CSE 586A **Xiwen Li**

Problem Set I

1. The problem is to compare solution on a differential equation using two different differential equations.

   Euler's approximation approximates both v_t and ϕ_t in each iteration

   (a) $\frac{dy}{dt} + 2y = 2 - e^{-4t}$

   Let $\mu(t) = e(2t)$, $\mu'(t) = 2e^{2t}$

   Multiply both sides by $\mu(t)$: $\frac{de^{2t}y}{dt} + 2e^{2t}y = 2e^{2t} - e^{-2t}$

   $\mu(t)\frac{dy}{dt} + \mu'(t)y = 2e^{2t} - e^{-2t}$

   $(\mu(t)y(t))' = 2e^{2t} - e^{-2t}$

   $y(t) = 1 + \frac{1}{2}e^{-4t} - \frac{C}{e^{2t}}$

   Since $y(0)$=1,

   $y(t) = 1 + \frac{1}{2}e^{-4t} - \frac{1}{2e^{2t}}$

   (b) We can rewrite the equation as:

   $\frac{dy}{dt} = -2y + 2 - e^{-4t}$

   Then the equation of Euler approximation is:

   $d_{n+1} = d_n + h\frac{dy}{dt}$
   $d_{n+1} = d_n + h(-2y + 2 - e^{-4t})$
   Approximated solution and exact solution are shown in Table 1. The step size h = 0.1.

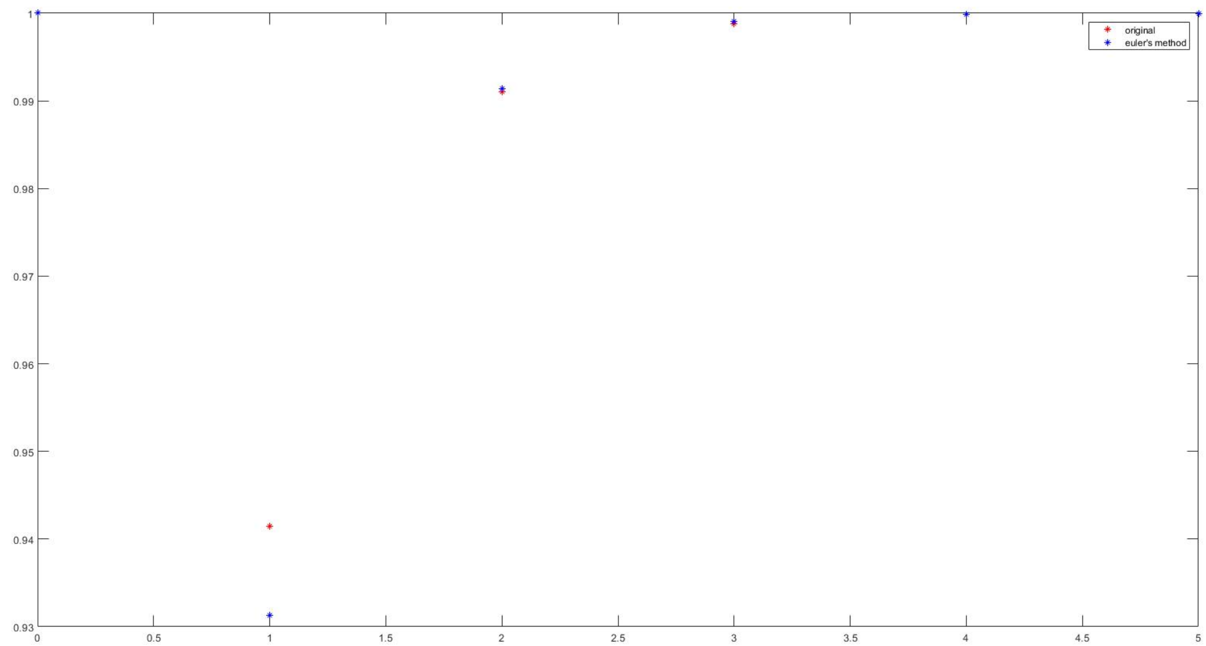

Figure 1: Comparison between Original Function and Euler's Approximation

step size	$\mathbf{t = 1}$	$\mathbf{t = 2}$	$\mathbf{t = 3}$	$\mathbf{t = 4}$	$\mathbf{t = 5}$
0.1	0.931324	0.991368	0.999050	0.999898	0.999989
original	0.941490	0.991010	0.998764	0.999832	0.999977

Table 1: Values by Euler's Method and by Original Function

(c) I plot five curves at different step size compared to the original curve. They are shown in Figure 1. From the picture, when $t = 0.001$, the approximation fits the original function best.

