Introduction to Computer Science

CSCI 109

Readings

St. Amant, Ch. 4, Ch. 8

Andrew Goodney

Fall 2017

"An **algorithm** (pronounced AL-go-rith-um) is a procedure or formula for solving a problem. The word derives from the name of the mathematician, Mohammed ibn-Musa al-Khwarizmi, who was part of the royal court in Baghdad and who lived from about 780 to 850."

Reminders

- Quiz 2 today (covers lecture material from 1/30 and 2/6)
- No lecture next week (Feb 20) due to Presidents' day
- Quiz 3 on Feb 27 (covers lecture material from today (2/13))
- HW2 due on 2/27
- Midterm on 3/20

Where are we?

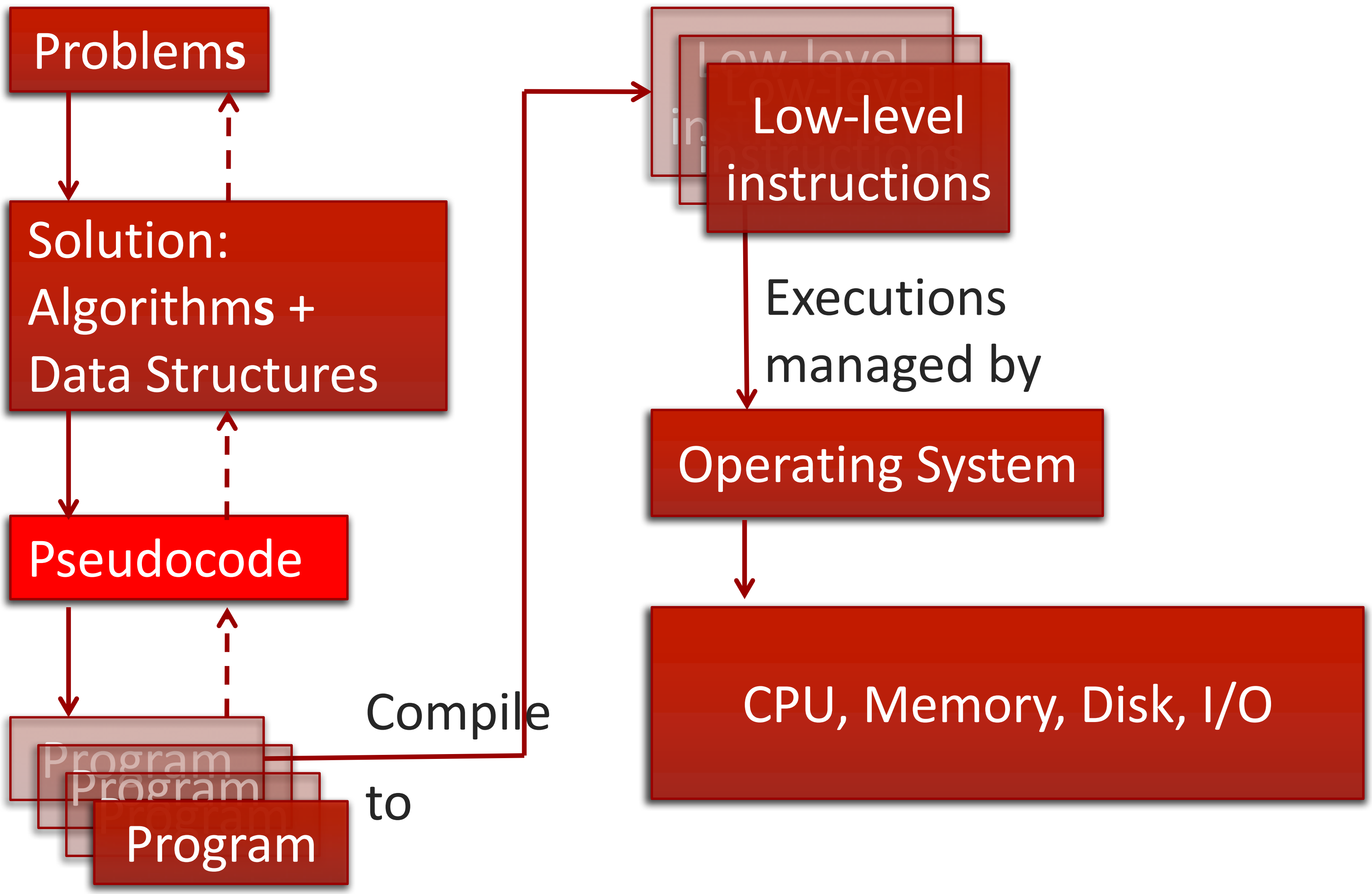
Date	Topic		Assigned	Due	Quizzes/Midterm/Final
21-Aug	Introduction	What is computing, how did computers come to be?			
28-Aug	Computer architecture	How is a modern computer built? Basic architecture and assembly	HW1		
4-Sep	Labor day				
11-Sep	Data structures	Why organize data? Basic structures for organizing data		HW1	
12-Sep	Last day to drop a Monday-only class without a mark of “W” and receive a refund or change to Pass/No Pass or Audit for Session 001				
18-Sep	Data structures	Trees, Graphs and Traversals	HW2		**Quiz 1** on material taught in class 8/21-8/28
25-Sep	More Algorithms/Data Structures	Recursion and run-time			
2-Oct	Complexity and combinatorics	How "long" does it take to run an algorithm.		HW2	**Quiz 2** on material taught in class 9/11-9/25
6-Oct	Last day to drop a course without a mark of “W” on the transcript				
9-Oct	Algorithms and programming	(Somewhat) More complicated algorithms and simple programming constructs			**Quiz 3** on material taught in class 10/2
16-Oct	Operating systems	What is an OS? Why do you need one?	HW3		**Quiz 4** on material taught in class 10/9
23-Oct	Midterm	Midterm			**Midterm** on all material taught so far.
30-Oct	Computer networks	How are networks organized? How is the Internet organized?		HW3	
6-Nov	Artificial intelligence	What is AI? Search, plannning and a quick introduction to machine learning			**Quiz 5** on material taught in class 10/30
10-Nov	Last day to drop a class with a mark of “W” for Session 001				
13-Nov	The limits of computation	What can (and can't) be computed?	HW4		**Quiz 6** on material taught in class 11/6
20-Nov	Robotics	Robotics: background and modern systems (e.g., self-driving cars)			**Quiz 7** on material taught in class 11/13
27-Nov	Summary, recap, review	Summary, recap, review for final		HW4	**Quiz 8** on material taught in class 11/20
8-Dec	Final exam 11 am - 1 pm in SAL 101				**Final** on all material covered in the semester

