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Stability analysis and state feedback control of continuous-time T–S fuzzy systems via anew switched fuzzy Lyapunov function approach

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ABSTRACT

This paper deals with stability analysis and control design problems for continuous-time Takagi–Sugeno (T–S) fuzzy systems. First, a new membership-function-dependent switching law is proposed and a relaxation parameter is introduced into this switching law to guarantee a minimal dwell time between two consecutive switching. Compared to the existing methods, the most important point is that with the help of the dwell time, the discretized Lyapunov function (DLF) technique can be adopted. Then a new stability criterion of the T–S fuzzy system with less conservatism is derived based on a fuzzy discretized Lyapunov function (FDLF). Second, to estimate the domain of attraction (DA), an algorithm with less iteration steps is proposed. Based on the proposed switching method, sufficient conditions for existence of the state feedback controllers are presented via the switched non-parallel distributed compensation (non-PDC) scheme. The effectiveness of the proposed method is illustrated through three simulation examples.

1. Introduction

Over the last decades, Takagi–Sugeno (T–S) fuzzy models [1] have been widely exploited in nonlinear system modeling and control since it is an effective tool in approximating most complex nonlinear systems [2]. The main feature of the T–S modeling is that the nonlinear systems is described by a weighted sum of linear time invariant systems, where each linear system is a rule of fuzzy implication and the weights of the sum are nonlinear functions, called membership functions, which satisfy the sum-convex property [3]. Then, the nonlinear systems described by T–S fuzzy models can be analyzed using linear matrix inequalities (LMIs) formulations.

In the past decades, considerable efforts have been contributed to the T–S fuzzy systems. For T–S fuzzy control systems, many researchers have presented the conventional quadratic Lyapunov function approaches to find a constant positive definite matrix of a quadratic Lyapunov function satisfying the stability conditions of all subsystems [4–7]. However, it is obvious that the common quadratic approach leads to considerable conservativeness in that a common Lyapunov matrix should be found for all subsystems of the T–S fuzzy systems [8]. For nonlinear Markov jump systems, Cheng [9] considers a kind of Markov jump Lyapunov function. However, this Lyapunov function has the same disadvantage as common quadratic Lyapunov function. Therefore, for reducing the conservativeness, a kind of fuzzy Lyapunov functions which depend on the same

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membership functions as the original T–S fuzzy system have been developed. Nowadays, there has been a lot of interest in the use of the fuzzy Lyapunov functions for both continuous-time systems [8,10,11] and discrete-time systems [13–15]. Besides, some other fuzzy Lyapunov function approaches have been proposed for the stability analysis of the T–S fuzzy systems, such as piecewise quadratic Lyapunov functions [17], line-integral fuzzy Lyapunov function approach [12].

For the convenience in dealing with the nonlinear systems, numerous problems have been studied based on T–S fuzzy model. For example, state feedback control design problems are studied in [18,19], designing static output feedback controllers and dynamic output feedback controllers are given in [20,21], robust control design methods are presented in [23], adaptive fuzzy decentralized control problems are studied in [22,25,43] and non-fragile filtering and control problem have been investigated in [16,24]. For the control problem, a kind of non-parallel distributed compensation (non-PDC) control laws are widely applied in [26–28]. Besides, to reduce the conservatism, a kind of switched controllers is proposed for the fuzzy systems [29–31]. Especially, a new type of switched parallel distributed compensation (PDC) controllers which are switched based on the values of membership functions are proposed in [29]. However, for the fact that the switching law in [29] is based on the membership functions and has no minimal dwell time, the famous Metzler matrix method [34] and average dwell time method [35,36] cannot be applied to the fuzzy systems with switching control. This is the main motivation of the present study.

On the other hand, how to estimate the domain of attraction (DA) is another important issue in stability analysis of nonlinear systems. For a Lyapunov function $V(x(t))$ which guarantees the local stability of the equilibrium, any sublevel set of the Lyapunov function is an inner estimate of the DA if the set belongs to the region where $V(x(t)) > 0$ and $\dot{V}(x(t)) < 0$ hold for all $x(t) \neq 0$[37]. Based on the Lyapunov theory, some preliminary results in obtaining such estimates are proposed in [37]. Nowadays, based on the fuzzy Lyapunov function method, some systematic approaches to estimate the DA for continuous-time T–S fuzzy systems have been investigated in [8,10,38,41,42].

Motivated by the aforementioned, in this paper, the problem of stability analysis and stabilization for a class of T–S fuzzy systems are investigated. First, for T–S fuzzy systems, a kind of membership-function-dependent switching law is proposed. Compared to the existing switched or piecewise controllers method in [29–31], it is proved that the proposed switching signal has a minimal dwell time. It is worth noting that the existence of minimal dwell time is one of the most important points in this paper. With the help of the minimal dwell time, discretized Lyapunov function (DLF) technique can be adopted for less conservatism. Second, a more effective algorithm to estimate the DA for continuous-time T–S fuzzy systems is obtained based on the new stability condition. Third, referring to the non-PDC control scheme in [27], the state feedback control problem for continuous-time T–S fuzzy systems is solved via a new switched non-PDC controller. Finally, three simulation examples are illustrated to show the effectiveness of the proposed method.

The paper unfolds as follows: in Section 2, the T–S fuzzy systems and some definitions are given. The stability analysis and state feedback control of continuous-time T–S fuzzy systems are considered in Section 3. The stability analysis and an algorithm estimating the DA are proposed in Section 3.1 and the switched non-PDC state feedback controller is obtained in Section 3.2. In Section 4, three simulation examples are given to illustrate the effectiveness of the new proposed method. Finally, conclusions are presented in Section 5.

Notation. For a matrix P, P^T denotes its transpose and $He(P) \triangleq P + P^T$. $P = [p_{ij}]_{m\times n}$ denotes $P \in \mathbb{R}^{m\times n}$ and p_{ij} is the element on row i column j. $P > 0$ and $P < 0$ denote positive definite and negative definite, respectively. $\mathbb{R}^n$ denotes the n-dimensional Euclidean space and $\mathbb{R}^{n\times m}$ is the set of all $n \times m$ real matrices. The notation $\|x\|_2$ refers to the Euclidean vector norm of vector $x \in \mathbb{R}^n$. Denote

$$P_{h(t)} = \textstyle\sum_{i=1}^{r} h_i(t)P_i, \quad P_{h(t)h(t)} = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(t)h_j(t)P_{ij} \tag{1}$$

2. Preliminaries and problem statement

Consider a class of continuous-time T–S fuzzy control system which can be described as follows:

$$\dot{x}(t) = \sum_{i=1}^{r} h_i(z(t))(A_i x(t) + B_i u(t)) \tag{2}$$

where $x(t) \in \mathbb{R}^{n_x}$ is the state, $u(t) \in \mathbb{R}^{n_u}$ is the control input. $\mathbb{I} = \{1, 2, \ldots, r\}$, r is the number of IF-THEN rules. A_i, B_i are constant matrices of the appropriate dimensions. $z(t) = [z_1(t), z_2(t), \ldots, z_q(t)]^T$ is the vector containing premise variables in the fuzzy inference rule. Besides, $h_i(t)$ are the normalized membership functions for ith rule fulfilling the following properties

$$h_i(t) \geq 0, \quad \sum_{i=1}^{r} h_i(z(t)) = 1, \quad \sum_{i=1}^{r} \dot{h}_i(z(t)) = 0, \quad i \in \mathbb{I} \tag{3}$$

where $h_i(z(t))$ are said to be the normalized membership functions. Considering the time derivative of the normalized membership functions, we assume that

$$|\dot{h}_i(z(t))| \leq \phi_i, \quad i \in \mathbb{I} \tag{4}$$

For the presentation convenience, denote $h(z(t)) = [h_1(z(t)), h_2(z(t)), \ldots, h_r(z(t))]^T$, besides $h(z(t))$ and $h_i(z(t))$ are denoted as $h(t)$ and $h_i(t)$, respectively.

