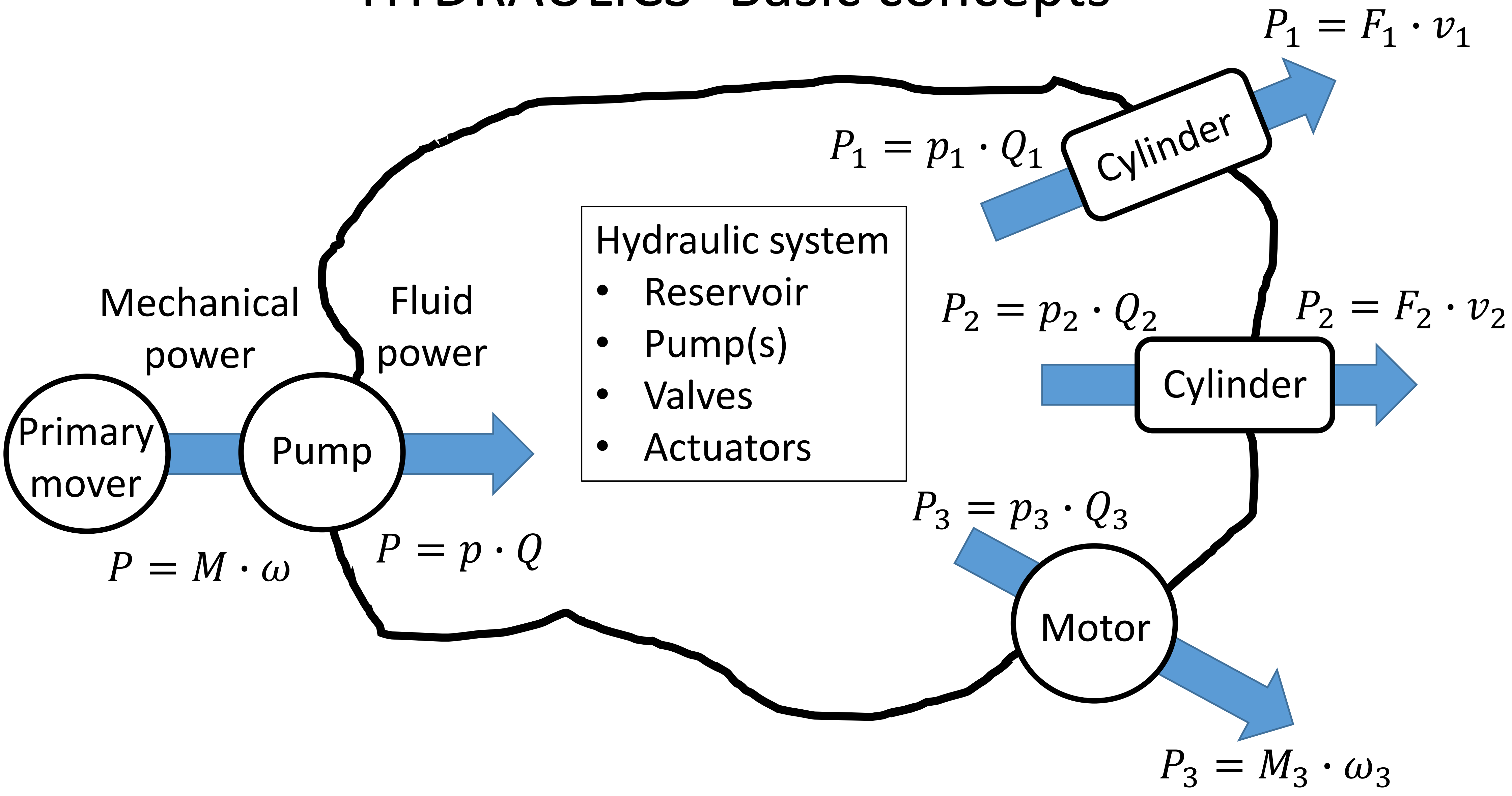
HYDRAULICS

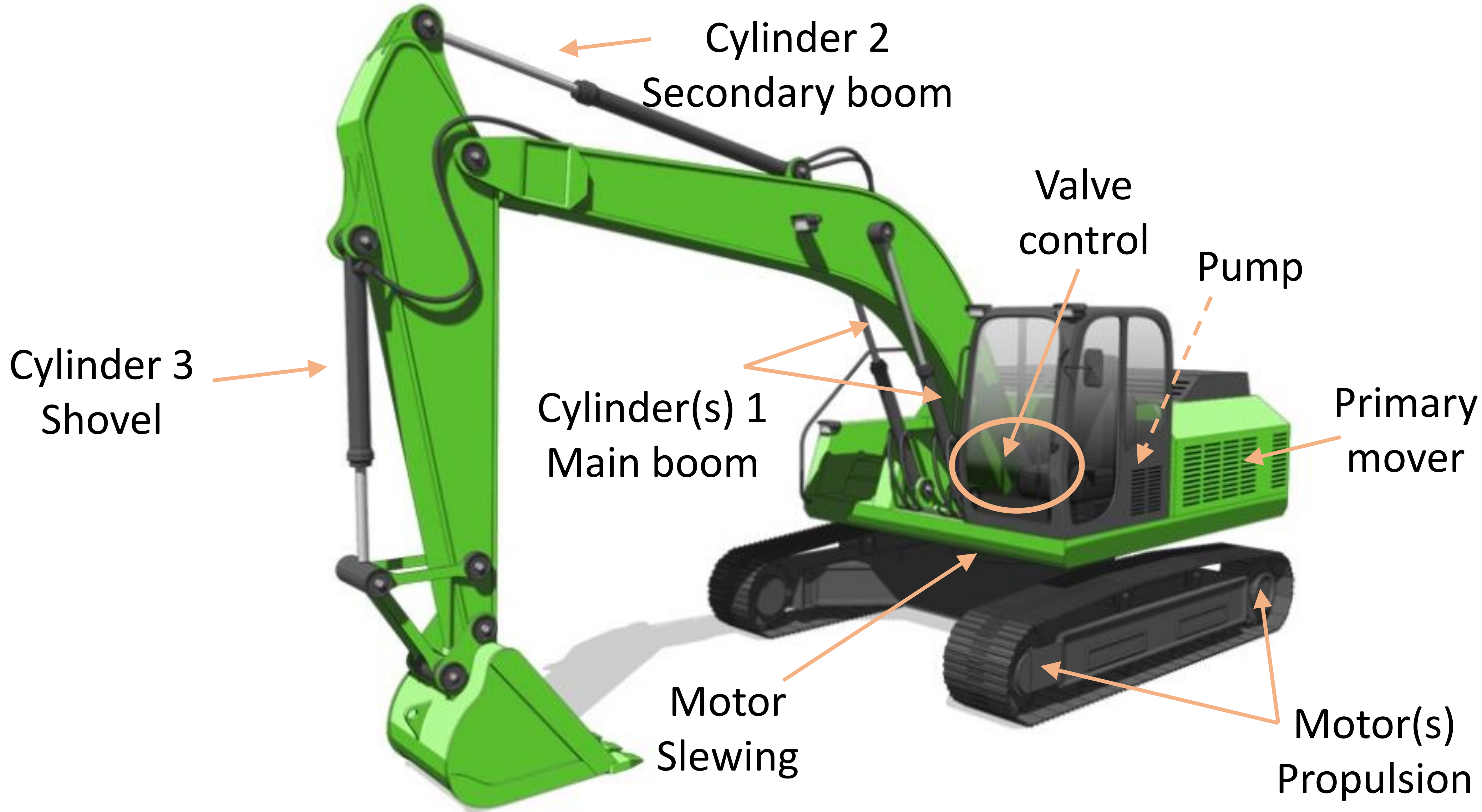
- Basic concepts
- Orifice equation
- Continuity equation, incompressible
- Continuity equation, compressible
- Examples

