

Assignment #2—Using ADTs

*The idea for Random Writer comes from Joe Zachary at the University of Utah.
Parts of this handout were written by Julie Zelenski and Jerry Cain.*

Due: Tuesday, July 5

Now that you've been introduced to the handy CS106 class library, it's time to put these objects to work! In your role as a client of these ADTs with the low-level details abstracted away, you can put your energy toward solving more interesting problems. In this assignment, your job is to write two short client programs that use these classes to do nifty things. The tasks may sound a little daunting at first, but given the power tools in your arsenal, each requires only a page or two of code. Let's hear it for abstraction!

The assignment has several purposes:

1. To let you experience more fully the joy of using powerful library classes.
2. To stress the notion of abstraction as a mechanism for managing data and providing functionality without revealing the representational details.
3. To increase your familiarity with using C++ class templates.
4. To give you some practice with classic data structures such as the stack, queue, vector, map, and lexicon.

Problem 1. Word ladders

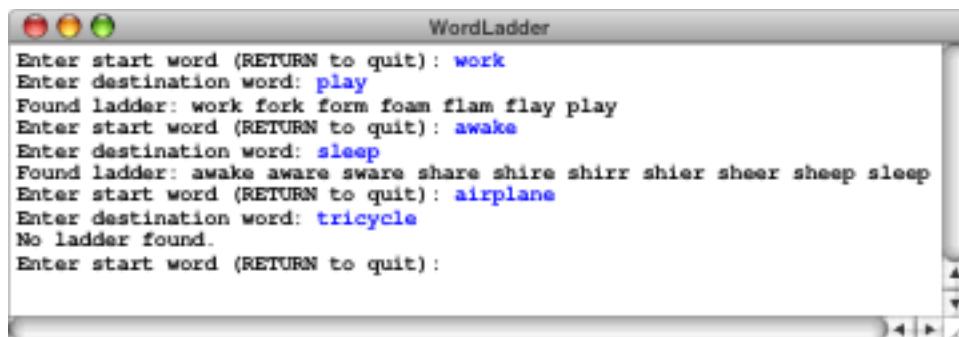
A **word ladder** is a connection from one word to another formed by changing one letter at a time with the constraint that at each step the sequence of letters still forms a valid word. For example, here is a word ladder connecting "code" to "data".

`code → core → care → dare → date → data`

That word ladder, however, is not the shortest possible one. Although the words may be a little less familiar, the following ladder is one step shorter:

`code → cade → cate → date → data`

Your job in this problem is to write a program that finds a minimal word ladder between two words. Your code will make use of several of the ADTs from Chapter 4, along with a powerful algorithm called breadth-first search to find the shortest such sequence. Here, for example, is a sample run of the word-ladder program in operation:



```
WordLadder
Enter start word (RETURN to quit): work
Enter destination word: play
Found ladder: work fork form foam flam flay play
Enter start word (RETURN to quit): awake
Enter destination word: sleep
Found ladder: awake aware sware share shire shirr shier sheer sheep sleep
Enter start word (RETURN to quit): airplane
Enter destination word: tricycle
No ladder found.
Enter start word (RETURN to quit):
```

A sketch of the word ladder implementation

Finding a word ladder is a specific instance of a *shortest-path problem*, in which the challenge is to find the shortest path from a starting position to a goal. Shortest-path problems come up in a variety of situations such as routing packets in the Internet, robot motion planning, determining proximity in social networks, comparing gene mutations, and more.

One strategy for finding a shortest path is the classic algorithm known as *breadth-first search*, which is a search process that expands outward from the starting position, considering first all possible solutions that are one step away from the start, then all possible solutions that are two steps away, and so on, until an actual solution is found. Because you check all the paths of length 1 before you check any of length 2, the first successful path you encounter must be as short as any other.

For word ladders, the breadth-first strategy starts by examining those ladders that are one step away from the original word, which means that only one letter has been changed. If any of these single-step changes reach the destination word, you're done. If not, you can then move on to check all ladders that are two steps away from the original, which means that two letters have been changed. In computer science, each step in such a process is called a *hop*.

The breadth-first algorithm is typically implemented by using a queue to store partial ladders that represent possibilities to explore. The ladders are enqueued in order of increasing length. The first elements enqueued are all the one-hop ladders, followed by the two-hop ladders, and so on. Because queues guarantee first-in/first-out processing, these partial word ladders will be dequeued in order of increasing length.

