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Event-triggered fuzzy adaptive compensation control for uncertain stochastic nonlinear systems with given transient specification and actuator failures

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Abstract

In this paper, an event-triggered Fuzzy adaptive compensation controller is constructed for uncertain stochastic nonlinear systems with given transient specification and actuator failures. The main characteristics of the systems are that they take into account the effect of actuator failures and given transient specification, system communication resource limitation simultaneously. It is still an arduous task and challenge to design for uncertain stochastic nonlinear system specially. To stabilize the uncertain nonlinear stochastic systems, the fuzzy logic systems are used to approximate the unknown functions, an event-triggered control mechanism is established to save the system communication resource and a transformed system control mechanism is developed to satisfy the given transient specification all the time, in normal mode or faulty mode. Based on the Lyapunov approach, it is proved that all the signals of the closed-loop system are bounded, and the tracking error is limited to a given transient specification. The simulation results illustrates the effectiveness of the proposed control approach.

Keywords: Adaptive control; Actuator failure; Event-triggered; Stochastic nonlinear systems; Transient performance

1. Introduction

In practice, especially in control systems, various system components such as actuators, sensors and processors may undergo abrupt failures individually or simultaneously during operation. The failures, which are often uncertain

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in time, value and pattern, cause system instability and even generate catastrophic accidents. So more and more researchers have been motivated to investigate the control schemes to compensate for the unknown actuator failures, such as sliding modes [1], adaptive control methods [2–6], fault detection methods [7–10], Backstepping Control methods [11–17], Fuzzy control methods [18–20], etc. In [8–10], adaptive fault-tolerant controllers are established, both SISO and MIMO. In [4], a fuzzy adaptive actuator failure compensation control for a class of uncertain stochastic nonlinear systems is developed. The Backstepping Control technique suffers from the explosion of complexity problem. So an adaptive dynamic surface control (DSC) algorithm was presented for nonlinear systems with parametric uncertainties [14–17]. The considered system contains the actuator failures of both loss of effectiveness and lock-in-place, unmodeled dynamics and without direct measurements of state variables. However, the aforementioned schemes do not all consider the effect of limitation of system network transmission resource between actuator and controller. In fact, the limitation exists in many physical systems. Hybrid filter is developed in [21,22], when the considered system is subjected to stochastic cyber attacks or deception attacks. Attacks are aggressive behaviors aiming at destroying communication systems, real sampling data, networked infrastructures and devices. The hybrid triggered scheme could reduce the pressure of network bandwidth. For uncertain stochastic nonlinear systems including FLS, controller design based on actuator failures and limitation of system network transmission resource is very challenging.

As well as we know, system needs large transmission resource between controller and actuator when system actuator undergo abrupt failures individually or simultaneously during operation. Recently, several specific control techniques, especial the invariant threshold or periodic event-triggered communication schemes, have been developed to saving system transmission resource, see in [23–25]. Though simplicity and convenience are unquestionable outstanding advantages of the invariant threshold or periodic event-triggered communication schemes, they would lead to a lot of redundant data transmissions. Thus some different event-triggered communication schemes are presented in [26–33]. Considering the effect of mismatched membership functions in the closed-loop system, the proposed controller is reducing the utilization of limited network bandwidth and handling network-induced delay in [30]. And a novel event-triggered fault detection filter subject to mismatched membership functions, data quantization and transmission delay is designed to reduce the utilization of limited transmission bandwidth in [31]. They extend the application scope of the event-triggered control scheme.

Under the adaptive event-triggered scheme, which is developed in [32], the releasing rate of the sampling data is dramatically reduced. See in [33], an event-triggered communication scheme is proposed to mitigate the utility of limited network bandwidth and the model of multiple sensor distortions is established, which is more general in practical system. However, they do not consider saving system communication resource when system was subjected to actuator failures based on given transient performance.

In the absence of unmodeled failures and limitation of communication bandwidth, tracking error could be converged to a bound by most adaptive control systems, including FLS and Neural Network, see in [34–52]. For uncertain stochastic nonlinear systems with actuator failures, controller design based on transient specification is very challenging, as it is not an easy task to guarantee the given transient response under the condition of limited resources. In other words, the most given specifications for systems are not considered transient performance in controller design. In fact, the resulting transient responses of adaptive control systems may be unacceptable due to not characterize the maximum overshoot of the tracking error. This clearly limits practical applications of adaptive control schemes. Thus many researchers have put lots of efforts to study and improve the transient performance for adaptive control systems. In [9], an adaptive control scheme for uncertain nonlinear systems is using to guarantee prescribed performance bounds (PPB). In [53], two backstepping-based adaptive fuzzy fault-tolerant control methods for a class of nonlinear strict-feedback systems with actuator failures are proposed by adjusting the PPB parameters of the errors. And [54] considers designing adaptive finite-time controllers for a class of single input single output (SISO) strict feedback nonlinear plants with parametric uncertainties based on given specifications. To date, there have been few completed results available for uncertain stochastic nonlinear systems with the limitation of communication resource between controller and actuator.

Motivated by the above observation, event-triggered Fuzzy adaptive compensation control for uncertain stochastic nonlinear systems with actuator failures and given transient specification is investigated based on saving the system transmission resource. The stability of the closed-loop system is proved and the tracking error is remained bounded. Compared with the existing literature, the main contributions of this paper are summarized as follows.

- The considered stochastic systems are subjected with actuator failures and the limitation of communication resource simultaneously. Generally speaking, owing to these practical faults, the extension of compensation controller design for stochastic systems in [4] is nontrivial. So an adaptive controller and an event-triggered control mechanism are established to make the system asymptotically stable and to save the communication resource simultaneously.
- It is worth mentioning that event-triggered control mechanisms has been raised in [26–31]. In general, a stochastic system has the limitation of communication resource with actuator failures and given transient performance, which results in the increasing difficulty and challenge of analysis and design. Thus a transformed system fuzzy control mechanism is established to ensure the transient performance based on the limitation of communication resource between controller and actuator. And all closed-loop signals remain bounded simultaneously.

The remaining part of this paper is organized as follows. In Section 2, the preliminaries and problem formulation are presented. In Section 3, a novel control scheme to guarantee transient performance and compensate for the failures of hysteretic actuator is proposed. In Section 4, some simulation examples are provided to illustrate the effectiveness of the proposed control scheme. Finally, we summarize this paper in Section 5.

2. Theoretical background and problem statement

2.1. Problem formulation and preliminaries

In this paper, we consider a family of stochastic nonlinear systems as follows.

$$\begin{cases} dx_j &= \left(x_{j+1} + f_j\left(\bar{x}_j\right)\right)dt + \psi_j^T\left(\bar{x}_j\right)dv \\ j &= 1, 2, ..., n-1 \\ dx_n &= \left(u(t) + f_n\left(\bar{x}_n\right)\right)dt + \psi_n^T\left(\bar{x}_n\right)dv \\ y &= x_1 \\ u(t) &= \sum\limits_{i=1}^{m} b_i\beta_i(x)u_{ci}(t) \end{cases} \tag{1}$$

where $x = [x_1, x_2, \cdots, x_n] \in R^n$, $y \in R$ and $u_{ci}(t) \in R$ $(i = 1, 2, \cdots, m)$ are system states, output and inputs, respectively, and $\bar{x}_j$ is defined as $\left[x_1, \cdots, x_j\right]$, $\beta_i(x) \in R$ $(i = 1, 2, ..., m)$ are known smooth nonlinear functions, $f_i \in R$, $\psi_j \in R^r$ $(j = 1, 2, \cdots, n)$ and $b_i \in R$ are smooth nonlinear functions and unknown constant parameters, respectively. v is an independent r-dimension standard Brownian motion.

We now consider the ith actuator which may fail but uncertain in occurrence time, pattern and value. The failure of the ith actuator at time instant t_{iF} can be modeled as

$$u_{ci}(t) = \rho_i u_{PFi}(t) + (1-\rho_i)u_{TFi}(t), \forall t \geq t_{iF} \tag{2}$$

where time instant t_{iF} is the time instant when the ith actuator fails, and $u_{PFi}(t)$ represents ith actuator input, $u_{ci}(t)$ represents ith actuator output. $\rho_i \in [0, 1]$, $u_{TFi}(t)$ and t_{iF} are unknown constants. When the constant $\rho_i = 1$, $u_{ci}(t) = u_{PFi}(t)$, the ith actuator works in the failure-free case. So we consider the following 2 patterns of failures.

1) Partial Loss of Effectiveness (PLOE): $u_{ci}(t) = \rho_i u_{PFi}(t)$, $0 < \rho_i < 1$, which indicates the systems lose partial performance during the operation.
2) Total Loss of Effectiveness (TLOE): $u_{ci}(t) = u_{TFi}$, $\rho_i = 0$, which implies that actuator output $u_{ci}(t)$ no longer affected by $u_{PFi}(t)$, but by u_{TFi}.

Considering the above 2 patterns of actuator failures, the system model (1) can be written as

$$\begin{cases} dx_j &= \left[x_{j+1} + f_j(\bar{x}_j)\right]dt + \psi_j^T(\bar{x}_j)dv \\ j &= 1, 2, \cdots, n-1 \\ dx_n &= [\sum\limits_{i=1}^{m} b_i\beta_i(x)(\rho_i u_{PFi}(t) + (1-\rho_i)u_{TFi}(t)) + f_n(\bar{x}_n)]dt + \psi_n^T(\bar{x}_n)dv \\ y &= x_1 \end{cases} \tag{3}$$

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Generally, the system model's communication resource between controller and actuator assumed to be sufficient, but it is limited. Due to save network communication resource, the event-triggered scheme has better features to solve the problem. Thus, the event-triggered scheme between the ith actuator and the ith controller is considered, which can be modeled as

$$\begin{cases} u_{PFi}(t) = w_i(t_k), \forall t \in [t_k, t_{k+1}) \\ t_{k+1} = \inf\{t \in R, |w_i(t) - u_{PFi}(t)| \geq f_E(u_{PFi}(t))\} \end{cases} \tag{4}$$

where $t_k, k \in R^+$ is the controller update time. $w_i(t_k)$ and $f_E(\cdot)$ are designed functions. Their detailed design procedures are presented next section. During the time $t \in [t_k, t_{k+1})$, the value of $u_{PFi}(t)$ holds as a constant $w_i(t_k)$. Whenever (4) is triggered, the time t_k will be marked as t_{k+1}, and the control value $u_{PFi}(t) = w(t_{k+1})$ will be applied to system.

