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Fuzzy Switched Hybrid Systems - Modeling and Identification

Rainer Palm
Siemens AG Corporate Technology
Information and Communications
81730 Munich, Germany

Dimiter Driankov
University of Linköping
Dept. of Computer Science
581 83 Linköping, Sweden

Abstract

In this paper the combination of hybrid systems and fuzzy multiple model systems is described. Further, a hierarchical identification of the resulting fuzzy switched hybrid system is outlined. The behavior of the discrete component is identified by black box fuzzy clustering and subsequent parameter identification taking into account some prior-knowledge about the discrete states. The identification of the continuous models for each discrete state is done based on local linear fuzzy models.

Keywords Hybrid systems, fuzzy control, multiple models, identification, fuzzy clustering.

1 Introduction

In the last few years two new trends in the modeling and control of complex systems have emerged: *hybrid systems* and *multiple model systems*. Hybrid systems, incorporate two distinct types of systems, continuous and discrete state that interact with each other [Stiver et al 97, Nerode and Kohn 91, Nicollin 91, Branicky et al 94, Lennartson et al 94].

Key features of these systems are the mixture of continuous and discrete variables, and the coupling of time-driven and event-driven dynamics of the continuous and discrete-event parts respectively. Multiple model systems are aimed at reducing the complexity of continuous nonlinear systems by means of a combination of local models [Takagi and Sugeno 85, Tanaka and Sugeno 92, Johansen 94, Palm et al 96] each describing the system in a certain operating region. The combination of the local models is done by, e.g., RBF networks and/or fuzzy control methods, using the universal approximation property of these methods [Wang 92].

The two trends from above meet in type of systems called *switched hybrid systems* [Branicky et al 94]. In such system the multiple model part is a simplification of the continuous part of the hybrid system. In the literature on switched hybrid systems a predominant number of studies concerns a multiple model part which consists of a collection of local linear models. In this paper we consider the case of a fuzzy multiple model system of the Takagi-Sugeno type which is a nonlinear system. Furthermore, most of the work done in the area of hybrid systems is related to analysis and design while the issue of hybrid system identification has received almost no attention. The present paper rectifies this situation by proposing an identification method for fuzzy switched hybrid systems.

In Sect. 2 we introduce the notion of a fuzzy switched hybrid system which is a combination of a hybrid system with multiple fuzzy models. Sect. 3 contains the proposed identification method. Sect. 4 illustrates the identification method by a robotics example. Sect.5 provides conclusions and future research.

2 Fuzzy switched hybrid systems

We consider here hybrid models introduced on a class of continuous-time systems with part continuous, part discrete state. At a time when the discrete state undergoes a change (a transition), the state vector is still continuous, though the vector field may change discontinuously. No jumps of the continuous state vector are allowed. Furthermore, transitions occur when and only when the continuous state vector satisfies a condition given for each transition. In the case of control inputs, these influence transitions only through the differential equations, never directly. In other words we are concerned here with Witsenhausen's type of a hybrid system [Witsenhausen 66]. It must be stressed that the hybrid system we are dealing with represents a hierarchy of discrete and continuous subsystems where the discrete part is atop of the hierarchy.

The description of the above type of a hybrid system starts with a continuous system given as

$$\dot{x} = f(x(t), u(t)), \tag{1}$$

where $x(t) \in \mathbf{R}^n$ and $u(t) \in \mathbf{R}^m$. The effect of the discrete components of the state on the above continuous components is accounted for by letting f depend on the discrete state, that is

$$\dot{x} = f(x(t), q(t), u(t)), \tag{2}$$

where $q(t) \in \mathbf{Q}$ and $\mathbf{Q}$ is a collection of discrete states. The coupling in the opposite direction, i.e., the effect of the continuous components on the discrete state, is described by conditions on the continuous state for which transitions of the discrete component occur. That is,

$$q(t) = g(x(t), q(t^-)), \tag{3}$$

where the notation t^- means that the discrete state is piecewise continuous from the right. Thus the state of the hybrid system is characterized at any time by the pair (x, q) ranging over $\mathbf{R}^n \times \mathbf{Q}$ (see Fig. 1).

