [-0.81233741, -1.27654624, -0.70335995],
    [-0.17129677, -1.18803311, -0.47310444],
    [-0.51590475, -1.01354314, -0.8504215 ],
    [-0.15419291, -0.48629638, -0.52901952],
    [-0.00618733, -0.12435261, -0.15226949]])
print(correct_scores)
print()

# The difference should be very small. We get < 1e-7
print('Difference between your scores and correct scores:')
print(np.sum(np.abs(scores - correct_scores)))

Your scores:
[[-0.81233741 -1.27654624 -0.70335995]
 [-0.17129677 -1.18803311 -0.47310444]
 [-0.51590475 -1.01354314 -0.8504215 ]
 [-0.15419291 -0.48629638 -0.52901952]
 [-0.00618733 -0.12435261 -0.15226949]]

correct scores:
[[-0.81233741 -1.27654624 -0.70335995]
 [-0.17129677 -1.18803311 -0.47310444]
 [-0.51590475 -1.01354314 -0.8504215 ]
 [-0.15419291 -0.48629638 -0.52901952]
 [-0.00618733 -0.12435261 -0.15226949]]
```



```
.....  
Difference between your scores and correct scores:  
3.6802720496109664e-08
```

Forward pass: compute loss

In the same function, implement the second part that computes the data and regularization loss.

```
In [4]: loss, _ = net.loss(X, y, reg=0.05)  
correct_loss = 1.30378789133  
  
# should be very small, we get < 1e-12  
print('Difference between your loss and correct loss:')  
print(np.sum(np.abs(loss - correct_loss)))  
  
Difference between your loss and correct loss:  
0.0303446713695934
```

Backward pass

Implement the rest of the function. This will compute the gradient of the loss with respect to the variables w_1 , b_1 , w_2 , and b_2 . Now that you (hopefully!) have a correctly implemented forward pass, you can debug your backward pass using a numeric gradient check:

```
In [5]: from cs682.gradient_check import eval_numerical_gradient  
  
# Use numeric gradient checking to check your implementation of the backward pass.  
# If your implementation is correct, the difference between the numeric and  
# analytic gradients should be less than 1e-8 for each of  $w_1$ ,  $w_2$ ,  $b_1$ , and  $b_2$ .  
  
loss, grads = net.loss(X, y, reg=0.05)  
  
# these should all be less than 1e-8 or so  
for param_name in grads:  
    f = lambda W: net.loss(X, y, reg=0.05)[0]  
    param_grad_num = eval_numerical_gradient(f, net.params[param_name], verbose=False)  
    print('%s max relative error: %e' % (param_name, rel_error(param_grad_num, grads[param_name])))  
  
w2 max relative error: 4.285714e-01  
b2 max relative error: 3.865091e-11  
w1 max relative error: 4.285714e-01  
b1 max relative error: 2.625087e-09
```

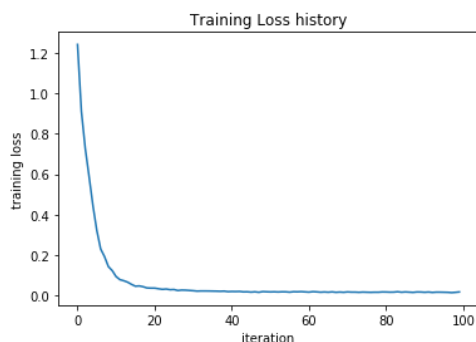
Train the network

To train the network we will use stochastic gradient descent (SGD), similar to the SVM and Softmax classifiers. Look at the function `TwoLayerNet.train` and fill in the missing sections to implement the training procedure. This should be very similar to the training procedure you used for the SVM and Softmax classifiers. You will also have to implement `TwoLayerNet.predict`, as the training process periodically performs prediction to keep track of accuracy over time while the network trains.

Once you have implemented the method, run the code below to train a two-layer network on toy data. You should achieve a training loss less than 0.2.