Consider the following continuous-time nonlinear system

$$\dot{x}(t) = f(x(t))x(t) + g(x(t))u(t) \tag{5}$$

where $f : \mathbb{R}^{n_x} \to \mathbb{R}^{n_x} \times \mathbb{R}^{n_x}$ and $g : \mathbb{R}^{n_x} \to \mathbb{R}^{n_x} \times \mathbb{R}^{n_u}$ are matrix-valued nonlinear functions. As mentioned in [10], a wide range of nonlinear systems in the form of (5) can be approximatively represented by T–S fuzzy system (2) in the set of the state variables:

$$\mathbf{C} := \{x(t) \in \mathbb{R}^{n_x} | \mid x_i(t) \mid \leq \bar{x}_i, i \in \mathbb{N}\} \tag{6}$$

where $\mathbb{N} = \{1, 2, \ldots, n_x\}$ and $\mathbf{C} \subseteq \mathbf{H} = \{x(t) \in \mathbb{R}^{n_x} | f(x(t)) = A_{h(t)},\ g(x(t)) = B_{h(t)}\}$. Consider the assumption (4), the following set is defined

$$\mathbf{R} := \{x(t) \in \mathbb{R}^{n_x} | \mid \dot{h}_i(t) \mid \leq \phi_i, i \in \mathbb{I}\} \tag{7}$$

Throughout this paper, the origin is assumed as the equilibrium point. Thus, DA is the set of initial conditions for which the state asymptotically converges to the origin:

$$\mathbf{D} := \left\{ x(0) \in \mathbb{R}^{n_x} | \lim_{t \to +\infty} x(t) = 0 \right\} \tag{8}$$

Suppose that $V(x(t))$ is a Lyapunov function for the origin in (2). For a domain $\mathbf{U} \subset \mathbb{R}^{n_x}$ contains the origin, $V(x(t)) : \mathbf{U} \to \mathbb{R}$ is a continuously differentiable function, $V(0) = 0$ and $V(x(t)) > 0, \forall x(t) \in \mathbf{U} - \{0\}$ such that the time derivative of $V(x(t))$ along the trajectories of (2) is locally negative definite. Then, for a real number $c > 0$, the sublevel set

$$\mathbf{\Omega}(c) := \{x(t) \in \mathbb{R}^{n_x} | V(x(t)) \leq c\} \tag{9}$$

is an inner estimate of the DA, for example, $\mathbf{\Omega}(c) \subseteq \mathbf{D}$ if $\mathbf{\Omega}(c) \subseteq \mathbf{U}$ [37]. Besides, $c \leq c^* = max\{c \in \mathbb{R} | \mathbf{\Omega}(c) \subseteq \mathbf{U}\}$.

Lemma 2.1. *[33] Let the symmetric matrices $\Upsilon_{ij}, i, j \in \mathbb{I}$. Inequality $\Upsilon_{h(t)h(t)} < 0$ holds if the following condition is fulfilled:*

$$\Upsilon_{ij} + \Upsilon_{ji} < 0, \quad i \leq j, \quad i, j \in \mathbb{I}$$

3. Main results

In this section, our goal is to obtain the stability conditions and the sufficient conditions for the solution of state feedback control problem for the T–S fuzzy system (2). Two sections are given to show the stability analysis and solution of stabilization problem, respectively.

Before continuing, it should be noted that instead of the traditional common Lyapunov function $V(x(t))$, in this paper, the switched Lyapunov function $V_{\sigma(t)}(x(t))$ is introduced for the T–S fuzzy systems. $\sigma(t) \in \mathbb{I}$ is a switching signal, which is a piecewise constant function. $\sigma(t) = l$ means that $V_l(x(t))$ is applied. The switching instants are expressed by a sequence $\{t_0, t_1, \ldots, t_k, \ldots\}$ where $t_0 = 0$ denotes the initial time, t_k denotes the kth switching instant and the l_kth subsystem is activated when $t \in [t_k,\ t_{k+1})$. Besides, it is assumed that t_k^- denotes the last time instant before the kth switching.

Remark 1. Although a membership-function-dependent switching method has been proposed in [29], this method cannot be applied to stability analysis of the T–S fuzzy systems for that only the common Lyapunov function is considered. While the switching law in [29] may be arbitrary, the switched Lyapunov function is unsuitable. To solve this problem, a new switching law is proposed first.

3.1. Stability analysis

In this part, setting $u(t) \equiv 0$, the T–S fuzzy system (2) becomes

$$\dot{x}(t) = A_{h(t)}x(t) \tag{10}$$

Consider the following membership-function-dependent switching law

$$\sigma(t) = \begin{cases} \arg\max_{i \in \mathbb{I}} h_i(0), & t = 0 \\ j, & \mu h_j(t) \geq \max_{i \in \mathbb{I}} h_i(t) \\ \arg\max_{i \in \mathbb{I}} h_i(t), & \mu h_j(t) < \max_{i \in \mathbb{I}} h_i(t) \end{cases} \tag{11}$$

where $j = \sigma(t^-)$, t^- represents the previous time step, $\mu \geq 1$ is a relaxation parameter.

Assumption 1. For any switching signal $\sigma(t)$, there exists a real number $T_d \geq 0$ such that,

$$\inf\{t_{k+1} - t_k\} \geq T_d \tag{12}$$

Remark 2. First, a minimal dwell time δt satisfying the following equation is introduced:

$$\delta t = \frac{\mu - 1}{r\phi(\mu + 1)} \tag{13}$$

where $\phi = \min_{i\in\mathbb{I}} \phi_i$. The switching signal (11) yields that $h_{\sigma(t_k)}(t_k) = \max_{i\in\mathbb{I}} h_i(t_k) \geq \frac{1}{r}$. Then, considering the switching signal (11) and assumption (4), if $t - t_k \leq \delta t$, we have

$$\begin{aligned}\mu h_{\sigma(t_k)}(t) &\geq \mu(h_{\sigma(t_k)}(t_k) - (t-t_k)\phi)\\ &\geq \max_{i\in\mathbb{I}} h_i(t_k) + (\mu-1)h_{\sigma(t_k)}(t_k) - \mu(t-t_k)\phi\\ &\geq \max_{i\in\mathbb{I}} h_i(t_k) + \phi(1+\mu)\frac{\mu-1}{r\phi(\mu+1)} - \mu(t-t_k)\phi\\ &\geq \max_{i\in\mathbb{I}} h_i(t_k) + \phi(1+\mu)(t-t_k) - \mu(t-t_k)\phi\\ &\geq \max_{i\in\mathbb{I}} h_i(t_k) + (t-t_k)\phi \geq \max_{i\in\mathbb{I}} h_i(t)\end{aligned}\tag{14}$$

which means no switching between $[t_k, t)$. It is obvious that $\inf\{t_{k+1} - t_k\} \geq T_d \geq \delta t$.

Remark 3. Obviously, the switching law (11) yields $\mu h_{\sigma(t)}(t) \geq h_i(t), \forall i \in \mathbb{I}$. If $\mu = 1$, the switching law (11) for the T–S fuzzy systems is reduced to the existing switching law in [29–31]. Obviously, $\mu > 1$ will ensure that $T_d \geq \delta t > 0$ which means that the switching signal (11) has a minimal dwell-time. Therefore, under this switching law, the average dwell time method or the DLF technique can be adopted. As shown in [32], DLF technique is a more effective way to deal with dwell time switching. Therefore, the DLF technique is adopted in this paper.