MAS 416, Lecture 4

HYDRAULICS- Basic concepts

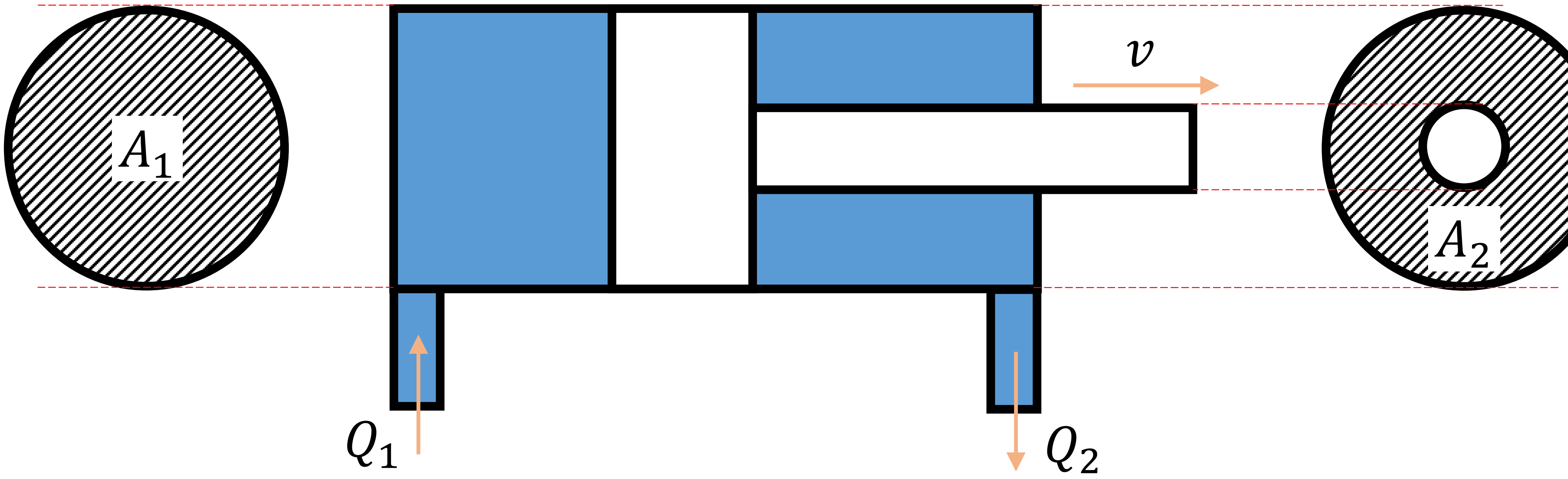


HYDRAULICS - Basic concepts



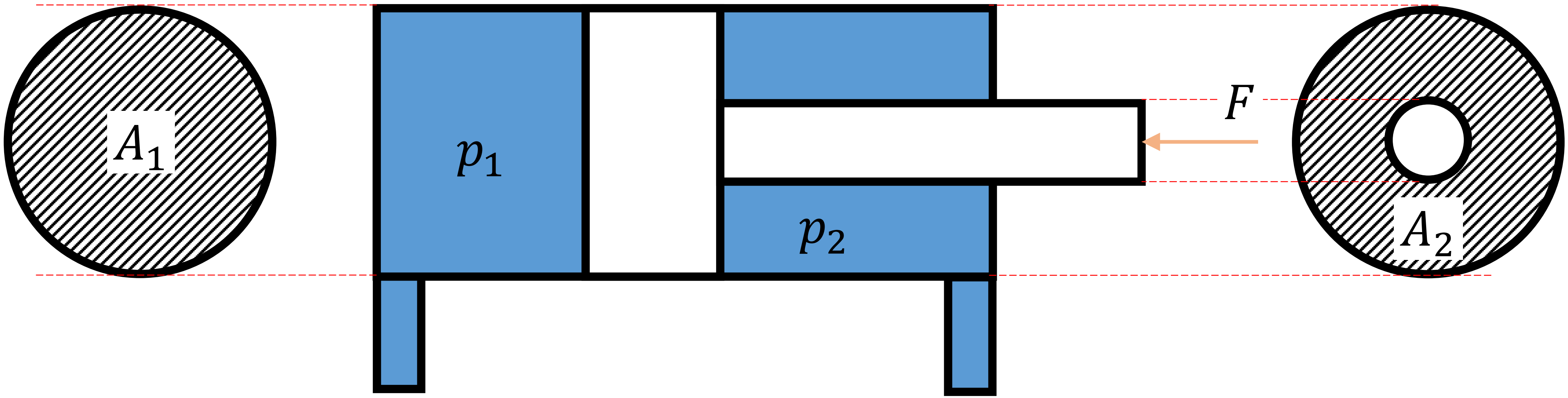
http://global.kawasaki.com/en/industrial_equipment/hydraulic/systems/excavator.html