Data Structures and Algorithms

- A problem-solving view of computers and computing
- Organizing information: sequences and trees
- Organizing information: graphs
- Abstract data types: recursion

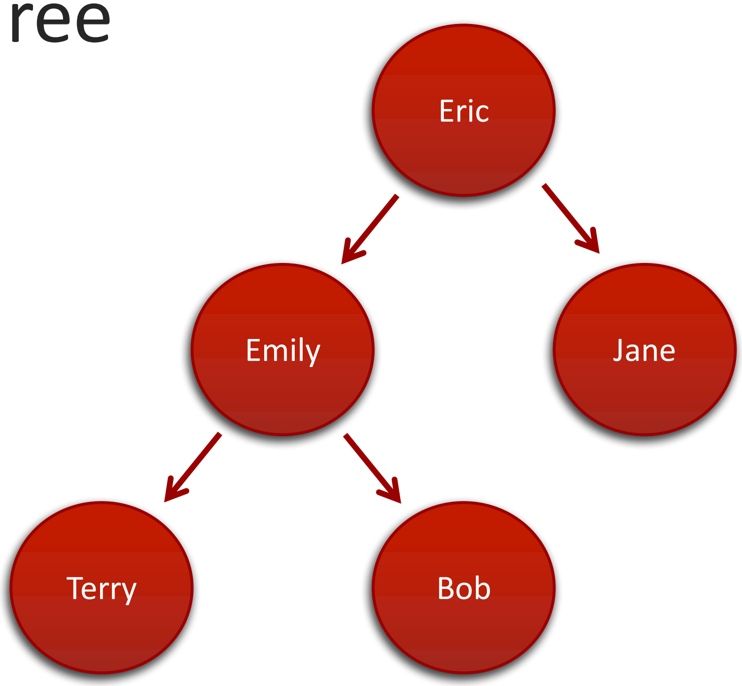
Reading:
St. Amant   Ch. 4
Ch. 8 (partial)

