Assignment 4: Priority Queue

Assignment idea and majority of handout by Julie Zelenski.

So, it's finally time for you to be implementing a class of your own: a priority queue, which is a variation on the standard queue described in the reader. The standard queue is a collection of elements managed in a first-in, first-out manner. The first element added to the collection is always the first element extracted; the second is second; so on and so on.

In some cases, a FIFO strategy may be too simplistic for the activity being modeled. A hospital emergency room, for example, needs to schedule patients according to priority. A patient who arrives with a more serious problem should pre-empt others even if they have been waiting longer. This is a *priority queue*, where elements are added to the queue in arbitrary order, but when time comes to extract the next element, it is the highest priority element in the queue that is removed. Such an object would be useful in a variety of situations. In fact, you can even use a priority queue to implement sorting; insert all the values into a priority queue and extract them one by one to get them in sorted order.

The main focus of this assignment is to implement a priority queue class in several different ways. You'll have a chance to experiment with arrays and linked structures, and in doing so you'll hopefully master the pointer gymnastics that go along with it.

Due: Thursday, July 29th at 4:00 p.m.

The `PQueue` Interface

The priority queue will be a collection of strings. Lexicographically smaller strings should be considered **higher** priority than lexicographically larger ones, so that **`"ping"`** is higher priority than **`"pong"`**, regardless of insertion order.

Here are the methods that make up the **`public`** interface of all priority queues:

```
class PQueue {
   public:
      void enqueue(string elem);
      string extractMin();
      string peek();
      static PQueue *merge(PQueue *one, PQueue *two);
   private:
      // implementation dependent member variables and helper methods
      int logSize;
};
```

`enqueue` is used to insert a new element to the priority queue. **`extractMin`** returns the value of highest priority (i.e., lexicographically smallest) element in the queue and removes it. **`merge`** destructively unifies the incoming queues and returns their union as a new queue. For the detailed descriptions on how these methods behave, see the **`pqueue.h`** interface file

included in the starter files. All PQueues inherit the logSize variable. In your implementations you will need to keep this value updated for the size call to work properly.

Implementing the priority queue

While the external representation may give the illusion that we store everything in sorted order behind the scenes at all time, the truth is that we have a good amount of flexibility on what we choose for an internal representation. Sure, all of the operations need to work properly, and we want them to be fast. But we might optimize not for speed but for ease of implementation, or to minimize memory footprint. Maybe we optimize for one operation at the expense of others.

This assignment is all about implementation and internal representation. Yes, you'll master arrays and linked lists in the process, but the takeaway point of the assignment—or the most important of the many takeaway points—is that you can use whatever machinery you deem appropriate to manage the internals of a new, object-oriented container.

You'll implement the priority queue in four different ways. Two are fairly straightforward, but the third is more difficult, and the optional fourth is downright difficult (although so neat and clever and beautiful that it's worth whatever discomfort you might endure while implementing it.)

Implementation 1:	Optimized for simplicity and for the **`enqueue`** method by backing your priority queue by an unsorted **`Vector<string>`**. **`merge`** is pretty straightforward, but **`peek`** and **`extractMin`** are expensive, but the expense might be worth it in cases where you need to get a version up and running really quickly for a prototype, or a proof of concept, or perhaps because you need to **`enqueue`** 50,000 elements and extract a mere 50. I don't provide much in terms of details on this one, as it's pretty straightforward.
Implementation 2:	Optimized for simplicity and for the **`extractMin`** operation by maintaining a sorted doubly linked (**`next`** and **`prev`** pointers required) list of strings behind the scenes. **`peek`** and **`extractMin`** will run super fast, but **`enqueue`** will be slow, because it needs to walk the list from front to back to find the insertion point (and that takes time that's linear in the size of the priority queue itself. **`merge`** can (and should) be implemented to run in linear time, for much the same reason Merge from merge sort can be.
Implementation 3:	Balance insertion and extraction times by implementing your priority queue in terms of a binary heap, discussed in detail below. When properly implemented, **`peek`** runs in O(1) time, **`enqueue`** and **`extractMin`** each run in O(lg n) time, and **`merge`** runs in O(n) time.

