CS270: LAB #10

Murder Mystery

You may work in teams of one or two people (three is acceptable in the event of an unscheduled absence). Unless stated otherwise, the lab is due to be submitted into Gradescope at the end of the day.
In order to receive credit, follow these instructions:

[a] Every team member should be discussing simultaneously the same problem – do NOT try to divvy up the labor and assign different problems to different students since the material is cumulative.

[b] Directly edit this lab PDF using Sedja/PDFescape with your answers (extra pages can be added in the rare event you need more than the allotted space)

[c] Each lab, rotate which member has the responsibility of being the Scribe. This is the person that is typing the answers and uploading the final PDF – note that only a single copy of the filled in PDF is turned into Gradescope. Only one lab needs to be submitted for the entire team, and all members receive the same score. Make sure to use a font that your PDF editor is compatible with (otherwise you might find your answers appear as weird shapes/sizes or simply disappear entirely!)

[d] The Gradescope submission must have each answer properly tagged with the appropriate question. Moreover, every member of the team must be listed as a submitter. Although it is the Scribe which executes these actions, it is still the responsibility of the entire team to make certain this is done properly (thus it is highly recommended that the Scribe share their screen so the entire team can witness it). Answers which are improperly tagged cannot be seen by the grader and thus cannot be scored.

[e] **FOR REMOTE ONLY**: Each lab, rotate which member has the responsibility of being the Recorder. This is the person who hits the Zoom Record button (once the technical permission is granted by the TA/RCF/Professor) and ensures that everyone has their camera/microphone on. They are also the member that is responsible to make sure the DrexelStream video is marked as viewable and entered into the https://tinyurl.com/VidLinkForm webform before 11:59pm (they should also email the rest of their team as confirmation.) Note that the video file doesn't get created/processed until after the Recorder has quit Zoom.

[f] Each lab, rotate which member has the responsibility of being the Manager. This is the person that ensures that everyone is participating equally and honestly, keeps the group on task, ensures that all team members understand a solution before going on to the next question, and presses the "hand up" button in Zoom to summon a TA or the professor (but they only do so after surveying the group to make sure everyone has the same question).

Team Name (CS pioneer): Marvin Minsky

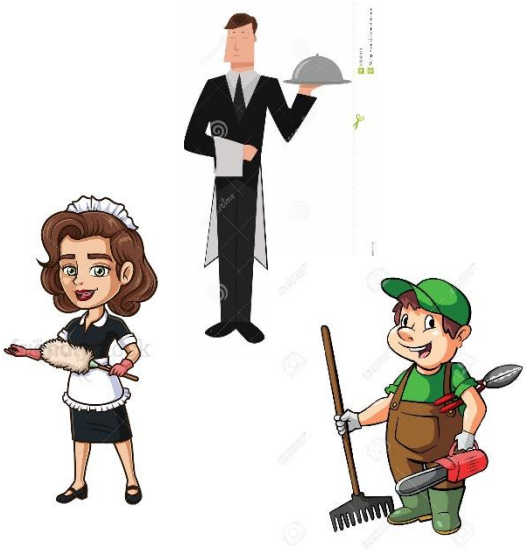
Scribe name: Lixiao Yang

Recorder name: Jerry Li

Manager name: Terie Ha

Other team member (if any): Evelyn Thai

MURDER MYSTERY



A person will lie if and only if they were involved in the murder of Mr. Body, and the three suspects present in the mansion at the time of the crime have made the following statements:

Maid: "The Butler is innocent but the Gardener is guilty"

Butler: "Exactly one of us is a murderer"

Gardener: "The Maid and I are both innocent"

The famed detective Sheerluck Homey gets involved in the case and makes the pronouncement:

"Arrest them all, because they each helped murder Mr. Body!"

When asked by his assistant how he knew they were all guilty, here was Sheerluck's reasoning:

"First, let me show that the Gardener cannot be innocent. If he were, that would mean his statement was true, and he claimed that both he and the maid were innocent. But if the maid were innocent, then HER statement would also be true, but she stated the Gardener was guilty, which contradicts our hypothesis of the Gardener's innocence. Thus, the Gardener is not innocent after all.

Consequently, that means his statement (proclaiming his and the maid's innocence) is false, hence both he and the maid are both guilty. But now look at the Butler, he said there was exactly one murderer, and we've now deduced there are at least two, thus the Butler's statement is a lie, and hence he must one of the murderers as well!"