Theorem 3.1. *For given $\mu > 1$, $T_d > 0$ and $K > 0$, if there exist symmetric positive definite matrices $P_{jl,n}$, $R_{ijl} = R_{jil}^T$, symmetric matrices $M_{il,n}$ such that*

$$P_{jl,0} < P_{jq,K}, \quad l \neq q \tag{15}$$

$$P_{jl,n} - M_{il,n} > 0 \tag{16}$$

$$J_{ijl,mm} + J_{jil,mm} < 0, \quad i \leq j \tag{17}$$

$$J_{ijl,(m+1)m} + J_{jil,(m+1)m} < 0, \quad i \leq j \tag{18}$$

$$J_{ijl,K} + J_{jil,K} < 0, \quad i \leq j \tag{19}$$

where $J_{ijl,nm} = He(P_{jl,n}A_i) + \sum_{q=1}^{r}\phi_q(P_{ql,n} - M_{il,n}) + \frac{K}{T_d}(P_{jl,m+1} - P_{jl,m}) + a_{il}\mu\sum_{q=1}^{r} R_{qjl} - R_{ijl}$, $J_{ijl,K} = He(P_{jl,K}A_i) + \sum_{q=1}^{r}\phi_q(P_{ql,K} - M_{il,K}) + a_{il}\mu\sum_{q=1}^{r} R_{qjl} - R_{ijl}$, $a_{il} = 0 (i \neq l)$ and $a_{il} = 1 (i = l)$. $i, j \in \mathbb{I}$, $n = 0, 1, \ldots, K$, $m = 0, 1, \ldots, K-1$. Then the T–S system (10) is asymptotically stable. Moreover, $\mathbf{\Omega}(c^*)$ is an inner estimate of the DA, where $c^* = \max\{c \in \mathbb{R} | \mathbf{\Omega}(c) \subseteq \mathbf{R} \cap \mathbf{C}\}$.

Proof. Please see Appendix A. □

Remark 4. With the help of the minimal dwell time of the switching law (11), the DLF technique is adopted for less conservatism in Theorem 3.1. Based on (11), "$a_{il}\mu\sum_{q=1}^{r} R_{qjl_k} - R_{ijl_k}$" are introduced for a relaxation. Compared to the existing switching method in [29] where the Lyapunov function is non-switched, in this paper, the switched Lyapunov function contributes to less conservatism.

In Theorem 3.1, the upper bounds of the time derivative of the membership functions ϕ_i should be available. However, sometimes, the bounds of the time derivative of the membership functions may be unavailable. Therefore, the following corollary is proposed for this case.

Corollary 3.2. *For given $\mu > 1$, $T_d > 0$ and $K > 0$, if there exist symmetric positive definite matrices $P_{l,n}$, R_{il} such that*

$$P_{l,0} < P_{q,K}, \quad l \neq q \tag{20}$$

$$J_{il,mm} < 0 \tag{21}$$

$$J_{il,(m+1)m} < 0 \tag{22}$$

$$J_{il,K} < 0 \tag{23}$$

where $J_{il,nm} = He(P_{l,n}A_i) + \frac{K}{T_d}(P_{l,m+1} - P_{l,m}) + a_{il}\mu\sum_{q=1}^{r} R_{ql} - R_{il}$, $J_{il,K} = He(P_{l,K}A_i) + a_{il}\mu\sum_{q=1}^{r} R_{ql} - R_{il}$, $a_{il} = 0 (i \neq l)$ and $a_{il} = 1 (i = l)$. $i \in \mathbb{I}$, $n = 0, 1, \ldots, K$, $m = 0, 1, \ldots, K-1$. Then the T-S system (10) is asymptotically stable. Moreover, $\mathbf{\Omega}(c^*)$ is an inner estimate of the DA, where $c^* = \max\{c \in \mathbb{R} | \mathbf{\Omega}(c) \subseteq \mathbf{C}\}$.

Proof. Let us consider the following DLF as a Lyapunov function candidate.

$$V(x(t)) = x^T(t)P_{\sigma(t)}(t)x(t) \tag{24}$$

Based on the proof of Theorem 3.1, it is easy to obtain Corollary 3.2. □

Remark 5. The new proposed method can provide less conservative results than the existing methods without considering the switched Lyapunov function. In fact, if choose $P_{jl,n} = P_j$, $R_{ijl} = 0$ in the condition of Theorem 3.1, it reduces to Theorem 1 in [10] which considers the fuzzy Lyapunov function. Besides, if choose $P_{l,n} = P$, $R_{il} = 0$ in the condition of Theorem 3.2, it reduces to Theorem 1 in [40] which considers the common quadratic Lyapunov function.

Usually the upper bounds for the time-derivatives of the membership functions over the global region are not easy to obtain. Therefore, to obtain the inner estimate of the DA $\mathbf{\Omega}(c^*)$, the values ϕ_i, $i \in \mathbb{I}$ are computed in a prescribed region such as the following hyper-rectangle in the state-space:

$$\mathbf{X}(\delta) = \{x(t) \in \mathbb{R}^{n_x} | \ | x_j(t) | \leq \delta_j, j \in \mathbb{N}\} \subseteq \mathbf{C}$$

where δ_i are positive real numbers and $\delta = [\delta_1, \ldots, \delta_{n_x}]^T$. Then the inner estimate of the DA $\mathbf{\Omega}(c^*)$ can be obtained by the following algorithm.

Algorithm 1.

Step 1: Set $k = 1$. Initialize parameters $\delta_j^k > 0$, $\Delta_j > 0$, $\mu > 1$ and $e > 0$ small enough. $\delta^k = [\delta_1^k, \ldots, \delta_{n_x}^k]^T$.

Step 2: Over the prescribed region $\mathbf{X}(\delta^k)$, compute the upper bounds of the time derivative of the membership functions $h_i(t)$ via a gridding procedure.

$$\phi_i^k = \max_{x(t) \in \mathbf{X}(\delta^k)} |\dot{h}_i(t)|, i \in \mathbb{I} \tag{25}$$

Step 3: Considering the switching signal (11), compute the minimal dwell time with $\phi^k = \max_{i \in \mathbb{I}} \phi_i^k$:

$$T_d^k \geq \delta t^k = \frac{\mu - 1}{r\phi^k(\mu + 1)} \tag{26}$$

Step 4: Based on the values ϕ_i^k and T_d^k obtained in Steps 2 and 3, solve the LMI problem in Theorem 3.1.

(i) If the LMI problem is feasible, set $k = k + 1$, $\delta_j^k = \delta_j^{k-1} + \Delta_j$.

- If $\exists j$ $\delta_j^k > \bar{x}_j$, go to Step 5.
- If $\forall j$ $\delta_j^k \leq \bar{x}_j$, return to Step 2.

(ii) If the LMI problem is infeasible, set $\Delta_j = \frac{1}{2}\Delta_j$, $\delta_j^k = \delta_j^k - \Delta_j$.

- If $\forall j$ $\Delta_j \leq e$, go to Step 5.
- If $\exists j$ $\Delta_j > e$, return to Step 2.

Step 5: $\phi_i^* = \phi_i^{k-1}$, $T_d^* \geq \delta t^* = \delta t^{k-1}$, $\delta_j^* = \delta_j^{k-1}$ and compute c^* via a gridding procedure

$$c^* = \max\{c \in \mathbb{R} | \mathbf{\Omega}(c) \subseteq \mathbf{X}(\delta^*)\} \tag{27}$$

Remark 6. Considering (6) and (7), for the fact that $|\dot{h}_i(t)| \leq \phi_i^*$ and $\delta_j^* \leq \bar{x}_j$, it is obvious that $\mathbf{X}(\delta^*) \subseteq \mathbf{C}$ and $\mathbf{X}(\delta^*) \subseteq \mathbf{R}$. Therefore, $\mathbf{X}(\delta^*) \subseteq \mathbf{R} \cap \mathbf{C}$ and $\mathbf{\Omega}(c^*)$ is an inner estimate of the DA.

3.2. Stabilization

In this part, the state-feedback control problem of the T–S fuzzy system (2) is considered. Based on the stability analysis in Section 3.1, a new switched non-PDC controller is introduced for less conservatism.

The switched non-PDC controller is designed as follows

$$u(t) = K_{\sigma(t)}(t)x(t) \tag{28}$$

and the closed-loop system of (2) becomes

$$\dot{x}(t) = \bar{A}_{h(t)\sigma(t)}(t)x(t) = (A_{h(t)} + B_{h(t)}K_{\sigma(t)}(t))x(t) \tag{29}$$

To stabilize the closed-loop system (29), the following theorem provides the sufficient conditions for existence of the desired controller gain $K_{\sigma(t)}(t)$.