Implementation 1: Unsorted `Vector`

This implementation is layered on top of our **`Vector`** class. **`enqueue`** simply appends the new element to the end. When it comes time to **`extractMin`** the minimum element (i.e. the one with the highest priority in our version), it performs a linear search to find it. This implementation is straightforward as far as layered abstractions go, and serves more as an introduction to the architecture of the assignment than it does as an interesting implementation. Do this one first.

Aside

As you implement each of the subclasses, you'll leave **`pqueue.h`** and **`pqueue.cpp`** alone, and instead be modifying the interface (**`.h`**) and implementation (**`.cpp`**) files for each of the subclasses. In the case of the unsorted vector version, you'll be concerned with **`pqueue-vector.h`** and **`pqueue-vector.cpp`**. **`pqueue-vector.h`** defines the public interface you're implementing, but its **`private`** section is empty:

```
class VectorPQueue : public PQueue {
public:
   VectorPQueue();
   ~VectorPQueue();

   static VectorPQueue *merge(VectorPQueue *one, VectorPQueue *two);

   void enqueue(string elem);
   string extractMin();
   string peek();

private:
   // update the private section with the list of
   // data members and helper methods needed to implement
   // the vector-backed version of the PQueue.
};
```

Not surprisingly, the **`private`** section shouldn't be empty, but instead should list the items that comprise your internal representation. You should erase the comment I've provided and insert the list of data members and private helper functions you think should be there.

The **`pqueue-vector.cpp`** file provides dummy, placeholder implementations of everything, just so that the project cleanly compiles. In a few cases, the dummy implementations actually do the right thing, but a large majority of them need to updates to include real code that does real stuff.

Note that the parent **`PQueue`** class defines a protected field called **`logSize`**, which means you have access to it. You should ensure that **`logSize`** is always maintained to house the logical size of the priority queue—both here and in the other three implementations. By unifying the **`logSize`** field to the parent class, we can implement size and **`isEmpty`** at the **`PQueue`** class level (I already did) and they work automatically for all subclasses.

As you advance through the implementations, understand that you'll be modifying different pairs of interface and implementation files (**`pqueue-heap.h`** and **`pqueue-heap.cpp`** for the binary heap version, etc). In all cases, the private sections of the interface are empty and need to be fixed, and in all cases the implementation files have placeholder implementations to sedate the compiler into being happy.

Implementation 2: Sorted doubly-linked list

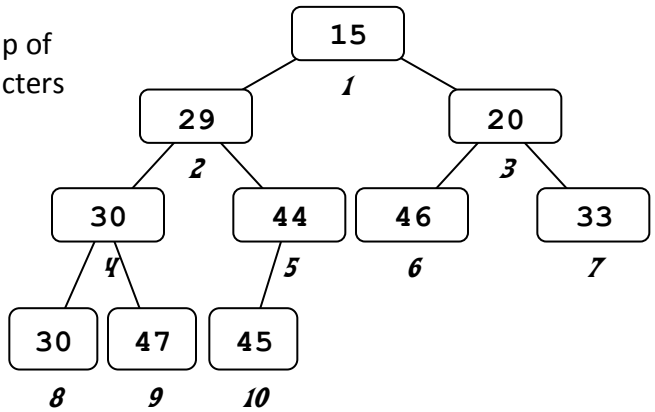
The linked list implementation is a doubly linked list of values, where the values are kept in sorted order (i.e., smallest to largest) to facilitate finding and removing the smallest element quickly. Insertion is a little more work, but made easier because of the decision to maintain both **`prev`** and **`next`** pointers. **`merge`** is conceptually simple, although the implementation can be tricky for those just learning pointers and linked lists for the first time.

Implementation 3: Binary Heap

Although the binary search trees we'll eventually discuss in lecture might make a good implementation of a priority queue, there is another type of binary tree that is an even better choice in this case. A *heap* is a binary tree that has these two properties:

- It is a *complete* binary tree, i.e. one that is full in all levels (all nodes have two children), except for possibly the bottom-most level which is filled in from left to right with no gaps.
- The value of each node is less than or equal to the value of its children.