For deriving suitable adaptive controllers, the following assumptions are made.

Assumption 1. $\beta_i(x) \neq 0, b_i \neq 0$, and the direct of b_i is known.

Assumption 2. The max quantity of total failed actuators is $m-1$, but partial failed actuators' quantity can be up to m. And the remaining actuation power can achieve the control objectives.

Let $T_0 = 0$, suppose that $m_j \left(0 \leq m_j \leq m\right)$ actuators are failure and no new faults occur in time interval $[T_j, T_{j+1})$, $j = 0, 1, ..., F$. Clearly, all actuators work normally in time interval $[T_0, T_1)$ and new failure will occur after time instant T_1. From Assumption 2, T_F is finite and T_{F+1} is infinite. Let the set Q_{TTj} denote the actuators of total failure in interval $[T_j, T_{j+1})$, $j = 0, 1, \cdots, F$ and the set Q_{PTj} represents other not total failure actuators. It is clear that $Q_{TTj} \cup Q_{PTj} = \{1, 2, \cdots, m\}$.

2.2. Stochastic systems stability

A class of stochastic nonlinear systems, which are considered in this paper, can be transformed into the following form

$$dx = \aleph(x,t)dt + H(x,t)dv \tag{5}$$

where $x \in \mathrm{R}^{\mathrm{n}}$ stands for the system states, $\aleph \in R^n$, $H \in R^{n\times r}$ are locally Lipschitz functions, v is denoted as (1).

Definition 1. (see in [55]): Define the operator LV as

$$LV = \frac{\partial V}{\partial t} + \frac{\partial V}{\partial x}\aleph + \frac{1}{2}T_r\{H^T \frac{\partial^2 V}{\partial x^2} H\} \tag{6}$$

where T_r represents the matrix trace.

Lemma 1. *(Young's Inequality, see in [56]): For $\forall (x_1, x_2) \in R^2$, the following inequality holds:*

$$x_1 x_2 \leq \frac{\varepsilon^\flat}{\flat}|x_1|^\flat + \frac{1}{\ell\, \varepsilon^\ell}|x_2|^\ell \tag{7}$$

where $\varepsilon > 0, \flat > 1, \ell > 1$, and $(\flat - 1)(\ell - 1) = 1$.

Lemma 2. *(see in [57]): Consider a dynamic system as*

$$\dot{\hat{\Phi}}(t) = -\gamma\hat{\Phi}(t) + \bar{k}\Upsilon(t) \tag{8}$$

where $\gamma > 0$ and $\bar{k} > 0$ and $\Upsilon(t)$ is a positive function. If $\hat{\Phi}(t_0) \geq 0$ is given bounded initial condition, $\hat{\Phi}(t) \geq 0$ for $\forall t \geq t_0$.

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Lemma 3. *(see in [43]): Suppose that there exist a function $V(x,t) \in C^{2.1}$, two positive constants c and b, k_∞-functions α_1 and α_2 such that*

$$\begin{cases} \alpha_1(\|x\|) \le V(x,t) \le \alpha_2(\|x\|) \\ LV \le -cV(x,t) + b \end{cases} \tag{9}$$

for $\forall x \in R^n$ and $\forall t > 0$. Then, there exists an unique strong solution of (5) for each $x_0 \in R^n$ and the system is bounded in probability.

2.3. Approximate function research

In this subsection, we will introduce that a continuous function $Y(\cdot)$ defined on some compact set Ω is approximated by a Fuzzy Logic System (FLS) [19,18,20]. See in [58], it can be got that

f_i^τ and g^l are fuzzy sets in R and N is the number of the rules. μ represents the membership function. The output of the fuzzy system is

$$Y(X) = \frac{\Sigma_{l=1}^{N} \Phi_l \Pi_{i=1}^{n} \mu_{f_i^l}(x_i)}{\Sigma_{l=1}^{N} \left[\Pi_{i=1}^{n} \mu_{f_i^l}(x_i)\right]} \tag{10}$$

where $\Phi_l = \max_{y \in R} \mu_{g^l}(y)$, $\Phi = (\Phi_1, \Phi_2, ..., \Phi_N)^T$, $Y \in R$ and $X = [x_1, x_2, \cdots, x_n]^T \in R^n$ are the output and input of the fuzzy system respectively.

Let $\xi_l(X) = \frac{\Pi_{i=1}^{n} \mu_{f_i^l}(x_i)}{\Sigma_{l=i}^{N}\left[\Pi_{i=1}^{n} \mu_{f_i^l}(x_i)\right]}$, and $\xi(x) = (\xi_1(X), \xi_2(X), \cdots, \xi_N(X))^T$.

Then, the unknown function can be written as

$$Y(X) = \Phi^T \xi(X) \tag{11}$$

Lemma 4. *(see in [58]): Let $Y(X)$ be a continuous function. Then, for $\forall \varepsilon > 0$, there exists a FLS such that*

$$\sup_{X \in \Omega} \left| Y(X) - \Phi^T \xi(X) \right| \le \varepsilon \tag{12}$$

The objective of this paper is to obtain a control scheme which is proposed to guarantee transient specifications when taking into account actuator failures and saving network communication resource simultaneously for uncertain stochastic nonlinear systems. To establish the control scheme, the following assumption is made.

Assumption 3. The nth order time derivatives of the reference output signal $y_r(t)$ are continuous and bounded.

3. Fuzzy adaptive controllers design

The objective in this paper is to design an event-triggered fuzzy adaptive compensation controller for considered system with actuator failures and given transient performance. To ensure that the tracking error $e(t) = y(t) - y_r(t)$ is preserved within a bound, it is necessary to propose new design methods based on backstepping technique, which involves n recursive steps.

3.1. Transformed system for given transient specification

To ensure the transient performance no matter actuator failures occur or not, it is necessary to choose firstly a performance function $\eta(t): R_+ \to R_+ \backslash \{0\}$ with $\lim_{t \to \infty} \eta(t) = \eta_\infty > 0$, which is a monotonous decreasing smooth function. Then, if the condition holds that:

$$-\delta_1 \eta(t) < e(t) < \delta_2 \eta(t), \forall t \ge 0 \tag{13}$$

where $0 < \delta_1, \delta_2 \le 1$ could be satisfied, it would be safely achieved the objective of given transient specification.

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From (13), we could achieve that:

(1) The decreasing function $\eta(t)$ introduces that $e(t)$ is convergence within a lower bound.

(2) When considered system suffers actuator failures during operation, unacceptable large overshooting will be no occur. It indicates that system has given transient specification. For instance, when $\eta(t)$ approaches η_∞ closely enough, an actuator failure occurs, $-\delta_1(\eta_\infty+\varepsilon_1) < e(t) < \delta_2(\eta_\infty+\varepsilon_1)$ will be satisfied, where $\varepsilon_1 > 0$ is sufficiently small.

To satisfying the "constrained" error condition, firstly, a strictly increasing function $G(p)$ is designed as the following properties:

$$-\delta_1 < G(p) < \delta_2 \tag{14}$$

$$\lim_{p\to+\infty} G(p) = \delta_2, \lim_{p\to-\infty} G(p) = -\delta_1 \tag{15}$$

$$G(0) = 0 \tag{16}$$

From properties (14)–(16) of $G(p)$, equation (13) can be expressed as follows

$$e(t) = \eta(t)\, G(p) \tag{17}$$

Since function $G(p)$ is the strict monotonic increasing function and $\eta(t) \neq 0$, there exists the function

$$p = G^{-1}\left(\frac{e(t)}{\eta(t)}\right) \tag{18}$$

If $-\delta_1\eta(0) < e(0) < \delta_2\eta(0)$ and designed controller ensures p bounded for $t \geq 0$, it is easy to achieve $-\delta_1 < \frac{e(t)}{\eta(t)} < \delta_2$. Furthermore, if $\lim\limits_{t\to\infty} p = 0$ is followed, the target of $\lim\limits_{t\to\infty} e(t) = 0$ can be achieved, which indicates that the tracking error is preserved within a given specification.

In this paper, we design function $G(p)$ as

$$G(p) = \frac{\delta_2 e^{(p+p_1)} - \delta_1 e^{-(p+p_1)}}{e^{(p+p_1)} + e^{-(p+p_1)}} \tag{19}$$

where $p_1 = \frac{1}{2}(\ln\delta_1 - \ln\delta_2)$. It can be easily shown that $G(p)$ has the properties (14)–(16). The function p is solved as

$$p = G^{-1}(\sigma(t)) = \frac{1}{2}\ln(\delta_2\sigma(t) + \delta_1\delta_2) - \frac{1}{2}\ln(\delta_1\delta_2 - \delta_1\sigma(t)) \tag{20}$$

where $\sigma(t) = e(t)\big/\eta(t)$.

We compute $\dot{p}$ as

$$\begin{aligned}\dot{p} &= \frac{\partial G^{-1}}{\partial\sigma}\dot{\sigma} = \frac{1}{2}\left[\frac{1}{\sigma+\delta_1} - \frac{1}{\sigma-\delta_2}\right]\left(\frac{\dot{e}}{\eta} - \frac{e\dot{\eta}}{\eta^2}\right) \\ &= \varsigma\left(\dot{e} - \frac{e\dot{\eta}}{\eta}\right) = \varsigma\left(\dot{y} - \dot{y}_r - \frac{e\dot{\eta}}{\eta}\right)\end{aligned} \tag{21}$$

where ς is defined as

$$\varsigma = \frac{1}{2\eta}\left[\frac{1}{\sigma+\delta_1} - \frac{1}{\sigma-\delta_2}\right] \tag{22}$$

Due to the property (14) and (17), ς is well defined and $\varsigma \neq 0$. The original nonlinear system (1) can be rewritten as

$$dp = \varsigma\left(x_2 + f_1 - \dot{y}_r - \frac{e\dot{\eta}}{\eta}\right)dt + {\psi_1}^T(\bar{x}_1)dv \tag{23}$$

$$dx_j = \left[x_{j+1} + f_j(\bar{x}_j)\right]dt + \psi_j^T(\bar{x}_j)dv,\ j = 2, 3, \cdots, n-1 \tag{24}$$

$$dx_n = [\sum_{i=1}^m b_i\beta_i u_i + f_n(\bar{x}_n)]dt + \psi_n^T(\bar{x}_n)dv \tag{25}$$

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3.2. Fuzzy adaptive controllers analysis and design

In this subsection, we elaborate the details of the backstepping design procedure which contains n steps.

