Neurocomputing 173 (2016) 1299–1309

Contents lists available at ScienceDirect

Neurocomputing

journal homepage: www.elsevier.com/locate/neucom

Hybrid synchronization behavior in an array of coupled chaotic systems with ring connection ☆

Xiangyong Chen [a,b], Jianlong Qiu [a,c,*], Jinde Cao [b,d], Haibo He [c]

[a] School of Science, Linyi University, Linyi, Shandong 276005, China
[b] Department of Mathematics, Southeast University, Nanjing, Jiangsu 210096, China
[c] Department of Electrical and Computer Engineering, University of Rhode Island, Kingston, USA
[d] Department of Mathematics, Faculty of Science, King Abdulaziz University, Jeddah, Saudi Arabia

ARTICLE INFO

Article history:
Received 8 April 2015
Received in revised form
9 August 2015
Accepted 2 September 2015
Communicated by Yang Tang
Available online 10 September 2015

Keywords:
Chaotic system
State coupling
Ring connection
Hybrid Synchronization behavior
Anti-synchronization
Complete synchronization

ABSTRACT

In this paper, we investigate the hybrid synchronization behavior in an array of coupled chaotic systems with ring connection, of which means complete synchronization (CS) and anti-synchronization (AS) could coexist. First, the anti-synchronization controllers are designed, which can transform the synchronization error dynamic system into a nonlinear system with an antisymmetric structure. Second, we investigate the complete synchronization behavior in such a chaotic system under the anti-synchronization control. After that, the stability conditions are given for reaching hybrid synchronization. Finally, numerical examples and simulation results are presented to verify and validate the hybrid synchronization behavior in coupled chaotic system.

1. Introduction

Since the synchronization behavior in chaotic systems was investigated in the literature [1], this problem has become an interesting and important topic because of its useful applications on communications, automatic control and so on [2–4]. Nowadays, some synchronization models [5–10] have been studied in chaotic dynamical systems. In last several years, a new chaos synchronization behavior, which is considered as hybrid synchronization [11–25,37,38], is found in chaotic systems. In the new synchronization mode, one part of chaotic systems is completely synchronized and the others belong to the anti-synchronization. The existence of hybrid synchronization can effectively enhance security in communication. Due to these reasons, to design the proper controller to reach the hybrid synchronization has become an important problem. Shi et al. [11,12] analyzed the hybrid synchronization problem in two Lorenz systems with unknown parameters and time delay. Zhang et al. [13] investigated the coexistence of the anti-phase synchronization and complete synchronization in chaotic systems. Sundarapandian et al. [14] used sliding control technology to propose the hybrid synchronization controllers of four-wing chaotic systems. Das et al. [15] studied the hybrid phase synchronization in two identical and non-identical Rikitake and Windmi systems. Li and Liu [16] derived a hybrid projective synchronization scheme of the chaotic dynamic systems with external disturbances, which can eliminate the influence of uncertainties effectively and reach the synchronization with a small error bound. Liu et al. [17] designed the hybrid synchronization controller in identical chaotic systems by considering the unknown parameters. In [18–21], some hybrid synchronization problems of several hyperchaotic systems were investigated. In [22,23], some hybrid synchronization behaviors of several fractional-order chaotic dynamic systems were studied. In [24,25], the hybrid synchronization control problems of complex networks were studied. With the mentioned above, although many research results have been investigated on the hybrid synchronization problems, most of the works were only pay attention to the synchronization control problems in two chaotic dynamic systems.

☆This work was supported by the National Natural Science Foundation of China (61403179, 61273012, 11301252 and 61304023) and by a Project of the Postdoctoral Sustentation Fund of Jiangsu Province (1402042B), and by the Applied Mathematics Enhancement Program of Linyi University.

* Corresponding author at: School of Science, Linyi University, Linyi, Shandong 276005, China.
E-mail address: qiujianlong@lyu.edu.cn (J. Qiu).

http://dx.doi.org/10.1016/j.neucom.2015.09.003
0925-2312/

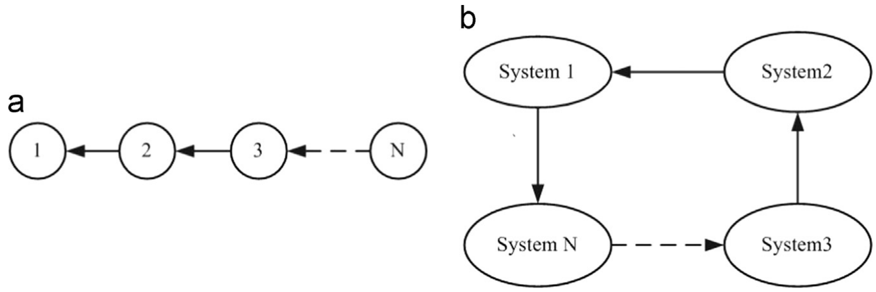

Fig. 1. Diagram of coupled chaotic systems with chain connection and ring connection, where $1, 2, \ldots, N$ mean the first chaotic system, the second chaotic system, …, the Nth chaotic systems in (a).

In the past few years, the synchronization of multiple chaotic dynamic systems has become an interesting topic in nonlinear science and control research area. The study on this kind of synchronization is not only more helpful for understanding the basic mechanism of collective behaviors, but also in favor of its potential applications in multilateral communications, secret signaling and many other engineering areas. Till now, many efforts have been reported in literatures. Several different types of chaos synchronization of multiple chaotic systems were investigated in [26–34]. That included global synchronization [26], complete synchronization [27–30], anti-synchronization [31], projective synchronization [32], etc. Lu et al. [33] designed a class of adaptive synchronization controllers in multi-Lorenz systems. Yang et al. [34] studied synchronization of an array of identical chaotic systems, and investigated the application on secure communication. The chaos synchronization problems in multiple coupled complex networks were studied in [35,36].

However, there is very little concern about hybrid synchronization control of multiple coupled chaotic systems. Yang and Cao [37] concerned a class of adaptive and impulsive synchronization problems of complex networks, and designed the hybrid synchronization controller, which make the complex network synchronize onto an isolate chaotic system. Ji [38] studied only hybrid synchronization problem in three unified coupled chaotic systems. Cao et al. [39] solved the global synchronization of several hybrid coupled neural networks. In fact, hybrid synchronization of multiple coupled chaotic dynamic systems is more useful in some practical situations. It is more effective to enhance the security in digital communication and lead a more bright future in multilateral communications than the single synchronization mechanism. Consequently, it is important to design the effective hybrid synchronization controllers for multiple coupled chaotic systems.

Currently, some researchers mainly considered the previous synchronization control problem of multiple chaotic dynamic systems with chain connection structure [27] as Fig. 1(a). In [27–29], the complete synchronization in N non-identical chaotic systems with chain connection was studied. In fact, we all know that a fault that one of that synchronization between two chaotic systems cannot be reached is inevitable in a real world. In the previous multi-systems model with chain connections, the occurrence of a fault will make multi-systems not to complete the synchronization. With the mentioned above, we consider a chaos synchronization mode in multiple coupled chaotic dynamic systems with ring connections structure as Fig. 1(b). In our chaotic systems model, the ring structure formed among multiple chaotic systems makes them correlative as Fig. 1(b). So all chaotic systems are coupled on a chain. Then the state variables of the first chaotic system couple the Nth system, and the state variables of the second system couple the first, ⋯ analogically, until the state variables of the Nth system couple those of the $(N-1)$ th. According to this mode, we can reach the synchronization of multi-systems through the other systems which ensure their synchronization. 1~2~3~··· N−1~ N→1

Motivated by the above discussions, it is important to propose the hybrid synchronization scheme for multi-systems with ring connection. However, it is much more difficult to realize this chaos synchronization, so we must to search an easier method to reach such synchronization. In [40,41], Liu and Zhang proposed a direct design method to synchronize the chaotic systems. This method has two important advantages: (1) the procedure is easy for selecting the synchronization controllers; (2) it is easily to implement the simple controllers . In [6,29–31], this method is used to reach some chaos synchronization control mode. The existing results show the validity of the proposed schemes than the earlier results [27]. Thus, the effective method is used to research the hybrid synchronization control problem of multiple coupled chaotic systems with the ring connection structure.

