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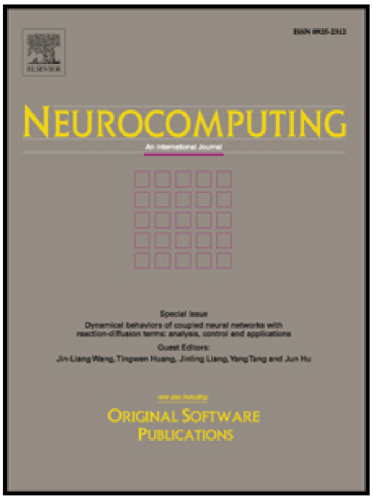

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Dissipativity and passivity analysis for memristor-based neural networks with leakage and two additive time-varying delays☆

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Abstract

In this paper, the problems of dissipativity and passivity analysis for memristor-based neural networks (MNNs) with both time-varying leakage delay and two additive time-varying delays are studied. By introducing an improved Lyapunov-Krasovskii functional (LKF) with triple integral terms, and combining the reciprocally convex combination technique, Wirtinger-based integral inequality with free-weighting matrices technique, some less conservative delay-dependent dissipativity and passivity criteria are obtained. The proposed criteria that depend on the upper bounds of the leakage and additive time-varying delays are given in terms of linear matrix inequalities (LMI), which can be solved by MATLAB LMI Control Toolbox. Meanwhile, the criteria for the system with a single time-varying delay are also provided. Finally, some examples are given to illustrate the effectiveness and superiority of the obtained results.

Keywords: memristor-based neural networks, leakage delays, additive time-varying delays, dissipativity, passivity

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1. Introduction

Memristor was postulated first by Chua in 1971 as the fourth basic element of electrical circuits [1]. About 37 years later, HP researchers showed that memristance arises naturally in nanoscale systems in which solid-state electronic and ionic transport are coupled under an external bias voltage [2]. Compared with ordinary resistor, the value of memristor is not unique and it hinges on the voltage applied to the corresponding history and current state [3]. Memristor owns a lot of characteristics, such as good scalability, a high density and low power [4]. Therefore, the nonvolatile temperament of memristor makes it a noticeable candidate for the next-generation memory technology [5, 6, 7]. By substituting resistors in artificial neural networks, a class of memristor-based neural networks (MNNs) can be constructed. This kinds of system can exhibit intelligent behaviors [8], such as cognition and simulate forgetting mechanism of neuromorphic system [9].

Recently most of the researchers have paid attention to know about the dynamical behaviors of MNNs [10, 11, 12, 13, 14]. In some practical applications, owing to the restricted speed of information processing, the reality of time delays frequently causes oscillation, divergence, or instability in neural networks [15]. Signals transmitted from one point to another may experience a few segments of networks, which can induce successive delays with different properties due to variable network transmission conditions [16]. A typical time delay called leakage delay may exist in the negative feedback terms of the system and these terms are variously known as forgetting or leakage terms. Since leakage delays have a destabilizing influence on the dynamical behaviors of MNNs, it is necessary and important to consider the leakage delay effects on the study of state estimation of MNNs, however to the best of our knowledge, there are only a few relative works on MNNs with leakage delay [17, 18, 19]. Thus, the stability problem of MNNs with leakage and additive time-varying delays has become a topic of great theoretical and practical importance [20].

The dissipativity and passivity problems for a spread of sensible systems are attracting researchers' attention for several years [21, 22, 23]. It is well-known that dissipativity analysis contains other dynamic analysis such as passivity analysis, H_∞ analysis, and so on [24, 25]. The problem of strict $(Q, S, R) - \gamma -$ *dissipativity* analysis for memristive neural networks with a constant leakage delay and a single time-varying delay is studied in [26]. The quadratical stability and extended dissipative conditions for the memristive neural networks with two additive time-varying delays have been proposed in [27]. The problem of passivity analysis of stochastic neural networks with leakage delay and Markovian jumping parameters is considered in [28]. X. Li and R. Rakkiyappan et al. [29] have investigated extensively the problem of dissipativity analysis for memristor-based complex-valued neural networks with time-varying delays. S. Ding and Z. Wang et al. [30] have investigated the dissipativity problem for a new array of discrete-time memristive neural networks with time-varying delays by defining a set of logical switched signals and utilizing robust analysis method.

This paper further investigates the dissipativity and passivity analysis for MNNs. The main contributions of this paper are summarized as follows:

1. The considered MNNs in this paper include not only additive time-varying delays, but also time-varying leakage delay, so it is firstly studied in the present paper, the model of MNNs is more general than ever.
2. By constructing a new Lypunov-Krasovskii functional (LKF) and using the second-order reciprocally convex combination technique, Wirtinger-based integral inequality with free-weighting matrices technique, some less conservative delay-dependent dissipativity and passivity criteria are obtained.
3. Our result is more general and it is valid for the usual neural networks model. Many relative works are included in this paper.

The remaining part of this paper is organized as follows. In Section 2, the model of MNNs is formulated by applying the theory of set-valued maps and functional differential inclusions, some predictable assumptions on the boundedness and Lipschitz continuity of activation functions are formulated base on the knowledge of memristor. In Section 3, some less conservative dissipativity and passivity criteria are established. Two numerical simulation examples are provided to validate all of the theoretical results in Section 4. Finally, the conclusion is drawn in Section 5.

Notation: The notations used throughout this paper are standard. $\mathbb{R}^n$ and $\mathbb{R}^{m\times n}$, respectively, denote the n-dimensional Euclidean space and the set of all $m \times n$ real matrices. $\|\cdot\|$ refers to the Euclidean vector norm. $diag\{\cdots\}$ is the block diagonal matrix. The superscript T is the transposition and the notation $P > 0 (P \geq 0)$ means that P is the real symmetric matrix and positive definite (semidefinite). $\boldsymbol{I}$ and $\boldsymbol{O}$ are the identity matrix and zero matrix with appropriate dimensions, respectively. The notation $\star$ is the symmetric block in one symmetric matrix, and $\mathbb{C}$ is the set of continuous functions. co$\{\Pi_1, \Pi_2\}$ represents closure of the convex hull generated by real matrices Π_1 and Π_2 or real numbers Π_1 and Π_2. $\mathcal{L}_2^n$ is the space of square integrable functions on $\mathbb{R}^+$ with values in $\mathbb{R}^n$. $\mathcal{L}_{2e}^n$ is the extended $\mathcal{L}_2^n$ space defined by $\mathcal{L}_{2e}^n = \{f : f$ is a measurable function on $\mathbb{R}^+$, $P_T f \in \mathcal{L}_2^n, \forall T \in \mathbb{R}^+ \}$, where $(P_T f)(t) = f(t)$ if $t \leq T$, and 0 if $t > T$. For any function $x = \{x(t)\}, y = \{y(t)\} \in \mathcal{L}_{2e}^n$, matrix Q, we define $\langle x, Qy\rangle_T = \int_0^T x^T(t)Qy(t)dt$.

2. Problem Description Preliminaries

Consider the following class of MNNs with leakage and two additive time-varying delays:

$$\begin{cases} \dfrac{dx_i(t)}{dt} = -\,d_i(t)x_i(t-\eta_i(t)) + \displaystyle\sum_{j=1}^{n} a_{ij}(t)f_j(x_j(t)) \\ \qquad\qquad + \displaystyle\sum_{j=1}^{n} b_{ij}(t)g_j(x_j(t-\tau_{j1}(t)-\tau_{j2}(t))) + u_i(t) \\ y_i(t) = f_i(x_i(t)), i = 1, 2, \cdots, n. \end{cases} \tag{1}$$

where n is the amount of neuronal cell in a neural networks; $x_i(t) \in \mathbb{R}$ is the ith neuronal cell state; $y_i(t) \in \mathbb{R}$ is the output of the ith neuronal cell;

$d_i(t) > 0$ represents the rate which the ith neuronal cell will reset its potential to the resting state in isolation when disconnected from the network and external inputs at time t; $\tau_{j1}(t)$ and $\tau_{j2}(t)$ are the two additive time varying delays that are assumed to satisfy the conditions $0 \leq \tau_{j1}(t) \leq \tau_1, 0 \leq \tau_{j2}(t) \leq \tau_2$, $|\dot{\tau}_{j1}(t)| \leq \mu_1$, $|\dot{\tau}_{j2}(t)| \leq \mu_2$; $\eta_i(t)$ denotes the leakage delay satisfying $0 \leq \eta_i(t) \leq \eta$, $|\dot{\eta}_i(t)| \leq \mu = \mu_1 + \mu_2$, $\eta, \tau_1, \tau_2, \mu_1, \mu_2, \mu$ are nonnegative constants; $f_j(\cdot)$ and $g_j(\cdot)$ are activation functions; $u_i(t)$ is the controller to be designed; $a_{ij}(t)$ and $b_{ij}(t)$ represent the memristive synaptic weights, denote the strengths of the jth neuronal cell on the ith neuronal cell at time t, which are defined as follows:

$$a_{ij}(t) = \begin{cases} \hat{a}_{ij}, & \dot{\kappa}^1_{ij}(t) < 0 \\ unchanged, & \dot{\kappa}^1_{ij}(t) = 0 \\ \check{a}_{ij}, & \dot{\kappa}^1_{ij}(t) > 0 \end{cases}$$

$$b_{ij}(t) = \begin{cases} \hat{b}_{ij}, & \dot{\kappa}^2_{ij}(t) < 0 \\ unchanged, & \dot{\kappa}^2_{ij}(t) = 0 \\ \check{b}_{ij}, & \dot{\kappa}^2_{ij}(t) > 0 \end{cases}$$

where $\kappa^1_{ij}(t) = f_{ij}(x_j(t)) - x_i(t)$, $\kappa^2_{ij}(t) = g_{ij}(x_j(t - \tau_{j1}(t) - \tau_{j2}(t))) - x_i(t)$, $\kappa^1_{ij}(t)$ and $\kappa^2_{ij}(t)$ represent the voltage of the corresponding memresistance. $\hat{a}_{ij}$, $\check{a}_{ij}$, $\hat{b}_{ij}$, $\check{b}_{ij}$ are constants for $i, j = 1, 2, \cdots, n$. The initial values associated with system (1) are $x_i(t) = \varphi_i(t) \in \mathbb{C}([-\tau^*, 0], \mathbb{R})$, $\tau^* = \max\{\eta, \tau_1 + \tau_2\}$ for $i = 1, 2, \cdots, n$.

Remark 1. In some existing literatures [18, 31, 32, 33, 34, 35], the memristor is just looked as a simple switch, its state switching depends on the potential $x_i(t)$ which is connected to one port of the memristor, if $x_i(t)$ is more or less than the switching jumps T_i, the memristive synaptic weights are switched. But the memristor states depend on the voltage of its two ports and the voltage variation trend [3, 36]. So we use the derivative of $\kappa^1_{ij}(t)$ and $\kappa^2_{ij}(t)$ to switch the $a_{ij}(t)$ and $b_{ij}(t)$, the model of MNNs is more general than ever.

