CS270: LAB #15

Mathematical Induction

You may work in teams of ideally one or two people (three is acceptable in the event of an unscheduled absence). Unless stated otherwise, the lab is due to be submitted into Gradescope at the end of the day (11:59pm).
In order to receive credit, follow these instructions:

[a] Every team member should be discussing simultaneously the same problem – do NOT try to divvy up the labor and assign different problems to different students since the material is cumulative.

[b] Directly edit this lab PDF with your answers (extra pages can be added in the rare event you need more than the allotted space)

[c] Each lab, rotate which member has the responsibility of being the Scribe. This is the person that is typing the answers and uploading the final PDF – note that only a single copy of the filled in PDF is turned into Gradescope. Only one lab needs to be submitted for the entire team, and all members receive the same score. Make sure to use a font that your PDF editor is compatible with (otherwise you might find your answers appear as weird shapes/sizes or simply disappear entirely!)

[d] The Gradescope submission must have each answer properly tagged with the appropriate question. Moreover, every member of the team must be listed as a submitter. Although it is the Scribe which executes these actions, it is still the responsibility of the entire team to make certain this is done properly (thus it is highly recommended that the Scribe share their screen so the entire team can witness it). Answers which are improperly tagged cannot be seen by the grader and thus cannot be scored.

REMOTE ONLY

[e] Each lab, rotate which member has the responsibility of being the Recorder. This is the person who hits the Zoom Record button (once the technical permission is granted by the TA/RCF/Professor) and ensures that everyone has their camera/microphone on. They are also the member that is responsible to make sure the DrexelStream video is marked as viewable and entered into the https://tinyurl.com/VidLinkForm webform before 11:59pm (they should also email the rest of their team as confirmation.) Note that the video file doesn't get created/processed until after the Recorder has quit Zoom.

[f] Each lab, rotate which member has the responsibility of being the Manager. This is the person that ensures that everyone is participating equally and honestly, keeps the group on task, ensures that all team members understand a solution before going on to the next question, and presses the "hand up" button in Zoom to summon a TA or the professor (but they only do so after surveying the group to make sure everyone has the same question).

Team Name (CS pioneer): Marvin Minsky

Scribe name: Jerry Li

Recorder name: Lixiao yang

Manager name: Evelyn Thai

Other team member (if any): Terie Ha

Make sure your proofs include written sentences like the ones demonstrated in the video
(that is, they should all start with a sentence reading "This proof proceeds by induction on ... " etc).
Your submissions should NOT just be a bunch of scribbled algebraic expressions!
This lab is worth a total of 100pts (each of the 5 proofs is worth 20 points)

#1. Prove by Induction that $\neg(A_1 \vee A_2 \vee \dots \vee A_n) \leftrightarrow (\neg A_1 \wedge \neg A_2 \wedge \dots \wedge \neg A_n)$

Base case: anchored at n=2.
$\neg(A1 \vee A2) \Leftrightarrow (\neg A1 \wedge \neg A2)$ was already proven by TruthTables

Leap: We obtain the IH be replacing the n in our claim with k; so in other words, we get to assume that
IH: $\neg(A1 \vee A2 \vee \dots \vee Ak) \Leftrightarrow (\neg A1 \wedge \neg A2 \wedge \dots \wedge \neg Ak)$
Now we just start with the new LHS and by using the base case with n=2, we get
$\neg((A1 \vee A2 \vee \dots \vee Ak) \vee Ak+1) \Leftrightarrow ((\neg A1 \wedge \neg A2 \wedge \dots \wedge \neg Ak) \wedge \neg Ak+1)$

by treating $A1 \vee A2 \vee \dots \vee Ak$ as a single A

$\Leftrightarrow (\neg A1 \wedge \neg A2 \wedge \dots \wedge \neg Ak) \wedge \neg Ak+1$, by using IH

Consequently, since both the base case and leap have being demonsted,
this proves by POMI that DeMorgan's Law works for any size clause.

