STOR 320 Modeling X

Lecture 33

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Final Presentation Time

- See Schedule via Group Assignment
- Submit slides via Sakai before Presentation Day.
- 5-7 minutes presentation.

Introduction

- Big Data
  - Large Sample Size
  - Large Number of Variables
  - Traditional Methods are Difficult to Implement
  - Depends on the Available Technology

- Goal: Explore Approaches for Quick Filtering of Predictors

- Tutorial 15
  - Download Rmd
  - Install Package `> library(glmnet)`
  - Knit the Document
  - Read the Introduction

Linear Models

- Consider the Following:

$$y_i = \beta_0 + X_{1i}\beta_1 + \ ... + X_{pi}\beta_p + \epsilon_i$$

where $i = 1,2,3, \dots, n$

- Matrix Representation

$$\boldsymbol{y} = \boldsymbol{\beta_0} + \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\epsilon}$$

where $\boldsymbol{y} = [y_1, y_2, \dots, y_n]'$,

$\boldsymbol{\beta} = [\beta_1, \beta_2, \dots, \beta_p]'$,

$\boldsymbol{\epsilon} = [\epsilon_1, \epsilon_2, \dots, \epsilon_n]'$,

and

$$\mathbf{X} = \begin{bmatrix} X_{11} & X_{21} & \cdots & X_{p1} \\ X_{12} & X_{22} & \cdots & X_{p2} \\ \vdots & \vdots & \ddots & \vdots \\ X_{1n} & X_{2n} & \cdots & X_{pn} \end{bmatrix}$$

Linear Model

- Information About Model Matrix

$$\mathbf{X} = \begin{bmatrix} X_{11} & X_{21} & \cdots & X_{p1} \\ X_{12} & X_{22} & \cdots & X_{p2} \\ \vdots & \vdots & \ddots & \vdots \\ X_{1n} & X_{2n} & \cdots & X_{pn} \end{bmatrix}$$

This Matrix Should Be Standardized

- Once Standardized, The Intercept $\boldsymbol{\beta_0}$ is Unnecessary in the Model

- For Interpretability, the Response Vector $\boldsymbol{y}$ Can Also Be Standardized

Part 1: Simulate and Mediate

- Run Chunk 1
  - Simulating Response From a Linear Model
  - All Predictor Variables in X are Standardized
  - What is n?
  - What is p?
  - What do We Know About the True Signal We Want to Detect?

> rnorm()

