CS270: LAB #13

last TFL Natural Deduction rules

You may work in teams of ideally one or two people (three is acceptable in the event of an unscheduled absence). Unless stated otherwise, the lab is due to be submitted into Gradescope at the end of the day.
In order to receive credit, follow these instructions:

[a] Every team member should be discussing simultaneously the same problem – do NOT try to divvy up the labor and assign different problems to different students since the material is cumulative.

[b] Directly edit this lab PDF with your answers (extra pages can be added in the rare event you need more than the allotted space)

[c] Each lab, rotate which member has the responsibility of being the Scribe. This is the person that is typing the answers and uploading the final PDF – note that only a single copy of the filled in PDF is turned into Gradescope. Only one lab needs to be submitted for the entire team, and all members receive the same score. Make sure to use a font that your PDF editor is compatible with (otherwise you might find your answers appear as weird shapes/sizes or simply disappear entirely!)

[d] The Gradescope submission must have each answer properly tagged with the appropriate question. Moreover, every member of the team must be listed as a submitter. Although it is the Scribe which executes these actions, it is still the responsibility of the entire team to make certain this is done properly (thus it is highly recommended that the Scribe share their screen so the entire team can witness it). Answers which are improperly tagged cannot be seen by the grader and thus cannot be scored.

REMOTE ONLY:
[e] Each lab, rotate which member has the responsibility of being the Recorder. This is the person who hits the Zoom Record button (once the technical permission is granted by the TA/RCF/Professor) and ensures that everyone has their camera/microphone on. They are also the member that is responsible to make sure the DrexelStream video is marked as viewable and entered into the https://tinyurl.com/VidLinkForm webform before 11:59pm (they should also email the rest of their team as confirmation.) Note that the video file doesn't get created/processed until after the Recorder has quit Zoom.

[f] Each lab, rotate which member has the responsibility of being the Manager. This is the person that ensures that everyone is participating equally and honestly, keeps the group on task, ensures that all team members understand a solution before going on to the next question, and presses the "hand up" button in Zoom to summon a TA or the professor (but they only do so after surveying the group to make sure everyone has the same question).

Team Name (CS Pioneer): ____________________________

Scribe name: ____________________________

Recorder name: ____________________________

Manager name: ____________________________

Other team member (if any): ____________________________

Question 1 : 15 points

Contradictions are an important Proof method.

Let us assume we want to prove the following mathematical property.

First, a little notation.

$\mathbb{Z}$ is the set of all integers $\mathbb{Z} = \{\cdots, -3, -2, -1, 0, 1, 2, 3, \cdots\}$.

$\in$ is the symbol for in. It means a variable is selected from a set.

Saying $a \in \mathbb{Z}$ is saying that a is an integer.

$$(a, b \in \mathbb{Z}) \implies a^2 - 4b \neq 2$$

The easiest way to prove this is to assume the argument is false.

Assume $a^2 - 4b = 2$.

(a) (5 points) Solve the equation $a^2 - 4b = 2$ for b.

(b) (5 points) Assume that a is an even number. That means $a = 2 * x$ where $x \in \mathbb{Z}$. Explain why b is not an integer in this case.

(c) (5 points) Assume that a is an odd number. That means $a = 2 * y + 1$ where $y \in \mathbb{Z}$. Explain why b is not an integer in this case.

Conclusion: This proves that $(a, b \in \mathbb{Z}) \implies a^2 - 4b \neq 2$. We know this must be true because we have shown it is impossible for $a^2 - 4b = 2$ to be true. Therefore, we know the opposite must be true.

Question 2 : 16 points

We can use Contradiction in Proofs by Deduction. This will introduce the last 4 basic rules of Truth Function Logical Deduction.

Here is an example Proof by Deduction.

The Argument is $P \implies Q, \neg Q \therefore \neg P$.

1.	$P \implies Q$	Premise
2.	$\neg Q$	Premise
3.1	P	Assume
3.2	Q	$\implies$E 1, 3.1
3.3	$\perp$	$\neg$ E 2, 3.2
4.	$\neg P$	$\neg$I 3

A written explanation for the above proof is written below. The explanation in not in the correct order. Write the line number from the proof next to the sentence explaining it.

