MA5251 Project 2

Li Xuanguang, A0154735B

April 29, 2023

1 Numerical Method Discussion

We are given the burgers equation

$$\frac{\partial u}{\partial t} + \frac{\partial}{\partial x}\left(\frac{u^2}{2}\right) = 0.02\frac{\partial^2 u}{\partial x^2}$$
$$u(-1,t) = 1, u(1,t) = -1$$
$$u(x,0) = u_0(x) = -\sin\left(\frac{5}{2}\pi x\right)$$

First, we take the approximation

$$u_N(x,t) = \sum_{k=0}^{N} \hat{u}_k(t) T_k(x)$$

As there is a nonlinear term in the equation, choose the right collocation points first and use interpolation to approximate the result. We use Gauss-Lobatto notes:

$$x_j = \cos\left(\frac{j\pi}{n}\right), \quad j = 0, 1, \ldots, n$$

After approximating the nonlinear term $\frac{u^2}{2}$, we deal with the approximation after taking derivative with respect to x. By lecture 19, we have the following recursion for approximation:

$$u_N'(x) = \sum_{k=0}^{N} \hat{u}_k^{(1)} T_k(x)$$

where $u_N^{(1)} = 0$, $u_{N-1}^{(1)} = 2N\hat{u}_N$, $\hat{u}_{k-1}^{(1)} = \frac{\hat{u}_{k+1}^{(1)} + 2k\hat{u}_k}{c_{k-1}}$.
For the term $0.02\frac{\partial^2 u}{\partial x^2}$, the differentiation is done twice.

Next, we use semi-discretisation with the following:

$$\frac{\partial u_N}{\partial t}(x_j, t) = \frac{\partial f_N}{\partial x}(x_j, t), \quad j = 1, \ldots, N-1$$

where

$$f_N = I_N\left(-\frac{u_N^2}{2}\right) + 0.02\frac{\partial u_N}{\partial x}$$

and

$$u_N(-1,t) = 1$$
$$u_N(1,t) = -1$$
$$u_N(x,0) = I_N u_0(x)$$
$$u_0(x) = -\sin\left(\frac{5}{2}\pi x\right)$$

We then apply the third-order RK method to get the results.

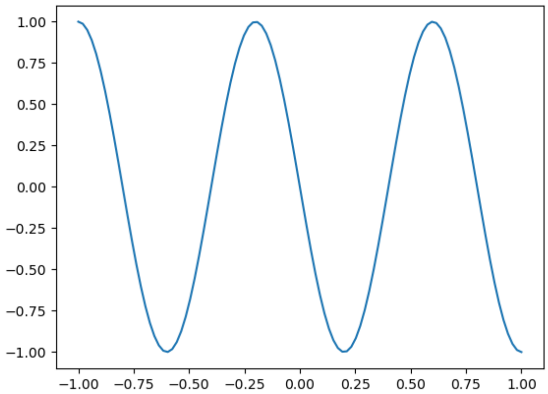
2 Numerical Results and Discussion

The code is written in Python, adapted off Prof Cai Zhenning's Matlab code in Lecture 19.

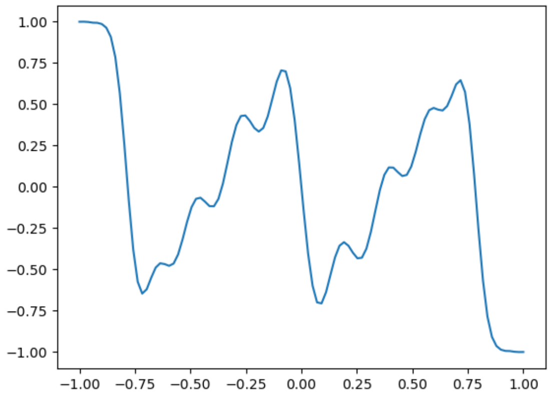
We run the code with the following parameters:

1. `N = 32`
2. `T = 5`
3. $\triangle$`t = 0.001`
4. $\triangle$`tp = 0.3` (timestep for print, used to print outputs)

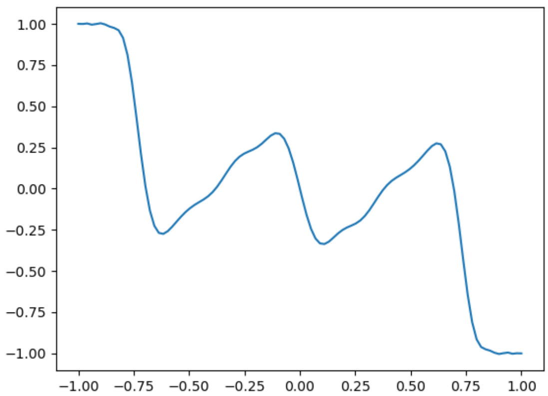
As seen from the figures (next page), the curve reaches steady state by $t = 3.3$.