Remark 2. The dissipativity and passivity problems of MNNs have received much of the focus in recent years. So far, a great many important results on dissipativity or passivity have been obtained for MNNs. Unfortunately, The references [29, 30] just considered a single time-varying delay. The references [37, 38] considered additive time-varying delays, but didn't include leakage delay. The references [39, 40] considered a leakage delay, but the leakage delay is a constant. Practically, the leakage delay and multiple signal transmission delays coexist in the system of MNNs.

For convenience, we introduce the following Definitions about set-valued map and differential inclusion.

Definition 1 [41]. Let $E \subset \mathbb{R}^n$, $x \mapsto F(x)$ is called a set-valued map form $E \hookrightarrow \mathbb{R}^n$, if to each point x of a set $E \subset \mathbb{R}^n$, there corresponds a nonempty set $F(x) \subset \mathbb{R}^n$.

Definition 2 [42]. A set-valued map F with nonempty values is said to be upper semi-continuous at $x_0 \in E \subset \mathbb{R}^n$, if for any open set N containing $F(x_0)$, there exists a neighborhood M of x_0 such that $F(M) \subset N$. $F(x)$ is said to have a closed (convex, compact) image if for each $x \in E$, $F(x)$ is closed (convex, compact).

By applying the above theories of set-valued maps and differential inclusions, the MNNs (1) can be written as the following differential inclusion:

$$\begin{cases} \dfrac{dx_i(t)}{dt} \in -d_i(t)x_i(t-\eta_i(t)) + \sum_{j=1}^{n} co[a_{ij}^-, a_{ij}^+]f_j(x_j(t)) \\ \qquad + \sum_{j=1}^{n} co[b_{ij}^-, b_{ij}^+]g_j(x_j(t-\tau_{j1}(t)-\tau_{j2}(t))) + u_i(t) \\ y_i(t) = f_i(x_i(t)), i = 1, 2, \cdots, n. \end{cases} \tag{2}$$

where $a_{ij}^- = \min\{\hat{a}_{ij}, \check{a}_{ij}\}$, $a_{ij}^+ = \max\{\hat{a}_{ij}, \check{a}_{ij}\}$, $b_{ij}^- = \min\{\hat{b}_{ij}, \check{b}_{ij}\}$, $b_{ij}^+ = \max\{\hat{b}_{ij}, \check{b}_{ij}\}$. There exist $\bar{a}_{ij}(t) \in co[a_{ij}^-, a_{ij}^+]$, $\bar{b}_{ij}(t) \in co[b_{ij}^-, b_{ij}^+]$ satisfy

$$\begin{cases} \dfrac{dx_i(t)}{dt} = -d_i(t)x_i(t-\eta_i(t)) + \sum_{j=1}^{n} \bar{a}_{ij}(t)f_j(x_j(t)) \\ \qquad + \sum_{j=1}^{n} \bar{b}_{ij}(t)g_j(x_j(t-\tau_{j1}(t)-\tau_{j2}(t))) + u_i(t) \\ y_i(t) = f_i(x_i(t)) \end{cases} \tag{3}$$

for a.e. $t \in [0, \infty), i = 1, 2, \cdots, n$.

Now the (3) can be rewritten as the following matrix form:

$$\begin{cases} \dot{x}(t) = -Dx(t-\eta(t)) + Af(x(t)) + Bg(x(t-\tau_1(t)-\tau_2(t))) + u(t) \\ y(t) = f(x(t)) \end{cases} \tag{4}$$

输出

where $x(t) = (x_1(t), x_2(t), \cdots, x_n(t))^T$, $D = diag\{d_1(t), d_2(t), \cdots, d_n(t)\}$, $A = [\bar{a}_{ij}(t)]_{n\times n}$, $B = [\bar{b}_{ij}(t)]_{n\times n}$, $f(x(t)) = (f_1(x_1(t)), f_2(x_2(t)), \cdots, f_n(x_n(t)))^T$, $x(t-\eta(t)) = (x_1(t-\eta_1(t)), x_2(t-\eta_2(t)), \cdots, x_n(t-\eta_n(t)))^T$, $g(x(t-\tau_1(t)-\tau_2(t))) = (g_1(x_1(t-\tau_{11}(t)-\tau_{12}(t))), g_2(x_2(t-\tau_{21}(t)-\tau_{22}(t))), \cdots, g_n(x_n(t-\tau_{n1}(t)-\tau_{n2}(t))))^T$, $u(t) = (u_1(t), u_2(t), \cdots, u_n(t))^T$.

The time-varying delay $\tau(t) = \tau_1(t) + \tau_2(t)$ satisfying the conditions:

$$0 \le \tau_1(t) \le \tau_1; 0 \le \tau_2(t) \le \tau_2;$$
$$|\dot{\tau}_1(t)| \le \mu_1; |\dot{\tau}_2(t)| \le \mu_2;$$

where $\tau_1, \tau_2, \mu_1, \mu_2$ are nonnegative constants, we denote $\tau = \tau_1 + \tau_2$, $\mu = \mu_1 + \mu_2$.

To derive the main results, we introduce the following assumptions, definitions and lemmas.

Assumption 1[43]. For all $\alpha, \beta \in \mathbb{R}$ and $\alpha \neq \beta$, $i \in \{1, 2, \cdots, n\}$, the activation functions f, g are bounded and there exist four constant matrices $L^- = diag\{l_1^-, l_2^-, \cdots, l_n^-\}$, $L^+ = diag\{l_1^+, l_2^+, \cdots, l_n^+\}$, $H^- = diag\{h_1^-, h_2^-, \cdots, h_n^-\}$, $H^+ = diag\{h_1^+, h_2^+, \cdots, h_n^+\}$ such that

$$l_i^- \le \frac{f_i(\alpha) - f_i(\beta)}{\alpha - \beta} \le l_i^+$$

这个条件比Lipschitz条件弱吗？

$$h_i^- \le \frac{g_i(\alpha) - g_i(\beta)}{\alpha - \beta} \le h_i^+$$

where l_i^-, l_i^+, h_i^- and h_i^+ are constants, $f_i(0) = 0$, $g_i(0) = 0$.

Remark 3. Assumption 1 limits the lower and upper bounds of the activation functions slope, it is weaker than the well-known Lipschitz-type conditions (i.e. $|f_i(\alpha) - f_i(\beta)| \le l_i|\alpha - \beta|$, $|g_i(\alpha) - g_i(\beta)| \le h_i|\alpha - \beta|$, $l_i = \max\{|l_i^-|, |l_i^+|\}$, $h_i = \max\{|h_i^-|, |h_i^+|\}$), which is used in [25, 29].

Definition 3 [39]. Given real symmetric matrices Q, R and S with appropriate dimensions, the system (4) is called strictly $(Q, S, R) - \gamma - dissipative$, if there exists a scalar $\gamma > 0$ such that the inequality

$$\langle y, Qy\rangle_T + 2\langle y, Su\rangle_T + \langle u, Ru\rangle_T \geq \gamma\langle u, u\rangle_T, \forall u \in \mathcal{L}_{2e}^n \tag{5}$$

hold for all $t \geq 0$ and under the zero initial condition.

Definition 4 [37]. The system (4) is said to be passive, if for all solutions of (4), there exists a scalar $\gamma > 0$ such that the inequality

$$2\int_0^T y^T(s)u(s)ds \geq -\gamma\int_0^T u^T(s)u(s)ds \tag{6}$$

is satisfied under the zero initial condition.

Lemma 1 [44]. Consider a given matrix $R > 0$. Then, for all continuous function ω in $[a, b] \to \mathbb{R}^n$ the following inequality holds:

$$\begin{aligned}\int_a^b \omega^T(u)R\omega(u)du &\geq \frac{1}{b-a}(\int_a^b \omega(u)du)^T R\int_a^b \omega(u)du + \frac{3}{b-a}\Omega^T R\Omega\\ &\geq \frac{1}{b-a}(\int_a^b \omega(u)du)^T R\int_a^b \omega(u)du\end{aligned} \tag{7}$$

where $\Omega = \int_a^b \omega(s)ds - \frac{2}{b-a}\int_a^b\int_a^s \omega(r)drds$.

Lemma 2 [45]. For any constant matrix $W \in \mathbb{R}^{n\times n}$, two scalars $b \geq a \geq 0$ and vector function $x(s) : [a, b] \to \mathbb{R}^n$, such that the following integration is well defined, then:

$$\begin{aligned}&-\frac{(b^2-a^2)}{2}\int_{-b}^{-a}\int_{t+\theta}^{t} x^T(s)Wx(s)dsd\theta\\ &\leq -(\int_{-b}^{-a}\int_{t+\theta}^{t} x(s)dsd\theta)^T W\int_{-b}^{-a}\int_{t+\theta}^{t} x(s)dsd\theta\end{aligned} \tag{8}$$

Lemma 3 [46]. Let $f_1, f_2, \cdots, f_N : \mathbb{R}^m \to \mathbb{R}$ have positive values in an open subset $\mathbf{D}$ of $\mathbb{R}^m$. Then, the reciprocally convex combination of f_i over $\mathbf{D}$ satisfies

$$\min_{\{a_i|a_i>0,\sum_i a_i=1\}} \frac{1}{a_i}f_i(t) = \sum_i f_i(t) + \max_{g_{i,j}(t)}\sum_{i\neq j} g_{i,j}(t) \tag{9}$$

Subject to

$$\{g_{i,j} : \mathbb{R}^m \to \mathbb{R}, g_{j,i} \triangleq g_{i,j}(t), \begin{bmatrix} f_i(t) & g_{i,j}(t)\\ g_{i,j}(t) & f_j(t)\end{bmatrix} \geq 0\}$$

Remark 4. Lemma 1 shows the Wirtinger-based integral inequality and Jensen inequality. The Wirtinger-based integral inequality delivers a more accurate lower bound of $\int_a^b w^T(u)Rw(u)du$ so it reduces the conservatism of the Jensen inequality. Lemma 2 gets from Jensen inequality and Schur Complements, which can be used to deal with multiple integral terms. Lemma 3 (The reciprocally convex combination technique) achieves performance behavior identical to approaches based on the integral inequality but also decreases the number of decision variables dramatically up to those based on the Jensen inequality.

3. Main results

In this section, by using the Lyapunov functional method and LMI technique, we will derive the delay-dependent dissipativity criterion for the MNNs (4) in the following theorem.

