Hauptseminar: Logjam

by Li Yang Wu

Content

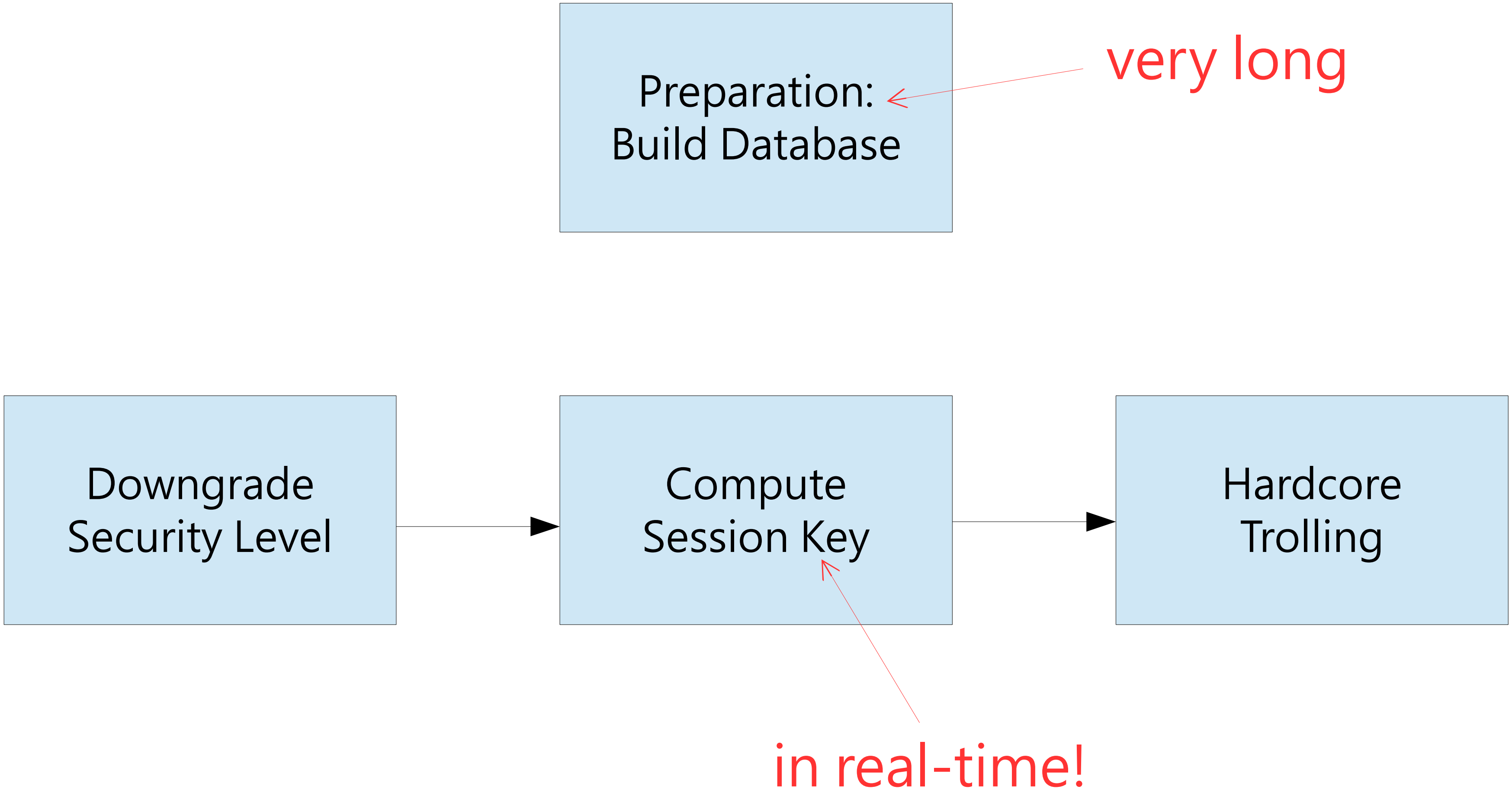
- Logjam
- Diffie-Hellman Key Exchange
- Number Field Sieve
- Transport Layer Security Handshake
- Logjam Summary
- Weak DH Parameters
- Breaking 1024 bit Groups

