Solving Nonlinear Equations System with Dynamic Repulsion-based Evolutionary Algorithms

Zuowen Liao, Wenyin Gong, Xuesong Yan, Ling Wang, and Chengyu Hu

***Abstract*—Nonlinear equations system arises commonly in science and engineering. Repulsion techniques are considered to be the effective methods to locate different roots of nonlinear equations system. In general, the repulsive radius needs to be given by the user before the run. However, its optimal parameter setting is difficult and problem-dependent. To alleviate this drawback, in this paper, we firstly propose a dynamic repulsion technique, and then a general framework based on the dynamic repulsion technique and evolutionary algorithms is presented to effectively solve nonlinear equations system. The major advantages of our framework are: i) the repulsive radius is controlled dynamically during the evolutionary process; ii) multiple roots of nonlinear equations system can be simultaneously located in a single run; iii) the diversity of the population is preserved due to the population re-initialization; and iv) different repulsion techniques and different evolutionary algorithms can be readily integrated into this framework. To extensively evaluate the performance of our framework, we choose 42 problems with diverse features as the test suite. In addition, some representative differential evolution and particle swarm optimization variants are incorporated into the framework. Our method is also compared with other state-of-the-art methods. Experimental results indicate that the dynamic repulsion technique can improve the performance of the original repulsion technique with static repulsive radius. Moreover, the proposed method is able to yield better results compared with other methods.**

***Index Terms*—Nonlinear equations system, repulsion technique, dynamic repulsive radius, evolutionary algorithms.**

I. Introduction

MANY real-world problems can be reduced to solve nonlinear equations systems (NESs) [1], [2], such as economics [3], physics [4], chemistry [5], system engineering [6], [7], signal processing [8], [9], and so on. Generally, a NES can be formulated as follows:

$$\mathbf{e}(\mathbf{x}) = \begin{cases} e_1(\mathbf{x}) & = & 0 \\ & \vdots & \\ e_n(\mathbf{x}) & = & 0 \end{cases} \tag{1}$$

where n is the number of equations; $\mathbf{x} = (x_1, \cdots, x_D)^T$ is a D-dimensional decision variable vector; $\mathbf{x} \in \mathcal{S}$, and $\mathcal{S} \subseteq \mathbb{R}^D$ is a compact set that denotes the feasible region of the search space. Usually,

$$\mathcal{S} = [\underline{x}_j, \overline{x}_j]^D$$

where $j = 1, \cdots, D$, $\underline{x}_j$ and $\overline{x}_j$ are the lower bound and upper bound of x_j, respectively. Note that, there exists at least one equation that is nonlinear.

To solve the NES problem shown in Equation (1), we require to locate the root(s) $\mathbf{x}^* \in \mathcal{S}$ to satisfy $\mathbf{e}(\mathbf{x}^*) = \mathbf{0}$. Due to the nonlinearity, it is perhaps one of the most difficult tasks in numerical computation to solve NESs [10]. More importantly, most of NESs have more than one equally important roots. For example, Figure 1 depicts a NES problem (*i.e.*, F23 in the supplementary material) that has two nonlinear equations and six roots. Because each root is equally important, the major task for solving NESs is to locate multiple roots. However, it causes the great challenge in numerical computation, especially to locate them in a single run simultaneously.

In the literature, there are different classical methods proposed for solving NESs to find different roots, which can be classified into three categories [11], [12]: i) Newton and quasi-Newton type methods, *e.g.*, [13], [14]; ii) embedding (homotopy continuation) methods, *e.g.*, [15], [16]; and iii) interval-Newton methods, *i.e.*, [17], [18]. However, these classical methods have some weaknesses. For example, the performance of Newton and quasi-Newton type methods is highly sensitive to the initial guess; the embedding methods cannot directly deal with variable bounds and inequality constraints; and the internal-Newton methods are computationally expensive. Moreover, most of the classical methods are single-point-based algorithms, which cannot find different roots in a single run.

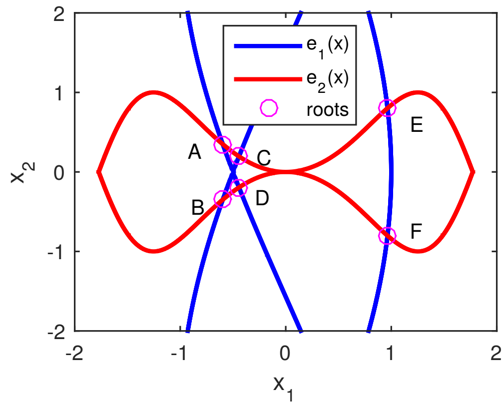

Fig. 1. An example of a NES problem with multiple roots, where the circles indicate the roots of the problem.

This work was partly supported by the National Key R&D Program of China under Grant No. 2016YFB0901900, National Natural Science Foundation of China under Grant Nos. 61573324 and 61673354, National Natural Science Fund for Distinguished Young Scholars of China under Grant No. 61525304, the State Key Laboratory of Intelligent Control and Decision of Complex Systems, and the Fundamental Research Funds for the Central Universities, China University of Geosciences (Wuhan) under Grant No. CUG160603. *(Corresponding author: Wenyin Gong.)*

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Recently, the *repulsion* techniques, which can create repulsion areas around previously found roots, were developed for solving NESs to find different roots [19], [20], [21]. Due to their simplicity and easy implementation, they have gained growing attention during the last few decades [22], [23], [24], [25]. In the literature, there are two representative repulsion techniques , *i.e.*, *multiplicative* techniques [20], [21] and *additive* techniques [19], [23]. Although different repulsion techniques were proposed, they have similar principle, *i.e.*, penalizing the solutions that lie in the neighborhood of previously found roots so as to drive the algorithm to search new roots. Therefore, it is vital to determine the repulsive radius in the repulsion techniques. Too large repulsive radius may lead to losing some roots that are very close to the obtained roots. In contrast, too small repulsive radius may result in reducing the probability to find new roots that are far from the obtained roots, since the search algorithm may waste more computational sources to seek the closer neighborhood of the found roots due to the low repulsion pressure. However, the optimal setting of the repulsive radius is difficult and problem-dependent. In addition, owing to the dynamic features of the search algorithm, it is not a good choice to set the fixed repulsive radius during the run.

Based on the above considerations, we make the first attempt to propose the dynamic repulsion technique for solving NESs, where the repulsive radius is dynamically controlled during the run. In this way, we can alleviate the trivial task to set the proper repulsive radius for different problems by trial-and-error. Moreover, the proposed dynamic repulsion technique is cooperated with evolutionary algorithms (EAs) to locate multiple roots of NESs in a single run simultaneously. Our proposed framework is referred to as DREA, *i.e.*, Dynamic Repulsion-based EA. In DREA, to maintain the diversity of the population, the population re-initialization is used. Additionally, an archive is employed to store the roots located during the run. DREA is a general framework, in which different EAs and different repulsion techniques can be easily integrated into DREA. To extensively evaluate the performance of our approach, we choose 42 NESs from the literature. Furthermore, systematical experiments have been conducted in this paper, including the effectiveness of the dynamic repulsion technique, comparison with other state-of-the-art algorithms, the effect of different repulsion techniques and different EAs, and the influence of different parameter settings in DREA.

The main contributions of this paper are as follows:

- The core contribution is that we make the first attempt to develop the dynamic repulsion technique, which is able to remedy the drawback of the current repulsion techniques with static repulsive radius. In addition, in this paper, four control strategies are presented to dynamically tune the repulsive radius during the run.
- A general framework, *i.e.*, DREA, is presented, where different EAs and different repulsion techniques can be easily used in this framework. In this paper, five EAs and three repulsion techniques are empirically studied to evaluate the effectiveness of our framework.
- Extensive experiments have been carried out on 42 NES problems with diverse features. In addition, the performance of our proposed method is compared with other state-of-the-art methods. The influences of different components in DREA and different parameter settings are also discussed in this paper.

The rest of the paper is organized as follows. Section II briefly reviews the related work for solving NESs. In Section III, we will present our proposed method in detail, followed by the experimental results and analysis in Section IV. In Section V, the effectiveness of the dynamic repulsion technique with other EAs and other repulsion techniques is discussed. In addition, the influence of different parameter settings in DREA is also empirically studied herein. Finally, Section VI concludes the paper and points out some possible future directions.

II. Related Work

Due to the significant importance for solving NESs, besides the classical methods as reviewed in [12], there are also other methods, such as the BFGS based trust-region [26], neural dynamics [27], [6], modified conjugate gradient algorithms [28], [29], GAs with sequential quadratic programming (GA-SQP) [30], niche cuckoo search algorithm (NCSA) [31], particle swarm optimization with Nelder-Mead (PSO-NM) [32], proposed in the literature. However, in this section, we mainly focus on the methods that were developed to locate multiple roots of NESs, which can be roughly divided into three groups: *i.e.*, repulsion-based methods, clustering-based methods, and multiobjective optimization-based methods.

The repulsion techniques penalize the solutions that lie within the neighborhood of the roots found previously, in this way, they can promote the optimization algorithm to seek new promising areas, and hence, to detect new roots of NESs. These techniques are simple and can be easily implemented. Therefore, they have been combined with different optimization algorithms for solving NESs in the literature. In [33], [22], the authors combined simulated annealing (SA) with repulsion technique to calculate critical points of phase transitions. Cooperated with the repulsion method, Hirsch *et al.* extended the C-GRASP method, which is a multi-start local search procedure, to solve NESs [19]. Henderson *et al.* [20] used the continuous SA to find multiple roots of double retrograde vaporization. In [21], Pourjafari and Mojallali presented a two-phase root-finder with invasive weed optimization (IWO) for locating more than one root of NESs. In [23], the authors presented a biased random-key genetic algorithm (BRKGA), in which BRKGA runs multiple times to locate different roots. In [24] and [25], combined with the repulsion techniques, the improved harmony search and Nelder-Mead were respectively used to solve NESs.

Clustering is the task of partitioning a set of objects into different groups [34]. Inspired by this, recently, some researchers employed the clustering techniques to group similar solutions into different clusters so as to find multiple roots of NESs. In [35], a clustering-based Multistart and Minfinder method was presented. Sacco and Henderson [36] proposed a hybrid metaheuristic with Fuzzy clustering means, where the Luss-Jaakola random search method, the Fuzzy clustering means, and the Nelder-Mead method were combined together to find multiple roots of NESs. In [21], a clustering method was applied at the exact search phase, then the IWO algorithm was used to each cluster to determine the exact location of the root.

In evolutionary multiobjective optimization, the main goal is to obtain a set of representative Pareto optimal solutions, which is similar to locate multiple roots of NESs. Taking this into consideration, solving NESs via multiobjective optimization-based methods obtains more attention recently. Grosan and Abraham [37] made the first attempt to solve NESs by transforming the NES problem into an n-objective optimization problem. In [38], the authors presented a bi-objective transformation technique for solving NESs, where the location function and the system function are respectively used as the two objective functions. Qin *et al.* [39] presented a $(D+1)$-objective transformation technique for NESs. Gong *et al.* [40] extended the work presented in [38] and developed the weighted bi-objective transformation technique (A-WeB), where all the decision variables of NES are considered in the location function. In [41], Naidu and Ojha presented a hybrid cooperative multiobjective optimization IWO for NESs, where the NES problem is converted into a constrained bi-objective optimization problem.

III. Our Approach

In this section, the motivations of our approach will be explained firstly. Then, the proposed dynamic repulsion technique is presented in Section III-B. In Section III-C, the framework, DREA, will be given in detail.

A. Motivations

In Section II, the methods to locate multiple roots of NESs were briefly described. However, there are some weaknesses in these methods. For example,

- For the repulsion-based methods, the repulsive radius should be given by the user and kept unchanged during the run. However, the optimal setting of the repulsive radius is difficult and problem-dependent.
- For the clustering-based methods, it might be difficult to determine the number of clusters in advance.
- For the multiobjective optimization-based methods, the approaches presented in [37] and [39] will suffer from the "curse of dimensionality" when $n > 3$ or $D > 2$. The methods in [38] and [40] may lose some roots because of the non one-to-one mapping between the roots and the location function. The method in [41] needs to carefully set the penalty factors; in addition, it can not guarantee the conflict between the two objectives.

In evolutionary computation community, the dynamic control of parameters is an effective technique to improve the performance of EAs [42], [43]. This technique may also be adapted to the control of the repulsive radius for different repulsion techniques.

Motivated by the above considerations, in this work, we make the first attempt to propose the dynamic repulsion technique, where the repulsive radius is dynamically changed during the run. In this way, we can remedy to set the proper repulsive radius for different NESs by trial-and-error.

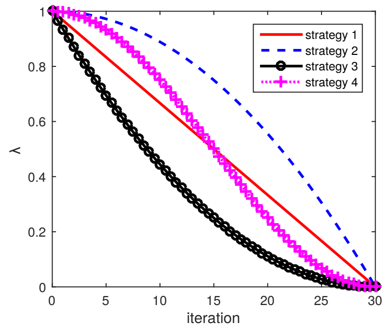


Fig. 2. Four dynamic control strategies used in this work.

B. Dynamic Repulsion Technique

As mentioned in Section I, the repulsive radius plays a vital role in the repulsion techniques. Unreasonable settings of the repulsive radius may result in poor performance of the algorithms. In this section, the dynamic control of the repulsive radius is proposed as follows:

$$\gamma_t = \gamma_{\min} + \lambda_t \times (\gamma_{\max} - \gamma_{\min}) \tag{2}$$

where γ_t is the current repulsive radius at iteration t; $\gamma_{\min}$ and $\gamma_{\max}$ are respectively the minimal and maximal values of the repulsive radius; and $\lambda_t \in [0,1]$ is dynamically changed during the run. In this work, the following four strategies are used for λ_t:

- Strategy 1:
$$\lambda_t = 1 - \frac{t}{t_{\max}} \tag{3}$$
- Strategy 2:
$$\lambda_t = 1 - \left(\frac{t}{t_{\max}}\right)^2 \tag{4}$$
- Strategy 3:
$$\lambda_t = \left(1 - \frac{t}{t_{\max}}\right)^2 \tag{5}$$
- Strategy 4:
$$\lambda_t = \frac{\left(1 + \cos\left(\frac{t \times \pi}{t_{\max}}\right)\right)}{2} \tag{6}$$

where t is the iteration counter; and $t_{\max}$ is the maximal number of iterations.

The dynamic behaviors of the four strategies are depicted in Figure 2. We can clearly see that λ_t gradually decreases from 1 to 0 during the run, in such a way that the repulsive radius γ_t gradually decreases from $\gamma_{\max}$ to $\gamma_{\min}$. The reason behind the decreasing radius is that:

- In the early stage, the repulsive radius is large so that it is able to promote the optimization algorithms to detect new roots that are far from the found roots. For example, as shown in Figure 1, suppose that root "A" is found and stored in the archive, due to the large repulsive radius, the areas that contain the roots "B", "C", and "D" may

Algorithm 1: The framework of DREA

```
Input: Control parameters: μ, t_max, NFEs_max
Output: The final archive 𝒜
1  Set t = 0 and s_𝒜 = 0;
2  t_EAmax = ⌊ NFEs_max / (μ × t_max) ⌋;
3  while t < t_max do
4      Calculate γ_t via Equation (2);
5      Randomly generate an initial population 𝒫;
6      Calculate f(x_i) and R(x_i) of each individual x_i in 𝒫;
7      Update the archive 𝒜 via each individual x_i in 𝒫;
8      Set t_EA = 1;
9      while t_EA < t_EAmax do
10         for i = 1 to μ do
11             Generate the offspring u_i using operators in
               specific EA;
12             Calculate f(u_i) of the offspring u_i;
13             if s_𝒜 ≠ 0 then
14                 Calculate R(x_i) and R(u_i) via the repulsion
                   technique;
15             Update the archive 𝒜 using u_i;
16         Selection between the parent population and the
           offspring population to form new parent population 𝒫;
17         t_EA = t_EA + 1;
18     t = t + 1;
```

Algorithm 2: Archive updating

```
Input: Solution x and ε > 0
Output: The updated archive 𝒜
if s_𝒜 = 0 then                         // The archive is empty
    if f(x) < 1e − 5 then
        𝒜 = 𝒜 ∪ x;
        s_𝒜 = s_𝒜 + 1;
else
    if R(x) < 1e − 5 then
        if |R(x) − R(x*_j)| < ε or ‖ x − x*_j ‖ < ε, for any
        j = 1, ⋯ , s_𝒜 then   // A new root is found
            𝒜 = 𝒜 ∪ x;
            s_𝒜 = s_𝒜 + 1;
```

be repulsed; in this way, the probabilities of finding the roots "E" and "F" are increased.

- As the search proceeds, the repulsive radius gradually reduces, which leads to gradually decreasing the repulsion pressure. In this way, the optimization algorithms can pay more attention to search the closer neighborhood of the found roots, and hence, the areas containing "B", "C", and "D" become the basin of attraction during the run.

Combined with the proposed dynamic control of the repulsive radius, we can easily develop the dynamic repulsion techniques from the existing repulsion techniques in the literature. As an example, the repulsion technique presented in [24] is illustrated:

$$\text{minimize} \quad R(\mathbf{x}) = f(\mathbf{x}) \times \prod_{j=1}^{s_{\mathcal{A}}} \zeta_\gamma(\rho, \xi_j) \tag{7}$$

where

$$f(\mathbf{x}) = \sum_{i=1}^{n} e_i^2(\mathbf{x}) \tag{8}$$

$$\xi_j = \| \mathbf{x} - \mathbf{x}_j^* \| \tag{9}$$

$$\zeta_\gamma(\rho, \xi_j) = \begin{cases} |\text{erf}(\rho \times \xi_j)|^{-1}, & \text{if } \xi_j \leq \gamma \\ 1, & \text{otherwise} \end{cases} \tag{10}$$

where $f(\mathbf{x})$ and $R(\mathbf{x})$ are respectively the objective and repulsion values of $\mathbf{x}$; $s_{\mathcal{A}}$ is the number of roots that have been found[1]; $\mathbf{x}_j^*$ is the j-th root in the archive $\mathcal{A}$; ξ_j is the Euclidean distance between $\mathbf{x}$ and $\mathbf{x}_j^*$; "erf" is the error function; the parameter $\rho > 0$ scales the penalty, $\rho = 0.1$ is used as suggested in [24] in this work; and γ adjusts the radius of the repulsion area.

[1] Note that if $s_{\mathcal{A}} = 0$, it means the archive is empty, then $R(\mathbf{x}) = f(\mathbf{x})$.