Our article extends the work reported in [38,39] and provides several new results. First, we discuss the anti-synchronization control problems, then the synchronization error dynamic system can be transformed into a nonlinear dynamic system with a special anti-symmetric structure [6,29]. And the complete synchronization behavior is also realized under the anti-synchronization controllers. It shows that the first chaotic system anti-synchronizes the second chaotic system, the second chaotic system anti-synchronizes the third system, the third system anti-synchronizes the fourth system, and the first system synchronizes the third system, the second system synchronizes the fourth system. We can find that the hybrid synchronization mode exists in such systems. Furthermore, we derive some sufficient asymptotical stability conditions. It shows that we realize the hybrid synchronization control problem. Research results will undoubtedly improve the performance of multilateral communications.

The paper is presented as follows. In Section 2, theoretically analysis about the hybrid synchronization problem was given. Several new stability conditions are given for reaching hybrid synchronization. Followed Section 3, we apply the proposed synchronization scheme to analyze the hybrid synchronization control of three and four coupled chaotic dynamic systems, respectively. The conclusions are given in Section 4.

2. Hybrid synchronization controllers design of chaotic systems

First, the multiple coupled chaotic systems with ring connections structure are described as

$$\begin{cases} \dot{x}_1 = A_1 x_1 + g_1(x_1) + D_1(x_N - x_1), \\ \dot{x}_2 = A_2 x_2 + g_2(x_2) + D_2(x_1 - x_2), \\ \vdots \\ \dot{x}_N = A_N x_N + g_N(x_N) + D_N(x_{N-1} - x_N), \end{cases} \tag{2.1}$$

where $x_1, x_2, \ldots, x_N$ are defined as the state vectors, and $x_i = (x_{i1}, x_{i2}, \ldots x_{in})^T$ $(i = 1, \ldots, N)$; $g_i(x_i)$ $(i = 1, \ldots, N)$ $(R^n \to R^n)$ is the continuous nonlinear function, $A_1, A_2, \ldots, A_N \in R^n \times R^n$ are constant matrices, $D_i = diag(d_{i1}, \ldots, d_{iN})$ are n dimensional diagonal matrices, and $d_{ij} \geq 0$ represent the coupled parameters of the diagonal matrices. If

$$A_i \neq A_j, \quad i, j = 1, \ldots, N, \quad i \neq j$$

and

$$g_i(\cdot) \neq g_j(\cdot), \quad i, j = 1, \ldots, N, \quad i \neq j,$$

then (2.1) is an array of non-identical chaotic dynamic systems. 不相同的混沌

Remark 1. According to (2.1), the chaotic systems are coupled on a chain [27], where the state variables of the first system couple the Nth, the second system couple the first, … analogically, until the Nth chaotic system couple those of the $(N-1)$th.

Now we apply the above coupling mode to propose the hybrid synchronization control problem, which reads

$$\begin{cases} \dot{x}_1 = A_1 x_1 + g_1(x_1) + D_1(x_N - x_1), \\ \dot{x}_2 = A_2 x_2 + g_2(x_2) + D_2(x_1 - x_2) + u_1, \\ \vdots \\ \dot{x}_N = A_N x_N + g_N(x_N) + D_N(x_{N-1} - x_N) + u_{N-1}. \end{cases} \tag{2.2}$$

Firstly, hybrid synchronization can be defined as follows.

Definition 1. For the chaotic dynamic systems (2.2), we define that there is hybrid synchronization if the controllers u_i $(i = 1, \ldots, N-1)$ exist such that all trajectories $x_1(t), \ldots, x_N(t)$ in (2.2) with any initial condition $(x_1(0), \ldots, x_N(0))$ satisfy

(1) For the anti-synchronization errors $e_i(t) = (e_{i1}, e_{i2}, \ldots e_{in})^T (i = 1, \ldots, N-1)$, we have that

$$\lim_{t\to\infty} \| e_i(t) \| = \lim_{t\to\infty} \| x_i(t) + x_{i+1}(t) \| = 0, \quad i = 1, \ldots, N-1.$$

(2) For the complete synchronization errors $e'_j(t) = (e'_{j1}, e'_{j2}, \ldots, e'_{jn})^T$ and $e''_k(t) = (e''_{k1}, e''_{k2}, \ldots, e''_{kn})^T$, if $N (N \geq 3)$ is odd, then $e'_j(t)$ and $e''_k(t)$ satisfy

$$\lim_{t\to\infty} \| e'_j(t) \| = \lim_{t\to\infty} \| x_{j+2}(t) - x_j(t) \| = 0 \quad (j = 1, 3, 5, \ldots, N-2), \lim_{t\to\infty} \| e''_k(t) \| = \lim_{t\to\infty} \| x_{k+2}(t) - x_k(t) \| = 0 \quad (k = 2, 4, 6, \ldots, N-3).$$

And if N $(N \geq 4)$ is even, then $e'_j(t)$ and $e''_k(t)$ satisfy

$$\lim_{t\to\infty} \| e'_j(t) \| = \lim_{t\to\infty} \| x_{j+2}(t) - x_j(t) \| = 0 \quad (j = 1, 3, 5, \ldots, N-3), \lim_{t\to\infty} \| e''_k(t) \| = \lim_{t\to\infty} \| x_{k+2}(t) - x_k(t) \| = 0 \quad (k = 2, 4, 6, \ldots, N-2).$$

From the above definition, we note easily that the error dynamical systems $e_i(t), e'_j(t),$ and $e''_k(t)$ are global asymptotically stable. Then we know that the hybrid synchronization control problem become a problem to choose the proper controller $u_i(t)$ to make $e_i(t), e'_j(t)$ and $e''_k(t)$ asymptotically converge in origin.

The anti-synchronization error dynamic system of (2.2) is to be as

$$\dot{e} = \begin{bmatrix} K_1 & K_2 & K_3 & \cdots & D_1 \\ D_2 & A_3 - D_3 & 0 & \cdots & 0 \\ 0 & D_3 & A_4 - D_4 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & K_4 \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ \vdots \\ e_{N-1} \end{bmatrix} + \begin{bmatrix} \begin{pmatrix} \big((-1-(-1)^N)D_1 + 2D_2 - (A_2 - A_1)\big)x_1 \\ + g_2(x_2) + g_1(x_1) + u_1 \end{pmatrix} \\ \begin{pmatrix} [-2(D_2 - D_3) - (A_3 - A_2)]x_2 \\ + g_3(x_3) + g_2(x_2) + u_2 + u_1 \end{pmatrix} \\ \vdots \\ \begin{pmatrix} [-2(D_{N-1} - D_N) - (A_N - A_{N-1})]x_{N-1} + \\ + g_N(x_N) + g_{N-1}(x_{N-1}) + u_{N-1} + u_{N-2} \end{pmatrix} \end{bmatrix}, \tag{2.3}$$

$e_1 =$
$e_2 =$
$\vdots$
$e_N =$

where $e = (e_1, e_2, \ldots, e_N)^T$, $K_1 = A_2 - (-1)^{N-1}D_1 - D_2$, $K_2 = -(-1)^{N-2}D_1$, $K_3 = -(-1)^{N-3}D_1$, $K_4 = A_N - D_N$.

Here we apply the direct design method to reach anti-synchronization between N chaotic dynamic systems. This method can easily select the proper controllers in synchronization of chaotic systems. So we use this method to transform the error dynamic system into a special stable dynamic system with an antisymmetric structure [6,29,30,40].

