Fast and Accurate Parameter Extraction for Different Types of Fuel Cells with Decomposition and Nature-inspired Optimization Method ☆

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Abstract

Fast and accurate parameter extraction of fuel cells is crucial to the control and performance analysis of fuel cell power systems. Unfortunately, due to the multi-variable and nonlinear features of fuel cell models, it is a difficult task to identify the parameters of the models. In this paper, we propose a decomposition technique, where the unknown parameters are divided into two groups: nonlinear and linear. The optimization techniques only need to optimize the nonlinear parameters, and then the linear parameters are determined based on the nonlinear ones. With the help of the decomposition technique, a generalized framework by using the nature-inspired optimization method is proposed to try to fast and accurately extract the parameters for different types of fuel cells. To test the performances of our approach, two widely used types of fuel cells are studied, i.e., proton exchange membrane fuel cell and solid oxide fuel cell. Extensive simulation tests with thirty-two instances are carried out for comparing our approach with existing approaches. The comparison demonstrates the efficiency of the decomposition technique. Moreover, the results show that our approach can not only significantly reduce the computational resources, but also yields high quality solutions.

Keywords: Fuel cells, parameter extraction, decomposition technique, nature-inspired optimization method.

1. Introduction

A fuel cell (FC) is an electrochemical device that directly converts the chemical energy of fuels (usually a gas rich in hydrogen) to electricity [1, 2]. Due to its noise-free, clean, sustainable, and reliable advantages, the power generated by FCs obtains increasing attention all over the world recently [3, 4, 5]. According to the type of electrolyte materials, there are several different types of FCs, such as alkaline FC, proton exchange membrane FC (PEMFC), solid oxide FC (SOFC), and direct methanol FC [6, 7]. Among various types of FCs, PEMFC and SOFC are widely used and developed [8, 9].

For the control, design, and performance analysis of PEMFC and SOFC, a variety of models have been developed during the last few decades, such as models for PEMFC [10, 11, 12, 13, 14], and models for SOFC [15, 16, 17, 18, 19]. Regarding different models of PEMFC and SOFC, there are several unknown parameters of the models that significantly affect the model results [20, 21, 22]. Therefore, to predict the dynamic behavior of FC power systems,

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these parameters need to be precisely extracted. However, because of the multi-variable, nonlinear, and strongly coupled features of the models, it is a challenging task to extract them by traditional approaches [23, 24, 25].

In recent years, the use of nature-inspired optimization methods for parameter extraction of FC models has become popular [5], such as genetic algorithms [26, 25], particle swarm optimization [27, 24, 28, 29], harmony search [30, 31], and differential evolution (DE) [32, 33, 34]. Most recently, some newly proposed swarm intelligence algorithms are also used to extract the parameters of FC models, *e.g.*, grey wolf optimizer [35], multi-verse optimizer [36], salp swarm optimizer [37], and so on. The advantages of the nature-inspired optimization methods are the insensitivity to the initial guess, flexibility, global search ability, etc. However, one disadvantage of these methods is that they require significant computational resources when extracting parameters from FC models. For example, the maximum number of function evaluations[1] (Max_NFEs) in [28] is 100,000; in [25], 90,000; in [36], 100,000; and in [33], 15,000.

To fast and accurately extract the parameters of FC models, firstly a decomposition technique is proposed, where the parameters of FC models are decomposed into the nonlinear group and linear group. Then, based on the decomposition technique, we develop a generalized framework for parameter extraction via a nature-inspired optimization method in order to obtain the nonlinear parameters. The nonlinear parameters are obtained by the nature-inspired optimization method. Then, the linear parameters are determined by the algebraic method and the corresponding nonlinear parameters. Benefit from the proposed decomposition technique, the search space can be reduced. Therefore, it can save the computational resources of the nature-inspired optimization method and thus accelerate the parameter extraction process greatly. Our proposed approach is extensively evaluated with 7 representative nature-inspired optimization methods and 32 instances of PEMFC and SOFC models. The results show that our approach can obtain high quality solutions only with Max_NFEs = 500.

The rest of the paper is organized as follows. In Section 2, the problem statement is described. Section 3 explains our proposed approach in detail. The results and analysis are given in Section 4. Finally, in Section 5, we conclude this paper and point out some future work.

2. PEMFC and SOFC Models

In this section, the mathematical formulations of both the PEMFC and SOFC stack models are described. Then, the objective function for parameter extraction of the PEMFC and SOFC models is given.

2.1. Mathematical Formulation of PEMFC Stack Model

In this work, the PEMFC stack model in [38] is used, which was also studied in [39, 40, 41]. If the internal current is considered, the model can be formulated as follows:

$$\begin{aligned} V &= N_{\text{cell}} \cdot V_{\text{cell}} \\ &= N_{\text{cell}} \cdot \{E_o - A \cdot \ln(i + i_n) \\ &\quad -R \cdot (i + i_n) - m \cdot \exp[n \cdot (i + i_n)]\}, \end{aligned} \tag{1}$$

where V is the voltage (in V) of the stack formed by N_{cell} cells connected in series; V_{cell} is the cell voltage (in V); E_o is the open circuit voltage (in V); A is the Tafel's parameter (in V); i is the current density (in A/cm^2); i_n is the internal current density (in A/cm^2); R is the resistance (in Ω), and m and n are the diffusion's parameters (in V and A^{-1}). In this model, there are six parameters (*i.e.*, E_o, A, R, m, n, and i_n) that must be extracted.

2.2. Mathematical Formulation of SOFC Stack Model

In this work, the SOFC stack model [33] is as follows:

$$\begin{aligned} V &= N_{\text{cell}} \cdot V_{\text{cell}} \\ &= N_{\text{cell}} \cdot \Big[E_o - i \cdot R - A \cdot \sinh^{-1}\left(\tfrac{i}{2i_{0,\text{a}}}\right) \\ &\quad -A \cdot \sinh^{-1}\left(\tfrac{i}{2i_{0,\text{c}}}\right) + B \cdot \ln\left(1 - \tfrac{i}{i_l}\right)\Big], \end{aligned} \tag{2}$$

where $i_{0,\text{a}}$ and $i_{0,\text{c}}$ are respectively the anode and cathode exchange current density (in A/cm^2); B is a constant that depends on the fuel cell and its operating state (in V), and i_l is the limiting current density (in A/cm^2). In this model, the parameters that must be extracted are $E_o, A, R, B, i_{0,\text{a}}, i_{0,\text{c}}$, and i_l.

[1] In this work, the number of function evaluations means the number of function calls of the objective function.

2.3. Objective Function

In order to extract the parameters of the PEMFC and SOFC models in Equations (1) and (2) based on the measured data, the parameter extraction problem is usually formulated into an optimization problem. In this work, the mean square error is used as the objective function for both the PEMFC and SOFC models [27, 25, 30]:

$$\begin{aligned} \min \quad f(\mathbf{x}, i_k) &= \frac{1}{N_{\text{data}}} \sum_{k=1}^{K} (V_k^{\text{m}} - V_k^{\text{s}})^2 \\ &= \frac{N_{\text{cell}}^2}{K} \sum_{k=1}^{K} (V_{\text{cell},k}^{\text{m}} - V_{\text{cell},k}^{\text{s}})^2, \end{aligned} \tag{3}$$

where $\mathbf{x}$ is the parameter vector[2]; for the PEMFC model, $\mathbf{x} = [E_o, A, R, m, n, i_n]^{\text{T}}$; and for the SOFC model, $\mathbf{x} = [E_o, A, R, B, i_{0,\text{a}}, i_{0,\text{b}}, i_l]^{\text{T}}$. Additionally, K is the number of the measured data; i_k is the k^{th} current density of the measured data; V_k^{m} is the k^{th} voltage of the measured data, and V_k^{s} is the k^{th} voltage of the simulated data. The objective of the parameter extraction problem is to minimize the error between the simulated data and the measured data. Note that for the SOFC model, the following constraint must be satisfied:

$$i_k < i_l, \tag{4}$$

where i_k is the k^{th} current density of the measured data.

