

# Protocol for Universal Gates in Optimally Biased Superconducting Qubits

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We present a new experimental protocol for performing universal gates in a register of superconducting qubits coupled by fixed on-chip linear reactances. The qubits have fixed, detuned Larmor frequencies and can remain, during the entire gate operation, biased at their optimal working point where decoherence due to fluctuations in control parameters is suppressed to first order. Two-qubit gates are performed by **simultaneously irradiating two qubits at their respective Larmor frequencies** with appropriate amplitude and phase, while one-qubit gates are performed by the usual single-qubit irradiation pulses.

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Single quantum bits displaying coherence in the time domain have now been implemented in various superconducting integrated electrical circuits [1]. Microwave spectroscopy [2], coherent temporal oscillations [3], and a conditional gate operation [4] have been reported in experiments on pairs of capacitively or inductively coupled qubits. In all these implementations, decoherence is by far the largest obstacle to be overcome for applications to quantum information processing. Yet, as Vion *et al.* (see Ref. [1(b)]) have demonstrated, symmetries in circuit architecture and bias conditions can be exploited for suppressing to first order decoherence due to fluctuations in control parameters.

The schemes for performing two-qubit gates proposed so far rely on dynamical tuning of either the qubit transition frequencies [2] or a subcircuit controlling the qubit-qubit interaction [5]. The former requires dc pulses that move the qubits away from their optimal bias points for coherence, while the latter requires additional control lines and nonlinear elements that inevitably introduce additional couplings to uncontrolled degrees of freedom in the environment. In this Letter, we present a novel scheme that minimizes decoherence by maintaining each qubit at its optimal bias point, and by **employing only noise-free fixed linear coupling reactances**. Furthermore, the spread of qubit frequencies occurring naturally in fabrication is in this case an advantage rather than a hindrance.

Our strategy consists of constructing circuits with fixed, detuned Larmor frequencies and fixed coupling strengths—sort of “artificial molecules”—and realizing gates with protocols inspired by those of nuclear magnetic resonance (NMR) quantum computation [6]. The essential difference between our “molecules” and those of NMR resides in the qubit-qubit couplings and the way they are exploited. In NMR, the secular terms in the coupling Hamiltonian (those that commute with the Zeeman Hamiltonian and thus act to first order) dominate the spin-spin interaction. Two-qubit gates are realized as the spins precess freely, while refocusing pulses are applied in order to do nothing. In our scheme, the coupling is purely

*nonsecular* and has no effect to first order. So, unlike in NMR, we must construct pulses to enhance the second-order effect of the coupling. We refer to this strategy with the (NMR style) nickname **FLICFORQ**: fixed linear couplings between fixed off-resonant qubits.

The superconducting register we have in mind could consist of **charge qubits** (controlled via **charges on gate capacitors**) interacting through on-chip capacitors or of **flux qubits** (controlled via **fluxes** through superconducting loops) interacting through mutual inductances. We focus for the moment on two-qubit registers (Fig. 1), the simplest that allow the realization of a universal set of quantum gates, leaving the extension to larger systems to the discussion. The optimal bias conditions for the circuits shown are  $N_1^g = N_2^g = 1/2$  for charge qubits, where  $N^g = C^g U/2e$  is the dimensionless gate charge, or  $N_1^\phi = N_2^\phi = 1/2$  for flux qubits, where  $N^\phi = \Phi^{\text{ext}}/\Phi_0$  is the flux frustration. Under these conditions, the systems become immune, to first order, to variations in the control parameters, such as  $1/f$  charge noise in the Josephson junctions or substrate or noise due to the motion of trapped flux [7].

At optimal bias (and in the two-level approximation), these two-qubit systems are described by the reduced Hamiltonian

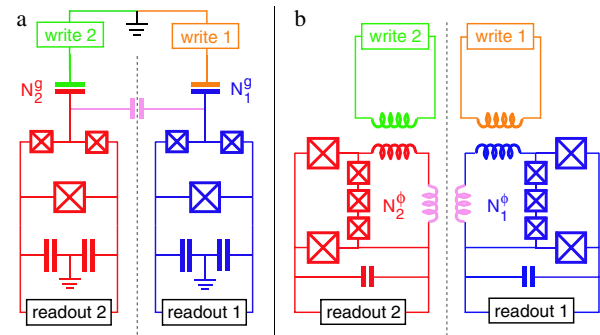


FIG. 1 (color online). Superconducting two-qubit circuits for performing universal quantum gates at optimal bias point. (a) Charge qubits (Saclay style) coupled by capacitor. (b) Flux qubits (Delft style) coupled by mutual inductance.