Sparse

Part 1-Chunk 2

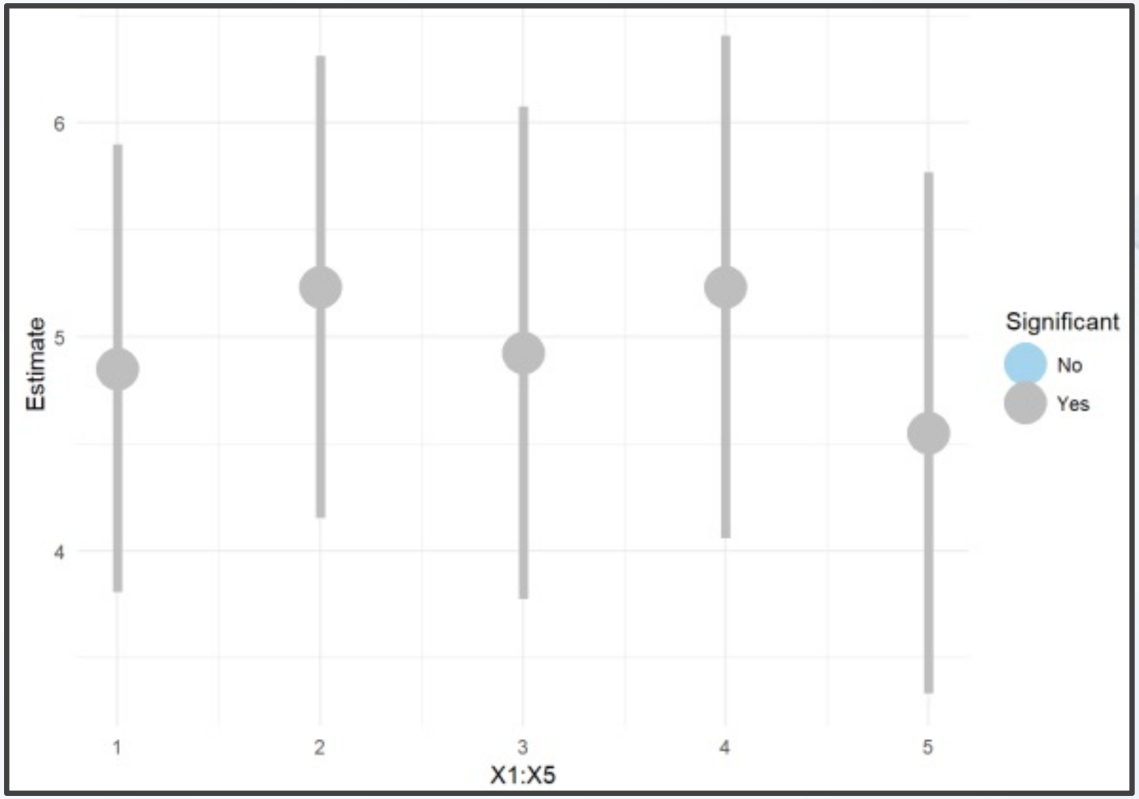
- Run Chunk 2
  - Fitting Naïve Linear Model
  - Obtaining Confidence Intervals for Parameters

```
> confint(lm.model)
```

  - Figure Info
    - Show the Estimated Coefficients of Linear Model
    - Show Confidence Intervals for These Coefficients
    - What Does the Color Aesthetic Being Used For?