3. Parameter Extraction based on Decomposition and Nature-inspired Optimization

In this section, we explain our approach in detail, including the motivations of this work, the decomposition technique for search space reduction, and the framework based on the nature-inspired optimization method and the proposed decomposition technique for parameter extraction of FC models.

3.1. Motivations

Although nature-inspired optimization methods have been widely used to extract parameters for the PEMFC and SOFC models, they cannot extract the parameters fast. The reason is that all unknown parameters need to be obtained by the optimization methods, in this way, the search space is very large. As a result, the optimization methods have to cost too many computational resources to minimize the error between the simulated data and the measured data.

In Equations (1) and (2), we observe that the parameters that must be extracted can be classified into two groups: *i.e.*, linear and nonlinear. For PEMFC, the linear parameters are E_o, A, R, and m; and the nonlinear ones are n and i_n. For SOFC, the linear parameters are E_o, A, R, and B; and the nonlinear ones are $i_{0,\text{a}}, i_{0,\text{c}}$, and i_l. By using the measured data, the linear parameters are determined by the nonlinear ones.

Based on the above observations, in order to reduce the computational resources, we propose a decomposition approach, where the optimization method only needs to obtain the nonlinear parameters. The linear parameters can be further determined by the obtained nonlinear parameters.

3.2. Decomposition of Search Space

For the sake of simple description, in this subsection, the term $\frac{N_{\text{cell}}^2}{K}$ in Equation (3) is omitted. Therefore, the objective function can be reformulated as

$$\begin{aligned} \min \quad f(\mathbf{x}, i_k) &= \sum_{k=1}^{K} (V_{\text{cell},k}^{\text{m}} - V_{\text{cell},k}^{\text{s}})^2 \\ &= (\mathbf{V}_{\text{cell}}^{\text{m}} - \mathbf{V}_{\text{cell}}^{\text{s}})^{\text{T}} (\mathbf{V}_{\text{cell}}^{\text{m}} - \mathbf{V}_{\text{cell}}^{\text{s}}), \end{aligned} \tag{5}$$

subject to

$$g(\mathbf{x}, i_k) - V_{\text{cell},k}^{\text{s}} = 0 \quad k = 1, \cdots, K$$

where:

[2]Hereinafter, vectors are assumed to be the column vectors unless otherwise noted.

- For PEMFC:

$$\begin{aligned} g(\mathbf{x}, i_k) &= E_o - A \cdot \ln(i_k + i_n) \\ &\quad -R \cdot (i_k + i_n) - m \cdot \exp[n \cdot (i_k + i_n)] \end{aligned}$$

- For SOFC:

$$\begin{aligned} g(\mathbf{x}, i_k) &= E_o - i_k \cdot R - A \cdot \sinh^{-1}\left(\frac{i_k}{2i_{0,\mathrm{a}}}\right) \\ &\quad -A \cdot \sinh^{-1}\left(\frac{i_k}{2i_{0,\mathrm{c}}}\right) + B \cdot \ln\left(1 - \frac{i_k}{i_l}\right) \end{aligned}$$

3.2.1. For the PEMFC Model

For the reformulation (5), the generalized Benders-like decomposition [42, 43] can be used with two decomposed parameter vectors:

$$\begin{aligned} \mathbf{x}_1 &= [n, i_n]^{\mathrm{T}} \\ \mathbf{x}_2 &= [E_o, A, R, m]^{\mathrm{T}} \end{aligned}$$

The nested form of Equation (5) is defined as:

$$\min_{\mathbf{x}_1, \mathbf{V}^{\mathrm{s}}_{\mathrm{cell}}} (\mathbf{V}^{\mathrm{m}}_{\mathrm{cell}} - \mathbf{V}^{\mathrm{s}}_{\mathrm{cell}})^{\mathrm{T}} (\mathbf{V}^{\mathrm{m}}_{\mathrm{cell}} - \mathbf{V}^{\mathrm{s}}_{\mathrm{cell}}) + \xi(\mathbf{x}_1, \mathbf{V}^{\mathrm{s}}_{\mathrm{cell}}), \tag{6}$$

subject to

$$g(\mathbf{x}_1, \mathbf{x}_2, i_k) - V^{\mathrm{s}}_{\mathrm{cell},k} = 0 \quad k = 1, \cdots, K$$

where

$$\xi(\mathbf{x}_1, \mathbf{V}^{\mathrm{s}}_{\mathrm{cell}}) = \min_{\mathbf{x}_2, \mathbf{V}^{\mathrm{s}}_{\mathrm{cell}}} (\mathbf{V}^{\mathrm{m}}_{\mathrm{cell}} - \mathbf{V}^{\mathrm{s}}_{\mathrm{cell}})^{\mathrm{T}} (\mathbf{V}^{\mathrm{m}}_{\mathrm{cell}} - \mathbf{V}^{\mathrm{s}}_{\mathrm{cell}}). \tag{7}$$

Suppose $\mathbf{x}_1$ is obtained by the nature-inspired optimization method, then the constraints in Equation (7) are linear. Thus, by solving the corresponding normal equation, the value of $\mathbf{x}_2$ can be obtained by a closed-form expression:

$$\overbrace{\begin{bmatrix} \mathbf{1}^{\mathrm{T}}\mathbf{V}^{\mathrm{m}}_{\mathrm{cell}} \\ \mathbf{M}^{\mathrm{T}}\mathbf{V}^{\mathrm{m}}_{\mathrm{cell}} \\ \mathbf{N}^{\mathrm{T}}\mathbf{V}^{\mathrm{m}}_{\mathrm{cell}} \\ \mathbf{O}^{\mathrm{T}}\mathbf{V}^{\mathrm{m}}_{\mathrm{cell}} \end{bmatrix}}^{\mathbf{Y}(\mathbf{x}_1)} = \overbrace{\begin{bmatrix} \mathbf{1}^{\mathrm{T}}\mathbf{1} & \mathbf{1}^{\mathrm{T}}\mathbf{M} & \mathbf{1}^{\mathrm{T}}\mathbf{N} & \mathbf{1}^{\mathrm{T}}\mathbf{O} \\ \mathbf{M}^{\mathrm{T}}\mathbf{1} & \mathbf{M}^{\mathrm{T}}\mathbf{M} & \mathbf{M}^{\mathrm{T}}\mathbf{N} & \mathbf{M}^{\mathrm{T}}\mathbf{O} \\ \mathbf{N}^{\mathrm{T}}\mathbf{1} & \mathbf{N}^{\mathrm{T}}\mathbf{M} & \mathbf{N}^{\mathrm{T}}\mathbf{N} & \mathbf{N}^{\mathrm{T}}\mathbf{O} \\ \mathbf{O}^{\mathrm{T}}\mathbf{1} & \mathbf{O}^{\mathrm{T}}\mathbf{M} & \mathbf{O}^{\mathrm{T}}\mathbf{N} & \mathbf{O}^{\mathrm{T}}\mathbf{O} \end{bmatrix}}^{\mathbf{Z}(\mathbf{x}_1)} \overbrace{\begin{bmatrix} E_o \\ A \\ R \\ m \end{bmatrix}}^{\mathbf{x}_2} \tag{8}$$

where $\mathbf{1}$ is a unit vector of size K, and $\mathbf{M}$, $\mathbf{N}$, and $\mathbf{O}$ are vectors of size K. Their elements are calculated as follows:

$$\begin{aligned} M_k &= M_k(\mathbf{x}_1) = -\ln(i_k + i_n) \\ N_k &= N_k(\mathbf{x}_1) = -(i_k + i_n) \\ O_k &= O_k(\mathbf{x}_1) = -\exp[n \cdot (i_k + i_n)], \end{aligned}$$

where $k = 1, \cdots, K$.