```
In [6]: net = init_toy_model()  
stats = net.train(X, y, X, y,  
                 learning_rate=1e-1, reg=5e-6,  
                 num_iters=100, verbose=False)  
  
print('Final training loss: ', stats['loss_history'][-1])  
  
# plot the loss history  
plt.plot(stats['loss_history'])  
plt.xlabel('iteration')  
plt.ylabel('training loss')  
plt.title('Training Loss history')  
plt.show()
```

Final training loss: 0.017138255651162324



Load the data

Now that you have implemented a two-layer network that passes gradient checks and works on toy data, it's time to load up our favorite CIFAR-10 data so we can use it to train a classifier on a real dataset.

```
In [7]: from cs682.data_utils import load_CIFAR10
```

```

In [7]: from cs682.data_utils import load_CIFAR10

def get_CIFAR10_data(num_training=49000, num_validation=1000, num_test=1000):
    """
    Load the CIFAR-10 dataset from disk and perform preprocessing to prepare
    it for the two-layer neural net classifier. These are the same steps as
    we used for the SVM, but condensed to a single function.
    """
    # Load the raw CIFAR-10 data
    cifar10_dir = 'cs682/datasets/cifar-10-batches-py'

    X_train, y_train, X_test, y_test = load_CIFAR10(cifar10_dir)

    # Subsample the data
    mask = list(range(num_training, num_training + num_validation))
    X_val = X_train[mask]
    y_val = y_train[mask]
    mask = list(range(num_training))
    X_train = X_train[mask]
    y_train = y_train[mask]
    mask = list(range(num_test))
    X_test = X_test[mask]
    y_test = y_test[mask]

    # Normalize the data: subtract the mean image
    mean_image = np.mean(X_train, axis=0)
    X_train -= mean_image
    X_val -= mean_image
    X_test -= mean_image

    # Reshape data to rows
    X_train = X_train.reshape(num_training, -1)
    X_val = X_val.reshape(num_validation, -1)
    X_test = X_test.reshape(num_test, -1)

    return X_train, y_train, X_val, y_val, X_test, y_test

# Cleaning up variables to prevent loading data multiple times (which may cause memory issue)
try:
    del X_train, y_train
    del X_test, y_test
    print('Clear previously loaded data.')
except:
    pass

# Invoke the above function to get our data.
X_train, y_train, X_val, y_val, X_test, y_test = get_CIFAR10_data()
print('Train data shape: ', X_train.shape)
print('Train labels shape: ', y_train.shape)
print('Validation data shape: ', X_val.shape)
print('Validation labels shape: ', y_val.shape)
print('Test data shape: ', X_test.shape)
print('Test labels shape: ', y_test.shape)

Train data shape: (49000, 3072)
Train labels shape: (49000,)
Validation data shape: (1000, 3072)
Validation labels shape: (1000,)
Test data shape: (1000, 3072)
Test labels shape: (1000,)

```

Train a network

To train our network we will use SGD. In addition, we will adjust the learning rate with an exponential learning rate schedule as optimization proceeds; after each epoch, we will reduce the learning rate by multiplying it by a decay rate.

```

In [8]: input_size = 32 * 32 * 3
hidden_size = 50
num_classes = 10
net = TwoLayerNet(input_size, hidden_size, num_classes)

# Train the network
stats = net.train(X_train, y_train, X_val, y_val,
                  num_iters=1000, batch_size=200,
                  learning_rate=1e-4, learning_rate_decay=0.95,
                  reg=0.25, verbose=True)

