MATH 180 - Homework 2

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Question 1

Part a

$f(3) = 3$

Part b

$n = 3$ since $f(3) = 3$.

Part c

$\because$ Each element in the codomain is in the range
$\therefore$ The function is a suejection
$\because$ Each element in the codomain is the image of at most one element of the domain
$\therefore$ The function is a injection
$\therefore$ The function is a bijection

Question 2

Part a

$\because$ The element 6 in codomain is not in the range
$\therefore$ The function is not a surjection
$\because$ $f(4) = f(6) = 1$
$\therefore$ The function is not a injection
$\therefore$ The function is none of the above functions

Part b

$\because$ Each element in the codomain is in the range
$\therefore$ The function is a suejection
$\because$ Each element in the codomain is the image of at most one element of the domain
$\therefore$ The function is a injection
$\therefore$ The function is a bijection

Part c

The function can be rewritten as $f(x) = \begin{pmatrix} 1 & 2 & 3 & 4 & 6 \\ 4 & 3 & 2 & 1 & 1 \end{pmatrix}$
$\because$ Range is not equal to codomain
$\therefore$ The function is not a suejection
$\because$ f(4)=f(6)=1
$\therefore$ The function is not a injection
$\therefore$ The function is none of the above functions

Question 3

$f_1(x) = \begin{pmatrix} 1 & 2 \\ a & a \end{pmatrix}$
$f_2(x) = \begin{pmatrix} 1 & 2 \\ a & b \end{pmatrix}$
$f_3(x) = \begin{pmatrix} 1 & 2 \\ a & c \end{pmatrix}$
$f_4(x) = \begin{pmatrix} 1 & 2 \\ b & a \end{pmatrix}$
$f_5(x) = \begin{pmatrix} 1 & 2 \\ b & b \end{pmatrix}$
$f_6(x) = \begin{pmatrix} 1 & 2 \\ b & c \end{pmatrix}$
$f_7(x) = \begin{pmatrix} 1 & 2 \\ c & a \end{pmatrix}$
$f_8(x) = \begin{pmatrix} 1 & 2 \\ c & b \end{pmatrix}$
$f_9(x) = \begin{pmatrix} 1 & 2 \\ c & c \end{pmatrix}$
There are 9 functions, in which 6 are injective, 0 are surjective, 0 are bijective.

Question 4

$f(6) = 3f(5) = 9f(4) = 27f(3) = 81f(2) = 243f(1) = 729f(0) = 1458$

Question 5

Part a

$\because$ There exists that $f : \{1\} \to \{1, 2\}$
$\therefore$ It is possible

Part b

$\because$ There exists that $f : \{1, 2\} \to \{1\}$
$\therefore$ It is possible

Part c

$\because$ There exists that $f : \mathbb{N} \to \mathbb{N}$ with $f(x) = x + 1$
$\therefore$ It is possible

Part d

$\because$ $\forall a \in Y$ is the image of at least one element from the domain
$\therefore$ $|X| \geq |Y|$ and when f is surjective, it must also be injective
$\therefore$ It is impossible

Part e

$\because$ If f is injective, range is euqal to codomain
$\therefore$ with finite sets X, Y, when f is injective, it must also be surjective
$\therefore$ It is impossible

Part f

$\because$ If f is surjective, domain and codomain must be equal
$\therefore$ with finite sets X, Y, when f is surjective, it must also be injective
$\therefore$ It is impossible

Question 6

$7 \times 5 \times 12 = 420$

Question 7

Let $A =$ watch The The Office, $B =$ watch Parks and Rec, $C =$ watch Superstore
Then $|A| = 30, |B| = 23, |C| = 18, |A \cap B| = 14, |B \cap C| = 12, |A \cap C| = 11, |A \cap B \cap C| = 7$
$\therefore |A| + |B| + |C| - |A \cap B| - |B \cap C| - |A \cap C| + |A \cap B \cap C| = 41$

Question 8

Let $A =$ multiple of 2, $B =$ multiple of 3, $C =$ multiple of 5
Then $|A| = 400, |B| = 266, |C| = 160, |A \cap B| = 133, |B \cap C| = 53, |A \cap C| = 80, |A \cap B \cap C| = 26$
$\therefore |A| + |B| + |C| - |A \cap B| - |B \cap C| - |A \cap C| + |A \cap B \cap C| = 586$

Question 9

Part a

$10^6 = 1000000$

Part b

$10 \times 9 \times 8 \times 7 \times 6 \times 5 = 151200$

Part c

$10^3 = 1000$

Part d

$10^3 + 10^3 - 1 = 1999$

Part e

Words with no repeat: $10 \times 9 \times 8 \times 7 \times 6 \times 5 = 151200$
Words with no sub-word "hid": $7 \times 6 \times 5 \times 4 = 840$
$151200 - 840 = 150360$

Question 10

$\because$ The sum of the digits is even
$\therefore$ The number of the two digits are either odd or even
When digits are odd, let $A = \{1, 3, 5, 7, 9\}, B = \{1, 3, 5, 7, 9\}$, we can get the cartesian

product $A \times B = 25$
When digits are even, let $A = \{2, 4, 6, 8\}, B = \{0, 2, 4, 6, 8\}$, we can get the cartesian product $A \times B = 20$
$\therefore$ There are 45 possibilities.