MATH 180 - Homework 7

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Question 1

x	y	$x \Leftrightarrow y$	$x \Rightarrow y$	$y \Rightarrow x$	$(x \Rightarrow y) \wedge (y \Rightarrow x)$
T	T	T	T	T	T
T	F	F	F	T	F
F	T	F	T	F	F
F	F	T	T	T	T

Question 2

x	y	z	$(x \vee y) \Leftrightarrow z$	$x \Rightarrow z$	$y \Rightarrow z$	$(x \Rightarrow z) \wedge (y \Rightarrow z)$
T	T	T	T	T	T	T
T	T	F	F	F	F	F
T	F	T	T	T	T	T
T	F	F	F	F	T	F
F	T	T	T	T	T	T
F	T	F	F	T	F	F
F	F	T	T	T	T	T
F	F	F	T	T	T	T

Question 3

x	y	$x \Rightarrow y$	$x \wedge (x \Rightarrow y)$	$(x \wedge (x \Rightarrow y)) \Rightarrow y$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

Question 4

Let m, n be an interger
$\therefore x, y$ can be written as $x = 2m + 1, y = 2n$.
$\therefore x + y = 2m + 2n + 1 = 2(m + n) + 1$
$\therefore$ For $x, y \in \mathbb{Z}, x + y$ is an odd integer.

Question 5

Let m, n be an interger
$\therefore x, y$ can be written as $x = 2m + 1, y = 2n + 1$.
$\therefore x \times y = (2m + 1)(2n + 1) = 4mn + 2m + 2n + 1 = 2(2mn + m + n) + 1$
$\therefore$ The product of two odd integers is odd.

Question 6

Let $k \in \mathbb{Z}$
$\therefore a|b$ can be written as $b = ka$
$\therefore bc = (ka)c = a(kc)$
$\therefore$ If $a|b$ then $a|bc$.

Question 7

Let $j, k \in \mathbb{Z}$
$\therefore b = ja, d = kc$
$\therefore bd = (ja)(kc) = jk(ac)$
$\therefore$ If $a|b$ and $c|d$, then $ac|bd$.

Question 8

Let $n = m + k$ for m^2, n^2.
$\therefore n^2 - m^2 = 2mk + k^2$ for some $k \geqslant 2$.
$\because 2mk + k^2 = k(2m + k)$
$\therefore$ The difference between distinct, nonconsecutive perfect squares is composite.

Question 9

The statement is equal to: $(a, b \in \mathbb{R}) \wedge (ab = 0) \Rightarrow (a = 0) \vee (b = 0)$
Its contradiction: $\neg(a, b \in \mathbb{R}) \vee \neg(ab = 0) \Rightarrow (a = 0) \vee (b = 0)$.
If $\neg(a, b \in \mathbb{R})$, then $a, b \neq 0$.
If $ab \neq 0$, then $a, b \neq 0$.
So the contradiction is false, the statement is true.

Question 10

Part a

Suppose not. Then $\sqrt{3}$ is equal to a fraction $\frac{a}{b}$.
Without loss of generality, assume $\frac{a}{b}$ is in lowest terms. So,

$$3 = \frac{a^2}{b^2}$$

$$3b^2 = a^2$$

Thus a^2 is divisible by 3. So $a = 3k$ for some integer k, and $a^2 = 9k^2$. We then have,
$3b^2 = 9k^2, b^2 = 3k^2$ which is a contradiction.
$\therefore \sqrt{3}$ is irrational.

Part b

Suppose not. Then $\sqrt[3]{2}$ is equal to a fraction $\frac{a}{b}$.
Without loss of generality, assume $\frac{a}{b}$ is in lowest terms. So,

$$2 = \frac{a^3}{b^3}$$

$$2b^3 = a^3$$

Thus a^3 is even. So $a = 2k$ for some integer k, and $a^3 = 8k^3$. We then have,
$2b^3 = 8k^3, b^3 = 2k^3$ which is a contradiction.
$\therefore \sqrt[3]{2}$ is irrational.