CS270: LAB #11

Natural Deduction

This Lab will NOT be done during class time due to the midterm. Instead, it will be released right after lab10 on Fri July 28th at 11:59pm. It can be done independently or in groups, and due on Fri Aug 4th at 11:59pm

You may work in teams of 1-2 people (three is acceptable in the event of an unscheduled absence). Unless stated otherwise, the lab is due to be submitted into Gradescope at the end of the day at 11:59pm
In order to receive credit, follow these instructions:

[a] Every team member should be discussing simultaneously the same problem – do NOT try to divvy up the labor and assign different problems to different students since the material is cumulative.

[b] Directly edit this lab PDF with your answers (extra pages can be added in the rare event you need more than the allotted space)

[c] Each lab, rotate which member has the responsibility of being the Scribe. This is the person that is typing the answers and uploading the final PDF – note that only a single copy of the filled in PDF is turned into Gradescope. Only one lab needs to be submitted for the entire team, and all members receive the same score. Make sure to use a font that your PDF editor is compatible with (otherwise you might find your answers appear as weird shapes/sizes or simply disappear entirely!)

[d] The Gradescope submission must have each answer properly tagged with the appropriate question. Moreover, every member of the team must be listed as a submitter. Although it is the Scribe which executes these actions, it is still the responsibility of the entire team to make certain this is done properly (thus it is highly recommended that the Scribe share their screen so the entire team can witness it). Answers which are improperly tagged cannot be seen by the grader and thus cannot be scored.

[f] Each lab, rotate which member has the responsibility of being the Manager. This is the person that ensures that everyone is participating equally and honestly, keeps the group on task, ensures that all team members understand a solution before going on to the next question, and presses the "hand up" button in Zoom to summon a TA or the professor (but they only do so after surveying the group to make sure everyone has the same question).

Team CS pioneer: Marvin Minsky

Scribe name: Jerry Li

Recorder name: Lixiao Yang

Manager name: Terie Ha

Other team member (if any): Evelyn Thai

Question 1 : 11 points

Proof by Deduction is a proof technique using formal Logic.

In this lab, we will look at a few of the rules of Deduction.

First, we will look at the meaning of some of the symbols in Deduction.

An **argument** is a statement we want to prove correct.

An **argument** has a format.

$$\phi_1, \phi_2, \cdots, \phi_3 \therefore \psi \tag{1}$$

Each ϕ_n is a **premise**. These are known to be true at the start of the proof. They are separated by commas.

There is one ψ. The ψ value is called the **conclusion**.

The goal of a **Proof by Deduction** is to use rules to show the **conclusion** must be true if the **premises** are all true.

$$A \wedge B, A \implies B \therefore C \tag{2}$$

$$A, B, (A \wedge B) \implies X \therefore X \vee Y \tag{3}$$

(a) (2 points) Argument 2 has two **premises**. Write them below.

A ∧ B, A ⇒ B

(b) (2 points) Argument 2 has a **conclusion**. Write it below.

c

(c) (2 points) How many **premises** does Argument 3 have?

3

(d) (3 points) Write all the **premises** of Argument 3 below.

A, B, (A ∧ B) ⇒ X

(e) (2 points) Write the **conclusion** of Argument 3 below.

X ∨ Y

Question 2 : 2 points

We want to make an argument.

Let a be an even number. If a number is even, the following number is odd. Prove that $a + 1$ is odd.

First, we will assign variables to each premise and the conclusion.

Variable	Meaning
E	The number a is even.
$E \implies O$	If a number is even, the next number is odd.
O	The next number is odd.

Write this argument in **Proof by Deduction** notation. (Don't prove it, just write it out.)

E, E ⇒ O ∴O

Question 3 : 6 points

The previous argument seems like it is obviously true. (Hint: it is!)

In **Proof by Deduction**, there are rules to justify things that are true. By using multiple simple rules, we can prove complex arguments.

Conjunction Introduction

If we know two premises are true independently, we know they are true together.

The below is a proof of the argument $A, B \therefore A \wedge B$.