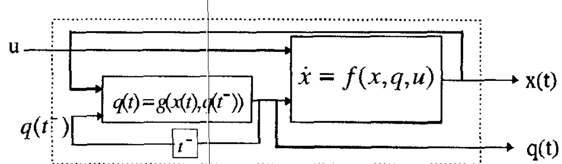

Figure 1. Block scheme of the hybrid system.

A transition from one discrete state q^k to another one q^j $(k \neq j)$ is triggered when the continuous state x satisfies a given transition condition.Thus starting at say, (x_0, q^k) the system develops according to, say $\dot{x} = f^k(x(t), q_k(t), u(t))$. If x satisfies a transition condition, say $\dot{x} < \dot{x}_{max}$, at time t_1 then the state of the hybrid system becomes $(x(t_1), q^j)$ from which the process continues according to $\dot{x} = f^j(x(t), q^j(t), u(t))$. Thus the development of the continuous component of the state at any time point can be represented as

$$\dot{x} = \sum_{k=1}^{L} w^k(q^k(t)) f^k(x(t), q^k(t), u(t)) \tag{4}$$

where L is the number of discrete states elements of $\mathbf{Q}$, $w^k \in \{0, 1\}$, and $\sum_{k=1}^{L} w^k = 1$. Furthermore, $w^k = 1$ if and only if the discrete state $q^k(t)$ is actually present at time t. The development of the discrete component is given by functions $g^k(x(t), q^k(t))$ and each such function can be explicitly represented as a crisp IF-THEN rule of the form

$$\begin{aligned} &\text{IF} \quad x(t) \quad \text{is} \quad P(x) \quad \text{and} \quad q(t^-) = q^k \\ &\text{THEN} \quad q(t) = q^j \end{aligned} \tag{5}$$

where $P(x)$ is a transition condition. For example, a set of such rules can be illustrated by the state transition graph in Fig. 2. The set of transition rules can be

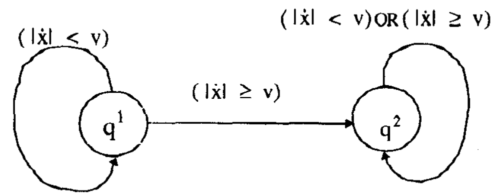

Figure 2. The transition graph of the discrete component.

represented in the compact form

$$q(t) = \sum_{k=1}^{L} w^k(q^k(t^-)) g^k(x(t), q^k(t^-)), \tag{6}$$

where $w^k(q^k(t^-))$ is equal to 1 if and only if the IF-part of the k-th rule is satisfied. A black box model of (6) will compute for any point in time a $q(t)$ given $x(t)$ and without taking into account $q(t^-)$ and the corresponding transition condition. Such a black box model can only imitate the behavior of $q(t)$ in time, but cannot reflect the structure of the discrete component, i.e, the transition conditions and the direction of the transitions. Such a black box model for the discrete component is identified in Sections 3 and 4.

Now let us concentrate on the multiple model representation of a particular nonlinear $f^k(x(t), q^k(t), u(t))$. For ease of representation we will omit the upper index k which means that the fuzzy multiple model described below can be used for representing any f^k. The model consists of a set of fuzzy IF-THEN rules where the IF-part of each fuzzy rule describes a particular fuzzy region in the state space of f (different f's may have different state spaces) and its corresponding THEN-part contains a linear model. Thus, the i-th fuzzy rule describes the fuzzy model's dynamics within the fuzzy region LX^i specified in the rule's IF-part and is written as

$$\text{IF} \quad x \quad \text{is} \quad LX^i \quad \text{THEN} \quad \dot{x} = A^i x + B^i u, \tag{7}$$

