STOR 320 Modeling VII

Lecture 30

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Introduction

- Now We Consider
  - Categorical Response Variables
  - Numerical/Categorical Explanatory Variables
- Focus is on Classification
- Read Chapter 4 in ISLR

Introduction

- Basic Case: Binary Response
  - Variable Has Two Possible Outcomes
  - Typically, Yes or No Responses to a Question
  - Example
    - Y = Who Will Win the 2024 Presidential Election?
    - Y = Did You Pass Your STOR 320 Class?
    - Y = What Factors Influence the Admission into Graduate School?

Scenario

- Question: Are Students Who Get Good Grades Likely to be Admitted to Graduate School?
    - Y = Would the Student be Admitted to a Graduate School?
    - X = College GPA
- Why is Linear Regression Inappropriate?

$$P(Admission|X) = \beta_0 + \beta_1 X$$

Problem Setting

- Bernouilli Random Variable

$$Y = \begin{cases} 1 & if\ Yes \\ 0 & if\ No \end{cases}$$

$$p = E(Y) = P(Y = 1)$$

- Sample n Students

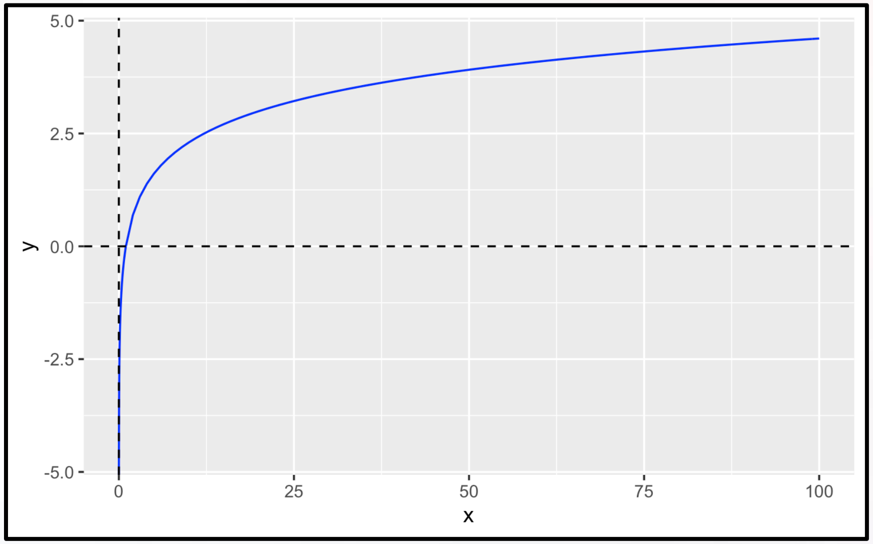
$$\mathrm{Y}' = \sum Y_i \sim Binomial(n, p)$$

$$\hat{p} = \frac{\sum y_i}{n}$$

Estimated Probability that a Student Would be Admitted to a Graduate School

- Analyze the Effect of X on p: $p = E(Y|X) \neq \beta_0 + \beta_1 X$

Logit Link



- Modeling the Mean
  - Logit Link Function

$$\log\left(\frac{p}{1-p}\right) = \beta_0 + \beta_1 X$$

Odds of Admission

  - Understanding Odds
    - Odds of Admission = 1
    - Odds of Admission < 1
    - Odds of Admission > 1

Model Construction

- Solving for $\frac{p}{1-p}$

$$\log\left(\frac{p}{1-p}\right) = \beta_0 + \beta_1 X$$

$$\frac{p}{1-p} = e^{\beta_0+\beta_1 X}$$

→ Odds of Admission Given the Student’s GPA

- Solving for p

$$p = e^{\beta_0+\beta_1 X} - pe^{\beta_0+\beta_1 X}$$

$$p(1 + e^{\beta_0+\beta_1 X}) = e^{\beta_0+\beta_1 X}$$

$$p = \frac{e^{\beta_0+\beta_1 X}}{1 + e^{\beta_0+\beta_1 X}}$$

→ Probability of Admission Given the Student’s GPA

Logistic Regression for Classification

- Recall: $Y = \begin{cases} 1 & if\ Yes \\ 0 & if\ No \end{cases}$

- After Getting Data, We Estimate
  - $\hat{\beta}_0$
  - $\hat{\beta}_1$
  - $\hat{p} = \frac{e^{\hat{\beta}_0 + \hat{\beta}_1 X}}{1 + e^{\hat{\beta}_0 + \hat{\beta}_1 X}}$ ➡ Estimated Probability of Admission Given the Student's GPA

- Two Scenarios
  - $\hat{p} < 0.5$ ➡ $\hat{Y} = 0$
  - $\hat{p} > 0.5$ ➡ $\hat{Y} = 1$

Evaluating the LR Model

- Two Methods
  - Leave Out Data Intentionally
  - Use Cross-Validation

- Positives and Negatives
  - True Positive = Predicted an Admission and the Student Got Admitted
  - False Positive=Predicted an Admission and the Student Didn’t Get Admitted
  - False Negative = Predicted a Student Wouldn’t be Admitted and They Did Get Admitted
  - True Negative = Predicted a Student Wouldn’t be Admitted and They Didn’t Get Admitted

Confusion Matrix

- Confusion Matrix

	Predicted	
Actual	*Will be Admitted*	*Won’t be Admitted*
Admission	n_{11}	n_{12}
Isn’t Admitted	n_{21}	n_{22}

  - Sensitivity:

$$n_{11}/(n_{11} + n_{12})$$

  - Specificity:

$$n_{22}/(n_{21} + n_{22})$$

  - False Positive Rate:

$$n_{21}/(n_{21} + n_{22})$$

  - False Negative Rate:

$$n_{12}/(n_{11} + n_{12})$$

Titanic: Data

- Titanic Survival Data

```
> library(titanic)
```

  - Response Variable

$$Y = \begin{cases} 1 & if\ \ Survived \\ 0 & if\ \ Did\ Not\ Survive \end{cases}$$

