How Much Control is Enough for Network Connectivity Preservation and Collision Avoidance?

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***Abstract*—For a multiagent system in free space, the agents are required to generate sufficiently large cohesive force for network connectivity preservation and sufficiently large repulsive force for collision avoidance. This paper gives an energy function based approach for estimating the control force in a general setting. In particular, the force estimated for network connectivity preservation and collision avoidance is separated from the force for other collective behavior of the agents. Moreover, the estimation approach is applied in three typical collective control scenarios including swarming, flocking, and flocking without velocity measurement.**

***Index Terms*—Collective control, collision avoidance, cooperative control, multiagent systems, network control.**

I. Introduction

COLLECTIVE control of multiagent systems has attracted much attention among researchers in physics, biology, mathematics, and control engineering for decades. Its broad applications include mobile sensor networks, formation of unmanned aerial vehicles (UAVs) or mobile robots, attitude alignment of clusters of satellites, and so on [1]–[4]. One of the most remarkable characteristics of multiagent control is its decentralized and cooperative nature. To facilitate the cooperative mechanism, maintenance of network connectivity is always a necessary condition for multiagent control.

One of the simplest methods for preservation of network connectivity is to specify one fixed topology or a finite number of time switching topologies for the network. In this situation, the preservation of network connectivity is automatically guaranteed by using connected or jointly connected topologies. Over the past years, there were a number of existing collective algorithms studied in this simple scenario (see [5], [6]). However, this scenario is not always practically applicable. On one hand, the agents should be labeled to match specified topologies. This setting lacks robustness against newly added agents or agent failure. On the other hand, two topologically "connected" agents might not be physically adjacent. As a result, long-distance communication is required between them. However, from the aspect of engineering applications, it often happens that robots, agents, or sensors just favor short-distance communication capability as long-distance communication is generally costly. It is also true in biological systems due to the limitation of individual vision range.

The other method for preservation of network connectivity is to externally apply a connectivity condition. It is assumed that the condition is satisfied for some specific initial distributions. Along this research line, a so-called joint connectivity condition was proposed in [7] and [8] that there exist infinitely many consecutive bounded time intervals such that the union of graphs over every interval is completely connected. The joint connectivity condition has been widely used in [9]–[13]. Besides the joint connectivity condition, some other connectivity mechanisms are also used in literature. For example, the conditions given in [7] and [8] were alleviated into a weaker one in [14] which requires the existence of a spanning tree over some temporal intervals. A kind of switching joint connectivity condition was studied in [15] which is of more practical potentials in engineering applications. However, such connectivity conditions are not possible to guarantee when a group of agents moves in free space. Moreover, there is no effective tool to examine the connectivity conditions *a prior* other than checking the agent trajectories for the entire evolution course. One exception is for a multiagent system in a bounded space equipped with a certain bouncing or periodic boundaries. In this case, it has been proved in [16] and [17] that the joint connectivity condition is almost always satisfied.

The research in this paper is along with another research line for preservation of network connectivity. On one hand, it does not apply fixed topologies for labeled agents. On the other hand, it does not externally impose connectivity conditions for agents either by ad hoc specifying their initial distribution

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or restricting them in a bounded space equipped with proper boundaries. In this situation, a certain cohesion ability must be added to each agent for maintaining network connectivity internally. For example, in [18] and [19], the interagent attractive interactions are intensified so as to prevent each agent from escaping the influence of others. In [20] and [21], each agent is equipped with an attractive function which preserves the network connectivity when the initial network is connected. The research along this line can also be found in [22]–[24]. More specifically, when two "connected" agents move in opposite direction, the cohesive force works to prevent them from separating. A dual scenario considered in this paper is collision avoidance; when two agents move to each other, the repulsive force is activated to prevent them from collision. In particular, when the distance of two agents is approaching to zero or the threshold for connection, an infinitely large force is required. However, the equipment of infinitely large force for an agent lacks practical applicability. It motivates us to develop an effective strategy such that an infinitely large force can be avoided for both network connectivity preservation and collision avoidance. Moreover, we aim to propose an effective approach for estimating the control force boundary.

An interesting feature of this paper is that the control force estimated for network connectivity preservation and collision avoidance is separated from the force for other collective behavior of the agents. In other words, the estimation approach can be applied in various collective control scenarios, among which three typical ones are examined in this paper including swarming, flocking, and flocking without velocity measurement. The swarming scenario is relatively simple where no specific state agreement is required other than network connectivity preservation and collision avoidance. The flocking behavior further requires velocity agreement among agents. A theoretical framework for design and analysis of distributed flocking was presented in [25] based on the three famous Reynolds' rules [26]. A number of researchers have studied the flocking problems from various perspectives [6], [9], [27], [28]. Most of the aforementioned literatures assume that both position and velocity states of the agents are measurable from their neighbors in the group. However, the velocity-free control method is of particular value for multiagent systems not equipped with velocity sensors or in the situation where the precise measurement of velocities is unavailable. From the engineering point of view, the intragroup communication cost can be substantially saved if the measurement requirement on neighboring velocities is eliminated. These years have witnessed some efforts on such velocity-free flock control methods [22], [29]–[32]. In this paper, the control force for network connectivity preservation and collision avoidance is also estimated in this scenario.

The reminder of this paper is organized as follows. The problem description and some preliminaries are given in Section II. The main control force estimation approach for network connectivity preservation and collision avoidance is proposed in Section III in a general setting. The estimation approach is examined in some typical collective control scenarios including swarming, flocking, and flocking without velocity measurement in Section IV. Numerical simulations are given in Section V. This paper is finalized in Section VI with some concluding remarks.

II. Problem Description and Preliminaries

We consider a network of n agents with the following double-integrator dynamics:

$$\begin{aligned} \dot{q}_i &= p_i \\ \dot{p}_i &= u_i,\ i \in \mathcal{V} := \{1, 2, \dots, n\} \end{aligned} \tag{1}$$

where $q_i,\ p_i,\ u_i \in \mathbb{R}^m,\ i \in \mathcal{V}$, are the position, velocity, and control (acceleration) of the agent i, respectively. Denote $q_{ij} := q_i - q_j$ and $p_{ij} := p_i - p_j$. The network connectivity is represented by a time-varying symmetric adjacency matrix $A(t) = [a_{ij}(t)]_{n\times n}$ with

$$a_{ii}(t) = 0,\ a_{ij}(t) \in \{0, 1\},\ i \neq j$$

where $a_{ij}(t) = 1$ describes the existence of a link between the agents i and j at time t, and $a_{ij}(t) = 0$ otherwise. The initial adjacency matrix $A(0)$ is defined by

$$a_{ij}(0) = \begin{cases} 0, & \left\|q_{ij}(t)\right\| \geq R \\ 1, & \left\|q_{ij}(t)\right\| < R \end{cases} \tag{2}$$

for $R > 0$ being the sensing radius of the agents. For $t > 0$, the time varying matrix is dynamically defined as follows:

$$a_{ij}(t) = \begin{cases} 0, a_{ij}(t^-) = 0 \text{ and } & \left\|q_{ij}(t)\right\| \geq R - \varepsilon \\ 0, a_{ij}(t^-) = 1 \text{ and } & \left\|q_{ij}(t)\right\| \geq R \\ 1, a_{ij}(t^-) = 0 \text{ and } & \left\|q_{ij}(t)\right\| < R - \varepsilon \\ 1, a_{ij}(t^-) = 1 \text{ and } & \left\|q_{ij}(t)\right\| < R \end{cases} \tag{3}$$

for a constant $0 \leq \varepsilon < R$. Subsequently, the neighborhood $\mathcal{N}_i(t)$ of the agent i can be defined as

$$\mathcal{N}_i(t) = \left\{j \mid a_{ij}(t) = 1,\ j \neq i,\ j = 1, \dots, n\right\}.$$