HYDRAULICS - Basic concepts



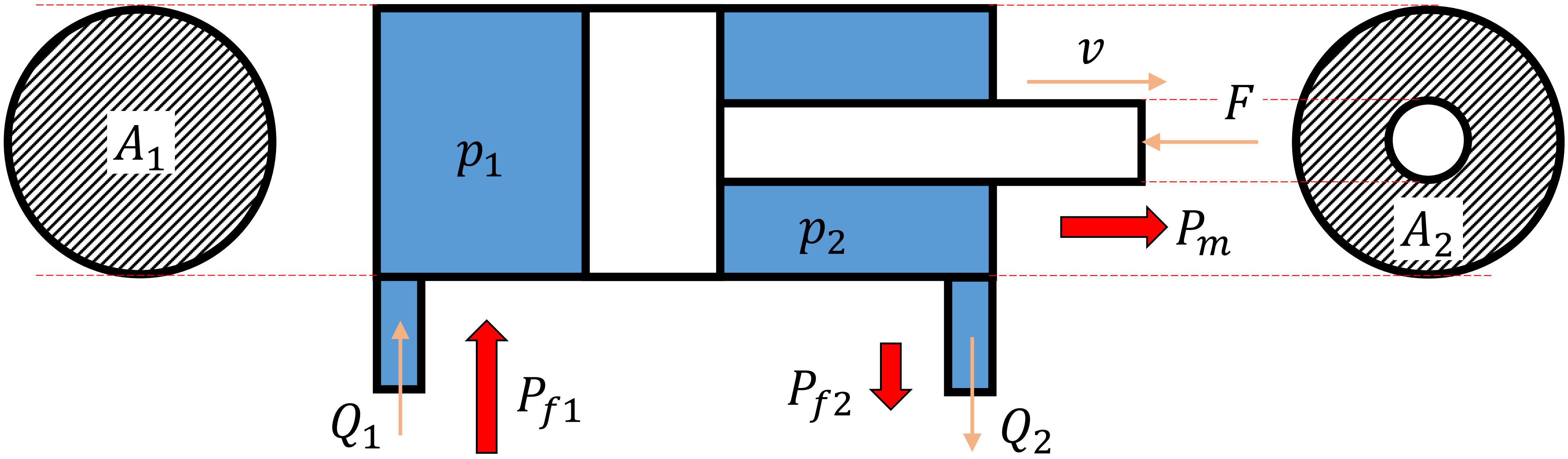
$$v = \frac{Q_1}{A_1} = \frac{Q_2}{A_2} \qquad Q_1 = A_1 \cdot v \qquad Q_2 = A_2 \cdot v \qquad \frac{Q_2}{Q_1} = \frac{A_2}{A_1}$$

HYDRAULICS - Basic concepts



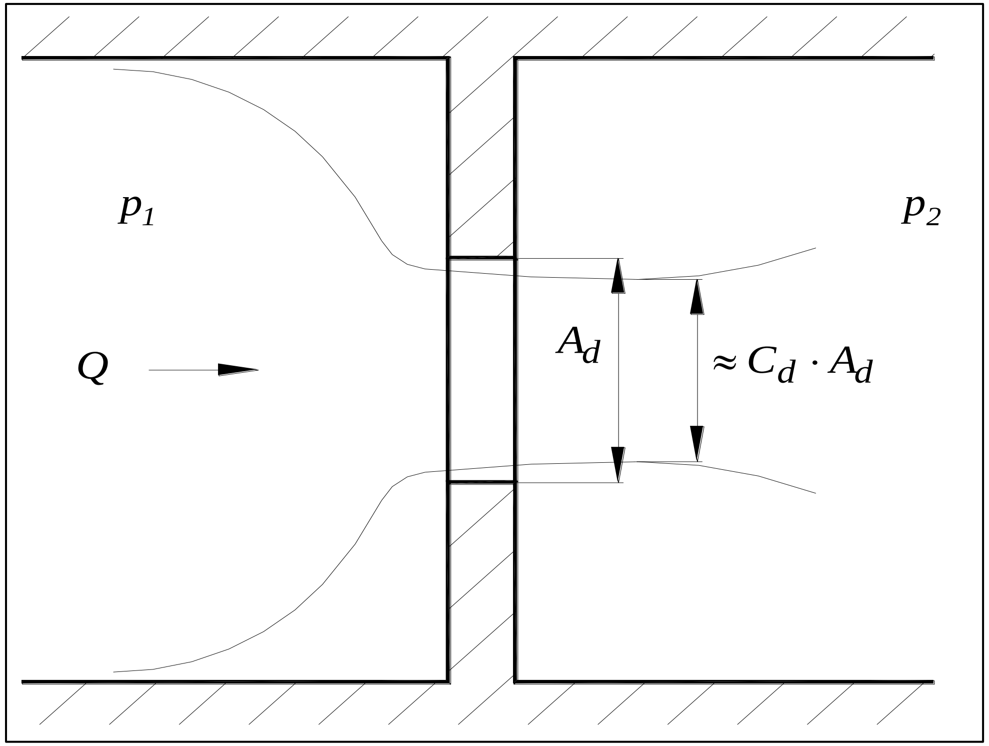
$$F = p_1 \cdot A_1 - p_2 \cdot A_2$$

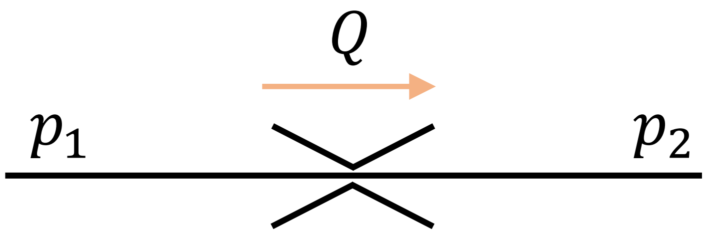
HYDRAULICS - Basic concepts



$$P_m = F \cdot v = (p_1 \cdot A_1 - p_2 \cdot A_2) \cdot v = p_1 \cdot Q_1 - p_2 \cdot Q_2 = P_{f1} - P_{f2}$$

