STOR 320 Modeling V

Lecture 18

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Introduction

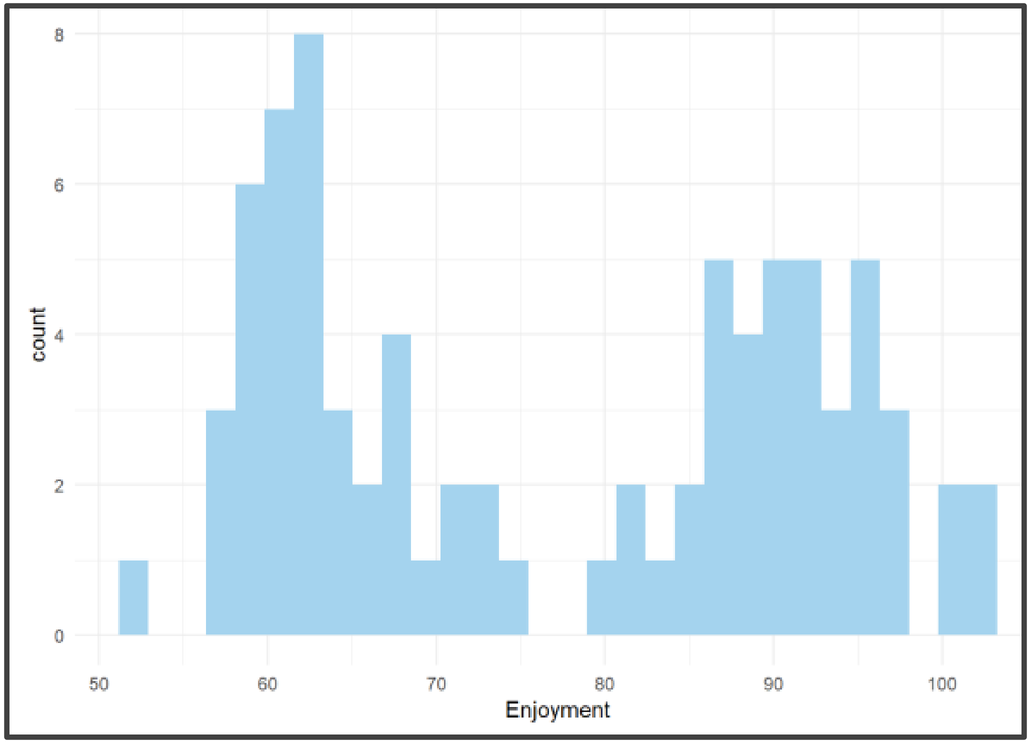
- Read Chapter 23 (R4DS)
- Previously: Numeric Variables
- New Focus
  - Categorical Predictor Variables
  - Interaction Effects
- Understand Using Multiple Datasets and Visualizations

Example 1: Data

- Data Overview
  - Enjoyment (E)
  - Food (F)
  - Condiment (C)
  - 80 Observations

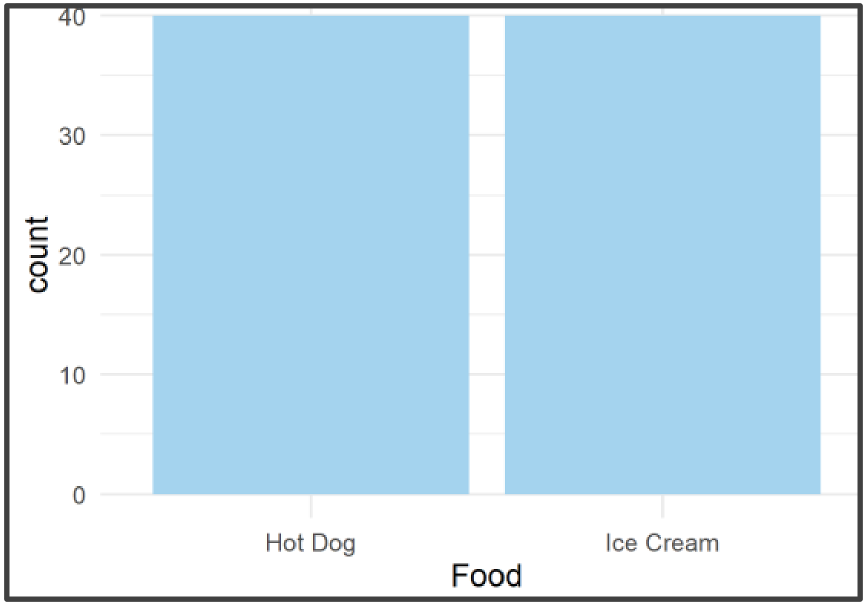
Enjoyment <dbl>	Food <chr>	Condiment <chr>
81.92696	Hot Dog	Mustard
84.93977	Hot Dog	Mustard
90.28648	Hot Dog	Mustard
89.56180	Hot Dog	Mustard
97.67683	Hot Dog	Mustard

- Enjoyment Visualized

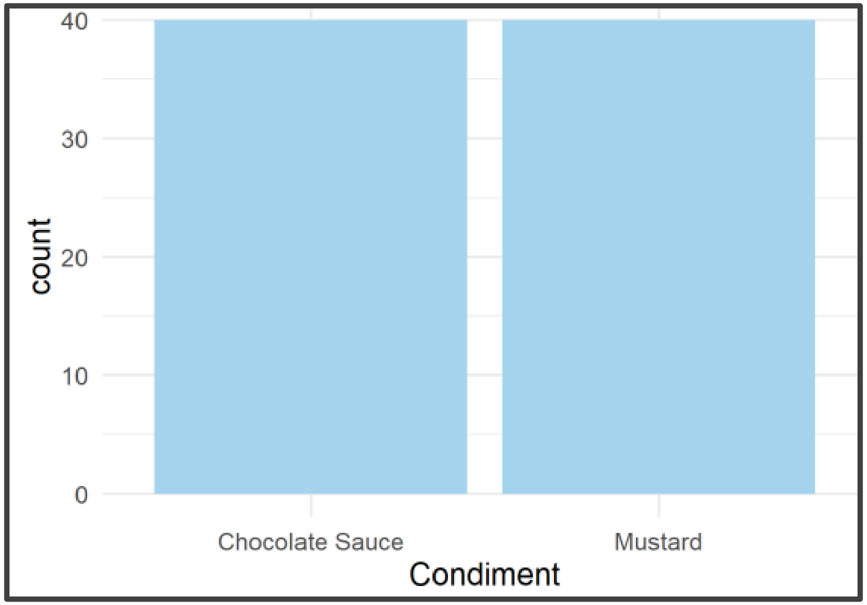


Example 1: Data

- Food Visualized



- Condiment Visualized



Example 1: Question

- Question of Interest

Can We Predict a Person's Culinary Enjoyment if…

We Serve Them a Particular Item:
- *Hot Dog*
- *Ice Cream*

With a Particular Condiment
- *Mustard*
- *Chocolate Sauce*

?

