A Nonparametric Copula Based Bias Correction Method for Statistical Downscaling

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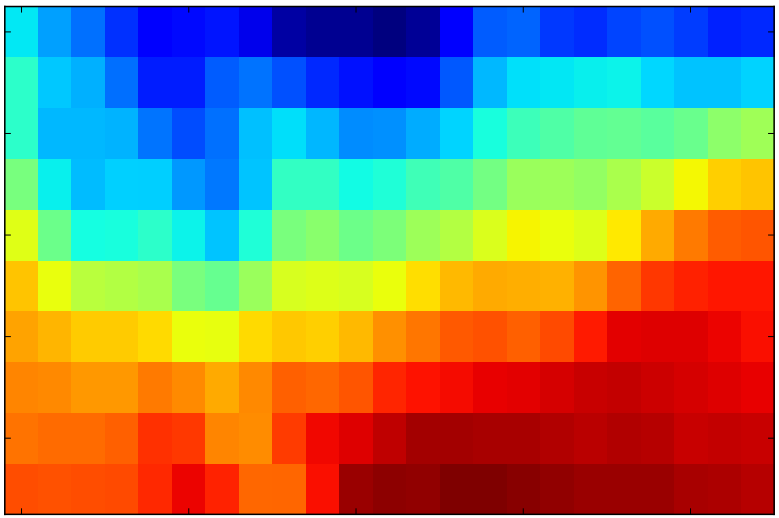
Statistical Downscaling

A)



B)