# Predict on the validation set
val_acc = (net.predict(X_val) == y_val).mean()
print('Validation accuracy: ', val_acc)

iteration 0 / 1000: loss 2.302572
iteration 100 / 1000: loss 2.302168
iteration 200 / 1000: loss 2.297208
iteration 300 / 1000: loss 2.258635
iteration 400 / 1000: loss 2.202487
iteration 500 / 1000: loss 2.116021
iteration 600 / 1000: loss 2.048671
iteration 700 / 1000: loss 1.984284
iteration 800 / 1000: loss 2.002012
iteration 900 / 1000: loss 1.946067
Validation accuracy: 0.287

```

Debug the training

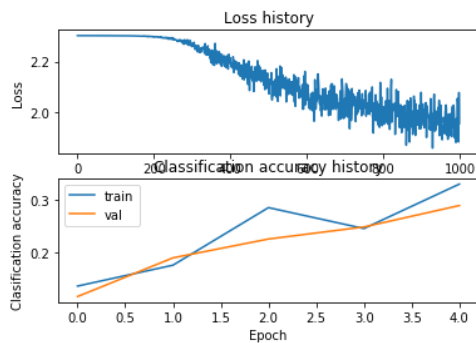
With the default parameters we provided above, you should get a validation accuracy of about 0.29 on the validation set. This isn't very good.

One strategy for getting insight into what's wrong is to plot the loss function and the accuracies on the training and validation sets during optimization.

Another strategy is to visualize the weights that were learned in the first layer of the network. In most neural networks trained on visual data, the first layer weights typically show some visible structure when visualized.

```
In [9]: # Plot the loss function and train / validation accuracies
plt.subplot(2, 1, 1)
plt.plot(stats['loss_history'])
plt.title('Loss history')
plt.xlabel('Iteration')
plt.ylabel('Loss')

plt.subplot(2, 1, 2)
plt.plot(stats['train_acc_history'], label='train')
plt.plot(stats['val_acc_history'], label='val')
plt.title('Classification accuracy history')
plt.xlabel('Epoch')
plt.ylabel('Classification accuracy')
plt.legend()
plt.show()
```

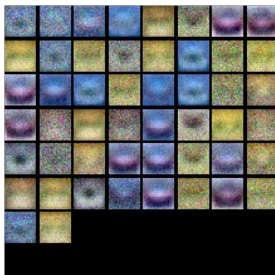


```
In [10]: from cs682.vis_utils import visualize_grid

# Visualize the weights of the network

def show_net_weights(net):
    W1 = net.params['W1']
    W1 = W1.reshape(32, 32, 3, -1).transpose(3, 0, 1, 2)
    plt.imshow(visualize_grid(W1, padding=3).astype('uint8'))
    plt.gca().axis('off')
    plt.show()

show_net_weights(net)
```



Tune your hyperparameters

What's wrong?. Looking at the visualizations above, we see that the loss is decreasing more or less linearly, which seems to suggest that the learning rate may be too low. Moreover, there is no gap between the training and validation accuracy, suggesting that the model we used has low capacity, and that we should increase its size. On the other hand, with a very large model we would expect to see more overfitting, which would manifest itself as a very large gap between the training and validation accuracy.

Tuning. Tuning the hyperparameters and developing intuition for how they affect the final performance is a large part of using Neural Networks, so we want you to get a lot of practice. Below, you should experiment with different values of the various hyperparameters, including hidden layer size, learning rate, number of training epochs, and regularization strength. You might also consider tuning the learning rate decay, but you should be able to get good performance using the default value.

Approximate results. You should aim to achieve a classification accuracy of greater than 48% on the validation set. Our best network gets over 52% on the validation set.

Experiment: Your goal in this exercise is to get as good of a result on CIFAR-10 as you can, with a fully-connected Neural Network. Feel free to implement your own techniques (e.g. PCA to reduce dimensionality, or adding dropout, or adding features to the solver, etc.).

```
In [35]: best_net = None # store the best model into this

#####
# TODO: Tune hyperparameters using the validation set. Store your best trained #
# model in best_net.                                                         #
#                                                                             #
# To help debug your network, it may help to use visualizations similar to the #
```

```

# ones we used above; these visualizations will have significant qualitative
# differences from the ones we saw above for the poorly tuned network.
#
# Tweaking hyperparameters by hand can be fun, but you might find it useful to
# write code to sweep through possible combinations of hyperparameters
# automatically like we did on the previous exercises.
#####
best_val = 0
best_stats = None
input_size = 32 * 32 * 3
hiddenSize = 200
learning_rates = [1e-4, 1e-3]
regularization_strengths = [0.01, 0.1, 0.2]
results = {}
iters = 1000

for lr in learning_rates:
    for reg in regularization_strengths:

        net = TwoLayerNet(input_size, hiddenSize, num_classes)
        stats = net.train(X_train, y_train, X_val, y_val, num_iters = iters,
                           batch_size = 200, learning_rate = lr,
                           learning_rate_decay = 0.95, reg = reg)

        y_train_pred = net.predict(X_train)
        train_accuracy = np.mean(y_train == y_train_pred)
        y_val_pred = net.predict(X_val)
        val_accuracy = (net.predict(X_val) == y_val).mean()

        results[(lr, reg)] = (train_accuracy, val_accuracy)

        if val_accuracy > best_val:
            best_stats = stats
            best_val = val_accuracy
            best_net = net

for lr, reg in sorted(results):
    train_accuracy, val_accuracy = results[(lr, reg)]
    print('lr %e reg %e train accuracy: %f val accuracy: %f' % (
        lr, reg, train_accuracy, val_accuracy))

print('best validation accuracy achieved during cross-validation: %f' % best_val)

#####
#                               END OF YOUR CODE                               #
#####

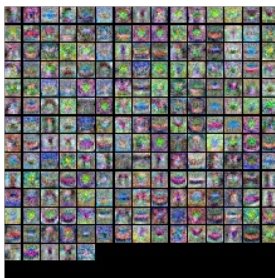
lr 1.000000e-04 reg 1.000000e-02 train accuracy: 0.309673 val accuracy: 0.305000
lr 1.000000e-04 reg 1.000000e-01 train accuracy: 0.309898 val accuracy: 0.310000
lr 1.000000e-04 reg 2.000000e-01 train accuracy: 0.310918 val accuracy: 0.310000
lr 1.000000e-03 reg 1.000000e-02 train accuracy: 0.511367 val accuracy: 0.496000
lr 1.000000e-03 reg 1.000000e-01 train accuracy: 0.514837 val accuracy: 0.478000
lr 1.000000e-03 reg 2.000000e-01 train accuracy: 0.511796 val accuracy: 0.480000
best validation accuracy achieved during cross-validation: 0.496000

```

```

In [36]: # visualize the weights of the best network
show_net_weights(best_net)

```



Run on the test set

When you are done experimenting, you should evaluate your final trained network on the test set; you should get above 48%.

```

In [37]: test_acc = (best_net.predict(X_test) == y_test).mean()
print('Test accuracy: ', test_acc)

```

Test accuracy: 0.505

Inline Question

Now that you have trained a Neural Network classifier, you may find that your testing accuracy is much lower than the training accuracy. In what ways can we decrease this gap? Select all that apply.

1. Train on a larger dataset.
2. Add more hidden units.
3. Increase the regularization strength.
4. None of the above.

Your answer: 1,3

Your explanation: If the training dataset is increasing, the more probability that testing image could match the training image. When we increase the regularization strength, the value of loss function would decrease.

Image features exercise

Complete and hand in this completed worksheet (including its outputs and any supporting code outside of the worksheet) with your assignment submission. For more details see the [assignments page](#) on the course website.

We have seen that we can achieve reasonable performance on an image classification task by training a linear classifier on the pixels of the input image. In this exercise we will show that we can improve our classification performance by training linear classifiers not on raw pixels but on features that are computed from the raw pixels.

All of your work for this exercise will be done in this notebook.

```
In [7]: import random
import numpy as np
from cs682.data_utils import load_CIFAR10
import matplotlib.pyplot as plt

from __future__ import print_function

%matplotlib inline
plt.rcParams['figure.figsize'] = (10.0, 8.0) # set default size of plots
plt.rcParams['image.interpolation'] = 'nearest'
plt.rcParams['image.cmap'] = 'gray'

# for auto-reloading extenrnal modules
# see http://stackoverflow.com/questions/1907993/autoreload-of-modules-in-ipython
%load_ext autoreload
%autoreload 2
```

Load data

Similar to previous exercises, we will load CIFAR-10 data from disk.