where

- $LX^i = (LX_1^i, \ldots, LX_n^i)^T$, and LX_k^i denotes the fuzzy value which x_k (the k-th component of the state vector) takes in the i-th fuzzy region. Each LX_k^i is determined by a fuzzy set $\int_X LX_k^i(x_k)/x_k$ of a standard triangular, trapezoidal, or bell-shaped type. The membership functions of any one of the previously mentioned types are nonlinear functions of x_k,
- $\dot{x} = A^i x + B^i u$ is a linear autonomous model corresponding to the i-th fuzzy region. The state vector x is a $n \times 1$ linear vector-function of time and its components $x_1, x_2, \cdots, x_n$ denote time dependent state variables. The control input vector u is a $m \times 1$ vector function of time with components $u_1, u_2, \cdots, u_n$. The entries of the matrices A_i $(n \times n)$ and B_i $(n \times m)$ are constant.

The system's dynamics corresponding to computation with a single fuzzy rule is given as

$$\dot{x} = LX^i(x)(A^i x + B^i u), \tag{8}$$

where $LX^i(x)$ is the degree of satisfaction of the i-th fuzzy region by the crisp state vector x. The results computed for each individual rule via (8) are aggregated by simply taking their weighted average, that is,

$$\dot{x} = \frac{\sum_i LX^i(x)(A^i x + B^i u)}{\sum_i LX^i(x)}. \tag{9}$$

The expression (9) can be simplified by normalizing the degrees of satisfaction $LX^i(x)$ and using instead their normalized counterparts $w^i(x)$ that are obtained as follows: for any $LX^1(x), LX^2(x), \ldots, LX^M(x)$

$$w^i(x) = \frac{LX^i(x)}{\sum_i LX^i(x)}. \tag{10}$$

Thus, $\sum_i w^i(x) = 1$ and (9) can be rewritten in the more simple form,

$$\dot{x} = \sum_i w^i(x)(A^i x + B^i u). \tag{11}$$

From (11) it is easily seen that the set of all fuzzy rules defines linear dynamics for all points that belong to the center of an arbitrary fuzzy region in the fuzzy state space. Take, for example, the i-th fuzzy region. For every center, i.e., $x = x^i$, we have that $w^i(x) = w^i(x^i) = 1$ while for each $j \neq i$ we have that $w^j(x) = 0$, since $\sum_i w^i(x) = 1$. Thus we have that (11) becomes (for each $x = x^i$),

$$\dot{x} = w^i(x^i)(A^i x^i + B^i u) = A_i x^i + B_i u.$$

On the other hand, (11) defines nonlinear dynamics for all points $x \neq x^i$. This is so, because in this case there is no $w^i(x) = 1$, and thus each linear part $(A^i \cdot x + B^i u)$ in (11) is multiplied by the nonlinear function $w^i(x)$.
In the context of (11) the fuzzy multiple model representation of $f^k(x(t), q^k(t), u(t)$ is then given as

$$f^k(x, q^k, u) = \sum_i w^{ik}(x)(A^{ik} x + B^{ik} u). \tag{12}$$

where the upper index k denotes the fact that the above fuzzy multiple model is related to the discrete component q^k.

3 Identification of the discrete and the continuous components

Identification described here is based on input/output data and some prior knowledge about the structure of the continuous and discrete components of the hybrid system. We assume that both the continuous and discrete states are measurable. We also assume knowledge about the possible discrete states q^l $(l = 1, \ldots, L)$ and that the order of the continuous component f^k for each discrete state is known and f^k is smooth.

In this context, the identification of the hybrid system is done in a hierarchical manner. First, we identify a black box model for the discrete component given by (6). The identification of this black box model is presented in detail in the next section. Second, having identified (6) we identify the continuous component f^k for each state q^k as a fuzzy multiple Takagi-Sugeno model. Here we will just outline the identification of the f^k's. The reader is referred to [Hellendoorn and Driankov 97] for details.

