MATH 180 - Homework 5

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Question 1

First differences: $1, 3, 5, 7, 9, 11$
Second differences: $2, 2, 2, 2, 2$
$\therefore$ The sequence is Δ^2-constant.
Let $a_n = an^2 + bn + c$, since $a_0 = 4, a_1 = 5, a_2 = 8$, we have

$$\begin{cases} c = 4 \\ a + b + c = 5 \\ 4a + 2b + c = 8 \end{cases} \tag{1}$$

$\therefore a = 1, b = 0, c = 4$ and the formula is $a_n = n^2 + 4$.

Question 2

First differences: $3, 4, 5, 6, 7, 8$
Second differences: $1, 1, 1, 1, 1$
$\therefore$ The sequence is Δ^2-constant.
Let $a_n = an^2 + bn + c$, since $a_0 = 2, a_1 = 5, a_2 = 9$, we have

$$\begin{cases} c = 2 \\ a + b + c = 5 \\ 4a + 2b + c = 9 \end{cases} \tag{2}$$

$\therefore a = \frac{1}{2}, b = \frac{5}{2}, c = 2$ and the formula is $a_n = \frac{1}{2}n^2 + \frac{5}{2}n + 2$.

Question 3

First differences: $2, 3, 7, 14, 24, 37$
Second differences: $1, 4, 7, 10, 13$

Third differences: $3, 3, 3, 3$
$\therefore$ The sequence is Δ^3-constant.
Let $a_n = an^3 + bn^2 + cn + d$, since $a_0 = 0, a_1 = 2, a_2 = 5, a_3 = 12$, we have

$$\begin{cases} d = 0 \\ a + b + c + d = 2 \\ 8a + 4b + 2c + d = 5 \\ 27a + 9b + 3c + d = 12 \end{cases} \tag{3}$$

$\therefore a = \frac{1}{2}, b = -1, c = \frac{5}{2}, d = 0$ and the formula is $a_n = \frac{1}{2}n^3 - n^2 + \frac{5}{2}n$.

Question 4

Part a

$P(3) = 10, P(4) = 20, P(5) = 35$

Part b

Sequence: 1,4,10,20,35,56,...
First differences: $3, 6, 10, 15, 21$
Second differences: $3, 4, 5, 6$
Third differences: $1, 1, 1$
$\therefore$ The sequence is Δ^3-constant.
Let $P(n) = an^3 + bn^2 + cn + d$, since $P(0) = 0, P(1) = 1, P(2) = 4, P(3) = 10$, we have

$$\begin{cases} d = 0 \\ a + b + c + d = 1 \\ 8a + 4b + 2c + d = 4 \\ 27a + 9b + 3c + d = 10 \end{cases} \tag{4}$$

$\therefore a = \frac{1}{6}, b = \frac{1}{2}, c = \frac{1}{3}, d = 0$ and the formula is $P(n) = \frac{1}{6}n^3 + \frac{1}{2}n^2 + \frac{1}{3}n$.

Part c

$P(15) = 680$

Part d

For every layer added, the number of cannonballs are the former layer number adds the current layer's number.

Question 5

$\because a_n - a_{n-1} = 2^n$ with $a_0 = 3$.
By iteration, we can get $a_n - a_0 = 2 + \cdots 2^n$.
$\therefore a_n = 3 + 2 + 2^2 + \cdots 2^n$ so that $-2a_n = -6 - 2^2 - 2^3 - \cdots - 2^{n+1}$
$\therefore a_n = 2^{n+1} + 1$

Question 6

Let $a_n = r^n$, we have $r^n = 5r^{n-1} + 6r^{n-2}$
$\therefore r^2 - 5r - 6 = 0$
By solving the equation we have $r = 6, r = -1$.
$\therefore a_n$ can be written as $a_n = A6^n + B(-1)^n$
$\because a_0 = 1, a_1 = 13$
$\therefore$ We can get $A = 2, B = -1$
$\therefore a_n = 2 \cdot 6^n + (-1)^{n+1}$

Question 7

Let $a_n = r^n$, we have $r^n = 5r^{n-1} - 6r^{n-2}$
$\therefore r^2 - 5r + 6 = 0$
By solving the equation we have $r = 2, r = 3$.
$\therefore a_n$ can be written as $a_n = A \cdot 2^n + B \cdot 3^n$
$\because a_0 = 4, a_1 = 11$
$\therefore$ We can get $A = 1, B = 3$
$\therefore a_n = 2^n + 3^{n+1}$

Question 8

Let $a_n = r^n$, we have $r^n = 8r^{n-1} - 16r^{n-2}$
$\therefore r^2 - 8r + 16 = 0$
By solving the equation we have $r = 4$.
$\therefore a_n$ can be written as $a_n = A \cdot 4^n + Bn \cdot 4^n$
$\because a_0 = 3, a_1 = 10$
$\therefore$ We can get $A = 3, B = -\frac{1}{2}$
$\therefore a_n = 3 \cdot 4^n - \frac{n}{2} \cdot 4^n$

Question 9

Part a

Recursive formula: $a_n = 4a_{n-1}$ with $a_0 = 1$
Closed formula: $a_n = 4^n$

Part b

Let $a_n = r^n$, we have $r^n = 4r^{n-1}$
$\therefore r - 4 = 0$
By solving the equation we have $r = 4$.
$\therefore a_n$ can be written as $a_n = 4^n$