Remark 1. It is a challenge to design a controller for stochastic systems, which has the limitation of communication resource with actuator failures and given transient performance. Compared with [34–43,47–50], it is designed a transformed system to achieve that the tracking error is preserved within a given bound.

Firstly, we define

$$z_1 = p \tag{26}$$

$$z_j = x_j - \alpha_{j-1} - y_r^{(j-1)},\ j = 2, 3, \cdots, n \tag{27}$$

where α_{j-1} is a virtual controller. In order to elaborate more clearly for each step, it is defined a vector function as $\bar{y}_r^{(i)} \doteq \left[y_r, y_r^{(1)}, ..., y_r^{(i)}\right]^T, i = 1, 2, ...n$, where $y_r^{(i)}$ denotes as the ith time derivative of y_r.

Step 1: From (27), obviously, $z_2 = x_2 - \alpha_1 - \dot{y}_r$. And noting from (26),

$$dz_1 = dp = \varsigma\left(x_2 + f_1 - \dot{y}_r - \frac{e\dot{\eta}}{\eta}\right)dt + {\psi_1}^T(\bar{x}_1)dv \tag{28}$$

To stabilize (28), it is chosen a stochastic Lyapunov function V_1 as

$$V_1 = \frac{z_1^4}{4} + \frac{\tilde{\vartheta}_1^2}{2\lambda_1} \tag{29}$$

The design constant $\lambda_1 > 0$ and the parameter error $\tilde{\vartheta}_1 \doteq \vartheta_1 - \hat{\vartheta}_1$, where $\vartheta_1 \doteq \|\Phi_1\|^2$ and $\hat{\vartheta}_1$ is the estimate of ϑ_1. By (5), (26), and (27), we have

$$LV_1 = z_1^3\varsigma(z_2 + \alpha_1 + f_1 - \frac{e\dot{\eta}}{\eta}) + \frac{3}{2}z_1^2\Psi_1^T\Psi_1 - \frac{\tilde{\vartheta}_1\dot{\hat{\vartheta}}_1}{\lambda_1} \tag{30}$$

Applying Lemma 1, the following inequalities can be obtained as

$$\frac{3}{2}z_1^2\Psi_1^T\Psi_1 \le \frac{3}{4}\tau_1^{-2}z_1^4\|\Psi_1\|^4 + \frac{3}{4}\tau_1^2 \tag{31}$$

$$z_1^3z_2 \le \frac{3}{4}z_1^4 + \frac{1}{4}z_2^4 \tag{32}$$

where $\tau_1 > 0$. Substituting (31) and (32) into (30) yields

$$LV_1 \le z_1^3\varsigma\left(\frac{3z_1}{4} + \alpha_1 + f_1 - \frac{e\dot{\eta}}{\eta} + \frac{3}{4}\tau_1^{-2}z_1\|\Psi_1\|^4\right) + \frac{\varsigma z_2^4}{4} + \frac{3}{4}\tau_1^2 - \frac{\tilde{\vartheta}_1\dot{\hat{\vartheta}}_1}{\lambda_1} \tag{33}$$

Define a new function $\bar{f}_1 \doteq \varsigma f_1 + (3/4)\varsigma z_1 + (3/4)\tau_1^{-2}\varsigma z_1\|\Psi_1\|^4 - \varsigma\frac{e\dot{\eta}}{\eta}$, then (33) can be rewritten as

$$LV_1 \le z_1^3(\varsigma\alpha_1 + \bar{f}_1) + \frac{3}{4}\tau_1^2 + \frac{1}{4}\varsigma z_2^4 - \frac{\tilde{\vartheta}_1}{\lambda_1}\dot{\hat{\vartheta}}_1 \tag{34}$$

According to Lemma 4,

$$\overline{f}_1 = \Phi_1^T\xi_1(X_1) + \delta_1(X_1), |\delta_1(X_1)| \le \varepsilon_1 \tag{35}$$

where $X_1 \doteq (x_1, y_r, \eta, \dot{\eta}, \varsigma)^T$.

Remark 2. In each step, a FLS is utilized to approximate the function $\bar{f}_i$. It needs too many updated laws to estimate parameters for FLS. It is difficulty for practical applications. So it is defined a constant $\vartheta_i \doteq \|\Phi_i\|^2, i = 1, 2, \cdots, n$. The number of the adaptive parameters for FLS are reduced significantly.

According to Lemma 1,

$$z_1^3\overline{f}_1 \leq \frac{1}{2a_1^2} z_1^6 \vartheta_1 \xi_1^T \xi_1 + \frac{1}{2}a_1^2 + \frac{3}{4}z_1^4 + \frac{1}{4}\varepsilon_1^4 \tag{36}$$

The virtual controller α_1 is designed as

$$\alpha_1 = -\left(\frac{c_1}{\varsigma} + \frac{3}{4\varsigma}\right) z_1 - \frac{1}{2\varsigma a_1^2} z_1^3 \hat{\vartheta}_1 \xi_1^T \xi_1 \tag{37}$$

where $c_1 > 0$ and $a_1 > 0$.

Remark 3. From (37), it can be designed different ς to improve the transient specifications by adjusting the design parameters of function $\eta(t)$ and the scalars δ_1, δ_2.

The adaptive law is taken as

$$\dot{\hat{\vartheta}}_1 = \frac{\lambda_1}{2a_1^2} z_1^6 \xi_1^T \xi_1 - \gamma_1 \hat{\vartheta}_1, \hat{\vartheta}_1(0) \geq 0 \tag{38}$$

where $\gamma_1 > 0$.

Substituting (35)–(38) into (34), we have

$$LV_1 \leq -c_1 z_1^4 + \frac{\gamma_1}{\lambda_1}\tilde{\vartheta}_1\hat{\vartheta}_1 + \frac{\varsigma z_2^4}{4} + \frac{3}{4}\tau_1^2 + \frac{1}{2}a_1^2 + \frac{1}{4}\varepsilon_1^4 \tag{39}$$

It is noted that

$$\frac{\gamma_1}{\lambda_1}\tilde{\vartheta}_1\hat{\vartheta}_1 \leq -\frac{\gamma_1}{2\lambda_1}\tilde{\vartheta}_1^2 + \frac{\gamma_1}{2\lambda_1}\vartheta_1^2 \tag{40}$$

Equation (39) can be rewritten as

$$LV_1 \leq -c_1 z_1^4 - \frac{\gamma_1}{2\lambda_1}\tilde{\vartheta}_1^2 + e_1 + \frac{\varsigma z_2^4}{4} \tag{41}$$

where $e_1 \doteq \frac{3}{4}\tau_1^2 + \frac{1}{2}a_1^2 + \frac{1}{4}\varepsilon_1^4 + \frac{\gamma_1}{2\lambda_1}\vartheta_1^2$, and the term $\frac{\varsigma z_2^4}{4}$ will be handled in ***Step 2***.

Step 2: Noting that $z_2 = x_2 - \alpha_1 - y_r^{(1)}$, it could be obtained that

$$dz_2 = (z_3 + f_2 + \alpha_2 - L\alpha_1)dt + \left(\Psi_2 - \frac{\partial \alpha_1}{\partial x_1}\Psi_1\right)^T dv \tag{42}$$

Then choose a stochastic Lyapunov function V_2 as

$$V_2 = V_1 + \frac{z_2^4}{4} + \frac{\tilde{\vartheta}_2^2}{2\lambda_2} \tag{43}$$

where $\lambda_2 > 0$ and $\tilde{\vartheta}_2 \doteq \vartheta_2 - \hat{\vartheta}_2$. From (6), (42), and (43), we have that

$$LV_2 = LV_1 + z_2^3(z_3 + \alpha_2 + f_2 - L\alpha_1) + \frac{3}{2}z_2^2\left(\Psi_2 - \frac{\partial \alpha_1}{\partial x_1}\Psi_1\right)^T\left(\Psi_2 - \frac{\partial \alpha_1}{\partial x_1}\Psi_1\right) - \frac{\tilde{\vartheta}_2\dot{\hat{\vartheta}}_2}{\lambda_2} \tag{44}$$

where

$$L\alpha_1 = \frac{\partial \alpha_1}{\partial x_1}[f_1 + x_2] + \frac{\partial \alpha_1}{\partial \hat{\vartheta}_1}\dot{\hat{\vartheta}}_1 + \sum_{j=0}^{1}\frac{\partial \alpha_1}{\partial y_r^{(j)}} y_r^{(j+1)} + \frac{1}{2}\frac{\partial^2 \alpha_1}{\partial x_1^2}\Psi_1^T\Psi_1 + \sum_{j=0}^{1}\frac{\partial \alpha_1}{\partial \eta^{(j)}}\eta^{(j+1)} \tag{45}$$

By using Lemma 1, the following inequalities get:

$$\frac{3}{2}z_2^2\left\|\Psi_2 - \frac{\partial \alpha_1}{\partial x_1}\Psi_1\right\|^2 \leq \frac{3}{4}\tau_2^{-2}z_2^4\left\|\Psi_2 - \frac{\partial \alpha_1}{\partial x_1}\Psi_1\right\|^4 + \frac{3}{4}\tau_2^2 \tag{46}$$

$$z_2^3 z_3 \le \frac{3}{4} z_2^4 + \frac{z_3^4}{4} \tag{47}$$

where $\tau_2 > 0$. $\bar{f}_2$ is defined as $\bar{f}_2 \doteq f_2 - L\alpha_1 + \frac{3}{4}\tau_2^{-2} z_2 \left\| \Psi_2 - \frac{\partial \alpha_1}{\partial x_1}\Psi_1 \right\|^4 + z_2(\frac{3}{4} + \frac{1}{4}\varsigma)$.

