Math 180

Homework 6

Question 1

Use a truth table to prove that the following Boolean expressions are logically equivalent.

$$x \iff y$$
$$(x \implies y) \wedge (y \implies x)$$

Question 2

Use a truth table to prove that the following Boolean expressions are logically equivalent.

$$(x \vee y) \implies z$$
$$(x \implies z) \wedge (y \implies z)$$

Question 3

A *tautology* is a Boolean expression that evaluates to true for all possible values of its variables. For example, $x \vee \neg x$ is True whether x is True or False:

x	$\neg x$	$x \vee \neg x$
T	F	T
F	T	T

Prove that the following is a tautology:

$$(x \wedge (x \implies y)) \implies y$$

Question 4

Prove that if x is an odd integer and y is an even integer, then $x + y$ is and odd integer.

Question 5

Prove that the product of two odd integers is odd.

Question 6

Suppose that a, b, and c. Prove that if $a|b$ then $a|bc$.

Question 7

Suppose that a, b, c, and d are integers. Prove that if $a|b$ and $c|d$, then $ac|bd$.

Question 8

Prove that the difference between distinct, nonconsecutive perfect squares is composite.

Hints:

- We can represent nonconsecutive squares as m^2 and n^2 where $n = m + k$, for some $k \geq 2$.
- Composite means *not prime*. To show that a number is composite, we have to show that it is a multiple of a number other than 1 and itself.

Question 9

Prove by contradiction: If a and b are real numbers and $ab = 0$, then $a = 0$ or $b = 0$.

Question 10

Prove each statement by contradiction.

Part a

Prove $\sqrt{3}$ is irrational.

Part b

Prove $\sqrt[3]{2}$ is irrational.