1.	A	Premise
2.	B	Premise
3.	$A \wedge B$	$\wedge$ I 1,2

(a) (2 points) Line 3 has a justification $\wedge$ I 1,2. What does this justification mean in plain English? (Hint: See section title)

If 1. and 2. are true, their conjunction introduction will also be true

(b) (4 points) Write a **Proof by Deduction** for the following argument.

X	Number a is even
Y	Number b is odd

Prove: $X, Y \therefore Y \wedge X$

You **must** justify each line.

1. X Premise
2. Y Premise
3. Y ∧ X ∧ I 2, 1

Question 4 : 12 points

Conjunction Elimination

If we know two things are true together, we can also prove they are true independently.

Argument: $A \wedge B \therefore B$

1.	$A \wedge B$	Premise
2.	B	$\wedge$ E 1

(a) (2 points) Line 2 has a justification $\wedge$ E 1. What does this justification mean in plain English?

If 1. is true, then it's conjunction elimination(2.) is also true

(b) (2 points) Why do you think the justification for **Conjunction Introduction** has more line numbers in it than the one for **Conjunction Elimination**?

Because Conjunction Introduction need to all the argument in the conjunction are ture, instead, if we know a conjuction is true, all the argument in the Conjunction Elimination are ture.

(c) (4 points) Manually write out a Proof by Natural Deduction for the following argument.

X	Number a is even
Y	Number b is odd

Prove: $X \wedge Y \therefore X$

You **must** justify each line.

1. X ∧ Y Premise
2. X ∧ E 1

(d) (4 points) Use the two rules you have learned to manually prove the following. Remember to justify every line. Your proof should have 5 total lines.

X ∧ Y ; Y ∧ Z ∴ X ∧ Z

1. X ∧ Y Premise
2. Y ∧ Z Premise
3. X ∧ E 1
4. Z ∧ E 2
5. X ∧ Z ∧ I 3, 4

Question 5 : 7 points

Online Proof Checker

You can use the online proof checker called Proof Buddy to verify your proofs (link is in bbLearn).

The Proof Checker has its own language for entering expressions. (For example, premises are separated by semi-colons)

(a) (2 points) Read the instructions. Enter the premises and conclusion for the following argument. Click Create Problem. Make sure your argument was interpreted correctly.

$$X \wedge Y \,;\, Y \wedge Z \therefore X \wedge Z$$

(b) (5 points) Complete the Proof.
To add a new line, click on `New Line`. Fill in the left box with the next statement. Fill in the right box with the justification.
Click `Check Proof` to verify if you proof is correct.
You can check your proof after each line to see if you made any mistakes.
Your proof is complete when the last line matches your conclusion. The proof checker will give you a "valid and complete" result statement highlighted in green when done properly.
Take a screenshot of your proof and paste the image below. Be sure to include the green statement that your proof is valid and complete. Points may be deducted if you leave off the result statements

Line #	Expression	Rule
1	$X \wedge Y$	Premise
2	$Y \wedge Z$	Premise
3	X	$\wedge$E 1
4	Z	$\wedge$E 2
5	$X \wedge Z$	$\wedge$ I 3, 4

Check Proof Save

Great job! Don't forget to save your work by clicking "Save"! This will allow you to view this correct proof later!

Result: The proof is valid and complete!

Question 6 : 5 points

Copy the following attempted proof into the Proof Checker.

1.	A	Premise
2.	$A \implies C$	Premise
3.	C	$\implies$ E 1,2

Does it work? Explain why or why not. If it doesn't, fix the error and then take a screenshot of your proof and paste the image below. Be sure to include the green sentence showing that it is valid and complete.

It is not working. The order of 1. and 2. is wrong. After switch their position, it works.

Line #	Expression	Rule
1	A → C	Premise
2	A	Premise
3	C	→ E 1, 2

Check Proof Save

Great job! Don't forget to save your work by clicking "Save"! This will allow you to view this correct proof later!

Result: The proof is valid and complete!

Question 7: 3 points

Disjunction Introduction

We know from the Truth Table of the Disjunction that True $\vee A =$ True.