Theorem 1: Given real symmetric matrices Q, R and S with appropriate dimensions, under Assumption 1, the MNNs (4) is dissipative in the sense of Definition 3, if there exist symmetric matrices $P > 0$, $Q_i > 0$, $R_i > 0$, $T_i > 0$, $(i = 1, 2, 3, 4)$, $U_j > 0$ $(j = 1, 2, 3)$, $Z > 0$, diagonal matrices $M = diag\{m_1, m_2, \cdots, m_n\} > 0$, $W = diag\{w_1, w_2, \cdots, w_n\} > 0$, $\Lambda_1 = diag\{\lambda_{11}, \lambda_{12}, \cdots, \lambda_{1n}\} > 0$, $\Lambda_2 = diag\{\lambda_{21}, \lambda_{22}, \cdots, \lambda_{2n}\} > 0$, and matrices G_i $(i = 1, 2, 3, 4)$, V_j $(j = 1, 2, 3)$ and scalar $\gamma > 0$, such that the following LMIs hold:

$$\Phi_l = \Xi - \Upsilon_l^T \begin{bmatrix} U_1 & V_1 & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \star & U_1 & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \star & \star & U_2 & V_2 & \mathbf{0} & \mathbf{0} \\ \star & \star & \star & U_2 & \mathbf{0} & \mathbf{0} \\ \star & \star & \star & \star & U_3 & V_3 \\ \star & \star & \star & \star & \star & U_3 \end{bmatrix} \Upsilon_l \le 0, \tag{10}$$

$l = (1, 2, 3, 4)$.

$$\begin{bmatrix} T_i & G_i \\ \star & T_i \end{bmatrix} \geq 0, (i = 1, 2, 3, 4). \tag{11}$$

$$\begin{bmatrix} U_j & V_j \\ \star & U_j \end{bmatrix} \geq 0, (j = 1, 2, 3). \tag{12}$$

where

$$\Xi = \begin{bmatrix} \mathcal{F}_{11} & \mathcal{F}_{12} & \mathcal{F}_{13} & \mathcal{F}_{14} \\ \star & \mathcal{F}_{22} & \mathcal{F}_{23} & \mathcal{F}_{24} \\ \star & \star & \mathcal{F}_{33} & \mathcal{F}_{34} \\ \star & \star & \star & \mathcal{F}_{44} \end{bmatrix},$$

$$\mathcal{F}_{11} = \begin{bmatrix} \Psi_{1,1} & \Psi_{1,2} & \Psi_{1,3} & \Psi_{1,4} & -2G_1^T & \Psi_{1,6} \\ \star & \Psi_{2,2} & \Psi_{2,3} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \star & \star & \Psi_{3,3} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \star & \star & \star & \Psi_{4,4} & \Psi_{4,5} & \boldsymbol{0} \\ \star & \star & \star & \star & \Psi_{5,5} & \boldsymbol{0} \\ \star & \star & \star & \star & \star & \Psi_{6,6} \end{bmatrix},$$

$$\mathcal{F}_{12} = \begin{bmatrix} -2G_2^T & \Psi_{1,8} & -2G_4^T & \Psi_{1,10} & PB \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \Psi_{2,10} & \Psi_{2,11} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \Psi_{6,7} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \end{bmatrix},$$

$$\mathcal{F}_{13} = \begin{bmatrix} 6T_3 & 6G_3^T & 6T_1 & 6G_1^T & 6T_2 \\ \Psi_{2,12} & \Psi_{2,13} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ 6G_3 & 6T_3 & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \Psi_{4,14} & \Psi_{4,15} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & 6G_1 & 6T_1 & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \Psi_{6,16} \end{bmatrix},$$

$$\mathcal{F}_{14} = \begin{bmatrix} 6G_2^T & 6T_4 & 6G_4^T & MPM & P \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & DPM & \Psi_{2,21} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & -MPM & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \Psi_{6,17} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \end{bmatrix},$$

$$\mathcal{F}_{22} = \begin{bmatrix} \Psi_{7,7} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \star & \Psi_{8,8} & \Psi_{8,9} & \boldsymbol{0} & \Psi_{8,11} \\ \star & \star & \Psi_{9,9} & \boldsymbol{0} & \boldsymbol{0} \\ \star & \star & \star & \Psi_{10,10} & \Psi_{10,11} \\ \star & \star & \star & \star & \Psi_{11,11} \end{bmatrix},$$

$$\mathcal{F}_{23} = \begin{bmatrix} \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & 6G_2 \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \end{bmatrix},$$

$$\digamma_{24}=\begin{bmatrix}6T_2 & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0}\\ \boldsymbol{0} & \Psi_{8,18} & \Psi_{8,19} & \boldsymbol{0} & \boldsymbol{0}\\ \boldsymbol{0} & 6G_4 & 6T_4 & \boldsymbol{0} & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \Psi_{10,20} & \Psi_{10,21}\\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \Psi_{11,20} & \Psi_{11,21}\end{bmatrix},$$

$$\digamma_{33}=\begin{bmatrix}-12T_3 & -12G_3^T & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0}\\ \star & -12T_3 & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0}\\ \star & \star & -12T_1 & -12G_1^T & \boldsymbol{0}\\ \star & \star & \star & -12T_1 & \boldsymbol{0}\\ \star & \star & \star & \star & -12T_2\end{bmatrix},$$

$$\digamma_{34}=\begin{bmatrix}\boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0}\\ -12G_2^T & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0}\end{bmatrix},$$

$$\digamma_{44}=\begin{bmatrix}-12T_2 & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0}\\ \star & -12T_4 & -12G_4^T & \boldsymbol{0} & \boldsymbol{0}\\ \star & \star & -12T_4 & \boldsymbol{0} & \boldsymbol{0}\\ \star & \star & \star & -Z & -MP\\ \star & \star & \star & \star & \Psi_{21,21}\end{bmatrix},$$

$\Psi_{1,1}=-PM-MP+Q_1+Q_2+Q_3+Q_4+R_1+R_2+R_3+R_4+\eta^2Z-4T_1-4T_2-4T_3-4T_4-2L^-\Lambda_1L^+,\ \Psi_{1,2}=-PD-2T_3-4G_3^T,\ \Psi_{1,3}=PM-2G_3^T,\ \Psi_{1,4}=-2T_1-4G_1^T,\ \Psi_{1,6}=-2T_2-4G_2^T,\ \Psi_{1,8}=-2T_4-4G_4^T,\ \Psi_{1,10}=PA+(L^-+L^+)\Lambda_1,\ \Psi_{2,2}=-(1-\mu)R_4+\tau_1^2DT_1D+\tau_2^2DT_2D+\eta^2DT_3D+\tau^2DT_4D+\frac{\tau_1^4}{4}DU_1D+\frac{\tau_2^4}{4}DU_2D+\frac{\tau^4}{4}DU_3D-8T_3-2(G_3+G_3^T),\ \Psi_{2,3}=-2T_3-4G_3^T,\ \Psi_{2,10}=-DW-\tau_1^2DT_1A-\tau_2^2DT_2A-\eta^2DT_3A-\tau^2DT_4A-\frac{\tau_1^4}{4}DU_1A-\frac{\tau_2^4}{4}DU_2A-\frac{\tau^4}{4}DU_3A,\ \Psi_{2,11}=-\tau_1^2DT_1B-\tau_2^2DT_2B-\eta^2DT_3B-\tau^2DT_4B-$

$\frac{\tau_1^4}{4}DU_1B - \frac{\tau_2^4}{4}DU_2B - \frac{\tau^4}{4}DU_3B$, $\Psi_{2,12} = 6T_3 + 6G_3$, $\Psi_{2,13} = 6T_3 + 6G_3^T$, $\Psi_{2,21} = -\tau_1^2DT_1 - \tau_2^2DT_2 - \eta^2DT_3 - \tau^2DT_4 - \frac{\tau_1^4}{4}DU_1 - \frac{\tau_2^4}{4}DU_2 - \frac{\tau^4}{4}DU_3$, $\Psi_{3,3} = -Q_4 - 4T_3$, $\Psi_{4,4} = -(1-\mu_1)R_1 - 8T_1 - 2(G_1 + G_1^T)$, $\Psi_{4,5} = -2T_1 - 4G_1^T$, $\Psi_{4,14} = 6T_1 + 6G_1$, $\Psi_{4,15} = 6T_1 + 6G_1^T$, $\Psi_{5,5} = -Q_1 - 4T_1$, $\Psi_{6,6} = -(1-\mu_2)R_2 - 8T_2 - 2(G_2 + G_2^T)$, $\Psi_{6,7} = -2T_2 - 4G_2^T$, $\Psi_{6,16} = 6T_2 + 6G_2$, $\Psi_{6,17} = 6T_2 + 6G_2^T$, $\Psi_{7,7} = -Q_2 - 4T_2$, $\Psi_{8,8} = -(1-\mu)R_3 - 2H^-\Lambda_2H^+ - 8T_4 - 2(G_4 + G_4^T)$, $\Psi_{8,9} = -2T_4 - 4G_4^T$, $\Psi_{8,11} = (H^- + H^+)\Lambda_2$, $\Psi_{8,18} = 6T_4 + 6G_4$, $\Psi_{8,19} = 6T_4 + 6G_4^T$, $\Psi_{9,9} = -Q_3 - 4T_4$, $\Psi_{10,10} = AW + WA^T + \tau_1^2A^TT_1A + \tau_2^2A^TT_2A + \eta^2A^TT_3A + \tau^2A^TT_4A + \frac{\tau_1^4}{4}A^TU_1A + \frac{\tau_2^4}{4}A^TU_2A + \frac{\tau^4}{4}A^TU_3A - 2\Lambda_1 - Q$, $\Psi_{10,11} = WB^T + \tau_1^2A^TT_1B + \tau_2^2A^TT_2B + \eta^2A^TT_3B + \tau^2A^TT_4B + \frac{\tau_1^4}{4}A^TU_1B + \frac{\tau_2^4}{4}A^TU_2B + \frac{\tau^4}{4}A^TU_3B$, $\Psi_{10,20} = -A^TPM$, $\Psi_{10,21} = W + \tau_1^2A^TT_1 + \tau_2^2A^TT_3 + \eta^2A^TT_3 + \tau^2A^TT_4 + \frac{\tau_1^4}{4}A^TU_1 + \frac{\tau_2^4}{4}A^TU_2 + \frac{\tau^4}{4}A^TU_3 - S$, $\Psi_{11,11} = \tau_1^2B^TT_1B + \tau_2^2B^TT_2B + \eta^2B^TT_3B + \tau^2B^TT_4B + \frac{\tau_1^4}{4}B^TU_1B + \frac{\tau_2^4}{4}B^TU_2B + \frac{\tau^4}{4}B^TU_3B - 2\Lambda_2$, $\Psi_{11,20} = -B^TPM$, $\Psi_{11,21} = \tau_1^2B^TT_1 + \tau_2^2B^TT_2 + \eta^2B^TT_3 + \tau^2B^TT_4 + \frac{\tau_1^4}{4}B^TU_1 + \frac{\tau_2^4}{4}B^TU_2 + \frac{\tau^4}{4}B^TU_3$, $\Psi_{21,21} = \tau_1^2T_1 + \tau_2^2T_2 + \eta^2T_3 + \tau^2T_4 + \frac{\tau_1^4}{4}U_1 + \frac{\tau_2^4}{4}U_2 + \frac{\tau^4}{4}U_3 - (R - \gamma I)$,