Theorem 3.3. *For given $\mu > 1$, $T_d > 0$ and $K > 0$, if there exist symmetric positive definite matrices $X_{jl,n}$, $\bar{R}_{ijl} = \bar{R}_{jil}^T$, symmetric matrices $\bar{M}_{il,n}$, matrices $S_{jl,n}$ such that*

$$X_{jl,K} < X_{jq,0}, \quad l \neq q \tag{30}$$

$$X_{jl,n} - \bar{M}_{il,n} > 0 \tag{31}$$

$$\bar{J}_{ijl,mm} + \bar{J}_{jil,mm} < 0 \tag{32}$$

Table 1
Max value λ_* of λ for $\phi_i = 1.5$.

	Theorem 1 in [40]	Corollary 3.2	Theorem 1 in [41]	Theorem 3.1
λ^*	2.49	$T_d = 2$: 4.09 $T_d = 1$: 3.57 $T_d = \delta t$: 2.71	4.64	$T_d = 2$: 5.15 $T_d = 1$: 5.12 $T_d = \delta t$: 5.09

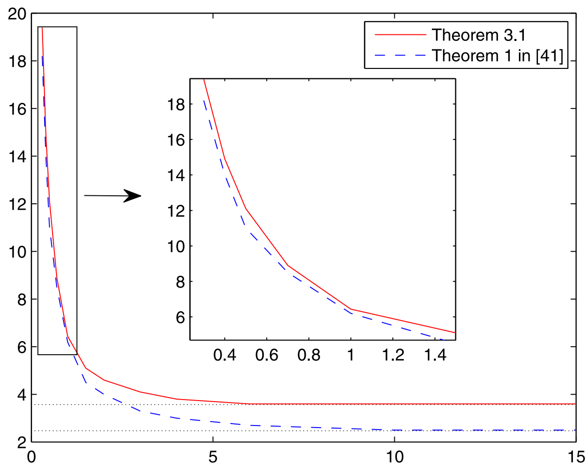


Fig. 1. Allowable upper bounds of λ for different ϕ_i.

$$\bar{J}_{ijl,(m+1)m} + \bar{J}_{jil,(m+1)m} < 0 \tag{33}$$

$$\bar{J}_{ijl,K} + \bar{J}_{jil,K} < 0 \tag{34}$$

where $\bar{J}_{ijl,nm} = He(A_iX_{jl,n} + B_iS_{jl,n}) + \sum_{q=1}^{r}\phi_q(X_{ql,n} - \bar{M}_{il,n}) - \frac{K}{T_d}(X_{jl,m+1} - X_{jl,m}) + a_{il}\mu\sum_{q=1}^{r}\bar{R}_{qjl} - \bar{R}_{ijl}$, $\bar{J}_{ijl,K} = He(A_iX_{jl,K} + B_iS_{jl,K}) + \sum_{q=1}^{r}\phi_q(X_{ql,K} - \bar{M}_{il,K}) + a_{il}\mu\sum_{q=1}^{r}\bar{R}_{qjl} - \bar{R}_{ijl}$, $a_{il} = 0(i \neq l)$ *and* $a_{il} = 1(i = l)$. $i, j \in \mathbb{I}$, $n = 0, 1, \ldots, K$, $m = 0, 1, \ldots, K-1$. *Then the closed-loop T–S system (29) with the switched non-PDC controller (28) is asymptotically stable under the switching signal (11).*

$$X_{h(t)\sigma(t)}(t) = \begin{cases} (1-\beta)X_{h(t)\sigma(t),m} + \beta X_{h(t)\sigma(t),m+1}, & t \in [t_k + \tau_m, t_k + \tau_{m+1}) \\ X_{h(t)\sigma(t),K}, & t \in [t_k + \tau_K, t_{k+1}) \end{cases}, \quad k = 0, 1, 2, \ldots$$

$$S_{h(t)\sigma(t)}(t) = \begin{cases} (1-\beta)S_{h(t)\sigma(t),m} + \beta S_{h(t)\sigma(t),m+1}, & t \in [t_k + \tau_m, t_k + \tau_{m+1}) \\ S_{h(t)\sigma(t),K}, & t \in [t_k + \tau_K, t_{k+1}) \end{cases}, \quad k = 0, 1, 2, \ldots$$

$$K_{\sigma(t)}(t) = S_{h(t)\sigma(t)}(t)X_{h(t)\sigma(t)}^{-1}(t) \tag{35}$$

Moreover, $\boldsymbol{\Omega}(c^*)$ *is an inner estimate of the DA, where* $c^* = \max\{c \in \mathbb{R} | \boldsymbol{\Omega}(c) \subseteq \mathbf{R} \cap \mathbf{C}\}$.

Proof. Please see Appendix B. □

Remark 7. Theorem 3.3 presents an LMI-based condition for designing state feedback non-PDC controller for continuous time T–S fuzzy systems. In fact, if choose $X_{jl,n} = X_{jq,m}(l, q \in \mathbb{I}, 0 \le n, m \le K)$, $S_{jl,n} = S_{jq,m}(l, q \in \mathbb{I}, 0 \le n, m \le K)$ and $\bar{R}_{ijl} = 0$, $i, j \in \mathbb{I}$ in the condition of Theorem 3.3, it reduces to the LMI-based controller design approach via the non-PDC method in [27].

4. Examples

In this section, three examples are presented to illustrate the effectiveness of the proposed method in this paper.

Example 1. Consider the T–S fuzzy systems consisting of two subsystems [39,41] described by

$$A_1 = \begin{bmatrix} 0 & 2 \\ -2 & -1 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0 & 2 \\ -2-\lambda & -1 \end{bmatrix}$$

In order to show the advantage of the proposed method, the maximum value λ^* of λ such that the stability is guaranteed is computed. It is assumed that $\phi_i = 1.5$. The allowable upper bounds of λ by using Theorem 3.1 and Corollary 3.2 with $\mu = 1.1$, $\delta t = \frac{\mu-1}{r\Phi(\mu+1)} = 0.0159$ are compared with the Theorem 1 in [41] and Theorem 1 in [40] as shown in Table 1.

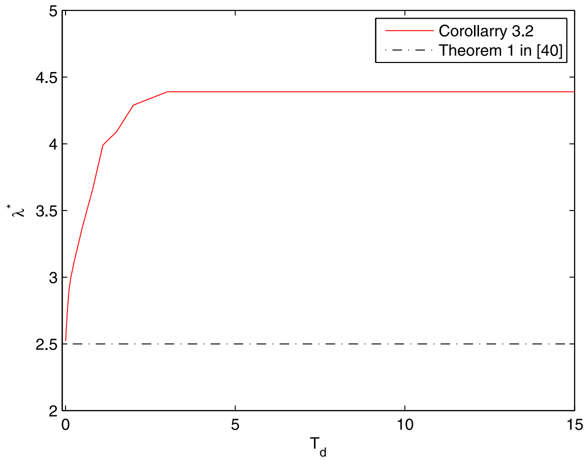


Fig. 2. Allowable upper bounds of λ for different T_d.

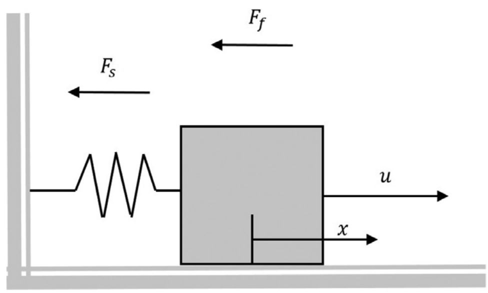


Fig. 3. Mass-spring-damping system.

Fig. 1 shows that the switched fuzzy Lyapunov function method is less conservative than the norm fuzzy Lyapunov function, especially the membership-function-dependent switching law method leads to more improvement when ϕ_i is larger. From Table 1 and Fig. 2, compared to the common Lyapunov function method in [40], using the proposed switched Lyapunov function method offers the more relaxed results. Besides, the switched Lyapunov function method provides the less conservatism with the larger T_d.