With $\epsilon = 0$, the definition of $A(t)$ and hence $\mathcal{N}_i(t)$ represents the traditional position-based ball shaped neighborhood with the radius R. The nonzero $\varepsilon > 0$ endows the definition (3) hysteresis, which is important in controller design for network connectivity preservation. The idea was originally used in [20], [21], and [23]. In this paper, we propose a simpler dynamic definition for the adjacency matrix $A(t)$ with

$$a_{ij}(t) = \begin{cases} 1, & a_{ij}(t^-) = 1 \text{ and } \left\|q_{ij}(t)\right\| < R \\ 0, & \text{otherwise.} \end{cases} \tag{4}$$

Later, we will see that the neighborhood definition (4) has the advantage over (3) in terms of saving control force for network connectivity preservation and collision avoidance.

It is obvious that $A(t)$ is always symmetric in both definitions (3) and (4). For the adjacency matrix $A(t)$, the corresponding Laplacian matrix $L(t) = [l_{ij}(t)]_{n\times n}$ is

$$l_{ij} = \begin{cases} \displaystyle\sum_{k=1,k\neq i}^{n} a_{ik} & j = i \\ -a_{ij} & j \neq i. \end{cases} \tag{5}$$

For the network with the adjacency matrix $A(t)$, we define an edge between the agents i and j at time t when $a_{ij}(t) = a_{ji}(t) = 1$. So, the edges of the network are represented by a set

$\mathcal{E}(t) \subseteq \{(i,j) \mid i,j \in \mathcal{V}, i \neq j\}$. Hence, the pair $\mathcal{G}(t) = \{\mathcal{V}, \mathcal{E}(t)\}$ defines an undirected proximity graph, consisting of the set of vertices $\mathcal{V}$ and the set of edges $\mathcal{E}(t)$. Obviously, with the $A(t)$ defined in (3), the proximity graph has up to $n(n-1)/2$ edges when the proximity graph is complete (all-to-all connected). But, with the $A(t)$ defined in (4), the proximity graph has edges no more than those of $A(0)$. This is another advantage of (4) in terms of saving communication. Throughout this paper, it is assumed that the initial network, represented by $\mathcal{G}(0)$, is connected.

Next, for the purpose of collision avoidance, we define another adjacency matrix $B(t) = [b_{ij}(t)]_{n\times n}$ with

$$b_{ii}(t) = 0,\ b_{ij}(t) = \begin{cases} 0, & \left\|q_{ij}(t)\right\| \geq r \\ 1, & \left\|q_{ij}(t)\right\| < r \end{cases} \tag{6}$$

for $0 < r < R$ being the radius of collision avoidance region. Also, the collision avoidance region $\mathcal{C}_i(t)$ of the agent i can be defined as

$$\mathcal{C}_i(t) = \left\{j \mid b_{ij}(t) = 1,\ j \neq i,\ j = 1, \ldots, n\right\}.$$

Now we consider a control algorithm as follows:

$$\begin{aligned} u_i(t) = &- \sum_{j\in\mathcal{C}_i(t)} k_1\left(\left\|q_{ij}(t)\right\|\right) \frac{q_{ij}(t)}{\left\|q_{ij}(t)\right\|} \\ &- \sum_{j\in\mathcal{N}_i(t)\setminus\mathcal{C}_i(t)} k_2\left(\left\|q_{ij}(t)\right\|\right) \frac{q_{ij}(t)}{\left\|q_{ij}(t)\right\|} + c_i(t) \end{aligned} \tag{7}$$

which consists of three components. The first component $-\sum_{j\in\mathcal{C}_i(t)} k_1(\|q_{ij}(t)\|)q_{ij}(t)/\|q_{ij}(t)\|$ with the function $k_1(\cdot) \leq 0$ provides a positive repulsive force for the agents in the neighborhood $\mathcal{C}_i(t)$ to avoid collision. The second component $-\sum_{j\in\mathcal{N}_i(t)\setminus\mathcal{C}_i(t)} k_2(\|q_{ij}(t)\|)q_{ij}(t)/\|q_{ij}(t)\|$ with the function $k_2(\cdot) \geq 0$ provides a negative adhesive force for the agents in $\mathcal{N}_i(t)\setminus\mathcal{C}_i(t)$ to preserve connectivity (noting those agents in $\mathcal{C}_i(t)$ have been dealt with for collision avoidance). The third control force $c_i(t)$ is used for additional collective control in various scenarios, which will be discussed on a case by case basis.

The objective of this paper shows the following.

1) How do the controllers of the form (7) work effectively for connectivity preservation and collision avoidance?
2) How much control force is enough for connectivity preservation and collision avoidance, for specific initial energy?

To answer these two questions, the main results follow in the next section.

III. Main Results

In this section, we will give two theorems which answer the questions raised in the previous section. The first theorem is given as follows.

Theorem 1: Consider the system (1) with the controller (7) where the continuous and monotonic functions k_1 and k_2 satisfy

$$\begin{aligned} &k_1 : (0, r] \mapsto (-\infty, 0],\ k_1(r) = 0,\ \lim_{x\to 0} k_1(x) = -\infty \\ &k_2 : [r, R) \mapsto [0, +\infty),\ k_2(r) = 0,\ \lim_{x\to R} k_2(x) = +\infty. \end{aligned}$$

The neighborhood $\mathcal{N}_i(t)$ is defined with (2) and (4) and $\mathcal{C}_i(t)$ with (6). Suppose the input $c_i(t)$ satisfies

$$\begin{aligned} &\int_0^t \sum_{i=1}^n p_i^{\mathsf{T}}(\tau)c_i(\tau)d\tau \leq V_0 \\ &\|c_i(t)\| \leq c^*,\ \forall t \geq 0, i \in \mathcal{V} \end{aligned} \tag{8}$$

for two constants $V_0, c^* \geq 0$. Then the following hold.