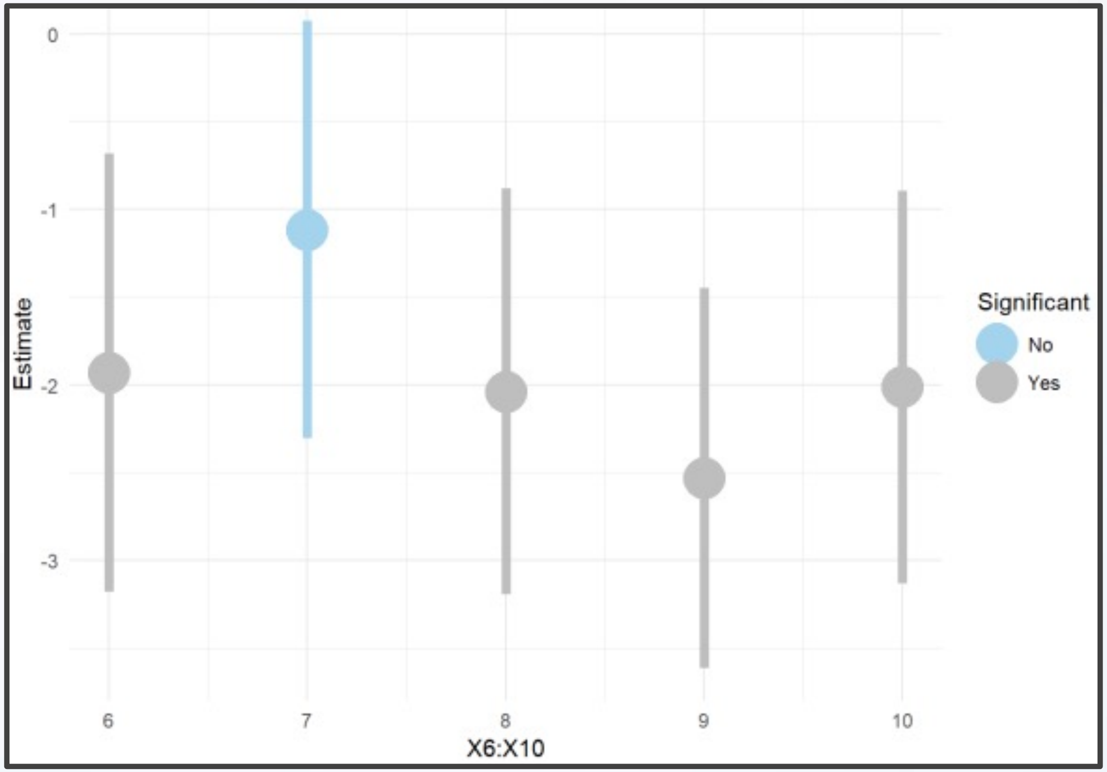
Part 1-Chunk 2

- Chunk 2 (Continued)
  - Knit the Document and Observe the 3 Graphics
  - Figure 1



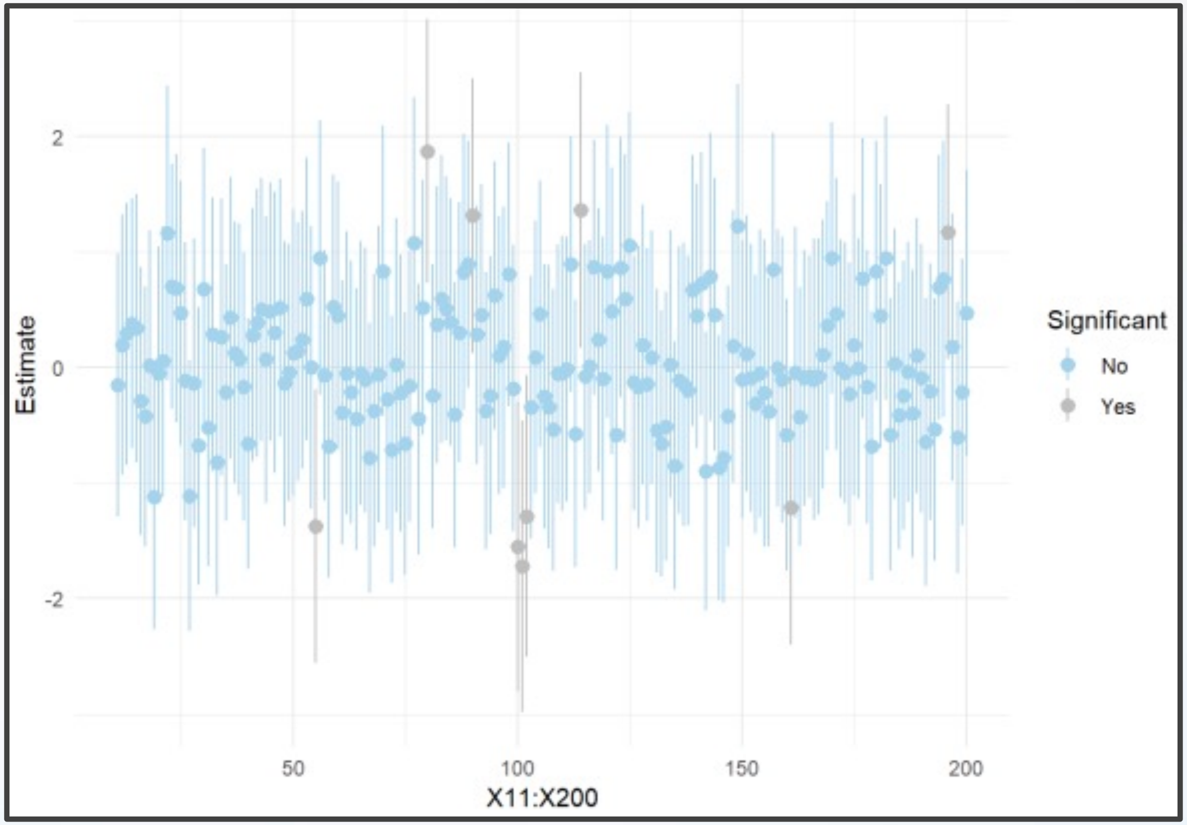
Part 1-Chunk 2

- Chunk 2 (Continued)
  - Figure 2
  - What is the Problem?