HYDRAULICS – Orifice equation





Classical presentation

$$Q = C_d \cdot A_d \cdot \sqrt{\frac{2}{\rho} \cdot (p_1 - p_2)}$$

Numerical presentation

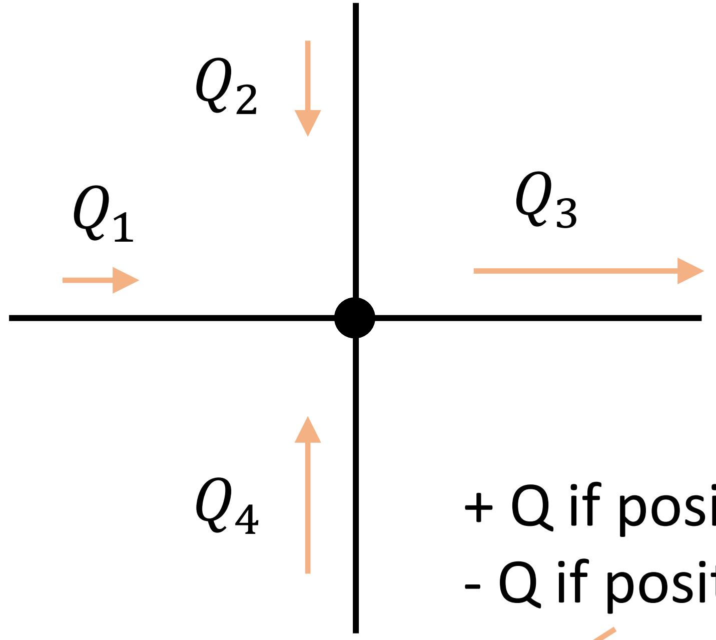
$$Q = K_v \cdot SIGN(\Delta p) \cdot \sqrt{|\Delta p|}$$

$$\Delta p = p_1 - p_2$$

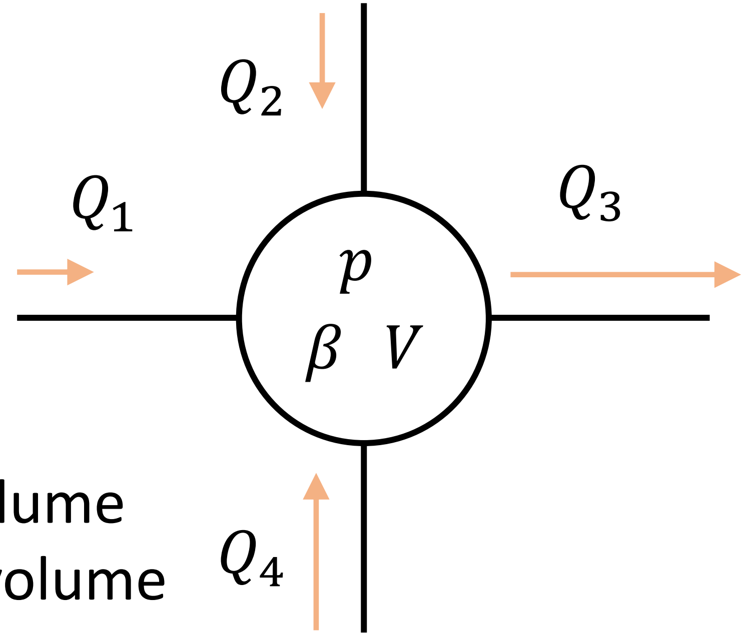
$$K_v = C_d \cdot A_d \cdot \sqrt{\frac{2}{\rho}}$$

HYDRAULICS – Continuity

Incompressible



Compressible, cst. volume



+ Q if positive direction into volume
- Q if positive direction out of volume

$$\Sigma Q_i = Q_1 + Q_2 - Q_3 + Q_4 = 0$$

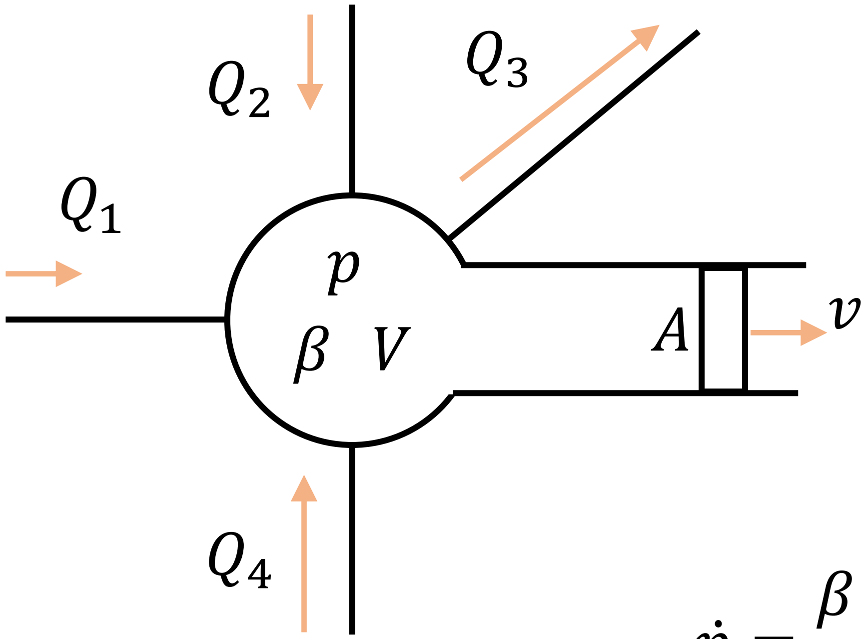
$$\Sigma Q_i = \frac{V}{\beta} \cdot \dot{p}$$