To get the process started, you simply add a ladder consisting of only the start word to the queue. From then on, the algorithm operates by dequeuing the ladder from the front of the queue and determining whether it ends at the goal. If it does, you have a complete ladder, which must be minimal. If not, you take that partial ladder and extend it to reach words that are one additional hop away, and enqueue those extended ladders, where they will be examined later. If you exhaust the queue of possibilities without having found a completed ladder, you can conclude that no ladder exists.

It is possible to make the algorithm considerably more concrete by implementing it in *pseudocode*, which is simply a combination of actual code and English. The pseudocode for the word-ladder problem appears in Figure 1.

As is generally the case with pseudocode, several of the operations that are expressed in English need to be fleshed out a bit. For example, the loop that reads

for (each word in the lexicon of English words that differs by one letter)

is a *conceptual* description of the code that belongs there. It is, in fact, unlikely that this idea will correspond to a single **for** loop in the final version of the code. The basic idea, however, should still make sense. What you need to do is iterate over all the words that differ from the current word by one letter. One strategy for doing so is to use two nested loops; one that goes through each character position in the word and one that loops through the letters of the alphabet, replacing the character in that index position with each

Figure 1. Pseudocode implementation of the word-ladder algorithm

```

Create an empty queue.
Add the start word to the end of the queue.
while (the queue is not empty) {
    Dequeue the first ladder from the queue.
    if (the final word in this ladder is the destination word) {
        Return this ladder as the solution.
    }
    for (each word in the lexicon of English words that differs by one letter) {
        if (that word has not already been used in a ladder) {
            Create a copy of the current ladder.
            Add the new word to the end of the copy.
            Add the new ladder to the end of the queue.
        }
    }
}
Report that no word ladder exists.

```

of the 26 letters in turn. Each time you generate a word using this process, you need to look it up in the lexicon of English words to make sure that it is actually a legal word.

Another issue that is a bit subtle is the restriction that you not reuse words that have been included in a previous ladder. One advantage of making this check is that doing so reduces the need to explore redundant paths. For example, suppose that you have previously added the partial ladder

cat → cot → cog

to the queue and that you are now processing the ladder

cat → cot → con

One of the words that is one hop away from **con**, of course, is **cog**, so you might be tempted to enqueue the ladder

cat → cot → con → cog

Doing so, however, is unnecessary. If there is a word ladder that begins with these four words, then there must be a shorter one that, in effect, cuts out the middleman by eliminating the unnecessary word **con**. In fact, as soon as you've enqueued a ladder ending with a specific word, you never have to enqueue that word again.

The simplest way to implement this strategy is to keep track of the words that have been used in any ladder (which you can easily do using another lexicon) and ignore those words when they come up again. Keeping track of what words you've used also eliminates the possibility of getting trapped in an infinite loop by building a circular ladder, such as

cat → cot → cog → bog → bag → bat → cat

One of the other questions you will need to resolve is what data structure you should use to represent word ladders. Conceptually, each ladder is just an ordered list of words,

which should make your mind scream out “Vector!” (Given that all the growth is at one end, stacks are also a possibility, but vectors will be more convenient when you are trying to print out the results.) The individual components of the `Vector` are of type `string`.

Implementing the application

At this point, you have everything you need to start writing the actual C++ code to get this project done. It’s all about leveraging the class library—you’ll find your job is just to coordinate the activities of various different queues, vectors, and lexicons necessary to get the job done. The finished assignment requires less than a page of code, so it’s not a question of typing in statements until your fingers get tired. It will, however, certainly help to think carefully about the problem before you actually begin that typing.

As always, it helps to plan your implementation strategy in phases rather than try to get everything working at once. Here, for example, is one possible breakdown of the tasks:

- *Task 1—Try out the demo program.* Play with the demo just for fun and to see how it works from a user’s perspective.
- *Task 2—Read over the descriptions of the classes you’ll need.* For this part of the assignment, the classes you need from Chapter 4 are `Vector`, `Queue`, and `Lexicon`. If you have a good sense of how those classes work before you start coding, things will go *much* more smoothly than they will if you try to learn how they work on the fly.
- *Task 3—Think carefully about your algorithm and data-structure design.* Be sure you understand the breadth-first algorithm and what the various data types you will be using.
- *Task 4—Play around with the lexicon.* The starter project for this problem includes a copy of the `EnglishWords.dat` file described in the reader. Before you write the word ladder application, you might experiment with a simpler program that uses the lexicon in simpler ways. For example, you might write a program that reads in a word and then prints out all the English words that are one letter away.
- *Task 5—Implement the breadth-first search algorithm.* Now you’re ready for the meaty part. The code is not long, but it is dense, and all those templates will conspire to trip you up. We recommend writing some test code to set up a small dictionary (with just ten words or so) to make it easier for you to test and trace your algorithm while you are in development. Test your program using the large dictionary only after you know it works in the small test environment.