$$\mathcal{H}/\hbar = \frac{1}{2}\omega_1^L \sigma_1^z + \frac{1}{2}\omega_1^R \cos(\omega_1^{\text{rf}} t + \phi_1) \sigma_1^x + \frac{1}{2}\omega_2^L \sigma_2^z + \frac{1}{2}\omega_2^R \cos(\omega_2^{\text{rf}} t + \phi_2) \sigma_2^x + \frac{1}{2}\omega^{xx} \sigma_1^x \sigma_2^x, \quad (1)$$

where  $\omega_a^L/2\pi$  is the Larmor frequency of qubit  $a$ ;  $\omega_a^R$  and  $\omega_a^{\text{rf}}/2\pi$  are, respectively, the amplitude and frequency of the signal applied to the write port of qubit  $a$ , and  $\omega^{xx}/2\pi = (t^{\text{ent}})^{-1}$  is the entangling frequency (if only the  $\sigma_1^x \sigma_2^x$  term were present in  $\mathcal{H}$ , the time needed to go from a computational basis state to a maximally entangled state would be  $t^{\text{ent}}/4$ ). The Larmor frequencies are **detuned** from one another, as occurs naturally during fabrication, by  $\delta = \omega_1^L - \omega_2^L$ , and are static throughout. The entangling frequency is set by circuit parameters at the time of fabrication. The present results are valid for parameters satisfying  $\omega^{xx} \ll \delta \ll \omega_{1,2}^L$ . In practice, there will also be some cross coupling of write signals between the qubits, but the difference between  $\omega_1^L$  and  $\omega_2^L$  suppresses this effect, a crucial practical advantage of FLICFORQ.

For simplicity, we limit this discussion to resonant rf pulses obeying  $\omega_1^{\text{rf}} = \omega_1^L$  and  $\omega_2^{\text{rf}} = \omega_2^L$ . In what follows, we first demonstrate that dynamic control of the irradiation amplitudes  $\omega_{1,2}^R$  and phases  $\phi_{1,2}$  allows deterministic entanglement of the two qubits; we then present a specific pulse sequence for performing the two-qubit  $\pi/2$  rotation  $(Y_1 Y_2)^{1/2} = (\mathbf{1} + \sigma_1^y \sigma_2^y)/\sqrt{2}$  which, when accompanied by one-qubit gates, is universal for quantum computation.

The mechanism allowing the very weak nonsecular interqubit coupling  $\omega^{xx}$  to produce maximally entangled two-qubit states is easily understood in the dressed atom picture of quantum optics [8]. When the rf fields and qubits are uncoupled, each qubit + field system has an infinite discrete ladder of doubly degenerate energy levels, labeled by the qubit state  $|1\rangle$  or  $|0\rangle$  and the photon number  $|N\rangle$ , and separated by  $\omega_{1,2}^{\text{rf}} = \omega_{1,2}^L$  (Fig. 2, outer levels). Taking the qubit-field coupling into account lifts the degeneracy, splitting the two states in each manifold by the field strength (Rabi frequency)  $\omega_{1,2}^R$  (Fig. 2, inner levels). The two dressed qubits may then absorb and emit energy at frequencies  $\omega_1^L \pm \omega_1^R$  and  $\omega_2^L \pm \omega_2^R$ , respectively. The irradiation thus splits the single-mode qubit spectra into two sidebands. Choosing  $\omega_1^R + \omega_2^R = \delta$  causes the upper sideband of qubit 1 to overlap the lower sideband of qubit 2, and the qubits can then exchange photons of energy  $\hbar(\omega_1^L - \omega_1^R) = \hbar(\omega_2^L + \omega_2^R)$  through the coupling reactance.

