The Solution of Model Predictive Control: Theory, Computation, and Design

lixc21

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Chapter 1

Getting Started with Model Predictive Control

1.1 Brief Review

In this section, we just consider state space linear time invariant system with zero steady state.

Lemma 1.3 (LQR convergence). For (A, B) controllable, the infinite LQR gives a convergent closed-loop system.

Proof. Because (A, B) is controllable, there exists a sequence of n inputs that transfers the state to zero. When $k > n$, we let $u = 0$, then the objective function $V(x, u) = \sum_{k=0}^{\infty} x_k^\top Q x_k + u^\top R u$ is finite, which implies the optimization problem is feasible. On the other hand, the solution is unique since $R > 0$ and the objective function is strict convex with u.

So the solution of the LQR problem exists and is unique. This imples to that the objective funtion is non-increasing with time, and we have $x \to 0$, $u \to 0$ as $k \to 0$. $\square$

Remark. *The optimal solution can be calculate from Riccati equation, which is from backward dynamic programming similar to Kalman filter.*

$$
\begin{aligned}
K &= -(B^\top P B + R)^{-1} B^\top P A \\
P &= Q + A^\top P A - A^\top P B (B^\top P B + R)^{-1} B^\top P A
\end{aligned}
$$

1.2 The Solution of Exercises