This means if we know something is true, we can create a disjunction with it.

$$A \therefore A \vee X$$

1.	A	Premise
2.	$A \vee X$	$\vee$ I 1

We created a new variable X. Does X's value affect the fact that $A \vee X$ is true?

No. Because A is true, A ∨ X ≡ True ∨ X≡ True, any disjunction introductions of True should be true.

Question 8 : 5 points

Prove the following argument. Complete your proof in the proof checker.

$$A \wedge C \, ; \, B \wedge X \therefore (A \wedge B) \vee Q$$

Take a screenshot of your proof and paste the image below. Be sure to include the result statement showing it is correct.

Line #	Expression	Rule
1	A ∧ C	Premise
2	B ∧ X	Premise
3	A	∧ E 1
4	B	∧ E 2
5	A ∧ B	∧ I 3, 4
6	(A ∧ B) ∨ Q	∨ I 5

Check Proof Save

Great job! Don't forget to save your work by clicking "Save"! This will allow you to view this correct proof later!

Result: The proof is valid and complete!

Question 9 : 10 points

Conditional Elimination

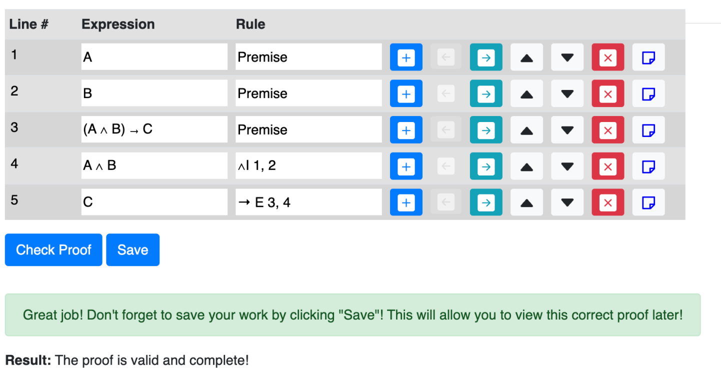
The conditional (implies) statement tells us that if one thing is true, another must also be true. If we know the left side of a conditional statement is true, we also know the right side is true.

$$A\ ;\ A \rightarrow B \therefore B$$

1.	A	Premise
2.	$A \implies B$	Premise
3.	B	$\implies$ E 2,1 [what would happen if you cited the order 1,2 ?]

(a) (5 points) Prove the following argument. Complete your proof in the proof checker. Paste a screenshot below including the green statement verifying correctness.

$$A\ ;\ B\ ;\ (A \wedge B) \rightarrow C \therefore C$$



Line #	Expression	Rule
1	A	Premise
2	B	Premise
3	(A ∧ B) → C	Premise
4	A ∧ B	∧I 1, 2
5	C	→ E 3, 4

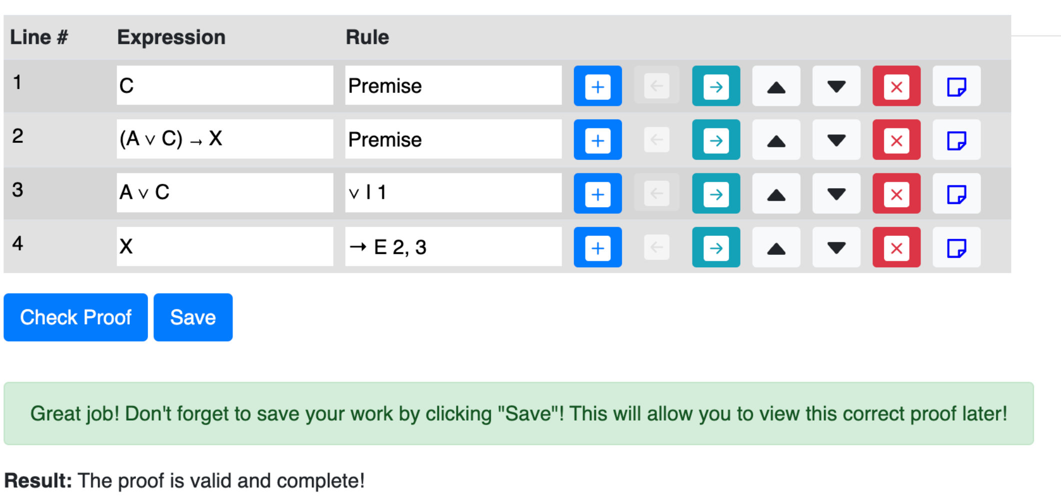
Check Proof Save

Great job! Don't forget to save your work by clicking "Save"! This will allow you to view this correct proof later!