Example 2. Consider the mass-spring-damping system [37] which is shown in Fig. 3 and according to Newton's law, it can be described by

$$m\ddot{x} + F_f + F_s = u(t)$$

where m stands for the mass, F_f is the friction force, F_s and u denote the restoring force of the spring and the external control input, respectively. In practice, F_f and F_s are the nonlinear or uncertain terms. In this paper, we assume that $u(t) = 0$, $F_f = c\dot{x}$ and $F_s = bx + ba^2x^3$. Then the dynamic equation can be written as

$$m\ddot{x} + c\dot{x} + bx + ba^2x^3 = 0$$

Consider the numerical values: $m = 4$, $c = 1$, $b = 4$ and $a = 0.5$. With $x_1 = x$ and $x_2 = \dot{x}$ as state variables, the nonlinear state-space equation is given by

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{b}{m} - \frac{ba^2}{m}x_1^2 & -\frac{c}{m} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Throughout simple algebraic manipulations, this system can be described by the T–S fuzzy system (10) in the set $\mathbf{C} = \{x \in \mathbb{R}^{n_x} \,|\, |x_1| \le \beta\}$, and

$$A_1 = \begin{bmatrix} 0 & 1 \\ -\frac{b}{m} - \frac{ba^2}{m}\beta^2 & -\frac{c}{m} \end{bmatrix}, \quad A_1 = \begin{bmatrix} 0 & 1 \\ -\frac{b}{m} & -\frac{c}{m} \end{bmatrix}, \quad h_1(t) = \frac{x_1^2}{\beta^2}, \quad h_2(t) = \frac{\beta^2 - x_1^2}{\beta^2}$$

Setting $\beta = 2$, compute the inner estimate of the DA $\mathbf{\Omega}(c^*)$ by using Algorithm 1.

Step 1: $k = 1$, $\mu = 1.1$, $\delta_1^1 = \delta_2^1 = 0.5$, $\Delta_1 = \Delta_2 = 0.2$ and $e = 0.005$.

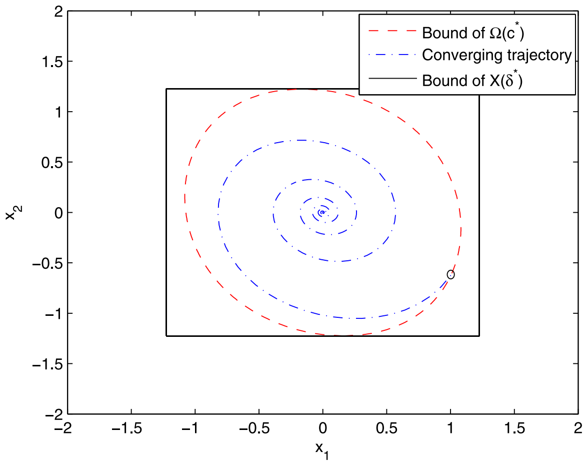


Fig. 4. The estimate of the DA obtained by Theorem 1 in [10].

Step 2: Compute the upper bounds of the time derivative of the membership functions $h_i(t)$ via a gridding procedure.

$$\phi_i^k = \max_{x \in \mathbf{X}(\delta^k)} |\dot{h}_i(t)|, i \in \mathbb{I}$$

where

$$\dot{h}_1(t) = \frac{2x_1\dot{x}_1}{\beta^2} = \frac{2x_1}{\beta^2}[1\ 0]\sum_{i=1}^{2} h_i(t)A_i\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \frac{2x_1x_2}{\beta^2}$$

$$\dot{h}_2(t) = -\frac{2x_1\dot{x}_1}{\beta^2} = -\frac{2x_1}{\beta^2}[1\ 0]\sum_{i=1}^{2} h_i(t)A_i\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = -\frac{2x_1x_2}{\beta^2}$$

For example, $k = 1$, then $\mathbf{X}(\delta^k) = \{x \in \mathbb{R}^{n_x} | |x|_j \le \delta_j^1, j = 1, 2\}$, $\phi_1^1 = \max_{x \in \mathbf{X}(\delta^k)} |\dot{h}_i(t)| = 0.125$ and $\phi_2^1 = \max_{x \in \mathbf{X}(\delta^k)} |\dot{h}_i(t)| = 0.125$.

Step 3: Compute the minimal dwell time with $\phi^k = \max_{i \in \mathbb{I}} \phi_i^k$:

$$T_d^k \ge \delta t^k = \frac{\mu - 1}{r\phi^k(\mu + 1)} \tag{36}$$

For example, $k = 1$, then $\phi^1 = 0.125$ and set $T_d^1 = \delta t^1 = 0.1905$.

Step 4: Based on the values ϕ_i^k and T_d^k obtained in Steps 2 and 3, solve the LMI problem in Theorem 3.1. Then based on the different results, update δ_j^k or Δ_j and go to Step 2 or Step 5.

For example, $k = 1$, the LMI problem in Theorem 3.1 is feasible. Then set $k = k + 1 = 2$, $\delta_j^2 = \delta_j^1 + \Delta_j = 0.7$, $j = 1, 2$. For $\delta_1^2 \le \beta$ which means that $\mathbf{X}(\delta^2) \subseteq \mathbf{C}$, return to Step 2.

$k = 5$, $\delta_j^5 = 1.3$, $(j = 1, 2)$, $\phi_i^5 = 0.854$, $T_d^5 = 0.0282$, the problem in Theorem 3.1 is infeasible. Then set $\Delta_j = \frac{\Delta_j}{2} = 0.1 > e$, $(j = 1, 2)$, $\delta_j^5 = \delta_k^5 - \Delta_j = 1.2$, return to Step 2.

$k = 9$, $\delta_j^9 = 1.2937$, $(j = 1, 2)$, $\phi_i^5 = 0.841$, $T_d^5 = 0.0283$, the problem in Theorem 3.1 is infeasible. Then set $\Delta_j = \frac{\Delta_j}{2} = 0.0031 < e$, $(j = 1, 2)$, $\delta_j^5 = \delta_k^5 - \Delta_j = 1.2906$, go to Step 5.

Step 5: $\phi_i^* = \phi_i^{k-1}$, $T_d^* > \delta t^* = \delta t^{k-1}$, $\delta_j^* = \delta_j^{k-1}$ and compute c^* via a gridding procedure.

$k = 9$, $\delta_j^* = \delta_j^8 = 1.2875$, $\phi_i^* = \phi_i^8 = 0.8329$, $T_d^* > \delta t^* = \delta t^8 = 0.0286$. Besides, denoting $\bar{\mathbf{X}}(\delta^*)$ as the bound of $\mathbf{X}(\delta^*)$ and $\bar{\mathbf{C}}$ as the bound of $\mathbf{C}$, it is obvious that $\bar{\mathbf{X}}(\delta^*) \cap \bar{\mathbf{C}} \ne \varnothing$. Therefore (27) is equivalent to

$$C^* = \min\{V_{\sigma(t)(x)} | x \in \bar{\mathbf{X}}(\delta^*)\} \tag{37}$$

With (37), via a gridding procedure, $c^* = 0.4180$.

The boundaries of $\mathbf{x}(\delta)$(solid line), $\mathbf{\Omega}(c)$(dash line), and a converging trajectory (dashed-dotted line) that are initialized at the "∘" mark are shown in Figs. 4 and 5. Fig. 5 shows the effectiveness of Algorithm 1. The inner estimate of the DA $\mathbf{\Omega}(c)$ increases with k and when $k = 8$, the largest boundary of $\mathbf{\Omega}(c^*)$ is obtained. Based on the aforementioned, $\mathbf{X}(\delta^*) = \{x \in \mathbb{R}^{n_x} | |x|_j \le 1.2875, j = 1, 2\}$ using Algorithm 1 and $\mathbf{X}(\delta^*) = \{x \in \mathbb{R}^{n_x} | |x|_j \le 1.225, j = 1, 2\}$ using the algorithm with Theorem 1 in [10]. Besides, from Figs. 4 and 5(d), it is obvious that the bound of $\mathbf{\Omega}(c^*)$ in Fig. 5(d) is larger than Fig. 4.

