

Math 180: Homework 1

Due: Wednesday, January 18th

Question 1

Let $A = \{1, 3, 5, 7, 11\}$ and $B = \{4, 5, 6, 7, 8\}$.

Find each of the following sets (list their elements).

1. $A \cap B$
2. $A \cup B$
3. $A \setminus B$
4. $B \setminus A$

Question 2

Find the cardinality of each set.

1. $A = \{x \in \mathbb{N} : 5 \leq x \leq 31\}$
2. $B = \{x \in \mathbb{Z} : x^2 = 36\}$
3. $C = \{x \in \mathbb{N} : 10 \leq x \leq 60 \text{ and } x \text{ is prime}\}$

Question 3

Find a set with the largest possible size (cardinality) that is a subset of both $\{2, 4, 6, 8, 10, 12\}$ and $\{1, 2, 3, 4, 5, 6, 7\}$

Question 4

Find a set with the smallest possible size (cardinality) that has both $\{2, 4, 6, 8, 10, 12\}$ and $\{1, 2, 3, 4, 5, 6, 7\}$ as subsets.

Question 5

Find an example of sets A and B such that $|A| = 3$, $|B| = 4$, and $|A \cup B| = 5$.

Question 6

Are there sets A and B such that $|A| = |B|$, $|A \cup B| = 10$, and $|A \cap B| = 5$?

Explain.

Hint: $|A \cup B| = |A| + |B| - |A \cap B|$.

Question 7

Let $A = \{2, 4, 6, 7\}$. Suppose B is a set such that $|B| = 5$.

1. What are the smallest and largest possible values of $|A \cup B|$?
2. What are the smallest and largest possible values of $|A \cap B|$?
3. What are the smallest and largest possible values of $|A \times B|$?

Question 8

Let $X = \{n \in \mathbb{N} : 10 \leq n \leq 20\}$.

Find examples of sets with the properties below and explain why your examples work.

1. A set $A \subseteq \mathbb{N}$ with $|A| = 10$ such that $X \setminus A = \{10, 12, 14\}$
2. A set $B \in \mathcal{P}(X)$ with $|B| = 5$ (here, $B \in \mathcal{P}(X)$ is just another way of writing that B is a subset of X)
3. A set $E \subseteq X$ such that $|E| = E$