Overview

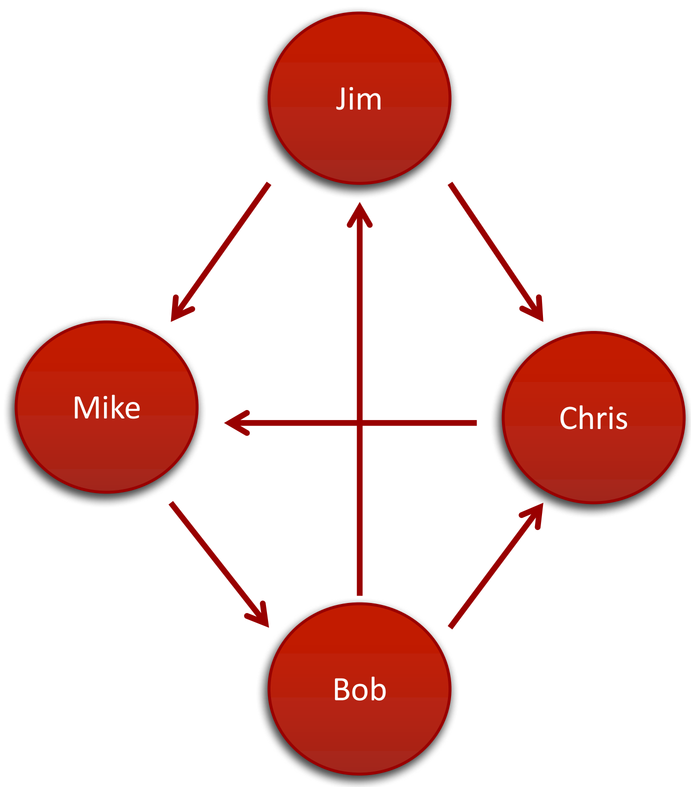

Problems
Solution:
Algorithms +
Data Structures
Pseudocode
Program
Compile
to
Low-level
instructions
Executions
managed by
Operating System
CPU, Memory, Disk, I/O

Sequences, Trees and Graphs

- Sequence: a list
  - Items are called elements
  - Item number is called the index
- Tree



- Graph

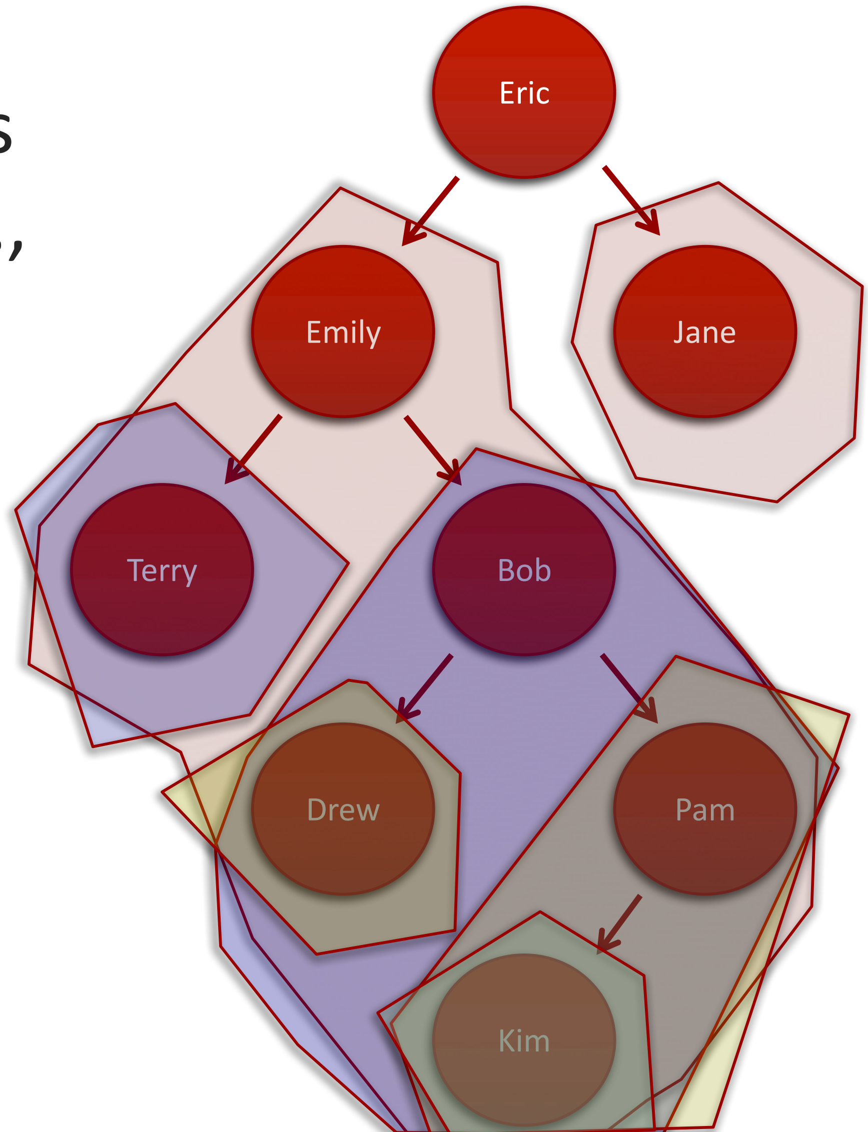


Recursion: abstract data types

- Defining abstract data types in terms of themselves (e.g., trees contain trees)
- So a tree is

