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Global exponential convergence of fuzzy complex-valued neural networks with time-varying delays and impulsive effects

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Abstract

In this paper, the global exponential convergence of T–S fuzzy complex-valued neural networks with time-varying delays and impulsive effects is discussed. By employing Lyapunov functional method and matrix inequality technique, we analyze a type of activation functions with Lipschitz function, and sufficient conditions in terms of complex-valued linear matrix inequality are obtained to ensure the global exponential convergence. Moreover, the framework of the exponential convergence ball in the state space of the considered neural networks and the exponential convergence rate index are also given out. Here, the existence and uniqueness of the equilibrium points need not be considered and the results improve existing results on the Lyapunov exponential stability as special cases. Finally, one numerical example with simulations is given to illustrate the effectiveness of our theoretical results.

Keywords: Complex-valued neural network; Convergence; T–S fuzzy model; Time-varying delay; Impulsive effect; Complex-valued linear matrix inequality

1. Introduction

In the past decades, stability analysis of various classes of neural network models, such as Hopfield neural network, Cohen–Grossberg neural network, cellular neural network and BAM neural network, has been extensively investigated due to their extensive applications in many fields such as pattern recognition, intelligent robot, predictive estimate, biology, signal and image processing. However, in electronic and biological neural systems, the time delays inevitably exist due to the communication between the neurons. Therefore, it is of significant importance to consider stability analysis of neural networks with time delays. Recently, so much attention have been paid to search sufficient conditions to verify the asymptotical or exponential stability of neural networks with delays [1–6]. Meanwhile, in practical applications, impulsive phenomenon exists universally in various fields such as chemical technology, population dynamics and economics, where the state is changed abruptly at certain moments of time. Examples of impulsive

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phenomena can be also found in automatic control system, artificial intelligence, robotics, etc. Therefore, impulsive neural network model belongs to new category of dynamical systems, which are neither continuous nor discrete ones. It is necessary to consider both impulsive effects and delays in the study of stability of neural networks. Recently, many researchers [7–9] have done extensive works on asymptotic behavior of impulsive neural networks with delays.

However, besides impulse effects and delays, in mathematical modeling of real world problems, we encounter some other inconveniences, for example, the complexity, the approximation and the vagueness. Fuzzy logic systems or neural networks have been proved to be universal approximators, i.e., they can approximate any nonlinear functions. Therefore, fuzzy logic systems and neural networks have been widely adopted for nonlinear systems [10–12]. In recent years, fuzzy logic theory has been efficiently applied to many applications and it is an effective approach to model a complex nonlinear system and deal with its stability. Takagi and Sugeno [13] first introduced fuzzy models and then the T–S fuzzy model is successfully and effectively used in complex nonlinear systems [14]. T–S fuzzy systems are nonlinear systems described by a set of IF-THEN rules. It has been shown that the T–S model method can give an effective way to represent complex nonlinear systems by some simple local linear dynamic systems with their linguistic description. Some nonlinear dynamic systems can be approximated by the overall fuzzy linear T–S models for the purpose of dynamical analysis [15]. In addition, the fuzzy neural networks [16] are also a kind of important neural networks. Recently, the authors [17–21] studied the stability problem of T–S fuzzy neural networks with time-varying delays by using the Lyapunov functional theory and LMI technique. Lagrange exponential stability of T–S fuzzy Cohen–Grossberg neural networks with time-varying delays was also discussed in [22].

On the other hand, as an extension of real-valued neural networks, complex-valued neural networks (CVNNs) with complex-valued state, input, connection weight and activation function become strongly desired because of their practical applications in physical systems dealing with electromagnetic, light, ultrasonic and quantum waves. For example, the XOR problem and the detection of symmetry [23] can not be solved with a single real-valued neuron but be solved by a single complex-valued neuron with the orthogonal decision boundaries, which reveals the potential computational power of complex-valued neurons. However, it was known that the main challenge is the choice of the activation functions in study of properties of CVNNs. As we all know, activation functions can be expressed by separating their real and imaginary parts, some results were given for various CVNNs in this way [24–28]. On the other hand, when the activation functions can not be separated into their real and imaginary parts, some stability criteria of CVNNs were also obtained under Lipschitz continuity condition in the complex domain [29–33].

Because there is no equilibrium point, chaos attractor, periodic state or almost periodic state in the neural networks outside the globally attracting set [34], many scholars have studied globally attractive sets of neural networks [35–40]. However, to the best of our knowledge, there is hardly any paper that considers the global convergence of fuzzy complex-valued neural networks with time-varying delays and impulsive effects. These constitute the motivation for the present research.

Motivated by the above analysis, we will generalize the ordinary T–S fuzzy models to express a class of CVNNs with time-varying delays and impulsive effects. The main purpose of this paper is to study the global exponential convergence for T–S fuzzy CVNNs with time-varying delays and impulsive effects, not discussing the existence and uniqueness of the equilibrium point. The main contributions of this paper are the following aspects: (i) The present paper is one of the first papers that attempts to study the global exponential convergence of T–S fuzzy CVNNs with time-varying delays and impulsive effects. (ii) The activation functions have not be separated into their real and imaginary parts. (iii) The established sufficient conditions are expressed in terms of complex-valued linear matrix inequalities to ensure global exponential convergence of T–S fuzzy CVNNs, which can be checked numerically using the effective Yalmip toolbox in Matlab and the convergence ball domains for the discussed networks are also given. (iv) Compared with the existed results, our results include the results in [33] as special cases and are more general.

The rest of this paper is organized as follows: Section 2 describes some preliminaries including some necessary definitions and lemmas. The main results are obtained in Section 3. In section 4, one numerical example is given to confirm the validity of our results. Finally, concluding remarks are presented in Section 5.

Notation: Throughout this paper, the superscripts Q^{-1}, Q^T and Q^* stand for the inverse, transpose and conjugate transpose of matrix Q, respectively. $\mathbb{C}$ is the set of complex numbers. The notation $X > Y$ means that X and Y are Hermitian matrices and that $X - Y$ is positive definite. $\mathbb{C}^n$ and $\mathbb{C}^{n\times n}$ denote, respectively, the set of all n-dimensional complex-valued vectors and all $n \times n$ complex-valued matrices. i shows the imaginary unit, i.e., $i = \sqrt{-1}$. $|a|$ denotes the module of $a \in \mathbb{C}$, and $\|z\|$ denotes the norm of $z \in \mathbb{C}^n$, i.e., $\|z\| = \sqrt{z^*z}$. $\lambda_{\max}(\lambda_{\min})$ refers to maximal(minimal) eigenvalue. N_+ stands for the set of positive integers. Let $\Gamma = \{1, 2, \dots, n\}$.

2. Preliminaries

In this paper, we consider the following CVNNs with time-varying delays

$$\frac{dz(t)}{dt} = -Cz(t) + Af(z(t)) + Bf(z(t-\tau(t))) + u(t), \tag{1}$$

where $z(t) = (z_1(t), z_1(t), \dots, z_n(t))^T \in \mathbb{C}^n$ is the state vector, $f(z(t)) = (f_1(z_1(t)), f_2(z_1(t)), \dots, f_n(z_n(t)))^T \in \mathbb{C}^n$ and $f(z(t-\tau(t))) = (f_1(z_1(t-\tau_1(t))), f_2(z_1(t-\tau_2(t))), \dots, f_n(z_n(t-\tau_n(t))))^T \in \mathbb{C}^n$ are the vector-valued activation functions without and with time delays whose elements consist of complex-valued nonlinear functions. $A = (a_{ij})_{n\times n} \in \mathbb{C}^{n\times n}$ and $B = (b_{ij})_{n\times n} \in \mathbb{C}^{n\times n}$, respectively, are the connection weight matrices without and with time delays, $C = \text{diag}\{c_1, c_2, \dots, c_n\} \in \mathbb{C}^{n\times n}$ is the self-feedback connection weight matrix, $\tau(t) = (\tau_1(t), \tau_2(t), \dots, \tau_n(t))^T$ is time delay vector with $0 \le \tau_j(t) \le \tau_j$ for $j \in \Gamma$, $u(t) = (u_1(t), u_2(t), \dots, u_n(t))^T$ denotes external input vector with $|u_j(t)| \le \bar{u}_j$ for $j \in \Gamma$. Denote $\bar{u} = (\bar{u}_1, \bar{u}_2, \dots, \bar{u}_n)^T$, $\tau = \max\limits_{1\le j\le n}\{\tau_j\}$.

The initial condition of neural network (1) is

$$z(s) = \varphi(s), \tag{2}$$

where $\varphi(s) \in \mathbb{C}^n$ is complex-valued continuous vector functions on $[t_0 - \tau, t_0]$.

The continuous fuzzy system was proposed to represent a nonlinear system [13]. From the T–S fuzzy model concept, the model of T–S fuzzy CVNNs with time-varying delays is described as follows:

Plant Rule r:
If $\{\theta_1(t)$ is $M_{r1}\}$,$\{\theta_2(t)$ is $M_{r2}\}$,...,$\{\theta_g(t)$ is $M_{rg}\}$.
Then $\begin{cases} \frac{dz(t)}{dt} = -C_r z(t) + A_r f(z(t)) + B_r f(z(t-\tau(t))) + u(t), \\ z(s) = \varphi(s), \quad s \in [t_0-\tau, t_0], \end{cases}$

where $\theta_l(t) (l = 1, 2, \dots, g)$ are the premise variables, $M_{rl} (r = 1, 2, \dots, m)$ are the fuzzy sets and m is the number of **If–Then** rules.