Here's a conceptual picture of a small heap of integer strings (i.e. strings where all characters are digits)

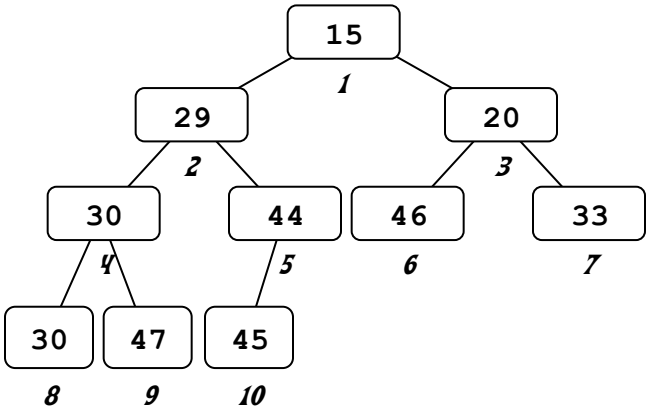


Note that a heap differs from a binary search tree in two significant ways. First, while a binary search tree keeps all the nodes in a sorted arrangement, a heap is ordered in a much weaker sense. Conveniently, the manner in which a heap is ordered is actually enough for the efficient performance of the priority queue operations. The second important difference is that while binary search trees come in many different shapes, a heap must be a complete binary tree, which means that every heap containing ten elements is the same shape as every other heap of ten elements.

Representing a heap using an array

One way to manage a heap would be to use a standard binary tree node definition and wire up left and right children pointers to all nodes. We can exploit the completeness of the tree and create a simple array representation and avoid the pointers.

Consider the nodes in the heap to be numbered level by level like this:



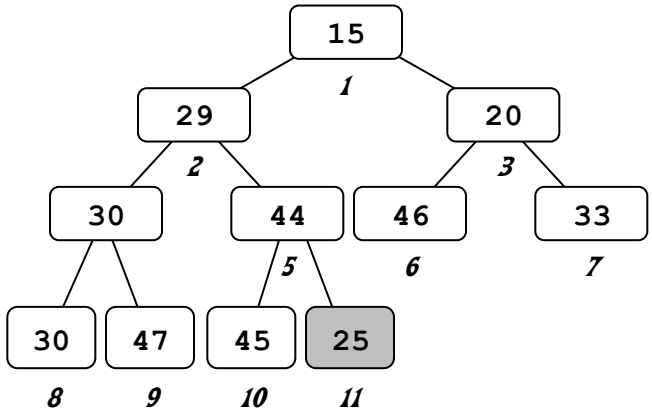
and now check out this array representation of the same heap:

15	29	20	30	44	46	33	30	47	45
1	2	3	4	5	6	7	8	9	10

You can divide any node number by 2 (discarding the remainder) to get the node number of its parent. For example, the parent of node 9 is node 4. The two children of node *i* are 2*i* and 2*i* + 1, e.g. node 3's two children are 6 and 7. Although many of the drawings in this handout use a tree diagram for the heap, keep in mind you will actually be representing the heap as a string array.