Logjam

- Attack on network security protocol
  - Secure client-server communication
- Exploits TLS 1.2 handshake flaw
- Downgrade security level in Diffie-Hellman
- In theorie: easy to avoid
- In practice: high success

Logjam

- Approach:



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DH Key Exchange

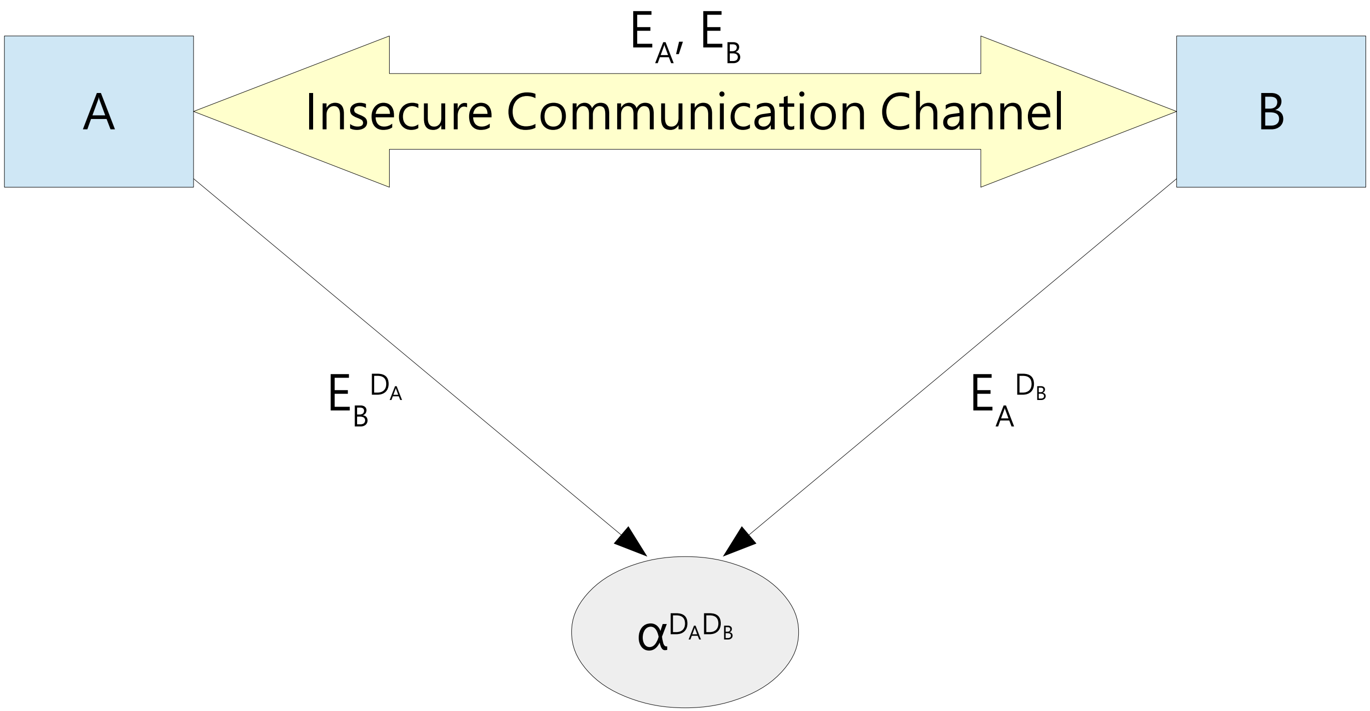
- Given:
  - Communication partners A and B
  - Insecure communication channel
  - Message space
- Determine:
  - Public key cryptosystem
  - Key space
  - … with exponential ciper-cryptanalyst ratio

DH Key Exchange

- Define key space:
  - Choose prime q
  - Define number field GF(q) = $\mathbb{Z}/q\mathbb{Z}$
  - Choose basis α in GF(q)
- Draw keys:
  - Choose private key D in GF(q)
  - Calculate public key $E = \alpha^D \bmod q$
- We have: $D_{A'}$ $D_{B'}$ $E_{A'}$ E_B