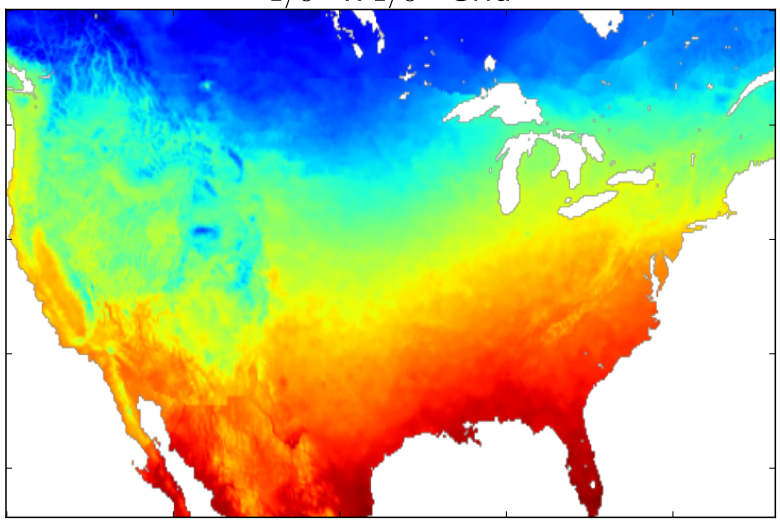


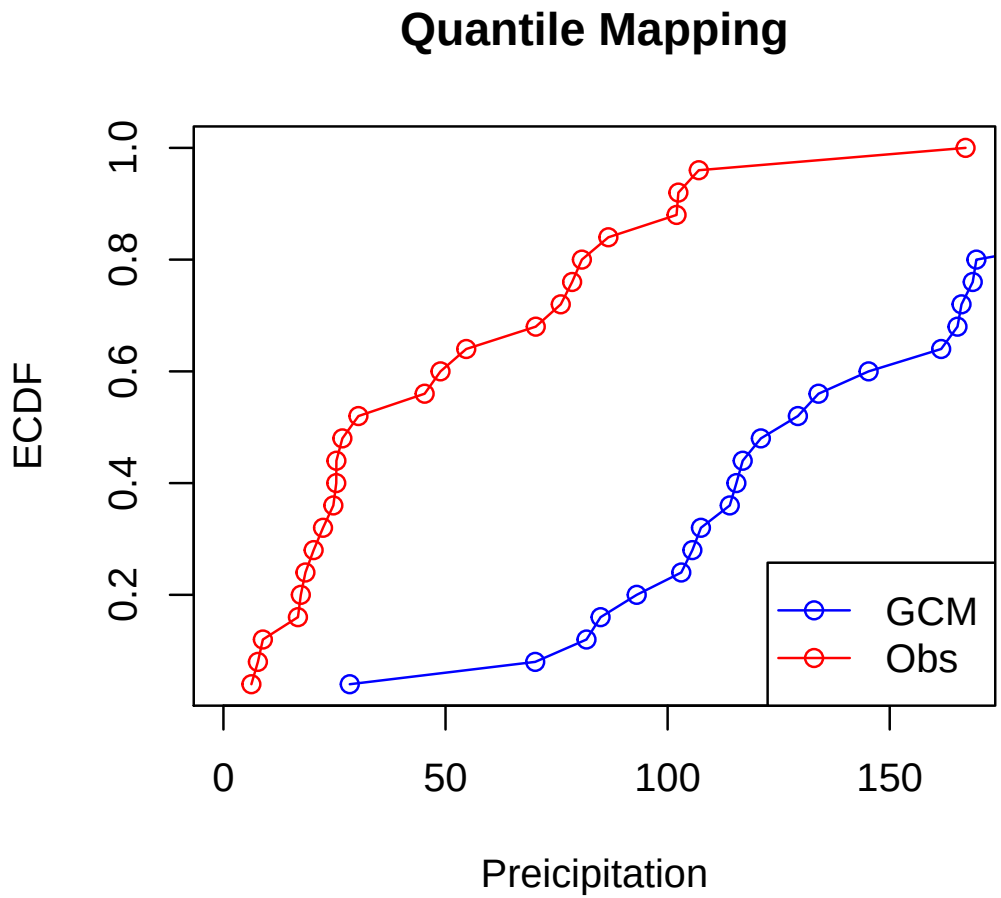
Figure: GCM

Observations

- Coarsely resolved climate models vs Local-scale climate information

BCSD

- Bias correction with Quantile Mapping (our main focus)



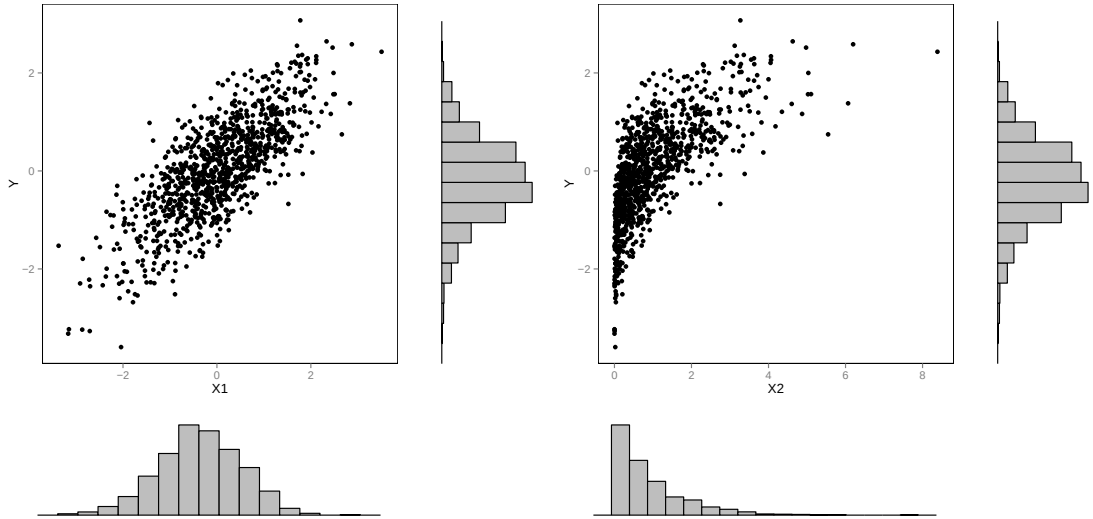
- Spatial disaggregation with historical local scaling factor

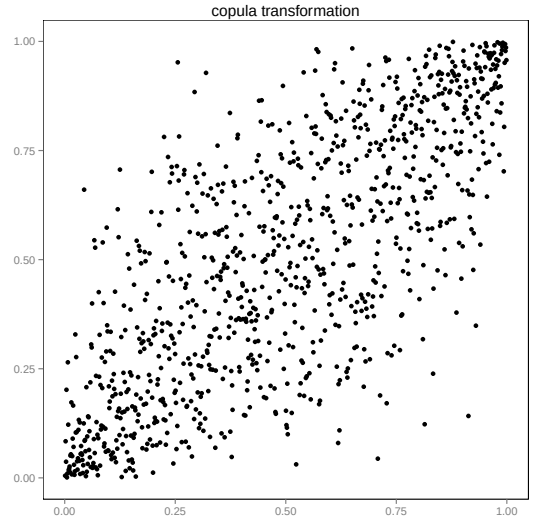
Copula

For d random variables $X = (X_1, \cdots, X_d)$,

$$\text{Sklar's Theorem:} \quad F(x_1, \cdots, x_d) = C[F_1(x_1), \cdots, F_d(x_d)], \qquad (1)$$

