STOR566: Introduction to Deep Learning

Lecture 7: Convolutional Neural Networks

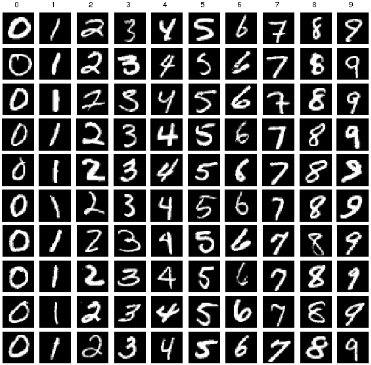
Yao Li
UNC Chapel Hill

Sep 8, 2022

Materials are from *Learning from data (Caltech)* and *Deep Learning (UCLA)*

MNIST

- Hand-written digits (0 to 9)
- Total $60,000$ samples, 10-class classification.



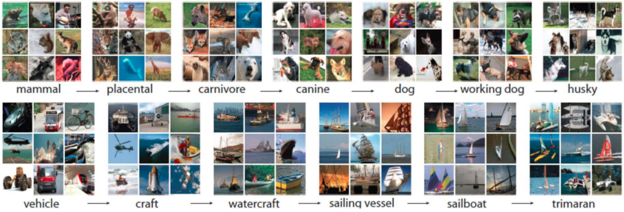
MNIST Classification Accuracy

- See the website by Yann LeCun:

  `http://yann.lecun.com/exdb/mnist/`

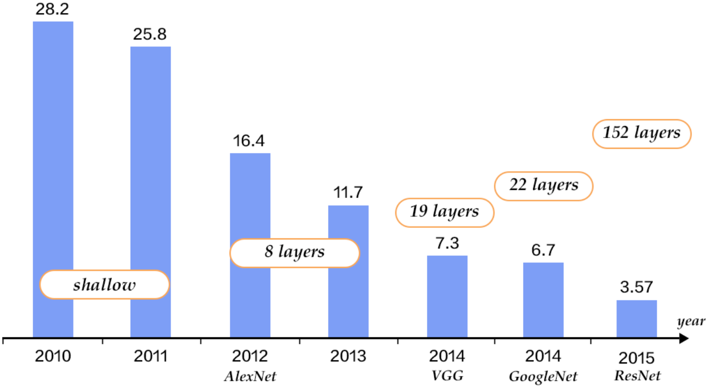
Classifier	Test Error
Linear classifier	12.0 %
SVM, Gaussian kernel	1.4%
SVM, degree 4 polynomial	1.1%
Best SVM result	0.56%
2-layer NN	$\sim$ 3.0%
3-layer NN	$\sim$ 2.5%
CNN, LeNet-5 (1998)	0.85%
Larger CNN (2011, 2012)	$\sim$ 0.3%

ImageNet Data


mammal → placental → carnivore → canine → dog → working dog → husky
vehicle → craft → watercraft → sailing vessel → sailboat → trimaran

- ILSVRC competition: 1000 classes and about 1.2 million images
- Full imagenet: $> 20,000$ categories, each with about a thousand images.

ImageNet Results



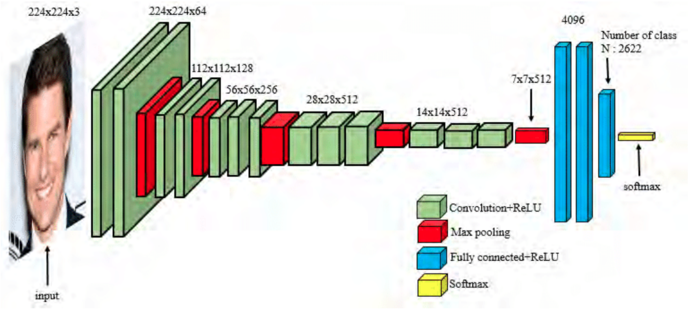
Top-5 error rates on ILSVRC image classification

picture from `http://www.paddlepaddle.org/documentation/book/en/0.14.0/03.image_classification/index.html`

