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Mean-square admissibility for stochastic T–S fuzzy singular systems based on extended quadratic Lyapunov function approach

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Abstract

The mean-square admissibility problem of stochastic T–S fuzzy singular systems via an extended quadratic Lyapunov function is investigated in this paper. Comparing with the existing quadratic Lyapunov function method, the extended quadratic Lyapunov function method can relax stabilization conditions. Firstly, the sufficient condition is given for the mean-square admissibility of stochastic T–S fuzzy singular systems based on an extended quadratic Lyapunov function approach. Secondly, two sufficient conditions for mean-square admissibility of closed-loop systems via the parallel distributed compensation (PDC) fuzzy controller and non-parallel distributed compensation (non-PDC) fuzzy controller are proposed. Furthermore, through the extended quadratic Lyapunov function method and non-PDC fuzzy controller, the less conservative mean-square admissibility conditions on solving fuzzy controllers are derived in terms of linear matrix inequalities (LMIs). Finally, some simulation examples are given to show the effectiveness and merits of the proposed fuzzy controller design methodology.
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1. Introduction

As is well known, many practical stochastic behaviors are inevitable. In this situation, the systems to be controlled are always modeled by stochastic systems. Itô stochastic differential-algebraic equations, also called stochastic singular systems [1], have received much attention recently since stochastic modelings have come to play an important role in many branches of science and engineering applications. Many important contributions in this direction have been reported in the literature (see [2–8] and references therein). For example, the filtering and linear quadratic Gaussian (LQG) problems for discrete-time stochastic singular systems were discussed in [2], where the linear unbiased

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least-squares state estimation algorithm and the optimal control law for the LQG problem were proposed. Gao and Shi [5] discussed the observer-based controller design for stochastic singular systems with Brownian motions, where the observer and control gains were designed by using a complete separation design technique. Boukas [6] studied the stabilization problem of stochastic singular nonlinear hybrid systems, the results of which on stochastic stability and stochastic stabilization were developed under some appropriate assumptions.

In recent years, there have been some attempts to employ Takagi–Sugeno (T–S) fuzzy model based control technique for stochastic nonlinear systems by interpolation of numerous local linear models in terms of If–Then fuzzy rules (see [9–15] and references therein). The problems of stabilizability and robust stabilization for stochastic fuzzy systems were investigated in [9,10]. The problems of finite-time stability and control for a class of stochastic singular biological economic systems were discussed in [11]. The robust stochastic fuzzy controller with H_∞ performance was designed for a class of Markovian jump nonlinear systems in [12]. Tseng [13] designed a robust fuzzy filter for a class of stochastic nonlinear systems, where the H_∞ fuzzy filtering design for stochastic nonlinear systems was given via using fuzzy approach. Wang [14] proposed sufficient conditions on robust stability for uncertain stochastic fuzzy systems with delays.

In the previously researched literature, most of the stability analysis was based on a single quadratic Lyapunov function (see [15–18] and references therein). By the convex optimization algorithm, the stability and stabilization problems of fuzzy systems can be converted into LMI problems. Nevertheless, restriction to the class of quadratic Lyapunov functions may lead to significant conservativeness. And then, some scholars also have managed to reduce the conservatism by the fuzzy Lyapunov function (see [19–22] and references therein). However, for continuous-time case, the fuzzy Lyapunov function will produce the time derivatives of membership functions, and their upper bounds have to be known prior to design a fuzzy controller. Since the time derivatives are proportional to not only states but also inputs, it is practically difficult to evaluate the upper bounds prior. As a result, the solution to the stability conditions cannot be simply solved numerically by some convex programming techniques. Motivated by the above analysis, the mean-square admissibility problem of stochastic T–S fuzzy singular systems based on an extended quadratic Lyapunov function approach is investigated in this paper.

The extended quadratic Lyapunov function concepts are employed in this paper to develop novel relaxed mean-square admissibility conditions for stochastic T–S fuzzy singular systems. By solving these relaxed mean-square admissibility conditions, the mean-square admissibility of closed-loop systems can be achieved by the designed PDC fuzzy controller. In addition, a non-PDC fuzzy controller design is also developed in this paper. Using the quadratic (non-quadratic) Lyapunov function, some scholars have derived the relaxed stability conditions for T–S fuzzy systems (see [22–33] and references therein), but the Brownian motions were not considered for the systems [19–33]. Compared with the approaches of [19–33], PDC and non-PDC fuzzy control design methods with relaxed stability conditions for continuous-time multiplicative noised fuzzy systems were proposed in [34]. However, to the best of our knowledge, the mean-square admissibility problem of stochastic T–S fuzzy singular systems has not yet been fully investigated. New study method will be adopted in this paper, which will reduce the conservativeness in mean-square admissibility analysis of continuous-time stochastic T–S fuzzy singular systems based on the extended quadratic Lyapunov functions. The main contribution of this paper is to develop the PDC-based fuzzy controller designed method and non-PDC-based fuzzy controller designed method by deriving the relaxed mean-square admissibility conditions for T–S fuzzy models with Brownian motions. Compared with the previous results, new relaxed methods have been proposed, which avoid time derivatives of membership functions.

In this paper, the PDC and non-PDC fuzzy controllers with the relaxed mean-square admissibility conditions are designed. In contrast to the PDC fuzzy controller, the non-PDC fuzzy controller can provide a better methodology with regard to relaxing the conservativeness of mean-square admissibility problem for T–S fuzzy singular systems with Brownian motions. To illustrate the validity and applicability of the proposed method, two design examples are provided. The first example shows that the LMI conditions based on the non-PDC fuzzy controller are less conservative than those based on the PDC fuzzy controller. The second example is a practical application of stochastic singular single-species bio-economic models, which demonstrates that the proposed method provides the effectiveness of the theoretical results. In general, we present some novel results which are theoretically beneficial to maintaining the sustainable development of the stochastic bio-economic system as well as keeping the economic interest of harvesting at an ideal level.

This paper is organized as follows. In Section 2, some necessary preliminaries are presented. In Section 3, the mean-square admissibility of stochastic T–S fuzzy singular systems is studied, which presents an important theorem for such systems based on an extended quadratic Lyapunov function approach. Furthermore, two sufficient conditions for mean-square admissibility of closed-loop systems via the PDC fuzzy controller and non-PDC fuzzy controller are proposed. Some simulation examples are provided in Section 4. Conclusions are given in Section 5.

Notations. Throughout this paper, the following notations will be used. The symbol $*$ represents transpose terms in a symmetric matrix and $diag\{\ldots\}$ stands for a block-diagonal matrix. $\Re^n$ denotes the n-dimensional Euclidean space. $\Re^{m\times n}$ is the set of all $m \times n$ real matrices. A^{T} is the transpose of matrix A. $X \geq 0$ $(X > 0)$ means X is a positive semi-definite (positive definite) matrix. Matrices $\{X_i\}_{i=1}^r$ denote matrices $X_1, X_2, \ldots, X_r$. $\mathrm{E}(x)$ denotes the expectation operator. C^1 represents a one order differentiable function, I is used to denote an identity matrix with proper dimension. Matrices, if their dimensions are not explicitly stated, are assumed to be compatible for algebraic operations.

