Automatica 109 (2019) 108509

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

Connectivity-preserving flocking for networked Lagrange systems with time-varying actuator faults☆

Zhi Feng, Guoqiang Hu *

School of Electrical and Electronic Engineering, Nanyang Technological University, 639798, Singapore

ARTICLE INFO

Article history:
Received 20 June 2018
Received in revised form 30 May 2019
Accepted 5 July 2019
Available online 8 August 2019

Keywords:
Multi-agent system
Fault-tolerant coordination
Swarming
Connectivity preservation
Collision avoidance

ABSTRACT

This paper studies fault-tolerant coordination with connectivity preservation and collision avoidance for unknown Lagrange agent systems subject to actuator faults. In contrast to the existing works, the actuator faults may change the states of the agents to violate the network connectivity and the existence of the loss of effectiveness faults makes control coefficients unknown and time-varying. Furthermore, the leader's dynamics are allowed to be nonlinear instead of being linear and generated by a marginally stable system. The aforementioned setting improves the practical relevance of the problem to be addressed in the paper and meanwhile, it poses technical challenges to flocking controller design and asymptotic stability analysis. Consequently, for a class of nonlinear leader systems, a novel distributed adaptive scheme is proposed by introducing a distributed estimator and a Nussbaum gain technique to provide a fully distributed solution with guaranteed fault-tolerant coordination, connectivity preservation, and collision avoidance simultaneously. Numerical examples are given to illustrate the effectiveness of the design.

1. Introduction

Multi-agent coordination problems including consensus (Feng, Hu, Ren, Dixon, & Mei, 2018), rendezvous (Feng, Sun, & Hu, 2017), flocking (Wen, Duan, Su, Chen, & Yu, 2012), and formation tracking (Sun, Hu, Xie, & Egerstedt, 2018) are interesting due to potential applications in search and rescue, autonomous surveillance, and environmental monitoring. The existing works in Cai and Huang (2016), Chen, Feng, Liu, and Ren (2015), Feng et al. (2018, 2017), Mei, Ren, and Ma (2011) and Wen et al. (2012) assume that the agents are under a static network keeping its connectivity during the motion evolution. While in practice it is difficult to verify this assumption and the network is always dynamic. To solve this problem, an attractive method is to design distributed controllers to preserve the connectivity of network.

Existing works in Dong (2016), Dong and Huang (2013, 2017, 2018), Feng et al. (2017), Ghapani, Mei, Ren, and Song (2016), Su and Huang (2015), Wen et al. (2012), Xiao, Wang, and Chen (2012), Zavlanos, Egerstedt, and Pappas (2011) and Zavlanos, Jadbabaie, and Pappas (2007) considered rendezvous/flocking in an ideal situation (e.g., no uncertainties, known control coefficients of each agent). By using an output regulation method, distributed controllers were proposed in Dong and Huang (2013) and Su and Huang (2015) for connectivity-preserving rendezvous of multi-agent systems. A discontinuous sliding mode technique was adopted in Dong (2016) for connectivity-preserving rendezvous with disturbance rejection, where disturbances were upper bounded by known constants. The results were extended in Dong and Huang (2017) and Ghapani et al. (2016) to consider networked Euler–Lagrange dynamics in which the designs required global graph information or known upper bounds of the derivatives of leader's states. Our previous work (Feng et al., 2017) developed distributed control laws to achieve robust connectivity-preserving rendezvous compensating for uncertainties and external disturbances.

In reality, the increasingly equipped actuators, sensors, and other components require the reliable operation of multi-agent systems. The main goal is to maintain state awareness and acceptable performance no matter components are healthy or faulty. One way to enhance fault tolerability is to design control algorithms that are directly robust to the effects of anomalies caused by faults (Chen, Song, & Lewis, 2017; Feng & Hu, 2017; Semsar-Kazerooni & Khorasani, 2010; Shen, Jiang, Shi, & Zhao, 2013; Wang, Song, & Lewis, 2015). Another way is to develop distributed monitoring schemes to detect, identify, and then

☆ This work was supported in part by Singapore Ministry of Education Academic Research Fund Tier 1 RG180/17(2017-T1-002-158), and in part by Singapore Economic Development Board under EIRP grant S14-1172-NRF EIRP-IHL. The material in this paper was partially presented at the IEEE International Conference on Robotics and Biomimetics, December 5–8, 2017, Macau, China. This paper was recommended for publication in revised form by Associate Editor Shreyas Sundaram under the direction of Editor Christos G. Cassandras.

* Corresponding author.
E-mail addresses: zhifeng@ntu.edu.sg (Z. Feng), gqhu@ntu.edu.sg (G. Hu).

https://doi.org/10.1016/j.automatica.2019.108509
0005-1098/

design fault-tolerant controller to tolerate faults (Filippo, Marino, & Pierri, 2015; Ma & Yang, 2016). The latter method in general allows more effective responses to anomalies, while it brings complicated and costly processes in fault detection and identification for controller design and implementation (Wang et al., 2015). The authors in Filippo et al. (2015), Ma and Yang (2016), Semsar-Kazerooni and Khorasani (2010) and Shen et al. (2013) considered the bias faults, while the loss of effectiveness actuation faults were studied in Chen et al. (2017) and Wang et al. (2015). However, uniformly ultimately bounded (UUB) results were obtained via robust control approaches for constant or time-varying faults. To the best of our knowledge, few efforts are made on fault-tolerant connectivity-preserving flocking of networked Lagrange systems, which can: (1) compensate for the loss of effectiveness actuation faults that make control coefficients unknown and time-varying; (2) address coupling impacts from limited sensing ranges and unknown control coefficients. The major difficulties in incorporating a Nussbaum gain technique for a connectivity-preserving flocking problem of Lagrange agent systems are twofold: (1) due to network constraints, not only the control laws but also the parameter updating laws are required to be distributed and independent of any global graph information or leader's dynamics; and (2) unlike adopting this technique for a single agent system in Sam Ge and Wang (2003), Yang, Ge, and Sun (2015) and Ye and Jing (1998) or multi-agent systems in Chen, Li, Ren, and Wen (2014) with a static communication graph keeping its connectivity for all time, the problem in this paper requires the graph to be time-varying and state-dependent which brings more challenges in designing distributed control laws for networked Lagrange systems.

Contribution: this paper studies fault-tolerant flocking for networked Lagrange systems under a limited sensing range. A distributed adaptive control framework is proposed for the fault-tolerant flocking and connectivity preservation. In contrast to the existing results in Cai and Huang (2016), Chen et al. (2015, 2017), Dong (2016), Dong and Huang (2013, 2017, 2018), Feng and Hu (2017), Feng et al. (2018, 2017), Filippo et al. (2015), Ghapani et al. (2016), Ma and Yang (2016), Mei et al. (2011), Semsar-Kazerooni and Khorasani (2010), Shen et al. (2013), Su and Huang (2015), Sun et al. (2018), Wang et al. (2015), Wen et al. (2012), Xiao et al. (2012) and Zavlanos et al. (2011, 2007), the main contributions of this paper are summarized as follows. (1) A fully distributed solution to the fault-tolerant flocking problem is provided for Lagrange systems under a limited sensing range and time-varying actuator faults. This solution is independent of any global graph information or leader's states/dynamics, and only depends on local agent states and relative neighborhood information; (2) To the best of our knowledge, this is the first distributed control scheme capable of simultaneously achieving fault-tolerant coordination, connectivity preservation, and collision avoidance. The Nussbaum function is employed to handle unknown control coefficients caused by actuator faults, while the potential function is used to enable connectivity-preserving flocking. Due to the requirements of designing distributed protocols and handling unknown coefficients in a dynamic communication topology, the synthesized design is nontrivial. In addition, the proposed controller enables asymptotic convergence, whereas the works in Chen et al. (2017), Feng and Hu (2017), Filippo et al. (2015), Ma and Yang (2016), Semsar-Kazerooni and Khorasani (2010), Shen et al. (2013) and Wang et al. (2015) only provide UUB convergence; (3) In the absence of faults, it will recover the consensus/rendezvous/flocking problems in Cai and Huang (2016), Chen et al. (2015), Dong (2016), Dong and Huang (2013, 2017, 2018), Feng et al. (2018, 2017), Ghapani et al. (2016), Mei et al. (2011), Su and Huang (2015), Sun et al. (2018), Wen et al. (2012), Xiao et al. (2012) and Zavlanos et al. (2011, 2007). However, the proposed design is different at least from the following three aspects: (a) remove the need of having access to leader's states for each follower in Chen et al. (2015); (b) avoid two-hop communication issues in Mei et al. (2011) and Sun et al. (2018); and (c) apply to a nonlinear leader trajectory instead of a marginally stable linear leader system in Cai and Huang (2016), Dong and Huang (2013, 2017) and Su and Huang (2015). Thus, the proposed design makes implementation of algorithms more practical and feasible.

Organization: Section 2 gives the model of Lagrange agents, actuator faults and formulated problem, respectively. The main results are provided in Section 3. Section 4 provides numerical examples followed by the conclusion in Section 5.

Notation: $\mathbb{R}$ and $\mathbb{R}^{N\times N}$ denote the sets of the reals and $N \times N$ matrices, respectively. 0_N (1_N) denotes the $N \times 1$ vector with zeros (ones). Let $\mathrm{col}(x_1, \ldots, x_N)$ and $\mathrm{diag}\{a_1, \ldots, a_N\}$ be a column vector and a diagonal matrix, respectively, with their entries x_i and a_i, $i = 1, \ldots, N$ representing a scalar (or a vector). $\otimes$ and $\|\cdot\|$ denote, respectively, the Kronecker product and Euclidean norm. For a real symmetric matrix M, $\lambda_{\min}(M)$ and $\lambda_{\max}(M)$ denote its minimum and maximum eigenvalues, respectively.

2. Problem formulation

2.1. Networked multi-agent model

Consider a group of N agents, each of which is governed by the following Euler–Lagrange dynamics (Cai & Huang, 2016; Dong & Huang, 2017; Feng et al., 2018; Ghapani et al., 2016)

$$M_i(q_i)\ddot{q}_i + C_i(q_i, \dot{q}_i)\dot{q}_i + G_i(q_i) = \tau_i + d_i, \ \ i = 1, \ldots, N, \tag{1}$$

where $q_i \in \mathbb{R}^n$ denotes the vector of generalized configuration coordinate, $M_i(q_i) \in \mathbb{R}^{n\times n}$ is the inertia matrix, $C_i(q_i, \dot{q}_i)\dot{q}_i \in \mathbb{R}^n$ is the vector of Coriolis and centrifugal torques, $G_i(q_i) \in \mathbb{R}^n$ is the vector of gravitational torque, $\tau_i \in \mathbb{R}^n$ is the control input, and $d_i \in \mathbb{R}^n$ is the disturbance.