By inferring from the fuzzy models, the final output of T–S fuzzy CVNNs can be obtained by

$$\begin{cases} \frac{dz(t)}{dt} = \sum\limits_{r=1}^{m} \phi_r(\theta(t))\{-C_r z(t) + A_r f(z(t)) + B_r f(z(t-\tau(t))) + u(t)\}, \\ z(s) = \varphi(s), \quad s \in [t_0-\tau, t_0], \end{cases} \tag{3}$$

where $\theta(t) = (\theta_1(t), \theta_2(t), \dots, \theta_g(t))^T$, $\omega_r(\theta(t)) = \prod\limits_{l=1}^{g} M_{rl}(\theta(t))$ and $\phi_r(\theta(t)) = \frac{\omega_r(\theta(t))}{\sum_{r=1}^{m}\omega_r(\theta(t))}$ denote the weight and normalized weight of each fuzzy rule, respectively. The term $M_{rl}(\theta(t))$ is the grade membership of $\theta_l(t)$ in M_{rl}. We assume that $\omega_r(\theta(t)) \ge 0$ for $r \in \{1, 2, \dots, m\}$ and $\sum\limits_{r=1}^{m} \phi_r(\theta(t)) = 1$ for all $t \ge t_0$.

Considering the impulse effects, the impulsive T–S fuzzy CVNNs can be obtained in the following form:

$$\begin{cases} \frac{dz(t)}{dt} = \sum\limits_{r=1}^{m} \phi_r(\theta(t))\{-C_r z(t) + A_r f(z(t)) + B_r f(z(t-\tau(t))) + u(t)\}, \quad t \ne t_k, \\ z(t_k^+) = E_k g_k(z(t_k^-)) + F_k r_k(z(t_k^- - \tau(t_k^-))) + w_k(t_k^-), \quad t = t_k, \\ z(s) = \varphi(s), \quad s \in [t_0-\tau, t_0], \end{cases} \tag{4}$$

where $\{t_1, t_2, \cdots\}$ is a sequence of strictly increasing impulsive moments, $E_k, F_k \in \mathbb{C}^{n\times n}$ represents the impulses gain matrices. $w_k(t_k^-) = (w_{k1}(t_k^-), w_{k2}(t_k^-), \dots, w_{kn}(t_k^-))^T \in \mathbb{C}^n$ represents external impulsive input vector, $g_k(z(t_k^-)) = (g_{k1}(z_1(t_k^-)), g_{k2}(z_2(t_k^-)), \dots, g_{kn}(z_n(t_k^-)))^T \in \mathbb{C}^n$, $r_k(z(t_k^- - \tau(t_k^-))) = (r_{k1}(z(t_k^- - \tau_1(t_k^-))), r_{k2}(z(t_k^- - \tau_2(t_k^-))), \dots, r_{kn}(z(t_k^- - \tau_n(t_k^-))))^T \in \mathbb{C}^n$ represent impulsive perturbations. We assume that $z(t)$ is right continuous at $t = t_k$, i.e. $z(t_k) = z(t_k^+)$. Hence, the solutions of (4) are piecewise right-hand continuous functions with discontinuities at $t = t_k$ for $k \in N_+$.

Assumption 1. There exist positive constants $L_j (j \in \Gamma)$ such that $|f_j(x) - f_j(y)| \leq L_j|x - y|$, $f_j(0) = 0$ for $\forall x, y \in \mathbb{C}$, and denote $L = \mathrm{diag}\{L_1, L_2, \dots, L_n\}$.

Remark 1. Activation functions satisfying Assumption 1 are actually the extension of the real-valued functions satisfying the Lipschitz continuity condition. In fact, it is easy to verify that Assumption on decomposing of complex-valued activation function into its real part and imaginary part in [24–26] is a strong constraint and it is a special case of Assumption 1 here. In other words, if activation functions satisfy Assumption 1 in [24–26], then Assumption 3 here also holds. When activation functions are not explicitly expressed by separating real and imaginary parts but satisfy the Lipschitz continuity condition, then we can use similar approach to the real-valued neural networks. For example, the function $f(z) = \frac{1}{1+\exp(-\bar{z})}$ satisfies Assumption 1. Meanwhile, Assumption 1 also removes the boundedness and differentiability assumption for activation functions.

Assumption 2. Assume that $g_k(0) = 0$, $r_k(0) = 0$, there exist positive diagonal matrices $G_k = \mathrm{diag}\{G_{k1}, G_{k2}, \dots, G_{kn}\}$ and $R_k = \mathrm{diag}\{R_{k1}, R_{k2}, \dots, R_{kn}\}$ such that the following inequalities hold for $\forall x, y \in \mathbb{C}$, $j \in \Gamma$, $k \in N_+$

$$|g_{kj}(x) - g_{kj}(y)| \leq G_{kj}|x - y|, |r_{kj}(x) - r_{kj}(y)| \leq R_{kj}|x - y|.$$

Assumption 3. There exist positive constants $\bar{w}_{kj}$ such that $|w_{kj}(t_k^-)| \leq \bar{w}_{kj}$ for $\forall k \in N_+$, $j \in \Gamma$. Let $\bar{w}_k = (\bar{w}_{k1}, \bar{w}_{k2}, \dots, \bar{w}_{kn})^T$.

Definition 1. Impulsive network (4) is said to be globally exponentially convergent to a ball $\mathcal{B}(r) = \{z(t) \in \mathbb{C}^n | \|z(t)\| \leq r\}$ with a convergence rate $\alpha > 0$, if for any given positive number $h \geq \sup\limits_{t_0-\tau \leq s \leq t_0} \|\varphi(s)\|$, there exists a positive number $K = K(h)$ such that for $\forall t \geq t_0$

$$||z(t)|| \leq r + Ke^{-\alpha(t-t_0)}.$$

Remark 2. According to Definition 1, when $r = 0$, then the equilibrium point $\tilde{z}$ of system (4) is globally exponentially stable in Lyapunov sense with a convergence rate $\alpha > 0$. In addition, Definition 1 also indicates that system (4) is ultimate bounded.

Lemma 1 *([33]). Let $x, y \in \mathbb{C}^n$ and $P \in \mathbb{C}^{n\times n}$ be a positive definite Hermitian matrix, then*

$$x^*y + y^*x \leq x^*P^{-1}x + y^*Py.$$

Lemma 2. *Let $x, y \in \mathbb{C}^n$ and $P \in \mathbb{C}^{n\times n}$ be a positive definite Hermitian matrix, then*

$$x^*Py + y^*Px \leq x^*Px + y^*Py.$$

Proof. $P \in \mathbb{C}^{n\times n}$ is a positive definite Hermitian matrix, so there exists a matrix $A \in \mathbb{C}^{n\times n}$ such that $P = A^*A$, then

$$x^*Py + y^*Px = (Ax)^*(Ay) + (Ay)^*(Ax) \leq (Ax)^*(Ax) + (Ay)^*(Ay) = x^*Px + y^*Py.$$

Lemma 3 *([32]). Let $x, y \in \mathbb{C}^n$ and $P \in \mathbb{C}^{n\times n}$ be a Hermitian matrix, then*

$$\lambda_{\min}(P)x^*x \leq x^*Px \leq \lambda_{\max}(P)x^*x.$$

Lemma 4 *([38]). Consider the following impulsive differential inequality*

$$\begin{cases} D^+f(t) \leq -\alpha_1 f(t) + \alpha_2 \sup\limits_{t-\tau \leq s \leq t} f(s) + \alpha_3, & t \neq t_k,\ t \geq t_0, \\ f(t_k) \leq a_k f(t_k^-) + b_k \sup\limits_{t_k-\tau \leq t < t_k} f(t) + c_k, & t_k \geq t_0,\ k \in N_+, \\ f(t) = \phi(t), & t \in [t_0 - \tau, t_0], \end{cases}$$

where $f(t) \geq 0$ *is continuous at* $t \neq t_k$, α_1, α_2 *and* α_3 *are positive constants. If* $0 < \alpha_2 < \alpha_1$, $t_k - t_{k-1} > \delta\tau$, $\delta > 1$, *then*

$$\begin{cases} f(t) \leq m_0 + \eta e^{-\lambda(t-t_0)}, & t \geq t_0,\ t \neq t_k, \\ f(t_k) \leq m_k + n_k \eta e^{-\lambda(t_k-t_0)}, & t_k \geq t_0, \end{cases}$$

or

$$f(t) \leq m_k + n_k \eta e^{-\lambda(t-t_0)}, \quad t \geq t_0,$$

where

$$\begin{cases} m_k = \max\{m_{k-1}, m_{k-1}(|a_k| + |b_k|) + c_k\}, & m_0 = \frac{\alpha_3}{\alpha_1-\alpha_2}, \\ n_k = \max\{n_{k-1}, n_{k-1}(|a_k| + |b_k| e^{\lambda\tau})\}, & n_0 = 1,\ k \in N_+, \end{cases}$$

$\eta \geq \sup\limits_{t_0-\tau \leq t < t_0} f(t)$ *and* $\lambda > 0$ *is a unique solution of the equality* $\lambda = \alpha_1 - \alpha_2 e^{\lambda\tau}$.