- For the static repulsion technique, γ is set by the user and kept fixed during the run. For example, in [24], $\gamma = 0.1 \times \min_{i=1}^{D}(\overline{x}_i - \underline{x}_i)$ is used.
- For the proposed dynamic repulsion technique, γ is simply replaced by γ_t as shown in Equation (2). All other settings are the same as the static repulsion technique.

Therefore, compared with the static repulsion technique, the proposed dynamic repulsion technique is very simple and does not increase the complexity. Moreover, it can be easily integrated into other existing repulsion techniques, such as [19], [21], for the control of the repulsive radius.

C. The Proposed Framework: DREA

With the aim of locating multiple roots of NESs in a single run simultaneously, in this work, the proposed dynamic repulsion technique is cooperated with EAs. The reasons are three-fold: i) the repulsion technique can create the repulsion regions around the found roots, and hence, it is able to assist the optimization algorithms to detect new roots; ii) EAs are population-based optimization algorithms, which contain a group of individuals; therefore, it is a reasonable choice to find different roots of NESs via EAs; and iii) EAs have been successfully used to solve different optimization problems [44], thus they might be also useful for solving NESs.

The framework of our proposed DREA is shown in Algorithm 1, where μ is the population size of EA; $NFEs_{\max}$ is the maximal number of function evaluations; $t_{\max}$ and $t_{\text{EA}_{\max}}$ are the external and internal maximal number of iterations, respectively. Same as other repulsion-based algorithms, in DREA, there is an archive $\mathcal{A}$ to save the roots located during the run, and $s_{\mathcal{A}}$ is the archive size. DREA works as follows:

- In order to locate multiple roots of NESs in a single run, the diversity of the population is important. Based on this consideration, two cycles are used in DREA. In line 5, the population is re-initialized at each external iteration, hence the diversity can be maintained.
- In line 4, the repulsive radius is dynamically changed using Equation (2).
- In lines 7 and 15, once a new root is found, the archive will be updated via Algorithm 2. In Algorithm 2, $\mathbf{x}_j^*$ is the j-th root in $\mathcal{A}$, and ϵ is a small positive value to

make the root not be the same as the existing roots in $\mathcal{A}$. $\epsilon = 0.01$ is used in this work.

- In lines 9-17, the population evolves using EA operators, such as crossover, mutation, and selection[2]. Note that, in line 14, the repulsion values of $R(\mathbf{x}_i)$ and $R(\mathbf{u}_i)$ are evaluated via the repulsion technique. Since their objective functions are calculated in lines 6 and 12, we do not need to re-calculate the objective functions in line 14.

It is worth pointing out that DREA is a general framework, where different EAs can be used in lines 9-17; in addition, different repulsion techniques are also able to be used in line 14 to calculate the repulsion values of the solutions.

1) On the Complexity of DREA: In DREA, there are three major aspects, *i.e.*, fitness calculation with repulsion technique, archive updating, and offspring generating. Their complexity is as follows:

- On the fitness calculation, the complexity is $O(\mu \cdot s_{\mathcal{A}} \cdot n)$, where $s_{\mathcal{A}}$ is the size of the archive.
- On the archive updating, the complexity is $O(\mu \cdot s_{\mathcal{A}} \cdot n)$.
- On the offspring generating, the total complexity is $O(\mu \cdot n)$.

Therefore, on the whole, the complexity of DREA is $O(t_{\max} \cdot t_{EA_{\max}} \cdot \mu \cdot s_{\mathcal{A}} \cdot n)$, *i.e.*, $O(NFEs_{\max} \cdot s_{\mathcal{A}} \cdot n)$.

IV. Experimental Results and Analysis

As above mentioned, DREA is a general framework, in which different repulsion techniques and different EAs can be used. As an illustration and substantiation, firstly the repulsion technique shown in Section III-B and JADE[3] proposed in [45] are integrated into DREA, and the resultant method is referred to as DR-JADE[4]. The influence of other repulsion techniques and EAs will be discussed in Section V.

A. Test Problems

To extensively evaluate the performance of different algorithms, in this work, we choose 42 NES problems with diverse features from the literature. Table I briefly describes the test problems used in this work. Their detailed information is given in the supplementary material. Note that, in Table I, the maximal number of function evaluations ($NFEs_{\max}$) are different for different problems owing to their different difficulties. In addition, there are also some algorithmic parameters that need to be set to begin the algorithm, such as μ, $t_{\max}$. The detailed parameter settings of different methods are given in Table S-R-I in the supplementary material.

TABLE I

Brief descriptions of the test problems used in this work, where "D" is the number of decision variables; "LE" and "NE" are respectively the number of linear and nonlinear equations; "NoR" is the number of roots; and "$NFEs_{\max}$" is the maximal number of function evaluations.

Prob.	D	LE	NE	NoR	$NFEs_{\max}$
F01	2	0	2	2	10 000
F02	2	0	2	3	50 000
F03	3	0	3	2	100 000
F04	2	1	1	2	20 000
F05	2	0	2	3	100 000
F06	3	1	2	2	50 000
F07	2	0	2	3	50 000
F08	2	1	1	2	50 000
F09	20	0	2	2	50 000
F10	2	1	1	11	50 000
F11	2	0	2	15	50 000
F12	2	0	2	13	50 000
F13	10	0	10	1	50 000
F14	2	1	1	8	50 000
F15	4	0	4	1	50 000
F16	2	0	2	7	50 000
F17	5	4	1	3	100 000
F18	6	0	6	1	50 000
F19	2	0	2	10	50 000
F20	2	0	2	9	50 000
F21	2	0	2	13	50 000
F22	8	0	8	16	100 000
F23	2	0	2	6	50 000
F24	20	19	1	2	200 000
F25	3	0	3	7	50 000
F26	2	0	2	4	50 000
F27	2	0	2	6	50 000
F28	3	0	3	8	100 000
F29	3	0	3	2	50 000
F30	3	0	3	12	50 000
F31	2	0	2	2	50 000
F32	2	0	2	4	50 000
F33	2	0	2	4	50 000
F34	2	0	2	2	50 000
F35	3	0	3	1	50 000
F36	2	0	2	2	50 000
F37	3	0	3	1	50 000
F38	2	0	2	3	50 000
F39	2	0	2	2	50 000
F40	3	0	3	5	50 000
F41	2	0	2	4	50 000
F42	2	0	2	2	50 000

B. Performance Criteria

To solve the NES problem via the optimization algorithms, the NES problem is firstly converted into an optimization problem as shown in Equation (8). Essentially, this optimization problem is a multimodal problem. Taking this into account, to compare the performance of different algorithms, two performance criteria proposed in [47] for multimodal optimization are borrowed in this work.

1) Root ratio (RR): It measures the average ratio of all known roots found over multiple runs within $NFEs_{\max}$.

$$RR = \frac{\sum_{i=1}^{N_r} N_{f,i}}{NoR \cdot N_r} \tag{11}$$

where N_r is the number of runs; $N_{f,i}$ is the number of roots found in the i-th run; and NoR is the number of known roots of a NES. In this work, for a solution $\mathbf{x}$, if its repulsion value $R(\mathbf{x}) < 1e-5$, it can be viewed as a root. To make the comparison meaningful, each algorithm is executed over $N_r = 30$ independent runs for each NES.

[2]In Algorithm 1, we suppose each parent $\mathbf{x}_i$ only generates one offspring $\mathbf{u}_i$ using EA operators. However, other operators that generate multiple offsprings for one parent can also be used in DREA.

[3]JADE is a differential evolution (DE) variant, where the parameters CR and F are adaptively controlled. It has obtained very promising results compared with other state-of-the-art EAs [45], [46]. In this work, due to the tight space limitation, we omit the description of JADE.

[4]For the sake of the brevity, the pseudo-code of DR-JADE is only provided in Section S-II in the supplementary material.

2) Success rate (SR): It is used to evaluate the ratio of successful runs[5] out of all runs:

$$SR = \frac{N_{r,s}}{N_r} \tag{12}$$

where $N_{r,s}$ is the number of successful runs.

TABLE II
AVERAGE RANKINGS OBTAINED BY THE FRIEDMAN TEST FOR BOTH RR AND SR CRITERIA.

Algorithm	Ranking (RR)	Ranking (SR)
$\delta = 0.01$	7.7738	7.4881
$\delta = 0.02$	7.5000	7.5595
$\delta = 0.03$	6.8214	6.9762
$\delta = 0.04$	**6.7381**	**7.0119**
$\delta = 0.05$	6.8095	7.0238
$\delta = 0.06$	7.2143	7.2500
$\delta = 0.07$	7.2738	7.6548
$\delta = 0.08$	7.1786	7.3333
$\delta = 0.09$	7.0476	7.1905
$\delta = 0.1$	7.2262	7.3452
$\delta = 0.2$	8.4881	8.2143
$\delta = 0.3$	8.5476	8.2143
$\delta = 0.4$	8.1667	7.9405
$\delta = 0.5$	8.2143	7.7976

C. Influence of Static Settings of Repulsive Radius

Firstly, we are interested in verifying the influence of the static settings of the repulsive radius. Therefore, the dynamic control technique proposed in Section III-B is removed from DR-JADE, and the resultant method is named as R-JADE, *i.e.*, repulsion-based JADE. In R-JADE, JADE is used as the optimization algorithm and the repulsion technique presented in [24] with static repulsive radius is adopted. Originally, the repulsive radius is set to be $\gamma = \delta \times \min_{i=1}^{D}(\overline{x}_i - \underline{x}_i)$ and $\delta = 0.1$ in [24]. In this subsection, we set $\delta = \{0.01, 0.02, \cdots, 0.09, 0.1, 0.2, 0.3, 0.4, 0.5\}$ to study its influence. All other parameters are the same as DR-JADE shown in Table S-R-I in the supplementary material.

The detailed results of RR and SR values are respectively reported in Tables S-R-II and S-R-III in the supplementary material. All results are averaged over 30 runs. The average rankings calculated by the Friedman test[6] are shown in Table II. In addition, both of the p-values of the Bonferroni-Dunn's procedure and the Holm's procedure are less than $3.85E-3$ for both RR and SR.

From the results in Tables S-R-II and S-R-III, we see that the promising results are obtained with $\delta \in [0.01, 0.1]$, and $\delta = 0.04$ gets the best rankings for RR and SR by the Friedman test as shown in Table II. Moreover, the p-values obtained by the Bonferroni-Dunn's procedure and the Holm's procedure signify that the results are significantly different for different settings of the repulsive radius. Therefore, we can conclude that the repulsive radius has significant influence, and its optimal setting is usually difficult and problem-dependent.

[5]A successful run is defined as a run where all known roots of a NES are found.

[6]The statistical results reported in this work are calculated by the KEEL software [48].

D. Principle of Dynamic Repulsion Technique

In this subsection, the principle of our proposed dynamic repulsion technique is empirically analyzed. For this purpose, DR-JADE is compared with R-JADE. Because the repulsion technique proposed in [24] is used, we set $\gamma_{\max} = 0.5 \times \min_{i=1}^{D}(\overline{x}_i - \underline{x}_i)$ and $\gamma_{\min} = 0.01 \times \min_{i=1}^{D}(\overline{x}_i - \underline{x}_i)$ for DR-JADE. Other parameters of DR-JADE are given in Table S-R-I in the supplementary material. Figures 3 - 5 respectively show the search behaviors of R-JADE with $\delta = 0.01$, R-JADE with $\delta = 0.5$, and DR-JADE-s1 over a typical run on F23. In DR-JADE-s1, strategy 1 shown in Equation (3) is used.

From these figures, we can observe that:

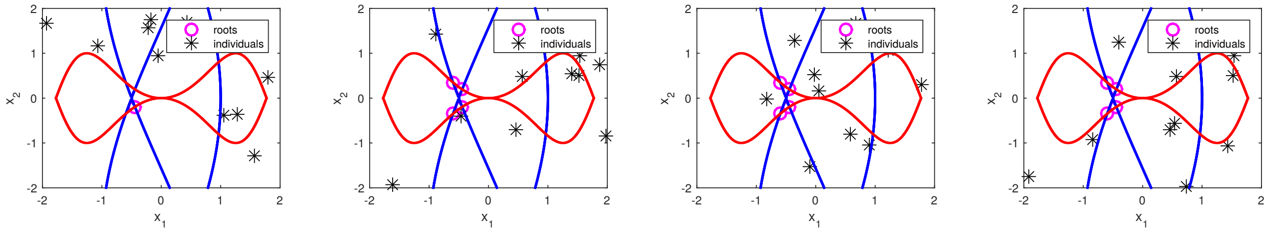
- Since the three algorithms start from the same initial population, after the first external generation, each of them locates a same root.
- Figure 3 shows that at $t = 5$ R-JADE finds other three roots, which lie in the closer areas of the firstly found root. However, in the subsequent iterations, R-JADE cannot detect the other two roots that are far from the located roots. The reason is that the small repulsive radius makes R-JADE waste computational resources to search closer areas of the found roots, and hence, it loses the opportunities to find the farther roots.
- On the contrary, if the repulsive radius is large, Figure 4 clearly depicts that finally R-JADE finds four roots that distribute sparsely, whereas two roots that lie within closer areas of the found roots are lost.
- When the dynamic repulsive radius is used, from Figure 5, we see that at $t = 5$ other two roots that are far from the found root are detected. At $t = 15$ another new root is found, and finally all roots are located at $t = 30$. The reason is that, at the early stage the repulsive radius is large, it drives DR-JADE-s1 to search a wide range, and thus DR-JADE-s1 can locate the roots that are far from the found roots. As the search proceeds, the repulsive radius gradually decreases, which results in the decrease of the repulsion pressure. In this way, the roots that lie within the closer areas of the found roots are obtained during the run.

TABLE III
AVERAGE RANKINGS OF DIFFERENT DR-JADE VARIANTS OBTAINED BY THE FRIEDMAN TEST FOR BOTH RR AND SR CRITERIA.

Algorithm	Ranking (RR)	Ranking (SR)
DR-JADE-s1	3.1071	3.0476
DR-JADE-s2	3.4643	3.4405
DR-JADE-s3	**2.5476**	**2.5595**
DR-JADE-s4	2.7857	2.8333
$\delta = 0.04$	3.0952	3.1190

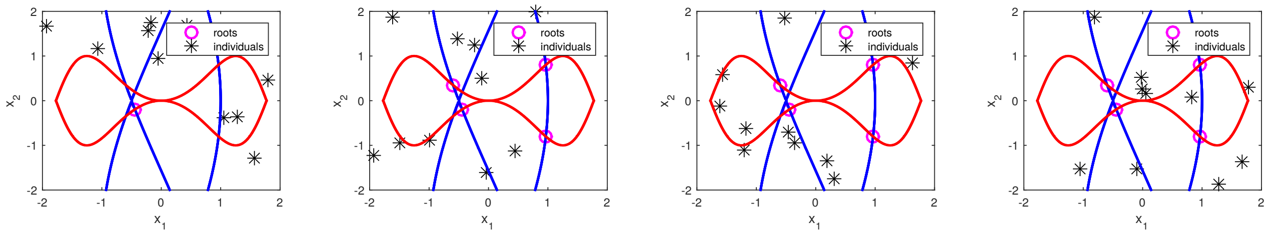
TABLE IV
RESULTS OBTAINED BY THE WILCOXON TEST FOR ALGORITHM DR-JADE-S3 IN TERMS OF RR AND SR.

VS	RR			SR		
	R^+	R^-	p-value	R^+	R^-	p-value
DR-JADE-s1	639.5	263.5	**1.84E-02**	616.5	286.5	**3.19E-02**
DR-JADE-s2	674.5	228.5	**3.72E-03**	674.5	228.5	**3.24E-03**
DR-JADE-s4	583.5	319.5	*8.54E-02*	582.0	321.0	*8.70E-02*
$\delta = 0.04$	601.0	302.0	*6.07E-02*	597.0	306.0	*6.66E-02*



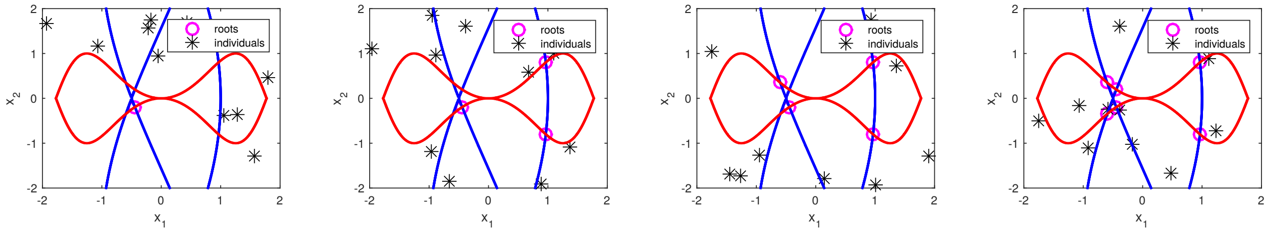
(a) $t = 1$ (b) $t = 5$ (c) $t = 15$ (d) $t = 30$

Fig. 3. Search behavior of R-JADE with fixed repulsive radius with $\gamma = 0.01 \times \min_{i=1}^{D}(\overline{x}_i - \underline{x}_i)$ over a typical run on F23. Stars denote the individuals in the population, and circles denote the obtained roots (the same below).



(a) $t = 1$ (b) $t = 5$ (c) $t = 15$ (d) $t = 30$

Fig. 4. Search behavior of R-JADE with fixed repulsive radius with $\gamma = 0.5 \times \min_{i=1}^{D}(\overline{x}_i - \underline{x}_i)$ over a typical run on F23.



(a) $t = 1$ (b) $t = 5$ (c) $t = 15$ (d) $t = 30$

Fig. 5. Search behavior of DR-JADE-s1 with dynamic repulsive radius over a typical run on F23.

To further understand the performance of our proposed dynamic technique, the four strategies presented in Section III-B are integrated into DR-JADE. The four DR-JADE variants are referred to as DR-JADE-s1 with strategy 1, DR-JADE-s2 with strategy 2, DR-JADE-s3 with strategy 3, and DR-JADE-s4 with strategy 4. These four methods are compared with R-JADE using $\delta = 0.04$ because of its overall best results shown in Table II.

The detailed results of RR and SR are respectively reported in Tables S-R-IV and S-R-V in the supplementary material. Additionally, Table III provides the average rankings of different algorithms obtained by the Friedman test, and the results of the Wilcoxon test for these algorithms are shown in Table IV.