1) The connectivity of $\mathcal{G}(t)$ is preserved for $t \geq 0$, that is
$$\left\|q_{ij}(t)\right\| \leq R^* < R,\ \forall t \geq 0,\ (i,j) \in \mathcal{E}(0). \tag{9}$$
2) Interagent collision is avoided, i.e., $\|q_{ij}(t)\| \geq r^* > 0$, $\forall t \geq 0$, $i,j \in \mathcal{V}$, $i \neq j$.
3) The control force is bounded, that is
$$\|u_i(t)\| \leq u^* + c^*,\ \forall t \geq 0,\ i \in \mathcal{V}. \tag{10}$$

Moreover, the constants r^*, R^*, and u^* are explicitly calculated as follows. Let

$$E_0 = \frac{1}{2}\sum_{i=1}^n \left(\sum_{j\in\mathcal{N}_i(0)} \int_r^{\|q_{ij}(0)\|} k(s)ds + p_i^{\mathsf{T}}(0)p_i(0)\right)$$

with $k(\cdot)$ being the combination of $k_1(\cdot)$ and $k_2(\cdot)$, that is

$$k(x) = \begin{cases} k_1(x), & x \in (0, r] \\ k_2(x), & x \in [r, R). \end{cases}$$

Then, $r^* \in (0, r]$ and $R^* \in [r, R)$ are determined by

$$\int_r^{r^*} k(s)ds = \int_r^{R^*} k(s)ds = E_0 + V_0 \tag{11}$$

and hence

$$u^* = (n-1)\max\left\{-k_1\left(r^*\right), k_2\left(R^*\right)\right\}.$$

Proof: For the convenience of proof, we define the following functions:

$$\begin{aligned} \psi_1(x) &= \int_r^x k_1(s)ds,\ x \in (0, r] \\ \psi_2(x) &= \int_r^x k_2(s)ds,\ x \in [r, R) \\ \psi(x) &= \begin{cases} \psi_1(x), & x \in (0, r] \\ \psi_2(x), & x \in [r, R) \end{cases} \end{aligned}$$

and

$$\begin{aligned} \bar{k}(x) &= \begin{cases} k_1(x), & x \in (0, r] \\ 0, & x \in [r, \infty) \end{cases} \\ \bar{\psi}(x) &= \begin{cases} \psi_1(x), & x \in (0, r] \\ 0, & x \in [r, \infty). \end{cases} \end{aligned}$$

Under the condition (8), there exists a function $\beta(t) \geq 0$ such that

$$\int_0^t \left(\beta(\tau) + \sum_{i=1}^n p_i^{\mathsf{T}}(\tau)c_i(\tau)\right) d\tau \leq V_0,\ \forall t \geq 0. \tag{12}$$

(A trivial but conservative choice is $\beta(t) = 0$.) Hence, we define a function

$$V(t) = V_0 - \int_0^t \left(\beta(\tau) + \sum_{i=1}^n p_i^{\mathsf{T}}(\tau)c_i(\tau)\right) d\tau$$

which satisfies $V(t) \geq 0$ for $t \geq 0$.

Next, we define an energy function

$$\Phi(t) = \frac{1}{2}\sum_{i=1}^{n}\sum_{j\in\mathcal{N}_i(t)} \psi\left(\left\|q_{ij}(t)\right\|\right) + \frac{1}{2}\sum_{i=1}^{n}\sum_{j\in\mathcal{N}_i^c(t)} \bar{\psi}\left(\left\|q_{ij}(t)\right\|\right) + \frac{1}{2}\sum_{i=1}^{n} p_i^{\mathsf{T}}(t)p_i(t) + V(t) \tag{13}$$

with $\mathcal{N}_i^c(t) = \mathcal{V}\backslash(\mathcal{N}_i(t)\cup\{i\})$. Let t_1 be the first time the graph $\mathcal{G}(t)$ changes. In other words, $\mathcal{N}_i(t)$ and $\mathcal{N}_i^c(t)$ are constant for $t \in (0, t_1)$. Therefore, for $t \in (0, t_1)$, the derivative of $\Phi(t)$ is

$$\begin{aligned}\dot{\Phi}(t) &= \frac{1}{2}\sum_{i=1}^{n}\sum_{j\in\mathcal{N}_i(t)} \frac{\partial\psi\big(\|q_{ij}(t)\|\big)}{\partial\left\|q_{ij}(t)\right\|}\frac{\dot{q}_{ij}^{\mathsf{T}}(t)q_{ij}(t)}{\left\|q_{ij}(t)\right\|}\\ &\quad+ \frac{1}{2}\sum_{i=1}^{n}\sum_{j\in\mathcal{N}_i^c(t)} \frac{\partial\bar{\psi}\big(\left\|q_{ij}(t)\right\|\big)}{\partial\left\|q_{ij}(t)\right\|}\frac{\dot{q}_{ij}^{\mathsf{T}}(t)q_{ij}(t)}{\left\|q_{ij}(t)\right\|}\\ &\quad+ \sum_{i=1}^{n} p_i^{\mathsf{T}}(t)\dot{p}_i(t) + \dot{V}(t).\end{aligned}$$

Next, by noting $\dot{\psi}(x) = k(x)$ and $\dot{\bar{\psi}}(x) = \bar{k}(x)$

$$\begin{aligned}\dot{\Phi}(t) &= \frac{1}{2}\sum_{i=1}^{n}\sum_{j\in\mathcal{N}_i(t)} k\left(\left\|q_{ij}(t)\right\|\right)\frac{p_{ij}^{\mathsf{T}}(t)q_{ij}(t)}{\left\|q_{ij}(t)\right\|}\\ &\quad+ \frac{1}{2}\sum_{i=1}^{n}\sum_{j\in\mathcal{N}_i^c(t)} \bar{k}(\|q_{ij}(t)\|)\frac{p_{ij}^{\mathsf{T}}(t)q_{ij}(t)}{\left\|q_{ij}(t)\right\|}\\ &\quad+ \sum_{i=1}^{n} p_i^{\mathsf{T}}(t)u_i(t) + \dot{V}(t).\end{aligned}$$

On one hand, by noting

$$\begin{aligned}(\mathcal{C}_i \setminus \mathcal{N}_i) \cup (\mathcal{C}_i \cap \mathcal{N}_i) &= \mathcal{C}_i\\ (\mathcal{C}_i \setminus \mathcal{N}_i) \cap (\mathcal{C}_i \cap \mathcal{N}_i) &= \emptyset,\end{aligned}$$

one has

$$\sum_{j\in\mathcal{C}_i} k_1\left(\left\|q_{ij}\right\|\right)\frac{q_{ij}}{\left\|q_{ij}\right\|} = \sum_{j\in\mathcal{C}_i\setminus\mathcal{N}_i} k_1\left(\left\|q_{ij}\right\|\right)\frac{q_{ij}}{\left\|q_{ij}\right\|} + \sum_{j\in\mathcal{C}_i\cap\mathcal{N}_i} k_1\left(\left\|q_{ij}\right\|\right)\frac{q_{ij}}{\left\|q_{ij}\right\|}.$$

On the other hand, by noting

$$\begin{aligned}(\mathcal{N}_i \setminus \mathcal{C}_i) \cup (\mathcal{C}_i \cap \mathcal{N}_i) &= \mathcal{N}_i\\ (\mathcal{N}_i \setminus \mathcal{C}_i) \cap (\mathcal{C}_i \cap \mathcal{N}_i) &= \emptyset\end{aligned}$$

and the definition of k, one has

$$\sum_{j\in\mathcal{C}_i\cap\mathcal{N}_i} k_1\big(\left\|q_{ij}\right\|\big)\frac{q_{ij}}{\left\|q_{ij}\right\|} + \sum_{j\in\mathcal{N}_i\setminus\mathcal{C}_i} k_2\big(\left\|q_{ij}\right\|\big)\frac{q_{ij}}{\left\|q_{ij}\right\|} = \sum_{j\in\mathcal{N}_i} k\big(\left\|q_{ij}\right\|\big)\frac{q_{ij}}{\left\|q_{ij}\right\|}.$$