Convolutional Neural Network

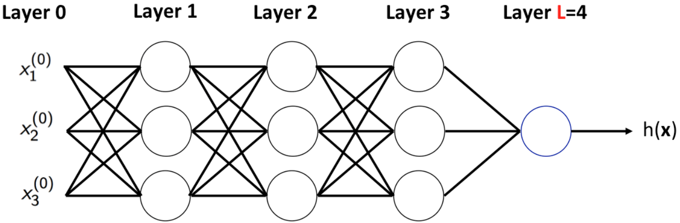
The structure of CNN

- Structure of VGG



- Two important layers:
    - Convolution
    - Pooling

Neural Networks



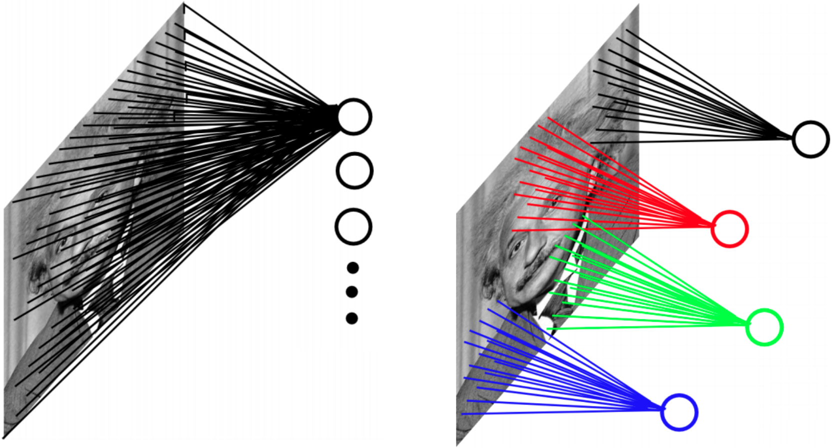
Number of parameters in the network?

Convolution Layer

- Fully connected layers have too many parameters

  $\Rightarrow$ poor performance
- Example: VGG first layer
  - Input: $224 \times 224 \times 3$
  - Output: $224 \times 224 \times 64$
  - Number of parameters if we use fully connected net: $(224 \times 224 \times 3) \times (224 \times 224 \times 64) =$ 483 billion
- Convolution layer leads to:
  - Local connectivity
  - Parameter sharing

Local connectivity



(Figure from Salakhutdinov 2017)

Parameter Sharing

- Making a reasonable assumption:

If one feature is useful to compute at some spatial position (x, y), then it should also be useful to compute at a different position (x_2, y_2)

- Using the convolution operator

Convolution

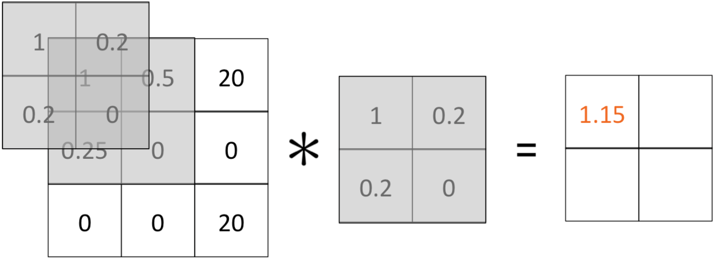
- The convolution of an image x with a kernel k is computed as

$$(x * k)_{ij} = \sum_{pq} x_{i+p,j+q} k_{p,q}$$

$$\begin{array}{|c|c|c|}\hline 1 & 0.5 & 20 \\ \hline 0.25 & 0 & 0 \\ \hline 0 & 0 & 20 \\ \hline \end{array} * \begin{array}{|c|c|}\hline 1 & 0.5 \\ \hline 0.25 & 0 \\ \hline \end{array} = \begin{array}{|c|c|}\hline & \\ \hline & \\ \hline \end{array}$$