- Tables S-R-IV and S-R-V show that, for both RR and SR, DR-JADE-s1, DR-JADE-s3, and DR-JADE-s4 obtain better results than R-JADE, while DR-JADE-s2 is slightly worse than R-JADE. On the whole, DR-JADE-s3 is able to obtain the best average results in both RR and SR.
- According to the Friedman test, the results in Table III show that DR-JADE-s3 and DR-JADE-s4 respectively get the best and second best rankings in terms of RR and SR among all the five methods. DR-JADE-s1 performs similarly to R-JADE. However, DR-JADE-s2 provides the worst results.
- From Table IV, we can see that DR-JADE-s3 is better than other four methods, since it can provide higher R^+ values than R^- values in all cases. Especially, DR-JADE-s3 significantly outperforms DR-JADE-s1 and DR-JADE-s2 at $\alpha = 0.05$ for RR and SR; and it can also provide significantly better results than DR-JADE-s4 and R-JADE at $\alpha = 0.1$ for RR and SR.

In general, the proposed dynamic repulsion technique is effective, especially with strategies 3 and 4. More importantly, it can alleviate the tedious task to set the proper values of the repulsive radius for different NESs. On the whole, DR-JADE-s3 obtains the best results, therefore, it is used in the subsequent experiments. For the sake of brevity, it is called as DR-JADE hereinafter.

E. Comparison with Other State-of-the-Art Methods

This subsection dedicates to comparison between DR-JADE and other state-of-the-art methods to demonstrate the promising performance of DR-JADE. Eight methods are compared

TABLE V
AVERAGE RANKINGS OF DR-JADE, A-WEB, NCDE, NSDE, MONES, I-HS,GA-SQP, PSO-NM, AND NCSA OBTAINED BY THE FRIEDMAN TEST FOR BOTH RR AND SR.

Algorithm	Ranking (RR)	Ranking (SR)
DR-JADE	**2.8214**	**2.8571**
A-WeB	3.7619	3.7381
MONES	4.7857	4.7143
I-HS	4.9762	5.1429
NCDE	4.5357	4.4524
NSDE	4.0000	4.1190
GA-SQP	8.0714	7.7500
PSO-NM	5.6429	5.7500
NCSA	6.4048	6.4762

TABLE VI
RESULTS OBTAINED BY THE WILCOXON TEST FOR ALGORITHM DR-JADE IN TERMS OF RR AND SR COMPARED WITH A-WEB, NCDE, NSDE, MONES, I-HS, GA-SQP, PSO-NM, AND NCSA .

VS	RR			SR		
	R^+	R^-	p-value	R^+	R^-	p-value
A-WeB	676.0	227.0	**4.70E-03**	655.5	247.5	**1.01E-02**
MONES	692.5	210.5	**2.33E-03**	663.5	239.5	**2.85E-03**
I-HS	777.5	125.5	**3.31E-05**	770.5	132.5	**2.41E-05**
NCDE	658.5	244.5	**6.98E-03**	645.5	257.5	**1.24E-02**
NSDE	661.0	242.0	**8.23E-03**	657.0	246.0	**4.11E-03**
GA-SQP	877.5	25.5	**0.00E+00**	872.5	30.5	**0.00E+00**
PSO-NM	817.5	85.5	**2.01E-06**	815.0	88.0	**1.01E-06**
NCSA	841.5	61.5	**1.01E-06**	869.0	34.0	**0.00E+00**

with DR-JADE, *i.e.*, A-WeB [40], MONES [38], I-HS [24], NCDE [49], NSDE [49], GA-SQP [30], PSO-NM [32], and NCSA [31]. Among these methods, A-WeB and MONES are the multiobjective optimization-based methods; and I-HS is a repulsion-based harmony search method. NCDE and NSDE are neighborhood mutation-based DE methods originally proposed for the multimodal optimization problems (MMOPs) [49], which have obtained very promising results compared with other methods for MMOPs [50]. GA-SQP and PSO-NM are the meta-heuristics hybrid with the local search methods[7]. NCSA is a modified cuckoo search algorithm with niche strategy. Due to the similarities between NESs and MMOPs, these two methods are also chosen for comparison. The parameter settings are given in Table S-R-I in the supplementary material. Note that, to make a fair comparison, the parameters of A-WeB, MONES, I-HS, NCDE, NSDE, GA-SQP, PSO-NM, and NCSA are set to be the same as used in their original literature.

The detailed results are respectively shown in Tables S-R-VI and S-R-VII for RR and SR in the supplementary material, where all results are averaged over 30 runs. Additionally, the summarized results obtained by the Friedman test and the Wilcoxon test are reported in Tables V and VI, respectively. We can clearly observe that DR-JADE obtains the best average values in both RR and SR. It can also get the best average rankings compared with other eight methods by the Friedman test as shown in Table V. Meanwhile, Table VI indicates that DR-JADE significantly outperforms other eight methods in terms of both RR and SR by the Wilcoxon test, since all p values are less than 0.05. Therefore, from these results we can conclude that the proposed DR-JADE method can be an effective alternative to simultaneously locate multiple roots of NESs in a single run. However, it is worth noting that the performance of DR-JADE deteriorates when $D \geq 20$ (F09 and F24). The reason might be that the search ability of JADE decreases when D increases. This motivates us to develop more powerful EAs for NESs in the future work.

[7]In GA-SQP and PSO-NM, there are respectively 12 and 16 versions presented in [30] and [32]. In this work, only their first versions are chosen for comparison.

TABLE VII
RESULTS OBTAINED BY THE WILCOXON TEST IN TERMS OF RR AND SR BETWEEN THE DYNAMIC AND STATIC REPULSION-BASED EAS.

DR-$*$ VS R-$*$	RR			SR		
	R^+	R^-	p-value	R^+	R^-	p-value
$*$: jDE	613.0	290.0	**3.60E-02**	580.5	322.5	*9.62E-02*
$*$: SHADE	605.5	297.5	*5.43E-02*	580.0	323.0	*9.75E-02*
$*$: SaDE	631.0	272.0	**2.41E-02**	585.0	318.0	*8.48E-02*
$*$: CLPSO	710.0	193.0	**9.85E-04**	688.0	215.0	**2.54E-03**

V. DISCUSSIONS

In the previous experimental studies, the superiority of the proposed dynamic repulsion technique and DR-JADE is verified. In this section, the effectiveness of the dynamic repulsion technique on other EAs and other repulsion techniques used in DREA is studied. In addition, the influence of different parameter settings is also discussed herein. Furthermore, the effectiveness of our approach on four benchmark NES models are also evaluated in this section.

A. Different Components in DREA

As described in Algorithm 1, DREA mainly contains two components, *i.e.*, dynamic repulsion technique and EA. Since DREA is a general framework, in this subsection, other EAs and other repulsion techniques are integrated into DREA to investigate the effectiveness of the dynamic repulsion technique.

1) Study on Other EAs: The results in Section IV-D indicated that our proposed dynamic repulsion technique can improve the performance of the static repulsion technique when JADE is used in DREA as the search engine. In this subsection, we will answer the question: "Can the dynamic repulsion technique be also effective for other EAs in the DREA framework?" With this aim, four advanced EAs, *i.e.*, jDE [51], SHADE [52], SaDE [53], and CLPSO [54][8] are integrated into DREA. The resultant methods are named as DR-jDE, DR-SHADE, DR-SaDE, and DR-CLPSO. They are respectively compared with their static repulsion-based methods, *i.e.*, R-jDE, R-SHADE, R-SaDE, and R-CLPSO. Note that, similar to DR-JADE, in the four dynamic repulsion-based methods, the strategy 3 shown in Equation (5) is used, and $\gamma_{\max} = 0.5 \times \min_{i=1}^{D}(\overline{x}_i - \underline{x}_i)$, $\gamma_{\min} = 0.01 \times \min_{i=1}^{D}(\overline{x}_i - \underline{x}_i)$. In the four static repulsion-based methods, $\gamma = 0.04 \times \min_{i=1}^{D}(\overline{x}_i - \underline{x}_i)$ is used. All other parameters are shown in Table S-R-I.

The detailed RR and SR results are given in Tables S-R-VIII and S-R-IX in the supplementary material, respectively.

[8]For the four EAs, jDE, SHADE, and SaDE are three DE variants, and CLPSO is a enhanced particle swarm optimization (PSO) method. All of the four methods obtained appealing results in the literature.

All results are averaged over 30 runs. The summarized results by the Wilcoxon test are provided in Table VII. From the results, we can observe that, regardless of different EAs used in DREA, the proposed dynamic repulsion-based methods obtain better results compared with their corresponding static repulsion-based methods in terms of both RR and SR as shown in Tables S-R-VIII and S-R-IX. Moreover, they can also provide significantly better results in all cases according the Wilcoxon test at $\alpha = 0.1$ shown in Table VII. Hence, the proposed dynamic repulsion technique can be similarly benefit to performance enhancement of other static repulsion-based EAs.

TABLE VIII
RESULTS OBTAINED BY THE WILCOXON TEST IN TERMS OF RR AND SR BETWEEN THE DYNAMIC AND STATIC REPULSION-BASED JADE VARIANTS WITH DIFFERENT REPULSION TECHNIQUES.

DR-$*$ VS R-$*$	RR			SR		
	R^+	R^-	p-value	R^+	R^-	p-value
$*$: JADE1	546.5	356.5	≥ 0.2	630.5	272.5	**2.45E-02**
$*$: JADE2	611.5	291.5	**4.53E-02**	602.5	300.5	*5.94E-02*

2) Study on Other Repulsion Techniques: As mentioned in Section I, in the literature, there are also other repulsion techniques proposed to find different roots of NESs. In this subsection, the influence of other repulsion techniques in DREA is investigated. For this purpose, two simple repulsion techniques presented in [19] and [21] are used to replace the repulsion technique previously used in DR-JADE. Therefore, two DR-JADE variants, *i.e.*, DR-JADE1 and DR-JADE2, are developed[9].

In DR-JADE1, the following repulsion technique [19] is used:

$$\text{minimize} \quad R(\mathbf{x}) = f(\mathbf{x}) + \beta \sum_{j=1}^{s_{\mathcal{A}}} \exp\left(-\xi_j\right)\chi_\gamma(\xi_j) \tag{13}$$

where

$$\chi_\gamma(\xi_j) = \begin{cases} 1, & \text{if } \xi_j \leq \gamma \\ 0, & \text{otherwise} \end{cases} \tag{14}$$

γ is a small constant to adjust the radius of repulsion areas, and β is a large constant to control the penalty scale. Originally, $\gamma = 0.01$ and $\beta = 1000$ are used in [19]. In DR-JADE1, γ_t in Equation (2) is used to replace λ in Equation (13), and $\gamma_{\max} = 0.5, \gamma_{\min} = 0.001$.

In DR-JADE2, the repulsion technique presented in [21] is used:

$$\text{minimize} \quad R(\mathbf{x}) = \big(f(\mathbf{x}) + \varepsilon\big) \prod_{j=1}^{s_{\mathcal{A}}} |\coth\left(\gamma \xi_j\right)| \tag{15}$$

where $\gamma \geq 1$ is used to adjust the radius of the repulsion region, and ε is a very small positive constant or equal to 0 [21]. Originally, $\varepsilon = 1e{-}10$ and $\gamma = 10$ are set in [21]. In DR-JADE2, $\gamma_{\max} = 100$ and $\gamma_{\min} = 1$ are used in Equation (2).

[9]The only difference between DR-JADE and DR-JADE1 (DR-JADE2) is the repulsion techniques, all other settings are kept unchanged.

DR-JADE1 and DR-JADE2 are respectively compared with their corresponding static versions, *i.e.*, R-JADE1 and R-JADE2. The detailed results are reported in Table S-R-X for both RR and SR in the supplementary material. Moreover, the statistical results obtained by the Wilcoxon test are shown in Table VIII. The results clearly demonstrate that DR-JADE1 and DR-JADE2 obtain better results than their corresponding R-JADE1 and R-JADE2 in both RR and SR, which signify that the proposed dynamic repulsion technique is also capable of improving the performance of other static repulsion techniques.

B. Different Parameter Settings in DREA

In this subsection, the influence of different parameter settings of μ and $t_{\max}$ is studied empirically. As an illustration, DR-JADE is used.

TABLE IX
AVERAGE RANKINGS OF DR-JADE WITH DIFFERENT POPULATION SIZE OBTAINED BY THE FRIEDMAN TEST FOR BOTH RR AND SR.

Parameter Setting	Ranking (RR)	Ranking (SR)
$\mu = 5$	3.5238	3.5476
$\mu = 8$	3.0238	3.0595
$\mu = 10$	**2.8095**	**2.9167**
$\mu = 20$	3.2143	3.3929
$\mu = 30$	4.0357	4.0595
$\mu = 40$	5.1310	5.0833
$\mu = 50$	6.2619	5.9405

1) Influence of Different Population Size: Previously, the population size $\mu = 10$ is used in DR-JADE. In this subsection, we set $\mu = \{5, 8, 20, 30, 40, 50\}$ to investigate its influence on DR-JADE. All other parameters are kept unchanged. Tables S-R-XI and S-R-XII respectively report the detailed results of RR and SR in the supplementary material. Meanwhile, the average rankings by the Friedman test are shown in Table IX. The results show that DR-JADE with relatively small population size, *i.e.*, $\mu \in [5, 20]$, gets better results than the large population size in both RR and SR. The reason is that in DREA the population is re-initialized at each external iteration to maintain the diversity, the large population size may make the algorithm over-explore the search range, and hence, degenerates the performance.

TABLE X
AVERAGE RANKINGS OF DR-JADE WITH DIFFERENT $t_{\max}$ OBTAINED BY THE FRIEDMAN TEST FOR BOTH RR AND SR.

Parameter Setting	Ranking (RR)	Ranking (SR)
$t_{\max} = 1$	8.4048	8.2500
$t_{\max} = 10$	6.2976	6.3214
$t_{\max} = 20$	4.8690	4.9405
$t_{\max} = 30$	3.9286	4.0833
$t_{\max} = 40$	3.9167	3.9048
$t_{\max} = 50$	**3.7857**	**3.8571**
$t_{\max} = 60$	4.2500	4.2976
$t_{\max} = 70$	4.5000	4.4524
$t_{\max} = 80$	5.0476	4.8929

2) Influence of Different $t_{\max}$: Another important parameter in DREA is the external maximal number of iterations $t_{\max}$, which controls the population re-initialization. Herein, its influence on DR-JADE is studied, and we set $t_{\max} =$

TABLE XI
COMPARISON AMONG DR-JADE, GA-SQP-1, AND PSO-NM-1 FOR THE FOUR BENCHMARK NES MODELS, WHERE "NUMBER OF OBTAINED ROOTS" INDICATES THE OBTAINED ROOTS OF DR-JADE IN EACH SINGLE RUN, AND THE MEAN VALUES ARE AVERAGED OVER 30 RUNS. "NA" MEANS NOT AVAILABLE.

Model	Number of obtained roots				Value of objective function ($f(\mathbf{x}) = \sum_{i=1}^{n} e_i^2(\mathbf{x})$)								
	DR-JADE				DR-JADE			GA-SQP-1 [30]			PSO-NM-1 [32]		
	Best	Worst	Mean	Std	Best	Mean	Std	Best	Mean	Std	Best	Mean	Std
F43	12	7	8.93	1.86	1.66E-25	3.45E-06	3.27E-06	NA	1.55E-07	5.70E-08	1.87E-33	1.69E-02	7.00E-03
F44	28	22	24.10	1.54	3.85E-28	2.06E-12	7.70E-12	NA	2.34E-21	1.00E-22	0.00E+00	1.90E-14	3.17E-14
F45	30	30	30.00	0.00	8.45E-17	4.07E-13	1.36E-12	NA	1.20E-14	1.30E-15	1.53E-33	9.34E-16	1.90E-16
F46	4	3	3.13	0.12	2.49E-29	1.33E-27	1.40E-27	NA	4.85E-31	6.20E-32	1.24E-32	3.38E-31	7.05E-31

$\{1, 10, 20, 40, 50, 60, 70, 80\}$, and all other parameters are kept unchanged. Note that when $t_{\max} = 1$, the population only initializes once.

The detailed results are respectively given in Tables S-R-XIII and S-R-IV for RR and SR in the supplementary material. In addition, Table X reports the average rankings obtained by the Friedman test. From the results, we observe that:

- DR-JADE with $t_{\max} = 30$ gets the best average RR value, followed by $t_{\max} = 40$. For SR, $t_{\max} = 40$ obtains the best average value, followed by $t_{\max} = 50$.
- According to the average rankings by the Friedman test, DR-JADE gets promising results with $t_{\max} \in [30, 60]$, and the best rankings in terms of both RR and SR are obtained with $t_{\max} = 50$. However, $t_{\max}$ with small or large values degenerates the performance of DR-JADE.

The reason of the above phenomenon is that: on one hand, when $t_{\max}$ is small, the diversity of DREA will be reduced due to the fewer times of population re-initialization. On the other hand, when $t_{\max}$ is large, $t_{\text{EA}_{\max}}$ becomes small as shown in line 2, Algorithm 1, and hence, the search ability of EA will be reduced. Thus, the moderate values of $t_{\max}$ for DR-JADE are promising choices.

C. On the Effectiveness of DR-JADE in Benchmark NES Models

In the previous sections, the performance of DR-JADE is verified through 42 NES problems with known roots. To further evaluate the effectiveness of our approach, in this section, four benchmark NES models with unknown roots are used. These four models, which are widely used in the literature [37], [30], [32], are: i) chemical equilibrium application model, ii) neurophysiology application model, iii) combustion theory application model, and iv) economics modeling system. More details for these models can be found in the supplementary material (F43 - F44). For DR-JADE, the $NEFs_{\max} = 200,000$, which is the same as used in [32]. All other parameter settings are kept unchanged.

The results of DR-JADE are reported in Table XI. Because the roots of these four models are unknown, the RR and SR criteria cannot be used. In Table XI, the number of obtained roots of DR-JADE and the values of the objective function for DR-JADE, GA-SQP-1, and PSO-NM-1[10] are shown. Note that, since DR-JADE is able to obtain multiple roots in each run, we only report the values of the objective function for DR-JADE in a typical run. The obtained roots for F43 - F46 are given in Tables S-R-XV - S-R-VIII in the supplementary material, respectively.

[10]The results of GA-SQP-1 and PSO-NM-1 are borrowed from their original literature in [30] and [32], respectively. The mean values only convert into $f(\mathbf{x}) = \sum_{i=1}^{n} e_i^2(\mathbf{x})$ to make a fair comparison with the results of DR-JADE.