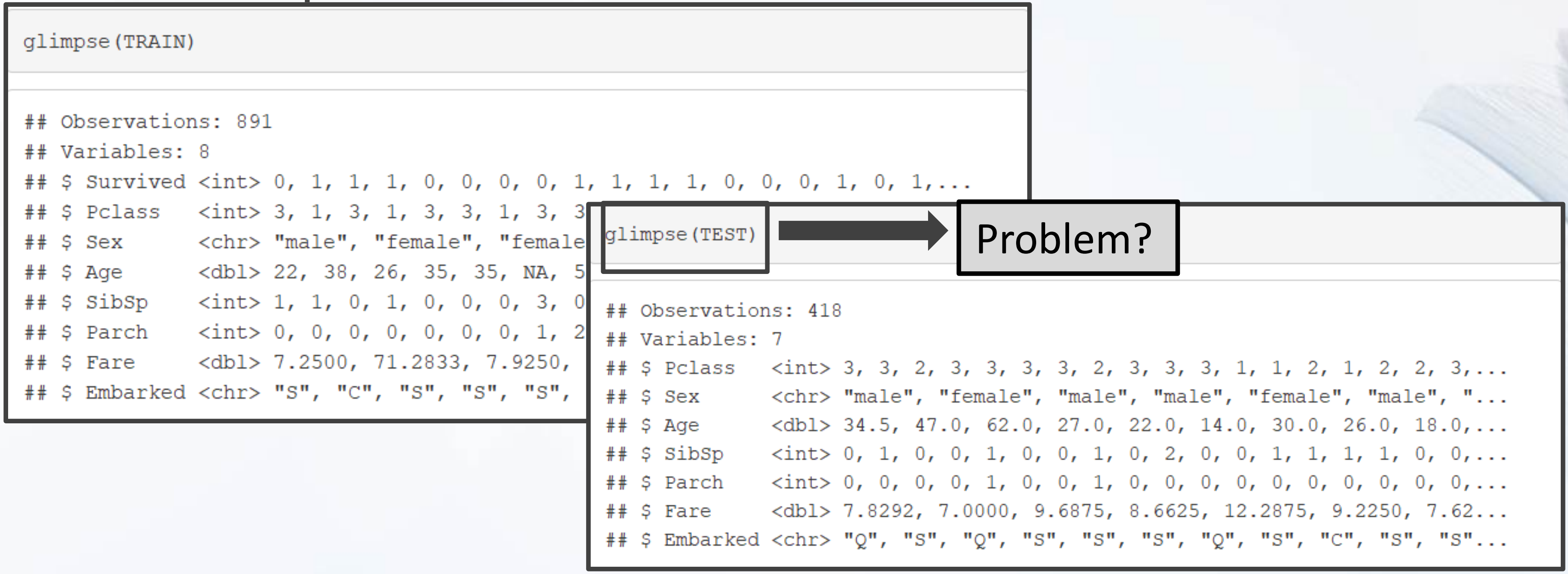
  - Explanatory Variables
    - Passenger Class
    - Sex
    - Age
    - Siblings/Spouses Aboard
    - Parents/Children Aboard
    - Passenger Fare
    - Port of Embarkation

Titanic: Data

- Titanic Survival Data (Continued)
  - Selecting Variables of Interest

```
> TRAIN=titanic_train[,c(2,3,5,6,7,8,10,12)]
> TEST=titanic_test[,c(2,4,5,6,7,9,11)])
```

  - Glimpse of Data



```
glimpse(TRAIN)

## Observations: 891
## Variables: 8
## $ Survived <int> 0, 1, 1, 1, 0, 0, 0, 0, 1, 1, 1, 1, 0, 0, 0, 1, 0, 1,...
## $ Pclass   <int> 3, 1, 3, 1, 3, 3, 1, 3, 3
## $ Sex      <chr> "male", "female", "female
## $ Age      <dbl> 22, 38, 26, 35, 35, NA, 5
## $ SibSp    <int> 1, 1, 0, 1, 0, 0, 0, 3, 0
## $ Parch    <int> 0, 0, 0, 0, 0, 0, 0, 1, 2
## $ Fare     <dbl> 7.2500, 71.2833, 7.9250,
## $ Embarked <chr> "S", "C", "S", "S", "S",
```

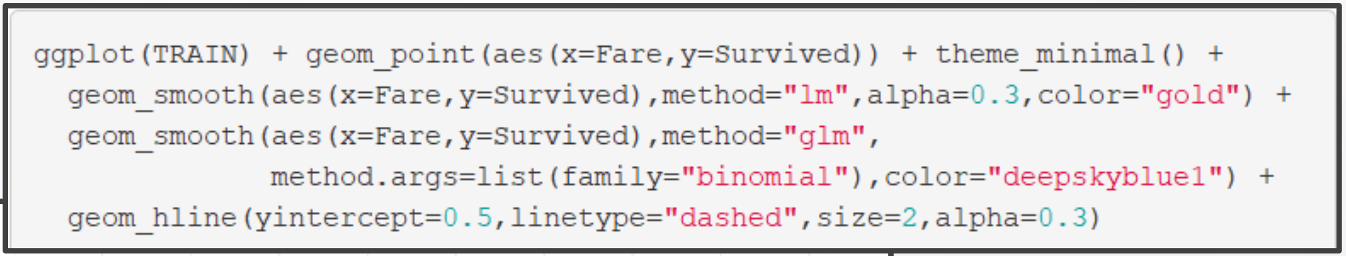
```
glimpse(TEST)

## Observations: 418
## Variables: 7
## $ Pclass   <int> 3, 3, 2, 3, 3, 3, 3, 2, 3, 3, 3, 1, 1, 2, 1, 2, 2, 3,...
## $ Sex      <chr> "male", "female", "male", "male", "female", "male", "...
## $ Age      <dbl> 34.5, 47.0, 62.0, 27.0, 22.0, 14.0, 30.0, 26.0, 18.0,...
## $ SibSp    <int> 0, 1, 0, 0, 1, 0, 0, 1, 0, 2, 0, 0, 1, 1, 1, 1, 0, 0,...
## $ Parch    <int> 0, 0, 0, 0, 1, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,...
## $ Fare     <dbl> 7.8292, 7.0000, 9.6875, 8.6625, 12.2875, 9.2250, 7.62...
## $ Embarked <chr> "Q", "S", "Q", "S", "S", "S", "Q", "S", "C", "S", "S"...
```

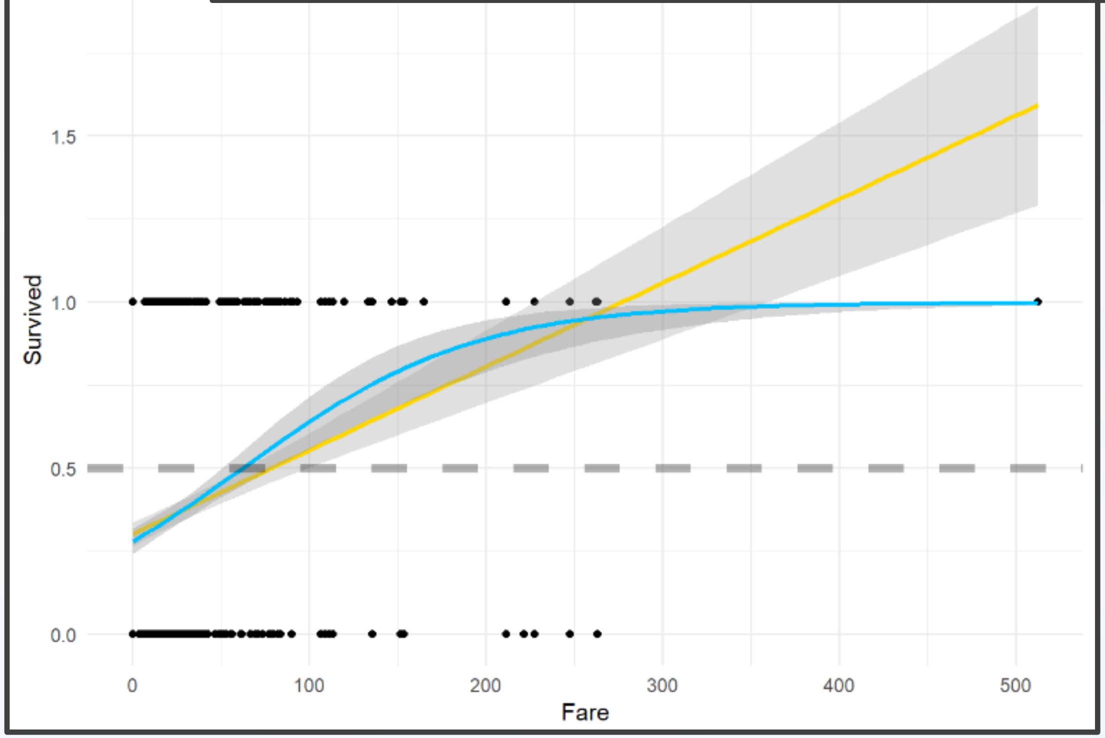
Problem?