2. System description and preliminaries

In this section, let us consider the stochastic T–S fuzzy singular system with Itô-type defined in a fundamental probability space $(\Omega, \mathcal{F}, \mathrm{P})$. The ith rule of the system is described by the following If–Then form:

Plant Rule i: If $z_1(t)$ is M_1^i, $z_2(t)$ is $M_2^i, \ldots,$ and $z_p(t)$ is M_p^i, Then

$$Edx(t) = A_i x(t)dt + B_i u(t)dt + J_i x(t)dW(t) \quad i = 1, 2, \ldots, r \tag{1}$$

where A_i, B_i, J_i are known constant matrices with appropriate dimensions. $x(t) \in \Re^n$ is the state vector, $u(t) \in \Re^m$ is the control input vector, M_j^i is the fuzzy set, r is the number of If–Then rules, $z_j(t)$ $(j = 1, 2, \ldots, p)$ is the premise variable. $x_0 \in \Re^n$ is the initial state. $E \in \Re^{n\times n}$ is a known singular matrix with $rank(E) = q \leq n$. Without loss of generality, throughout this paper, we assume that $W(t)$ is one-dimensional standard Brownian motion defined on the probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, \mathrm{P})$, where Ω is the sample space, $\mathcal{F}$ is the σ-algebra of subsets of the sample space and P is the probability measure on $\mathcal{F}$.

By using a standard fuzzy singleton inference method, a singleton fuzzifier to produce a fuzzy inference and weighted center-average defuzzifier, the final fuzzy model of the system is inferred as follows:

$$Edx(t) = \sum_{i=1}^{r} h_i\big(z(t)\big)A_i x(t)dt + \sum_{i=1}^{r} h_i\big(z(t)\big)B_i u(t)dt + \sum_{i=1}^{r} h_i\big(z(t)\big)J_i x(t)dW(t) \tag{2}$$

where $z(t) = [z_1(t), z_2(t), \ldots, z_p(t)]$. And for any $i = 1, 2, \ldots, r$, we have

$$h_i\big(z(t)\big) = \frac{\omega_i(z(t))}{\sum_{i=1}^{r}\omega_i(z(t))}, \qquad \omega_i\big(z(t)\big) = \prod_{j=1}^{p} M_j^i\big(z_j(t)\big) \tag{3}$$

in which $M_j^i(z_j(t))$ is the grade of membership of $z_j(t)$ in the fuzzy set M_j^i. Membership function $h_i(z(t))$ satisfies

$$h_i\big(z(t)\big) \geq 0, \qquad \sum_{i=1}^{r} h_i\big(z(t)\big) = 1$$

In this paper, we require $h_1(z(t)), h_2(z(t)), \ldots, h_r(z(t))$ to be C^1 functions. It should be noted that the assumption is satisfied for fuzzy models constructed from smooth nonlinear systems by using a sector nonlinearity approach [35].

Consider the following unforced stochastic T–S fuzzy singular system

$$Edx(t) = \tilde{A}_h x(t)dt + \tilde{J}_h x(t)dW(t) \tag{4}$$

where $\tilde{A}_h = \sum_{i=1}^{r} h_i(z(t))A_i$, $\tilde{J}_h = \sum_{i=1}^{r} h_i(z(t))J_i$.

Definition 1. (I) System (4) is said to be regular if $\det(sE - \tilde{A}_h)$ is not identically zero.

(II) System (4) is said to be impulse-free if $\deg(\det(sE - \tilde{A}_h)) = rank(E)$.

(III) System (4) is said to be mean-square stable if, for any $\varepsilon > 0$, there exists a $\delta(\varepsilon) > 0$ such that $\mathrm{E}\|x(t)\|^2 < \varepsilon$, $t > 0$ when $\mathrm{E}\|x(0)\|^2 < \delta(\varepsilon)$.

(IV) System (4) is said to be mean-square admissible, if the system (4) is regular, impulse-free and mean-square stable.

Lemma 1. *(See [36].) Let $x(t)$ be an n-dimensional Itô process on $t \geq 0$ with respect to the linear stochastic singular system*

$$Edx(t) = Ax(t)dt + Jx(t)dW(t)$$

Define $V(x(t)) = x^{\mathrm{T}}(t)E^{\mathrm{T}}Xx(t)$, where X is a nonsingular matrix such that $E^{\mathrm{T}}X = X^{\mathrm{T}}E \geq 0$. Then $V(x(t))$ is also an Itô process with the stochastic singular differential given by

$$dV\big(x(t)\big) = x^{\mathrm{T}}(t)\big(A^{\mathrm{T}}X + X^{\mathrm{T}}A + J^{\mathrm{T}}\big(E^{+}\big)^{\mathrm{T}}E^{\mathrm{T}}XE^{+}J\big)x(t)dt + 2x^{\mathrm{T}}(t)X^{\mathrm{T}}Jx(t)dW(t)$$

where E^{+} is the Moore–Penrose inverse matrix of the matrix E.

Define a weak infinitesimal operator $\mathcal{L}$ of random process $\{x(t), t \geq 0\}$

$$\mathcal{L}V\big(x(t)\big) = x^{\mathrm{T}}(t)\big(A^{\mathrm{T}}X + X^{\mathrm{T}}A + J^{\mathrm{T}}\big(E^{+}\big)^{\mathrm{T}}E^{\mathrm{T}}XE^{+}J\big)x(t) \tag{5}$$

Then, the Itô formula can be described as

$$dV\big(x(t)\big) = \mathcal{L}V\big(x(t)\big)dt + 2x^{\mathrm{T}}(t)X^{\mathrm{T}}Jx(t)dW(t) \tag{6}$$

Lemma 2. *(See [21].) Suppose that for a given piecewise continuous matrix $A(t) \in R^{n\times n}$, if there exist a bounded time-varying matrix $P(t) \in R^{n\times n}$ and a scalar $\alpha > 0$ satisfying the following inequality*

$$A^{\mathrm{T}}(t)P(t) + P^{\mathrm{T}}(t)A(t) \leq -\alpha I$$

for all t, then the following conclusions hold.

(i) *$A(t)$ is invertible;*
(ii) *$A^{-1}(t)$ is bounded.*

Lemma 3. *(See [37].) Let the symmetrical matrix $G \in \Re^{n\times n}$ and matrix $B \in \Re^{n\times m}$ be given. If there exist a scalar $\alpha > 0$, nonsingular matrix $P \in \Re^{n\times n}$, matrices $L \in \Re^{m\times n}$, $\hat{U} \in \Re^{n\times n}$ and positive definite matrix $Y \in \Re^{n\times n}$ such that the LMI*

$$\begin{bmatrix} G + BL + L^{\mathrm{T}}B^{\mathrm{T}} + Y & BL & 0 \\ * & -\hat{U} - \hat{U}^{\mathrm{T}} & \alpha P - \hat{U} \\ * & * & -Y \end{bmatrix} < 0 \tag{7}$$

holds, then $\hat{U}$ is nonsingular, and $K = \alpha L\hat{U}^{-1}$ satisfies the following nonlinear matrix inequality

$$G + BKP + K^{\mathrm{T}}P^{\mathrm{T}}B^{\mathrm{T}} < 0 \tag{8}$$

Lemma 4. *The following items are true.*

(i) Assume that $\mathrm{rank}(E) = q$, there exist two orthogonal matrices U and V such that

$$E = U\begin{bmatrix} \Sigma_q & 0 \\ * & 0 \end{bmatrix}V^{T} = U\begin{bmatrix} I_q & 0 \\ * & 0 \end{bmatrix}\tilde{V}^{T} \tag{9}$$

where $\Sigma_q = diag\{\delta_1, \delta_2, \ldots, \delta_q\}$ with $\delta_k > 0$ for all $k = 1, 2, \ldots, q$. Partition $U = [U_1\ U_2]$, $V = [V_1\ V_2]$ and $\tilde{V} = [V_1\Sigma_q\ V_2]$ with $EV_2 = 0$ and $U_2^{\mathrm{T}}E = 0$.