From the results in Table XI, it can be seen that

- DR-JADE is capable of locating multiple roots for these models in each single run.
- With respect to the values of the objective function $f(\mathbf{x})$, the mean values of DR-JADE are worse than GA-SQP-1 and PSO-NM-1 in the four models. However, it is worth emphasizing that the aim of DR-JADE is to locate multiple roots of NESs in a single run simultaneously, therefore, the population re-initialization is used to maintain the diversity of the population. In this way, this may result in sacrificing the quality of the roots more or less. Whereas GA-SQP-1 and PSO-NM-1 are only to find one root, thus they can obtain better values of $f(\mathbf{x})$. By carefully checking the best values of $f(\mathbf{x})$ obtained by DR-JADE, they are less than 10^{-15} for the four models, which means that our approach is also able to get the high quality values of $f(\mathbf{x})$ in some roots in a single run.
- In general, from the results it can be seen that DR-JADE can not only locate multiple roots simultaneously in a single run, but obtains acceptable quality of the roots.

VI. CONCLUSIONS AND FUTURE WORK

In this paper, to remedy the drawback of traditional repulsion techniques that need to set the repulsive radius by the user, we propose the dynamic repulsion technique to dynamically control the repulsive radius during the run. Based on the dynamic repulsion technique and EA, a general framework, DREA, is developed to locate multiple roots of NESs in a single run simultaneously. To evaluate the effectiveness of our approach, five EAs and three repulsion techniques are integrated into DREA. In addition, 42 NESs are chosen from the literature as the test suite. Experimental results verify our expectation that the proposed dynamic repulsion technique is effective and it can improve the performance of the static repulsion technique. Furthermore, different EAs can be readily used in DREA to simultaneously find multiple roots of NESs in a single run, such as DR-JADE, DR-SHADE. Therefore, they can be an effective alternative to solve NESs.

The optimization algorithms in DREA play an important role. In the near future, we will try to develop other powerful EAs for NESs. Additionally, the use of the proposed method to solve the complex real-world NES problems, such as hypersurface exploration of the potential energy landscape [55], simulation of nonlinear resistive circuits [56], parameter estimation for nonlinear signal processing systems [57], [58], [59], time-varying NESs [60], [27], is another possible future direction.

The source code can be downloaded from Dr. Gong's homepage at: http://www.escience.cn/people/wygong.

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Supplementary Material for "Solving Nonlinear Equations System with Dynamic Repulsion-based Evolutionary Algorithms"

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S-I. Test Problems

1) F01:

$$\begin{cases} x_1^2 - x_2^2 = 0 \\ 1 - |x_1 - x_2| = 0 \end{cases} \tag{1}$$

where $x_i \in [-3, 3]$, $i = 1, \cdots, D$, and $D = 2$. It has two roots: (-0.5, 0.5) and (0.5, -0.5) [1].

2) F02:

$$\begin{cases} x_1^2 - x_2 - 2 = 0 \\ x_1 + \sin(\frac{\pi x_2}{2}) = 0 \end{cases} \tag{2}$$

where $x_i \in [-3, 3]$, $i = 1, \cdots, D$, and $D = 2$. It has three roots: (1, -1), (0, -2), and (0.7075, -1.5) [1].

3) F03:

$$\begin{cases} x_1 x_2 + (x_1 - 2x_3)(x_2 - 2x_3) - 165 = 0 \\ \frac{(x_1 x_2^3)}{12} - \frac{(x_1 - 2x_3)(x_2 - 2x_3)^3}{12} - 9369 = 0 \\ \frac{2(x_2 - x_3)^2 (x_1 - x_3)^2 x_3}{x_1 + x_2 - 2x_3} - 6835 = 0 \end{cases} \tag{3}$$

where $x_i \in [0, 50]$, $i = 1, \cdots, D$, and $D = 3$. It has two roots: (43.155566, 10.128950, 12.944048) and (7.602995, 24.541982, 11.576716) [1].

4) F04:

$$\begin{cases} x_1 + x_2 - 3 = 0 \\ x_1^2 + x_2^2 - 9 = 0 \end{cases} \tag{4}$$

where $x_i \in [-3, 3]$, $i = 1, \cdots, D$, and $D = 2$. It has two roots: (0, 3) and (3, 0) [1].

5) F05:

$$\begin{cases} x_1 - \sin(2x_1 + 3x_2) - \cos(3x_1 - 5x_2) = 0 \\ x_2 - \sin(x_1 - 2x_2) + \cos(x_1 + 3x_2) = 0 \end{cases} \tag{5}$$

where $x_i \in [-3, 3]$, $i = 1, \cdots, D$, and $D = 2$. It has three roots: (-0.173346, -0.256091), (0.838835, 0.537119) and (0.792693, 0.138017) [1].

6) F06:

$$\begin{cases} e^{x_1^2} - 8x_1 \sin(x_2) = 0 \\ x_1 + x_2 - 1 = 0 \\ (x_3 - 1)^3 = 0 \end{cases} \tag{6}$$

where $x_i \in [0, 1]$, $i = 1, \cdots, D$, and $D = 3$. It has two roots: (0.673584, 0.326416, 1) and (0, 1, 1) [1].

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7) F07:

$$\begin{cases} x_1^3 - 3x_1 x_2^2 - 1 = 0 \\ 3x_1^2 x_2 - x_2^3 + 1 = 0 \end{cases} \tag{7}$$

where $x_i \in [-1, 2]$, $i = 1, \cdots, D$, and $D = 2$. It has three roots: (-0.290515, 1.084215), (1.084215, -0.290515) and (-0.793701, -0.793701) [1].

8) F08:

$$\begin{cases} x_1^2 + x_2^2 - 1 = 0 \\ x_1 - x_2 = 0 \end{cases} \tag{8}$$

where $x_i \in [-1, 1]$, $i = 1, \cdots, D$, and $D = 2$. It has two roots: (-0.707107, -0.707107) and (0.707107, 0.707107) [2].

9) F09:

$$\begin{cases} \sum_{i=1}^{D} x_i^2 - 1 = 0 \\ |x_1 - x_2| + \sum_{i=3}^{D} x_i^2 = 0 \end{cases} \tag{9}$$

where $x_i \in [-1, 1]$, $i = 1, \cdots, D$, and $D = 20$. It has two roots: (-0.707107, -0.707107, 0, $\cdots$, 0) and (0.707107, 0.707107, 0, $\cdots$, 0) [2].

10) F10:

$$\begin{cases} x_1 - \sin(5\pi x_2) = 0 \\ x_1 - x_2 = 0 \end{cases} \tag{10}$$

where $x_i \in [-1, 1]$, $i = 1, \cdots, D$, and $D = 2$. It has 11 roots as shown in Table S-I [2].

TABLE S-I
THE ROOTS OF F10.

x_1	x_2
-0.924840	-0.924840
-0.866760	-0.866760
-0.562010	-0.562010
-0.428168	-0.428168
-0.187960	-0.187960
0.000000	0.000000
0.187960	0.187960
0.428168	0.428168
0.562010	0.562010
0.866760	0.866760
0.924840	0.924840

11) F11:

$$\begin{cases} x_1 - \cos(4\pi x_2) = 0 \\ x_1^2 + x_2^2 - 1 = 0 \end{cases} \tag{11}$$

where $x_i \in [-1, 1]$, $i = 1, \cdots, D$, and $D = 2$. It has 15 roots as shown in Table S-II [2].

12) F12:

$$\begin{cases} \cos(2x_1) - \cos(2x_2) - 0.4 = 0 \\ 2(x_2 - x_1) + \sin(2x_2) - \sin(2x_1) - 1.2 = 0 \end{cases} \tag{12}$$

where $x_i \in [-10, 10]$, $i = 1, \cdots, D$, and $D = 2$. It has 13 roots as shown in Table S-III [3].

TABLE S-II
THE ROOTS OF F11.

x_1	x_2
0.416408	-0.909178
-0.561364	-0.827569
-0.724322	-0.689462
0.837812	-0.545959
0.886984	-0.461799
-0.962322	-0.271914
-0.972855	-0.231415
1.000000	0.000000
-0.972855	0.231416
-0.962322	0.271914
0.886984	0.461799
0.837812	0.545959
-0.724322	0.689462
-0.561364	0.827569
0.416408	0.909178

TABLE S-III
THE ROOTS OF F12.

x_1	x_2
-9.268258	-8.931402
-8.744542	-7.164787
-6.126665	-5.789809
-5.602950	-4.023195
-2.985073	-2.648216
-2.461357	-0.881602
0.156520	0.493376
0.680236	2.259991
3.298113	3.634969
3.821828	5.401583
6.439705	6.776562
6.963421	8.543176
9.581298	9.918154

13) F13:

$$\begin{cases} x_1 - 0.25428722 - 0.18324757 x_4 x_3 x_9 = 0 \\ x_2 - 0.37842197 - 0.16275449 x_1 x_{10} x_6 = 0 \\ x_3 - 0.27162577 - 0.16955071 x_1 x_2 x_{10} = 0 \\ x_4 - 0.19807914 - 0.15585316 x_7 x_1 x_6 = 0 \\ x_5 - 0.44166728 - 0.19950920 x_7 x_6 x_3 = 0 \\ x_6 - 0.14654113 - 0.18922793 x_8 x_5 x_{10} = 0 \\ x_7 - 0.42937161 - 0.21180486 x_2 x_5 x_8 = 0 \\ x_8 - 0.07056438 - 0.17081208 x_1 x_7 x_6 = 0 \\ x_9 - 0.34504906 - 0.19612740 x_{10} x_6 x_8 = 0 \\ x_{10} - 0.42651102 - 0.21466544 x_4 x_8 x_1 = 0 \end{cases} \tag{13}$$

where $x_i \in [-2, 2]$, $i = 1, \cdots, D$, and $D = 10$. It has one root: (0.257833, 0.381097, 0.278745, 0.200669, 0.445251, 0.149184, 0.432010, 0.073403, 0.345967, 0.427326) [3].

14) F14:

$$\begin{cases} 100(x_1 - 0.25) = 0 \\ 100(x_1 \sin(4\pi x_2^2) + 0.75 x_1 - 0.25) = 0 \end{cases} \tag{14}$$

where $x_i \in [-1, 1]$, $i = 1, \cdots, D$, and $D = 2$. It has eight roots as shown in Table S-IV [4].

TABLE S-IV
THE ROOTS OF F14.

x_1	x_2
0.250000	-0.854337
0.250000	-0.721185
0.250000	-0.479471
0.250000	-0.141801
0.250000	0.141801
0.250000	0.479471
0.250000	0.721185
0.250000	0.854337

15) F15:

$$\begin{cases} 3.0 - x_1 x_3^2 = 0 \\ x_3 \sin(\pi / x_2) - x_3 - x_4 = 0 \\ -x_2 x_3 \exp(1.0 - x_1 x_3) + 0.2707 = 0 \\ 2x_1^2 x_3 - x_2^4 x_3 - x_2 = 0 \end{cases} \tag{15}$$

where $x_i \in [0, 5]$, $i = 1, \cdots, D$, and $D = 4$. It has one root: (3, 2, 1, 0) [5].

16) F16:

$$\begin{cases} (1-R)\left[\left(\frac{H}{10(1+\beta_1)} - x_1\right) \cdot \exp\left(\frac{10x_1}{1+\frac{10x_1}{\gamma}}\right)\right] - x_1 = 0 \\ (1-R)\left[\left(\frac{H}{10} - \beta_1 x_1 - (1+\beta_2)x_2\right) \cdot \exp\left(\frac{10x_2}{1+\frac{10x_2}{\gamma}}\right)\right] + x_1 - (1+\beta_2)x_2 = 0 \end{cases} \tag{16}$$

where $x_i \in [0, 1]$, $i = 1, \cdots, D$, $D = 2$, $R = 0.96$, $H = 22$, $\gamma = 1000$, and $\beta_1 = \beta_2 = 2$. It has seven roots as shown in Table S-V [5], [6], [7], [8].

TABLE S-V
THE ROOTS OF F16.

x_1	x_2
0.042100	0.061813
0.042100	0.268723
0.266600	0.178430
0.266600	0.327267
0.266600	0.461111
0.042318	0.686779
0.719074	0.244197

17) F17:

$$\begin{cases} 2x_1 + x_2 + x_3 + x_4 + x_5 - 6.0 = 0 \\ x_1 + 2x_2 + x_3 + x_4 + x_5 - 6.0 = 0 \\ x_1 + x_2 + 2x_3 + x_4 + x_5 - 6.0 = 0 \\ x_1 + x_2 + x_3 + 2x_4 + x_5 - 6.0 = 0 \\ x_1 x_2 x_3 x_4 x_5 - 1.0 = 0 \end{cases} \tag{17}$$

where $x_i \in [-10, 10]$, $i = 1, \cdots, D$, and $D = 5$. It has three roots: (1, 1, 1, 1, 1), (0.916355, 0.916355, 0.916355, 0.916355, 1.418227), and (-0.579043, -0.579043, -0.579043, -0.579043, 8.895215) [9], [10].

18) F18:

$$\begin{cases} x_1 + x_2^4 x_4 x_6 / 4 + 0.75 = 0 \\ x_2 + 0.405 \exp(1 + x_1 x_2) - 1.405 = 0 \\ x_3 - x_4 x_6 / 2 + 1.5 = 0 \\ x_4 - 0.605 \exp(1 - x_3^2) - 0.395 = 0 \\ x_5 - x_2 x_6 / 2 + 1.5 = 0 \\ x_6 - x_1 x_5 = 0 \end{cases} \tag{18}$$

where $x_i \in [-1, 1]$, $i = 1, \cdots, D$, and $D = 6$. It has one root: (-1, 1, -1, 1, -1, 1) [10], [11].

19) F19:

$$\begin{cases} \sin(x_1^3) - 3x_1 x_2^2 - 1 = 0 \\ \cos(3x_1^2 x_2) - |x_2^3| + 1 = 0 \end{cases} \tag{19}$$

where $x_i \in [-2, 2]$, $i = 1, \cdots, D$, $D = 2$. It has 10 roots as shown in Table S-VI. This problem is modified from [12].

TABLE S-VI
THE ROOTS OF F19.

x_1	x_2
-1.810885	-0.349092
-1.810885	0.349092
-1.502221	-0.409077
-1.502221	0.409077
-1.791302	0.301926
-1.791302	-0.301926
-0.947268	0.785020
-0.947268	-0.785020
-0.213057	1.256845
-0.213057	-1.256845

20) F20:

$$\begin{cases} 4x_1^3 + 4x_1x_2 + 2x_2^2 - 42x_1 - 14 = 0 \\ 4x_2^3 + 2x_1^2 + 4x_1x_2 - 26x_2 - 22 = 0 \end{cases} \tag{20}$$

where $x_i \in [-5, 5]$, $i = 1, \cdots, D$, and $D = 2$. It has 9 roots as shown in Table S-VII [13], [14].

TABLE S-VII
THE ROOTS OF F20.

x_1	x_2
-0.127961	-1.953715
-0.270845	-0.923039
0.086678	2.884255
3.385154	0.073852
3.584428	-1.848127
3.000000	2.000000
-3.779310	-3.283186
-3.073026	-0.081353
-2.805118	3.131313

21) F21:

$$\begin{cases} -\sin(x_1)\cos(x_2) - 2\cos(x_1)\sin(x_2) = 0 \\ -\cos(x_1)\sin(x_2) - 2\sin(x_1)\cos(x_2) = 0 \end{cases} \tag{21}$$

where $x_i \in [0, 2\pi]$, $i = 1, \cdots, D$, and $D = 2$. It has 13 roots as shown in Table S-VIII [7], [13].

TABLE S-VIII
THE ROOTS OF F21.

x_1	x_2
0.000000	0.000000
3.141593	0.000000
1.570796	1.570796
6.283185	0.000000
0.000000	3.141593
4.712389	1.570796
3.141593	3.141593
1.570796	4.712389
6.283185	3.141593
0.000000	6.283185
4.712389	4.712389
3.141593	6.283185
6.283185	6.283185

22) F22:

$$\begin{cases} \begin{aligned} x_1^2 + x_2^2 - 1.0 &= 0 \\ x_3^2 + x_4^2 - 1.0 &= 0 \\ x_5^2 + x_6^2 - 1.0 &= 0 \\ x_7^2 + x_8^2 - 1.0 &= 0 \\ 4.731 \cdot 10^{-3} x_1 x_3 - 0.3578 x_2 x_3 - 0.1238 x_1 + x_7 & \\ -1.637 \cdot 10^{-3} x_2 - 0.9338 x_4 - 0.3571 &= 0 \\ 0.2238 x_1 x_3 + 0.7623 x_2 x_3 + 0.2638 x_1 - x_7 & \\ -0.07745 x_2 - 0.6734 x_4 - 0.6022 &= 0 \\ x_6 x_8 + 0.3578 x_1 + 4.731 \cdot 10^{-3} x_2 &= 0 \\ -0.7623 x_1 + 0.2238 x_2 + 0.3461 &= 0 \end{aligned} \end{cases} \tag{22}$$

where $x_i \in [-1, 1]$, $i = 1, \cdots, D$, and $D = 8$. It has 16 roots as shown in Table S-IX [9], [13], [14].

TABLE S-IX
THE ROOTS OF F22.

x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8
0.1644	-0.9864	-0.9471	-0.3210	-0.9982	-0.0594	0.4110	0.9116
0.1644	-0.9864	-0.9471	-0.3210	-0.9982	0.0594	0.4110	-0.9116
0.1644	-0.9864	-0.9471	-0.3210	0.9982	-0.0594	0.4110	0.9116
0.1644	-0.9864	-0.9471	-0.3210	0.9982	0.0594	0.4110	-0.9116
0.1644	-0.9864	0.7185	-0.6956	-0.9980	-0.0638	-0.5278	0.8494
0.1644	-0.9864	0.7185	-0.6956	-0.9980	0.0638	-0.5278	-0.8494
0.1644	-0.9864	0.7185	-0.6956	0.9980	-0.0638	-0.5278	0.8494
0.1644	-0.9864	0.7185	-0.6956	0.9980	0.0638	-0.5278	-0.8494
0.6716	0.7410	-0.6516	-0.7586	-0.9625	-0.2711	-0.4376	0.8992
0.6716	0.7410	-0.6516	-0.7586	-0.9625	0.2711	-0.4376	-0.8992
0.6716	0.7410	-0.6516	-0.7586	0.9625	-0.2711	-0.4376	0.8992
0.6716	0.7410	-0.6516	-0.7586	0.9625	0.2711	-0.4376	-0.8992
0.6716	0.7410	0.9519	-0.3064	-0.9638	-0.2666	0.4046	0.9145
0.6716	0.7410	0.9519	-0.3064	-0.9638	0.2666	0.4046	-0.9145
0.6716	0.7410	0.9519	-0.3064	0.9638	0.2666	0.4046	-0.9145
0.6716	0.7410	0.9519	-0.3064	0.9638	-0.2666	0.4046	0.9145

23) F23:

$$\begin{cases} 4x_1^3 - 3x_1 - \cos(x_2) = 0 \\ \sin(x_1^2) - |x_2| = 0 \end{cases} \tag{23}$$

where $x_i \in [-2, 2]$, $i = 1, \cdots, D$, and $D = 2$. It has 6 roots as shown in Table S-X. This problem is modified from [15].

