Class 2 Notes – does not need to be submitted

Math reminder: Solving with exponents.

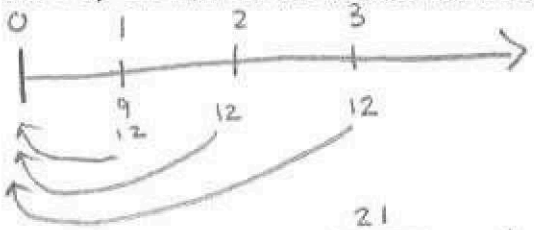
$100 = (1+r)^2 \rightarrow r = \sqrt{100} - 1 = 9 = 900\%$

$100 = (1+r)^{10} \rightarrow \sqrt[10]{100} = (1+r)$ or $100^{\frac{1}{10}} = (1+r)$

$1.58 = 1+r$

$r = 0.58 = 58\%$

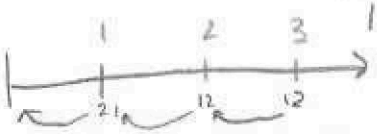
Example 1: The patriots need a new wide receiver. They are estimating that they will have to pay a $9 million signing bonus next year, as well as $12 million every year for the next three years (at t=1, t=2, and t=3). The interest rate is 6.1%. How much for they need to allocate now to be able to pay for this?



PV

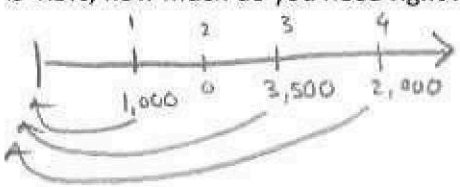
$$\frac{21}{1.061} + \frac{12}{1.061^2} + \frac{12}{1.061^3} = \$40.5 \text{ mill}$$

or



$\frac{12}{1.061} = 11.31$ $\quad \frac{11.31+12}{1.061} = 21.97$ $\quad \frac{21.97+21}{1.061} = 40.5$

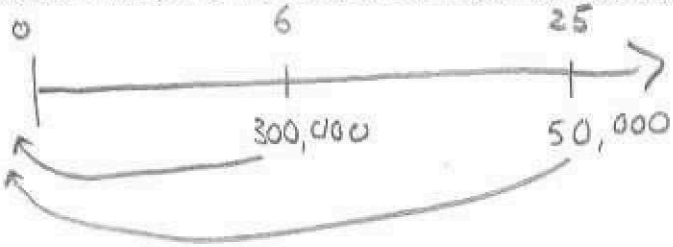
Example 2: You want to go on vacation in one year, which costs $1,000. Then you want to get married in three years, which will cost $3,500, and the honeymoon in four years will cost $2,000. If the interest rate is 4.5%, how much do you need right now to be able to pay for all these expenses?



$$\frac{1,000}{1.045} + \frac{3,500}{1.045^3} + \frac{2,000}{1.045^4}$$

$$= \$5,701.10$$

Example 3: You want to withdraw $300,000 for a house in 6 years, and $50,000 for your child's college education in 25 years. The interest rate is 7%. How much do you need to save right now?

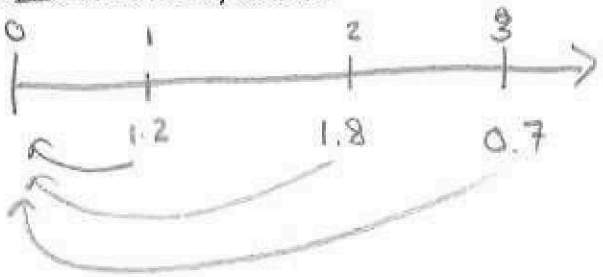


$$\frac{300,000}{1.07^6} + \frac{50,000}{1.07^{25}} = \$209,115.13$$

Example 4: A university wants to estimate the impact of the Covid-19 pandemic on its tuition revenue. The financial office believes that the school lost $1.2 million in 2022 (at the end of this year) and will lose $1.8 million in 2023 and $0.7 million in 2024. The prevailing rate of return is 5.9%.

a)How much does the school need to set aside right now to cover those losses?

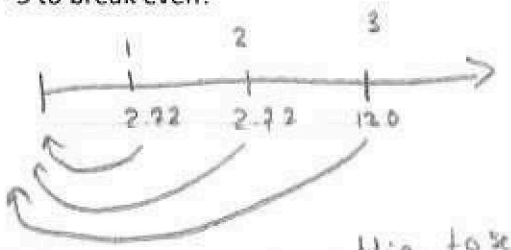
b)In three years, what was the total economic impact for the school? Assume that they could have invested the money at 5.9%.



$$\frac{1.2}{1.059} + \frac{1.8}{1.059^2} + \frac{0.7}{1.059^3}$$

$$= 3.33 \text{ mil}$$

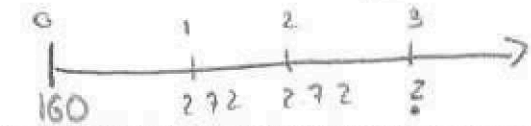
Example 5: Target will pay a dividend of $2.72 at the end of the year (t=1) and next year (t=2). Then, you think you can sell the stock for $120 at t = 3 (in three years). If your discount rate is 6.2%, how much are you willing to pay? If the price is $160 right now, what is the minimum you have to sell the stock for at t = 3 to break even?