Visualization: Survival vs. Fare

- Visualizing the Data

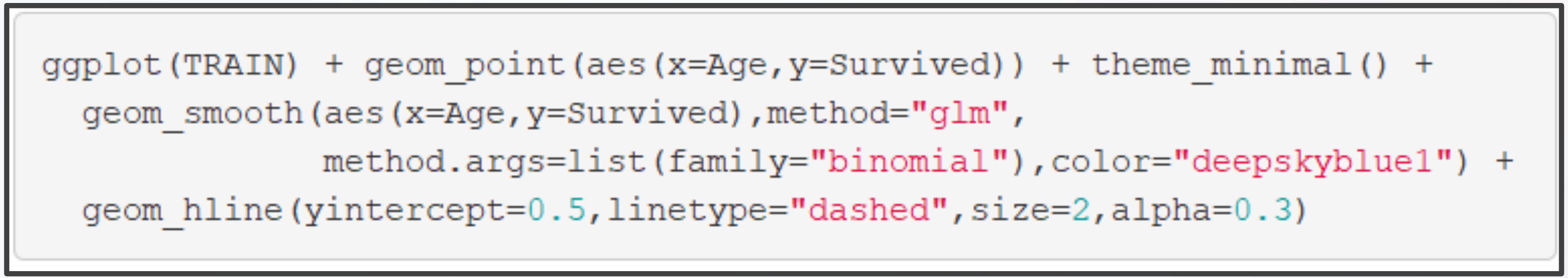


```
ggplot(TRAIN) + geom_point(aes(x=Fare,y=Survived)) + theme_minimal() +
  geom_smooth(aes(x=Fare,y=Survived),method="lm",alpha=0.3,color="gold") +
  geom_smooth(aes(x=Fare,y=Survived),method="glm",
              method.args=list(family="binomial"),color="deepskyblue1") +
  geom_hline(yintercept=0.5,linetype="dashed",size=2,alpha=0.3)
```

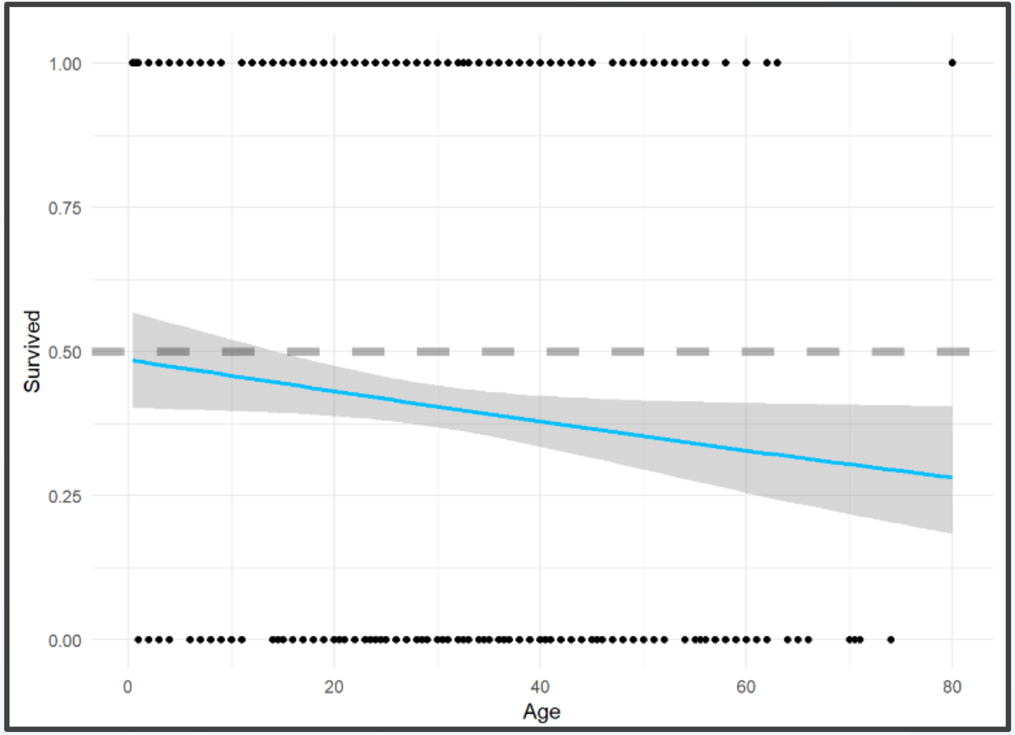


Visualization: Survival vs. Age

- Visualizing the Data (Continued)

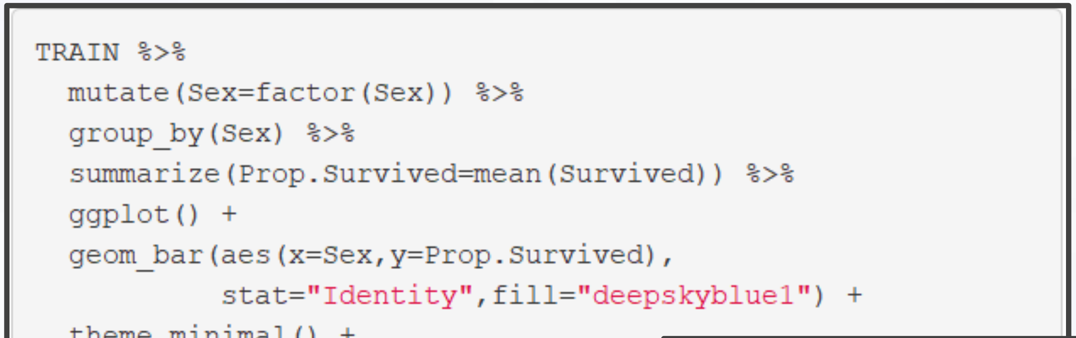


```
ggplot(TRAIN) + geom_point(aes(x=Age,y=Survived)) + theme_minimal() +
  geom_smooth(aes(x=Age,y=Survived),method="glm",
              method.args=list(family="binomial"),color="deepskyblue1") +
  geom_hline(yintercept=0.5,linetype="dashed",size=2,alpha=0.3)
```