TABLE S-X
THE ROOTS OF F23.

x_1	x_2
-0.597167	-0.349098
-0.597167	0.349098
-0.442758	-0.194781
-0.442758	0.194781
0.964499	-0.801774
0.964499	0.801774

24) F24:

$$\begin{cases} x_i + \sum_{j=1}^{D} x_j - (D+1) = 0 \quad i = 1, \cdots, D-1 \\ \left[\prod_{j=1}^{D} x_j\right] - 1 = 0 \end{cases} \tag{24}$$

where $x_i \in [-2, 2]$, $i = 1, \cdots, D$, and $D = 20$. It has two roots: (1, $\cdots$, 1) and (0.994922, $\cdots$, 0.994922, 1.101551) [15].

25) F25:

$$x_i - \cos\left(2x_i - \sum_{j=1}^{D} x_j\right) = 0, \quad i = 1, \cdots, D \tag{25}$$

where $x_i \in [-1, 1]$, $i = 1, \cdots, D$, and $D = 3$. It has 7 roots as shown in Table S-XI [16].

TABLE S-XI
THE ROOTS OF F25

x_1	x_2	x_3
0.810561	0.810561	-0.625687
0.810561	-0.625687	0.810561
-0.625687	0.810561	0.810561
0.543850	0.995778	0.543850
0.543850	0.543850	0.995778
0.995778	0.543850	0.543850
0.739086	0.739086	0.739086

26) F26:

$$\begin{cases} x_1^2 + x_2^2 - 2 = 0 \\ x_1^2 + \frac{x_2^2}{4} - 1 = 0 \end{cases} \tag{26}$$

where $x_i \in [-2, 2]$, $i = 1, \cdots, D$, and $D = 2$. It has 4 roots as shown in Table S-XII. This problem is modified from [17].

TABLE S-XII
THE ROOTS OF F26.

x_1	x_2
-0.816497	-1.154701
0.816497	-1.154701
-0.816497	1.154701
0.816497	1.154701

27) F27:

$$\begin{cases} \exp\left(x_1^2 + x_2^2\right) - 3 = 0 \\ |x_2| + x_1 - \sin\left(3(|x_2| + x_1)\right) = 0 \end{cases} \tag{27}$$

where $x_i \in [-2, 2]$, $i = 1, \cdots, D$, and $D = 2$. It has 6 roots as shown in Table S-XIII. This problem is modified from [17].

TABLE S-XIII
THE ROOTS OF F27.

x_1	x_2
-0.741152	-0.741152
-0.741152	0.741152
-0.256625	1.016246
-0.256625	-1.016246
-1.016246	-0.256625
-1.016246	0.256625

28) F28:

$$\begin{cases} -3.84x_1^2 + 3.84x_1 - x_2 = 0 \\ -3.84x_2^2 + 3.84x_2 - x_3 = 0 \\ -3.84x_3^2 + 3.84x_3 - x_1 = 0 \end{cases} \tag{28}$$

where $x_i \in [0, 1]$, $i = 1, \cdots, D$, and $D = 3$. It has 8 roots as shown in Table S-XIV [18].

TABLE S-XIV
THE ROOTS OF F28.

x_1	x_2	x_3
0.000000	0.000000	0.000000
0.488122	0.959435	0.149452
0.540304	0.953754	0.169399
0.959447	0.149373	0.487917
0.149440	0.488092	0.959440
0.953781	0.169343	0.540157
0.169254	0.539937	0.953788
0.739584	0.739584	0.739574

29) F29:

$$\begin{cases} 3x_1^2 + \sin(x_1x_2) - x_3^2 + 2.0 = 0 \\ 2x_1^3 - x_2^2 - x_3 + 3.0 = 0 \\ \sin(2x_1) + \cos(x_2x_3) + x_2 - 1.0 = 0 \end{cases} \tag{29}$$

where $x_1 \in [-5, 5]$, $x_2 \in [-1, 3]$, and $x_3 \in [-5, 5]$. It has two roots: (-0.064417, 2.090440, -1.370473) and (-0.032759, 1.264629, 1.400644) [19].

30) F30:

$$\begin{cases} 5x_1^9 - 6x_1^5x_2^2 + x_1x_2^4 + 2x_1x_3 = 0 \\ -2x_1^6x_2 + 2x_1^2x_2^3 + 2x_2x_3 = 0 \\ x_1^2 + x_2^2 - 0.265625 = 0 \end{cases} \tag{30}$$

where $x_1 \in [-0.6, 6]$, $x_2 \in [-0.6, 0.6]$, and $x_3 \in [-5, 5]$. It has 12 roots as shown in Table S-XV [15], [13].

31) F31:

$$\begin{cases} x_1^2 - x_2 - 2 = 0 \\ x_1 + \sin\left(\frac{\pi}{2}x_2\right) = 0 \end{cases} \tag{31}$$

where $x_1 \in [0, 1]$ and $x_2 \in [-10, 0]$. It has two roots: (0, -2) and (0.707660, -1.5) [5].

TABLE S-XV
THE ROOTS OF F30.

x_1	x_2	x_3
0.279855	0.432789	-0.014189
0.279855	-0.432789	-0.014189
-0.279855	0.432789	-0.014189
-0.279855	-0.432789	-0.014189
0.466980	0.218070	0.000000
-0.466980	0.218070	0.000000
0.466980	-0.218070	0.000000
-0.466980	-0.218070	0.000000
0.000000	0.515388	0.000000
0.000000	-0.515388	0.000000
0.515388	0.000000	-0.012446
-0.515388	0.000000	-0.012446

32) F32:

$$\begin{cases} x_1^2 + x_2^2 + x_1 + x_2 - 8 = 0 \\ x_1|x_2| + x_1 + |x_2| - 5 = 0 \end{cases} \tag{32}$$

where $x_1 \in [0, 2.5]$ and $x_2 \in [-4, 6]$. It has 4 roots: (0.404634, -3.271577), (2.403604, -0.762837), (1, 2), and (2, 1). This problem is modified from [20].

33) F33:

$$\begin{cases} x_1^2 - |x_2| + 1 + \frac{1}{9}|x_1 - 1| = 0 \\ x_2^2 + 5x_1^2 - 7 + \frac{1}{9}|x_2| = 0 \end{cases} \tag{33}$$

where $x_1 \in [-1, 1]$ and $x_2 \in [-10, 10]$. It has 4 roots: (-0.814326, -1.864719), (0.861828, -1.758100), (-0.814326, 1.864719), and (0.861828, 1.758100). This problem is modified from [20].

34) F34:

$$\begin{cases} 0.5\sin(x_1x_2) - \frac{0.25}{\pi}x_2 - 0.5x_1 = 0 \\ \left(1 - \frac{0.25}{\pi}\right)\left[\exp(2x_1) - e\right] + \frac{e}{\pi}x_2 - 2ex_1 = 0 \end{cases} \tag{34}$$

where $x_1 \in [0.25, 1]$ and $x_2 \in [1.5, 2\pi]$. It has two roots: (0.299465, 2.836948) and (0.499966, 3.141589) [14], [21].

35) F35:

$$\begin{cases} x_1^{x_2} + x_2^{x_1} - 5x_1x_2x_3 - 85 = 0 \\ x_1^3 - x_2^{x_3} - x_3^{x_2} - 60 = 0 \\ x_1^{x_3} + x_3^{x_1} - x_2 - 2 = 0 \end{cases} \tag{35}$$

where $x_1 \in [3, 5]$, $x_2 \in [2, 4]$, and $x_3 \in [0.5, 2]$. It has one root: (4, 3, 1) [22].

36) F36:

$$\begin{cases} x_1^3 - 3x_1x_2^2 - 1 = 0 \\ 3x_1^2x_2 - x_2^3 + 1 = 0 \end{cases} \tag{36}$$

where $x_1 \in [-1, -0.1]$ and $x_2 \in [-2, 2]$. It has two roots: (-0.793701, -0.793701) and (-0.290515, 1.084215) [11].

37) F37:

$$\begin{cases} 0.1x_1 + \cos(2x_2) + 0.09240 = 0 \\ \sin(3x_3) + \sin\left(\frac{10x_1}{3}\right) + \log(2x_2) - 2.52x_3 + 0.08805 = 0 \\ 2(x_1 - 0.75)^2 + \sin\left(16\pi x_2 - \frac{\pi}{2}\right) - 3.26815 = 0 \end{cases} \tag{37}$$

where $x_1 \in [1, 2.5]$, $x_2 \in [0.2, 2]$, and $x_3 \in [0.1, 3]$. It has one root: (1.852100, 0.926050, 0.617370) [23].

38) F38:

$$\begin{cases} 4x_1^3 - 3x_1 - x_2 = 0 \\ x_1^2 - x_2 = 0 \end{cases} \tag{38}$$

where $x_1 \in [-5, 1.5]$ and $x_2 \in [0, 5]$. It has three roots: (-0.75, 0.5625), (0, 0), and (1, 1) [15].

39) F39:

$$\begin{cases} x_1^3 - 3x_1x_2^2 + a_1(2x_1^2 + x_1x_2) + b_1x_2^2 + c_1x_1 + a_2x_2 = 0 \\ 3x_1^2x_2 - x_2^3 - a_1(4x_1x_2 - x_2^2) + b_2x_1^2 + c_2 = 0 \end{cases} \quad (39)$$

where $a_1 = 25, b_1 = 1, c_1 = 2, a_2 = 3, b_2 = 4, c_2 = 5$, $x_1 \in [0, 2]$, and $x_2 \in [10, 30]$. It has two roots: (1.6359718, 13.8476653) and (0.6277425, 22.2444123) [17].

40) F40:

$$\begin{cases} x_1^2 - x_1 - x_2^2 - x_2 + x_3^2 = 0 \\ \sin(x_2 - \exp(x_1)) = 0 \\ x_3 - \log|x_2| = 0 \end{cases} \quad (40)$$

where $x_1 \in [0, 2]$, $x_2 \in [-10, 10]$, and $x_3 \in [-1, 1]$. It has 5 roots shown in Table S-XVI. This problem is modified from [24].

TABLE S-XVI
THE ROOTS OF F40.

x_1	x_2	x_3
0.825297	-0.859034	-0.151946
1.299490	0.525835	-0.642769
1.533662	-1.648068	0.499604
1.981360	-2.172180	0.775731
1.983283	0.983378	-0.016762

41) F41:

$$\begin{cases} x_1^4 + 4x_2^4 - 6.0 = 0 \\ x_1^2x_2 - 0.6787 = 0 \end{cases} \quad (41)$$

where $x_1 \in [-2, 2]$ and $x_2 \in [0, 1.1]$. It has 4 roots: (-1.563533, 0.277628), (-0.789706, 1.088295), (1.563533, 0.277628), and (0.789706, 1.088295). This problem is modified from [25].

42) F42:

$$\begin{cases} (0.25/\pi)x_2 + 0.5x_1 - 0.5\sin(x_1x_2) = 0 \\ (e/\pi)x_2 - 2ex_1 + (1 - 0.25/\pi)(e^{2x_1} - e) = 0 \end{cases} \quad (42)$$

where $x_1 \in [0.25, 1]$ and $x_2 \in [1.5, 2\pi]$. It has two roots: (0.5, π), and (0.2995, 2.8369) [1].

43) F43 - Chemical equilibrium application model:

$$\begin{cases} x_1x_2 + x_1 - 3x_5 = 0 \\ 2x_1x_2 + x_1 + x_2x_3^2 + R_8x_2 - Rx_5 + \\ \quad 2R_{10}x_2^2 + R_7x_2x_3 + R_9x_2x_4 = 0 \\ 2x_2x_3^2 + 2R_5x_3^2 - 8x_5 + R_6x_3 + R_7x_2x_3 = 0 \\ R_9x_2x_4 + 2x_4^2 - 4Rx_5 = 0 \\ x_1(x_2 + 1) + R_{10}x_2^2 + x_2x_3^2 + R_8x_2 + \\ \quad R_5x_3^2 + x_4^2 - 1 + R_6x_3 + R_7x_2x_3 + R_9x_2x_4 = 0 \end{cases} \quad (43)$$

where $x_1, x_5 \in [0, 1]$, $x_2 \in [0, 60]$, $x_3, x_4 \in [-1, 1]$, and $R = 10.0$, $R_5 = 0.193$, $R_6 = 0.002597/\sqrt{40}$, $R_7 = 0.003448/\sqrt{40}$, $R_8 = 0.00001799/40$, $R_9 = 0.0002155/\sqrt{40}$, $R_{10} = 0.00003846/40$ [3], [26].

44) F44 - Neurophysiology application model:

$$\begin{cases} x_1^2 + x_3^2 - 1 = 0 \\ x_2^2 + x_4^2 - 1 = 0 \\ x_5x_3^3 + x_6x_4^3 - c_1 = 0 \\ x_5x_1^3 + x_6x_2^3 - c_2 = 0 \\ x_5x_1x_3^2 + x_6x_2x_4^2 - c_3 = 0 \\ x_5x_3x_1^2 + x_6x_4x_2^2 - c_4 = 0 \end{cases} \quad (44)$$

where $c_j = 0, j = 1, \cdots, 4$, $x_i \in [-10, 10]$, $i = 1, \cdots, n$, and $n = 6$ [3], [26].

45) F45 - Combustion theory application model:

$$\begin{cases} x_2 + 2x_6 + x_9 + 2x_{10} - 10^{-5} = 0 \\ x_3 + x_8 - 3 \cdot 10^{-5} = 0 \\ x_1 + x_3 + 2x_5 + 2x_8 + x_9 + x_{10} - 5 \cdot 10^{-5} = 0 \\ x_4 + 2x_7 - 10^{-5} = 0 \\ 0.5140437 \cdot 10^{-7}x_5 - x_1^2 = 0 \\ 0.1006932 \cdot 10^{-6}x_6 - 2x_2^2 = 0 \\ 0.7816278 \cdot 10^{-15}x_7 - x_4^2 = 0 \\ 0.1496236 \cdot 10^{-6}x_8 - x_1x_3 = 0 \\ 0.6194411 \cdot 10^{-7}x_9 - x_1x_2 = 0 \\ 0.2089296 \cdot 10^{-14}x_{10} - x_1x_2^2 = 0 \end{cases} \quad (45)$$

where $x_i \in [-1, 1]$, $i = 1, \cdots, n$, and $n = 10$ [3], [26].

46) F46 - Economics modeling system:

$$\begin{cases} (x_k + \sum_{i=1}^{n-k-1} x_ix_{i+k})x_n - c_k = 0 \quad 1 \le k \le n-1 \\ \sum_{i=1}^{n-1} x_i + 1 = 0 \end{cases} \quad (46)$$

where $x_i \in [-10, 10]$, $i = 1, \cdots, n$, $n = 5$, $c_k = 0$, and $k = 1, \cdots, n-1$ [3], [26].

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S-II. Pseudo Code of DR-JADE

Algorithm 1: The pseudo code of DR-JADE

Input: Control parameters: μ, $t_{\max}$, $NFEs_{\max}$
Output: The final archive $\mathcal{A}$

```
Set t = 0 and s_A = 0;
t_EAmax = ⌊ NFEs_max / (μ × t_max) ⌋;
while t < t_max do
    Calculate λ_t and γ_t;
    Set u_CR = 0.5, u_F = 0.5;
    Randomly generate an initial population P;
    Calculate f(x_i) and R(x_i) of each individual x_i in P;
    Update the archive A via each individual x_i in P;
    Set t_EA = 1;
    while t_EA < t_EAmax do
        Set S_CR = φ, S_F = φ;
        for i = 1 to μ do
            Generate CR_i and F_i by the Gaussian and Cauchy random generators, respectively;
            Generate the offspring u_i using operators in JADE;
            Calculate f(u_i) of the offspring u_i;
            if s_A ≠ 0 then
                Calculate R(x_i) and R(u_i) via the repulsion technique;
            Update the archive A using u_i;
        for i = 1 to μ do
            if R(u_i) ≤ R(x_i) then
                Replace x_i with u_i;
                CR_i → S_CR; F_i → S_F;
        Update the u_CR and u_F;
        t_EA = t_EA + 1;
    t = t + 1;
```

S-III. Parameter Settings

TABLE S-R-I
Parameter settings for different methods. Note that all other parameters used in different EAs are kept the same as used in their original literature.

Method	Parameter settings
DR-$*$, R-$*$	$\mu = 10, t_{\max} = 30$
$*$: JADE	$u_{CR} = 0.5, u_F = 0.5, c = 0.1$
$*$: jDE	$\tau_1 = 0.1, \tau_2 = 0.1$
$*$: SHADE	$H_m = \mu$
$*$: SaDE	$\mathrm{CRm}_k = 0.5, LP = 50$
$*$: CLPSO	$m = 7, c = 2.0$
A-WeB	$\mu = 100, H_m = \mu$
MONES	$\mu = 100, H_m = \mu$
I-HS	$\mu = 10, \mathrm{HMCR} = 0.95, \mathrm{PAR}_{\min} = 0.35,$ $\mathrm{PAR}_{\max} = 0.99, \mathrm{BW}_{\min} = 10^{-6}, \mathrm{BW}_{\max} = 5$
NCDE	$\mu = 100, F = 0.9, CR = 0.1$
NSDE	$\mu = 100, F = 0.9, CR = 0.1$
GA-SQP	$\mu = 100$
PSO-NM	$\mu = 25, c_1 = c_2 = 2.5, w_{\max} = 0.9, w_{\min} = 0.4$
NCSA	$\mu = 30, P_\alpha = 0.25$

S-IV. SUPPLEMENTAL RESULTS

TABLE S-R-II
INFLUENCE OF DIFFERENT SETTINGS OF STATIC REPULSIVE RADIUS IN R-JADE WITH RESPECT TO THE ROOT RATIO.