Property 1. *$M_i(q_i)$ is symmetric and positive definite.*

Property 2. *$\dot{M}_i(q_i) - 2C_i(q_i, \dot{q}_i)$ is skew symmetric.*

Property 3. *For $x, y \in \mathbb{R}^n$, $M_i(q_i)y + C_i(q_i, \dot{q}_i)x + G_i(q_i) = Y_i(q_i, \dot{q}_i, x, y)\theta_i$, where $Y_i(\cdot) \in \mathbb{R}^{n\times p}$ is a known regression matrix, and $\theta_i \in \mathbb{R}^p$ is an unknown parameter vector.*

In (1), the external disturbances $d_i = \mathrm{col}(d_{i1}, d_{i2}, \ldots, d_{in}) \in \mathbb{R}^n$ may come in many forms, and here are described by

$$d_{ik}(t) = \phi_{ik} + \sum_{j=1}^{\mathbf{m}_{ik}} \psi_{ikj}\sin(\omega_{ikj}t + \gamma_{ikj}) = \psi_{ik}^T\gamma_{ik}, \tag{2}$$

where ϕ_{ik}, ψ_{ikj} are unknown amplitudes, γ_{ikj} are unknown phase angles, ω_{ikj} are known frequencies, $\psi_{ik} = \mathrm{col}(1, \ \sin\omega_{ik1}t \cdots \sin\omega_{ik\mathbf{m}_{ik}}t, \ \cos\omega_{ik1}t \cdots \cos\omega_{ik\mathbf{m}_{ik}}t) \in \mathbb{R}^{2n\mathbf{m}_{ik}+1}$ is a known vector, and $\gamma_{ik} = \mathrm{col}(\phi_{ik}, \ \psi_{ik1}\cos\gamma_{ik1} \cdots \psi_{ik\mathbf{m}_{ik}}\cos\gamma_{ik\mathbf{m}_{ik}}, \ \psi_{ik1}\sin\gamma_{ik1} \cdots \psi_{ik\mathbf{m}_{ik}}\sin\gamma_{ik\mathbf{m}_{ik}}) \in \mathbb{R}^{2n\mathbf{m}_{ik}+1}$ is an unknown vector with constant elements.

Let $\Psi_i = \mathrm{diag}\{\psi_{i1}, \ldots, \psi_{in}\}$ and $\Upsilon_i = \mathrm{col}(\gamma_{i1}, \ldots, \gamma_{in})$. The external disturbances d_i can be rewritten as

$$d_i(t) = \Psi_i^T\Upsilon_i, \ \ i = 1, \ldots, N. \tag{3}$$

Remark 1. The external disturbances in (2) are in the form of harmonic or periodic signals that represent a class of sinusoidal functions of arbitrary amplitudes, initial phases and frequencies. For networked Lagrange systems in (1), many practical disturbances are modeled as harmonic signals include the wave and current disturbances for underwater robots and wind disturbances for aerial robots.

2.2. Actuator fault representation

In contrast to the existing works of distributed coordination based on healthy actuation of multi-agent systems, actuators with undetectable faults are considered in this paper. When actuation faults occur, there exists a discrepancy between the actual control input τ_{ai} and the designed control input τ_i of the ith actuator (e.g., Chen et al., 2017; Wang et al., 2015; Yang et al., 2015):

$$\tau_{ai} = b_i(t, t_{bi})\tau_i + o_i(\upsilon(t), t_{oi}),\ i = 1, \dots, N, \tag{4}$$

where $b_i = \operatorname{diag}\{b_{i1}l_{i1}, \dots, b_{in}l_{in}\} \in \mathbb{R}^{n\times n}$ is an unknown time-varying matrix with $0 < b_{ij}(t) \le 1$ $(j = 1, 2, \dots, n)$ reflecting the actuation effectiveness of the ith agent and $l_{ij} \ne 0$ being an unknown parameter with unknown signs; $o_i(\cdot) \in \mathbb{R}^n$ denotes a known $\mathcal{C}^1$ function reflecting the additive bias faults, where $\upsilon(t) \in \mathbb{R}^{n_\upsilon}$ is a time-varying signal. In (4), $t_{bi} = \operatorname{col}(t_{bi1}, \dots, t_{bin})$ and $t_{oi} = \operatorname{col}(t_{oi1}, \dots, t_{oin})$ with t_{bij} and t_{oij} denoting, respectively, the time instants at which the loss of actuation effectiveness faults and the bias faults occur in the jth actuator of the ith agent.

Remark 2. The actuator fault representation in (4) leads to the time-varying control coefficients with unknown signs which make it difficult to design fault-tolerant flocking algorithms. Similar fault formulation was also studied in Yang et al. (2015) for a single agent system. Note that the Nussbaum gain technique and sliding model control scheme are two widely used methods in the existing works (Ton, Mehta, & Kan, 2017; Yang et al., 2015) to deal with unknown control coefficients. However, when it comes to multi-agent systems, the designs in Ton et al. (2017) and Yang et al. (2015) might not be directly applied due to the requirements of designing protocols in a distributed fashion and the challenges of handling agent control coefficients in a time-varying communication graph.

2.3. Communication network description

Suppose that each agent can communicate with its neighboring agents under a limited sensing range $R > 0$. On the other hand, once the distance between agents falls below a threshold $r \in (0, R)$, collision avoidance maneuvers must be studied. Let $\mathcal{G}(t) = \{\mathcal{V},\ \mathcal{E}(t)\}$ be an undirected graph where $\mathcal{V} = \{1, 2, \dots, N\}$ is a set of vertices whose elements represent the agents, and $\mathcal{E}(t) \subseteq \mathcal{V} \times \mathcal{V}$. Then, the neighbor set for agent i is $\mathcal{N}_i(t) = \{j : \|q_i - q_j\| < R\}$. Let $x_0(t) = \operatorname{col}(q_0(t), \dot{q}_0(t))$ be a desired reference trajectory (viewed as a virtual leader $\mathcal{V}_0$). Thus, for the leader-following network, denote $\bar{\mathcal{G}}(t) = (\bar{\mathcal{V}}, \bar{\mathcal{E}}(t))$ with $\bar{\mathcal{V}} = \{0, 1, 2, \dots, N\}$ and $\bar{\mathcal{E}}(t) \subseteq \bar{\mathcal{V}} \times \bar{\mathcal{V}}$. The node 0 is associated with the leader. $(i, j) \in \bar{\mathcal{E}}(t)$ if and only if agent j can use the information of agent i for control. Let $\bar{\mathcal{N}}_i(t) = \{j \ne i, (j, i) \in \bar{\mathcal{E}}(t)\}$ be the neighbor set of agent i. Clearly, $\mathcal{G}(t) = (\mathcal{V}, \mathcal{E}(t))$ defines a subgraph of $\bar{\mathcal{G}}(t)$ where $\mathcal{E}(t)$ is obtained from $\bar{\mathcal{E}}(t)$ by removing all the edges between the node 0 and nodes in $\bar{\mathcal{V}}$.

To introduce our problem, we first characterize the network connectivity setting (Dong & Huang, 2017, 2018; Feng et al., 2017; Zavlanos et al., 2007). Given any $R > 0$, $\epsilon \in (0, R)$, $r \in [0, R - \epsilon)$, the edge set $\bar{\mathcal{E}}(t) = \{(i, j)|i, j \in \bar{\mathcal{V}}\}$ is defined:

(1) $\bar{\mathcal{E}}(0) = \{(i, j) : r < \|q_i(0) - q_j(0)\| < R - \varepsilon, i, j = 1, \dots, N\} \cup \{(0, j) : r < \|q_0(0) - q_j(0)\| < R - \varepsilon\}$;

(2) If $\|q_i(t) - q_j(t)\| > R$, then $(i, j) \notin \bar{\mathcal{E}}(t)$;

(3) $(i, 0) \notin \bar{\mathcal{E}}(t)$, for $i = 0, 1, \dots, N$;

(4) For $i = 0, 1, \dots, N$, $j = 1, \dots, N$, if $(i, j) \notin \bar{\mathcal{E}}(t^-)$ and $r < \|q_i(t) - q_j(t)\| < R - \varepsilon$, then $(i, j) \in \bar{\mathcal{E}}(t)$;

(5) For $i = 0, 1, \dots, N$, $j = 1, \dots, N$, if $(i, j) \in \bar{\mathcal{E}}(t^-)$ and $r < \|q_i(t) - q_j(t)\| < R$, then $(i, j) \in \bar{\mathcal{E}}(t)$.

2.4. Control objective

The control objective is to ensure that the states of agent i move cohesively with the leader to preserve connectivity and avoid collision, and eventually achieve velocity alignment under actuator faults, as described in Fig. 1.

Problem 1. Given systems (1), (2), (4), find a distributed fault-tolerant control law τ_i such that for any initial conditions that makes $\bar{\mathcal{G}}(0)$ contain a spanning tree with the node 0 as the root, the closed-loop system has the following properties:
(i) connectivity preservation: $\bar{\mathcal{G}}(t)$ is connected for all $t \ge 0$;
(ii) collision avoidance and cohesive moving: agents move cohesively, while $\|q_i - q_j\| > r,\ i = 1, \dots, N, j = 0, 1, \dots, N$;
and (iii) velocity alignment: $\lim_{t\to\infty}(\dot{q}_i - \dot{q}_0) = 0_n,\ i = 1, \dots, N$.

Remark 3. Although this problem formulation is similar to those in Dong (2016), Dong and Huang (2013, 2017, 2018), Ghapani et al. (2016), Su and Huang (2015) and Zavlanos et al. (2007), it is more challenging at least from the following aspects. (1) Agent dynamics: the Lagrange plants are allowed to be subject to time-varying actuator faults and uncertainties. (2) Nonlinear leader: the leader's trajectory does not need to be generated by a marginally stable linear system in Dong and Huang (2013, 2017) and Su and Huang (2015) or a constant/varying velocity in Ghapani et al. (2016). (3) Design requirements: it aims to propose the connectivity-preserving flocking algorithm with a fault-tolerant feature in a time-varying communication graph and a fully distributed nature that is independent of any global graph information or leader's states. Due to the aforementioned challenges, the proposed algorithms in Dong (2016), Dong and Huang (2013, 2017, 2018), Ghapani et al. (2016), Su and Huang (2015) and Zavlanos et al. (2007) cannot be directly applied.

To solve Problem 1, we make the following assumptions.

Assumption 1. The states $x_0(t)$ and $\upsilon(t)$ are smooth such that $x_0^{(i)}(t), \upsilon^{(i)}(t),\ \forall i = 1, 2, 3$ exist and are upper bounded by certain unknown constants (Feng et al., 2018; Ghapani et al., 2016; Shen et al., 2013).

Assumption 2. For $i = 1, 2, \dots, N$, the nonzero elements of $b_i(t)$ are unknown and time-varying, having same signs and taking values in a set $I_i := [b_i^-, b_i^+]$, $0 \notin I_i$ (Chen et al., 2014; Sam Ge & Wang, 2003; Ton et al., 2017; Yang et al., 2015; Ye & Jing, 1998).

Remark 4. Assumption 1 is made for a time-varying trajectory that only satisfies mild smoothness and boundedness properties. In practice, many guidance and navigation applications utilize smooth high-order differentiable trajectories (Feng et al., 2018; Ghapani et al., 2016; Shen et al., 2013). The unknown upper bounds on any derivatives of $x_0(t), \upsilon(t)$ will not be used in the controller. Assumption 2 is a standard assumption in Chen et al. (2014), Sam Ge and Wang (2003), Ton et al. (2017), Yang et al. (2015) and Ye and Jing (1998), which indicates that $b_i(t)$ are unknown and time-varying in comparison with those in Cai and Huang (2016), Chen et al. (2015, 2017), Dong (2016), Dong and Huang (2013, 2017, 2018), Feng and Hu (2017), Feng et al. (2018, 2017), Filippo et al. (2015), Ghapani et al. (2016), Ma and Yang (2016), Mei et al. (2011), Semsar-Kazerooni and Khorasani (2010), Shen et al. (2013), Su and Huang (2015), Sun et al. (2018), Wang et al. (2015), Wen et al. (2012), Xiao et al. (2012) and Zavlanos et al. (2011, 2007) that require all the control coefficients are either known or unknown but constant. In conclusion, Assumptions 1 and 2 are mild assumptions.