A more quantitative picture of the qubit-qubit interaction follows from an analysis using rotating reference frames. First, note that measurement in the computational basis commutes with the  $\sigma_{1,2}^z$  terms in  $\mathcal{H}$ , and that one-qubit rf pulses perform pure  $\sigma^x$  and  $\sigma^y$  rotations in the doubly Larmor-precessing frame  $\mathfrak{R}^{(2)}$  rotating at  $\omega_{1,2}^L$  about  $\sigma_{1,2}^z$ , respectively. This allows us to define all gates in  $\mathfrak{R}^{(2)}$ , where the qubits are static in the absence of irradiation pulses.

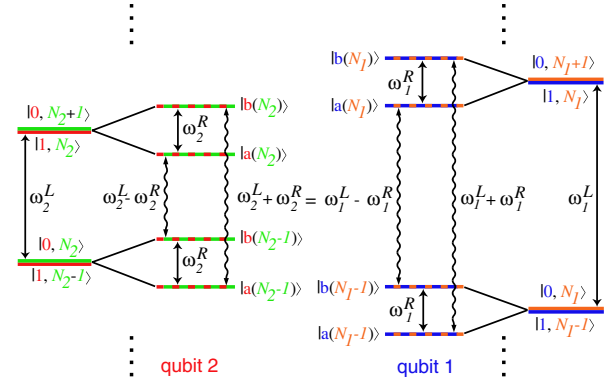


FIG. 2 (color online). Energy levels of qubit + rf photon systems with (inner levels) and without (outer levels) qubit-photon coupling. Outer: Systems have an infinite ladder of doubly degenerate levels corresponding to products of a photon number state (green, orange) and a qubit state (red, blue). Inner: Photon-qubit coupling lifts degeneracy in each manifold by Rabi frequency  $\omega_{1,2}^R$ . Transitions between adjacent manifolds (wavy arrows) correspond to absorption or emission of a photon from a dressed qubit system. The off-resonant qubits can be put on speaking terms by adjusting Rabi frequencies such that  $\omega_1^R + \omega_2^R = \delta$ . Shown:  $\omega_{1,2}^R = \delta/2$ .

Then an effective Hamiltonian for a FLICFORQ system under doubly resonant irradiation may be obtained by moving from  $\mathfrak{R}^{(2)}$  to a quadruply rotating frame  $\mathfrak{R}^{(4)}$  which, in addition, rotates at  $\omega_{1,2}^R$  about  $\sigma_{1,2}^x$ , respectively [9]. We now focus on the case where  $\omega_{1,2}^R = \delta/2$ , as depicted in Fig. 2, for which one obtains in  $\mathfrak{R}^{(4)}$  in the rotating wave approximation [10]

$$\mathcal{H}'/\hbar = \frac{\omega^{xx}}{16} \{(\sigma_1^y \sigma_2^y - \sigma_1^z \sigma_2^z) \cos(\phi_1 - \phi_2) + (\sigma_1^z \sigma_2^y + \sigma_1^y \sigma_2^z) \sin(\phi_1 - \phi_2)\}, \quad (2)$$

which is a universal Hamiltonian.

The evolution operator  $\mathcal{U}$  in  $\mathfrak{R}^{(2)}$  is related to the evolution operator  $\mathcal{U}'$  in  $\mathfrak{R}^{(4)}$  according to

$$\mathcal{U} = \mathcal{R}(t) \mathcal{U}'(t - t_0) \mathcal{R}^{-1}(t_0), \quad (3)$$

where

$$\mathcal{R}(t) = \exp\{it(\omega_1^R \sigma_1^x + \omega_2^R \sigma_2^x)/4\} \quad (4)$$

is the one-qubit rotation operator connecting  $\mathfrak{R}^{(2)}$  and  $\mathfrak{R}^{(4)}$ , so, in general, a two-qubit gate must be accompanied by the appropriate one-qubit rotations. However, for  $\omega_{1,2}^R = \delta/2$ ,  $\mathcal{R}(t)$  reduces to the identity operation at times  $t_m^{\text{sync}} = 4\pi m/\delta$  when the two frames  $\mathfrak{R}^{(2)}$  and  $\mathfrak{R}^{(4)}$  coincide. So one can always do away with the initial one-qubit rotation  $\mathcal{R}^{-1}(t_0)$  by choosing to initiate two-qubit gates only at times  $t_0 = t_m^{\text{sync}}$ . Figure 3(a) shows an explicit pulse sequence that uses this scheme to implement the universal two-qubit gate  $(Y_1 Y_2)^{1/2}$ .