From above, it can be achieved the following result:

$$LV_2 \le -c_1 z_1^4 - \frac{\gamma_1}{2\lambda_1}\tilde{\vartheta}_1^2 + e_1 + \frac{3}{4}\tau_2^2 + z_2^3 \bar{f}_2 + z_2^3 \alpha_2 + \frac{1}{4} z_3^4 - \frac{1}{\lambda_2}\tilde{\vartheta}_2 \dot{\hat{\vartheta}}_2 \tag{48}$$

Similarly, a FLS $\Phi_2^T \xi_2 (X_2)$ is utilized to approximate $\bar{f}_2$, where $X_2 \doteq \left[\bar{x}_2^T, \bar{\hat{\vartheta}}_1^T, \bar{y}_r^{(1)T}, \dot{\eta}, \ddot{\eta} \right]^T \in \Omega_{z_2}$ with $\bar{\hat{\vartheta}}_1 \doteq \hat{\vartheta}_1$. According to Lemma 4, $\bar{f}_2$ can be expressed as

$$\bar{f}_2 = \Phi_2^T \xi_2 (X_2) + \delta_2 (X_2), |\delta_2 (X_2)| \le \varepsilon_2 \tag{49}$$

where $\forall \varepsilon_2 > 0$. Furthermore, it can be obtained:

$$z_2^3 \bar{f}_2 \le \frac{1}{2a_2^2} z_2^6 \vartheta_2 \xi_2^T \xi_2 + \frac{1}{2} a_2^2 + \frac{3}{4} z_2^4 + \frac{1}{4}\varepsilon_2^4 \tag{50}$$

The virtual controller is designed as

$$\alpha_2 = -\left(c_2 + \frac{3}{4} \right) z_2 - \frac{1}{2a_2^2} \hat{\vartheta}_2 z_2^3 \xi_2^T \xi_2 \tag{51}$$

where $c_2 > 0$ and $a_2 > 0$.

The adaptive law $\dot{\hat{\vartheta}}_2$ is chosen as

$$\dot{\hat{\vartheta}}_2 = \frac{\lambda_2}{2a_2^2} z_2^6 \xi_2^T \xi_2 - \gamma_2 \hat{\vartheta}_2, \hat{\vartheta}_2 (0) \ge 0 \tag{52}$$

where $\gamma_2 > 0 \in R$. Similarly, it can be achieved the following inequality:

$$\frac{\gamma_2}{\lambda_2}\tilde{\vartheta}_2 \hat{\vartheta}_2 \le -\frac{\gamma_2}{2\lambda_2}\tilde{\vartheta}_2^2 + \frac{\gamma_2}{2\lambda_2}\vartheta_2^2 \tag{53}$$

Substituting (50)–(53) into (48) yields

$$LV_2 \le -\sum_{j=1}^{2} \left(c_j z_j^4 + \frac{\gamma_j}{2\lambda_j}\tilde{\vartheta}_j^2 \right) + \sum_{j=1}^{2} e_j + \frac{1}{4} z_3^4 \tag{54}$$

where $e_2 \doteq (\gamma_2/2\lambda_2)\,\vartheta_2^2 + (1/2)\,a_2^2 + (3/4)\,\tau_2^2 + (1/4)\,\varepsilon_2^4$.

Step i $(2 < i \le n-1)$**:** From (27), $z_j = x_j - \alpha_{j-1} - y_r^{(j-1)}$, it can be achieved

$$dz_i = \left(x_{i+1} + f_i - L\alpha_{i-1} - y_r^{(i)} \right) dt + \left(\psi_i - \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_j}\psi_j \right)^T dv \tag{55}$$

where

$$L\alpha_{i-1} = \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_j} \left[f_j + x_{j+1} \right] + \sum_{j=1}^{i-1} \dot{\hat{\vartheta}}_j \frac{\partial \alpha_{i-1}}{\partial \hat{\vartheta}_j} + \sum_{j=0}^{i-1} y_r^{(j+1)} \frac{\partial \alpha_{i-1}}{\partial y_r^{(j)}} + \frac{1}{2} \sum_{p,q=1}^{i-1} \frac{\partial^2 \alpha_{i-1}}{\partial x_p \partial x_q} \Psi_p^T \Psi_q.$$

Choose a stochastic Lyapunov function V_i as

$$V_i = V_{i-1} + \frac{1}{4} z_i^4 + \frac{1}{2\lambda_i}\tilde{\vartheta}_i^2 \tag{56}$$

where $\forall \lambda_i > 0$ and $\tilde{\vartheta}_i \doteq \vartheta_i - \hat{\vartheta}_i$. And then, straightforward derivation gives the following result:

$$LV_i = LV_{i-1} + z_i^3\left(z_{i+1}+\alpha_i+f_i-L\alpha_{i-1}\right)+\frac{3}{2}z_i^2\left(\Psi_i-\sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}\Psi_j\right)^T\left(\Psi_i-\sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}\Psi_j\right)$$
$$-\frac{1}{\lambda_i}\tilde{\vartheta}_i\dot{\hat{\vartheta}}_i \tag{57}$$

where $\forall\tau_i>0$. It can be arrived the following result:

$$LV_i \le -\sum_{j=1}^{i-1}\left(c_jz_j^4+\frac{\gamma_j}{2\lambda_j}\tilde{\vartheta}_j^2\right)+\sum_{j=1}^{i-1}e_j+\frac{3}{4}\tau_i^2+z_i^3\bar{f}_i+z_i^4+z_i^3\alpha_i+\frac{1}{4}z_{i+1}^4-\frac{1}{\lambda_i}\tilde{\vartheta}_i\dot{\hat{\vartheta}}_i \tag{58}$$

where $\bar{f}_i$ is alternated as

$\bar{f}_i \doteq f_i - L\alpha_{i-1}+\frac{3}{4}z_i\left\|\left(\psi_i-\sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}\psi_j\right)\right\|^4\tau_i^{-2}$, and $\forall c_j>0$.

Similarly, a FLS $\Phi_i^T\xi_i(X_i)$ is utilized to approximate $\bar{f}_i$, where $X_i \doteq \left[\bar{x}_i^T,\bar{\hat{\vartheta}}_{i-1}^T,\bar{y}_r^{(i-1)T}\right]^T \in \Omega_{z_i}\subset R^{3i}$ with $\bar{\hat{\vartheta}}_{i-1}\doteq\left[\hat{\vartheta}_1,\hat{\vartheta}_2,...,\hat{\vartheta}_{i-1}\right]^T$.

$\bar{f}_i$ can be expressed as

$$\overline{f}_i=\Phi_i^T\xi_i(X_i)+\delta_i(X_i),\ |\delta_i(X_i)|\le\varepsilon_i \tag{59}$$

where $\varepsilon_i>0$. Furthermore, applying Lemma 1, it can be obtained that

$$z_i^3\overline{f}_i\le\frac{1}{2a_i^2}z_i^6\vartheta_i\xi_i^T\xi_i+\frac{1}{2}a_i^2+\frac{3}{4}z_i^4+\frac{1}{4}\varepsilon_i^4. \tag{60}$$

The virtual controller α_i is designed as

$$\alpha_i=-\left(c_i+\frac{7}{4}\right)z_i-\frac{1}{2a_i^2}z_i^3\hat{\vartheta}_i\xi_i^T\xi_i \tag{61}$$

where $\forall c_i>0$ and $\forall a_i>0$.

The adaptive law $\dot{\hat{\vartheta}}_i$ is designed as

$$\dot{\hat{\vartheta}}_i=\frac{\lambda_i}{2a_i^2}z_i^6\xi_i^T\xi_i-\gamma_i\hat{\vartheta}_i,\ \hat{\vartheta}_i(0)\ge0 \tag{62}$$

where $\forall\gamma_i>0$.

Similarly, it can be obtained that

$$\frac{\gamma_i}{\lambda_i}\tilde{\vartheta}_i\hat{\vartheta}_i\le-\frac{\gamma_i}{2\lambda_i}\tilde{\vartheta}_i^2+\frac{\gamma_i}{2\lambda_i}\vartheta_i^2. \tag{63}$$

Substituting (60)–(63) into (58) yields

$$LV_i\le-\sum_{j=1}^{i}\left(c_jz_j^4+\frac{\gamma_j}{2\lambda_j}\tilde{\vartheta}_j^2\right)+\sum_{j=1}^{i}e_j+\frac{1}{4}z_{i+1}^4 \tag{64}$$

where $e_j\doteq\left(\gamma_j/2\lambda_j\right)\vartheta_j^2+(1/2)a_j^2+(3/4)\tau_j^2+(1/4)\varepsilon_j^4$, $j=3,4,...,i$.

Step n: The detailed design procedure for adaptive scheme of controllers and updating strategy of saving network communication resource is presented next subsection. Due to demonstration, it can be arrived the result as

$$LV_{nj}\le-\sum_{i=1}^{n}\left(c_iz_i^4+\frac{\gamma_i}{2\lambda_i}\tilde{\vartheta}_i^2\right)-\sum_{i\in Q_{PTj}}\frac{|b_i|\rho_i\gamma_k}{2}\tilde{k}^T\Gamma_k{}^{-1}\tilde{k}+\sum_{i=1}^{n}e_i \tag{65}$$

3.3. Event-triggered controller analysis and design

Nowadays, the safety and reliability for networked control system (NCS) is a hot topic. However, how to saving communication resource, especially for uncertain stochastic nonlinear systems with actuator failures, is an open issue both in theory and in practice. Based on this consideration, we assume that the network between controller and actuator has not network-induced delay and packet dropout. The function $w_i(t)$ and $f_E(\cdot)$ in equation (4) are established as follows.

$$\begin{cases} w_i(t) = -(1+\delta)(\bar{u}_{ci} tanh(\frac{z_n^3 sgn(b_i)\beta_i(x)\bar{u}_{ci}}{\varepsilon}) + \bar{m} tanh(\frac{z_n^3 sgn(b_i)\beta_i(x)\bar{m}}{\varepsilon})) \\ u_{PFi}(t) = w_i(t_k), \forall t \in [t_k, t_{k+1}) \\ e(t) = w_i(t) - u_{PFi}(t) \\ t_{k+1} = \inf\{t \in R \,|\, |e(t)| \geq \delta |u_{PFi}(t)| + m_1\}, \delta > 0 \end{cases} \tag{66}$$

where $t_k, k \in R^+, \varepsilon, 0 < \delta < 1, m_1, \bar{m} > \frac{m_1}{1-\delta}$ are all positive design parameters.