For each q^k the system to be identified can be appropriately modeled by a well defined number of submodels

$$\dot{x} = f^{jk}(x, q^j, u); \quad j = 1, ..., N. \tag{13}$$

Fuzzy identification of f^k is then done as follows. Using the C-means method [Bezdek 87] the input/output data (the pairs (x, u)) are partitioned in a number of fuzzy clusters (fuzzy regions). Based on the fuzzy cluster centers, (x^{jk}, u^{jk}), the membership functions representing each cluster are obtained using weights w^{jk}. The submodels f^{jk} are identified at the center of each fuzzy cluster.

The weights w^{jk} are computed as

$$w^{jk} = \frac{1}{\sum_{i=1}^{N} (\frac{d^{jk}}{d^{ik}})^{\frac{1}{m-1}}}, \tag{14}$$

where $d^{jk} = ||z - z^{jk}||$, $||\cdot||$ is an Euclidian norm, $z = (x^T, u^T)^T$, z^{jk} is the center of the j-th fuzzy cluster, and $m > 1$ is a fuzzification parameter. The bigger m is the more gradual is the transition from one cluster to another. Then we identify a linear model at each center of a fuzzy cluster

$$\dot{x} = A^{jk}x + B^{jk}u + const^{jk}, \tag{15}$$

where $A^{jk} = \frac{\partial f}{\partial x}|_{x^{jk},u^{jk}}$, $B^{jk} = \frac{\partial f}{\partial u}|_{x^{jk},u^{jk}}$, and $const^{jk} = -(A^{jk}x^{jk} + B^{jk}u^{jk})$. Since we do not linearize around zero but around the cluster centers x^{jk}, u^{jk} there is, in contrast to (12), a constant $const^{jk}$ in (15). For an arbitrary point (x, u) we then obtain the fuzzy approximation

$$\dot{x} = f^k = \sum_{j=1}^{N} w^{jk}(x,u)(A^{jk}x + B^{jk}u + const^{jk}). \tag{16}$$

4 Simulation example

In this example we simulate a single link robot arm given by the dynamic equation

$$\ddot{x} = -\frac{m^k gl}{J^k}\sin x - \frac{D^k}{J^k}\dot{x} + \frac{1}{J^k}u, \tag{17}$$

where x is the angle of the arm, u is the control input (manipulated variable). What makes the robot arm a hybrid system is that its mass m^k, inertia J^k, and damping D^k form a discrete state $q^k = (m^k, J^k, D^k)$ are changing depending on the angle x. This can physically happen if the robot works under different environmental conditions and with changing payloads. The discrete states q^k employed in the simulation are
$q^1 = (1, 1, 2)$,
$q^2 = (5, 5, 2)$,
$q^3 = (10, 10, 2)$,
The transition graph is shown in Fig. 3.

The crisp rules used to generate the transition graph are

IF $x(t) < 0.523$ AND $q(t^-) = q^1$ THEN $q(t) = q^1$
IF $x(t) \geq 0.523$ AND $q(t^-) = q^1$ THEN $q(t) = q^2$
IF $x(t) < 0.523$ AND $q(t^-) = q^2$ THEN $q(t) = q^1$
IF $0.523 \leq x(t) < 1.047$ AND $q(t^-) = q^2$ THEN $q(t) = q^2$
IF $x(t) \geq 1.047$ AND $q(t^-) = q^2$ THEN $q(t) = q^3$

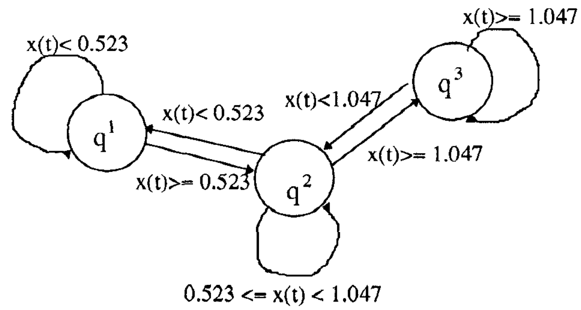

Figure 3. The transition graph of the discrete component.