DH Key Exchange

- Session key:



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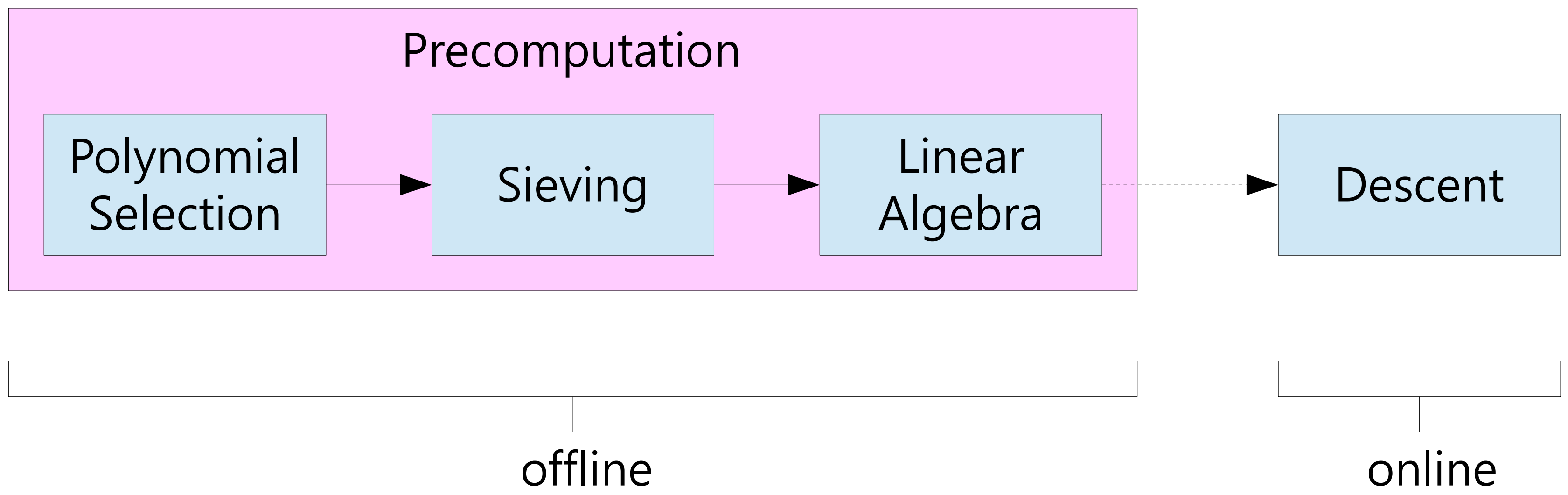
Number Field Sieve

- Approach:



Number Field Sieve

- Polynomial Selection:
  - Find polynomial f and choose m s.t. $f(m) = 0 \bmod q$
  - Based on f, find a ring of integers O and homomorphism s.t. $\varphi : \mathbb{Z}[\gamma] \rightarrow \mathbb{Z}/q\mathbb{Z}$, where γ is some root of f
- Benefit: Express knowledge about logs of factors of α with linear equations.

Number Field Sieve

- Sieving:
  - Define the set of "good" prime ideals B in O
  - Sieve through pairs of integers (c,d) that are related to elements in B
  - Form matrices from these relations
  - Modify matricies to express only information about log factors of α
- Output: Some matrices A_i