$\Upsilon_l^T = [\Gamma_{1l}^T, \Gamma_{2l}^T, \Gamma_{3l}^T, \Gamma_{4l}^T, \Gamma_{5l}^T, \Gamma_{6l}^T]$, $(l = 1, 2, 3, 4)$,

$\Gamma_{11}^T = \Gamma_{12}^T = \tau_1(e_1 - e_{15})$, $\Gamma_{13}^T = \Gamma_{14}^T = \boldsymbol{0}$,

$\Gamma_{21}^T = \Gamma_{22}^T = \boldsymbol{0}$, $\Gamma_{23}^T = \Gamma_{24}^T = \tau_1(e_1 - e_{14})$,

$\Gamma_{31}^T = \Gamma_{33}^T = \tau_2(e_1 - e_{17})$, $\Gamma_{32}^T = \Gamma_{34}^T = \boldsymbol{0}$,

$\Gamma_{41}^T = \Gamma_{43}^T = \boldsymbol{0}$, $\Gamma_{42}^T = \Gamma_{44}^T = \tau_2(e_1 - e_{16})$,

$\Gamma_{51}^T = \tau(e_1 - e_{19})$, $\Gamma_{52}^T = \tau_1(e_1 - e_{19})$, $\Gamma_{53}^T = \tau_2(e_1 - e_{19})$, $\Gamma_{54}^T = \boldsymbol{0}$,

$\Gamma_{61}^T = \boldsymbol{0}$, $\Gamma_{62}^T = \tau_2(e_1 - e_{18})$, $\Gamma_{63}^T = \tau_1(e_1 - e_{18})$, $\Gamma_{64}^T = \tau(e_1 - e_{19})$,

$e_i = [\boldsymbol{0}_{n\times(i-1)n}, \boldsymbol{I}_{n\times n}, \boldsymbol{0}_{n\times(21-i)n}]$, $(i = 1, 2, \cdots, n)$.

Proof. We construct the Lyapunov-Krasovskii functional as follows:

$$V(t) = \sum_{l=1}^{7} V_l(t) \tag{13}$$

where

$$V_1(t) = [x(t) - M\int_{t-\eta}^{t} x(s)ds]^T P[x(t) - M\int_{t-\eta}^{t} x(s)ds],$$

$$V_2(t) = 2\sum_{i=1}^{n} w_i \int_{0}^{x_i(t)} f_i(s)ds,$$

$$
\begin{aligned}
V_3(t) = & \int_{t-\tau_1}^{t} x^T(s)Q_1x(s)ds + \int_{t-\tau_2}^{t} x^T(s)Q_2x(s)ds + \\
& \int_{t-\tau}^{t} x^T(s)Q_3x(s)ds + \int_{t-\eta}^{t} x^T(s)Q_4x(s)ds, \\
V_4(t) = & \int_{t-\tau_1(t)}^{t} x^T(s)R_1x(s)ds + \int_{t-\tau_2(t)}^{t} x^T(s)R_2x(s)ds + \\
& \int_{t-\tau(t)}^{t} x^T(s)R_3x(s)ds + \int_{t-\eta(t)}^{t} x^T(s)R_4x(s)ds, \\
V_5(t) = & \tau_1 \int_{-\tau_1}^{0} \int_{t+\theta}^{t} \dot{x}^T(s)T_1\dot{x}(s)dsd\theta + \tau_2 \int_{-\tau_2}^{0} \int_{t+\theta}^{t} \dot{x}^T(s)T_2\dot{x}(s)dsd\theta + \\
& \eta \int_{-\eta}^{0} \int_{t+\theta}^{t} \dot{x}^T(s)T_3\dot{x}(s)dsd\theta + \tau \int_{-\tau}^{0} \int_{t+\theta}^{t} \dot{x}^T(s)T_4\dot{x}(s)dsd\theta, \\
V_6(t) = & \frac{\tau_1^2}{2} \int_{-\tau_1}^{0} \int_{\theta}^{0} \int_{t+\lambda}^{t} \dot{x}^T(s)U_1\dot{x}(s)dsd\lambda d\theta + \\
& \frac{\tau_2^2}{2} \int_{-\tau_2}^{0} \int_{\theta}^{0} \int_{t+\lambda}^{t} \dot{x}^T(s)U_2\dot{x}(s)dsd\lambda d\theta + \\
& \frac{\tau^2}{2} \int_{-\tau}^{0} \int_{\theta}^{0} \int_{t+\lambda}^{t} \dot{x}^T(s)U_3\dot{x}(s)dsd\lambda d\theta, \\
V_7(t) = & \eta \int_{-\eta}^{0} \int_{t+\theta}^{t} x^T(s)Zx(s)dsd\theta.
\end{aligned}
$$

Let

$$
\begin{aligned}
\zeta^T(t) = & [x^T(t), x^T(t-\eta(t)), x^T(t-\eta), x^T(t-\tau_1(t)), x^T(t-\tau_1), x^T(t-\tau_2(t)), \\
& x^T(t-\tau_2), x^T(t-\tau(t)), x^T(t-\tau), f^T(x(t)), g^T(x(t-\tau(t))), \\
& \frac{1}{\eta(t)} \int_{t-\eta(t)}^{t} x^T(s)ds, \frac{1}{\eta-\eta(t)} \int_{t-\eta}^{t-\eta(t)} x^T(s)ds, \frac{1}{\tau_1(t)} \int_{t-\tau_1(t)}^{t} x^T(s)ds, \\
& \frac{1}{\tau_1-\tau_1(t)} \int_{t-\tau_1}^{t-\tau_1(t)} x^T(s)ds, \frac{1}{\tau_2(t)} \int_{t-\tau_2(t)}^{t} x^T(s)ds, \frac{1}{\tau_2-\tau_2(t)} \int_{t-\tau_2}^{t-\tau_2(t)} x^T(s)ds, \\
& \frac{1}{\tau(t)} \int_{t-\tau(t)}^{t} x^T(s)ds, \frac{1}{\tau-\tau(t)} \int_{t-\tau}^{t-\tau(t)} x^T(s)ds, \int_{t-\eta}^{t} x^T(s)ds, u^T(t)]
\end{aligned}
$$

Calculating the upper right Dini derivative of $V(t)$ along the trajectory of system (4), we have

$$
\begin{aligned}
D^+V_1(t) = & 2[x^T(t) - \int_{t-\eta}^{t} x^T(s)ds \times M]P[-Dx(t-\eta(t)) + \\
& Af(x(t)) + Bg(x(t-\tau(t))) + u(t) - Mx(t) + Mx(t-\eta)],
\end{aligned} \tag{14}
$$

$$\begin{aligned} D^+V_2(t) =& 2[-Dx(t-\eta(t)) + Af(x(t)) + \\ & Bg(x(t-\tau(t))) + u(t)]^T W f(x(t)), \end{aligned} \tag{15}$$

$$\begin{aligned} D^+V_3(t) =& x^T(t)(Q_1+Q_2+Q_3+Q_4)x(t) - x^T(t-\tau_1)Q_1x(t-\tau_1) \\ & - x^T(t-\tau_2)Q_2x(t-\tau_2) - x^T(t-\tau)Q_3x(t-\tau) \\ & - x^T(t-\eta)Q_4x(t-\eta) \end{aligned} \tag{16}$$

$$\begin{aligned} D^+V_4(t) \le& x^T(t)(R_1+R_2+R_3+R_4)x(t) - \\ & x^T(t-\tau_1(t))R_1x(t-\tau_1(t))(1-\mu_1) - \\ & x^T(t-\tau_2(t))R_2x(t-\tau_2(t))(1-\mu_2) - \\ & x^T(t-\tau(t))R_3x(t-\tau(t))(1-\mu) - \\ & x^T(t-\eta(t))R_4x(t-\eta(t))(1-\mu) \end{aligned} \tag{17}$$

$$\begin{aligned} D^+V_5(t) =& \tau_1^2\dot{x}^T(t)T_1\dot{x}(t) - \tau_1\int_{t-\tau_1}^{t}\dot{x}^T(s)T_1\dot{x}(s)ds + \\ & \tau_2^2\dot{x}^T(t)T_2\dot{x}(t) - \tau_2\int_{t-\tau_2}^{t}\dot{x}^T(s)T_2\dot{x}(s)ds + \\ & \eta^2\dot{x}^T(t)T_3\dot{x}(t) - \eta\int_{t-\eta}^{t}\dot{x}^T(s)T_3\dot{x}(s)ds + \\ & \tau^2\dot{x}^T(t)T_4\dot{x}(t) - \tau\int_{t-\tau}^{t}\dot{x}^T(s)T_4\dot{x}(s)ds \end{aligned} \tag{18}$$

$$\begin{aligned} D^+V_6(t) =& \frac{\tau_1^4}{4}\dot{x}^T(t)U_1\dot{x}(t) - \frac{\tau_1^2}{2}\int_{-\tau_1}^{0}\int_{t+\theta}^{t}\dot{x}^T(s)U_1\dot{x}(s)dsd\theta + \\ & \frac{\tau_2^4}{4}\dot{x}^T(t)U_2\dot{x}(t) - \frac{\tau_2^2}{2}\int_{-\tau_2}^{0}\int_{t+\theta}^{t}\dot{x}^T(s)U_2\dot{x}(s)dsd\theta + \\ & \frac{\tau^4}{4}\dot{x}^T(t)U_3\dot{x}(t) - \frac{\tau^2}{2}\int_{-\tau}^{0}\int_{t+\theta}^{t}\dot{x}^T(s)U_3\dot{x}(s)dsd\theta \end{aligned} \tag{19}$$

$$D^+V_7(t) \le \eta^2 x^T(t)Zx(t) - \int_{t-\eta}^{t}x^T(s)dsZ\int_{t-\eta}^{t}x(s)ds \tag{20}$$

For any matrix T_1 with $\begin{bmatrix} T_1 & G_1 \\ \star & T_1 \end{bmatrix} \geq 0$, by using Lemma 1 and Lemma 3, we can get:

$$
\begin{aligned}
&-\tau_1 \int_{t-\tau_1}^{t} \dot{x}^T(s) T_1 \dot{x}(s) ds \\
&= -\tau_1 [\int_{t-\tau_1}^{t-\tau_1(t)} \dot{x}^T(s) T_1 \dot{x}(s) ds + \int_{t-\tau_1(t)}^{t} \dot{x}^T(s) T_1 \dot{x}(s) ds] \\
&\leq -\frac{\tau_1}{\tau_1 - \tau_1(t)} [\alpha_1^T(t) T_1 \alpha_1(t) + 3\alpha_2^T(t) T_1 \alpha_2(t)] \\
&\quad - \frac{\tau_1}{\tau_1(t)} [\alpha_3^T(t) T_1 \alpha_3(t) + 3\alpha_4^T(t) T_1 \alpha_4(t)] \\
&\leq -\alpha_1^T(t) T_1 \alpha_1(t) - 3\alpha_2^T(t) T_1 \alpha_2(t) - \alpha_3^T(t) T_1 \alpha_3(t) - 3\alpha_4^T(t) T_1 \alpha_4(t) \\
&\quad - 2\alpha_1^T(t) G_1 \alpha_3(t) - 6\alpha_2^T(t) G_1 \alpha_4(t)
\end{aligned}
\tag{21}
$$

where

$\alpha_1(t) = x(t - \tau_1(t)) - x(t - \tau_1)$;

$\alpha_2(t) = x(t - \tau_1(t)) + x(t - \tau_1) - \frac{2}{\tau_1 - \tau_1(t)} \int_{t-\tau_1}^{t-\tau_1(t)} x(s) ds$;

$\alpha_3(t) = x(t) - x(t - \tau_1(t))$;

$\alpha_4(t) = x(t) + x(t - \tau_1(t)) - \frac{2}{\tau_1(t)} \int_{t-\tau_1(t)}^{t} x(s) ds$.