With the above discussion, the control input u_i is selected as

$$\begin{cases} u_1 = v_1 - \left(\left(\begin{matrix} (-1-(-1)^N)D_1 \\ +2D_2-(A_2-A_1) \end{matrix} \right) x_1 \atop +g_2(x_2)+g_1(x_1) \right), \\ u_2 = v_2 - \left(\left[\begin{matrix} -2(D_2-D_3)- \\ (A_3-A_2) \end{matrix} \right] x_2 \atop +g_3(x_3)+g_2(x_2)-u_1 \right), \\ \vdots \\ u_{N-1} = \left(v_{N-1} - \left[\begin{matrix} -2(D_{N-1}-D_N)- \\ (A_N-A_{N-1}) \end{matrix} \right] x_{N-1} \atop -g_N(x_N)-g_{N-1}(x_{N-1})-u_{N-2} \right). \end{cases} \tag{2.4}$$

Then, the error system is to be as

$$\dot{e} = \Xi \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ \vdots \\ e_{N-1} \end{bmatrix} + \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ \vdots \\ v_{N-1} \end{bmatrix}, \tag{2.5}$$

where

$$\Xi = \begin{bmatrix} K_1 & K_2 & K_3 & \cdots & D_1 \\ D_2 & A_3-D_3 & 0 & \cdots & 0 \\ 0 & D_3 & A_4-D_4 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & K_4 \end{bmatrix} \tag{2.6}$$

and $[v_1 \cdots v_{N-1}]^T = H[e_1 \cdots e_{N-1}]^T$, we define H as a coefficient matrix. Then system (2.5) can be rewritten by

$$\dot{e} = L(e)e, \tag{2.7}$$

where $L(e) = \Xi + H$.

Theorem 1. *Consider system (2.7) with $L(e) = L_1(e) + L_2$. If $L_1(e)$ and L_2 satisfy*

$$L_1^T(e) = -L_1(e), L_2 = diag(l_1, \ldots, l_n), \quad l_i < 0 \ (i = 1, \ldots, n),$$

then (2.7) is asymptotically stable. So we realize the anti-synchronization under the framework of the chaotic systems (2.2).

Proof. Choose a proper Lyapunov function as

$$V = \tfrac{1}{2} e^T e,$$

then, $\dot{V}$ can be get as

$$\dot{V} = \tfrac{1}{2}(\dot{e}^T e + e^T \dot{e}) = \tfrac{1}{2} e^T \left(L(e)^T + L(e) \right) e,$$

where $L_1^T(e) = -L_1(e)$ and $L_2 = diag(l_1, \ldots, l_n)$, $l_i < 0$ $(i = 1, \ldots, n)$. And we readily obtain

$$\dot{V} = e^T L_2 e < 0$$

Then the equilibrium e=0 of (2.7) is global asymptotically stable, so we achieve the anti-synchronization of (2.2).□

Remark 2. We construct the coefficient matrix H to obtain the adjustable controllers u_i for (2.3), then (2.3) can be transformed into $\dot{e} = L(e)e$ under u_i, where L possesses a special antisymmetric structure, which can ensure that we reach the anti-synchronization of (2.2).

In addition, the complete synchronization behavior for such chaotic systems under the anti-synchronization controllers designed is investigated. We divide the discussion of two kinds of circumstances by considering the number of the chaotic systems.

(1) If the number of chaotic systems N $(N \geq 3)$ is odd, the complete synchronization error is described as

$$e_j'(t) = x_{j+2}(t) - x_j(t) \quad (j = 1, 3, 5, \ldots, N-2), e_k''(t) = x_{k+2}(t) - x_k(t) \quad (k = 2, 4, 6, \ldots, N-3). \tag{2.8}$$

Thus, the result for the complete synchronization is established as follows.

Theorem 2. *For the errors (2.8), if there exist the controllers $u_i(t)$ $(i = 1, \ldots, N-1)$ such that (2.3) is asymptotically stable, then we can get that*

$$\lim_{t\to\infty} \| e_j'(t) \| = \lim_{t\to\infty} \| x_{j+2}(t) - x_j(t) \| = 0 \quad (j = 1, 3, 5, \ldots, N-2), \lim_{t\to\infty} \| e_k''(t) \| = \lim_{t\to\infty} \| x_{k+2}(t) - x_k(t) \| = 0 \quad (k = 2, 4, 6, \ldots, N-3).$$

That is, the errors $e_j'(t)$ *and* $e_k''(t)$ *converge to* 0 *as time* t *approaches to infinity, so the complete synchronization of* $\dot{x}_q$ *and* $\dot{x}_{q+2}$ $(q=j,k)$ *is realized.*

Proof. By Theorem 1, there exists the controllers $u_i(t)$ $(i=1,\dots,N-1)$ such that

$$\lim_{t\to\infty}\|e_i(t)\| = \lim_{t\to\infty}\|x_i(t)+x_{i+1}(t)\| = 0.$$

鉴于

In view of (2.8), we have

$$e_j'(t)=x_{j+2}(t)-x_j(t)=x_{j+2}(t)-x_{j+1}(t)+x_{j+1}(t)-x_j(t), e_k''(t)=x_{k+2}(t)-x_k(t)=x_{k+2}(t)-x_{k+1}(t)+x_{k+1}(t)-x_k(t).$$

Therefore

$$\lim_{t\to\infty}\|e_j'(t)\| = \lim_{t\to\infty}\|x_{j+2}(t)-x_j(t)\| = \lim_{t\to\infty}\|x_{j+2}(t)-x_{j+1}(t)+x_{j+1}(t)-x_j(t)\| = \lim_{t\to\infty}\|x_{j+2}(t)-x_{j+1}(t)\| + \lim_{t\to\infty}\|x_{j+1}(t)-x_j(t)\| = 0+0=0,$$

and

$$\lim_{t\to\infty}\|e_k''(t)\| = \lim_{t\to\infty}\|x_{k+2}(t)-x_k(t)\| = \lim_{t\to\infty}\|x_{k+2}(t)-x_{k+1}(t)+x_{k+1}(t)-x_k(t)\| = \lim_{t\to\infty}\|x_{k+2}(t)-x_{k+1}(t)\| + \lim_{t\to\infty}\|x_{k+1}(t)-x_k(t)\| = 0+0=0.$$

So, $e_j'(t)$ and $e_k''(t)$ converge to 0. We can obtain that $\dot{e}_j'(t)$ and $\dot{e}_k''(t)$ are asymptotically stable, then the complete synchronization of $\dot{x}_q$ and $\dot{x}_{q+2}$ $(q=j,k)$ is realized. This completes the proof.□

(2) If the number of the chaotic systems N $(N\ge 3)$ is even, the complete synchronization error is formulated as

$$e_j'(t)=x_{j+2}(t)-x_j(t) \quad (j=1,3,5,\dots,N-3), e_k''(t)=x_{k+2}(t)-x_k(t) \quad (k=2,4,6,\dots,N-2). \tag{2.9}$$

Thus, the result for the complete synchronization behavior is established.

Theorem 3. *For the state errors (2.9), if there exist the controllers* $u_i(t)$ $(i=1,\dots,N-1)$ *such that (2.3) is asymptotically stable, then we can get that*

$$\lim_{t\to\infty}\|e_j'(t)\| = \lim_{t\to\infty}\|x_{j+2}(t)-x_j(t)\| = 0 \quad (j=1,3,5,\dots,N-3), \lim_{t\to\infty}\|e_k''(t)\| = \lim_{t\to\infty}\|x_{k+2}(t)-x_k(t)\| = 0 \quad (k=2,4,6,\dots,N-2).$$

That is, the errors $e_j'(t)$ and $e_k''(t)$ *converge to as time* t *goes to infinity, which means that the complete synchronization of systems* $\dot{x}_p$ *and* $\dot{x}_{p+2}$ $(p=j,k)$ *is realized.*

Proof. We omit the proof since it is similar to that of Theorem 2.□

In view of Theorems 1–3, we know that the controllers $u_i(t)$ can be got to make $\dot{e}_i(t)$, $\dot{e}_j'(t)$ and $\dot{e}_k''(t)$ asymptotically converge to origin. Then we reach the hybrid synchronization control of systems (2.2).