From Equation (8), the linear parameter vector $\mathbf{x}_2$ can be obtained for a given nonlinear parameter vector $\mathbf{x}_1$ as follows:

$$\mathbf{x}_2 = \mathbf{Z}^{-1}(\mathbf{x}_1)\mathbf{Y}(\mathbf{x}_1). \tag{9}$$

3.2.2. For the SOFC Model

Similarly, the original parameter vector can also be decomposed into two vectors as:

$$\begin{aligned} \mathbf{x}_1 &= [i_{0,\mathrm{a}}, i_{0,\mathrm{c}}, i_l]^{\mathrm{T}} \\ \mathbf{x}_2 &= [E_o, A, R, B]^{\mathrm{T}} \end{aligned}$$

For a given $\mathbf{x}_1$, $\mathbf{x}_2$ can be similarly determined as:

$$\begin{bmatrix} E_o \\ A \\ R \\ B \end{bmatrix} = \begin{bmatrix} \mathbf{1}^{\mathrm{T}}\mathbf{1} & \mathbf{1}^{\mathrm{T}}\mathbf{M} & \mathbf{1}^{\mathrm{T}}\mathbf{N} & \mathbf{1}^{\mathrm{T}}\mathbf{O} \\ \mathbf{M}^{\mathrm{T}}\mathbf{1} & \mathbf{M}^{\mathrm{T}}\mathbf{M} & \mathbf{M}^{\mathrm{T}}\mathbf{N} & \mathbf{M}^{\mathrm{T}}\mathbf{O} \\ \mathbf{N}^{\mathrm{T}}\mathbf{1} & \mathbf{N}^{\mathrm{T}}\mathbf{M} & \mathbf{N}^{\mathrm{T}}\mathbf{N} & \mathbf{N}^{\mathrm{T}}\mathbf{O} \\ \mathbf{O}^{\mathrm{T}}\mathbf{1} & \mathbf{O}^{\mathrm{T}}\mathbf{M} & \mathbf{O}^{\mathrm{T}}\mathbf{N} & \mathbf{O}^{\mathrm{T}}\mathbf{O} \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{1}^{\mathrm{T}}\mathbf{V}^{\mathrm{m}}_{\mathrm{cell}} \\ \mathbf{M}^{\mathrm{T}}\mathbf{V}^{\mathrm{m}}_{\mathrm{cell}} \\ \mathbf{N}^{\mathrm{T}}\mathbf{V}^{\mathrm{m}}_{\mathrm{cell}} \\ \mathbf{O}^{\mathrm{T}}\mathbf{V}^{\mathrm{m}}_{\mathrm{cell}} \end{bmatrix} \tag{10}$$

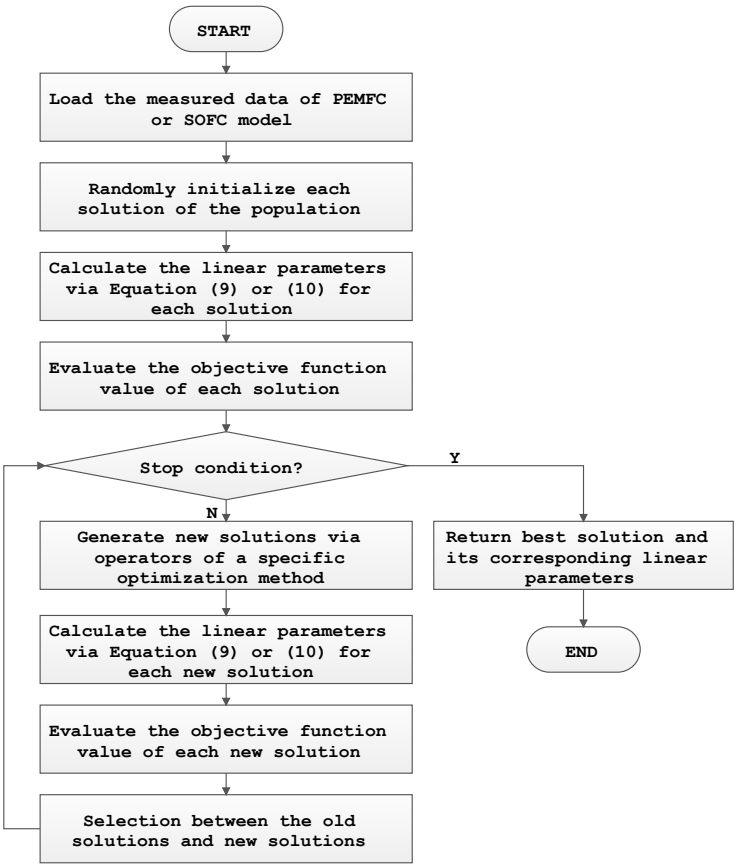

Figure 1: The framework of parameter extraction of FC models with decomposition technique and nature-inspired optimization method.

where

$$\begin{aligned} M_k &= M_k(\mathbf{x}_1) = -\sinh^{-1}\left(\frac{i_k}{2i_{0,\mathrm{a}}}\right) - \sinh^{-1}\left(\frac{i_k}{2i_{0,\mathrm{c}}}\right) \\ N_k &= N_k(\mathbf{x}_1) = -i_k \\ O_k &= O_k(\mathbf{x}_1) = \ln\left(1 - \frac{i_k}{i_l}\right), \end{aligned}$$

where $k = 1, \cdots, K$.

Remark 1: In Equations (9) and (10), the inversion of the matrix may not exist. In addition, some of the linear parameters in $\mathbf{x}_2$ may violate the physical meaning, for example, $E_o < 0$. In each of the situations in Equations (9) and (10), we set a very large value of the objective function (*e.g.*, 10^{20}) to penalize the invalid solution. Therefore, the invalid solutions will be eliminated during the optimization progress.

Remark 2: With the proposed decomposition technique, the parameters that need to be optimized by the optimization method are reduced to 2 for the PEMFC model and 3 for the SOFC model. Thus, the search space is reduced, and hence, it can accelerate the optimization process for parameter extraction of the PEMFC and SOFC models.

3.3. Nature-Inspired Optimization-based Parameter Extraction

Due to the powerful search ability of nature-inspired optimization methods, they are used to obtain the nonlinear parameters. With the help of the proposed decomposition technique, the framework of parameter extraction is shown in Figure 1. In this framework, different nature-inspired optimization methods can be used.

First, the measured data of the PEMFC or SOFC model is loaded. Then, each solution $X_i, i = 1, \cdots, N_{\mathrm{pop}}$ is randomly initialized within the search range of each nonlinear parameter, where N_{pop} is the population size and

$X_i = \{\mathbf{x}_1\}$. After that, the linear parameters are calculated via Equation (9) or Equation (10) for each solution. If the solution is invalid, it is penalized, otherwise, its objective function value is evaluated via Equation (3). If the number of function evaluations (NFEs) is less than Max_NFEs, the algorithm will be repeated as follows: i) Generate new solutions by the operators of a specific nature-inspired optimization method, such as mutation and crossover in DE [44]; ii) Determine the linear parameters for each new solution; iii) Evaluate the objective function value of each new solution; and iv) Apply the selection operator to form the new population from the old solutions and new solutions. Finally, once NFEs $\geq$ Max_NFEs, output the best solution and its corresponding linear parameters.

4. Experimental Results and Analysis

To extensively evaluate the performance of the proposed approach, seven representative nature-inspired optimization methods: *i.e.*, real-coded genetic algorithm (rcGA) [45], artificial bee colony (ABC) [46], comprehensive learning PSO (CLPSO) [47], jDE [48], SaDE [49], SHADE [50], and IJADE [33], were used in the proposed framework. Furthermore, 32 instances (including 18 instances of the PEMFC model and 14 instances of the SOFC model) were tested.