Part 1-Chunk 2

- Chunk 2 (Continued)
  - Figure 3
  - What is the Problem?



Part 1-Chunk3

- Run Chunk 3
  - Regression for Each Predictor
  - Obtaining Coefficients

```
> coef(individual.mod)
(Intercept)       X.200
  0.1257668  -0.3200960
```

Save

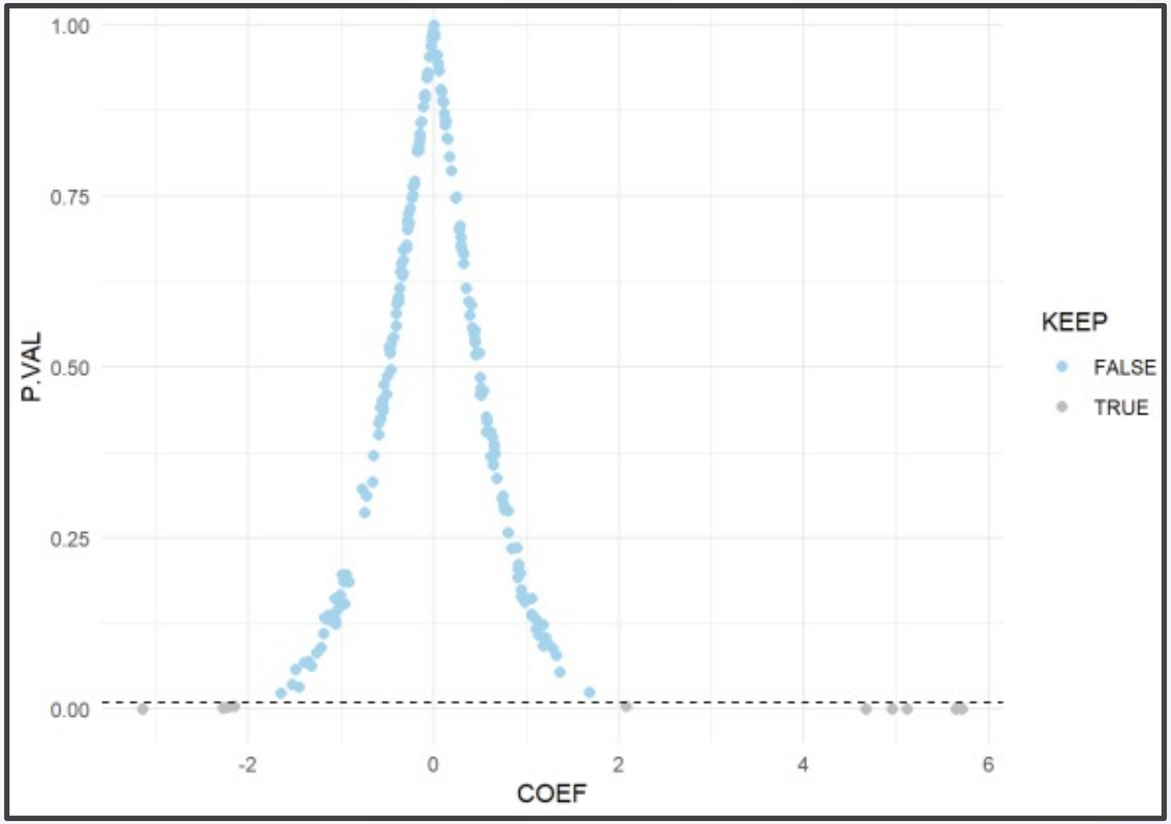
  - Obtaining P-Values

```
> summary(individual.mod)

Call:
lm(formula = y ~ ., data = SIM.DATA[, c(1, j + 1)])

Residuals:
    Min      1Q  Median      3Q     Max 
-47.252 -11.318   0.035  10.759  45.336 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)   0.1258     0.7021   0.179    0.858
X.200        -0.3201     0.7230  -0.443    0.658

Residual standard error: 15.66 on 498 degrees of freedom
Multiple R-squared:  0.0003934, Adjusted R-squared:  -0.001614 
F-statistic: 0.196 on 1 and 498 DF,  p-value: 0.6582
```

