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HW#4: Proof Buddy

Every question of this assignment will be completed in Proof Buddy – you can find the link to Proof Buddy on the bbLearn course page in the lefthand navigation panel. Do NOT hit the "Submit" button until you are completely finished with ALL the problems, since you will not be able to re-enter it again. (Frequent use of the Save button to protect your work in progress is certainly encouraged though, just don't "Submit" until fully complete). Because the Proof Buddy server can be a bit finicky, as a backup, we are also requiring you to submit your solution into bbLearn: create a pdf with a full screenshot of your solution, one question per page. Your screenshot must include the green sentence reading "Result: The proof is valid and complete!" or the error message. If you cannot get your answer to work, partial credit might be given, but you must include at least one error message in order to earn partial credit.

Note that if there's nothing before the ∴ symbol, then it means that problem has NO premises at all, and that the expression shown is the conclusion you need to arrive at. Be aware that efficiency, while not the priority, could be a factor in your score if your proof takes unnecessary steps.

For the first eight questions, you may use the Basic Rules of TFL only. You will get NO CREDIT for a problem if you utilize an unpermitted rule. For questions #9: you may utilize every possible rule, including FOL and derived rules or your own created lemmas. Each problem is worth seven pts (98 pts total) and you get two points for writing your name at the top of this cover sheet.

Question1: $\therefore$ A ∨ ¬A [doable in 8 steps]

Line #	Expression	Rule
1.1	¬(A ∨ ¬A)	Assumption
1.2.1	A	Assumption
1.2.2	A ∨ ¬A	∨I 1.2.1
1.2.3	⊥	¬E 1.1, 1.2.2
1.3	¬A	¬I 1.2
1.4	A ∨ ¬A	∨I 1.3
1.5	⊥	¬E 1.1, 1.4
2	A ∨ ¬A	IP 1

Check Proof Save

Great job! Don't forget to save your work by clicking "Save"! This will allow you to view this correct proof later!

Result: The proof is valid and complete!

Question2: $\therefore A \leftrightarrow \neg\neg A$ [doable in 9 steps]

Line #	Expression	Rule
1.1	A	Assumption
1.2.1	¬A	Assumption
1.2.2	⊥	¬E 1.2.1, 1.1
1.3	¬¬A	¬I 1.2
2.1	¬¬A	Assumption
2.2.1	¬A	Assumption
2.2.2	⊥	¬E 2.1, 2.2.1
2.3	A	IP 2.2
3	A ↔ ¬¬A	↔I 1, 2

Check Proof Save

Great job! Don't forget to save your work by clicking "Save"! This will allow you to view this correct proof later!

Result: The proof is valid and complete!

Question3: $\therefore (A \vee B) \leftrightarrow (B \vee A)$ [doable in 13 steps]

Line #	Expression	Rule
1.1	A ∨ B	Assumption
1.2.1	A	Assumption
1.2.2	B ∨ A	∨I 1.2.1
1.3.1	B	Assumption
1.3.2	B ∨ A	∨I 1.3.1
1.4	B ∨ A	∨E 1.1, 1.2, 1.3
2.1	B ∨ A	Assumption
2.2.1	A	Assumption
2.2.2	A ∨ B	∨I 2.2.1
2.3.1	B	Assumption
2.3.2	A ∨ B	∨I 2.3.1
2.4	A ∨ B	∨E 2.1, 2.2, 2.3
3	(A ∨ B) ↔ (B ∨ A)	↔I 1, 2

Check Proof Save

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Result: The proof is valid and complete!

