MATH 180 - Homework 4

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Question 1

Part a

$\because$ Each term differs from the nth power of 2 by 3.
$\therefore a_n = 2^n + 3$

Part b

$\because$ Each number is one less than the integer square.
$\therefore a_n = (n+1)^2 - 1$

Part c

$\because$ Each element is the product of two adjacent numbers.
$\therefore a_n = (n+2)(n+3) = n^2 + 5n + 6.$

Part d

$\because$ The difference between each two items is increased by 3 each time.
$\therefore a_n = \frac{n(3n+1)}{2}$

Question 2

$a_0 = 0, a_1 = 1, a_2 = 3, a_3 = 7, a_4 = 15, a_5 = 31$
Closed formula: $a_n = 2^n - 1$

Question 3

$a_0 = 0, a_1 = 2, a_2 = 6, a_3 = 12, a_4 = 20, a_5 = 30$
Recursive formula: $a_n = a_{n-1} + 2n$ with $a_0 = 0$

Question 4

$7a_{n-1} - 10a_{n-2}$
$= 7(2^{n-1} + 5^{n-1}) - 10(2^{n-2} + 5^{n-2})$
$= 4 \cdot 2^{n-2} - 25 \cdot 5^{n-2}$
$= 2^n - 5^n = a_n$
$\therefore$ The initial conditions are $a_0 = 0$ and $a_1 = -3$.

Question 5

$(2 + 59) \times 10 = 610$

Question 6

$\frac{(77-3) \times 21}{2} = 777$

Question 7

$\because a_0 = 1$
$\therefore S_n = n + 2n(n+1) + 1 = 2n^2 + 3n + 1$

Question 8

$S - 2S = -S = 3 - 3 \cdot 2^{n+1}$
$S_n = 3 \cdot 2^{n+1} - 3$

Question 9

Let $N = 0.\overline{37}$
$\because 0.99N = 0.37$
$\therefore 0.\overline{37} = \frac{37}{99}$

Question 10

Let $N = 0.\overline{213}$
$\because 0.999N = 0.213$
$\therefore 0.\overline{213} = \frac{213}{999}$