MATH 180 - Homework 6

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Question 1

For P(0): $0 - 0$ is obviously divisible by 3.
For P(k): Let $k^3 - k = 3m$
$\therefore (k+1)^3 - (k+1) = k^3 + 3k^2 + 3k + 1 - k - 1 = 3m + 3(k^2 + k) = 3(k^2 + k + m)$
$\therefore P(k) \to P(k+1)$
$\because P(0)$ is true
$\therefore$ P(n) is true for $\forall n \in \mathbb{N}$.

Question 2

For P(4): $4! = 24 > 16 = 2^4$.
For P(k): Let $k! > 2^n$
$\therefore (k+1)! = (k+1)k! > (k+1)2^k$
$\because k + 1 \geq 5$
$\therefore (k+1)2^k > 4 \cdot 2^k = 2^{k+2} > 2^{k+1}$
$\therefore P(k) \to P(k+1)$
$\because P(0)$ is true
$\therefore$ P(n) is true for $\forall n \in \mathbb{N}$ with $n \geq 4$.

Question 3

For P(2): $2! = 2 < 2^2 = 4$.
For P(k): Let $k! < k^k$
$\therefore (k+1)! = (k+1)k! < (k+1)k^k \because k \geq 2$
$\therefore (k+1)k^k < (k+1)^{k+1}$
$\therefore (k+1)! < (k+1)^{k+1}$
$\because P(0)$ is true
$\therefore$ P(n) is true for $\forall n \in \mathbb{N}$ with $n > 4$.

Question 4

P(1) is not true.
Let the two sets be S_1, S_2, the horse color in each set is same. However, there are no overlap in the two sets when $n = 1$. Thus we can not conclude that S_1 has the same color as S_2.