Moreover, by the definition of $\bar{k}$, one has

$$\sum_{j\in\mathcal{C}_i\setminus\mathcal{N}_i} k_1\big(\left\|q_{ij}\right\|\big)\frac{q_{ij}}{\left\|q_{ij}\right\|} = \sum_{j\in\mathcal{N}_i^c} \bar{k}\big(\left\|q_{ij}\right\|\big)\frac{q_{ij}}{\left\|q_{ij}\right\|}.$$

From above, u_i in (7) can be rewritten as

$$\begin{aligned}u_i(t) &= -\sum_{j\in\mathcal{N}_i(t)} k\big(\left\|q_{ij}(t)\right\|\big)\frac{q_{ij}(t)}{\left\|q_{ij}(t)\right\|}\\ &\quad- \sum_{j\in\mathcal{N}_i^c(t)} \bar{k}\big(\left\|q_{ij}(t)\right\|\big)\frac{q_{ij}(t)}{\left\|q_{ij}(t)\right\|} + c_i(t).\end{aligned} \tag{14}$$

Next

$$\begin{aligned}\sum_{i=1}^{n} p_i^{\mathsf{T}}(t)u_i(t) &= -\sum_{i=1}^{n}\sum_{j\in\mathcal{N}_i(t)} k\big(\left\|q_{ij}(t)\right\|\big)\frac{p_i^{\mathsf{T}}(t)q_{ij}(t)}{\left\|q_{ij}(t)\right\|}\\ &\quad-\sum_{i=1}^{n}\sum_{j\in\mathcal{N}_i^c(t)} \bar{k}\big(\left\|q_{ij}(t)\right\|\big)\frac{p_i^{\mathsf{T}}(t)q_{ij}(t)}{\left\|q_{ij}(t)\right\|}\\ &\quad+ \sum_{i=1}^{n} p_i^{\mathsf{T}}(t)c_i(t)\\ &= -\frac{1}{2}\sum_{i=1}^{n}\sum_{j\in\mathcal{N}_i(t)} k\big(\left\|q_{ij}(t)\right\|\big)\frac{p_{ij}^{\mathsf{T}}(t)q_{ij}(t)}{\left\|q_{ij}(t)\right\|}\\ &\quad-\frac{1}{2}\sum_{i=1}^{n}\sum_{j\in\mathcal{N}_i^c(t)} \bar{k}\big(\left\|q_{ij}(t)\right\|\big)\frac{p_{ij}^{\mathsf{T}}(t)q_{ij}(t)}{\left\|q_{ij}(t)\right\|}\\ &\quad+ \sum_{i=1}^{n} p_i^{\mathsf{T}}(t)c_i(t).\end{aligned}$$

As a result

$$\dot{\Phi}(t) = \sum_{i=1}^{n} p_i^{\mathsf{T}}(t)c_i(t) - \left(\beta(t) + \sum_{i=1}^{n} p_i^{\mathsf{T}}(t)c_i(t)\right) = -\beta(t) \le 0.$$

By noting $\bar{\psi}(\|q_{ij}(0)\|) = 0$ for $j \in \mathcal{N}_i^c(0)$ and $V(0) = V_0$, one has

$$\Phi(0) = E_0 + V_0.$$

For each $i \in \mathcal{V}$, if there exists an agent $j \in \mathcal{N}_i(t)$ and a finite time $t^* < t_1$, such that $\|q_{ij}(t^*)\| > R^*$, then, it implies

$$\Phi(t^*) \ge \psi_2\big(\left\|q_{ij}\left(t^*\right)\right\|\big) > \psi_2\left(R^*\right) = E_0 + V_0$$

and a contradiction. Therefore, t_1 is infinite which implies that the connectivity of $\mathcal{G}(t)$ is preserved for $t \ge 0$. The conclusion 1) is thus proved.

Similarly, for each $i \in \mathcal{V}$, if there exists an agent j and a finite time $t^* < t_1$, such that $\|q_{ij}(t^*)\| < r^*$, then, it implies

$$\Phi\left(t^*\right) \ge \psi_1\left(\left\|q_{ij}\left(t^*\right)\right\|\right) > \psi_1\left(r^*\right) = E_0 + V_0$$

and a contradiction. The conclusion 2) is thus proved.

From the above proof, it has been shown that

$$r^* \le \left\|q_{ij}(t)\right\| \le R^*, \forall i, j \in \mathcal{V},\ i \neq j, t \ge 0.$$

Then, from (14)

$$\begin{aligned}\|u_i(t)\| &\le \sum_{j\in\mathcal{N}_i(t)} \left\|k\big(\left\|q_{ij}(t)\right\|\big)\right\| + \sum_{j\in\mathcal{N}_i^c(t)} \left\|\bar{k}\big(\left\|q_{ij}(t)\right\|\big)\right\| + c^*\\ &\le (n-1)\max\left\{-k_1\big(r^*\big), k_2\big(R^*\big)\right\} + c^* = u^* + c^*.\end{aligned}$$

The conclusion 3) is thus proved. ■

Remark 1: It is shown in Theorem 1 that for the dynamic neighborhood $\mathcal{N}_i(t)$ defined with (2) and (4), the connectivity of $\mathcal{G}(t)$ is preserved. In particular, for two agents in each other's neighborhood, the distance is always less than $R^* < R$. Because of the adhesive force caused by the k_2 function, sufficiently large energy is required to separate the two agents. The total energy the system achieved is the initial energy E_0 (including the potential energy $(1/2)\sum_{i=1}^{n}\sum_{j\in\mathcal{N}_i(0)}\int_r^{\|q_{ij}(0)\|}k(s)ds$ and the kinetic energy $(1/2)\sum_{i=1}^{n}p_i^{\mathsf{T}}(0)p_i(0)$) and the input energy caused by the force $c_i(t)$ (for a collective behavior to be specified), i.e., $\int_0^t\sum_{i=1}^{n}p_i^{\mathsf{T}}(\tau)c_i(\tau)d\tau$, bounded by V_0 as in (8). So, the largest distance for any neighbored agents separated by the energy E_0+V_0 is R^* determined by $\int_r^{R^*}k(s)s = E_0+V_0$. For any two agents in the neighborhood $\mathcal{C}_i$ defined with (6), because of the repulsive force caused by the k_1 function, sufficiently large energy is required to pull the two agents closer to each other. So, the closest distance for any two agents pulled by the energy E_0+V_0 is r^* determined by $\int_r^{r^*}k(s)s = E_0+V_0$.

Remark 2: The control force is bounded by u^*+c^* where c^* is the boundary of $c_i(t)$, the force for a collective behavior and u^* is the boundary of the force for both connectivity preservation and collision avoidance. The estimation u^* given in Theorem 1 is sometimes too conservative in practice, in particular, when the agent number is high. In fact, in the last inequality of the proof, when $q_{il}(t)$ is close to r^* or R^* for the agent l, to guarantee $\Phi(t)\le E_0+V_0$, all the terms in the right hand side of (13) are zero, except for $\psi(\|q_{il}(t)\|)=E_0+V_0$ or $\bar{\psi}(\|q_{il}(t)\|)=E_0+V_0$. In other words, $q_{ij}(t)$, for $j\neq l$, should be such that 1) $\psi(\|q_{ij}(t)\|)=0$ for any $j\in\mathcal{N}_i(t)$, which implies $k(\|q_{ij}(t)\|)=0$ or 2) $\bar{\psi}(\|q_{ij}(t)\|)=0$ for any $j\in\mathcal{N}_i^c(t)$, which implies $\bar{k}(\|q_{ij}(t)\|)=0$. As a result

$$\begin{aligned}\sum_{j\in\mathcal{N}_i(t)}\left\|k\big(\|q_{ij}(t)\|\big)\right\| &+ \sum_{j\in\mathcal{N}_i^c(t)}\left\|\bar{k}\big(\|q_{ij}(t)\|\big)\right\|\\ &\le \max\left\{\|k(\|q_{il}(t)\|)\|,\ \left\|\bar{k}(\|q_{il}(t)\|)\right\|\right\}\\ &\le \max\left\{-k_1\left(r^*\right),k_2\left(R^*\right)\right\}.\end{aligned}$$

Therefore, a more practically less conservative estimation is $u^*=\max\{-k_1(r^*),k_2(R^*)\}$.