Example 1: Model 1

- Regressing E on F

```
EvsF.Model=lm(Enjoyment~Food,data=CONDIMENT)
tidy(EvsF.Model)
```

```
## # A tibble: 2 x 5
##   term           estimate std.error statistic  p.value
##   <chr>             <dbl>     <dbl>     <dbl>    <dbl>
## 1 (Intercept)      77.5        2.39   32.4     5.82e-47
## 2 FoodIce Cream   -0.283       3.39   -0.0835 9.34e- 1
```

- $\hat{E} = 77.5 - 0.283F$

- Questions:
  - What Does 77.5 Represent?
  - What About -0.283?

Example 1: Model 1

- What is R Doing?

```
CONDIMENT$Food[1:6]
```

```
## [1] "Hot Dog" "Hot Dog" "Hot Dog" "Hot Dog
" "Hot Dog" "Hot Dog"
```

```
head(model_matrix(CONDIMENT, Enjoyment~Food))
```

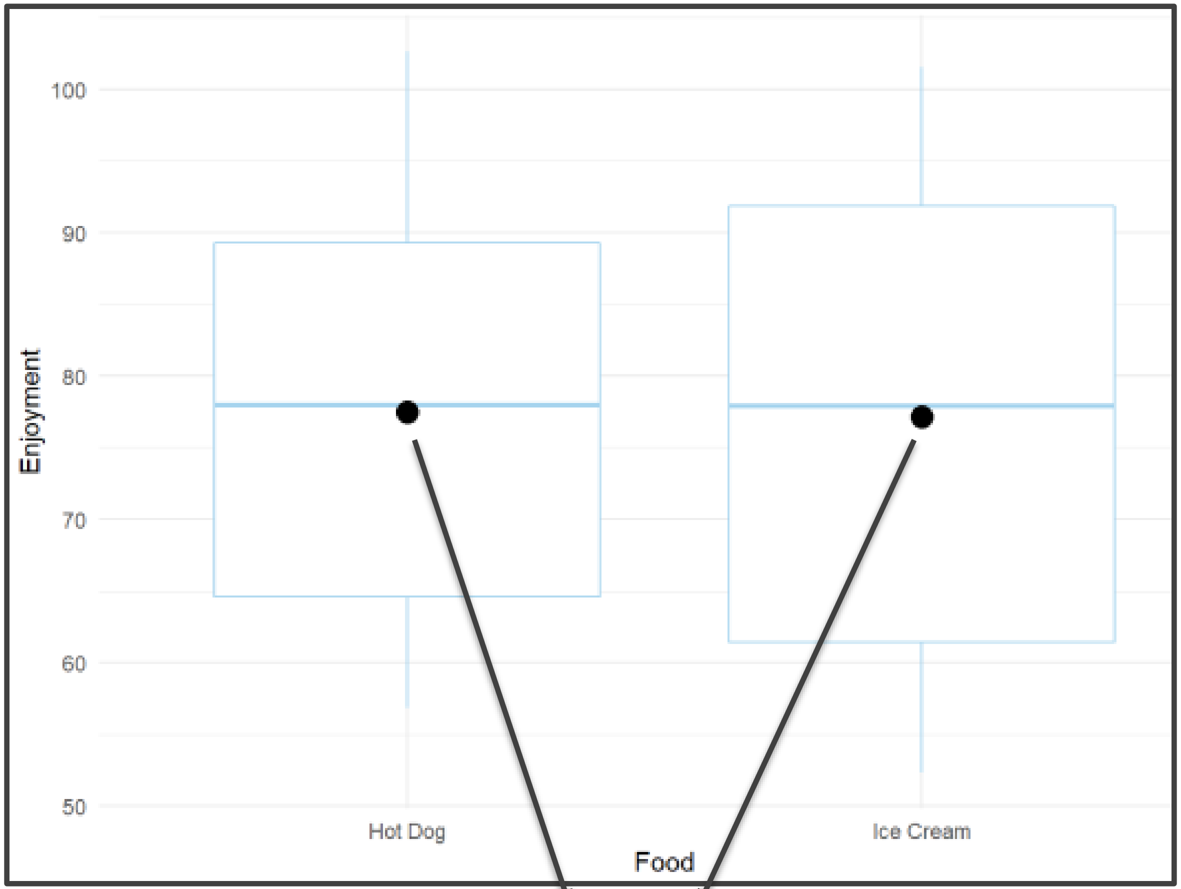
```
## # A tibble: 6 x 2
##   `(Intercept)` `FoodIce Cream`
##           <dbl>           <dbl>
## 1             1               0
## 2             1               0
## 3             1               0
## 4             1               0
## 5             1               0
## 6             1               0
```

Example 1: Interpretation

- Regressing E on F
  - $\hat{E} = 77.5 - 0.283F$
  - $F = \begin{cases} 0 & if\ Hot\ Dog \\ 1 & if\ Ice\ Cream \end{cases}$
  - If You Eat a Hot Dog,
    $\hat{E} = 77.5 - 0.283(0) = 77.5$
  - If You Eat Ice Cream,
    $\hat{E} = 77.5 - 0.283(1) = 77.217$
  - P-value = 0.934 for the Parameter Estimated by 0.283 (Not Statistically Significant)

Example 1: Interpretation

- Understanding This Visually



Means of Two Distributions

Example 1: Model 2

- Regressing E on C

```
EvsC.Model=lm(Enjoyment~Condiment,data=CONDIMENT)
tidy(EvsC.Model)
```

```
## # A tibble: 2 x 5
##   term              estimate std.error statistic  p.value
##   <chr>                <dbl>     <dbl>     <dbl>    <dbl>
## 1 (Intercept)          79.2       2.38      33.3  6.67e-48
## 2 CondimentMustard     -3.73      3.36     -1.11  2.71e- 1
```

Not Significant: P-value > 0.05

- $\hat{E} = 79.2 - 3.73C$
- $C = \begin{cases} 0 & if\ Chocolate\ Sauce \\ 1 & if\ Mustard \end{cases}$

Example 1: Model 3

- Regressing E on C + F

```
EvsCF.Model=lm(Enjoyment~Food+Condiment,data=CONDIMENT)
tidy(EvsCF.Model)
```

```
## # A tibble: 3 x 5
##   term              estimate std.error statistic  p.value
##   <chr>                <dbl>     <dbl>     <dbl>    <dbl>
## 1 (Intercept)         79.3        2.93   27.1     4.07e-41
## 2 FoodIce Cream       -0.283      3.38   -0.0836 9.34e- 1
## 3 CondimentMustard    -3.73       3.38   -1.10    2.74e- 1
```

- $\hat{E} = 79.3 - 0.283F - 3.73C$
- $F = \begin{cases} 0 & \textit{if Hot Dog} \\ 1 & \textit{if Ice Cream} \end{cases}$
- $C = \begin{cases} 0 & \textit{if Chocolate Sauce} \\ 1 & \textit{if Mustard} \end{cases}$
- What does 79.3 Represent?

