ARTHUR LI

ABSTRACT ALGEBRA

Introduction

THIS COLLECTION of notes serve as a guide to mastering abstract algebra with content from undergraduate to graduate level course. The notes combine knowledge from different sources, including course notes and textbooks used in the courses.

Prerequisites

These notes will assume no familiarity with any aspects of abstract algebra, and builds upon the foundation from Group Theory to more abstract topics such as Categories and Commutative Algebra. A good starting point will be the series on Visual Group Theory by Professor Matthew Macauley.

Familiarity with basic styles of proof is assumed (contradiction, contrapositive, etc.).

Organization and Sources

This section will be edited as the notes progress towards completion.

Contents

Preliminaries 7

Preliminaries

Introductory Ideas and Definitions

Definition 0.0.1. *Class* is a collection A of objects (elements) such that given any object x it is possible to determine if x is a member of A.

Axiom of extensionality asserts that two classes with the same elements are equal. (Formally, $[x \in A \iff x \in B] \Rightarrow A = B$).

A class is defined to be a *set* if and only if there exists a class B such that $A \in B$.

A class that is not a set is called a *proper set*.

Axiom of class formation asserts that for any statement $P(y)$ in the first predicate calculus involve a variable y, there exists a class A such that $x \in A$ if and only if x is a set and the statement $P(x)$ is true. The class is denoted $\{x|P(x)\}$.

A class A is a *subclass* of class B ($B \subset A$) provided $\forall x \in A, x \in A \iff x \in B$.

A subclass A of a class B that is itself a set is called a *subset* of B.

The *empty or null set* (denoted $\emptyset$) is the set with no elements.

Power axiom asserts that for every set A the class $P(A)$ of all subsets of A is itself a set. $P(A)$ is the *power set* of A, denoted 2^A.

A *family of sets* indexed by (nonempty) class I is a collection of sets A_i, one for each $i \in I$ (denoted $\{A_i|i \in I\}$).

The *union* is defined as $\bigcup_{i \in I} A_i = \{x|x \in A_i forsomei \in I\}$.

The *intersection* is defined as $\bigcap_{i \in I} A_i = \{x|x \in A_i foreveryi \in I\}$.

If $A \cap B = \emptyset$, then A and B are disjoint.

The *relative complement* of A in B is the following subclass of B: $B - A = \{x|x \in Bandx \notin A\}$.

If all classes under discussion are subsets of some fixed set U (the universe of discussion), then $U - A = A'$ is the *complement* of A.

Definition 0.0.2. Given classes A and B, a *function / map / mapping* f from A to B (written $f : A \to B$ assigns to each $a \in A$ exactly one element $b \in B$.

Then b is the value of function at a, or the *image* of a, written $f(a)$.

A is the *domain* of the function, written $domf$, and B is the *range* or *codomain*.

Two functions are equal if they have