Save

Part 1-Chunk3

- Run Chunk 3
  - Figure Plots P-Values Against Coefficients



Part 1-Chunk 3

- Run Chunk 3
  - Suppose We Were to Keep Only the Predictor Variables that Had P-Values<0.01
  - Observe the Table

	P-Val > 0.01	P-Val < 0.01
Non-Zero	1%	4%
Zero	94%	1%

- 95% of Variables Ignored
- 5% of Variables Included
- Errors (What is Worse?)
  - We Will Ignore Variables that Are Important
  - We Will Include Variables that Are Irrelevant

Part 1-Chunk 4

- Chunk 4
  - Try to Find the Smallest Cutoff Value So That We are Not Missing Important Variables
  - To Ensure We are Not Missing Important Variables, Should we Increase or Decrease the Original Cutoff (0.01)
  - What Cutoff Works?
  - Try Multiple Cutoffs and Observe the Table
  - Run the Code Inside the Chunk Until All 10 Important Variables are Retained for the Future

Part 1-Chunk 4

- Chunk 4 (Continued)
  - Traditional Choice: 0.20
  - Output in Table

	P-Val > 0.01	P-Val < 0.01
Non-Zero	0%	5%
Zero	71%	24%

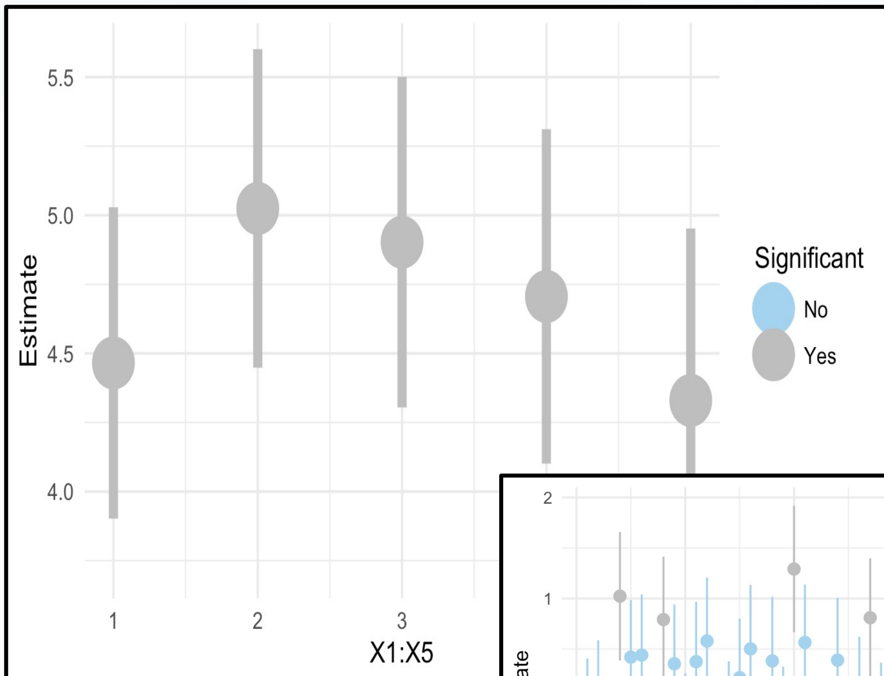
None of the Non-Zero Parameters Will Be Ignored