3. Application examples

In this section, the hybrid synchronization is simulated and realized for different numbers of chaotic systems (N=3,4), respectively.

Case I: When the number of the chaotic systems N=3, three different chaotic systems are selected to discuss the hybrid synchronization, which include the Chen system, Lorenz system and Lü system, respectively. They are described as follows:

$$\begin{cases}\dot{x}_{11}=-35x_{11}+35x_{12}+d_{11}(x_{31}-x_{11}),\\ \dot{x}_{12}=-7x_{11}+28x_{12}-x_{11}x_{13}+d_{12}(x_{32}-x_{12}),\\ \dot{x}_{13}=-3x_{13}+x_{11}x_{12}+d_{13}(x_{33}-x_{13}),\end{cases} \tag{3.1}$$

1 e1 : e11 e12 e13

$$\begin{cases}\dot{x}_{21}=-36x_{21}+36x_{22}+d_{21}(x_{11}-x_{21})+u_{11},\\ \dot{x}_{22}=20x_{22}-x_{21}x_{23}+d_{22}(x_{12}-x_{22})+u_{12},\\ \dot{x}_{23}=-3x_{23}+x_{21}x_{22}+d_{23}(x_{13}-x_{23})+u_{13},\end{cases} \tag{3.2}$$

2 e1 e2 两个误差

and

$$\begin{cases}\dot{x}_{31}=-10x_{31}+10x_{32}+d_{31}(x_{21}-x_{31})+u_{21},\\ \dot{x}_{32}=28x_{31}-x_{22}-x_{31}x_{33}+d_{32}(x_{22}-x_{32})+u_{22},\\ \dot{x}_{33}=-\frac{8}{3}x_{33}+x_{31}x_{32}+d_{33}(x_{23}-x_{33})+u_{23},\end{cases} \tag{3.3}$$

3

where

$$A_1=\begin{bmatrix}-35 & 35 & 0\\ -7 & 28 & 0\\ 0 & 0 & -3\end{bmatrix},\quad A_2=\begin{bmatrix}-36 & 36 & 0\\ 0 & 20 & 0\\ 0 & 0 & -3\end{bmatrix},$$

$$A_3=\begin{bmatrix}-10 & 10 & 0\\ 28 & -1 & 0\\ 0 & 0 & -\frac{8}{3}\end{bmatrix},\quad g_s(x_s)=\begin{bmatrix}0\\ -x_{s1}x_{s3}\\ x_{s1}x_{s2}\end{bmatrix},$$

and $s=1,2,3$, $D_1=diag(d_{11},d_{12},d_{13})$, $D_2=diag(d_{21},d_{22},d_{23})$, $D_3=diag(d_{31},d_{32},d_{33})$ are the coupled matrices, $u_1=[u_{11},u_{12},u_{13}]^T$, $u_2=[u_{21},u_{22},u_{23}]^T$ are the control inputs.

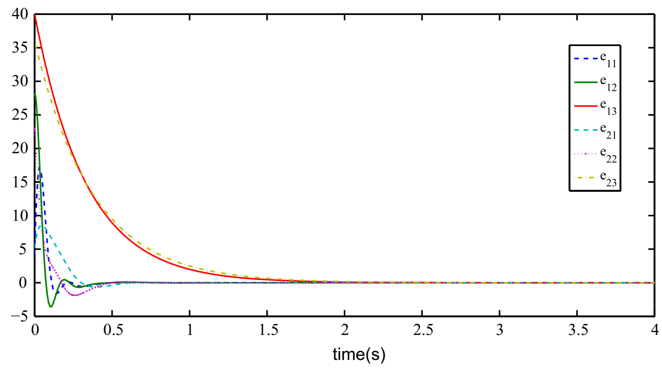

Fig. 2. Error state trajectories of e_{11}, e_{12}, e_{13}, e_{21}, e_{22}, and e_{23} of the error systems e_1 and e_2.

Let the anti-synchronization state error be $e_i = x_i + x_{i+1}$ $(i=1,2)$, where e_1 and e_2 are the three state errors. We can obtain that

$$\dot{e} = \begin{bmatrix} A_2 - D_1 - D_2 & D_1 \\ D_2 & A_3 - D_3 \end{bmatrix} e + \begin{bmatrix} (2D_2 - (A_2 - A_1))x_1 + g_2(x_2) + g_1(x_1) + u_1 \\ \begin{pmatrix} -2(D_2 - D_3) - \\ -(A_3 - A_2) \end{pmatrix} x_2 + g_3(x_3) + g_2(x_2) + u_2 + u_1 \end{bmatrix}, \tag{3.4}$$

where e contains six state errors.

The controllers u_1 and u_2 are designed to be

$$\begin{cases} u_1 = v_1 - (2D_2 - (A_2 - A_1))x_1 - g_2(x_2) - g_1(x_1), \\ u_2 = v_2 + \begin{pmatrix} 2(D_2 - D_3) + \\ +(A_3 - A_2) \end{pmatrix} x_2 - g_3(x_3) - g_2(x_2) - u_1, \end{cases}$$

where

$$v_1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ -36 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} e, v_2 = \begin{bmatrix} K_{11} & K_{12} \end{bmatrix} e, K_{11} = \begin{bmatrix} -d_{11} - d_{21} & 0 & 0 \\ 0 & -d_{12} - d_{22} & 0 \\ 0 & 0 & -d_{13} - d_{23} \end{bmatrix}, K_{12} = \begin{bmatrix} 0 & 0 & 0 \\ -38 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

As a result, (3.4) is rewritten as

$$\dot{e} = \begin{bmatrix} \Gamma_1^* & \Gamma_2^* \\ \Gamma_3^* & \Gamma_4^* \end{bmatrix} e, \tag{3.5}$$

where

$$\Gamma_1^* = \begin{bmatrix} K_5 & 36 & 0 \\ -36 & K_6 & 0 \\ 0 & 0 & K_7 \end{bmatrix}, \quad \Gamma_2^* = \begin{bmatrix} d_{11} & 0 & 0 \\ 0 & d_{12} & 0 \\ 0 & 0 & d_{13} \end{bmatrix}, \Gamma_3^* = \begin{bmatrix} -d_{11} & 0 & 0 \\ 0 & -d_{12} & 0 \\ 0 & 0 & -d_{13} \end{bmatrix}, \quad \Gamma_4^* = \begin{bmatrix} K_8 & 10 & 0 \\ -10 & K_9 & 0 \\ 0 & 0 & K_{10} \end{bmatrix},$$

and $K_5 = -36 - d_{11} - d_{21}$, $K_6 = 20 - d_{12} - d_{22}$, $K_7 = -3 - d_{13} - d_{23}$, $K_8 = -10 - d_{31}$, $K_9 = -1 - d_{32}$, $K_{10} = -\frac{8}{3} - d_{33}$.

If the conditions

$$-36 - d_{11} - d_{21} < 0, \quad 20 - d_{12} - d_{22} < 0, -3 - d_{13} - d_{23} < 0, \quad -10 - d_{31} < 0, -1 - d_{32} < 0, \quad -8/3 - d_{33} < 0$$

are satisfied. Theorem 1 induces that (3.5) is asymptotically stable under the operations of controllers u_1 and u_2, which means that we realize the anti-synchronization between multi-systems.