3. Main results

Theorem 1. *Under Assumptions 1–3, if there exist a positive definite Hermitian matrix* $P \in \mathbb{C}^{n\times n}$, *a real-valued positive definite matrix* S, *two real-valued positive diagonal matrices* K, Y, *three scalars* $a > b > 0$, $\delta > 1$ *such that the following conditions are satisfied*

(i) $t_k - t_{k-1} \geq \delta\tau$, $k \in N_+$;

(ii) $LKL < bP$;

(iii) $\Upsilon = \begin{bmatrix} -PC_r - C_r^* P + aP + LYL & PA_r & PB_r & P \\ A_r^* P & -Y & 0 & 0 \\ B_r^* P & 0 & -K & 0 \\ P & 0 & 0 & -S \end{bmatrix} < 0, \quad r = 1, 2, \ldots, m.$

Then, network (4) is globally exponentially convergent, i.e.

$$\begin{cases} \|z(t)\| \leq \sqrt{\frac{m_0}{\lambda_{\min}(P)}} + \sqrt{\frac{\eta}{\lambda_{\min}(P)}} e^{-\frac{\lambda}{2}(t-t_0)}, & t \geq t_0, t \neq t_k, \\ \|z(t_k)\| \leq \sqrt{\frac{m_k}{\lambda_{\min}(P)}} + \sqrt{\frac{n_k\eta}{\lambda_{\min}(P)}} e^{-\frac{\lambda}{2}(t_k-t_0)}, & t = t_k \geq t_0. \end{cases}$$

Meanwhile, network (4) is globally uniformly exponentially convergent to the ball $\mathscr{B}_1 = \{z(t) \in R^n | \|z(t)\| \leq \sqrt{\frac{m_k}{\lambda_{\min}(P)}}\}$ *with a rate* $\frac{\lambda}{2}$, *where*

$$\begin{cases} m_k = \max\{m_{k-1}, m_{k-1}(|a_k| + |b_k|) + c_k\}, \quad m_0 = \frac{\bar{u}^T S \bar{u}}{a-b}, \\ n_k = \max\{n_{k-1}, n_{k-1}(|a_k| + |b_k| e^{\lambda\tau})\}, \quad n_0 = 1,\ k \in N_+, \\ a_k = 3\frac{\lambda_{\max}(E_k^* P E_k)}{\lambda_{\min}(P)} \max\limits_{1\leq i\leq n}\{G_{ki}\}, b_k = 3\frac{\lambda_{\max}(F_k^* P F_k)}{\lambda_{\min}(P)} \max\limits_{1\leq i\leq n}\{R_{ki}\}, c_k = 3\lambda_{\max}(P)\bar{w_k}^T \bar{w}_k, \end{cases}$$

$\sup\limits_{t_0-\tau\leq s\leq t_0} V(s) \leq \eta$, $\lambda > 0$ *is a unique solution of the equality* $\lambda = a - be^{\lambda\tau}$, *and* $V(t)$ *is defined as follows*

$$V(t) = z^*(t)Pz(t). \tag{5}$$

Proof. For the case of $t \neq t_k$. Calculate the derivative of $V(t)$ described by (5) along the solutions of (4), we have

$$\begin{aligned} \dot{V}(t) &= z^*(t)P\dot{z}(t) + \dot{z}^*(t)Pz(t) \\ &= \sum_{r=1}^{m} \phi_r(\theta(t))\{z^*(t)(-PC_r - C_r^* P)z(t) + z^*(t)PA_r f(z(t)) + f^*(z(t))A_r^* Pz(t) \\ &\quad + z^*(t)PB_r f(z(t-\tau(t))) + f^*(z(t-\tau(t)))B_r^* Pz(t) + z^*(t)Pu(t) + u^*(t)Pz(t)\}. \end{aligned} \tag{6}$$

Since Y and K are real-valued positive diagonal matrices and S is a real-valued positive definite matrix, we can get from Assumption 1 and Lemma 1

$$z^*(t)Pu(t)+u^*(t)Pz(t)\le z^*(t)PS^{-1}Pz(t)+u^*(t)Su(t)\le z^*(t)PS^{-1}Pz(t)+\bar{u}^TS\bar{u}, \tag{7}$$

$$f^*(z(t))Yf(z(t))\le z^*(t)LYLz(t), \tag{8}$$

$$\begin{aligned}&z^*(t)PB_rf(z(t-\tau(t)))+f^*(z(t-\tau(t)))B_r^*Pz(t)\\&\quad\le z^*(t)PB_rK^{-1}B_r^*Pz(t)+f^*(z(t-\tau(t)))Kf(z(t-\tau(t)))\\&\quad\le z^*(t)PB_rK^{-1}B_r^*Pz(t)+z^*(t-\tau(t))LKLz(t-\tau(t)).\end{aligned} \tag{9}$$

Using the Schur complement, $\Upsilon<0$ is equivalent to the following LMI

$$\begin{aligned}\tilde{\Upsilon}=&\begin{bmatrix}-PC_r-C_r^*P+aP+LYL & PA_r\\ A_r^*P & -Y\end{bmatrix}\\&+\begin{bmatrix}PB_r & P\\0&0\end{bmatrix}\begin{bmatrix}K&0\\0&S\end{bmatrix}^{-1}\begin{bmatrix}PB_r & P\\0&0\end{bmatrix}^*<0.\end{aligned} \tag{10}$$

Combining with (6)–(10), it is easy to get

$$\begin{aligned}\dot{V}(t)\le&\sum_{r=1}^m\phi_r(\theta(t))\{z^*(t)(-PC_r-C_r^*P+LYL+PB_rK^{-1}B_r^*P+PS^{-1}P)z(t)\\&+z^*(t)PA_rf(z(t))+f^*(z(t))A_r^*Pz(t)-f^*(z(t))Yf(z(t))\\&+z^*(t-\tau(t))LKLz(t-\tau(t))+\bar{u}^TS\bar{u}\}\\=&\sum_{r=1}^m\phi_r(\theta(t))\begin{bmatrix}z(t)\\f(z(t))\end{bmatrix}^*\tilde{\Upsilon}\begin{bmatrix}z(t)\\f(z(t))\end{bmatrix}\\&-aV(t)+z^*(t-\tau(t))LKLz(t-\tau(t))+\bar{u}^TS\bar{u}\\\le&-aV(t)+b\sup_{t-\tau\le s\le t}V(s)+\bar{u}^TS\bar{u}.\end{aligned} \tag{11}$$

When $t=t_k$, by Lemmas 2–3 and Assumptions 2–3, we get

$$\begin{aligned}V(t_k^+)=&z^*(t_k^+)Pz(t_k^+)\\=&[E_kg_k(z(t_k^-))+F_kr_k(z(t_k^--\tau(t_k^-)))+w_k(t_k^-)]^*P[E_kg_k(z(t_k^-))\\&+F_kr_k(z(t_k^--\tau(t_k^-)))+w_k(t_k^-)]\\\le&3g_k^*(z(t_k^-))E_k^*PE_kg_k(z(t_k^-))+3r_k^*(z(t_k^--\tau(t_k^-)))F_k^*PF_kr_k(z(t_k^--\tau(t_k^-)))\\&+3w_k^*(t_k^-)Pw_k(t_k^-)\\\le&a_kV(t_k^-)+b_k\sup_{t_k-\tau\le t<t_k}V(t)+c_k.\end{aligned} \tag{12}$$

Combining with (11), (12) and Lemma 4, we can obtain

$$\begin{cases}V(t)\le m_0+\eta e^{-\lambda(t-t_0)}, & t\ge t_0, t\ne t_k,\\ V(t_k)\le m_k+n_k\eta e^{-\lambda(t_k-t_0)}, & t=t_k\ge t_0,\end{cases} \tag{13}$$

where $\lambda>0$ is a unique solution of the equality $\lambda=a-be^{\lambda\tau}$, it is easy to get

$$\begin{cases}\|z(t)\|\le\sqrt{\frac{m_0}{\lambda_{\min}(P)}}+\sqrt{\frac{\eta}{\lambda_{\min}(P)}}e^{-\frac{\lambda}{2}(t-t_0)}, & t\ge t_0, t\ne t_k,\\ \|z(t_k)\|\le\sqrt{\frac{m_k}{\lambda_{\min}(P)}}+\sqrt{\frac{n_k\eta}{\lambda_{\min}(P)}}e^{-\frac{\lambda}{2}(t_k-t_0)}, & t=t_k\ge t_0.\end{cases} \tag{14}$$

So, by Definition 1, the inequality (14) implies that network (4) is globally exponentially convergent. Therefore, the conclusion holds and the proof is completed. □

Remark 3. When the impulsive effects are not considered, under Assumption 1 and the other conditions of Theorem 1 hold, then model (4) without impulsive effects or model (1) is globally exponentially convergent to the ball $\tilde{\mathscr{B}}_1 = \{z(t) \in \mathbb{C}^n | \|z(t)\| \leq \sqrt{\frac{\bar{u}^T S \bar{u}}{\lambda_{\min}(P)(a-b)}}$ with a rate $\frac{\lambda}{2}$.