4.1. Parameter Settings and Search Ranges

Unless a change is mentioned, in this work, the parameter settings for the 7 methods that were used are listed in Table 1. Except for the population size, all algorithmic parameters were set the same as in their original literature. The Max_NFEs was set as 500 for all instances. Since all 7 methods are stochastic methods, in order to make the comparison meaningful, each instance was tested in 100 independent runs. The programs were executed on the following platform: CPU: Intel Xeon(R) E5-2620 2.40GHz; RAM: 32.00 GB; Operating system: Microsoft Windows 10 Professional Edition; Compiler: Microsoft Visual C++ 6.0. All programs used in this work were coded in standard C++.

Table 1: Parameter settings for the seven nature-inspired optimization methods.

Algorithm	Parameter settings
rcGA [45]	$N_{\text{pop}} = 20, p_c = 0.9, p_m = 0.15$
ABC [46]	$N_{\text{pop}} = 20, limit = 500$
CLPSO [47]	$N_{\text{pop}} = 20, m = 7, c = 1.49445$
jDE [48]	$N_{\text{pop}} = 20, \tau_1 = 0.1, \tau_2 = 0.1$
SaDE [49]	$N_{\text{pop}} = 20, LP = 50$
SHADE [50]	$N_{\text{pop}} = 20, H_m = 10, p = 0.1$
IJADE [33]	$N_{\text{pop}} = 20, \mu_{CR} = 0.9, \mu_F = 0.5, p = 0.05$

Table 2 lists the search ranges for the parameters of the PEMFC and SOFC models. Note that, the decomposed model (denoted as model/d) is compared with the original model (denoted as model/o). If model/d was used, the optimization method only needed to obtain the nonlinear parameters as mentioned in Section 3. Otherwise, all model parameters had to be obtained by the optimization method.

Table 2: Search ranges for each parameter.

PEMFC model							
Parameter	E_o (V)	A (V)	R (Ω)	m (V)	n (A^{-1})	i_n (A/cm^2)	
lower bound	0	0	0	0	0	0	
upper bound	2	1	1	1	1	10	
SOFC model							
Parameter	E_o (V)	A (V)	R (Ω)	B (V)	$i_{0,a}$ (mA/cm^2)	$i_{0,c}$ (mA/cm^2)	i_l (mA/cm^2)
lower bound	0	0	0	0	0	0	0
upper bound	1.2	1	1	1	30	30	200

Table 3: Brief description of 32 instances used in this work, where N_{cell} is the number of cells in the FC stack, A_{area} is the cell active area, and K denotes the number of measured data.

Instance	Fuel cell	N_{cell}	A_{area} (cm^2)	K	Instance	Fuel cell	N_{cell}	A_{area} (cm^2)	K
P1	Ballard	1	232	14	P17	FCLAB-PEMFC	5	100	1346
P2	BCS	32	64	7	P18	FCLAB-PEMFC	5	100	1335
P3	Geem-Noh	1	50.6	13	P19	HUST-SOFC	1	1000	21
P4	SR-12	48	62.5	37	P20	HUST-SOFC	1	1000	21
P5	Temasek	20	150	50	P21	HUST-SOFC	1	1000	21
P6	WCS-PEMFC	24	320	144	P22	HUST-SOFC	1	1000	21
P7	WCS-PEMFC	24	320	147	P23	WCS-SOFC	96	1000	1673
P8	WCS-PEMFC	24	320	147	P24	WCS-SOFC	96	1000	1585
P9	WCS-PEMFC	24	320	147	P25	WCS-SOFC	96	1000	1586
P10	WCS-PEMFC	24	320	148	P26	WCS-SOFC	96	1000	1586
P11	FCLAB-PEMFC	5	100	1229	P27	WCS-SOFC	96	1000	1586
P12	FCLAB-PEMFC	5	100	1129	P28	WCS-SOFC	96	1000	1586
P13	FCLAB-PEMFC	5	100	1054	P29	WCS-SOFC	96	1000	1585
P14	FCLAB-PEMFC	5	100	1030	P30	WCS-SOFC	96	1000	1584
P15	FCLAB-PEMFC	5	100	1314	P31	WCS-SOFC	96	1000	1584
P16	FCLAB-PEMFC	5	100	1310	P32	WCS-SOFC	96	1000	1586

4.2. Instance Description

Table 3 describes the 32 instances used for tests. For the PEMFC model, P1-P2 and P4 were obtained from [30], P3 was obtained from [51], P5 was from [52], P6-P10 were generated by the WCS-PEMFC MATLAB/SIMULINK [53] at different temperatures, and P11-P18 were from [54]. For the SOFC models, P19-P22 were provided by the SOFC Research & Development Center of Huazhong University of Science and Technology, which was generated by a planar single cell (HUST-SOFC) at different temperatures, and P23-P32 were generated by the WCS-SOFC MATLAB/SIMULINK [53] at different temperatures and different pressures.

Table 4: Comparison on the objective function values of different optimization methods for parameter extraction of model/o and model/d in the simulated instances of the PEMFC and SOFC models, where "m±s" denotes "mean ± standard deviation" over 100 runs. "+" shows that the method for model/d performs significantly better than the method for model/o according to the Wilcoxon's test at $\alpha = 0.05$.

Algorithm	PEMFC			SOFC		
	model/o	model/d	sig.	model/o	model/d	sig.
rcGA	1.71E+02 ± 1.51E+02	**2.93E-05 ± 3.37E-05**	+	2.05E+06 ± 3.27E+06	**1.18E-02 ± 1.72E-02**	+
ABC	1.97E+00 ± 4.68E+00	**1.75E-06 ± 3.57E-06**	+	4.02E+05 ± 2.73E+06	**5.42E-04 ± 8.18E-04**	+
CLPSO	2.80E+00 ± 8.07E+00	**7.95E-06 ± 1.91E-05**	+	5.67E+04 ± 3.46E+05	**6.22E-03 ± 1.63E-02**	+
jDE	1.44E+00 ± 2.37E+00	**4.56E-07 ± 1.31E-06**	+	3.98E+04 ± 8.87E+04	**2.09E-04 ± 5.01E-04**	+
SaDE	9.37E-01 ± 2.39E+00	**2.92E-07 ± 7.21E-07**	+	3.47E+04 ± 5.35E+04	**2.42E-04 ± 3.37E-04**	+
SHADE	2.14E-01 ± 8.74E-02	**1.80E-08 ± 3.49E-08**	+	4.60E+02 ± 1.70E+03	**4.44E-04 ± 3.76E-03**	+
IJADE	1.36E-01 ± 8.83E-02	**5.63E-12 ± 1.56E-11**	+	1.27E+03 ± 3.95E+03	**2.14E-05 ± 6.75E-05**	+

4.3. Effectiveness of Decomposition Technique

To evaluate the effectiveness of our proposed decomposition technique, we generated two simulated instances for the PEMFC and SOFC models with the given parameters.

- For the PEMFC model, the parameter vector was set to be $[E_o, A, R, m, n, i_n]^{\mathrm{T}} = [1.04, 0.05, 0.001, 0.08, 0.21, 1.26]^{\mathrm{T}}$, $N_{\text{cell}} = 10$, $A_{\text{area}} = 10$ cm^2. The current was set from 0 A to 10.1 A, and $K = 50$.
- For the SOFC model, the parameter vector was $[E_o, A, R, B, i_{0,\text{a}}, i_{0,\text{c}}, i_l]^{\mathrm{T}} = [1.15, 0.015, 0.0003, 0.02, 2.5, 10.5, 160]^{\mathrm{T}}$, $N_{\text{cell}} = 96$, $A_{\text{area}} = 1000$ cm^2. The current was set from 0.1 A to 158 A, and $K = 50$.