Example 1: Model 3

- Obtaining Predicted Values

```
GRID=CONDIMENT %>%
      data_grid(
        Food=unique(Food),
        Condiment=unique(Condiment)
      )
print(GRID)
```

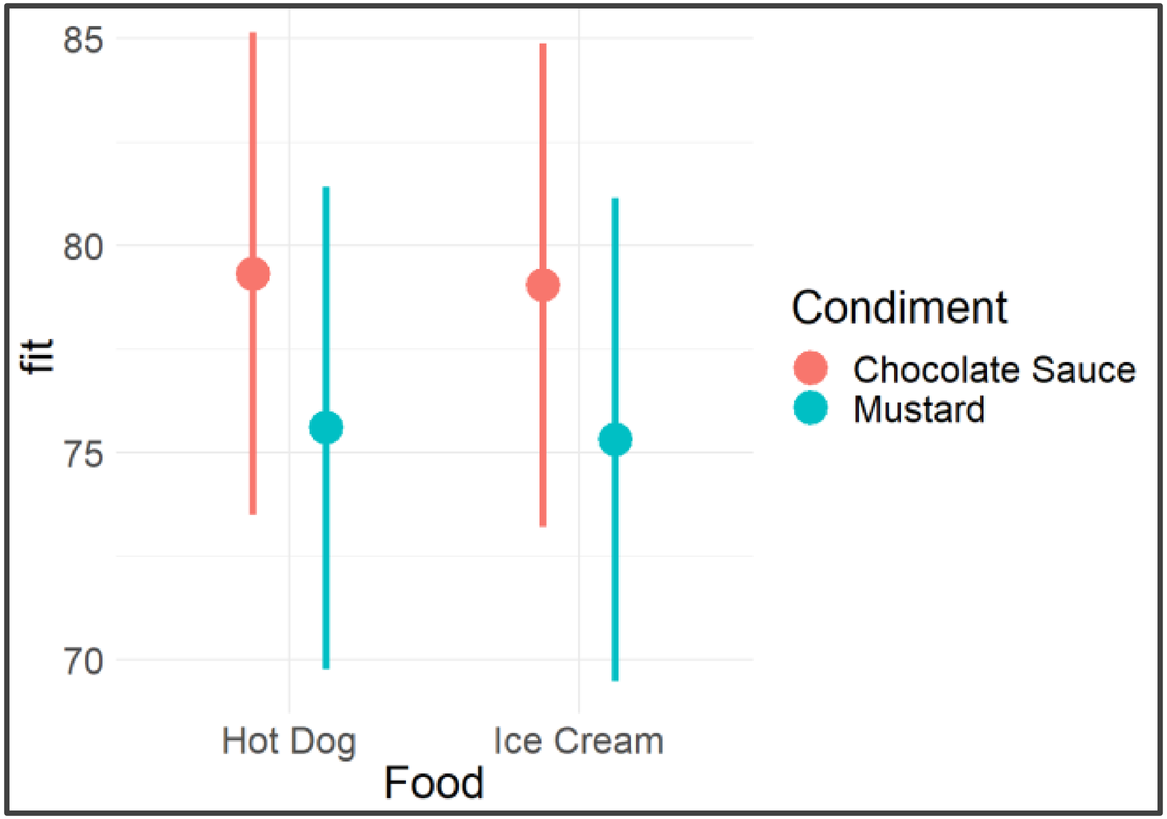
```
## # A tibble: 4 x 2
##   Food      Condiment
##   <chr>     <chr>
## 1 Hot Dog   Chocolate Sauce
## 2 Hot Dog   Mustard
## 3 Ice Cream Chocolate Sauce
## 4 Ice Cream Mustard
```

```
GRID2=cbind(GRID,predict(EvsCF.Model,
                          newdata=GRID,
                          interval="confidence"))
print(GRID2)
```

```
##        Food       Condiment      fit      lwr      upr
## 1   Hot Dog Chocolate Sauce 79.32368 73.49373 85.15363
## 2   Hot Dog         Mustard 75.59862 69.76867 81.42857
## 3 Ice Cream Chocolate Sauce 79.04103 73.21108 84.87098
## 4 Ice Cream         Mustard 75.31598 69.48603 81.14593
```

Example 1: Model 3

- Prediction Visualization



Example 1: Model 4

- Interaction Effect

```
EvFC.Full.Model=lm(Enjoyment~Food+Condiment+Food*Condiment,data=CONDIMENT)
tidy(EvFC.Full.Model)
```

```
## # A tibble: 4 x 5
##   term                             estimate std.error statistic  p.value
##   <chr>                               <dbl>     <dbl>     <dbl>    <dbl>
## 1 (Intercept)                          65.3      1.12      58.3 7.18e-65
## 2 FoodIce Cream                        27.7      1.58      17.5 2.11e-28
## 3 CondimentMustard                     24.3      1.58      15.3 5.58e-25
## 4 FoodIce Cream:CondimentMustard      -56.0      2.24     -25.0 1.95e-38
```

$$\hat{E} = 65.32 + 27.73F + 24.29C - 56.03FC$$

- $F = \begin{cases} 0 & if\ Hot\ Dog \\ 1 & if\ Ice\ Cream \end{cases}$
- $C = \begin{cases} 0 & if\ Chocolate\ Sauce \\ 1 & if\ Mustard \end{cases}$
- $FC = \begin{cases} 0 & otherwise \\ 1 & if\ Ice\ Cream\ and\ Mustard \end{cases}$

Example 1: Model 4

- Interaction Effect

$$\hat{E} = 65.32 + 27.73F + 24.29C - 56.03FC$$

- $F = \begin{cases} 0 & if\ Hot\ Dog \\ 1 & if\ Ice\ Cream \end{cases}$
- $C = \begin{cases} 0 & if\ Chocolate\ Sauce \\ 1 & if\ Mustard \end{cases}$
- $FC = \begin{cases} 0 & otherwise \\ 1 & if\ Ice\ Cream\ and\ Mustard \end{cases}$