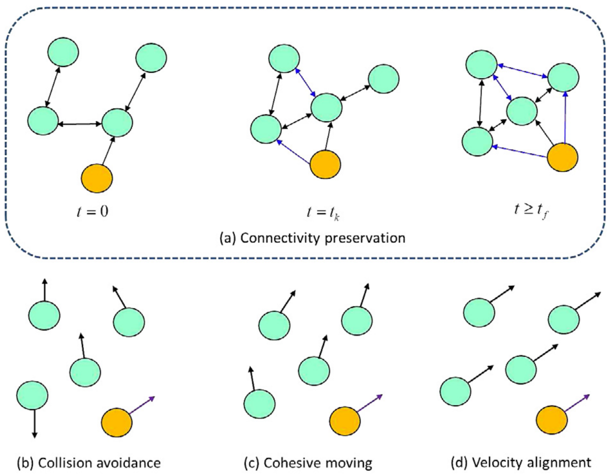


Fig. 1. Graphical representation of connectivity-preserving flocking. The blue line denotes added edges, while the yellow and green circles represent the leader and followers, respectively. (a)–(d) describe the properties.

3. Main results

The main design includes: (1) a distributed smooth estimator that estimates the states $\dot{q}_0(t)$ and $\upsilon(t)$; and (2) a distributed controller that guarantees fault-tolerant coordination with connectivity preservation and collision avoidance. By incorporating a Nussbaum function based adaptive control scheme, we show that Problem 1 can be solved via a novel graph-independent distributed controller.

Distributed Estimator Design: for $i = 1, 2, \ldots, N$, the following distributed estimator is developed

$$\dot{\hat{v}}_i = -\int_0^t [\alpha_i(\tau)\xi_i(\tau) + \beta_i(\tau)\text{sgn}(\xi_i(\tau))]d\tau - 2\hat{v}_i, \tag{5}$$

$$\xi_i = \sum_{j=1}^{N} a_{ij}(t)(\hat{v}_i - \hat{v}_j) + a_{i0}(t)(\hat{v}_i - v_0),\ \hat{v}_i(0) = \hat{v}_{iO}, \tag{6}$$

where $\hat{v}_i = \text{col}(\hat{v}_{1i}, \hat{v}_{2i})$ is used to estimate $v_0 = \text{col}(\dot{q}_0, \upsilon)$, $\hat{v}_{iO}$ is an initial condition, and the adaptive gains are

$$\alpha_i = \frac{1}{2}k_{\alpha i}\xi_i^T\xi_i + k_{\alpha i}\int_0^t \xi_i^T(\tau)\xi_i(\tau)d\tau, \tag{7}$$

$$\beta_i = k_{\beta i}\|\xi_i\|_1 + k_{\beta i}\int_0^t \|\xi_i(\tau)\|_1 d\tau, \tag{8}$$

where $k_{\alpha i}, k_{\beta i} > 0$, $\|\cdot\|_1$ is the 1-norm, and for $i \in \mathcal{V}, j \in \bar{\mathcal{V}}$,

$$a_{ij}(t) = \begin{cases} 1, & (j, i) \in \bar{\mathcal{E}}(t), \\ 0, & \text{otherwise}. \end{cases} \tag{9}$$

Let $\tilde{v}_i = \hat{v}_i - v_0$ and $\Delta_{di} = -2\dot{v}_0 - \ddot{v}_0$. Then, by (5) and (6),

$$\ddot{\tilde{v}}_i = -2\dot{\tilde{v}}_i - \alpha_i \sum_{j=1}^{N} h_{ij}(t)\tilde{v}_j - \beta_i \text{sgn}(\sum_{j=1}^{N} h_{ij}(t)\tilde{v}_j) + \Delta_{di}, \tag{10}$$

where $h_{ij}(t)$ is the (i, j)th entry of $H(t)$ defined as $h_{ij}(t) = -a_{ij}(t)$ for $i \neq j$ and $h_{ii}(t) = \sum_{j=0, j\neq i}^{N} a_{ij}(t)$.

Lemma 1. *Assume that there exists $t^* \in [0, +\infty)$ so that $\bar{\mathcal{G}}(0) \subseteq \bar{\mathcal{G}}(t)$ and $\bar{\mathcal{G}}(t)$ contains a spanning tree with the node 0 as the root for all $t \in [0, t^*)$. Then, under Assumption 1, the following function $\mathcal{P}(t)$ is positive semi-definite*

$$\mathcal{P}(t) = \sum_{i=1}^{N}[k_\beta\|\xi_i(0)\|_1 - (\sum_{j=1}^{N} h_{ij}(0)\tilde{v}_j(0))^T \Delta_{di}(0)] - \mathcal{S}(t),$$

where $\mathcal{S}(t)$ is the Filippov solution to the following equation

$$\dot{\mathcal{S}}(t) = \sum_{i=1}^{N}(\sum_{j=1}^{N} h_{ij}\rho_j)^T[\Delta_{di} - k_\beta sgn(\xi_i)],\ \ \mathcal{S}(0) = 0, \tag{11}$$

provided that $\rho_j = \tilde{v}_j + \dot{\tilde{v}}_j$ and k_β defined in (25) satisfies

$$k_\beta > \sup_{t\in[0,\infty)} \{2\|\dot{v}_0(t)\|_\infty + 3\|\ddot{v}_0(t)\|_\infty + \|\dddot{v}_0(t)\|_\infty\}. \tag{12}$$

Proof. It is similar to our works in Feng et al. (2018, 2017), and is omitted. □

Remark 5. In (5)–(9), a distributed estimator has been proposed to estimate the global information $v_0 = \text{col}(\dot{q}_0, \upsilon)$ with a dynamic communication graph. The development of this distributed estimator is significant because it can decouple the information interaction between the followers and the leader. Then, each agent can utilize the local estimator state $\hat{v}_i$ to facilitate the following distributed controller development. In comparison with the existing works, this estimator brings several advantages: (1) provide a fully distributed design because the adaptive gains α_i and β_i are utilized to remove the requirement of any global graph information and the unknown upper bounds of leader's dynamics; (2) remove the need of leader's states for each follower in Chen et al. (2015) and avoid two-hop communication issues in Mei et al. (2011) and Sun et al. (2018); and (3) estimate a nonlinear leader instead of a marginally stable linear leader system in Cai and Huang (2016), Dong and Huang (2013, 2017) and Su and Huang (2015). Thus, the proposed design makes the implementation of algorithms more practical and feasible.

To get connectivity preservation and collision avoidance, define the following bounded potential function[1]

$$\varphi(s) = \frac{s^2 - r^2}{R^2 - s^2 + \frac{R^2}{Q}} + \frac{R^2 - s^2}{s^2 - r^2 + \frac{R^2}{Q}},\ \ r \leq s \leq R, \tag{13}$$

where $Q > 0$, $s = \|q_{ij}\|_\sigma$, $q_{ij} = q_i - q_j$, and the σ-norm (defined as $\|\chi\|_\sigma = (\sqrt{1 + \varsigma\|\chi\|^2} - 1)/\varsigma$ for a vector χ and a scalar $\varsigma > 0$) is used to have a smooth potential (Feng et al., 2017).

Lemma 2. *$\varphi(s)$ is a nonnegative function of $\|q_{ij}\|_\sigma$, and it is differentiable with respect to $\|q_{ij}\|_\sigma \in (r, R)$ so that*

(i) $d\varphi(s)/ds$ has a unique zero at $r_0 = \sqrt{(R^2 + r^2)/2}$;

(ii) $\varphi(s) \to (1 - r^2/R^2)Q = \bar{Q} \in [Q_{\max}, +\infty)$ as $s \to R$ or r. For any $k_1 > 0$, $k_2 \geq 0$, $\epsilon_1 \in (r, r_0)$, and $\epsilon_2 \in [r, R - r_0)$, there exists $Q > 0$ such that $\varphi(R) = \varphi(r) = \bar{Q} > Q_{max} = k_1 \max\{\varphi(\epsilon_1), \varphi(R - \epsilon_2)\} + k_2$.

Proof. Based on certain calculations, the proof can be done by following a similar way in Dong and Huang (2017, 2018), Ghapani et al. (2016), Zavlanos et al. (2007), and is thus omitted. □

[1] Motivated by Dong and Huang (2017, 2018), Ghapani et al. (2016) and Zavlanos et al. (2007), (13) is defined to achieve flocking. It observes that this generalized potential function is bounded and smooth, while for $s = \|q_{ij}\|$, $r < s < R$, the ones in Dong and Huang (2018) and Zavlanos et al. (2007) with $\varphi(s) = \frac{1}{s^2} + \frac{1}{R^2 - s^2}$ and in Dong and Huang (2017) and Ghapani et al. (2016) with $\varphi(s) = \frac{1}{2(s^2 - r^2)} + \frac{1}{2(R^2 - s^2)}$ are unbounded. If collision avoidance is not required, it is reduced to $\varphi(s) = \frac{s^2}{R^2 - s^2 + \frac{R^2}{Q}}$, $0 \leq s \leq R$, which recovers the connectivity-preserving rendezvous results in Dong and Huang (2013, 2017, 2018), Feng et al. (2017) and Su and Huang (2015).

To handle unknown and time-varying $b_i(t)$, the Nussbaum function $\mathcal{N}(k)$ is introduced, which is smooth and satisfies: $\lim\limits_{k\to\infty}\sup\frac{1}{k}\int_0^k \mathcal{N}(s)ds = +\infty$ and $\lim\limits_{k\to\infty}\inf\frac{1}{k}\int_0^k \mathcal{N}(s)ds = -\infty$ (Chen et al., 2014; Sam Ge & Wang, 2003; Ye & Jing, 1998). Then, denote $\mathcal{N}_i(\chi_i) = \text{diag}\{\mathcal{N}_i(\chi_{i1}), \ldots, \mathcal{N}_i(\chi_{in})\}$ as a diagonal matrix with the main diagonal being the Nussbaum function. We choose $\mathcal{N}_i(\chi_{ik}) = \exp(\chi_{ik}^2)\cos((\pi/2)\chi_{ik}) + 1$, $k = 1, \ldots, n$ throughout the paper.

To facilitate the controller design, we define the following error signal s_i via estimated information $\hat{\upsilon}_i = \text{col}(\hat{\upsilon}_{1i}, \hat{\upsilon}_{2i})$

$$s_i = \dot{q}_i - \dot{q}_{ri},\ \dot{q}_{ri} = \hat{\upsilon}_{1i} - \alpha_{\varrho i}\sum_{j\in\bar{\mathcal{N}}_i(t)} \frac{\partial\varphi(\|q_{ij}\|_\sigma)}{\partial q_i}, \tag{14}$$

where $\dot{q}_{ri}$ is the virtual reference velocity, $\hat{\upsilon}_{1i}$ from $\hat{\upsilon}_i = \text{col}(\hat{\upsilon}_{1i}, \hat{\upsilon}_{2i})$ in (5) is the estimation of $\dot{q}_0(t)$, and $\alpha_{\varrho i} > 0$.

