CS270: LAB #6

DubNums

You may work in teams of 3 people (2 or 4 is acceptable in the event of an unscheduled absence). Unless stated otherwise, the lab is due to be submitted into Gradescope at the end of the day if not finished during class time. In order to receive credit, follow these instructions:

[a] Every team member should be discussing simultaneously the same problem – do NOT try to divvy up the labor and assign different problems to different students since the material is cumulative.

[b] Directly edit this lab PDF using Sedja/PDFescape with your answers (extra pages can be added in the rare event you need more than the allotted space)

[c] Each lab, rotate which member has the responsibility of being the Scribe. This is the person that is typing the answers and uploading the final PDF – note that only a single copy of the filled in PDF is turned into Gradescope. Only one lab needs to be submitted for the entire team, and all members receive the same score. Make sure to use a font that your PDF editor is compatible with (otherwise you might find your answers appear as weird shapes/sizes or simply disappear entirely!)

[d] The Gradescope submission must have each answer properly tagged with the appropriate question. Moreover, every member of the team must be listed as a submitter. Although it is the Scribe which executes these actions, it is still the responsibility of the entire team to make certain this is done properly (thus it is highly recommended that the Scribe share their screen so the entire team can witness it). Answers which are improperly tagged cannot be seen by the grader and thus cannot be scored.

[e] **FOR REMOTE ONLY**: Each lab, rotate which member has the responsibility of being the Recorder. This is the person who hits the Zoom Record button (once the technical permission is granted by the TA/RCF/Professor) and ensures that everyone has their camera/microphone on. They are also the member that is responsible to make sure the DrexelStream video is marked as viewable and entered into the https://tinyurl.com/VidLinkForm webform before 11:59pm (they should also email the rest of their team as confirmation.) Note that the video file doesn't get created/processed until after the Recorder has quit Zoom.

[f] Each lab, rotate which member has the responsibility of being the Manager. This is the person that ensures that everyone is participating equally and honestly, keeps the group on task, ensures that all team members understand a solution before going on to the next question, and presses the "hand up" button in Zoom to summon a TA or the professor (but they only do so after surveying the group to make sure everyone has the same question).

Team Name (CS pioneer): _____

Scribe name: _____

Recorder name: _____

Manager name: _____

Other team member (if any): _____

Question 1 : 18 points

Binary Values

You are probably familiar with the idea of Binary Numbers.

In this exercise, we will look at binary numbers from a different perspective. We will create a **representation** of binary numbers using Racket lists.

Numbers are made from three symbols:

Expression	Human meaning
dubzero	represents 0
(D X)	represents 2x, where X represents x.
(DP1 X)	represents 2x+1.

We will refer to numbers written using these Racket symbols as "dubnums" (short for "double-notation numbers.")

Note: to avoid confusion with the Peano representation, we shall use the fully spelled out 'zero rather than just 'z

Write each of the following numbers using only double-notation.

Integer	DubNum expression
0	dubzero
1	(DP1 dubzero)
2	(D (DP1 dubzero))
3	(DP1 (DP1 dubzero))

Note: when entering these into Dr Racket, you should use a ' in front of these lists and in front of the symbol dubzero to prevent evaluation. (but it's not necessary when writing it manually)

(a) (3 points) Give the dubnum representation of 4

(b) (3 points) Give the dubnum representation of 5

(c) (3 points) Give the dubnum representation of 6

(d) (3 points) Give the dubnum representation of 7

(e) (3 points) Give the dubnum representation of 8

(f) (3 points) Give the dubnum representation of 9

Question 2 : 17 points

To Increment means to increase a value by 1.

For example $1 + 1 = 2$ is shown below using the increment command.

(inc (DP1 dubzero)) = (D (DP1 dubzero))

We need to figure out how to implement the increment command.

(a)(3points)What is the return value of (inc dubzero)in dubnum format?

(b)(4points)What is the return value of (inc(D(DP1 dubzero)))in dubnum format?

