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Extra Credit: Doing all of these questions correctly adds 5 points (2.5%) to your Exam 1 score. Due Monday, April 24th at midnight (end of day).

1. Derive the PV(annuity formula) from the sum of individual present values. Use the derivation to explain why the PV(annuity) formula always brings you to the "year before the first cash flow"

The present value of the entire annuity can be calculated as:

$$PV = \frac{C}{1+r} + \frac{C}{(1+r)^2} + \cdots + \frac{C}{(1+r)^n}$$, factoring out C and using the sum of geometric series

$$PV = C\,\frac{\frac{1}{1+r}\left[1-\frac{1}{(1+r)^n}\right]}{1-\frac{1}{1+r}} = C\left[\frac{1-\frac{1}{(1+r)^n}}{r}\right]$$

2. Show that when t=1 and g=0, the growing annuity formula can be used to discount a single cash flow from t=1 to t=0. You may use an example to convince yourself first, but you must show the algebra to convert the formula to PV = CF/(1+r)

$$PV = C\left[\frac{1-\frac{1+0}{1+r}}{r-0}\right] = C\left[\frac{1-\frac{1}{1+r}}{r}\right] = C\left(\frac{1}{r}-\frac{1}{r(1+r)}\right) = C\left[\frac{1+r-1}{r(1+r)}\right] = \frac{C}{1+r}$$

which is equal to the annuity formula when t=1.

3. Show how to arrive at the perpetuity formula, given the PV (<u>annuity</u>) formula and t = infinity.

$$\because \frac{1}{(1+r)^n} \to 0 \text{ when } n \to \infty$$

$$\therefore PV = C\left[\frac{1-\frac{1}{(1+r)^\infty}}{r}\right] \approx C\left[\frac{1-0}{r}\right] = \frac{C}{r}$$

4. Show how to arrive at the growing perpetuity formula, given the PV(<u>growing annuity</u>) formula, and t = infinity.

$\because$ g is usually smaller than r

$$\therefore \left(\frac{1+g}{1+r}\right)^n \to 0 \text{ when } n \to \infty$$

$$\therefore PV = C\left[\frac{1-\left(\frac{1+g}{1+r}\right)^\infty}{r-g}\right] \approx C\left(\frac{1-0}{r-g}\right) = \frac{C}{r-g}$$