Note that breadth-first search is not the most efficient algorithm for generating minimal word ladders. As the lengths of the partial word ladders increase, the size of the queue grows exponentially, leading to exorbitant memory usage when the ladder length is long and tying up your computer for quite a while examining them all. Later this quarter, we will touch on improved search algorithms, and in advanced courses such as CS161 you will learn even more efficient alternatives.

Problem 2. Random language generation using models of English text

In the past few decades, computers have revolutionized student life. In addition to providing no end of entertainment and distractions, computers also have also facilitated

much productive student work. However, one important area of student labor that has been painfully neglected is the task of filling up space in papers, Ph.D. dissertations, grant proposals, and recommendation letters with important sounding and somewhat sensible random sequences. Come on, you know the overworked TA/professor/reviewer doesn't have time to read too closely. . . .

To address this burning need, the random writer is designed to produce somewhat sensible output by generalizing from patterns found in the input text. When you're coming up short on that 10-page paper due tomorrow, feed in the eight pages you already have written into the random writer, and—*Voila!*—another couple of pages appear. You can even feed your own `.cpp` files back into your program and have it build a new random program on demand.

How does random writing work?

Random writing is based on an idea advanced by Claude Shannon in 1948 and subsequently popularized by A. K. Dewdney in his *Scientific American* column in 1989. Shannon's famous paper introduces the idea of a Markov model for English text. A **Markov model** is a statistical model that describes the future state of a system based on the current state and the conditional probabilities of the possible transitions. Markov models have a wide variety of uses, including recognition systems for handwriting and speech, machine learning, and bioinformatics. Even Google's PageRank algorithm has a Markov component to it. In the case of English text, the Markov model is used to describe the possibility of a particular character appearing given the sequence of characters seen so far. The sequence of characters within a body of text is clearly not just a random rearrangement of letters, and the Markov model provides a way to discover the underlying patterns and, in this case, to use those patterns to generate new text that fits the model.

First, consider generating text in total randomness. Suppose you have a monkey at the keyboard who is just as likely to hit any key as another. While it is theoretically possible—given enough monkeys, typewriters, and ages of the universe—that this sort of random typing would produce a work of Shakespeare, most output will be gibberish that makes pretty unconvincing English:

| | |
|-----------------|--|
| No model | <code>aowk fh4.s8an zp[q;1k ss4o2mai/</code> |
|-----------------|--|

A simple improvement is to gather information on character frequency and use that as a weight for choosing the next letter. In English text, not all characters occur equally often. Better random text would mimic the expected character distribution. Read some input text (such as Mark Twain's *Tom Sawyer*, for example) and count the character frequencies. You'll find that spaces are the most common, that the character `e` is fairly common, and that the character `q` is rather uncommon. Suppose that space characters represent 16% of all characters in *Tom Sawyer*, `e` just 9%, and `q` a mere .04% of the total. Using these weights, you could produce random text that exhibited these same probabilities. It wouldn't have a lot in common with the real *Tom Sawyer*, but at least the characters would tend to occur in the proper proportions. In fact, here's an example of what you might produce:

| | |
|----------------|--|
| Order 0 | <code>rla bsht eS ststofo hhfosdsdewno oe wee h .mr ae irii ela</code> |
|----------------|--|

```
iad o r te u t mnyto onmalysnce, ifu en c fDwn oee iteo
```

This is an *order-0 Markov model*, which predicts that each character occurs with a fixed probability, independent of previous characters.

We're getting somewhere, but most events occur in context. Imagine randomly generating a year's worth of Fahrenheit temperature data. A series of 365 random integers between 0 and 100 wouldn't fool the average observer. It would be more credible to make today's temperature a random function of yesterday's temperature. If it is 85 degrees today, it is unlikely to be 15 degrees tomorrow. The same is true of English words: If this letter is a **q**, then the following letter is quite likely to be a **u**. You could generate more realistic random text by choosing each character from the ones likely to follow its predecessor.