Similarly, applying Lemma 1 and Lemma 3 in fourth, sixth and eighth term of Equ.(18), we can obtain the following: 4 6 8

$$
\begin{aligned}
&-\tau_2 \int_{t-\tau_2}^{t} \dot{x}^T(s) T_2 \dot{x}(s) ds \\
&\leq -\alpha_5^T(t) T_2 \alpha_5(t) - 3\alpha_6^T(t) T_2 \alpha_6(t) - \alpha_7^T(t) T_2 \alpha_7(t) \\
&\quad - 3\alpha_8^T(t) T_2 \alpha_8(t) - 2\alpha_5^T(t) G_2 \alpha_7(t) - 6\alpha_6^T(t) G_2 \alpha_8(t)
\end{aligned}
\tag{22}
$$

$$
\begin{aligned}
&-\eta \int_{t-\eta}^{t} \dot{x}^T(s) T_3 \dot{x}(s) ds \\
&\leq -\alpha_9^T(t) T_3 \alpha_9(t) - 3\alpha_{10}^T(t) T_3 \alpha_{10}(t) - \alpha_{11}^T(t) T_3 \alpha_{11}(t) \\
&\quad - 3\alpha_{12}^T(t) T_3 \alpha_{12}(t) - 2\alpha_9^T(t) G_3 \alpha_{11}(t) - 6\alpha_{10}^T(t) G_3 \alpha_{12}(t)
\end{aligned}
\tag{23}
$$

$$
\begin{aligned}
&-\tau\int_{t-\tau}^{t}\dot{x}^T(s)T_4\dot{x}(s)ds \\
&\leq -\alpha_{13}^T(t)T_4\alpha_{13}(t)-3\alpha_{14}^T(t)T_4\alpha_{14}(t)-\alpha_{15}^T(t)T_4\alpha_{15}(t) \\
&\quad -3\alpha_{16}^T(t)T_4\alpha_{16}(t)-2\alpha_{13}^T(t)G_4\alpha_{15}(t)-6\alpha_{14}^T(t)G_4\alpha_{16}(t)
\end{aligned} \tag{24}
$$

where

$\alpha_5(t) = x(t-\tau_2(t)) - x(t-\tau_2)$;

$\alpha_6(t) = x(t-\tau_2(t)) + x(t-\tau_2) - \frac{2}{\tau_2-\tau_2(t)}\int_{t-\tau_2}^{t-\tau_2(t)} x(s)ds$;

$\alpha_7(t) = x(t) - x(t-\tau_2(t))$;

$\alpha_8(t) = x(t) + x(t-\tau_2(t)) - \frac{2}{\tau_2(t)}\int_{t-\tau_2(t)}^{t} x(s)ds$;

$\alpha_9(t) = x(t-\eta(t)) - x(t-\eta)$;

$\alpha_{10}(t) = x(t-\eta(t)) + x(t-\eta) - \frac{2}{\eta-\eta(t)}\int_{t-\eta}^{t-\eta(t)} x(s)ds$;

$\alpha_{11}(t) = x(t) - x(t-\eta(t))$;

$\alpha_{12}(t) = x(t) + x(t-\eta(t)) - \frac{2}{\eta(t)}\int_{t-\eta(t)}^{t} x(s)ds$;

$\alpha_{13}(t) = x(t-\tau(t)) - x(t-\tau)$;

$\alpha_{14}(t) = x(t-\tau(t)) + x(t-\tau) - \frac{2}{\tau-\tau(t)}\int_{t-\tau}^{t-\tau(t)} x(s)ds$;

$\alpha_{15}(t) = x(t) - x(t-\tau(t))$;

$\alpha_{16}(t) = x(t) + x(t-\tau(t)) - \frac{2}{\tau(t)}\int_{t-\tau(t)}^{t} x(s)ds$;

Now, the second term of Equ.(19) can be written as

$$
\begin{aligned}
&-\frac{\tau_1^2}{2}\int_{-\tau_1}^{0}\int_{t+\theta}^{t}\dot{x}^T(s)U_1\dot{x}(s)dsd\theta \\
=&-\frac{\tau_1^2}{2}\int_{-\tau_1}^{-\tau_1(t)}\int_{t+\theta}^{t}\dot{x}^T(s)U_1\dot{x}(s)dsd\theta-\frac{\tau_1^2}{2}\int_{-\tau_1(t)}^{0}\int_{t+\theta}^{t}\dot{x}^T(s)U_1\dot{x}(s)dsd\theta
\end{aligned} \tag{25}
$$

Further, using Lemma 2, the above equation becomes

$$
\begin{aligned}
&-\frac{\tau_1^2}{2}\int_{-\tau_1}^{0}\int_{t+\theta}^{t}\dot{x}^T(s)U_1\dot{x}(s)dsd\theta \\
&\leq -\frac{\tau_1^2}{\tau_1^2-\tau_1^2(t)}[\int_{-\tau_1}^{-\tau_1(t)}\int_{t+\theta}^{t}\dot{x}(s)dsd\theta]^TU_1[\int_{-\tau_1}^{-\tau_1(t)}\int_{t+\theta}^{t}\dot{x}(s)dsd\theta] \\
&\quad -\frac{\tau_1^2}{\tau_1^2(t)}[\int_{-\tau_1(t)}^{0}\int_{t+\theta}^{t}\dot{x}(s)dsd\theta]^TU_1[\int_{-\tau_1(t)}^{0}\int_{t+\theta}^{t}\dot{x}(s)dsd\theta]
\end{aligned} \tag{26}
$$

For any matrix V_1 with $\begin{bmatrix} U_1 & V_1 \\ \star & U_1 \end{bmatrix} \geq 0$, from Lemma 3, the above inequality becomes

$$
\begin{aligned}
& -\frac{\tau_1^2}{2}\int_{-\tau_1}^{0}\int_{t+\theta}^{t}\dot{x}^T(s)U_1\dot{x}(s)dsd\theta \\
\leq & -[\int_{-\tau_1}^{-\tau_1(t)}\int_{t+\theta}^{t}\dot{x}(s)dsd\theta]^T U_1[\int_{-\tau_1}^{-\tau_1(t)}\int_{t+\theta}^{t}\dot{x}(s)dsd\theta] \\
& -[\int_{-\tau_1(t)}^{0}\int_{t+\theta}^{t}\dot{x}(s)dsd\theta]^T 2V_1[\int_{-\tau_1}^{-\tau_1(t)}\int_{t+\theta}^{t}\dot{x}(s)dsd\theta] \\
& -[\int_{-\tau_1(t)}^{0}\int_{t+\theta}^{t}\dot{x}(s)dsd\theta]^T U_1[\int_{-\tau_1(t)}^{0}\int_{t+\theta}^{t}\dot{x}(s)dsd\theta] \\
= & -\Delta_1^T U_1\Delta_1 - 2\Delta_1^T V_1\Delta_2 - \Delta_2^T U_1\Delta_2 \\
= & \zeta^T(t)[-\Gamma_1^T(t)U_1\Gamma_1(t) - 2\Gamma_1^T(t)V_1\Gamma_2(t) - \Gamma_2^T(t)U_1\Gamma_2(t)]\zeta(t)
\end{aligned}
\tag{27}
$$

where $\Delta_1 = (\tau_1 - \tau_1(t))x(t) - \int_{t-\tau_1}^{t-\tau_1(t)} x(s)ds$, $\Delta_2 = \tau_1(t)x(t) - \int_{t-\tau_1(t)}^{t} x(s)ds$, $\Gamma_1(t) = (\tau_1 - \tau_1(t))(e_1 - e_{15})$, $\Gamma_2(t) = \tau_1(t)(e_1 - e_{14})$.

Similarly, applying Lemma 3 in fourth and sixth term of Equ.(19), we can obtain the following:

$$
\begin{aligned}
& -\frac{\tau_2^2}{2}\int_{-\tau_2}^{0}\int_{t+\theta}^{t}\dot{x}^T(s)U_2\dot{x}(s)dsd\theta \\
\leq & -\Delta_3^T U_2\Delta_3 - 2\Delta_3^T V_2\Delta_4 - \Delta_4^T U_2\Delta_4 \\
= & \zeta^T(t)[-\Gamma_3^T(t)U_2\Gamma_3(t) - 2\Gamma_3^T(t)V_2\Gamma_4(t) - \Gamma_4^T(t)U_2\Gamma_4(t)]\zeta(t)
\end{aligned}
\tag{28}
$$

$$
\begin{aligned}
& -\frac{\tau^2}{2}\int_{-\tau}^{0}\int_{t+\theta}^{t}\dot{x}^T(s)U_3\dot{x}(s)dsd\theta \\
\leq & -\Delta_5^T U_3\Delta_5 - 2\Delta_5^T V_3\Delta_6 - \Delta_6^T U_3\Delta_6 \\
= & \zeta^T(t)[-\Gamma_5^T(t)U_3\Gamma_5(t) - 2\Gamma_5^T(t)V_3\Gamma_6(t) - \Gamma_6^T(t)U_3\Gamma_6(t)]\zeta(t)
\end{aligned}
\tag{29}
$$

where $\Delta_3 = (\tau_2 - \tau_2(t))x(t) - \int_{t-\tau_2}^{t-\tau_2(t)} x(s)ds$, $\Delta_4 = \tau_2(t)x(t) - \int_{t-\tau_2(t)}^{t} x(s)ds$, $\Delta_5 = (\tau - \tau(t))x(t) - \int_{t-\tau}^{t-\tau(t)} x(s)ds$, $\Delta_6 = \tau(t)x(t) - \int_{t-\tau(t)}^{t} x(s)ds$, $\Gamma_3(t) = (\tau_2 - \tau_2(t))(e_1 - e_{17})$, $\Gamma_4(t) = \tau_2(t)(e_1 - e_{16})$, $\Gamma_5(t) = (\tau - \tau(t))(e_1 - e_{19})$, $\Gamma_6(t) = \tau(t)(e_1 - e_{18})$.