Prob.	δ=0.01	δ=0.02	δ=0.03	δ=0.04	δ=0.05	δ=0.06	δ=0.07	δ=0.08	δ=0.09	δ=0.1	δ=0.2	δ=0.3	δ=0.4	δ=0.5
F01	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F02	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	0.9889	0.9556	0.9111	0.9000	0.9333
F03	0.7833	0.7333	**0.9000**	0.8000	0.8167	0.7667	0.7833	0.8000	0.7167	0.8000	0.7167	0.7833	0.7833	0.7833
F04	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F05	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	0.7778	0.6778	0.6889	0.6667	0.6556	0.6111	0.7778	0.7778
F06	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F07	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F08	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F09	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
F10	0.9939	**1.0000**	**1.0000**	**1.0000**	0.8182	0.8182	0.8212	0.8182	0.8182	0.6364	0.4818	0.4000	0.3879	0.5576
F11	0.9756	**0.9956**	0.8667	0.8667	0.7333	0.7333	0.7333	0.7333	0.7333	0.7333	0.6000	0.4089	0.3267	0.8711
F12	0.9795	0.9949	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	0.8513	**1.0000**	0.9949	0.9923	0.9949
F13	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F14	0.9167	0.9708	0.9917	0.9958	**1.0000**	**1.0000**	0.7500	0.7500	0.7500	0.7500	0.4000	0.4417	0.4292	0.7083
F15	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	0.9667	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F16	0.7095	0.7333	0.7381	0.7333	0.7333	0.7333	0.7524	0.7571	0.8048	0.8333	0.8095	0.6571	**0.8476**	0.6619
F17	**1.0000**	**1.0000**	0.6667	0.6778	0.6778	0.7333	0.7333	0.7222	0.7556	0.7889	0.8778	1.0000	1.0000	1.0000
F18	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F19	**0.9967**	0.7967	0.8000	0.8000	0.8000	0.8000	0.8000	0.6033	0.6000	0.6033	0.4767	0.3133	0.3533	0.3767
F20	0.8333	0.9037	0.9481	0.9519	0.9889	0.9815	0.9926	**1.0000**	**1.0000**	**1.0000**	0.8963	0.9815	**1.0000**	**1.0000**
F21	0.9436	0.9282	0.9436	0.9590	0.9744	0.9821	0.9846	0.9846	0.9923	0.9949	**1.0000**	**1.0000**	0.9821	0.9821
F22	0.8208	0.8229	0.8188	0.8354	0.8229	0.8271	0.8125	0.8354	0.8458	0.8521	0.8396	0.8438	0.8979	**0.9563**
F23	0.7889	0.8778	0.9611	**1.0000**	0.9944	0.6556	0.6889	0.6833	0.6778	0.6833	0.6722	0.6889	0.7889	0.8500
F24	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
F25	0.9143	0.9429	0.9810	0.9524	0.9810	0.9810	0.9905	**0.9952**	1.0000	**0.9952**	0.6762	0.6286	0.5762	0.5762
F26	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F27	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	0.5111	0.4944	0.5833	0.7611
F28	0.8833	0.8792	0.8917	0.9000	0.9042	0.8917	0.5250	0.5167	0.5250	0.5208	0.5208	0.6000	0.6250	0.6333
F29	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F30	0.9417	0.9500	0.9694	0.9806	**0.9972**	0.9889	0.9944	**0.9972**	0.9972	1.0000	0.7444	0.4306	0.3944	0.2694
F31	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F32	0.9917	0.9917	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F33	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F34	0.4667	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.1333
F35	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F36	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F37	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F38	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	0.9778	**1.0000**	**1.0000**
F39	**1.0000**	**1.0000**	**1.0000**	**1.0000**	0.9833	0.9833	**1.0000**	0.9833	**1.0000**	0.9833	0.9833	**1.0000**	**1.0000**	**1.0000**
F40	0.7800	0.7800	0.8200	0.7933	0.8067	0.8667	0.8600	0.8867	0.8267	0.8667	0.9467	**0.9533**	0.8000	0.8000
F41	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F42	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	0.9500
Avg.	0.8981	**0.9000**	0.8999	0.8987	0.8936	0.8867	0.8683	0.8634	0.8627	0.8583	0.8158	0.8005	0.8082	0.8233

TABLE S-R-III

INFLUENCE OF DIFFERENT SETTINGS OF STATIC REPULSIVE RADIUS IN R-JADE WITH RESPECT TO THE SUCCESS RATE.

Prob.	δ=0.01	δ=0.02	δ=0.03	δ=0.04	δ=0.05	δ=0.06	δ=0.07	δ=0.08	δ=0.09	δ=0.1	δ=0.2	δ=0.3	δ=0.4	δ=0.5
F01	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F02	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	1.0000	0.9667	0.8667	0.7333	0.7000	0.8000
F03	0.5667	0.4667	**0.8000**	0.6333	0.6333	0.5333	0.5667	0.6000	0.4333	0.6000	0.5000	0.6000	0.5667	0.5667
F04	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F05	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	0.3333	0.0333	0.0667	0.0000	0.0333	0.0333	0.3333	0.3333
F06	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F07	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F08	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F09	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
F10	0.9333	**1.0000**	**1.0000**	**1.0000**	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
F11	0.7000	0.9333	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.1000
F12	0.8000	0.9333	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	0.0000	**1.0000**	0.9333	0.9000	0.9667
F13	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F14	0.4333	0.8333	0.9333	0.9667	**1.0000**	**1.0000**	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
F15	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	0.9667	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F16	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0333	0.0333	**0.1667**	0.0000	0.0000	0.1000	0.0667
F17	**1.0000**	**1.0000**	0.0333	0.0333	0.1000	0.2333	0.2000	0.2000	0.2667	0.3667	0.6333	**1.0000**	**1.0000**	**1.0000**
F18	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F19	**0.9667**	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
F20	0.0333	0.2667	0.5667	0.5667	0.9000	0.8333	0.9333	**1.0000**	**1.0000**	**1.0000**	0.4000	0.8667	**1.0000**	**1.0000**
F21	0.4333	0.3333	0.4000	0.6000	0.6667	0.7667	0.8333	0.8000	0.9000	0.9333	**1.0000**	**1.0000**	0.7667	0.7667
F22	0.0000	0.0000	0.0000	0.0000	0.0000	0.0667	0.0000	0.0000	0.0333	0.0333	0.0667	0.0000	0.1333	**0.4000**
F23	0.1000	0.3667	0.7667	1.0000	**0.9667**	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0333	0.3333
F24	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
F25	0.5000	0.7000	0.9000	0.7000	0.8667	0.8667	0.9333	**0.9667**	1.0000	**0.9667**	0.0000	0.0000	0.0000	0.0000
F26	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F27	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	0.0000	0.0000	0.0000	0.0000
F28	0.1667	0.0333	0.2000	0.2000	0.2333	0.1667	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
F29	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F30	0.4667	0.5333	0.6333	0.7667	0.9667	0.8667	0.9333	0.9667	0.9667	**1.0000**	0.0000	0.0000	0.0000	0.0000
F31	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F32	0.9667	0.9667	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F33	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F34	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
F35	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F36	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F37	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F38	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	0.9333	**1.0000**	**1.0000**
F39	**1.0000**	**1.0000**	**1.0000**	**1.0000**	0.9667	0.9667	**1.0000**	0.9667	**1.0000**	0.9667	0.9667	**1.0000**	**1.0000**	**1.0000**
F40	0.1667	0.1667	0.3667	0.2667	0.2667	0.4667	0.4000	0.5667	0.3000	0.5000	0.7333	**0.7667**	0.0000	0.0000
F41	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F42	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	0.9000
Avg.	0.7198	0.7270	0.7286	**0.7317**	0.7278	0.7087	0.6690	0.6698	0.6667	0.6548	0.6000	0.6159	0.6079	0.6246

TABLE S-R-IV
COMPARISON OF THE DIFFERENT DYNAMIC CONTROL STRATEGIES IN DR-JADE WITH RESPECT TO THE ROOT RATIO.

Prob.	DR-JADE-s1	DR-JADE-s2	DR-JADE-s3	DR-JADE-s4	$\delta = 0.04$
F01	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F02	**1.0000**	0.9889	**1.0000**	**1.0000**	**1.0000**
F03	0.8000	0.7667	0.8167	**0.8500**	0.8000
F04	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F05	0.9333	0.8778	**1.0000**	0.9778	**1.0000**
F06	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F07	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F08	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F09	0.0000	0.0000	0.0000	0.0000	0.0000
F10	0.9273	0.8091	0.9970	0.9576	**1.0000**
F11	0.8067	0.7600	**0.9578**	0.8911	0.8667
F12	**1.0000**	0.9974	**1.0000**	**1.0000**	**1.0000**
F13	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F14	0.8875	0.8417	0.9583	0.9083	**0.9958**
F15	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F16	1.0000	0.9952	**1.0000**	0.9952	0.7333
F17	**1.0000**	**1.0000**	**1.0000**	**1.0000**	0.6778
F18	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F19	0.8033	0.7400	**0.8767**	0.8333	0.8000
F20	**1.0000**	**1.0000**	**1.0000**	**1.0000**	0.9519
F21	**1.0000**	**1.0000**	**1.0000**	**1.0000**	0.9590
F22	0.8417	**0.8792**	0.8396	0.8438	0.8354
F23	0.9944	0.9944	**1.0000**	0.9944	**1.0000**
F24	0.0000	0.0000	0.0000	0.0000	0.0000
F25	**1.0000**	**1.0000**	**1.0000**	**1.0000**	0.9524
F26	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F27	**1.0000**	0.9889	**1.0000**	**1.0000**	**1.0000**
F28	0.8375	0.7500	0.9125	**0.9250**	0.9000
F29	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F30	0.8194	0.7444	0.9167	0.8389	**0.9806**
F31	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F32	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F33	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F34	0.2333	**0.5000**	**0.5000**	0.3333	**0.5000**
F35	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F36	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F37	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F38	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F39	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F40	0.9067	0.8800	0.9533	0.9200	0.7933
F41	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F42	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
Avg.	0.8998	0.8932	**0.9221**	0.9112	0.8987

TABLE S-R-V
COMPARISON OF THE DIFFERENT DYNAMIC CONTROL STRATEGIES IN DR-JADE WITH RESPECT TO THE SUCCESS RATE.

Prob.	DR-JADE-s1	DR-JADE-s2	DR-JADE-s3	DR-JADE-s4	$\delta = 0.04$
F01	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F02	**1.0000**	0.9667	**1.0000**	**1.0000**	**1.0000**
F03	0.6000	0.5333	0.6667	**0.7000**	0.6333
F04	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F05	0.8000	0.6667	**1.0000**	0.9333	**1.0000**
F06	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F07	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F08	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F09	0.0000	0.0000	0.0000	0.0000	0.0000
F10	0.3000	0.0333	**0.9667**	0.5667	**1.0000**
F11	0.0000	0.0000	**0.4667**	0.0333	0.0000
F12	**1.0000**	0.9667	**1.0000**	**1.0000**	**1.0000**
F13	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F14	0.3333	0.0667	0.6667	0.3667	**0.9667**
F15	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F16	**1.0000**	0.9667	**1.0000**	0.9667	0.0000
F17	**1.0000**	**1.0000**	**1.0000**	**1.0000**	0.0333
F18	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F19	0.0000	0.0000	**0.0333**	0.0000	0.0000
F20	**1.0000**	**1.0000**	**1.0000**	**1.0000**	0.5667
F21	**1.0000**	**1.0000**	**1.0000**	**1.0000**	0.6000
F22	**0.0667**	**0.0667**	0.0333	0.0333	0.0000
F23	0.9667	0.9667	**1.0000**	0.9667	**1.0000**
F24	0.0000	0.0000	0.0000	0.0000	0.0000
F25	**1.0000**	**1.0000**	**1.0000**	**1.0000**	0.7000
F26	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F27	**1.0000**	0.9333	**1.0000**	**1.0000**	**1.0000**
F28	0.1667	0.0000	0.4333	**0.5000**	0.2000
F29	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F30	0.0333	0.0000	0.2667	0.1000	**0.7667**
F31	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F32	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F33	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F34	0.0000	0.0000	0.0000	0.0000	0.0000
F35	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F36	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F37	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F38	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F39	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F40	0.5333	0.4000	**0.7667**	0.6000	0.2667
F41	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F42	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
Avg.	0.7571	0.7278	**0.8167**	0.7802	0.7317

TABLE S-R-VI
COMPARISON BETWEEN DR-JADE AND OTHER STATE-OF-THE-ART METHODS WITH RESPECT TO ROOT RATIO.

Prob	DR-JADE	A-WeB	NCDE	NSDE	MONES	I-HS	GA-SQP	PSO-NM	NCSA
F01	**1.0000**	0.7250	**1.0000**	**1.0000**	**1.0000**	**1.0000**	0.7667	**1.0000**	0.9833
F02	**1.0000**	0.8800	**1.0000**	0.9778	0.9778	0.9667	0.7000	0.9556	0.8444
F03	0.8167	0.5450	0.9897	0.9256	**1.0000**	0.7000	0.2000	0.4000	0.3000
F04	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	0.6667	**1.0000**	**1.0000**
F05	**1.0000**	0.9900	0.6556	0.9222	0.8111	0.5667	0.4556	0.8000	0.8444
F06	**1.0000**	0.8300	**1.0000**	**1.0000**	**1.0000**	0.9000	0.9000	0.8333	0.0500
F07	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	0.7667	0.9111	0.9667
F08	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	0.8000	**1.0000**	0.9667
F09	0.0000	0.6200	0.9833	**1.0000**	0.9833	0.0000	0.0000	0.0000	0.5667
F10	0.9970	**1.0000**	0.9848	0.9485	0.9758	0.6970	0.2424	0.8182	0.6061
F11	0.9578	0.9573	**0.9778**	0.9644	0.9422	0.4844	0.2000	0.6844	0.4444
F12	**1.0000**	**1.0000**	0.4641	0.5436	0.5923	0.8821	0.1513	0.3846	0.4872
F13	**1.0000**	**1.0000**	**1.0000**	0.9333	**1.0000**	0.2667	**1.0000**	0.8667	**1.0000**
F14	0.9583	0.9400	0.8708	0.8833	**1.0000**	0.8208	0.1875	0.8750	0.5375
F15	**1.0000**	0.4200	0.0667	0.1333	0.0333	0.0333	0.6000	**1.0000**	**1.0000**
F16	**1.0000**	0.8371	0.9190	0.9238	0.7762	0.6762	0.4381	0.8952	0.5333
F17	**1.0000**	0.8933	0.0000	0.0111	0.0000	0.8778	0.6333	0.6444	0.5444
F18	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	0.9333	0.7333	**1.0000**
F19	0.8767	0.8880	**0.9933**	0.9533	0.8333	0.7933	0.1133	0.5933	0.5200
F20	**1.0000**	0.9733	0.9185	0.9333	0.1963	0.7667	0.2259	0.3926	0.5222
F21	**1.0000**	**1.0000**	0.9821	0.9359	**1.0000**	0.5077	0.1154	0.3538	0.2231
F22	0.8396	0.6688	**0.9917**	0.9896	0.0833	0.8063	0.1375	0.0667	0.2125
F23	**1.0000**	0.9433	0.9944	**1.0000**	**1.0000**	0.8944	0.4889	0.8333	0.7722
F24	0.0000	**0.6200**	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0333
F25	**1.0000**	0.9514	0.9762	0.9952	0.7190	0.6857	0.5286	0.8952	0.6762
F26	**1.0000**	0.9950	**1.0000**	**1.0000**	**1.0000**	**1.0000**	0.5000	0.6833	0.9000
F27	**1.0000**	**1.0000**	**1.0000**	0.9944	0.9889	0.9833	0.4389	0.7778	0.7222
F28	0.9125	0.8550	**1.0000**	**1.0000**	0.7333	0.4750	0.4000	0.7500	0.5500
F29	**1.0000**	**1.0000**	**1.0000**	0.9833	0.5000	0.9833	0.6167	0.8000	0.9000
F30	0.9167	0.0933	0.9750	0.9389	**1.0000**	0.9222	0.2833	0.4778	0.4278
F31	**1.0000**	**1.0000**	0.7833	0.9167	0.8333	0.9833	0.6333	**1.0000**	0.9167
F32	**1.0000**	**1.0000**	0.9167	0.9917	0.8583	**1.0000**	0.4667	0.8833	0.8833
F33	**1.0000**	**1.0000**	0.4500	0.9250	**1.0000**	**1.0000**	0.4583	0.9000	0.8583
F34	0.5000	**1.0000**	0.0000	0.0000	**1.0000**	0.9167	0.5500	**1.0000**	0.4333
F35	**1.0000**	**1.0000**	0.0000	0.0000	0.0000	**1.0000**	0.1333	**1.0000**	**1.0000**
F36	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	0.6500	**1.0000**	0.9833
F37	**1.0000**	0.8800	0.3667	0.8667	0.1667	**1.0000**	0.1667	0.8000	**1.0000**
F38	**1.0000**	**1.0000**	0.7889	0.9889	**1.0000**	0.9556	0.7222	0.9778	0.8556
F39	**1.0000**	0.9400	0.3000	0.5000	0.0000	0.9333	0.0000	**1.0000**	0.7500
F40	0.9533	0.9320	0.8067	0.8600	0.1400	**0.9933**	0.2200	0.8400	0.7200
F41	**1.0000**	**1.0000**	0.9917	**1.0000**	**1.0000**	0.9833	0.5250	0.9000	0.8167
F42	**1.0000**	0.9900	0.9833	**1.0000**	**1.0000**	**1.0000**	0.5667	**1.0000**	0.9833
Avg.	**0.9221**	0.8897	0.7888	0.8319	0.7415	0.7966	0.4424	0.7554	0.6985

TABLE S-R-VII
COMPARISON BETWEEN DR-JADE AND OTHER STATE-OF-THE-ART METHODS WITH RESPECT TO SUCCESS RATE.