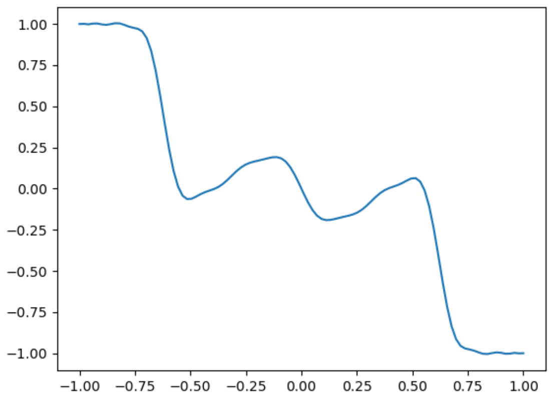
State at $t = 0.0$



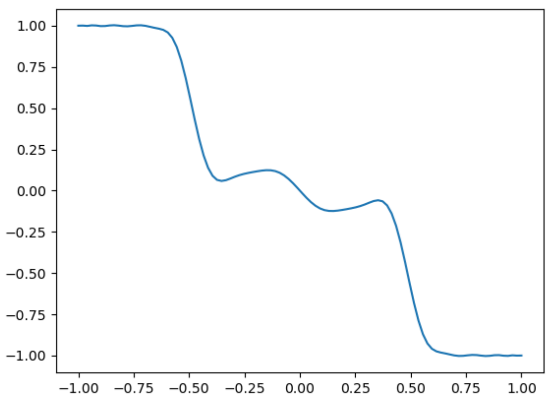
State at $t = 0.3$



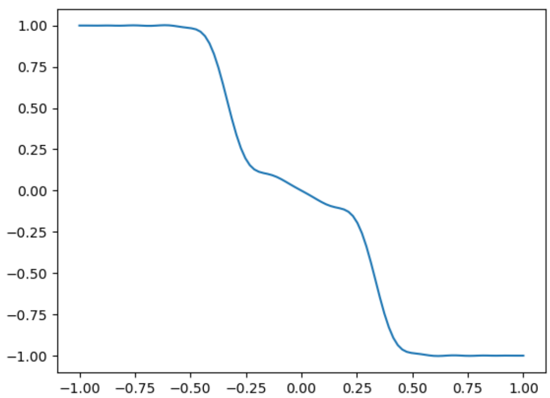
State at $t = 0.6$



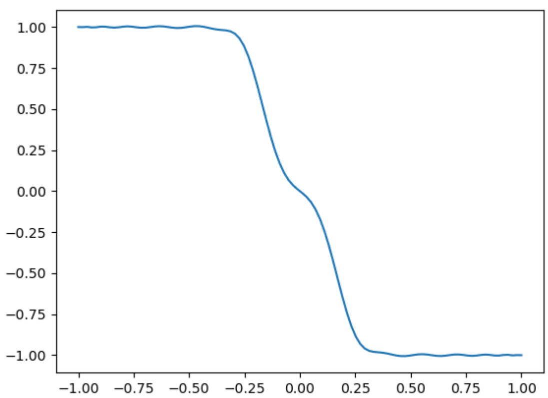
State at $t = 0.9$



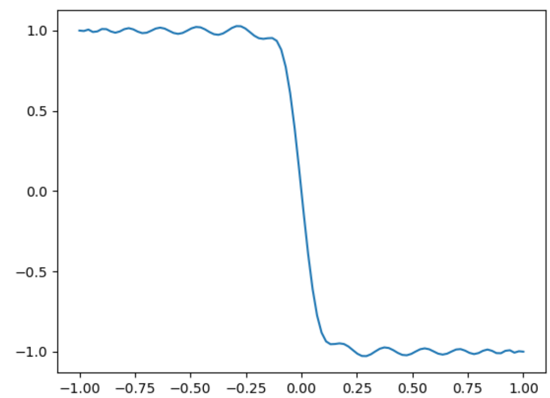
State at $t = 1.2$



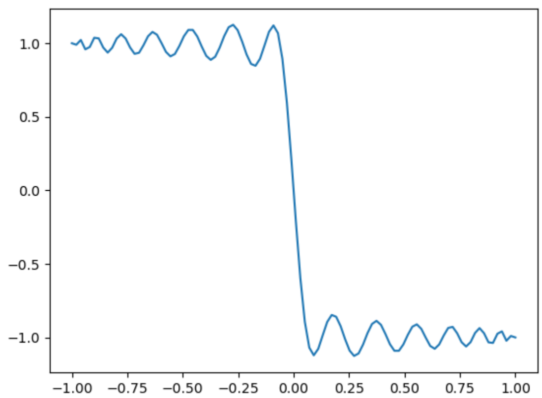
State at $t = 1.5$



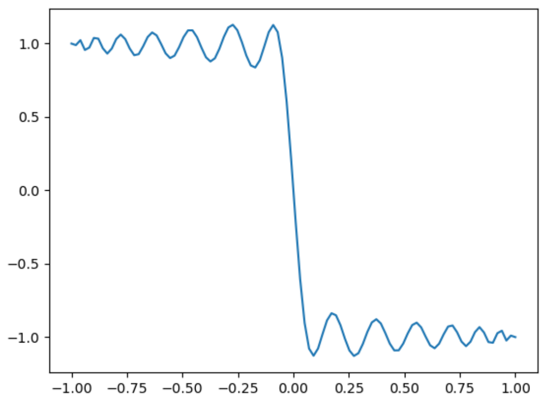
State at $t = 1.8$