Hot dog with Chocolate$= 65.32$

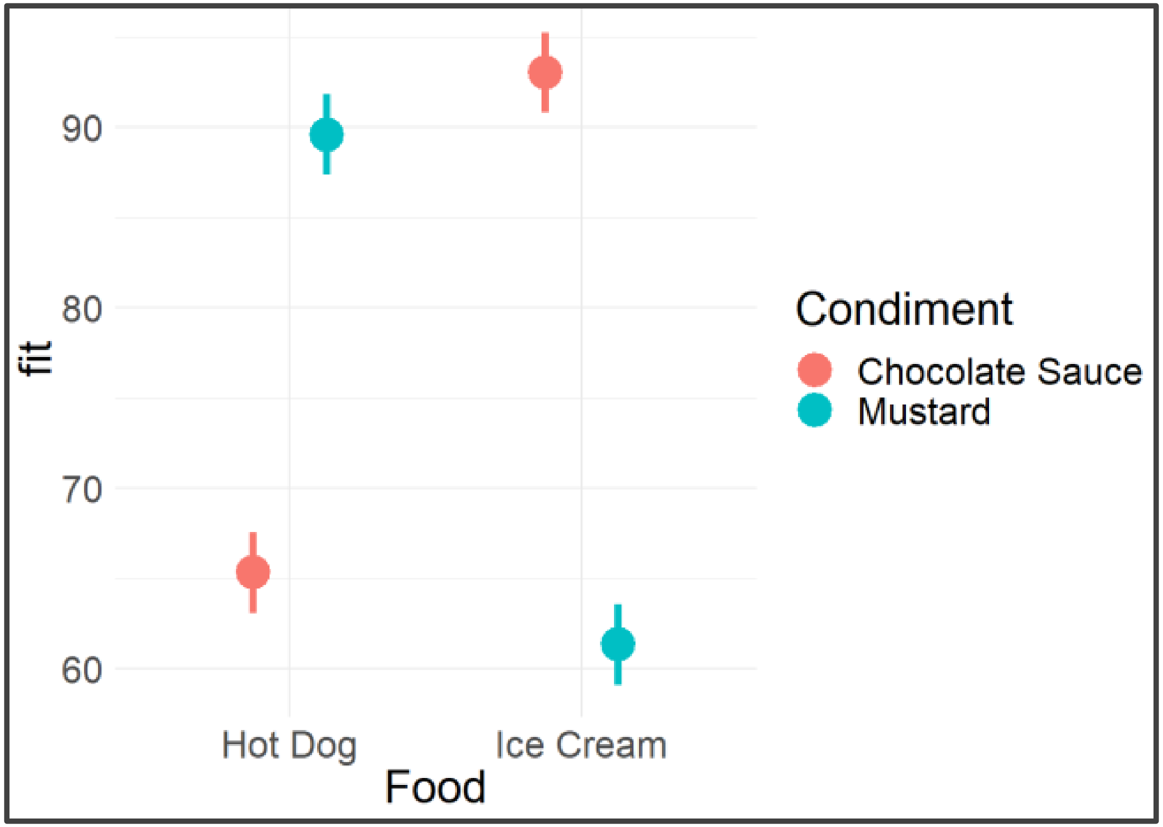
Hot dog with Mustard$= 65.32 + 24.29$

Ice cream with Chocolate$= 65.32 + 27.73$

Ice cream with Mustard$= 65.32 + 27.73 + 24.29 - 56.03$

Example 1: Model 4

- Understanding This Visually
  - What Is Different?



Example 1: Summary

- Summary
  - Categorical Predictors
  - Purpose:
    - Generalize t-test
    - Estimate Difference in Means Between Groups

Example 2: Data

- Data Overview
  - Popular Built-in Data
    - Sepal.Width (W)
    - Sepal.Length (L)
    - Species (S)
    - 150 Observations

```
IRIS=iris[,c(1,2,5)]
names(IRIS)=c("L","W","S")
head(IRIS)
```

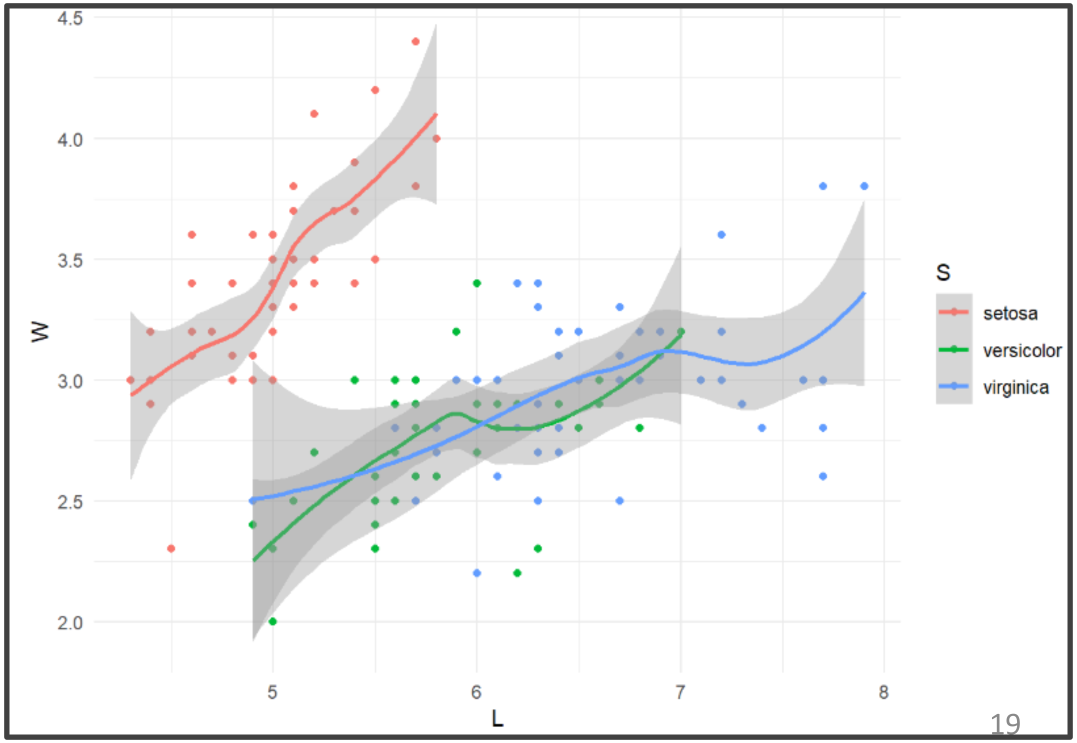
```
##     L   W      S
## 1 5.1 3.5 setosa
## 2 4.9 3.0 setosa
## 3 4.7 3.2 setosa
## 4 4.6 3.1 setosa
## 5 5.0 3.6 setosa
## 6 5.4 3.9 setosa
```

Example 2: Question

- Question of Interest

Can We Explain the Variation in Sepal Width Using Sepal Length and Species (setosa,versicolor,virginica)?