Distributed Controller Design: to make the full use of the Nussbaum function and local estimates $\hat{\upsilon}_i$ in (5), we propose a novel distributed fault-tolerant adaptive controller:

$$\tau_i = \mathcal{N}_i(\chi_i)u_i,\ u_i = Y_i\hat{\theta}_i - o_i(\hat{\upsilon}_{2i}) - \kappa_{si}(t)s_i - \hat{d}_i, \tag{15}$$

$$s_i = \dot{q}_i - \hat{\upsilon}_{1i} + \alpha_{\varrho i}\sum_{j\in\bar{\mathcal{N}}_i(t)} \frac{\partial\varphi(\|q_{ij}\|_\sigma)}{\partial q_i},\ i = 1, \ldots, N, \tag{16}$$

where $\mathcal{N}_i(\chi_i)$ is a Nussbaum function, $Y_i = Y_i(q_i, \dot{q}_i, \dot{q}_{ri}, \ddot{q}_{ri})$, $\hat{\theta}_i$ denotes the estimate of θ_i, $\hat{d}_i = \Psi_i^T\hat{\Upsilon}_i$ denotes the estimate of d_i in (3), $\hat{\upsilon}_i = \text{col}(\hat{\upsilon}_{1i}, \hat{\upsilon}_{2i})$ is the estimated state of $\upsilon_0 = \text{col}(\dot{q}_0, \upsilon)$ defined in (5), and the adaptive control gains are designed for $k_{\chi i}, k_{si}, k_{\theta i}, k_{\Psi i} > 0$, $i = 1, \ldots, N$,

$$\dot{\chi}_i = -k_{\chi i}\text{diag}(s_{i1}, \ldots, s_{in})u_i,\ \dot{\kappa}_{si} = k_{si}s_i^Ts_i, \tag{17}$$

$$\dot{\hat{\theta}}_i = -k_{\theta i}Y_i^T(q_i, \dot{q}_i, \dot{q}_{ri}, \ddot{q}_{ri})s_i,\ \dot{\hat{\Upsilon}}_i = k_{\Psi i}\Psi_i s_i. \tag{18}$$

Remark 6. In (14), $\dot{q}_{ri}(t)$ introduces the partial derivatives of the potential function in the controller, and it is a key to the flocking problem. Although $\dot{q}_{ri}(t)$ appears to have a similar form with those in Cai and Huang (2016), Chen et al. (2015), Feng et al. (2018) and Mei et al. (2011) where the partial derivatives are replaced by the position synchronization term, it actually differs fundamentally from those works mainly in that $\dot{q}_{ri}(t)$ relies on $\alpha_{\varrho i}\sum_{j\in\bar{\mathcal{N}}_i(t)}\frac{\partial\varphi(\|q_{ij}\|_\sigma)}{\partial q_i}$ to achieve flocking. On the other hand, to deal with unknown control coefficients, the Nussbaum gain technique is employed for rejecting actuator faults and enabling asymptotic convergence, whereas works in Chen et al. (2017), Feng and Hu (2017), Filippo et al. (2015), Ma and Yang (2016), Semsar-Kazerooni and Khorasani (2010), Shen et al. (2013) and Wang et al. (2015) on actuator faults only achieve UUB results. From (15)–(17), $\mathcal{N}_i(\chi_i(t))$, $s_i(t)$, and $\dot{q}_{ri}(t)$ are properly designed to construct an estimator-based controller that is fully distributed and fault-tolerant so that connectivity preservation, collision avoidance and velocity matching can be simultaneously achieved under faults and uncertainties.

Remark 7. By incorporating the estimator $\hat{\upsilon}_i(t)$ and Nussbaum function, a novel distributed scheme is developed in (15)–(18) to solve Problem 1. Unlike (Cai & Huang, 2016; Chen et al., 2015, 2017; Dong, 2016; Dong & Huang, 2013, 2017, 2018; Feng & Hu, 2017; Feng et al., 2018, 2017; Filippo et al., 2015; Ghapani et al., 2016; Ma & Yang, 2016; Mei et al., 2011; Semsar-Kazerooni & Khorasani, 2010; Shen et al., 2013; Su & Huang, 2015; Sun et al., 2018; Wang et al., 2015; Wen et al., 2012; Xiao et al., 2012; Zavlanos et al., 2011, 2007), this distributed estimator-based controller is more general, explained as

- The controller can account for unknown and time-varying control coefficients thanks to the adopted Nussbaum gain technique. The proposed design removes the assumption in Cai and Huang (2016), Chen et al. (2015, 2017), Dong (2016), Dong and Huang (2013, 2017, 2018), Feng and Hu (2017), Feng et al. (2018, 2017), Filippo et al. (2015), Ghapani et al. (2016), Ma and Yang (2016), Mei et al. (2011), Semsar-Kazerooni and Khorasani (2010), Shen et al. (2013), Su and Huang (2015), Sun et al. (2018), Wang et al. (2015), Wen et al. (2012), Xiao et al. (2012) and Zavlanos et al. (2011, 2007) that all control coefficients are available for controller design, and it can handle the actuator faults.
- In the absence of faults (i.e., $b_i = 1$), it reduces to the leaderless or leader-following rendezvous/flocking designs in Dong (2016), Dong and Huang (2013, 2017, 2018), Su and Huang (2015) and Zavlanos et al. (2007). Moreover, it can be applied to a nonlinear leader case, while works in Cai and Huang (2016), Dong and Huang (2013, 2017) and Su and Huang (2015) require that the leader system is generated by an autonomous linear system with its system matrix S being available to all followers and having semi-simple eigenvalues with zero real parts.
- The proposed distributed controller guarantees that zero-error coordinated tracking can be achieved even when the leader is nonlinear and time-varying. Therefore, this improves the UUB convergence results in Chen et al. (2017), Ghapani et al. (2016), Mei et al. (2011), Shen et al. (2013) and Wang et al. (2015).
- The proposed controller is fully distributed in the sense that the control gains rely on only local information interactions and are independent of any global graph information or the leader's states/dynamics required in Cai and Huang (2016), Chen et al. (2015, 2017), Dong (2016), Dong and Huang (2013, 2017, 2018), Feng and Hu (2017), Feng et al. (2018, 2017), Filippo et al. (2015), Ghapani et al. (2016), Ma and Yang (2016), Mei et al. (2011), Semsar-Kazerooni and Khorasani (2010), Shen et al. (2013), Su and Huang (2015), Sun et al. (2018), Wang et al. (2015), Wen et al. (2012), Xiao et al. (2012) and Zavlanos et al. (2011, 2007).

Subtracting $Y_i\theta_i$ from both sides of (1) and using Property 3 yield the following open-loop error system

$$M_i(q_i)\dot{s}_i + C_i(q_i, \dot{q}_i)s_i = b_i\tau_i + o_i(\upsilon) + d_i - Y_i\theta_i. \tag{19}$$

Submitting the developed distributed controller (15) into (19) gives the following closed-loop error system

$$\begin{aligned} M_i(q_i)\dot{s}_i &= -C_i(q_i, \dot{q}_i)s_i + (b_i(t)\mathcal{N}_i(\chi_i) - I_n)u_i + Y_i\tilde{\theta}_i \\ &\quad + \tilde{o}_i(\tilde{\upsilon}_{2i}, \upsilon) - \kappa_{si}(t)s_i + \Psi_i^T\tilde{\Upsilon}_i, \end{aligned} \tag{20}$$

$$\dot{\tilde{\theta}}_i = -k_{\theta i}Y_i^T(q_i, \dot{q}_i, \dot{q}_{ri}, \ddot{q}_{ri})s_i,\ \dot{\tilde{\Upsilon}}_i = -k_{\Psi i}\Psi_i s_i, \tag{21}$$

$$\dot{\chi}_i = -k_{\chi i}\text{diag}(s_{i1}, \ldots, s_{in})u_i,\ \dot{\kappa}_{si} = k_{si}s_i^Ts_i, \tag{22}$$

where $\tilde{\theta}_i = \hat{\theta}_i - \theta_i$, $\tilde{\upsilon}_{2i} = \upsilon - \hat{\upsilon}_{2i}$, $\tilde{o}_i(\tilde{\upsilon}_{2i}, \upsilon) = o_i(\upsilon) - o_i(\hat{\upsilon}_{2i})$, and $\tilde{\Upsilon}_i = \Upsilon_i - \hat{\Upsilon}_i$. Note that $\tilde{o}_i(0, \upsilon) = 0_n$ for all $\upsilon \in \mathbb{R}^{n_\upsilon}$.

As υ is bounded, we have $\upsilon \in \Delta$ for all $t \geq 0$ with Δ being some compact subset of $\mathbb{R}^{n_\upsilon}$. Together with the condition that $o_i(\cdot)$ is a $\mathcal{C}^1$ function, by Lemma 11.1 of Chen and Huang (2015), there exists a smooth function $\bar{o}_i(\tilde{\upsilon}_{2i}, \upsilon) \geq 1$ so that for all $\upsilon \in \Delta$,

$$\tilde{o}_i^2(\tilde{\upsilon}_{2i}, \upsilon) \leq \bar{o}_i(\tilde{\upsilon}_{2i}, \upsilon)\|\tilde{\upsilon}_{2i}\|^2. \tag{23}$$

Now, we first establish the following lemma.

Lemma 3. *Assume that there exists $t^* \in [0, +\infty)$ such that $\bar{\mathcal{G}}(0) \subseteq \bar{\mathcal{G}}(t)$ and $\bar{\mathcal{G}}(t)$ contains a spanning tree with the node 0 as the root for all $t \in [0, t^*)$. Then, under Assumption 1, there exists* *a subsequent nonnegative Lyapunov function $V(t)$ in (25) such that along the trajectory of the closed-loop system (20), it satisfies for all $t \in [0, t^*)$,*

$$
\begin{aligned}
V(t) \le V(0) &- \int_0^t (\zeta_1^T(s)\Xi(s)\zeta_1(s) + \zeta_2^T(s)\Omega(s)\zeta_2(s))ds \\
&- \sum_{i=1}^N \sum_{k=1}^n \frac{1}{k_{\chi i}} \int_0^t (b_{ik}(s) l_{ik} \mathcal{N}_i(\chi_{ik}(s)) - 1)\dot{\chi}_{ik}(s)ds \\
&+ \sum_{i=1}^N \int_0^t \bar{o}_i(\tilde{v}_{2i}(s), \upsilon(s))\|\tilde{v}_{2i}(s)\|^2 ds, \qquad (24)
\end{aligned}
$$

where $\zeta_1 = col(\rho, \tilde{v})$, $\zeta_2 = col(\mathbf{s}, \dot{\tilde{q}}, \tilde{v}_1)$, $\rho = col(\rho_1, \ldots, \rho_N)$, $\tilde{v} = col(\tilde{v}_1, \tilde{v}_2)$, $\tilde{v}_1 = col(\tilde{v}_{11}, \ldots, \tilde{v}_{1N})$, $\tilde{v}_{1i} = \hat{v}_{1i} - \dot{q}_0$, $\tilde{v}_2 = col(\tilde{v}_{21}, \ldots, \tilde{v}_{2N})$, $\mathbf{s} = col(s_1, \ldots, s_N)$, $\dot{\tilde{q}} = col(\dot{\tilde{q}}_1, \ldots, \dot{\tilde{q}}_N)$, $\dot{\tilde{q}}_i = \dot{q}_i - \dot{q}_0$, $\Xi(t) = \begin{bmatrix} \bar{H} & \frac{-\bar{H}}{2} \\ \frac{-\bar{H}}{2} & \tilde{k}\bar{H}^2 \end{bmatrix}$, $\Omega(t) = \begin{bmatrix} \bar{k}_\kappa I_{Nn} & \frac{-I_{Nn}}{2} & 0_{Nn} \\ \frac{-I_{Nn}}{2} & I_{Nn} & \frac{-I_{Nn}}{2} \\ 0_{Nn} & \frac{-I_{Nn}}{2} & k\tilde{H}^2 \end{bmatrix}$, $\bar{H} = H(t) \otimes I_{n+n_\upsilon}$, $\tilde{H} = H(t) \otimes I_n$, and for $k_\alpha, k, k_\kappa, \bar{k}_\kappa > 0$, $\tilde{k} = k_\alpha - k$ with $k_\alpha > k + \frac{1}{4\lambda_{min}(\bar{H}(t))}$, $k > \frac{1}{4\lambda_{min}^2(\tilde{H}(t))}$, and $k_\kappa = \bar{k}_\kappa + \frac{1}{4}$ with $\bar{k}_\kappa > \frac{k\lambda_{min}^2(\tilde{H}(t))}{4k\lambda_{min}^2(\tilde{H}(t))-1}$ ensure that $\Xi(t)$ and $\Omega(t)$ are positive definite by the Schur complement lemma.