Let $(x_{11}(0), x_{12}(0), x_{13}(0)) = (10, 20, 30), (x_{21}(0), x_{22}(0), x_{23}(0)) = (-5.8, 8, 10)$ and $(x_{31}(0), x_{32}(0), x_{33}(0)) = (11, 15, 26)$. Meanwhile, we choose $d_{11} = d_{21} = d_{13} = d_{23} = d_{31} = d_{33} = 0, d_{12} = 10, d_{22} = 11$ and $d_{32} = 1$. We use the fourth order Runge–Kutta integration method to simulate the numerical example. The state trajectories of (3.4) and $\dot{e}'_1(t) = \dot{x}_3(t) - \dot{x}_1(t)$ are shown in Figs. 2 and 3, respectively. They imply that the synchronization and anti-synchronization are reached. Figs. 4, 5 and 6 show the state trajectories of (3.1), (3.2) and (3.3), under u_1 and u_2. We can get that $x_1(t)$ and $x_2(t)$, $x_2(t)$ and $x_3(t)$ achieve the anti-synchronization, and $x_1(t)$ and $x_3(t)$ achieve the complete synchronization. Then the hybrid synchronization is achieved.

Case II: When the number of chaotic systems $N=4$, four Lorenz chaotic systems are selected to investigate the hybrid synchronization.

$$\begin{cases} \dot{x}_{11} = -10x_{11} + 10x_{12} + d'_{11}(x_{41} - x_{11}), \\ \dot{x}_{12} = 28x_{11} - x_{12} - x_{11}x_{13} + d'_{12}(x_{42} - x_{12}), \\ \dot{x}_{13} = -\frac{8}{3}x_{13} + x_{11}x_{12} + d'_{13}(x_{43} - x_{13}), \end{cases} \tag{3.6}$$

1

$$\begin{cases} \dot{x}_{21} = -10x_{21} + 10x_{22} + d'_{21}(x_{11} - x_{21}) + u'_{11}, \\ \dot{x}_{22} = 28x_{21} - x_{22} - x_{21}x_{23} + d'_{22}(x_{12} - x_{22}) + u'_{12}, \\ \dot{x}_{23} = -\frac{8}{3}x_{23} + x_{21}x_{22} + d'_{23}(x_{13} - x_{23}) + u'_{13}, \end{cases} \tag{3.7}$$

2

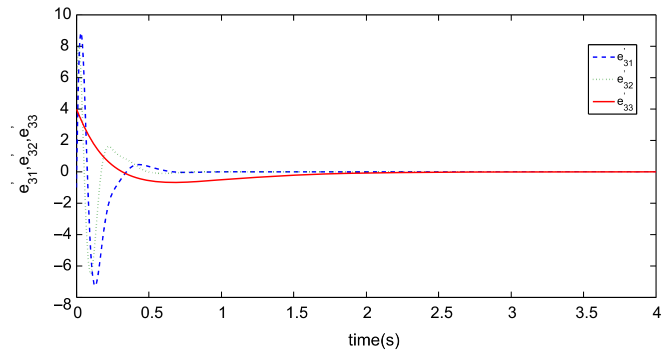

Fig. 3. Error state trajectories of e'_{31}, e'_{32}, e'_{33} of $\dot{e}'_1(t)$.

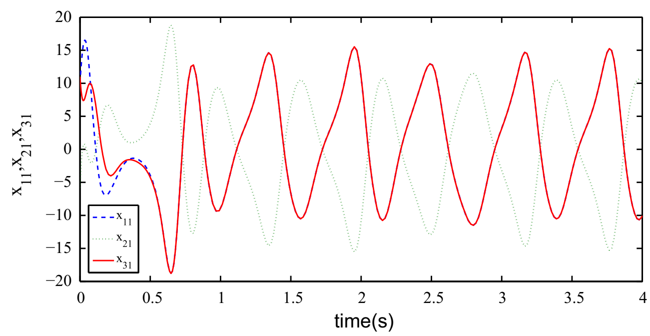

Fig. 4. The state trajectories x_{11}, x_{21} and x_{31} of the Lorenz systems.

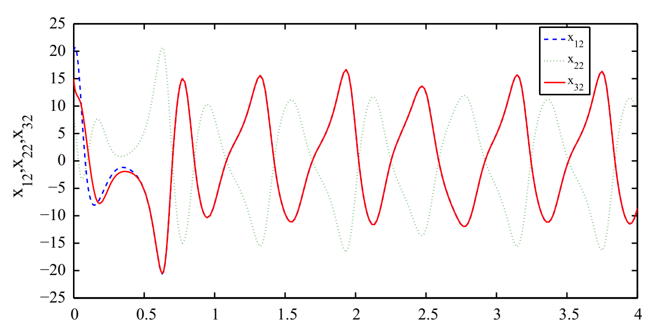

Fig. 5. The state trajectories x_{12}, x_{22} and x_{32} of the Lorenz systems.

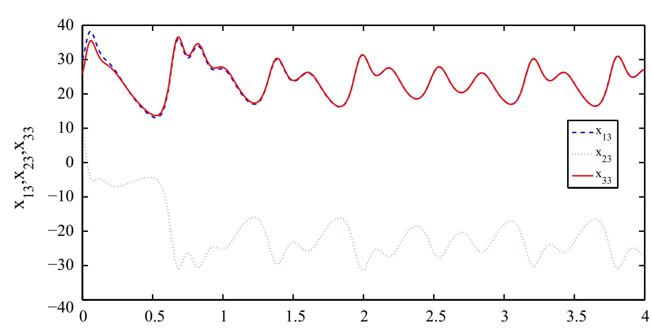

Fig. 6. The state trajectories x_{13}, x_{23} and x_{33} of the Lorenz systems with time t.

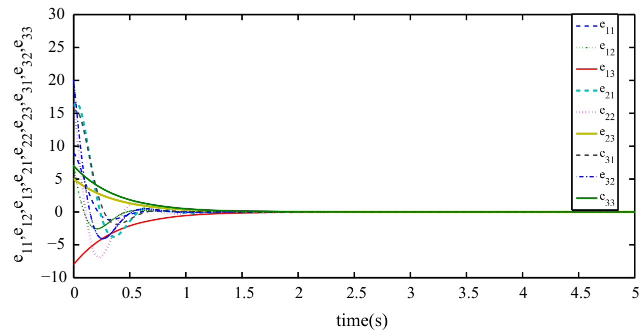

Fig. 7. Error state trajectories of e_{11}, e_{12}, e_{13}, e_{21}, e_{22}, e_{23}, e_{31}, e_{32} and e_{33} of e_1, e_2 and e_3.

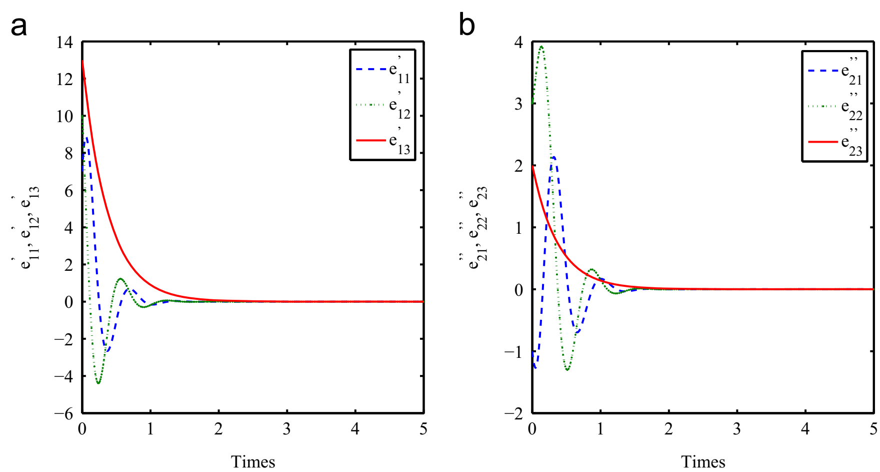

Fig. 8. Error state trajectories of e'_{11}, e'_{12}, e'_{13}, e''_{21}, e''_{22}, e''_{23} of $\dot{e}'_1(t)$ and $\dot{e}''_2(t)$.