IF $x(t) < 1.047$ AND $q(t^-) = q^3$ THEN $q(t) = q^2$
IF $x(t) \geq 1.047$ AND $q(t^-) = q^3$ THEN $q(t) = q^3$

The system input $u(t)$ was chosen to be $u = 60 \cdot \sin 10t$ in order to excite the system so that enough data are available for model identification. The goal of the identification is to reconstruct the behavior of both the continuous and the discrete components of the above simulation model. Since the identification of the continuous component was already outlined in the previous section here we only deal with the discrete component in terms of its black box model. This model is constructed in two steps (see Fig. 4).

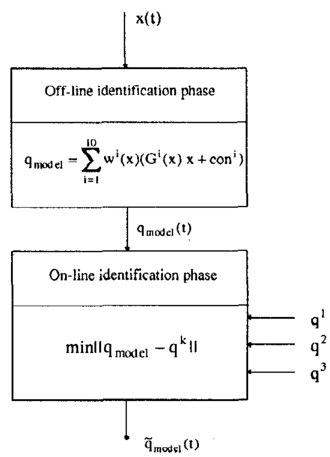

Figure 4. Two-step identification of the discrete component

Step 1: Off-line identification phase
The discontinuous behavior of the discrete state $q(t)$ is approximated and smoothed by means of a fuzzy multiple model with 10 linear submodels (clusters) which is

identified off-line by using as input only the continuous state $x(t)$. That is

$$q_{model} = \sum_{i=1}^{10} w^i(x)(G^i(x)x + con^i), \tag{18}$$

where $G^i(x)$ is a gain matrix and con^i is a constant vector. Fig. 5 shows the identification results using fuzzy clustering.

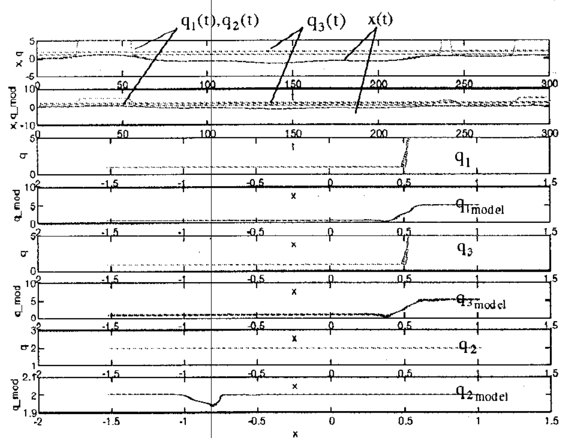

Figure 5. Identification results at Step 1.

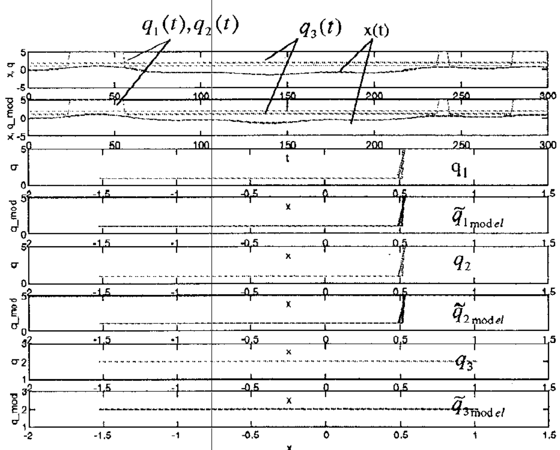

Figure 6. Identification results at Step 2.

$q_1(t)$, $q_2(t)$, and $q_3(t)$ are the three components of the discrete state $q^k = (q_1, q_2, q_3)^k$. q^k denotes a particular state for which the q_i's acquire different values, e.g., $q^1 = (1, 1, 2)$. Fig. 5c, e, and g show the q_i's of the simulated system. Fig. 5d, f, and h show the q_{imodel}'s of the identified system. It can be seen that, although fuzzy clustering learns the step of $q_1(t)$ and $q_3(t)$, the transition from one discrete state to another is smooth instead of being crisp like in the simulated system. This drawback will be rectified at Step 2.