The seven methods mentioned in Section 4.1 were used to extract the parameters for both model/d and model/o. The mean and standard deviation values of the objective function are reported in Table 4, where all results are averaged over 100 runs, and Max_NFEs = 500.

The simulation results in Table 4 clearly show that, regardless of the optimization methods, the proposed decomposition technique is very effective. It can significantly improve the accuracy of the objective function compared with the original models for both the PEMFC and SOFC models. In addition, Table 4 also reveals that, with the help of the

decomposition technique, different nature-inspired optimization methods can be easily incorporated into our proposed framework to accelerate parameter extraction of both the PEMFC and SOFC models. Since our proposed decomposition technique is able to reduce the search space, it can benefit the optimization method to extract the parameters of FC models efficiently.

Among the 7 nature-inspired optimization methods, on the whole, IJADE yielded the best results. Therefore, IJADE is used for further study in the following subsections.

Table 5: Extracted parameters by IJADE for model/o and model/d of PEMFC. All results are averaged over 100 runs.

Parameter	Real value	model/o	model/d
E_o	1.0400	1.5007 ± 2.83E-01	1.0400 ± 5.38E-05
A	0.0500	0.0587 ± 8.88E-02	0.0500 ± 3.60E-05
R	0.0010	0.0303 ± 2.72E-02	0.0010 ± 1.38E-05
m	0.0800	0.2432 ± 1.98E-01	0.0800 ± 1.59E-05
n	0.2100	0.0968 ± 6.08E-02	0.2100 ± 1.79E-05
i_n	1.2600	3.9691 ± 2.41E+00	1.2601 ± 7.66E-04

Table 6: Extracted parameters by IJADE for model/o and model/d of SOFC. All results are averaged over 100 runs.

Parameter	Real value	model/o	model/d
E_o	1.1500	1.0568 ± 1.72E-01	1.1500 ± 1.83E-04
A	0.0150	0.0234 ± 5.71E-02	0.0160 ± 2.12E-03
R	0.0003	0.0005 ± 4.64E-04	0.0003 ± 4.25E-05
B	0.0200	0.1438 ± 1.87E-01	0.0202 ± 4.69E-04
$i_{0,\mathrm{a}}$	2.5000	12.0263 ± 6.44E+00	2.6124 ± 2.47E-01
$i_{0,\mathrm{c}}$	10.5000	20.9700 ± 6.10E+00	13.4765 ± 6.69E+00
i_l	160.0000	181.2230 ± 1.19E+01	160.0393 ± 8.51E-02

Table 7: Comparison on the objective function values of IJADE for parameter extraction of model/o and model/d on all instances.

Instance	model/o	model/d	sig.	Instance	model/o	model/d	sig.
P1	4.21E-02 ± 2.38E-01	**3.87E-05 ± 6.33E-11**	+	P17	6.44E-01 ± 3.76E+00	**6.53E-05 ± 4.97E-11**	+
P2	6.63E-01 ± 6.71E-01	**5.16E-02 ± 1.29E-04**	+	P18	9.70E-01 ± 4.87E+00	**3.18E-05 ± 2.00E-07**	+
P3	3.51E-02 ± 2.02E-01	**1.27E-05 ± 6.09E-12**	+	P19	3.33E+02 ± 1.52E+03	**2.01E-04 ± 4.91E-05**	+
P4	8.10E+00 ± 6.39E+01	**7.99E-02 ± 3.60E-07**	+	P20	2.35E+02 ± 1.05E+03	**1.98E-04 ± 3.91E-05**	+
P5	1.90E+01 ± 1.13E+02	**1.77E-04 ± 1.25E-11**	+	P21	2.46E+02 ± 1.07E+03	**2.01E-04 ± 5.86E-05**	+
P6	7.28E+00 ± 6.80E+01	**1.34E-03 ± 2.16E-08**	+	P22	2.58E+02 ± 1.09E+03	**2.01E-04 ± 5.27E-05**	+
P7	5.63E-01 ± 3.40E+00	**1.44E-03 ± 1.44E-08**	+	P23	2.67E+02 ± 1.11E+03	**1.99E-04 ± 3.92E-05**	+
P8	1.40E+00 ± 7.34E+00	**1.56E-03 ± 3.00E-08**	+	P24	3.51E-03 ± 1.05E-02	**9.76E-06 ± 2.98E-06**	+
P9	1.14E+00 ± 5.15E+00	**1.78E-03 ± 1.84E-08**	+	P25	4.74E-03 ± 1.95E-02	**2.83E-07 ± 2.91E-07**	+
P10	1.29E+00 ± 5.48E+00	**2.03E-03 ± 2.82E-08**	+	P26	4.64E-03 ± 2.16E-02	**5.81E-06 ± 1.27E-06**	+
P11	1.60E+00 ± 1.12E+01	**5.93E-05 ± 4.58E-09**	+	P27	4.40E-03 ± 2.14E-02	**6.89E-06 ± 4.92E-06**	+
P12	3.00E+00 ± 1.47E+01	**9.40E-06 ± 5.77E-12**	+	P28	2.35E+02 ± 1.05E+03	**1.93E-04 ± 4.82E-05**	+
P13	2.38E+00 ± 1.10E+01	**1.53E-05 ± 6.59E-09**	+	P29	2.05E+02 ± 9.68E+02	**6.53E-04 ± 2.77E-04**	+
P14	2.52E+00 ± 1.20E+01	**5.05E-05 ± 1.64E-10**	+	P30	2.35E+02 ± 1.05E+03	**1.93E-04 ± 4.82E-05**	+
P15	8.18E-01 ± 4.43E+00	**8.88E-05 ± 1.12E-08**	+	P31	2.67E+02 ± 1.11E+03	**8.15E-05 ± 2.42E-05**	+
P16	1.44E+00 ± 1.02E+01	**5.38E-05 ± 2.71E-11**	+	P32	2.75E+02 ± 1.15E+03	**1.86E-04 ± 1.26E-03**	+

Table 8: Comparison between the reported results and the results obtained by IJADE with model/d.

Fuel cell	IGHS [30]	BSABCM-3 [55]	rank-MADE [56]	TRADE [34]	IJADE model/d
Ballard	4.90E-05 ± 2.33E-05	2.78E-05 ± 1.05E-06	4.75E-05 ± 3.41E-07	4.28E-05 ± 2.70E-11	**3.87E-05 ± 6.33E-11**
BCS	1.07E-01 ± 1.19E-02	NA ± NA	8.06E-02 ± 8.05E-06	8.19E-02 ± 8.70E-09	**5.16E-02 ± 1.29E-04**
SR-12	1.33E-01 ± 2.72E-02	9.86E-02 ± 1.45E-02	1.56E-01 ± 1.28E-07	8.71E-02 ± 7.00E-06	**7.99E-02 ± 3.60E-07**
Temasek	NA ± NA	NA ± NA	5.35E-03 ± 3.46E-06	5.97E-04 ± 1.70E-06	**1.77E-04 ± 1.25E-11**

4.4. Validation of Our Approach

In subsection 4.3, the effectiveness of the decomposition technique was verified based on objective function values. In this subsection, we validate the accuracy of the extracted parameters by IJADE. For this purpose, the simulated

