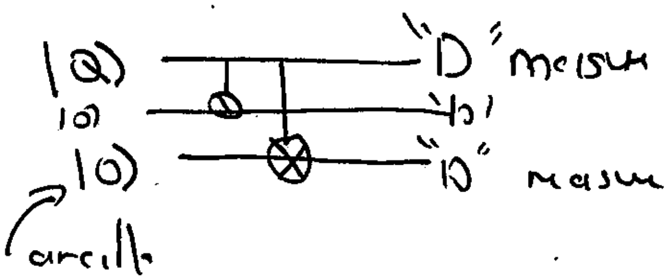
Superconducting applications

Measurement

- preliminary:
  - We need to know from randomized bench marking what the fidelities are for single qubit and two qubit operation (eg controlled Z gate)
  - We also need to determine the measurement fidelity
    - Start with qubit in $|0\rangle$ & measurement its state. Repeat to determine probability $P(0)$ of measuring in ground state if started in "0". Also determine probability of measuring in excited state ($P(1)$). Maybe even determine probability no result
    - Repeat the above measurement with qubit in $|e\rangle$ state. Determine $P_e(|e\rangle)$, $P_g(|e\rangle)$, $P_?(|e\rangle)$.
  - Now if $P(\text{measurement}) < P_{\text{two qubit}} \cdot P^2_{\text{single qubit}}$

then we can improve the measurement probability / fidelity

- Key idea is the circuit.



If our qubit is in the state $|0\rangle \rightarrow$ then after the circuit above we should have

$$|Q\rangle|0\rangle|0\rangle \Rightarrow |0\rangle|0\rangle|0\rangle \longrightarrow \text{detection}$$

prob error

$$\text{prob}(0,0,0) = (1-p_e)^3 \quad \leftarrow \text{good signal}$$

$$\text{prob}(0,0,1) = p_e(1-p_e)^2 \quad \leftarrow \text{good signal}$$

$$\text{prob}(0,1,1) = p_e^2(1-p_e) \quad \leftarrow \text{bad}$$

$$\text{prob}(1,1,1) = p_e^3 \quad \leftarrow \text{bad}$$

Using a majority vote, we can say

$$P(0 \text{ result}) = P(0,0,0) + P(0,0,1) + P(0,1,0) + P(1,0,\dots$$

$$= (1-p_3)^3 + 3p_e(1-p_e)^2$$

$$= (1-p_e)^2(1-p_e+3p_e) \quad = (1-p_e)^2(1+2p_e)$$

$$= (1-2p_e+p_e^2)(1+2p_e)$$

$$\sim (1 - 0p_e - 3p_e^2 \quad -2p_e^3)$$

$$P(\text{"}0\text{"}) \sim 1 - 3p_e^2$$

c.f for direct measure $1-p_e$

So we can make a more efficient detector.

Metrology

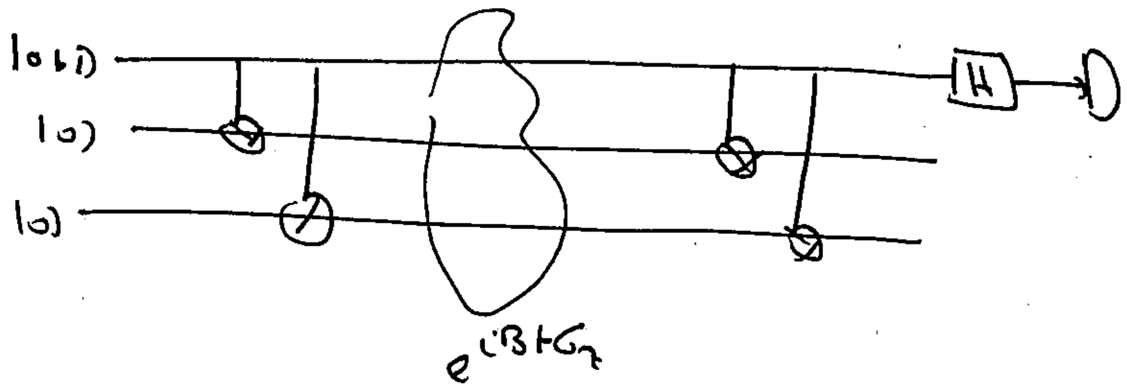
Consider that we can generate a state of the form

$$|\psi\rangle = |\underset{g}{000}\rangle + |\underset{e}{111}\rangle \qquad \text{(3 qubit case)}$$