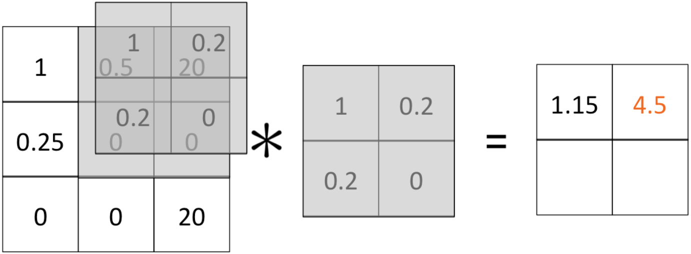
Convolution

1*1 + 0.5*0.2 + 0.25*0.2 + 0*0 = 1.15



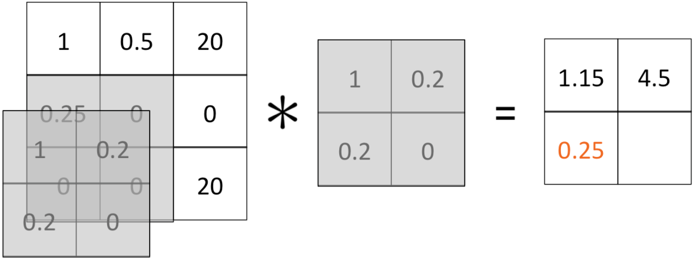
Convolution

0.5*1 + 20*0.2 + 0*0.2 + 0*0 = 4.5



Convolution

0.25*1 + 0*0.2 + 0*0.2 + 0*0 = 0.25



1 0.5 20

0.25 0 0

1 0.2

0 0 20

0.2 0

*

1 0.2

0.2 0

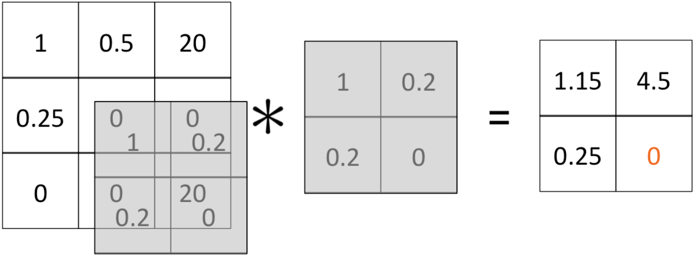
=

1.15 4.5

0.25

Convolution

0*1 + 0*0.2 + 0*0.2 + 20*0 = 0



Multiple Channels

- Multiple input channels:

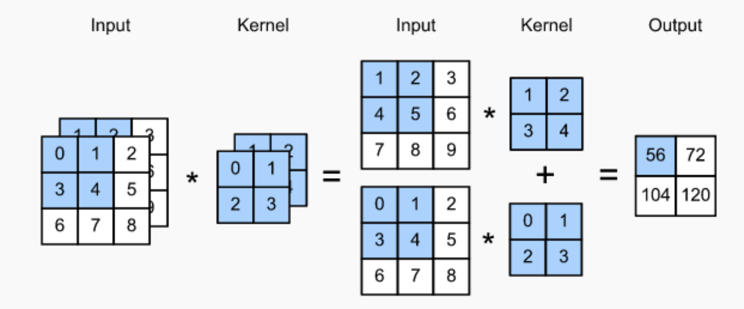
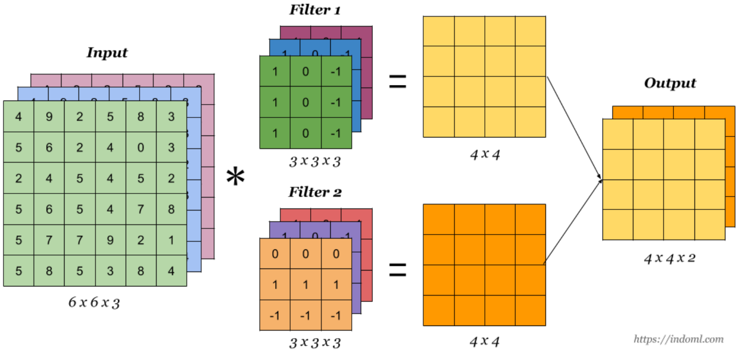