```
In [8]: from cs682.features import color_histogram_hsv, hog_feature

def get_CIFAR10_data(num_training=49000, num_validation=1000, num_test=1000):
    # Load the raw CIFAR-10 data
    cifar10_dir = 'cs682/datasets/cifar-10-batches-py'

    X_train, y_train, X_test, y_test = load_CIFAR10(cifar10_dir)

    # Subsample the data
    mask = list(range(num_training, num_training + num_validation))
    X_val = X_train[mask]
    y_val = y_train[mask]
    mask = list(range(num_training))
    X_train = X_train[mask]
    y_train = y_train[mask]
    mask = list(range(num_test))
    X_test = X_test[mask]
    y_test = y_test[mask]

    return X_train, y_train, X_val, y_val, X_test, y_test

# Cleaning up variables to prevent loading data multiple times (which may cause memory issue)
try:
    del X_train, y_train
    del X_test, y_test
    print('Clear previously loaded data.')
except:
    pass

X_train, y_train, X_val, y_val, X_test, y_test = get_CIFAR10_data()
```

Extract Features

For each image we will compute a Histogram of Oriented Gradients (HOG) as well as a color histogram using the hue channel in HSV color space. We form our final feature vector for each image by concatenating the HOG and color histogram feature vectors.

Roughly speaking, HOG should capture the texture of the image while ignoring color information, and the color histogram represents the color of the input image while ignoring texture. As a result, we expect that using both together ought to work better than using either alone. Verifying this assumption would be a good thing to try for your interests.

The `hog_feature` and `color_histogram_hsv` functions both operate on a single image and return a feature vector for that image. The `extract_features` function takes a set of images and a list of feature functions and evaluates each feature function on each image, storing the results in a matrix where each column is the concatenation of all feature vectors for a single image.

```
In [9]: from cs682.features import *

num_color_bins = 10 # Number of bins in the color histogram
feature_fns = [hog_feature, lambda img: color_histogram_hsv(img, nbin=num_color_bins)]
X_train_feats = extract_features(X_train, feature_fns, verbose=True)
X_val_feats = extract_features(X_val, feature_fns)
X_test_feats = extract_features(X_test, feature_fns)

# Preprocessing: Subtract the mean feature
mean_feat = np.mean(X_train_feats, axis=0, keepdims=True)
X_train_feats -= mean_feat
X_val_feats -= mean_feat
```

```

X_test_feats -= mean_feat

# Preprocessing: Divide by standard deviation. This ensures that each feature
# has roughly the same scale.
std_feat = np.std(X_train_feats, axis=0, keepdims=True)
X_train_feats /= std_feat
X_val_feats /= std_feat
X_test_feats /= std_feat

# Preprocessing: Add a bias dimension
X_train_feats = np.hstack([X_train_feats, np.ones((X_train_feats.shape[0], 1))])
X_val_feats = np.hstack([X_val_feats, np.ones((X_val_feats.shape[0], 1))])
X_test_feats = np.hstack([X_test_feats, np.ones((X_test_feats.shape[0], 1))])

```

```

Done extracting features for 1000 / 49000 images
Done extracting features for 2000 / 49000 images
Done extracting features for 3000 / 49000 images
Done extracting features for 4000 / 49000 images
Done extracting features for 5000 / 49000 images
Done extracting features for 6000 / 49000 images
Done extracting features for 7000 / 49000 images
Done extracting features for 8000 / 49000 images
Done extracting features for 9000 / 49000 images
Done extracting features for 10000 / 49000 images
Done extracting features for 11000 / 49000 images
Done extracting features for 12000 / 49000 images
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Done extracting features for 41000 / 49000 images
Done extracting features for 42000 / 49000 images
Done extracting features for 43000 / 49000 images
Done extracting features for 44000 / 49000 images
Done extracting features for 45000 / 49000 images
Done extracting features for 46000 / 49000 images
Done extracting features for 47000 / 49000 images
Done extracting features for 48000 / 49000 images

```

Train SVM on features

Using the multiclass SVM code developed earlier in the assignment, train SVMs on top of the features extracted above; this should achieve better results than training SVMs directly on top of raw pixels.

