Homework 2

Due: Wednesday, January 25th

Question 1

Consider the function $f : \{1, 2, 3, 4, 5\} \to \{1, 2, 3, 4, 5\}$ given by

$$f(n) = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 5 & 3 & 1 & 4 \end{pmatrix}$$

Part a

Find $f(3)$

Part b

Find an element n in the domain for which $f(n) = n$

Part c

Is this function a bijection? Explain.

Question 2

Each of the following functions has $\{1, 2, 3, 4, 5, 6\}$ as its domain and co-domain.

For each, determine whether it is (only) injective, (only) surjective, bijective, or none of the above.

Part a

$$f(n) = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 5 & 3 & 1 & 4 & 1 \end{pmatrix}$$

Part b

$$f(n) = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 5 & 3 & 6 & 2 & 1 & 4 \end{pmatrix}$$

1

Part c

$$f(x) = |5 - x|$$

Question 3

Write out all of the functions $f : \{1, 2\} \to \{a, b, c\}$ (using two-line notation). - How many functions are there? - How many are injective? - How many are surjective? - How many are bijective?

Question 4

Consider the function $f : \mathbb{N} \to \mathbb{N}$ given recursively by $f(0) = 2$ and $f(n+1) = 3 \cdot f(n)$. Find $f(6)$.

Question 5

Suppose $f : X \to Y$ is a function. Which of the following are possible? Explain.

Part a

f is injective but not surjective.

Part b

f is surjective but not injective.

Part c

$|X| = |Y|$ and f is injective but nor surjective.

Note: X and Y could be infinite sets.

Part d

$|X| = |Y|$ and f is surjective but nor injective.

Note: X and Y could be infinite sets.

Part e

$|X| = |Y|$, X and Y are finite, and f is injective but not surjective.

Part e

$|X| = |Y|$, X and Y are finite, and f is surjective but not injective.

Question 6

Your wardrobe consists of 7 shirts, 5 pairs of pants, and 12 hats. How many different outfits can you make?

Question 7

A group of college students were asked about their TV watching habits. Of those surveyed, 30 students watch The The Office, 23 watch Parks and Rec, and 18 watch Superstore. Additionally, 14 watch The Office and Parks and Rec, 12 watch Parks and Rec and Superstore, and 11 watch The Office and Superstore. There are 7 students who watch all three shows. How many students surveyed watched at least one of the shows?

Question 8

For how many $n \in \{1, 2, 3, \ldots, 800\}$ is n a multiple of 2, 3, or 5? Explain your answer using the Principle of Inclusion/Exclusion.

Question 9

Consider all 6 letter "words" made from the letters a through j. (In this context words are just strings of letters, not necessarily actual English words.)

Part a

How many of these words are there total?

Part b

How many of these words contain no repeated letters?

Part c

How many of these words start with the sub-word "kid"?

Part d

How many of these words either start with "aha" or end with "bah" or both?

Part e

How many of the words containing no repeats also do not contain the sub-word "hid"?

Question 10

For how many two-digit numbers (10 - 99) is the *sum of the digits* even? Explain your solution.