According to Assumption 1, we can get

$$
\sum_{i=1}^{n} 2\lambda_{1i}[l_i^- x_i(t) - f_i(x_i(t))][f_i(x_i(t)) - l_i^+ x_i(t)] \geq 0 \tag{30}
$$

$$\begin{aligned}\sum_{i=1}^{n} 2\lambda_{2i}[h_i^- x_i(t-\tau_{i1}(t)-\tau_{i2}(t)) - g_i(x_i(t-\tau_{i1}(t)-\tau_{i2}(t)))]\times \\ [g_i(x_i(t-\tau_{i1}(t)-\tau_{i2}(t))) - h_i^+ x_i(t-\tau_{i1}(t)-\tau_{i2}(t))] \geq 0\end{aligned} \tag{31}$$

Then Equations (30) - (31) can be written in the following vector form:

$$\begin{aligned}-2x^T(t)L^-\Lambda_1 L^+ x(t) - 2f^T(x(t))\Lambda_1 f(x(t)) \\ +2x^T(t)(L^- + L^+)\Lambda_1 f(x(t)) \geq 0\end{aligned} \tag{32}$$

$$\begin{aligned}-2x^T(t-\tau(t))H^-\Lambda_2 H^+ x(t-\tau(t)) - 2g^T(x(t-\tau(t)))\times \\ \Lambda_2 g(x(t-\tau(t))) + 2x^T(t-\tau(t))(H^- + H^+)\Lambda_2 g(x(t-\tau(t))) \geq 0\end{aligned} \tag{33}$$

From (14) - (29) and adding (32) and (33), we have

$$\begin{aligned}D^+V(t) - f^T(x(t))Qf(x(t)) - 2f^T(x(t))Su(t) - u^T(t)(R-\gamma I)u(t) \\ \leq \zeta^T(t)\Phi(\tau_1(t),\tau_2(t))\zeta(t)\end{aligned} \tag{34}$$

230 where

$\Phi(\tau_1(t),\tau_2(t)) = \Xi - \Gamma_1^T(t)U_1\Gamma_1(t) - 2\Gamma_1^T(t)V_1\Gamma_2(t) - \Gamma_2^T(t)U_1\Gamma_2(t) - \Gamma_3^T(t)U_2\Gamma_3(t) - 2\Gamma_3^T(t)V_2\Gamma_4(t) - \Gamma_4^T(t)U_2\Gamma_4(t) - \Gamma_5^T(t)U_3\Gamma_5(t) - 2\Gamma_5^T(t)V_3\Gamma_6(t) - \Gamma_6^T(t)U_3\Gamma_6(t)$.

Letting $\tau_1(t) = 0$, $\tau_1(t) = \tau_1$ and $\tau_2(t) = 0$, $\tau_2(t) = \tau_2$ respectively, we can get

$$\begin{cases}\Phi_1 = \Phi(0,0), \\ \Phi_2 = \Phi(0,\tau_2), \\ \Phi_3 = \Phi(\tau_1,0), \\ \Phi_4 = \Phi(\tau_1,\tau_2)\end{cases} \tag{35}$$

235 According to the LMIs (10), we have

$$D^+V(t) - f^T(x(t))Qf(x(t)) - 2f^T(x(t))Su(t) - u^T(t)(R-\gamma I)u(t) \leq 0 \tag{36}$$

Under the zero initial conditon, $V(0) = 0$. For any $T > 0$, $V(T) \geq 0$. Integrating Equ.(36) with respect to t over the time period from 0 to T, we have

$$\begin{aligned}\int_0^T [-f^T(x(t))Qf(x(t)) - 2f^T(x(t))Su(t) - u^T(t)(R - \gamma I)u(t)]dt \\ \leq -V(T) \leq 0\end{aligned} \tag{37}$$

So Equ. (5) holds, which implies MNNs (4) is strictly $(Q, S, R) - \gamma -$ *dissipative*. This completes the proof.

Remark 5. In Theorem 1, not only free-weighting matrices technique but also the reciprocally convex combination technique, Wirtinger-based integral inequality are employed. Also, double integral form in $V_5((t)$ and novel triple integral form in $V_6(t)$ are introduced by considering both time-varying leakage delay and two additive time-varying delays, which has not been addressed yet in the previous literature [37, 39, 47]. Construction this form of double and triple integral terms in the LKF is a recent tool for getting less conservative results.

Remark 6. According to Definition 3 and Definition 4, we can obtain the passivity conditions for the system (4) by substituting $Q = \boldsymbol{0}$, $S = \boldsymbol{I}$ and $R = 2\gamma\boldsymbol{I}$ in Theorem 1. In this case, we can get Theorem 2 in the same way as Theorem 1.

Theorem 2. Under Assumption 1, the MNNs (4) is passive in the sense of Definition 4, if there exist symmetric matrices $P > 0$, $Q_i > 0$, $R_i > 0$, $T_i > 0$, $(i = 1, 2, 3, 4)$, $U_j > 0$ $(j = 1, 2, 3)$, $Z > 0$, diagonal matrices $M = diag\{m_1, m_2, \cdots, m_n\} > 0$, $W = diag\{w_1, w_2, \cdots, w_n\} > 0$, $\Lambda_1 = diag\{\lambda_{11}, \lambda_{12}, \cdots, \lambda_{1n}\} > 0$, $\Lambda_2 = diag\{\lambda_{21}, \lambda_{22}, \cdots, \lambda_{2n}\} > 0$, and matrices G_i $(i = 1, 2, 3, 4)$, V_j $(j = 1, 2, 3)$ and scalar $\gamma > 0$,such that the following LMIs hold:

$$\tilde{\Phi}_l = \tilde{\Xi} - \Upsilon_l^T \begin{bmatrix} U_1 & V_1 & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \star & U_1 & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \star & \star & U_2 & V_2 & \mathbf{0} & \mathbf{0} \\ \star & \star & \star & U_2 & \mathbf{0} & \mathbf{0} \\ \star & \star & \star & \star & U_3 & V_3 \\ \star & \star & \star & \star & \star & U_3 \end{bmatrix} \Upsilon_l \leq 0, \tag{38}$$

$l = (1, 2, 3, 4)$.

$$\begin{bmatrix} T_i & G_i \\ \star & T_i \end{bmatrix} \geq 0, (i = 1, 2, 3, 4). \tag{39}$$

$$\begin{bmatrix} U_j & V_j \\ \star & U_j \end{bmatrix} \geq 0, (j = 1, 2, 3). \tag{40}$$

where $\tilde{\Xi} = (\tilde{\Psi}_{p,q})_{21\times 21}$, $\tilde{\Psi}_{p,q} = \Psi_{p,q}$, $\forall p, q = 1, 2, \cdots, 21$, excepet $\tilde{\Psi}_{10,10} = AW + WA^T + \tau_1^2 A^T T_1 A + \tau_2^2 A^T T_2 A + \eta^2 A^T T_3 A + \tau^2 A^T T_4 A + \frac{\tau_1^4}{4} A^T U_1 A + \frac{\tau_2^4}{4} A^T U_2 A + \frac{\tau^4}{4} A^T U_3 A - 2\Lambda_1$, $\Psi_{10,21} = W + \tau_1^2 A^T T_1 + \tau_2^2 A^T T_3 + \eta^2 A^T T_3 + \tau^2 A^T T_4 + \frac{\tau_1^4}{4} A^T U_1 + \frac{\tau_2^4}{4} A^T U_2 + \frac{\tau^4}{4} A^T U_3 - I$, $\Psi_{21,21} = \tau_1^2 T_1 + \tau_2^2 T_2 + \eta^2 T_3 + \tau^2 T_4 + \frac{\tau_1^4}{4} U_1 + \frac{\tau_2^4}{4} U_2 + \frac{\tau^4}{4} U_3 - \gamma I$.

The remaining coefficients are same as in Theorem 1.

Proof. The proof is same as that of Theorem 1 and hence it is omitted.

Remark 7. The less conservative of the proposed delay-dependent dissipativity and passivity criterion over [22, 29] relies on how to handle the terms $-\tau_1 \int_{t-\tau_1}^{t} \dot{x}^T(s) T_1 \dot{x}(s) ds$, $-\tau_2 \int_{t-\tau_2}^{t} \dot{x}^T(s) T_2 \dot{x}(s) ds$, $-\eta \int_{t-\eta}^{t} \dot{x}^T(s) T_3 \dot{x}(s) ds$ and $-\tau \int_{t-\tau}^{t} \dot{x}^T(s) T_4 \dot{x}(s) ds$. Lemma 1 and Lemma 3 is employed to handle these terms instead of using Jensen Inequality in [22, 29].

Remark 8. When $u(t) = 0$ and $Q = 0$, it can be seen from Equ.(36) that $D^+V(t) \leq 0$. So it implies the asymptotic stability of the system (4). So the dissipativity and passivity analysis for MMNs is more general and applicable.

4. Illustrative examples

In this section, two numerical examples are given to illustrate the effectiveness of the obtained results.