Owing to (2) and (27), it can be obtained that

$$dz_n = (\sum_{i=1}^{m} b_i \beta_i(x)(\rho_i u_{PFi}(t) + u_{TFi}) + f_n - L\alpha_{n-1} - y_r^{(n)})dt + \left(\psi_n - \sum_{j=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial x_j}\psi_j\right)^T dv \tag{67}$$

where

$$L\alpha_{n-1} = \sum_{j=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial x_j}\left[f_j(\bar{x}_j) + x_{j+1}\right] + \sum_{j=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial \hat{\vartheta}_j}\dot{\hat{\vartheta}}_j + \sum_{j=0}^{n-1} \frac{\partial \alpha_{n-1}}{\partial y_d^{(j)}} y_d^{(j+1)} + \frac{1}{2}\sum_{p,q=1}^{n-1} \frac{\partial^2 \alpha_{n-1}}{\partial x_p \partial x_q}\Psi_p^T \Psi_q. \tag{68}$$

From (66), we safely have $w_i(t) = (1+\lambda_1(t)\delta)u_{PFi}(t) + \lambda_2(t)m_1$ in the interval $[t_k, t_{k+1})$, where $\lambda_1(t)$ and $\lambda_2(t)$ are time-varying parameters satisfying $|\lambda_1(t)| \leq 1$ and $|\lambda_2(t)| \leq 1$. Thus we obtain $u_{PFi}(t) = \frac{w_i(t) - \lambda_2(t)m_1}{1+\lambda_1(t)\delta}$.

Now, we make some analysis on the control law $\bar{u}_{ci}$. The control law can be designed as

$$\bar{u}_{ci} = sgn(b_i)\frac{1}{\beta_i(x)}k_j^T \bar{w},\ i = 1, 2, \ldots, m \tag{69}$$

where k_j and $\bar{w}$ are denoted in interval $[T_j, T_{j+1})$ as

$$k_j = (k_{j,1}, k_{j,21}, \ldots, k_{j,2m})^T, \bar{w} = (\bar{w}_1, \bar{w}_{21}, \ldots, \bar{w}_{2m})^T \tag{70}$$

where $\bar{w}_{2i} = \beta_i(x), i \in 1, 2, \cdots, m$.

In the interval $[T_j, T_{j+1})$, it is satisfied that

$$\sum_{i \in Q_{PTj}} |b_i|\rho_i {k_j}^T \bar{w} = \alpha_n - \sum_{i \in Q_{TTj}} b_i \beta_i(x) u_{TFi} \tag{71}$$

when k_j and $\bar{w}$ are chosen as

$$\begin{cases} k_{j,1} = \frac{1}{\sum\limits_{i \in Q_{PTj}} |b_i|\rho_i}, k_{j,2i} = \frac{-b_i u_{TFi}}{\sum\limits_{i \in Q_{PTj}} |b_i|\rho_i},\ \text{for } i \in Q_{TTj}, \\ k_{j,2i} = 0,\ \text{for } i \in Q_{PTj}; \\ \bar{w}_1 = \alpha_n, \bar{w}_{2i} = \beta_i(x),\ \text{for } i \in 1, 2, \cdots, m \end{cases} \tag{72}$$

Owing to the unknown stochastic system parameters and actuator failure mode, k_j is can not know in advance and they are needed to be estimated. So k_j is replaced by its estimate $\hat{k}$ for all $j = 0, 1, \ldots, \ell$.

The control law is rewritten as

$$\bar{u}_{ci} = sgn(b_i)\frac{1}{\beta_i(x)}\hat{k}^T \bar{w} \tag{73}$$

Remark 4. It is worth mentioning that some algorithms (see in [26–29]), for addressing the problem of redundant data transmissions for nonlinear systems without actuator failures. In general, for compensating the actuator failures and

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saving communication resource simultaneously, it would result in the increasing difficulty and challenge of controller analysis and design. Note that, an adaptive controller based on event-triggered control mechanism is established as above.

In the interval $[T_j, T_{j+1})$, we define $d = (0.557\varepsilon \sum_{i=1}^{m} \rho_i |b_i|)^2$, $\hat{d}$ is the estimate of d and define a positive definite stochastic Lyapunov function as follows:

$$V_{nj} = V_{n-1} + \frac{1}{4} z_n^4 + \frac{1}{2\lambda_n} \tilde{\vartheta}_n^2 + + \frac{1}{2\lambda_d} \tilde{d}^2 + \sum_{i \in Q_{PTj}} \frac{|b_i| \rho_i}{2\lambda_n} \tilde{k}^T \Gamma_k{}^{-1} \tilde{k} \tag{74}$$

where $\lambda_n > 0$, $\lambda_d > 0$, Γ_k is a non-singular positive definite matrix, $\tilde{\vartheta}_n \doteq \vartheta_n - \hat{\vartheta}_n$, $\tilde{d} \doteq d - \hat{d}$, and $\tilde{k} \doteq k_j - \hat{k}$.

From (6) and (67), we have

$$\begin{aligned} LV_{nj} &= LV_{n-1} + z_n^3 \Big(\sum_{i=1}^{m} b_i \beta_i(x) \left(\rho_i u_{PFi} + u_{TFi}\right) + f_n - L\alpha_{n-1}\Big) \\ &\quad + \frac{3}{2} z_n^2 \left\| \psi_n - \sum_{j=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial x_j} \psi_j \right\|^2 - \frac{1}{\lambda_n} \tilde{\vartheta}_n \dot{\hat{\vartheta}}_n - \frac{1}{\lambda_d} \tilde{d} \dot{\hat{d}} - \sum_{i=1, i \in Q_{PTj}}^{m} \frac{\rho_i |b_i|}{\lambda_n} \tilde{k}^T \Gamma_k^{-1} \dot{\hat{k}} - z_n^3 y_r^{(n)} \end{aligned} \tag{75}$$

By Young's inequality, it can be got the following inequality:

$$\begin{aligned} &\frac{3}{2} z_n^2 \left\| \psi_n - \sum_{j=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial x_j} \psi_j \right\|^2 \\ &\quad \le \frac{3}{4} \tau_n^2 + \frac{3}{4} \tau_n^{-2} z_n^4 \left\| \psi_n - \sum_{j=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial x_j} \psi_j \right\|^4 \end{aligned} \tag{76}$$

where $\tau_n > 0$.

And $\bar{f}_n$ is defined as

$$\bar{f}_n \doteq f_n - L\alpha_{n-1} + \frac{3}{4} \tau_n^{-2} z_n \left\| \psi_n - \sum_{j=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial x_j} \psi_j \right\|^4 \tag{77}$$

Similarly, $\bar{f}_n$ can be expressed as

$$\bar{f}_n = \Phi_n^T \xi_n (X_n) + \delta_n(X_n), |\delta_n(X_n)| \le \varepsilon_n \tag{78}$$

Thus

$$\begin{aligned} LV_{nj} &\le -\sum_{i=1}^{n-1} \left(c_i z_i^4 + \frac{\gamma_i}{2\lambda_i} \tilde{\vartheta}_i^2 \right) + \sum_{i=1}^{n-1} e_i + \frac{1}{4} z_n^4 + \frac{3}{4} \tau_n^2 + z_n^3 \left(\sum_{i=1}^{m} b_i \beta_i \left(\rho_i u_{PFi} + u_{TFi}\right) + \bar{f}_n \right) - \frac{1}{\lambda_n} \tilde{\vartheta}_n \dot{\hat{\vartheta}}_n \\ &\quad - \frac{1}{\lambda_d} \tilde{d} \dot{\hat{d}} - \sum_{i=1, i \in Q_{PTj}}^{m} \rho_i |b_i| \tilde{k}^T \Gamma_k^{-1} \dot{\hat{k}} - z_n^3 y_r^{(n)} \end{aligned} \tag{79}$$

Using Lemma 1, one has

$$z_n^3 \bar{f}_n \le \frac{1}{2a_n^2} z_n^6 \vartheta_n \xi_n^T \xi_n + \frac{1}{2} a_n^2 + \frac{3}{4} z_n^4 + \frac{1}{4} \varepsilon_n^4 \tag{80}$$

Substituting (80) into (79) yields

$$LV_{nj} \le -\sum_{i=1}^{n-1}\left(c_i z_i^4 + \frac{\gamma_i}{2\lambda_i}\tilde{\vartheta}_i^2\right) + \sum_{i=1}^{n-1} e_i + z_n^4 + \frac{3}{4}l_n^2 + \frac{1}{2}a_n^2 + \frac{1}{4}\varepsilon_n^4 + \frac{1}{2a_n^2}z_n^6\vartheta_n\xi_n^T\xi_n + z_n^3\sum_{i=1}^{m} b_i\beta_i(x)\left(\rho_i u_{PFi} + u_{TFi}\right) - \frac{1}{\lambda_n}\tilde{\vartheta}_n\dot{\hat{\vartheta}}_n - \frac{1}{\lambda_d}\tilde{d}\dot{\hat{d}} - \sum_{i=1, i\in Q_{PTj}}^{m} \rho_i |b_i| \tilde{k}^T\Gamma_k^{-1}\dot{\hat{k}} - z_n^3 y_r^{(n)} \tag{81}$$

Then the adaptive laws are chosen as

$$\dot{\hat{\vartheta}}_n = \frac{\lambda_n}{2a_n^2}z_n^6\xi_n^T\xi_n - \gamma_n\hat{\vartheta}_n,\ \hat{\vartheta}_n(0) \ge 0 \tag{82}$$

Substituting (82) into (81), we can get

$$LV_{nj} \le -\sum_{i=1}^{n-1}\left(c_i z_i^4 + \frac{\gamma_i}{2\lambda_i}\tilde{\vartheta}_i^2\right) + \sum_{i=1}^{n-1} e_i + z_n^4 + \frac{3}{4}\tau_n^2 + \frac{1}{2}a_n^2 + \frac{1}{4}\varepsilon_n^4 + z_n^3\sum_{i=1}^{m} b_i\beta_i(x)\left(\rho_i u_{PFi}(t) + u_{TFi}\right) - \frac{\gamma_n}{2\lambda_n}\tilde{\vartheta}_n^2 + \frac{\gamma_n\vartheta_n^2}{2\lambda_n} - \frac{1}{\lambda_d}\tilde{d}\dot{\hat{d}} - \sum_{i=1, i\in Q_{PTj}}^{m} \frac{\rho_i |b_i|}{\lambda_n} \tilde{k}^T\Gamma_k^{-1}\dot{\hat{k}} + \frac{1}{2a_n^2}z_n^6\hat{\vartheta}_n\xi_n^T\xi_n - z_n^3 y_r^{(n)} \tag{83}$$

Now, we make some analysis on one term in (83). Obviously, see in [28], it can be got following property

$$0 \le |\varrho| - \varrho tanh\left(\frac{\varrho}{\varepsilon}\right) \le 0.2785\varepsilon \tag{84}$$

where $\varepsilon \in R^+$ and $\varrho \in R$.