  Either a single vertex, or

  a vertex that is the parent of one or more trees



Recursion: algorithms

- Defining algorithms in terms of themselves (e.g., quicksort)

  Check whether the sequence has just one element. If it does, stop

  Check whether the sequence has two elements. If it does, and they are in the right order, stop. If they are in the wrong order, swap them, stop.

  Choose a pivot element and rearrange the sequence to put lower-valued elements on one side of the pivot, higher-valued elements on the other side

  Quicksort the lower elements

  Quicksort the higher elements

Recursion: algorithms

- How do you write a selection sort recursively ?
- How do you write a breadth-first search of a tree recursively ? What about a depth-first search ?

Analysis of algorithms

- How long does an algorithm take to run?
  time complexity
- How much memory does it need?
  space complexity

Estimating running time

- How to estimate algorithm running time?
  - Write a program that implements the algorithm, run it, and measure the time it takes
  - Analyze the algorithm (independent of programming language and type of computer) and calculate in a general way how much work it does to solve a problem of a given size
- Which is better?

Analysis of binary search

Problem 2: Binary Search
[10 points]

You are given a list of *n* numbers in sorted order: the number at position *1* is the smallest; the number at position *n* is the largest. You need to find if a particular number (call it *a*) is in the sorted list. Write an algorithm to perform a binary search on this list to perform the task of finding whether *a* is in the list. If *a* is in the list, the algorithm should report its position in the list. If *a* is not in the list, the algorithm should report this fact.

When *n=8*, how many steps does it take the algorithm to find the answer to whether *a* is in the list?

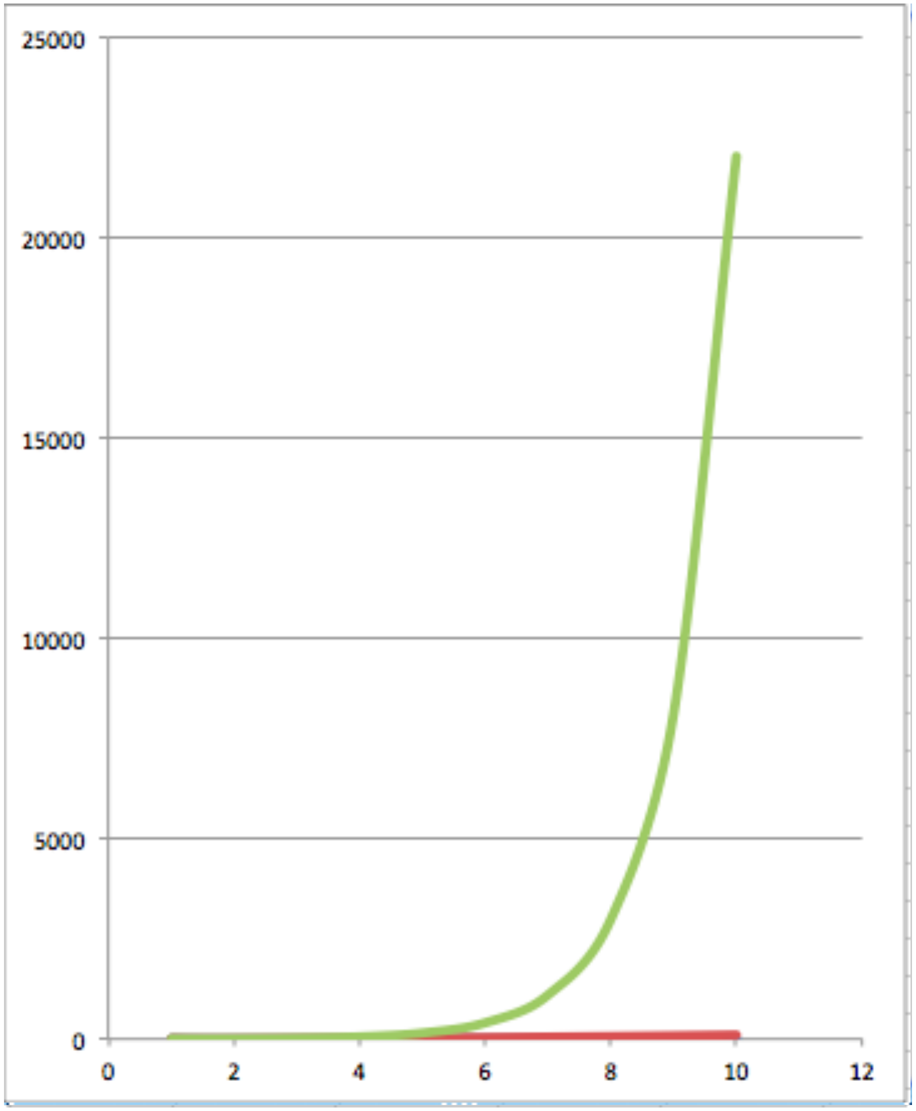
When *n=32*, how many steps does it take the algorithm to find the answer to whether *a* is in the list?