```

In [18]: # Use the validation set to tune the learning rate and regularization strength

from cs682.classifiers.linear_classifier import LinearSVM

learning_rates = [0.9e-9, 0.9e-8, 0.9e-7]
regularization_strengths = [2e4, 1.9e5, 2.2e6]

results = {}
best_val = -1
best_svm = None

#####
# TODO:
# Use the validation set to set the learning rate and regularization strength.
# This should be identical to the validation that you did for the SVM; save
# the best trained classifier in best_svm. You might also want to play
# with different numbers of bins in the color histogram. If you are careful
# you should be able to get accuracy of near 0.44 on the validation set.
#####
for lr in learning_rates:
    for reg in regularization_strengths:
        svm = LinearSVM()
        svm.train(X_train_feats, y_train, learning_rate = lr, reg = reg, num_iters = 5000, verbose = False)
        y_train_pred = svm.predict(X_train_feats)
        train_accuracy = np.mean(y_train == y_train_pred)
        y_val_pred = svm.predict(X_val_feats)
        val_accuracy = np.mean(y_val == y_val_pred)
        results[(lr, reg)] = (train_accuracy, val_accuracy)
        if val_accuracy > best_val:

```

```

        best_val = val_accuracy
        best_svm = svm
#####
#                               END OF YOUR CODE                               #
#####

# Print out results.
for lr, reg in sorted(results):
    train_accuracy, val_accuracy = results[(lr, reg)]
    print('lr %e reg %e train accuracy: %f val accuracy: %f' % (
        lr, reg, train_accuracy, val_accuracy))

print('best validation accuracy achieved during cross-validation: %f' % best_val)

lr 9.000000e-10 reg 2.000000e+04 train accuracy: 0.076755 val accuracy: 0.086000
lr 9.000000e-10 reg 1.900000e+05 train accuracy: 0.103633 val accuracy: 0.105000
lr 9.000000e-10 reg 2.200000e+06 train accuracy: 0.398184 val accuracy: 0.383000
lr 9.000000e-09 reg 2.000000e+04 train accuracy: 0.134714 val accuracy: 0.114000
lr 9.000000e-09 reg 1.900000e+05 train accuracy: 0.409286 val accuracy: 0.410000
lr 9.000000e-09 reg 2.200000e+06 train accuracy: 0.415571 val accuracy: 0.426000
lr 9.000000e-08 reg 2.000000e+04 train accuracy: 0.415388 val accuracy: 0.413000
lr 9.000000e-08 reg 1.900000e+05 train accuracy: 0.417224 val accuracy: 0.411000
lr 9.000000e-08 reg 2.200000e+06 train accuracy: 0.397918 val accuracy: 0.392000
best validation accuracy achieved during cross-validation: 0.426000

```

```

In [19]: # Evaluate your trained SVM on the test set
y_test_pred = best_svm.predict(X_test_feats)
test_accuracy = np.mean(y_test == y_test_pred)
print(test_accuracy)

0.421

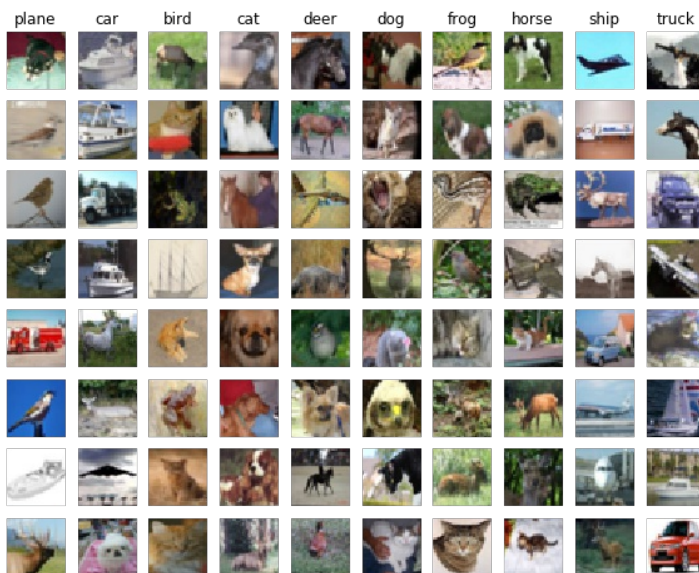
```

```

In [12]: # An important way to gain intuition about how an algorithm works is to
# visualize the mistakes that it makes. In this visualization, we show examples
# of images that are misclassified by our current system. The first column
# shows images that our system labeled as "plane" but whose true label is
# something other than "plane".