Result: The proof is valid and complete!

(b) (5 points) Prove the following argument. Complete your proof in the proof checker. Paste a screenshot below including the green statement verifying correctness.

$$C\ ;\ (A \vee C) \rightarrow D \therefore D$$



Line #	Expression	Rule
1	C	Premise
2	(A ∨ C) → X	Premise
3	A ∨ C	∨ I 1
4	X	→ E 2, 3

Check Proof Save

Great job! Don't forget to save your work by clicking "Save"! This will allow you to view this correct proof later!

Result: The proof is valid and complete!

Question 10 : 8 points

Conditional Introduction

In **Conditional Introduction**, we prove that something being true implies something else. This rule requires a subproof. A subproof is when we assume something is true (or false) to see what will happen.

A→B ; B→C ∴ A→C

1.	$A \implies B$	Premise
2.	$B \implies C$	Premise
3.1	A	Assumption
3.2	B	$\implies$ E 1,3.1
3.3	C	$\implies$ E 2,3.2
4.	$A \implies C$	$\implies$ I 3

(a) (2 points) Why do you think we assumed A was true?

In Conditional Introduction, we prove that something being TRUE implies something else, and we implies A to B, so A should be TRUE.

(b) (2 points) Why does the justification for line4 refer to line3, but there is no line called "3"?

The subproof in line 3 is to prove the relationship between A to C. It does not proof A, B, or C, only proves the interrelationship between them.

(c) (4 points) Copy this Proof into the online proof checker. Paste a screenshot below.

Line #	Expression	Rule
1	A → B	Premise
2	B → C	Premise
3.1	A	Assumption
3.2	B	→E 1, 3.1
3.3	C	→E 2, 3.2
4	A → C	→ I 3

Check Proof Save

Great job! Don't forget to save your work by clicking "Save"! This will allow you to view this correct proof later!

Result: The proof is valid and complete!

Prove each of the following. Check your answers in the Proof checker. Paste a screenshot as your answer.

Question 11 : 11 points

$C \therefore M \implies (C \vee X)$

Line #	Expression	Rule
1	C	Premise
2.1	M	Assumption
2.2	C ∨ X	∨I 1
3	M → (C ∨ X)	→I 2

Check Proof Save

Great job! Don't forget to save your work by clicking "Save"! This will allow you to view this correct proof later!

Result: The proof is valid and complete!

Question 12 : 10 points

$(A \wedge B) \wedge C \therefore (A \vee X) \wedge (C \vee X)$

Line #	Expression	Rule
1	(A ∧ B) ∧ C	Premise
2	A ∧ B	∧ E 1
3	C	∧ E 1
4	A	∧ E 2
5	A ∨ X	∨ I 4
6	C ∨ X	∨ I 3
7	(A ∨ X) ∧ (C ∨ X)	∧ I 5, 6

Check Proof Save

Great job! Don't forget to save your work by clicking "Save"! This will allow you to view this correct proof later!

Result: The proof is valid and complete!

Question 13 : 10 points

$A \implies (B \implies C) \therefore (A \land B) \implies C$

Line #	Expression	Rule
1	A → (B → C)	Premise
2.1	A ∧ B	Assumption
2.2	A	∧ E 2.1
2.3	B	∧ E 2.1
2.4	B → C	→E 1, 2.2
2.5	C	→E 2.4, 2.3
3	(A ∧ B) → C	→I 2

Check Proof Save

Great job! Don't forget to save your work by clicking "Save"! This will allow you to view this correct proof later!

Result: The proof is valid and complete!