Prob	DR-JADE	A-WeB	NCDE	NSDE	MONES	I-HS	GA-SQP	PSO-NM	NCSA
F01	**1.0000**	0.5300	**1.0000**	**1.0000**	**1.0000**	**1.0000**	0.5333	**1.0000**	0.9667
F02	**1.0000**	0.6800	**1.0000**	0.9333	0.9333	0.9000	0.3667	0.8667	0.5667
F03	0.6667	0.2000	0.8667	0.3667	**1.0000**	0.5300	0.0000	0.0667	0.0000
F04	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	0.3333	**1.0000**	**1.0000**
F05	**1.0000**	0.9700	0.2667	0.8000	0.4333	0.0700	0.0000	0.4000	0.5333
F06	**1.0000**	0.6500	**1.0000**	**1.0000**	**1.0000**	0.8300	0.8000	0.6667	0.0000
F07	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	0.4000	0.7333	0.9000
F08	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	0.6000	**1.0000**	0.9333
F09	0.0000	0.3600	**0.9667**	**1.0000**	0.9667	0.0000	0.0000	0.0000	0.1333
F10	0.9667	**1.0000**	0.8333	0.5667	0.7667	0.0000	0.0000	0.0000	0.0000
F11	0.4667	0.5800	**0.8000**	0.6333	0.4333	0.0000	0.0000	0.0000	0.0000
F12	**1.0000**	**1.0000**	0.0000	0.0000	0.0000	0.1000	0.0000	0.0000	0.0000
F13	**1.0000**	**1.0000**	**1.0000**	0.9333	**1.0000**	0.2667	**1.0000**	0.8667	**1.0000**
F14	0.6667	0.6000	0.3000	0.2667	**1.0000**	0.1667	0.0000	0.2667	0.0000
F15	**1.0000**	0.4200	0.0667	0.1333	0.0333	0.0333	0.6000	**1.0000**	**1.0000**
F16	**1.0000**	0.1200	0.4333	0.5000	0.0000	0.0000	0.0000	0.3333	0.0000
F17	**1.0000**	0.6800	0.0000	0.0000	0.0000	0.6667	0.0667	0.0667	0.1000
F18	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	0.9333	0.7333	**1.0000**
F19	0.0333	0.2800	**0.9333**	0.8000	0.2333	0.0000	0.0000	0.0000	0.0000
F20	**1.0000**	0.7600	0.4000	0.4667	0.0000	0.0000	0.0000	0.0000	0.0000
F21	**1.0000**	**1.0000**	0.7667	0.4333	**1.0000**	0.0000	0.0000	0.0000	0.0000
F22	0.0333	0.0000	0.8667	**0.9000**	0.0000	0.0000	0.0000	0.0000	0.0000
F23	**1.0000**	0.6600	0.9667	**1.0000**	**1.0000**	0.4667	0.0000	0.2000	0.0667
F24	0.0000	**0.2400**	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
F25	**1.0000**	0.7000	0.8333	0.9667	0.0333	0.0333	0.0000	0.3333	0.0000
F26	**1.0000**	0.9800	**1.0000**	**1.0000**	**1.0000**	**1.0000**	0.0333	0.2000	0.6333
F27	**1.0000**	**1.0000**	**1.0000**	0.9667	0.9667	0.9000	0.0000	0.2667	0.0667
F28	0.4333	0.1400	**1.0000**	**1.0000**	0.0333	0.0000	0.0000	0.1333	0.0000
F29	**1.0000**	**1.0000**	**1.0000**	0.9667	0.0333	0.9667	0.2333	0.6000	0.8000
F30	0.2667	0.0000	0.7667	0.4667	**1.0000**	0.2667	0.0000	0.0000	0.0000
F31	**1.0000**	**1.0000**	0.5667	0.8333	0.6667	0.9667	0.3667	**1.0000**	0.8333
F32	**1.0000**	**1.0000**	0.6667	0.9667	0.4667	**1.0000**	0.0000	0.5333	0.5667
F33	**1.0000**	**1.0000**	0.0333	0.7667	**1.0000**	**1.0000**	0.0000	0.6000	0.5000
F34	0.0000	**1.0000**	0.0000	0.0000	**1.0000**	0.8333	0.3667	**1.0000**	0.0000
F35	**1.0000**	**1.0000**	0.0000	0.0000	0.0000	**1.0000**	0.1333	**1.0000**	**1.0000**
F36	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	0.4333	**1.0000**	0.9667
F37	**1.0000**	0.8800	0.3667	0.8667	0.1667	1.0000	0.1667	0.8000	**1.0000**
F38	**1.0000**	**1.0000**	0.4333	0.9667	**1.0000**	0.8667	0.2333	0.9333	0.5667
F39	**1.0000**	0.8800	0.0000	0.0000	0.0000	0.8667	0.0000	**1.0000**	0.5000
F40	0.7667	0.6600	0.3333	0.5000	0.0000	**0.9667**	0.0000	0.4000	0.1667
F41	**1.0000**	**1.0000**	0.9667	**1.0000**	**1.0000**	0.9333	0.0333	0.6667	0.3333
F42	**1.0000**	0.9900	0.9666	**1.0000**	**1.0000**	**1.0000**	**0.4000**	**1.0000**	0.9667
Avg.	**0.8167**	0.7371	0.6524	0.6905	0.5992	0.5626	0.1913	0.4921	0.4071

TABLE S-R-VIII
INFLUENCE OF DIFFERENT OPTIMIZATION ALGORITHMS IN DREA IN TERMS OF THE ROOT RATIO.

Prob.	DR-jDE	R-jDE	DR-SHADE	R-SHADE	DR-SaDE	R-SaDE	DR-CLPSO	R-CLPSO
F01	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	**0.5000**	0.2667
F02	1.0000	1.0000	1.0000	1.0000	0.9333	**1.0000**	**1.0000**	0.9889
F03	0.2000	0.2000	0.8667	**0.8833**	0.0500	0.0167	0.0000	0.0000
F04	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
F05	1.0000	1.0000	0.9778	**1.0000**	0.9889	**1.0000**	1.0000	1.0000
F06	**0.9667**	0.8833	1.0000	1.0000	**1.0000**	0.9333	1.0000	1.0000
F07	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
F08	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
F09	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
F10	0.9758	**1.0000**	0.9909	**1.0000**	0.9818	**0.9970**	**0.6485**	0.5788
F11	**0.9178**	0.8667	**0.9222**	0.8667	**0.9333**	0.8667	**0.9867**	0.9289
F12	0.9974	**1.0000**	0.9974	**1.0000**	0.9974	**1.0000**	**0.9897**	0.9667
F13	1.0000	1.0000	**1.0000**	0.9333	**0.9333**	0.9000	0.3000	0.3000
F14	0.9667	**1.0000**	**0.9708**	0.9958	0.9792	**1.0000**	0.8917	**0.9375**
F15	0.5667	0.6000	**1.0000**	0.9333	0.8000	**0.9000**	**0.4000**	0.3667
F16	**0.9952**	0.7333	**0.9905**	0.7476	**1.0000**	0.7476	**1.0000**	0.7476
F17	**0.6111**	0.4444	**1.0000**	0.6000	**0.5111**	0.3111	**0.5000**	0.3222
F18	1.0000	1.0000	1.0000	1.0000	**0.8667**	0.7667	0.8000	0.8000
F19	**0.8333**	0.8000	**0.8567**	0.8000	**0.8767**	0.8000	**0.8833**	0.8067
F20	**1.0000**	0.9593	**1.0000**	0.9519	**1.0000**	0.9296	**0.9556**	0.8296
F21	**0.9718**	0.9359	**0.9949**	0.9538	**0.9897**	0.9385	**0.6513**	0.3872
F22	**0.1500**	0.1479	**0.8417**	0.7083	**0.2146**	0.1938	0.3500	**0.3750**
F23	1.0000	1.0000	**1.0000**	0.9833	**1.0000**	0.9944	**0.9778**	0.9500
F24	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
F25	**1.0000**	0.9048	**1.0000**	0.9810	**0.9952**	0.8190	**0.9857**	0.9762
F26	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
F27	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
F28	**0.8583**	0.8542	**0.9125**	0.8917	0.7917	**0.8458**	**0.9667**	0.8875
F29	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	**0.9500**	0.8833
F30	**0.9000**	0.8728	0.9028	**0.9722**	**0.8611**	0.8472	**0.0583**	0.0500
F31	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9667	0.9667
F32	1.0000	1.0000	1.0000	1.0000	**1.0000**	0.9750	1.0000	1.0000
F33	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
F34	**0.5000**	0.4833	**0.5000**	0.4833	**0.5000**	0.4833	**1.0000**	0.7500
F35	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.0000	0.0000
F36	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
F37	1.0000	1.0000	1.0000	1.0000	0.8333	0.8333	0.7667	**0.8333**
F38	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	**0.9778**	0.9333
F39	1.0000	1.0000	1.0000	1.0000	**1.0000**	0.8500	0.0000	0.0000
F40	**0.9200**	0.8400	**0.9000**	0.7467	**0.9000**	0.7467	**0.0267**	0.0200
F41	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9583	0.9583
F42	1.0000	1.0000	1.0000	1.0000	0.9833	**1.0000**	1.0000	1.0000
Avg.	**0.8650**	0.8459	**0.9196**	0.8912	**0.8553**	0.8261	**0.7260**	0.6860

TABLE S-R-IX
INFLUENCE OF DIFFERENT OPTIMIZATION ALGORITHMS IN DREA IN TERMS OF THE SUCCESS RATE.

Prob.	DR-jDE	R-jDE	DR-SHADE	R-SHADE	DR-SaDE	R-SaDE	DR-CLPSO	R-CLPSO
F01	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	**0.2667**	0.1000
F02	1.0000	1.0000	1.0000	1.0000	**0.8000**	1.0000	**1.0000**	0.9667
F03	0.0333	0.0333	0.7333	**0.7667**	0.0000	0.0000	0.0000	0.0000
F04	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
F05	1.0000	1.0000	0.9333	**1.0000**	0.9667	**1.0000**	1.0000	1.0000
F06	**0.9333**	0.7667	1.0000	1.0000	**1.0000**	0.8667	1.0000	1.0000
F07	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
F08	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
F09	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
F10	0.7333	**1.0000**	0.9000	**1.0000**	0.8667	**0.9667**	**0.0333**	0.0000
F11	**0.0667**	0.0000	**0.2333**	0.0000	**0.3000**	0.0000	**0.8333**	0.2333
F12	0.9667	**1.0000**	0.9667	**1.0000**	0.9667	**1.0000**	**0.8667**	0.6333
F13	1.0000	1.0000	**1.0000**	0.9333	**0.9333**	0.9000	0.3000	0.3000
F14	0.7667	**1.0000**	**0.8000**	0.9667	0.8333	**1.0000**	0.3000	**0.5667**
F15	0.5667	**0.6000**	**1.0000**	0.9333	0.8000	**0.9000**	**0.4000**	0.3667
F16	**0.9667**	0.0000	**0.9333**	0.0000	**1.0000**	0.0000	**1.0000**	0.0000
F17	**0.1667**	0.0000	**1.0000**	0.0333	**0.1000**	0.0000	**0.0667**	0.0000
F18	1.0000	1.0000	1.0000	1.0000	**0.8667**	0.7667	0.8000	0.8000
F19	**0.0667**	0.0000	**0.1000**	0.0000	**0.1333**	0.0000	**0.1000**	0.0000
F20	**1.0000**	0.6333	**1.0000**	0.6000	**1.0000**	0.3667	**0.6667**	0.0000
F21	**0.7000**	0.3000	**0.9333**	0.4667	**0.8667**	0.3000	0.0000	0.0000
F22	0.0000	0.0000	**0.0333**	0.0000	0.0000	0.0000	0.0000	0.0000
F23	**1.0000**	1.0000	**1.0000**	0.9000	**1.0000**	0.9667	**0.8667**	0.7000
F24	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
F25	**1.0000**	0.5333	**1.0000**	0.8667	**0.9667**	0.2000	**0.9333**	0.8333
F26	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
F27	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
F28	**0.1667**	0.0000	**0.4333**	0.1667	0.0333	**0.0667**	**0.7333**	0.1333
F29	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	**0.9000**	0.8000
F30	**0.2667**	0.1667	0.2667	**0.7000**	**0.1333**	0.1000	0.0000	0.0000
F31	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9333	0.9333
F32	1.0000	1.0000	1.0000	1.0000	**1.0000**	0.9000	1.0000	1.0000
F33	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
F34	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	**1.0000**	0.5333
F35	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.0000	0.0000
F36	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
F37	1.0000	1.0000	1.0000	1.0000	0.8333	0.8333	0.7667	**0.8333**
F38	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	**0.9333**	0.8000
F39	1.0000	1.0000	1.0000	1.0000	**1.0000**	0.7000	0.0000	0.0000
F40	**0.6000**	0.3000	**0.5333**	0.1333	**0.6000**	0.1667	0.0000	0.0000
F41	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.8333	0.8333
F42	1.0000	1.0000	1.0000	1.0000	0.9667	**1.0000**	1.0000	1.0000
Avg.	**0.7381**	0.6746	**0.8048**	0.7254	**0.7373**	0.6429	**0.6079**	0.5087

TABLE S-R-X
INFLUENCE OF DIFFERENT REPULSION TECHNIQUES IN DREA IN TERMS OF RR AND SR.

Prob.	RR		SR		RR		SR	
	DR-JADE1	R-JADE1	DR-JADE1	R-JADE1	DR-JADE2	R-JADE2	DR-JADE2	R-JADE2
F01	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
F02	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
F03	0.7167	**0.7500**	0.5000	**0.5667**	**0.7500**	0.7167	**0.5000**	0.4667
F04	0.9667	**1.0000**	0.9333	**1.0000**	1.0000	1.0000	1.0000	1.0000
F05	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
F06	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
F07	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
F08	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
F09	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
F10	**1.0000**	0.9939	**1.0000**	0.9333	1.0000	1.0000	1.0000	1.0000
F11	0.9533	0.9533	0.5333	0.5333	0.9867	**1.0000**	0.8000	**1.0000**
F12	**0.9077**	0.8436	**0.2333**	0.0333	**0.9513**	0.9077	**0.5333**	0.2000
F13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
F14	**0.9833**	0.8458	**0.8667**	0.2000	**0.9750**	0.9375	**0.8333**	0.6000
F15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
F16	**0.9524**	0.7143	**0.6667**	0.0000	**0.8619**	0.7286	**0.2000**	0.0000
F17	**0.9889**	0.9778	**0.9667**	0.9333	0.9889	**1.0000**	0.9667	**1.0000**
F18	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
F19	0.9267	**0.9733**	0.3333	**0.7667**	0.9400	**0.9767**	0.5333	**0.8000**
F20	**0.8481**	0.7222	**0.1000**	0.0000	**0.9556**	0.7667	**0.6333**	0.0333
F21	**0.9385**	0.8615	**0.4000**	0.1333	**0.9718**	0.9103	**0.6667**	0.3000
F22	0.8208	**0.8229**	0.0000	0.0000	**0.8854**	0.8396	**0.1333**	0.0333
F23	**0.9889**	0.7722	**0.9333**	0.0667	**1.0000**	0.7778	**1.0000**	0.1333
F24	**0.0167**	0.0667	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
F25	**0.9952**	0.9429	**0.9667**	0.6333	**1.0000**	0.9476	**1.0000**	0.7000
F26	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
F27	**1.0000**	0.9833	**1.0000**	0.9000	1.0000	1.0000	1.0000	1.0000
F28	0.8667	**0.8875**	**0.2667**	0.1333	**1.0000**	0.8875	**1.0000**	0.1000
F29	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
F30	**0.9333**	0.9194	**0.4000**	0.3000	0.9806	**0.9889**	0.8000	**0.8667**
F31	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
F32	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
F33	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
F34	0.4167	**0.5000**	0.0000	0.0000	0.5000	0.5000	0.0000	0.0000
F35	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
F36	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
F37	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
F38	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
F39	0.9833	0.9833	0.9667	0.9667	**1.0000**	0.9833	**1.0000**	0.9667
F40	**0.8533**	0.7133	**0.4333**	0.0667	**0.9200**	0.7933	**0.7000**	0.3000
F41	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
F42	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
Avg.	**0.9061**	0.8864	**0.7500**	0.6706	**0.9206**	0.8967	**0.8167**	0.7262

TABLE S-R-XI
INFLUENCE OF DIFFERENT POPULATION SIZE IN DR-JADE IN TERMS OF THE ROOT RATIO.

Prob.	$\mu = 5$	$\mu = 8$	$\mu = 10$	$\mu = 20$	$\mu = 30$	$\mu = 40$	$\mu = 50$
F01	**1.0000**	**1.0000**	**1.0000**	0.2000	0.0667	0.0333	0.0000
F02	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	0.9889	**1.0000**
F03	**0.9333**	0.8500	0.8167	0.4000	0.0167	0.0000	0.0000
F04	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	0.2333	0.0500
F05	0.9556	**1.0000**	**1.0000**	**1.0000**	**1.0000**	0.9889	0.7333
F06	**1.0000**	**1.0000**	**1.0000**	0.9833	0.9833	0.7667	0.5167
F07	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F08	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F09	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
F10	0.9576	0.9848	**0.9970**	0.9879	0.8485	0.5667	0.3455
F11	0.8800	0.9222	**0.9578**	0.9511	0.8889	0.6911	0.2378
F12	0.9949	**1.0000**	**1.0000**	0.9949	0.3231	0.0538	0.0179
F13	**1.0000**	**1.0000**	**1.0000**	**1.0000**	0.0000	0.0000	0.0000
F14	**0.9583**	0.9542	**0.9583**	0.8417	0.4375	0.0875	0.0000
F15	**1.0000**	**1.0000**	**1.0000**	0.9667	0.5667	0.0000	0.0000
F16	0.9857	**1.0000**	**1.0000**	0.9952	0.9714	0.9667	0.9619
F17	0.8889	**1.0000**	**1.0000**	**1.0000**	**1.0000**	0.9667	0.3222
F18	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F19	0.7900	0.8533	**0.8767**	0.8467	0.6800	0.2567	0.0633
F20	**1.0000**	**1.0000**	**1.0000**	**1.0000**	0.5074	0.0185	0.0000
F21	0.9897	0.9923	**1.0000**	**1.0000**	0.9692	0.5077	0.2590
F22	0.7604	0.8333	0.8396	**0.8542**	0.8250	0.8083	0.5188
F23	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	0.7278
F24	0.0000	0.0000	0.0000	**0.0667**	0.0500	0.0167	0.0000
F25	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	0.7238
F26	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	0.8000
F27	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	0.8944	0.1667
F28	0.9000	0.9042	0.9125	0.9167	0.9333	0.9208	**0.9583**
F29	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	0.7500	0.1000
F30	0.9111	**0.9444**	0.9167	0.9222	0.9083	0.7917	0.3806
F31	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	0.9167
F32	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	0.6333
F33	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	0.9833	0.2750
F34	**0.5000**	**0.5000**	**0.5000**	0.4667	0.3333	0.2833	0.1167
F35	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	0.2000	0.0000
F36	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F37	**1.0000**	**1.0000**	**1.0000**	**1.0000**	0.6333	0.0333	0.0000
F38	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	0.7667
F39	**1.0000**	**1.0000**	**1.0000**	0.9333	0.6333	0.3167	0.0667
F40	0.9400	0.9067	**0.9533**	0.9000	0.6200	0.2733	0.2067
F41	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	0.8417
F42	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
Avg.	0.9130	0.9201	**0.9221**	0.8864	0.7666	0.6047	0.4216

TABLE S-R-XII
INFLUENCE OF DIFFERENT POPULATION SIZE IN DR-JADE IN TERMS OF THE SUCCESS RATE.