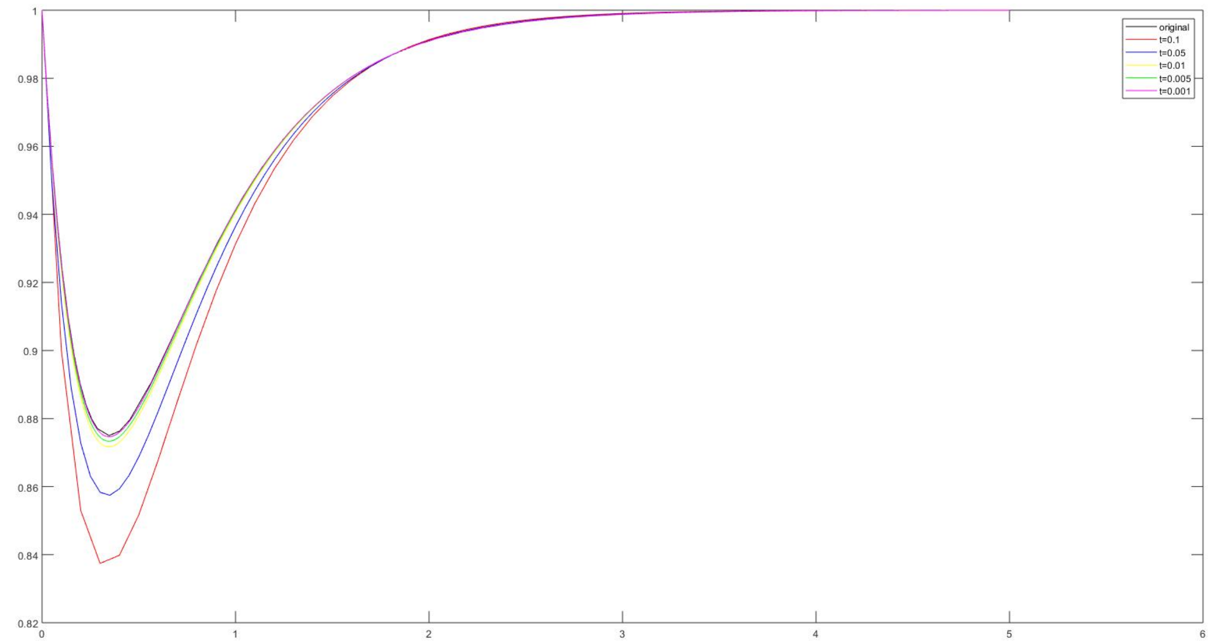

Figure 2: Final Image Transformed by (a)

2. The problem is to use Euler's approximation to estimate the transformation at t = 1.

   At the very beginning, v_0 and ϕ_0 are initialized. v_0 is loaded from a velocity field. ϕ_t consists of both x coordinate matrix and y coordinate matrix of all pixel locations. Their sizes are both 100 by 100. The initial image is defined by I_0. The algorithm iteratively approximates v_t and ϕ_t. At each iteration, I approximate v_t and ϕ_t using equation (1) and (2) with step size h = 0.001.

$$v_{t+1} = v_t + h * \frac{dv_t}{dt} \tag{1}$$

$$\phi_{t+1} = \phi_t + h * \frac{d\phi_t}{dt} \tag{2}$$

   Once I get final transformation, final image is interpolated by as equation (3). Result image using differential equations in part (a) is shown in Figure 2, while result image using equations in part (b) is shown in Figure 3.