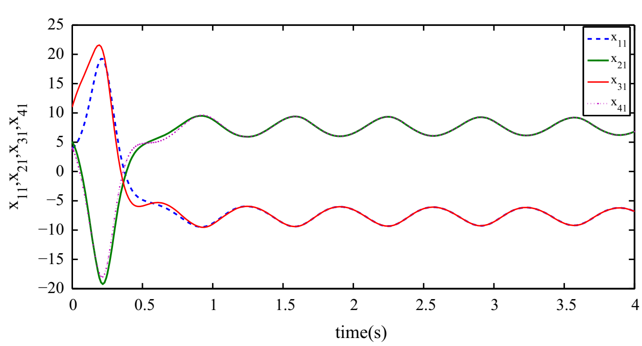

Fig. 9. State trajectories of x_{11}, x_{21}, x_{31} and x_{41} of the Lorenz systems.

$$\begin{cases} \dot{x}_{31} = -10x_{31} + 10x_{32} + d'_{31}(x_{21} - x_{31}) + u'_{21}, \\ \dot{x}_{32} = 28x_{31} - x_{22} - x_{31}x_{33} + d'_{32}(x_{22} - x_{32}) + u'_{22}, \\ \dot{x}_{33} = -\frac{8}{3}x_{33} + x_{31}x_{32} + d'_{33}(x_{23} - x_{33}) + u'_{23}, \end{cases} \tag{3.8}$$

3

and

$$\begin{cases} \dot{x}_{41} = -10x_{41} + 10x_{42} + d'_{41}(x_{31} - x_{41}) + u'_{31}, \\ \dot{x}_{42} = 28x_{41} - x_{42} - x_{41}x_{43} + d'_{42}(x_{32} - x_{42}) + u'_{32}, \\ \dot{x}_{43} = -\frac{8}{3}x_{43} + x_{41}x_{42} + d'_{43}(x_{33} - x_{43}) + u'_{33}, \end{cases} \tag{3.9}$$

4

where

$$A'_1 = A'_2 = A'_3 = A'_4 = A = \begin{bmatrix} -10 & 10 & 0 \\ 28 & -1 & 0 \\ 0 & 0 & -\frac{8}{3} \end{bmatrix}, g_s(x_s) = \begin{bmatrix} 0 \\ -x_{s1}x_{s3} \\ x_{s1}x_{s2} \end{bmatrix}, \quad s = 1,2,3,4$$

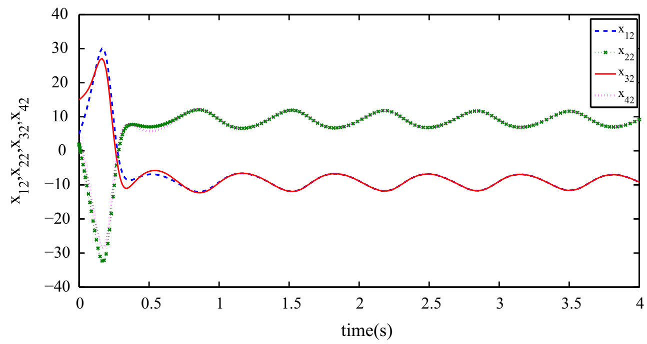

Fig. 10. State trajectories x_{12}, x_{22}, x_{32} and x_{42} of the Lorenz systems.

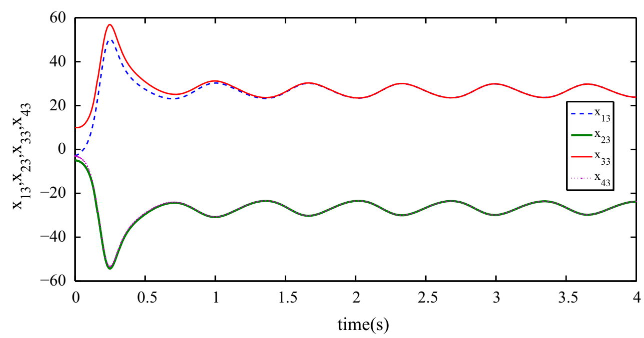

Fig. 11. State trajectories x_{13}, x_{23}, x_{33} and x_{43} of the Lorenz systems.

and $D_1' = diag(d_{11}', d_{12}', d_{13}')$, $D_2' = diag(d_{21}', d_{22}', d_{23}')$, $D_3' = diag(d_{31}', d_{32}', d_{33}')$ and $D_4' = diag(d_{41}', d_{42}', d_{43}')$ are defined as the coupled matrices, $u_1' = [u_{11}', u_{12}', u_{13}']^T$, $u_2' = [u_{21}', u_{22}', u_{23}']^T$ and $u_3' = [u_{31}', u_{32}', u_{33}']^T$ are the control inputs.

With the same techniques in Case I, the anti-synchronization error is $\dot{e}_i = \dot{x}_i + \dot{x}_{i+1}$ $(i = 1, 2, 3)$. Then we can get that

$$\dot{e} = \begin{bmatrix} \Gamma_1 & -\Gamma_2 & \Gamma_2 \\ \Gamma_3 & \Gamma_4 & 0 \\ 0 & \Gamma_5 & \Gamma_6 \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} + \begin{bmatrix} (-2D_1' + 2D_2')x_1 + g_2(x_2) + g_1(x_1) + u_1' \\ [-2(D_2' - D_3')]x_2 + g_3(x_3) + g_2(x_2) + u_2' + u_1' \\ [-2(D_3' - D_4')]x_3 + g_4(x_4) + g_3(x_3) + u_3' + u_2' \end{bmatrix} \tag{3.10}$$

where e contains nine state errors, and

$$\Gamma_5 = \begin{bmatrix} d_{31}' & 0 & 0 \\ 0 & d_{32}' & 0 \\ 0 & 0 & d_{33}' \end{bmatrix}, \Gamma_6 = \begin{bmatrix} -10 - d_{41}' & 10 & 0 \\ 28 & -1 - d_{42}' & 0 \\ 0 & 0 & -\frac{8}{3} - d_{43}' \end{bmatrix},$$

we can get the controllers u_1', u_2' and u_3' as

$$\begin{cases} u_1' = v_1' + 2(D_1' - D_2')x_1 - g_2(x_2) - g_1(x_1), \\ u_2' = v_2' + 2(D_2' - D_3')x_2 - g_3(x_3) - g_2(x_2) - u_1', \\ u_3' = v_3' + 2(D_3' - D_4')x_3 - g_4(x_4) - g_3(x_3) - u_2', \end{cases}$$

where

$$v_1' = [K_{12} \;\; 0 \;\; 0]e, \quad v_2' = [K_{13} \;\; K_{12} \;\; K_{14}]e, K_{12} = \begin{bmatrix} 0 & 0 & 0 \\ -38 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, K_{13} = \begin{bmatrix} d_{11}' - d_{21}' & 0 & 0 \\ 0 & d_{12}' - d_{22}' & 0 \\ 0 & 0 & d_{13}' - d_{23}' \end{bmatrix},$$

$$K_{14} = \begin{bmatrix} -d_{31}' & 0 & 0 \\ 0 & -d_{32}' & 0 \\ 0 & 0 & -d_{33}' \end{bmatrix}, v_3' = [K_{15} \;\; 0 \;\; K_{12}]e, K_{15} = \begin{bmatrix} -d_{11}' & 0 & 0 \\ 0 & -d_{12}' & 0 \\ 0 & 0 & -d_{13}' \end{bmatrix}.$$

