The Gross-Pitaevskii equation: Dynamics of solitons and vortices

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The Gross-Pitaevskii equation is a mean field equation that has been incredibly successful in describing the dynamics of Bose-Einstein condensates. The equation features a non-linear term and thus allows for stable soliton solutions in 1D and additional topological defects such as vortices in 2D. The goal of this project is to study these phenomena using the split-step Fourier method.

This report summarizes the results of the final project we conducted as part of the lecture course on Computational Quantum Dynamics, held by Dr. Martin Gärttner. All numerical work was done using the Python programming language.

1 Introduction

In the first part of this report we want to present the physical concepts needed for the understanding of the performed calculations. At the end, the Split-Step Fourier method, we used to analyze the dynamics of the different quantum systems, is introduced.

1.1 The Gross-Pitaevskii equation

$$i\hbar\frac{\partial\psi(\mathbf{r},t)}{\partial t} = \left[-\frac{\hbar^2}{2m}\nabla^2 + V(\mathbf{r},t) + g\|\psi\|^2\right]\psi(\mathbf{r},t) \tag{1}$$

1.2 Solitons

Solitons are non-dispersive wave solutions of the Gross-Pitaevskii equation. In the following we distinguish between bright solitons, describing attractive interactions and dark solitons for repulsive interactions. The latter ones will be the objects of interest for our studies. They are characterized by their so called *greyness* $\nu = \frac{v_s}{c_s}$, with the Bogoliubov speed of sound $c_s = \sqrt{\frac{ng}{m}}$ and the velocity v_s of the solitons movement inside the gas.
If we restrict ourselves to the case of repulsive interactions and vanishing external potential ($V = 0$) the analytic solution of a single solitonic excitation reads

$$\phi_\nu^{(1)}(z,t) = \sqrt{n}\left[i\nu + \gamma^{-1}\tanh\left(\frac{z-(z^0+\nu c_s t)}{\sqrt{2}\xi\gamma}\right)\right]\mathrm{e}^{i\mu t} \tag{2}$$

with the Lorentz factor $\gamma^{-1} = \sqrt{1-\nu^2}$, the healing length $\xi = \frac{1}{\sqrt{mng}}$, the homogenous background density n and the chemical potential μ.

1.3 Other topological defects: Vortices

1.4 The Split-Step Fourier method

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2 Evolution of dark solitons in a homogeneous 1D Bose gas

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3 Dynamics of solitons in a homogeneous 2D Bose gas

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4 Visualizing the dynamics of vortices in a 2D Bose gas

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5 Conclusion

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