or

$$\dot{p} = \frac{\beta}{V} \cdot \Sigma Q_i$$

HYDRAULICS – Continuity

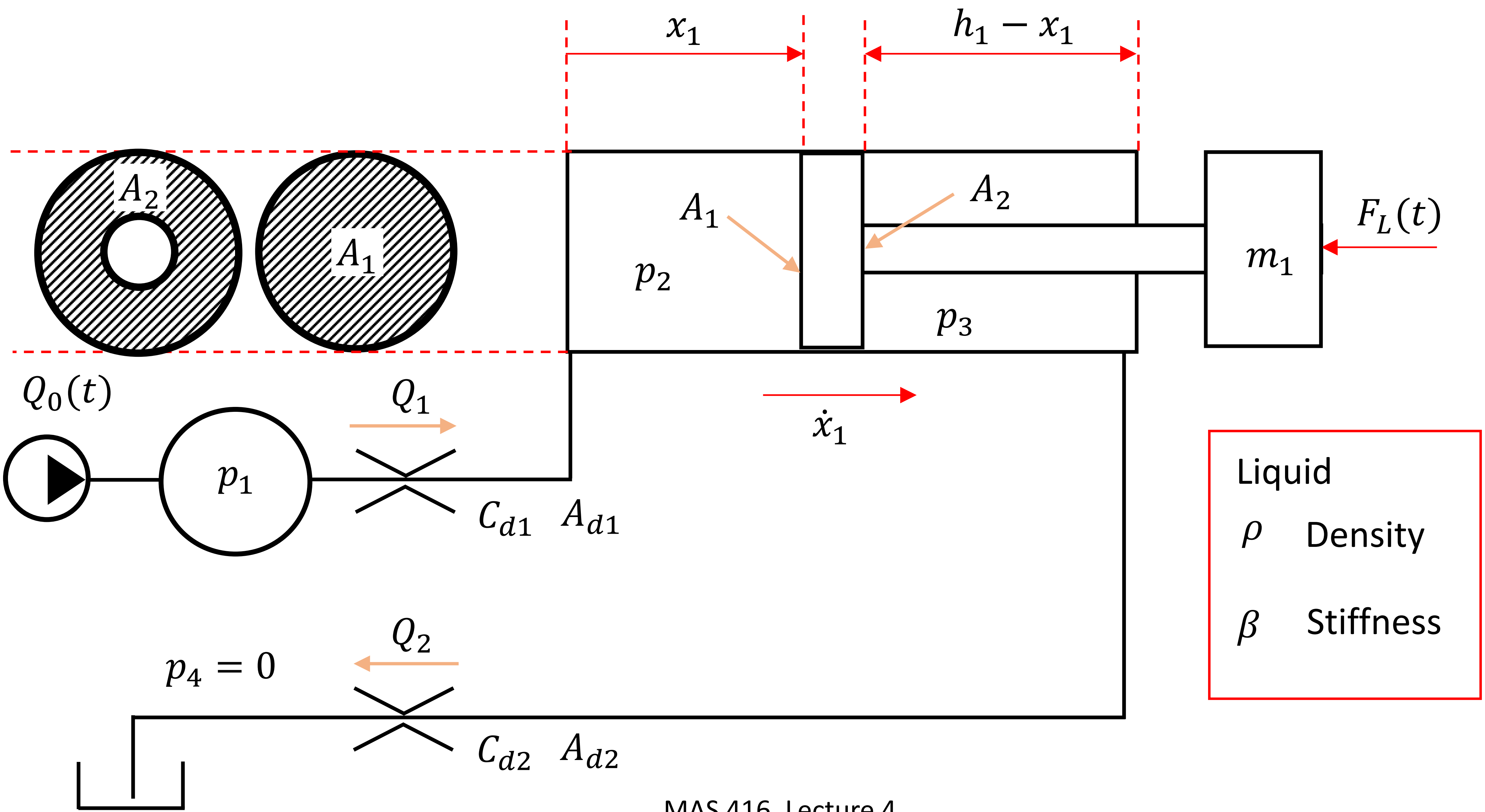
Compressible, changing volume

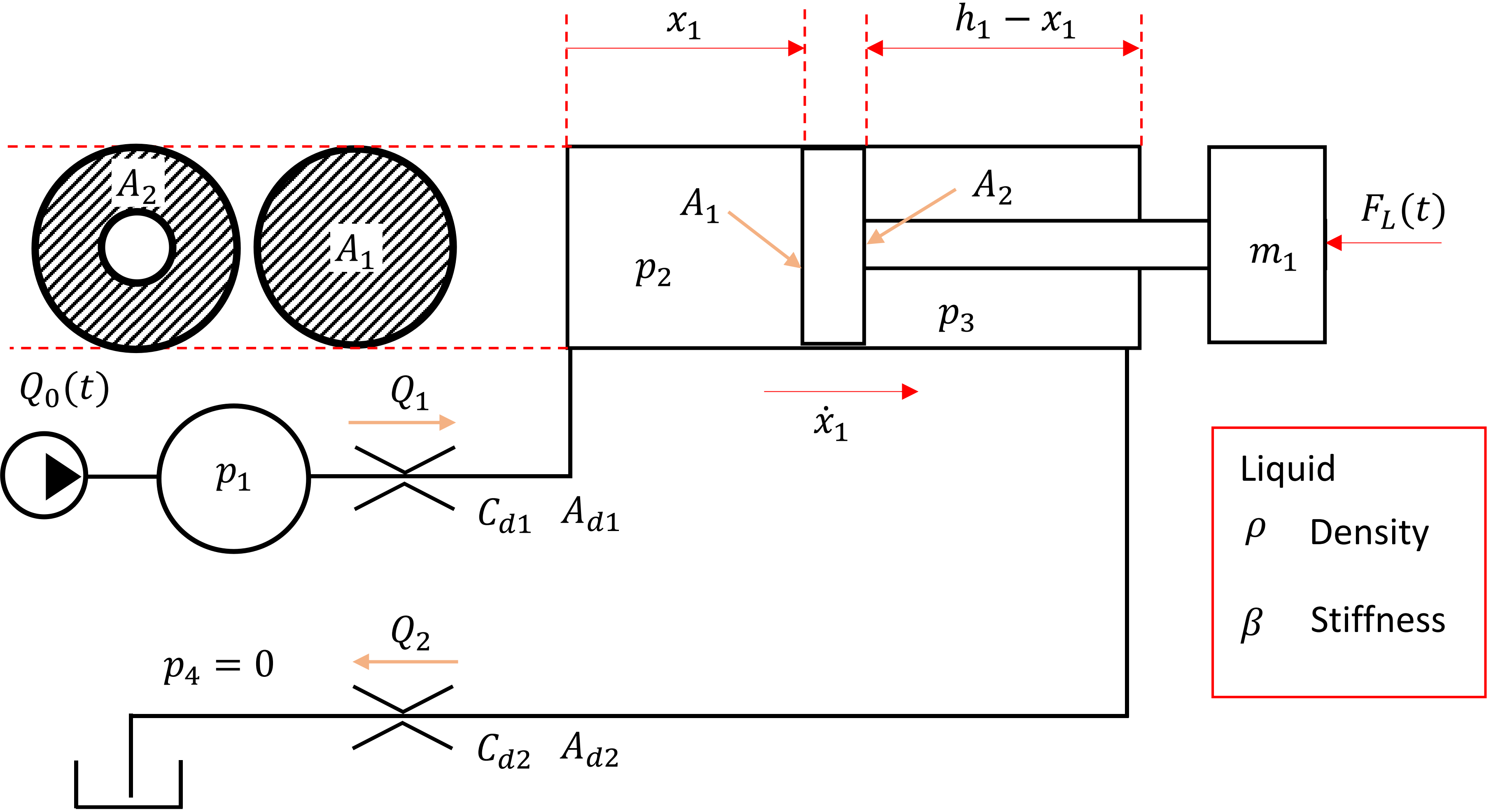


$$\dot{p} = \frac{\beta}{V} \cdot \left(\Sigma Q_i - \dot{V}\right)$$

$\dot{V}$ is positive if volume is increasing
$\dot{V}$ is negative if volume is decreasing

