Homework 4

Question 1

For each of the following sequences, find a closed formula for a_n, the n^{th} term of the sequence. Assume the first term is a_0. Briefly show how you got your answer.

Part a

$$4, 5, 7, 11, 19, 35, \ldots$$

Part b

$$0, 3, 8, 15, 24, 35, \ldots$$

Part c

$$6, 12, 20, 30, 42, \ldots$$

Part d

$$0, 2, 7, 15, 26, 40, 57, \ldots$$

Hint: These are sometimes called *house numbers*.

Question 2

Write out the first five terms, starting with a_0, for the sequence below. Then, provide a closed formula.

$$a_n = 3a_{n-1} - 2a_{n-2}, \text{ with } a_0 = 0,\ a_1 = 1$$

Question 3

Write out the first five terms, starting with a_0, for the sequence below. Then, provide a recursive formula.

$$a_n = n^2 + n$$

Hint: Consider 2.1.3a from the book.

Question 4

Show that $a_n = 2^n - 5^n$ is a solution to the recurrence relation $a_n = 7a_{n-1} - 10a_{n-2}$. What would the initial conditions need to be for this to be the closed formula for the sequence?

Hint: Consider 3.1.9

Question 5

Compute the sum $2 + 5 + 8 + \cdots + 59$.

Question 6

Compute the sum $-3 + 1 + \cdots + 77$.

Question 7

Find a closed form for the sum of the sequence $a_n = 1 + 4n$ for $n \geq 0$.

Question 8

Compute the sum of the sequence $3 \cdot 2^n$ for $n \geq 0$.

Question 9

Express the repeating decimal $0.37373737373737\ldots$ as a fraction (without decimals, of course).

Question 10

Express the repeating decimal $0.213213213213\ldots$ as a fraction (without decimals, of course).