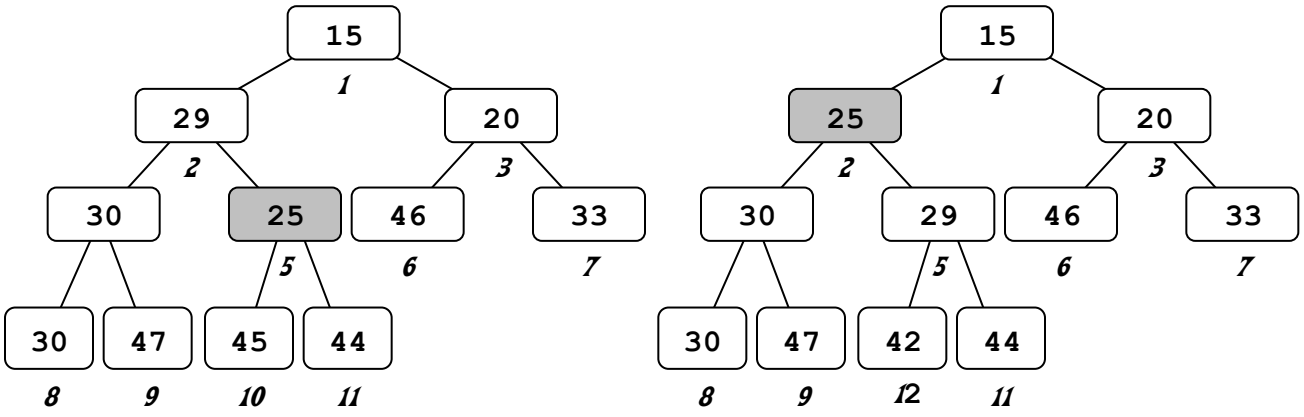
Heap insert

Inserting into a heap is done differently than its functional counterpart in a binary search tree. A new element is added to the very bottom of the heap and it rises up to its proper place. Suppose, for example, we want to insert **"25"** into our heap. We add a new node at the bottom of the heap (the insertion position is equal to the size of the heap):



15	29	20	30	44	46	33	30	47	45	25
1	2	3	4	5	6	7	8	9	10	11

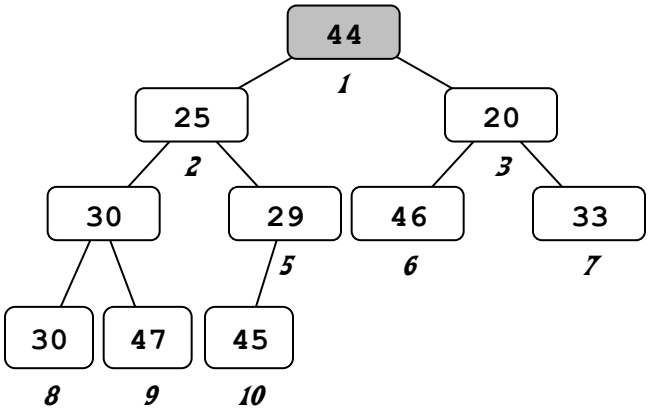
We compare the value in this new node with the value of its parent and, if necessary, exchange them. Since our heap is actually laid out in an array, we "exchange" the nodes by swapping array values. From there, we compare the moved value to its new parent and continue moving the value upward until it needs to go no further. This is sometimes called the *bubble-up* operation.



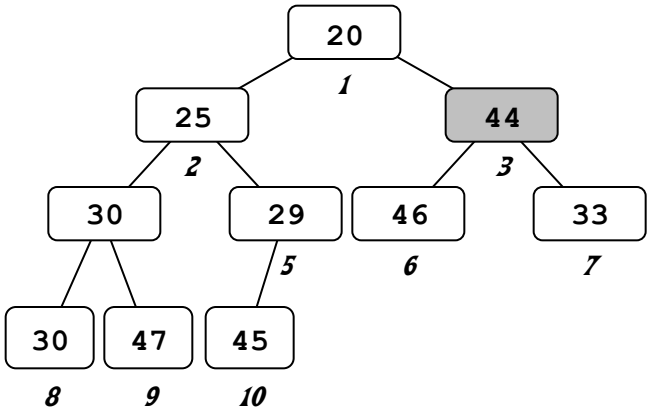
Heap remove

Where is the smallest value in the heap? Given heap ordering, the smallest value resides in the root, where it can be easily accessed. However, after removing this value, you must re-configure the remaining nodes back into a heap. Remember the completeness property dictates the shape of the heap, and thus it is the bottommost node that needs to be removed from the structure. Rather than re-arranging everything to fill in the gap left by the root node, we can

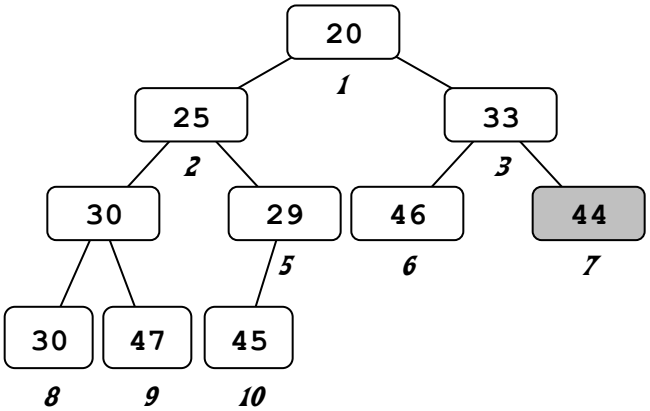
leave the root node where it is, copy the **value** from the last node to the root node, and remove the last node.