Theorem 1 is given for the dynamic neighborhood $\mathcal{N}_i(t)$ defined with (2) and (4). A slightly different version of the theorem is given below for the dynamic neighborhood $\mathcal{N}_i(t)$ defined with (2) and (3).

Theorem 2: In Theorem 1, if the neighborhood $\mathcal{N}_i(t)$ is defined with (2) and (3), the results still hold with (11) replaced by

$$\int_r^{r^*}k(s)ds=\int_r^{R^*}k(s)ds=E_0+V_0+F_0 \tag{15}$$

for

$$F_0=\left[\frac{n(n-1)}{2}-\gamma\right]\int_r^{R-\varepsilon}k_2(s)ds$$

where γ is the number of edges of $\mathcal{G}(0)$ and ε is the parameter used in (3).

Proof: Let t_1 be the first time the graph $\mathcal{G}(t)$ changes. For $t\in(0,t_1)$, one has $\dot{\Phi}(t)=-\beta(t)\le 0$ following the proof of Theorem 1. If there exists an agent $j\in\mathcal{N}_i(t)$ and a finite time $0<t^*<t_1$, such that $\|q_{ij}(t^*)\|>R^*$. Then, it implies

$$\Phi\left(t^*\right)\ge\psi_2\big(\left\|q_{ij}\left(t^*\right)\right\|\big)>\psi_2\big(R^*\big)=E_0+V_0+F_0\ge\Phi(0)$$

which leads to a contradiction. Therefore, no edge of $\mathcal{G}(t)$ is lost at $t\in(0,t_1)$. In other words, the graph $\mathcal{G}(t)$ changes at t_1 only because a new edge is added. Based on the neighborhood definition (3) and the definition of $\Phi(t)$, one has

$$\Phi\left(t_1+\right)=\Phi(t_1-)+\int_r^{R-\varepsilon}k_2(s)ds.$$

Next, let $t_2>t_1$ be the first time after t_1 the graph $\mathcal{G}(t)$ changes. For $t\in(t_1,t_2)$, one has $\dot{\Phi}(t)=-\beta(t)\le 0$ following the same procedure. If there exists an agent $j\in\mathcal{N}_i(t)$ and a finite time $t_1<t^*<t_2$, such that $\|q_{ij}\left(t^*\right)\|>R^*$. Then, it implies

$$\begin{aligned}\Phi(t^*)&\ge\psi_2\big(\left\|q_{ij}\left(t^*\right)\right\|\big)>\psi_2\big(R^*\big)=E_0+V_0+F_0\\ &\ge\Phi(0)+\int_r^{R-\varepsilon}k_2(s)ds\\ &\ge\Phi\left(t_1-\right)+\int_r^{R-\varepsilon}k_2(s)ds=\Phi\left(t_1+\right)\end{aligned}$$

which leads to a contradiction again. Therefore, no edge of $\mathcal{G}(t)$ is lost at $t\in(t_1,t_2)$. In other words, the graph $\mathcal{G}(t)$ changes at t_2 only because a new edge is added.

A network of n agents has the maximal $n(n-1)/2$ edges and the initial network $\mathcal{G}(0)$ has γ edges. Therefore, the above procedure can be repeated for up to $n(n-1)/2-\gamma$ times, after which $\mathcal{G}(t)$ does not change any longer. During the whole procedure, no edge of $\mathcal{G}(0)$ is lost. In particular

$$\left\|q_{ij}(t)\right\|\le R^*,\ \forall t\ge 0,\ (i,j)\in\mathcal{E}(0) \tag{16}$$

is always satisfied. The conclusion 1) is thus proved. The proof for conclusions 2) and 3) follows that in Theorem 1. ■

Remark 3: By comparing the two theorems, we can see that the neighborhood definition (4) has the advantage over (3) in terms of saving control force, with $F_0=0$ in Theorem 1 but $F_0>0$ in Theorem 2, for network connectivity preservation and collision avoidance.

Remark 4: The control force estimation for network connectivity preservation and collision avoidance is separated from the collective control $c_i(t)$ as long as the function $c_i(t)$ satisfies the bounded conditions in (8). The specific design of $c_i(t)$ for various collective behaviors will be discussed in the next section. In particular, the saturation control design approach will be applied. Some relevant research on saturation control design of multiagent systems can be found in [29], [30], and [33]–[35].

IV. Case Studies

In this section, the estimation approach, Theorem 1, is examined in some typical collective control scenarios including swarming, flocking, and flocking without velocity measurement.

The following theorem is the simplest scenario (including $c_i(t)=0$ as the special case) where $c_i(t)$ does not aim to achieve any significant collective behavior. This scenario only requires that network connectivity be preserved during the entire evolution and collision always avoided. This class of behavior is called a swarming.

Theorem 3: Suppose the control $c_i(t)$ in (7) is

$$c_i(t) = C\frac{p_i(t)}{\|p_i(t)\| + \epsilon} \tag{17}$$

for a constant $\epsilon > 0$ and a skew-symmetric matrix C, i.e., $C+C^{\mathsf{T}} = 0$. Then, Theorem 1 holds for $V_0 = 0$ and $c^* = \|C\|$.

Proof: It is easy to verify that

$$\int_0^t \sum_{i=1}^{n} p_i^{\mathsf{T}}(\tau)c_i(\tau)d\tau = \int_0^t \sum_{i=1}^{n} \frac{p_i^{\mathsf{T}}(\tau)Cp_i(\tau)}{\|p_i(\tau)\| + \epsilon}d\tau = 0$$

as $p_i^{\mathsf{T}}(\tau)Cp_i(\tau) = 0$ for a skew-symmetric matrix C. Also

$$\|c_i(t)\| = \left\|C\frac{p_i(t)}{\|p_i(t)\| + \epsilon}\right\| \le \frac{\|C\|\,\|p_i(t)\|}{\|p_i(t)\| + \epsilon} \le \|C\|.$$

Therefore, Theorem 1 holds for $V_0 = 0$ and $c^* = \|C\|$. ■

In the following theorem, $c_i(t)$ is designed using the relative velocity feedback $p_{ij} = p_i - p_j$ to achieve the consensus of velocity, called flocking. The flocking controller has been studied in literature as discussed in Introduction. The feature of the theorem given in this paper is the additional estimation of control force in the presence of connectivity preservation and collision avoidance. In this theorem, $\mathrm{sat}(\cdot)$ is an element-wise saturation function

$$\mathrm{sat}(s) = \min(1, \max(-1, s))\,,\ \ s \in \mathbb{R}.$$

Theorem 4: Suppose the control $c_i(t)$ in (7) is

$$c_i(t) = -\epsilon \sum_{j\in\mathcal{N}_i} \mathrm{sat}\big(p_{ij}\big). \tag{18}$$

Then, Theorem 1 holds for $V_0 = 0$ and $c^* = (n-1)\epsilon$. Moreover, the group of agents (1) asymptotically converges to a rigid flock with a common velocity, i.e., $\lim_{t\to\infty} p_i(t) = p_o$, $i \in \mathcal{V}$, for a constant p_o.