Proof. Select a nonnegative Lyapunov functional candidate $V(t) = V_1(t) + V_2(t) + V_3(t)$, where

$$
\begin{aligned}
V_1 &= \frac{1}{2}\sum_{i=1}^N s_i^T M_i(q_i)s_i + \sum_{i=1}^N \frac{1}{2k_{\theta i}}\tilde{\theta}_i^T\tilde{\theta}_i + \sum_{i=1}^N \frac{1}{2k_{si}}(\kappa_{si} - k_\kappa)^2 \\
&+ \frac{1}{2}\sum_{i=1}^N [\frac{1}{k_{\alpha i}}(\alpha_i - k_\alpha)^2 + \frac{1}{k_{\beta i}}(\beta_i - k_\beta)^2 + \frac{1}{k_{\Psi i}}\tilde{\Upsilon}_i^T\tilde{\Upsilon}_i], \\
V_2 &= \frac{1}{2}\sum_{i=1}^N \alpha_{\varrho i}[\sum_{j=1}^N \varphi(\|q_{ij}\|_\sigma) + 2\varphi(\|q_{i0}\|_\sigma)], \\
V_3 &= \frac{1}{2}k_\alpha(\bar{H}\tilde{v})^T\bar{H}\tilde{v} + \frac{1}{2}\rho^T\bar{H}\rho + \mathcal{P}(t), \qquad (25)
\end{aligned}
$$

where $k_\kappa, k_\alpha, k_\beta > 0$ are estimated constants of adaptive gains and $\mathcal{P}(t)$ has been given in Lemma 1.

Under Filippov's framework, the time derivative of $V(t)$ exists almost everywhere (a.e.), i.e., $\dot{V} \overset{a.e.}{\in} \dot{\tilde{V}}$ (Feng et al., 2018, 2017). Thus, it follows from (16), (20)–(22), (25), and Property 2 that the time derivative of V_1 along the trajectory of (20) is

$$
\begin{aligned}
\dot{\tilde{V}}_1 \subset &\sum_{i=1}^N s_i^T \left(Y_i\tilde{\theta}_i + \Psi_i^T\tilde{\Upsilon} - \kappa_{si}s_i + (b_i(t)\mathcal{N}_i(\chi_i) - I_n)u_i\right) \\
&- \sum_{i=1}^N(\tilde{\theta}_i^T Y_i^T + \tilde{\Upsilon}_i^T\Psi_i)s_i + \sum_{i=1}^N(\kappa_{si} - k_\kappa)s_i^T s_i + \sum_{i=1}^N s_i^T \\
&\times \tilde{o}_i(\tilde{v}_{2i}, \upsilon) + \sum_{i=1}^N(\alpha_i - k_\alpha)(\sum_{j=1}^N h_{ij}(t)\tilde{v}_j)^T(\sum_{j=1}^N h_{ij}(t)\rho_j) \\
&+ \sum_{i=1}^N(\beta_i - k_\beta)(\sum_{j=1}^N h_{ij}(t)\rho_j)^T \mathbb{k}[\mathrm{sgn}(\sum_{j=1}^N h_{ij}(t)\tilde{v}_j)], \qquad (26)
\end{aligned}
$$

where for $\delta = \mathrm{col}(\delta_1, \ldots, \delta_n)$, $\mathbb{k}[\mathrm{sgn}(\delta)] = \mathrm{SGN}(\delta)$ is a set-valued function defined by $\mathrm{SGN}(\delta) = \mathrm{col}(\mathrm{SGN}_1(\delta_1), \ldots, \mathrm{SGN}_n(\delta_n))$ with each element being a function defined as $\mathrm{SGN}_i(\delta_i) = 1$ if $\delta_i > 0$, $[-1, 1]$ if $\delta_i = 0$, and -1 if $\delta_i < 0$ (Feng et al., 2018).

Similarly, it follows from (10) and Lemma 1 that

$$
\begin{aligned}
\dot{\tilde{V}}_3 \subset &k_\alpha\tilde{v}^T\bar{H}^2(\rho - \tilde{v}) + \sum_{i=1}^N(\sum_{j=1}^N h_{ij}(t)\rho_j)^T(-\alpha_i\sum_{j=1}^N h_{ij}(t) \\
&\times \tilde{v}_j - \dot{\hat{v}}_i + \tilde{v}_i - \tilde{v}_i - \beta_i\mathbb{k}[\mathrm{sgn}(\sum_{j=1}^N h_{ij}(t)\tilde{v}_j)] + \Delta_{di}) \\
&- \sum_{i=1}^N(\sum_{j=1}^N h_{ij}(t)\rho_j)^T\left(\Delta_{di} - k_\beta\mathbb{k}[\mathrm{sgn}(\xi_i)]\right). \qquad (27)
\end{aligned}
$$

Since $\frac{\partial\varphi(\|q_{ij}\|_\sigma)}{\partial q_i} = -\frac{\partial\varphi(\|q_{ij}\|_\sigma)}{\partial q_j}$ and $\sum_{i=1}^N\sum_{j=1}^N \dot{q}_0^T\frac{\partial\varphi(\|q_{ij}\|_\sigma)}{\partial q_i} = \dot{q}_0^T\sum_{i=1}^N\sum_{j=1}^N\frac{\partial\varphi(\|q_{ij}\|_\sigma)}{\partial q_i} = 0$, it is obtained that

$$
\begin{aligned}
\dot{V}_2 &= \sum_{i=1}^N\frac{\alpha_{\varrho i}}{2}\sum_{j=1}^N\left(\dot{q}_i^T\frac{\partial\varphi(\|q_{ij}\|_\sigma)}{\partial q_i} + \dot{q}_j^T\frac{\partial\varphi(\|q_{ij}\|_\sigma)}{\partial q_j}\right) \qquad (28) \\
&+ \sum_{i=1}^N\alpha_{\varrho i}\left(\dot{q}_i^T\frac{\partial\varphi(\|q_{i0}\|_\sigma)}{\partial q_i} - \dot{q}_0^T\frac{\partial\varphi(\|q_{i0}\|_\sigma)}{\partial q_i}\right) \\
&= \sum_{i=1}^N\alpha_{\varrho i}\dot{\tilde{q}}_i^T\sum_{j\in\bar{\mathcal{N}}_i(t)}\frac{\partial\varphi(\|q_{ij}\|_\sigma)}{\partial q_i} = \sum_{i=1}^N\dot{\tilde{q}}_i^T(s_i - \dot{\tilde{q}}_i + \tilde{v}_{1i}).
\end{aligned}
$$

Let $k_\kappa = \bar{k}_\kappa + 1/4$. Combining (26) with (27) and (28) gives

$$
\begin{aligned}
\dot{V} &\overset{a.e.}{=} \sum_{i=1}^N s_i^T(-k_\kappa s_i + \tilde{o}_i(\tilde{v}_{2i}, \upsilon)) - k_\alpha\sum_{i=1}^N(\sum_{j=1}^N h_{ij}(t)\tilde{v}_j)^T\sum_{j=1}^N \\
&\times h_{ij}(t)\rho_j + k_\alpha\tilde{v}^T\bar{H}^2(\rho - \tilde{v}) + \sum_{i=1}^N(\sum_{j=1}^N h_{ij}(t)\rho_j)^T(\tilde{v}_i - \rho_i) \\
&+ \sum_{i=1}^N\dot{\tilde{q}}_i^T(s_i - \dot{\tilde{q}}_i + \tilde{v}_{1i}) + \sum_{i=1}^N s_i^T(b_i(t)\mathcal{N}_i(\chi_i) - I_n)u_i \\
&\overset{a.e.}{\le} -\bar{k}_\kappa\mathbf{s}^T\mathbf{s} - (k_\alpha - k)\tilde{v}^T\bar{H}^2\tilde{v} - k\tilde{v}^T\bar{H}^2\tilde{v} + \rho^T\bar{H}\tilde{v} \\
&- \rho^T\bar{H}\rho + \dot{\tilde{q}}^T(\mathbf{s} - \dot{\tilde{q}} + \tilde{v}_1) + \sum_{i=1}^N\bar{o}_i(\tilde{v}_{2i}, \upsilon)\|\tilde{v}_{2i}\|^2 \\
&- \sum_{i=1}^N\sum_{k=1}^n\frac{1}{k_{\chi i}}(b_{ik}(t)l_{ik}\mathcal{N}_i(\chi_{ik}) - 1)\dot{\chi}_{ik} \\
&\overset{a.e.}{\le} -\begin{bmatrix}\rho \\ \tilde{v}\end{bmatrix}^T\Xi(t)\begin{bmatrix}\rho \\ \tilde{v}\end{bmatrix} - \begin{bmatrix}\mathbf{s} \\ \dot{\tilde{q}} \\ \tilde{v}_1\end{bmatrix}^T\Omega(t)\begin{bmatrix}\mathbf{s} \\ \dot{\tilde{q}} \\ \tilde{v}_1\end{bmatrix} + \sum_{i=1}^N\sum_{k=1}^n\frac{1}{k_{\chi i}} \\
&\times [\dot{\chi}_{ik} - b_{ik}(t)l_{ik}\mathcal{N}_i(\chi_{ik})\dot{\chi}_{ik}] + \sum_{i=1}^N\bar{o}_i(\tilde{v}_{2i}, \upsilon)\|\tilde{v}_{2i}\|^2, \qquad (29)
\end{aligned}
$$

which implies (24), and the proof is completed. □

Remark 8. In Lemma 3, k_α, k_κ, k related to the eigenvalue of global graph information are only used for Lyapunov analysis, and not employed in the proposed algorithm, which implies that the controller does not need the global graph information. Similarly, the unknown upper bounds of the derivatives of $v_0(t)$ for k_β in (12) are not required. In addition, the requirement of a marginally stable linear leader system in Cai and Huang (2016), Dong (2016), Dong and Huang (2013, 2017), Feng et al. (2018) and Su and Huang (2015) is removed as the designs allow for any leader trajectory that only satisfies mild properties.

Combining Lemmas 1–3 yields the solution to Problem 1, which can be summarized in the following theorem.

Theorem 1. *Suppose that Assumption 1 and 2 hold, and at the initial time, there is no collision among the agents. For systems (1), (3), (4), Problem 1 is solvable under the proposed controller.*

Proof. The proof consists of two steps as follows.