(c) (3 points) To write a recursive function, we need to think about general cases. For example, many numbers in double-notation will look like (DP1 X) where X is some other double-notation number (i.e. a "dubnum") They follow the pattern "A list starting with DP1". These represent the odd numbers.
What other type of dubnum will our program need to handle as input?

(d) (3 points) Next, we need to determine what to do for each case.
What should happen what we execute (inc (DP1 X))?
We are adding a 1 to a value x that has already been doubled and had 1 added to it. Algebraically, we can think of this as meaning $(2x + 1) + 1$. Doubling a number and adding 2 is the same as adding 1 to the number then doubling it $((2x + 1) + 1 = 2(x + 1)$. We can describe this equality in double-notation and use it in our Racket function design.
(inc (DP1 X)) = (D (inc X))
Algebraically, (inc (DP1 X)) and (D (inc X)) are exactly the same. They are not the same from a programming perspective. One of the expressions is computationally less complex then the other. Which is computationally less complex? why?

(e) (4points) what would be the return value of (inc (DX)) be in dubnum notation?

You will be using these answers in parts A-E to form your Racket implementation of inc in the next question.

Question 3 : 8 points

Implement **inc** for double-notation numbers.

```
; input-contract: X is a dubnum representing the integer x
;output-contract: (incX) is the dubnum representing x+1
(define (inc X)
        ...
)
```

Your function should pass the following tests.

```
(inc dubzero);Returns '(DP1 dubzero)
(inc(inc dubzero));Returns '(D (DP1 dubzero))
(inc(inc(inc dubzero)));Returns '(DP1 (DP1 dubzero))
(inc(inc(inc(inc dubzero))));Returns '(D (D (DP1 dubzero)))
(inc(inc(inc(inc(inc dubzero)))));Returns '(DP1 (D (DP1 dubzero)))
(inc(inc(inc(inc(inc(inc dubzero))))));Returns '(D (DP1 (DP1 dubzero)))
```

Question 4 : 20 points

We want to write a function for addition of numbers in double-notation.

We will start by thinking about the problem algebraically. This will let us get a handle on what our representations should look like.

(a) (4 points) If x is an integer what is $x + 0$?

(b) (4 points) If y is an integer what is $0 + y$?

(c) (4 points) If x and y are integers, can you algebraically rewrite $(2 * x) + (2 * y)$ into an expression containing $(x + y)$.

(d) (4 points) If x and y are integers, can you algebraically rewrite $(2 * x + 1) + (2 * y)$ into an expression containing $(x + y)$.

(e) (4 points) If x and y are integers, can you algebraically rewrite $(2 * x + 1) + (2 * y + 1)$ into an expression containing $(x + y)$.

Question 5 : 16 points

Now that we have some ideas about how the addition could work, we can come up with the cases of our function. You may use the inc command from the previous page in addition to D, DP1, and zero in your answers. For each part below, express the result of (binadd X Y) where X,Y are as follows.

Your answer may have X,Y in it, but should not contain any As or Bs. You may use binadd recursively in your answers.

(a) (4 points) X=(DP1 A), Y=(D B)

(b) (4 points) X = (D A), Y = (DP1 B)

(c) (4 points) X=(DP1 A), Y = (DP1 B)

(d) (4points) X=(DA), Y=(DB)

Question 6 : 16 points

In this question, you will come up with a method to represent multiplication with dubnums.

You **may** use the `inc` and `binadd` command from the previous page in addition to D, DP1, and zero.

You may also use binmult recursively in the below answers. You answer may use X,Y but should not have As or Bs in it.

(a) (4 points) In plain English, what are the base case(s) of multiplication?

(b) (6points) If X and Y are dubnums, what would be the result of (binmult X Y) when Y=(DA)

(c) (6points) If X and Y are dubnums, what would be the result of (binmult X Y) when Y = (DP1 A)

Question 7 : 5 points

Classical Binary is written with 0 and 1. In plain English, explain how you would convert from dubnum notation to binary.