MATH 180 - Homework 3

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Question 1

Part 1

$|\mathcal{P}(S)| = 2^8 = 256$

Part 2

$\binom{8}{6} = 28$.

Part 3

$\because$ The only possible number for the last 2 elements are {1, 3, 5, 8}.
$\therefore \binom{4}{2} = 6$.

Question 2

Part 1

$\because$ There are 8 bits left with starting substring 1101.
$\therefore 2^8 = 256$

Part 2

$\because$ There are three 1's in 1101.
$\therefore$ The weight for the rest string is 3.
$\therefore \binom{8}{3} = 56$

Question 3

Part 1

$\because$ The paths need to go up 11 times and go right 8 times.
$\therefore$ There are 19 moves total and 8 of them is go right.
$\therefore \binom{19}{8} = 75582$

Part 2

$\because$ The paths need to go up 4 times and go right 4 times to $(5, 6)$.
$\therefore \binom{8}{4} = 70$
$\because$ The paths need to go up 7 times and go right 4 times to $(9, 13)$.
$\therefore \binom{11}{4} = 330$
$\therefore$ There are $330 \times 70 = 23100$ paths in total.

Part 3

$75582 - 23100 = 52482$
$\therefore$ There are 52482 paths.

Question 4

$\binom{19}{14} \times 3^5 = 11628 \times 243 = 2825604$

Question 5

$\because$ Each circle equals to 8 line permutations.
$\therefore \frac{8!}{8} = 5040$

Question 6

1. $5! = 120$
2. $\frac{7!}{2! \times 2! \times 2!} = 630$
3. $\frac{11!}{3!} = 6652800$

Question 7

For pants: $\binom{4}{2} = 6$
For shirts: $\binom{7}{3} = 35$

For sweaters: $\binom{3}{1} = 3$
$\therefore$ There are $6 \times 35 \times 3 = 630$ ways for him.

Question 8

Part 1

There are $15^8 = 2562890625$ functions.

Part 2

There are $15 \times 14 \times 13 \times 12 \times 11 \times 10 \times 9 \times 8 = 259459200$ injective functions.