$$I_{final} = I_0 \circ \phi_{t=1} \tag{3}$$

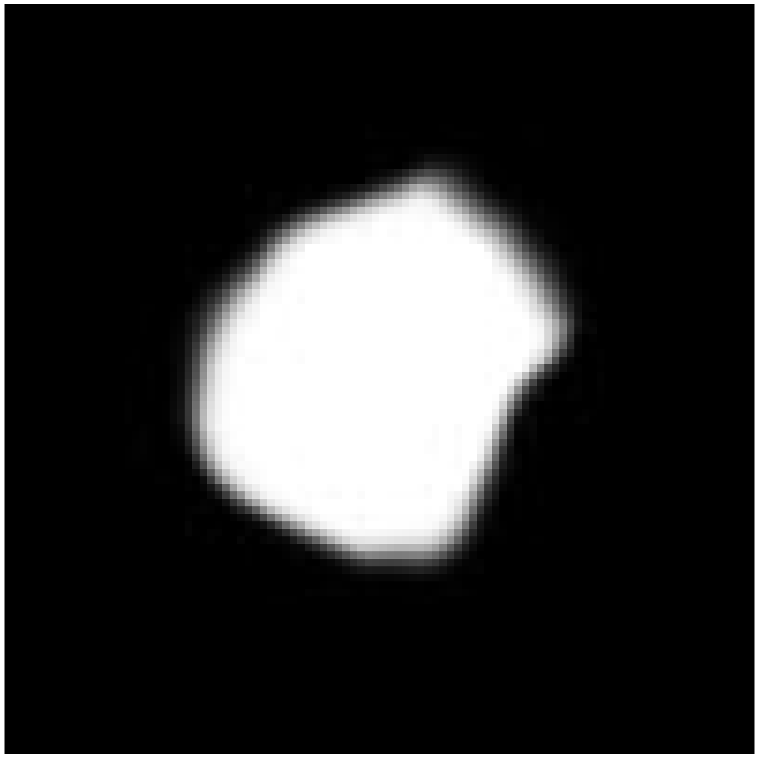

Figure 3: Final Image Transformed by (a)

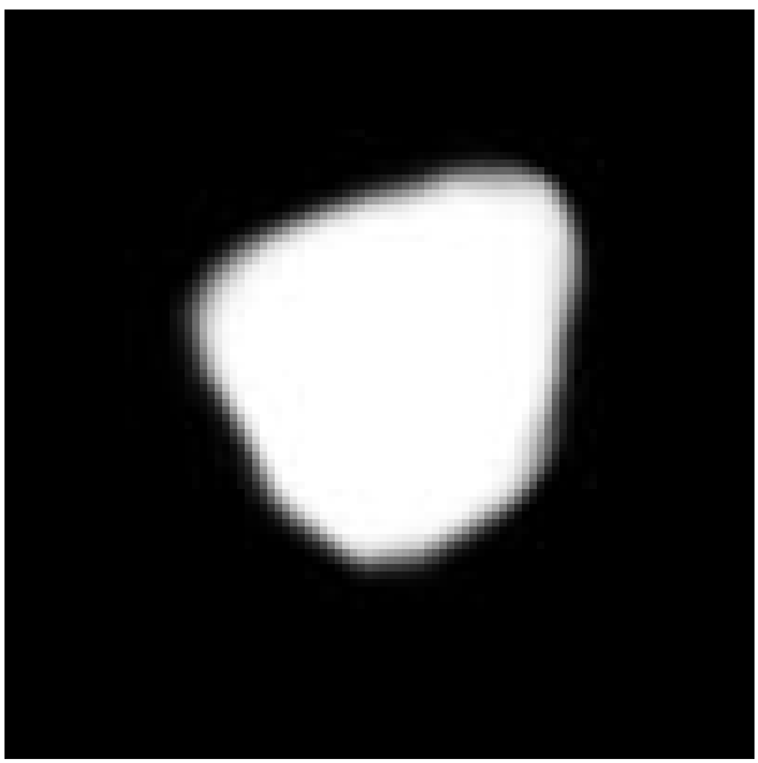

Figure 4: Final Image Transformed by (b)