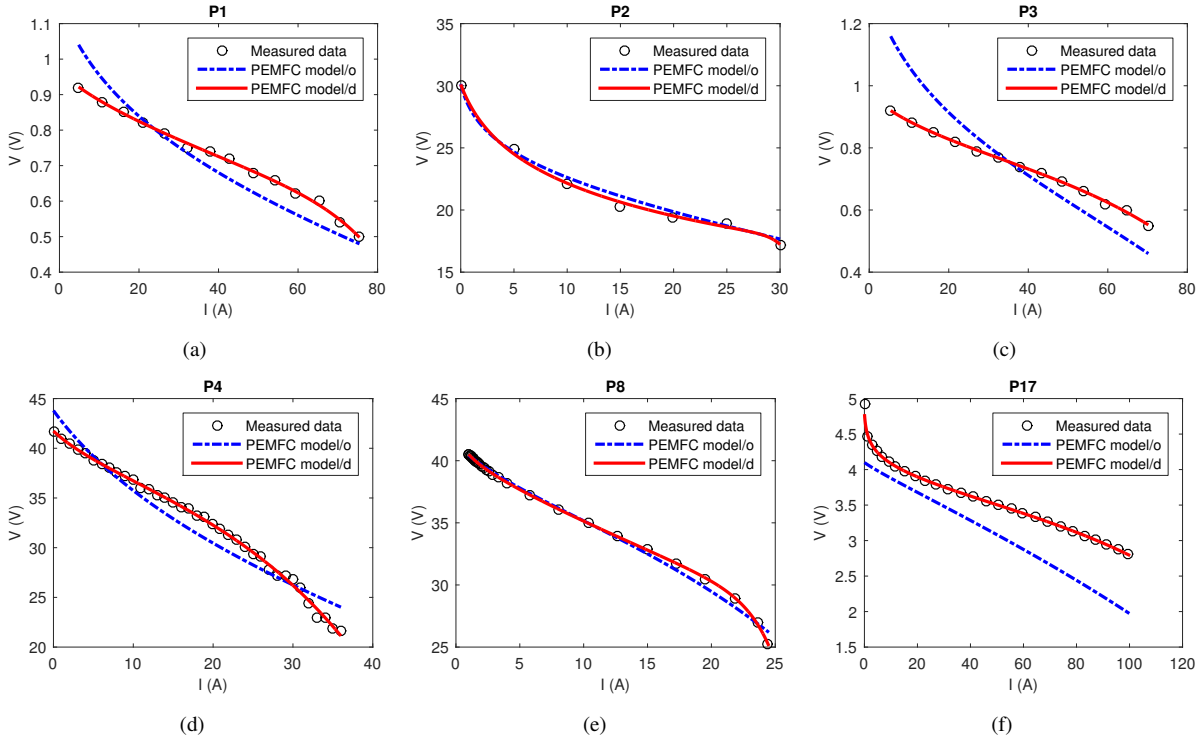

Figure 2: The *V-I* characteristics of PEMFC models obtained by IJADE for model/o and model/d. (a)-(d): P1-P4; (e): P8; (f): P17.

instances in Section 4.3 are also used. The extracted parameters by IJADE for the PEMFC and SOFC models are given in Tables 5 and 6, respectively.

For the PEMFC model, from Table 5 it can be observed that IJADE can exactly extract the parameters by using model/d with Max_NFEs = 500. Each of the extracted parameters is very close to its real value. Moreover, the standard deviation values are very small, which means that IJADE is robust when combined with the proposed decomposition technique. However, if model/o was used, the results deteriorate remarkably.

For the SOFC model, Table 6 shows that, by using model/d, IJADE can also extract more accurate parameters compared with model/o.

Therefore, based on these results, it can be concluded that our approach can extract the parameters for the PEMFC and SOFC models accurately and fast.

4.5. On the Quality of Final Solutions

To further evaluate the superiority of our proposed decomposition technique, 32 instances listed in Table 3 were used. The mean and standard deviation values of the final solutions by IJADE are reported in Table 7, where the results of IJADE with model/d are compared with those of IJADE with model/o. All results are averaged over 100 runs. From the results, it can be seen that:

- IJADE with model/d consistently provided better mean objective function values than IJADE with model/o in all instances. According to the results of the Wilcoxon's test, it is clear that IJADE with model/d significantly outperformed IJADE with model/o at $\alpha = 0.05$.
- With respect to the standard deviation values, IJADE with model/d was also more robust than IJADE with model/d in all instances.

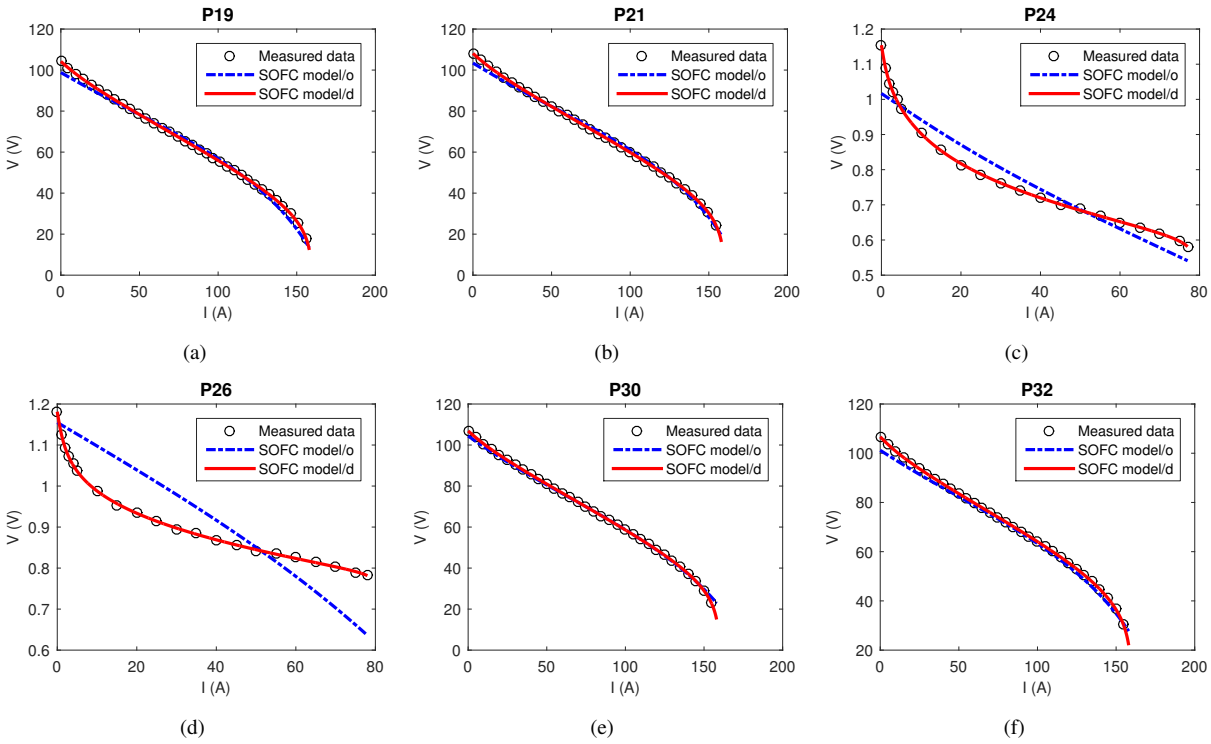

Figure 3: The *V-I* characteristics of SOFC models obtained by IJADE for model/o and model/d. (a): P19; (b): P21; (c): P24; (d): P26; (e) P30; (f): P32.

4.6. On the V-I Characteristics

It is important to verify the parameter extraction process, therefore, the representative parameters[3] extracted by IJADE were put into the PEMFC and SOFC mathematical models. In Figures 2 and 3, the *V-I* characteristics of 12 out of 32 instances are plotted. Note that, if the number of measured data is too many, only selected data points are plotted. From Figures 2 and 3, it can be seen that the *V-I* characteristics obtained by IJADE with model/d agree well with the measured data in all instances. However, if the original model (model/o) is used, the shapes of the fitted curves are far way from the measured data. The reason is that the search space is large if the original model is used, which makes the optimization method require more computational resources to obtain the promising parameters. However, in this work, Max_NFEs = 500, which is rather small for IJADE with model/o.