Step 1 (Connectivity preservation and collision avoidance)

(1) Assume that there exists $t^* \in [0, +\infty)$ so that $\bar{\mathcal{G}}(t)$ contains a spanning tree with the node 0 as the root for all $t \in [0, t^*)$. We now show $V(t) \le Q_{max} < \bar{Q}$. By the definition of $\bar{\mathcal{E}}(0)$ in Section 2.3, $H(0)$ is positive definite (Feng et al., 2017). Hence, there exist k_α, k_κ, k, $\bar{k}_\kappa$ in Lemma 3 such that $\Xi(0)$ and $\Omega(0)$ are positive definite. Then, it guarantees $\Xi(t) \ge \Xi(0) > 0$ and $\Omega(t) \ge \Omega(0) > 0$ for all $t \in [0, t^*)$. Therefore, by Lemma 3, it obtains that for all $t \in [0, t^*)$, $V(t) = \sum_{i=1}^{N} V_i(t)$, $V_i(t) \le V_i(0) + \int_0^t \bar{o}_i(\tilde{v}_{2i}(s), \upsilon(s))\|\tilde{v}_{2i}(s)\|^2 ds + \sum_{k=1}^{n} \frac{1}{k_{\chi i}} \int_0^t (-b_{ik} l_{ik} \mathcal{N}_i(\chi_{ik}(s)) + 1)\dot{\chi}_{ik}(s) ds$. It is noted that the second term is upper bounded by Assumption 1 and the convergence of (10) for all $t \in [0, t^*)$. With a similar spirit of the argument in Ye and Jing (1998), Lemma 1), the last term is upper bounded for all $t \in [0, t^*)$ by Assumption 2. On the other hand, it follows from Lemma 2 that $\bar{Q} > Q_{max} = k_1 \max\{\varphi(\epsilon_1), \varphi(R - \epsilon_2)\} + k_2$, where $k_1 = \alpha_\varrho N(N+1)/2$, $\alpha_\varrho = \max\{\alpha_{\varrho i}\}$, and $k_2 = V_1(0) + V_3(0) + c_N$, where c_N is the upper bound of the sum of the last two terms in (24). That is, $\bar{Q} > Q_{max} \ge V(0) + c_N$. As $\varphi(\|q_{ij}\|_\sigma)$ is a nonnegative and bounded function, $V(0)$ is bounded. Let $V_N(0) = V(0) + c_N$. Our choice of Q is so that $V_N(0) \le Q_{max} < \bar{Q}$. By Lemma 3, we obtain $V(t) \le V_N(0) \le Q_{max} < \bar{Q}$.

(2) Next, we show $\bar{\mathcal{G}}(0) \subseteq \bar{\mathcal{G}}(t)$, and there is no collision among agents for all $t \ge 0$ by contradiction (Dong & Huang, 2013, 2017; Feng et al., 2017).

Based on the continuity of the solution of (20), there exists $0 < t_1 \le \infty$ such that for $t \in [0, t_1)$, $\bar{\mathcal{G}}(t) = \bar{\mathcal{G}}(0)$. If $t_1 = \infty$, then $\bar{\mathcal{G}}(t)$ is connected for all $t \ge 0$. Otherwise, there must exist some $t_1 \ge 0$ such that $\bar{\mathcal{G}}(t) = \bar{\mathcal{G}}(0)$, $t \in [0, t_1)$ and $\bar{\mathcal{G}}(t_1) \ne \bar{\mathcal{G}}(0)$. We now claim $\bar{\mathcal{G}}(0) \subseteq \bar{\mathcal{G}}(t_1)$. Suppose that $\bar{\mathcal{G}}(t)$ switches at time t_i, $i = 1, 2, \ldots$ and it is fixed on $[t_i, t_{i+1})$. Since $\bar{\mathcal{G}}(t)$ is connected for $t \in [0, t_1)$, our choice of gains ensures $V(t) \le V_N(0) \le Q_{max} < \bar{Q}$ for all $t \in [0, t_1)$. Assume, for some edge $(i, j) \in \bar{\mathcal{E}}(0)$, $(i, j) \notin \bar{\mathcal{E}}(t_1)$. Then, $\lim_{t \to t_1^-} \|q_{ij}\|_\sigma = R$ which implies that $\lim_{t \to t_1^-} V(t) \ge \bar{Q}$ and this contradicts $V(t) \le V_N(0) \le Q_{max} < \bar{Q}$. That is, no existing edges will be lost at time t_1. Thus, $\bar{\mathcal{G}}(0) \subseteq \bar{\mathcal{G}}(t_1)$. Meanwhile, we can also show that $\lim_{t \to t_1^-} \|q_{ij}\|_\sigma \nrightarrow r$ since if otherwise, $\lim_{t \to t_1^-} V(t) \ge \bar{Q}$, which also contradicts $V(t) \le V_N(0) \le Q_{max} < \bar{Q}$. That is, no collision can occur for $t \in [0, t_1)$. By (24), $V(t)$ is bounded for all $t \in [0, t^*)$ as long as t^* is so that $\bar{\mathcal{G}}(t)$ is connected for all $t \in [0, t^*)$. Since $\bar{\mathcal{G}}(t)$ can only have finitely many edges, by repeating the above synthesis, we can conclude that $t^* = +\infty$, and there exists a $z > 0$ such that $\bar{\mathcal{G}}(t) = \bar{\mathcal{G}}(0)$, $t \in [0, t_1)$, $\bar{\mathcal{G}}(t) = \bar{\mathcal{G}}(t_i) \supseteq \bar{\mathcal{G}}(t_{i-1})$, $t \in [t_i, t_{i+1})$, $i = 1, \ldots, z-1$, $\bar{\mathcal{G}}(t) = \bar{\mathcal{G}}(t_z) \supseteq \bar{\mathcal{G}}(t_{z-1})$, $t \in [t_z, \infty)$. Similarly, $V(t) \le V_N(0) \le Q_{max} < \bar{Q}$, for all $t \ge 0$. Hence, $\bar{\mathcal{G}}(t)$ contains a spanning tree with the node 0 for all $t \ge 0$. That is, the graph $\bar{\mathcal{G}}(0)$ is preserved, and there is no collision, i.e., $\|q_i - q_j\| > r$, $i \in \mathcal{V}$, $j \in \bar{\mathcal{V}}$.

Step 2 (Velocity matching, i.e., $\lim_{t\to\infty} (\dot{q}_i - \dot{q}_0) = 0_n$).

Since $\bar{\mathcal{G}}(t)$ contains a spanning tree with the node 0 as the root by Step 1, $V(t)$ is upper bounded for all $t \ge t_z$ by (24)), (25), and Lemmas 1–3. Hence, s_i, $\tilde{\theta}_i$, $\tilde{\Upsilon}_i$, ρ_i, $\tilde{v}_i$, and $\tilde{\dot{q}}_i$ are all bounded for all $t \ge t_z$. Since $\bar{o}_i(\cdot)$ is smooth, $\bar{o}_i(\tilde{v}_{2i}, \upsilon)\tilde{v}_{2i}$ is bounded for all $t \ge t_z$. By (13), $\sum_{j \in \bar{\mathcal{N}}_i(t)} \frac{\partial \varphi(\|q_{ij}\|_\sigma)}{\partial q_i}$ and its time derivative are bounded for all $t \ge t_z$. By (14)–(22), $\dot{s}_i$, $\dot{\rho}_i$, $\dot{\tilde{v}}_i$, and $\dot{\tilde{\dot{q}}}_i$ are bounded for all $t \ge t_z$ (Dong & Huang, 2017, 2018). From (29), we have $\int_0^t W(s) ds \le V(0) - V(t) + \sum_{i=1}^{N} \int_0^t \bar{o}_i(\tilde{v}_{2i}(s), \upsilon(s))\|\tilde{v}_{2i}(s)\|^2 ds + \sum_{i=1}^{N} \sum_{k=1}^{n} \frac{1}{k_{\chi i}} \int_0^t (-b_{ik} l_{ik} \mathcal{N}_i(\chi_{ik}(s)) + 1)\dot{\chi}_{ik}(s) ds$ with $W(t) = \begin{bmatrix} \rho \\ \tilde{v} \end{bmatrix}^T \Xi(t) \begin{bmatrix} \rho \\ \tilde{v} \end{bmatrix} + \begin{bmatrix} \mathbf{s} \\ \tilde{\dot{q}} \\ \tilde{v}_1 \end{bmatrix}^T \Omega(t) \begin{bmatrix} \mathbf{s} \\ \tilde{\dot{q}} \\ \tilde{v}_1 \end{bmatrix}$. Therefore, $\int_0^t W(s) ds$ is bounded for all $t \ge t_z$ by using the similar boundness analysis in Step 1 for the last two terms. Then, the existence of $\lim_{t\to\infty} \int_0^t W(s) ds$ can be guaranteed and it is finite. That is, the errors s_i, $\tilde{\dot{q}}_i$, ρ_i, and $\tilde{v}_i$ are square integrable. Then, the Barbalat's Lemma (Khalil, 2002), Lemma 8.2) is used to conclude $\lim_{t\to\infty} W(t) = 0$. Hence, $\lim_{t\to\infty} \tilde{\dot{q}}_i(t) = 0_n$. Velocity matching is achieved, i.e., $\lim_{t\to\infty} (\dot{q}_i - \dot{q}_0) = 0_n$. □

Corollary 1. *Suppose that Assumptions 1 and 2 hold. The controller in (15)–(18) enables connectivity-preserving rendezvous in the sense that $\bar{\mathcal{G}}(t)$ is connected for all $t \ge 0$ and $\lim_{t\to\infty} (q_i - q_0) = 0_n$ and $\lim_{t\to\infty} (\dot{q}_i - \dot{q}_0) = 0_n$.*

Proof. Similar to the proof of Theorem 1, it can be obtained by using the first term of (13) only (Feng & Hu, 2017). □

Discussion: In Theorem 1, a fully distributed solution to the fault-tolerant connectivity-preserving flocking problem is provided. The obtained results are more general than those works in Dong (2016), Dong and Huang (2013, 2017, 2018), Ghapani et al. (2016), Mei et al. (2011), Su and Huang (2015), Xiao et al. (2012) and Zavlanos et al. (2011, 2007) from the following aspects:

- *Problem formulation:* in contrast to Dong (2016), Dong and Huang (2013, 2017, 2018), Ghapani et al. (2016), Mei et al. (2011), Su and Huang (2015), Xiao et al. (2012) and Zavlanos et al. (2011, 2007), a more practical fault-tolerant flocking problem is addressed for Lagrange agent systems under a limited sensing range. The actuator faults can violate the network connectivity and make control coefficients unknown and time-varying. It is more challenging to design distributed schemes capable of simultaneously achieving fault-tolerant coordination, connectivity preservation and collision avoidance.
- *Leader trajectory:* the studied nonlinear leader-following flocking problem is technically more challenging than a leaderless case in Dong (2016), Dong and Huang (2018), Xiao et al. (2012) and Zavlanos et al. (2011, 2007) or a leader-following case in Cai and Huang (2016), Dong and Huang (2013, 2017) and Su and Huang (2015) where a leader system is generated by an autonomous linear system with its system matrix S being available to all followers and assumed to have semi-simple eigenvalues with zero real parts. In contrast, the studied nonlinear and time-varying leader is more general.
- *Algorithms:* in the absence of actuator faults, the design recovers consensus/rendezvous/flocking results in Dong (2016), Dong and Huang (2013, 2017, 2018), Ghapani et al. (2016), Mei et al. (2011), Su and Huang (2015), Xiao et al. (2012) and Zavlanos et al. (2011, 2007). In contrast to the output regulation method in Dong and Huang (2013) and Su and Huang (2015), the proposed scheme does not depend on the solution of regulator equations. Moreover, it removes some design limitations in Dong (2016), Dong and Huang (2013, 2017, 2018), Ghapani et al. (2016), Mei et al. (2011), Su and Huang (2015), Xiao et al. (2012) and Zavlanos et al. (2011, 2007): (a) the design in Mei et al. (2011) is subject to two-hop communication; (b) protocols in Cai and Huang (2016), Dong and Huang (2013, 2017) and Su and Huang (2015) require the leader's dynamics to be generated by a marginally stable linear system; and (c) gains in Dong (2016), Dong and Huang (2013, 2017, 2018), Ghapani et al. (2016), Mei et al. (2011), Su and Huang (2015), Xiao et al. (2012) and Zavlanos et al. (2011, 2007) rely on global graph information and leader's states.