Substituting $u_{PFi}(t) = \frac{w_i(t) - \lambda_2(t)m_1}{1+\lambda_1(t)\delta}$ into $z_n sgn(b_i)\beta_i(x)u_{PFi}$, we can get

$$\begin{aligned} z_n sgn(b_i)\beta_i(x)u_{PFi}(t) &= z_n sgn(b_i)\beta_i(x)\frac{w_i(t) - \lambda_2(t)m_1}{1+\lambda_1(t)\delta} \\ &= \frac{z_n sgn(b_i)\beta_i(x)w_i(t)}{1+\lambda_1(t)\delta} - \frac{z_n sgn(b_i)\beta_i(x)\lambda_2(t)m_1}{1+\lambda_1(t)\delta} \\ &\le \frac{z_n sgn(b_i)\beta_i(x)w_i(t)}{1+\lambda_1(t)\delta} + \left|\frac{z_n sgn(b_i)\beta_i(x)\lambda_2(t)m_1}{1+\lambda_1(t)\delta}\right| \\ &\le z_n sgn(b_i)\beta_i(x)\bar{u}_{ci}(t) - |z_n sgn(b_i)\beta_i(x)\bar{m}| + \left|\frac{z_n sgn(b_i)\beta_i(x)m_1}{1-\delta}\right| + 0.557\varepsilon \\ &\le z_n sgn(b_i)\beta_i(x)\bar{u}_{ci}(t) + 0.557\varepsilon \end{aligned} \tag{85}$$

Substituting (85) into (83), we can find that

$$LV_{nj} \le -\sum_{i=1}^{n-1}\left(c_i z_i^4 + \frac{\gamma_i}{2\lambda_i}\tilde{\vartheta}_i^2\right) + \sum_{i=1}^{n-1} e_i + z_n^4 + \frac{3}{4}l_n^2 + \frac{1}{2}a_n^2 + \frac{1}{4}\varepsilon_n^4 + \frac{\gamma_n\vartheta_n^2}{2\lambda_n} + z_n^3\sum_{i=1}^{m} b_i\beta_i(x)\left(\rho_i\bar{u}_{ci}(t) + u_{TFi}\right) + z_n^2 d^{\frac{1}{2}} - z_n^3 y_r^{(n)} - \frac{\gamma_n}{2\lambda_n}\tilde{\vartheta}_n^2 - \frac{1}{\lambda_d}\tilde{d}\dot{\hat{d}} - \sum_{i=1, i\in Q_{PTj}}^{m} \frac{\rho_i |b_i|}{\lambda_n} \tilde{k}^T\Gamma_k^{-1}\dot{\hat{k}} + \frac{1}{2a_n^2}z_n^6\hat{\vartheta}_n\xi_n^T\xi_n \tag{86}$$

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From $\tilde{k} = k_j - \hat{k}$ and (71), we find that

$$z_n^3 \sum_{i=1}^{m} b_i \beta_i(x) \left(\rho_i \bar{u}_{ci}(t) + u_{TFi}\right)$$
$$= z_n^3 \left(\alpha_n - \sum_{i \in Q_{PTj}} |b_i| \rho_i \tilde{k}^T \bar{w}\right) \tag{87}$$

so

$$z_n^3 \sum_{i \in Q_{PTj}} |b_i| \rho_i \hat{k}^T \bar{w} + z_n^3 \sum_{i \in Q_{TTj}} b_i \beta_i(t) u_{TFi} - \sum_{i \in Q_{PTj}} \frac{\rho_i |b_i|}{\lambda_n} \tilde{k}^T \Gamma_k{}^{-1} \dot{\hat{k}}$$
$$= z_n^3 \alpha_n - \sum_{i \in Q_{TTj}} \frac{\rho_i |b_i|}{\lambda_n} \tilde{k}^T \Gamma_k{}^{-1} \left(\dot{\hat{k}} + \lambda_n \Gamma_k \bar{w} z_n^3\right)$$

The parameter k update law is obtained as

$$\dot{\hat{k}} = -\lambda_n \Gamma_k \bar{w} z_n^3 - \gamma_n \hat{k} \tag{88}$$

Furthermore, applying Lemma 1, it can be obtained that

$$z_n^2 d^{\frac{1}{2}} \leq \frac{1}{2a_d^2} z_n^4 d + \frac{1}{2} a_d^2 \tag{89}$$

Then, the adaptation law is taken as

$$\dot{\hat{d}} = \frac{\lambda_d}{2a_d} z_n^4 - \gamma_d \hat{d} \tag{90}$$

Based on the inequality

$$\frac{\gamma_n}{\lambda_n} \tilde{\vartheta}_n \hat{\vartheta}_n \leq -\frac{\gamma_n}{2\lambda_n} \tilde{\vartheta}_n^2 + \frac{\gamma_n}{2\lambda_n} \vartheta_n^2 \tag{91}$$

$$\tilde{k}^T \Gamma_k^{-1} \hat{k} \leq -\frac{1}{2} \tilde{k}^T \Gamma_k^{-1} \tilde{k} + \frac{1}{2} k_j{}^T \Gamma_k^{-1} k_j \tag{92}$$

$$\frac{\gamma_d}{\lambda_d} \tilde{d} \hat{d} \leq -\frac{\gamma_d}{2\lambda_d} \tilde{d}^2 + \frac{\gamma_d}{2\lambda_d} d^2 \tag{93}$$

and set $e_n = \frac{\varepsilon_n^4}{4} + \frac{a_n^2}{2} + \frac{3}{4}\tau_n^2 + \frac{a_d^2}{2} + \frac{\gamma_n}{2\lambda_n}\vartheta_n^2 + \frac{\gamma_d}{2\lambda_d} d^2 + \sum_{i \in Q_{PTj}} \frac{|b_i| \rho_i \gamma_n}{2\lambda_n} k_j{}^T \Gamma_k^{-1} k_j$, the following result holds:

$$LV_{nj} \leq -\sum_{i=1}^{n} \left(c_i z_i^4 + \frac{\gamma_i}{2\lambda_i} \tilde{\vartheta}_i^2\right) + \sum_{i=1}^{n} e_i - z_n^3 y_r^{(n)} + (c_n+1) z_n^4 + \frac{1}{2a_n^2} z_n^6 \hat{\vartheta}_n \xi_n^T \xi_n + z_n^3 \alpha_n + \frac{1}{2a_d^2} z_n^4 \hat{d} - \frac{\gamma_d}{2\lambda_d} \tilde{d}^2$$
$$- \sum_{i \in Q_{PFj}} \frac{\rho_i |b_i|}{\lambda_n} \tilde{k}_k^T \Gamma_k^{-1} (\dot{\hat{k}} + \lambda_n \Gamma_k \bar{w} z_n^3) \tag{94}$$

Choosing the virtual control α_n as follows

$$\alpha_n = -(c_n+1) z_n - \frac{1}{2a_d^2} z_n \hat{d} - \frac{1}{2a_n^2} z_n^3 \hat{\vartheta}_n \xi_n^T \xi_n + y_r^{(n)} \tag{95}$$

Substituting (95) into (94), we can get the result as

$$LV_{nj} \leq -\sum_{i=1}^{n} \left(c_i z_i^4 + \frac{\gamma_i}{2\lambda_i} \tilde{\vartheta}_i^2\right) - \sum_{i \in Q_{PTj}} \frac{|b_i| \rho_i \gamma_k}{2} \tilde{k}^T \Gamma_k{}^{-1} \tilde{k} + \sum_{i=1}^{n} e_i \tag{96}$$

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3.4. System stability analysis

Owing to previous design formulation, the main result will be summarized to Theorem 1.

Theorem 1. *Consider the stochastic system consisting of plant (1), the controller (73) with the parameter update laws (38), (52), (62), (82), (88), (90), the intermediate virtual controllers (37), (51), (61), (95), and the event-triggered control strategy (66) in the presence of uncertain actuator failures (2) and the limitation of communication resource under Assumptions 1∼3. All the signals are bound. Furthermore, the tracking error of the system is preserved within a given transient specification all the time.*

Proof. Design the $V = V_n$, and denote

$$\rho = min\{4c_i, \gamma_i \ (i = 1, 2, \cdots, n), \gamma_k\}$$

and

$$\varrho = \sum_{i=1}^{n} e_i.$$

From (96), it can be safely arrived that

$$LV \leq -\rho V + \varrho, t \geq 0 \tag{97}$$

From (97) and Lemma 1, z_i and $\tilde{\vartheta}_i$ are bounded. ϑ_i is a constant, hence $\hat{\vartheta}_i$ is also bounded. Also $z_1 = x_1 - y_r$ and y_r is bounded, which indicates that x_1 is bounded. Considering that α_1 is consist of z_1 and $\hat{\vartheta}_1$, therefore α_1 is bounded, which implies that x_2 is also bounded. By the same token, x_i, $i = 3, ..., n$ are bounded. And it is implied that $\bar{u}_{ci}(t)$ is also bounded from (73). Thus, all the signals of the closed-loop system are bounded, and the tracking error is made within a given transient specification all the time. □

4. Simulation results

Many practical systems such as ball and beam systems are in nonstrict-feedback form. In addition, as practical systems, the existence of stochastic disturbance is inevitable. Therefore, the controlled ball on a beam model can be written in the form defined in equation (1) if the actuator failures are considered. In this section, the numerical examples are considered to demonstrate the utility of the proposed scheme for considered systems with actuator failures and given transient specification.