For a general value of *n*, how many steps does it take the algorithm to find the answer to whether *a* is in the list?

- n = 8, the algorithm takes 3 steps
- n = 32, the algorithm takes 5 steps
- For a general n, the algorithm takes $\log_2 n$ steps

Growth rates of functions

- Linear
- Quadratic
- Exponential



Big O notation

- Characterize functions according to how fast they grow
- The growth rate of a function is called the **order of the function**. (hence the O)
- Big O notation usually only provides an upper bound on the growth rate of the function
- Asymptotic growth

$f(x) = O(g(x))$ as $x \to \infty$ if and only if there exists a positive number M such that $f(x) \leq M * g(x)$ for all $x > x_0$

Examples

- $f(n) = 3n^2 + 70$
  - We can write $f(n) = O(n^2)$
  - What is a value for M?
- $f(n) = 100n^2 + 70$
  - We can write $f(n) = O(n^2)$
  - Why?
- $f(n) = 5n + 3n^5$
  - We can write $f(n) = O(n^5)$
  - Why?
- $f(n) = n \log n$
  - We can write $f(n) = O(n \log n)$
  - Why?
- $f(n) = \pi n^n$
  - We can write $f(n) = O(n^n)$
  - Why?
- $f(n) = (\log n)^5 + n^5$
  - We can write $f(n) = O(n^5)$
  - Why?

Examples

- $f(n) = log_a n$ and $g(n) = log_b n$ are both asymptotically $O(log\ n)$
  - The base doesn't matter because $log_a n = log_b n / log_b a$
- $f(n) = log_a n$ and $g(n) = log_a(n^c)$ are both asymptotically $O(log\ n)$
  - Why?
- $f(n) = log_a n$ and $g(n) = log_b(n^c)$ are both asymptotically $O(log\ n)$
  - Why?
- What about $f(n) = 2^n$ and $g(n) = 3^n$?
  - Are they both of the same order?

Conventions

- $O(1)$ denotes a function that is a constant
  - $f(n) = 3$, $g(n) = 100000$, $h(n) = 4.7$ are all said to be $O(1)$

- For a function $f(n) = n^2$ it would be perfectly correct to call it $O(n^2)$ or $O(n^3)$ (or for that matter $O(n^{100})$)
- However by convention we call it by the smallest order namely $O(n^2)$

Complexity

- (Binary) search of a sorted list: $O(log_2 n)$
- Selection sort
- Quicksort
- Breadth first traversal of a tree
- Depth first traversal of a tree
- Prim’s algorithm to find the MST of a graph
- Kruskal’s algorithm to find the MST of a graph
- Dijkstra’s algorithm to find the shortest path from a node in a graph to all other nodes

Selection sort

- Putting the smallest element in place requires scanning all *n* elements in the list (and *n-1* comparisons)
- Putting the second smallest element in place requires scanning *n-1* elements in the list (and *n-2* comparisons)
- …
- Total number of comparisons is
  - $(n-1) + (n-2) + (n-3) + \ldots + 1$
  - $n(n-1)/2$
  - $O(n^2)$
- There is no difference between the best case, worst case and average case

Quicksort

- Best case:
  - Assume an ideal pivot
  - The average depth is $O(\log n)$
  - Each level of processes at most n elements
  - The total amount of work done on average is the product, $O(n \log n)$
- Worst case:
  - Each time the pivot splits the list into one element and the rest
  - So, (n-1) + (n-2) + (n-3) + … (1)
  - $O(n^2)$
- Average case:
  - $O(n \log n)$ [but proving it is a bit beyond CS 109]

BF and DF traversals of a tree

- A breadth first traversal visits the vertices of a tree level by level
- A depth first traversal visit the vertices of a tree by going deep down one branch and exhausting it before popping up to visit another branch
- What do they have in common?