Willing to pay

$$\frac{2.72}{1.062} + \frac{2.72}{1.062^2} + \frac{120}{1.062^3} = 105.14$$

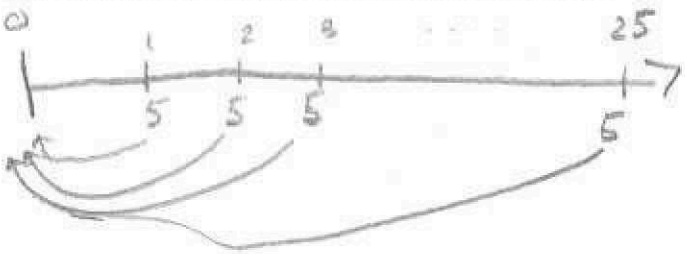
Min. to sell:



$$160 = \frac{2.72}{1.062} + \frac{2.72}{1.062^2} + \frac{x}{1.062^3}$$

$$155.03 = \frac{x}{1.062^3} \Rightarrow x = 185.69$$

Example 6: A firm just decided to buy a new plant. The expenses (paying employees, pay insurance, etc.) will be exactly $5,000,000 every year for the next 25 years, starting at the end of this year. The interest rate is 2.5%. What is the present value of the costs?



$$PV = \frac{5}{1.025} + \frac{5}{1.025^2} + \dots + \frac{5}{1.025^{25}}$$

$$= 92.12$$

Annuities

Adding up so many cash flows is possible, but annoying.

There is a quicker way to calculate the PV and FV: The annuity formula.

Use when you have the *same* cash flows, *evenly spaced*.

Words to look for: "every year for x years."

CAUTION: The annuity formula "assumes" that the first cash flow is at t=1.

When the first cash flow is after t=1, the annuity is called "deferred". → keyword: "starting at"

When the first cash flow is at t=0, the annuity is called "due".

Annuity PV:

$$PV = C\left[\frac{1-\frac{1}{(1+r)^t}}{r}\right]$$

"putting all CFs into a bowl"

number of CFs you are adding / saving "for t years"

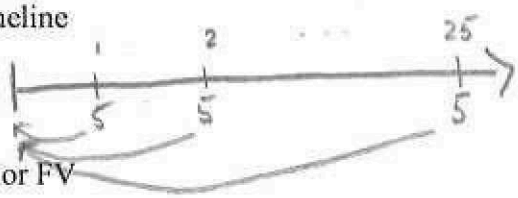
Annuity FV:

$$FV = C\left[\frac{(1+r)^t - 1}{r}\right]$$

Example 1:

A firm just decided to buy a new plant. The expenses (paying employees, pay insurance, etc.) will be exactly $5,000,000 every year for the next 25 years, starting at the end of this year. The interest rate is 2.5%. What is the present value of the costs?

1) Timeline



2) PV or FV

PV

3) Calculate

$$PV = 5\left[\frac{1-\frac{1}{1.025^{25}}}{0.025}\right] = 92.12$$

Example 2: You are currently paying a mortgage of $7,200 every year. You have 10 more years left to pay. You are thinking about refinancing. The new payments would be $6,000 annually, also for 10 years, with the first payment also being in one year (at t =1). Your discount rate is 5%. Should you refinance? Yes, refinance

currently:

$$PV = 7{,}200\left[\frac{1-\frac{1}{1.05^{10}}}{0.05}\right]$$

$$= 55{,}596.49$$

new:

$$PV = 6{,}000\left[\frac{1-\frac{1}{1.05^{10}}}{0.05}\right] = \$46{,}330.41$$

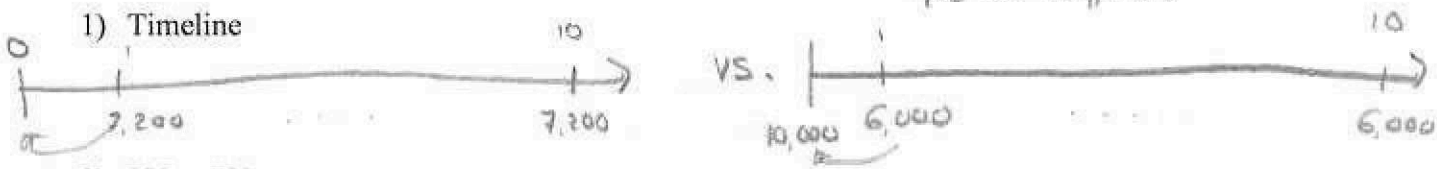
1) Timeline

2) PV or FV

3) Calculate

Example 3: You are currently paying a mortgage of $7,200 every year. You have 10 more years left to pay. You are thinking about refinancing. The new payments would be $6,000 annually, also for 10 years, with the first payment also being in one year (at t =1). Your discount rate is 5%. You also need to pay a fee of $10,000 right now if you accept the offer. Should you refinance? what's the max fee so that you are indifferent?