4.1. Dynamic model

Now we consider a 2nd order system described as

$$\begin{aligned} dx_1 &= (x_2 + f_1(\bar{x}_1))dt + \psi_1^T(\bar{x}_1)dv \\ dx_2 &= (\sum_{i=1}^{2} b_i \beta_i(x)(\rho_i u_{PFi}(t) + u_{TFi}) + f_2(\bar{x}_2))dt + \psi_2^T(\bar{x}_2)dv \\ y &= x_1 \end{aligned}$$

where $u_{PFi}(t), u_{TFi}, \bar{x}_1, \bar{x}_2$ is denoted as Section 2, x_1 and x_2 are state variables. The reference signal is chosen as $y_r = sin(t)$.

For illustrating the effectiveness of the aforementioned proposed controllers, we choose $f_1(\bar{x}_1)$, $f_2(\bar{x}_2)$, $\psi_1(\bar{x}_1)$ and $\psi_2(\bar{x}_2)$ respectively as $(1 - sin^2(x_1))x_1$,$-3.5x_2 + x_1x_2^2$, $0.5cos(x_1)$ and $0.1x_1sin(2x_1x_2)$, and $\beta_1(x) = 1.9 + 0.1sin(x_1)$, $\beta_2(x) = 1.9 + 0.1sin(x_2)$.

And the fuzzy membership functions are chosen as follows:

$$\mu_{f_i^l}(x) = e^{(0.5(x+3-(l-1)(6/(15-1)))^2/3^2}, l = 1, 2, \cdots, 15.$$

4.2. Adaptive control design

By Theorem 1, the virtual controllers α_1 and α_2, the controller $\bar{u}_{ci}$, and the parameter adaptive laws $\dot{\hat{\vartheta}}_1$, $\dot{\hat{\vartheta}}_2$ and $\dot{\hat{k}}$, are designed as

$$\alpha_1 = -\left(\frac{c_1}{\varsigma} + \frac{3}{4\varsigma}\right) z_1 - \frac{1}{2\varsigma a_1^2} z_1^3 \hat{\vartheta}_1 \xi_1^T \xi_1$$

$$\alpha_2 = -(c_2 + \frac{3}{4}) z_2 - \frac{1}{2a_d^2} z_2 \hat{d} - \frac{1}{2a_2^2} z_2^3 \hat{\vartheta}_2 \xi_2^T \xi_2 + y_r^{(2)}$$

$$\bar{u}_{ci} = sgn(b_i) \frac{1}{\beta_i(x)} \hat{k}^T \bar{w}, \ i = 1, 2$$

$$\dot{\hat{\vartheta}}_1 = \frac{\lambda_1}{2a_1^2} z_1^6 \xi_1^T \xi_1 - \gamma_1 \hat{\vartheta}_1$$

$$\dot{\hat{\vartheta}}_2 = \frac{\lambda_2}{2a_2^2} z_2^6 \xi_2^T \xi_2 - \gamma_2 \hat{\vartheta}_2$$

$$\dot{\hat{k}} = -\lambda_n \Gamma_k \bar{w} z_2^3 - \gamma_n \hat{k}$$

$$\dot{\hat{d}} = \frac{\lambda_d}{2a_d} z_2^4 - \gamma_d \hat{d}$$

where $X_1 = [x_1, y_r, \eta, \dot{\eta}, \varsigma]^T$ and $X_2 = [\bar{x}_2^T, \bar{\hat{\vartheta}}_1^T, \bar{y}_r^{(1)T}, \dot{\eta}, \ddot{\eta}]^T$.

The initial conditions are all designed as $x_1(0) = 0.15$, $x_2(0) = 0$, $\hat{\vartheta}_1(0) = 0.1$, $\hat{\vartheta}_2(0) = 0.2$, $\hat{k}(0) = [-1.5, 0, 0]^T$, $\bar{u}_{c1}(0) = 0$ and $\bar{u}_{c2}(0) = 0$.

The parameters for proposed scheme are chosen as $b_1 = 1$, $b_2 = 1$, $a_1 = 0.5$, $a_2 = 0.5$, $a_d = 0.5$, $c_1 = 8$, $c_2 = 8$, $\lambda_1 = 25$, $\lambda_2 = 20$, $\lambda_d = 15$, $\gamma_1 = 1$, $\gamma_2 = 2$, $\gamma_d = 2$, $\tau_1 = 0.5$, $\tau_2 = 0.5$.

4.3. Simulation examples and analysis

The PLOE and TLOE failure modes are simulated respectively.

Example 1. Actuator $u_1 = 3$ and actuator u_2 works in the failure-free case from $t = 8$ s, and actuator $u_1 = 2$ and actuator u_2 loses 50% of its effectiveness from $t = 15$ s.

The tracking error $e(t)$, which is under the Given Transient Specifications control (GTS), is illustrated in Fig. 1(a), and the tracking error $e(t)$, which is not under the Given Transient Specifications control (NGTS), is illustrated in Fig. 2(a). It is indicated that the tracking error is bounded with the given transient performance under transform system control. The state x_1 and the reference output y_r are plotted in Fig. 1(b) and Fig. 2(b) under GTS and NGTS respectively, which remain bounded. The triggering events is illustrated in Fig. 1(c) and Fig. 2(c) and the communication bandwidth percentage is shown in Fig. 1(d) and Fig. 2(d) under GTS and NGTS respectively. The communication bandwidth percentage is under 50% all the time. The controller outputs $\bar{u}_{c1}$ and $\bar{u}_{c2}$ and the actuator outputs u_1 and u_2 are obtained in Fig. 1(e)–Fig. 1(f) and Fig. 2(e)–Fig. 2(f) under GTS and NGTS respectively.

In order to illustrate the above results clearly, we list Table 1 which include the number of triggering events including GTS and NGTS.

Seeing from Table 1, when actuator u_1 is stuck at $u_1 = 3$ from $t = 8$ s, the tracking error has increased from $t = 9$ s to $t = 10$. So the number of triggering event of u_1 and u_2 have increased. After the controllers compensate actuator failures, the tracking error will decrease, so the triggering event number of u_1 and u_2 begin to decrease at $t = 11 \sim 14$ s. Later, actuator u_1 is stuck at $u_1 = 2$ and u_2 loses 50% of its effectiveness from $t = 15$ s, which is a more severe failure, so the number of triggering event of u_1 and u_2 increase to higher value at $t = 15$. Similarly, the number of triggering event of u_1 and u_2 are decreasing at $t = 16 \sim 20$ s as controllers compensate actuator failures. From Table 1, it also shows that the triggering event number will not signally increase under GTS, which could achieve the tracking error within a given performance.

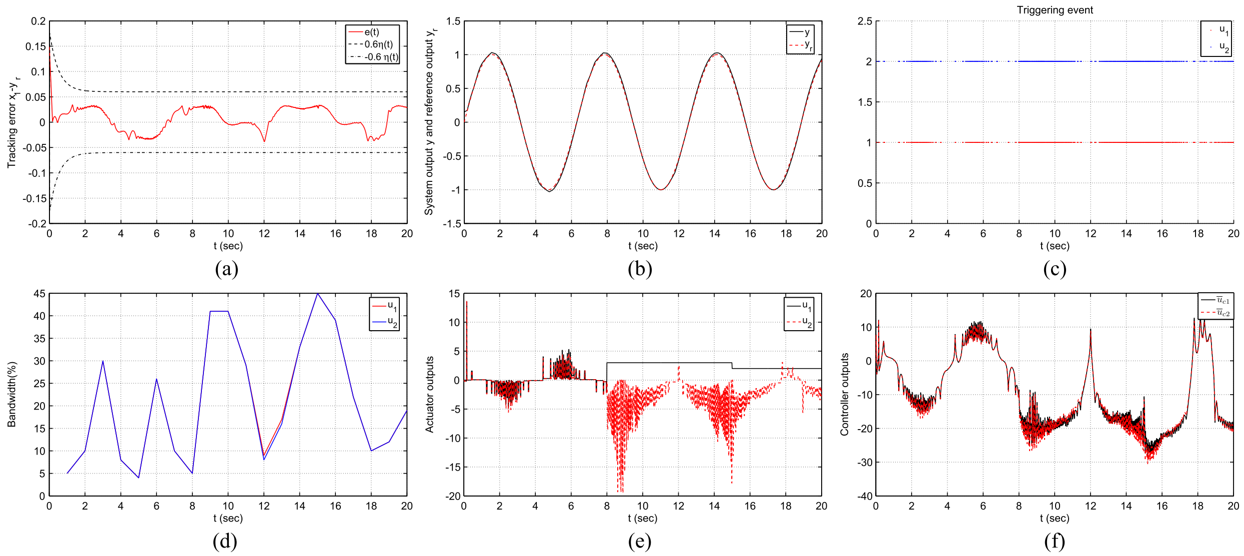

Fig. 1. Example 1 with GTS: (a) Tracking error $e(t)$; (b) System output y and reference output y_r; (c) Triggering events of u_1 and u_2; (d) Communication bandwidth; (e) Actuator outputs u_1 and u_2; (f) Controller outputs $\bar{u}_{c1}$ and $\bar{u}_{c2}$. (For interpretation of the colors in the figure(s), the reader is referred to the web version of this article.)