As a result, (3.10) can be rewritten as

$$\dot{e} = \begin{bmatrix} \Delta_1^* & \Delta_2^* & \Delta_3^* \\ \Delta_4^* & \Delta_5^* & \Delta_6^* \\ \Delta_7^* & \Delta_8^* & \Delta_9^* \end{bmatrix} e, \tag{3.11}$$

where

$$\Delta_1^* = \begin{bmatrix} K_{16} & 10 & 0 \\ -10 & K_{17} & 0 \\ 0 & 0 & K_{18} \end{bmatrix}, K_{16} = -10 + d'_{11} - d'_{21}, \quad K_{17} = -1 + d'_{12} - d'_{22},$$

$$K_{18} = -\frac{8}{3} + d'_{13} - d'_{23}, \quad \Delta_8^* = -\Delta_6^* = -K_{14}, \Delta_3^* = \Delta_4^* = -\Delta_2^* = -\Delta_7^* = -K_{15},$$

$$\Delta_5^* = \begin{bmatrix} -10 - d'_{31} & 10 & 0 \\ -10 & -1 - d'_{32} & 0 \\ 0 & 0 & -\frac{8}{3} - d'_{33} \end{bmatrix}, \Delta_9^* = \begin{bmatrix} -10 - d'_{41} & 10 & 0 \\ -10 & -1 - d'_{42} & 0 \\ 0 & 0 & -\frac{8}{3} - d'_{43} \end{bmatrix}.$$

If the conditions

$$-10 + d'_{11} - d'_{21} < 0, \quad -1 + d'_{12} - d'_{22} < 0, -8/3 + d'_{13} - d'_{23} < 0, \quad -10 - d'_{31} < 0, -1 - d'_{32} < 0, \quad -8/3 - d'_{33} < 0, -10 - d'_{41} < 0,$$
$$-1 - d'_{42} < 0, \quad -8/3 - d'_{43} < 0$$

are satisfied, then Theorem 1 induces that (3.11) is asymptotically stable under the operations of controllers u'_1, u'_2 and u'_3, which means that we reach the anti-synchronization for four chaotic systems.

Let $(x_{11}(0), x_{12}(0), x_{13}(0)) = (4, 5, -3)$, $(x_{21}(0), x_{22}(0), x_{23}(0)) = (5, 2, -5)$, $(x_{31}(0), x_{32}(0)h, \quad x_{33}(0)) = (11, 15, 10)$ and $(x_{41}(0), x_{42}(0), x_{43}(0)) = (4, 5, -3)$, respectively. Meanwhile, we choose $d'_{12} = d'_{22} = d'_{32} = d'_{13} = d'_{23} = d'_{33} = d'_{42} = d'_{43} = 0$, $d'_{11} = d'_{21} = 1$, $d'_{31} = d'_{41} = -1$. We apply same solving method to prepare numerical simulations.

Then the state trajectories of (3.10), $\dot{e}'_1(t) = \dot{x}_3(t) - \dot{x}_1(t)$ and $\dot{e}''_2(t) = \dot{x}_4(t) - \dot{x}_2(t)$ are depicted in Figs. 7 and 8(a) and (b). These imply that the synchronization and anti-synchronization are realized. According to Definition 1, the hybrid synchronization is realized. Figs. 9, 10 and 11 give the state of chaotic systems under the controllers u_1 and u_2. It is easy to obtain that $x_1(t)$ and $x_2(t)$, $x_2(t)$ and $x_3(t)$, $x_3(t)$ and $x_4(t)$ achieve the anti-synchronization, and $x_1(t)$ and $x_3(t)$, $x_2(t)$ and $x_4(t)$ achieve the complete synchronization. So the hybrid synchronization is reached.

4. Conclusions

Hybrid synchronization behavior of multiple chaotic systems is analyzed in this paper. We reached the anti-synchronization in the multiple coupled chaotic systems, and the complete synchronization behavior under the anti-synchronization controllers proposed. Several new synchronization criteria are given for the coexistence of CS and AS of the chaotic dynamic systems. This technology will possess better theory and application value in engineering practice. Furthermore, our presented synchronization control strategy can ensure to achieve the strict synchronization for such chaotic systems. To design the proper controller to reach the other synchronization in multiple coupled chaotic systems with impulsive effects [42,43], time delay [44] and uncertainty is our further research work.

References

[1] L.M. Pecora, T.L. Carroll, Synchronization in chaotic systems, Phys. Rev. Lett. 64 (8) (1990) 821–824.
[2] K.Y. Lian, T.S. Chiang, C.S. Chiu, P. Liu, Synthesis of fuzzy model-based designs to synchronization and secure communications for chaotic systems, IEEE Trans. Syst. Man Cybern. B, Cybern. 31 (1) (2001) 66–83.
[3] S. Das, U. Halder, D. Maity, Chaotic dynamics in social foraging swarms: an analysis, IEEE Trans. Syst. Man Cybern. B, Cybern. 42 (4) (2012) 1288–1293.
[4] S. Tang, J. Liu, Chaos synchronization in semiconductor lasers with optoelectronic feedback, IEEE J. Quant. Electron. 39 (6) (2003) 708–715.
[5] W. He, J. Cao, Generalized synchronization of chaotic systems: the auxiliary system approach via matrix measure, Chaos 19 (1) (2009) 013118.
[6] N. Cai, Y. Jing, S. Zhang, Generalized projective synchronization of different chaotic systems based on antisymmetric structure, Chaos Solitons Fractals 42 (2) (2009) 1190–1196.
[7] J. Sun, Y. Shen, G.D. Zhang, Transmission projective synchronization of multi-systems with non-delayed and delayed coupling via impulsive control, Chaos 22 (2012) 043107.
[8] J. Lu, J. Cao, Daniel W.C. Ho, Adaptive stabilization and synchronization for chaotic Lur'e systems with time-varying delays, IEEE Trans. Circuits Syst.—I 55 (5) (2008) 1347–1356.
[9] J. Sun, Y. Shen, Q. Yin, C.J. Xu, Compound synchronization of four memristor chaotic oscillator systems and secure communication, Chaos 23 (2013) 013140.
[10] J. Sun, Y. Shen, G.D. Zhang, et al., Combination–combination synchronization among four identical or different chaotic systems, Nonlinear Dyn. 73 (2013) 1211–1222.
[11] X. Shi, L. Zuo, Hybrid synchronization phenomenon in two coupled delay hyperchaotic Lorenz systems with unknown parameters via a single linear controller, J. Vib. Control 19 (7) (2013) 1051–1060.
[12] X. Shi, L. Zuo, The alternating between complete synchronization and hybrid synchronization of hyperchaotic Lorenz system with time delay, Nonlinear Dyn. 69 (3) (2012) 1177–1190.
[13] Q. Zhang, J. Lu, S. Chen, Coexistence of anti-phase and complete synchronization in the generalized Lorenz system, Commun. Nonlinear Sci. Numer. Simul. 15 (10) (2010) 3067–3072.
[14] V. Sundarapandian, S. Sivaperumal, Sliding controller design of hybrid synchronization of four-wing chaotic systems, Int. J. Soft Comput. 6 (2011) 224–231.
[15] S. Das, M. Srivastava, A.Y.T. Leung, Hybrid phase synchronization between identical and nonidentical three-dimensional chaotic systems using the active control method, Nonlinear Dyn. 73 (4) (2013) 2261–2272.
[16] P. Li, S. Liu, Robust adaptive full state hybrid synchronization of chaotic complex systems with unknown parameters and external disturbances, Nonlinear Dyn. 70 (1) (2012) 585–599.
[17] H. Liu, Y. Li, H. Yu, L. Zhu, Hybrid synchronization in unified chaotic systems with unknown parameters, J. Northeastern Univ. 32 (10) (2011) 1394–1397.
[18] P. Li, R. Dong, B. Lu, Hybrid synchronization of Lü hyperchaotic system with disturbances by sliding mode control, Procedia Eng. 15 (8) (2011) 23–27.
[19] G. Xu, S. Chen, Hybrid synchronization of a Chen hyper-chaotic system with two simple linear feedback controllers, Appl. Math. 4 (11B) (2013) 13–17.
[20] Y. Lan, Q. Li, Hybrid synchronization in hyperchaotic systems, J. Comput. Inf. Syst. 8 (18) (2012) 7699–7707.
[21] M. Sabir, S.K. Sebastian, Hybrid synchronization of hyperchaotic Lu system, Pramana—J. Phys. 73 (4) (2009) 781–786.
[22] X. Liu, J. Jiang, Hybrid projective synchronization of fractional-order chaotic systems with time delay, Discrete Dyn. Nat. Soc. (2013) 8 pp., Article ID 459801.
[23] S.K. Agrawal, M. Srivastava, S. Das, Hybrid synchronization between different fractional order hyperchaotic systems using active control method, J. Nonlinear Syst. Appl. 4 (3) (2013) 70–76.