- Visual of Relationship



Example 2: Models

- Multiple Models

```
model1=lm(W~L,IRIS)
tidy(model1)
```

```
## # A tibble: 2 x 5
##   term        estimate std.error statistic  p.value
##   <chr>          <dbl>     <dbl>     <dbl>    <dbl>
## 1 (Intercept)   3.42      0.254      13.5  1.55e-27
## 2 L            -0.0619    0.0430     -1.44 1.52e- 1
```

$$\hat{E} = 3.42 - 0.06L$$

```
model2=lm(W~L+S,IRIS)
tidy(model2)
```

```
## # A tibble: 4 x 5
##   term        estimate std.error statistic  p.value
##   <chr>          <dbl>     <dbl>     <dbl>    <dbl>
## 1 (Intercept)    1.68     0.235       7.12 4.46e-11
## 2 L              0.350    0.0463      7.56 4.19e-12
## 3 Sversicolor   -0.983    0.0721    -13.6  7.62e-28
## 4 Svirginica    -1.01     0.0933    -10.8  2.41e-20
```

Setosa: $\hat{E} = 1.68 + 0.35L$

Versicolor: $\hat{E} = 1.68 + 0.35L - 0.983$

Virginica: $\hat{E} = 1.68 + 0.35L - 1.01$

Example 2: Models

- Full Model Estimated

```
model3=lm(W~L+S+L*S,IRIS)
tidy(model3)
```

```
## # A tibble: 6 x 5
##   term          estimate std.error statistic  p.value
##   <chr>            <dbl>     <dbl>     <dbl>    <dbl>
## 1 (Intercept)     -0.569     0.554     -1.03 3.06e- 1
## 2 L                0.799     0.110      7.23 2.55e-11
## 3 Sversicolor      1.44      0.713      2.02 4.51e- 2
## 4 Svirginica       2.02      0.686      2.94 3.85e- 3
## 5 L:Sversicolor   -0.479     0.134     -3.58 4.65e- 4
## 6 L:Svirginica    -0.567     0.126     -4.49 1.45e- 5
```

Adjustment In Mean

Adjustment In Slope

Setosa: $\hat{E} = 0.799L - 0.569$

Versicolor: $\hat{E} = (0.799 - 0.479)L + 1.44 - 0.569$

Virginica: $\hat{E} = (0.799 - 0.567)L + 2.02 - 0.569$

Example 2: Predictions

- Gathering Predictions

```
IRIS %>%
  gather_predictions(model1,model2,model3)%>%
  glimpse()
```

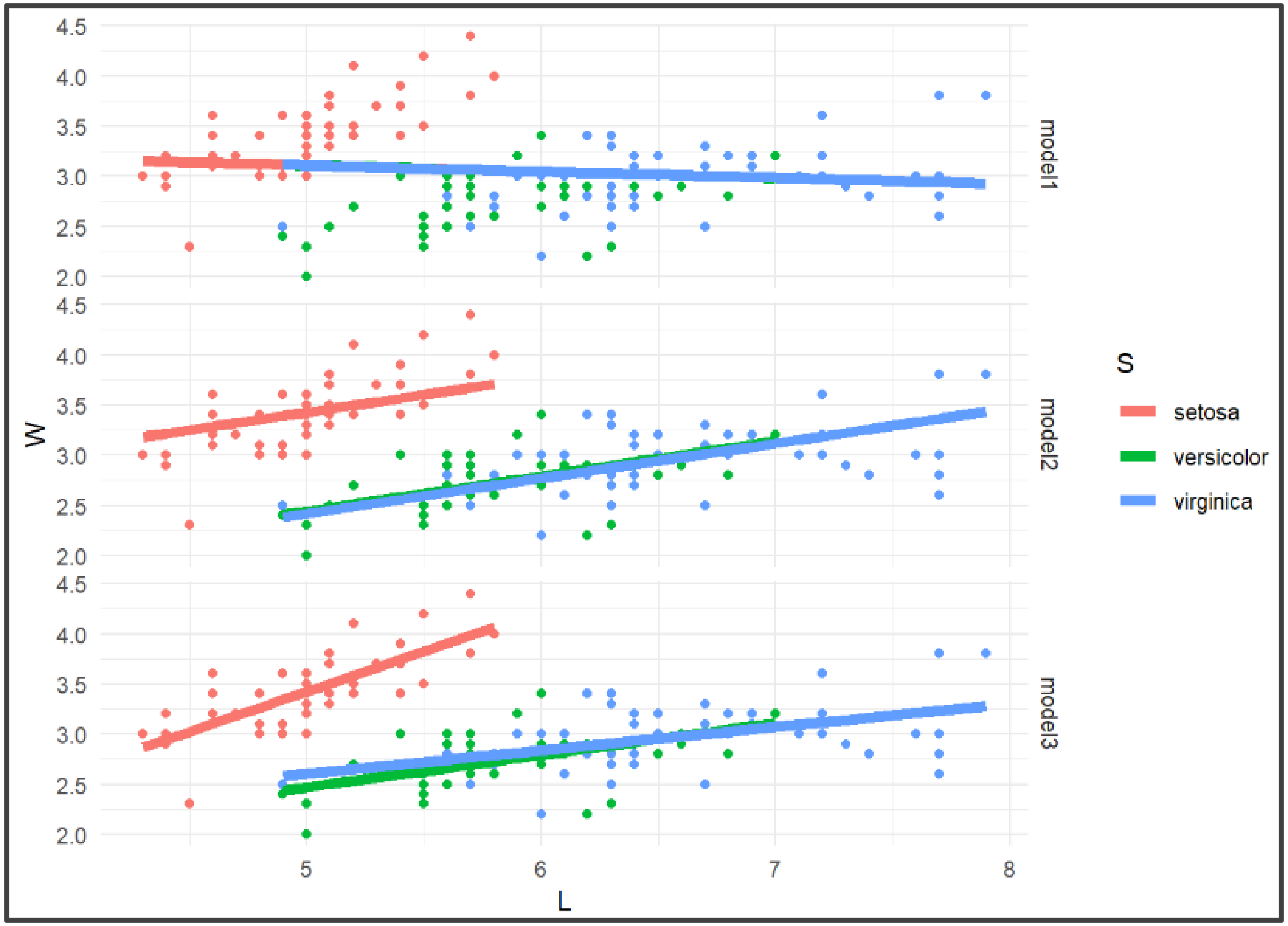
```
## Observations: 450
## Variables: 5
## $ model <chr> "model1", "model1", "model1", "model1", "model1", "model...
## $ L     <dbl> 5.1, 4.9, 4.7, 4.6, 5.0, 5.4, 4.6, 5.0, 4.4, 4.9, 5.4, 4...
## $ W     <dbl> 3.5, 3.0, 3.2, 3.1, 3.6, 3.9, 3.4, 3.4, 2.9, 3.1, 3.7, 3...
## $ S     <fct> setosa, setosa, setosa, setosa, setosa, setosa, setosa, ...
## $ pred  <dbl> 3.103334, 3.115711, 3.128088, 3.134277, 3.109523, 3.0847...
```