It is important to note that this scheme allows us to implement any two-qubit gate with strictly no dc excursions from the optimal bias point of either qubit, and with no need for a tunable coupling between the two qubits.

For example, the standard two-qubit rotation  $U^{\text{CNOT}}$  may be constructed by the sequence of  $\pi/2$  rotations, with time running left to right,

$$X_1^{1/2} Z_2^{1/2} (Y_1 Y_2)^{1/2} Z_2^{-1/2} X_1^{-1/2} Z_1^{1/2} X_2^{1/2}. \quad (5)$$

We note in passing that this particular decomposition of  $U^{\text{CNOT}}$  can, with simple algebra, be adapted to systems where the core two-qubit gate is other than  $(Y_1 Y_2)^{1/2}$  [11].

We have performed simulations of the protocol generating  $(Y_1 Y_2)^{1/2}$  as shown in Fig. 3. Our simulations are performed by numerically solving a set of 15 coupled differential equations describing the evolution of each component of the two-qubit density operator under the full time-dependent Hamiltonian (1) [12]. The simulations are exact in that they do not rely in any way on the approximations leading to (2) or on a perturbative expansion of the time-dependent Hamiltonian. However, we have used a two-level approximation and square rf pulses; see the discussion of errors below.

What will be the dominant sources of fidelity loss in this gate scheme? As with other gate schemes for superconducting qubits, there will be errors due to leakage to states outside the computational subspace. The fast Rabi flopping used here to bring two detuned qubits into resonance—the essential new “technology” our scheme requires—may make this more significant. The leakage will depend strongly on the anharmonicity of the qubit systems, which is typically set by fabrication and bias parameters. For example, for charge qubits at optimal bias our simulations indicate that the ratio of the effective Josephson energy  $E_J$  to the Cooper pair charging energy  $E_{\text{CP}}$  should satisfy  $E_J/E_{\text{CP}} \lesssim 2$  if the leakage probability due to strong driving ( $\omega^R \approx 0.1\omega^L$ ) is to be  $\lesssim 1\%$ .

There will also be errors due to the Bloch-Siegert shift [13], as we have made a rotating wave approximation in deriving the effective Hamiltonian (2). This effect, which is proportional to  $(\omega^R)^2/\omega^L$ , will have minimal impact on one-qubit rotations, since  $\omega^R$  can be chosen to be small. For the two-qubit gates, however,  $\omega_{1,2}^R$  are set by  $\delta$ , which is constrained by  $\delta \gg \omega^{xx}$ , so the effect could be more troublesome. There will also be errors  $\sim (\omega^{xx})^2/\delta$  in the one-qubit operations due to the presence of the nonsecular coupling. There is thus a trade-off in selecting system parameters for optimal gate fidelity. Parameters in the general range of those used in our simulations approximately balance these two effects.

In practice, there will also be gate errors due to imperfect rf pulses. Since one-qubit rotations are sensitive to the integrated applied irradiation power, constructing pulses to perform high-fidelity one-qubit gates should be straightforward. Producing the two-qubit pulses, however, will be

more difficult: the strength of the effective qubit-qubit interaction depends strongly on the amplitude of both rf signals, so the gates will be very sensitive to variations in the pulse amplitudes. For the parameters used in simulations and a qubit linewidth  $\approx 2$  MHz, the uncertainty in the pulse amplitudes should not be more than about 0.5%. Though challenging, this level of stability is possible with commercially available electronics.

Finally, cross coupling of write signals between the two qubits will also lead to gate errors, though in practice this can be actively compensated with supplementary pulses.