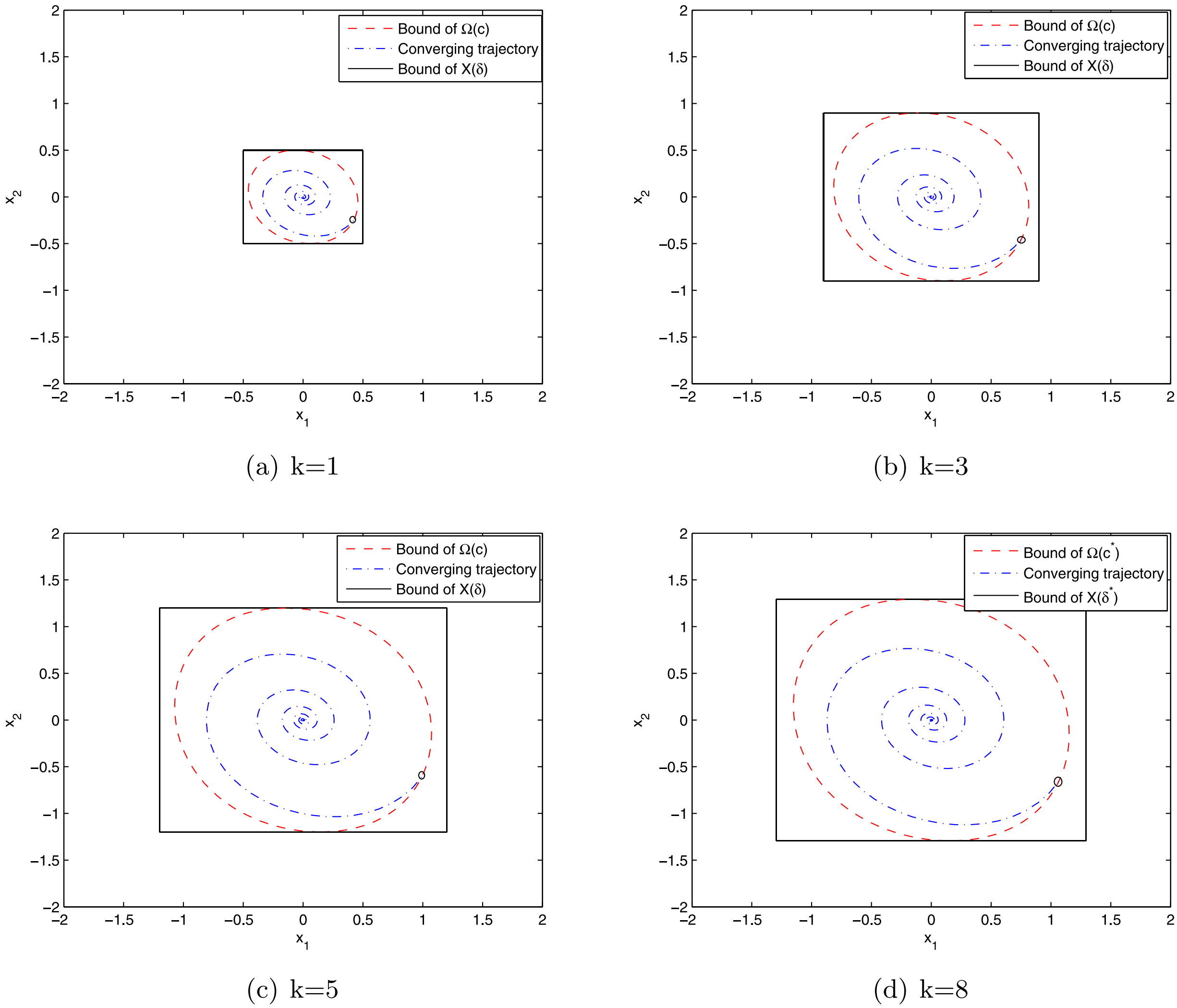


Fig. 5. The estimate of the DA obtained by Algorithm 1.

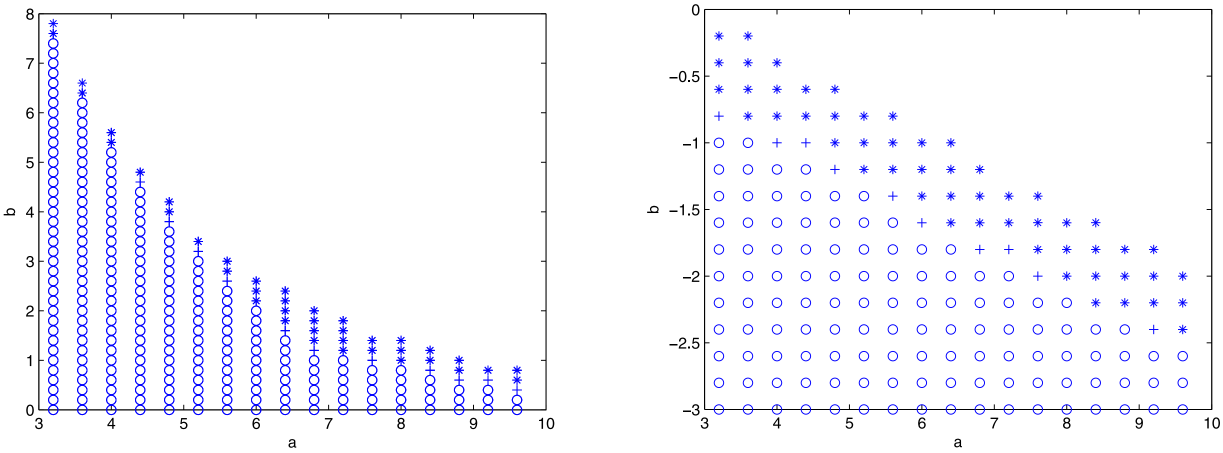
Remark 8. Compared to the existing algorithm in [10], Algorithm 1 has the adjustable step length. The major advantage of the adjustable step length is that the proposed algorithm has much less iterative steps than the existing algorithm. For this example, Algorithm 1 needs 9 steps while the existing algorithm in [10] with $\Delta = 0.01$ needs 81 steps.

Example 3. Consider the T–S fuzzy system (2) with the following system matrices [10]

$$A_1 = \begin{bmatrix} a & -4 \\ -1 & -2 \end{bmatrix}, \quad A_2 = \begin{bmatrix} -2 & 4 \\ 20 & -2 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 1 \\ 10 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 1 \\ -b \end{bmatrix}$$
$$h_1(t) = \frac{1 + \sin x_1(t)}{2}, \quad h_2(t) = 1 - h_1(t), \quad \mathbf{C} = \left\{ x(t) \in \mathbb{R}^n \middle| |x_i(t)| \le \frac{\pi}{10}, i \in \mathbb{N} \right\} \tag{38}$$

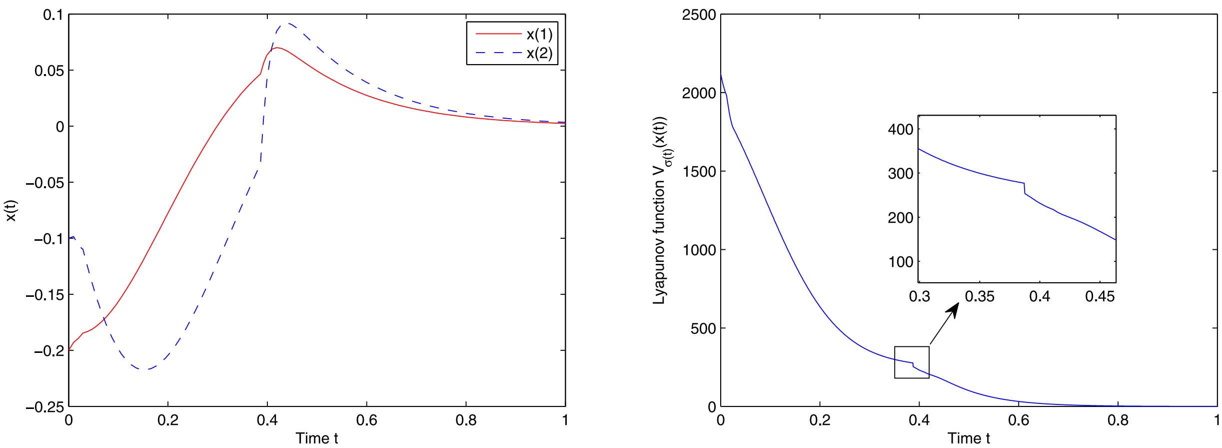
The stability of the closed-loop system was checked for several values of pairs (a, b), $a \in [3, 10]$ and $b \in [-3, 8]$. Assume that $\phi_i = 1, i = 1, 2$ and set $\mu = 1.1$ and $T_d = \delta t = \frac{\mu - 1}{r\phi(\mu+1)} = 0.0238$. The results of Theorem 5 in [10], switched fuzzy PDC control method in [3] and Theorem 3.1 are shown in Fig. 6(a). The results of PDC control method in [4], switched PDC control method in [3] and Corollary 3.2 are shown in Fig. 6(b). As can be seen, the methods involving the switched controllers provide larger feasible regions than the existing method. Besides, the method with the switched Lyapunov function proposed in this paper obtain much less conservative results than the results in [3].