For this, process the input and build an order-1 model that determines the probability with which each character follows another character. It turns out that **s** is much more likely to be followed by **t** than **y** and that **q** is almost always followed by **u**. You could now produce some randomly generated *Tom Sawyer* by picking a starting character and then choosing the character to follow according to the probabilities of what characters followed in the source text. Here's some output produced from an order-1 model:

Order 1

```
"Shand tucthiney m?" le ollds mind Theybooure He, he s
whit Pereg lenigabo Jodind alllld ashanthe ainofevids tre
lin--p asto oun theanthadomoere
```

This idea extends to longer sequences of characters. An order-2 model generates each character as a function of the two-character sequence preceding it. In English, the sequence **sh** is typically followed by the vowels, less frequently by **r** and **w**, and rarely by other letters. An order-5 analysis of *Tom Sawyer* reveals that **leave** is often followed by **s** or space but never **j** or **q**, and that **Sawye** is always followed by **r**. Using an order-*k* model, you generate random output by choosing the next character based on the probabilities of what followed the previous *k* characters in the input text. This string of characters preceding the current point is called the *seed*.

At only a moderate level of analysis (say, orders 5 to 7), the randomly generated text begins to take on many of the characteristics of the source text. It probably won't make complete sense, but you'll be able to tell that it was derived from *Tom Sawyer* as opposed to, say, *Pride and Prejudice*. At even higher levels, the generated words tend to be valid and the sentences start to scan. Here are some more examples:

Order 2

```
"Yess been." for gothin, Tome oso; ing, in to weliss of
an'te cle -- armit. Papper a comeasione, and smomenty,
fropeck hinticer, sid, a was Tom, be suck tied. He sis
tred a youck to themen
```

Order 4

```
en themself, Mr. Welshman, but him awoke, the balmy
shore. I'll give him that he couple overy because in
the slated snufflindeed structure's kind was rath. She
said that the wound the door a fever eyes that WITH him.
```

| | |
|----------|--|
| Order 6 | Come -- didn't stand it better judgment; His hands and bury it again, tramped herself! She'd never would be. He found her spite of anything the one was a prime feature sunset, and hit upon that of the forever. |
| Order 8 | look-a-here -- I told you before, Joe. I've heard a pin drop. The stillness was complete, how- ever, this is awful crime, beyond the village was sufficient. He would be a good enough to get that night, Tom and Becky. |
| Order 10 | you understanding that they don't come around in the cave should get the word "beauteous" was over-fondled, and that together" and decided that he might as we used to do -- it's nobby fun. I'll learn you." |

A sketch of the random writer implementation

Your program is to read a source text, build an order- k Markov model for it, and generate random output that follows the frequency patterns of the model.

First, you prompt the user for the name of a file to read for the source text and reprompt as needed until you get a valid name. (And you probably have a function like this lying around somewhere.) Now ask the user for what order of Markov model to use (a number from 1 to 10). This will control what seed length you are working with.

Use simple character-by-character reading on the file. As you go, track the current seed and observe what follows it. Your goal is to record the frequency information in such a way that it will be easy to generate random text later without any complicated manipulations.

Once the reading is done, your program should output 2000 characters of random text generated from the model. For the initial seed, choose the sequence that appears most frequently in the source (e.g., if you are doing an order-4 analysis, the four-character sequence that is most often repeated in the source is used to start the random writing). If there are several sequences tied for most frequent, any of them can be used as the initial seed. Output the initial seed, then choose the next character based on the probabilities of what followed that seed in the source. Output that character, update the seed, and the process repeats until you have 2000 characters.

For example, consider an order-2 Markov model built from this sentence from Franz Kafka's *Metamorphosis*:

As Gregor Samsa awoke one morning from uneasy dreams he found himself transformed in his bed into a gigantic insect.

Here is how the first few characters might be chosen:

- The most commonly occurring sequence is the string "in", which appears four times. This string therefore becomes the initial seed.
- The next character is chosen based on the probability that it follows the seed "in" in the source. The source contains four occurrences of "in", one followed by g, one

followed by **t**, one followed by **s**, and one followed by a space. Thus, there should be a 1/4 chance each of choosing **g**, **t**, **s**, or space. Suppose space is chosen this time.

- The seed is updated to "**n** ". The source contains one occurrence of "**n** ", which is followed by **h**. Thus the next character chosen is **h**.
- The seed is now " **h**". The source contains three occurrences of " **h**", once followed by **e**, and twice followed by **i**. Thus, there is a 1/3 chance of choosing **e** and 2/3 for **i**. Imagine **i** is chosen this time.
- The seed is now "**hi**". The source contains two occurrences of "**hi**", once followed by **m**, the other by **s**. For the next character, there is 1/2 chance of choosing **m** and 1/2 chance for **s**.