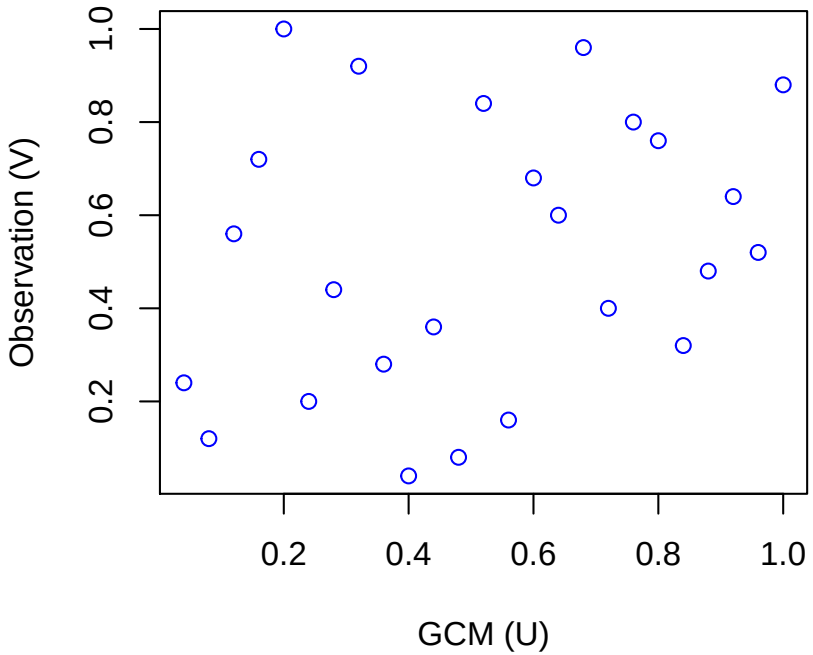
- $F_j(x) = \mathbb{P}(X_j \leq x)$. C is a copula, where $C(u_1, ..., u_d)$ is the CDF of $U_j = F_j(X_j)$.
- This decomposition separates the dependence structure in the data from the marginals.

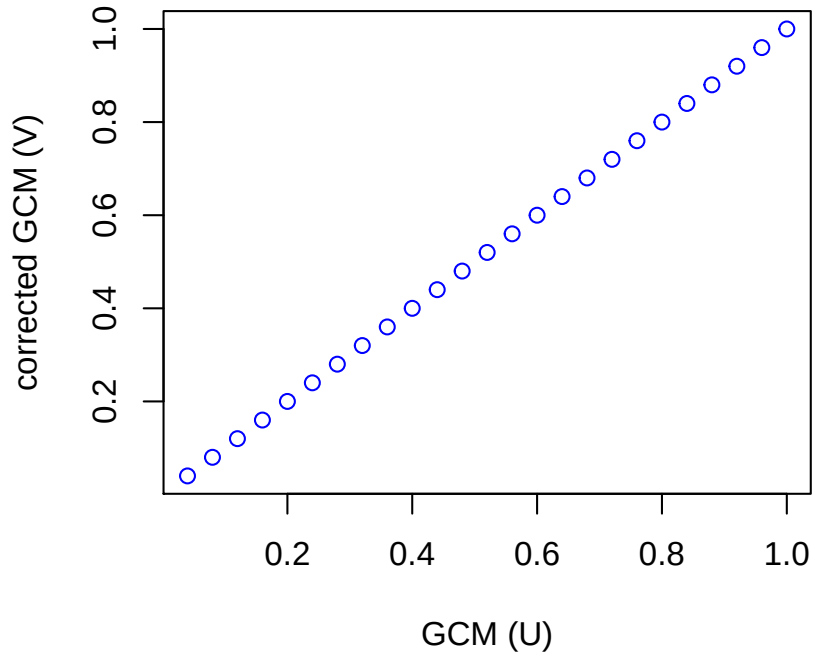




NCBCSD

- Quantile mapping is using $C(u, v) = \min(u, v)$ (Fréchet-Hoeffding upper bound), which assumes the strongest dependency.