Set $a = 8$, $b = 1.3$, by Theorem 3.1, the results with $x(0) = [-0.2 \quad -0.1]$ are shown in Fig. 7. We can see that with the proposed switched non-PDC controller, system (2) with the system matrices in (38) is asymptotically stable under the switching signal (11). Besides, Fig. 7(c) shows membership functions and the switching instant is marked. The switched Lyapunov function is shown in Fig. 7(b). We can find that although the Lyapunov function is discontinuous at the switching

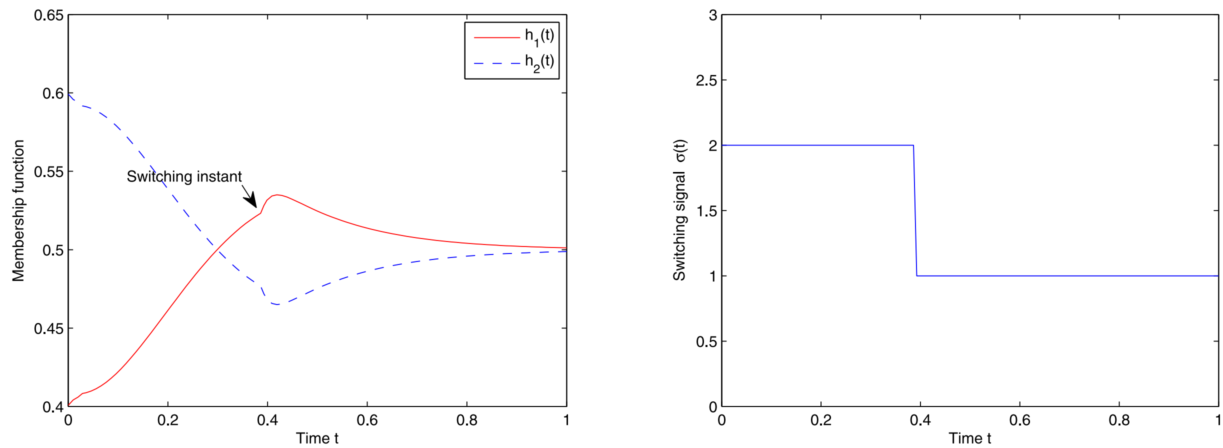


(a) Fuzzy Lyapunov function method (b) Common Lyapunov function method

Fig. 6. (a) Stabilization region based on Theorem 3.1(*,+,○), [3](+,○) and [10](○), (b) stabilization region based on Corollary 3.2(*,+,○), [3](+,○) and [4](○).



(a) State responses (b) The switched Lyapunov function



(c) Membership functions (d) Switching signal

Fig. 7. The closed-loop system with the switched non-PDC controller.

instant, the switching leads to decrease of the Lyapunov function. Therefore, the Lyapunov function decreases strictly overall the running time.

5. Conclusions

In this paper, the problem of stability analysis and designing state feedback controllers for the T–S fuzzy systems has been studied. For stability analysis, based on a new membership-function-dependent switching law, a kind of switched DLF has been introduced for less conservatism. It is proved that there exists a minimal dwell time between two consecutive switching. Based on the switched Lyapunov function, a new stability criterion is obtained, and an algorithm to estimate the DA is presented. Then a switched non-PDC controller is proposed based on the DLF technique, and the design conditions are given in terms of LMIs. Two numerical examples and a real system example have been provided to demonstrate the effectiveness of the proposed method. It is shown that the new method is less conservative than the corresponding stability analysis and controller design methods. In our future work, with the proposed dwell-time constrained switching law, some other techniques for the switched systems, such as the dwell time method, can be applied on the T–S fuzzy system.

Appendix A

Proof. Let us consider the following fuzzy discretized Lyapunov function(FDLF) as a Lyapunov function candidate.

$$V_{\sigma(t)}(x(t)) = x^T(t)P_{h(t)\sigma(t)}(t)x(t) \tag{39}$$

Based on the aforementioned, $t_{k+1} - t_k \geq T_d$, $k = 0, 1, 2, \ldots$. Then divide the interval $[t_k, t_k + T_d)$ into K segments described as $[t_k + \tau_m, t_k + \tau_{m+1})$ where $\tau_m = m\Delta T$ and $\Delta T = \frac{T_d}{K}$, $m = 0, 1, 2, \ldots, K-1$. Letting $P_{h(t)l_k}(t_k + \tau_m) = P_{h(t)l_k,m}$ and using a linear interpolation formula, it is obtained that

$$P_{h(t)l_k}(t) = P_{h(t)l_k}(t_k + \tau_m + \beta\Delta T) = (1-\beta)P_{h(t)l_k,m} + \beta P_{h(t)l_k,m+1}, \quad t \in [t_k + \tau_m, t_k + \tau_{m+1}) \tag{40}$$

where $\beta = \frac{t - t_k - \tau_m}{\Delta T}$. Besides, $P_{h(t)l_k}(t) = P_{h(t)l_k,K}$ for $t \in [t_k + \tau_K, t_{k+1})$. Then (39) becomes

$$V_{\sigma(t)}(x(t)) = \begin{cases} x^T(t)((1-\beta)P_{h(t)\sigma(t),m} + \beta P_{h(t)\sigma(t),m+1})x(t), & t \in [t_k + \tau_m, t_k + \tau_{m+1}) \\ x^T(t)P_{h(t)\sigma(t),K}x(t), & t \in [t_k + \tau_K, t_{k+1}) \end{cases} \quad k = 0, 1, 2, \ldots \tag{41}$$

In order to show that the time-derivative of (39) along (2) is negative:

$$\begin{aligned} \dot{V}_{\sigma(t)}(x(t)) &= 2\dot{x}^T(t)P_{h(t)\sigma(t)}(t)x(t) + x^T(t)\dot{P}_{h(t)\sigma(t)}(t)x(t) \\ &= x^T(t)(He(P_{h(t)\sigma(t)}(t)A_{h(t)}) + \dot{P}_{h(t)\sigma(t)}(t))x(t) < 0 \end{aligned} \tag{42}$$

the following proof is divided into three parts.