Number Field Sieve

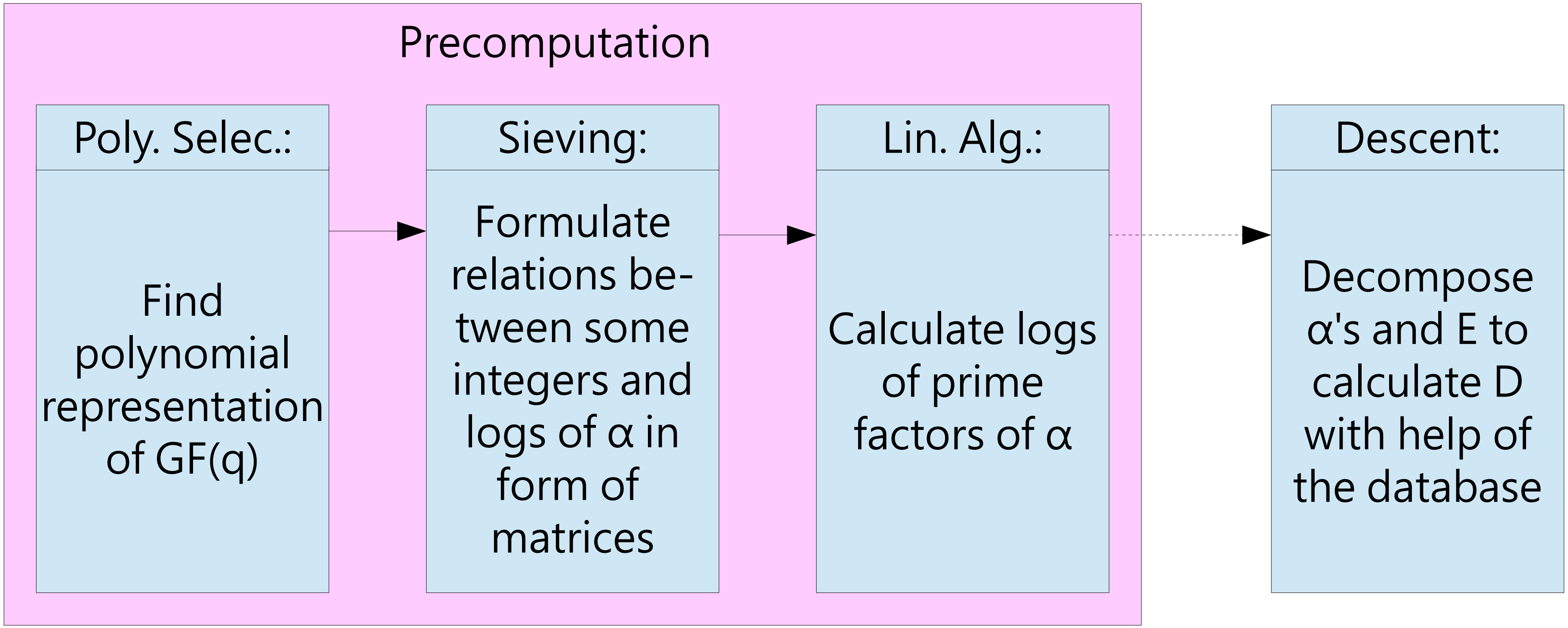
- Linear Algebra:
  - Take matrices A_i
  - Compute rank r and extract matrices A_i' with sizes of its ranks
  - Find set P of primes p_i with bounded
  - Compute det(A) mod p for each p_i in P
  - Find relations between the r+1th row and A_i'
  - Calculate logs of prime factors of α

Number Field Sieve

- Descent:
  - Find l s.t. $\alpha^l E \equiv p_1 p_2 \ldots p_t \bmod q$, for small p_i's
  - Find logs of p_i's with database
  - Compute D from these logs

Number Field Sieve

- Refined Approach:



