HW2

Xitong Li

September 21, 2024

Matlab code:
https://github.com/lixit/CV_3D/tree/main/HW2
This report online:
https://www.overleaf.com/read/fsbbgpmbwsbb#9ffa3b

Question 1. Affine rectification: (rectification up to affine properties can be measured)

Solution. The line at infinity l_∞ is preserved iff under an affine transformation, but not under projective transformation.

Suppose in a projective distorted image, $l = (l_1 \, l_2 \, l_3)^T$ corresponds to the line at infinity $l_\infty = (0\,0\,1)^T$

We know the line transformation is: $l' = H^{-T} l$, and we want to find a point transformation H that transform l to l_∞

$$H^{-T}(l_1 \, l_2 \, l_3)^T = (0\,0\,1)^T \tag{1}$$

Find the vanishing line l in an image:

1. Find 2 pairs of supposed parallel lines in an image. These 2 pairs of supposed parallel lines intersect at 2 points.
2. These 2 points is on the imaged line at infinity.
3. Get the homogeneous coordinate of the 2 points
4. The line through these 2 points can be computed as: $l = x \times x'$

If the imaged line at infinity is $l = (l_1 l_2 l_3)^T$, a suitable point transformation H that map l back to l_∞ is

$$H = H_A \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ l_1 & l_2 & l_3 \end{bmatrix} \tag{2}$$

We can specify any H_A as long as it's an affine transformation. So that: $H^{-T}(l_1 l_2 l_3)^T = (0\,0\,1)^T$
Finally, we use H to transform every points on the image to get rectified image up to affine properties preserved.
□

Question 2. Metric rectification.

Solution. We know the circular points, I, J, and their conic dual C^*_∞ are fixed under similarity transformation:

$$I^{'} = H_s I$$
$$J^{'} = H_s J$$
$$C^{*'}_\infty = H_s C^*_\infty H_s^T = C^*_\infty$$

We can recover metric properties by transferring circular points to their canonical position
The conic dual to the circular points:

$$C^*_\infty = IJ^T + JI^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{3}$$

If a point transformation according to: $x^{'} = Hx$,
Then:

$$C^{*'}_\infty = \begin{bmatrix} kk^T & kk^T v \\ v^T kk^T v^T & kk^T v \end{bmatrix} \tag{4}$$

Given the image has been affine rectified, so v = 0.
And we know that if 2 lines l and m are orthogonal on the the world plane. After affine transform, they become l' and m', then

$$l^{'T} C^{*'}_\infty m^{'} = 0 \tag{5}$$

Then we can find 2 pairs of such lines to solve K and get the homography H. □

Result 3. Please see all result in the pictures folder.

The tile.jpg picture I took of the first floor in the house works petty good.

Figure 1: Original Image

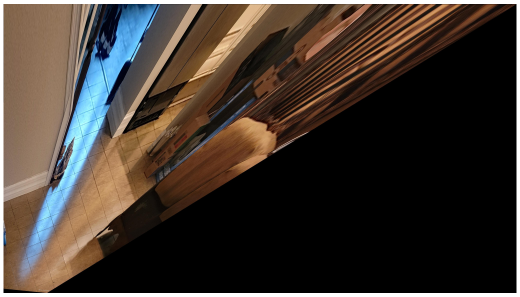

Figure 2: Affine rectification

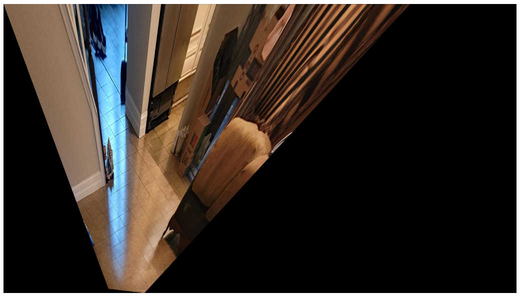

Figure 3: Metric rectification

Problem 4.

1. While other test images works well, the Building.jpg image is unable to be rectified.
2. The metric rectification can also be done by identifying an ellipse in the image. I have included the code for it. But the code has bugs and is not working right now.

Conclusion 5. .

By using prior knowledge and identifying parallel lines or orthogonal lines or circles on the world plane, we can do various rectifications.
That is very useful and fun. A further improvement might be doing this in an autonomous way.