examples_per_class = 8
classes = ['plane', 'car', 'bird', 'cat', 'deer', 'dog', 'frog', 'horse', 'ship', 'truck']
for cls, cls_name in enumerate(classes):
    idxs = np.where((y_test != cls) & (y_test_pred == cls))[0]
    idxs = np.random.choice(idxs, examples_per_class, replace=False)
    for i, idx in enumerate(idxs):
        plt.subplot(examples_per_class, len(classes), i * len(classes) + cls + 1)
        plt.imshow(X_test[idx].astype('uint8'))
        plt.axis('off')
        if i == 0:
            plt.title(cls_name)
plt.show()

```



Inline question 1:

Describe the misclassification results that you see. Do they make sense?

The misclassification results do make sense. The images have similar features to the predicted object make them look like each other. The feature of HOG is bad to differentiate those images.

Neural Network on image features

Earlier in this assignment we saw that training a two-layer neural network on raw pixels achieved better classification performance than linear classifiers on raw pixels. In this notebook we have seen that linear classifiers on image features outperform linear classifiers on raw pixels.

For completeness, we should also try training a neural network on image features. This approach should outperform all previous approaches: you should easily be able to achieve over 55% classification accuracy on the test set; our best model achieves about 60% classification accuracy.


```
In [13]: # Preprocessing: Remove the bias dimension
# Make sure to run this cell only ONCE
print(X_train_feats.shape)
X_train_feats = X_train_feats[:, :-1]
X_val_feats = X_val_feats[:, :-1]
X_test_feats = X_test_feats[:, :-1]

print(X_train_feats.shape)

(49000, 155)
(49000, 154)
```

```
In [14]: from cs682.classifiers.neural_net import TwoLayerNet

input_dim = X_train_feats.shape[1]
hidden_dim = 500
num_classes = 10

net = TwoLayerNet(input_dim, hidden_dim, num_classes)
best_net = None

#####
# TODO: Train a two-layer neural network on image features. You may want to #
# cross-validate various parameters as in previous sections. Store your best #
# model in the best_net variable. #
#####
stats = net.train(X_train_feats, y_train, X_val_feats, y_val, num_iters = 2000,
                  batch_size = 200, learning_rate = 1,
                  learning_rate_decay = 0.95, reg = 0, verbose = True)
val_accuracy = (net.predict(X_val_feats) == y_val).mean()
print('Validation accuracy: ', val_accuracy)

best_net = net
#####
#                               END OF YOUR CODE                               #
#####

iteration 0 / 2000: loss 2.302585
iteration 100 / 2000: loss 1.512682
iteration 200 / 2000: loss 1.332612
iteration 300 / 2000: loss 1.230554
iteration 400 / 2000: loss 1.227534
iteration 500 / 2000: loss 1.058418
iteration 600 / 2000: loss 1.104683
iteration 700 / 2000: loss 1.077495
iteration 800 / 2000: loss 1.096989
iteration 900 / 2000: loss 0.994719
iteration 1000 / 2000: loss 0.766091
iteration 1100 / 2000: loss 0.954954
iteration 1200 / 2000: loss 0.859290
iteration 1300 / 2000: loss 0.738803
iteration 1400 / 2000: loss 0.919005
iteration 1500 / 2000: loss 0.716359
iteration 1600 / 2000: loss 0.779091
iteration 1700 / 2000: loss 0.841313
iteration 1800 / 2000: loss 0.874381
iteration 1900 / 2000: loss 0.594293
Validation accuracy: 0.562
```

```
In [15]: # Run your best neural net classifier on the test set. You should be able
# to get more than 55% accuracy.

test_acc = (best_net.predict(X_test_feats) == y_test).mean()
print(test_acc)

0.545
```