Corollary 1. *Under Assumptions 1–3, if there exist a positive definite Hermitian matrix P, four scalars $a > b > 0$, $\delta > 1$ and $\varepsilon > 0$ such that the following conditions are satisfied:*

(i) $t_k - t_{k-1} \geq \delta\tau$, $k \in N_+$;
(ii) $-PC_r - C_r^* P + \varepsilon^{-1} P A_r A_r^* P + \varepsilon^{-1} P B_r B_r^* P + \varepsilon^{-1} P^2 + \varepsilon L^2 \leq -aP$, $r = 1, 2, \ldots, m$;
(iii) $\varepsilon L^2 \leq bP$.

Then network (4) is globally exponentially convergent, i.e.

$$\begin{cases} \|z(t)\| \leq \sqrt{\frac{m_0}{\lambda_{\min}(P)}} + \sqrt{\frac{\eta}{\lambda_{\min}(P)}} e^{-\frac{\lambda}{2}(t-t_0)}, & t \geq t_0, t \neq t_k, \\ \|z(t_k)\| \leq \sqrt{\frac{m_k}{\lambda_{\min}(P)}} + \sqrt{\frac{n_k \eta}{\lambda_{\min}(P)}} e^{-\frac{\lambda}{2}(t_k-t_0)}, & t = t_k \geq t_0. \end{cases}$$

Meanwhile, network (4) is globally exponentially convergent to the ball $\mathscr{B}_2 = \{z(t) \in \mathbb{C}^n | \|z(t)\| \leq \sqrt{\frac{m_k}{\lambda_{\min}(P)}}\}$ with a rate $\frac{\lambda}{2}$, where

$$\begin{cases} m_k = \max\{m_{k-1}, m_{k-1}(|a_k| + |b_k|) + c_k\}, \quad m_0 = \frac{\varepsilon \bar{u}^T \bar{u}}{a-b}, \\ n_k = \max\{n_{k-1}, n_{k-1}(|a_k| + |b_k| e^{\lambda\tau})\}, \quad n_0 = 1, \; k \in N_+, \\ a_k = 3\frac{\lambda_{\max}(E_k^* P E_k)}{\lambda_{\min}(P)} \max\limits_{1 \leq i \leq n}\{G_{ki}\}, b_k = 3\frac{\lambda_{\max}(F_k^* P F_k)}{\lambda_{\min}(P)} \max\limits_{1 \leq i \leq n}\{R_{ki}\}, c_k = 3\lambda_{\max}(P) \bar{w_k}^T \bar{w_k}, \end{cases}$$

$\sup\limits_{t_0-\tau \leq s \leq t_0} V(s) \leq \eta$, *$\lambda > 0$ is a unique solution of the equality $\lambda = a - be^{\lambda\tau}$, and $V(t)$ is defined in (5).*

Remark 4. Corollary 1 provides a sufficient condition of global exponential convergence of (4), which is simple and easily verified for low dimension fuzzy complex-valued neural networks. For high dimension complex-valued neural networks, the condition (ii) in Corollary 1 is nonlinear matrix inequality with respect to P. The feasible solutions of the nonlinear matrix inequality can not be easily solved, especially for calculation by manual. So the condition (ii) has some limitations in calculation for high dimension fuzzy complex-valued neural networks. In order to calculate conveniently, sometimes one may take $P = I$, then (ii) and (iii) become as follows:

$$-C_r - C_r^* + \varepsilon^{-1} A_r A_r^* + \varepsilon^{-1} B_r B_r^* + \varepsilon^{-1} I + \varepsilon L^2 \leq -aI; \quad \varepsilon L^2 \leq bI.$$

Remark 5. In general, it is very difficult to directly solve the complex-valued matrix P in Corollary 1 satisfying the nonlinear inequality (ii) and the solutions of (ii) are not unique. But, the complex-valued matrix inequality (ii) of Corollary 1 can be translated into complex-valued linear matrix inequality problem, which can be easily solved numerically by using the effective interior point algorithms in convex optimization technique and Yalmip toolbox in Matlab. Making use of Schur decomposition method, then the complex-valued matrix inequality (ii) of Corollary 1 is equivalent to the following LMIs on the complex-valued matrix P:

$$\begin{bmatrix} -PC_r - C_r^* P + aP + \varepsilon L^2 & PA_r & PB_r & P \\ A_r^* P & -\varepsilon I & 0 & 0 \\ B_r^* P & 0 & -\varepsilon I & 0 \\ P & 0 & 0 & -\varepsilon I \end{bmatrix} \leq 0, \quad r = 1, 2, \ldots, m.$$

Remark 6. When $w_k(t_k^-) = 0$ and $u(t) = 0$, network (4) turns into the following fuzzy complex-valued neural networks

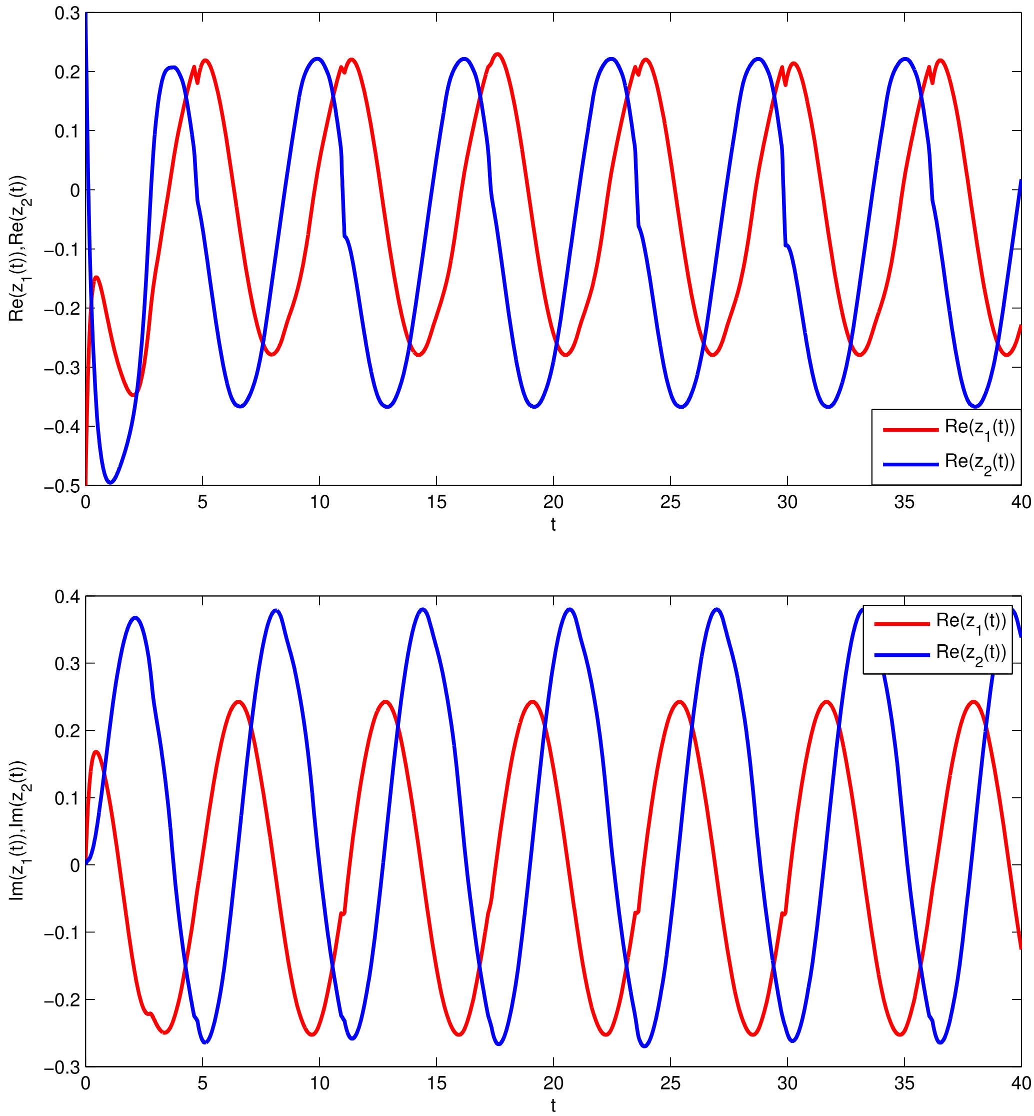

Fig. 1. State trajectories for (17) without impulse.

$$\begin{cases} \frac{dz(t)}{dt} = \sum_{r=1}^{m} \phi_r(\theta(t))\{-C_r z(t) + A_r f(z(t)) + B_r f(z(t-\tau(t)))\}, \quad t \neq t_k, \\ z(t_k^+) = E_k g_k(z(t_k^-)) + F_k r_k(z(t_k^- - \tau(t_k^-))), \quad t = t_k, \\ z(s) = \varphi(s), \quad s \in [t_0 - \tau, t_0]. \end{cases} \tag{15}$$

From the process of derivation of Theorem 1, we have the following result.