First, for $t \in [t_k + \tau_m, t_k + \tau_{m+1})$, $k = 0, 1, 2, \ldots$, considering (41), the following equation can be derived from $\dot{\beta} = \frac{1}{\Delta T} = \frac{K}{T_d}$:

$$\dot{P}_{h(t)l_k}(t) = \frac{K}{T_d}(P_{h(t)l_k,m+1} - P_{h(t)l_k,m}) + \sum_{q=1}^{r}\dot{h}_q(t)P_{ql_k}(t) \tag{43}$$

where $P_{ql_k}(t) = (1-\beta)P_{ql_k,m} + \beta P_{ql_k,m+1}$. Then (42) is guaranteed by

$$\sum_{i=1}^{r}\sum_{j=1}^{r}h_i(t)h_j(t)\left(He(P_{jl_k}(t)A_i) + \sum_{q=1}^{r}\dot{h}_q(t)P_{ql_k}(t) + \frac{K}{T_d}(P_{jl_k,m+1} - P_{jl_k,m})\right) < 0 \tag{44}$$

Setting that $M_{il_k}(t)$ has the same structure as $P_{il_k}(t)$, then (16) yields $P_{ql_k}(t) - M_{il_k}(t) > 0$. Considering (3), we have

$$\sum_{q=1}^{r}\dot{h}_q(t)P_{ql_k}(t) = \sum_{q=1}^{r}\dot{h}_q(t)(P_{ql_k}(t) + M_{il_k}(t)) \leq \sum_{q=1}^{r}\phi_q(P_{ql_k}(t) + M_{il_k}(t)) \tag{45}$$

Besides, considering (11) and the fact that $R_{ijl_k} > 0$, one can obtain

$$\sum_{i=1}^{r}\sum_{j=1}^{r} h_i(t)h_j(t)(a_{il_k}\mu\sum_{q=1}^{r}R_{qjl_k} - R_{ijl_k}) = \begin{bmatrix} h_1(t)I_{r\times r} \\ \vdots \\ h_r(t)I_{r\times r}\end{bmatrix}^T (\mu E^T_{r\otimes r,l_k} - I_{r\otimes r})[R_{ijl_k}]_{r\times r}\begin{bmatrix} h_1(t)I_{r\times r} \\ \vdots \\ h_r(t)I_{r\times r}\end{bmatrix}$$
$$= \begin{bmatrix} (\mu h_{l_k}(t) - h_1(t))I_{r\times r} \\ \vdots \\ (\mu h_{l_k}(t) - h_r(t))I_{r\times r}\end{bmatrix}^T [R_{ijl_k}]_{r\times r}\begin{bmatrix} h_1(t)I_{r\times r} \\ \vdots \\ h_r(t)I_{r\times r}\end{bmatrix} > 0 \tag{46}$$

where $I_{r\times r} \in \mathbb{R}^{r\times r}$ is unit matrix and

$$I_{r\otimes r} = \begin{bmatrix} I_{r\times r} & \cdots & 0 \\ * & \ddots & \vdots \\ * & * & I_{r\times r}\end{bmatrix} \in \mathbb{R}^{r^2\times r^2},$$

$$E_{r\otimes r,l_k} = \begin{array}{c}\begin{bmatrix} 0 & \cdots & I_{r\times r} & \cdots & 0 \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & \cdots & I_{r\times r} & \cdots & 0 \end{bmatrix} \\ \begin{matrix} \downarrow \\ l_k \end{matrix}\end{array} \in \mathbb{R}^{r^2\times r^2}.$$

Thus (44) holds if the following inequality holds

$$\sum_{i=1}^{r}\sum_{j=1}^{r} h_i(t)h_j(t)\Bigg(He(P_{jl_k}(t)A_i) + \sum_{q=1}^{r}\phi_q(P_{ql_k}(t) - M_{il_k}(t)) + \frac{K}{T_d}(P_{jl_k,m+1} - P_{jl_k,m})$$
$$+ a_{il_k}\mu\sum_{q=1}^{r}R_{qjl_k} - R_{ijl_k}\Bigg) = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(t)h_j(t)\big((1-\beta)J_{ijl_k,mm} + \beta J_{ijl_k,(m+1)m}\big) < 0 \tag{47}$$

Then by using Lemma 2.1, (47) is guaranteed by (17) and (18). Therefore, (42) is guaranteed during $t \in [t_k + \tau_m, t_k + \tau_{m+1})$.

Second, for $t \in [t_k + \tau_K, t_{k+1})$, $k = 0, 1, 2, \ldots,$ we have

$$\dot{P}_{h(t)l_k}(t) = \sum_{q=1}^{r}\dot{h}_q(t)P_{ql_k,K} \tag{48}$$

By the similar way as before, (42) is obtained from (19).

Third, considering (15), one can deduce that

$$V_{\sigma(t_{k+1})}(x(t_{k+1})) < V_{\sigma(t_k)}(x(t^-_{k+1})) \tag{49}$$

Combining (49) and aforementioned, (42) is guaranteed for $t \geq 0$. Therefore, the T–S system (10) is asymptotically stable. □

Appendix B

Proof. First, for $t \in [t_k + \tau_m, t_k + \tau_{m+1})$, $k = 0, 1, 2, \ldots,$ the following equation can be derived.

$$\dot{X}_{h(t)l_k}(t) = \frac{K}{T_d}(X_{h(t)l_k,m+1} - X_{h(t)l_k,m}) + \sum_{q=1}^{r}\dot{h}_q(t)X_{ql_k}(t) \tag{50}$$

where $X_{ql_k}(t) = (1-\beta)X_{ql_k,m} + \beta X_{ql_k,m+1}$. Based on (35), from (32) and (33), it is obtained that

$$\sum_{i=1}^{r}\sum_{j=1}^{r} h_i(t)h_j(t)(He(A_iX_{jl_k}(t) + B_iS_{jl_k}(t)) + \sum_{q=1}^{r}\phi_q\Big(X_{ql_k}(t) - \bar{M}_{il_k}(t)) - \frac{K}{T}(X_{jl_k,m+1} - X_{jl_k,m})\Big)$$
$$+ \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(t)h_j(t)\Bigg(a_{il_k}\mu\sum_{q=1}^{r}\bar{R}_{qjl_k} - \bar{R}_{ijl_k}\Bigg) < 0 \tag{51}$$

Similar to the aforementioned, $\sum_{i=1}^{r}\sum_{j=1}^{r}h_i(t)h_j(t)(a_{il_k}\mu\sum_{q=1}^{r}\bar{R}_{qjl_k}-\bar{R}_{ijl_k})>0$, therefore, considering assumption (3) and (4), one can deduce that

$$\begin{aligned}&\sum_{i=1}^{r}\sum_{j=1}^{r}h_i(t)h_j(t)\left(He(A_iX_{jl_k}(t)+B_iS_{jl_k}(t))-\sum_{q=1}^{r}\dot{h}_q(t)X_{ql_k}(t)-\frac{K}{T}(X_{jl_k,m+1}-X_{jl_k,m})\right)\\ &\quad = He(A_{h(t)}X_{h(t)l_k}(t)+B_{h(t)}S_{h(t)l_k}(t))-\dot{X}_{h(t)l_k}(t)<0\end{aligned} \tag{52}$$

Setting $P_{l_k}(t)=X^{-1}_{h(t)l_k}(t)$, we have $\dot{P}_{l_k}(t)=-X^{-1}_{h(t)l_k}(t)\dot{X}_{h(t)l_k}(t)X^{-1}_{h(t)l_k}(t)$. Then substituting (35) into (52) and pre- and post-multiplying (52) by $P_{l_k}(t)$ and its transpose yields

$$He(P_{l_k}(t)(A_{h(t)}+B_{h(t)}K_{l_k}(t)))+\dot{P}_{l_k}(t)<0,\quad t\in[t_k+\tau_m,t_k+\tau_{m+1}) \tag{53}$$

Second, by the similar way used in the first step, it is obtained form (34) that

$$He(P_{l_k}(t)(A_{h(t)}+B_{h(t)}K_{l_k}(t)))+\dot{P}_{l_k}(t)<0,\quad t\in[t_k+\tau_K,t_{k+1}) \tag{54}$$

Third, choose Lyapunov function as follows

$$V_{\sigma(t)}(x(t))=x^T(t)P_{\sigma(t)}(t)x(t) \tag{55}$$

Considering the time derivative of (55) along (29)

$$\dot{V}_{\sigma(t)}(x(t))=x^T(t)(He(P_{h(t)\sigma(t)}(t)\bar{A}_{h(t)})+\dot{P}_{h(t)\sigma(t)}(t))x(t) \tag{56}$$

(53) and (54) yield

$$\dot{V}_{\sigma(t)}(x(t))<0,\quad \forall t\in[t_k,t_{k+1}) \tag{57}$$

Finally, (30) yields

$$V_{\sigma(t_{k+1})}(x(t_{k+1}))<V_{\sigma(t_k)}(x(t^-_{k+1})) \tag{58}$$

Combining (58) with (57), we can conclude that the T–S fuzzy system (29) is asymptotically stable under the switched controller (28) with the switching signal (11). □

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