150 Predictions for 3 Models

- Variable Named “model”
- Allows Us To Quickly Create Graphics That Compare Models

Example 2: Visualization

- Visualizing Models



Example 2: Summary

- Summary
  - Numerical Response Variable
  - Categorical & Numerical Explanatory Variables

Example 3: Data

- Data Overview
  - Advertising Data
    - Sales
    - TV
    - Radio
    - 200 Observations

```{r, message=F}
Ad = read_csv("Advertising.txt")[,c(2,3,5)]
head(Ad)
```

TV	Radio	Sales
<dbl>	<dbl>	<dbl>
230.1	37.8	22.1
44.5	39.3	10.4
17.2	45.9	9.3
151.5	41.3	18.5
180.8	10.8	12.9
8.7	48.9	7.2

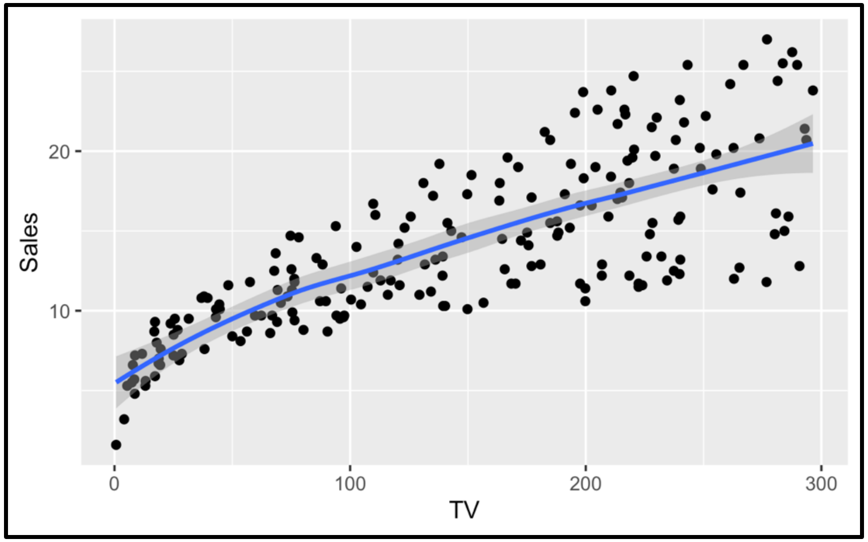
    - Numbers in thousands

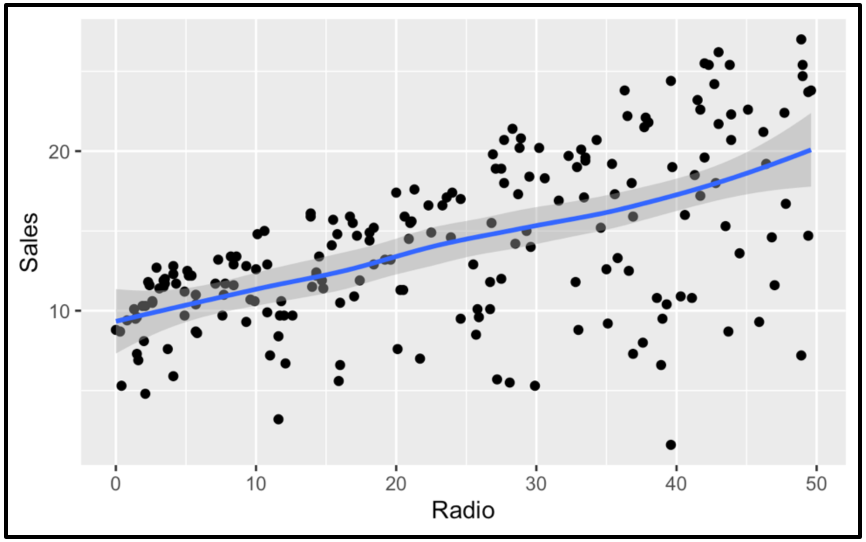
Example 3: Question

- Question of Interest

Can We Explain the Variation in Sales Using TV and Radio advertising budget?