Corollary 2. *Under Assumptions 2–3, if there exist a positive definite Hermitian matrix P, two real-valued positive diagonal matrices K, Y, three scalars $a > b > 0$ and $\delta > 1$ such that the following conditions hold for $k \in N_+$, $r = 1, 2, \ldots, m$*

$$t_k - t_{k-1} \geq \delta\tau, \quad LKL < bP; \quad \breve{\Upsilon} = \begin{bmatrix} -PC_r - C_r^* P + aP + LYL & PA_r & PB_r \\ A_r^* P & -Y & 0 \\ B_r^* P & 0 & -K \end{bmatrix} < 0.$$

Then, the zero solution of network (15) is globally exponentially stable in Lyapunov sense.

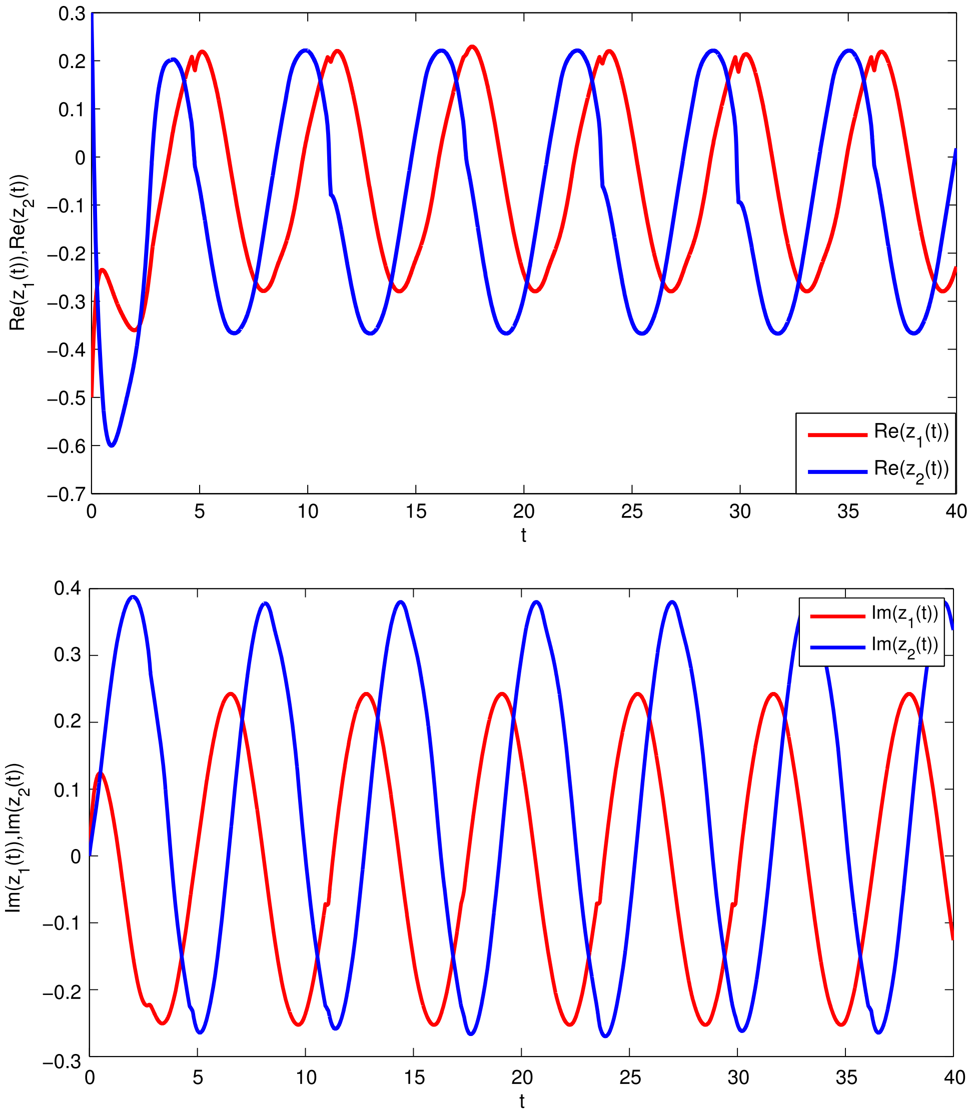

Fig. 2. State trajectories for (18) without impulse.

Remark 7. When $m = 1$, we can also obtain globally exponential stability condition of (15) and the formula (15) with $m = 1$ is the formula (26) in [33]. In addition, when $a = b = 1$, one can see that Corollary 2 here becomes Theorem 1 given in [33] as a special case. So the results here are more extensive.

Remark 8. When the impulsive effects are not considered and $u(t) = 0$, network (4) turn into the following fuzzy complex-valued neural networks

$$\frac{dz(t)}{dt} = \sum_{r=1}^{m} \phi_r(\theta(t))\{-C_r z(t) + A_r f(z(t)) + B_r f(z(t - \tau(t)))\}. \tag{16}$$

Similar to the proof of Theorem 1, we can obtain the following result.

Corollary 3. *Under Assumption 1, if there exist a positive definite Hermitian matrices P, two real-valued positive diagonal matrices K, Y, two scalars $a > b > 0$ such that the following conditions are satisfied*

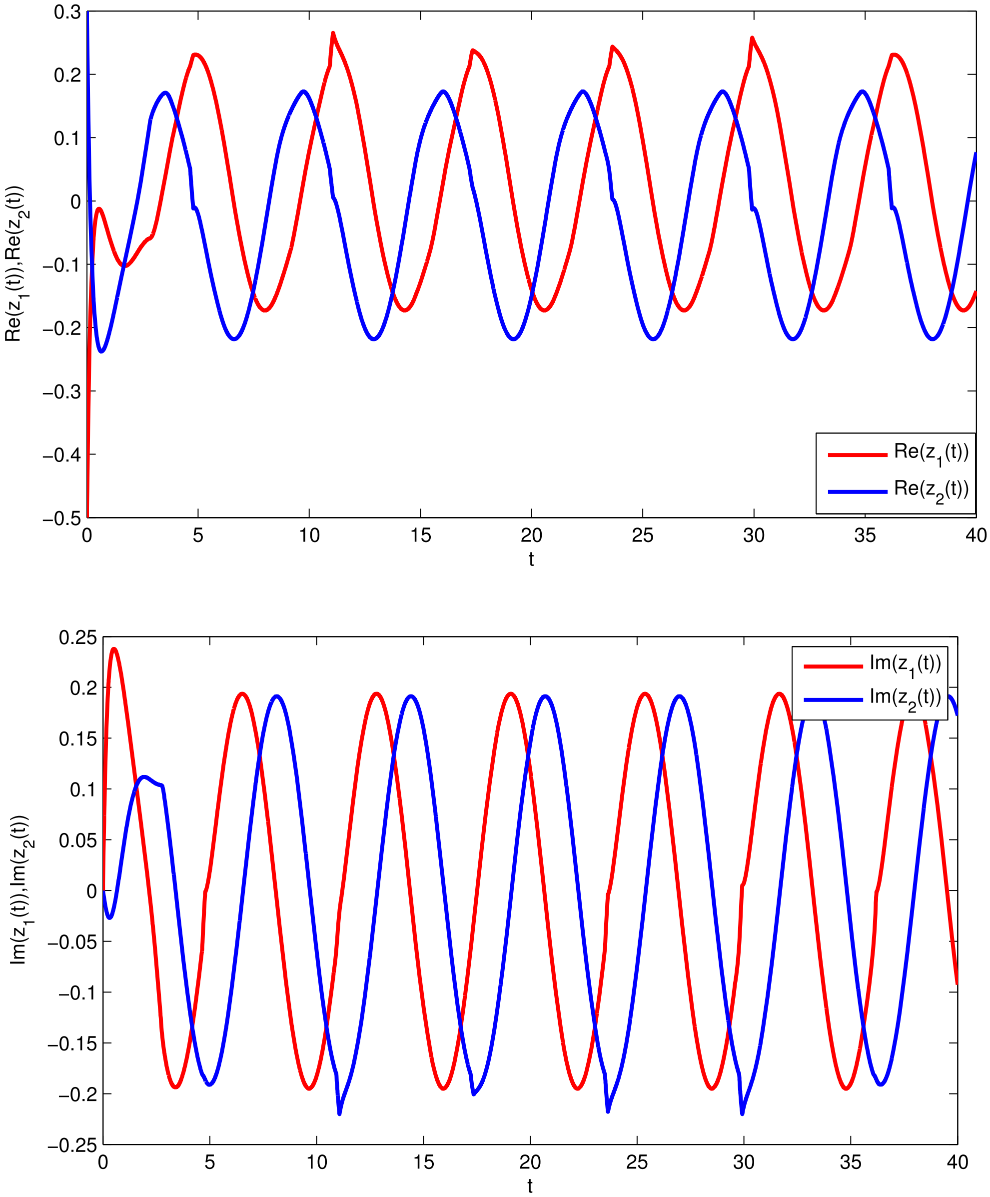

Fig. 3. State trajectories for (19) without impulse.

$$LKL < bP; \quad \check{\Upsilon} = \begin{bmatrix} -PC_r - C_r^* P + aP + LYL & PA_r & PB_r \\ A_r^* P & -Y & 0 \\ B_r^* P & 0 & -K \end{bmatrix} < 0.$$

Then, the zero solution of network (16) is globally exponentially stable in Lyapunov sense.

