Math 180

Homework 6: Proof by Induction

Question 1

Prove by induction that for all natural numbers n, $n^3 - n$ is divisible by 3.

Question 2

Prove by induction that $n! > 2^n$ for all natural numbers $n \geq 4$

Question 3

Prove by induction that $n! < n^n$ for $n > 1$

Question 4

What is wrong with the following proof that *all horses have the same color*?

> Let $P(n)$ be the proposition that all the horses in a set of n horses are the same color. Base case: Clearly, $P(1)$ is true. Now assume that $P(n)$ is true. That is, assume that all the horses in any set of n horses are the same color. Consider any $n + 1$ horses; number these as horses $1, 2, 3, ..., n, n + 1$. Now the first n of these horses all must have the same color, and the last n of these must also have the same color. Since the set of the first n horses and the set of the last n horses overlap, all n + 1 must be the same color. This shows that $P(n + 1)$ is true and finishes the proof by induction.