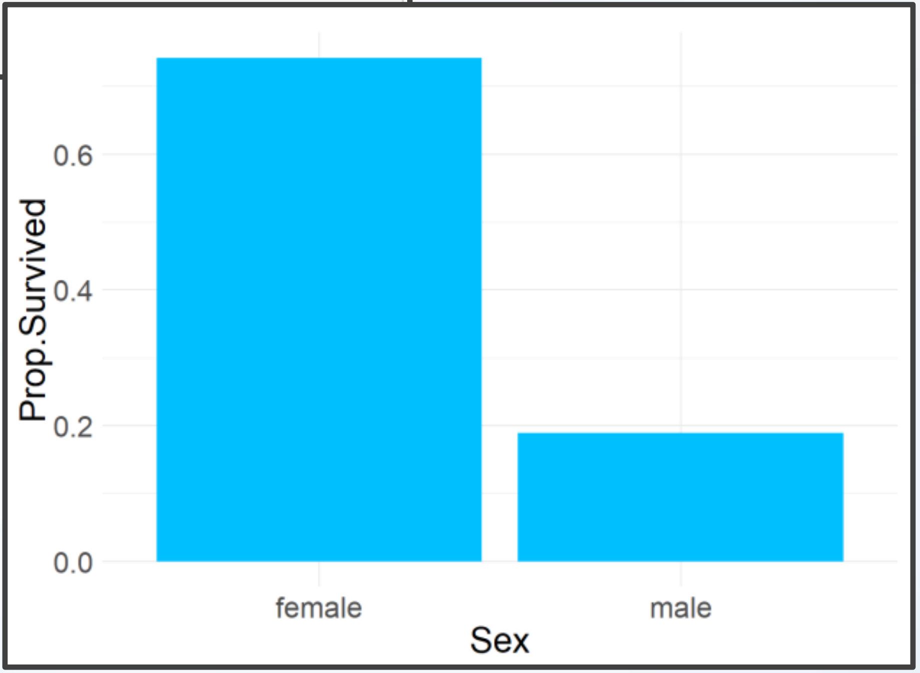


Visualization: Survival vs. Sex

- Visualizing the Data (Continued)



```
TRAIN %>%
  mutate(Sex=factor(Sex)) %>%
  group_by(Sex) %>%
  summarize(Prop.Survived=mean(Survived)) %>%
  ggplot() +
  geom_bar(aes(x=Sex,y=Prop.Survived),
           stat="Identity",fill="deepskyblue1") +
  theme_minimal() +
  theme(text=element_text(size=20))
```



Data Splitting

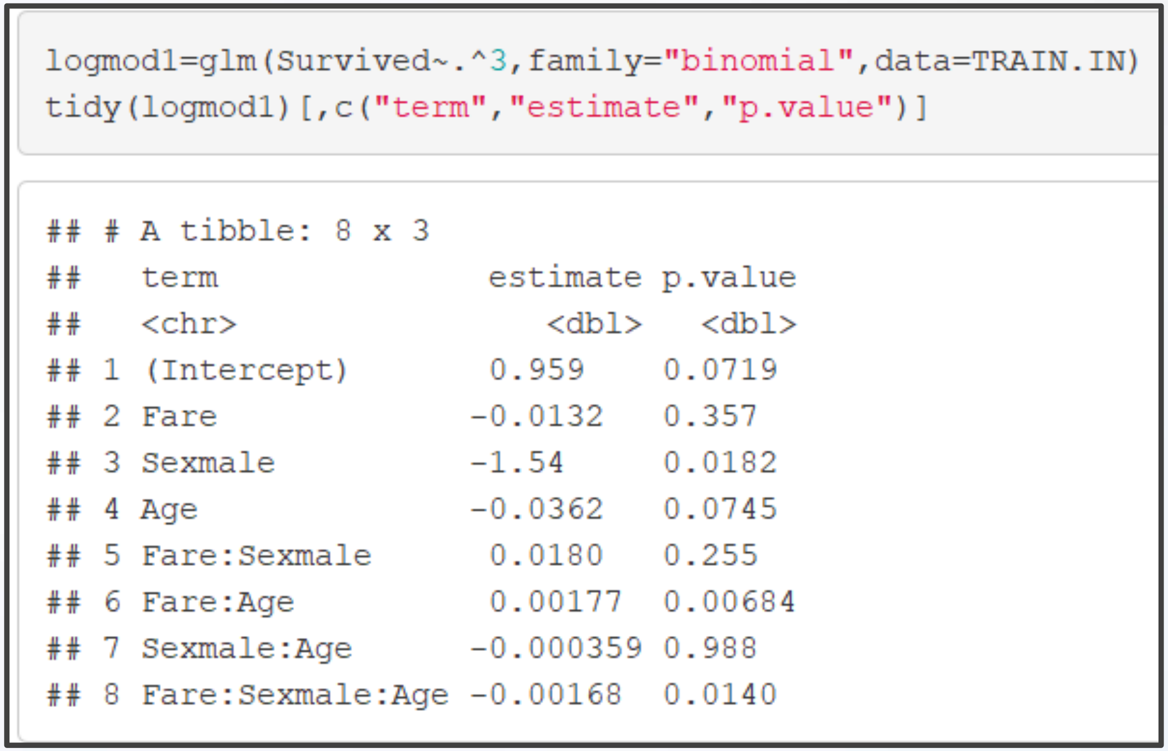
- Logistic Regression Models
  - Split Training Set Up

```
> set.seed(216)
> sample.in=sample(1:dim(TRAIN)[1],
            size=floor(0.8*dim(TRAIN)[1]))
> TRAIN.IN=TRAIN[sample.in,
            c("Survived","Fare","Sex","Age")]
> TRAIN.OUT=TRAIN[-sample.in,
              c("Survived","Fare","Sex","Age")]
```

  - Modeling the Probability of Survival Given the Ticket Fare, the Sex of the Passenger, and the Age of the Passenger

Model 1

- Logistic Regression Models (Cont.)
  - Including 3-Way Interaction

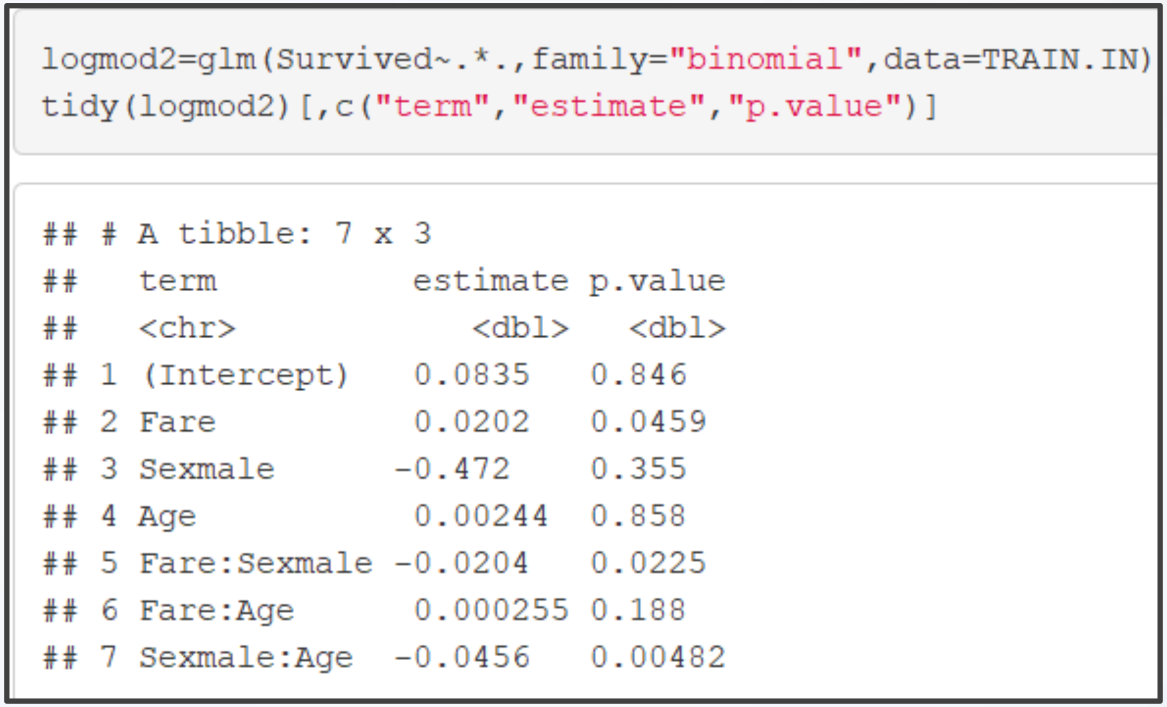


```
logmod1=glm(Survived~.^3,family="binomial",data=TRAIN.IN)
tidy(logmod1)[,c("term","estimate","p.value")]
```

```
## # A tibble: 8 x 3
##   term              estimate p.value
##   <chr>                <dbl>   <dbl>
## 1 (Intercept)        0.959    0.0719
## 2 Fare              -0.0132   0.357
## 3 Sexmale           -1.54     0.0182
## 4 Age               -0.0362   0.0745
## 5 Fare:Sexmale       0.0180   0.255
## 6 Fare:Age           0.00177  0.00684
## 7 Sexmale:Age       -0.000359 0.988
## 8 Fare:Sexmale:Age  -0.00168  0.0140
```