Remark 9. When $m = 1$, the conditions with $a = b = 1$ in Corollary 3 are equivalent to the conditions of Corollary 1 in [33], that is to say, the globally exponential stability of the zero equilibrium point of (16) with $m = 1$ is same as the globally exponential stability of the equilibrium point of the models (45) and (46) in [33].

Remark 10. It should be noted that in this paper the boundedness of activation functions is not assumed. According to the Liouville's theorem, if $f(z)$ is analytic and bounded for all $z \in \mathbb{C}$, then $f(z)$ is a constant function. It is obvious that

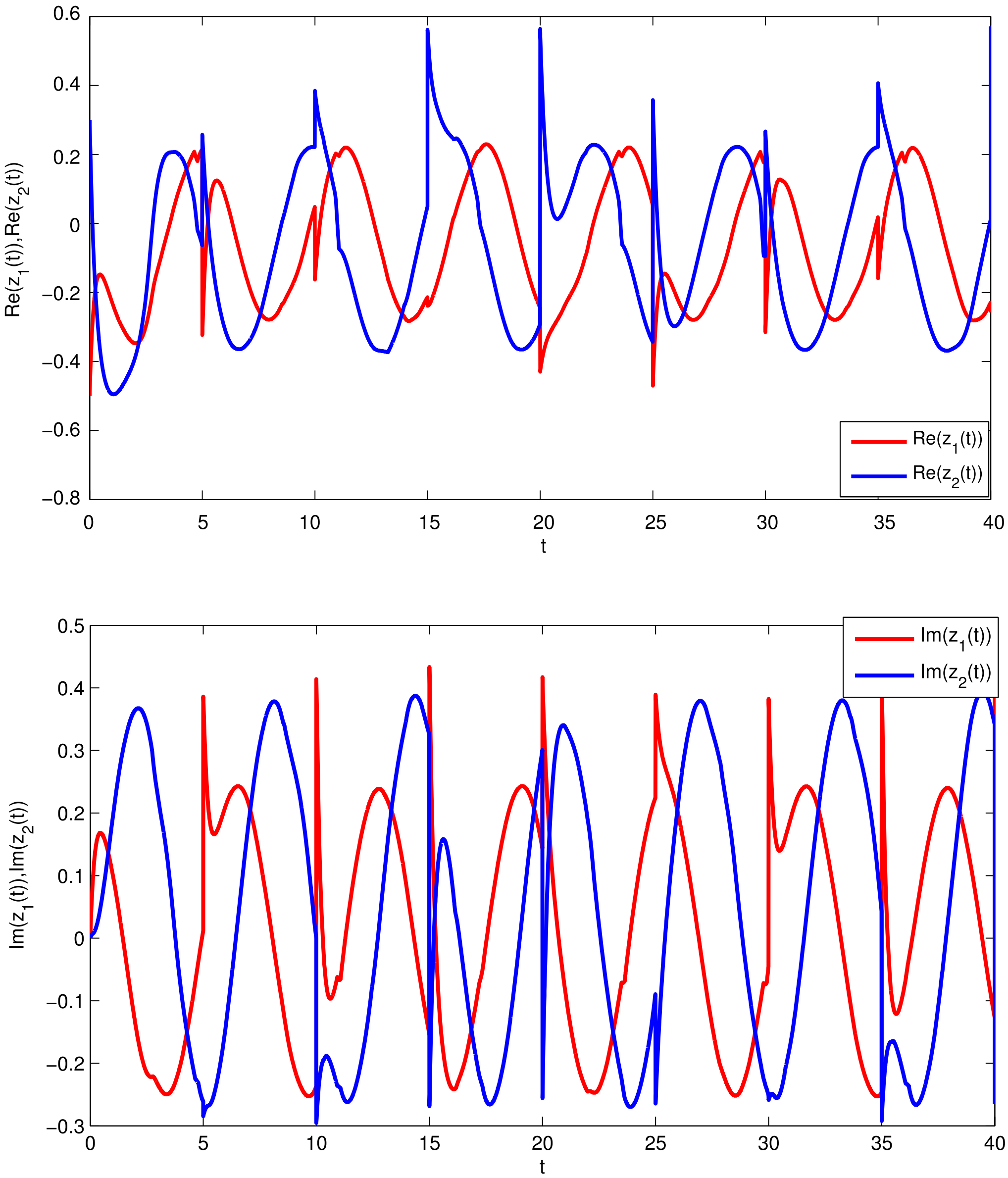

Fig. 4. State trajectories for (19) with impulse.

a bounded and analytic function is a trivial case and it is usually not suitable for complex-valued activation functions. However, Theorem 4 in [24] assumes that the activation functions are bounded besides the Lipschitz condition, and thus the activation functions are not analytic. This leads to limitations in choosing activation functions. Therefore, compared with the existing work in [24], our study serves a wider class of complex-valued neural networks.

4. An illustrative example

Example 1. Consider a two-neuron T–S fuzzy CVNN and the plant rule with $m=2$.

$$\frac{dz(t)}{dt} = \sum_{r=1}^{2} \phi_r(\theta(t))\{-C_r z(t) + A_r f(z(t)) + B_r f(z(t-\tau(t))) + u(t)\}. \tag{17}$$

Now we give the **Plant Rule**:

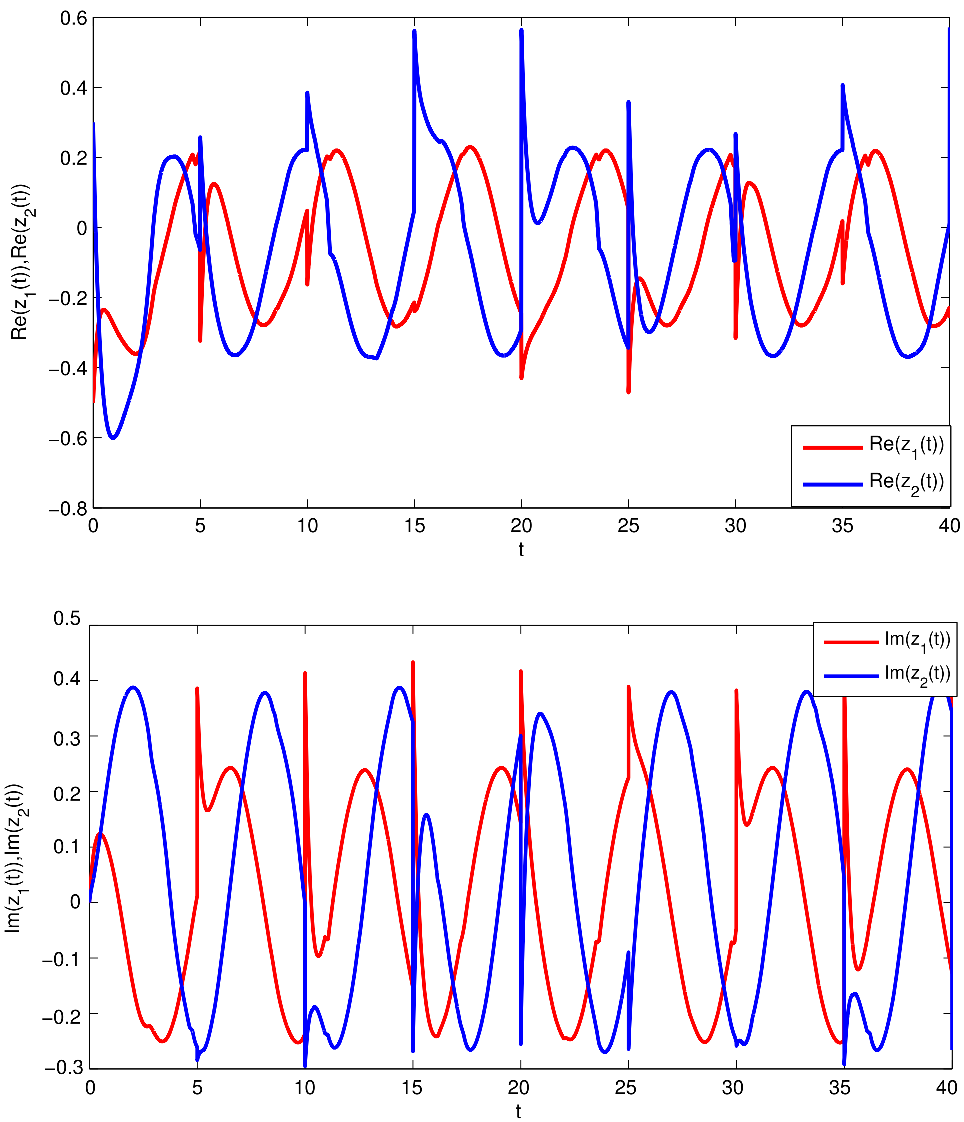

Fig. 5. State trajectories for (18) with impulse.