Image from Dive into Deep Learning

- $(1 \times 1 + 2 \times 2 + 4 \times 3 + 5 \times 4) + (0 \times 0 + 1 \times 1 + 3 \times 2 + 4 \times 3) = 56$

Multiple Channels

- Multiple input channels and output channels:



4	9	2	5	8	3
5	6	2	4	0	3
2	4	5	4	5	2
5	6	5	4	7	8
5	7	7	9	2	1
5	8	5	3	8	4

1	0	-1
1	0	-1
1	0	-1

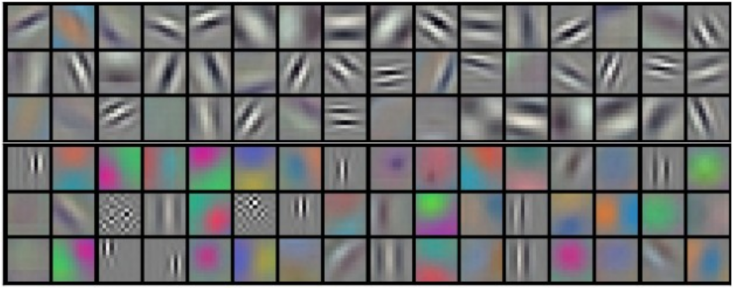
0	0	0
1	1	1
-1	-1	-1

https://indoml.com

- Number of parameters: $k_1 \times k_2 \times d_{in} \times d_{out} + d_{out}$

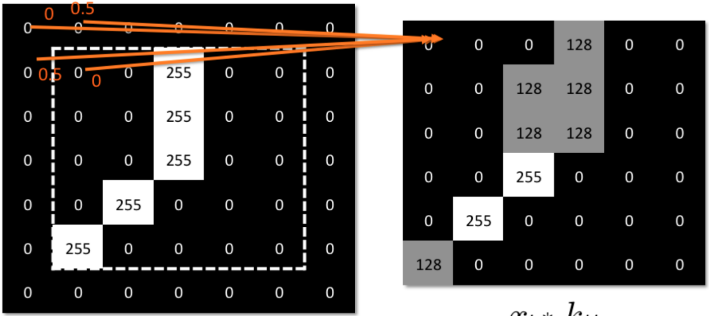
Learned Kernels

- Example kernels learned by AlexNet



Padding

- Use zero padding to allow going over the boundary
    - Easier to control the size of output layer



0	0	0	0	0	0	0
0	0	0	255	0	0	0
0	0	0	255	0	0	0
0	0	0	255	0	0	0
0	0	255	0	0	0	0
0	255	0	0	0	0	0
0	0	0	0	0	0	0

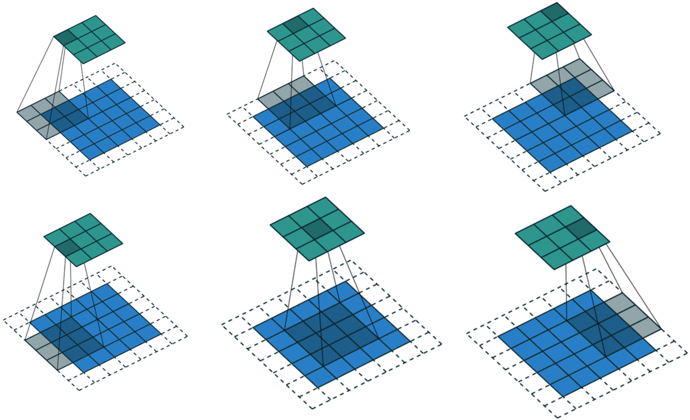
x_i

0	0	0	128	0	0
0	0	128	128	0	0
0	0	128	128	0	0
0	0	255	0	0	0
0	255	0	0	0	0
128	0	0	0	0	0