- Fit Linear Model for Variables Kept in Consideration

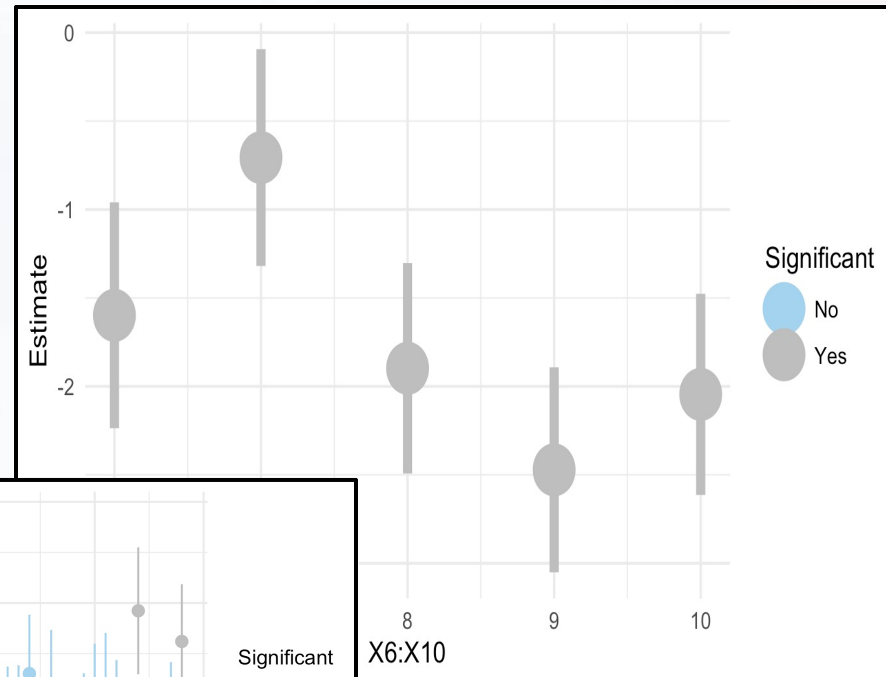
```
> lm(y~.,data=SIM.DATA[,c(1,which(KEEP)+1)])
```

Part 1-Chunk 4

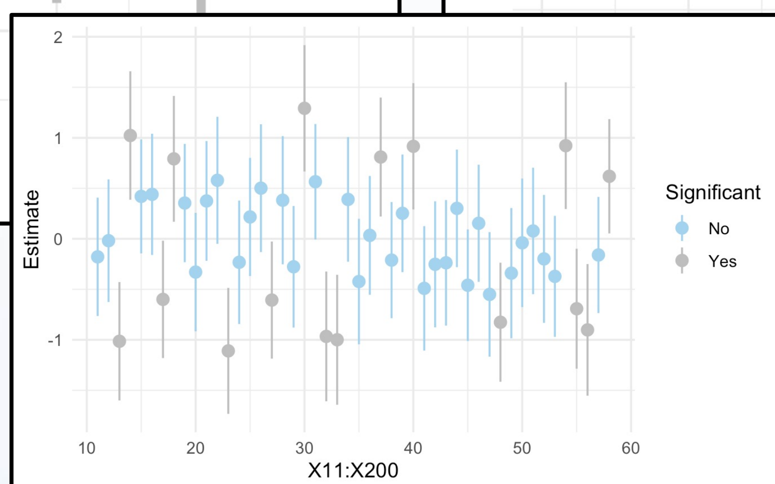
- Suppose Cutoff is 0.2
  - Figure 1



- Figure 2



- Figure 3



Part 1: Recap

- Recap
  - Before Building Complex Models We are Performing a Simple Screening Procedure

  - Problems
    - We May Lose Variables with Significant Interactions
    - We May Still Have Too Many
    - We May Retain Variables that are Highly Correlated