If $\{\theta_1(t) \text{ is } M_{11}\}$

Then

$$\begin{cases} \frac{dz(t)}{dt} = -C_1 z(t) + A_1 f(z(t)) + B_1 f(z(t-\tau(t))) + u(t), \\ z(s) = \varphi(s), \quad s \in [-\tau, 0], \end{cases} \tag{18}$$

If $\{\theta_2(t) \text{ is } M_{22}\}$

Then

$$\begin{cases} \frac{dz(t)}{dt} = -C_2 z(t) + A_2 f(z(t)) + B_2 f(z(t-\tau(t))) + u(t), \\ z(s) = \varphi(s), \quad s \in [-\tau, 0]. \end{cases} \tag{19}$$

The parameters are as follows

$$C_1 = \begin{bmatrix} 4-0.1i & 0 \\ 0 & 3-0.2i \end{bmatrix}, \; A_1 = B_1 = \begin{bmatrix} 1i & -1 \\ 1i & 2i \end{bmatrix},$$

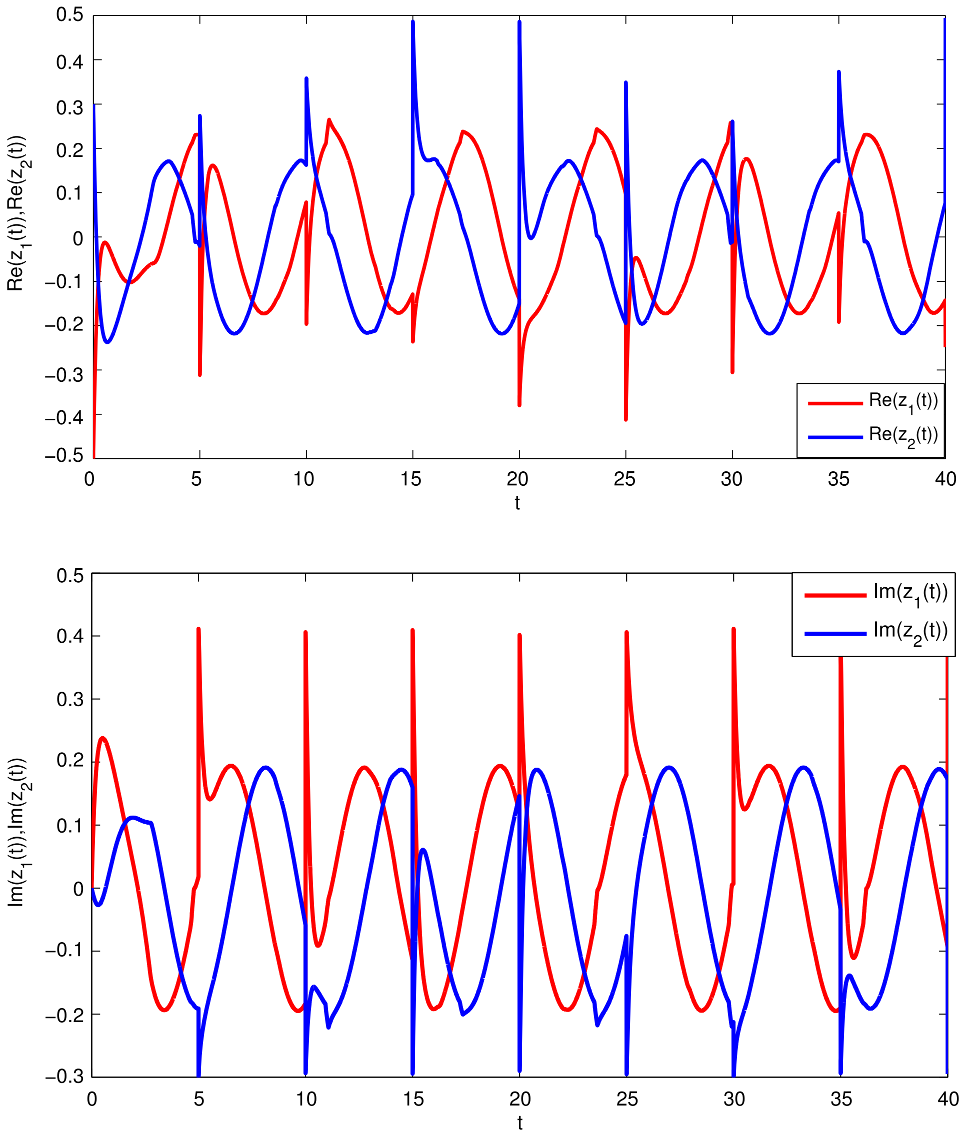

Fig. 6. State trajectories for (19) with impulse.

$$C_2 = \begin{bmatrix} 5+0.2i & 0 \\ 0 & 5-0.4i \end{bmatrix}, \; A_2 = B_2 = \begin{bmatrix} 1 & -1i \\ 1i & -1 \end{bmatrix},$$

$$\tau_j(t) = 2 + 2\sin(t), \; f_j(z_j(t)) = \frac{1}{5}(|Re(z_j)| + |Im(z_j)|i), \; j = 1, 2,$$

$$u(t) = (-\sin(t) + \cos(t)i, -\cos(t) + \sin(t)i)^T.$$

Let the initial conditions $z_1(s) = -0.5\cos(0.6t) - 2\sin(0.9t)i$, $z_2(t) = 0.3\cos(0.6t) + 2\sin(0.7t)i$ with $s \in [-4, 0]$ for networks (17)–(19).

We define membership functions as follows

$$\phi_1(\theta(t)) = \frac{1}{1+e^{-2t}}, \quad \phi_2(\theta(t)) = 1 - \phi_1(\theta(t)).$$

Hence, Assumption 1 is satisfied with $L = \mathrm{diag}\{\frac{1}{5}, \frac{1}{5}\}$. Let $a = 1$, $b = 0.5$, by using the effective Yalmip toolbox in Matlab, we can find appropriate matrices P, K, S, Y presented as follows which satisfy the conditions (ii)(iii) of Theorem 1:

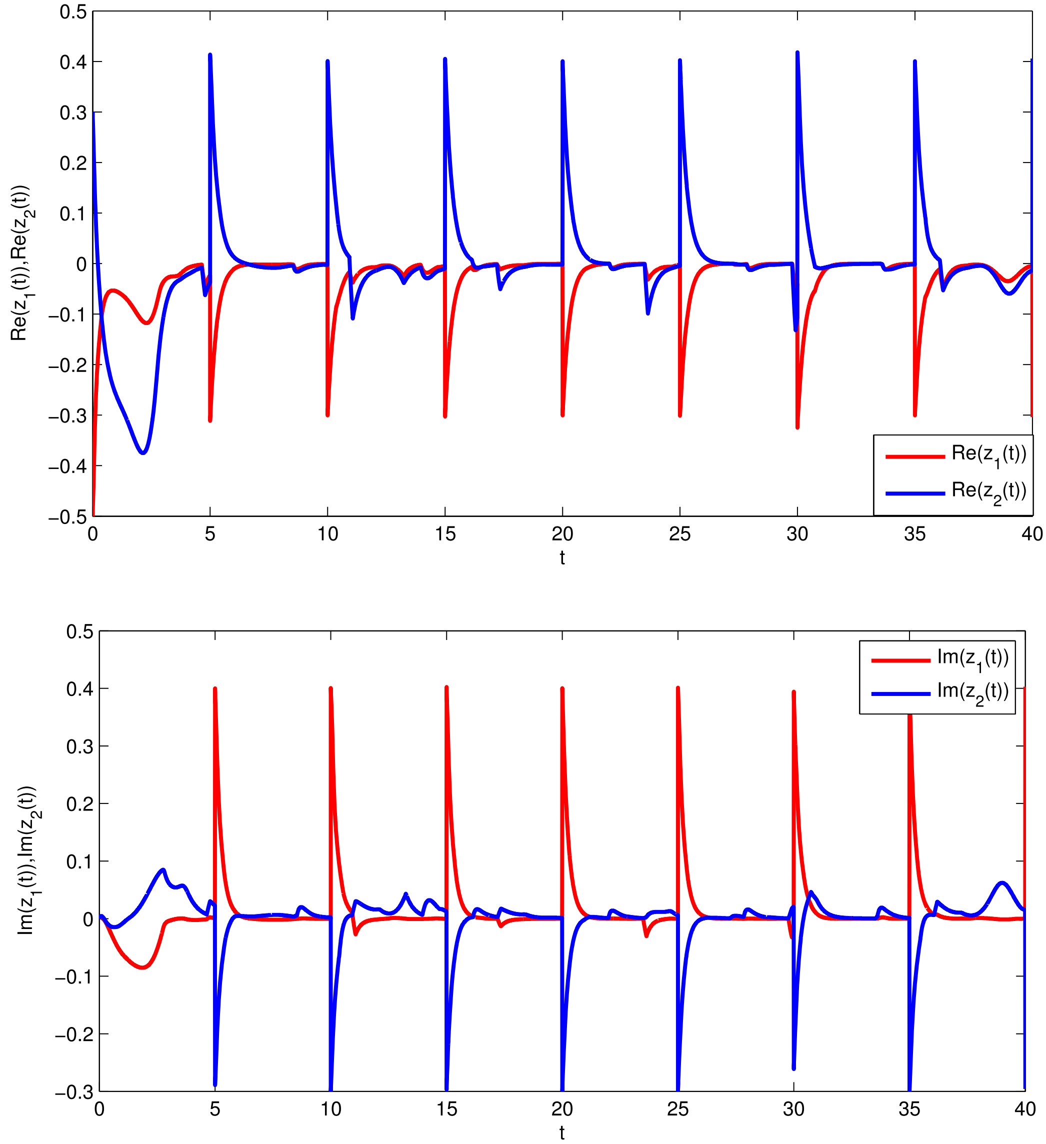

Fig. 7. State trajectories for (17) with impulse and $u(t) = 0$, $w_k \neq 0$.