4.7. Compared With Reported Results

Table 8 compares our results to four nature-inspired optimization methods: improved global harmony search (IGHS) [30], backtracking search based on Burger's chaotic map (BSABCM-3) [55], ranking-based multi-strategy adaptive DE (rank-MADE) [56], and transferred adaptive DE (TRADE) [34] for P1, P2, P4, and P5.

The results show that our approach obtained better results than the other four methods. It is worth emphasizing that the Max_NFEs was set to be $5,030$, $60,000$, $10,000$, and $30,000$ for IGHS, BSABCM-3, rank-MADE, and TRADE in the corresponding literature, respectively. However, IJADE with model/d only cost Max_NFEs = 500, which is several order of magnitudes less than the other methods. Hence, it can be claimed that our proposed decomposition technique is effective and efficient to extract the parameters for the FC models.

5. Conclusion

In this paper, in order to fast and accurately extract the unknown parameters of different types of FC models, we first proposed a decomposition technique. It is able to reduce the search space, and hence, to accelerate the optimization process for parameter extraction. Using this decomposition technique, a general framework that also uses the nature-inspired optimization method was developed. Seven nature-inspired optimization methods were incorporated into the framework, and 32 instances of the PEMFC and SOFC models were used to extensively test the performance of our approach. Comparative results showed that the decomposition technique is very effective. In particular, IJADE with model/d can extract the parameters of the PEMFC and SOFC models fast and accurately. Therefore, IJADE with model/d can be an effective and efficient alternative for parameter extraction of different types of FC models.

Since our approach only requires few computational resources, in addition to the application to static models, this approach might be applied to the dynamic FC models under different operation conditions [57, 58]. Additionally, the use of the proposed approach for online or real-time FC systems can be studied in future work [59].

The source code and the uncommercial instances used in this paper can be obtained from the authors upon request.

References

[1] J. Larminie, A. Dicks, Fuel Cell Systems Explained (Second Edition), Wiley, 2003.

[2] G. Liu, Z. Yang, M. Halim, X. Li, M. Wang, J. Y. Kim, Q. Mei, X. Wang, J. K. Lee, A gradient activation method for direct methanol fuel cells, Energy Conversion and Management 138 (2017) 54 – 60.

[3] G. D'Andrea, M. Gandiglio, A. Lanzini, M. Santarelli, Dynamic model with experimental validation of a biogas-fed SOFC plant, Energy Conversion and Management 135 (2017) 21 – 34.

[4] M. Wang, G. Liu, Z. Tian, Y. Shao, L. Wang, F. Ye, M. X. Tran, Y. Yun, J. K. Lee, Microstructure-modified proton exchange membranes for high-performance direct methanol fuel cells, Energy Conversion and Management 148 (2017) 753 – 758.

[5] M. Secanell, J. Wishart, P. Dobson, Computational design and optimization of fuel cells and fuel cell systems: A review, Journal of Power Sources 196 (8) (2011) 3690 – 3704.

[6] G. Liu, H. Zhou, X. Ding, X. Li, D. Zou, X. Li, X. Wang, J. K. Lee, Effect of fabrication and operating parameters on electrochemical property of anode and cathode for direct methanol fuel cells, Energy Conversion and Management 122 (2016) 366 – 371.

[7] M. Wang, M. Chen, Z. Yang, Y. Wang, Y. Wang, G. Liu, J. K. Lee, X. Wang, A study on fuel additive of methanol for room temperature direct methanol fuel cells, Energy Conversion and Management 168 (2018) 270 – 275.

[3] In this work, the representative parameters are related to the *median* objective function value of an optimization method over 100 runs.

[8] S. Peighambardoust, S. Rowshanzamir, M. Amjadi, Review of the proton exchange membranes for fuel cell applications, International Journal of Hydrogen Energy 35 (17) (2010) 9349 – 9384.
[9] F. Ramadhani, M. Hussain, H. Mokhlis, S. Hajimolana, Optimization strategies for Solid Oxide Fuel Cell (SOFC) application: A literature survey, Renewable and Sustainable Energy Reviews 76 (2017) 460 – 484.
[10] A. Bıyıkoğlu, Review of proton exchange membrane fuel cell models, International Journal of Hydrogen Energy 30 (11) (2005) 1181 – 1212.
[11] C. Wang, M. Nehrir, S. Shaw, Dynamic models and model validation for PEM fuel cells using electrical circuits, IEEE Transactions on Energy Conversion 20 (2) (2005) 442 – 451.
[12] Q. Niu, L. Zhang, K. Li, A biogeography-based optimization algorithm with mutation strategies for model parameter estimation of solar and fuel cells, Energy Conversion and Management 86 (2014) 1173 – 1185.
[13] S. L. Chavan, D. B. Talange, Modeling and performance evaluation of PEM fuel cell by controlling its input parameters, Energy 138 (2017) 437 – 445.
[14] R. Kaur, V. Krishnasamy, K. Muthusamy, P. Chinnamuthan, A novel proton exchange membrane fuel cell based power conversion system for telecom supply with genetic algorithm assisted intelligent interfacing converter, Energy Conversion and Management 136 (2017) 173 – 183.
[15] S. Chan, K. Khor, Z. Xia, A complete polarization model of a solid oxide fuel cell and its sensitivity to the change of cell component thickness, Journal of Power Sources 93 (1 - 2) (2001) 130 – 140.
[16] C. Wang, M. Nehrir, A Physically Based Dynamic Model for Solid Oxide Fuel Cells, IEEE Transactions on Energy Conversion 22 (4) (2007) 887–897.
[17] A. Gebregergis, P. Pillay, D. Bhattacharyya, R. Rengaswemy, Solid Oxide Fuel Cell Modeling, IEEE Transactions on Industrial Electronics 56 (1) (2009) 139–148.
[18] S. A. Hajimolana, M. A. Hussain, W. A. W. Daud, M. Soroush, A. Shamiri, Mathematical modeling of solid oxide fuel cells: A review, Renewable and Sustainable Energy Reviews 15 (4) (2011) 1893 – 1917.
[19] O. J. Shariatzadeh, A. Refahi, M. Rahmani, S. Abolhassani, Economic optimisation and thermodynamic modelling of SOFC tri-generation system fed by biogas, Energy Conversion and Management 105 (2015) 772 – 781.
[20] J. Corrêa, F. Farret, V. Popov, M. Simoes, Sensitivity analysis of the modeling parameters used in Simulation of proton exchange membrane fuel cells, IEEE Transactions on Energy Conversion 20 (1) (2005) 211 – 218.
[21] M. Ni, M. K. Leung, D. Y. Leung, Parametric study of solid oxide fuel cell performance, Energy Conversion and Management 48 (5) (2007) 1525 – 1535.
[22] K. Tseronis, I. Bonis, I. Kookos, C. Theodoropoulos, Parametric and transient analysis of non-isothermal, planar solid oxide fuel cells, International Journal of Hydrogen Energy 37 (1) (2012) 530 – 547.
[23] M. Outeiro, R. Chibante, A. Carvalho, A. de Almeida, A parameter optimized model of a Proton Exchange Membrane fuel cell including temperature effects, Journal of Power Sources 185 (2) (2008) 952 – 960.
[24] M. Ye, X. Wang, Y. Xu, Parameter identification for proton exchange membrane fuel cell model using particle swarm optimization, International Journal of Hydrogen Energy 34 (2) (2009) 981 – 989.
[25] J. Yang, X. Li, J. H. Jiang, L. Jian, L. Zhao, J. G. Jiang, X. G. Wu, L. H. Xu, Parameter optimization for tubular solid oxide fuel cell stack based on the dynamic model and an improved genetic algorithm, International Journal of Hydrogen Energy 36 (10) (2011) 6160 – 6174.
[26] I. Mohamed, N. Jenkins, Proton exchange membrane (PEM) fuel cell stack configuration using genetic algorithms, Journal of Power Sources 131 (1) (2004) 142 – 146.
[27] Z.-J. Mo, X.-J. Zhu, L.-Y. Wei, G.-Y. Cao, Parameter optimization for a PEMFC model with a hybrid genetic algorithm, International Journal of Energy Research 30 (8) (2006) 585–597.
[28] Q. Li, W. Chen, Y. Wang, S. Liu, J. Jia, Parameter Identification for PEM Fuel-Cell Mechanism Model Based on Effective Informed Adaptive Particle Swarm Optimization, IEEE Transactions on Industrial Electronics 58 (6) (2011) 2410 – 2419.
[29] B. Jiang, N. Wang, L. Wang, Parameter identification for solid oxide fuel cells using cooperative barebone particle swarm optimization with hybrid learning, International Journal of Hydrogen Energy 39 (1) (2014) 532 – 542.
[30] A. Askarzadeh, A. Rezazadeh, An Innovative Global Harmony Search Algorithm for Parameter Identification of a PEM Fuel Cell Model, IEEE Transactions on Industrial Electronics 59 (9) (2012) 3473 – 3480.
[31] M. Karimi, A. Askarzadeh, A. Rezazadeh, Using Tournament Selection Approach to Improve Harmony Search Algorithm for Modeling of Proton Exchange Membrane Fuel Cell, Int. J. Electrochem. Sci. 7 (2012) 6426 – 6435.
[32] U. K. Chakraborty, T. E. Abbott, S. K. Das, PEM fuel cell modeling using differential evolution, Energy 40 (1) (2012) 387 – 399.
[33] W. Gong, Z. Cai, J. Yang, X. Li, J. Jian, Parameter identification of an SOFC model with an efficient, adaptive differential evolution algorithm, International Journal of Hydrogen Energy 39 (10) (2014) 5086–5096.
[34] W. Gong, X. Yan, X. Liu, Z. Cai, Parameter extraction of different fuel cell models with transferred adaptive differential evolution, Energy 86 (2015) 139 – 151.
[35] M. Ali, M. El-Hameed, M. Farahat, Effective parameters' identification for polymer electrolyte membrane fuel cell models using grey wolf optimizer, Renewable Energy 111 (2017) 455 – 462.
[36] A. Fathy, H. Rezk, Multi-verse optimizer for identifying the optimal parameters of PEMFC model, Energy 143 (2018) 634 – 644.
[37] A. A. El-Fergany, Extracting optimal parameters of PEM fuel cells using Salp Swarm Optimizer, Renewable Energy 119 (2018) 641 – 648.
[38] J. Kim, S. Lee, S. Srinivasan, C. E. Chamberlin, Modeling of Proton Exchange Membrane Fuel Cell Performance with an Empirical Equation, Journal of The Electrochemical Society 142 (8) (1995) 2670–2674.
[39] F. Laurencelle, R. Chahine, J. Hamelin, K. Agbossou, M. Fournier, T. K. Bose, A. Laperriére, Characterization of a Ballard MK5-E Proton Exchange Membrane Fuel Cell Stack, Fuel Cells 1 (1) (2001) 66–71.
[40] A. Saadi, M. Becherif, A. Aboubou, M. Ayad, Comparison of proton exchange membrane fuel cell static models, Renewable Energy 56 (2013) 64 – 71.
[41] Y. Akimoto, K. Okajima, Semi-Empirical Equation of PEMFC Considering Operation Temperature, Energy Technology & Policy 1 (1) (2014) 91–96.
[42] A. M. Geoffrion, Generalized Benders Decomposition, Journal of Optimization Theory and Applications 10 (4) (1972) 237–260.