Model 2

- Logistic Regression Models (Cont.)
  - Only 2-Way Interactions

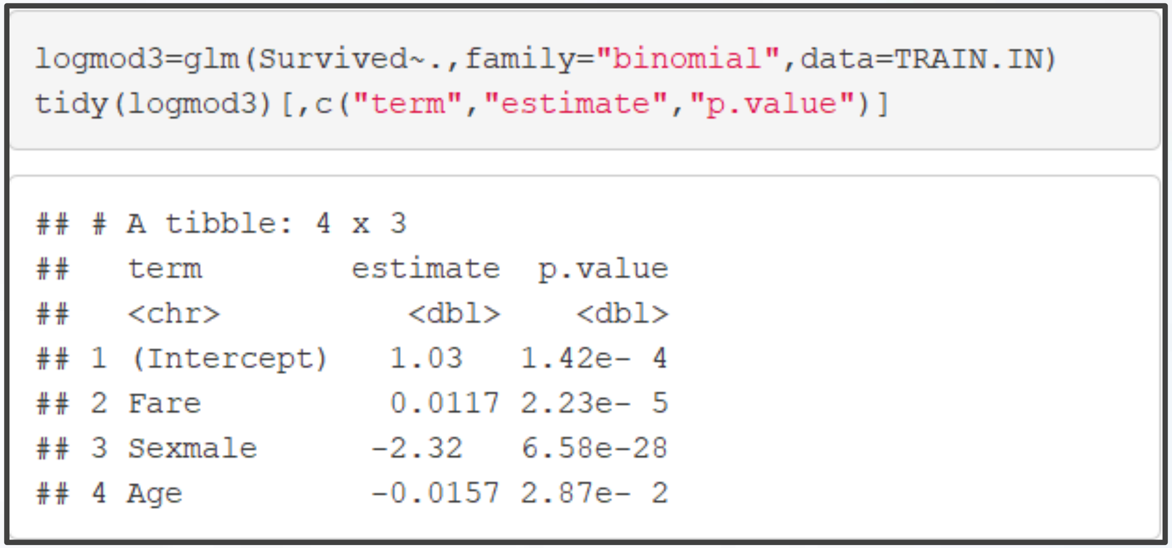


```
logmod2=glm(Survived~.*.,family="binomial",data=TRAIN.IN)
tidy(logmod2)[,c("term","estimate","p.value")]
```

```
## # A tibble: 7 x 3
##   term          estimate p.value
##   <chr>            <dbl>   <dbl>
## 1 (Intercept)   0.0835   0.846
## 2 Fare          0.0202   0.0459
## 3 Sexmale      -0.472    0.355
## 4 Age           0.00244  0.858
## 5 Fare:Sexmale -0.0204   0.0225
## 6 Fare:Age      0.000255 0.188
## 7 Sexmale:Age  -0.0456   0.00482
```

Model 3

- Logistic Regression Models (Cont.)
  - No Way Interactions

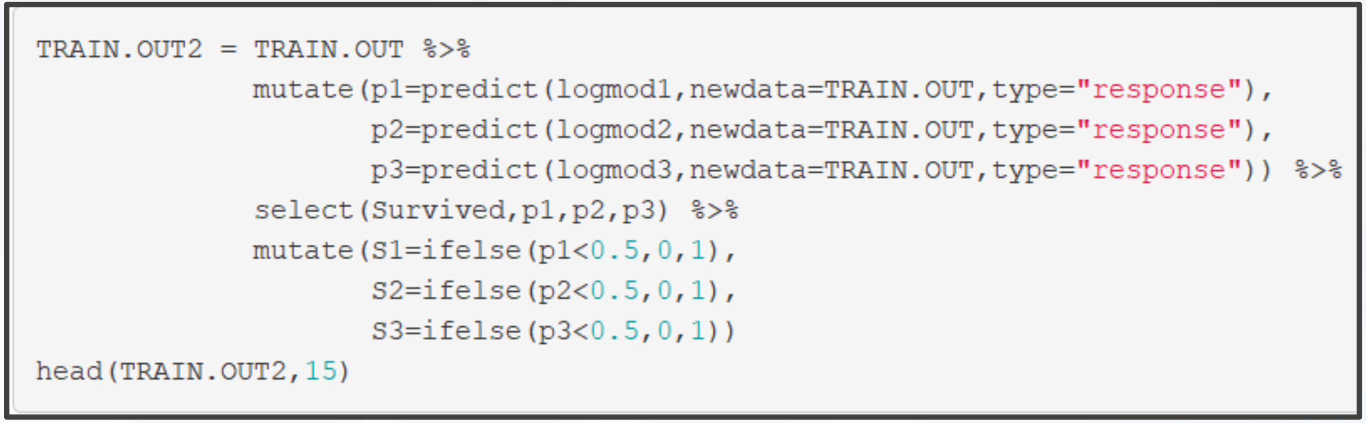


```
logmod3=glm(Survived~.,family="binomial",data=TRAIN.IN)
tidy(logmod3)[,c("term","estimate","p.value")]
```

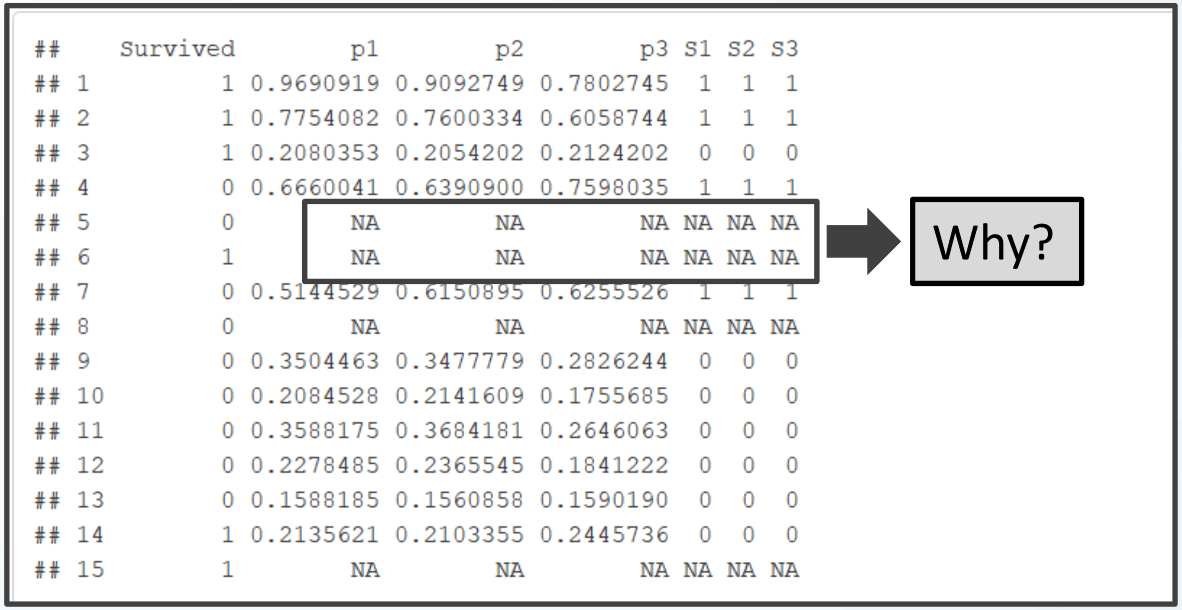
```
## # A tibble: 4 x 3
##   term        estimate  p.value
##   <chr>          <dbl>    <dbl>
## 1 (Intercept)   1.03   1.42e- 4
## 2 Fare          0.0117 2.23e- 5
## 3 Sexmale      -2.32   6.58e-28
## 4 Age          -0.0157 2.87e- 2
```