BF and DF traversals of a tree

- A breadth first traversal visits the vertices of a tree level by level
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- What do they have in common?
- Both visit **all** the vertices of a tree
- If a tree has *V* vertices, then both BF and DF are *O(V)*

Prim’s algorithm

- Initialize a tree with a single vertex, chosen arbitrarily from the graph
- Grow the tree by adding one vertex. Do this by adding the minimum-weight edge chosen from the edges that connect the tree to vertices not yet in the tree
- Repeat until all vertices are in the tree
- How fast it goes depends on how you store the vertices of the graph
- If you don’t keep the vertices of the graph in some readily sorted order then the complexity is $O(V^2)$ where the graph has V vertices

Kruskal's algorithm

- Initialize a tree with a single edge of lowest weight
- Add edges in increasing order of weight
- If an edge causes a cycle, skip it and move on to the next highest weight edge
- Repeat until all edges have been considered
- Even without much thought on how the edges are stored (as long as we sort them once in the beginning), the complexity is *O(E log E)* where the graph has *E* edges

Dijkstra's algorithm

- At each iteration we refine the distance estimate through a new vertex we're currently considering
- In a graph with *V* vertices, a loose bound is $O(V^2)$

Recap

- (Binary) search of a sorted list: $O(\log_2 n)$
- Selection sort: $O(n^2)$
- Quicksort: $O(n \log n)$
- Breadth first traversal of a tree: O(V)
- Depth first traversal of a tree: O(V)
- Prim's algorithm to find the MST of a graph: $O(V^2)$
- Kruskal's algorithm to find the MST of a graph: $O(E \log E)$
- Dijkstra's algorithm to find the shortest path from a node in a graph to all other nodes: $O(V^2)$

What do they have in common?

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A knapsack problem

- You have a knapsack that can carry 20 lbs
- You have books of various weights
- Is there a collection of books whose weight adds up to exactly 20 lbs?
- Can you enumerate all collections of books that are 20 lbs

Book	Weight
Book 1	2
Book 2	3
Book 3	13
Book 4	7
Book 5	10
Book 6	6

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How many combinations are there?

# of books	Combinations	Combinations
0	{}	1
1	{2} {3} {13} {7} {10} {6}	6
2	{2,3} {2,13} {2,7} {2,10} {2,6} {3,13} {3,7} {3,10} {3,6} {13,7} {13,10} {13,6} {7,10} {7,6} {10,6}	15
3	{2,3,13} {2,13,7} {2,7,10} {2,10,6} {2,3,7} {2,3,10} {2,3,6} {2,13,10} {2,13,6} {2,7,6} {3,13,7} {3,13,10} {3,13,6} {3,7,10} {3,7,6} {3,10,6} {13,7,10} {13,10,6} {13,7,6} {7,10,6}	20
4	{2,3,13,7} {2,3,13,10} {2,3,13,6} {2,3,7,10} {2,3,7,6} {2,3,10,6} {2,13,7,10} {2,13,10,6} {2,13,7,6} {2,7,10,6} {3,13,7,10} {3,13,10,6} {3,13,7,6} {3,7,10,6} {13,7,10,6}	15
5	{2,3,13,7,10} {3,13,7,10,6} {13,7,10,6,2} {7,10,6,2,3} {10,6,2,3,13} {6,2,3,13,7}	6
6	{2,3,13,7,10,6}	1
	TOTAL	64

How many combinations are there?

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3	{2,3,13} {2,13,7} {2,7,10} {2,10,6} {2,3,7} {2,3,10} {2,3,6} {2,13,10} {2,13,6} {2,7,6} {3,13,7} {3,13,10} {3,13,6} {3,7,10} {3,7,6} {3,10,6} {13,7,10} {13,10,6} {13,7,6} {7,10,6}	20
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6	{2,3,13,7,10,6}	1
	TOTAL	64

Subset sum problem

- Given a set of integers and an integer *s*, does any non-empty subset sum to *s*?
- {1, 4, 67, -1, 42, 5, 17} and $s = 24$ *No*
- {4, 3, 17, 12, 10, 20} and $s = 19$ *Yes* {4, 3, 12}

- If a set has N elements, it has 2^N subsets.
- Checking the sum of each subset takes a maximum of N operations
- To check all the subsets takes $2^N N$ operations
- Some cleverness can reduce this by a bit (2^N becomes $2^{N/2}$, but all known algorithms are exponential – i.e. $O(2^N N)$

Big O notation

- Characterize functions according to how fast they grow
- The growth rate of a function is called the **order of the function**. (hence the O)
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What do they have in common?