(ii) If X satisfies

$$EX = X^{\mathrm{T}}E^{\mathrm{T}} \geq 0 \tag{10}$$

Then $\tilde{X} = \tilde{V}^{\mathrm{T}}XU$ with U and $\tilde{V}$ satisfying (9) if and only if

$$\tilde{X} = \begin{bmatrix} X_{11} & 0 \\ X_{21} & X_{22} \end{bmatrix} \tag{11}$$

with $X_{11} \geq 0 \in \Re^{q\times q}$. *In addition, when* X *is nonsingular, we have* $X_{11} > 0$ *and* $\det(X_{22}) \neq 0$. *Furthermore,* X *satisfying (10) can be parameterized as*

$$X = \tilde{V}^{-\mathrm{T}} P \tilde{V}^{-1} E^{\mathrm{T}} + V_2 Y U^{\mathrm{T}} \tag{12}$$

where $P = diag\{X_{11}, \Psi\}$, $Y = [\, X_{21} \;\; X_{22} \,]$, *and* $\Psi \in \Re^{(n-q)\times(n-q)}$ *is an arbitrary parameter matrix.*

(iii) If X *is a nonsingular matrix,* Ψ *is a positive definite matrix,* X *and* E *satisfy (10),* P *is a diagonal matrix from (12), and the following equality holds:*

$$E^{\mathrm{T}} X^{-1} = E^{\mathrm{T}} Q E \tag{13}$$

Then the positive definite matrix $Q = U P^{-1} U^{\mathrm{T}}$ *is a solution of (13).*

Proof. The proof of this lemma is similar to that the proof of Lemma 3 in the reference [38] and is omitted.

The following assumption is useful for the investigation of this paper. □

Assumption 1. (See [6].) The pair $(E, \tilde{A}_h)$ is regular and $rank([E \;\; \tilde{J}_h]) = rank(E)$.

3. Main results

3.1. Mean-square admissibility of the stochastic T–S fuzzy singular system

In this section, the mean-square admissibility condition is firstly derived for system (4). Based on the extended quadratic Lyapunov function approach and stochastic analysis approach, the following theorem is derived.

Theorem 1. *System (4) is mean-square admissible if there exist positive definite matrices* $X \in \Re^{q\times q}$, $\Psi_i \in \Re^{(n-q)\times(n-q)}$ *and matrices* $\{Y_i\}_{i=1}^{r} \in \Re^{(n-q)\times n}$ *such that*

$$\tilde{G}_{ii} < 0, \quad i = 1, 2, \ldots, r \tag{14}$$

$$\tilde{G}_{ij} + \tilde{G}_{ji} < 0, \quad 1 \leq i < j \leq r \tag{15}$$

where

$$\tilde{G}_{ij} = \begin{bmatrix} X_i^{\mathrm{T}} A_j^{\mathrm{T}} + A_j X_i & X_i^{\mathrm{T}} J_j^{\mathrm{T}} (E^{+})^{\mathrm{T}} E^{\mathrm{T}} \\ * & -\tilde{Q}_i \end{bmatrix}$$

$$X_i = \tilde{V}^{-\mathrm{T}} P_i \tilde{V}^{-1} E^{\mathrm{T}} + V_2 Y_i U^{\mathrm{T}}, \qquad P_i = diag\{X, \Psi_i\}, \qquad \tilde{Q}_i = U^{-\mathrm{T}} P_i U^{\mathrm{T}}$$

with orthogonal matrix U *and nonsingular matrix* $\tilde{V}$ *satisfying* $U^{\mathrm{T}} E \tilde{V}^{-\mathrm{T}} = \left[\begin{smallmatrix} I_q & 0 \\ 0 & 0 \end{smallmatrix}\right]$, *here,* $\tilde{V} = [\tilde{V}_1 \;\; V_2]$ *and* $EV_2 = 0$.

Proof. Under Assumption 1, the pair $(E, \tilde{A}_h)$ is regular, by the condition (i) in Lemma 4, there exist orthogonal matrix U and nonsingular matrix $\tilde{V}$ such that

$$U^{\mathrm{T}} E \tilde{V}^{-\mathrm{T}} = \begin{bmatrix} I_q & 0 \\ 0 & 0 \end{bmatrix}, \qquad U^{\mathrm{T}} A_i \tilde{V}^{-\mathrm{T}} = \begin{bmatrix} A_{11}^i & A_{12}^i \\ A_{21}^i & A_{22}^i \end{bmatrix}$$

$$U^{\mathrm{T}} J_i \tilde{V}^{-\mathrm{T}} = \begin{bmatrix} J_{11}^i & J_{12}^i \\ 0 & 0 \end{bmatrix}, \qquad \tilde{V}^{\mathrm{T}} x(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}, \quad i = 1, 2, \ldots, r \tag{16}$$

Since $X_i = \tilde{V}^{-\mathrm{T}} P_i \tilde{V}^{-1} E^{\mathrm{T}} + V_2 Y_i U^{\mathrm{T}}$, we have

$$E X_i = X_i^{\mathrm{T}} E^{\mathrm{T}} \geq 0 \tag{17}$$

From (16) and (17), one gets

$$\begin{bmatrix} I_q & 0 \\ 0 & 0 \end{bmatrix} \tilde{V}^{\mathrm{T}} X_i U^{-\mathrm{T}} = U^{\mathrm{T}} X_i^{\mathrm{T}} \tilde{V} \begin{bmatrix} I_q & 0 \\ 0 & 0 \end{bmatrix} \geq 0 \tag{18}$$

From (18), we can obtain

$$\tilde{V}^{\mathrm{T}} X_i U^{-\mathrm{T}} = \begin{bmatrix} X & 0 \\ X_{21}^i & X_{22}^i \end{bmatrix} \tag{19}$$

$$U^{\mathrm{T}} E X_i U^{-\mathrm{T}} = \begin{bmatrix} X & 0 \\ 0 & 0 \end{bmatrix} \tag{20}$$

The conditions (14) and (15) imply

$$\left(\sum_{j=1}^{r} h_j\big(z(t)\big) A_j\right)\left(\sum_{i=1}^{r} h_i\big(z(t)\big) X_i\right) + \left(\sum_{i=1}^{r} h_i\big(z(t)\big) X_i\right)^{\mathrm{T}} \left(\sum_{j=1}^{r} h_j\big(z(t)\big) A_j\right)^{\mathrm{T}} < 0 \tag{21}$$

which is equivalent to

$$\begin{aligned} &\left(U^{\mathrm{T}} \sum_{j=1}^{r} h_j\big(z(t)\big) A_j \tilde{V}^{-\mathrm{T}}\right)\left(\tilde{V}^{\mathrm{T}} \sum_{i=1}^{r} h_i\big(z(t)\big) X_i U^{-\mathrm{T}}\right) \\ &\quad + \left(\tilde{V}^{\mathrm{T}} \sum_{i=1}^{r} h_i\big(z(t)\big) X_i U^{-\mathrm{T}}\right)^{\mathrm{T}} \left(U^{\mathrm{T}} \sum_{j=1}^{r} h_j\big(z(t)\big) A_j \tilde{V}^{-\mathrm{T}}\right)^{\mathrm{T}} < 0 \end{aligned} \tag{22}$$

Applying (16) and (19) into (22), one can get

$$\begin{bmatrix} \nabla_1 & \nabla_2 \\ * & \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(z(t)) h_j(z(t)) (A_{22}^j X_{22}^i + (X_{22}^i)^{\mathrm{T}} (A_{22}^j)^{\mathrm{T}}) \end{bmatrix} < 0 \tag{23}$$

Since ∇_1 and ∇_2 are irrelevant to the results of the following discussion, the real expressions of these two variables are omitted here. From (23), it is easy to see that $\sum_{i=1}^{r}\sum_{j=1}^{r} h_i(z(t)) h_j(z(t)) (A_{22}^j X_{22}^i + (X_{22}^i)^{\mathrm{T}} (A_{22}^j)^{\mathrm{T}}) < 0$, this implies that $\sum_{j=1}^{r} h_j(z(t)) A_{22}^j$ is nonsingular. Therefore, system (4) is impulse-free.

