Denote $v = (v_1, v_2, ..., v_n)$ to be the ability of k individuals. For each game $t = 1, 2, ..., T$, we assume I_t^+ is the set of positive team members, I_t^- is the set of negative team members. $I = \bigcup_{t=1}^{T} (I_t^+ \cup I_t^-)$, $k_s = |\{s : s \in I\}|$, and the objective function can be simply written as

$$\begin{aligned}
l(v) &= -\sum_{t=1}^{m} (\log \frac{e^{T_t^+}}{e^{T_t^+} + e^{T_t^-}} + \log \frac{e^{T_t^-}}{e^{T_t^+} + e^{T_t^-}}) + \lambda \sum_{s=1}^{k} (e^{v_s} + e^{-v_s}) \\
&= \sum_{t=1}^{m} \{-(\log \frac{e^{T_t^+}}{e^{T_t^+} + e^{T_t^-}} + \log \frac{e^{T_t^-}}{e^{T_t^+} + e^{T_t^-}}) + \lambda \sum_{s \in I_t^+ \cup I_t^-} \frac{e^{v_s} + e^{-v_s}}{k_s}\} \\
l_t(v) &= -(\log \frac{e^{T_t^+}}{e^{T_t^+} + e^{T_t^-}} + \log \frac{e^{T_t^-}}{e^{T_t^+} + e^{T_t^-}}) + \lambda \sum_{s \in I_t^+ \cup I_t^-} \frac{e^{v_s} + e^{-v_s}}{k_s}
\end{aligned}$$

The gradient of each component in game t:

- If $s \in I_t^+$

$$\nabla_{v_s} l_t(v) = -1 + \frac{2e^{T_t^+}}{e^{T_t^+} + e^{T_t^-}} + \lambda \frac{e^{v_s} - e^{-v_s}}{k_s}$$

- If $s \in I_t^-$

$$\nabla_{v_s} l_t(v) = -1 + \frac{2e^{T_t^-}}{e^{T_t^+} + e^{T_t^-}} + \lambda \frac{e^{v_s} - e^{-v_s}}{k_s}$$

SGD algorithm:

- Input: I_t^+ and I_t^-, initial values of v
- Each time we randomly pick a game t, and update the relevant $v_s \in I_t^+ \cup I_t^-$ in the following way:

$$v_s \leftarrow v_s - \eta \nabla_{v_s} l_t(v)$$