

# The Solution of Model Predictive Control: Theory, Computation, and Design

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# Chapter 1

## Getting Started with Model Predictive Control

### 1.1 Brief Review

In this section, we just consider state space linear time invariant system with zero steady state.

**Lemma 1.3** (LQR convergence). For  $(A, B)$  controllable, the infinite LQR gives a convergent closed-loop system.

*Proof.* Because  $(A, B)$  is controllable, there exists a sequence of  $n$  inputs that transfers the state to zero. When  $k > n$ , we let  $u = 0$ , then the objective function  $V(x, u) = \sum_{k=0}^{\infty} x_k^T Q x_k + u^T R u$  is finite, which implies the optimization problem is feasible. On the other hand, the solution is unique since  $R > 0$  and the objective function is strict convex with  $u$ .

So the solution of the LQR problem exists and is unique. This implies that the objective function is non-increasing with time, and we have  $x \rightarrow 0, u \rightarrow 0$  as  $k \rightarrow \infty$ .  $\square$

**Remark.** The optimal solution can be calculated from Riccati equation, which is from backward dynamic programming similar to Kalman filter.

### 1.2 The Solution of Exercises