Example 1. Consider the following MNNs with leakage and two additive time-varying delays:

$$\begin{cases} \frac{dx_i(t)}{dt} = -\, d_i x_i(t-\eta_i(t)) + \sum_{j=1}^{2} a_{ij}(t) f_j(x_j(t)) \\ \qquad\qquad + \sum_{j=1}^{2} b_{ij}(t) g_j(x_j(t-\bar{\tau}_j(t))) + u_i(t) \\ y_i(t) = f_i(x_i(t)), i = 1, 2. \end{cases} \tag{41}$$

where

$\bar{\tau}_j(t) = \tau_{j1}(t) + \tau_{j2}(t)$, $d_1 = 1.10, d_2 = 1.20$, $u_1(t) = sin(0.5t), u_2(t) = cos(2t)$,

$$a_{11}(t) = \begin{cases} 0.60, & \dot{\kappa}_{11}^1 < 0 \\ unchanged, & \dot{\kappa}_{11}^1 = 0 \\ 0.80, & \dot{\kappa}_{11}^1 > 0 \end{cases}, \; a_{12}(t) = \begin{cases} 0.10, & \dot{\kappa}_{12}^1 < 0 \\ unchanged, & \dot{\kappa}_{12}^1 = 0 \\ 0.15, & \dot{\kappa}_{12}^1 > 0 \end{cases},$$

$$a_{21}(t) = \begin{cases} 0.12, & \dot{\kappa}_{21}^1 < 0 \\ unchanged, & \dot{\kappa}_{21}^1 = 0 \\ 0.16, & \dot{\kappa}_{21}^1 > 0 \end{cases}, \; a_{22}(t) = \begin{cases} 0.70, & \dot{\kappa}_{22}^1 < 0 \\ unchanged, & \dot{\kappa}_{22}^1 = 0 \\ 0.80, & \dot{\kappa}_{22}^1 > 0 \end{cases},$$

$$b_{11}(t) = \begin{cases} 0.30, & \dot{\kappa}_{11}^2 < 0 \\ unchanged, & \dot{\kappa}_{11}^2 = 0 \\ 0.35, & \dot{\kappa}_{11}^2 > 0 \end{cases}, \; b_{12}(t) = \begin{cases} 0.08, & \dot{\kappa}_{12}^2 < 0 \\ unchanged, & \dot{\kappa}_{12}^2 = 0 \\ 0.10, & \dot{\kappa}_{12}^2 > 0 \end{cases},$$

$$b_{21}(t) = \begin{cases} 0.07, & \dot{\kappa}_{21}^2 < 0 \\ unchanged, & \dot{\kappa}_{21}^2 = 0 \\ 0.12, & \dot{\kappa}_{21}^2 > 0 \end{cases}, \; b_{22}(t) = \begin{cases} 0.40, & \dot{\kappa}_{22}^2 < 0 \\ unchanged, & \dot{\kappa}_{22}^2 = 0 \\ 0.42, & \dot{\kappa}_{22}^2 > 0 \end{cases},$$

$\kappa_{ij}^1 = f_{ij}(x_j(t)) - x_i(t)$, $\kappa_{ij}^2 = g_{ij}(x_j(t-\bar{\tau}_j(t))) - x_i(t)$, $i = 1$ or 2, $j = 1$ or 2.

Let us consider the nonlinear activation functions as $f(x(t)) = g(x(t)) = \sin(x(t))$, it can be verified that the activation functions satisfy Assumption

1 with $l_i^- = h_i^- = -1$, $l_i^+ = h_i^+ = 1$, $(i = 1, 2)$. We shall assume that the leakage delay to be $\eta(t) = 0.2 + 0.2sin(2t)$ and the time-varying delays to be $\tau_1(t) = 0.1 + 0.1sin(t)$, $\tau_2(t) = 0.1 + 0.1cos(t)$. So $\eta = 0.4$, $\tau_1 = 0.2$, $\tau_2 = 0.2$, $\mu_1 = 0.1$, $\mu_2 = 0.1$, $\mu = 0.2$, Then a set of corresponding feasible solution to the LMIs in Theorem 1 can be obtained by the Matlab LMI control toolbox as following:

$$P = \begin{bmatrix} 0.0475 & -0.0291 \\ -0.0291 & 0.0287 \end{bmatrix}, Q_1 = \begin{bmatrix} 2.3090 & -1.3697 \\ -1.3697 & 1.4282 \end{bmatrix},$$
$$Q_2 = \begin{bmatrix} 2.2678 & -1.3111 \\ -1.3111 & 1.4202 \end{bmatrix}, R_1 = \begin{bmatrix} 3.2735 & -1.9551 \\ -1.9551 & 2.0149 \end{bmatrix},$$
$$T_1 = \begin{bmatrix} 0.0746 & -0.0445 \\ -0.0445 & 0.0459 \end{bmatrix}, U_1 = \begin{bmatrix} 15.4356 & -9.0026 \\ -9.0026 & 9.6499 \end{bmatrix},$$

Due to the page limit, we omit the remaining feasible matrices. Fig.1 shows phase trajectories of MNNs (41) under the zero initial condition. Fig.2 shows trajectories of neuron state $x_1(t)$ and $x_2(t)$ of MNNs (41). Fig.3 shows 3-D trajectories of neuron state $x_1(t)$ and $x_2(t)$ of MNNs (41). It can be seen that neuron state $x_1(t)$ and $x_2(t)$ are tending to periodic when the outputs of MNNs (41) controllers are designed to periodic signals. According to Theorem 1 and Definition 3, the system (41) is dissipative.

Example 2. Consider a two-neuron MNNs (4) with the following parameters:

$$D = \begin{bmatrix} 1.50 & 0 \\ 0 & 1.80 \end{bmatrix}, u(t) = \begin{bmatrix} cost \\ sin(2t) \end{bmatrix},$$
$$A = \begin{bmatrix} 1.8 & 0.10 \\ 0.12 & 2.00 \end{bmatrix}, B = \begin{bmatrix} 0.80 & 0.07 \\ 0.05 & 1.20 \end{bmatrix},$$

Let us consider the nonlinear activation functions as $f(x(t)) = g(x(t)) = 0.01(|x+1| - |x-1|)$. We shall assume that the leakage delay to be $\eta(t) = 0.1 + 0.1sin(2t)$ and the time-varying delays to be $\tau_1(t) = 0.05 + 0.05sin(t)$,

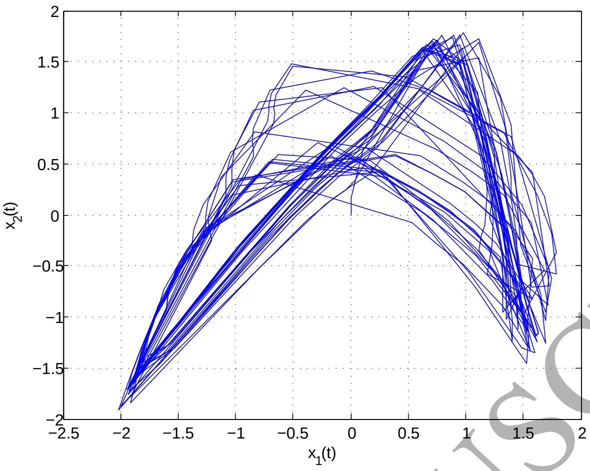

Figure 1: Phase trajectories of MNNs (41).

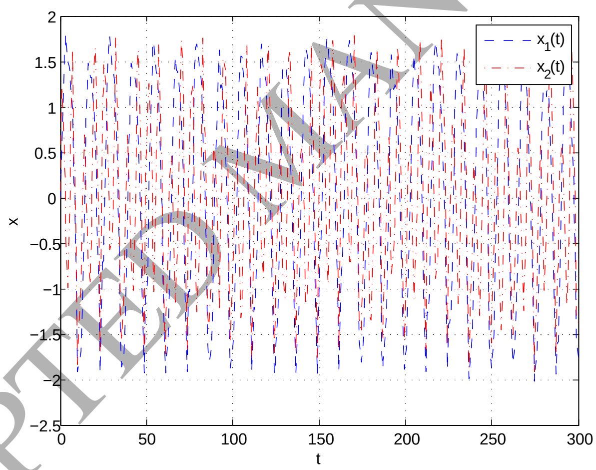

Figure 2: Trajectories of neuron state $x_1(t)$ and $x_2(t)$ of MNNs (41).

$\tau_2(t) = 0.05 + 0.05cos(t)$. So $\eta = 0.2$, $\tau_1 = 0.1$, $\tau_2 = 0.1$, $\mu_1 = 0.05$, $\mu_2 = 0.05$, $\mu = 0.1$, Then a set of corresponding feasible solution to the LMIs in Theorem 2 can be obtained by the Matlab LMI control toolbox as following:

$$P = \begin{bmatrix} 0.0026 & -0.0001 \\ -0.0001 & 0.0016 \end{bmatrix}, Q_1 = \begin{bmatrix} 1.1953 & -0.0439 \\ -0.0439 & 0.7844 \end{bmatrix},$$

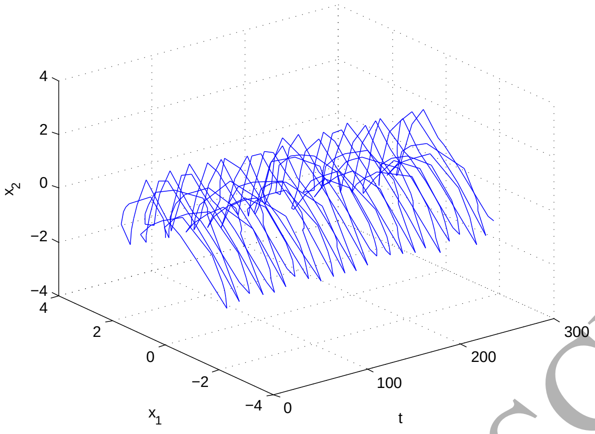

Figure 3: 3-D trajectories of neuron state $x_1(t)$ and $x_2(t)$ of MNNs (41).

$$Q_2 = \begin{bmatrix} 1.1574 & -0.0423 \\ -0.0423 & 0.7586 \end{bmatrix}, \ R_1 = \begin{bmatrix} 1.6760 & -0.0614 \\ -0.0614 & 1.1008 \end{bmatrix},$$

$$T_1 = \begin{bmatrix} 0.0396 & -0.0014 \\ -0.0014 & 0.0260 \end{bmatrix}, \ U_1 = \begin{bmatrix} 29.7285 & -1.0527 \\ -1.0527 & 19.8825 \end{bmatrix},$$

Due to the page limit, we omit the remaining feasible matrices. Fig.4 shows the time respond trajectories of MNNs (4) with 20 random initial conditions and without external input respectively, it is clear that $x_1(t)$ and $x_2(t)$ are asymptotic stability. Therefore, we conclude that system (4) is passive.

In order to demonstrate the improvement of our results, we compare our results with those in [26, 48, 49]. Table 1 shows the comparison results for allowable upper bounds of τ for different μ. From the table, the results calculated based on the criteria given in this paper are less conservative than the ones reported in the existing literatures.

5. Conclusion

In this paper, the problem of dissipativity and passivity has been analysed for MNNs in the case of both leakage and two additive time-varying delays. MNNs

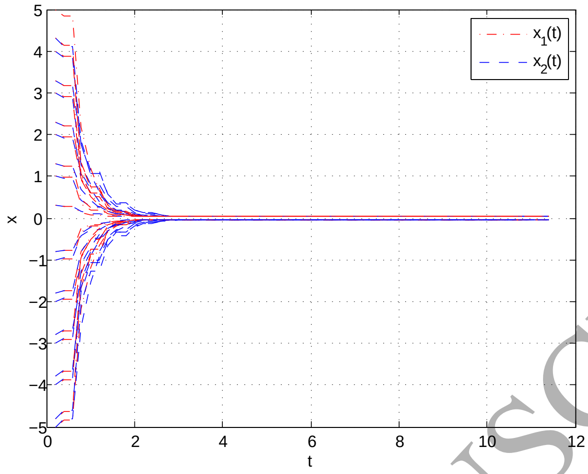

Figure 4: Time respond trajectories of MNNs (4).($u_1(t) = u_2(t) = 0$)

Table 1: Allowable upper bounds of τ for different μ.