(a) (2 points) If P is true, then we know that Q must also be true.

(a) ______________

(b) (2 points) We assume that P is true.

(b) ______________

(c) (2 points) We know that Q is false.

(c) ______________

(d) (2 points) We have shown it is impossible for P to be true, therefore P must be false.

(d) ______________

(e) (2 points) Q is know to be True. This is because P is true.

(e) ______________

(f) (2 points) It is impossible for Q to be both true and false.

(f) ______________

(g) (2 points) The symbol $\perp$ is called an **up tack** or **falsum**. It is used to represent the constant false. Explain why the justification **¬ E** 2,3.2 needs to reference two lines.

(h) (2 points) Explain what the "3" in the justification " ¬ I 3" refers to

Question 3 : 16 points

When we assume a positive and reach a contradiction, this proves the opposite is true. This was the **Negative Introduction** rule.

A different rule is used when we assume a negative to prove a positive. It is the **Indirect Proof** written as IP.

1.	$\neg P \implies Q$	Premise
2.	$\neg Q$	Premise
3.1	$\neg P$	Assume
3.2	Q	$\implies$ E 1, 3.1
3.4	$\bot$	¬ E 2, 3.2
4.	P	IP 3

(a) (3 points) If we assume A then reach a contradiction to prove $\neg A$, which rule is used?

- ◯ Negative Introduction
- ◯ Indirect Proof

(b) (3 points) If we assume $\neg A$ then reach a contradiction to prove A, which rule is used?

- ◯ Negative Introduction
- ◯ Indirect Proof

(c) (10 points) Prove the following argument using the Proof Buddy tool linked on bbLearn
Paste a screenshot of your proof below.

$$P \to \neg P \therefore \neg P$$

Question 4 : 23 points

There is one final rule we will need. It is known as **The Principle of Explosion**.

The **Principle of Explosion** says *from falsehood, anything*. If a subproof reaches a contradiction, we can say anything is true.

1.	$P \wedge \neg P$	Premise
2.	P	$\wedge$E 1
3.	$\neg P$	$\wedge$E1
4.	$\perp$	¬ E 2,3
5.	Q	X 4

This proof is correct. Our argument is $P \wedge \neg P \therefore Q$. The symbol $\therefore$ (therefore) is another way to say implies. This argument is really saying $(P \wedge \neg P) \implies Q$.

(a)(8points)Fill in the Truth Table for the expression below (Red boxes=inputs, Blue=operations, Green=output)

(P	∧	¬ P)	→	Q

Note that Proof Buddy has a "Construct Counter Example" button
which enables you to show that a particular claim can not be p
which enables you to show that a particular claim can not be proven
(i.e. the conclusion does not logically follow from the premises

(b) (15 points)

Decide if the following claim is logically valid. If it is, use the tool to prove it and provide a screen shot.

If it's not, then provide a counter example (i.e. an assignment of True/False values to each of the propositional variables such that all the premises are true but the conclusion is false)

$$P \wedge (Q \vee R) \; ; \; P \rightarrow \neg R \;\therefore\; Q \vee E$$

Question 5 : 15 points

Decide if the following claim is logically valid. If it is, use the tool to prove it and provide a screen shot. If it's not, then provide a counter example (i.e. an assignment of True/False values to each of the propositional variables such that all the premises are true but the conclusion is false)

$$Q \to (Q \land \neg Q) \quad \therefore \quad \neg Q$$

Question 6 : 10 points

Decide if the following claim is logically valid. If it is, use the tool to prove it and provide a screen shot. If it's not, then provide a counter example (i.e. an assignment of True/False values to each of the propositional variables such that all the premises are true but the conclusion is false)

$$\neg F \to G \; ; \; F \to H \quad \therefore \quad G$$

Question 7: 5 points

Decide if the following claim is logically valid. If it is, use the tool to prove it and provide a screen shot. If it's not, then provide a counter example (i.e. an assignment of True/False values to each of the propositional variables such that all the premises are true but the conclusion is false)

$$\neg F \to G \; ; \; F \to H \quad \therefore \quad G \lor H$$