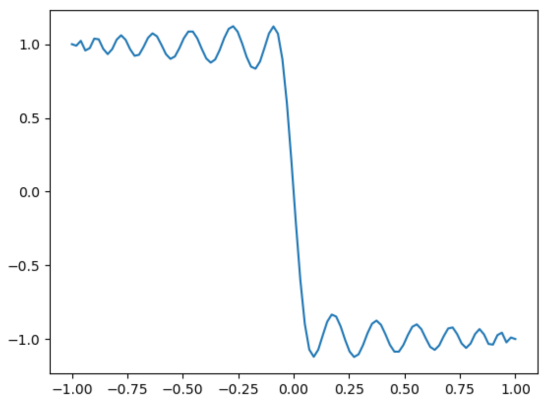
State at $t = 2.1$



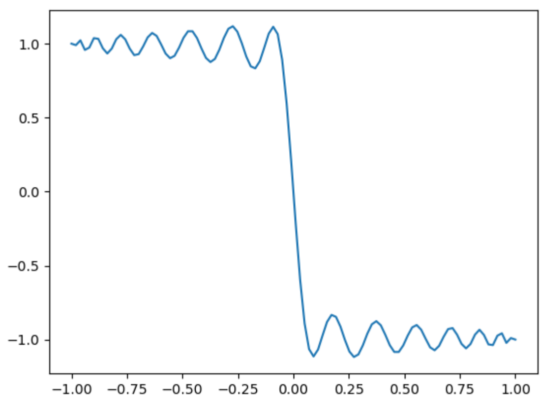
State at $t = 2.4$



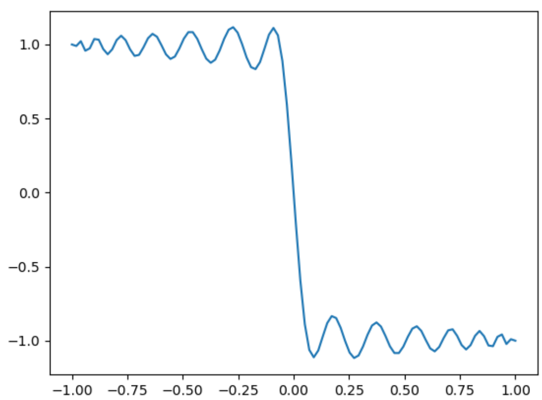
State at $t = 2.7$



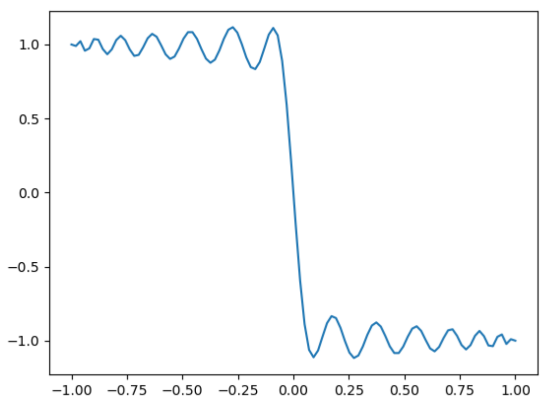
State at $t = 3.0$



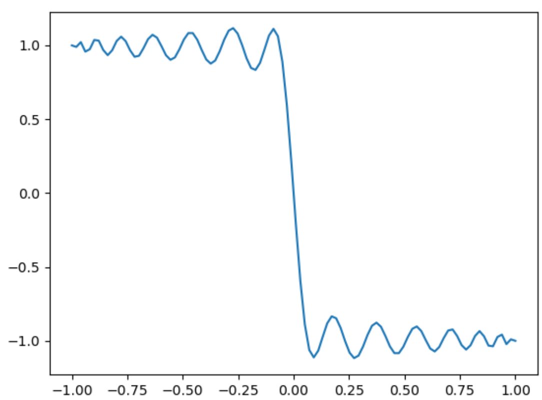
State at $t = 3.3$



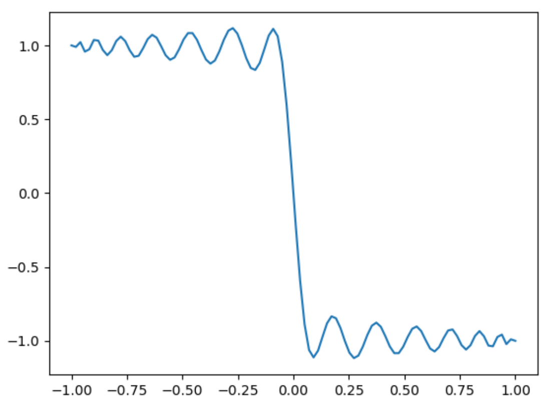
State at $t = 3.6$



State at $t = 3.9$



State at $t = 4.2$



State at $t = 4.5$