Predictions

- Getting Predictions



```
TRAIN.OUT2 = TRAIN.OUT %>%
             mutate(p1=predict(logmod1,newdata=TRAIN.OUT,type="response"),
                    p2=predict(logmod2,newdata=TRAIN.OUT,type="response"),
                    p3=predict(logmod3,newdata=TRAIN.OUT,type="response")) %>%
             select(Survived,p1,p2,p3) %>%
             mutate(S1=ifelse(p1<0.5,0,1),
                    S2=ifelse(p2<0.5,0,1),
                    S3=ifelse(p3<0.5,0,1))
head(TRAIN.OUT2,15)
```

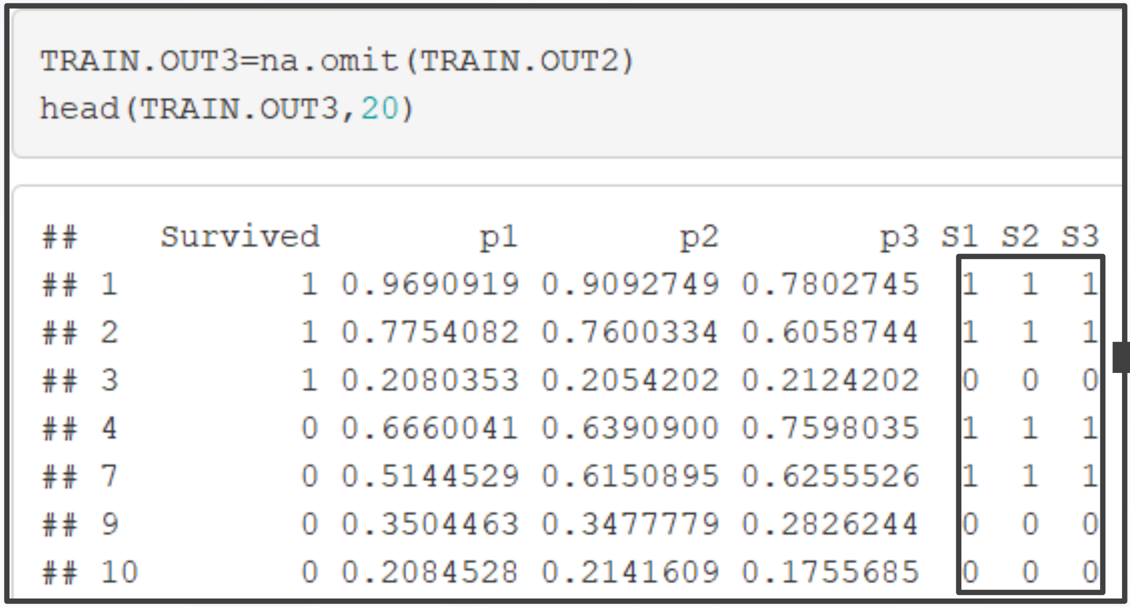


```
##    Survived        p1        p2        p3 S1 S2 S3
## 1         1 0.9690919 0.9092749 0.7802745  1  1  1
## 2         1 0.7754082 0.7600334 0.6058744  1  1  1
## 3         1 0.2080353 0.2054202 0.2124202  0  0  0
## 4         0 0.6660041 0.6390900 0.7598035  1  1  1
## 5         0        NA        NA        NA NA NA NA
## 6         1        NA        NA        NA NA NA NA
## 7         0 0.5144529 0.6150895 0.6255526  1  1  1
## 8         0        NA        NA        NA NA NA NA
## 9         0 0.3504463 0.3477779 0.2826244  0  0  0
## 10        0 0.2084528 0.2141609 0.1755685  0  0  0
## 11        0 0.3588175 0.3684181 0.2646063  0  0  0
## 12        0 0.2278485 0.2365545 0.1841222  0  0  0
## 13        0 0.1588185 0.1560858 0.1590190  0  0  0
## 14        1 0.2135621 0.2103355 0.2445736  0  0  0
## 15        1        NA        NA        NA NA NA NA
```

Why?

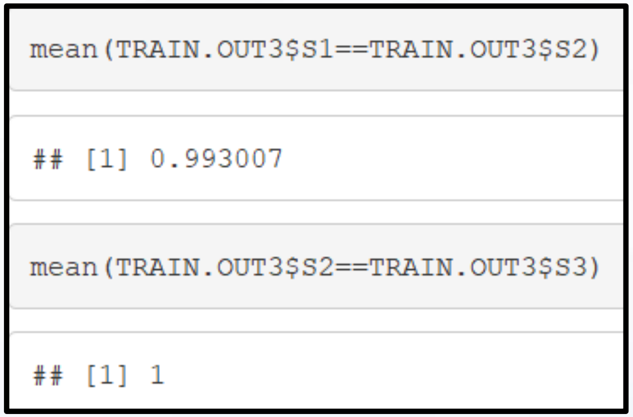
Predictions

- Getting Predictions



```
TRAIN.OUT3=na.omit(TRAIN.OUT2)
head(TRAIN.OUT3,20)
```

```
##     Survived        p1        p2        p3 S1 S2 S3
## 1          1 0.9690919 0.9092749 0.7802745  1  1  1
## 2          1 0.7754082 0.7600334 0.6058744  1  1  1
## 3          1 0.2080353 0.2054202 0.2124202  0  0  0
## 4          0 0.6660041 0.6390900 0.7598035  1  1  1
## 7          0 0.5144529 0.6150895 0.6255526  1  1  1
## 9          0 0.3504463 0.3477779 0.2826244  0  0  0
## 10         0 0.2084528 0.2141609 0.1755685  0  0  0
```



```
mean(TRAIN.OUT3$S1==TRAIN.OUT3$S2)
```

```
## [1] 0.993007
```