1) Timeline



2) PV or FV

PV

3) Calculate

$7{,}200\left[\frac{1-\frac{1}{1.05^{10}}}{0.05}\right]$ vs. $6{,}000\left[\frac{1-\frac{1}{1.05^{10}}}{0.05}\right]+10{,}000$

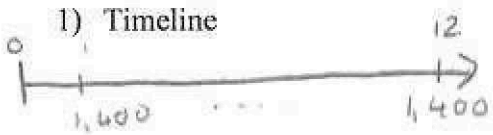
55,964.91 vs. 46,330.41 + 10,000 = 56,330.41

Do not refinance

Example 4:

Your co-op agrees to hire you part-time. You are offered a $1,400 monthly salary for 12 months. Your discount rate is 1.5% monthly. What is the PV of this offer?

1) Timeline



2) PV or FV

PV

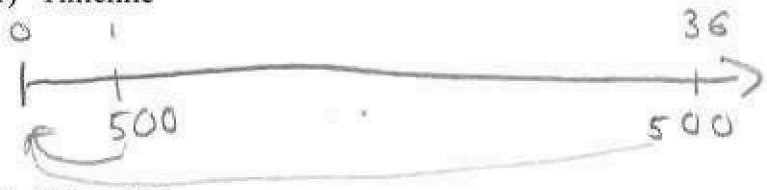
3) Calculate

$1{,}400\left[\frac{1-\frac{1}{1.015^{12}}}{0.015}\right] = \$15{,}270.51$

Example 5:

Alternatively, you could work a part-time job on campus for the full remaining 3 years. The campus job offers $500 per month for 36 months. Your discount rate is still 1.5% monthly. What is the PV of this offer?

1) Timeline



2) PV or FV

PV

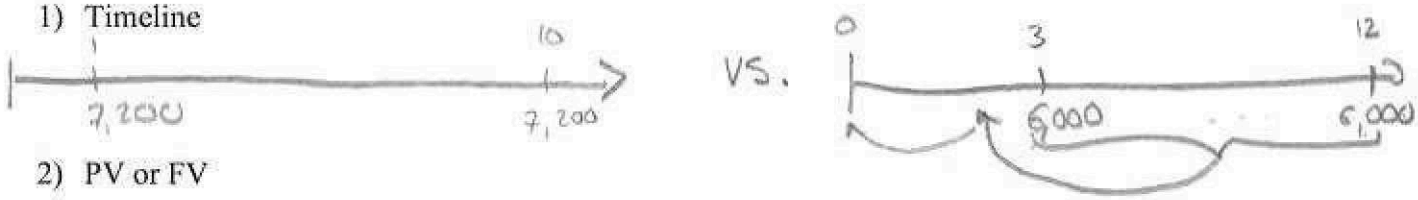
3) Calculate

$$500\left[\frac{1-\frac{1}{1.015^{36}}}{0.015}\right] = 13,830.34$$

Example 6:

You are currently paying a mortgage of $7,200 every year. You have 10 more years left to pay. You are thinking about refinancing. The new payments would be $6,000 annually, also for 10 years, with the first payment **starting in three years (at t =3)**. Your discount rate is 5%. Should you refinance?

1) Timeline



2) PV or FV

PV

3) Calculate

$$7,200\left[\frac{1-\frac{1}{1.05^{10}}}{0.05}\right] \quad \text{VS.} \quad \frac{6,000\left[\frac{1-\frac{1}{1.05^{10}}}{0.05}\right]}{1.05^{2}}$$

step 1 annuity

step 2 go back to t=0

$= 55,596.49$ vs. $42,023.05$

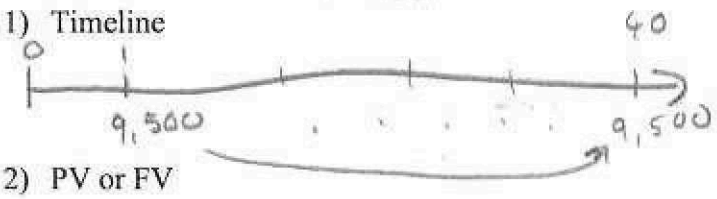
Yes refinance

0.1 · 95,000 = 9,500

Example 7:

You anticipate that you will earn $95,000 annually starting next year. You want to save 10% of this every year for 40 years. If you can earn an average return of 5% on your investment annually, how much will you have in 40 years?

1) Timeline



2) PV or FV

FV

3) Calculate

$$FV = 9{,}500\left[\frac{1.05^{40}-1}{0.05}\right]$$

$$= \$\ 1{,}147{,}597.86$$