We have a complete tree again, and the left and right sub-trees are heaps. The only potential problem: a violation of the heap ordering property, localized at the root. In order to restore the min-heap property, we need to trickle that value down to the right place. We will use an inverse to the strategy that allows us to float up the new value during the **`enqueue`** operation. Start by comparing the value in the root to the values of its two children. If the root's value is larger than the values of either child, swap the value in the root with that of the smaller child:



This step fixes the heap ordering property for the root node, but at the expense of possibly breaking the sub-tree of the child we switched with. The sub-tree is now another heap where only the root node is out of order, so we can iteratively apply the same re-ordering on the sub-tree to fix it up and so-on down through its sub-trees.



You stop trickling downwards when the value sinks to a level such that it is smaller than both of its children or it has no children at all. This recursive action of moving the out-of-place root value down to its proper place is often called *heapify*-ing.

You should implement the priority queue as a heap array using the strategy shown above. This array should start small and grow dynamically as needed.

Merging two heaps

The merge operation—that is, destroying two heaps and creating a new one that's the logical union of the two originals—can be accomplished via the same heapify operation discussed above. Yes, you could just insert elements from the second into the first, one at a time, until the second is depleted and the first has everything. But it's actually faster—asymptotically so, in fact—to do the following:

- Create an array that's the logical concatenation of the first heap's array and the second heap's array, without regard for the heap ordering property. The result is a complete, array-backed binary tree that in all likelihood isn't even close to being a heap.
- Recognize that all of the leaf nodes—taken in isolation—respect the heap property.
- Heapify all sub-heaps rooted at the parents of all the leaves.
- Heapify all sub-heaps rooted at the grandparents of all the leaves.
- Continue heapify increasingly higher ancestors until you reach the root, and heapify that as well.

Binary Heap Implementation Notes

Manage your own raw memory. It's tempting to just use a **`Vector<string>`** to manage the array of elements. But using the vector introduces an extra layer of code in between your **`HeapPQueue`** and the memory that actually store the elements, and in practice, a core container class like the **`HeapPQueue`** would be implemented without that extra layer. Make it a point to implement your **`HeapPQueue`** in terms of raw, dynamically allocation arrays of **`string`**s instead of a **`Vector<string>`**.

Freeing memory. You are responsible for freeing heap-allocated memory. Your implementation should not orphan any memory during its operations and the destructor should free all of the internal memory for the object. We recommend getting the entire data structure working without deleting anything, and then going back and taking care of it.

Think before you code. The amount of code necessary to complete the implementation work is not large, but you will find it requires a bit of thinking getting it to work correctly. It will help to sketch things on paper and work through the boundary cases carefully before you write any code.

Test thoroughly. I know we've already said this, but it never hurts to repeat it a few times. You don't want to be surprised when our grading process finds a bunch of lurking problems that you didn't discover because of inadequate testing. The code you write has some complex interactions and it is essential that you take time to identify and test all the various cases. I've provided you with a minimal test harness to ensure that the **`HeapPQueue`** works in simple scenarios, but it's your job to play villain and try to break your implementation, knowing that you're done when you fail to do so.

Accessing files

On the class web site, there are two starter projects: one for XCode and a one for Visual Studio. Each project contains these files:

`qqueue-test.cpp`	Test harness to assist in development and testing.
`pqueue`	Interface and partial implementation of base **PQueue** class. The primary purpose of the **`PQueue`** class is to define the interface that all concrete priority queue implementations should agree on.
`pqueue-vector`	Interface and implementation file housing the unsorted vector version of the priority queue.
`pqueue-linked-list`	Interface and implementation file housing the sorted, doubly linked list version of the priority queue.
`pqueue-heap`	Interface and implementation file housing the version of the priority queue that's backed by the array-packed binary heap.
`pqueue-binomial-heap`	Interface and implementation file housing the version of the priority queue that's backed by the binomial heap. You can ignore these files unless you elect to do the extra credit portion of the assignment.