- (Binary) search of a sorted list: $O(\log_2 n)$
- Selection sort: $O(n^2)$
- Quicksort: $O(n \log n)$
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Travelling salesperson problem

- Given a list of cities and the distances between each pair of cities, what is the shortest possible route that visits each city exactly once and returns to the origin city?
- Given a graph where edges are labeled with distances between vertices. Start at a specified vertex, visit all other vertices exactly once and return to the start vertex in such a way that sum of the edge weights is minimized
- There are $n!$ routes (a number on the order of n^n - much bigger than 2^n)
- $O(n!)$

Enumerating permutations

- List all permutations (i.e. all possible orderings) of *n* numbers
- What is the order of an algorithm that can do this?

Enumerating permutations

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- What is the order of an algorithm that can do this?
- *O(n!)*

Analysis of problems

- Study of algorithms illuminates the study of <u>classes</u> of problems
- If a polynomial time algorithm exists to solve a problem then the problem is called *tractable*
- If a problem cannot be solved by a polynomial time algorithm then it is called *intractable*
- This divides problems into #? groups:

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- If a polynomial time algorithm exists to solve a problem then the problem is called *tractable*
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- This divides problems into three groups:
  - Problems with known polynomial time algorithms
  - Problems that are proven to have no polynomial-time algorithm
  - Problems with no known polynomial time algorithm but not yet proven to be intractable

Tractable and Intractable

- Tractable problems (**P**)
  - Sorting a list
  - Searching an unordered list
  - Finding a minimum spanning tree in a graph
- Intractable
  - Listing all permutations (all possible orderings) of n numbers
- Might be (in)tractable
  - Subset sum: given a set of numbers, is there a subset that adds up to a given number?
  - Travelling salesperson: n cities, n! routes, find the shortest route

These problems have no known polynomial time solution

However no one has been able to prove that such a solution does not exist

Tractability and Intractability

- 'Properties of problems' (*NOT* 'properties of algorithms')
- Tractable: problem can be solved by a polynomial time algorithm (or something more efficient)
- Intractable: problem cannot be solved by a polynomial time algorithm (all solutions are proven to be more inefficient than polynomial time)
- Unknown: not known if the problem is tractable or intractable (no known polynomial time solution, no proof that a polynomial time solution does not exist)

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P and NP

- **P**: set of problems that can be solved in polynomial time

Easy to solve

- Consider subset sum
  - No known polynomial time algorithm
  - However, if you give me a solution to the problem, it is easy for me to check if the solution is correct – i.e. I can write a polynomial time algorithm to check if a given solution is correct
- **NP**: set of problems for which a solution can be checked in polynomial time

Easy to check

P=NP?

- All problems in **P** are also in **NP**
- Are there any problems in **NP** that are not also in **P**?
- In other words, is

$$\boxed{\mathbf{P = NP\ ?}}$$

- Central open question in Computer Science

Data Structures and Algorithms

- A problem-solving view of computers and computing
- Organizing information: sequences and trees
- Organizing information: graphs
- Abstract data types: recursion