14pts [a] Do you find any flaws with the 1st paragraph of the detective's reasoning? If so, what is it?

If the Maid's statement and the Gardener's statement contradict each other, you can't infer the the Gardener is not innocent simply based on the contradiction. Based on the contradiction, the Maid and the Gardener can both or one of them is lying, the conclusion of such inference can't be decided.

14pts [b] Do you find any flaws with the 2nd paragraph of the detective's reasoning? If so, what is it?

If the Gardener's statement is false, it doesn't mean the truth is the negation of it. If a statement is partially incorrect, it is false. So based on his statement is false, we can't infer whether the Maid is guilty or not.

14pts [c] If the entire list of axioms and theorems were allowed, do you think it would be possible to derive Sheerluck's conclusion using the formal technique of Equational Reasoning? Explain why or why not. [Note: you do not actually have to try to write up that proof at this time]

Yes, since the statement of an innocent person can be expressed in equational reasoning and logical expressions can be constructed based on their statement, and use "if and only if" to connect the each suspects and their statement, because if a person's statement is true if and only if he/she is innocent.

14pts [d] Let **M**, **B**, **G** represent the propositional variables that the Maid, Butler, and Gardener are innocent, respectively. Convert each of the suspects' statements into a boolean expression using the appropriate symbols, and then write an equivalent expression in CNF. Note that it would *not* be correct to simply claim **M ∧ G** because of the Gardener's statement – explain why that translation *doesn't* fully express our given scenario.

Maid: $M \Leftrightarrow (B \wedge \neg G) \equiv (\neg B \vee G \vee M) \wedge (B \vee \neg M) \wedge (\neg G \vee \neg M)$
Butler: $B \Leftrightarrow ((\neg M \wedge B \wedge G) \vee (M \wedge \neg B \wedge G) \vee (M \wedge B \wedge \neg G)) \equiv (\neg M \vee B \vee \neg G) \wedge (M \vee G \vee \neg B) \wedge (\neg B \vee G \vee M) \wedge (\neg B \vee \neg M \vee \neg G)$
Gardener: $G \Leftrightarrow (M \wedge G) \equiv \neg G \vee M$

Because if a person's statement is true if and only if the person is innocent so such a situation should be considered as one of the conditions in the expression. So a person's statement should be connected with "equivalent" to "xxx is innocent".

14pts [e] Use our Python or Racket resources to create a DIMACS expression representing our entire scenario and use the SAT solver to solve it. Include a screenshot of the result. Explain what this output means in the context of our scenario. Does it jibe with Sheerluck's conclusion? Does this mean that Sheerluck's reasoning was logically proper afterall?

Verdict: *SATISFIABLE*

```
SATISFIABLE

-1 -2 -3 0
```

This result means that everyone is not innocent and is lying. It is the same result as Sheerluck's conclusion. However, this does not mean Sheerluck's reasoning is logically proper since the same conclusion does not guarantee the correct logical inference process.

14pts [f] Suppose it turns out that the Butler has an ironclad alibi. How would you adjust the input to to now include a presumption of the Butler's innocence? Include a screenshot of your input and the output. Explain what this output means in the context of our scenario. Does this still jibe with Sheerluck's conclusion? Explain how it is possible for the satisfiable response to "change" answers.

```
p cnf 3 7
-2 1 3 0
2 -1 0
-1 -3 0
-3 1 0
-1 2 -3 0
1 -2 3 0
-1 -2 -3 0
2 0
```

Verdict: *SATISFIABLE*

```
SATISFIABLE

1 2 -3 0
```

If the Butler is innocent, then the conclusion would be only the Gardener is guilty and both the Maid and the Butler are innocent. This situation can satisfy the two's statement are true and the Butler's statement has contradiction to the rest's statement, who is lying as the suspect.

16pts [g] Now instead of the Butler's innocence, throw into the DIMACS input a presumption of the Gardener's innocence and provide a screenshot of your input and the output. Explain what this output would mean in the context of our scenario.

```
p cnf 3 7
-2 1 3 0
2 -1 0
-1 -3 0
-3 1 0
-1 2 -3 0
1 -2 3 0
-1 -2 -3 0
3 0
```

Verdict: *UNSATISFIABLE*

```
UNSATISFIABLE
```

This output means the Gardener can't be innocent since such presumption doesn't have any satisfiable situation.