Conditions	[48]	[49]	[26]	Theorem 2
$\mu = 0$	0.4528	0.7340	Unknown	Large
$\mu = 0.5$	0.3638	0.6834	1.0760	1.2803
$\mu = 0.7$	0.3575	0.6355	1.0704	1.2207

have been converted into the conventional neural networks by applying differential inclusions and set-valued maps. Wirtinger-based integral inequality, reciprocally convex combination technique and free-weighting matrices technique have been used to obtain less conservative dissipativity and passivity criteria, which is based on proper Lyapunov functional approach. The effectiveness of the corresponding criteria have been shown via two numerical examples. The employed methods can be extended to other problems of MNNs with leakage and time-varying delays, such as stabilization analysis, H_∞ analysis, and so on. MNNs is a new and challenging topic, looking for more complex and practical MNNs model and more advanced mathematical method is our further work. In future, we shall consider the dissipativity and passivity for stochastic memristive complex-values neural networks with two delay components based on dynamic delay interval method.

References

[1] L. Chua, Memristor-the missing circuit element, IEEE Transactions on circuit theory 18 (5) (1971) 507–519.

[2] D. B. Strukov, G. S. Snider, D. R. Stewart, R. S. Williams, The missing memristor found, nature 453 (7191) (2008) 80–83.

[3] L. Chua, Resistance switching memories are memristors, Applied Physics A 102 (4) (2011) 765–783.

[4] D. Liu, S. Zhu, W. Chang, Input-to-state stability of memristor-based complex-valued neural networks with time delays, Neurocomputing 221 (2017) 159–167.

[5] L. Wang, H. Li, S. Duan, T. Huang, H. Wang, Pavlov associative memory in a memristive neural network and its circuit implementation, Neurocomputing 171 (2016) 23–29.

[6] S. Hu, Y. Liu, Z. Liu, T. Chen, Q. Yu, L. Deng, Y. Yin, S. Hosaka, Synaptic long-term potentiation realized in pavlov's dog model based on a niox-based memristor, Journal of Applied Physics 116 (21) (2014) 214502.

[7] H. Kim, M. P. Sah, C. Yang, T. Roska, L. O. Chua, Neural synaptic weighting with a pulse-based memristor circuit, IEEE Transactions on Circuits and Systems I: Regular Papers 59 (1) (2012) 148–158.

[8] X. Hu, S. Duan, G. Chen, L. Chen, Modeling affections with memristor-based associative memory neural networks, Neurocomputing 223 (2017) 129–137.

[9] P. Zhang, C. Li, T. Huang, L. Chen, Y. Chen, Forgetting memristor based neuromorphic system for pattern training and recognition, Neurocomputing 222 (2016) 47–53.

[10] M. Itoh, L. O. Chua, Memristor cellular automata and memristor discrete-time cellular neural networks, International Journal of Bifurcation and Chaos 19 (11) (2009) 3605–3656.

[11] R. Sakthivel, R. Anbuvithya, K. Mathiyalagan, Y.-K. Ma, P. Prakash, Reliable anti-synchronization conditions for bam memristive neural networks with different memductance functions, Applied Mathematics and Computation 275 (2016) 213–228.

[12] M. S. Ali, Stability of markovian jumping recurrent neural networks with discrete and distributed time-varying delays, Neurocomputing 149 (2015) 1280–1285.

[13] A. Chandrasekar, R. Rakkiyappan, Impulsive controller design for exponential synchronization of delayed stochastic memristor-based recurrent neural networks, Neurocomputing 173 (2016) 1348–1355.

[14] G. Jie, P. Zhu, A. Alsaedi, F. E. Alsaadi, T. Hayat, A new switching control for finite-time synchronization of memristor-based recurrent neural networks , Neural Networks 86 (2017) 1–9.

[15] Y. Liu, B. Z. Guo, J. H. Park, S. M. Lee, Nonfragile exponential synchronization of delayed complex dynamical networks with memory sampled-data control, IEEE Transactions on Neural Networks and Learning Systems PP (99) (2016) 1–11.

[16] H. Zhang, Q. Shan, Z. Wang, Stability analysis of neural networks with two delay components based on dynamic delay interval method, IEEE Transactions on Neural networks and learning systems 28 (2) (2017) 259–267.

[17] A. Chandrasekar, R. Rakkiyappan, X. Li, Effects of bounded and unbounded leakage time-varying delays in memristor-based recurrent neural networks with different memductance functions , Neurocomputing 202 (2016) 67–83.

[18] P. Jiang, Z. Zeng, J. Chen, On the periodic dynamics of memristor-based neural networks with leakage and time-varying delays, Neurocomputing 219 (2017) 163–173.

[19] C. Xu, P. Li, Y. Pang, Exponential stability of almost periodic solutions for memristor-based neural networks with distributed leakage delays, Neural Computation 28 (12) (2016) 2726–2756.

[20] Y. Liu, S. M. Lee, O. M. Kwon, H. P. Ju, New approach to stability criteria for generalized neural networks with interval time-varying delays, Neurocomputing 149 (2015) 1544–1551.

[21] S. Wen, Z. Zeng, T. Huang, Y. Chen, Passivity analysis of memristor-based recurrent neural networks with time-varying delays , Journal of the Franklin Institute 350 (8) (2013) 2354–2370.

[22] R. Rakkiyappan, K. Sivaranjani, G. Velmurugan, Passivity and passification of memristor-based complex-valued recurrent neural networks with interval time-varying delays, Neurocomputing 144 (1) (2014) 391–407.

[23] J. Liu, R. Xu, Passivity analysis and state estimation for a class of memristor-based neural networks with multiple proportional delays, Advances in Difference Equations 2017 (1) (2017) 34.

[24] M. Syed Ali, R. Saravanakumar, J. Cao, New passivity criteria for memristor-based neutral-type stochastic bam neural networks with mixed time-varying delays, Neurocomputing 171 (C) (2016) 1533–1547.

[25] Y. Huang, S. Ren, Passivity and passivity-based synchronization of switched coupled reaction-diffusion neural networks with state and spatial diffusion couplings, Neural Processing Letters (2017) 1–17.

[26] J. Xiao, S. Zhong, Y. Li, Relaxed dissipativity criteria for memristive neural networks with leakage and time-varying delays, Neurocomputing 171 (C) (2016) 708–718.

[27] H. Wei, R. Li, C. Chen, Z. Tu, Extended dissipative analysis for memristive neural networks with two additive time-varying delay components , Neurocomputing 216 (2016) 429–438.

[28] M. S. Ali, S. Arik, M. E. Rani, Passivity analysis of stochastic neural networks with leakage delay and markovian jumping parameters , Neurocomputing 218 (2016) 139–145.

[29] X. Li, R. Rakkiyappan, G. Velmurugan, Dissipativity analysis of memristor-based complex-valued neural networks with time-varying delays, Information Sciences 294 (2015) 645 – 665.

[30] S. Ding, Z. Wang, H. Zhang, Dissipativity analysis for stochastic memristive neural networks with time-varying delays: A discrete-time case, IEEE Transactions on Neural Networks and Learning Systems PP (99) (2016) 1–13.

[31] A. Wu, S. Wen, Z. Zeng, Synchronization control of a class of memristor-based recurrent neural networks, Information Sciences 183 (1) (2012) 106–116.

[32] Y. Song, S. Wen, Synchronization control of stochastic memristor-based neural networks with mixed delays, Neurocomputing 156 (2015) 121–128.

[33] N. Li, J. Cao, Lag synchronization of memristor-based coupled neural networks via-measure, IEEE transactions on neural networks and learning systems 27 (3) (2016) 686–697.

[34] H. Li, H. Jiang, C. Hu, Existence and global exponential stability of periodic solution of memristor-based bam neural networks with time-varying delays, Neural Networks 75 (2016) 97–109.

[35] H. Bao, J. H. Park, J. Cao, Exponential synchronization of coupled stochastic memristor-based neural networks with time-varying probabilistic delay coupling and impulsive delay, IEEE transactions on neural networks and learning systems 27 (1) (2016) 190–201.

[36] S. P. Adhikari, M. P. Sah, H. Kim, L. O. Chua, Three fingerprints of memristor, IEEE Transactions on Circuits and Systems I: Regular Papers 60 (11) (2013) 3008–3021.

[37] G. Nagamani, S. Ramasamy, Dissipativity and passivity analysis for uncertain discrete-time stochastic markovian jump neural networks with additive time-varying delays, Neurocomputing 174 (2016) 795–805.

[38] R. Rakkiyappan, A. Chandrasekar, J. Cao, Passivity and passification of memristor-based recurrent neural networks with additive time-varying delays, IEEE transactions on neural networks and learning systems 26 (9) (2015) 2043–2057.

[39] J. Xiao, S. Zhong, Y. Li, Relaxed dissipativity criteria for memristive neural networks with leakage and time-varying delays, Neurocomputing 171 (C) (2015) 708–718.

[40] J. Liu, R. Xu, Delay-dependent passivity and stability analysis for a class of memristor-based neural networks with time delay in the leakage term, Neural Processing Letters (2017) 1–19.

[41] J. P. Aubin, A. Cellina, Differential Inclusions, Springer Berlin Heidelberg, 1984.

[42] J. P. Aubin, Set-Valued Analysis, Birkhäuser Boston, 1999.

[43] S. Wen, T. Huang, Z. Zeng, Y. Chen, P. Li, Circuit design and exponential stabilization of memristive neural networks, Neural Networks 63 (2015) 48–56.

[44] A. Seuret, F. Gouaisbaut, Wirtinger-based integral inequality: Application to time-delay systems , Automatica 49 (9) (2013) 2860–2866.

[45] O. M. Kwon, S. M. Lee, H. P. Ju, E. J. Cha, New approaches on stability criteria for neural networks with interval time-varying delays, Applied Mathematics and Computation 218 (19) (2012) 9953–9964.

[46] P. Park, J. W. Ko, C. Jeong, Reciprocally convex approach to stability of systems with time-varying delays, Automatica 47 (1) (2011) 235–238.

[47] L. Duan, L. Huang, Periodicity and dissipativity for memristor-based mixed time-varying delayed neural networks via differential inclusions, Neural Networks 57 (9) (2014) 12–22.

[48] H. B. Zeng, Y. He, M. Wu, S. P. Xiao, Passivity analysis for neural networks with a time-varying delay, Neurocomputing 74 (5) (2011) 730–734.

[49] B. Zhang, S. Xu, J. Lam, Relaxed passivity conditions for neural networks with time-varying delays, Neurocomputing 142 (142) (2014) 299–306.

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