$$\dot{p} = \frac{\beta}{V} \cdot (Q_1 + Q_2 - Q_3 + Q_4 - A \cdot v)$$


x_1
$h_1 - x_1$
A_2
A_1
A_1
A_2
p_2
p_3
m_1
$F_L(t)$
$Q_0(t)$
Q_1
$\dot{x}_1$
p_1
C_{d1} A_{d1}
Liquid
ρ Density
β Stiffness
Q_2
$p_4 = 0$
C_{d2} A_{d2}



STATE VARIABLES

p_1 p_2 p_3

x_1 $\dot{x}_1$

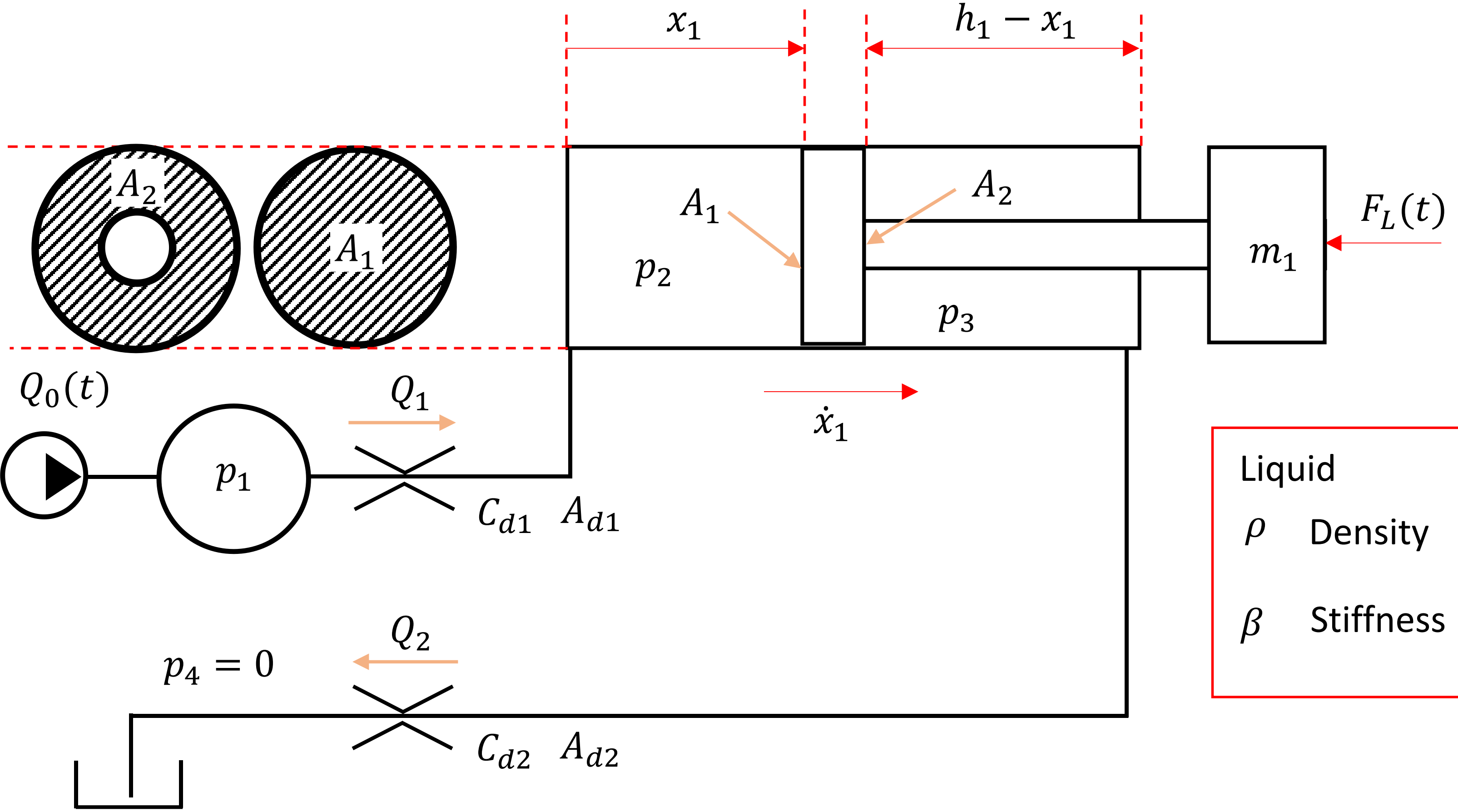
INITIAL CONDITIONS

$$p_1^{(init)} = p_2^{(init)} = \frac{F_{L,0}}{A_1}$$

$$p_3^{(init)} = 0$$

$$x_1^{(init)} = x_0$$

$$\dot{x}_1^{(init)} = 0$$



Liquid

ρ Density

β Stiffness

ALGEBRAIC EQS.