Next, we prove system (4) is mean-square stable. Construct the following extended quadratic Lyapunov function

$$V\big(x(t)\big) = x^{\mathrm{T}}(t) E^{\mathrm{T}} \left(\sum_{k=1}^{r} h_k\big(z(t)\big) X_k\right)^{-1} x(t) = x^{\mathrm{T}}(t) E^{\mathrm{T}} \tilde{X}_h^{-1} x(t) \tag{24}$$

where $\tilde{X}_h = \sum_{k=1}^{r} h_k(z(t)) X_k$.

Inequality (21) can be simplified to $\tilde{A}_h \tilde{X}_h + \tilde{X}_h^{\mathrm{T}} \tilde{A}_h^{\mathrm{T}} < 0$. By Lemma 2, $\tilde{X}_h$ is nonsingular. From (17), $E^{\mathrm{T}} \tilde{X}_h^{-1} = \tilde{X}_h^{-\mathrm{T}} E \geq 0$ holds.

By Lemma 1, the stochastic derivative of $V(x(t))$ along the trajectory of system (4) can be obtained as follows:

$$dV\big(x(t)\big) = \mathcal{L}V\big(x(t)\big)dt + 2x^{\mathrm{T}}(t) \tilde{X}_h^{-\mathrm{T}} \tilde{J}_h x(t) dW(t) \tag{25}$$

where

$$\mathcal{L}V\big(x(t)\big) = x^{\mathrm{T}}(t)\left(E^{\mathrm{T}} \frac{d\tilde{X}_h^{-1}}{dt} + \tilde{A}_h^{\mathrm{T}} \tilde{X}_h^{-1} + \tilde{X}_h^{-\mathrm{T}} \tilde{A}_h + \tilde{J}_h^{\mathrm{T}} \big(E^{+}\big)^{\mathrm{T}} E^{\mathrm{T}} \tilde{X}_h^{-1} E^{+} \tilde{J}_h\right) x(t)$$

From $\tilde{X}_h \tilde{X}_h^{-1} = I$, we have $\frac{d}{dt}[\tilde{X}_h \tilde{X}_h^{-1}] = 0$. Then,

$$\frac{d\tilde{X}_h^{-1}}{dt} = -\tilde{X}_h^{-1} \frac{d}{dt} \sum_{k=1}^{r} h_k\big(z(t)\big) X_k \tilde{X}_h^{-1} = -\tilde{X}_h^{-1} \sum_{k=1}^{r} \dot{h}_k\big(z(t)\big) X_k \tilde{X}_h^{-1}$$

With this we then have

$$\begin{aligned}\mathcal{L}V\big(x(t)\big) &= x^{\mathrm{T}}(t)\Bigg(-E^{\mathrm{T}}\tilde{X}_h^{-1}\sum_{k=1}^{r}\dot{h}_k\big(z(t)\big)X_k\tilde{X}_h^{-1} + \tilde{A}_h^{\mathrm{T}}\tilde{X}_h^{-1} + \tilde{X}_h^{-\mathrm{T}}\tilde{A}_h + \tilde{J}_h^{\mathrm{T}}\big(E^+\big)^{\mathrm{T}}E^{\mathrm{T}}\tilde{X}_h^{-1}E^+\tilde{J}_h\Bigg)x(t)\\ &= x^{\mathrm{T}}(t)\Bigg(-\tilde{X}_h^{-\mathrm{T}}\sum_{k=1}^{r}\dot{h}_k\big(z(t)\big)EX_k\tilde{X}_h^{-1} + \tilde{A}_h^{\mathrm{T}}\tilde{X}_h^{-1} + \tilde{X}_h^{-\mathrm{T}}\tilde{A}_h + \tilde{J}_h^{\mathrm{T}}\big(E^+\big)^{\mathrm{T}}E^{\mathrm{T}}\tilde{X}_h^{-1}E^+\tilde{J}_h\Bigg)x(t)\end{aligned} \tag{26}$$

From the properties of membership functions, we have

$$\sum_{k=1}^{r}\dot{h}_k\big(z(t)\big) = 0$$

Therefore,

$$\begin{aligned}U^{\mathrm{T}}\sum_{k=1}^{r}\dot{h}_k\big(z(t)\big)EX_kU &= \sum_{k=1}^{r}\dot{h}_k\big(z(t)\big)U^{\mathrm{T}}E\big(\tilde{V}^{-\mathrm{T}}P_k\tilde{V}^{-1}E^{\mathrm{T}} + V_2Y_kU^{\mathrm{T}}\big)U\\ &= \sum_{k=1}^{r}\dot{h}_k\big(z(t)\big)U^{\mathrm{T}}E\tilde{V}^{-\mathrm{T}}P_k\tilde{V}^{-1}E^{\mathrm{T}}U\\ &= \sum_{k=1}^{r}\dot{h}_k\big(z(t)\big)\begin{bmatrix}I_q & 0\\ 0 & 0\end{bmatrix}\begin{bmatrix}X & 0\\ 0 & \Psi_k\end{bmatrix}\begin{bmatrix}I_q & 0\\ 0 & 0\end{bmatrix}\\ &= \sum_{k=1}^{r}\dot{h}_k\big(z(t)\big)\begin{bmatrix}X & 0\\ 0 & 0\end{bmatrix}\\ &= \begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}\end{aligned}$$

From the above analysis, we can obtain

$$\sum_{k=1}^{r}\dot{h}_k\big(z(t)\big)EX_k = 0$$

So, $\mathcal{L}V(x(t)) < 0$ holds if and only if the following inequality holds.

$$\tilde{A}_h^{\mathrm{T}}\tilde{X}_h^{-1} + \tilde{X}_h^{-\mathrm{T}}\tilde{A}_h + \tilde{J}_h^{\mathrm{T}}\big(E^+\big)^{\mathrm{T}}E^{\mathrm{T}}\tilde{X}_h^{-1}E^+\tilde{J}_h < 0 \tag{27}$$

Multiplying the inequality (27) on the left and right by $\tilde{X}_h^{\mathrm{T}}$ and its transpose, respectively, we have

$$\tilde{X}_h^{\mathrm{T}}\tilde{A}_h^{\mathrm{T}} + \tilde{A}_h\tilde{X}_h + \tilde{X}_h^{\mathrm{T}}\tilde{J}_h^{\mathrm{T}}\big(E^+\big)^{\mathrm{T}}E^{\mathrm{T}}\tilde{X}_h^{-1}E^+\tilde{J}_h\tilde{X}_h < 0 \tag{28}$$

From $\tilde{Q}_i = U^{-\mathrm{T}}P_iU^{\mathrm{T}}$, we have

$$\tilde{Q}_i = U^{-\mathrm{T}}\begin{bmatrix}X & 0\\ 0 & \Psi_i\end{bmatrix}U^{\mathrm{T}}$$

Then,

$$\tilde{Q}_h = U^{-\mathrm{T}}\begin{bmatrix}X & 0\\ 0 & \sum_{i=1}^{r}h_i(z(t))\Psi_i\end{bmatrix}U^{\mathrm{T}}$$