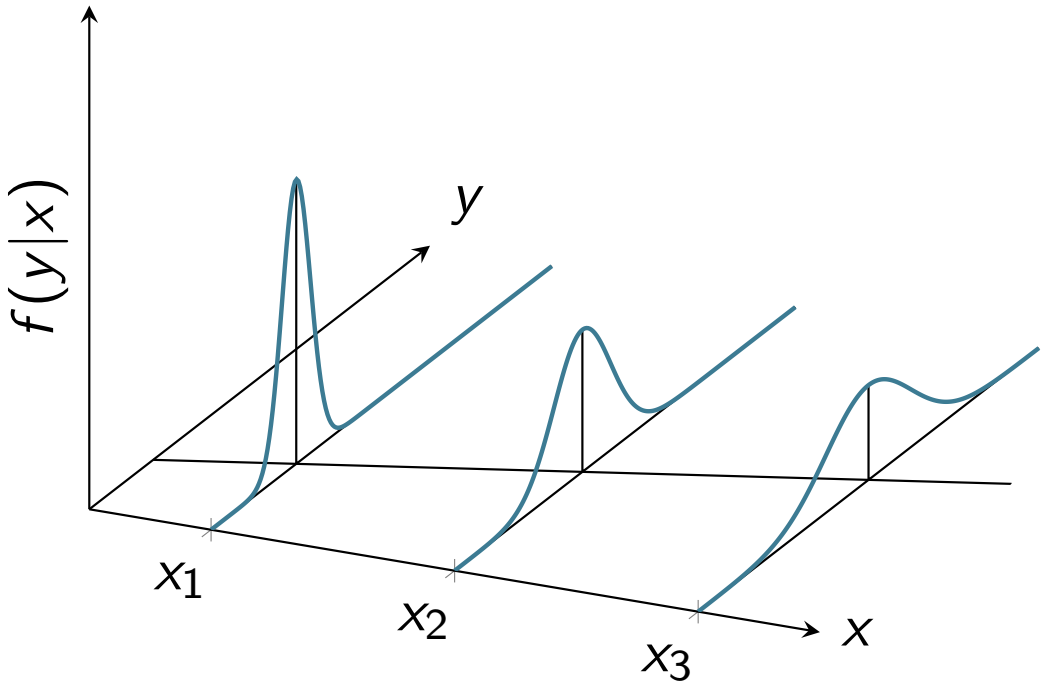




- Nonparametric copula based BCSD method (NCBCSD)