[24] J. Hu, J. Liang, J. Cao, Synchronization of hybrid-coupled heterogeneous networks: pinning control and impulsive control schemes, J. Frankl. Inst. 351 (5) (2014) 2600–2622.
[25] J. Wu, T. Lu, Hybrid synchronization of the general delayed and non-delayed complex dynamical networks via pinning control, Neurocomputing 89 (2012) 168–177.
[26] Y. Yu, S. Zhang, Global synchronization of three coupled chaotic systems with ring connection, Chaos Solitions Fractals 24 (5) (2005) 1233–1242.
[27] Y. Liu, L. Lv, Synchronization of N different coupled chaotic systems with ring and chain connections, Appl. Math. Mech. 29 (10) (2008) 1181–1190.
[28] Y. Tang, J. Fang, Synchronization of N-coupled fractional-order chaotic systems with ring connection, Commun. Nonlinear Sci. Numer. Simul. 15 (2) (2010) 401–412.
[29] X. Chen, J. Qiu, Synchronization of N different chaotic systems based on antisymmetric structure, Math. Probl. Eng. (2013) 6 pp., Article ID 742765.
[30] X. Chen, J. Qiu, Q. Song, A. Zhang, Synchronization of N coupled chaotic systems with ring connection based on special antisymmetric structure, Abstr. Appl. Anal. 680604 (2013) 10.
[31] X. Chen, C. Wang, J. Qiu, Synchronization and anti-synchronization of N different coupled chaotic systems with ring connection, Int. J. Mod. Phys. C 25 (4) (2014) 12.
[32] G. Grassi, Propagation of projective synchronization in a series connection of chaotic systems, J. Frankl. Inst. 347 (2) (2010) 438–451.
[33] J. Lu, X. Han, Y. Li, Adaptive coupled synchronization among multi-Lorenz systems family, Chaos Solitions Fractals 31 (4) (2007) 66–878.
[34] L. Yang, J. Zhang, Synchronization of three identical systems and its application for secure communication with noise perturbation, in: Proceedings of International Conference on Information Engineering and Computer Science, Wuhan, China, 2009, pp. 1–4.
[35] W. He, J. Cao, Global synchronization in arrays of coupled networks with one single time-varying delay coupling, Phys. Lett. A 373 (31) (2009) 2682–2694.
[36] J. Cao, Z. Wang, Y. Sun, Synchronization in an array of linearly stochastically coupled networks with time delays, Physica A 385 (2) (2007) 718–728.
[37] X. Yang, J. Cao, Hybrid adaptive and impulsive synchronization of uncertain complex networks with delays and general uncertain perturbations, Appl. Math. Comput. 227 (15) (2014) 480–493.
[38] X. Ji, Hybrid synchronization of three identical coupled chaotic systems using the direct design method, Adv. Mater. Res. 912–914 (2014) 695–699.
[39] J. Cao, G.R. Chen, P. Li, Global synchronization in an array of delayed neural networks with hybrid coupling, IEEE Trans. Syst. Man Cybern. B, Cybern. 38 (2) (2008) 488–498.
[40] B. Liu, M. Zhou, M. Jiang, K. Zhang, Synchronizing chaotic systems using control based on tridiagonal structure, Chaos Solitons Fractals 39 (2007) 2274–2281.
[41] B. Liu, K. Zhang, Stability of nonlinear systems with tridiagonal structure and its applications, Acta Autom. Sin. 33 (2007) 442–446.
[42] W. Zhang, Y. Tang, Q. Miao, W. Du, Exponential synchronization of coupled switched neural networks with mode-dependent impulsive effects, IEEE Trans. Neural Netw. Learn. Syst. 24 (8) (2013) 1316–1326.
[43] Y. Tang, H. Gao, W. Zhang, J. Kurths, Leader-following consensus of a class of stochastic delayed multi-agent systems with partial mixed impulses, Automatica 53 (1) (2015) 346–354.
[44] R. Rakkiyappan, S. Dharani, Q.X. Zhu, Synchronization of reaction–diffusion neural networks with time-varying delays via stochastic sampled-data controller, Nonlinear Dyn. 79 (1) (2015) 485–500.

Xiangyong Chen received the Master's degree and Ph.D. degree in control theory and control engineering from the Northeastern University, China, in 2008 and 2012 respectively. His current research interests include complex dynamic system and complex networks, differential game and optimal control, synchronization control of chaotic systems.

Jianlong Qiu received the Ph.D. degree from Southeast University, Nanjing, China, in 2007. He is a Professor of School of Sciences and Deputy Director of Department of Research Administration at the Linyi University. He has been engaged in collaborative research as a Visiting Scholar in Stevens Institute of Technology and University of Rhode Island. His current research interest involves genetic network, stability theory, neural network and applied mathematics.

Jinde Cao received the B.S. degree from Anhui Normal University, Wuhu, China, in 1986, the M.S. degree from Yunnan University, Kunming, China, in 1989, and the Ph.D. degree from Sichuan University, Chengdu, China, in 1998, respectively, all in mathematics/applied mathematics. From March 1989 to May 2000, he was with the Yunnan University, where he was a Professor during 1996–2000. In May 2000, he was in the Department of Mathematics, Southeast University, Nanjing, China. From July 2001 to June 2002, he was a Postdoctoral Research Fellow in the Department of Automation and Computer-aided Engineering, Chinese University of Hong Kong, Hong Kong. During 2006–2008, he was a Visiting Research Fellow and a Visiting Professor in the School of Information Systems, Computing and Mathematics, Brunel University, London, UK. He is currently a TePin Professor and a Doctoral Advisor at the Southeast University. He is the author or coauthor of more than 160 journal papers and five edited books and a reviewer of the Mathematical Reviews and the Zentralblatt-Math. He is an Associate Editor of the Neurocomputing, the Journal of The Franklin Institute, the Mathematics and Computers in Simulation, the Differential Equations and Dynamical Systems, the International Journal of Differential Equations, the Open Electrical and Electronic Engineering Journal, and IEEE Transactions on Neural Networks, etc. His research interests include neural networks, nonlinear systems, stability theory, complex systems and complex networks, and applied mathematics.

Haibo He received the B.S. and M.S. degrees in electrical engineering from Huazhong University of Science and Technology, Wuhan, China, in 1999 and 2002, respectively, and the Ph.D. degree in electrical engineering from Ohio University in 2006. He is currently the Robert Haas Endowed Professor in Electrical Engineering at the University of Rhode Island. His research interests include computational intelligence, self-adaptive systems, machine learning and data mining, hardware design for machine intelligence, and various application fields including smart grid, sensor networks, biomedical applications, and cognitive communication networks. He has published 1 research book with Wiley, edited 1 book entitled with Wiley-IEEE, edited 6 conference proceedings with Springer, and authored/co-authors over 120 peer-reviewed journal and conference papers, including one Cover Page Highlighted Paper in the IEEE Transactions on Information Forensics and Security, one Most-Cited Paper in the IEEE Transactions on Knowledge and Data Engineering, and several highly cited papers in the IEEE Transactions on Power Delivery and IEEE Transactions on Neural Networks. He served as the Co Founding-Editor-in-Chief of the Journal of Intelligent Learning Systems and Applications, as well as a Guest Editor and Editorial Board member for many international journals. Currently, he is an Associate Editor of the IEEE Transactions on Neural Networks and Learning Systems and IEEE Transactions on Smart Grid.