We note that these effects leading to gate errors are predominantly systematic and can thus, in principle, be

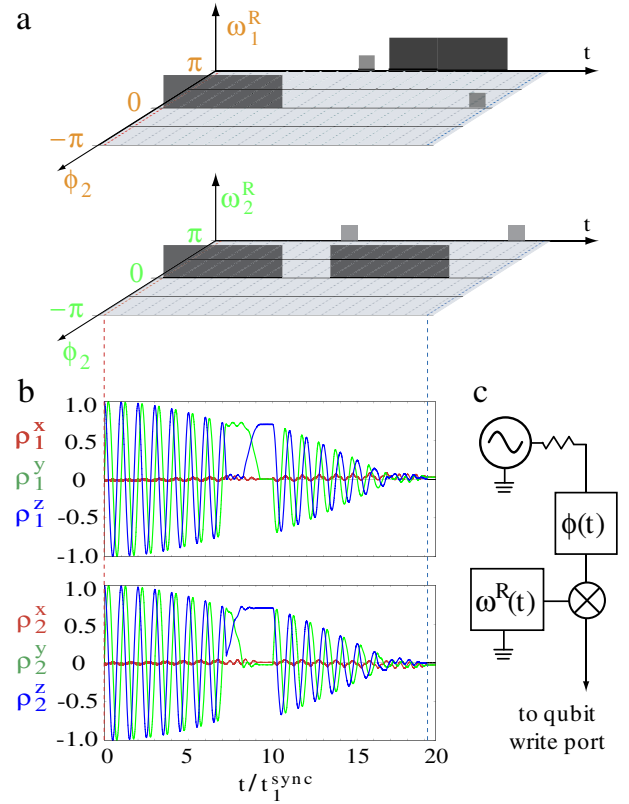


FIG. 3 (color online). (a) Polar representation of pulse sequence for universal two-qubit gate  $(Y_1 Y_2)^{1/2}$  using FLICFORQ. Two-qubit pulses (black) have amplitude  $\delta/2$  and duration  $t^{\text{ent}}$ , and are initiated only at times  $t_m^{\text{sync}} = 4\pi m/\delta$  (gray dashed lines) when the doubly and quadruply rotating frames coincide. One-qubit pulses (gray) have amplitude  $(\frac{\delta\pi}{\omega^{xx}} \bmod 2\pi)/t^{\text{sync}}$  and duration  $t^{\text{sync}}$ . (b) Sample simulation of pulse sequence using the full time-dependent Hamiltonian (1) and parameters  $\omega_1^L = 1.1060$ ,  $\omega_2^L = 1.0527$ ,  $\omega^{xx} = 0.003679$ . Initial state is  $|00\rangle$ . Simultaneous vanishing of each reduced density operator indicates generation of maximally entangled state. Final state is  $(|00\rangle - i|11\rangle)/\sqrt{2}$ ; gate fidelity is  $>99\%$ , with errors due to Bloch-Siegert shift. Plotted are the components of each reduced density operator ( $\rho_i^\alpha = \langle \sigma_i^\alpha \rangle$ ). (c) Schematic experimental set-up for producing pulses to implement universal gates with FLICFORQ.

reduced or eliminated. It is our hope that some of the techniques for combating gate errors that have been developed in NMR quantum computing can be adapted or extended to suit FLICFORQ systems. For example, the error rate due to the always-on coupling could be largely reduced by **dynamically decoupling** the qubits with refocusing pulses [6,14]. Here, this would be achieved by performing appropriately timed  $\pi$  rotations about  $\sigma^y$ , which anticommutes with the coupling term  $\sigma_1^x \sigma_2^x$ . The techniques of composite pulses, pulse shaping, and phase ramping could prove similarly useful. For one-qubit gates, some early steps have been taken in this direction [15].

We believe a main advantage of the gate scheme presented in this Letter is that it can be directly generalized to larger registers with **minimal extra hardware and control lines**. Producing arrays of qubits with well-separated Larmor frequencies is possible with present fabrication technologies, and a fixed linear coupling reactance between all *pairs* of qubits could easily be achieved by coupling each qubit to a **common superconducting island or loop, or cavity** in the dispersive regime [16]. Then pairwise application of the protocol generating  $(Y_1 Y_2)^{1/2}$  would allow the direct production of multiqubit entangled states of the form  $|\text{GHZ}\rangle = (|0\rangle^{\otimes n} + e^{i\phi}|1\rangle^{\otimes n})/\sqrt{2}$ , which, for  $n > 2$ , can display maximal violations of Bell-type inequalities [17].

A FLICFORQ register could likely be scaled directly to 10–20 qubits, at which point the system would become limited by the anharmonicity of superconducting qubits and the limited available working bandwidth. Registers larger than this could then be constructed by coupling multiple subregisters by a single tunable subcircuit, see, e.g., [5].

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