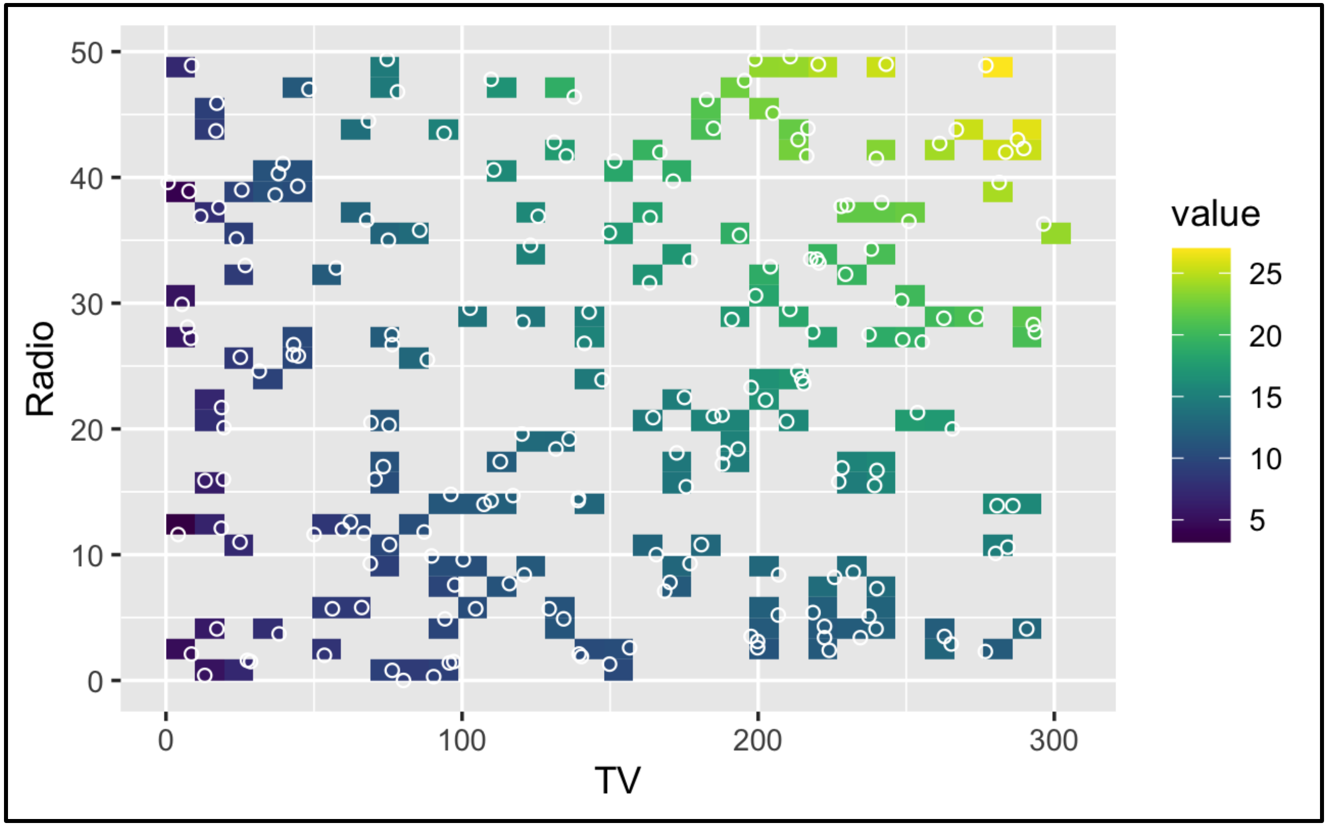
- Visual of Relationship





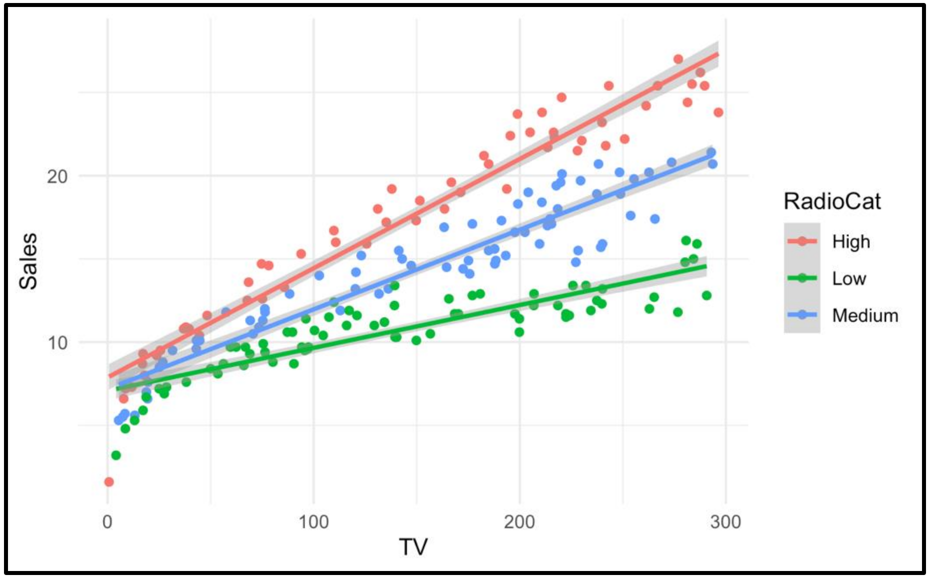
Example 3: Question

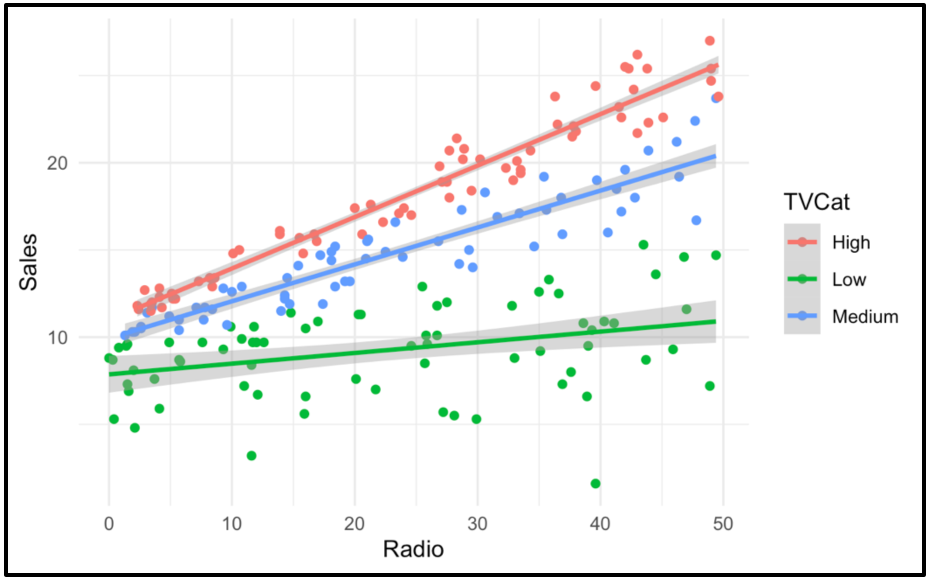
- Visual of Relationship



Example 3: Question

- Visual of Relationship





Example 3: Model1

- Model 1

```
model1=lm(Sales~TV+Radio,Ad)
tidy(model1)
```

```
## # A tibble: 3 x 5
##   term        estimate std.error statistic  p.value
##   <chr>          <dbl>     <dbl>     <dbl>    <dbl>
## 1 (Intercept)   2.92     0.294        9.92 4.57e-19
## 2 TV            0.0458   0.00139     32.9  5.44e-82
## 3 Radio         0.188    0.00804     23.4  9.78e-59
```

