CSE 586A **Xiwen Li**

Problem Set I

1. The problem is to compare solution on a differential equation using two different differential equations.

   Euler's approximation approximates both v_t and ϕ_t in each iteration

   (a) $\frac{dy}{dt} + 2y = 2 - e^{-4t}$

   Let $\mu(t) = e(2t)$, $\mu'(t) = 2e^{2t}$

   Multiply both sides by $\mu(t)$: $\frac{de^{2t}y}{dt} + 2e^{2t}y = 2e^{2t} - e^{-2t}$

   $\mu(t)\frac{dy}{dt} + \mu'(t)y = 2e^{2t} - e^{-2t}$

   $(\mu(t)y(t))' = 2e^{2t} - e^{-2t}$

   $y(t) = 1 + \frac{1}{2}e^{-4t} - \frac{C}{e^{2t}}$

   Since $y(0)$=1,

   $y(t) = 1 + \frac{1}{2}e^{-4t} - \frac{1}{2e^{2t}}$

   (b) We can rewrite the equation as:

   $\frac{dy}{dt} = -2y + 2 - e^{-4t}$

   Then the equation of Euler approximation is:

   $d_{n+1} = d_n + h\frac{dy}{dt}$
   $d_{n+1} = d_n + h(-2y + 2 - e^{-4t})$

step size	**t = 1**	**t = 2**	**t = 3**	**t = 4**	**t = 5**
0.1	0.931324	0.991368	0.999050	0.999898	0.999989
0.05	0.936470	0.991113	0.998898	0.999866	0.999984
0.01	0.940499	0.991019	0.998789	0.999839	0.999979
0.005	0.940996	0.991014	0.998776	0.999836	0.999978
0.001	0.941391	0.991011	0.998766	0.999833	0.999977
original	0.941490	0.991010	0.998764	0.999832	0.999977

Table 1: Values by Euler's Method

   (c)

2. The problem is to use Euler's approximation to estimate the transformation at t = 1.

At the very beginning, v_0 and ϕ_0 are initialized. The initial image is defined by I_0. At each iteration, I approximate v_t and ϕ_t using equation:

$$v_{t+1} = v_t + h * \frac{dv_t}{dt} \tag{1}$$

$$\phi_{t+1} = \phi_t + h * \frac{d\phi_t}{dt} \tag{2}$$

h is the step size. Once I get transformation, Final image is interpolated by as equation (3):

$$I_{final} = I_0 \circ \phi_{t=1} \tag{3}$$

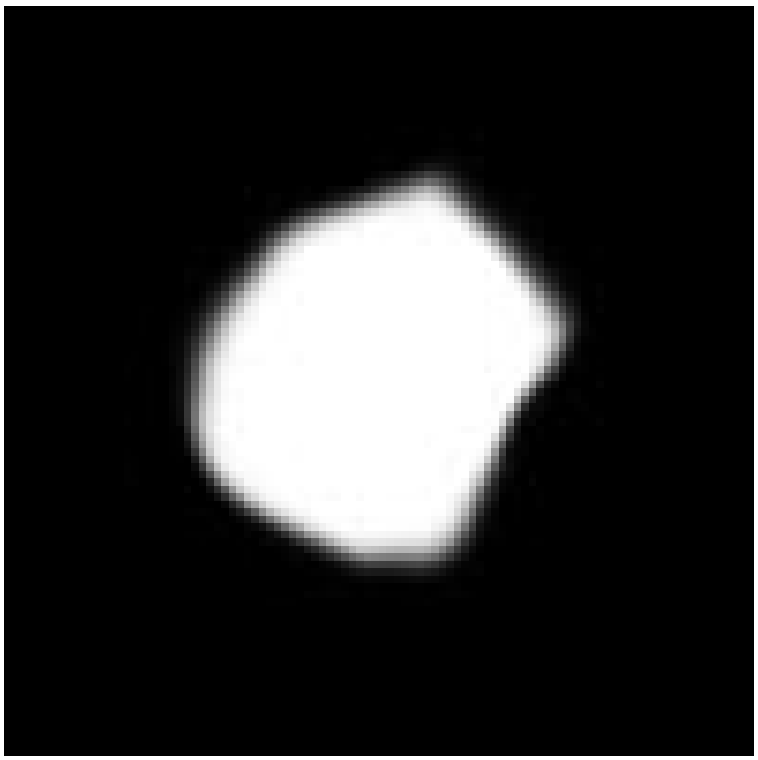
(a) 

Figure 1: Final Image Transformed by (a)

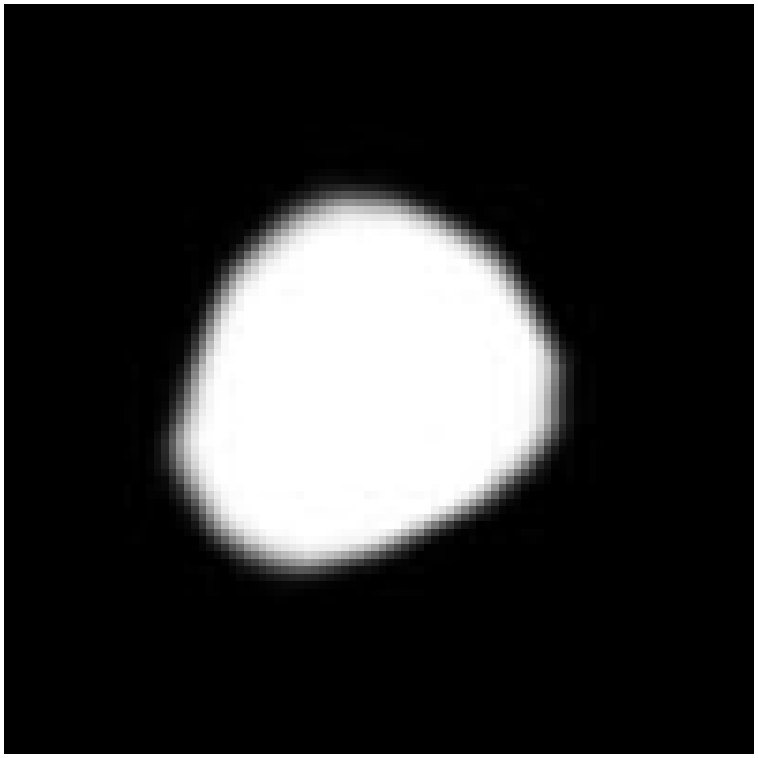
(b) 

Figure 2: Final Image Transformed by (b)