Question4: $\therefore (A \rightarrow B) \leftrightarrow (\neg A \vee B)$ [doable in 19 steps]

Question5: ∴ ¬(A ∨ B) ↔ (¬A ∧ ¬B) [doable in 21 steps]

Line #	Expression	Rule
1.1	¬(A ∨ B)	Assumption
1.2.1	A	Assumption
1.2.2	A ∨ B	∨I 1.2.1
1.2.3	⊥	¬E 1.1, 1.2.2
1.3	¬A	¬I 1.2
1.4.1	B	Assumption
1.4.2	A ∨ B	∨I 1.4.1
1.4.3	⊥	¬E 1.1, 1.4.2
1.5	¬B	¬I 1.4
1.6	¬A ∧ ¬B	∧I 1.3, 1.5
2.1	¬A ∧ ¬B	Assumption
2.2.1	A ∨ B	Assumption
2.2.2.1	A	Assumption
2.2.2.2	¬A	∧E 2.1
2.2.2.3	⊥	¬E 2.2.2.2, 2.2.2.1
2.2.3.1	B	Assumption
2.2.3.2	¬B	∧E 2.1
2.2.3.3	⊥	¬E 2.2.3.2, 2.2.3.1
2.2.4	⊥	∨E 2.2.1, 2.2.2, 2.2.3
2.3	¬(A ∨ B)	¬I 2.2
3	¬(A ∨ B) ↔ (¬A ∧ ¬B)	↔I 1, 2

Check Proof Save

Great job! Don't forget to save your work by clicking "Save"! This will allow you to view this correct proof later!

Result: The proof is valid and complete!

Question6: $\therefore (A \wedge (B \vee C)) \leftrightarrow ((A \wedge B) \vee (A \wedge C))$ [doable in 23 steps]

Line #	Expression	Rule
1.1	A ∧ (B ∨ C)	Assumption
1.2	A	∧E 1.1
1.3	B ∨ C	∧E 1.1
1.4.1	B	Assumption
1.4.2	A ∧ B	∧I 1.2, 1.4.1
1.4.3	(A ∧ B) ∨ (A ∧ C)	∨I 1.4.2
1.5.1	C	Assumption
1.5.2	A ∧ C	∧I 1.2, 1.5.1
1.5.3	(A ∧ B) ∨ (A ∧ C)	∨I 1.5.2
1.6	(A ∧ B) ∨ (A ∧ C)	∨E 1.3, 1.4, 1.5
2.1	(A ∧ B) ∨ (A ∧ C)	Assumption
2.2.1	A ∧ B	Assumption
2.2.2	A	∧E 2.2.1
2.2.3	B	∧E 2.2.1
2.2.4	B ∨ C	∨I 2.2.3
2.2.5	A ∧ (B ∨ C)	∧I 2.2.2, 2.2.4
2.3.1	A ∧ C	Assumption
2.3.2	A	∧E 2.3.1
2.3.3	C	∧E 2.3.1
2.3.4	B ∨ C	∨I 2.3.3
2.3.5	A ∧ (B ∨ C)	∧I 2.3.2, 2.3.4
2.4	A ∧ (B ∨ C)	∨E 2.1, 2.2, 2.3
3	(A ∧ (B ∨ C)) ↔ ((A ∧ B) ∨ (.	↔I 1, 2

Check Proof Save

Great job! Don't forget to save your work by clicking "Save"! This will allow you to view this correct proof later!

Result: The proof is valid and complete!

Question7: $\therefore (A \vee (B \wedge C)) \leftrightarrow ((A \vee B) \wedge (A \vee C))$ [doable in 24 steps]