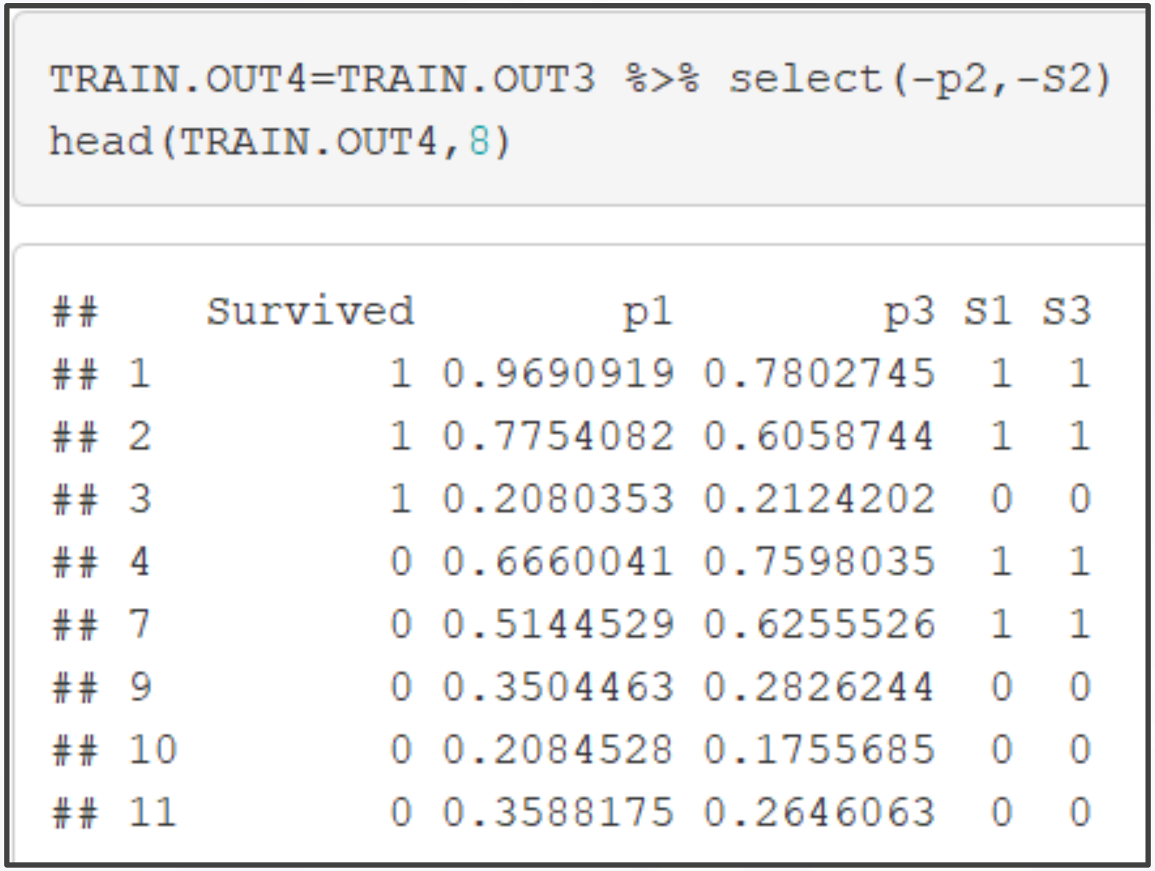
```
mean(TRAIN.OUT3$S2==TRAIN.OUT3$S3)
```

```
## [1] 1
```

What Do You Notice About the Predictions?

Predictions

- Getting Predictions



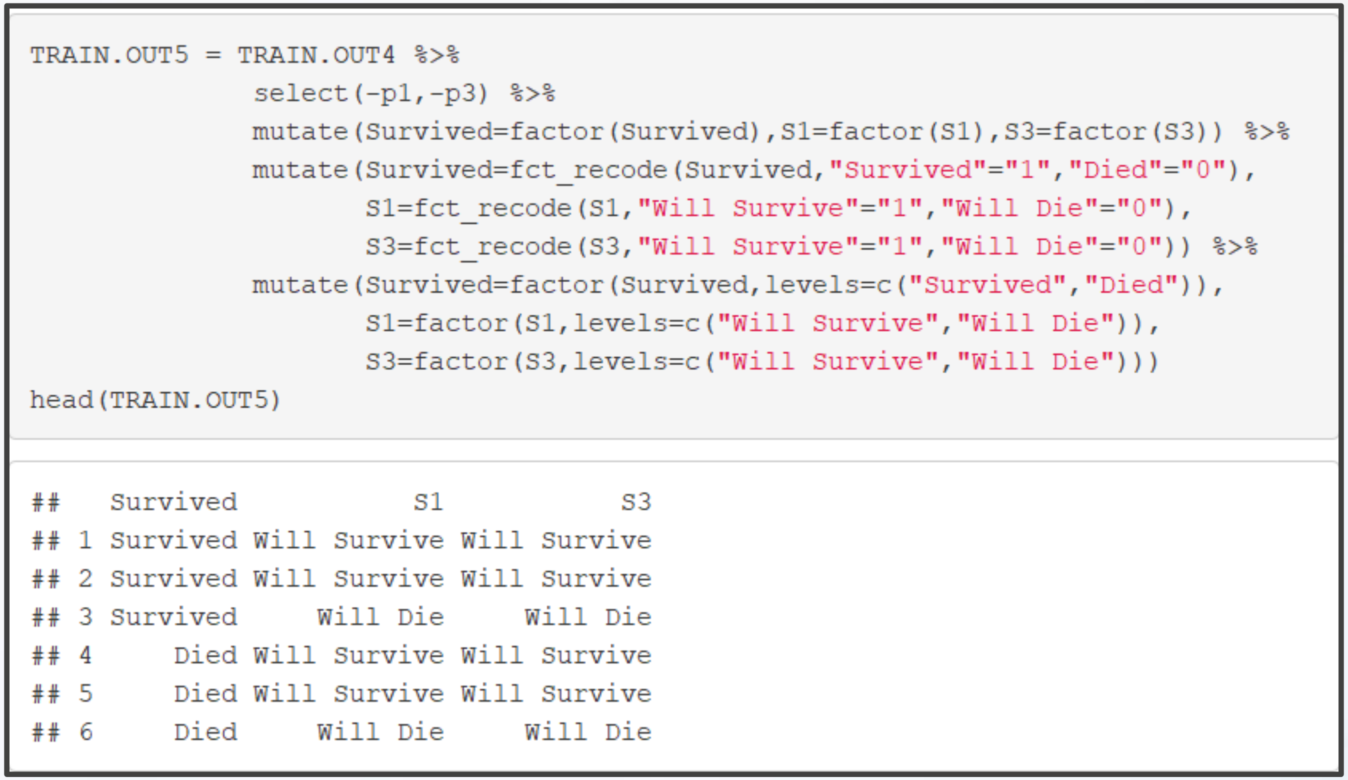
```
TRAIN.OUT4=TRAIN.OUT3 %>% select(-p2,-S2)
head(TRAIN.OUT4,8)
```

```
##      Survived        p1        p3 S1 S3
## 1           1 0.9690919 0.7802745  1  1
## 2           1 0.7754082 0.6058744  1  1
## 3           1 0.2080353 0.2124202  0  0
## 4           0 0.6660041 0.7598035  1  1
## 7           0 0.5144529 0.6255526  1  1
## 9           0 0.3504463 0.2826244  0  0
## 10          0 0.2084528 0.1755685  0  0
## 11          0 0.3588175 0.2646063  0  0
```

Where Do You See Error?