Reading:
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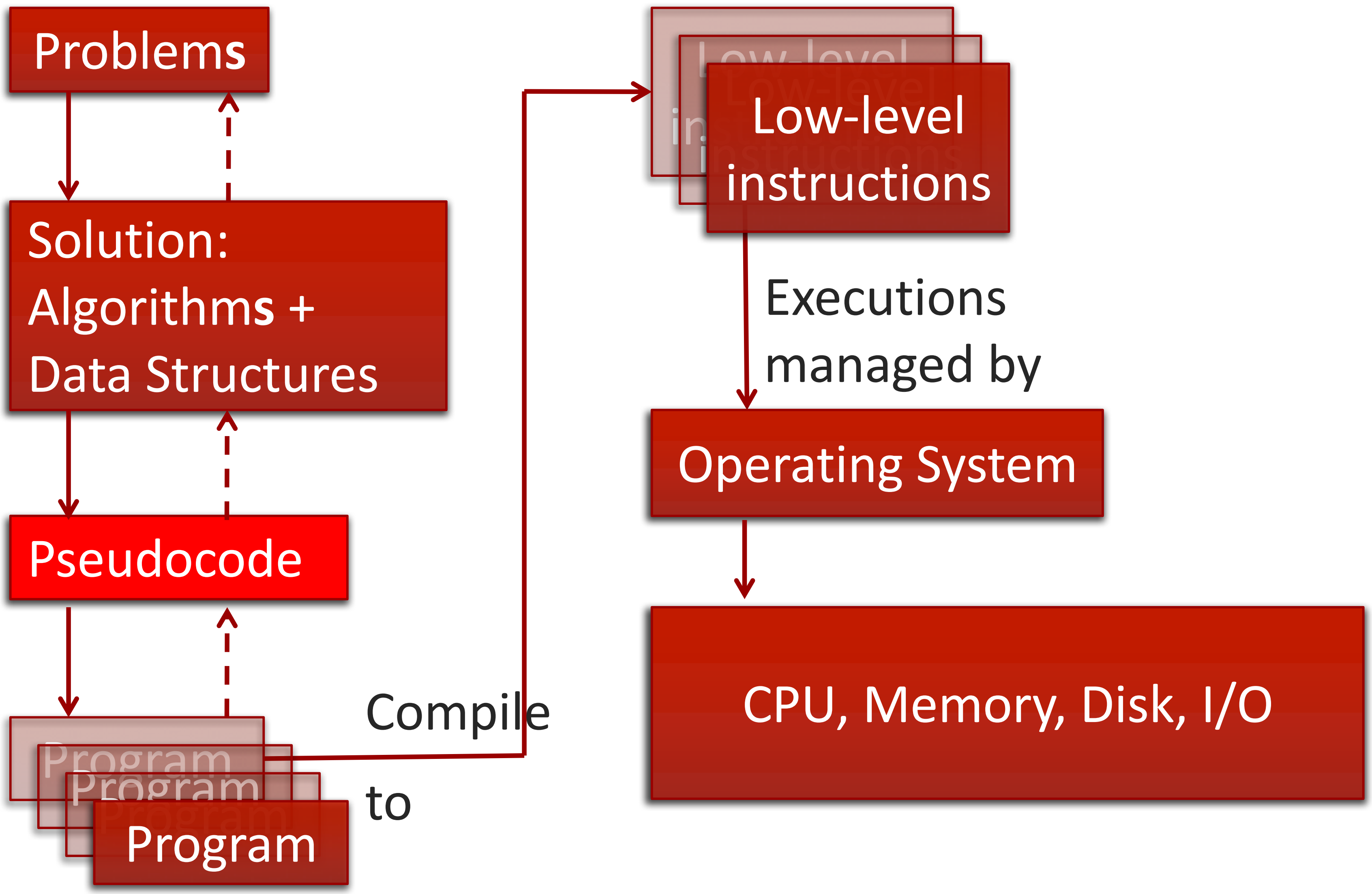
Ch. 8 (partially)

Overview



Problems
Solution:
Algorithms +
Data Structures
Pseudocode
Program
Compile
to
Low-level
instructions
Executions
managed by
Operating System
CPU, Memory, Disk, I/O

Quiz 2

1. Which statement is false?

(A) A graph may have multiple spanning trees

(B) A graph may have multiple minimum spanning trees

(C) Dijkstra's algorithm returns the same minimum spanning tree as Prim's algorithm

(D) A tree has no cycles

(E) Selection sort is a brute-force algorithm

2. Which statement is false?

(A) There may be multiple paths between two nodes in a tree

(B) Graphs may contain loops

(C) A tree may be weighted or unweighted

(D) Every tree is a graph

(E) Quicksort is a divide-and-conquer algorithm

3. When sorting a list of *n* unique numbers, the Quicksort algorithm chooses the number *a* as the first pivot, and places it in index *i* of the list. What will be the index of the number a when the algorithm finishes?

(A) 1 (B) *i* (C) *a* (D) *n* (E) Can't tell from the information provided

4. An unweighted, undirected tree has 16 edges. How many vertices does it have?

(A) 16 (B) 15 (C) 8 (D) 17 (E) None of the choices A-D is correct

5. An unweighted, undirected graph has 5 vertices. What is the maximum number of edges this graph could have (assuming each edge connects two distinct vertices)

(A) 5 (B) 14 (C) 10 (D) 25 (E) None of the choices A-D is correct