$x_i \underline{*} k_{ij}$

Strides

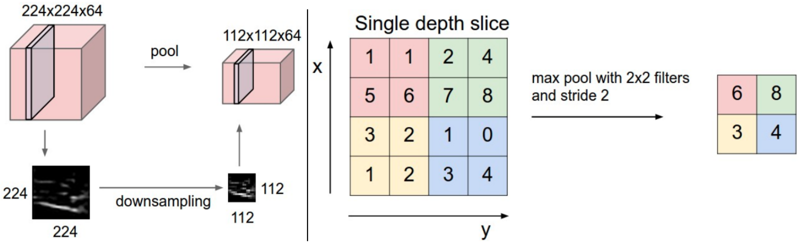
- Stride: The amount of movement between applications of the filter to the input image
- Strude $= (1, 1)$: no stride



stride = (2,2)

Pooling

- It's common to insert a pooling layer in-between successive convolutional layers
- Reduce the size of representation, down-sampling
- Example: Max Pooling



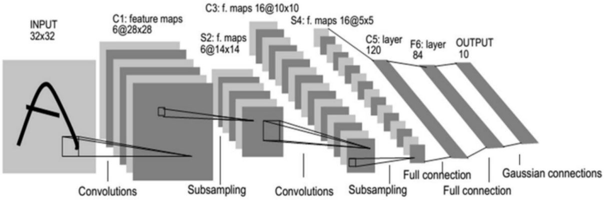
Single depth slice

1	1	2	4
5	6	7	8
3	2	1	0
1	2	3	4

max pool with 2x2 filters and stride 2

6	8
3	4

Example: LeNet5

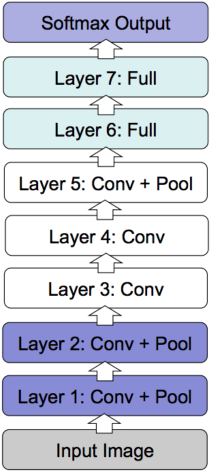


- Input: 32 × 32 images (MNIST)
- Convolution 1: 6 5 × 5 filters, stride 1
  - Output: 6 28 × 28 maps
- Pooling 1: 2 × 2 max pooling, stride 2
  - Output: 6 14 × 14 maps
- Convolution 2: 16 5 × 5 filters, stride 1
  - Output: 16 10 × 10 maps
- Pooling 2: 2 × 2 max pooling with stride 2
  - Output: 16 5 × 5 maps (total 400 values)
- 3 fully connected layers: 120 ⇒ 84 ⇒ 10 neurons

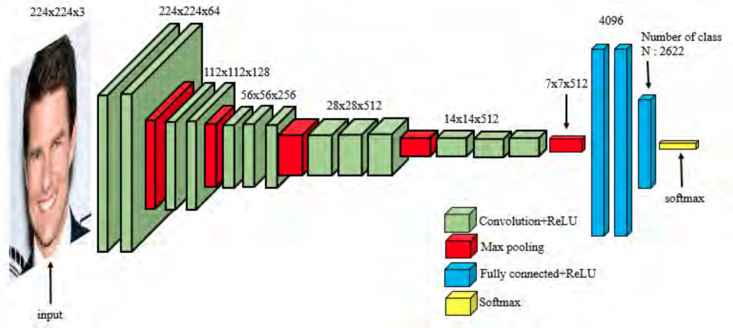
AlexNet

- 8 layers in total, about 60 million parameters and 650,000 neurons.
- Trained on ImageNet dataset

  "ImageNet Classification with Deep Convolutional Neural Networks", by Krizhevsky, Sustskever and Hinton, NIPS 2012.

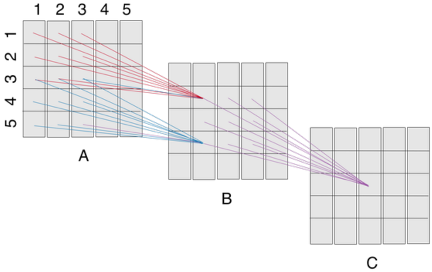


Example: VGG Network



What do the kernels learn?

- The receptive field of a neuron is the input region that can affect the neuron's output
- The receptive field for a first layer neuron is its neighbors (depending on kernel size) ⇒ capturing very local patterns
- For higher layer neurons, the receptive field can be much larger ⇒ capturing global patterns



Conclusions

- Convolution
- Pooling

Questions?