Shrinkage Estimation

- Classic Linear Model Estimation
  - Minimize Sum of Squared Error

$$SSE = \sum [y_i - (\beta_0 + \boldsymbol{x_i}'\boldsymbol{\beta})]^2$$

  - Optimization: Find $\widehat{\beta_0}$ and $\widehat{\boldsymbol{\beta}}$ that Make SSE as Small as Possible
  - $\widehat{\beta_0}$ and $\widehat{\boldsymbol{\beta}}$ are Easily Found Using Matrix Representation

- Regularized Estimation
  - Produces Biased Estimates
  - Shrinks Coefficients Toward 0
  - Favors Smaller Models
  - May Lead to a Better Model for Out-of-Sample Prediction

Shrinkage Estimation

- Three Popular Methods
  - Download R Package `> library(glmnet)`
  - Penalized SSE

$$PSSE = SSE + \lambda[(1-\alpha)\sum_{i=1}^{p}\beta_i^2 + \alpha\sum_{i=1}^{p}|\beta_i|]$$

  - Variations
    - Ridge (1970): $\lambda = 1 \,\&\, \alpha = 0$
    - Lasso (1996): $\lambda = 1 \,\&\, \alpha = 1$
    - Elastic Net (2005)

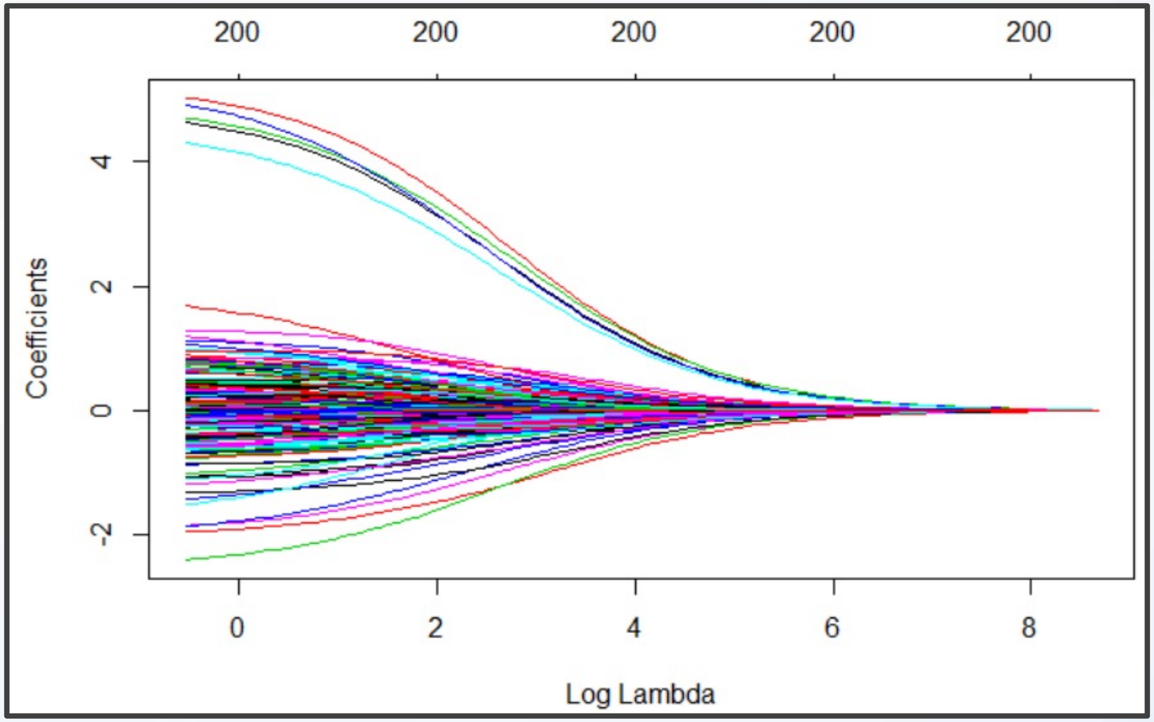
$$\lambda = 1 \ \&\ 0 < \alpha < 1$$

  - Notice When
    - $\lambda = 0$ PSSE=SSE
    - As λ Gets Bigger, the Coefficients Approach 0