Proof: Direct calculation shows that

$$\begin{aligned}\sum_{i=1}^{n} p_i^{\mathsf{T}}(t)c_i(t) &= -\,\epsilon \sum_{i=1}^{n}\sum_{j\in\mathcal{N}_i} p_i^{\mathsf{T}}(t)\mathrm{sat}\left(p_{ij}\right)\\ &= -\,\frac{1}{2}\epsilon \sum_{i=1}^{n}\sum_{j\in\mathcal{N}_i} p_i^{\mathsf{T}}(t)\mathrm{sat}\left(p_{ij}\right)\\ &\quad -\,\frac{1}{2}\epsilon \sum_{i=1}^{n}\sum_{j\in\mathcal{N}_i} p_j^{\mathsf{T}}(t)\mathrm{sat}\left(p_{ji}\right)\\ &= -\,\frac{1}{2}\epsilon \sum_{i=1}^{n}\sum_{j\in\mathcal{N}_i} p_{ij}^{\mathsf{T}}(t)\mathrm{sat}\left(p_{ij}\right)\end{aligned}$$

where the fact $\mathrm{sat}(p_{ji}) = -\mathrm{sat}(p_{ij})$ is used. Letting

$$\beta(t) = \frac{1}{2}\epsilon \sum_{i=1}^{n}\sum_{j\in\mathcal{N}_i} p_{ij}^{\mathsf{T}}(t)\mathrm{sat}\left(p_{ij}(t)\right) \ge 0$$

gives

$$\sum_{i=1}^{n} p_i^{\mathsf{T}}(t)c_i(t) + \beta(t) = 0.$$

Also, one has

$$\|c_i(t)\| = \left\|\epsilon \sum_{j\in\mathcal{N}_i} \mathrm{sat}\big(p_{ij}\big)\right\| \le (n-1)\epsilon.$$

We are ready to see that Theorem 1 holds for $V_0 = 0$ and $c^* = (n-1)\epsilon$.

From the proof of Theorem 1, one has

$$\dot{\Phi}(t) = -\beta(t) \le 0$$

for the energy function $\Phi(t) \ge 0$ defined in (13). Since $u_{ij}(t)$ and $p_{ij}(t)$ are bounded, so is the derivative

$$\begin{aligned}\dot{\beta}(t) &= \frac{1}{2}\epsilon \sum_{i=1}^{n}\sum_{j\in\mathcal{N}_i} u_{ij}^{\mathsf{T}}(t)\mathrm{sat}\big(p_{ij}(t)\big)\\ &\quad + \frac{1}{2}\epsilon \sum_{i=1}^{n}\sum_{j\in\mathcal{N}_i} p_{ij}^{\mathsf{T}}(t)\frac{\partial \mathrm{sat}\big(p_{ij}(t)\big)}{\partial p_{ij}}u_{ij}(t).\end{aligned}$$

It implies that $\beta(t)$ is uniformly continuous. By Barbalat's Lemma, one has

$$\lim_{t\to\infty} \beta(t) = 0$$

which implies

$$\lim_{t\to\infty} p_{ij}(t) = 0,\ \forall (i,j) \in \mathcal{E}(t).$$

From Theorem 1, one has $\mathcal{E}(t) = \mathcal{E}(0)$. As the network $\mathcal{G}(0)$ is connected, there exists p_o such that $\lim_{t\to\infty} p_i(t) = p_o$, $i \in \mathcal{V}$. ■

In the controller given in Theorem 4, it is assumed that both position and velocity states of the agents are measurable from their neighbors in the group. From the engineering point of view, the intragroup communication cost can be substantially saved if the measurement requirement on neighboring velocities is eliminated. Moreover, the velocity-free control method is of particular value for multiagent systems not equipped with velocity sensors or in the situation where the precise measurement of velocities is unavailable. This fact motivates the study on velocity-free controllers in the following theorem.

Theorem 5: Suppose the control $c_i(t)$ in (7) is as follows:

$$\begin{aligned}c_i^{\mathsf{T}}(t) &= -\sum_{j=1}^{n} l_{ij}(t)\sigma'\left(y_j(t)\right)\\ y_i(t) &= -\,x_i(t) + \sum_{j=1}^{n} l_{ij}(t)q_j(t)\\ \dot{x}_i(t) &= -\,x_i(t) + \sum_{j=1}^{n} l_{ij}(t)q_j(t)\end{aligned} \tag{19}$$

where $x_i,\ y_i \in \mathbb{R}^m,\ \ \forall i \in \mathcal{V}$. The function $\sigma(y_j)$ satisfies the following conditions.

1) $\sigma(y_j)$ is continuously differentiable, positive definite, and radially unbounded.
2) The first derivative $\sigma'(y_j)$ is uniformly continuous and satisfies $\|\sigma'(y_j)\| \le \sigma^*$ for a constant σ^*.
3) The function $\sigma'(y_j)y_j$ is positive definite.

Then, Theorem 1 holds for

$$V_0 = \sum_{i=1}^{n} \sigma\left(y_i(0)\right), \quad c^* = 2(n-1)\sigma^*.$$

Moreover, the group of agents (1) asymptotically converges to a rigid flock with a common velocity, i.e., $\lim_{t\to\infty} p_i(t) = p_o$, $i \in \mathcal{V}$, for a constant p_o.

Proof: First, we define the function

$$V(t) = \sum_{i=1}^{n} \sigma\left(y_i(t)\right) \geq 0 \tag{20}$$

where $\sigma(\cdot)$ is positive definite. Next, direct calculation shows that

$$\begin{aligned}
&\sum_{i=1}^{n} p_i^{\mathsf{T}}(t)c_i(t) + \dot{V}(t)\\
&\quad= -\sum_{i=1}^{n}\sum_{j=1}^{n} l_{ij}(t)\sigma'(y_j(t))p_i(t) + \sum_{i=1}^{n}\sigma'\left(y_i(t)\right)\dot{y}_i(t)\\
&\quad= -\sum_{i=1}^{n}\sum_{j=1}^{n} l_{ij}(t)\sigma'(y_j(t))p_i(t)\\
&\qquad+ \sum_{i=1}^{n}\sum_{j=1}^{n} l_{ij}(t)\sigma'\left(y_i(t)\right)p_j(t) - \sum_{i=1}^{n}\sigma'\left(y_i(t)\right)\dot{x}_i(t)\\
&\quad= -\sum_{i=1}^{n}\sigma'\left(y_i(t)\right)\dot{x}_i(t) = -\sum_{i=1}^{n}\sigma'\left(y_i(t)\right)y_i(t).
\end{aligned}$$