$$V_2 = A_1 \cdot x_1$$

$$V_3 = A_2 \cdot (h_1 - x_1)$$

$$Q_1 = C_{d1} \cdot A_{d1} \cdot \sqrt{\frac{2}{\rho}(p_1 - p_2)}$$

$$Q_2 = C_{d2} \cdot A_{d2} \cdot \sqrt{\frac{2}{\rho} p_3}$$

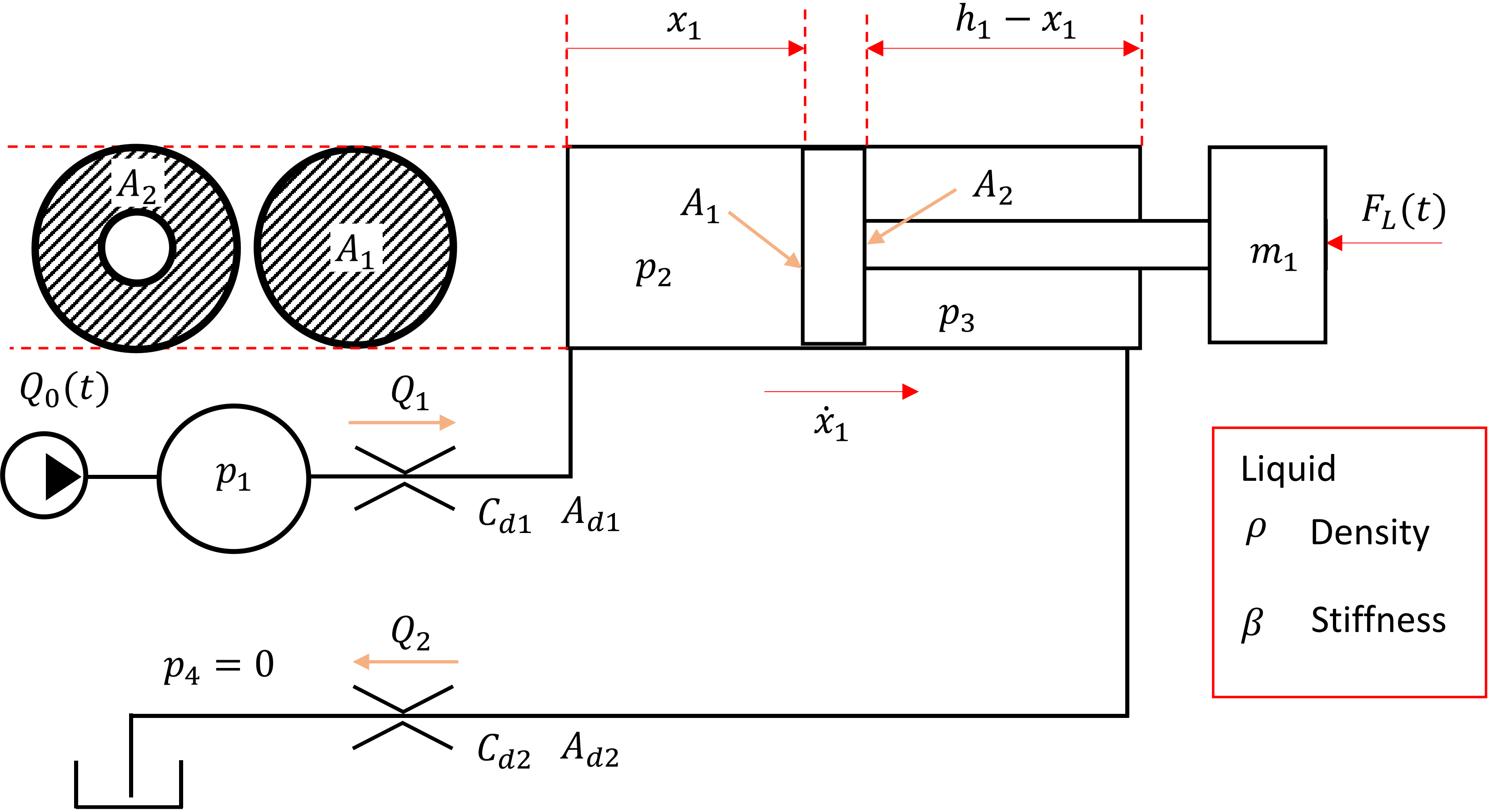
DIFFERENTIAL EQS.

$$\dot{p}_1 = \frac{\beta}{V_1} \cdot (Q_0(t) - Q_1)$$

$$\dot{p}_2 = \frac{\beta}{V_2} \cdot (Q_1 - A_1 \cdot \dot{x}_1)$$

$$\dot{p}_3 = \frac{\beta}{V_3} \cdot (A_2 \cdot \dot{x}_1 - Q_2)$$

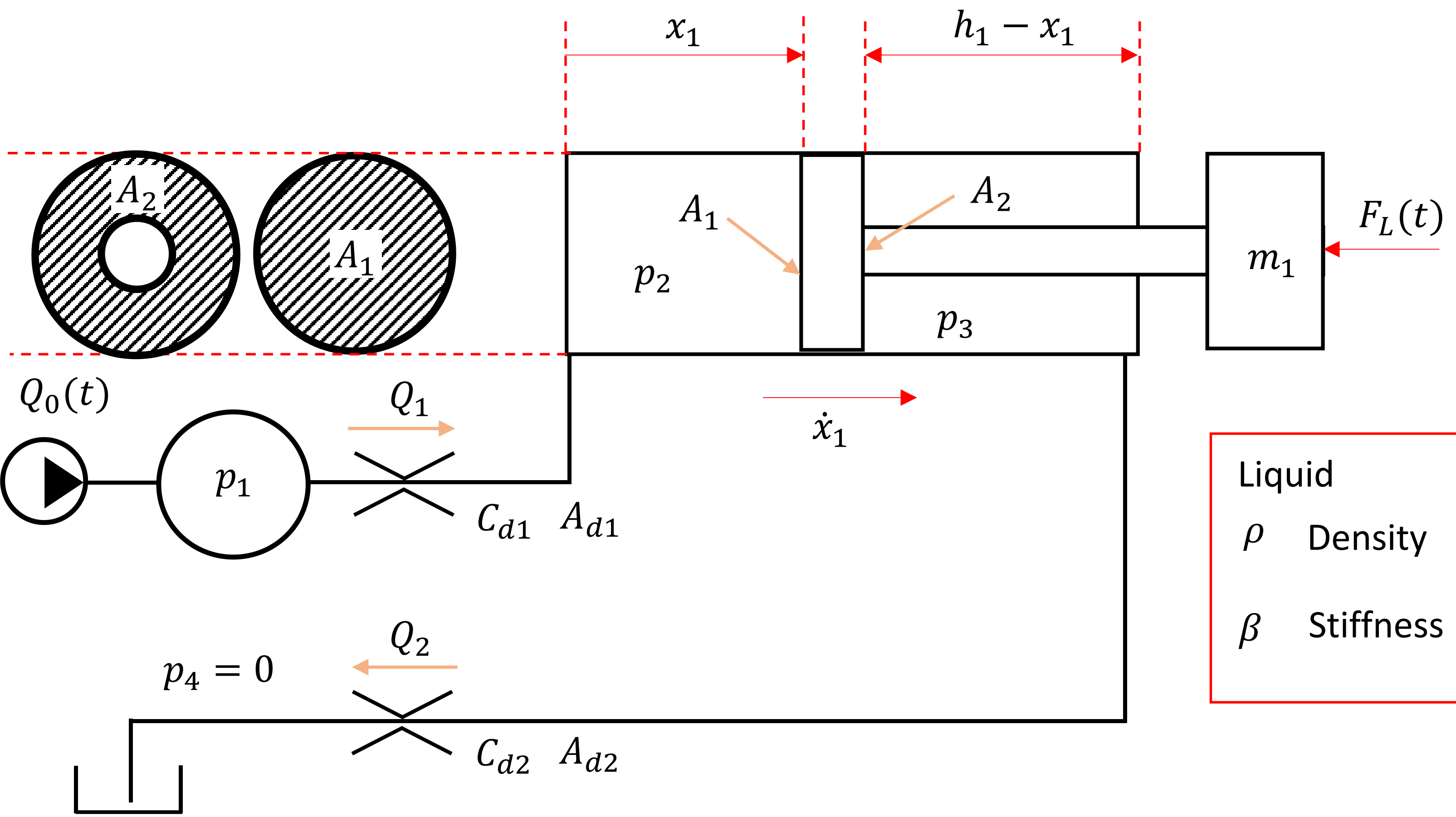
$$\ddot{x}_1 = \frac{1}{m_1} \cdot (A_1 \cdot p_2 - A_2 \cdot p_3)$$