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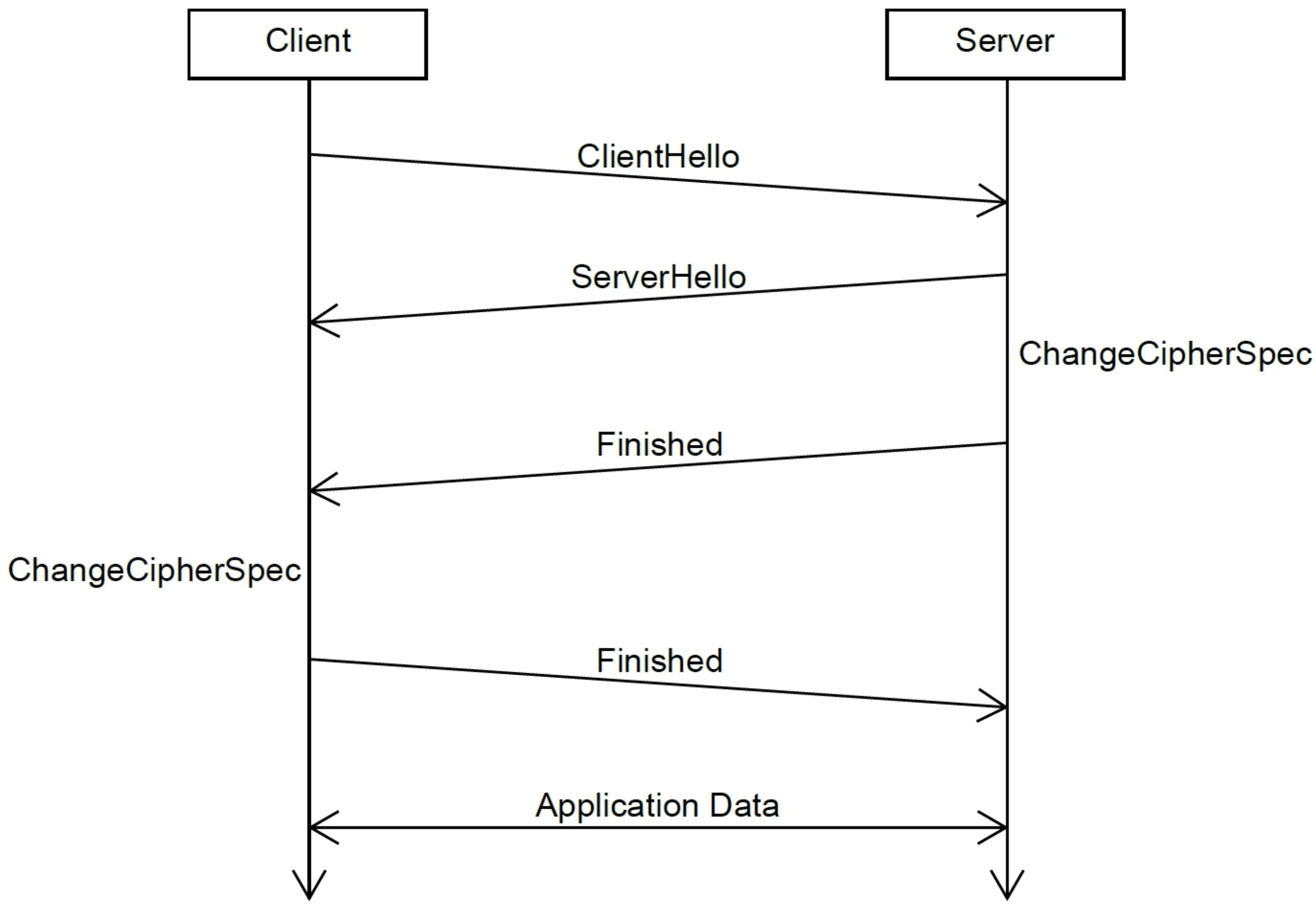
TLS Handshake