#2. Prove that $2^0 + 2^1 + 2^2 + 2^3 + \ldots + 2^n = 2^{n+1} - 1$

Base case: anchored at n=1
LHS: 2^0 + 2^1 = 3
RHS: 2^(1+1) - 1 = 3
Since the LHS=RHS (both came out to be 3), this establishes the base case.

Leap:
IH: 2^0 + 2^1 + 2^2 + ... + 2^k = 2^(k+1) - 1

When n = k+1
LHS:
2^0 + 2^1 + 2^2 + ... + 2^k + 2^(k+1)
=(2^0 + 2^1 + 2^2 + ... + 2^k) + 2^(k+1)
=2^(k+1) - 1 + 2^(k+1) by using IH
=2^(k+2) - 1 by using algebra
RHS:
2^((k+1)+1) - 1
=2^(k+2) - 1 by using algebra

Since the LHS=RHS both simplified to be the same answer, this establishes the leap.

Consequently, since both the base case and leap have been demonstrated, this proves by POMI the equation holds for all n.

#3. Prove that $5*8^n+2$ is always a multiple of 7 (note: there is no need to use mods with this problem. use induction)

∃m∈ℕ, 5*8^n + 2 = 7m

Base case: anchored at n=0
LHS: 5*8^0+2 = 7
RHS: choose m = 1, so 7*1 = 7

Since the LHS=RHS, this establishes the base case.

Leap: We may assume that 5*8^k + 2 = 7m is a multiple of 7 with 5*8^k + 2 = 7m(this is the IH)

Observe that:
5*8^(k+1) + 2
= 8*5*8^k + 2
= (5*7+5)8^k + 2
= 7*5*8^k + 5*8^k + 2 by algebra
= 7*5*8^k + 7m by IH
= 7(5*8^k + m) by algebra
Note that is 7*integer as desired, and thus the leap is established

Consequently, since both the base case and leap have been demonstrated,
this proves by POMI that 5*8^n + 2 is a multiple of 7 for all n ≥ 0

#4. Prove that factorial growth dominates exponential growth. In other words, show that $n! > 2^n$ provided $n \geq 4$

Base case: anchored at n = 4
LHS = 4*3*2*1 = 24
RHS = 2^4 = 16
Since 24>16, the LHS > RHS as claimed, and so this establishes the base case.

Leap: We may assume that k! > 2^k for some k >= 4 (that is our IH)

k! > 2^k IH
LEMMA: k > 3 so that k+1> 4 > 2
k! > 2^k, by IH
(k+1)k! > (k+1)*2^k, multiplying both sides by k+1
(k+1)! > (k+1)*2^k > 2*2^k, by the LEMMA
(k+1)! > 2*2^k = 2^(k+1), by algebra, and this is the desired result

Consequently, since both the base case and leap have been demonstrated, this proves by POMI that n! > 2^n, for n >= 4

#5. If tofu comes in 5-packs and 7-packs, show that for a "sufficiently large" amount of tofu, you can buy exactly the amount that you need.

(note: if you find the minimum amount N that is guaranteed to be done, you must also show mathematically that N-1 **cant** be done!)

block 0: 24, 25, 26, 27, 28
block 1: 29, 30, 31, 32, 33
block 2: 34, 35, 36, 37, 38

Base case: anchored at n = 0
24 = 5, 5, 7, 7
25 = 5, 5, 5, 5, 5
26 = 7, 7, 7, 5
27 = 5, 5, 5, 5, 7
28 = 7, 7, 7, 7

this establishes the base case

Leap: Assuming the IH that everything in block k is doable

We simply include one more 5-pack to each member of block k.

This has the effect of adding +5 to the total numbers of tofu, which is exactly numbers of the next block.

Consequently, since both the base case and leap have been demonstrated, this proves by POMI that provided n is "sufficiently large" (i.e. a dozen or more), it is possible to get exactly n tofu from packs of 5 and 7.

Since you cannot get 23 by combination of 5 and 7, 24 is the minimum amount