If we apply a small magnetic field to all qubit we would have

$$|\psi\rangle = e^{-inBt}|0\ldots\rangle + e^{inBt}|11\ldots\rangle$$

- we have n times the phase shift compared to one qubit



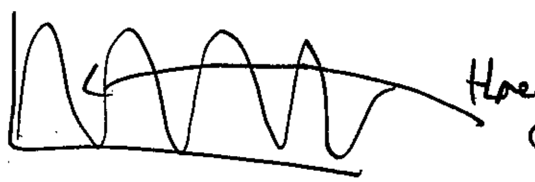
$$|0+1\rangle|00\rangle \xrightarrow{CNOT} |000\rangle + |111\rangle$$

$$\xrightarrow{Bt} e^{-3iBt}|000\rangle + e^{3iBt}|111\rangle$$

$$\xrightarrow{CNOT} \left\{e^{-3iBt}|0\rangle + e^{3iBt}|1\rangle\right\}|0\rangle|0\rangle$$

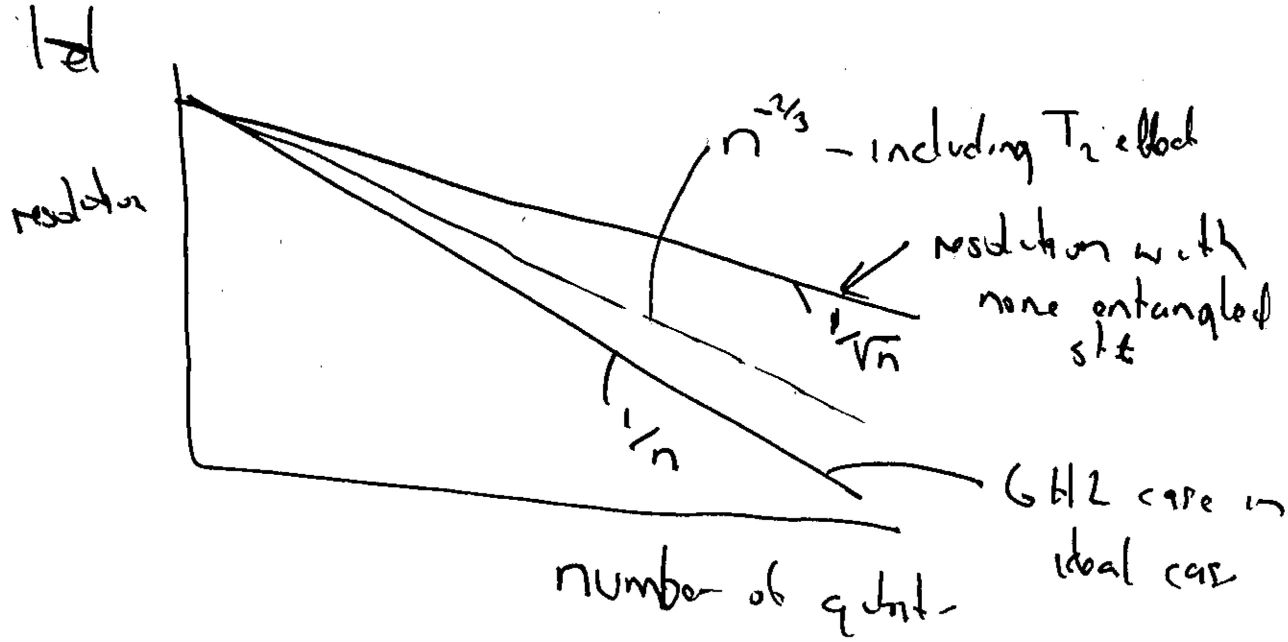
$$\xrightarrow{H} \left\{\cos 3Bt\,|0\rangle + i\sin 3Bt\,|1\rangle\right\}|00\rangle$$

Measure "0", "1"



these time gaps

So what this means is that we can detect a small magnetic field shift



So the ideal is use your GHZ state to show magnetic field sensitivity beyond the standard quantum limit.

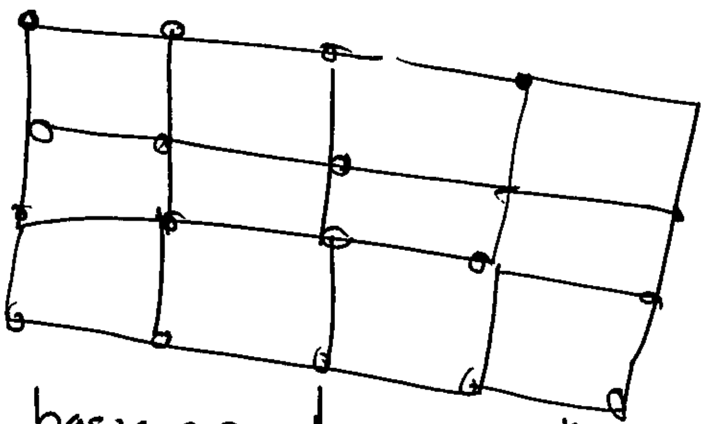
Error correction & computation

- If we have multiple qubits (<20) we want to look at the simple error correction codes

  - 3 qubit Bit flip code
  - 7 qubit CSS code
  - 9 qubit Shor code
  - small surface code.

- We also want to generate small cluster states

e.g.



and do basic operations on them.