Thus, we have $E^{\mathrm{T}}\tilde{Q}_h^{-1}E = E^{\mathrm{T}}\tilde{X}_h^{-1}$, inequality (28) is equivalent to

$$\tilde{X}_h^{\mathrm{T}}\tilde{A}_h^{\mathrm{T}} + \tilde{A}_h\tilde{X}_h + \tilde{X}_h^{\mathrm{T}}\tilde{J}_h^{\mathrm{T}}\big(E^+\big)^{\mathrm{T}}E^{\mathrm{T}}\tilde{Q}_h^{-1}EE^+\tilde{J}_h\tilde{X}_h < 0 \tag{29}$$

By the Schur complement, we have

$$\begin{bmatrix}\tilde{X}_h^{\mathrm{T}}\tilde{A}_h^{\mathrm{T}} + \tilde{A}_h\tilde{X}_h & \tilde{X}_h^{\mathrm{T}}\tilde{J}_h^{\mathrm{T}}(E^+)^{\mathrm{T}}E^{\mathrm{T}}\\ * & -\tilde{Q}_h\end{bmatrix} < 0 \tag{30}$$

The above inequality (30) can be arranged as

$$\sum_{i=1}^{r}\sum_{j=1}^{r} h_i\big(z(t)\big)h_j\big(z(t)\big)\begin{bmatrix} X_i^{\mathrm{T}}A_j^{\mathrm{T}}+A_jX_i & X_i^{\mathrm{T}}J_j^{\mathrm{T}}(E^{+})^{\mathrm{T}}E^{\mathrm{T}} \\ * & -\tilde{Q}_i \end{bmatrix}<0 \tag{31}$$

Inequality (31) can be rewritten as

$$\sum_{i=1}^{r} h_i^2\big(z(t)\big)\tilde{G}_{ii}+\sum_{i=1}^{r}\sum_{i<j}^{r} h_i\big(z(t)\big)h_j\big(z(t)\big)(\tilde{G}_{ij}+\tilde{G}_{ji})<0$$

From the conditions (14) and (15), we have $\mathcal{L}V(x(t))<0$. Furthermore, one has $\mathrm{E}\{\mathcal{L}V(x(t))\}<0$, and consequently, by using (III) in Definition 1 and [39], we can conclude that system (4) is mean-square stable. Therefore, according to (IV) in Definition 1, system (4) is mean-square admissible. This completes the proof. □

Remark 1. Because the fuzzy Lyapunov function will produce the time derivatives of membership functions, it is necessary to evaluate their upper bounds prior. However, the upper bounds are difficult to solve in practical problems. To avoid this case, the Lyapunov function (24) is employed. Different from the single quadratic Lyapunov function $V(x(t))=x(t)^{\mathrm{T}}E^{\mathrm{T}}Xx(t)$, in this paper, since $\tilde{V}^{-1}E^{\mathrm{T}}U^{-\mathrm{T}}U^{\mathrm{T}}X_h^{-1}\tilde{V}^{-\mathrm{T}}=\left[\begin{smallmatrix} X^{-1} & 0 \\ 0 & 0\end{smallmatrix}\right]$ and $\tilde{V}^{\mathrm{T}}X_iU^{-\mathrm{T}}=\left[\begin{smallmatrix} X & 0 \\ X_{21}^i & X_{22}^i\end{smallmatrix}\right]$, the Lyapunov function (24) is called an extended quadratic Lyapunov function.

Remark 2. For the solving problem of LMI (31), there are also other relaxation approaches [40–42], and the relaxed stability conditions are given as well.

3.2. Fuzzy controller design approach

In this section, we will design state-feedback fuzzy controllers to make the resulting closed-loop system mean-square admissible.

3.2.1. PDC-based fuzzy controller design

Consider the following state-feedback fuzzy controller

$$u(t)=\sum_{i=1}^{r} h_i\big(z(t)\big)F_ix(t) \tag{32}$$

The closed-loop system of system (2) under this controller is represented as

$$Edx(t)=(\tilde{A}_h+\tilde{B}_h\tilde{F}_h)x(t)dt+\tilde{J}_hx(t)dW(t) \tag{33}$$

where $\tilde{B}_h=\sum_{i=1}^{r}h_i(z(t))B_i$, $\tilde{F}_h=\sum_{i=1}^{r}h_i(z(t))F_i$.

With the aid of the mean-square admissibility analysis method presented in the last section, the result for the controller design is as follows.

Theorem 2. *System (33) is mean-square admissible if there exist a scalar $\alpha>0$, positive definite matrices $X\in\Re^{q\times q}$, $\Psi_i\in\Re^{(n-q)\times(n-q)}$, matrices $\{Y_i\}_{i=1}^{r}\in\Re^{(n-q)\times n}$, $\{L_i\}_{i=1}^{r}\in\Re^{m\times n}$, $Y_{ij}\in\Re^{n\times n}$, $G_{ij}\in\Re^{n\times n}$, $i,j=1,2,\ldots,r$ and matrix $\hat{U}\in\Re^{n\times n}$ such that*

$$\Pi_{ii}<0,\quad i=1,2,\ldots,r \tag{34}$$

$$\Pi_{ij}+\Pi_{ji}<0,\quad 1\le i<j\le r \tag{35}$$

$$\mathrm{M}_{ii}<0,\quad i=1,2,\ldots,r \tag{36}$$

$$\mathrm{M}_{ij}+\mathrm{M}_{ji}<0,\quad 1\le i<j\le r \tag{37}$$

where

$$\Pi_{ij} = \begin{bmatrix} \Omega_1 & X_i^{\mathrm{T}} J_j^{\mathrm{T}} (E^+)^{\mathrm{T}} E^{\mathrm{T}} \\ * & -\tilde{Q}_i \end{bmatrix}, \qquad \Omega_1 = X_i^{\mathrm{T}} A_j^{\mathrm{T}} + A_j X_i - G_{ij}$$

$$\mathrm{M}_{ij} = \begin{bmatrix} \Omega_2 & B_j L_i & 0 \\ * & -\hat{U} - \hat{U}^{\mathrm{T}} & \alpha X_i - \hat{U} \\ * & * & -Y_{ij} \end{bmatrix}, \qquad \Omega_2 = G_{ij} + B_j L_i + L_i^{\mathrm{T}} B_j^{\mathrm{T}} + Y_{ij}$$

$$X_i = \tilde{V}^{-\mathrm{T}} P_i \tilde{V}^{-1} E^{\mathrm{T}} + V_2 Y_i U^{\mathrm{T}}, \qquad P_i = diag\{X, \Psi_i\}, \qquad \tilde{Q}_i = U^{-\mathrm{T}} P_i U^{\mathrm{T}}$$

with orthogonal matrix U and nonsingular matrix $\tilde{V}$ satisfying $U^{\mathrm{T}} E \tilde{V}^{-\mathrm{T}} = \left[\begin{smallmatrix} I_q & 0 \\ 0 & 0 \end{smallmatrix}\right]$, here, $\tilde{V} = [\tilde{V}_1 \ V_2]$ and $EV_2 = 0$. In this case, F_i can be chosen as $F_i = \alpha L_i \hat{U}^{-1}$.