Prob.	$\mu = 5$	$\mu = 8$	$\mu = 10$	$\mu = 20$	$\mu = 30$	$\mu = 40$	$\mu = 50$
F01	**1.0000**	**1.0000**	**1.0000**	0.0333	0.0000	0.0000	0.0000
F02	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	0.9667	**1.0000**
F03	**0.8667**	0.7000	0.6667	0.1333	0.0000	0.0000	0.0000
F04	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	0.0667	0.0000
F05	0.8667	**1.0000**	**1.0000**	**1.0000**	**1.0000**	0.9667	0.3000
F06	**1.0000**	**1.0000**	**1.0000**	0.9667	0.9667	0.5667	0.2667
F07	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F08	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F09	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
F10	0.5333	0.8333	**0.9667**	0.8667	0.2000	0.0000	0.0000
F11	0.1333	0.1333	**0.4667**	0.4333	0.1000	0.0000	0.0000
F12	0.9333	**1.0000**	**1.0000**	0.9333	0.0000	0.0000	0.0000
F13	**1.0000**	**1.0000**	**1.0000**	**1.0000**	0.0000	0.0000	0.0000
F14	**0.7000**	**0.7000**	0.6667	0.1333	0.0000	0.0000	0.0000
F15	**1.0000**	**1.0000**	**1.0000**	0.9667	0.5667	0.0000	0.0000
F16	0.9000	**1.0000**	**1.0000**	0.9667	0.8000	0.7667	0.7333
F17	0.7000	**1.0000**	**1.0000**	**1.0000**	**1.0000**	0.9000	0.0000
F18	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F19	0.0000	**0.1333**	0.0333	0.0667	0.0000	0.0000	0.0000
F20	**1.0000**	**1.0000**	**1.0000**	**1.0000**	0.0667	0.0000	0.0000
F21	0.8667	0.9000	**1.0000**	**1.0000**	0.8667	0.0667	0.0000
F22	0.0000	0.0000	0.0333	0.0333	**0.0667**	0.0333	0.0000
F23	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	0.0333
F24	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
F25	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	0.0667
F26	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	0.5667
F27	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	0.7000	0.0000
F28	0.4000	0.4000	0.4333	0.4000	**0.5333**	0.4667	0.7333
F29	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	0.5000	0.0000
F30	0.2000	**0.5000**	0.2667	0.2667	0.3667	0.0667	0.0000
F31	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	0.8333
F32	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	0.2667
F33	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	0.9333	0.0000
F34	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
F35	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	0.2000	0.0000
F36	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F37	**1.0000**	**1.0000**	**1.0000**	**1.0000**	0.6333	0.0333	0.0000
F38	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	0.3000
F39	**1.0000**	**1.0000**	**1.0000**	0.8667	0.2667	0.0000	0.0000
F40	0.7000	0.5667	**0.7667**	0.5333	0.0000	0.0000	0.0000
F41	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	0.5667
F42	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
Avg.	0.7810	0.8063	**0.8167**	0.7524	0.6056	0.4579	0.2540

TABLE S-R-XIII
INFLUENCE OF DIFFERENT $t_{\max}$ IN DR-JADE IN TERMS OF THE ROOT RATIO.

Prob.	$t_{\max}=1$	$t_{\max}=10$	$t_{\max}=20$	$t_{\max}=30$	$t_{\max}=40$	$t_{\max}=50$	$t_{\max}=60$	$t_{\max}=70$	$t_{\max}=80$
F01	0.5000	**1.0000**	**1.0000**	**1.0000**	**1.0000**	0.6333	0.1667	0.1500	0.0167
F02	0.3333	**1.0000**	**1.0000**	**1.0000**	0.9889	**1.0000**	**1.0000**	**1.0000**	0.9889
F03	0.0667	0.3833	0.6167	0.8167	0.7333	**0.8667**	0.8500	0.7500	0.6500
F04	0.5000	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F05	0.2333	0.9222	0.9667	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F06	0.5000	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	0.9833	0.9833	0.8500
F07	0.3333	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F08	0.5000	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F09	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
F10	0.0909	0.8273	0.9818	0.9970	**1.0000**	0.9939	**1.0000**	**1.0000**	0.9939
F11	0.0667	0.6244	0.8933	0.9578	0.9689	**0.9867**	0.9644	0.9578	0.9689
F12	0.0769	0.7103	0.9872	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F13	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	0.7667	0.0333	0.0333	0.0000
F14	0.1250	0.8208	0.8875	0.9583	**0.9792**	0.9625	0.9500	0.8917	0.8750
F15	0.5333	**1.0000**	**1.0000**	**1.0000**	0.9000	0.9333	0.8333	0.8333	0.8000
F16	0.1429	0.7476	0.9762	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F17	0.3111	0.9556	0.9889	**1.0000**	**1.0000**	**1.0000**	0.9889	**1.0000**	0.9778
F18	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F19	0.1000	0.7400	0.8367	0.8767	0.8633	**0.8867**	0.8633	0.8300	0.8267
F20	0.1111	0.8185	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F21	0.0769	0.6795	0.9641	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F22	0.0625	0.4771	0.7063	0.8396	0.8958	0.9417	**0.9667**	0.9563	0.9333
F23	0.1667	0.9833	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F24	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
F25	0.1429	0.8952	0.9952	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F26	0.2500	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F27	0.1667	0.9778	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F28	0.1250	0.6833	0.7875	0.9125	0.9167	0.9542	**0.9750**	0.9500	**0.9792**
F29	0.4333	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F30	0.0833	0.6833	0.9250	0.9167	**0.9389**	0.8778	0.8722	0.8917	0.8556
F31	0.3833	0.9833	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F32	0.2500	0.9833	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F33	0.2500	0.9917	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F34	0.0333	0.4667	**0.5000**	**0.5000**	**0.5000**	**0.5000**	**0.5000**	0.4667	0.4167
F35	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F36	0.5000	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F37	0.9333	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	0.9667	**1.0000**
F38	0.3333	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F39	0.5000	0.9000	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F40	0.1933	0.6933	0.8467	0.9533	0.9533	**0.9867**	0.9400	0.9600	0.9133
F41	0.2500	0.9917	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F42	0.4500	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
Avg.	0.3121	0.8319	0.9014	**0.9221**	0.9200	0.9117	0.8783	0.8719	0.8582

TABLE S-R-XIV
INFLUENCE OF DIFFERENT $t_{\max}$ IN DR-JADE IN TERMS OF THE SUCCESS RATE.

Prob.	$t_{\max}=1$	$t_{\max}=10$	$t_{\max}=20$	$t_{\max}=30$	$t_{\max}=40$	$t_{\max}=50$	$t_{\max}=60$	$t_{\max}=70$	$t_{\max}=80$
F01	0.0000	**1.0000**	**1.0000**	**1.0000**	**1.0000**	0.4667	0.0333	0.0333	0.0000
F02	0.0000	**1.0000**	**1.0000**	**1.0000**	0.9667	**1.0000**	**1.0000**	**1.0000**	0.9667
F03	0.0000	0.1333	0.3333	0.6667	0.5333	**0.7333**	0.7000	0.5333	0.4000
F04	0.0000	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F05	0.0000	0.7667	0.9000	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F06	0.0000	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	0.9667	0.9667	0.7000
F07	0.0000	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F08	0.0000	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F09	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
F10	0.0000	0.0000	0.8333	0.9667	**1.0000**	0.9333	**1.0000**	**1.0000**	0.9333
F11	0.0000	0.0000	0.0667	0.4667	0.6667	**0.8000**	0.5667	0.5333	0.6000
F12	0.0000	0.0000	0.8333	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F13	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	0.7667	0.0333	0.0333	0.0000
F14	0.0000	0.0667	0.3333	0.6667	**0.8333**	0.7667	0.6667	0.3667	0.3000
F15	0.5333	**1.0000**	**1.0000**	**1.0000**	0.9000	0.9333	0.8333	0.8333	0.8000
F16	0.0000	0.0000	0.8333	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F17	0.0000	0.8667	0.9667	**1.0000**	**1.0000**	**1.0000**	0.9667	1.0000	0.9333
F18	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F19	0.0000	0.0000	0.0333	0.0333	0.1000	**0.1667**	0.0333	0.0000	0.0333
F20	0.0000	0.0333	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F21	0.0000	0.0000	0.5667	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F22	0.0000	0.0000	0.0000	0.0333	0.1000	0.3000	**0.6000**	0.5333	0.3333
F23	0.0000	0.9000	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F24	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
F25	0.0000	0.3000	0.9667	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F26	0.0000	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F27	0.0000	0.9000	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F28	0.0000	0.0333	0.0333	0.4333	0.5333	0.6667	**0.8333**	0.6333	**0.8333**
F29	0.0000	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F30	0.0000	0.0000	0.3333	0.2667	**0.4667**	0.2333	0.2000	0.3000	0.2000
F31	0.0000	0.9667	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F32	0.0000	0.9333	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F33	0.0000	0.9667	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F34	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
F35	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F36	0.0000	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F37	0.9333	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	0.9667	**1.0000**
F38	0.0000	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F39	0.0000	0.8000	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F40	0.0000	0.0333	0.3667	0.7667	0.7667	**0.9333**	0.7333	0.8000	0.5667
F41	0.0000	0.9667	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F42	0.0000	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
Avg.	0.1063	0.5873	0.7476	0.8167	**0.8302**	0.8262	0.7897	0.7746	0.7524

TABLE S-R-XV
THE OBTAINED ROOTS OF DR-JADE FOR F43 IN A TYPICAL RUN.

Item	$f(\mathbf{x})$	x_1	x_2	x_3	x_4	x_5
1	2.52E-06	0.0046	23.2599	0.0793	0.8592	0.0369
2	9.81E-06	0.0061	17.6346	0.0910	-0.8593	0.0369
3	5.57E-23	0.0028	39.2423	-0.0614	0.8597	0.0370
4	1.66E-25	0.0025	43.8792	0.0578	-0.8602	0.0370
5	4.87E-06	0.0049	22.0703	-0.0817	0.8594	0.0369
6	3.80E-06	0.0051	21.2273	0.0829	0.8592	0.0369
7	5.92E-06	0.0049	22.0197	0.0815	-0.8595	0.0369
8	4.10E-06	0.0051	21.0061	0.0833	0.8592	0.0369
9	5.57E-18	0.0031	34.5979	0.0650	0.8594	0.0370

TABLE S-R-XVI
THE OBTAINED ROOTS OF DR-JADE FOR F44 IN A TYPICAL RUN.

Item	$f(\mathbf{x})$	x_1	x_2	x_3	x_4	x_5	x_6
1	2.53E-14	-1.0000	1.0000	-0.0010	0.0010	0.1308	0.1308
2	1.51E-26	0.9182	-0.3376	-0.3962	-0.9413	0.0000	0.0000
3	2.86E-21	1.0000	-1.0000	0.0000	0.0000	-5.9134	5.9134
4	8.00E-26	-1.0000	1.0000	0.0000	0.0000	6.7063	6.7063
5	1.17E-11	0.9994	0.9994	0.0342	0.0343	4.7359	-4.7331
6	3.67E-25	-1.0000	-1.0000	0.0000	0.0000	-4.8628	-4.8628
7	3.55E-11	0.9989	0.9989	0.0463	0.0463	-2.3882	2.3857
8	6.90E-23	1.0000	-1.0000	0.0000	0.0000	1.9428	-1.9428
9	1.37E-24	-0.2470	-0.8124	0.9690	0.5831	0.0000	0.0000
10	2.76E-23	0.0429	-0.1196	-0.9991	0.9928	0.0000	0.0000
11	8.59E-21	0.8676	0.8438	0.4973	0.5367	0.0000	0.0000
12	3.60E-19	-0.1749	-0.2383	0.9846	-0.9712	0.0000	0.0000
13	2.80E-27	0.9603	-0.0535	-0.2791	0.9986	0.0000	0.0000
14	1.40E-21	0.0053	0.0053	1.0000	1.0000	0.0000	0.0000
15	4.08E-28	1.0000	-1.0000	0.0000	0.0000	3.5762	-3.5762
16	7.72E-19	0.4951	0.6080	-0.8689	0.7940	0.0000	0.0000
17	1.85E-17	0.1148	0.3883	0.9934	-0.9215	0.0000	0.0000
18	1.48E-23	0.8189	-0.5688	0.5740	0.8225	0.0000	0.0000
19	2.43E-22	0.7117	0.2477	0.7025	-0.9688	0.0000	0.0000
20	3.85E-28	0.8985	0.9970	0.4389	0.0769	0.0000	0.0000
21	4.54E-27	-0.9965	0.9984	0.0834	-0.0558	0.0000	0.0000
22	1.61E-22	0.9832	-0.7625	0.1826	0.6470	0.0000	0.0000
23	2.79E-23	-0.8298	0.8513	-0.5581	0.5247	0.0000	0.0000

TABLE S-R-XVII
THE OBTAINED ROOTS OF DR-JADE FOR F45 IN A TYPICAL RUN.

Item	$f(\mathbf{x})$	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}
1	8.45E-17	-1.50E-07	-7.91E-05	-0.3738	-8.84E-06	-0.0930	0.1247	9.42E-06	0.3738	-0.1265	-0.0614
2	1.89E-15	-1.50E-07	5.22E-05	0.4098	3.58E-06	0.3878	0.0537	3.21E-06	-0.4097	-0.6243	0.2584
3	1.80E-15	-1.47E-07	-1.28E-07	-0.0902	-1.36E-05	-0.1267	-0.3482	1.18E-05	0.0903	-0.3701	0.5333
4	1.40E-14	-1.49E-07	-7.10E-09	0.0898	-2.10E-05	-0.6745	-0.9546	1.55E-05	-0.0898	0.9684	0.4704
5	4.27E-15	-7.50E-07	-8.30E-05	-0.0548	-2.04E-04	0.1802	0.1226	1.07E-04	0.0548	-0.5853	0.1701
6	3.59E-15	-1.48E-07	-2.14E-06	-0.3287	1.40E-05	-0.4207	-0.0645	-1.99E-06	0.3288	0.8963	-0.3837
7	4.51E-15	-1.48E-07	-2.21E-04	-0.1598	4.04E-05	0.6407	0.9696	-1.52E-05	0.1599	-0.9433	-0.4979
8	1.08E-12	-2.76E-06	-5.20E-04	-0.0484	-8.68E-04	-0.4874	-0.5744	4.39E-04	0.0484	0.7037	0.2228
9	7.23E-12	-3.26E-06	6.77E-04	0.1871	1.29E-03	0.1306	-0.0839	-6.42E-04	-0.1871	-0.3151	0.2411
10	4.70E-15	2.41E-07	1.74E-05	0.0462	-2.05E-04	-0.1693	-0.3302	1.08E-04	-0.0462	0.1090	0.2757
11	8.61E-15	-2.15E-08	7.28E-05	-0.0706	1.85E-04	-0.0020	-0.3138	-8.76E-05	0.0706	-0.7606	0.6941
12	5.11E-15	-1.49E-07	-9.50E-08	0.5363	2.58E-05	0.1293	-0.5919	-7.90E-06	-0.5363	-0.6283	0.9060
13	1.05E-15	-1.51E-07	1.14E-04	0.5400	6.31E-05	0.4833	0.2692	-2.65E-05	-0.5400	-0.3145	-0.1120
14	7.15E-16	-1.52E-07	-7.31E-05	-0.3109	2.72E-05	0.0012	0.0982	-8.58E-06	0.3110	-0.4305	0.1171
15	2.64E-15	-1.34E-07	-1.24E-04	-0.0993	6.93E-05	0.3020	0.3109	-2.97E-05	0.0993	-0.7850	0.0817
16	1.99E-12	-6.67E-08	-6.59E-04	0.3120	9.90E-04	0.1055	-0.5487	-4.90E-04	-0.3120	-0.8963	0.9972
17	2.94E-15	-1.38E-07	-5.60E-06	-0.5615	-1.18E-04	-0.4720	-0.4325	6.38E-05	0.5615	-0.1002	0.4827
18	1.05E-12	-1.52E-05	6.32E-04	-0.0152	-5.73E-04	-0.5839	-0.7723	2.92E-04	0.0153	0.7610	0.3915
19	1.10E-14	-1.37E-07	2.15E-04	0.3415	1.46E-04	0.3442	0.0142	-6.81E-05	-0.3414	-0.6653	0.3184
20	4.52E-16	-1.50E-07	4.48E-05	0.4007	3.57E-05	0.2827	0.0400	-1.28E-05	-0.4007	-0.2496	0.0848
21	2.36E-15	-1.49E-07	9.15E-06	0.2409	2.22E-05	0.3016	-0.0090	-6.10E-06	-0.2409	-0.7426	0.3803
22	6.26E-15	-1.50E-07	-1.24E-06	-0.3076	2.80E-06	-0.5337	-0.7365	3.60E-06	0.3077	0.0463	0.7134
23	8.47E-15	-2.01E-07	3.55E-06	0.1246	-7.51E-05	-0.4663	-0.8290	4.26E-05	-0.1245	0.4561	0.6010
24	2.24E-15	-1.50E-07	1.42E-04	-0.6947	-7.92E-06	-0.3262	0.3991	8.96E-06	0.6947	0.7138	-0.7560
25	6.55E-13	-6.10E-07	-5.50E-04	0.1811	7.90E-05	-0.0089	-0.2167	-3.44E-05	-0.1810	-0.0361	0.2351
26	3.57E-15	-1.60E-07	8.17E-06	-0.3330	6.73E-05	-0.1902	-0.3931	-2.86E-05	0.3331	-0.6915	0.7389
27	3.27E-15	-1.61E-07	1.33E-05	-0.4898	-8.45E-05	-0.3721	-0.4547	4.73E-05	0.4898	-0.4005	0.6549
28	5.33E-16	-9.16E-08	-2.03E-04	0.0120	5.77E-05	0.4341	0.8183	-2.38E-05	-0.0120	-0.0757	-0.7804
29	1.81E-15	-1.52E-07	-8.46E-05	0.5620	1.03E-04	0.4066	0.0377	-4.64E-05	-0.5620	-0.4271	0.1759
30	1.06E-13	2.91E-05	2.40E-04	0.0030	-5.32E-04	-0.1280	0.2410	2.71E-04	-0.0030	1.0000	-0.7411

TABLE S-R-XVIII
THE OBTAINED ROOTS OF DR-JADE FOR F46 IN A TYPICAL RUN.

Item	$f(\mathbf{x})$	x_1	x_2	x_3	x_4	x_5
1	2.81E-27	-0.0524	2.5516	-1.7943	-1.7050	-0.5865
2	2.49E-29	1.3493	1.7490	2.1998	-6.2981	-0.1588
3	1.16E-27	1.0524	-1.4966	0.0307	-0.5865	-1.7050