Line #	Expression	Rule
1.1	$A \vee (B \wedge C)$	Assumption
1.2.1	A	Assumption
1.2.2	$A \vee B$	∨I 1.2.1
1.2.3	$A \vee C$	∨I 1.2.1
1.2.4	$(A \vee B) \wedge (A \vee C)$	∧I 1.2.2, 1.2.3
1.3.1	$B \wedge C$	Assumption
1.3.2	B	∧E 1.3.1
1.3.3	$A \vee B$	∨I 1.3.2
1.3.4	C	∧E 1.3.1
1.3.5	$A \vee C$	∨I 1.3.4
1.3.6	$(A \vee B) \wedge (A \vee C)$	∧I 1.3.3, 1.3.5
1.4	$(A \vee B) \wedge (A \vee C)$	∨E 1.1, 1.2, 1.3
2.1	$(A \vee B) \wedge (A \vee C)$	Assumption
2.2	$A \vee B$	∧E 2.1
2.3	$A \vee C$	∧E 2.1
2.4.1	A	Assumption
2.4.2	$A \vee (B \wedge C)$	∨I 2.4.1
2.5.1	B	Assumption
2.5.2.1	C	Assumption
2.5.2.2	$B \wedge C$	∧I 2.5.1, 2.5.2.1
2.5.2.3	$A \vee (B \wedge C)$	∨I 2.5.2.2
2.5.3	$A \vee (B \wedge C)$	∨E 2.3, 2.4, 2.5.2
2.6	$A \vee (B \wedge C)$	∨E 2.2, 2.4, 2.5
3	$(A \vee (B \wedge C)) \leftrightarrow ((A \vee B) \wedge ($	↔I 1, 2

Check Proof Save

Great job! Don't forget to save your work by clicking "Save"! This will allow you to view this correct proof later!

Result: The proof is valid and complete!

Question8: $\therefore\ \neg(A \wedge B) \leftrightarrow (\neg A \vee \neg B)$ [doable in 24 steps]

Line #	Expression	Rule
1.1	¬ (A ∧ B)	Assumption
1.2.1	¬(¬ A ∨ ¬ B)	Assumption
1.2.2.1	¬A	Assumption
1.2.2.2	¬ A ∨ ¬ B	∨I 1.2.2.1
1.2.2.3	⊥	¬E 1.2.1, 1.2.2.2
1.2.3	A	IP 1.2.2
1.2.4.1	B	Assumption
1.2.4.2	A ∧ B	∧I 1.2.3, 1.2.4.1
1.2.4.3	⊥	¬E 1.1, 1.2.4.2
1.2.5	¬B	¬I 1.2.4
1.2.6	¬ A ∨ ¬B	∨I 1.2.5
1.2.7	⊥	¬E 1.2.1, 1.2.6
1.3	¬ A ∨ ¬B	IP 1.2
2.1	¬ A ∨ ¬B	Assumption
2.2.1	A ∧ B	Assumption
2.2.2	A	∧E 2.2.1
2.2.3	B	∧E 2.2.1
2.2.4.1	¬A	Assumption
2.2.4.2	⊥	¬E 2.2.4.1, 2.2.2
2.2.5.1	¬B	Assumption
2.2.5.2	⊥	¬E 2.2.5.1, 2.2.3
2.2.6	⊥	∨E 2.1, 2.2.4, 2.2.5
2.3	¬ (A ∧ B)	¬I 2.2
3	¬ (A ∧ B)↔(¬ A ∨ ¬B)	↔I 1, 2

Check Proof Save

Great job! Don't forget to save your work by clicking "Save"! This will allow you to view this correct proof later!

Result: The proof is valid and complete!

QUESTION 9: [doable in 9 steps]

$$\therefore (\forall x{\in}U\ Fx) \vee (\exists y{\in}U\ \neg Fy)$$

Line #	Expression	Rule
1.1	∀x∈U Fx	Assumption
1.2	(∀x∈U Fx) ∨ (∃y∈U ¬Fy)	∨I 1.1
2.1	¬(∀x∈U Fx)	Assumption
2.2	∃x∈U ¬Fx	CQ 2.1
2.3.1	¬Fa	Assumption
2.3.2	∃y∈U ¬Fy	∃I 2.3.1
2.4	∃y∈U ¬Fy	∃E 2.2, 2.3
2.5	(∀x∈U Fx) ∨ (∃y∈U ¬Fy)	∨I 2.4
3	(∀x∈U Fx) ∨ (∃y∈U ¬Fy)	LEM 1, 2

Check Proof Save

Great job! Don't forget to save your work by clicking "Save"! This will allow you to view this correct proof later!

Result: The proof is valid and complete!