Model1: $\hat{E} = 2.92 + 0.046TV + 0.188Radio$

```
## # A tibble: 1 x 12
##   r.squared adj.r.squared sigma statistic  p.value    df logLik   AIC   BIC
##       <dbl>         <dbl> <dbl>     <dbl>    <dbl> <dbl>  <dbl> <dbl> <dbl>
## 1     0.897         0.896  1.68      860. 4.83e-98     2  -386.  780.  794.
## # … with 3 more variables: deviance <dbl>, df.residual <int>, nobs <int>
```

Example 3: Model Selection

- AIC = $-2\ln(\hat{L}) + 2p$
  - goodness of fit: $2\ln(\hat{L})$
  - $\hat{L}$: the maximized value of the likelihood of the model
  - p: number of parameters in the model

- BIC = $-2\ln(\hat{L}) + p\ln(n)$
  - n: number of observations in the data

Example 3: Model 2

```
model2=lm(Sales~TV*Radio,Ad)
tidy(model2)
```

```
## # A tibble: 4 x 5
##   term        estimate std.error statistic  p.value
##   <chr>          <dbl>     <dbl>     <dbl>    <dbl>
## 1 (Intercept)  6.75    0.248         27.2  1.54e-68
## 2 TV           0.0191  0.00150       12.7  2.36e-27
## 3 Radio        0.0289  0.00891        3.24 1.40e- 3
## 4 TV:Radio     0.00109 0.0000524     20.7  2.76e-51
```

Adjustment
In Slope

```
## # A tibble: 1 x 12
##   r.squared adj.r.squared sigma statistic   p.value    df logLik   AIC   BIC
##       <dbl>         <dbl> <dbl>     <dbl>     <dbl> <dbl>  <dbl> <dbl> <dbl>
## 1     0.968         0.967 0.944     1963. 6.68e-146     3  -270.  550.  567.
## # … with 3 more variables: deviance <dbl>, df.residual <int>, nobs <int>
```

Model2: $\hat{E} = 6.75 + 0.019TV + 0.029Radio + 0.001TV \times Radio$

$$\hat{E} = 6.75 + (0.019 + 0.001Radio) \times TV + 0.029Radio$$

$$\hat{E} = 6.75 + 0.019TV + (0.029 + 0.001TV) \times Radio$$

Example 3: Predictions

- Gathering Predictions

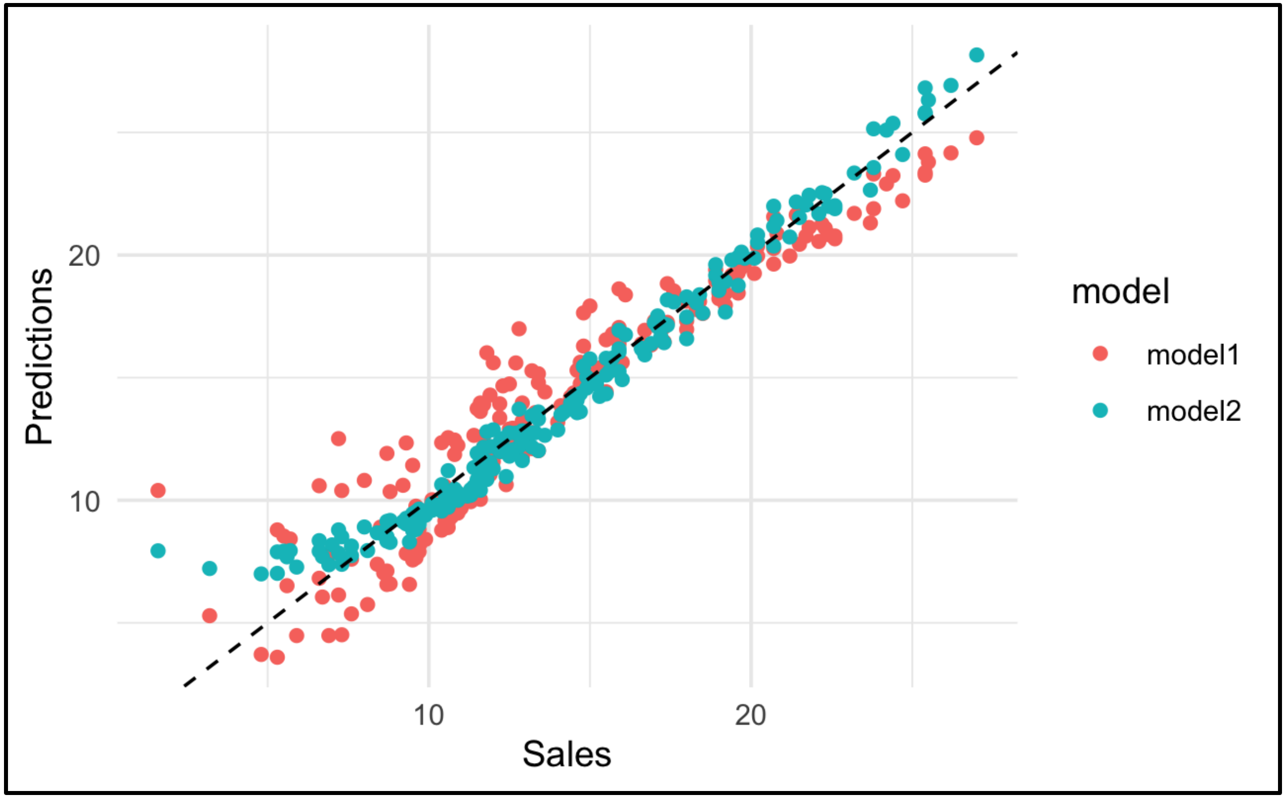
```{r}
Ad %>%
  gather_predictions(model1,model2)%>%
  glimpse()
```

```
Rows: 400
Columns: 5
$ model <chr> "model1", "model1", "model1", "model1", "model1", "…
$ TV    <dbl> 230.1, 44.5, 17.2, 151.5, 180.8, 8.7, 57.5, 120.2, …
$ Radio <dbl> 37.8, 39.3, 45.9, 41.3, 10.8, 48.9, 32.8, 19.6, 2.1…
$ Sales <dbl> 22.1, 10.4, 9.3, 18.5, 12.9, 7.2, 11.8, 13.2, 4.8, …
$ pred  <dbl> 20.555465, 12.345362, 12.337018, 17.617116, 13.2239…
```

200 Predictions for 2 Models

Example 3: Visualization

- Visualizing Prediction vs. True Value



Example 3: Summary

- Summary for Lectures on Categorical Predictor and Interactions
  - Numerical Response Variable
  - Categorical Predictor
  - Interaction between Two Categorical Predictors
  - Interaction between Two Categorical and Numerical Predictor
  - Interaction between Two Numerical Predictors