Step 2: On-line identification phase
The already identified (18) is used to produce on-line data $q_{model}(t)$. At every time step t the identified state q_{model} is compared to all possible three discrete states q^k. Then a state q^i with the smallest Euclidian norm $||q - q_{model}||$ is chosen to be the actual discrete state for the time step t. Figure 6 shows simulation results where the behavior of $\tilde{q}_{model}(t)$ is almost identical to that of the $q(t)$ used in the simulation.

The identification of the continuous component was carried out by fuzzy clustering of the x, u - space based on different data sets each of them belonging to a particular discrete state q^k $(k = 1, \ldots, 3)$. We used 10 clusters and identified the parameter matrices A^{jk},B^{jk}, and $const^{jk}$ $(j = 1, \ldots, 10; k = 1, \ldots, 3)$ for each local linear model in (19)

$$\dot{x} = f^k = \sum_{j=1}^{10} w^{jk}(x, u)(A^{jk}x + B^{jk}u + const^{jk}). \tag{19}$$

The identified model was verified by means of the control input $u = 61 \cdot \sin(25t) + u_d$ where $u_d = 50$ if $0 \leq t < 100$ OR $200 \leq t < 300$, and $u_d = 0$ if $100 < t < 200$. Figure 7 shows the behavior of the simulated system. Figure 8 shows encouraging results using the fuzzy switched hybrid system model showing only small differences in the dynamical behavior of the system and that of the hierarchical model.

5 Conclusions

The paper describes the combination of hybrid and fuzzy multiple model systems which results in so-called *fuzzy switched hybrid systems.* A crucial point is the interaction between the discrete and the continuous component of the system and their hierarchical identification. A special case is the identification of the discrete component which is carried out by black box fuzzy clustering and identification by means of continuous and discrete data. Then, it follows a so-called on-line identification step taking into account some prior-knowledge about the discrete states. Simulation results show the power of the identification procedure presented. Further research will concentrate on the identification of the structure of the discrete component, i.e., the transition graph, and on the analysis of fuzzy switched hybrid systems. The resulting methods will be applied to the field of automotive and traffic systems.

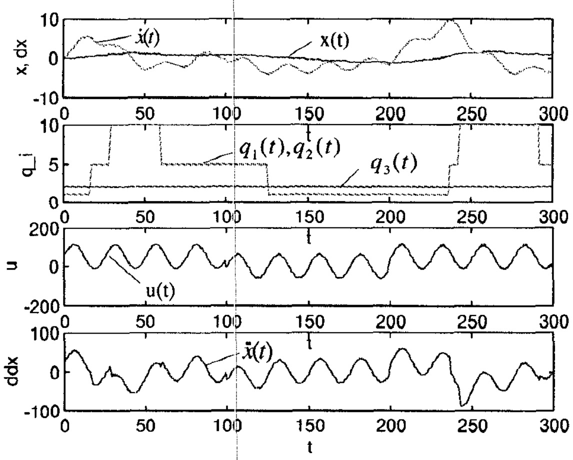

Figure 7. Evolution of the simulated system

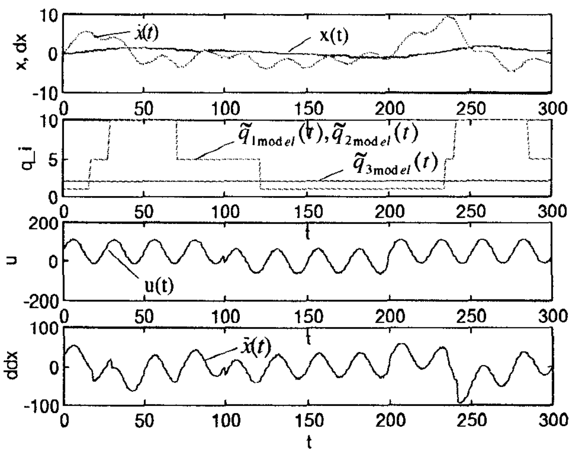

Figure 8. Evolution of the fuzzy switched hybrid system model

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