[43] R. Rahmaniani, T. G. Crainic, M. Gendreau, W. Rei, The Benders decomposition algorithm: A literature review, European Journal of Operational Research 259 (3) (2017) 801 – 817.
[44] R. Storn, K. Price, Differential Evolution–A Simple and Efficient Heuristic for Global Optimization Over Continuous Spaces, J. of Global Optim. 11 (4) (1997) 341–359.
[45] F. Herrera, M. Lozano, J. Verdegay, Tackling real-coded genetic algorithms: Operators and tools for behavioural analysis, Artificial Intelligence Review 12 (4) (1998) 265–319.
[46] D. Karaboga, B. Basturk, A powerful and efficient algorithm for numerical function optimization: Artificial bee colony (ABC) algorithm, J. of Global Optimization 39 (3) (2007) 459–471.
[47] J. J. Liang, A. K. Qin, P. N. Suganthan, S. Baskar, Comprehensive learning particle swarm optimizer for global optimization of multimodal functions, IEEE Trans. on Evol. Comput. 10 (3) (2006) 281–295.
[48] J. Brest, S. Greiner, B. Bošković, M. Mernik, V. Žumer, Self-Adapting Control Parameters in Differential Evolution: A Comparative Study on Numerical Benchmark Problems, IEEE Trans. on Evol. Comput. 10 (6) (2006) 646–657.
[49] A. K. Qin, V. L. Huang, P. N. Suganthan, Differential Evolution Algorithm With Strategy Adaptation for Global Numerical Optimization, IEEE Trans. on Evol. Comput. 13 (2) (2009) 398–417.
[50] R. Tanabe, A. Fukunaga, Success-History Based Parameter Adaptation for Differential Evolution, in: 2013 IEEE Congress on Evolutionary Computation (CEC), 71 – 78, 2013.
[51] Z. W. Geem, J.-S. Noh, Parameter Estimation for a Proton Exchange Membrane Fuel Cell Model Using GRG Technique, Fuel Cells 16 (5) (2016) 640–645.
[52] C. Dai, W. Chen, Z. Cheng, Q. Li, Z. Jiang, J. Jia, Seeker optimization algorithm for global optimization: A case study on optimal modelling of proton exchange membrane fuel cell (PEMFC), International Journal of Electrical Power & Energy Systems 33 (3) (2011) 369 – 376.
[53] C. Wang, M. H. Nehrir, Dynamic models for PEMFs and tubular SOFCs, URL `http://www.coe.montana.edu/ee/fuelcell/`, [Online; accessed 02-July-2018], 2018.
[54] FCLab, IEEE PHM 2014 Data Challenge, URL `http://eng.fclab.fr/ieee-phm-2014-data-challenge/`, [Online; accessed 02-July-2018], 2018.
[55] A. Askarzadeh, L. dos Santos Coelho, A backtracking search algorithm combined with Burger's chaotic map for parameter estimation of PEMFC electrochemical model, International Journal of Hydrogen Energy 39 (21) (2014) 11165 – 11174.
[56] W. Gong, Z. Cai, Parameter optimization of PEMFC model with improved multi-strategy adaptive differential evolution, Engineering Applications of Artificial Intelligence 27 (2014) 28 – 40.
[57] H. Fathabadi, Fuel cell/back-up battery hybrid energy conversion systems: Dynamic modeling and harmonic considerations, Energy Conversion and Management 103 (2015) 573 – 584.
[58] X. Lü, Y. Qu, Y. Wang, C. Qin, G. Liu, A comprehensive review on hybrid power system for PEMFC-HEV: Issues and strategies, Energy Conversion and Management 171 (2018) 1273 – 1291.
[59] D. Zhou, T. T. Nguyen, E. Breaz, D. Zhao, S. Clnet, F. Gao, Global parameters sensitivity analysis and development of a two-dimensional real-time model of proton-exchange-membrane fuel cells, Energy Conversion and Management 162 (2018) 276 – 292.