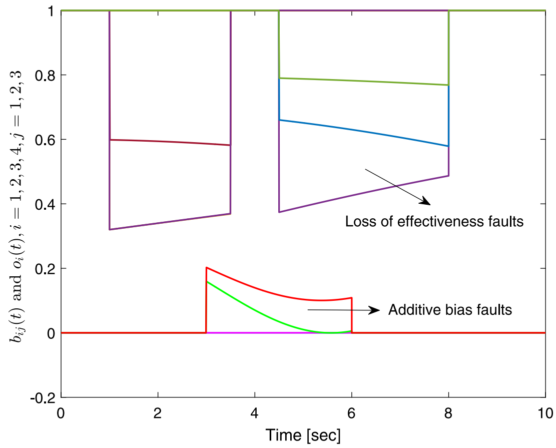


Fig. 2. The time-varying actuator faults $b_{ij}(t)$ and $o_i(t)$.

4. Numerical simulation

In this section, numerical simulation is presented to illustrate the effectiveness of the theoretical results. We consider the coordination problem of five Lagrange agents, where the relative dynamics of the ith Lagrange agent are (Ghapani et al., 2016)

$$m_i\ddot{x}_i - 2m_i n_0\dot{y}_i - m_i n_0^2 x_i + \frac{m_i\mu_e(r_0+x_i)}{r_i^3} - \frac{m_i\mu_e}{r_0^2} = \tau_{ix},$$

$$m_i\ddot{y}_i + 2m_i n_0\dot{x}_i - m_i n_0^2 y_i + \frac{m_i\mu_e y_i}{r_i^3} = \tau_{iy},$$

$$m_i\ddot{z}_i + \frac{m_i\mu_e z_i}{r_i^3} = \tau_{iz},\ i = 1, 2, 3, 4, \tag{30}$$

where m_i is the unknown but constant mass of the ith agent, μ_e is the gravitational constant of Earth, r_0 is the radius of the chief, $n_0 = \sqrt{\mu_e/r_0^3}$ is the angular velocity of the reference orbit, and $r_i = \sqrt{x_i^2 + (y_i + r_0)^2 + z_i^2}$; $q_i = \mathrm{col}(x_i, y_i, z_i)$ is the position of ith agent, and $\tau_i = \mathrm{col}(\tau_{ix}, \tau_{iy}, \tau_{iz})$ is the control input. Let $M_i = m_i I_3$, $C_i = 2m_i[0, -n_0, 0; n_0, 0, 0; 0, 0, 0]$, $G_i = m_i[-n_0^2 x_i + \frac{\mu_e(r_0+x_i)}{r_i^3} - \frac{\mu_e}{r_0^2}; -n_0^2 y_i + \frac{\mu_e y_i}{r_i^3}; \frac{\mu_e z_i}{r_i^3}]$, then, the dynamics in (30) can be written in the form of (1) where the disturbances in (2) are: $d_{ik} = d_{ik}^s \sin(\omega_{ik}^s t) + d_{ik}^c \cos(\omega_{ik}^c t)$, $d_{ik}^s, d_{ik}^c \in (0.1 * i, 0.6 * i)$ and $\omega_{ik}^s = \frac{\pi}{10*i}$, $\omega_{ik}^c = \frac{\pi}{20*i}$, $i = 1, \ldots, 4$, $k = 1, 2, 3$.

In this simulation, let $\tilde{q}_i = q_i - q_0 = \mathrm{col}(\tilde{x}_i, \tilde{y}_i, \tilde{z}_i)$ and the agents' initial positions are: $q_0(0) = \mathrm{col}(0.8, 2, 0)$, $q_1(0) = \mathrm{col}(-0.8, 0.7, 0)$, $q_2(0) = \mathrm{col}(1, 0.7, 0)$, $q_3(0) = \mathrm{col}(1.5, -1, 0)$, $q_4(0) = \mathrm{col}(-0.8, -1, 0)$. Let $R = 2$ and $r = 0.2$. The initial edge $\mathcal{E}(0)$ forms a connected graph. Choose $l_{i1} = l_{i2} = 1$ and $l_{i3} = 2$. We consider actuator faults depicted in Fig. 2. In particular, for $t_{bi} \in [1, 3.5]$, $b_{i1} = 0.i + 0.2 * \sin(t/10)$, $b_{i2} = 0.i + 0.3 * \cos(t/10)$, $b_{i3} = 0.i + 0.4 * \sin(t/20)$, $i = 3$, and then for $t_{bi} \in [4.5, 8]$, $b_{i1} = 0.2 + 0.i * \sin(t/10)$, $b_{i2} = 0.3 + 0.i * \cos(t/10)$, $b_{i3} = 0.4 + 0.i * \sin(t/20)$, $i = 4$; otherwise, $b_{ij} = 1$ for $i = 1, 2, j = 1, \ldots, 3$; for $t_{oi} \in [3, 6]$, $o_1 = o_2 = 0$, $o_3 = \upsilon_1^2 + \upsilon_1\upsilon_2\upsilon_3$, and $o_4 = \upsilon_1^2 + \upsilon_2^2 + \upsilon_3^3$. Further, the controller parameters are

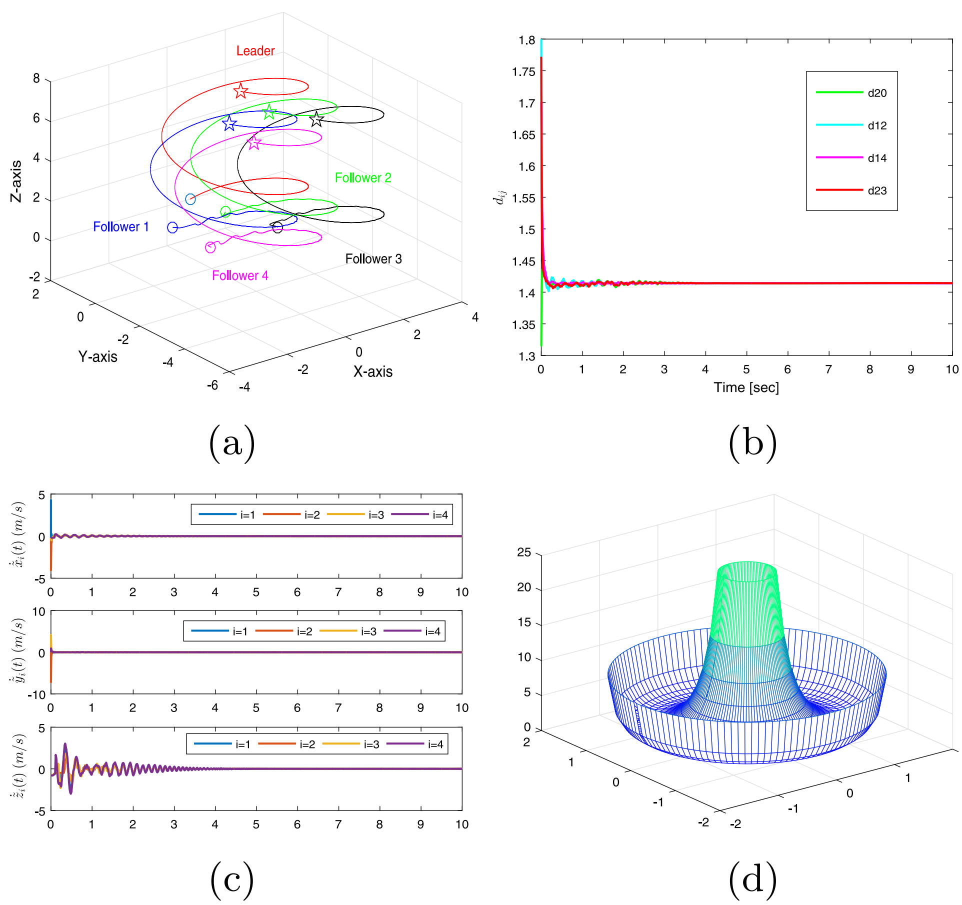


Fig. 3. Simulated connectivity-preserving flocking results for Case I using (15)–(16): (a) flocking trajectories of the followers and the leader from the initial positions (dots) to final positions (pentagrams); (b) distances among initially connected agents; (c) the velocity tracking errors; (d) plot of the potential function (13) with $Q = 20\,000$ and $\varsigma = 0.2$.

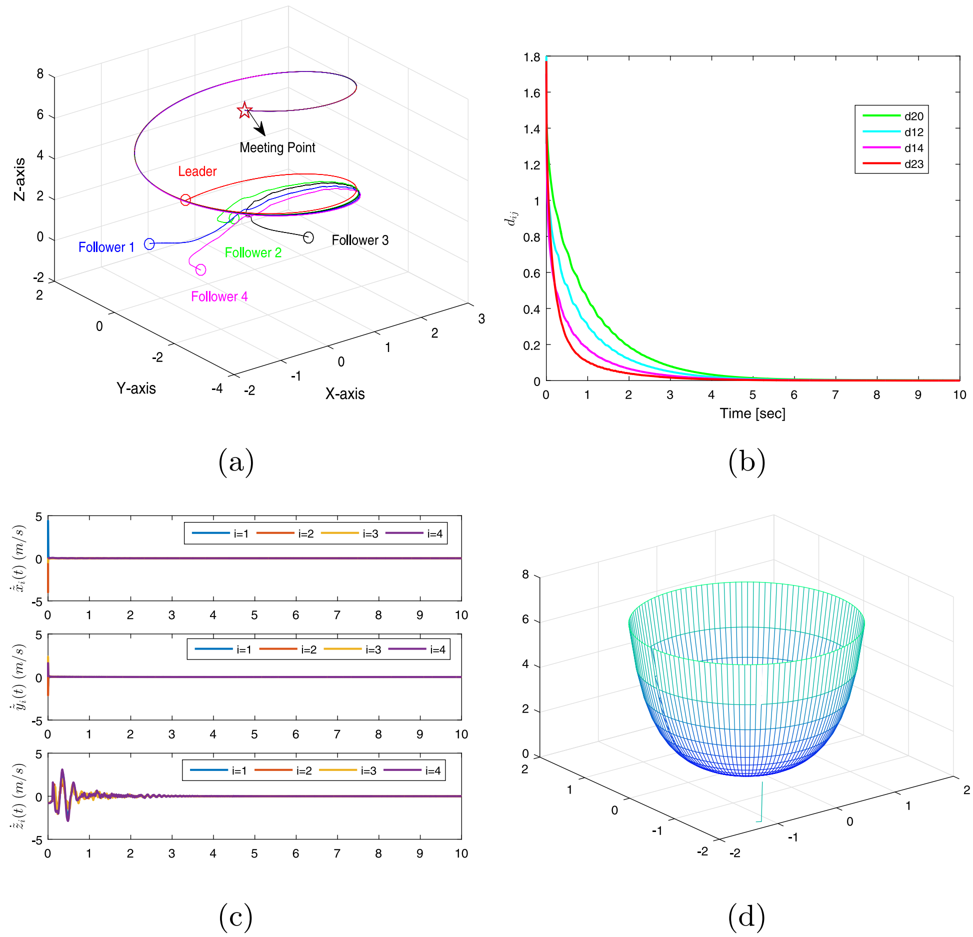


Fig. 4. Simulated connectivity-preserving rendezvous results for Case II using (15)–(16): (a) rendezvous trajectories; (b) distances among initially connected agents; (c) the velocity tracking errors; (d) plot of the first term of (13).