If your program ever gets into a situation in which there are no characters to choose from (which can happen if the only occurrence of the current seed is at the exact end of the source), your program can just stop writing early.

A few implementation hints

Although it may sound daunting at first glance, this task is supremely manageable with the bag of power tools you bring to the job site.

- **Map** and **Vector** are just what you need to store the model information. The keys into the map are the possible seeds (e.g., if the order is 2, each key is a 2-character sequence found in the source text). The associated value is a vector of all the characters that follow that seed in the source text. That vector can—and likely will—contain a lot of duplicate entries. Duplicates represent higher probability transitions from this Markov state. Explicitly storing duplicates is the easiest strategy and makes it simple to choose a random character from the correct frequency distribution. A more space-efficient strategy would store each character at most once, with its frequency count. However, it's a bit more awkward to code this way. You are welcome to do either, but if you choose the latter, please take extra care to keep the code clean.
- Determining which seed(s) occurs most frequently in the source can be done by iterating over the entries once you have finished the analysis.
- To read a file one character at a time, use the **get** member function for **ifstream**.

Random writer task breakdown

A suggested plan of attack that breaks the problem into the manageable phases with verifiable milestones:

- *Task 1—Try out the demo program.* Play with the demo just for fun and to see how it works from a user's perspective.
- *Task 2—Review the collection classes described in Chapter 4.* The primary tools you need for this problem are **vector** and **map** from the CS106 libraries. Once you're familiar with what functionality our classes provide, you're in a better position to figure out what you'll need to do for yourself.
- *Task 3—Design your data structure.* Think through the problem and map out how you will store the analysis results. It is vital that you understand how to construct the nested arrangement of string/vector/map objects that will properly represent the

information. Since the **Map** will contain values that are **Vectors**, a nested template type is in your future.

- *Task 4—Implement analyzing source text.* Implement the reading phase. Be sure to develop and test incrementally. Work with small inputs first. Verify your analysis and model storage is correct before you move on. There’s no point in trying to generate random sentences if you don’t even have the data read correctly!
- *Task 5—Implement random generation.* Now you’re ready to randomly generate. Since all the hard work was done in the analysis step, generating the random results is straightforward.

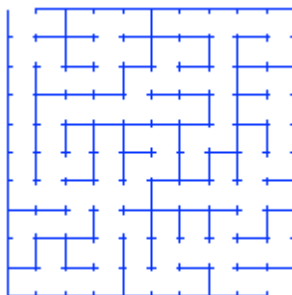
You can run the random writer program on any sort of input text, in any language! The web is a great place to find an endless variety of input material (blogs, slashdot, articles, etc.) When you’re ready for a large test case, Project Gutenberg maintains a library of thousands of full-length public-domain books. We’ve supplied you with files containing the text of Twain’s *Tom Sawyer*, William Shakespeare’s *Hamlet*, and George Eliot’s *Middlemarch*—all of which come from Project Gutenberg. At higher orders of analysis, the results the random writer produces from these sources can be surprisingly sensible and often amusing.

Possible extensions

There are many extensions you could think about introducing into the random writer program—most of which will also make it easier for you to generate a contest entry. One possibility, is to extend the Markov model so that the individual units are words rather than characters. Another it to write a program that runs the Markov model in the opposite direction, in which the goal is to recognize an author by his or her characteristic patterns. This requires building multiple Markov models, one for each candidate author, and comparing them to an unattributed text to find the best match. This sort of “literary forensic analysis” has been used to try to determine the correct attribution for texts where the authorship is unknown or disputed.

Problem 3. Maze Generation

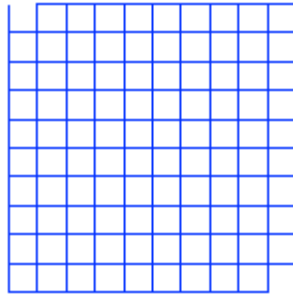
For the final task, you’re to write a program that generates a bona fide maze. Here’s an example of one your program might generate:



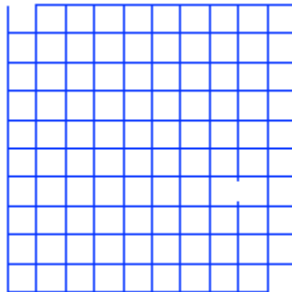
The goal is to create an **n** by **n** maze such that there’s one unique path from the upper left

corner to the lower right one. Our algorithm for generating such mazes is remarkably simple to explain, and given the services of the **Set** and the **Vector**, is almost as easy to implement. (By the way, it's not really our algorithm—it's a simplified version of Kruskal's minimal spanning tree algorithm, which we'll more formally discuss later on when we talk about graphs.)