Evaluation

- Evaluating Results
  - Helpful Modifications

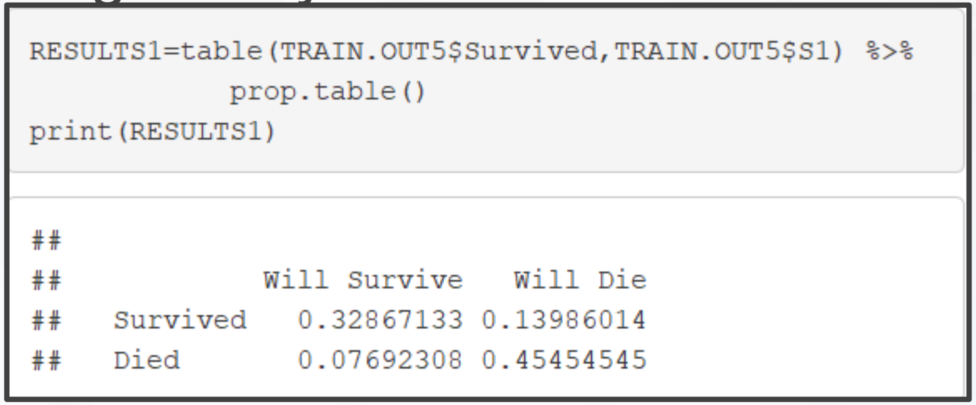


```
TRAIN.OUT5 = TRAIN.OUT4 %>%
              select(-p1,-p3) %>%
              mutate(Survived=factor(Survived),S1=factor(S1),S3=factor(S3)) %>%
              mutate(Survived=fct_recode(Survived,"Survived"="1","Died"="0"),
                     S1=fct_recode(S1,"Will Survive"="1","Will Die"="0"),
                     S3=fct_recode(S3,"Will Survive"="1","Will Die"="0")) %>%
              mutate(Survived=factor(Survived,levels=c("Survived","Died")),
                     S1=factor(S1,levels=c("Will Survive","Will Die")),
                     S3=factor(S3,levels=c("Will Survive","Will Die")))
head(TRAIN.OUT5)
```

```
##   Survived           S1           S3
## 1 Survived Will Survive Will Survive
## 2 Survived Will Survive Will Survive
## 3 Survived     Will Die     Will Die
## 4     Died Will Survive Will Survive
## 5     Died Will Survive Will Survive
## 6     Died     Will Die     Will Die
```

Evaluation: Confusion Matrix

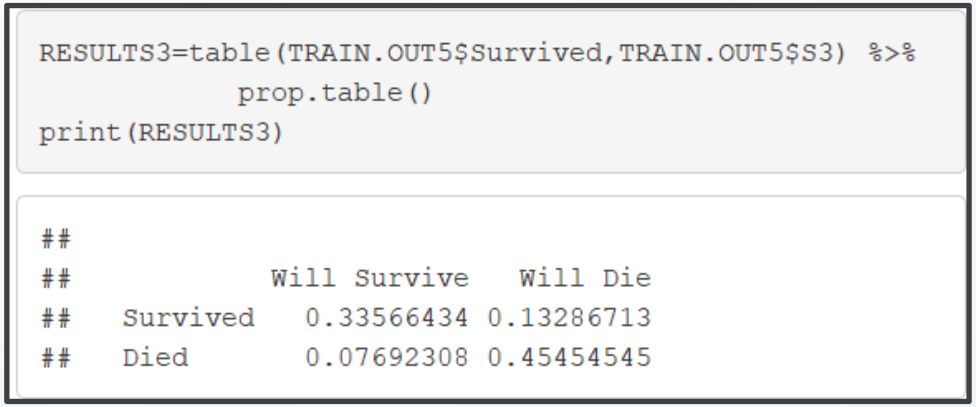
- Evaluating Results (Continued)
  - Confusion Matrix
    - Including 3-Way Interactions



```
RESULTS1=table(TRAIN.OUT5$Survived,TRAIN.OUT5$S1) %>%
          prop.table()
print(RESULTS1)
```

```
##
##            Will Survive   Will Die
##   Survived   0.32867133 0.13986014
##   Died       0.07692308 0.45454545
```

    - No Way Interactions

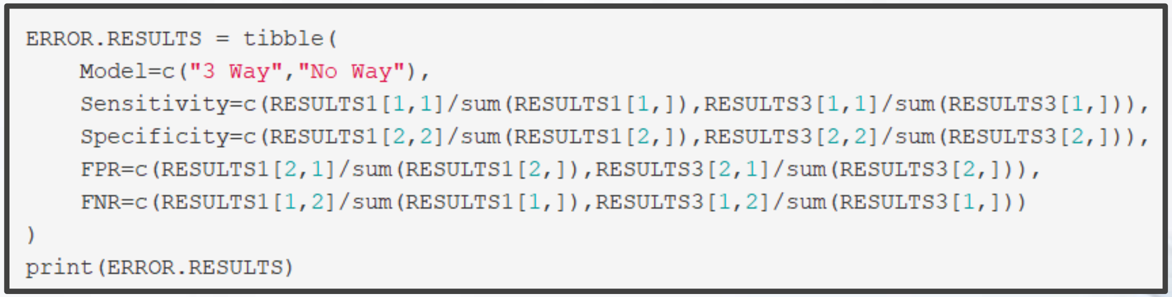


```
RESULTS3=table(TRAIN.OUT5$Survived,TRAIN.OUT5$S3) %>%
          prop.table()
print(RESULTS3)
```

```
##
##            Will Survive   Will Die
##   Survived   0.33566434 0.13286713
##   Died       0.07692308 0.45454545
```

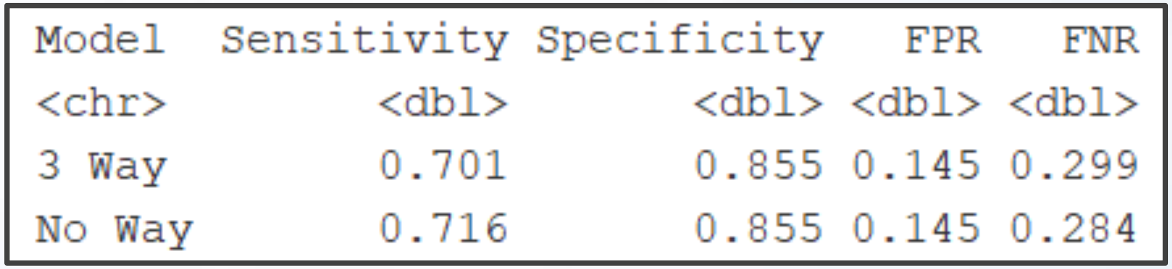
Evaluation: Rates

- Evaluating Results (Continued)
  - Error Statistics
    - Code



```
ERROR.RESULTS = tibble(
    Model=c("3 Way","No Way"),
    Sensitivity=c(RESULTS1[1,1]/sum(RESULTS1[1,]),RESULTS3[1,1]/sum(RESULTS3[1,])),
    Specificity=c(RESULTS1[2,2]/sum(RESULTS1[2,]),RESULTS3[2,2]/sum(RESULTS3[2,])),
    FPR=c(RESULTS1[2,1]/sum(RESULTS1[2,]),RESULTS3[2,1]/sum(RESULTS3[2,])),
    FNR=c(RESULTS1[1,2]/sum(RESULTS1[1,]),RESULTS3[1,2]/sum(RESULTS3[1,]))
)
print(ERROR.RESULTS)
```

    - Results



```
Model   Sensitivity Specificity   FPR   FNR
<chr>         <dbl>       <dbl> <dbl> <dbl>
3 Way         0.701       0.855 0.145 0.299
No Way        0.716       0.855 0.145 0.284
```