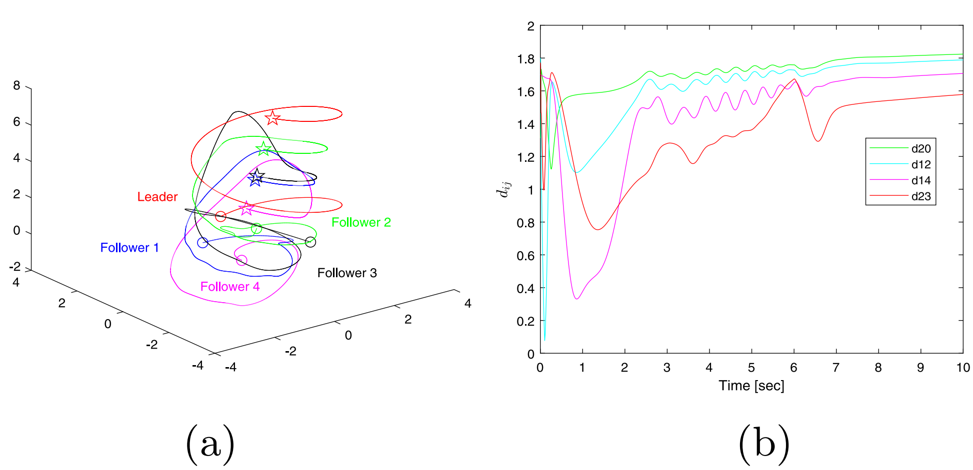


Fig. 5. Simulated results by the algorithms in Dong and Huang (2018) under the time-varying actuator faults: (a) trajectories of all agents; (b) distances among initially connected agents.

$\kappa_i = 3$, $k_{\chi i} = 0.1$, $k_{\theta i} = 0.2$, $k_{\Psi i} = 0.3$, $\alpha_{\varrho i} = 1$, and $k_{si} = 30$. The initial values of parameters are: $\hat{\chi}_i(0) = 0_3$, $\hat{\theta}_i(0) = 0$, $\hat{\Upsilon}_i(0) = 0_6$.

Case I: Connectivity-Preserving Flocking

Consider a nonlinear moving leader with $q_0(t) = \text{col}(2\sin(t) + 0.8, 2\cos(t), 0.8t)$. By Theorem 1, Fig. 3(a)–(d) shows the simulation results under the proposed controller. Fig. 3(a) depicts the 3D flocking trajectories of all agents. Fig. 3(b) shows the distances between initially connected agents, and they are all smaller than R, which implies that the connectivity of the initial network is maintained. By Fig. 3(a), inter-agent collision is avoided. Clearly, all the followers move cohesively with the leader without collision. Fig. 3(c) describes the velocity tracking errors between the agents and leader, and thus the velocity matching is obtained. Fig. 3(d) plots the potential function.

Thus, the connectivity-preserving flocking can be achieved under time-varying actuator faults.

Case II: Connectivity-Preserving Rendezvous

For the studied nonlinear leader in Case I, we further perform the distributed algorithm for Corollary 1, and the simulation results are provided in Fig. 4, where connectivity-preserving rendezvous is achieved under actuator faults. Furthermore, to compare with the results in existing works (Dong, 2016; Dong & Huang, 2013, 2017, 2018; Ghapani et al., 2016; Su & Huang, 2015; Zavlanos et al., 2007), the distributed scheme in Dong and Huang (2018) (for just an example) is conducted under the same environment. Fig. 5 shows the simulation results under the time-varying faults, where rendezvous cannot be achieved and connectivity is also lost. This illustrates the effectiveness of the proposed method.

5. Conclusion

In this paper, we investigated fault-tolerant connectivity-preserving flocking for networked Lagrange systems under time-varying actuator faults. For a class of nonlinear leader systems, the distributed estimator-based controller is proposed by incorporating a Nussbaum gain technique to ensure that all agents can achieve connectivity-preserving flocking under uncertainties and time-varying actuator faults.

References

Cai, H., & Huang, J. (2016). The leader-following consensus for multiple uncertain Euler–Lagrange systems with an adaptive distributed observer. *IEEE Transactions on Automatic Control*, *61*, 3152–3157.

Chen, F., Feng, G., Liu, L., & Ren, W. (2015). Distributed average tracking of Euler–Lagrange systems. *IEEE Transactions on Automatic Control*, *60*(2), 534–552.

Chen, Z., & Huang, J. (2015). *Stabilization and regulation of nonlinear systems: A robust and adaptive approach*. Springer.

Chen, W., Li, X., Ren, W., & Wen, C. (2014). Adaptive consensus of multi-agent systems with unknown identical control directions based on a novel nussbaum-type function. *IEEE Transactions on Automatic Control*, *59*(7), 1887–1892.

Chen, G., Song, Y. D., & Lewis, F. L. (2017). Distributed fault-tolerant control of networked uncertain Euler–Lagrange systems under actuator faults. *IEEE Transactions on Cybernetics*, *47*(7), 1706–1718.

Dong, J. (2016). Finite-time connectivity preservation rendezvous with disturbance rejection. *Automatica*, *71*, 57–61.

Dong, Y., & Huang, J. (2013). A leader-following rendezvous problem of double integrator multi-agent systems. *Automatica*, *49*(5), 1386–1391.

Dong, Y., & Huang, J. (2017). Leader-following consensus with connecti vity preservation of uncertain Euler-Lagrange multi-agent syste ms. *International Journal of Robust and Nonlinear Control*, *27*(18), 4772–4787.

Dong, Y., & Huang, J. (2018). Consensus and flocking with connectivity preservation of uncertain Euler–Lagrange multi-agent systems. *Transactions on ASME Journal of Dynamic Systems, Measurement, and Control*, *140*, 091011–7.

Feng, Z., & Hu, G. (2017). Connectivity-preserving rendezvous for Lagrange systems with actuator faults, the IEEE conference on robotics and biomimetics (pp 1363–1368).

Feng, Z., Hu, G., Ren, W., Dixon, W. E., & Mei, J. (2018). Distributed coordination of multiple Euler–Lagrange systems. *IEEE Transactions on Control of Network Systems*, *5*(1), 55–66.

Feng, Z., Sun, C., & Hu, G. (2017). Robust connectivity preserving rende zvous of multi-robot systems under unknown dynamics and dist urbance. *IEEE Transactions on Control of Network Systems*, *4*(1), 725–735.

Filippo, A., Marino, A., & Pierri, F. (2015). Observer-based decentralized fault detection and isolation strategy for multirobot systems. *IEEE Transactions on Control Systems Technologyl*, *23*(4), 1465–1476.

Ghapani, S., Mei, J., Ren, W., & Song, Y. (2016). Fully distributed flocking with a moving leader for Lagrange networks with parametric uncertainties. *Automatica*, *67*(1), 67–76.

Khalil, H. K. (2002). Nonlinear systems, 3rd ed., NJ, USA.

Ma, H., & Yang, G. (2016). Adaptive fault tolerant control of cooperative systems with actuator faults and unreliable interconnections. *IEEE Transactions on Automatic Control*, *61*(11), 3240–3255.

Mei, J., Ren, W., & Ma, G. (2011). Distributed coordinated tracking with a dynamic leader for multiple Euler-Lagrange systems. *IEEE Transactions on Automatic Control*, *56*(6), 1415–1421.

Sam Ge, S., & Wang, J. (2003). Robust adaptive tracking for time-varying uncertain nonlinear systems with unknown control coefficients. *IEEE Transactions on Automatic Control*, *48*(8), 1463–1469.

Semsar-Kazerooni, E., & Khorasani, K. (2010). Team consensus for a network of unmanned vehicles in presence of actuator faults. *IEEE Transactions on Control Systems Technologyl*, *18*(5), 1155–1161.

Shen, Q., Jiang, B., Shi, P., & Zhao, J. (2013). Cooperative adaptive fuzzy tracking control for networked unknown nonlinear multi-agent systems with time-varying actuator faults. *IEEE Transactions on Fuzzy Systems*, *99*(1), 1–11.

Su, Y., & Huang, J. (2015). Leader-following rendezvous with connectivity preservation and disturbance rejection via an internal model approach. *Automatica*, *57*(1), 203–212.

Sun, C., Hu, G., Xie, L., & Egerstedt, M. B. (2018). Robust finite-time connectivity preserving coordination of second-order multi-agent systems. *Automatica*, *89*, 21–27.

Ton, C., Mehta, S. S., & Kan, Z. (2017). Super-twisting control of double integrator systems with unknown constant control direction. *IEEE Transactions on Control Systems Letters*, *1*(2), 370–375.

Wang, Y., Song, Y., & Lewis, F. L. (2015). Robust adaptive fault-tolerant control of multiagent systems with undetectable actuation failures. *IEEE Transactions on Industrial Electronics*, *62*(2), 3978–3988.

Wen, G., Duan, Z., Su, H., Chen, G., & Yu, W. (2012). A connectivity-preserving flocking algorithm for multi-agent dynamic systems. *IET Control Theory & Applications*, *6*(6), 813–821.

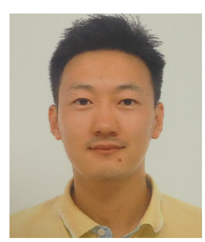
Xiao, F., Wang, L., & Chen, T. (2012). Connectivity preservation for multi-agent rendezvous with link failure. *Automatica*, *48*, 25–35.

Yang, Q., Ge, S. S., & Sun, Y. (2015). Adaptive actuator fault tolerant control for uncertain nonlinear systems with multiple actuators. *Automatica*, *60*(3), 92–99.

Ye, X., & Jing, J. (1998). Adaptive nonlinear design without a priori knowledge of control direction. *IEEE Transactions on Automatic Control*, *43*(11), 1617–1621.

Zavlanos, M. M., Egerstedt, M. B., & Pappas, G. J. (2011). Graph-theoretic connectivity control of mobile robot networks. *Proceedings of the IEEE*, *99*(9), 1525–1540.

Zavlanos, M. M., Jadbabaie, A., & Pappas, G. J. (2007). Flocking while preserving network connectivity, the 46th IEEE conference on decision and control (pp. 2919–2924).



Zhi Feng is currently a research fellow in the School of Electrical and Electronic Engineering at Nanyang Technological University, Singapore. He received the Ph.D. degree from Nanyang Technological University in 2017. His research interests include multi-agent systems, distributed control and optimization, and security and resilience with applications to energy and robotic systems. He was a recipient of the Best Paper in Automation Award in the 14th IEEE International Conference on Information and Automation, 2017.

Guoqiang Hu joined the School of Electrical and Electronic Engineering at Nanyang Technological University, Singapore in 2011, and is currently a tenured Associate Professor and the Director of the Centre for System Intelligence and Efficiency. He was an Assistant Professor at Kansas State University from 2008 to 2011. He received the B.Eng. degree in Automation from the University of Science and Technology of China in 2002, the M.Phil. degree in Automation and Computer-Aided Engineering from the Chinese University of Hong Kong in 2004, and the Ph.D. degree in Mechanical Engineering from the University of Florida in 2007. His research interests include distributed control, distributed optimization and game theory with applications to multi-robot systems and smart city systems. He was a recipient of the Best Paper in Automation Award in the 14th IEEE International Conference on Information and Automation, and a recipient of the Best Paper Award (Guan Zhao-Zhi Award) in the 36th Chinese Control Conference. He serves as Associate Editor for IEEE Transactions on Automatic Control, IEEE Transactions on Control Systems Technology and IEEE Transactions on Automation Science and Engineering, and Technical Editor for IEEE/ASME Transactions on Mechatronics.