Part 2: Ridge

- Run Chunk 1
  - Ridge Penalty

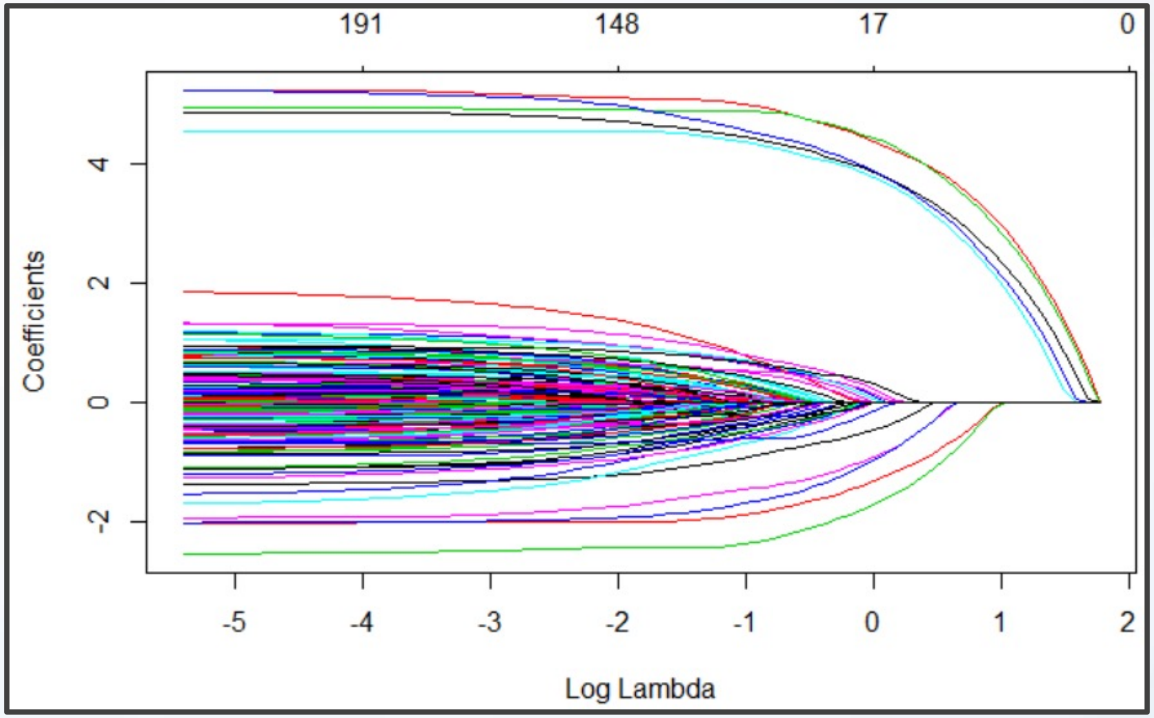
```
> ridge.mod=glmnet(x=as.matrix(SIM.DATA[,-1]),
+                  y=as.vector(SIM.DATA[,1]),
+                  alpha=0)
> plot(ridge.mod,xvar="lambda")
```



Part 2: Lasso

- Run Chunk 2
  - Lasso Penalty

```
> lasso.mod=glmnet(x=as.matrix(SIM.DATA[,-1]),
+                   y=as.vector(SIM.DATA[,1]),
+                   alpha=1)
> plot(lasso.mod,xvar="lambda")
```



Part 2: Elastic Net

- Run Chunk 3
  - Elastic Net Penalty

```
> enet.mod=glmnet(x=as.matrix(SIM.DATA[,-1]),
+                   y=as.vector(SIM.DATA[,1]),
+                   alpha=1/2)
> plot(enet.mod,xvar="lambda")
```