Proof. We apply the Lyapunov function as defined in the proof of Theorem 1. Define

$$\tilde{L}_h = \sum_{i=1}^{r} h_i\big(z(t)\big) L_i$$

$$\tilde{G}_{hh} = \sum_{i=1}^{r} \sum_{j=1}^{r} h_i\big(z(t)\big) h_j\big(z(t)\big) G_{ij}$$

$$\tilde{Y}_{hh} = \sum_{i=1}^{r} \sum_{j=1}^{r} h_i\big(z(t)\big) h_j\big(z(t)\big) Y_{ij}$$

From (36) and (37), it can be seen that

$$\begin{bmatrix} \tilde{\Omega}_2 & \tilde{B}_h \tilde{L}_h & 0 \\ * & -\hat{U} - \hat{U}^{\mathrm{T}} & \alpha \tilde{X}_h - \hat{U} \\ * & * & -\tilde{Y}_{hh} \end{bmatrix} = \sum_{i=1}^{r} h_i^2\big(z(t)\big) \mathrm{M}_{ii} + \sum_{i=1}^{r} \sum_{i<j}^{r} h_i\big(z(t)\big) h_j\big(z(t)\big) (\mathrm{M}_{ij} + \mathrm{M}_{ji}) < 0 \tag{38}$$

where

$$\tilde{\Omega}_2 = \tilde{G}_{hh} + \tilde{B}_h \tilde{L}_h + \tilde{L}_h^{\mathrm{T}} \tilde{B}_h^{\mathrm{T}} + \tilde{Y}_{hh}$$

By Lemma 3, we can obtain from (38) that

$$\tilde{B}_h \tilde{F}_h \tilde{X}_h + \tilde{X}_h^{\mathrm{T}} \tilde{F}_h^{\mathrm{T}} \tilde{B}_h^{\mathrm{T}} < -\tilde{G}_{hh} \tag{39}$$

From (34) and (35), one has

$$\begin{bmatrix} \tilde{X}_h^{\mathrm{T}} \tilde{A}_h^{\mathrm{T}} + \tilde{A}_h \tilde{X}_h - \tilde{G}_{hh} & \tilde{X}_h^{\mathrm{T}} \tilde{J}_h^{\mathrm{T}} (E^+)^{\mathrm{T}} E^{\mathrm{T}} \\ * & -\tilde{Q}_h \end{bmatrix}$$

$$= \sum_{i=1}^{r} h_i^2\big(z(t)\big) \Pi_{ii} + \sum_{i=1}^{r} \sum_{i<j}^{r} h_i\big(z(t)\big) h_j\big(z(t)\big) (\Pi_{ij} + \Pi_{ji}) < 0 \tag{40}$$

That is

$$\begin{bmatrix} \tilde{X}_h^{\mathrm{T}} \tilde{A}_h^{\mathrm{T}} + \tilde{A}_h \tilde{X}_h - \tilde{G}_{hh} & \tilde{X}_h^{\mathrm{T}} \tilde{J}_h^{\mathrm{T}} (E^+)^{\mathrm{T}} E^{\mathrm{T}} \\ * & -\tilde{Q}_h \end{bmatrix} < 0 \tag{41}$$

By the Schur complement, we then have

$$\tilde{X}_h^{\mathrm{T}} \tilde{A}_h^{\mathrm{T}} + \tilde{A}_h \tilde{X}_h + \tilde{X}_h^{\mathrm{T}} \tilde{J}_h^{\mathrm{T}} \big(E^+\big)^{\mathrm{T}} E^{\mathrm{T}} \tilde{Q}_h^{-1} E E^+ \tilde{J}_h \tilde{X}_h - \tilde{G}_{hh} < 0 \tag{42}$$

From (39) and (42), it follows that

$$\tilde{X}_h^{\mathrm{T}} (\tilde{A}_h + \tilde{B}_h \tilde{F}_h)^{\mathrm{T}} + (\tilde{A}_h + \tilde{B}_h \tilde{F}_h) \tilde{X}_h + \tilde{X}_h^{\mathrm{T}} \tilde{J}_h^{\mathrm{T}} \big(E^+\big)^{\mathrm{T}} E^{\mathrm{T}} \tilde{Q}_h^{-1} E E^+ \tilde{J}_h \tilde{X}_h < 0 \tag{43}$$

By Theorem 1, we can easily get $\mathcal{L}V(x(t)) < 0$ for $x(t) \neq 0$ and then system (33) is mean-square admissible. This completes the proof. □

3.2.2. Non-PDC-based fuzzy controller design

In general, the extended quadratic Lyapunov function is usually used to derive the stabilization conditions for PDC-based fuzzy control systems. However, the stabilization conditions developed based on the extended quadratic Lyapunov function remain more conservative characteristics. In order to obtain more relaxed mean-square admissibility conditions for continuous-time stochastic fuzzy singular systems, a non-PDC fuzzy controller is developed in this section.

Consider the following non-PDC fuzzy controller

$$u(t)=F\big(z(t)\big)x(t)=\sum_{i=1}^{r}h_i\big(z(t)\big)N_i\left(\sum_{i=1}^{r}h_i\big(z(t)\big)X_i\right)^{-1}x(t) \tag{44}$$

where $F(z(t))$ is the feedback gain depending on $z(t)$ and N_i is an LMI variable to design the controller.

The closed-loop system of system (2) is represented as

$$Edx(t)=\big(\tilde{A}_h+\tilde{B}_h\tilde{N}_h\tilde{X}_h^{-1}\big)x(t)dt+\tilde{J}_hx(t)dW(t) \tag{45}$$

where $\tilde{N}_h=\sum_{i=1}^{r}h_i(z(t))N_i$.

Theorem 3. *System (45) is mean-square admissible if there exist positive definite matrices $X\in\Re^{q\times q}$, $\Psi_i\in\Re^{(n-q)\times(n-q)}$, matrices $\{Y_i\}_{i=1}^{r}\in\Re^{(n-q)\times n}$ and $\{N_i\}_{i=1}^{r}\in\Re^{m\times n}$ such that*

$$\tilde{\Phi}_{ii}<0,\quad i=1,2,\dots,r \tag{46}$$

$$\tilde{\Phi}_{ij}+\tilde{\Phi}_{ji}<0,\quad 1\le i<j\le r \tag{47}$$

where

$$\tilde{\Phi}_{ij}=\begin{bmatrix}\Theta & X_i^{\mathrm{T}}J_j^{\mathrm{T}}(E^+)^{\mathrm{T}}E^{\mathrm{T}}\\ * & -\tilde{Q}_i\end{bmatrix},\qquad \Theta=X_i^{\mathrm{T}}A_j^{\mathrm{T}}+N_i^{\mathrm{T}}B_j^{\mathrm{T}}+A_jX_i+B_jN_i$$

$$X_i=\tilde{V}^{-\mathrm{T}}P_i\tilde{V}^{-1}E^{\mathrm{T}}+V_2Y_iU^{\mathrm{T}},\qquad P_i=diag\{X,\Psi_i\},\qquad \tilde{Q}_i=U^{-\mathrm{T}}P_iU^{\mathrm{T}}$$

with orthogonal matrix U and nonsingular matrix $\tilde{V}$ satisfying $U^{\mathrm{T}}E\tilde{V}^{-\mathrm{T}}=\begin{bmatrix}I_q&0\\0&0\end{bmatrix}$, here, $\tilde{V}=[\tilde{V}_1\ V_2]$ and $EV_2=0$.