TIME INTEGRATION

$$p_2 = p_2 + \dot{p}_2 \cdot dt \qquad x_1 = x_1 + \dot{x}_1 \cdot dt$$

$$p_1 = p_1 + \dot{p}_1 \cdot dt \qquad p_3 = p_3 + \dot{p}_3 \cdot dt \qquad \dot{x}_1 = \dot{x}_1 + \ddot{x}_1 \cdot dt$$

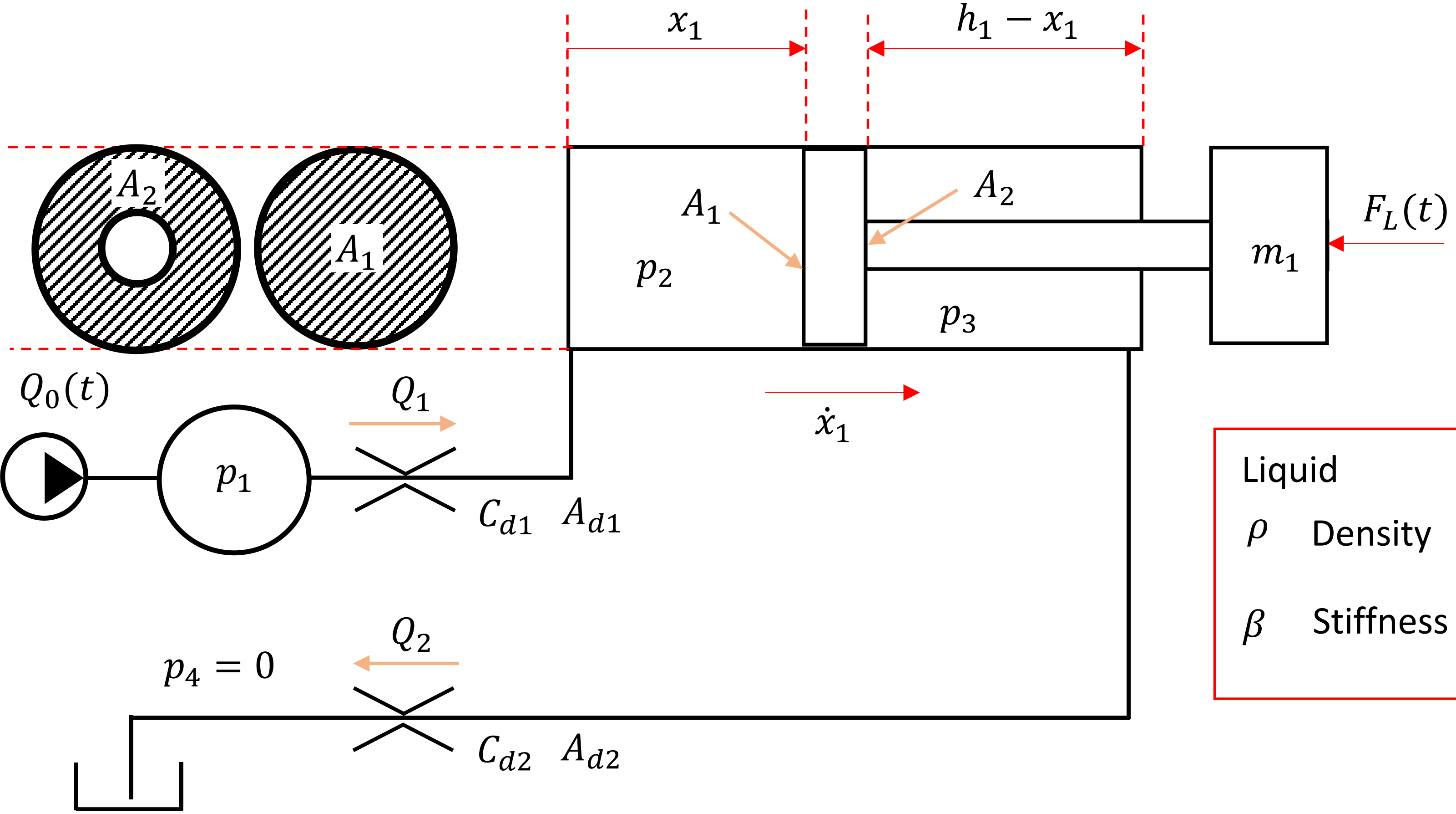


Liquid

ρ Density

β Stiffness

NUMERICAL TIPS:



Liquid

ρ Density

β Stiffness

NUMERICAL TIPS

If $x_1 < 0 \implies x_1 = 0 \;\&\; \dot{x}_1 = 0$

If $x_1 > h_1 \implies x_1 = h_1 \;\&\; \dot{x}_1 = 0$

If $p_i < -1e5 \implies p_i = -1e5$
$i = 1, 2, 3$

NUMERICAL TIPS

$$Q_1 = C_{d1} \cdot A_{d1} \cdot SIGN(p_1 - p_2)\sqrt{\frac{2}{\rho}|p_1 - p_2|} \qquad Q_2 = C_{d2} \cdot A_{d2} \cdot SIGN(p_3)\sqrt{\frac{2}{\rho}|p_3|}$$