NCBCSD as Modal Regression

$$\texttt{mode}(Y|X = x) = \texttt{argmax}_y f_{Y|X}(y|x), \tag{2}$$



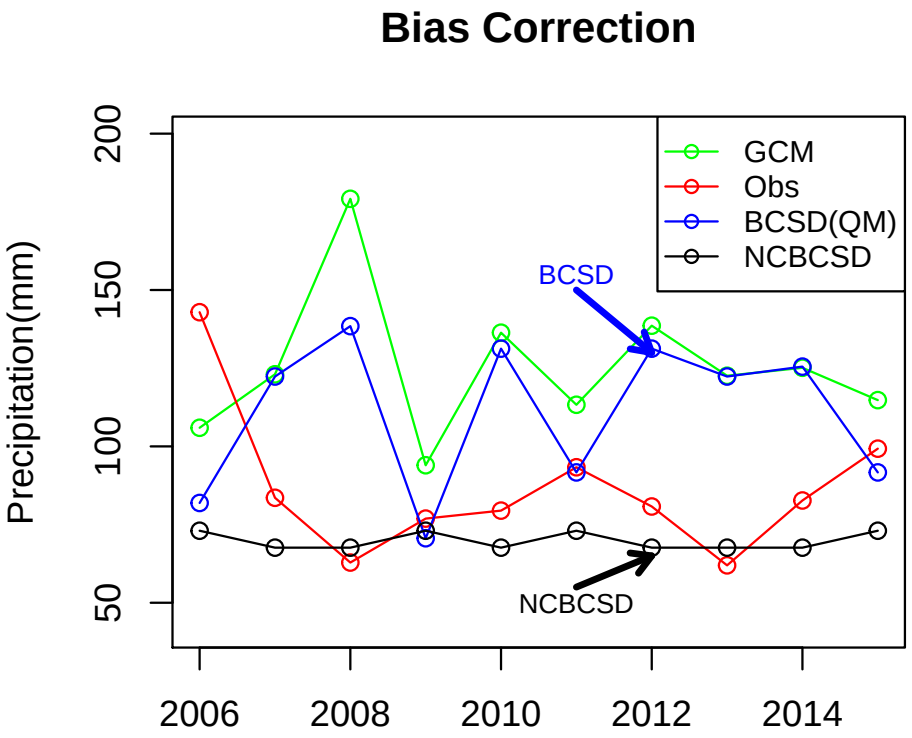
NCBCSD as Modal Regression

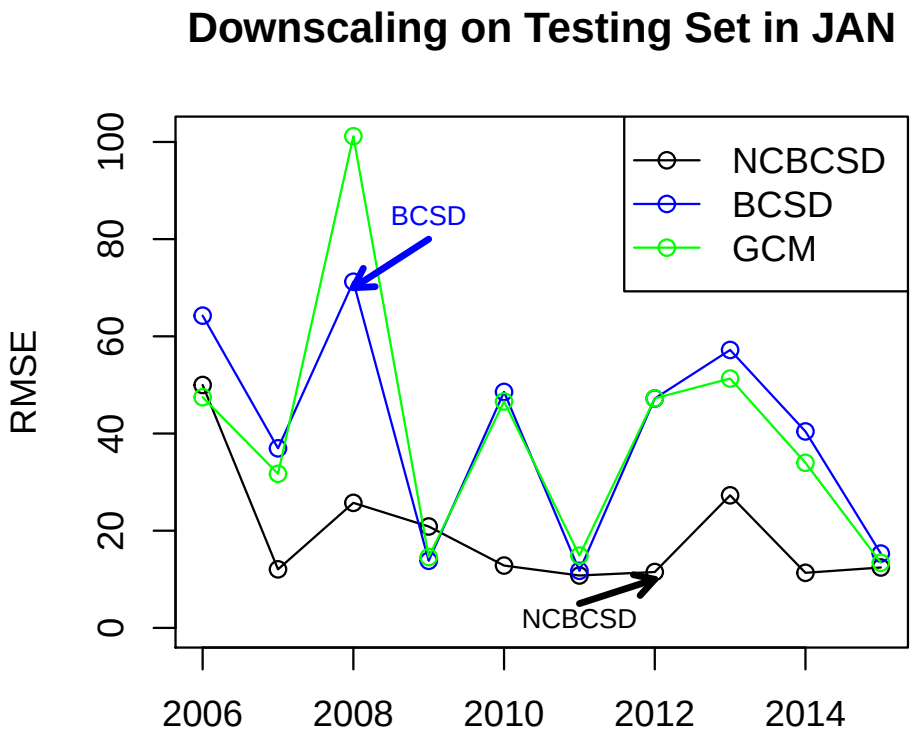
$$\begin{aligned} \texttt{mode}(Y|X=x) &= \texttt{argmax}_y c(u,v) f_Y(y), \\ &\approx \texttt{argmax}_y \hat{c}(u,v) \hat{f}_Y(y), \end{aligned} \tag{3}$$

where $U = F_X(X)$, $V = F_Y(Y)$ and with the (nonparametric) kernel density estimation (KDE):

$$\begin{aligned} \text{Copula:} \quad & \hat{c}_{kde}(u,v) = \frac{1}{nh^2} \sum_{i=1}^{n} K(\frac{u-u_i}{h}) K(\frac{v-v_i}{h}), \\ \text{Marginal:} \quad & \hat{f}_{Y_{kde}}(y) = \frac{1}{nh} \sum_{i=1}^{n} K(\frac{y-y_i}{h}). \end{aligned} \tag{4}$$

Results





- Climate Variables: Monthly precip. of south New England.
- GCM Data: GFDL's Coupled Physical Model (CM3).
- Local Data: Univ. of Idaho Gridded Surface Meteorological Data.

Thank you!