Proof. Consider the extended quadratic Lyapunov function (24), from inequality (30) of the Theorem 1, we have $\mathcal{L}V(x(t))<0$ for t if

$$\begin{bmatrix}(\tilde{X}_h^{\mathrm{T}}(\tilde{A}_h+\tilde{B}_h\tilde{N}_h\tilde{X}_h^{-1})^{\mathrm{T}}+(\tilde{A}_h+\tilde{B}_h\tilde{N}_h\tilde{X}_h^{-1})\tilde{X}_h) & \tilde{X}_h^{\mathrm{T}}\tilde{J}_h^{\mathrm{T}}(E^+)^{\mathrm{T}}E^{\mathrm{T}}\\ * & -\tilde{Q}_h\end{bmatrix}<0 \tag{48}$$

The above inequality (48) can be arranged as

$$\sum_{i=1}^{r}\sum_{j=1}^{r}h_i\big(z(t)\big)h_j\big(z(t)\big)\begin{bmatrix}(X_i^{\mathrm{T}}A_j^{\mathrm{T}}+N_i^{\mathrm{T}}B_j^{\mathrm{T}}+A_jX_i+B_jN_i) & X_i^{\mathrm{T}}J_j^{\mathrm{T}}(E^+)^{\mathrm{T}}E^{\mathrm{T}}\\ * & -\tilde{Q}_i\end{bmatrix}<0 \tag{49}$$

While inequality (49) is equivalent to

$$\sum_{i=1}^{r}h_i^2\big(z(t)\big)\tilde{\Phi}_{ii}+\sum_{i=1}^{r}\sum_{i<j}^{r}h_i\big(z(t)\big)h_j\big(z(t)\big)(\tilde{\Phi}_{ij}+\tilde{\Phi}_{ji})<0 \tag{50}$$

From the conditions (46) and (47), we have $\mathcal{L}V(x(t))<0$. Therefore, system (45) is mean-square admissible. This completes the proof. □

4. Simulation examples

In this section, we will present two examples to demonstrate the effectiveness and the merits of the results established in the paper.

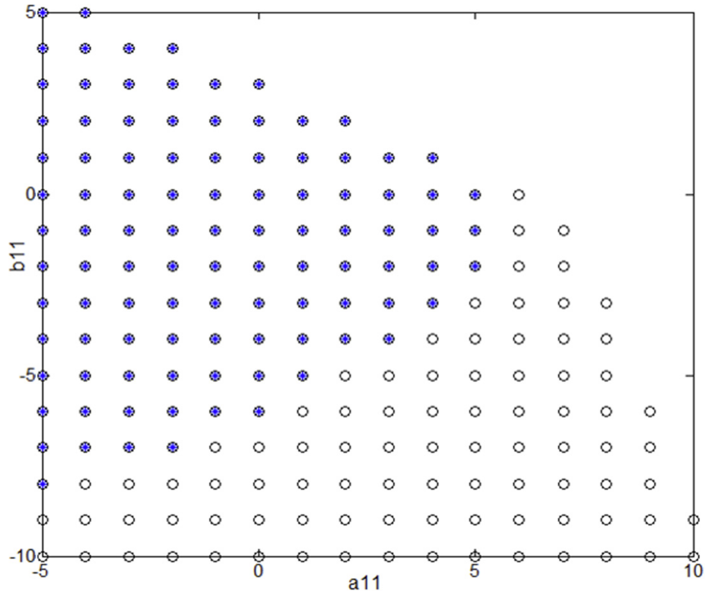

Fig. 1. Feasible regions of Theorem 2 and Theorem 3.

Example 1. Consider the following fuzzy model:

R^1: If $x_2(t)$ is M_1, Then

$$Edx(t) = \big(A_1x(t) + B_1u(t)\big)dt + J_1x(t)dW(t)$$

R^2: If $x_2(t)$ is M_2, Then

$$Edx(t) = \big(A_2x(t) + B_2u(t)\big)dt + J_2x(t)dW(t)$$

$h_i(x_2(t))$ represents the membership function of the fuzzy set M_i, $(i = 1, 2)$, which follows that

$$h_1\big(x_2(t)\big) = \frac{1}{2} + \frac{1}{2}\sin x_2(t), \qquad h_2\big(x_2(t)\big) = \frac{1}{2} - \frac{1}{2}\sin x_2(t)$$

and

$$E = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \qquad A_1 = \begin{bmatrix} a_{11} & 2 \\ -2 & 0 \end{bmatrix}, \qquad A_2 = \begin{bmatrix} 3 & 1 \\ -5 & -1 \end{bmatrix}, \qquad J_1 = \begin{bmatrix} 0.9 & 0.7 \\ 0 & 0 \end{bmatrix}$$
$$J_2 = \begin{bmatrix} 0.2 & 0.4 \\ 0 & 0 \end{bmatrix}, \qquad B_1 = \begin{bmatrix} b_{11} \\ 1 \end{bmatrix}, \qquad B_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Based on the extended quadratic Lyapunov function approach, a comparison between the PDC controller designed method and non-PDC controller designed method is provided in the sequel. Through applying Theorem 2 and Theorem 3, we can verify the system is mean-square admissible. Let a_{11}, b_{11} take values in intervals $[-5,\ 10]$ and $[-10,\ 5]$, respectively. Let "$*$", "¤" denote the points of (a_{11}, b_{11}) at which the closed-loop system can be determined to be stabilized via the PDC and non-PDC controllers, respectively.

Fig. 1 shows the feasible regions for several combinations of a_{11} and b_{11} using Theorem 2 and Theorem 3, respectively. It can be seen from the figures that the feasible regions for the non-PDC controller designed method based on the extended quadratic Lyapunov function approach has provided less conservative results than the PDC controller designed method.

In order to further verify the mean-square admissibility of closed-loop stochastic T–S fuzzy singular systems via the PDC and non-PDC fuzzy controllers, a singular single-species bio-economic practical model example is given in the following example.

Example 2. Consider the singular single-species bio-economic model [43] as follows:

$$\begin{aligned}
&\frac{dx(t)}{dt} = ay(t) - bx(t)\\
&\frac{dy(t)}{dt} = \delta x(t) - \beta y^2(t) - y(t)E(t)\\
&0 = E(t)\big(py(t) - c\big) - m
\end{aligned}$$

where $x(t)$ and $y(t)$ represent the densities of the juveniles and adults of the population, respectively. At any time ($t > 0$), the birth rate of the juvenile population is proportional to the density of existing adult with proportionality constant a, the death rate of the juvenile population and the rate of transformation of the adult population are proportional to the density of existing juvenile population with proportionality constant b, δ denotes the proportional transforming rate from the juvenile population to the adult population, the death rate of adult population is proportional to the square of the adult population size with proportionality constant β, all the parameters mentioned above are positive constants. $E(t)$ represents the harvest effort of harvested adult population, $p > 0$ is the unit price of the adult population, $c > 0$ is the fixed cost of harvesting per unit of effort, m is the economic profit.

Most of authors discussed the stability problem of such systems under all these populations are growing in closed homogeneous environment (see [44–46] and references therein). In fact, the white noise always exists in the environment (e.g., variation in intensity of sunlight, temperature, water level, etc.) and we cannot omit the influence of the white noise to many dynamical systems. Assume that lots of small random disturbance mainly impact the intrinsic growth rate of the population. Let $a \to a + \sigma \dot{W}(t)$, then we have the following stochastic singular single-species bio-economic model:

$$\begin{aligned}
&dx(t) = \big[ay(t) - bx(t)\big]dt + \sigma y(t)dW(t)\\
&dy(t) = \big[\delta x(t) - \beta y^2(t) - y(t)E(t)\big]dt\\
&0 = E(t)\big(py(t) - c\big) - m
\end{aligned}$$

Due to the unstable fluctuations, the impulsive phenomena have always been regarded as unfavorable ones from the ecological point of view. In order to plan harvesting strategies and maintain the sustainable development of the system, it is necessary to take action to stabilize biological population. So, we propose the following state feedback control method for the system as follows:

$$\begin{aligned}
&dx(t) = \big[ay(t) - bx(t)\big]dt + \sigma y(t)dW(t)\\
&dy(t) = \big[\delta x(t) - \beta y^2(t) - y(t)E(t) + u(t)\big]dt\\
&0 = E(t)\big(py(t) - c\big) - m + u(t)
\end{aligned}$$

where $u(t)$ is control variable, representing regulation control for a biological resource.