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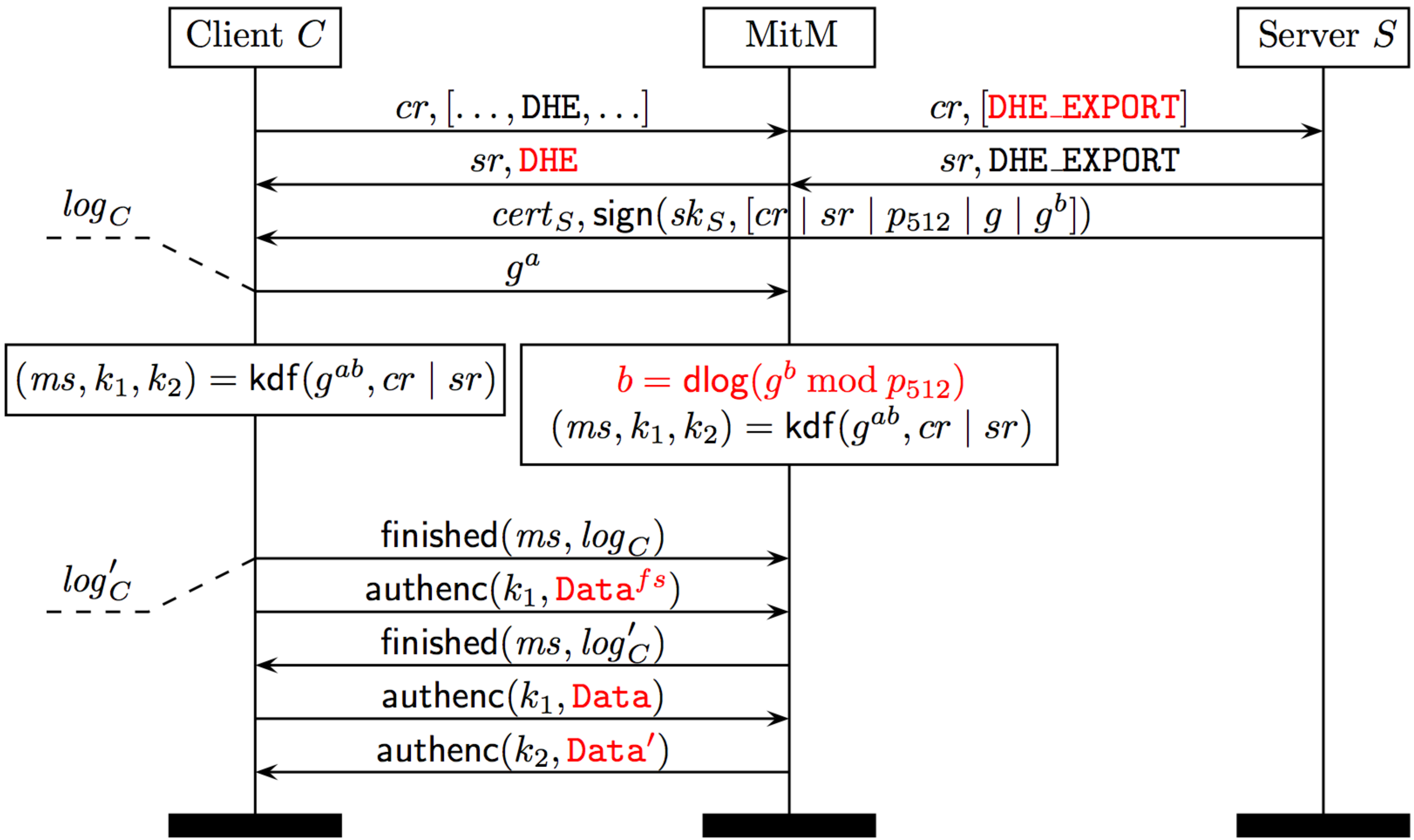
Logjam Summary



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Weak DH Parameters

- Use Pollard's lamda method and Pohlig-Hellman decomposition for an improved log calculation if E is chosen small and q is not chosen "safe".
  - Pollard's lambda method calculates logs efficiently, if it is known to lie in a fixed bound {b, ..., b+w}
  - Pohlig-Hellman decomposition extracts information about logs given some prime factors of q-1

Weak DH Parameters

- Improved attack:
  - Decompse q-1 in prime factors
  - Extract information of the log from factors for which the log is feasible to compute (Pohlig-Hellman)
  - Express the missing information as a new log problem with fixed bounds.
  - Solve with Pollard's lambda method

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Breaking 1024 bit Groups

- Only a cost estimation
- Motivation: Edward Snowden leaks
  - Assertion: NSA decripts all communication
- Assumptions:
  - Optimistic cost extrapolation of recent records in factorization and log computation
  - Existence of specialized hardware for certain tasks

Breaking 1024 bit Groups

- Result:
  - Total cost slightly over $11B
  - Budget for Consolidated Cryptographic Program plus some additional investments: over $11B