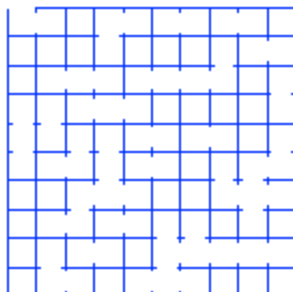
The basic idea has you start with a maze where all possible walls are present, as with:



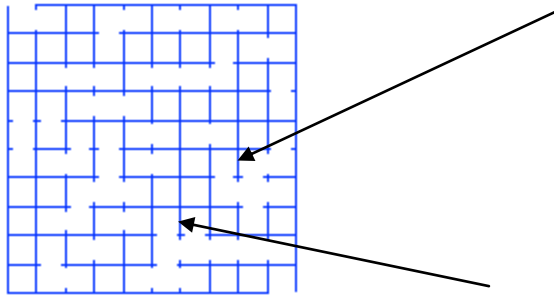
(Our example here happens to involve a 10 x 10 grid, but it generalizes to any dimension.) All walls are present, and the grid is divided into $10^2 = 100$ **chambers**. Each iteration of the algorithm considers a randomly selected wall that's not been considered before, and removes that wall if and only if it separates two chambers. The first randomly selected wall can always be removed, because all locations are initially their own chambers. That first wall might be the one that's now missing in the following:



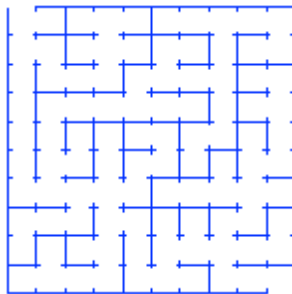
After a several more iterations, you'll see a good number of randomly selected walls have been removed:



The working maze pictured above includes a few walls that should not be removed, because the cells on either side of each of them are part of the same chamber. I've gone ahead and drawn arrows pointing towards some of them here (save for the arrows, it's the same picture you see above):



All walls are considered exactly once in some random order, and if as you consider each wall you notice that it separates two different chambers, you merge those two chambers into one by removing the wall. Initially, there are 100 chambers, so eventually 99 walls are erased to leave one big, daunting maze, like this:



Notice the two walls I drew arrows to in the previous snapshot are still present. We can't tell from the photos when they were considered, but we shouldn't be surprised they're still there in the final maze.

Maze Generation Pseduo-code

Here is a high-level description of a reasonable solution:

```
choose a dimension
generate all walls, and shuffle them
generate all chambers, where each chamber includes one unique cell
foreach wall in randomly-ordered-walls
    if wall separates two chambers
        remove wall
```

We're going to let you tackle this one all by yourself, relying on the pictures and the pseudo-code above to get through it. Here are a few other details worth mentioning:

- All of the graphics routines are documented in **maze-graphics.h**. Everything provided is pretty straightforward, but if you want to make changes to these files, then go for it.
- There's a small header file called **maze-types.h**, which defines a **cell** type (which uses some advanced C++ so that you can build Sets of them without having to define a comparator, and a **wall** type, which helps describe a wall. You can use these definitions, or you can use your own.
- Our solution uses the **Set** (to help model a chamber) and the **Vector** (to maintain an ordered list of walls and chambers). In particular, we didn't use a **Grid** at all, so you shouldn't be worried if you don't use one either.

Requirements and hints for all programs

- The programs will be a hybrid of the procedural and object-oriented paradigms. You will be creating and messaging lots of objects, but will write your code as a procedural algorithm, starting with the **main** function and decomposed into **ordinary** top-level functions.
- Be on your toes about making sure that your template types are always properly specialized and that all types match. Using **Vector** without specialization just won't fly, and a **Vector<int>** is not the same thing as a **Vector<double>**. The error messages you receive when you have mismatches can be cryptic and hard to interpret. Look carefully at the types and see if you can spot where they don't quite match. Also remember that a nested template type requires a space between the two closing angle bracket, e.g. **Vector<Vector<int> >**. If you're stuck on some template syntax, bring your code by the Lair and we can help you sort it out.

References

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