QUESTION 10: [doable in 7 steps]

$$\forall x \in U\ (Fx \Rightarrow Gx)\ ;\ \exists y \in U\ Fy \therefore \exists z \in U\ Gz$$

Line #	Expression	Rule
1	∀x∈U (Fx → Gx)	Premise
2	∃y∈U Fy	Premise
3.1	Fa	Assumption
3.2	Fa → Ga	∀E 1
3.3	Ga	→E 3.2, 3.1
3.4	∃z∈U Gz	∃I 3.3
4	∃z∈U Gz	∃E 2, 3

Check Proof Save

Great job! Don't forget to save your work by clicking "Save"! This will allow you to view this correct proof later!

Result: The proof is valid and complete!

QUESTION 11: [doable in 12 steps]

∀x∈U (¬Mx ∨ Lax) ; ∀y∈U (By ⇒ Lay) ; ∀z∈U (Mz ∨ Bz) ∴ ∀w∈U Law

Line #	Expression	Rule
1	∀x∈U (¬Mx ∨ Lax)	Premise
2	∀y∈U (By →Lay)	Premise
3	∀z∈U (Mz ∨ Bz)	Premise
4	¬Mc ∨ Lac	∀E 1
5	Bc → Lac	∀E 2
6	Mc ∨ Bc	∀E 3
7.1	¬Mc	Assumption
7.2	Bc	DS 6, 7.1
7.3	Lac	→E 5, 7.2
8.1	Lac	Assumption
9	Lac	∨E 4, 7, 8
10	∀w∈U Law	∀I 9

Check Proof Save

Great job! Don't forget to save your work by clicking "Save"! This will allow you to view this correct proof later!

Result: The proof is valid and complete!

QUESTION 12: [doable in 6 steps]

$$Pa \lor Qb \; ; \; Qb \Rightarrow (b = c) \; ; \; \neg Pa \therefore Qc$$

Line #	Expression	Rule
1	Pa v Qb	Premise
2	Qb → (b = c)	Premise
3	¬ Pa	Premise
4	Qb	DS 1, 3
5	b = c	→E 2, 4
6	Qc	=E 5, 4

Check Proof Save

Great job! Don't forget to save your work by clicking "Save"! This will allow you to view this correct proof later!

Result: The proof is valid and complete!

QUESTION 13: [doable in 8 steps]

$$\forall x{\in}U\ \forall y{\in}U\ (Rxy \Rightarrow (x = y)) \therefore\ Rab \Rightarrow Rba$$

Line #	Expression	Rule
1	∀x∈U ∀y∈U (Rxy → (x = y))	Premise
2	∀y∈U (Ray → (a = y))	∀E 1
3	Rab → (a = b)	∀E 2
4.1	Rab	Assumption
4.2	a = b	→E 3, 4.1
4.3	Rbb	=E 4.2, 4.1
4.4	Rba	=E 4.2, 4.3
5	Rab → Rba	→I 4

Check Proof Save

Great job! Don't forget to save your work by clicking "Save"! This will allow you to view this correct proof later!

Result: The proof is valid and complete!

QUESTION 14: [doable in 8 steps]

Suppose for a particular program the following two facts are known:
Fact1: Whenever any variable is out of bounds, then that variable creates an infinite loop.
Fact2: the loop terminated when initialized with the value stored in a specific variable named c.

We will be expressing these in first order logic. Let V be the set of possible variables. There are only two predicates you need the range over this set: let B(x) be a predicate stating "the variable x is in Bounds", and let `M(x)` be a predicate stating "the loop `terMinates` when initialized with the value stored in x".

[a] express Fact1 in FOL.

[b] express Fact2 in FOL

[c] a programmer claims that there must be a variable which is in bounds. Express this claim in FOL.

[d] Use the proof checker in FOL mode to show the programmer's claim is correct given the known facts. You may use any rule you wish; you are not limited to only the Basic rules, but it is not necessary to go beyond them if you don't wish to. Copy-Paste a screenshot of your proof in the space below