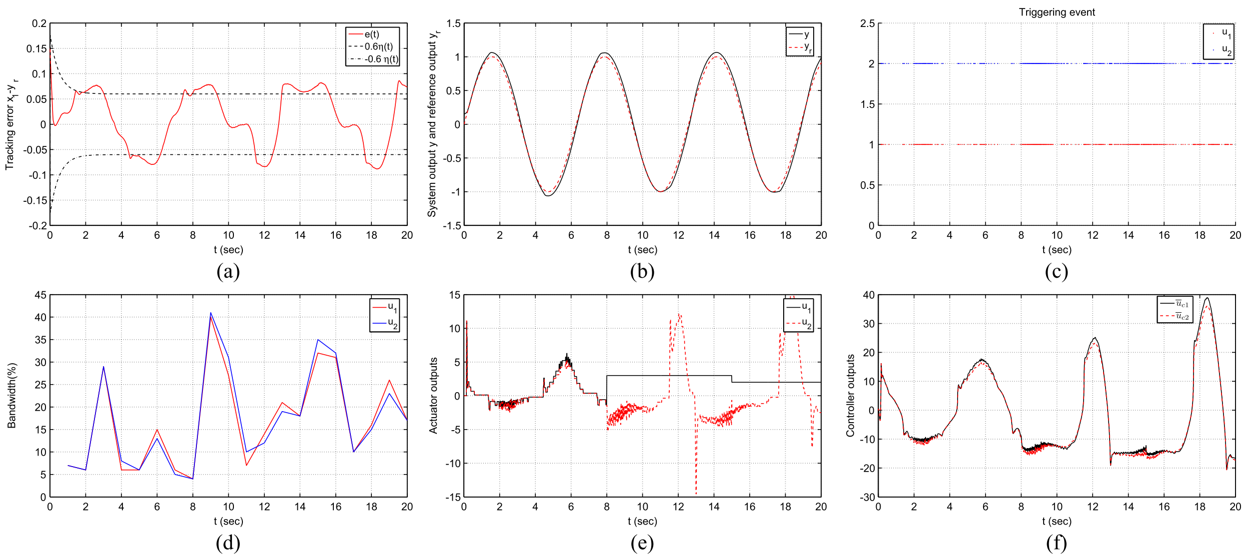

Fig. 2. Example 1 with NGTS: (a) Tracking error $e(t)$; (b) System output y and reference output y_r; (c) Triggering events of u_1 and u_2; (d) Communication bandwidth; (e) Actuator outputs u_1 and u_2; (f) Controller outputs $\bar{u}_{c1}$ and $\bar{u}_{c2}$.

Example 2. Actuator u_1 loses 70% of its effectiveness from $t = 8$ s and loses 80% of its effectiveness from $t = 15$ s, and actuator u_2 loses 60% of its effectiveness from $t = 15$ s.

The performances of tracking error, states, triggering events, controllers outputs and actuators outputs are obtained in Fig. 3 and Fig. 4 under GTS and NGTS respectively. The tracking error and the states x_1, x_2 are all bounded under example 2. Furthermore, the communication bandwidth percentage is under 50% all the time and the transient performance is within a given bound under GTS.

Similarly, we list Table 2, which include the number of triggering events at different time, for example 2.

Seeing from Table 2, we can obtain the same results as Table 1, i.e. when actuator u_1 loses 70% of its effectiveness from $t = 8$ s, the number of triggering event of u_1 and u_2 have increased at $t = 9$ s. After the compensation of

Table 1
Number of triggering events for Example 1.

u_i \ Time (s)		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
u_1	(GTS)	5	10	30	8	4	26	10	5	41	41	29	9	17	33	45	39	22	10	12	19
	(NGTS)	7	6	29	6	6	15	6	4	40	27	7	14	21	18	32	31	10	16	26	17
u_2	(GTS)	5	10	30	8	4	26	10	5	41	41	29	8	16	33	45	39	22	10	12	19
	(NGTS)	7	6	29	8	6	13	5	4	41	31	10	12	19	18	35	32	10	15	23	17

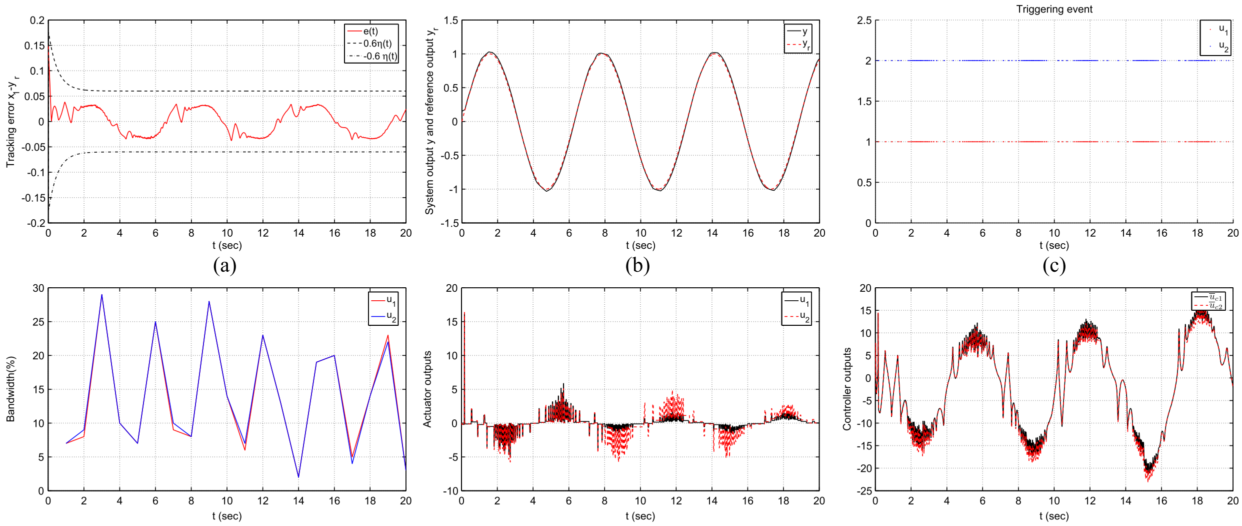

Fig. 3. Example 2 with GTS: (a) Tracking error $e(t)$; (b) System output y and reference output y_r; (c) Triggering events of u_1 and u_2; (d) Communication bandwidth; (e) Actuator outputs u_1 and u_2; (f) Controller outputs $\bar{u}_{c1}$ and $\bar{u}_{c2}$.

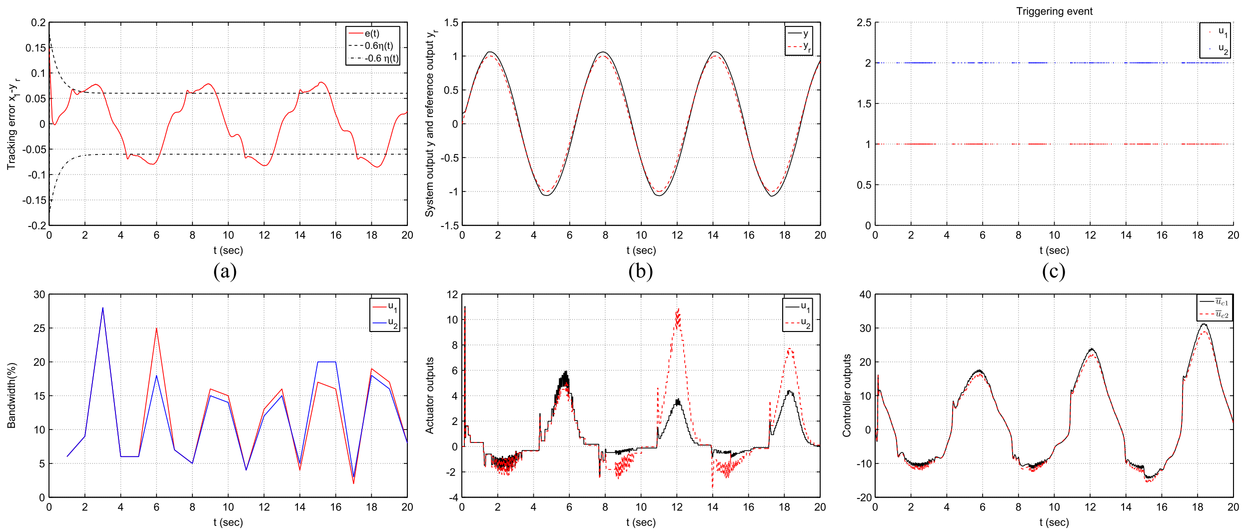

Fig. 4. Example 2 with NGTS: (a) Tracking error $e(t)$; (b) System output y and reference output y_r; (c) Triggering events of u_1 and u_2; (d) Communication bandwidth; (e) Actuator outputs u_1 and u_2; (f) Controller outputs $\bar{u}_{c1}$ and $\bar{u}_{c2}$.

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Table 2
Number of triggering events for Example 2.

u_i \ Time (s)		7	8	29	10	7	25	9	8	28	14	6	23	13	2	19	20	5	14	23	3
u_1	(GTS)	7	8	29	10	7	25	9	8	28	14	6	23	13	2	19	20	5	14	23	3
	(NGTS)	6	9	28	6	6	25	7	5	16	15	4	13	16	4	17	16	2	19	17	8
u_2	(GTS)	7	9	29	10	7	25	10	8	28	14	7	23	13	2	19	20	4	14	22	3
	(NGTS)	6	9	28	6	6	18	7	5	15	14	4	12	15	5	20	20	3	18	16	8

controller, the number of triggering event of u_1 and u_2 gradually decrease at $t = 10 \sim 14$ s. Then, the number of triggering event of u_1 and u_2 have increased at $t = 15$ s again when u_1 loses 80% of its effectiveness and actuator u_2 loses 60% of its effectiveness from $t = 15$ s. And the triggering event number of u_1 and u_2 are decreasing at $t = 16 \sim 20$ s.

The simulation results indicate that although the stochastic system is suffered from the actuator failures, which under consideration include TLOE and PLOE, and event trigger, the error between the system output and the given reference signal is ensured to be bounded.

5. Conclusion

In this paper, an event-triggered fuzzy controller for a class of uncertain stochastic nonlinear systems with actuator failures and given transient specification is developed. In the process of controller design, the adaptive fuzzy compensation control law is designed to effectively regulate the unknown parameters for nonlinear functions and actuator failures models online. Furthermore, for satisfying the given transient specification and the limitation of communication resource, event-triggered control mechanism and transformed system control mechanism are established. Simulation studies have verified the theoretical results by the proposed schemes. However, in order to handle the problem of the chattering effect for systems is challenge of analysis and design, the proposed control scheme does have certain conservation, which we will attempt to cope with in the future.

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