Let

$$\beta(t) = \sum_{i=1}^{n}\sigma'\left(y_i(t)\right)y_i(t) \geq 0$$

one has

$$-\sum_{i=1}^{n} p_i^{\mathsf{T}} c_i - \beta(t) = \dot{V}(t).$$

Furthermore

$$\begin{aligned}
&-\int_0^t \sum_{i=1}^{n} p_i^{\mathsf{T}}(\tau)c_i(\tau)d\tau\\
&-\int_0^t \beta(\tau)d\tau = V(t) - V(0) \geq -V(0)
\end{aligned}$$

and

$$\int_0^t \left(\beta(\tau) + \sum_{i=1}^{n} p_i^{\mathsf{T}}(\tau)c_i(\tau)\right)d\tau \leq V(0) = \sum_{i=1}^{n}\sigma\left(y_i(0)\right).$$

Another calculation shows

$$\|c_i(t)\| = \left\|\sum_{j=1}^{n} l_{ij}(t)\sigma'\left(y_j(t)\right)\right\| \leq 2(n-1)\sigma^*.$$

Therefore, Theorem 1 holds for

$$V_0 = \sum_{i=1}^{n} \sigma\left(y_i(0)\right), \quad c^* = 2(n-1)\sigma^*.$$

The function $\Phi(t)$ is bounded, so are $p_i(t)$ and $V(t)$. Moreover, $V(t)$ is bounded, which implies that $y_i(t)$ is bounded as $\sigma(y_i)$ is radially unbounded. From (19), one has

$$\dot{y}_i(t) = -y_i(t) + \sum_{j=1}^{n} l_{ij}(t)p_j(t)$$

which is bounded. From the proof of Theorem 1, one has

$$\dot{\Phi}(t) = -\beta(t) \leq 0$$

for the energy function $\Phi(t) \geq 0$ defined in (13). Since $y_i(t)$, $\dot{y}_i(t)$ and $\sigma''(y_i(t))$ are bounded (from the condition that $\sigma'(y_j)$ is uniformly continuous), so is the derivative

$$\dot{\beta}(t) = \sum_{i=1}^{n} y_i^{\mathsf{T}}(t)\sigma''\left(y_i(t)\right)\dot{y}_i(t) + \sigma'\left(y_i(t)\right)\dot{y}_i(t).$$

It implies that $\beta(t)$ is uniformly continuous. By Barbalat's Lemma, one has

$$\lim_{t\to\infty}\beta(t) = \lim_{t\to\infty}\sum_{i=1}^{n}\sigma'\left(y_i(t)\right)y_i(t) = 0$$

which implies

$$\lim_{t\to\infty} y_i(t) = 0, \ \forall i \in \mathcal{V}$$

using the condition 3). As

$$\ddot{y}_i(t) = -\dot{y}_i(t) + \sum_{j=1}^{n} l_{ij}(t)u_j(t)$$

is bounded, the function $\dot{y}_i(t)$ is uniformly continuous, and hence

$$\lim_{t\to\infty}\dot{y}_i(t) = 0, \ \forall i \in \mathcal{V}$$

using Barbalat's Lemma again. As a result

$$\lim_{t\to\infty}\sum_{j=1}^{n} l_{ij}(t)p_j(t) = 0, \ \forall i \in \mathcal{V}.$$

From Theorem 1, $\mathcal{G}(t) = \mathcal{G}(0)$ is connected. For the property of the Laplacian matrix L ([36]), there exists p_o such that $\lim_{t\to\infty} p_i(t) = p_o$, $i \in \mathcal{V}$. The proof is thus completed. ■

Remark 5: In Theorems 4 and 5, c^* is calculated as $c^* = (n-1)\epsilon$ and $c^* = 2(n-1)\sigma^*$, respectively. In fact, it is easy to see that less conservative boundaries can be calculated as $c^* = \bar{n}\epsilon$ and $c^* = 2\bar{n}\sigma^*$ where $\bar{n}$ is the maximum number of neighbors of an agent in terms of the neighborhood $\mathcal{N}_i$, $i = 1, \ldots, n$.

V. Numerical Simulation

In this section, we present two cases to illustrate the effectiveness of the proposed algorithms in Theorems 4 and 5. The following two functions are used in both cases:

$$k_1(x) = 1 - \frac{r}{x}, \ k_2(x) = \frac{x-r}{R-x}$$

with $r = 1.5$ and $R = 2$. The conditions for k_1 and k_2 in Theorem 1 are satisfied.

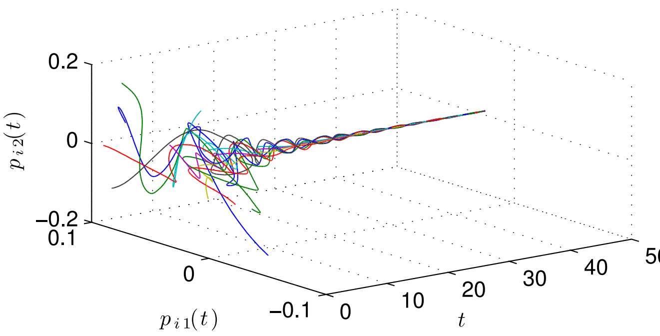

Fig. 1. Profile of velocity trajectories of ten agents (case 1).

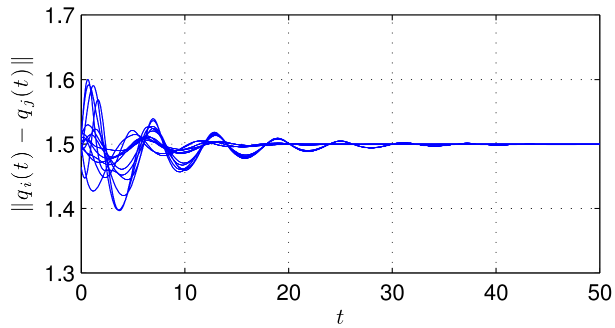

Fig. 2. Profile of distances between every two neighbored agents (case 1).

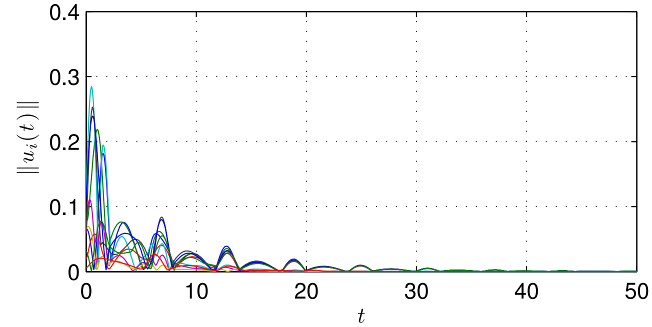

Fig. 3. Profile of control forces of ten agents (case 1).

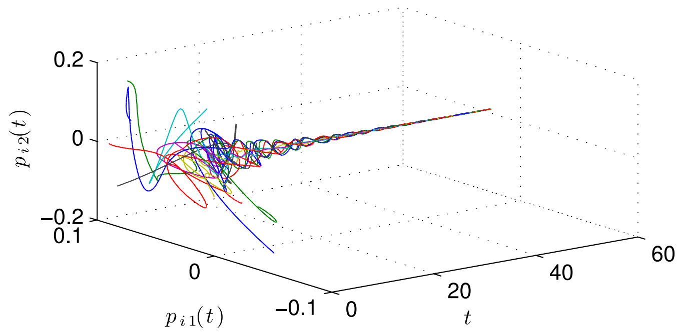

Fig. 4. Profile of velocity trajectories of ten agents (case 2).