Let $X(t) = [x(t)\ y(t)\ E(t)]^{\mathrm{T}}$, the above model can be expressed by fuzzy model, which consists of the following two rules:

R^1: If $y(t)$ is F_1, Then

$$\Xi dX(t) = \big[A_1X(t) + B_1u(t)\big]dt + J_1X(t)dW(t)$$

R^2: If $y(t)$ is F_2, Then

$$\Xi dX(t) = \big[A_2X(t) + B_2u(t)\big]dt + J_2X(t)dW(t)$$

where

$$\Xi = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \qquad A_1 = \begin{bmatrix} -b & a & 0 \\ \delta & \beta k & k \\ 0 & 0 & -pk - c \end{bmatrix}, \qquad A_2 = \begin{bmatrix} -b & a & 0 \\ \delta & -\beta k & -k \\ 0 & 0 & pk - c \end{bmatrix}$$

$$B_1 = B_2 = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}, \qquad J_1 = J_2 = \begin{bmatrix} 0 & \sigma & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

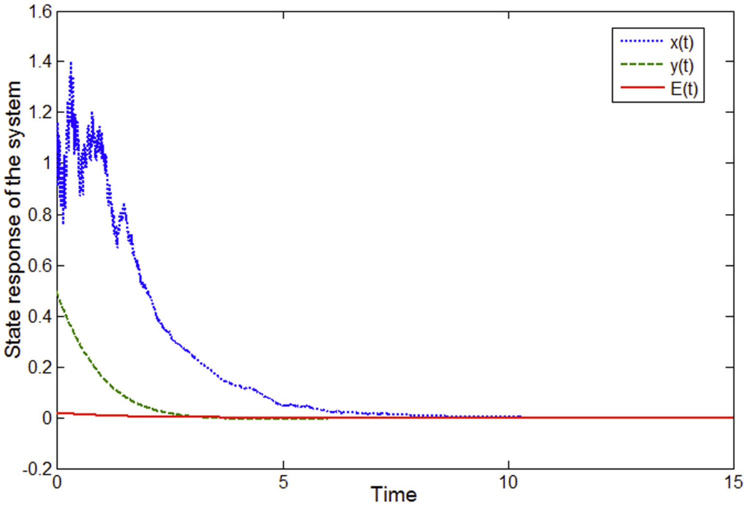

Fig. 2. State responses of the system based on the PDC fuzzy controller.

with $k > 0$ is the carrying capacity of the population. F_i $(i = 1, 2)$ is the fuzzy set, $h_i(y(t)) \geq 0$ is a membership function of F_i, and $h_1(y(t)) = \frac{1}{2}(1 - \frac{y(t)}{k})$, $h_2(y(t)) = \frac{1}{2}(1 + \frac{y(t)}{k})$, $\sum_{i=1}^{2} h_i(y(t)) = 1$.

By using a standard fuzzy singleton inference method, a singleton fuzzifier to produce a fuzzy inference and weighted center-average defuzzifier, we can obtain the global stochastic T–S fuzzy singular system as follows:

$$\Xi dX(t) = \left[A_h X(t) + B_h u(t)\right]dt + J_h X(t) dW(t)$$

where $A_h = \sum_{i=1}^{2} h_i(y(t))A_i$, $B_h = \sum_{i=1}^{2} h_i(y(t))B_i$, $J_h = \sum_{i=1}^{2} h_i(y(t))J_i$.

Based on marine fishery statistics data in Liaoning province [47], we properly process the data (nondimensional transformation, equal ratio simplification, approximation, and so on) according to the parameters in the model of this paper. In sequence, the values of parameters are as follows:

$$a = 0.2, \quad b = 0.7, \quad \delta = 0.05, \quad \beta = 0.1, \quad p = 1, \quad c = 30, \quad k = 5, \quad \sigma = 0.06$$

Firstly, through applying Theorem 2, the PDC fuzzy controller can be obtained to guarantee the considered system achieving mean-square admissibility. By solving the relaxed sufficient conditions (34)–(37), the PDC fuzzy controller can be constructed as follows:

$$u(t) = \sum_{i=1}^{2} h_i\big(y(t)\big)F_i x(t) = \big(h_1\big(y(t)\big)F_1 + h_2\big(y(t)\big)F_2\big)x(t)$$

where

$$F_1 = \begin{bmatrix} -0.1292 & -1.3406 & -0.1153 \end{bmatrix}, \qquad F_2 = \begin{bmatrix} -0.0892 & -0.3998 & 0.1180 \end{bmatrix}$$

The simulation results are shown in Fig. 2.

Next, through applying Theorem 3, the non-PDC fuzzy controller can be obtained to guarantee the considered system achieving mean-square admissibility. By solving the relaxed sufficient conditions (46)–(47), the non-PDC fuzzy controller can be constructed as follows:

$$\begin{aligned} u(t) &= \sum_{i=1}^{2} h_i\big(y(t)\big)N_i \left(\sum_{i=1}^{2} h_i\big(y(t)\big)X_i\right)^{-1} x(t) \\ &= \left\{\big(h_1\big(y(t)\big)N_1 + h_2\big(y(t)\big)N_2\big) \times \big(h_1\big(y(t)\big)X_1 + h_2\big(y(t)\big)X_2\big)^{-1}\right\}x(t) \end{aligned}$$

where

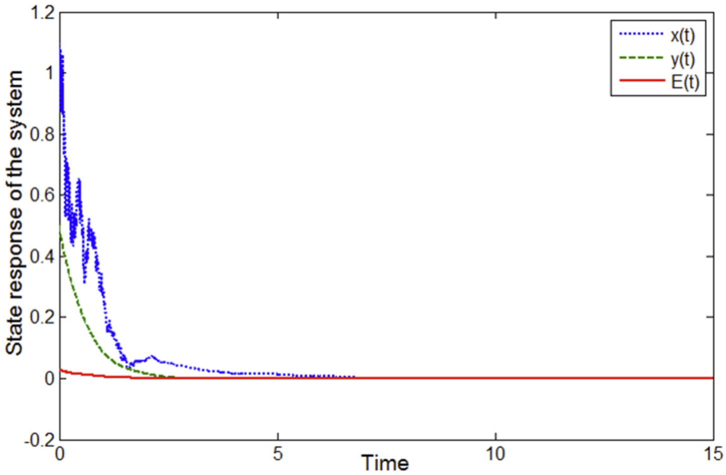

Fig. 3. State responses of the system based on non-PDC fuzzy controller.

$$N_1 = \begin{bmatrix} -0.2256 & -0.7212 & -1.5356 \end{bmatrix}, \qquad N_2 = \begin{bmatrix} -0.1449 & -0.0390 & 0.0000 \end{bmatrix}$$

$$X_1 = \begin{bmatrix} 0.6934 & 0.0199 & 0 \\ 0.0199 & 0.8217 & 0 \\ 0.0065 & -0.0191 & 0.0571 \end{bmatrix}, \qquad X_2 = \begin{bmatrix} 0.6934 & 0.0199 & 0 \\ 0.0199 & 0.8217 & 0 \\ 0.0059 & -0.0067 & 0.0193 \end{bmatrix}$$

Employing the above non-PDC fuzzy controller to control system, the simulation results are shown in Fig. 3.

It is shown that the system tends to stability through the designed PDC fuzzy controller or non-PDC fuzzy controller from the Fig. 2 and Fig. 3.

5. Conclusions

In this paper, the mean-square admissibility has been solved for T–S fuzzy singular systems with stochastic disturbances. Based on the extended quadratic Lyapunov function approach, we have obtained mean-square admissibility of stochastic T–S fuzzy singular systems. Furthermore, we have proposed the PDC fuzzy controller design method and the non-PDC fuzzy controller design method to fully take advantage of the extended quadratic Lyapunov function, respectively. Finally, two simulation examples have illustrated the utility of the proposed theoretical results.

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