$$P = \begin{bmatrix} 0.1232 & -0.0033 - 0.0067i \\ -0.0033 + 0.0067i & 0.1186 \end{bmatrix},$$

$$K = \begin{bmatrix} 1.0069 & 0 \\ 0 & 1.0068 \end{bmatrix}, \quad S = \begin{bmatrix} 1.0229 & 0 \\ 0 & 1.0229 \end{bmatrix}, \quad Y = \begin{bmatrix} 1.0204 & 0 \\ 0 & 1.0154 \end{bmatrix},$$

so, network (17) without impulse is globally exponentially convergent to the ball

$$\mathscr{B}_3 = \{z(t) \in \mathbb{C}^2 \mid |z_1(t)|^2 + |z_2(t)|^2 \leq 4.0916\}.$$

Fig. 1 shows the real parts and imaginary parts of state trajectories of network (17) without impulse. Fig. 2 exhibits the real parts and imaginary parts of state trajectories of (18) with the plant rule parameters C_1, A_1, B_1 and without impulse. Fig. 3 shows the real parts and imaginary parts of state trajectories of (19) with the plant rule parameters C_2, A_2, B_2 and without impulse.

On the other hand, considering the impulse effects for networks (17)–(19), the impulses gain parameter matrices and the impulsive perturbations are as follows

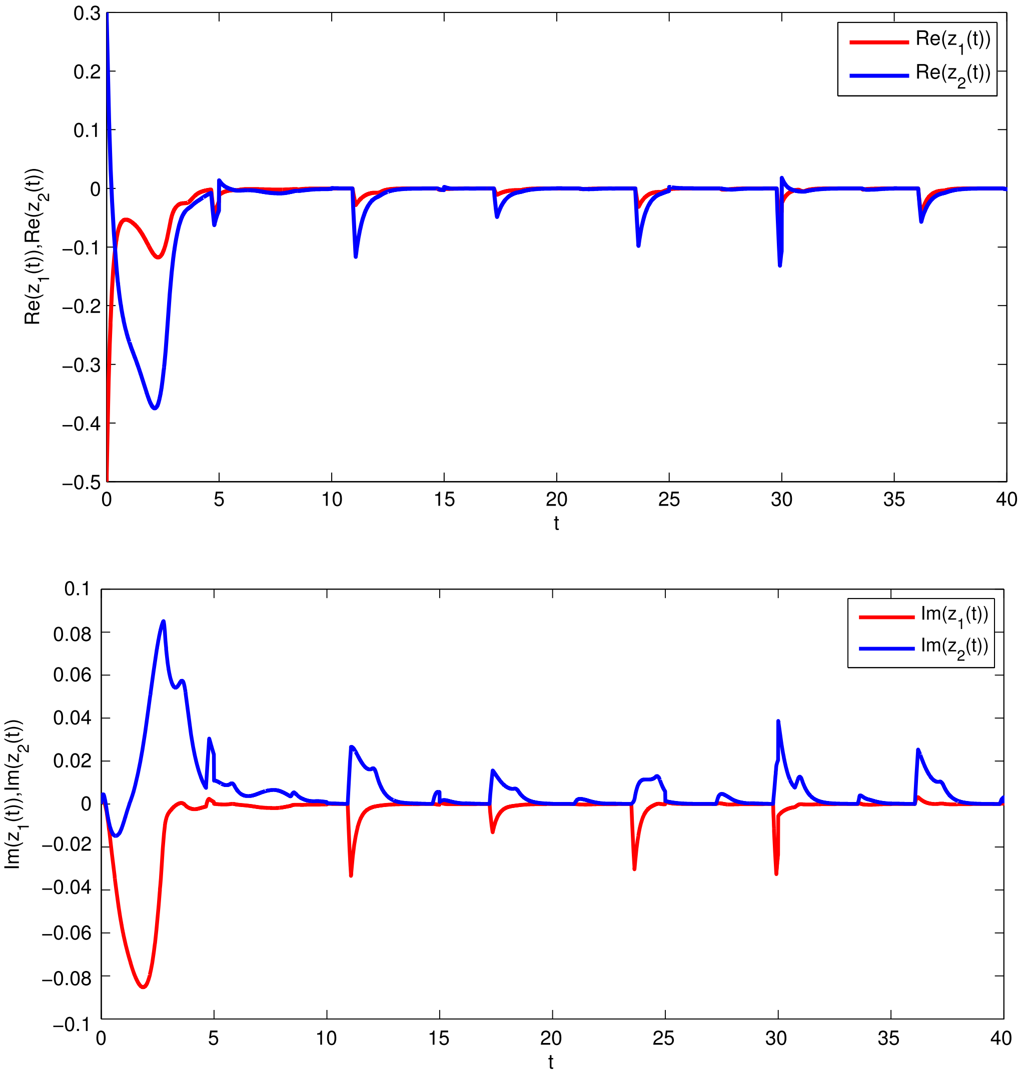

Fig. 8. State trajectories for (17) with impulse and $u(t) = 0$, $w_k = 0$.

$$E_k = \begin{bmatrix} 0.25 + 0.25i & 0.25 - 0.25i \\ -0.25 + 0.5i & -0.25 - 0.25i \end{bmatrix}, \; F_k = \begin{bmatrix} 0.25 & -0.25i \\ 0.25i & -0.25 \end{bmatrix}, \; w_k = \begin{bmatrix} -0.3 + 0.4i \\ 0.4 - 0.3i \end{bmatrix},$$
$$g_k(z) = (0.1 + 0.1i)z, \; r_k(z) = z \cdot i, \; t_k = 5k, \; t_k - t_{k-1} = 5 > \tau = 4, \; k \in N_+.$$

We can obtain $a_k = 0.1389$, $b_k = 0.4913$, $c_k = 0.2330$ and $m_k = m_0 = 4.0916$. So, network (17) with impulse is also globally exponentially convergent to the ball

$$\mathscr{B}_4 = \{z(t) \in \mathbb{C}^2 \mid |z_1(t)|^2 + |z_2(t)|^2 \leq 4.0916\}.$$

Fig. 4 shows the real parts and imaginary parts of state trajectories of network (17) with impulsive effects. Fig. 5 exhibits the real and imaginary parts of state trajectories of (18) with the plant rule parameters C_1, A_1, B_1 and impulsive effects. Fig. 6 shows the real and imaginary parts of state trajectories of (19) with the plant rule parameters C_2, A_2, B_2 and impulsive effects. If $u(t) = 0$ and $z(t_k^+) = E_k g_k(z(t_k^-)) + F_k r_k(z(t_k^- - \tau(t_k^-))) + w_k(t_k^-)$, Fig. 7 shows the real and imaginary parts of state trajectories of network (17). If $u(t) = 0$ and $z(t_k^+) = E_k g_k(z(t_k^-)) + F_k r_k(z(t_k^- - \tau(t_k^-)))$, the real parts and imaginary parts of state trajectories of network (17) are showed in Fig. 8. From Figs. 1–8, we can conclude that the time responses of states of (17)–(19) with and without T–S fuzzy rules or impulse effects are different, respectively. So, Figs. 1–7 testify the validity of the results for Theorem 1 and

Remark 3, and networks (17)–(19) with and without impulsive effects remain bounded. Fig. 8 indicates that the result of Corollary 2 is correct and the zero solution of T–S fuzzy CVNN (17) with $u(t) = 0$ and $w_k(t_k^-) = 0$ is globally exponentially Lyapunov stable.

5. Conclusions

In this paper, the global convergence of fuzzy complex-valued neural networks with time-varying delays and impulsive effects has been discussed. By employing Lyapunov functional method and inequality technique, some sufficient conditions on complex-valued linear matrix inequality form are obtained to ensure the global exponential convergence. Moreover, the detail estimations of the exponential convergence ball and the exponential convergence rate index are estimated. Our results improve existing results on the globally exponential stability in Lyapunov sense as special cases. Further, the method in this paper can also be extended to study the boundedness and convergence for other neural networks with impulse disturbance and parameters uncertainties. Finally, the effectiveness of the theoretical results is demonstrated via an illustrative example with simulations.

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