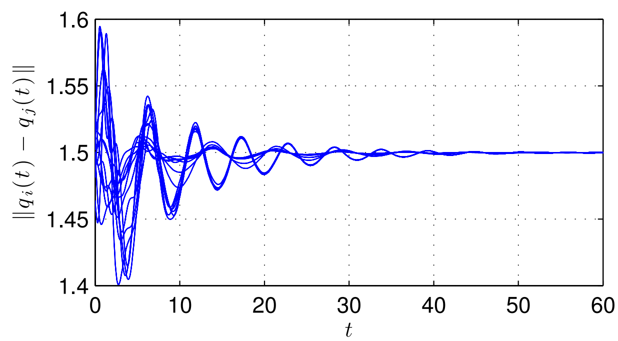

Fig. 5. Profile of distances between every two neighbored agents (case 2).

In both cases, simulation is performed for ten agents moving in a 2-D plane. With the initial position $q_i(0)$ and the initial velocity $p_i(0)$, the initial energy E_0 is calculated as

$$E_0 = \frac{1}{2}\sum_{i=1}^{n}\left(\sum_{j\in\mathcal{N}_i(0)}\int_r^{\|q_{ij}(0)\|} k(s)ds + p_i^{\mathsf{T}}(0)p_i(0)\right) = 0.070.$$

In this first case, we aim to achieve a flocking behavior using the controller $c_i(t)$ given in Theorem 4 with the parameter $\epsilon = 0.2$ and the element-wise saturation function $\mathrm{sat}(\cdot)$. One has $V_0 = 0$, $E_0 + V_0 = 0.070$, and hence $r^* = 1.08$ and $R^* = 1.72$ based on Theorem 1. Then, based on Remarks 2 and 5, we have the estimation $u^* = 0.79$ and $c^* = 0.2$.

In the second case, we aim to achieve a flocking behavior using the controller $c_i(t)$ given in Theorem 5 with the element-wise function

$$\sigma(s) = 0.2\ s \times \arctan(s),\ \ s \in \mathbb{R}.$$

Hence, $\sigma'(s) = 0.2\arctan(s) + 0.2\ s/(1+s^2)$ satisfies $\|\sigma'(s)\| \le 0.1(\pi+1)$. Obviously, all the conditions 1)–3) in Theorem 5 are satisfied. To simplify the control algorithm in Theorem 5, let $x_i(0) = \sum_{j=1}^{n} l_{ij}(0)q_j(0)$ such that $y_i(0) = 0$ and $V_0 = 0$. In this case, one has $E_0 + V_0 = 0.070$, and hence $r^* = 1.08$ and $R^* = 1.72$ based on Theorem 1. Then, based on Remarks 2 and 5, we have the estimation $u^* = 0.79$ and $c^* = 0.2(\pi+1) = 0.83$.

The simulation results for the first case are given in Figs. 1–3. As we can see in Fig. 1 that the group of agents asymptotically converges to a rigid flock with a common velocity. For each pair of agents in each other's neighborhood $\mathcal{N}_i$, their distance is plotted in Fig. 2. It is observed that the distance is within the range $(r^*, R^*) = (1.08, 1.72)$, so collision avoidance and connectivity preservation are effectively guaranteed during the flocking process. The profile of control inputs is plotted in Fig. 3 where all inputs are bounded by $u^* + c^* = 0.99$. The similar simulation results for the second case are given in Figs. 4–6, respectively.

Finally, to demonstrate the necessity of the repulsive force $-\sum_{j\in\mathcal{C}_i(t)} k_1(\|q_{ij}(t)\|)q_{ij}(t)/\|q_{ij}(t)\|$ and the adhesive force $-\sum_{j\in\mathcal{N}_i(t)\setminus\mathcal{C}_i(t)} k_2(\|q_{ij}(t)\|)q_{ij}(t)/\|q_{ij}(t)\|$ in (7) for network connectivity preservation and collision avoidance, we run simulation with $k_1 = 0$ and $k_2 = 0$ and the controller c_i used in Theorem 4. The results are shown in Figs. 7 and 8. In particular, the group of agents does not achieve flocking any more in Fig. 7. The network connectivity is broken as some pairs of initially neighbored agents are separated with their distances beyond $R = 2$ in Fig. 8. Also, one pair of agents gets close to each other with the distance close to 0.1, which might be practically regarded as collision.

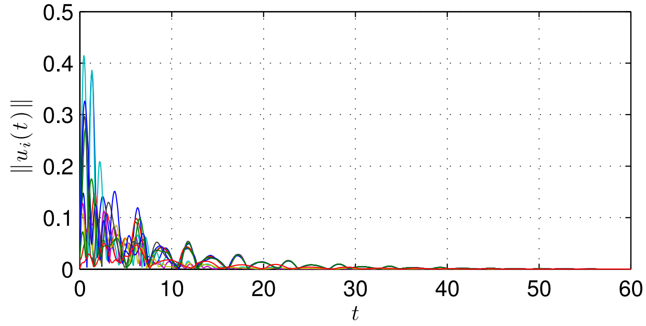

Fig. 6. Profile of control forces of ten agents (case 2).

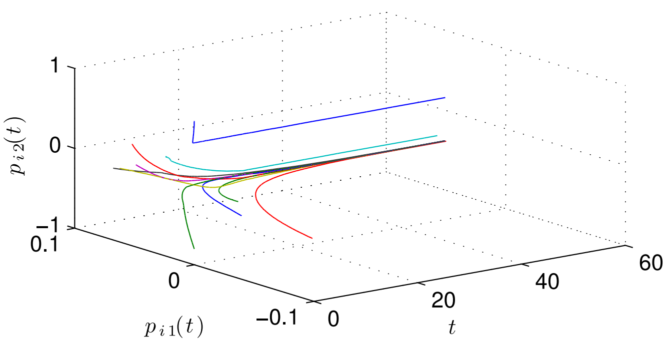

Fig. 7. Profile of velocity trajectories of ten agents (no flocking is achieved).

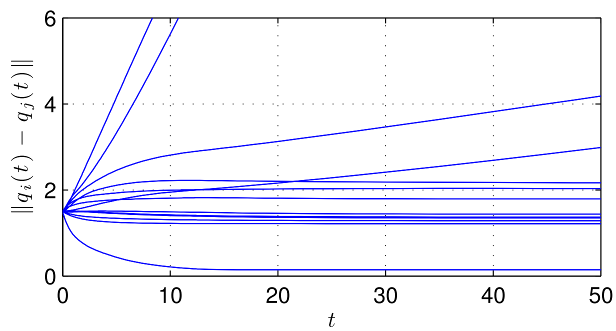

Fig. 8. Profile of distances between every two initially neighbored agents (network connectivity is broken and collision nearly occurs).

VI. Conclusion

This paper has given an estimation of the control force for network connectivity preservation and collision avoidance. It was quantitatively revealed that sufficient energy is required for connectivity break or collision when an adhesive/repulsive control algorithm is applied on a group of agents. The energy includes the initial potential energy determined by the distances of every two agents, the initial kinetic energy determined by agent velocities, and the input energy caused by the additional control force for more specific collective behaviors. Therefore, to avoid connectivity break or collision, sufficiently large force control is required to account for the energy and its boundary can be explicitly calculated. In particular, the approach has been applied in three typical collective control scenarios including swarming, flocking, and flocking without velocity measurement.

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